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ERRATA

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Page 330, Table 1, first and second lines, instead of "$\int (y-1)\sin\phi\cos\phi\,\frac{dw}{2}$," read, "$\int (y-1)\sin\phi\cos\phi\,dw$"; and instead of "$\int (\sin^2\phi + y\cos\phi)\,dw$," read, "$\int (\sin^2\phi + y\cos^2\phi)\,dw$."

Page 331, Equation (29) and Equation (30), should read:

$$\text{“}(56.26 + 1.263 \times 15)\,R_1 - 0.155 \times 15\,R_2 = 178.956 \ldots\ldots (29)\text{”}$$

$$\text{“}-\,0.155 \times 15\,R_1 + (56.26 + 3.434 \times 15)\,R_2 = 209.652 \ldots\ldots (30)\text{”}$$

Page 331, Line following Equation (30), should read, "$R_1 = 2\,440$ lb, and $R_2 = 1\,998$ lb."

Page 331, Third line below Equation (30), should read, "$M_c = 99\,377 - 0.585 \times 2\,440 \times 15 + 0.072 \times 1\,998 \times 15 = 80\,124$ ft-lb."

Page 331, Sixth line, below Equation (30), should read,

"$f_c = \frac{M}{S} = \frac{6 \times 80\,124 \times 12}{42 \times 42 \times 42} = 79$ lb per sq in., which is about 8%, etc."

Page 331, Eleventh line from bottom should read, "$M = R_2\,a = 1\,998 \times 15 = 29\,970$ ft-lb, which is about 85% of," etc.

Page 959, Fig. 14, for Member 4–7, change "$K = 596$," to read "$K = 5.96$."

Page 960, Table 2, for Member 2–1, Column (5), change "+ 185" to read "+ 135."

Page 960, Line 8, instead of "computed by Professor Schwalbe," etc., should read, "computed by Professor Schwalbe's method," etc.

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Page 98, Equation (35) should read:

$$Q_e = \sqrt[2.9]{\frac{q_1^{2.9} N_1 + q_2^{2.9} N_2 + q_3^{2.9} N_3 + \cdots + q_n^{2.9} N_n}{N_1 + N_2 + N_3 + \cdots + N_n}}$$

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SELECTION OF MATERIALS FOR ROLLED-FILL EARTH DAMS

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WITH DISCUSSION BY MESSRS. T. T. KNAPPEN, PAUL BAUMANN, WILLIAM C. HILL, A. FLORIS, FRED D. PYLE, JOEL B. COX, STANLEY M. DORE, JOHN E. FIELD, WILLIAM P. CREAGER, JOSEPH JACOBS, C. H. KADIE, JR., RALPH BENNETT, R. R. PROCTOR, L. F. HARZA, AND CHARLES H. LEE.

SYNOPSIS

The traditional manner of determining suitability of material for earth dams is to exercise judgment, based upon previous experience, aided by sense testimony, such as appearance, "feel" to the hand, and possibly grittiness between the teeth. In recent years, with increase in the number and size of earth dams and the occurrence of unexpected failure, it became apparent that a more scientific method of selection was needed. The first efforts in this direction were for hydraulic-fill dams where knowledge of particle size and other physical properties was found essential as a guide to safe and permanent construction.

The recent improvements in mechanical equipment for handling earth have so reduced the cost of rolled-fill construction that the rolled-fill dam is now an economic competitor of the hydraulic-fill dam. Due to this fact and to the freedom of the rolled-fill dam from the element of treachery during construction, it is becoming a more popular type of dam. The rolled-fill dam has this disadvantage, however, that if failure occurs, it does so when the structure is in service and after it has had full opportunity to absorb water. Due to this characteristic and to the fact that practical earth-testing methods have now become more or less standardized in other lines of technical activity, such as highway construction, the present tendency in rolled-fill construction is toward the use of scientific earth-testing methods. This tendency extends both to the selection of material and to control during construction.

This paper is confined to a discussion of earth testing as an aid to the selection of materials. Pertinent fundamental principles of soil technology

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[1] Cons. Hydr. Engr., San Francisco, Calif.

as developed from the latest research and practice are drawn upon as a basis for conclusions. These principles are supplemented by certain research findings in concrete technology. It is shown that particle size has a direct relation to the more important properties of earth material affecting its suitability for rolled-fill dam construction. Mechanical analysis, combined with microscopic examination and, in some cases, a simple mineralogical test, is proposed as a basis for determining suitability. Before making a final selection these factors may be confirmed by tests for compaction and permeability.

REQUIREMENTS FOR SUITABILITY

The principal requirements for suitability of material for the impervious part of a rolled-fill earth dam are:

(1) Permanent stability against sloughing when saturated;

(2) Reasonable water-tightness against the maximum head of water;

(3) Workability in construction operations, involving spreading and compacting in thin layers upon the top of the embankment by the use of mechanical equipment;

(4) Insolubility of the mineral constituents composing the individual particles of the material; and

(5) Reasonable cost of handling, including excavation, transportation, spreading, and compacting.

If the dam includes a permeable down-stream section, freedom of drainage must be considered for this part.

STABILITY

The essential requirement for stablization of earth is high shearing strength resulting from a proper combination of cohesion and internal friction. This can be secured in a well-graded material through mechanical compaction to the point of maximum density by tamping, rolling, or the tramping of animals. Internal friction in an earth material is produced by the rubbing together of bulky granular particles held in contact by the forces of cohesion. The presence of coarse material, such as gravel and sand, is essential for high internal friction.

Cohesion depends upon the permanent adhesive strength due to molecular attraction of minute films of water and moist clay particles between granular particles. As an illustration, C. A. Hogentogler, Assoc. M. Am. Soc. C. E., states[3] that an almost liquid clay with cohesion amounting to about 130 lb per sq ft, when combined with a cohesionless sand will increase the supporting power of the latter from 270 lb per sq ft to 2 500 lb per sq ft. Water films in liquid or plastic soils usually have the same characteristics as water in bulk, but when the thickness of the films is reduced by drying or mechanical compaction of the soil to a semi-solid or solid state, the boiling point of film water rises, the freezing point lowers, and the surface tension increases so that the films become tough. Charles Terzaghi, M. Am. Soc. C. E., states that in thicknesses of less than 0.000002 in., water films behave like semi-solid substances[3].

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[3] *Public Roads*, February, 1935.

The thickness of films on river sand passing a 10-mesh sieve has been estimated at 0.00002 in., and that passing a 60-mesh sieve at 0.000005 in.[4] This thickness decreases with the size of the particle, and in the case of clay is much less than for sand. As a result of this toughening of water films by the compaction of a plastic soil, the latter changes to a semi-solid or solid condition and acquires stability. If the size of the earth particles is sufficiently varied and a sufficiently high degree of compaction is obtained, this stability becomes permanent even when the material is submerged in water.

Active research in earth stabilization has been conducted during the past two decades, first in connection with highway construction, and more recently for rolled-fill dams. Research in the latter field has developed the fundamental principles of earth compaction[5], while the former among many other things has contributed a basis for the selection of materials for earth road surfaces[2]

Probably the most severe test for the stability of earth material is as a wearing surface for an earth road. Such a surface must remain unyielding and hard under pressure and the impact of the heaviest traffic; and it must be free from mud in wet weather or excessive dust in dry weather. Although the forces acting upon a dam are not concentrated as are those upon an earth road, they are of similar character. They consist principally of pressure from the weight of superimposed water and impact from waves and earthquake shock. An earth dam must also retain its stability under alternate submergence and atmospheric exposure. Because of the similarity in requirements for permanent stability in earth roads and in earth dams, the interchange of research data is admissible.

Without going into unnecessary detail, the conclusion may be stated that as the result of research, now thoroughly tested in practice, permanent stability in rolled-fill earth dams can be obtained with well-graded material compacted at a critical moisture content by use of appropriate mechanical equipment. The importance of this finding merits the following detailed consideration of both grading and compaction.

Grading.—A graded earth material is a mixture of earth particles regularly proportioned by size. The most desirable range of sizes and degree of grading differ with the type of construction and the service. For concrete aggregate a limited number of coarse particle sizes is used. For stabilized earth a wide variety of sizes, including both very coarse and very fine, is desirable. In earth-stabilization practice it is seldom possible from the economic standpoint to prepare artificially graded materials, so that natural materials must be used which conform as nearly as possible to an ideal grading.

The purpose of grading in earth stabilization is twofold: (1) To provide the mechanical properties of strength and hardness combined with cohesion and high internal friction; and (2) to secure maximum density.

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[3] *Physical Review*, Vol. 16, 1920, p. 56.

[4] *Journal*, Am. Chemical Soc., Vol. 41, 1919, p. 477.

[5] *Engineering News-Record*, August 31, September 7, 21, and 28, 1935.

The mechanical properties of a well-graded earth result from diversity in particle size. The coarser particles held on a No. 10 sieve, including gravel, pebbles, and cobbles, may be considered as the coarse aggregate and the finer particles passing the No. 10 sieve and consisting of fine gravel, sand, silt, and clay, combined with water, may be considered as the mortar. The function of the coarse aggregate and coarse sand is to provide structural strength and hardness and a high degree of internal friction. The fine sand provides a bedding for the coarse sand. Silt is a filler which prevents the granular particles from rocking. Clay occupies the remaining voids and reduces void spaces to such small dimensions that tough connecting water films are formed which produce strong cohesion. It is thus evident that fractions of all sizes from pebbles to clay are necessary to provide a full degree of strength, hardness, cohesion, and internal friction.

High density in a graded granular material results from the voids between larger particles being occupied by smaller particles. This agrees with the fact, well known to hydraulic engineers, that the granular materials of greatest porosity are those having uniform particle size such as filter sand or clay, whereas those of least porosity have a great variety of particle size. The effect of grading upon density is well illustrated by data published[6] in 1934 in a report giving mechanical analyses and porosities for several hundred undisturbed samples of alluvial material from the South Coastal Basin of Southern California. Data for eight typical samples from this report have been plotted in Fig. 1. Samples *G*–151, *G*–145, and *G*–107 con-

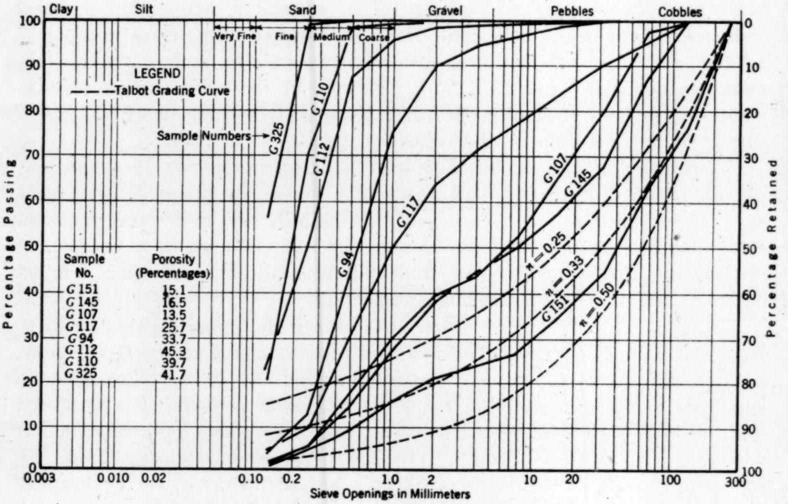

FIG. 1.—RELATION OF POROSITY TO GRADING.

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[6] *Bulletin No. 45*, Div. of Water Resources, State of California, South Coastal Basin Investigation, "Geology and Ground-Water Storage Capacity of Valley-Fill", 1934.

tain all sizes from large cobbles to fine sand, with a preponderance of coarse sizes and regular gradation of fine particles. The porosity of these three samples averages 15 per cent. Samples *G*–112, *G*–110, and *G*–325 are composed of fine and medium sand. These samples have porosities averaging 42%, and those with the intermediate ranges of sizes represented by Samples *G*–117 and *G*–94 have porosities between these two extremes.

Curves plotted to a semi-logarithmic scale which flatten out toward the fines and are concave upward represent well-graded material, whereas those which are flat in coarse material and become steep toward the fines, or are nearly straight and vertical, represent uniformity in particle size. The progressive decrease of porosity with grading is strikingly shown in Fig. 1.

The ideal grading for maximum density has been thoroughly investigated in the well-known experiments on aggregates and concrete by Sanford E. Thompson, M. Am. Soc. C. E., and the late William B. Fuller, M. Am. Soc. C. E., in 1903[7], supplemented by those of A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and F. E. Richart, M. Am. Soc. C. E., in 1919 to 1923[8]. These experiments show that regular grading produced greater density than irregular grading, and that maximum density resulted from a relatively coarse grading As a result of his experiments, Professor Talbot developed a general grading equation which may be written, as,

$$p = \left(\frac{d}{D}\right)^n \text{ ....(1)}$$

in which p = the proportion by weight passing a given screen or sieve opening; d = size of opening; D = maximum particle size; and n = an exponent. These experiments were conducted on graded mixtures with several maximum sizes of aggregates and values of n varying from 0.24 to 1.20, representing mixtures varying from extremely fine gradings to extremely coarse. Characteristics of gradings for typical values of n, as shown by these experiments, are as follows:

For $n = 1.0$, the curve of Equation (1) is a straight line when plotted to natural scale and represents a very coarse grading;

For $n = 0.5$, the curve is a parabola (natural scale) and represents a medium grading corresponding with the Fuller-Thompson parabolic grading (provided sieve holes are of the same shape); it produces maximum density of mixture for aggregates having a maximum size of approximately $\frac{1}{4}$ to $\frac{1}{2}$ in.

For $n = 0.33$, the curve is more nearly an ellipse (natural scale) and represents a fine grading with maximum density of mixture (porosity, 20 to 22%) for aggregates having a maximum size of from 1 in. to 2 in.; and,

For $n = 0.25$, the curve also resembles an ellipse and represents a very fine grading with maximum density of mixture for aggregates having a maximum size of from 4 to 6 in.

The Talbot grading equation, although developed for use with concrete aggregates, is applicable to earth mixtures and may be used as a criterion for density in the selection of natural earth material for rolled-fill dams This is shown by the following two examples:

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[7] *Transactions*, Am. Soc. C. E., Vol. LIX (1907), p. 67.

[8] *Bulletin No. 137*, Eng. Experiment Station, Univ. of Illinois, October 15, 1923.

Example 1.—Grading curves with 10-in. (256-mm) maximum size of particle, when plotted in Fig. 1 correspond in shape with the mechanical analysis curve for Sample *G*–151, the curve for $n = 0.33$ corresponding most closely in position. A similar plotting with a 5-in. (128-mm) maximum size shows that the grading curve for $n = 0.33$ also corresponds generally in shape and position with the mechanical analysis curves of Samples *G*–145 and *G*–107. These three samples all have high density (average porosity, 15%). Similar plotting against the mechanical analysis curves of five other samples, all of which have lower densities, shows no similarity in shape or position. In other words, the Talbot grading equation automatically designates the material of highest density.

Example 2.—Based upon extensive experience, the United States Bureau of Public Roads recommends the following size specification for mixtures having sufficient stability for earth road surfacing[9]:

Sieve.	Percentage passing by weight.	Sieve.	Percentage passing by weight.
1-in.	100	No. 10	40 to 65
¾-in.	85 to 100	No. 40	25 to 50
No. 4	55 to 85	No. 270	10 to 25

The soil mortar in this material passing the No. 10 sieve has a composition as shown in Table 1[10]. These limits have been plotted in Fig. 2, together with grading curves for the corresponding maximum particles sizes. Here, again, there is found to be a very close agreement in shape and position, the curve for $n = 0.33$ lying closest to the mechanical analysis curve for the mixture, and $n = 0.50$ closest to the corresponding curve for the soil mortar. This shows that the Talbot grading curves for maximum density designate an earth mixture found to be satisfactory for the surfacing of earth roads.

TABLE 1.

Fraction	Diameter, in millimeters	Percentage by weight
Clay	Less than 0.005	5–10
Silt	0.005 to 0.05	10–20
Sand, total	0.05 to 1.0	70–85
Sand, coarse	0.5 to 1.0	45–60
Average effective size	0.01	.....
Uniformity coefficient	Greater than 15	.....

From these and other studies it is concluded that the Talbot grading curve with values of n between 0.25 and 0.50 may be used as a criterion for high density in the selection of natural material for rolled-fill dams.

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[9] *Public Roads*, February, 1935, p. 277.

[10] *Loc. cit.*, July, 1931.

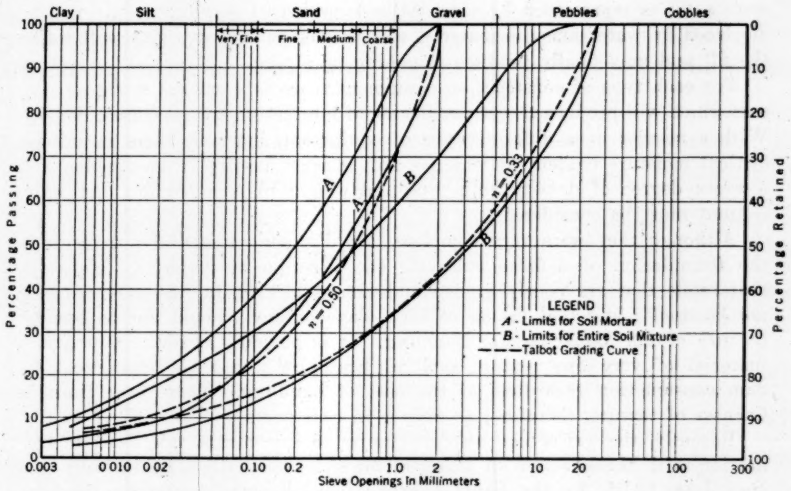

FIG. 2.—SIZE SPECIFICATION OF MIXTURES RECOMMENDED BY U. S. BUREAU OF PUBLIC ROADS FOR SURFACING EARTH ROADS.

COMPACTION

The purpose of compaction of earth material is to bring the soil particles into uniform close contact. This has three effects: First, it increases the mechanical bond by interlocking the individual grains; second (and, of much more importance), it decreases the thickness of connecting water films, thus greatly increasing their adhesive force; and third, it partly expels air which, if retained, would ultimately be replaced by water, thus increasing the thickness of water films and decreasing cohesion. The net effect of compaction is to increase density and produce permanent cohesion.

In open areas earth is compacted by spreading it in thin layers and rolling it with trucks and heavy flat rollers, or kneading and rolling it with sheepsfoot rollers. In restricted spaces the material is made compact by tamping.

The fundamental principles of soil compaction and their application to rolled-fill earth dams in the selection of materials and control during construction have been thoroughly developed by R. R. Proctor, M. Am. Soc. C. E.[11] His work shows that in a given earth material and for each particular method of compaction, there is a critical moisture content at which maximum density is attained. Higher water content results in increased plasticity, larger voids, and less density, until a point is reached at which equipment will not be supported. A lower content may produce a fill of greater apparent stability with little or no plasticity, but one having a greater void volume and less density. The decreased water content and increased void volume in such a fill is represented by air. After a period of service the air will be displaced by water and the material will become saturated. This will soften the fill and may finally produce sloughing or piping.

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[11] *Engineering News-Record*, August 31, September 7, 21, and 28, 1933.

For each type of construction equipment there is a critical moisture content which will produce the proper degree of lubrication for maximum density. With a heavier or a different type of equipment, however, there is a lower critical moisture content at which a still greater density is attainable. For a given degree of density both type and weight of construction and water content must be considered.

Although they were very complete, Mr. Proctor's tests did not result in the formulation of definite rules for the selection of material. He states that "soils that are poorly graded and almost lacking in fine particles cannot be used"; and that "those of high clay content" should not be selected because of the higher cost of handling. It is obvious, however, that a clay material of very fine texture such as bentonite would be impractical for dam construction regardless of the cost of handling. There is a limit in fineness of texture, therefore, as well as in coarseness.

Recently, the writer had an opportunity to investigate this limit in connection with the selection of material for the Coyote Dam being built near San Jose, Calif., by the Santa Clara Valley Water Conservation District. Among the materials selected for testing was one of ancient lacustrine origin. With the passage of time this material had become indurated to a siltstone, was subsequently elevated, and, finally, became softened by weathering, to a depth of more than 12 ft. It was located conveniently for handling and in preliminary field examination had been favorably considered for use in rolled-fill construction of the impervious section of the dam. When samples from the weathered zone were subjected to mechanical analysis by the sieve and hydrometer method, difficulty was experienced in breaking the material down to ultimate particle size during the ordinary period of stirring, even with pre-treatment. Microscopic examination of particles after stirring, as held by Tyler standard sieves from No. 48 to No. 270, showed numerous rounded aggregations of silt and clay representing particles of original siltstone. These particles were soft and more or less plastic, and after drying crushed easily under the thumb-nail. With long-continued further stirring these aggregates finally broke down into particles of silt and clay size. It was also found that larger unbroken pieces of the original material absorbed water rapidly and softened so as to crush easily between the fingers.

The question then arose as to whether this material would act in the completed fill as a mixed sand, silt, and clay, which was the constituency as indicated by field examination and standard mechanical analysis, or whether it would act in accordance with ultimate particle sizes as obtained by prolonged stirring. The latter indicated a material of finer texture than that of the borrow-pit samples from Alexander Dam, in Hawaii, and closely resembled the core samples from that dam. The experience with such fine material for rolled-fill construction both in the Islands and elsewhere has been very unsatisfactory[13]. This is illustrated by the clause often placed in specifications for rolled-fill dams, requiring exclusion of adobe soils. It was determined, therefore, to make a series of compaction tests, including materials from very fine to very coarse texture, and to observe especially the extent to which air remained in the compacted samples. Ten samples were selected for testing as described in Table 2. Mechanical analyses of samples were made by the standard hydrometer and sieve method as developed by the

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[13] *Engineering News-Record*, October 20, 1932, p. 468.

TABLE 2.—DESCRIPTION OF SAMPLES TESTED AT COYOTE DAM, IN CALIFORNIA

Sample	Classification	Type of material	Location	Project	True specific gravity
*L*11–1.....	Clay	Silt	Bed of reservoir	Zuni Reservoir, N. Mex.	2.89
*L*8–*A*.....	Clay	Antioch clay loam	Borrow-pit	Port Costa Dam, Calif.	2.88
*L*15–*CG*...	Silty clay	Siltstone	Sample pit	Coyote Dam, Calif.	2.88
*L*15–*A*1...	Sandy clay	Weathered sandstone	Sample pit	Almaden Dam, Calif.	2.75
*L*15–*C*3...	Sandy clay	Weathered sandstone	Sample pit	Calero Dam, Calif.	2.75
*L*15–*C*9...	Clay loam	Weathered Franciscan sandstone	Sample pit	Coyote Dam, Calif.	2.72
*L*15–*S*9....	Fine sandy clay	Alluvial sand	Sample pit	Stevens Dam, Calif.	2.71
*L*15–*S*6....	Coarse sandy loam	Disintegrated Santa Clara formation	Sample pit	Stevens Dam, Calif.	2.76
*L*15–*A*5...	Coarse sandy loam	Weathered sandstone	Sample pit	Almaden Dam, Calif.	2.73
*L*15–*S*10...	Sandy gravel	Weathered Santa Clara formation	Sample pit	Stevens Dam, Calif.	2.89
*L*1–1......	Fine sand	Beach sand	Cliff House Beach, San Francisco, Calif.	....	2.93

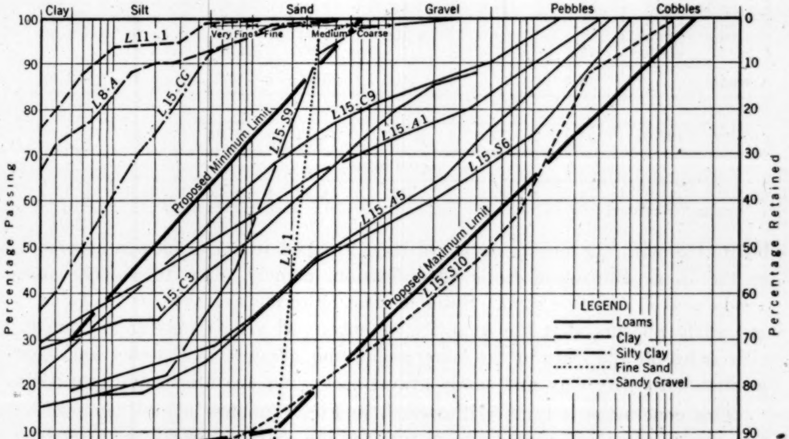

FIG. 3.—MECHANICAL ANALYSIS OF SAMPLES TESTED IN COMPACTION.

U. S. Bureau of Public Roads[13]. In the case of Sample *L*15-Coyote *G*, composed largely of siltstone, the suspension was shaken for 17 hr on a revolving wheel and correction was made after a microscopic examination and a count of undigested siltstone particles in the various screen fractions. The results of mechanical analyses are shown graphically in Fig. 3.

Compaction tests were made on all samples in conformity with the standard laboratory method developed by Mr. Proctor, the results being listed in Table 3. The samples have been arranged in this table in order of texture, beginning with the finest, and have been grouped into five classes according to the U. S. Bureau of Soils Classification, namely, clay, silty clay, loam, sandy gravel, and fine sand. Summarizing by group averages, as in Table 4, it clearly appears that the quantity of entrapped air at maximum compaction is by far the least in the loam group, and greatest in the clay and sand groups, with silty clay and sandy gravel occupying an intermediate position.

TABLE 3.—SUMMARY OF COMPACTION TESTS

Sample	AT MAXIMUM COMPACTION				AVERAGE AIR CONTENT, WITH WATER CONTENT GREATER THAN AT MAXIMUM COMPACTION		AIR CONTENT AT MAXIMUM COMPACTION	
	Dry weight, in pounds per cubic foot	Water content, percentage by weight	Porosity, percentage by volume	Needle penetration resistance, in pounds per square inch	Percentage by volume	Equivalent water, percentage by weight	Percentage by volume	Equivalent water, percentage by weight
CLAY								
*L*11-1..	100.5	20.8	44.5	1 490	6.03	4.21	10.9	6.80
*L*8-*A*...	88.7	27.0	51.1	1 520	8.06	6.40	12.5	8.80
Average	.....	23.9	....	.....	7.04	5.30	11.7	7.80
SILTY CLAY								
*L*15*C*–*G*	103.0	21.5	43.0	1 030	5.14	3.40	7.58	4.60
LOAM								
*L*15–*A*1.	116.0	15.7	32.9	750	3.25	1.86	3.25	1.86
*L*15–*C*3.	114.0	16.6	33.0	750	2.70	1.54	3.10	1.70
*L*15–*C*9.	120.0	13.8	29.6	840	2.50	1.35	2.86	1.40
*L*15–*S*9.	118.0	14.0	31.0	1 200	4.43	2.46	4.40	2.40
*L*15–*S*6.	123.0	12.3	29.3	1 460	3.90	2.10	4.70	2.20
*L*15–*A*5.	114.0	16.7	33.4	770	2.23	1.33	3.20	1.60
Average	.....	14.8	....	.....	3.17	1.77	3.58	1.86
SANDY GRAVEL								
*L*15–*S*10	127.0	11.1	30.0	2 300	5.13	2.70	7.35	3.60
FINE SAND								
*L*1–1...	111.0	11.0	40.2	570	14.50	8.45	20.3	12.3

This conclusion is further illustrated in Fig. 4, upon which all compaction test curves, after adjustment to one true specific gravity of 2.80, are assembled with the curve of zero air voids. The horizontal distance between any point on a compaction curve and the line of zero air voids represents air voids as

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[13] *Public Roads*, October, 1931.

TABLE 4.—COMPACTION TESTS, COYOTE DAM, IN CALIFORNIA

Class of material	(a) Percentage of Total Voids at Maximum Compaction Occupied by Air					(b) Percentage Increase in Water Content at Maximum Compaction if Air is Displaced by Water	
	Limits in proportion of clay, percentage	Water content, in percentage by weight	Porosity, in percentage by volume	Air content, in percentage by volume	Percentage of total voids represented by air	Air content; equivalent water as percentage by weight	Increase in water content if saturated; percentage by weight
Clay	30 to 100	23.9	47.8	11.7	24.5	7.80	32.6
Silty clay	30 to 50	21.5	43.0	7.6	17.6	4.60	21.4
Clay loam	20 to 30	14.8	31.5	3.6	11.4	1.86	12.6
Sandy gravel	15 to 0*	11.1	30.0	7.4	24.6	3.60	32.4
Fine sand	15 to 0*	11.0	40.2	20.3	50.5	12.30	112.0

* Silt and clay.

equivalent water content, in percentages of dry weight. Each texture group has a distinctive type of compaction curve and occupies a distinctive area upon the diagram, thus indicating differing physical properties. Compaction curves for clay, silty clay, and sand groups, are farthest from the line of zero air voids and lowest in dry weight at maximum compaction; and they have very flat peaks. The loam group is the closest to the line of zero air voids, has the greatest dry weight, and its curves have very sharp peaks. The sandy gravel curve, although having a sharp peak, and dry weight similar to those

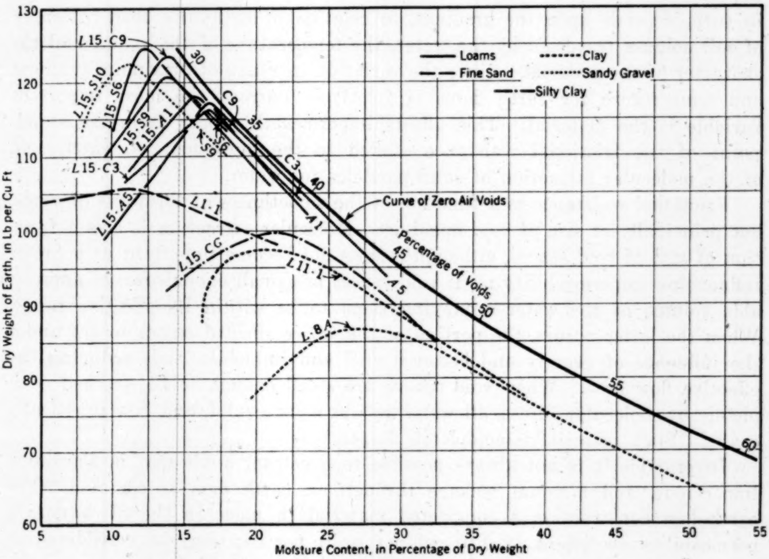

FIG. 4.—CURVES OF COMPACTION AND ZERO AIR VOIDS.

for loam, is farther from the curve of zero air voids. The superiority of loam from the standpoint of dry weight and minimum air voids is apparent.

Expressing this superiority in terms of mechanical analysis, the data in Figs. 3 and 4 indicate that for permanent stability of compacted earth material the clay fraction should not exceed 30 to 35%; nor should it be less than 3 to 15%, the latter limits depending on the degree to which the material is graded.

With regard to the disintegrated siltstone (Sample *L*15-*CG*), it appeared that the air content at maximum compaction expressed as equivalent water was three times that of the next nearest available material represented by Sample *L*15-*C*9 (Fig. 4) and that it lay in the clay group in Fig. 3. Due to the large percentage of air voids, even under ideal conditions of handling in the laboratory, and the probability of non-uniform compaction in the fill due to difficulty in handling, the siltstone was discarded for use in the main structure of the dam.

IMPERMEABILITY

The water-tightness of an earth dam is determined by the degree of impermeability of the material as compacted in the fill. This impermeability, in turn, depends upon the gradient, or relation of hydraulic head to length of soil column traversed by the water, the temperature of the water, and the character of the material. Since the variation in range of maximum gradient and temperature for earth dams is relatively narrow, the most important variable is the material. This affects impermeability both from the standpoint of the frictional resistance offered to the movement of water, and of the molecular attraction of solid particles for water.

Frictional resistance is a function of the smoothness of the walls of voids, but principally of size of void openings. Molecular attraction is also a function of size of void spaces, although it does not become important as a factor influencing impermeability until void spaces are small enough for an appreciable volume of the water occupying them to be within its effective range. When the latter occurs, the portion of the water affected ceases to act under the influence of gravity and becomes stiff and immobile, thus reducing the effective flow area. When void spaces are small enough to be spanned completely by molecular forces, all water acts as a semi-solid, and flow practically ceases. Such material is completely impervious.

In practice it is not always possible to construct a fill that is absolutely impervious, and nominal leakage through an earth dam is common. For earth-dam construction a compacted material in place in the fill having a permeability coefficient of 0.10 gal per sq ft per day may be considered as practically water-tight. The permeability coefficient is defined herein as the quantity of water, in gallons per day, flowing through a section of material perpendicular to the direction of flow and 1 sq ft in area under a hydraulic gradient of 100% and temperature of 60° F. A permeability coefficient of 0.1 as thus defined is equivalent to a percolation rate, as defined by Proctor, of 4.86 ft per yr. For an earth dam of uniform construction 200 ft in height, and 650 ft in average width, with a slope of 1 on 2.5, a top width of 20 ft, and a 10-ft free-board (assuming the water-table just below the surface of the ground at the down-stream toe), a permeability coefficient of 0.1 gal per sq ft per day represents a leakage, when full, of approximately 2 600 gal per day, or 0.004 cu ft per sec. Under certain conditions permeability coefficients exceeding 1.0 gal per sq ft per day might be allowable.

The recent standardizing of methods for measuring and expressing permeability make possible a correlation of mechanical analysis and permeability coefficients. Based upon the study of numerous permeability tests made by the Pacific Hydrologic Laboratory, and Hydrologic Laboratory, Water Resources Branch, United States Geological Survey, the statement can be made that regularly graded material having a degree of fineness sufficient to include at least 3% of clay, or ungraded material containing at least 25% of clay, when thoroughly compacted, is practically water-tight. Partly or poorly graded materials may be impervious if they contain less than 25% and

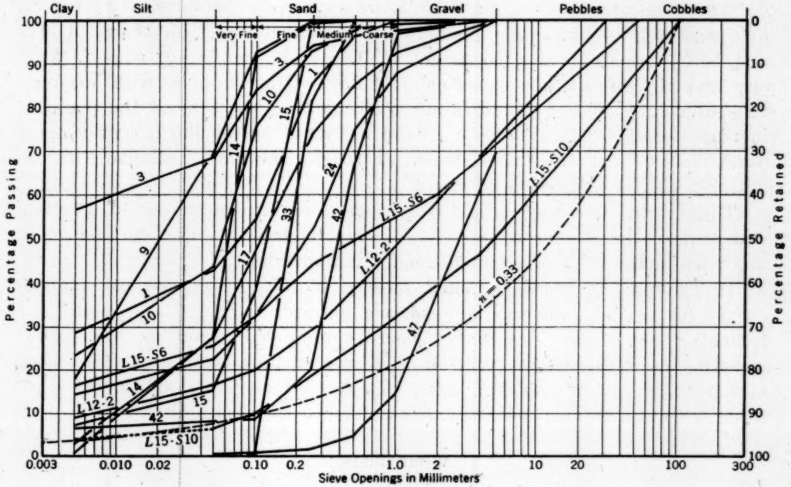

FIG. 5.—RELATION OF IMPERMEABILITY TO GRADING.

TABLE 5.—PERMEABILITY DATA PERTAINING TO FIG. 5

Sample	Permeability coefficient, in gallons per square foot per 24 hours (100% gradient)	Porosity (percentage of voids)	Sample	Permeability coefficient, in gallons per square foot per 24 hours (100% gradient)	Porosity (percentage of voids)
*L*15–*S*6*............	0.034	29.3	14..................	13	34.9
*L*15–*S*10*............	0.010	30.0	15..................	13	44.5
*L*12–2*..............	0.516	....	17..................	18	53.7
3..................	0	36.2	24..................	131	36.0
1..................	0	35.8	33..................	375	40.2
9..................	1.6	37.8	42..................	1 095	44.4
10..................	2.28	35.2	47..................	6 200	31.9

* The first three samples were graded approximately; the remainder were not graded.

more than 3% of clay, the quantity depending upon the departure from regular grading. These conclusions are illustrated by mechanical analysis and permeability data for a wide variety of samples as presented in Fig. 5 and Table 5.

Samples *L*15-*S*10 and *L*15-*S*6 closely approximate regular grading, the mechanical analysis curves being roughly parallel to the regular grading curve for $n = 0.33$ (see Fig. 5). Sample *L*12-2, which conforms more nearly to a straight line, also has a rough grading, although not to the same degree as the other two. Sample *L*15-*S*10, which approaches nearest to the ideal curve, contains only 6.5% of silt and clay and approximately 3% of clay. Its permeability coefficient is 0.01. Sample *L*15-*S*6, which is less well-graded, has a permeability coefficient of 0.034, and a clay content of 17. Sample *L*12-2, which is poorly graded and has only 9% of clay, has a permeability coefficient of 0.5.

All other samples listed in Table 5 are ungraded. Samples 3 and 1, with 57% and 29% of clay, are impermeable; but Sample 9, with 17.3% of clay and 53.1% of silt, has a coefficient of 1.6; and Sample 10, with 23% of clay and 21% of silt, has a coefficient of 2.28. Other samples with less than 25% of clay have larger and larger coefficients as the size of the particles increase. Sample 47, with no clay and 54.7% of gravel, has a coefficient of 6 200, as compared with the coefficient of 0.01 for the graded sample, *L*15-*S*10, with 3% of clay and 68.6% of gravel.

These data illustrate in a striking manner the importance of grading, and confirm the statement that 3% of clay in graded, and 25% of clay in ungraded, material is sufficient to render it impervious.

In passing, it should be noted in Fig. 5 that the mechanical analysis curves for graded materials are concave upward, steep in large sizes, and flattening out toward the fines; whereas ungraded materials are convex upward, flat in large sizes, very steep in intermediate sizes, and either steep or flattening out in very fine sizes. These characteristics are useful when interpreting mechanical analyses with reference to permeability.

WORKABILITY

A workable material is one that can be compacted readily into a fill of uniform texture and high density by the use of mechanical equipment. In order to fulfill this requirement the material must compact easily, and not be so harsh as to cause the equipment to wear out rapidly, nor so sticky as to delay the spreading and rolling operations. These qualities are controlled largely by the texture of the material.

Materials that compact most easily are composed of well-graded gravel and sand with a silt and clay percentage of from 10 to 25 to supply lubrication. The application of proper moisture percentage for maximum density also facilitates compaction.

Harshness is produced by coarseness of grading either with or without the maximum size of particles, such as large pebbles and cobble-stones. The maximum allowable size of stone varies with the thickness of the layers and the type of rolling equipment, whether flat or sheepsfoot. With 6-in. layers, cobbles should not exceed 5 in. in diameter. The allowable coarseness is also related to the type of equipment. As a general practice, values of n in Equation (1) should not exceed 0.50.

Stickiness is produced by high clay content. The allowable quantity can be varied, depending upon the type of equipment and time at which moisture is applied. In general, clayey material with a moisture percentage giving maximum density in compaction is not sticky in handling, provided the moisture is uniformly distributed through the mass. Clayey material is very slow to absorb water, however, and if worked immediately after sprinkling may be very sticky, even if the proper quantity of moisture has been added. This is due to the fact that the moisture content of the exposed surface with which equipment comes in contact is temporarily far greater than the critical moisture. If several hours are allowed after sprinkling for complete absorption and distribution of moisture, no difficulty may be experienced.

There are also other methods of overcoming stickiness which do not involve delay. The sprinkling of the rolled surface of the previous layer immediately in advance of dumping material for the new layer has proved successful. The exposed surface of the new material remains comparatively dry during the process of spreading and initial rolling, attaining an ultimate moisture content not in excess of the critical moisture after the water has moved slowly upward through the layer by capillarity. Another method is to sprinkle the material in the bank before excavation if natural moisture is much below the critical. In some instances, rainfall will moisten the soil in the bank or borrow-pit to a sufficient depth to bring the moisture content up to the percentage desired.

INSOLUBILITY

An obvious requirement for permanency in an earth dam is that the substances composing the individual earth particles must not be soluble. Gypsum and lime are frequently encountered in sedimentary rock formations and may appear in alluvial deposits either as incrustation or nodules occupying voids, or as particles of the original material. Although these substances are not as readily soluble as rock salt or alkali incrustations, percolating water will dissolve and remove appreciable quantities in time, rendering the fill more porous and threatening its stability and water-tightness.

The presence of soluble substances can usually be detected by simple observation, aided sometimes with a pocket microscope or an acid test.

COST OF HANDLING

Other requirements having been satisfied, the cost of handling the material is of final importance. Handling includes the operations of excavation, transportation, spreading, and rolling. Important factors affecting the cost of material taken from a given bank or borrow-pit are the distance and relative elevation from the dam, the quantity of satisfactory material available with reference to the volume of the dam, and the clay content of the material. The relation of these factors to cost is well understood and needs no discussion.

Mechanical Analysis as a Basis for Selection

Of the five requirements for judging the suitability of material for rolled-fill earth dam construction (cited previously), the three most important are stability, water-tightness, and workability. There is an intimate relation between each of them and the mechanical composition of the material. Stability and water-tightness are both secured as a result of regular grading of particle sizes and compaction of material; grading is entirely a matter of mechanical composition. In order to be effective permanently, compaction necessitates the establishment of limits for the clay and silt fractions. Workability also involves limits, both for fine and coarse fractions. This suggests the possibility for using mechanical analysis as a preliminary basis for the selection of material.

Examination of mechanical analysis curves for a variety of materials, such as those shown in Figs. 1 and 5, discloses great divergence in slopes, curvature, and terminal points. Even between fixed terminal points an almost infinite number of curves are possible. Such curves represent material varying from well-graded to uniform size of particle, from dense to very porous, and from impermeable to highly permeable. As these extremes result not only from variation in particle size, but also from variation in grading, it is apparent that if any method of selection based on mechanical analysis is to be successful, it must include limits of size as well as of departure from regular grading. If specified from the graphical standpoint, this means that in referring to the mechanical analysis diagrams, limits in shape and slope of curvature must be considered as well as limits in area. To establish such limits, it is proposed, first, to set up limits of particle size for coarse and for fine materials; and, second, to prescribe curvature limits based upon both degree and fineness of grading. The establishment of a size limit for coarse materials has already been attempted[14] by E. W. Lane, M. Am. Soc. C. E., but, to date, no attempt has been made to establish limits for size of fine materials, nor for degree and fineness of grading.

Before proceeding to details, it should be noted: First, that the terminal points of a mechanical analysis curve indicate size limits; second, that the slope of a mechanical analysis curve indicates the fineness of grading as numerically represented by the value of n in Equation (1); and, third, that the shape of a mechanical analysis curve indicates the degree of grading, if any, as numerically represented by departure from Equation (1). It thus appears that two sets of limiting curves are desirable, one applying to regularly graded materials and the other to ungraded materials. Between these limits will lie the poorly graded materials. Figs. 6 and 7, with Table 6, illustrate the range of the proposed specifications for limiting curves for graded and ungraded materials. (See, also, the revision of Fig. 6 presented as Fig. 23 in the closing discussion.)

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[14] *Engineering News-Record*, December 18, 1930, p. 961.

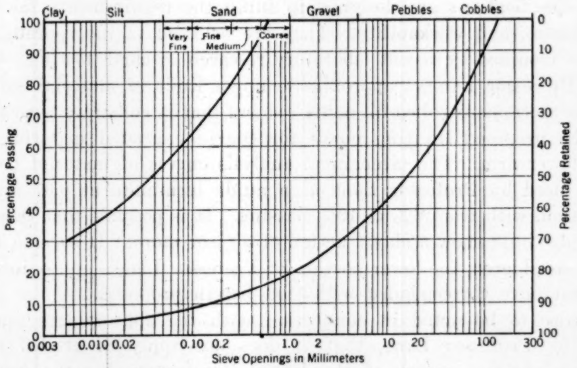

FIG. 6.—PROPOSED LIMITS IN MECHANICAL ANALYSIS FOR GRADED MATERIALS SUITABLE FOR IMPERVIOUS SECTIONS OF ROLLED-FILL EARTH DAMS

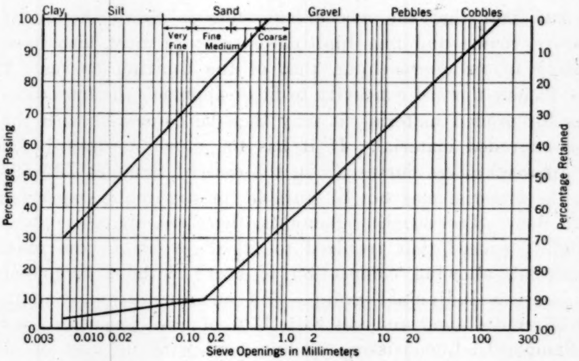

FIG. 7.—PROPOSED LIMITS FOR UNGRADED MATERIALS SUITABLE FOR IMPERVIOUS SECTIONS FOR ROLLED-FILL DAMS

TABLE 6.—PROPOSED LIMITS FOR GRADED AND UNGRADED MATERIALS

Class of material	Terminal Points for Curves (See Figs. 6 and 7)		Graded materials; values of n in Equation (1)	Ungraded materials; Remarks
	Maximum particle size, in millimeters	Minimum percentage of clay ‡		
Coarse	128*	3 to 5	0.33	Straight lines from terminal points to 10% less than 0.15 mm.
Intermediate	128* to 0.6†	3 to 30	0.33 to 0.25	Approximately parallel to the fine limit and the upper arm of the coarse limit, but not crossing the lower arm of the coarse limit.
Fine	0.6	30	0.25	Straight line.

* 5 in. † No. 28 mesh. ‡ Particle size, 0.005 mm.

These specifications are designed to fulfill the requirements for stability, water-tightness, and workability. They are believed to incorporate standard practice as modified by recent laboratory research in earth testing. Comparison with Professor Lane's "approximate upper limit of material suitable for impervious sections of rolled-fill dams", shows that it coincides with the upper arm of the proposed limiting curve for coarse ungraded material, but not with the lower arm. The mechanical analysis curves of material from existing dams used by Professor Lane as a guide terminate at 0.07 mm in the sand fraction, with from 6% to 10% passing. It is possible that these curves, if extended, would show a clay fraction of 3%, or more. If so, and this fraction were considered, the lower portion of Professor Lane's approximate upper limit might have corresponded with that herein proposed.

It is also to be noted in connection with the approximate upper limit proposed by Professor Lane, that it does not apply to graded materials. Mechanical analysis curves of the latter, satisfactory for the construction of rolled-fill dams, may lie beyond this limit in the sand, gravel, and pebble fractions, provided such material contains at least 3% of clay.

In applying the proposed limits to determine the suitability of a material, it is not sufficient that the mechanical analysis curve should lie between the respective coarse and fine limiting curves; it must also have a shape corresponding to, or approaching, that of the limiting curves. If it does not have sufficient upward concavity to fit or approach the curvature of limiting curves for graded material, it may then be compared with the limiting curves for ungraded material. If it has too much downward concavity to follow or approach these curves, as in the case of Samples *L*15-*S*9 and *L*1-1 in Fig. 3, the material may not be suitable for use in the impervious part of a rolled-fill dam. The deciding factor is, therefore, the clay content, which should slightly exceed that required to fill the voids in the granular part of the material, allowing compaction to occur without completely closing the gaps between individual grains. As an illustration, Sample *L*15-*S*9, composed of graded sand and silt with 17% of clay, is entirely satisfactory, whereas Sample *L*1-1, consisting of fine sand with no clay or silt, is not suitable.

The selection of material from mechanical analysis data, although believed to be practical, is more or less preliminary in character at present and subject to modification as the result of more positive tests, such as compaction, permeability, etc.

These criteria for selection of earth material for rolled-fill dam construction are presented in the hope that they will bring forth discussion and additional data, from which may be evolved a basis for preliminary selection generally acceptable to the profession.

DISCUSSION

T. T. **Knappen**,[15] M. Am. Soc. C. E. (by letter).—There are certain fundamental conceptions essential to the design of a dam. At a particular site an engineer may consider some form of concrete structure or a structure built of earthen materials, which may include rock. In considering an earth dam, the engineer has a choice between dry and hydraulic methods of construction. For the dry method practice has standardized on the placement of material in thin layers, with suitable compaction by rollers of various types, or by power-tamping or vibrating equipment. For the economical design of an earth dam, it is essential, first, to utilize all materials from required excavations; and, second, to construct the remainder of the dam from the materials available in the vicinity. The greatest economy generally results from the use of materials closest to the site.

In this paper, the author has based his selection to a large extent on grain-size distribution. This is a logical method in the preliminary stages. In Figs. 6 and 7, he has given the proposed limits for materials suitable for impervious sections of rolled-fill earth dams. The writer agrees that, in general, material falling within these limits will be satisfactory. However, it is entirely possible to use materials that do not come within these limits. First, considering materials that are finer than the upper curves on these two diagrams, although such materials may be somewhat difficult to handle and may be structurally somewhat unstable, they may furnish, nevertheless, an excellent impervious core for a dam embankment. In the computation for such cases the outer shell must be depended upon for structural strength. The dam may be analyzed by a method recently outlined by the writer.[16]

Considering the two lower curves which represent the limit of permeability, it is perfectly possible to use materials of greater permeability than those indicated. It may be conceded that an earth dam can be safely designed and built, provided it is properly designed, with the expectation of a large leakage. The approximate upper limit, indicated by the author, of 1 gal per sq ft per day, with an hydraulic gradient of 1 is extremely conservative. This is equivalent to a value of k in Darcy's formula,

$$Q = k\,i\,A\,t \qquad (2)$$

of 0.4×10^{-4} cm per sec. In the case illustrated, the author deduced that the leakage would be 0.004 cu ft per sec with a coefficient of 0.1 gal per sq ft per day. From a structural standpoint, a leakage of a thousand times this amount is not necessarily excessive with proper design. In other words, a permeability coefficient of 100 gal per sq ft per day might be satisfactory in centimeter-gram-second units (40×10^{-4} cm per sec.) It would seem to the writer that the upper limit of permissible seepage is largely a matter of economics. The cost of reducing the leakage should be balanced against the value of the water saved.

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[15] Associate, Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

[16] "Calculation of the Stability of Earth Dams", presented before the World Power Conference, Washington, D. C., September, 1936.

In the case of flood-control dams, or dams at which constant releases are required, it may be of no economic consideration whatever. In such cases, it becomes merely a structural problem of designing the embankment to pass the leakage safely. Recently, the writer had occasion to design a dam for which the estimated leakage through the foundations, with water at spillway level, amounted to 50 cu ft per sec. In such a case, it would be unwarranted to go to great expense to obtain an impermeable embankment.

The writer cannot agree with the author in eliminating the siltstone (Sample *L*15–*CG*, Table 2) for use in the Coyote Dam on the basis of the information set forth in the paper. It would appear that this material could have been used advantageously at least in the center one-fourth of the embankment, with resulting economy. Certainly, more exhaustive tests to determine its structural nature in terms of cohesion and angle of internal friction under saturated conditions in the embankment should have been made before discarding its use in the center section of the dam, and there is a possibility that it might have been used safely for a large percentage of the embankment.

In the paragraph following Fig. 4 the author makes the statement that "the data in Figs. 3 and 4 indicate that for permanent stability of compacted earth material the clay fraction should not exceed 30 to 35%; nor should it be less than 3 to 15%, the latter limits depending on the degree to which the material is graded." Based on this statement, the engineer having such a material as Sample *L*15–*CG* would be precluded from constructing a dam, whereas it can be used with proper design. A combination of Sample *L*15–*CG*, or even Sample *L*8–*A* for a core and Sample *L*15–*S*10 (or, preferably, a coarser material) for the shells will result in an excellent and economical dam, possibly a better one than that built uniformly with Sample *L*15–*C*9.

There are few materials that cannot be used successfully, to some extent, in the construction of earth dams. The materials approved by the author may be used with little additional investigation, but for true economy in difficult designs every analytical resource of the science of soil mechanics should be called upon to produce a design utilizing the closest available materials to produce a safe and enduring structure. The writer knows of more than one dam that could never have been built had the author's limitations been applied.

PAUL BAUMANN,[17] M. AM. SOC. C. E. (by letter).—Due to the rapid exhaustion of suitable dam sites it is believed that, in the future, rolled-fill dams will gradually exceed masonry dams in number. Hence, this paper is particularly timely and valuable.

The author points out five principal requirements for suitability of material for the impervious part of a rolled-fill earth dam. He further stresses the importance of grading the material to satisfy these requirements. This tends to imply that unless material which conforms to the specifications of gradation is available in the vicinity of a site chosen for a rolled-fill dam, such a dam cannot be built. The writer's experience, acquired during the construction of San Gabriel Dam No. 1, does not substantiate this contention.

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[17] Junior Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

Material that is not initially suitable for rolled-fills, because of gradation defects, can be made suitable by proper compaction.

Two widely different types of materials have been placed and compacted in San Gabriel Dam No. 1: A clayey sand or loam in Zone 2; and quarry-run material which has passed through a 6 by 9-in. grill in Zone 3. The location of these zones is shown in Fig. 8 which is composite cross-section of the dam.

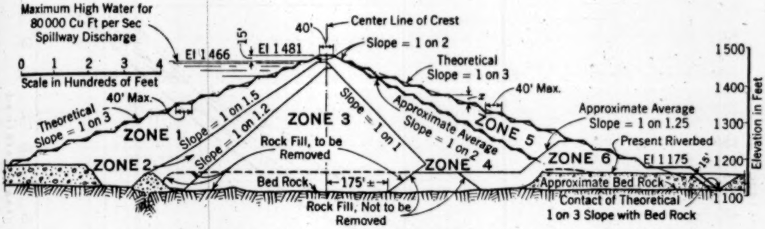
Maximum High Water for 80 000 Cu Ft per Sec Spillway Discharge
0 1 2 3 4
Scale in Hundreds of Feet
El 1466
15'
El 1481
40'
Center Line of Crest
Slope = 1 on 2
Theoretical Slope = 1 on 3
40' Max.
Theoretical Slope = 1 on 3
40' Max.
ZONE 1
ZONE 2
Slope = 1 on 1.5
Slope = 1 on 1.2
Slope
ZONE 3
Slope = 1 on 1
Approximate Average Slope = 1 on 2
ZONE 5
Approximate Average Slope = 1 on 1.25
Rock Fill, to be Removed
Bed Rock
175' ±
ZONE 4
ZONE 6
Present Riverbed
El 1175
15'
Approximate Bed Rock
Rock Fill, Not to be Removed
Contact of Theoretical 1 on 3 Slope with Bed Rock
1500
1400
1300
1200
1100
Elevation in Feet

FIG. 8.—COMPOSITE MAXIMUM SECTION, SAN GABRIEL DAM NO. 1.

In order to illustrate the comparative initial and final gradation, a typical screen analysis is shown for each of the two materials in Figs. 9 and 10. Whereas, the fineness modulus of the wet clayey sand is 1.17 for the average sample before compaction and is not changed materially through compaction, the average fineness modulus of the quarry-run material is 5.72 before compaction and 5.06 after compaction; and, although the material in Zone 2 conforms approximately to what might be called standard specifications for

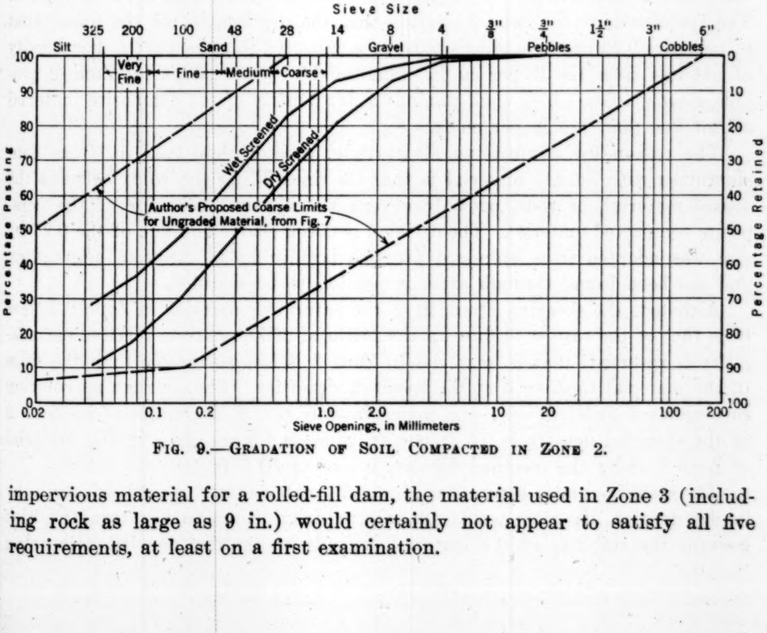

FIG. 9.—GRADATION OF SOIL COMPACTED IN ZONE 2.

impervious material for a rolled-fill dam, the material used in Zone 3 (including rock as large as 9 in.) would certainly not appear to satisfy all five requirements, at least on a first examination.

As a matter of fact, with the rolling equipment used at San Gabriel Dam No. 1, which was specially designed for the job, material in Zone 3 requires less effort for compaction than material in Zone 2; that is, whereas twelve trips are required to produce the specified compaction in Zone 3, between sixteen and twenty trips are required to produce such compaction in Zone 2. The specifications for Zone 3 provide that the dry density of the fines (that is, particles 0.25 in., or smaller) must be compacted so as to have a dry density of not less than 120 lb per cu ft., whereas the specifications for Zone 2 provide a compaction which shall produce a dry density of the composite material of not less than 115 lb per cu ft.

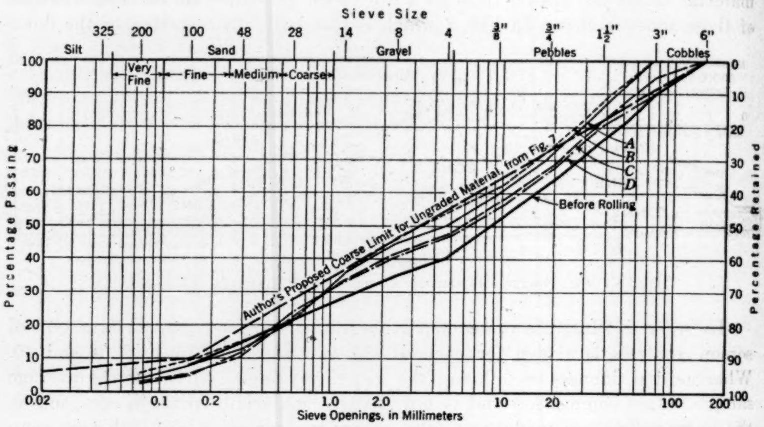

FIG. 10.—EFFECT OF ROLLING (TEN TRIPS WITH A 12.1-TON ROLLER) ON GRADATION OF MATERIAL PASSING A 6 BY 9-FOOT GRILL.

The reason for eliminating all particles greater than 0.25 in. from consideration with Zone 3 material is that the fines filling the voids between the coarse material, or rock, actually govern the physical behavior of the composite compacted material. The division between coarse and fine of the 0.25-in. size was adopted from standard practice pertaining to concrete aggregates, and has been found through experimentation to be justified.

Although the shearing strength of the saturated material in Zone 2 is less than that of the saturated fines in the material of Zone 3, the relative permeability is reversed; that is, material in Zone 2 is less permeable than the fines in the material in Zone 3 at the specified densities. The average percolation rate under a gradient of 1 and approximately 60° F for material in Zone 2 at the specified density is 0.5 ft per yr, whereas for the fines in the material of Zone 3 under the specified density, it averages 1.5 ft per yr.

The stability under saturation of any fill material is governed primarily by its dry density. In other words, the internal friction or shearing strength which governs the stability of the material depends on the dry density. Shearing tests on saturated fines of the material in Zone 3 have shown shearing strengths varying from 22 lb per sq in. for a vertical load of 25 lb per sq in., to a shearing strength of 525 lb per sq in. for a vertical load of 1 200 lb per sq in. Due to this governing factor (that is, the dry density of the material), it is quite obvious that the conditions for stability of a hydraulic-fill dam under saturation and the conditions for stability of a rolled-fill dam under saturation are necessarily at variance. With the hydraulic-fill process it is not possible to create an initial dry density of the material in excess of a relatively low limit, whereas with the rolled-fill, or mechanically compacted-fill process, the attainable dry density is approaching that of the solid material. The composite fill in Zone 3, for example, in place, has an average weight of 150 lb per cu ft., which indicates that it is approaching the physical state of an artificial stone. The specific gravity of the material used in Zone 3 averages 2.78, which means a solid weight of 173.5 lb per cu ft. The average dry weight of the composite material in place was found to be 143 lb which would mean that the porosity of the composite fill in Zone 3 is approximately 17.5 per cent. Typical results of density tests are shown in Table 7.

TABLE 7.—SOIL CHARACTERISTICS SUPPORTING THE CURVES IN FIG. 12

Sample taken from Zone: (1)	FIELD CONDITIONS					COMPACTION DATA	
	Wet weight, in place, in pounds per cubic foot (2)	Dry weight, in pounds per cubic foot (3)	Percentage of moisture in fines (4)	Percentage of rock (5)	Number of trips by a 12.1-ton roller (6)	Specific gravity (7)	Percentage passing a No. 200 sieve (8)
3....	151.5	128.7*	9.6	42.4	12	2.78	25.4
2....	133.9	118.2†	13.3	3.2	20	2.79	36.3

* Fines. † Composite.

The reason for the small porosity and, consequently, the small permeability of material in Zone 3, is the breakdown that occurs during the rolling of the material as is evident from Fig. 10. The effect of compaction relative to breakdown for variable moisture is further illustrated in Fig. 11. Fig. 11(*a*) shows the increase in breakdown due to twenty-five hard tamps with a 5.5-lb tamper over the breakdown due to five 4-in. drops. The maximum, additional breakdown coincides with the peak of compaction.

Furthermore, the initial compaction of granular material and the breaking strength of the particles are governing factors in regard to its stability under saturation, because the water which, in a saturated condition, completely fills the voids, cannot escape in case of sudden compression of the material, but will, itself, be put under stress. Hence, if the compression is of sufficient magnitude to crush the parts in contact between particles, the shearing strength may readily drop to a critical value due to load being transmitted through water on the one hand and to the production of very fine particles or flour caused by the crushing on the other. These fine particles, after crushing, will naturally mix with the water and will form slime which tends to lubricate the contact surfaces and minimize internal friction.

In regard to cohesion due to the surface tension of water which is always present in relatively fine material with one or more free surfaces exposed to air, such cohesion cannot develop under saturation. This is the principal cause of the softening up of granular soils, as well as clays, under saturation. Hence, all tests bearing on the stability of a relatively impervious fill should be conducted on saturated material in order to determine the safety of such a structure in its critical condition.

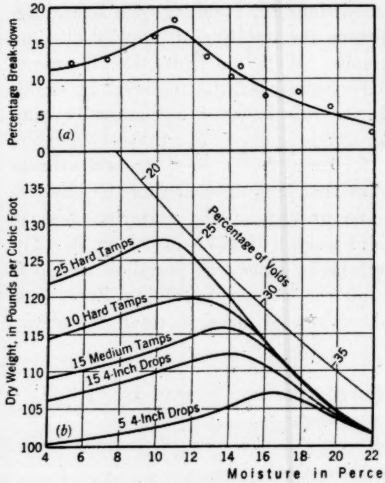

FIG. 11.—COMPARISON OF EFFECT OF MOISTURE CONTENT ON COMPACTION AND ON BREAKDOWN.

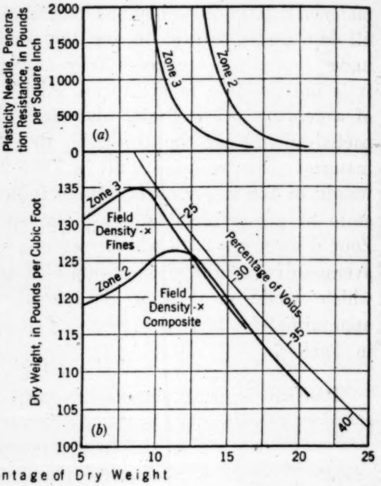

FIG. 12.—SOIL CHARACTERISTICS.

The author does not mention the penetration needle in connection with soil analysis and gives no reason for this omission. Based on tests (as yet incomplete) conducted at the laboratory at San Gabriel Dam No. 1, it is believed that the significance of the penetration or plasticity needle has been misinterpreted in the past. These tests tend to show that the penetration needle, although an excellent moisture indicator on the wet side of the peak of compaction (see Fig. 12) is not reliable as an indicator of stability of a fill if the pore water is under stress.

The safety of a dam fill composed of insoluble material depends directly on the shearing strength of its grain structure as long as over-topping is prevented. The shearing strength for a given density, in turn, depends on the rate of percolation and the pore-water stress. Although the rate of percolation primarily depends on the grain size, the pore-water stress depends on the condition of loading and particularly the rate at which load is being applied. Hence, the influence of the sudden application of load on a saturated dam fill, due to earthquake, for example, is an important part of the analysis of safety.

William C. Hill,[18] Assoc. M. Am. Soc. C. E. (by letter).—Engineers concerned with highway construction and soils will agree with the author that "probably the most severe test for the stability of earth material is as a wearing surface for an earth road." During the last few years a wearing course of sand and gravel stabilized with a binder soil has been developed for low-cost, dustless, road surfaces and bases for bituminous mats.

In Minnesota, the gravel to be stabilized in this manner is mixed with binder soils ranging from a sandy loam to a clay with plasticity indexes from 18 to 45. Care is taken that the grading of this mixture falls within the limits of a narrow band developed through the efforts of the soils engineers of the Highway Department from observations on a large number of existing gravel roads.

These roads were examined to determine their surface performance under varying traffic and climatic conditions. Samples were taken and analyzed in the laboratory to determine the gradation of those mixtures which were yielding most satisfactory performance. On the basis of these results, together with subsequent observation on some of the earlier projects stabilized by the Maintenance Department, the present specification was developed. This band differs from that of the United States Bureau of Public Roads given by the author, as shown in Table 8. (In a later report[19] than that cited by the author,[9] the Bureau has discontinued the use of the No. 270 sieve in favor of the No. 200, for practical purposes).

TABLE 8.—Percentage Passing by Weight

Sieve	U. S. Bureau of Public Roads grading	Minnesota grading
1-in.	100	....
¾-in.	85 -100	100
⅝-in.	....	95 -100
⅜-in.	....	70 - 95
No. 4	55 - 85	....
No. 10	40 - 65	35 - 65
No. 40	25 - 50	20 - 40
No. 100	....	15 - 30
No. 270	10 - 25	10 - 20

Fig. 13 shows the relation of the Minnesota band to the theoretical Talbot curve obtained by use of Equation (1) with the exponent, *n*, equal to 0.33. This curve indicates the theoretical grading to give maximum densities for aggregates having a maximum size of 1 in. to 2 in. It will be noted that the Talbot curve falls along the middle of the band. Where a close control of the mixing of the gravel with the binder soil has kept the grading near the middle of the band, a hard dense road surface has resulted.

The author points out that there is an optimum moisture content for maximum density. C. A. Hogentogler, Assoc. M. Am. Soc. C. E., has shown[20] that temperature influences this optimum point. In a series of experiments Mr. Hogentogler used three fine-grained soils and demonstrated that the higher the temperature the greater would be the density with the lower moisture content and equal compaction. He follows this with a demonstration proving that the stability of soils varies inversely with the temperature and is a reversible action.

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[18] Junior Engr., State Dept. of Highways, Brainerd, Minn.

[19] *Public Roads*, May, 1936, p. 50.

[20] *Loc. cit.*, pp. 48–50.

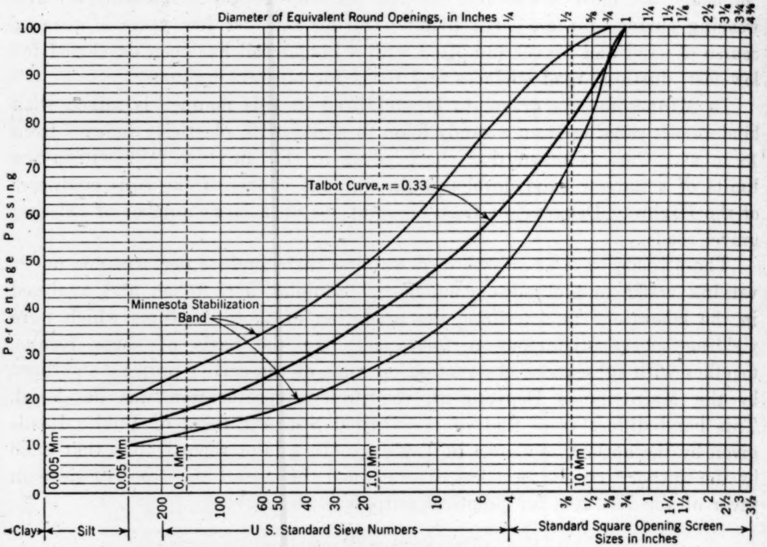

FIG. 13.—MECHANICAL ANALYSIS

A. FLORIS,[21] ESQ. (by letter).—In this scholarly paper the author discusses earth testing as an aid to the selection of materials for rolled-fill earth dams and establishes five principal requirements regarding suitability of material for this type of dams; Stability, water-tightness, workability, insolubility, and reasonable cost.

The first of these requirements is essentially one of statics and, according to the author, can be secured in a well-graded material through mechanical compaction. In other words, the author, in line with other authorities on the design and construction of earth dams, believes that the stability of a rolled-fill earth dam can be best assured, if a properly graded material is used. By analogy one might eventually conclude, that the safety of concrete dams can be taken for granted if the material is properly prepared. This is not correct, of course, because in addition to the quality of the concrete the forces acting on the dam must be investigated. This holds equally true for the earth dams treated by the author. Under the action of water pressure and dead weight, concrete dams can be analyzed quite satisfactorily. No such analysis is generally available for earth dams, and, unfortunately, the scope of the paper does not embrace this phase of the subject.

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[21] Dipl.-Ing., Los Angeles, Calif.

Too much emphasis has been laid upon the material and very little attention, if any, has been paid to the statical condition caused by the forces acting upon earth-fill dams. As far as the writer is aware the first attempt to remedy this unfavorable condition was made by S. Ehrenberg[22], whose analysis is based upon new concepts of soil mechanics.

FRED D. PYLE,[23] M. AM. SOC. C. E. (by letter).—In discussing the factors to be considered in examining materials for the construction of rolled-fill dams, Mr. Lee has performed a notable service to engineers. Personal observations made many years ago of priming of canal banks constructed by teams, indicated that where material is reasonably well graded no trouble is encountered, but where it is of uniform size (whether clay, sand, gravel, or rock) there is a leakage with resultant sloughing on the outside of the bank where the materials are sufficiently fine to prevent free drainage. Except for the top 2 or 3 ft, satisfactory highway fills may be constructed of ungraded materials, sand, gravel, or rock, provided there is sufficient compaction, drainage, and lateral support to prevent excessive settlement.

It would appear that the following factors, in addition to those discussed by Mr. Lee, might well be considered:

(*a*) Consolidation by weight, which may be determined by testing and which will become evident in the completed dam by the rate and amount of settlement;

(*b*) Allowable percentages of colloidal and clay materials; and

(*c*) Capacity of materials, when placed in rolled-fill dams, to adjust themselves to settlement without cracking or formation of planes of lessened resistance to percolation of water.

Referring to Factor (*a*), consolidation tests should include, in addition to tests of normal material, tests of such material with the introduction of water after a consolidation head equal to one-third the height of the dam is reached. The weight load of the fill tends to increase density with corresponding reductions in the voids, and permeability. More information should be secured by engineers on the rate and amount of settlement of all dams constructed of earth and rock, both during and after construction.

An excess of fine materials (see Factor (*b*)) often means a large capacity for water which may result in excessive cracking and checking on loss of moisture content and excessive settlement.

Concise, brief definitions of certain words or terms used by the author and by others would be of assistance to those interested in studying dams constructed of earth, whether by rolled or hydraulic methods.

In order that comparisons may be made quickly with a minimum of effort it is believed advisable that gradation graphs be in millimeters, with inches and sieve sizes shown at the top and the classifications—clay, silt, sand, gravel, etc.—shown across the middle of the sheet. It is interesting to note that material having diameters between 0.04 in. and 0.2 in. is classified as gravel on gradation graphs.

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[22] "Grundlagen der Berechnung von Staudaemmen," von S. Ehrenberg, *Wasserkraft und Wasserwirtschaft*, 1929 No. 23.

[23] Hydr. Engr., City of San Diego, San Diego, Calif.

As it is practically impossible, in the construction of rolled-fill dams, to mix materials from several sources to secure a modified material having or approaching the ideal characteristics, it becomes necessary to determine the suitability of local materials by inspection and by applying the laboratory tests, suggested by Mr. Lee.

Under the heading, "Impermeability", the author discusses permeability coefficients without mention of the degree of pressure or loading on the materials when tested. As the pressure has an effect on the voids, there must be some effect on the permeability coefficient.

JOEL B. COX,[24] ASSOC. M. AM. SOC. C. E. (by letter).—This paper is a greatly needed contribution to the literature on earth dams. Especially to be commended is the clear statement of the requirements of earth suitable for rolled-fill earth dams. The discussion of these requirements is, of necessity, unequal in its satisfaction. The study of grading is an important contribution to current knowledge of the subject, but it must be emphasized that the close relationship between porosity and grading shown by Fig. 1 is obtained from a single series of soils from a comparatively uniform source. The importance of this will be recognized on comparing such an example of Hawaiian laterite as Sample No. 233-*B* in Table 9 which gives a porosity, with the most careful packing, of 64.8 per cent. Its grading curve plotted on Fig. 1 certainly does not indicate the grading as a cause of the high porosity. This porosity is due to the peculiar expanded character of the fine colloidal material in all such soils.

Mechanical analysis cannot be expected to shed much light on this and similar conditions, largely because, by present methods, it is restricted to particle sizes of 0.005 mm and larger.

The excellent treatment of compaction would be improved by a critical discussion of the permissible air content at maximum compaction, which, according to the writer's observation, is a most important quality of suitable soils for rolled-fill construction.

The relationship between permeability and permanence and satisfaction of an earth-filled dam is discussed in this paper insufficiently. The effect of permeability is twofold: (1) The economic loss due to leakage from the reservoir; and (2) the effect on the dam itself. The latter is dependent largely on the relationship between the finest fraction of the soil in the dam and the velocity of percolating water. A far higher permeability is safe with a soil composed of sand and gravel than one containing silt and clay.

The author's main proposal that mechanical analysis be used as a preliminary basis for the selection of material, must be carefully covered by reservations, as Mr. Lee does. There is an intimate relationship, of course, between porosity and permeability and the mechanical composition of the material. In any soil containing an appreciable proportion of material finer than 0.005 mm, however, the qualities of the soil are so governed by the finer, colloidal fraction in its composition that mechanical analysis is of little value in interpreting or estimating the observed data. Since permeability and porosity are measured with an ease entirely comparable to that of making accurate mechanical analyses, it is not clear why there is an advantage in using the indirect and remote measurement rather than the direct one as a basis for selection. Table 9 and Fig. 14 give mechanical composition data, porosity, and permeability for a selected group of seven typical samples of Hawaiian soils, which illustrate the difficulties of using mechanical analysis in comparing materials of unlike origin. The low degree of compaction possible with such soils, the high air content, and the extraordinary elasticity under rolling which makes compaction in an earth dam exceedingly difficult, the impermeability which prevents proper drainage of the down-stream face, and the ease with which the finer fraction is removed by any percolating water, all render these soils extremely hazardous in rolled-fill dam construction, even if some of them fall within the mechanical analysis limits proposed as satisfactory. The lack of relationship between mechanical analysis and permeability is shown not only by these samples, but by such a sample as *L*15–*S*10 of Table 5, the permeability of which is not more than 0.0001th as much as would be estimated from its grading curve in comparison with the others shown.

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[24] Civ. Engr., McBryde Sugar Co., Ltd., Eleele, Kauai, Hawaii.

TABLE 9.—GRADING AND PERMEABILITY OF HAWAIIAN SOILS

Sample No.	Sizes, in Millimeters			Porosity (percentage of voids)	Permeability, in gallons per square foot per 24 hours	Sample No.	Sizes, in Millimeters			Porosity (percentage of voids)	Permeability, in gallons per square foot per 24 hours
	10% passing	30% passing	60% passing				10% passing	30% passing	60% passing		
239	(?)	0.003	0.052	64.5	0.00131	248–*A*	......	0.0008	0.013	62.6	0.0102
129	0.0013	0.0046	0.0135	65.6	0.00258	296	0.008	0.0390	0.670	66.0	0.0607
258	0.008	0.0132	0.026	66.5	0.00364	233–*B*	0.022	0.136	0.900	64.8	0.240
333	(?)	0.0143	0.800	60.4	0.00403	.....	......	......	......	....	....

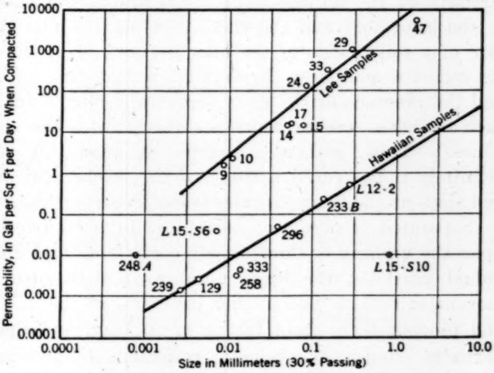

FIG. 14.—RELATIONS BETWEEN PERMEABILITY AND PARTICLE SIZE

Fig. 14 shows graphically the enormous difference between soils of entirely different character but with somewhat similar mechanical analysis curves. The fact that the two average lines representing the Lee samples and the

Hawaiian samples differ by ratios of from 1 000 or 10 000 to 1 in permeability clearly indicates the danger of too great dependence on such data. It is true that Mr. Lee points out that grading is more important than particle size in governing permeability, but no satisfactory measurement of different degrees of grading appears as yet available. The writer believes that although a mechanical analysis may be developed into a useful tool, furnishing criteria for estimating relative differences between soils of a homogeneous series, it will always be necessary to fall back on direct measurements of permeability, porosity, and compaction in comparing widely different soils.

Mr. Lee's experience with a siltstone soil and his recognition of the undesirability of adobe and lateritic soils for rolled-fill dams is important to the engineer engaged in such work. It is to be hoped that the discussion of this paper will bring forth a practicable proposal for a measurement of the workability of various soils.

STANLEY M. DORE,[25] M. AM. SOC. C. E. (by letter).—The use of soil laboratory results as aids in determining the suitability of materials for the construction of earth dams is well descrbed in this paper. The stability of an earth embankment depends primarily upon the shearing strength of the materials of which that embankment is built and, as the author points out, high shearing strength in soils exists when there is a proper combination of cohesion and internal friction. Whether or not the cohesion is due to the strength of the minute films of water (as the author contends) many tests in the laboratory and elsewhere bear out the fact that the larger the proportion of a soil that is of the finer grained, or of the clay, size, the greater are its cohesive qualities. Especially in materials of a glacial origin, cohesion and the presence of the finer particles go "hand in hand", and the value that such cohesion reaches depends directly upon the sizes and quantity of these "fines." Thus, without a sample at hand, the cohesion of a material is definitely indicated by a study of the mechanical analysis curve, the number and sizes of the finer particles being shown on the curve.

Separated from the cohesion, the internal friction of a material depends upon the grading or "uniformity coefficient" and upon the shapes of the individual particles, the large number of sizes furnishing more resistance to movement because the smaller particles fill the voids between the larger ones and prevent them from tilting or rocking, and the sharp, angular, cut-clear particles offering much more resistance to movement than rounded, worn, smooth particles. An examination of the mechanical analysis curve would indicate the extent of the grading, and microscopic examinations would reveal the predominating shapes; and from these examinations the internal friction of a material is indicated Mechanical analysis curves that are concaved upward show graded material of relatively higher shearing strengths, and curves that are convex upward show ungraded materials of lower shearing strengths. Thus, from the mechanical analysis curves and the microscopic examinations, both factors of the shearing strength can be estimated—namely, the cohesion and the internal friction—and the judgment of the degree of stability of a given material based upon visual examination and "feel" to the hand can be augmented considerably by these estimates.

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[25] Associate Civ. Engr., Massachusetts Metropolitan Dist. Water Supply Comm., Boston, Mass.

The shearing strength of a material is increased by compaction. Vibration is the most important factor in securing satisfactory compaction and density, and the author has not directly mentioned vibration as a means for securing compaction. The success of tamping, rolling, or compacting by other mechanical means usually depends to some extent upon the vibration attending these operations, and because of its great value in obtaining high degrees of density and compaction, various types of mechanical vibration equipment are supplanting, and will supplant, many of the "pressing or loading" types of compacting machinery now (1936) in use.

Without doubt, the best materials for use in rolled impervious embankments are those which are well graded and which contain fractions of all sizes from pebbles to clay; but the engineer must not lose sight of the fact that rolled fills, satisfactory for the purposes intended, can be built of certain uniform or ungraded materials and that it is often a question of economics whether the more, or the less, preferred materials are used.

In Fig. 4 the compaction curves for loam seem to give the highest densities. Such loams must be well graded, must contain appreciable quantities of pebbly or gravelly materials, and must be consolidated by heavy equipment in order to obtain such high densities. If these loams had contained large quantities of ungraded silts and sand in addition to a small percentage (say, 5%), of organic materials, such high densities would not have been obtained. When compacted in 6-in. layers, with a 5-ton, 40-in., sheepsfoot roller, top soils have dry weights generally within the limits defined by the stipled area in Fig. 15. These values cannot be plotted as a single curve, due to the effect of the variable organic content of the different top soils used on the critical

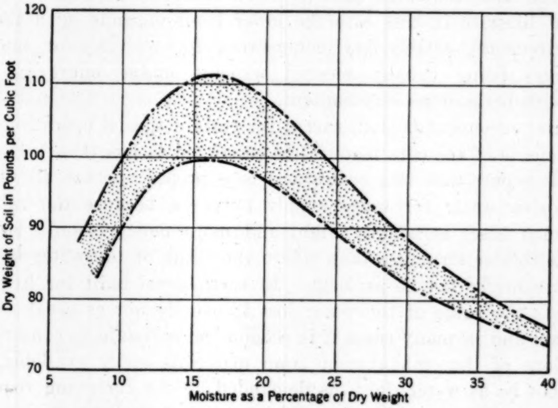

Fig. 15.—Foundations Top-Soils for Impervious Refills, Quabbin Dams

moisture content needed for maximum limits. The limiting curves in Fig. 15 show the general average conditions that yield best results. For instance, in Fig. 15 is indicated the moisture-content, density-compaction curves of loams consisting of a mixture of top soils and subsoils. The organic content in these materials averages 2 to 5% and the diagram shows that with very satisfactory compaction the densities were not greater than about 110 lb per cu ft.

The recommendation of the author for the upper limit of the clay fraction in materials for use in compacted fills is well chosen. Materials that have a clay fraction of about 30 to 35% can be compacted only with considerable difficulty because they cannot be worked satisfactorily due to the fact that in material of this nature enough of the "superfine" and colloidal particles are present to make the mass "sticky", "jelly-like", and resilient. However, the lower limit is not quite so definite. Materials containing little or no clay can be compacted satisfactorily, provided they are otherwise suitable and that the quantity of the clay fraction necessary is really dependent upon the impermeability requirements and not upon the ease or difficulty with which such material compacts. Of course, the compaction is somewhat easier when a small percentage of clay is present because, when moist, this clay lubricates the mass somewhat; but with sufficiently heavy equipment and the proper moisture contents, materials with little or no clay can be compacted satisfactorily and without undue difficulty or expense. For uniform or ungraded materials Mr. Lee indicates that a lower limit of perhaps 15% may be needed; but likewise in this case the lower limit depends upon the permeability requirements, satisfactory compaction for stability of materials of glacial origin being usually secured without undue effort in ungraded materials with little or no clay content.

The writer recommends that, except for certain special conditions, permeability coefficients of the core material be restricted to less than 1 gal per sq ft per day. It seems that this restriction is a proper one for all dams to be used in storing water for water supply purposes, because the value of the water in such cases is usually high, and more money should be spent in obtaining a tighter structure than where the value of the water lost through the structure might not be as large. In earth dams built for hydro-electric developments the value of the water lost is usually not as great as in water supply dams; and in many cases it is economical and safe to construct impervious portions of the embankment from materials easily available, although they may not be as water-tight, as demanded by the foregoing requirements. In such cases the permeability coefficient could exceed 2 gal or even 3 gal per sq ft per day; and the stability of the structure, if proper drainage of the down-stream section of the dam were provided, would not be endangered.

The author's proposal to use the mechanical analysis curves in choosing the limits for suitable materials and, arbitrarily, to establish such limits for graded and ungraded materials, is a good one. However, in the curve limits suggested, the parts representing these limits are not useful above the 40 or 50% size, and the most important parts are those between the 0 and 30% size.

Fig. 16 shows the Kendorco[26] classification in reference to the limits established by the author. In this classification, the words, "uniform" and "variable", are used in the same manner that the author uses "ungraded" and "graded." The finer of Classes 6 and 7 and all of Classes 8 and 9 represent materials suitable for impervious sections of the rolled-fill dams in accordance with the author's limitations. Classes 2, 3, 4, and 5, and possibly the coarser of Classes 6 and 7, represent materials that are probably unsuitable, although the curves for them fall outside the prescribed limits only at the smaller sizes. Thus, it seems that the important part of the curve for material of glacial origin is in the vicinity of the 5 to 10% sizes only, and the location of the limits elsewhere is immaterial. Although the limits chosen by the author will be satisfactory in nearly all cases, they must not be used too rigidly as it is very probable that materials perfectly suitable for the pur-

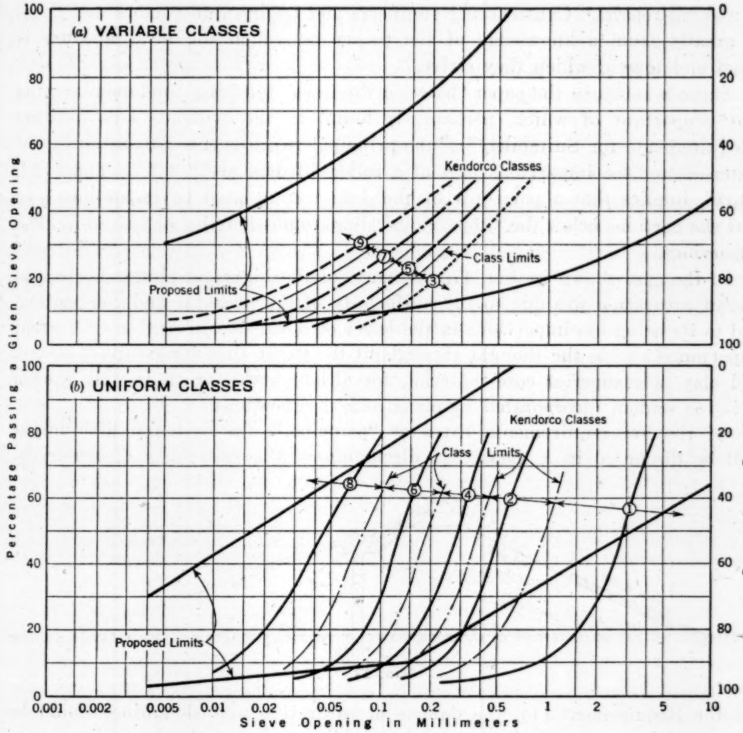

FIG. 16.—IMPERVIOUS SECTIONS OF ROLLED-FILL DAMS: COMPARISON OF PROPOSED LIMITS IN MECHANICAL ANALYSIS WITH GRADED AND UNGRADED MATERIALS OF GLACIAL ORIGIN.

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[26] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 689–690.

poses intended will be found, which, in some individual cases, will have mechanical analysis curves that will lie outside of one of the limits shown for a short distance.

John E. Field,[27] M. Am. Soc. C. E. (by letter).—Engineers will welcome a technical treatment of the selection of materials for the more impervious parts of an earth dam, as a guide to design and construction of hydraulic and rolled-fill structures, and in masonry and rock-fill dams, where the necessity of blanketing portions of the reservoir basin sometimes arises, particularly in making repairs. Constructing engineers and engineer inspectors will benefit greatly from a knowledge of the theory on which the ideal mixture is based and toward which they strive.

There is much in the paper "between the lines" touching fundamentals, the most important of which, perhaps, is found in the sentence (see heading "Requirements for Suitability"), "the principal requirements for suitability of materials for the impervious part of a rolled-fill dam are * * *." The word, "part", implies that a part only of the dam section need be impervious and that the portions below the "impervious" diaphragms may be, or should be, less impervious.

In the case shown in Fig. 17, the cause of the high and almost horizontal line of saturation was due to the uniformity of the material and compaction, and to its being too impervious in the lower or down-stream section. Of great importance also is the thought throughout the paper that a mixture of gravel and clay is a superior core material, for although this truth is well known, it is too seldom incorporated in plans and specifications.

Of the five requirements listed as "principal", the first and fifth might well be discarded in a theoretical discussion of the suitability of materials.

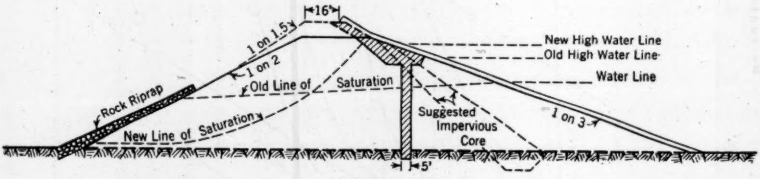

Fig. 17

As for Requirement (1), the danger of saturation and sloughing should be prevented by proper drainage and by rigidly confining the impervious core. Cost (see Requirement (5)) should not be considered in arriving at an ideal; it should be considered only in the practical application to a specific problem after the ideal, toward which it is desirable to strive, has been fixed.

The ideal mixture, local materials considered, and the methods of attaining it, has use in many structures other than dams and where the quantities involved are small, although unit costs may be high as, for example, back-fill around culverts and siphons under large canals, the lining of canals passing over cavernous limestone or through very porous sections, and the back-filling of trenches in abutments and cut-off trenches of earth-fill dams.

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[27] Cons. Engr., Denver, Colo.

The use of a nearly ideal mixture at considerable unit cost is shown in Fig. 17, a case in which the writer repaired an earth-fill dam. The dam, then about ten years old, had threatened to slough on the lower slope whenever the water rose to within 10 ft of the high-water line, and heavy riprap had been placed on the lower slope to enable the owner to use one-half the reservoir's capacity. The material in the fill was almost wholly of disintegrated granite containing some loam, clay, mica, and sand. It is unnecessary to give the details of the work, except to state that the mixture was 33% clay, with a small mixture of sand and loam, and 67% of the disintegrated granite excavated from the dam. The moisture content was about 10%, giving a stiff product easily handled and tamped; the width of the vertical trench was only 5 ft.

The repairs were successful in stopping all tendency to slough, the lower slope dried up and after five years' use no weakness has appeared. The cost was amply repaid by the gain in capacity above the old, theoretical, high-water line, the increased depth being about 7 ft. Notable are the very thin "impervious" vertical diaphragm, the more porous material back of it, and the change in the line of saturation.

With impermeability secured in a diaphragm, it is not always required that it have hardness, permanence, or stability. In the Necaxa Dam, and in many others of the hydraulic-fill type, the cores are probably of a consistency that would permit a lead weight to sink through their entire depth, but such a condition requires that the fluid core be firmly confined.

In all dams, weight is required to oppose the horizontal water pressures, and a high line of saturation and upward pressures decrease the effective weight. An impervious diaphragm and adequate drainage below it are effective and practical in lowering the line of saturation, and with these provided, the compaction of the body of the dam merely adds some weight and a more rigid support to the core, but adds nothing to its impermeability.

Therefore, it appears that a light diaphragm, made as impervious as possible with the ideal mixtures of sand, gravel, and clay, is more to be desired than a thicker section with an approximation to the ideal. Mechanical mixers of large capacity can be built and, in the writer's opinion, will be used in the not distant future to furnish the ideal material for the diaphragm, and at that time the formula and practice which the paper seeks to develop, will be used.

In his "Synopsis", the author mentions "the traditional manner of determining suitability of material" and intimates its inferiority to scientific methods; but the use of scientific methods and the blind adoption of methods and formulas are equally dangerous, particularly if the cost is considered.

The late James Dix Schuyler, M. Am. Soc. C. E., states[28] that he had seen more mistakes in the ordinary earth-fill dam than in all other engineering structures coming under his observation. In this statement the writer heartily concurs, but wishes to add that the most numerous and serious mistakes he has observed in earth dams have been made by engineers of high technical attainments, particularly in the matter of cost, where the engineer failed to appreciate and take advantage of local conditions and materials, but adhered to theoretical formulas.

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[28] "Earth Dams", by James Dix Schuyler.

There is one desirable quality in fills of mixed clay and gravel not mentioned by the author, namely, that shrinkage cracks are lessened or obviated. In flood-control reservoirs, particularly in the arid regions, the dams may stand for years, become thoroughly dried out, and shrinkage cracks become a menace when floods fill the reservoir.

Referring to the use of tools and clogging (see heading "Workability"), the writer feels that the disk harrow is a valuable tool in securing uniformity of mixture and of moisture; that the mere stirring of the fill material gives opportunity for the smaller particles to find and fill the voids much more certainly than even the sheepsfoot roller; and that thorough mixing by the harrow obviates the danger of clogging the rollers and other tools.

Movement seems to be essential to thorough compacting and the moisture content should give sufficient fluidity to the material for it to move under the roller. A slight wave just ahead of the wheels of heavily loaded trucks, or a shallow rut pressed into the surface of the fill, indicates that the material is sufficiently fluid, whereas deep ruts indicate too much moisture and no ruts at all indicate too little.

Insolubility.—The writer has found as high as 24% of soluble matter in soils and in the case of one dam that failed he found, by the use of distilled water as a solvent, 8%, and by the addition of 1% of hydrochloric acid, 15%, of soluble matter. In every case, especially in the arid regions, solubility should be one of the first considerations and a knowledge of the geologic source of the material should be ascertained.

Neither clay nor adobe is essential to impermeability. Glacial flour or the slimes from stamp-mills are quite as impermeable and are of greater stability and higher frictional resistance. A search for, and recognition of, these materials and their use should be kept in mind in all glacial-deposit and mining regions.

In practice, the engineer first determines the ideal mixture of clay, gravel, and water. In construction, the percentage varies from time to time in spite of the most careful supervision and willing co-operation, and the author (see heading "Stability: Grading") advances the same idea: "In earth-stabilization practice * * * natural materials must be used, which conform as nearly as possible to an ideal grading."

Through "traditional" methods, the engineer-inspector and his assistants must be able to recognize material variations instantly and to apply the corrective measures at once, without waiting, for sampling and the results of laboratory tests; and eternal vigilance, and immediate corrective action is the price that must be paid for good work and proper progress. For quick inspection, the writer has found the use of the compression needle valuable, especially in impressing the dump boss and foreman with the idea that a covered-up fault would certainly be detected.

In the paragraph that introduces Table 2 is found mention of specifications "requiring exclusion of adobe soils." What is locally known as "adobe soil" varies widely in different localities, and a definition covering all varieties would probably be "a soil from which sun-dried brick can be made." It often contains considerable loam (another term extremely indefinite), and sand. In specifications, the writer would probably state that "adobe mixed with sand and gravel shall be used in the impervious core section", and add that "shales and indurated materials, although partly decomposed, shall not be used."

William P. Creager,[29] M. Am. Soc. C. E. (by letter).—In the writer's opinion, the author has done very well with the limited available data. He has advanced a step farther toward the establishment of definite limits beyond which it would be inadvisable to go in the use of soils for rolled dam construction; but, as he states, his hope is that his criteria will bring forth additional data from which may be evolved a basis for preliminary selection.

It is significant that his hope does not extend beyond criteria for a "preliminary" selection of materials. In this, he has shown himself conversant with the fact that many characteristics of soils, such as grain shape and grain composition, are not indicated by mechanical analysis but that they have much influence on the behavior of the material. Thus, at best, only a rough approximation of the essential characteristics of materials can be obtained from mechanical analyses.

His comparisons of mechanical analysis curves with stability are especially pertinent. Recognizing the well-known fact that the greatest density and, hence, greatest stability, results from a well graded mixture, he has shown in Fig. 1 the shape which curves of well and poorly graded mixtures assume on the usual chart of mechanical analysis. Fig. 1, together with his comparison of mechanical analysis with compaction curves, Figs. 3 and 4, brings out vividly that the best materials not only must be well graded either coarse or fine particles being less suitable.

Thus, in Fig. 4, the "fine sand" (Sample *L*1-1) has no very fine particles and hence is lacking in cohesion. Therefore, it possesses insufficient compressibility and squeezing properties to expel the entrained air effectively. It has low density and many air voids. Next above in Fig. 4 is the "sandy gravel" (Sample *L*15-*S*10), which contains a small percentage of fines. This is a fairly well graded mixture and hence has high density. However, it has insufficient fines for the proper compressibility and squeezing effect to expel a large part of its entrained air.

Closest to the curve of zero air voids are the loams, shown by full lines in Fig. 4. These loams are the ideal materials containing between 16% and 33% of clay accompanied by abundant coarse particles. They have the fewest air voids.

Finally, the silty clays and clays, Samples *L*15-*CG*, *L*11-1, and *L*-8*A*, have a large percentage of fines, providing excellent squeezing properties; but they possess such low permeability that little if any draining of water and air can take place. Hence, they are of low density and have many air voids. It is difficult to account for the fact that Sample *L*11-1 shows up better than Sample *L*-8*A*. This is one of the inconsistencies that creep into the best of tests.

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[29] Cons. Engr., Buffalo, N. Y.

The writer feels that Fig. 6, defining proposed limits for graded materials is a great help in preliminary studies, provided: (1) That its use is tempered with experience and judgment; (2) that it is remembered that some materials outside these limits might be used; and (3) that some materials within such limits may be entirely unsuitable. Sample 42, for instance, lies wholly within the limiting curves, yet it has a compacted porosity of as much as 44.4%, and is quite permeable. This material is eliminated under the author's somewhat indefinite specification: "* * * it is not sufficient that the mechanical analysis curve should lie between the respective coarse and fine limiting curves; it must also have a shape corresponding to, or approaching, that of the limiting curves."

In order to assist in a graphical indication of this specification and more nearly define reasonable requirements, the writer suggests the possibility of using an additional curve, *AB*, Fig. 18, which would be the fine limiting curve for all materials having less than 15% clay content; that is, the curve for material having less than 15% clay content must not cross Curve *AB*. Thus, Sample 42, having less than 15% clay, lies wholly within the author's limiting lines, but it crosses the fine limiting line, *AB*, and, therefore, is undesirable, whereas Sample *L*15-*S*9 is on the order between good and bad material.

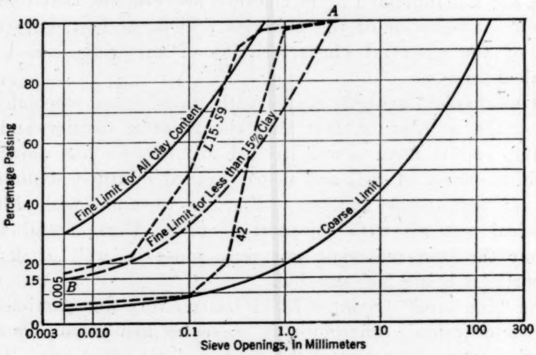

FIG. 18

It is not claimed that this additional curve will supplant judgment, but it is just one more step to assist in visualizing requirements. The writer's suggestion is not confined to one additional curve. Other fine limiting curves might be used for materials having less than, say, 22% and 8% clay content.

In conclusion, the writer can see no practical need for Fig. 7 showing proposed limits for ungraded materials. Comparing the coarse limiting curves of Figs. 6 and 7 for graded and ungraded materials, any material having a curve which lies outside the ungraded limiting curve, but inside the graded limiting curve, automatically departs from the ungraded class, and Fig. 6 would apply. Referring to the corresponding fine limiting curves for graded and ungraded materials, it is the writer's belief that the difference between these curves is far less than is justified by present knowledge of the relation between suitability and mechanical analysis.

JOSEPH JACOBS,[30] M. AM. SOC. C. E. (by letter).—(1) The demand and need for a material expansion of water storage facilities throughout the United States, the economic importance of safety for the heavy investments involved in potential developments, and the fact that much of the required new storage will be effected by means of earth dams, renders this paper of timely interest.

(2) The earth dam is the oldest type known to Man. Where truly adapted to the site it is probably the best and most permanent type and, in fact, there are now existent more dams of that type than of all others combined. Despite these facts there is yet much to be learned concerning earth dam construction: How to select and effect the most economic mixture of available materials, the best degree and best methods of compaction, the stability of section, economic slopes, core-walls, and the best location and depth of cut-off walls, the density and permeability of various mixtures of earth materials, etc. It is mainly within the past decade that intensive study has been given to the problems of soil mechanics, the formulation of their basic principles, and the application of these principles to practical engineering. The entire realm of earth dam construction is still a fair field for research and this research should include not only detailed laboratory investigations, but also an intensive study of existing dams in respect of the aforementioned elements.

(3) In this connection it is of interest to contrast the densities that have been attained in compacted earth mixtures with those of concrete mixtures. In compiling certain data from engineering literature on earth dams, the writer finds that of about thirty actual tests of earth mixtures, for which mechanical analyses were given, the dry weight of the mixtures, after compaction, ranged from 96.7 to 132.5 lb, and averaged 115.4 lb per cu ft, or only 67.8% of the like weight of the constituent material. He also estimated, for each of these thirty tests, the theoretical maximum density that would obtain if all particles of the mix were placed so as to effect maximum density—that is, if the voids in each size of material were filled successively with smaller sized materials as far as these were available. This estimate indicated that the densities of the actual mixtures ranged from 71 to 96%, and averaged 82%, of their respective theoretical maximum densities. These latter, depending on the particular proportions of the mix, ranged from 120 to 165 lb, and averaged 141 lb per cu ft, or about 83% of the weight of an equal solid of the constituent material.

(4) The foregoing results compare rather unfavorably with those obtained in concrete mixtures. In good, modern concrete, densities of from 155 to 157 lb per cu ft are readily obtainable if desired. Of this weight, about 6% is water, thus indicating, for the mixture, a dry weight of about 147 lb per cu ft, which is to be compared with the average of about 115 lb per cu ft for com-

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[30] Cons. Engr., Seattle, Wash.

pacted earth embankments. Other comparative figures are: A voids content of only 13.5% for the concrete mixture as against 32.2% for the earth mixture; and a 6% moisture content for the former as against two to three times that proportion for the latter. It should be noted, too, that this modern concrete is compacted not by rolling or tamping processes but only by vibration.

(5) It may be contended that an earth embankment and mass concrete are two entirely different things and that they are, therefore, not directly comparable. However, in the initial condition of the mix, and in respect of density, a comparison may not be unfair for they are both composed of exactly the same types of materials, namely, earth and rock, the cement content of the one merely replacing a portion of the silt and clay content of the other. They differ only in their relative degrees of synthetic proportioning and mixing and in method of compaction, with a final result that one attains a density which on the average is one-fourth greater than the other. The contrast is striking and, at least, suggests the query as to whether engineers are now securing the maximum economic densities in earth mixtures and as to what may be the economic limits of proportioning, mixing, and compacting mixtures for earth dam construction. The legitimacy of this query will be appreciated when it is realized how materially seepage is reduced by an increase in the density of the earth mixture.

(6) Of the many considerations to be kept in mind in the selection of borrow-pit materials for earth dam construction, and in determining their proper mix proportions, particularly for the impervious part of the dam, it is believed that the following are among the more important:

(*a*) That the material shall be of sufficient weight, and shall be sufficiently graded as to particle sizes, to ensure a high density upon compaction, and thus to afford stability to the dam structure without imposing the necessity of extremely flat embankment slopes. Safe dams can be built of relatively light materials, but the heavier material is a desideratum in order that the expense involved in providing the extremely flat slopes, otherwise required, may be avoided.

(*b*) That the material shall afford a sufficient gradation of particle sizes, and particularly that there be a sufficient proportion of fines, to ensure against percolation velocities that might cause such erosion and piping within the body of the dam as to threaten its ultimate failure.

(*c*) That the material shall afford a sufficient gradation of particle sizes, the same as Item (*b*), but to ensure against a seepage volume deemed prohibitive from considerations of economic water value alone, independent of possible physical damage to the dam by reason of such seepage.

(7) Assuming a satisfactory foundation, the stability of the dam proper, against imposed loads and attacking forces, depends mainly upon density of the embankment material in place; upon the position and gradient of the plane of saturation within the body of the dam and the volume of dam submergence below that plane; upon frictional resistance to displacement; and upon the cross-sectional form of the dam. The desired safety factor for sta-

bility, and the slopes and cross-section required therefor, are determinable from the other factors named. Most failures of earth dams have resulted either from their being overtopped by water, due to a lack of adequate spillway capacity, or to super-saturation followed by sloughing and progressive undermining toward the center from the outer slope or down-stream toe. There are few if any cases of earth dam failures that have been attributed directly to sliding. Such a failure, however, is conceivable. Under a condition of maximum loading as to water level above the dam, when maximum saturation obtains, the coefficient of friction of the embankment material is at a minimum, and, at such times, unless the base of the dam is adequately broad, a sliding movement would be possible as an incipient disturbance—and if sliding once began, disruption, subsidence, overtopping, and final failure would be likely to follow.

(8) Unless the embankment material is of reasonably good weight and grading, and well compacted, so that the plane of saturation is thereby well depressed toward the base of the dam, a condition obtains that makes for high porosity, high permeability, a high plane of saturation, a lowered resistance to movement under maximum load, and the necessity for flatter slopes and a broader base than otherwise would be required. This is due not only to the lesser basic density of the embankment and the greater volume of submerged material that loses weight by buoyancy, but also to the fact that a lesser coefficient of friction obtains under these conditions. This is illustrated in Table 10, which shows the required ratios, $\frac{\text{base width}}{\text{height of dam}}$, for the single consideration of stability against sliding. Other considerations may, and in many cases will, require base widths different from those indicated.

TABLE 10.—Ratio, $\frac{\text{Base Width}}{\text{Height of Dam}}$, for Stability Against Sliding

Weight of embankment material in place after compaction and under full water load, in pounds	Height of Dam, in Feet:							
	50		100		200		300	
	Coefficient of Friction							
(1)	0.25 (2)	0.50 (3)	0.25 (4)	0.50 (5)	0.25 (6)	0.50 (7)	0.25 (8)	0.50 (9)
120	4.9	2.4	5.1	2.5	5.2	2.6	5.2	2.6
100	6.3	3.0	6.5	3.2	6.6	3.3	6.6	3.3
80	8.7	4.2	9.0	4.4	9.0	4.5	9.2	4.5

(9) The assumptions upon which the data in Table 10 were developed, were as follows: A stability factor of two against sliding; dam heights ranging from 50 to 300 ft; a top width of 25 ft; a down-stream slope (horizontal ÷ vertical) two-thirds that of the up-stream slope; a mean weight of embankment material, above and below the plane of saturation, ranging from 80 to 120 lb per cu ft; and a coefficient of friction ranging from 0.25 to 0.50. Although greater unit cohesion and shear strengths than are indicated by these friction coefficient values are quite possible for dense, well com-

pacted materials, it is believed that, for conservative design, values of less than 0.50, and probably not to exceed 0.25, should generally be assumed. It will be noted from Table 10 how materially the required base width increases with a decrease in value of the coefficient of friction and a decrease in the density of the embankment material. Regardless of the showing of Table 10, however, or of stability computations in general, the writer would not favor earth dam slopes steeper than 1 on 2, nor base width ratios of less than 4.2 to 1, except possibly for very low dams.

(10) In this connection it is of interest to note the early British practice in India where so many excellent earth dams have been built. In 1910, Mr. A. Hill, Chief Engineer, Irrigation, presented data concerning some of these dams in a report entitled "Saturation of High Embankments of Storage Tanks in the Bombay Presidency." Although this report (which dealt with twelve dams from 31 to 81 ft in height) unfortunately contained no detailed data as to the density and mechanical analyses of the embankment materials, it did indicate that they were of average good quality, and it also contained detailed data as to embankment slopes and planes of saturation. From this report, the writer deduced the information contained in Table 11. There were some short sections of embankment slope much steeper and some much flatter than the averages shown in Table 11. Some were as steep as 1 on 1 (near the top) and some as flat as 1 on 14 (near the bottom). Furthermore, there were some short sections of saturation slopes as steep as 1 on 0.5 and as flat as 1 on 60.

TABLE 11.—DATA CONCERNING TWELVE EARTH DAMS IN INDIA

Item No.	Ratios	AVERAGES FOR INDIVIDUAL DAMS		Mean for all dams
		Maximum	Minimum	
1	$\frac{\text{Base width}}{\text{Height of dam}}$	6.7	4.9	5.70
2	Up-stream slope	3.6	2.5	3.04
3	Down-stream slope	3.9	2.1	2.52
4	Plane of saturation	7.6	1.9	3.6

(11) The author suggests that the qualitative desirability of materials for earth dam construction, both as to permeability and as to density, after compaction, may be judged by determining, from mechanical analyses of the materials, their value of *n* in the Talbot formula (Equation (1)). As to permeability, the writer questions the validity of this criterion, but believes that it is a fairly dependable index of potential density. Assuming a maximum particle size of 6 in. (152.4 mm), and a uniform value of *n* throughout for any particular mix, as the Talbot formula provides, the writer determined the theoretical proportions of the various particle sizes for that mix. He also considered a series of arbitrary mixes wherein the value of *n* was variable and for these the mean value of *n* was determined for each particular mix. Probably few natural mixes have a uniform value of *n* throughout. If it were deemed important (and the writer does not so regard it), a uniform value of *n* could be attained by extensive artificial proportioning; but such

extensive proportioning might be quite infeasible economically. Generally, the engineer must utilize, to the best advantage possible, the materials immediately at hand.

(12) For each of the mixes studied a theoretical maximum density was estimated, this being on the assumption that the various sized particles all found that proper place in the mix which would effect maximum density—a condition not fully attainable in practice. As already stated in Paragraph (3), of about thirty or more definite tests of mixtures and densities, for which data were available, the actual densities ranged from 71 to 96%, and averaged 82% of the theoretical maximum density. An 80% average would seem to be quite readily attainable. The theoretical mix of a uniform value of *n*, and the arbitrary mix of an equal mean value of *n*, are found to be not entirely accordant as to resultant densities, but they seem to be sufficiently in agreement to permit adjustment to a common density curve There is less accord as to permeability.

(13) From the computed *n*-values and densities for various hypothetical mixes, and from the recorded tests of numerous actual mixes, a graph (see Fig. 19), was developed showing density variation with variation in value of *n*. Curve *A*, Fig. 19, is for the theoretical maximum densities, assuming that all particles are so disposed in the mix as to effect maximum densities; and Curve *B* is for practically attainable densities taken as 80% of the theoretical maximum. The maximum particle size, *D*, is assumed to be 6 in. (= 152.4 mm). These curves are fairly close compromise adjustments to the platted actual computations. It will be noted from Curve *B* that maximum densities obtain for *n*-values ranging from 0.30 to 0.50, the actual maxi-

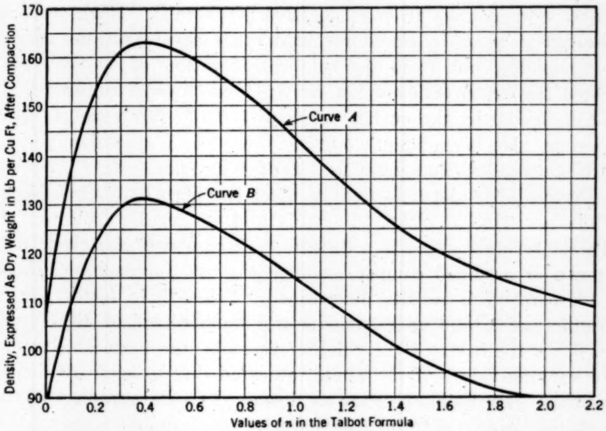

FIG. 19.—ESTIMATED DENSITIES OF EARTH MIXTURES FOR VARYING VALUES OF *n* IN THE TALBOT FORMULA (EQUATION (1)): $p = \left(\frac{d}{D}\right)^n$.

mum appearing to result for $n = 0.38$. It will be noted, too, that for densities as low as 115 lb per cu ft (which, as already indicated, is about the average now attained in actual practice) the value of n may range from 0.14 to 1.0. Although these values, both of which apply to 115 lb per cu ft, may indicate mixtures acceptable as to density, the latter also indicates a mixture entirely prohibitive as to seepage volume. Therefore, n alone, unless it is rigidly restricted within narrow limits, will not suffice as a criterion of all requirements for a good earth mixture for dam construction.

(14) The author states that a seepage loss of 0.1 gal per day per sq ft and, under certain conditions, in excess of 1.0 gal per day per sq ft, might be permissible. Ignoring the question of possible piping, and considering only the economic value of the water loss, it is not a simple matter to determine permissible seepage. It depends upon many factors: Primarily, upon the actual value of water in the particular locality and for the particular project involved, but also upon the relation of the monetary value of the annual seepage loss to the total annual cost of the project; upon its comparison with the extra cost of alternate methods of water securement that might avoid the seepage; upon whether or not the seepage water can again be utilized farther down stream, etc. Water values may range from \$500 to \$50 000 per cu ft per sec and storage costs from \$2 to \$200 per acre-ft. High seepage loss is to be avoided if possible, of course, and other considerations may make it entirely untenable, but from the sole consideration of the economic value of the water, the writer believes that seepage losses several hundred-fold greater than that indicated by the author (perhaps 500 to 1 000 times as great) may be permissible. He is confident, too, that there are many entirely stable earth dams with seepage losses as great as 5 gal per day per sq ft.

(15) Permeability depends mainly upon the porosity and "effective size" of the dam material in place. For mixtures of regular grading, according to the Talbot formula, there is a definite "effective size" for every value of n. From known or assumed compaction densities, as, for example, those shown in Fig. 19, porosity is readily determined. From these factors, an assumed water gradient of 1, and an assumed water temperature of 60° F, the writer determined, on the basis of the Slichter formula, the permeability of various mixes for varying values of n. These indicated that to keep within the maximum limits of permissible seepage as discussed in Paragraph (14) the value of n should not exceed 0.4 and the "effective size" should not exceed 0.5 mm. It also indicated, for the seepage limit designated by the author, an n-value of 0.26 and an "effective size" of only 0.025 mm.

(16) Such computations, however, can at best be accepted only as rough guides. In the first place, mixtures of irregular grading, but of the same mean value of n, give "effective sizes" and permeabilities that differ somewhat (and, in some cases, appreciably), from those of the regular grading determined by the Talbot formula. In the second place there is much uncertainty as to the dependable accuracy of any of the present empirical formulas for ground-water movement as applied to the type of earth mixtures considered herein. It should go without saying that in actual dam construction, empirical formulas, to which have been applied the mechanical analysis factors of

the earth mixture, should not be the main reliance for permeability determinations. There should be a comprehensive program of permeability tests of the actual materials, mixed and compacted to a condition as nearly that which will obtain in the final dam, as is possible.

(17) Seepage velocities that may cause piping should be avoided, of course. Generally, piping is a slow, continuing process that leaches out particles, progressively from fine to coarse, and which, unless halted, means ultimate structural collapse. The earth mixture, therefore, should be such as will permit no seepage velocities capable of dislodging and moving out the smallest sized particles which the dam material may contain in measurable quantity. A certain percentage of fines is an essential of good earth dam construction and some of these fines may be clay particles as small as 0.0005 mm in diameter. The writer estimates that for a safety factor of two against movement of such a particle, considering weight only and ignoring friction, the limiting water-jet velocity would be 0.00545 ft per sec, and that is the desirable limit of seepage velocity if practically obtainable. However, because there is a material friction and surface tension between particles, and because the 1 on 1 gradient upon which permeability factors are based is greater than obtains in a well built dam, an appreciably greater velocity than that stated would be permissible as far as piping is concerned. The seepage volume factor, however, intrudes itself and prevents any very material increase of that velocity. For a seepage velocity of 0.00545 ft per sec, through regularly graded Talbot mixes, the writer finds a limiting n-value of about 0.4 and a limiting "effective size" of about 0.5 mm.

(18) Seepage loss is so important a factor in connection with storage problems, and in relation to dam design and construction, that, unquestionably, there is an urgent need for further research concerning the laws governing ground-water movement. Every engineer who has had contact with the problem is aware of the glaring inconsistencies that obtain in the recorded data relating to soil permeability and to ground-water movement in general. These inconsistencies are revealed not only in the comparison of actual tests with estimates based on empirical formulas, but also in a comparison of the actual tests themselves, independent of any estimates by formula. Numerous instances can be cited of great disparities in actual tests of seepage through materials practically identical as to density, porosity, and "effective size"—and even greater disparities between actual tests and formula estimates. From a number of actual permeability tests of materials for which the writer computed discharge by one of the generally accepted empirical formulas, he found disparities as shown in Table 12. The degree of accuracy of the actual tests is not known but, presumably, they were made with care and are reasonably accurate. In any event, it is clear that either the tests or the formula as applied to graded earth mixtures is grossly in error, and it is confidently believed that it is the latter.

(19) Most of the permeability formulas have been devised for a more or less uniform and a relatively fine material and were not intended, primarily, for such graded mixtures as are customary for earth dams. They seem,

particularly, to lack applicability to coarse media, giving impossibly high velocities. They may be serviceable indicators as to the relative permeabilities of different mixtures, but should be interpreted with great caution as to absolute permeability. It may be that in the tests from which these formulas have been developed adequate cognizance was not given to the factors of entry and exit head losses; there may be error in the assumption that velocity varies as the first power of the head (and the writer believes there is error in that assumption as applied to coarse media); or there may be other factors in the problem that have not yet been discovered. Wherever the difficulty may lie, however, it is evident from the great disparities in results, to which reference has already been made, that further research is badly needed. It is the writer's judgment that this research should contemplate, in addition to laboratory tests, an intensive investigation of existing dams as to density, seepage, planes of saturation, mechanical analyses of the materials of the dam, etc. Such an investigation would involve considerable expense, but it is the safest procedure for the development of dependable formulas for the permeability of earth dams.

TABLE 12.—PERMEABILITY COMPARISONS, SHOWING INCONSISTENCIES

Case	MECHANICAL ANALYSES OF MATERIALS				Ratio: $\frac{\text{Actual test discharge}}{\text{Formula discharge}}$
	Dry weight per cubic foot, in pounds	Percentage of voids	Effective size, in millimeters	Uniformity coefficient	
1	96.7	45.0	0.250	4.0	0.96
2	119.4	34.0	0.559	9.2	2.14
3	109.3	38.0	0.066	3.0	0.0405
4	119.4	34.0	0.760	14.7	0.0149
5	117.5	34.0	0.102	18.1	0.414
6	127.0	30.0	0.100	95.0	0.000284
7	123.0	29.3	0.00085	2355.	14.35
8	113.4	31.9	0.730	5.35	2.70

(20) As final comment and, in part, as résumé of what has preceded, the writer submits the following:

(*a*) To satisfy best all requirements, that is, to secure an acceptable density and to keep within permissible limits of seepage velocity and seepage volume, the earth mixtures should preferably be such as to have an *n*-value within the range, 0.25 to 0.40, and an "effective size" not to exceed 0.5 mm. On account of the better workability of the material, due to the larger percentage of fines, the lesser values of *n* are the more desirable although it means, also, a slight sacrifice of density.

(*b*) It is permissible, and often necessary, to exceed the limits indicated in Item (*a*) but the embankment design, of course, must be adapted thereto. It may mean flatter slopes, a greater provision for drainage, the need for a core-wall, or of additional cut-off walls, etc., depending upon the degree of departure from the ideal material. If, for example, the value of *n* exceeds 1, the dry weight density will fall below 115 lb per cu ft and a base width greater than five times the height of dam will be required for assured stability.

(*c*) For the limits designated in Item (*a*), the percentage of fines, namely, material of less than 0.05 mm size, will generally not exceed 15% and may be somewhat less. With the present better knowledge of mixing and compaction methods, these relatively small percentages of fines are not only permissible, but they actually give a final density exceeding that of mixtures containing larger percentages of fines. It contrasts rather sharply with the one-quarter to one-third, or more, proportion of clay formerly deemed essential.

(*d*) To what extent economic considerations will permit increased expenditure to secure greater density of embankment materials by means of more thoroughy proportioning, mixing, and compacting than is now customary, depends mainly upon the number and proximity of available borrow-pits and upon their character in respect of the amount of additional processing that may be required. The fact that additional mixing may be rather expensive, and that the denser mixture requires more borrow-pit material for the same embankment volume, tends to limit the extent to which extra processing can be economically utilized. Generally, the available materials must be taken as they are, with no more processing than results from the excavation operations in the borrow-pit and from the spreading and harrowing or disking on the dam. The simultaneous use of borrow-pits of somewhat divergent composition may permit a nearer approach to the desired mixture, without undue synthetic proportioning, than would otherwise be possible

(*e*) An intensive research as to the permeability of earth dam embankments under varying conditions is urgently needed. It is a project that should be undertaken by the Federal Government through one of its regular technical agencies.

C. H. Kadie, Jr.,[31] Jun. Am. Soc. C. E. (by letter).—Valuable information on the selection of materials suitable for earth dams is presented by Mr. Lee. The author has developed a criterion for the selection of materials which will give excellent results within the stated limits. However, these limits seem to be too narrow. The writer believes that the design of the dam is predicated upon the available material, and that a structure may be designed and built to meet all necessary requirements by using materials which fall well outside the limits presented by Mr. Lee.

The design of an earth dam and the type of construction equipment selected to build it will vary with the materials used. To establish limits of satisfactory material would be to destroy one of the most attractive features of an earth dam, its absolute flexibility. Limiting the type of earth dam under discussion to rolled fill does not materially affect the limits of usable material. Having selected a borrow-pit, it is only necessary to design the structure and specify the construction equipment in order to obtain a thoroughly satisfactory dam. The shortest haul will not necessarily result in the most economical construction. The greatest economy will be found in the least number of station yards and the absence of any unusual construction equipment.

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[31] Asst. Engr., U. S. Bureau of Reclamation, Yuma, Ariz.

The material which will facilitate the construction of an economical earth dam lies within a small and known radius of the dam site. These materials should be investigated thoroughly for those desirable characteristics completely summarized by Mr. Lee under the heading "Requirements for Suitability". Materials much more harsh than those shown in Fig. 7 may be used by selecting a very heavy sheepsfoot roller. In fact, the coarse limit established by Mr. Lee would probably require a roller weighing more than 2 000 lb per lin ft, which is somewhat heavier than the usual design.

As shown by Mr. Lee, Talbot's general grading formula, Equation (1), is very well suited as a criterion in selecting materials. The more nearly a material conforms in mechanical analysis to the curve established by this equation, the greater will be its compacted density. This will be found especially useful in making a determination of the depth of cut for the most desirable material in a stratified borrow-pit. It must be remembered, however, that the present power shovels are seldom able to mix materials satisfactorily during excavation, from a cut that is more than 20 ft deep. It is a fairly simple matter to prepare mechanical analysis curves of composite samples to various depths. By solving for n in Equation (1) for a series of points, the actual size of the screens used is to be preferred, the curve which most nearly corresponds to some one value of n may be determined. It will be found that many materials are quite erratic in the part of the curve referring to material larger than 1 in. If the percentage of material greater than this size is small, it will have little effect on the result and may be ignored. Having selected the best depth of cut for scattered test pits, the same system may be used in choosing the best pits.

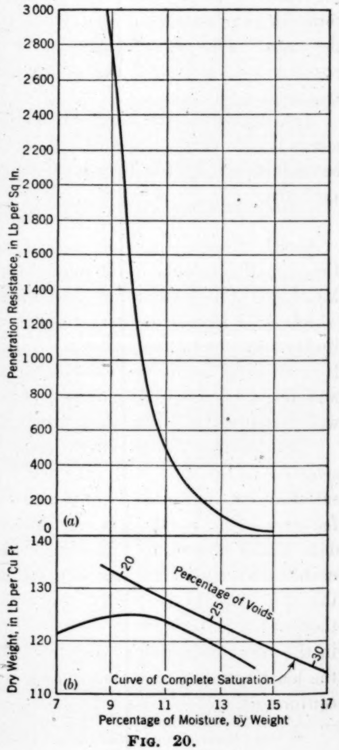

Fig. 20.

Mr. Lee has shown very clearly the known soil characteristics which may be determined by test. A knowledge of these characteristics is essential in choosing the material for a rolled-fill earth dam. One characteristic only mentioned briefly by him has proved extremely useful in the control of moisture during construction. Quoting Mr. Lee (see "Compaction"):

> "Higher water content [above critical] results in increased plasticity, larger voids, and less density, until a point

is reached at which equipment will not be supported. A lower content [below critical] may produce a fill of greater apparent stability with little or no plasticity, but one having a greater void volume and less density."

This is a relation always found in soil compaction. For any material the critical moisture is defined by the peak of the compaction curve. Fig. 20 shows a typical density-moisture (compaction) curve with a typical resistance to penetration curve (plasticity) measured in pounds per square inch. Since it is essential to obtain the correct moisture content, a knowledge of this relationship is necessary because it affords a convenient and rapid method of determining and controlling the moisture content. The details of making this test and the apparatus necessary have been presented in a series of papers by R. R. Proctor, M. Am. Soc. C. E.[32]

In the foregoing quotation the fact is mentioned that very high (above critical) moisture contents will result in a fill which will not support equipment. For satisfactory construction conditions the hauling units should not be so heavy that they will not be supported by the fill at a moisture content found to be critical for the rollers in use. Fig. 21 shows a series of compaction curves developed from the same material at various degrees of work. As the work increases (measured in blows per 2-in. compacted layer delivered by dropping a 5.5-lb rod, 18 in.) the density increases and the critical moisture content decreases. The additional work can be obtained by increased rolling or, best, by imposing additional weight on the rollers. With the present tendency toward large hauling units, and the necessity of using matched equipment (rollers and trucks), it would be best to specify a very heavy roller. This would make it possible to construct the fill at a lower moisture content and would avoid a condition of either too much moisture to support the hauling equipment, or a too little moisture content for the compaction units.

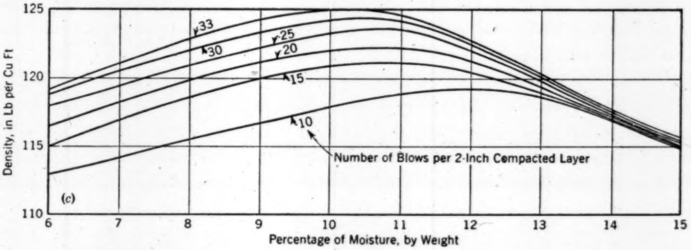

FIG. 21.—NUMBER OF BLOWS PER 2-INCH COMPACTED LAYER

RALPH BENNETT,[33] M. AM. SOC. C. E. (by letter).—In a discussion of earth dams it is usually assumed that only untreated materials are to be used. Raw fill is to be dug in convenient pits and consolidated by purely mechanical means. In other arts, there has developed a considerable use of earth which has been consolidated by the addition of imported media.

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[32] *Engineering News-Record*, August 31, September 7, 21, and 28, 1933.

[33] Cons. Engr., Los Angeles, Calif.

For sands the use of chemicals as a binder results in a concrete for which only a limited quantity of chemical is required. The result is an impervious, hard material made in place, and without requiring the elaborate and tedious handlings of materials necessary for the making of Portland cement concrete. Although already extensively used to form underground water cut-offs and to prevent infiltration into deep excavations, there does not seem as yet to have been any chemical binder used to produce a strong and impervious face on dams, levees, or ditch banks.

For clays an extensive use of water-emulsion oil admixtures has resulted, during 1936, in the production of a water-proof material which will carry loads up to and in excess of those which the same clay, hard and dry but untreated, would carry. Clays when treated with the emulsion rolled in place, and allowed to dry out, do not again absorb moisture. They are permanently cemented by the microscopic oil film which has replaced the water film that originally held the particles. The quantity of emulsion used is determined by the extent of the fines in the raw material as the binder acts only on them. The total percentage on ordinary clays is very low and must not be exceeded, or it will lose effectiveness.

R. R. Proctor,[34] M. Am. Soc. C. E. (by letter).—Five important requirements for soils that are to be used for the construction of rolled-fill earth dams are recorded in this paper. The requirements for water-tightness and stability are of the utmost importance; the dam may fail if the soils are not sufficiently compacted to resist excessive softening or percolation that may cause erosion or piping. Mr. Lee has shown that the grading of a soil determines the minimum porosity that can be secured by use of a particular method of compaction.

The writer does not make use of grading curves, as described by Mr. Lee, because his experience has indicated that laboratory and field experiments require less time and expense, and that such results are more readily adapted to the control of construction methods.[35] This is an absolutely essential requirement for any method of soil analysis or classification; no amount of detailed laboratory investigation of soils is of much value unless there is certainty that the soils will be compacted in the dam as the engineer requires.

Until a better method is developed, the writer prefers to use the plasticity needle penetration resistance when the compacted soil is fully saturated, for a guide to stability. The results of percolation tests, first made at the minimum soil density required for stability, are used as a guide for safety from excessive percolation. It has been found that a given method of compaction may produce a soil density of 12 lb per cu ft more than that required for a saturated penetration resistance of 300 lb per sq in., whereas, in other soils, such a saturated penetration resistance cannot be secured by this method of compaction. No other explanation than variation in grading has been found; however, the writer feels that the suitability of such soils is better established by actual tests.

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[34] Field Engr., Dept. of Water and Power, City of Los Angeles, Los Angeles, Calif.

[35] *Engineering News-Record*, August 31, September 7, 21, and 28, 1933.

An illustration of this fact is seen in the design of a large earth-fill dam proposed by the writer. The proposed borrow-pit was once selected for a rock quarry for a rock-fill dam. Without going into geologic details, subsequently revealed by studies of thin sections of the apparent rock under a petrographic microscope, the apparently hard and durable rock was a metamorphosed dioritic gneiss, so badly fractured that it readily broke down into an apparent soil as it was excavated and loaded into trucks. The writer had the usual compaction-plasticity curves prepared, which indicated that successful compaction of this material was possible. Several small compacted test fills were then made, using progressively heavier rollers until a roller weight and number of roller trips adequate to accomplish a soil density sufficient for a saturated penetration resistance of 300 lb per sq in. were determined.

A typical compaction curve for this material is shown in Fig. 22. Of this specimen, 43.8% was retained on a No. 4 sieve and 23% passed a No. 200 sieve. The specific gravity was 2.79. The laboratory compaction curve was prepared from the 57.2% that passed through the No. 4 screen. The extent of the field compaction was determined by measuring the density of the compacted material in place, and then calculating the density of the fines by deducting the weight and volume of all the material retained on the No. 4 screen. It will be noted that the field compaction was only 1 lb per cu ft in excess of the required minimum. This was caused by the 43.8% of rock (material retained on the No. 4 screen); other samples of similar material, but containing less rock, gave results comparable to the laboratory compaction curve. It will be noted that a compacted penetration resistance

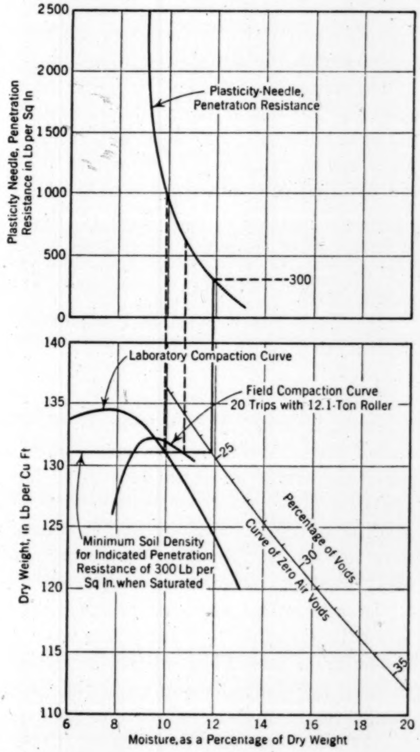

FIG. 22.

of 600 lb per sq in. represents the maximum allowable moisture content for compaction of this material; the addition of more water limits the compaction to less than the established minimum. It will also be noted that the soil can be compacted at penetration resistances greater than 2 500 lb per sq in. It is desirable to compact this soil at a compacted penetration resistance range from 2 000 to 3 000 lb per sq in., in order to reduce the cost of truck haul over the fill and to provide for consolidation during the construction period without removal of water, thus eliminating most of the slow settlement that often occurs after a dam is finished.

It is interesting to note the effect of compacting this material to a lower dry weight as, for example, that of 120 lb per cu ft. The laboratory compacted sample at this dry weight had a penetration resistance of 100 lb per sq in. as compacted, which, in the writer's opinion, is too low a value for safety in a large dam. The moisture content of this soil could increase, without change in dry weight, from 13.1% to 16.3% of the dry weight, upon saturation. The plasticity of this saturated material may be compared to that of a concrete mix of 1:2.46:2.95, dry rodded volume, with 7.1 gal of water per sack of cement. This is obviously too soft for use in a dam. This discussion clearly indicates the need for more than soil-grading curves to determine the suitability of some soils for use in earth dams.

The writer has had sufficient experience with various types of soils to appreciate the fact that the methods proposed by Mr. Lee are adequate to determine the suitability of most soils, but actual tests are needed for doubtful types Tests are also required to determine the moisture content before compaction, the required compacted dry weight of soil, and to check the results of construction operations, regardless of the degree of suitability the soils may possess. This work can be started a short time before actual construction begins, if lack of funds or other causes prohibit much preliminary experimental work.

Mr. Lee's paper required considerable effort and time to prepare, and a continuation of these efforts should be made to develop more data of this nature.

L. F. Harza,[36] M. Am. Soc. C. E. (by letter).—Much credit is due the author for crystallizing the technique of selecting ideal material for the impervious part of an earth dam in so far as this can be done by mechanical analysis. The paper is excellent with reference to the one problem of ideal grading of the impervious region, without relation to the remainder of the dam nor to its foundation nor abutments, upon the assumption that such ideal grading is consistent with other attendant facts and conditions.

In a practical application, the ideal material is seldom available in the immediate vicinity. Usually, a large part, if not all, of the dam must be built of less ideal materials in order that, for reasons of economy, it may be obtained from necessary excavations or near-by sources. One would rarely find a foundation upon which to build that would conform to the ideal grading and degree of compaction called for by the author.

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[36] Cons. Engr. and Pres., (Harza Eng. Co.), Chicago, Ill.

Furthermore, it may be contended that the word, "stability," in the sense in which it is used when applied to a given material, is an entirely different word from "stability" when applied to the dam as a whole, which is almost synonomous with "safety"; and the two uses of the word may have little in common. The unstable core of a hydraulic fill is made stable with relation to the dam as a whole by sufficiently heavy and free draining slopes. The same principle can be applied to a rolled fill to protect the up-stream slope from damage by rapid lowering of the reservoir level. Whether or not the ideal grading should be struggled for with much consequent increase in cost depends upon the purpose of the dam, the foundation and abutment conditions, and the value of water, drainage provisions, and other factors.

It is quite natural that the author's insistence upon tightness should be emphasized in California where water conservation is of such importance, and because soluble elements are common in soils of arid regions; but a vast number of flood-control dams are likely to be built in the East where tightness beyond the requirements of safety may have no importance whatever. Obviously, it would be unnecessary to struggle for high impermeability, for example, of mere retarding dams for flood-control purposes, such as those of the Miami Conservancy District. Dams of high permeability may be just as safe and stable as very tight dams: (*a*) If the materials of different character are disposed in proper relation so that they become progressively more permeable (and amply so) toward the toe; (*b*) if each material will not filter through the next material in succession; (*c*) if the materials are not soluble; and (*d*) if ample natural drainage is provided on the filter principle to insure the escape of leakage without the carrying out of material. Each material, of course, should be compacted as much as its character permits.

It would be superfluous to struggle for the ideal grading and tightness of a dam if the foundation itself were subject to leakage, dwarfing that of the dam. This is the usual case with reference to dams in the Southern Peninsula of Michigan, some of which rest on as much as 1 500 ft of porous sand and gravel, and depend for stability upon proper drainage. Seepage is dangerous only to the extent that it is not properly handled. The art or science of draining engineering structures, especially dams, is of such importance as to deserve recognition as almost a distinct field of engineering which has received altogether too little attention.

The logical design of a dam and the selection of materials, even for the impervious part, is inseparable from its foundation and abutment conditions, drainage possibilities (whether natural or artificial), importance of water conservation, and the purpose and location of the dam. It is well to lean toward the ideal as far as consistent with these conditions.

CHARLES H. LEE,[37] M. AM. SOC. C. E. (by letter).—The generous response in discussion has fulfilled the writer's hope that the subject-matter of this paper might be expanded. All aspects of the subject have received attention, including a number of points not listed in the paper. It is regretted that the scope of the latter precluded consideration of such broader features of earth dam design and construction as were mentioned by Mr. Floris. This subject has been covered recently, however, in a very able manner by T. T. Knappen, M. Am. Soc. C. E.[16] With regard to the penetration needle mentioned in several discussions, the writer is familiar with it and considers it a very effective and useful tool.

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[37] Cons. Hydr. Engr., San Francisco, Calif.

The five principal requirements for suitability of impervious material, as listed by the writer, were accepted with more or less amplification by all who discussed the paper. Additional requirements pertaining to shape of particles, colloidal material, and cracking characteristics have been contributed. The discussions will be commented upon in the order that the subject-matter appears in the paper.

Cohesion.—It was pointed out by Mr. Baumann that cohesion due to the surface tension of water is absent under conditions of saturation where no particle surface is exposed to air. He recommends that all stability tests on impervious fill be conducted upon saturated material. The writer confirms this rule and would add that all samples that are small enough so that the density is affected by surface tension, should be submerged when under test. This is the practice in making the Terzaghi consolidation test. It might also apply under certain conditions to shear tests.

Although open-minded with respect to the relation of thin films of water to cohesion, Mr. Dore agrees that the latter is always accompanied by the presence of "fines" and that its degree varies with the degree of fineness of particles and the percentage of "fines". In this connection, it may be stated that at all atmospheric temperatures water is present on the surface of "fines", in the form by hygroscopic moisture. Such moisture is in equilibrium with, and is derived from, water vapor in the air. Water, therefore, goes "hand in hand" with "fines", and there is the presumption that they both have some relation to cohesion.

Grading.—There is general agreement that well graded materials are ideal for the construction of the impervious sections of rolled-fill earth dams. The Talbot grading formula is also recognized as a satisfactory criterion for selection, provided n is confined to certain narrow limits. Mr. Jacobs submits statistical analyses from which he concludes that the value of n should be within the range of 0.25 to 0.40. This slightly widens the limit proposed by the writer, which was 0.25 to 0.33. As stated by Mr. Jacobs, the lesser values of n are more desirable on account of better workability of material. With the present tendency to increase the weight of rolling equipment and effectiveness of processing in compaction, however, increasing the value of the upper limit of n to 0.40 may not be inadvisable. Mr. Hill points out that the Talbot curve with $n = 0.33$ falls along the middle of the stabilization band defined by the Minnesota Highway Department for wearing surfaces for gravel roads.

Mr. Jacobs believes that the degree of conformity of a mechanical analysis curve with the Talbot grading curve within the specified limits of n is a dependable indication of potential density. Mr. Cox disagrees with this conclusion. He contributes interesting information regarding the abnormally high porosity of Hawaiian laterite soils which he attributes to the expanded character of the colloidal fraction. The condition described by Mr. Cox probably results from a high degree of flocculation in the soil induced by the chemical constituents of the local soil water. This condition is not confined to laterite soils or equatorial regions, but is encountered locally wherever soils of fine texture are highly flocculated. It may result from a variety of causes. One well known example is the soft mud deposited by fresh-water streams where discharging into salt water. Loess (wind-deposited) soils of fine texture may also have abnormally high porosity as deposited, although this may be readily reduced by heavy rolling. The latter is also true of highly porous cloudburst débris. Porous volcanic ash beds, on the other hand, may resist compaction. All such highly porous soils represent special conditions which must be taken into account, whether in dam, foundation, or other types of earth construction. The soils for which porosity and mechanical analyses were presented in Fig. 1, and also those with which Mr. Jacobs worked, are of the deflocculated type most common in earthwork in temperate zones.

Mr. Jacobs and Mr. Cox both question the value of the grading equation as an indicator of relative permeability. This subject will be discussed under the heading, "Permeability". Mr. Creager well describes the functions of the various fractions of a well graded soil mixture.

Compaction.—Mr. Jacobs compares the densities attained at present in compacted earth embankment and in mass concrete, and finds that as an average the latter attains a density 25% greater. He points out that they are composed of the same types of material and differ only in the degree of proportioning and mixing and in the method of compaction. His query as to whether maximum economic densities are being attained in earth mixtures, is a challenging one. The present tendency toward use of heavier rolling equipment, as illustrated by the work described by Mr. Baumann and referred to by Messrs. Kadie and Dore, will doubtless aid in reducing this difference. A more effective means will probably be found in the greater use of mechanical vibration equipment, as emphasized by Mr. Dore. It is also probable that, as pointed out by Mr. Pyle, there is a progressive compaction after completion of the dam resulting from the superimposed weight of material and expressed in settlement. No information is available as to the extent of such increase in density, but it is unquestionably appreciable especially in the lower courses. Mr. Pyle does well to suggest that compaction tests include loading commensurate with the weight of the dam. Mr. Hill's reference to the influence of higher temperature in producing greater density, as shown by Mr. Hogentogler's experiments, also has possibilities. This effect doubtless results from the lower viscosity of attached water resulting in increased lubrication.

Another field for investigation is the possibility of facilitating expulsion of air during the compacting process, thus reducing the air content at maximum compaction. The effectiveness of vibration in increasing density may be attributed in part to this.

Drainage.—Messrs. Knappen and Harza have contributed to this subject by emphasizing its importance in dams for flood control and other uses where impermeability is not essential. The permanence and stability of such dams depend largely upon adequate drainage. Natural drainage may be provided by use of materials that are progressively more permeable down stream. To be effective, drainage must prevent the building up of hydraulic pressure in any part of the dam to the point where the load is transferred from the solid material to water. It must also prevent the washing out of fine material. Mr. Harza does well to emphasize the importance of drainage for engineering structures.

Shape of Particles.—Messrs. Dore and Creager explain the function of individual particle shapes in providing resistance to movement in earth embankment. Sharp, angular particles develop greater internal friction than smooth, rounded ones and are distinctly advantageous. This is a characteristic to be taken advantage of, if possible.

Colloidal Fraction.—Messrs. Pyle and Cox emphasize the influence of appreciable quantities of colloidal material upon a soil in destroying the value of mechanical analysis as an index of character. Mr. Cox illustrates this phenomenon by reference to the Hawaiian laterite soils. This point is well taken and shows the value of extending mechanical analyses down at least to maximum clay size (0.005 mm) and further if possible.

Cracking Characteristics.—Messrs. Pyle and Field both refer to the undesirability of using material with an excess of fines, due to its tendency to form shrinkage cracks or planes of lessened resistance to water. Such material has a large capacity for water, and if the dam is unwatered for any length of time, cracks may form which will have disastrous effects upon refilling. This possibility is especially important in semi-arid and arid

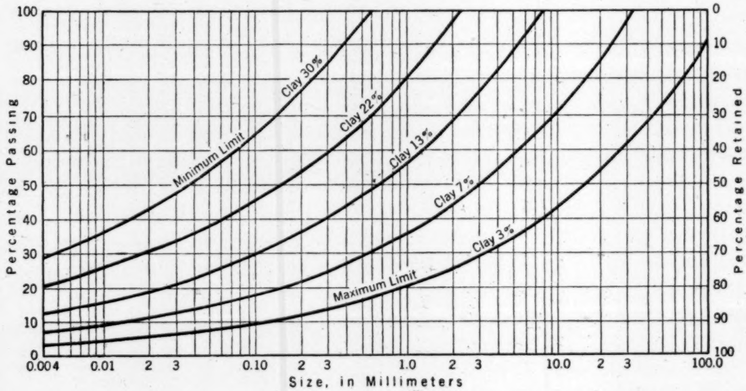

FIG. 23.—PROPOSED LIMITS IN MECHANICAL ANALYSIS FOR GRADED MATERIALS SUITABLE FOR IMPERVIOUS SECTIONS OF ROLLED-FILL DAMS (REVISED FORM OF FIG. 6).

regions. The writer's minimum limiting curve is designed to exclude such material; but if a given material approaches this limit, has an appreciable colloidal fraction, or a high degree of flocculation, it is recommended that the standard Atterberg shrinkage tests be made before final selection.

Air Content at Maximum Compaction.—From his experience with Hawaiian soils, Mr. Cox considers the establishment of a permissible air-content at maximum compaction as a very important feature of selection of suitable materials for rolled-fill dams. His need for such a limit results from the prevalence in the Islands of highly flocculated soils with appreciable colloidal fraction. In Continental United States it is believed that, except in special cases, the choice of materials in conformity with the writer's limiting curves in Fig. 23 (which is a revision of Fig. 6) will result in securing a permissible air-content. As a tentative value, the writer uses 5% by volume, or 3.5% expressed as equivalent water in terms of dry weight. (See Fig. 4 and supporting tabulations.)

Impermeability.—This subject has received discussion from a variety of experiences and viewpoints. As pointed out by Mr. Harza, water conservation is of the greatest importance in the West and minimum leakage is there essential. In the more humid Eastern States, however, water has little value, and water-tightness is not essential. This is especially true of flood-control dams, of which many have been, or are being, built.

The following is a summary of stated quantitative limits for leakage through rolled-fill earth dams:

Writer: For water-tight dams, 0.10 gal per sq ft per day is stated as a limit; the maximum allowable leakage for any dam is 1.00 gal per sq ft per day.

Knappen: A satisfactory maximum allowable limit is 100 gal per sq ft per day, the value in any given case being largely a matter of economics.

Cox: The limiting value is a question of economic loss from leakage, and the safety of the structure.

Dore: For core material in water supply dams, 1 gal per sq ft per day is an acceptable limit; for hydro-electric power dams, 2 to 3 gal per sq ft per day is acceptable.

Jacobs: The limiting value is a question of the economic value of water loss and the possibility of piping. There are many entirely stable dams with seepage loss as great as 5 gal per sq ft per day. From economic value alone losses as great as 50 to 100 gal per sq ft per day, may be permissible.

Messrs. Baumann and Field describe ingenious uses of differing types of material to produce water-tightness with stability. These represent normal practices where limited material is available for impermeable construction. Where ample satisfactory material has been available, many impervious dams in the West have been built with uniform section.

Mr. Bennett refers to the use of imported media to produce water-tightness, such as chemicals, as a binder for sand and water emulsions, and oil admixtures for application to clay. The writer might add to this list the use of sodium carbonate or sodium chloride with loam soils as deflocculants to improve water-tightness of the back-fill in core trenches or core-walls. Considerable experimental work has been done and actual use of such chemicals for this purpose has been made in several instances.

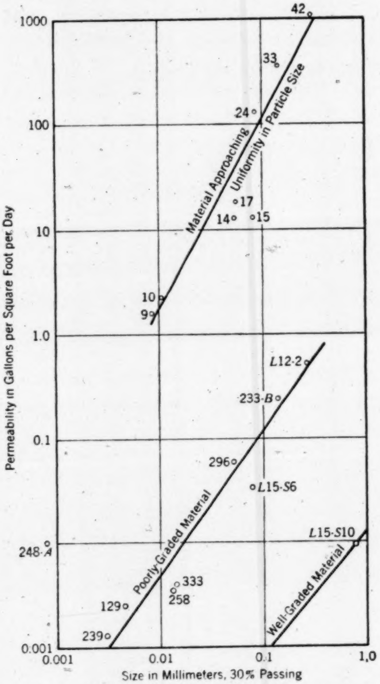

FIG. 24.—RELATIONS BETWEEN PERMEABILITY AND PARTICLE SIZE (RE-ARRANGEMENT OF FIG. 14).

Messrs. Cox and Jacobs take issue with the writer's conclusion that there is a correlation between mechanical analysis and the permeability coefficient as determined from actual test. Mr. Cox submits data for seven samples of Hawaiian soils (Table 9) and in Fig. 14 compares diagrammatically the permeability and size, in millimeters (30% passing), including in the latter study both these samples and twelve samples for which data are given by the writer (see Fig. 5). To supplement the mechanical analysis data shown in Table 9, Mr. Cox has kindly furnished the complete curves. Fig. 24 is a re-arrangement of Fig. 14, using the complete data. Examination of the mechanical analysis curves for the nineteen samples indicates the classification shown in Table 13.

With the exception of Sample Nos. 248-*A* and 258 these same groupings appear on Fig. 24 and lines designated correspondingly are shown thereon. It is apparent from this diagram that for each grading group there is a relationship between mechanical analysis and permeability and that the reason Sample No. *L*15–*S*10 has such low permeability in comparison with others is that it is the only sample having well graded material. Further-

TABLE 13.—CLASSIFICATION OF NINETEEN PERMEABILITY TEST SAMPLES

Type of material	Lee Sample Nos.	Hawaiian Sample Nos.
Uniform texture	9, 10, 14, 15, 17, 24, 33, 42	258
Poorly graded	*L*15–*S*6, *L*12–2	333, 296, 233*B*, 239, 248-*A*, 129
Well graded*	*L*15–*S*10	. . . .

* Conforming with Talbot equation for $n = 0.25$ to $n = 0.33$.

more, the diagram does not show "enormous difference between soils of entirely different character, but with somewhat similar mechanical analysis curves." It does show, however, enormous differences between dissimilar groups of soils having similar mechanical analysis curves. The differences in permeability, as shown by the grouping, are apparently due to degree of grading and not to character of soil. Within any group, the differences in permeability are due to character of soil as controlled by particle size. The eccentricity of Sample Nos. 248–*A* and 258 may be due to observational errors.

Mr. Jacobs states the writer's suggestion (which he questions) to be that the "qualitative desirability of materials for earth dam construction, * * * as to permeability * * * may be judged by determining, from mechanical analyses of the materials, their value of n in the Talbot formula." The writer's view, restated, is that materials which are shown by mechanical analysis to be graded in conformity with the Talbot equation, with values of n from 0.25 to $n = 0.33$ (curves concave upward), have much lower permeability coefficients than poorly graded materials (sloping curves straight or irregular) and very much lower than uniform textured materials (curves convex upward or vertical). This statement, it is believed, is confirmed by Fig. 24. With regard to Mr. Jacob's proposed limits of $n = 0.25$ to $n = 0.40$ and "effective size" not to exceed 0.5 mm, this condition would be fulfilled by Fig. 6 or Fig. 23 if the maximum limiting curve were drawn for $n = 0.40$ instead of $n = 0.33$. As previously stated, such a change may be desirable in the near future in view of the improvement in field compaction methods.

The writer agrees with Mr. Jacobs in questioning the accuracy of present empirical formulas for ground-water flow as applied to earth mixtures used in construction. As he well states, "permeability formulas have been devised for a more or less uniform and a relatively fine material and were not intended for such graded mixtures as are customary for earth dams." For many years the writer has depended upon actual permeability tests rather than the use of formulas.

Mechanical Analysis.—Almost every discussion included comment upon the writer's proposed limits for grading materials suitable for impervious sections of rolled-fill earth dams, as set forth in Fig. 6 and Fig. 7 and the supporting tabulations. These comments are summarized as follows:

Mr. Knappen prefaced his comments by the statement that "greatest economy generally results from the use of materials closest to the site." He considers that materials finer than the writer's minimum limit may be used for impervious core construction, dependence being placed upon the outer shell for structural strength; also that materials coarser than the maximum limit may be used where the water has little value, where large leakage is permissible, and where care is taken in design to pass the water without injury to the structure.

Mr. Baumann states that material which is not initially suitable because of gradation defects can be made so by proper compaction. The writer would point out, however, that this statement applies principally to material, the large pieces of which are readily crushed in rolling, such as that used in Zone 3 of San Gabriel Dam No. 1.

Mr. Dore considers the minimum limit satisfactory, but that curves of suitable materials especially of glacial origin, with little clay, may lie outside the maximum limit for short distances.

Mr. Creager thinks that the limits are helpful for preliminary studies provided their use is guided by experience and judgment. He believes that some materials outside the limits might be suitable and that other materials within the limits might be unsuitable. For a more accurate definition of requirements for graded materials, he suggests intermediate lower limits with varying clay content, and would eliminate entirely the diagram for ungraded materials (see Fig. 7).

Mr. Jacobs defines limiting maximum curves as those with *n* values in the Talbot equation between 0.25 and 0.40 and an effective size not to exceed 0.5 mm. The percentage of fines (0.05 mm) under these limits would generally not exceed 15 per cent. The maximum limit ($n = 0.40$) for the 5-in. maximum size of particle would slightly exceed the writer's maximum limit (see Fig. 23).

Mr. Kadie considers the limits as too narrow, believing that the design of a dam should be predicated upon the available material. Mr. Proctor states that he does not use limits, placing dependence upon laboratory compaction and permeability tests, and plasticity needle penetration tests in the field. Mr. Harza thinks that the need for attaining the ideal grading depends upon a number of factors and that no fixed limits can be established.

The use of limiting curves as a guide in studying mechanical analysis data is thus little favored, being generally confined to those familiar with dam construction for water supply storage.

Mr. Proctor and others point out that selection on the basis of mechanical analysis is at best preliminary and that other considerations, including the results of compaction and permeability tests, must form the basis for final selection. This is in accord with the writer's views as expressed in the paper. The principal field for selection by mechanical analysis, however, is in reconnaissance, preliminary investigation, general studies, small projects, etc., or where limited funds are available for investigation. Mechanical analysis is a widely known standard test of broad usefulness which can be performed at moderate cost by a variety of

TABLE 14.—CONTROL DATA FOR INTERMEDIATE FINE LIMITING CURVES (SEE FIG. 23)

Curve	Maximum size of particle, in millimeters	Value of *n* in the Talbot equation	Percentage of clay
Minimum limit: No. 28 sieve	0.6	0.25	30
Minimum limit: No. 8 sieve	2.36	0.25	22
Minimum limit: $\frac{5}{16}$ in.	8	0.28	13
Minimum limit: $1\frac{1}{4}$ in.	32	0.30	7
Maximum limit: 5 in.	127	0.33	3

established laboratories. For these reasons the writer believes that there is utility for a set of limiting curves, as shown in Fig. 23.

In preparing these revised curves, the suggestion of Mr. Creager has been followed. Three intermediate fine limiting curves have been drawn from the control data listed in Table 14. In using Fig. 23 it will be found that the most suitable materials are those which have mechanical analyses curves that lie entirely between adjacent pairs of curves.

With the addition of the intermediate curves in Fig. 23, the writer agrees with Mr. Creager that there is no practical need for Fig. 7, giving limits for ungraded materials, such limits being provided by the spread between the adjacent intermediate curves in Fig. 23.

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INTERACTION BETWEEN RIB AND SUPERSTRUCTURE IN CONCRETE ARCH BRIDGES

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With Discussion by Messrs. A. Floris, Frank M. Baron, A. A. Eremin, Hardy Cross, and Nathan M. Newmark.

Synopsis

Certain phases of the interaction between the rib and spandrel structure of concrete arch bridges are discussed herein with a view toward relating the action of the structure to the system of classification of indeterminate structures presented in 1935 by Hardy Cross[2], M. Am. Soc. C. E.

Certain limiting cases of the arch with integral superstructure are normal and can be designed directly from the results of a preliminary analysis. Structures in which either the rib or the deck girder develop all the flexural resistance are normal. A third type of structure having normal characteristics for most cases of loading is similar to a Vierendeel truss, with rigid columns and with floor and rib of approximately equal stiffness.

Other structures are hybrid and cannot be designed for given working stresses by the traditional procedure of repeated analysis and design. However, where the columns are flexible the action of the structure can be predetermined and the stresses can be predicted and controlled within reasonable limits.

In the case of the hybrid structure, the same degree of economy and safety can generally be obtained by designing the rib to carry all the load, and by neglecting the effect of interaction on the deck, as can be obtained by taking account of interaction by the traditional procedure of repeated analysis and design.

Introduction

The usual treatment of an open-spandrel arch bridge as a rib without restraint due to the superstructure is clearly incomplete. Interaction between

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Note.—Published in September, 1936, *Proceedings*.

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[2] "The Relation of Analysis to Structural Design," Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363.

rib and spandrel structure is inevitable as long as the parts of the structure remain continuous; but the fact that arch bridges designed by a procedure neglecting deck participation have apparently given good service, seems to indicate that interaction is relatively unimportant. However, arches so designed have been primarily "dead load" structures, carrying relatively light live loads and resisting live load flexural stresses effectively by virtue of the heavy dead load compression in the rib. In such an arch, when built of the proper shape, the greater part of the stress is as definitely determinate as for a statically determinate structure. It seems scarcely necessary to add that this type of arch bridge is entirely different in essential structural characteristics from a structure of the same shape designed to carry relatively heavy live loads, particularly when the size of the rib is reduced by shifting material from the rib to the posts and floor.

The Final Report of the Special Committee of the Society on Concrete and Reinforced Concrete Arches[3] has called attention to some phases of the significance of interaction. For example, in Section I, Article 3, Conclusion 7(*b*) suggests that live load stresses be determined considering the structure as a whole and not the rib alone, "because of the effect of the deck participation upon the stresses in the deck and columns rather than because of its effect upon the stresses in the rib"; and, again, Conclusion 9 points out that,

> "A deck without intermediate expansion points, constructed so as to act with the rib in carrying loads, reduces the magnitude of the live load moment at the springing where the deck does not increase the strength of the structure; and it increases the moment over the central portion of the span where the strengthening effect of the deck is greatest."

There seem to be two not very clearly defined and sometimes conflicting points of view in the Committee's report and elsewhere in the technical literature on the subject of interaction: (1) That interaction is a source of danger and should be avoided if possible by means of joints and hinges or by other means of articulation; and (2) that interaction presents an opportunity to effect marked economies in design by taking advantage of its favorable aspects. This latter point of view is best shown by a review of a paper[4] which states, "French engineers have shown Americans that remarkably light and efficient concrete bridges can be designed by considering the spandrel columns and floors as integral with the ribs."

Previous Investigations.—Any of a number of analytical methods may be used for the determination of stresses in an arch with superstructure. Most of them were developed primarily for trusses without diagonals and for building frames. In general, they will be found to consist of variations in either of two fundamental procedures:

(1) Maxwell's procedure in which cuts or hinges are introduced at enough points in the structure to make it statically determinate and then forces are found to produce continuity.

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[3] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1427.

[4] "Integral Arch Action" by George E. Beggs, M. Am. Soc. C. E., on file in Engineering Societies Library; reviewed in *Civil Engineering*, December, 1933, p. 697.

(2) A method used by Hardy Cross,[5] similar to Ostenfeld's deformation method, in which various independent sets of joint displacements are produced and the forces involved are determined. These independent configurations are then combined in such proportions as to give the forces acting on the structure.

Several approximate analytical methods[6] have been suggested but are not generally available. Various simplifying assumptions permit a fairly rapid solution of the problem. The writer has made studies of different types of arch bridges using methods of analysis[7] that he has adapted, in order to test the validity of the simplifying assumptions that have been suggested. In general, none of the approximate solutions is applicable to a wide range of structural types. Exact methods of analysis require almost as great an expenditure of time and effort as that necessary for the completion of an accurate model analysis.

Analyses by means of models using the deformeter gage introduced by George E. Beggs[8], M. Am. Soc. C. E., and by models using the polarized light method[9] have been made. Deformeter model analyses are reported in articles describing the arch viaduct, at Ashtabula, Ohio,[10] the George Westinghouse Bridge[11], in Pittsburgh, Pa., and the Raritan River Bridge, in New Brunswick, N. J[12]. In these cases it should be noted that revision in design was not always made from the results of the analyses.

A model analysis was used to check a simplified theoretical analysis of the Grandfey Viaduct[13], in Switzerland. It is not clear from the article whether any advantage was taken of interaction in the design of the rib. From its size it would appear not. It may be noted that[14] "greater stresses are conceded for the superstructure than those for the main arches" due to the deformation of the arches resulting in a greater loading of the superstructure than that contributed by direct loading alone.

The experimental method may be summarized briefly as the determination of stresses in a prototype loaded in a manner similar to the actual structure.

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[5] See, for example, "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., pp. 105, 228, and 236.

[6] "Statically Indeterminate Structures," by Hardy Cross M. Am. Soc. C. E., pub. in mineographed form, 1926, p. 167; see, also, "The Effect of Temperature Stresses in Open-Spandrel Arches of Reinforced Concrete," by M. F. Lindeman, Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering, Univ. of Illinois, 1930.

[7] "Interaction Between Rib and Superstructure in Concrete Arch Bridges", by N. M. Newmark, Jun. Am. Soc. C. E., Thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Univ. of Illinois, 1934 (prepared under the supervision of Professor Cross).

[8] "Model Analysis of a Reinforced Concrete Arch" (Yadkin River Bridge), by J. T. Thompson, M. Am. Soc. C. E., *Public Roads,* Vol. 9, No. 11, January 1929 p. 209; also, "Deck Participation in Concrete Arch Bridges," by A. H. Finlay, Assoc. M. Am. Soc. C. E., *Civil Engineering,* November, 1932, p. 685.

[9] "Cours de Beton Armé," by A. Mesnager, Paris, 1921, p. 223.

[10] "Features of an Open-Spandrel Arch Viaduct," by J. R. Burkey, *Engineering News-Record,* Vol. 101, No. 25 (December 20, 1928), p. 919.

[11] "A Concrete Arch of 460-ft. Span," by G. S. Richardson, M. Am. Soc. C. E., *Engineering News-Record,* Vol. 106, No. 17 (April 23, 1931), p. 680.

[12] "Two High Traveling Towers with Chutes Place Concrete on New Brunswick Multiple Arch Bridge," by Morris Goodkind, M. Am. Soc. C. E., *Engineering News-Record,* Vol. 104, No. 8 (February 20, 1930), p. 317.

[13] "Construction of the Grandfey Viaduct," by Adolph Bühler, *Proceedings,* Am. Concrete Inst., Vol. 23 (1927), p. 489.

[14] *Loc. cit.*, p. 526.

The most notable examples of experiments on arches with decks have been those conducted on the Yadkin River Bridge[15] and the series of experiments[16] by W. M. Wilson, M. Am. Soc. C. E., at the University of Illinois, on concrete models. It is important to note that in these experiments cracking and disintegration of the superstructure occurred in tests to destruction, the superstructure being affected almost to the same extent as the rib, although the loading was chosen so as to produce maximum stress in the rib.

Object of Present Study.—This paper relates certain phases of the behavior of arches with integral spandrel structure to the general philosophy of design presented recently by Hardy Cross[2]. It is not concerned with methods of analyzing stresses, nor with the details of design; it develops what is believed to be a rational point of view of the problem, and presents material leading to a possible basis for design.

The variables involved in this problem are numerous. The mere compilation of data, whether from experience, experiment, or analysis, does not always assist in clarifying a problem in which the variables are so numerous and difficult to separate; but with the classification of indeterminate structures developed by Professor Cross, one has available the basis for classifying and interpreting data bearing on the subject, or on related problems concerning other types of structures.

It is necessary to supplement experimental and analytical data with information pertaining to experience. What evidence, if any, is there of overstress in the floors, columns, or ribs of arches that have been constructed? Where have cracks occurred, and in what type of structure? That is, what are the relative stiffnesses of floor, columns, and rib, and where are the expansion joints (if any) located? The writer hopes that those who have information along these lines will contribute to the discussion. He would ask a further question: Has an arch bridge actually been designed and constructed taking interaction into account, and what changes in floor, columns, or rib were thereby made possible, or became necessary?

INTERPRETATION OF STRUCTURAL ACTION

Classification of Indeterminate Structures.—In revising the design of a member in an indeterminate structure from the results of an analysis, it is necessary to know how the stress in the member will be affected by revision in its proportions, and by revision in the dimensions of other members in the structure. Professor Cross has defined qualitative types of structural action in terms of the response of the member to changes in design[2,17].

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[15] "Loading Tests on a Reinforced Concrete Arch," by A. L. Gemeny and W. F. Hunter, *Public Roads,* Vol. 9, No. 10, December, 1928, p. 185.

[16] "Laboratory Tests of Reinforced Concrete Arches with Decks", by W. M. Wilson, *Bulletin 226,* Eng. Experiment Station, Univ. of Illinois, 1931; "Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges," by W. M. Wilson, *Transactions,* Am. Soc. C. E., Vol. 100 (1935), p. 424; and "Laboratory Tests of Three-Span Reinforced Concrete Arch Bridges with Decks on Slender Piers", by W. M. Wilson and R. W. Kluge, Jun. Am Soc. C. E., *Bulletin 270,* Eng. Experiment Station, Univ. of Illinois, 1935.

[17] *Transactions,* Am. Soc C. E., Vol. 101 (1936), p. 1395.

Briefly, the following types of action are defined, resulting either from external fixed loads or from imposed fixed deformations on the structure:

(1) Participation defines the action in which non-load-carrying or nuisance stresses are developed in a member due to very nearly fixed strains resulting from the deformation of the load-carrying members;

(2) Normal action corresponds to a very nearly fixed magnitude of the forces and moments acting on a member, and is developed when the forces acting on the member are practically independent of the size of the members in the structure; and,

(3) Hybrid action is best defined as action in a range intermediate between participation and normal action, where forces, moments, and strains are dependent on the size of the member considered, or on the proportions of other members in the structure.

It follows that, in redesigning a member found by analysis to be overstressed, participation stress can only be lessened by reducing the strain the member must undergo; for example, linear strain due to angle changes is easily reduced by decreasing the depth of the member. Normal stress can be reduced by increasing the size of the member, but hybrid stress may be either increased, decreased, or unchanged when the size of the member is increased. Where maximum stress arises from a combination of circumstances it is necessary to keep in mind the relative importance of the different factors contributing to the stress in a member in arriving at a design.

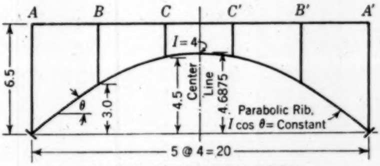
Load-Carrying and Nuisance Stress.—In considering the significance of over-stress, it is convenient to make a distinction between stresses depended upon to "carry the load" and secondary or nuisance stresses resulting from the action of the structure but having no useful function. Over-stress in normal action is serious. Over-stress due to participation is dangerous only to the extent that the ability of the member to perform its primary function is impaired. Over-stress in hybrid action may or may not be serious depending on whether the structure may "carry the load" safely in another way when the over-stress becomes large. In this connection, the significance of chance factors in the properties of the material becomes important.

The fact that the superstructure of an arch bridge is slightly overstressed is not always serious if the superstructure is not depended upon to reduce the stress in the rib. If it is possible to reduce the size of the rib of an arch bridge because of interaction, but if at the same time the superstructure must be strengthened and if the resulting structure is neither appreciably more economical nor certainly more safe than the structure as designed neglecting interaction, then the process is futile.

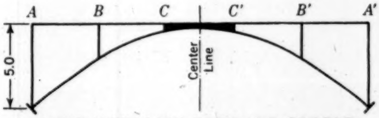
SOME PHASES OF INTERACTION BETWEEN RIB AND SPANDREL STRUCTURE

Results of Analyses; Moment Diagrams.—The writer has made analyses of different arch structures but, for the reasons stated previously, finds it difficult to draw empirical conclusions concerning interaction from the results of these analyses. However, some of the data obtained are of interest in showing, qualitatively, the effect of interaction, and are presented for that purpose.

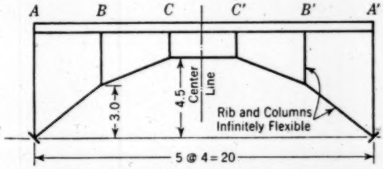
One of the series of arches analyzed consisted of a set of five-panel structures having different decks, all with practically the same rib. Sketches of the structures are shown in Fig. 1. The rib is parabolic in shape with the moment of inertia varying directly as the secant of the angle of inclination with the horizontal. Several different values of moment of inertia of deck and columns were considered. In Fig. 1, all the structures are symmetrical about the center line. The dimensions denote relative distances and the relative moment of inertia is given as *I*. The structure with a high deck is shown in Fig. 1(*a*), the structure with a low deck and an infinitely stiff saddle at the crown is shown in Fig. 1(*b*), and the structure consisting of a flexible polygonal rib with a stiffening girder is shown in Fig. 1(*c*). Except for the saddle, the rib in Fig. 1(*b*) is the same as that in Fig. 1(*a*).



(*a*) RIB WITH HIGH DECK



(*b*) RIB WITH LOW DECK AND SADDLE



(*c*) FLEXIBLE RIB WITH STIFFENING GIRDER

FIG. 1.—TYPES OF ARCH STRUCTURES ANALYZED.

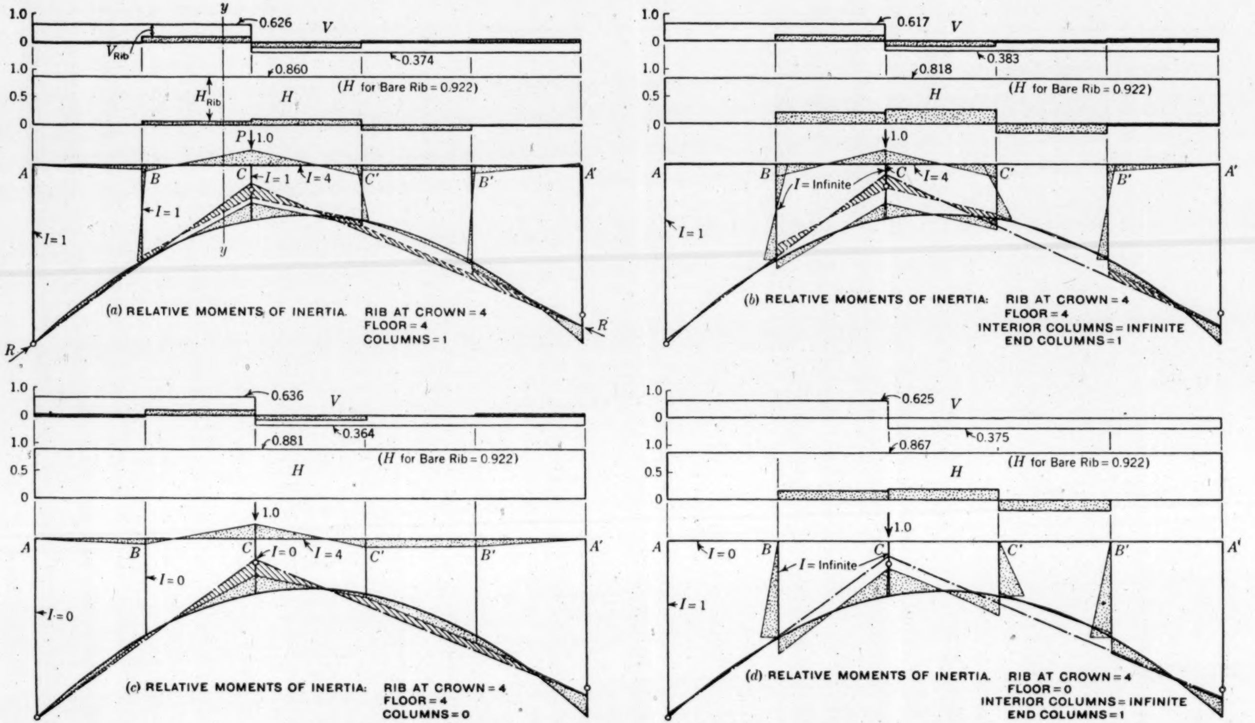
Consider the particular structure shown in Fig. 2(*a*) loaded with a force, *P*, and having reactions, *R*. Let *H* denote the horizontal component of the reactions, and let *V* denote the vertical components. At any vertical section through rib and deck, such as *y-y*, the sum of the horizontal components of the forces in rib and floor must equal the horizontal component of the reactions:

$$\text{Total } H = H \text{ in rib} + H \text{ in floor} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Furthermore, the sum of the vertical components of the forces in rib and floor must equal the total vertical shear, *V*, at the section:

$$\text{Total } V = V \text{ in rib} + V \text{ in floor} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Curves of *V* and *H* are shown in Fig. 2(*a*) with the numerical values of the total forces given on the diagram. The part of *V* and *H* resisted in the floor is shown by the stippled areas, and the remaining areas denote the part of *V* and *H* resisted in the rib. The change in the part of *H* resisted in the floor at the column points represents the shear in the column.


1.0
0
1.0
0.5
0
y
0.626
V
V_{Rib}
0.374
0.860
(H for Bare Rib = 0.922)
H_{Rib}
H
P
1.0
A
B
C
C'
B'
A'
I = 1
I = 4
R
0.617
0.383
0.818
I = Infinite
0.636
0.364
0.881
I = 0
0.625
0.375
0.867

(a) RELATIVE MOMENTS OF INERTIA. RIB AT CROWN = 4
FLOOR = 4
COLUMNS = 1

(b) RELATIVE MOMENTS OF INERTIA: RIB AT CROWN = 4
FLOOR = 4
INTERIOR COLUMNS = INFINITE
END COLUMNS = 1

(c) RELATIVE MOMENTS OF INERTIA: RIB AT CROWN = 4
FLOOR = 4
COLUMNS = 0

(d) RELATIVE MOMENTS OF INERTIA. RIB AT CROWN = 4
FLOOR = 0
INTERIOR COLUMNS = INFINITE
END COLUMNS = 1

FIG. 2.—MOMENT DIAGRAMS FOR A UNIT LOAD AT POINT *C* ON RIB WITH HIGH DECK.

The total moment, M, about the centroid of the rib at the section considered is resisted by flexure in the rib, by flexure in the floor, and by the moment of the horizontal thrust in the floor about the centroid of the rib:

$$\text{Total } M = M \text{ in rib} + M \text{ in floor} + H \text{ in floor} \times (\text{distance between floor and rib}) \ldots\ldots\ldots\ldots\ldots (3)$$

The curve of total moment about the centroid of the rib is shown in Fig. 2(*a*) as the vertical ordinate between the pressure line for the loads (the heavy dot-dash line) and the axis of the rib, since the value of H is constant for the entire length of the structure. To this same scale, the shaded areas plotted on the rib, columns, and floor as bases, represent the diagrams of moment for these members, plotted on their compression side. The moment in the floor is replotted on the pressure line as a base, and is shown by the cross-hatched areas in the diagram. The remaining area of total moment in Fig. 2 is then the moment about the centroid of the rib due to horizontal thrust in the floor. The three parts of the total moment diagram show, then, the relative values of the three sources of resisting moment.

The moment diagrams for the arch with high deck of Fig 1(*a*) for a unit load at Point C is shown in Fig. 2 for relative moments of inertia of rib at crown, floor, and columns of 4, 4, and 1 (Fig. 2(*a*)), 4, 4, and infinite (Fig. 2(*b*)), 4, 4, and zero (Fig. 2(*c*)), and 4, zero, and infinite (Fig. 2(*d*)), respectively. For the structures with columns of infinite stiffness, the end columns have relative moments of inertia of 1 compared to the rib. The moment diagram for the arch with a low deck of Fig. 1(*b*) for a unit load at Point C is shown in Fig. 3 for relative moments of inertia of rib (except for the infinitely stiff saddle at the crown), floor, and columns of 4, 4, and 1, respectively.

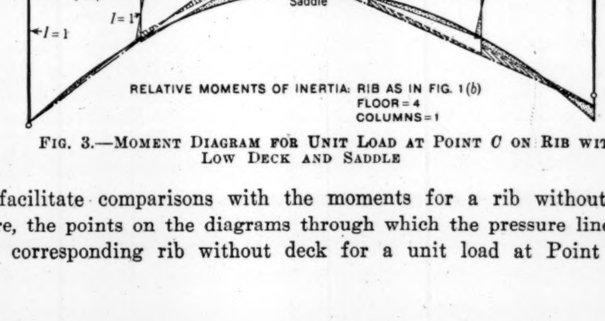


FIG. 3.—MOMENT DIAGRAM FOR UNIT LOAD AT POINT C ON RIB WITH LOW DECK AND SADDLE

To facilitate comparisons with the moments for a rib without superstructure, the points on the diagrams through which the pressure line passes for the corresponding rib without deck for a unit load at Point C, are indicated by the small circles at the springing and at the load point. The value of H for the bare rib is indicated on the diagrams.

Typical Influence Lines.—Typical influence lines for moment and flexural stress in rib and superstructure are shown in Fig. 4. They were computed for the single-span structure having a high deck without expansion joints tested by Professor Wilson, a drawing of which is shown in the Final Report of the Special Committee on Concrete and Reinforced Concrete Arches.[18]

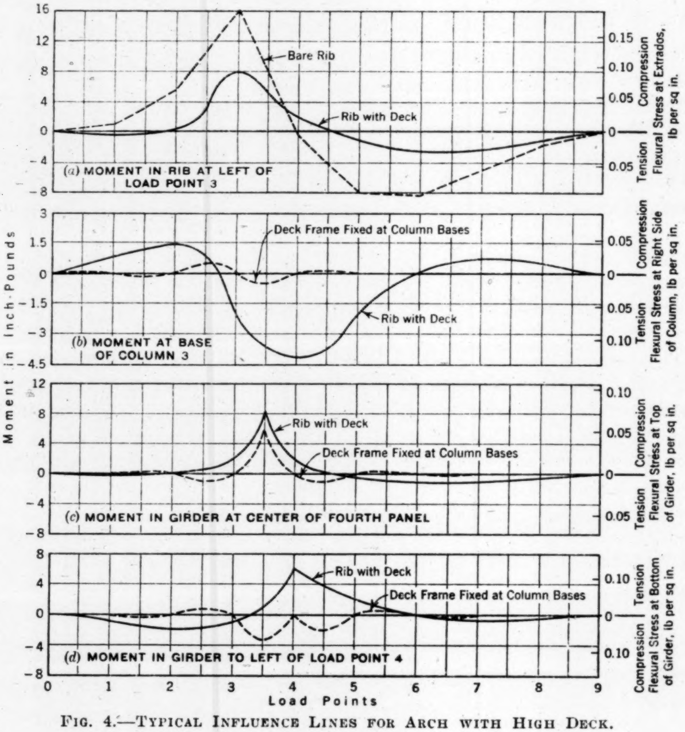


FIG. 4.—TYPICAL INFLUENCE LINES FOR ARCH WITH HIGH DECK.

The broken lines in Fig. 4(*b*), Fig. 4(*c*), and Fig. 4(*d*) show the influence lines for "primary stress" in the superstructure, defined as the stress with the deck and columns considered acting as a continuous frame fixed at the column bases. The differences between the full and broken lines in these diagrams indicate the relative effect of interaction on the moments. It is noted that the interaction effect in the floor at the end of a panel is so great that the sign of the moment for live loading is changed from that corresponding to primary action.

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[18] *Transactions*, Am. Soc. C. E., Vol 100 (1935), p. 1486, Fig. 41(*a*).

The scales for flexural and direct stress shown in the diagrams are computed on the basis of an uncracked concrete section, neglecting the steel reinforcement. The values obtained show the relative effects of moment in the various members considered. It appears that for a single concentrated load on the structure for which the influence lines are shown in Fig. 4, the maximum live load flexural stress at the base of Column 3 is approximately 1.4 times the live load flexural stress in the rib at the same load point, when the concrete is assumed to be capable of taking tension. A comparison may also be made of the flexural stress at the base of Column 3 with the maximum stress in the corresponding bare rib at the same point due to a single concentrated load. One finds the relative values of these stresses to be 0.67 in the column to 1.0 in the rib.

Stresses Due to External Imposed Deformations.—The effects of temperature change, spread of abutments, or rotation of abutments, are qualitatively similar. As an indication of the effect of such influences on the moments in the structure, comparisons have been made for structures of the type shown

TABLE 1.—MOMENTS DUE TO A UNIT CHANGE IN SPAN FOR RIB WITH VARIOUS TYPES OF DECK (SEE FIG. 1).

(Sign of moments corresponds to a shortening of span; positive moment produces compression at top of rib and girder, and compression on left side of column.)

Structure	Bare rib	Bare rib with saddle	Rib with high deck				Rib with low deck	Rib with stiffening girder	Stiff rib with girder
Moments of Inertia:									
Rib at crown	4	*	4	4	4	4	*	0	4
Girder	....	....	4	4	0	0	4	4	4
End columns	....	....	1	1	1	1	1	0	0
Interior columns	....	....	1	Infinite	1	Infinite	1	0	0
Height to neutral point	3.125	2.750	2.290	1.451	2.914	1.826	2.278	0	2.814
Springing Reactions:									
H	0.1024	0.1439	0.2134	0.5224	0.1341	0.3662	0.2461	0.0183	0.1384
M	0.3200	0.3957	0.4887	0.7578	0.3908	0.6685	0.5607	0	0.3894
Rib Moments:									
M_{AB}	0.3200	0.3957	0.4552	0.7363	0.3554	0.6330	0.4949	....	0.3894
M_{BA}	0.0103	−0.0360	−0.1303	−0.5310	−0.0306	−0.4491	−0.1611	....	−0.0532
M_{BC}	0.0103	−0.0360	0.0045	0.4526	0.0453	0.5091	0.0029	....	−0.0532
$M_{CB'}$	−0.1408	−0.2518	−0.1335	−0.2804	−0.1802	−0.4426	−0.1845	....	−0.0983
$M_{CC'}$	−0.1408	....	−0.0242	0.1732	−0.0960	0.1124	....	....	−0.0983
M_{cr}	−0.1600	....	−0.0980	−0.0901	−0.1321	−0.0586	....	....	−0.1242
Girder Moments:									
M_{AB}	....	....	0.0040	0.0279	0	0	−0.0116	0	0
M_{BA}	....	....	−0.0371	−0.2749	0	0	−0.0475	−0.0548	0.0274
M_{BC}	....	....	0.0800	0.3304	0	0	0.0843	−0.0548	0.0274
$M_{CB'}$	....	....	−0.2033	−0.4026	0	0	−0.2961	−0.0822	−0.1351
$M_{CC'}$	....	....	−0.0872	−0.0023	0	0	....	−0.0822	−0 1351
M_{cr}	....	....	−0.0872	−0.0023	0	0	....	−0.0822	−0.1351
Column Moments:									
Top: M_A	....	....	−0.0040	−0.0279	0	0	0.0116	....	0
M_B	....	....	−0.1171	−0.6054	0	0	−0.1318	....	0
M_C	....	....	−0.1161	−0.4002	0	0	....	....	0
Base: M_A	....	....	−0.0335	−0.0216	−0.0355	−0.0355	−0 0658	....	0
M_B	....	....	0.1348	0.9836	0.0759	0.9583	0.1640	....	0
M_C	....	....	0.1092	0.4536	0.0842	0.5550	....	....	0

* Infinitely stiff saddle at crown; otherwise, rib same as bare rib.

in Fig. 1. Data are shown in Table 1 for springing reactions and moments in all members due to a unit change in span for an arch rib with various types of deck. The effect on the rib of interaction with the deck is to increase the temperature moments considerably.

Consider the deformations of an arch structure with deck due to a given change in span. If the columns have little stiffness and the floor, at all sections, has the same relative stiffness to that of the rib, the change in shape of the rib will not be altered by the superstructure. Consequently, in such a structure, the temperature flexural stress in the rib will be the same as in the bare rib. However, there will be temperature flexural stresses in the floor, which will be to the temperature stress in the rib at any section as the depth of the floor is to the depth of the rib at that section, since the deflections (and, consequently, angle changes), of the rib and floor are the same. Note that the direct temperature stresses will be higher in the composite structure than in the free rib, but the direct stresses are comparatively insignificant.

If the floor has, say, a constant moment of inertia and that of the rib varies, and the columns still have very little stiffness, the change in shape of the rib due to a given abutment movement will be altered somewhat from the change in shape of a free rib since the composite structure is relatively more stiff at the crown than at the springing compared with the free rib. Hence, there will be a greater distortion in the haunch and near the springing with a relatively higher temperature stress in the composite structure at those points than in the bare rib.

Consider now the effect of the columns. Since, in general, the columns stiffen the structure and the given displacement must take place, there will be correspondingly greater distortions in the rib with stiffer columns; and, hence, there will be greater deformation stress in rib and floor with stiffer columns. The columns also will have high moments due to deformations.

The increase in relative stiffness of the crown of the composite structure over the crown of the free rib causes a reduction in maximum live load moment at the springing, but this same cause produces inevitably an increase in temperature flexural strain at the springing. The fact that one effect cannot exist without the other is worth some emphasis.

The significance of high deformation stresses is a matter of judgment. They are almost certainly not as serious in concrete as stresses due to loads. Attempts have been made to reduce stresses due to these causes by expansion joints and other articulations in the deck of the bridge. These attempts are not always successful, sometimes merely localizing the effect, particularly in the case of expansion joints at the ends of a saddle in a low deck bridge.

It must be noted that for structures in which interaction is depended upon to reduce stresses in the rib the stresses in the deck due to deformations and temperature changes are important in so far as they tend to cause cracking of the superstructure, for cracking of the superstructure reduces its stiffness and thereby the aid which it can render the rib.

Summary of Results of Analyses.—A study of a number of influence lines and moment diagrams seems to indicate the following conclusions:

(1) The maximum live load moment at the springing line and at intermediate column points on the rib of the integral structure is considerably reduced by the interaction with the deck. For intermediate column points the reduction may be of the order of 50% when the moment of inertia of the floor is equal to that of the rib. The moment between column points in the rib is reduced considerably more than the moment at the column points if the columns are stiff.

(2) The interaction between rib and deck for live load adds to the primary moment in the floor at the center of the panel, computed as for a frame fixed at the column bases. At the ends of the panels, the interaction moment in the floor is opposite in sign to, and may be much greater than, the primary moment.

(3) The interaction live load moments in the columns are considerably greater than the primary live load moments, even for slender columns. The interaction column moments are greatly increased by making the columns stiff.

(4) Where the columns are quite stiff and the rib and floor have approximately equal moments of inertia, points of inflection or points of zero moment are close to the center of all members for most conditions of loading.

(5) Temperature and deformation strains at the springing in the rib of the integral structure are much greater than in a bare rib. Temperature strains in the deck are likely to be of the same order as those in the rib. The temperature strains are increased by making the columns stiff.

Treatment of the Arch with Superstructure as a Flitch-Beam.—If the spandrel columns of an arch bridge are quite flexible and closely spaced, it is evident that the deflections, slopes, and angular changes along the arch rib must be equal to those along the floor (except for the change in length of the columns, which effect will be neglected, but can be taken into account if necessary). Then all direct stresses due to thrust will be carried by the rib, and the relation between moments and flexural stresses in rib and floor can be predicted and controlled by varying their relative depths and moments of inertia.

In other words, the structure can be designed as if it were composed of two members placed side by side as in a flitch-beam. This concept dates back at least to Rankine,[19] and has been used apparently by Considére.[20] Professor Cross[3] has discussed several new aspects of this treatment in connection with his system of classification.

An analysis for the purpose of design may be developed along the lines of Professor Cross' treatment as follows: The deck girder will have primary stresses due to its action as a continuous beam. These stresses are due to the dead load of the deck only, and the entire live load. The arch rib will take all the thrust, most of which is due to dead load. Then the difference between the working stress and the primary girder stress is available to resist interaction flexure in the girder. The difference between the working stress and the stress due to direct thrust in the arch rib is available to resist flexure in the rib. Temperature and deformation stresses may also be discounted from the working stress.

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[19] See "Civil Engineering," Seventh Edition, 1871, pp. 313–314; and other editions at approximately the same page numbers.

[20] See, for example, "Cours de Beton Armé," by A. Mesnager, Paris, 1921, p. 198.

The stress due to interaction flexure in the girder is to the flexural stress in the rib approximately as the depth of the girder is to the depth of the rib. The rib and deck resist the moment (except for primary moments) approximately in proportion to their moment of inertia.

A few rough figures will enable an estimate to be made as to whether a tentative combination of depths of girder and rib is efficient or inefficient, and will give some idea as to what can be done about it; for example, if the girder is found to be overstressed, it must be remembered that increasing the depth of the girder may increase interaction stress, although it decreases the primary stress.

Analyses of several structures by the writer have indicated that for loads applied at column points, the approximation involved in the assumption that the columns are closely spaced is very good. This approximation is strictly true for the particular case of a two-hinged arch rib polygonal in shape with a deck composed of straight prismatic segments between column points, in which the ratio, $\frac{(EI\ \cos\theta)_{\text{deck}}}{(EI\ \cos\theta)_{\text{rib}}}$, has the same value in every panel. For loads between column points it is necessary, for accuracy, to find, first, the column reactions, considering the floor as a continuous beam, and then to apply these reactions to the composite structure.

Where there are expansion joints in the floor the total moment must be taken by the rib. Where the floor is not continuous over the springing the moment at the springing is resisted entirely by the rib.

For certain cases it is possible that neglect of the columns will not be on the safe side. If the columns are stiff and the floor flexible there is considerable local flexure of the floor and the moments in the floor may be several times as great as those assumed on the basis of equal angle changes, unless the stiffness of rib and floor is of about the same order.

Relative Importance of Sources of Stress.—The principal sources of stress for which an arch bridge is designed are dead load, live load, and deformations such as those due to temperature change, shrinkage, abutment movements, etc.

Neglecting the effect of rib-shortening there will be no interaction stress for dead load if the pressure line for the weight of the rib and the dead load reactions of the deck acting as a continuous frame fixed at the column bases fits the arch axis. With some reservations,[21] rib-shortening may be considered as deformation stress. The manner of constructing the structure must be considered in evaluating the effect of rib-shortening; for example, if the arch centering is struck before the deck is constructed, rib-shortening due to the weight of the rib will not cause interaction stress.

The dead load and live load moments in the floor considered acting as a frame fixed at the column bases (defined as the primary moments in the floor) depend mainly on the panel length. The interaction moments in the floor are a function of the relative stiffness of floor and rib as well as of the span length of the rib. Adjustments of the depth of floor and rib, therefore, may make the interaction stress in the rib almost anything that is desired within reasonable limits. If it is desired to reduce interaction stress in the floor to a minimum, it is most easily done by reducing the depth of the floor by a proper choice of details and arrangement of framing.

For very short arches, say, spans up to 50 ft perhaps, the dead load stress is relatively small compared to the live load stress for an open spandrel arch, but such short spans are usually constructed more economically with filled spandrels. For arches of perhaps 150 ft, or more, in length, the dead load is generally more important than the live load. For long-span bridges interaction has no effect on the major part of the stress in the arch rib. In this connection it is interesting to note what has been written[11] concerning the design of the George Westinghouse Bridge of 412-ft span length:

> "A model study of the main span was made with the Beggs Deformeter to determine the stiffening effect of the superstructure. This study showed that the negative live load moment at the springing line would not exceed 60 per cent of the moment calculated for the bare rib. When this moment is reduced to terms of fiber stress the reduction is only from 780 to 730 lb. per sq. in."

A description[12] of the Raritan River Bridge, in New Brunswick, N. J., with a span length of 202 ft, contains the following comment:

> "Each of the main arch spans consists of three hingeless ribs, 12 ft wide, designed as free ribs by the application of the elastic theory without taking into account the restraining action of the superstructure. The design was checked by the use of deformeter gages on a celluloid model, and the agreement with the analytical method was practically exact. A determination of the stresses in the rib was also made on a complete model of the rib and superstructure, and marked reductions in the moments, thrusts and shears were noted due to the stiffening effect of the spandrels and floor. However, since the superstructure was poured after the centering was struck, it does not relieve the rib of any of its dead load stresses. The braced condition of the rib is applicable under live load, but as the live-load stresses comprise but a small proportion of the total, no advantage was taken of the reduced stresses in a redesign of the rib."

In even shorter arch bridges it may be noted that a decrease in live load stress in the rib is to some extent offset by an increase in deformation and temperature stress.

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[21] "Relation of Analysis to Structural Design," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363, especially p. 1365.

Studies made of arch bridges in Allegheny County, Pennsylvania, for both springing and crown stresses for spans between 180 ft and 400 ft, led to the following conclusions[22]:

> "For the extreme condition considered, the moment due to temperature and shrinkage was doubled to indicate the possible effect of additional moment from any cause such as a temperature drop or shrinkage greater than the assumed or movement of the supports. This investigation leads to interesting conclusions. Considering the extreme condition of combined stress, which gives an index to the ultimate safety of the rib, it develops that:
>
> "(1) The live load stress is a relatively small proportion of the total ranging from about 15 per cent for spans approximating 400 ft. to about 25 per cent for short spans. These percentages apply to shallow ribs, and are smaller for deep ribs.
>
> "(2) The arch shortening stresses may become more than 50 per cent of the total for deep ribs."

The stresses in the columns cannot be estimated very closely. The columns are ordinarily not designed for moment in any case, and usually not even for direct stress. Their least dimension depends upon their height, and when all the columns are made the same width, this dimension depends on the height of the longest column. The columns may possibly be considerably overstressed in flexure due to interaction. This is not necessarily serious unless the columns are depended upon to reduce the stress in the rib.

Various studies made by the writer seem to indicate that the column action is generally always hybrid. The effect on the column moments of increasing the size of the column, is demonstrated in Figs. 2 and 3. Moreover, even when the columns are so stiff that further increase in their section does not attract more moment to them, the column moments are still sensitive to variations in the ratio between rib and floor stiffness.

CLASSIFICATION OF ARCHES WITH INTEGRAL SPANDREL STRUCTURE

Rib with Flexible Deck.—If the superstructure is very flexible compared with the rib the interaction stresses, due to movement of the column bases, will be small compared with the primary stress in the deck, considering the column bases fixed. This type of structure is clearly normal with participation stress in the deck.

It is worth while to examine the possibility of reducing interaction stresses by detailing the superstructure so that it has a high degree of flexibility. Jack or spandrel arches and fascia beams may be omitted and the slabs framing into the floor-beams at each panel may be used to secure flexible construction. A structure so designed may possibly be overstressed in the deck, but the over-stress will not be serious since the deck is not depended upon to carry load. Any cracking in the deck will relieve the stress, but will not cause over-stress in the rib because the rib is proportioned to carry the entire load.

A method for determining interaction stresses in a structure of this kind has been suggested by Hardy Cross.[6] The method is akin to that commonly used to determine secondary stresses in bridge trusses and is as follows: For the given condition of loading determine the deflections of the column points in the arch bridge. In general, these deflections will be a horizontal and a vertical translation and a rotation. For these given deflections find the stresses in the superstructure. If the forces at the column base are small compared to the loads acting on the arch, no revision is necessary. If they are not negligible a revision may be made by computing the additional deflection of the arch rib due to these forces and revising the figures. If this revision is large, the method will not work. It may be used, however, to determine whether or not the secondary stresses are serious in any particular case.

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[22] "The Design of Concrete Arches in Allegheny County, Pennsylvania," by G. S. Richardson, M. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. XXVII (1932), p. 650.

The flexibility of the deck depends mainly on the panel length, and it is evident that decks of the same type and panel length in structures of different length will be practically the same in stiffness. The longer the arch bridge, the stiffer will be the rib. For structures of short and moderate lengths the deck, of necessity, will be fairly stiff as compared to the rib. The flexible deck type of structure will usually be found, therefore, in long spans.

The Flitch Arch.—If it is necessary or seems desirable to make the floor fairly heavy, it is possible to make use of the flitch-beam concept in the design of the structure, and depend on the floor, but not on the columns, in interaction. Where both the rib and the floor are depended on to carry the load, the structure is hybrid, but the controlling relations are comparatively clear; the designer can "tell the structure what to do" with reasonable assurance that it will act in the way he designs it.

The particular case of a flexible flitch arch with stiffening girder is of interest. Several arches of this type have been built in Switzerland, Sweden, and Finland. The Landquart Bridge, in Switzerland, is of this type, has a polygonal rib about 10 in. deep connected by flexible columns to a floor supported by girders approximately 3 ft in depth. Stresses in this type of structure may be developed directly from the flitch-beam concept by considering a two-hinged arch rib having the shape of the given rib and a moment of inertia variation, in which $\frac{ds}{I}$ is equal to $\frac{dx}{I}$ for the girder. The analogy holds for flexural stresses due to all causes, including rib-shortening. Of course, all direct stresses are carried by the arch rib, including direct stress due to rib-shortening and deformations. The secondary flexural stresses in the rib may be obtained from the relation that the flexural stress in the rib is to the interaction flexural stress in the girder approximately as the depth of the rib is to the depth of the girder. This type of flitch arch is a normal structure with participation stresses in the rib adding to the primary stress due to thrust.

The flitch-beam concept may be applied to the structure with a saddle connecting the deck and rib, providing the columns outside the saddle are fairly flexible. The section through the saddle may be considered as having an infinite moment of inertia. It should be pointed out that stresses in such a saddle, even for given moments, shears, and thrusts, cannot be computed accurately by methods of elementary mechanics. Usually, it will not be necessary to compute the stresses since the resistance of the saddle to flexure is very high. If the deck outside the saddle is quite flexible it may be convenient to consider the section common to rib and deck as a part of the rib alone, and neglect any other effect of the deck. The method of computing the participation stress in the floor suggested in the preceding section may then be used conveniently.

The flitch-beam concept applies also to the through arch bridge having closely spaced steel hangers supporting the roadway. Where the roadway is used as a tie, the procedure with slight modifications may still be used. If the tie and the roadway have no flexural resistance the structure is what is ordinarily known as the bowstring arch, which may be analyzed conveniently by the usual methods. The secondary flexural stresses in the roadway may be approximated as shown previously for similar structures.

There is nothing in the flitch-beam concept requiring that the structure be symmetrical. Unsymmetrical arches may be treated in the same manner as symmetrical arches.

Arch with Stiff Spandrel Columns.—The structure in which the effect of the columns is important is, in general, in the hybrid class. There are certain exceptions. If the columns are quite stiff, and if the rib and floor have approximately equal moments of inertia, the structure has certain normal characteristics. The corresponding simply supported Vierendeel truss is a normal structure and can readily be designed. For the Vierendeel truss an approximate analysis preliminary to design may be made by assuming points of contraflexure in the center of all members and also that at any section the moments in rib and floor are equal. For the arch, it is necessary to take into account the fact that the supports are fixed. It is noted in this connection that the arched structure is hybrid to the same extent that a bare arch rib is hybrid for live loads.

For the general case of the structure with moderately stiff columns, where the rib and floor are quite different in stiffness, the structure is certainly hybrid. It may be possible in certain cases to choose, arbitrarily, proportions which analysis will prove to be fairly satisfactory. In general, however, no definite rules can be given for the revision of such a design from the results of an analysis; nor is the stress in such a structure known certainly, even for given dimensions, because of the effect of variations in physical properties of the different members. The hybrid structure, particularly the complicated hybrid structure of this type, may be quite sensitive to variations within itself. It may be designed, of course, by choosing to neglect the uncertain factors. In any case it must be ascertained whether the neglect of these factors is on the safe side if any advantage is to be taken of interaction in the design.

EFFECT OF EXPANSION JOINTS AND OTHER TYPES OF ARTICULATION

The articulation of the structure by means of expansion joints in the floor and hinges, rollers, and sliding joints in the columns, affects principally the columns. As far as the rib is concerned, due to articulation of the deck, the dead load stress is unaffected, the decrease in live load stress over that for the plain rib is less, and the increase in temperature stress at the springing over that for the bare rib is less, than for the structure without articulation. In the floor both the interaction live load stress and the temperature stress are decreased by articulation of the floor, but in the case of the live load stress the decrease is not appreciable several panels away from the articulation.

Live load stresses in the columns are not always decreased by expansion joints in the floor. The expansion joint tends to localize the rib deformation, and, as a consequence, the neighboring columns may suffer. Temperature stresses in the columns are probably reduced in all cases by expansion joints because of the decreased stiffness of the entire structure.

Articulating the columns rather than using expansion joints in the floor is more effective in reducing the column moments, but the details of such articulations usually become quite troublesome.

SUMMARY

The most important points to be emphasized in this paper may be summarized as follows:

(1) The net effect of interaction on the total stress for which an arch rib must be designed is normally quite small. The ordinary method of designing arch structures neglecting interaction is sound, in general, since structures so designed have apparently given good service.

(2) The significance of over-stress in the superstructure depends upon whether the deck is counted upon to assist the rib in carrying the load. It may be possible to obtain greater economy by detailing floor and columns to reduce their rigidity, and hence their interaction stress, than by designing the superstructure to assist the rib. The conclusion that the deck always strengthens the rib must not be drawn too hastily. The possibility of a load cracking the deck and destroying part of the interaction must be considered. Furthermore, stresses due to temperature and movements of foundations are high in the integral structure.

(3) Certain limiting cases of the arch with integral spandrel structure are "normal," and can be designed directly, but practically all such structures of the proportions usually built are hybrid, and cannot be designed for given working stresses by the traditional procedure of repeated analysis and design.

(4) An arch with a comparatively heavy rib and flexible columns and deck supporting relatively light live loads is a normal structure with participation stress in deck and columns added to the normal stress in these members due to their action in the continuous deck frame.

(5) Another "normal" type is the structure consisting of a flexible rib with a stiffening girder. There will be flexural participation stress in the rib, in addition to the normal stress due to the direct thrust in the rib.

(6) A third type of structure having "normal" characteristics in certain cases may be described as an arched Vierendeel truss. This structure must have quite rigid columns, and floor and rib of approximately equal stiffness. The corresponding simply supported Vierendeel truss would be normal, but fixing the supports introduces complications. In this connection, it is important to note that the fixed arch rib has hybrid characteristics for live loads.

(7) Other "normal" arch structures may be secured in various ways by the manner of construction or design involving the use of articulations and flexible sections. Such structures have well marked participation stresses at such articulations and pseudo-hinges. Of course, not all structures with hinges and joints are "normal."

(8) In special cases applying to certain other arch structures which have hybrid characteristics the controlling relations are comparatively clear. The flitch-beam concept applying to the structure with flexible columns has considerable value and leads to a definite design procedure.

(9) For the general case where the columns must be of intermediate stiffness the structure is clearly hybrid, and there seems to be no convenient way of estimating in advance whether it can act as it is assumed to act in a preliminary analysis. Use of the traditional procedure of successive analysis (usually by means of a model) and design does not appear to be very promising. Such a procedure might easily be misleading, particularly where a re-design is based on an analysis which is not repeated for the new design. It is open to question whether a better structure is obtained by such a procedure than by neglecting interaction entirely.

DISCUSSION

A. Floris,[23] Esq. (by letter).—In his broad discussion of the interaction between arch and superstructure in concrete bridges, the author fails to include the arch-barrel type, often used in practice. The rib being a special case of the barrel, it is obvious that a general theory on the subject should start from this point. The analysis is greatly facilitated by the fact that the statically indeterminate quantities of the arch barrel as well as the superstructure in space, can be divided into two independent planar groups.

In the first group, the unknowns are influenced by moments and forces acting in the longitudinal plane of symmetry, whereas, in the second group, they are functions of moments and forces acting outside this plane. The proof of this theorem which reduces the space problem into a planar one has been given by Fr. Engesser[24] and later utilized by W. Schachenmeier in the analysis of arch bridges in which the action of the superstructure is taken into account[25]. In this elaborate analysis (the first of its kind as far as the writer knows), Schachenmeier investigates the complicated structure by choosing as a base system the fixed arch rib without deck. The analysis is complete and not difficult to apply, although the numerical work (as would be reasonable to expect) is considerable.

The aim of this discussion is merely to call attention to these two important contributions pertaining to the analysis of arched bridges. It does not constitute a criticism of the author's meritorious efforts to clarify the interaction between rib and superstructure in arch bridges.

Frank M. Baron,[26] Esq. (by letter).—A contribution to engineering literature which attempts to present and use a designer's viewpoint is to be highly commended. Many readers may fail to grasp the intrinsic value of Mr. Newmark's paper. The method of attack, as applied to the problem at hand, is certainly as valuable as his conclusions. It is an approach of this kind that yields fruitful information to an engineer as to the probable action of a structure. Too much of the literature recently is being deluged with methods of analysis, and sight is being lost of every engineer's goal—that of a successful design. The interpretation of analyses and tests, and the significance of factors governing a design are usually more important than the means of obtaining the required data.

A designer usually wants two types of answers to a problem—qualitative and quantitative. A qualitative study of the probable action of a structure will generally lead to the quickest estimate of the quantities required. A quantitative study gives only a quantitative answer, and does not lead into possibilities of structural control, unless one is to carry it to limits of comparative analysis. Analysis of any kind involves some assumptions, and, as a result, judgment must be trained as to the interpretation of the resulting data. One of the greatest follies in thinking is the acceptance of analytical results without properly weighing the assumptions and the probable action of the structure involved. This is where the true worth of the author's paper is distinguished. The method of attack is applicable not only to arches, but also to any type of continuous frame.

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[23] Dipl.-Ing., Los Angeles, Calif.

[24] "Das elastische Tönnengewölbe als räumliches System betrachtet," von Fr. Engesser. *Zeitschrift für Bauwesen*, 1909, pp. 107–118.

[25] "Ueber mehrfache elastische Gewölbe," von Wilhelm Schachenmeier, Leipzig, 1910.

[26] Asst. Research Engr., Chicago Bridge & Iron Co., Chicago, Ill.

In relating the action of an arch in terms of the classification of indeterminate structures presented in 1935 by Hardy Cross, M. Am. Soc. C. E.[3], the author has presented his viewpoints on arch action analogically. Exhaustive research was involved, as one can see that the necessary data must have been gathered through the analysis and compilation of the experiences of various engineers before any interpretation of results was possible.

In considering the significance of over-stress, the author makes a distinction between stresses depended on to "carry the load" and participation stresses. It stands to reason that participation and deformation stresses should be discounted if they do not impair a member from carrying the load, or are of localized nature. In several cases the author seems to have attached too much importance to over-stress due to participation action. Such over-stress does not necessarily mean that a structure will fail. Consider the case of over-stress in the column sections, for example. What if the over-stress due to interaction is so great that the column section should crack? The member may crack, but then the structure would not fall, as the columns would still perform their primary purpose of supporting the deck, and transmitting the deck loads to the rib. If cracking is considered as a failure, then some means of articulation may be adopted at times to prevent it.

It is true that, if the column is relied upon to help the rib, and the column should crack, it might prove disastrous to the rib; but this raises the question as to whether one would try to reduce the rib section in an attempt to obtain a more economical design due to help from participation stress. A few figures can be attached to the relative costs of the elements of the structure to show the futility of this choice[27]. In some cases, the deck is probably the only element that might justify consideration of interaction help from the columns or from the rib.

As to the matter of change in the sign of the moment at the end of the panel due to interaction effect, the author has lost sight of a point that needs further emphasis. It is the importance of dead load and live load ratio upon the consideration of this effect. Then, also, is this the stress that governs the depth or cross-section of the deck? It may be important at times only from consideration of the proper placing of steel.

The selection of the type of arch and the importance of interaction effect must also be made from the standpoint of dead load and live load ratio. A long span arch is usually a dead load structure, and may be shaped so that the pressure line follows the arch axis for dead load. Interaction would not be important in this case, as the ratio of dead load to live load is usually high. In the case of a short-span, open-spandrel arch, live load may predominate, and participation becomes a problem. It is preferable to have a high dead load ratio, and the selection of a filled spandrel arch might be the answer, provided other factors remain equal. In the medium span arch then, apparently, is where the problem of interaction must usually be decided.

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[27] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., p. 306.

The flitch-beam concept is outstanding in the possibilities that it affords a designer. Through its use, the engineer is able to predict the degrees of stress, but what is even more important is its adaptability to structural control. The effect of one member upon another may be studied, and a choice made as to the final structure desired. Instead of having the moment curve virtually "wag" the structure, proper selection of $\frac{\text{depth}}{\text{length}}$ ratios enables the designer to "wag" the moment curve. Structures try to do what the designer wants them to do as much as they can. Judgment, therefore, is required as to whether the structure can act as "instructed."

The writer wishes to raise the question as to what a consideration of interaction effect does to the safety factor of the entire structure. The author hints at it as applied to a column section. The importance of interaction in the members of an arch may often be determined through considerations of comparative safety factors. If interaction effect is depended upon to help the rib, then a much lower safety factor would result, and probably forbid the use of a smaller rib section. In considering participation stress in the column, the writer believes the difference in total safety factor would be so slight that a designer may easily neglect interaction. The difference involved in the deck would depend entirely upon the ratio of dead load to live load, as well as the judgment upon the discounting of participation stresses.

Concluding Remarks.—The paper is one of the few in which the arch is considered as a whole structure. Judging by the time spent in the analysis and design of the rib, many designers seem inclined to assume that this is the only part of an arch, whereas the rib usually has the smallest effect on the total cost of the structure. The paper is further enlightening upon the need of interpretation as applied to results and data obtained in the field of structures.

A. A. Eremin,[28] Assoc. M. Am. Soc. C. E. (by letter).—An interesting study of the interaction stresses in the rib and superstructure of concrete arch bridges is presented in this paper. However, for conclusive results the arch bridges shown in Fig. 1 seem to have spans that are too short. The theoretical and experimental study should be extended to arch bridges with larger spans and a greater number of panels. In practice, it is seldom that an open-spandrel arch bridge is constructed with less than ten spandrel panels.

In Item 5 of "Summary of Results of Analyses", the author refers to temperature and deformation strains instead of stresses as it seems he should have done. Apparently, temperature stresses in the deck may be of the same order as those in the arch only if the relative stiffness of rib, spandrel colums, and the spandrel girders in the arch bridge are approximately the same and the bridge can be considered as a Vierendeel arched truss.

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[28] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

The author's statement (see heading "Treatment of the Arch with Superstructure as a Flitch-Beam") that the stresses "due to interaction flexure in the girder is to the flexural stress in the rib approximately as the depth of the girder is to the depth of the rib", needs clearer interpretation. Theoretical and experimental studies available at present are not sufficient to support it. It seems evident that the relative stiffness of the straight spandrel girders and the arched rib cannot be compared in terms of a depth ratio only, without reference to the ratio of the arch rib rise to the span length. For complete study of such interaction the analysis of stresses and a comprehensive inspection of existing arch bridges may produce much useful information.

The interaction stresses in open-spandrel arch bridges vary with: (1) The ratio of the live load to the dead load considered in the bridge; (2) the ratio of the arch rib rise to its span length; (3) the height of the spandrel columns at the crown of the arch rib; and (4) the degree of restraint imposed to avoid variations in the span of the arch.

In long-span arch bridges the live load stresses are generally small as compared with dead load stresses. Therefore, the former have a negligible effect on displacements in the spandrel girders and arch ribs. In the Plougastel (arch) Bridge, in France[29], with a span length of 611.39 ft, a rigid double-deck, reinforced concrete spandrel truss was used. However, it had only two expansion joints, 116.28 ft from the center of the span. Evidently, the effect of the live load and temperature deformations in the arch rib on the stresses in the spandrel trusses was negligible.

In arch bridges with shorter spans the ratio of live load stresses to dead load stresses in ribs is larger. Therefore, the effect on stresses in the superstructure, of displacements in the rib produced by live load and temperature stresses, is greater. Various devices have been used to reduce the participating stresses in superstructures. In the Broakpark (arch) Viaduct, at Cleveland, Ohio,[30] with a span length of 192 ft 3 in. from center to center of piers, a flexible roadway deck slab served this purpose, whereas in the Wandau Enns (arch) Bridge, in Germany,[31] with a span length of 242.06 ft, expansion joints were provided at each spandrel column.

As the span length is further decreased more difficulties are encountered in making the superstructure sufficiently flexible to accommodate a possible displacement in the rib. An interesting case of a flexible roadway deck slab, with overhanging ends cut by expansion joints, is that of the arch bridge near Donner Lake, in California.[32] This structure has a span of 110 ft, divided into eleven spandrel panels. The roadway deck is on a grade of 7% and the center line of the roadway is curved to a radius of 360 ft. Nevertheless, the slab is in excellent condition and reveals no visible cracks.

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[29] *Le Genie Civil,* October, 1930, p. 317.

[30] *Engineering News-Record,* October 11, 1934, p. 467.

[31] *Beton und Eisen,* March 5, 1931, p. 73.

[32] *Engineering News-Record,* March 4, 1926, p. 357.

In concrete arch bridges of short span lengths, and with heavy railroad loading, the ratio of live load stresses to dead load stresses in the ribs is large. Therefore, it is practically impossible to make the deck slab sufficiently flexible. In such structures, rigidity is generally governed, not by the ribs but by the spandrel girders and spandrel columns. Ribs are made flexible enough to reduce the participating stresses in the connecting spandrel columns. A fine illustration of this type of bridge is the Voltschiel (arch) Bridge, near Andeer, in Switzerland,[33] in which the rigidity of the spandrel columns was increased by constructing massive abutment piers founded on rock. It is more difficult to provide sufficient rigidity in the spandrel columns in arch bridges with high abutment piers. Generally, in this case, the spandrel girders are rigidly connected to arch ribs at the crown similar to that used in the Landquart (arch) Bridge, at Klosters, Switzerland.[33] Of course, the tractive forces in this case are thrown in the arch and produce unsymmetrical stresses in the ribs. Lateral forces should be included among the principal sources of stress indicated by the author. The major lateral forces in arch bridges are generally produced by wind pressures which are negligible only in arch bridges with short spans. They cannot safely be neglected in arch bridges with large spans. The difficulty of analyzing interaction stresses when the spandrel girders are rigid, when the columns must resist combined forces in the plane of the rib, and when the structure is subjected to lateral forces, is a serious one. The stresses in the ribs can be determined more readily if the spandrel girders are broken by expansion joints. The lateral forces from the spandrel girders are then transmitted immediately to the arch ribs and the lateral stresses in the ribs can be computed by a simple method presented by the writer elsewhere.[34] In arch bridges with continuous, rigid, spandrel girders and columns, the lateral stresses are carried to the high abutment piers and this design could scarcely be considered as the most economical one.

The following tentative conclusions may be made in the light of this discussion:

(1) Open-spandrel arch bridges with spans greater than 120 ft, should be designed with a flexible roadway deck slab, or the spandrel girders should have expansion joints.

(2) Rigid spandrel girders and columns may prove the most economical for arch bridges with spans less than 120 ft. (If the structure is to be designed for railroad loading the span may be increased to 200 ft); and,

(3) If it is anticipated that foundations may settle or that elastic piers may be otherwise displaced enough to change the span length, spandrel girders and the arch rib should each have approximately the same degree of rigidity, for the most economic design. In such a case the structure will be the same as that generally called the Vierendeel arched truss.

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[33] *Engineering News-Record*, January 11, 1934, p. 44.

[34] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 324.

HARDY CROSS,[35] M. AM. SOC. C. E. (by letter).—This paper is worth study by those interested in interaction of rib and deck, a phase of arch design that in recent years has attracted some attention, especially in connection with the work of the Special Committee on Concrete and Reinforced Concrete Arches.[36] The paper raises the question as to whether analysis which includes the effect of the deck structure is really worth while. The Committee recommended that such analyses be made, but failed to indicate in any way what was to be done with them. Whether they should be made certainly depends largely on the relative importance of live load, dead load, and temperature distortions as affecting the total stress. Usually the ribbed arch of concrete is chiefly a dead load structure; the live load effect has become relatively important only with modern loading. Interaction is chiefly important in short spans because live load stresses may be relatively greater and also because there is more pronounced interaction. Dead load stresses are primarily compressive; live load stresses are chiefly flexural. If the live load stress becomes important it is much more useful to ask how this flexure should be provided for than merely to analyze a design more or less arbitrarily chosen.

Assuming that an analysis is to be made, the question is, then, whether it will furnish a satisfactory guide as to what should be done to improve the design. Thus, if over-stress is indicated in the columns, the question at once arises whether it would be wiser to make the columns larger, or to make them smaller; either answer may be satisfactory in some cases. Similarly, over-stress in the deck may indicate either that the deck should be made stronger or possibly that it should be made more shallow. In general, any hope of real improvement of design will depend a great deal on whether or not the designer can foresee clearly some simple way in which a structure is to carry its load; complicated analyses, unless interpreted very judiciously, rarely lead to simple pictures of structural action. This paper does furnish some simple pictures helpful to the designer.

The paper has value in drawing attention to types of open-spandrel arches designed to resist flexure in other ways than in the rib alone. It is possible to carry all flexure in the deck, that in the rib producing participation stress; it is possible (and may sometimes be desirable) to carry the flexure partly in the deck and partly in the rib. The paper makes clear that any attempt to divide the flexure between rib and deck will depend for its success upon the relative depths. Structures designed for flexural resistance in the deck have been used on the continent of Europe; they are worth some study.

It may be worth while to analyze such structures in the hope of predicting the type of interaction that can be secured, assuming that such interaction is desirable, or it may be worth while to analyze to determine whether, for a more or less arbitrarily assumed layout, such interaction is dangerous. There does not seem to be now any satisfactory way of telling from the analysis whether the condition is really dangerous. Mere computations of stress alone do not indicate the degree of safety any more than they do when secondary stresses are included in analyzing steel trusses.

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[36] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1427.

Thus, as Mr. Newmark points out, over-stress in a column due to participating flexure does not very seriously threaten the safety of the column and does not threaten at all the safety of the arched structure as a whole.

The writer is gratified to see a paper which diverts attention from algebraic manipulation to a consideration of the objectives of such manipulation.

NATHAN M. NEWMARK,[37] ASSOC. M. AM. SOC. C. E. (by letter).—The comments and criticism offered in the discussions of the paper are appreciated. Many of the points raised are of great interest.

Attention is called by Mr. Floris to methods of analysis for the arch with superstructure. The writer purposely did not discuss at any length the matter of analytical procedures. The paper was concerned with the question of why to make analyses rather than how to make them.

Mr. Baron and Professor Cross have stated the main point, namely, that it is necessary to decide what the structure can be made to do, qualitatively, before any detailed quantitative study can profitably be undertaken.

If the rib provides all the necessary flexural resistance to live load, as is usually the case, over-stress in the deck due to participation in the action of the rib may be no more serious than over-stress due to secondary stress in steel trusses. It is certainly better in such a case to detail the structure so as to make the deck flexible, and thereby reduce the participation flexure, than to attempt blindly to analyze the stress in the deck and increase its size and stiffness to provide for indicated over-stress. Such a procedure may make matters worse unless the designer knows qualitatively what the effect of his revisions will be on the structure.

On the other hand, if the superstructure is counted on to provide flexural resistance, over-stress in the deck and columns may be serious.

In certain cases where the controlling relations are relatively clear, it is possible to foretell the action of the structure. The flitch-beam concept enables one to provide a proper division of flexural resistance between rib and deck by choice of their depths.

The writer agrees with Mr. Eremin that the structures in Fig. 1 are too short to be representative of arch bridges in general. However, the results obtained from analyses of these structures showed essentially the same characteristics as the results of analyses of nine-panel arches similar to those tested by Professor Wilson,[18] influence lines for one of which are shown in Fig. 4. With the shorter structures it was possible to study a greater number of variables. The results were not intended to be used quantitatively in any case. The results of a small number of experiments, whether analytical or laboratory tests, have little statistical value when the number of variables involved is large.

Reference to temperature and deformation strains, instead of stresses, was made purposely. A temperature change or a change in span requires a corresponding change in length of the elements of the structure, and causes a distortion of the arch rib. This distortion must take place, although it can be localized, or it can be distributed by the details of the structure. The stiffer the structure and the higher the modulus of elasticity of the material, the greater will be the stresses corresponding to the more or less fixed deformations. Referring to temperature strains instead of stresses, serves to keep this point in mind. Moreover, the temperature strain is particularly significant in any consideration of ultimate strength of the structure. When combined with high stresses due to loading, the additional strain due to temperature effects and other fixed deformations accounts for only a small increase in stress on account of the nature of the stress-strain curve for concrete.

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It does not seem necessary to enter into a detailed presentation of the implications of the flitch-beam concept regarding the effect of relative depth on relative stress of two members so connected as to have the same deflection curve. If the floor and the rib of an arch bridge are connected by closely spaced flexible columns, it is evident that they will have the same deflection curves, and consequently the same angular changes per unit length along the horizontal. Therefore they will have unit flexural strains, and consequently flexural stresses, roughly in proportion to their depths. If the columns are flexible but not very closely spaced, the relation is still approximately true for loads at the column points. When the columns are not flexible, there may be considerable local flexure of the more flexible member, whether it be rib or floor. Obviously, a simplified approximate relation has limitations. However, that does not make it useless.

The interaction stress in the floor is that due to the deflection of the arch rib. The writer's statement: "The stress due to interaction flexure in the girder is to the flexural stress in the rib approximately as the depth of the girder is to the depth of the rib," is an attempt to interpret in a simple manner the behavior of the structure. The less stiff the columns, and the shorter the panel length, the more nearly true will the approximation be. The rise ratio of the arch is not involved in the development of this relation.

The Landquart Bridge at Klosters, Switzerland, referred to by Mr. Eremin, was mentioned in the paper as an example of the structure consisting of a flexible rib with stiffening girder supported by flexible columns. Practically all the flexural resistance in this case is centered in the floor. The arch rib is polygonal in shape and carries the dead load thrust. The rib has actually participation flexural stresses approximately one-fourth the interaction flexural stress in the floor since the depth of the rib is about 10 in. and that of the floor is about 3 ft.

Mr. Eremin has stated some of the variables on which the interaction stresses depend. One might add other variables having to deal with: (1) The sources of stress (including the dead load—live load ratio, lateral forces, shrinkage, and temperature); (2) sizes of the members; (3) shape of the structure; (4) properties of the material, and variations in properties; and (5) conditions of support. It is evident that the problem is not a simple one. For this reason, the writer attempted to develop simplified "pictures" of the action of the structure rather than to indicate how an exact analysis might be made or to report statistical results of analyses or tests. Professor Cross has indicated how such "pictures" of structural action may be useful.

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ECONOMIC DIAMETER OF STEEL PENSTOCKS

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With Discussion by Messrs. R. A. Monroe, William E. Rudolph, Peter Bier, Adolpho Santos, Jr., Joseph D. Lewin, F. Knapp, Ralph W. Powell, H. K. Barrows, Robert W. Angus, and M. H. Fresen.

Synopsis

Formulas are developed in this paper for determining the economic diameter of steel penstocks for hydro-electric power plants and pumping plants based on a minimum total annual cost. The formulas developed differ from those published heretofore, in that:

(*a*) The Scobey formula for flow of water in riveted steel and analogous pipes is used;

(*b*) Factors are introduced to take care of entrance, bend, and other hydraulic losses which vary approximately as a ratio of the velocity head; and,

(*c*) A loss factor is introduced to take care of the effect of the load factor, and the shape of the load curve under which the power or pumping plant connected to the penstock operates.

The economic diameter of a penstock for a certain large power development as computed by formulas developed in this paper shows a difference of only 3.25% as compared to the diameter determined from detailed studies.

Definitions of Terms

Some writers consider the "pipe line" to be the pipe between the forebay and the surge tank, and the "penstock" to be the pipe between the surge tank and the turbines. Other writers use the term, "penstock", as applying to any pipe or pipes between intake or forebay at the dam and the turbines at the power-house of a water-power development, and "pipe line" as applying to a pipe used for other than water-power development, such as water supply.

The latter distinction between the terms, "pipe line" and "penstock", is implied herein, since the formulas are equally applicable to the pipe from forebay to surge tank, as from surge tank to turbines.

Note.—Published in November, 1936, *Proceedings*.

[1] Hydro-Elec. Engr., U. S. Bureau of Reclamation, Denver, Colo. Mr. Voetsch died on February 7, 1935.

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Notation.—The algebraic symbols used in this paper are defined where they are first introduced in the text, and are arranged for convenience of reference in the Appendix. Essentially, they conform to the American Tentative Standard Symbols for Hydraulics[3] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929.

ANNUAL COST OF POWER LOST IN FRICTION

In 1930, Fred C. Scobey, M. Am. Soc. C. E., introduced a formula[4] for head lost in friction, h_f, per 1 000 ft of steel pipe, as follows:

$$h_f = K_s \frac{V^{1.9}}{D^{1.1}} \quad \text{.........................(1)}$$

in which K_s is a general coefficient for head lost due to friction, in feet; V = the velocity of water in the penstock, in feet per second, corresponding to Q, the rate of discharge expressed in cubic feet per second; and D = the economic inside diameter of the penstock, in feet. Substituting for V its equivalent, $\frac{Q}{A} = \frac{Q}{0.7854\ D^2}$, and simplifying, Equation (1) becomes:

$$h_{f_1} = \frac{K_s\ Q^{1.9}}{631.93\ D^{4.9}} \quad \text{.........................(2)}$$

in which h_{f_1} = the head lost, in feet, due to friction in a penstock, 1 ft long; and A = the cross-sectional area.

The annual loss of power, in kilowatt-hours, at a load factor, F, per foot of pipe, is:

$$P_{f_1} = Q\ h_{f_1} \times \frac{62.5}{550} \times e \times 0.746 \times 8\,766 \times f \quad \text{..........(3)}$$

in which e = the over-all efficiency of the plant to the point at which power is sold (in the case of a power plant), or to the point at which power is purchased (in the case of a pumping plant); and f = a loss factor, expressed as a decimal, corresponding to the load factor, F, under which a plant operates (average load ÷ maximum load). Substituting for h_{f_1} its value as expressed by Equation (2), Equation (3) becomes:

$$P_{f_1} = \frac{1.17631\ K_s\ Q^{2.9}\ e\,f}{D^{4.9}} \quad \text{..................(4)}$$

The annual cost of the lost power, therefore, may be expressed as:

$$E_{f_1} = P_{f_1}\ b = \frac{1.17631\ K_s\ Q^{2.9}\ e\,f\,b}{D^{4.9}} \quad \text{..............(5)}$$

in which E_{f_1} = the annual cost of the lost power due to friction in a penstock, 1 ft long; and b = the value of the power lost by friction and other hydraulic factors expressed in dollars per kilowatt-hour.

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[3] A. S. A.—Z 10 *b*—1929.

[4] "The Flow of Water in Riveted Steel and Analogous Pipes", by Fred C. Scobey, *Technical Bulletin No. 150*, U. S. Dept. of Agriculture, January, 1930, Equation (9), p. 10.

ANNUAL COST OF A PENSTOCK

If a penstock is operating under a head, H, an expression may be written for the thickness of the steel[5], t, as follows:

$$t = \frac{0.434\ H\ D \times 12}{2\ s_g\ e_j} \qquad (6)$$

in which s_g = the gross allowable tension in the steel penstock, in pounds per square inch; and e_j = the efficiency of the joints of the penstock, expressed as a decimal. The weight of a section, 1 ft long, W_1, in pounds, is:

$$W_1 = \frac{w\ \pi\ D\ t\ (1 + i)}{12} \qquad (7)$$

in which w = the weight of 1 cu ft of steel (= 490 lb); and i = the percentage by which the steel in the penstock is over-weight, due to laps, cover-plates, rivets, welds, etc., expressed as a decimal. Substituting Equation (6) in Equation (7) (with w = 490), and simplifying:

$$W_1 = \frac{334\ H\ D^2\ (1 + i)}{s_g\ e_j} \qquad (8)$$

The annual cost, E_{p1}, of a 1-ft length of penstock is:

$$E_{p1} = W_1\ a\ r = \frac{334\ H\ D^2\ a\ r\ (1 + i)}{s_g\ e_j} \qquad (9)$$

in which a = the unit cost of steel in the penstock, in dollars per pound; and r = the ratio of the annual fixed, operating and maintenance charges to the construction cost of the penstock.

ECONOMIC DIAMETER ANALYSES

Friction Loss and Annual Cost of Penstock Considered.—The annual cost, E_{t1}, of a 1-ft section of penstock is the sum of Equations (5) and (9). To determine the economic diameter, write out this sum and equate the first derivative with respect to D, to zero; thus,

$$\frac{dE_{t1}}{dD} = 1.17631\ K_s\ Q^{2.9}\ e\ f\ b\ (-4.9)\ D^{-5.9} + \frac{668\ H\ D\ a\ r\ (1 + i)}{s_g\ e_j} = -\frac{5.76392\ K_s\ Q^{2.9}\ e\ f\ b}{D^{5.9}} + \frac{668\ H\ D\ a\ r\ (1 + i)}{s_g\ e_j} = 0 \qquad (10)$$

Solving Equation (10) for D, and simplifying:

$$D = 0.50218 \sqrt[6.9]{\frac{K_s\ Q^{2.9}\ e\ f\ b\ s_g\ e_j}{a\ H\ r\ (1 + i)}} \qquad (11)$$

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[5] "Water Power Engineering", by H. K. Barrows, M. Am. Soc. C. E., McGraw-Hill Book Co., 1927, p. 347.

Considering Losses Other Than Those Due to Friction.—Hydraulic losses, h_h, other than those due to friction, which vary approximately as a ratio of the velocity head corresponding to the velocity in the penstock, are defined by the relation:

$$h_h = K_T \frac{V^2}{2\,g} \dots\dots\dots\dots\dots\dots\dots\dots (12)$$

in which K_T = the sum of all constants for hydraulic losses, or,

$$K_T = k_b \sqrt{\frac{\Delta}{90}} + K_e + K_t \dots\dots\dots\dots\dots\dots (13)$$

in which K_e is a constant ruled by entrance losses; K_t is a constant to take care of all other hydraulic losses that vary as a ratio of the velocity head; k_b is a constant expressing the additional head lost due to the introduction of a bend in an otherwise straight penstock (usual approximate value = 0.25); $K_b = k_b \sqrt{\frac{\Delta}{90}}$; and, Δ = the angle of bend, in degrees.

When a bend is introduced in an otherwise straight penstock, the head lost, h_b, is expressed by:

$$h_b = k_b \sqrt{\frac{\Delta}{90}} \times \frac{V^2}{2\,g} = K_b \frac{V^2}{2\,g} \dots\dots\dots\dots (14)$$

If there is more than one bend in the pipe line under consideration, K_b will be the total of the values for all bends. The loss, h_e, at the entrance to a penstock is usually expressed as:

$$h_e = K_e \frac{V^2}{2\,g} \dots\dots\dots\dots\dots\dots\dots\dots (15)$$

Assuming a penstock of uniform diameter, with $V = \frac{Q}{A} = \frac{Q}{0.7854\,D^2}$, the head loss (Equation (12)) will be:

$$h_h = K_T \frac{V^2}{2\,g} = \frac{K_T\,Q^2}{2\,(0.7854)^2\,g\,D^4} \dots\dots\dots\dots (16)$$

Substituting Equation (16) in Equation (3), P_h for P_{f_1}, and simplifying, the annual power loss, in kilowatt-hours, at the load factor, F, due to hydraulic factors, will be approximately:

$$P_h = \frac{18.70719\ K_T\,Q^3\,e\,f}{D^4} \dots\dots\dots\dots\dots (17)$$

and, consequently, the annual cost of power loss, E_h, due to hydraulic factors other than friction (compare Equation (5)) will be:

$$E_h = P_h\,b = \frac{18.70719\ K_T\,Q^3\,e\,f\,b}{D^4} \dots\dots\dots\dots (18)$$

Considering Friction Loss, Other Hydraulic Losses, and Annual Cost of Penstock.—For this case it will be necessary to determine all hydraulic losses.

including those due to friction, and the annual cost, E_p, of the entire penstock of length, L, in feet. The annual cost of the friction loss per foot of penstock is expressed by Equation (5). Let the length of penstock used for computing friction loss be L_f, and the annual cost of friction loss in the entire line will be:

$$E_f = E_{f_1} L_f = \frac{1.17631 \, K_s \, Q^{2.9} \, e \, f \, b \, L_f}{D^{4.9}} \quad \ldots\ldots\ldots\ldots (19)$$

Likewise, the annual cost of steel per foot of penstock is expressed by Equation (9). Let the length of penstock used for computing total weight be L_p, and the annual cost of the steel in the line will be:

$$E_p = E_{p_1} L_p = \frac{334 \, H \, D^2 \, a \, r \, L_p \, (1 + i)}{s_g \, e_j} \quad \ldots\ldots\ldots\ldots (20)$$

The total annual cost, E_t, of the entire penstock will be the sum of Equations (18), (19), and (20), or:

$$E_t = E_f + E_h + E_p \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

To determine the economic diameter equate the derivative of the total annual cost, E_t, with respect to D, to zero; thus:

$$\frac{dE_t}{dD} = -\frac{5.76392 \, K_s \, Q^{2.9} \, e \, f \, b \, L_f}{D^{5.9}} - \frac{74.82876 \, K_T \, Q^3 \, e \, f \, b}{D^5}$$

$$+ \frac{668 \, H \, D \, a \, r \, L_p \, (1 + i)}{s_g \, e_j} = 0 \quad \ldots\ldots\ldots\ldots (22)$$

In Equation (22), let,

$$G = 5.76392 \; K_s \; Q^{2.9} \; L_f \ldots\ldots\ldots\ldots\ldots\ldots (23a)$$

$$M = 74.82876 \; K_T \; Q^3 \ldots\ldots\ldots\ldots\ldots\ldots (23b)$$

$$J = e \; f \; b \ldots\ldots\ldots\ldots\ldots\ldots (23c)$$

and,

$$B = \frac{668 \, H \, a \, r \, L_p \, (1 + i)}{s_g \, e_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23d)$$

Then, substituting and solving for B, Equation (22) becomes of the form:

$$B = \frac{G \, J}{D^x} + \frac{M \, J}{D^y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

in which x and y are different exponents of D. Because of this difference, Equation (24) cannot be solved directly; but since the difference between them is only 13%, it is possible to assume that $x = y$, so that $B = \dfrac{J \, (G + M)}{D^x}$;

or, approximately:

$$D = \sqrt[x]{\frac{J\,(G + M)}{B}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Substituting $x = 6.9$, Equation (25) can be written as:

$$D = \sqrt[6.9]{\frac{J\,(G + M)}{B}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

approximately, or, in terms of all the fundamental factors involved:

$$D = \sqrt[6.9]{\frac{e\,f\,b\,(5.76392\,K_s\,Q^{2.9}\,L_f + 74.82876\,K_T\,Q^3)\,s_g\,e_j}{668\,H\,a\,r\,L_p\,(1 + i)}} \quad \ldots\ldots (27)$$

and, for greater accuracy, Equation (24) (with $x = 6.9$ and $y = 6.0$) can be solved for D by a trial-and-error method.

Considering Thickness of Shell.—In some instances, the thickness of the shell or of the plates of a penstock will be given instead of the head. As before (see Equation (9)) the annual cost of a 1-ft length of penstock is $E_{p1} = W_1\ a\ r$. Substituting the value of W_1 from Equation (7) (with $w = 490$), and simplifying:

$$E_{p1} = 128.282\ t\ D\ a\ r\ (1 + i) \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The annual cost of a penstock of length, L_p, is:

$$E_p = E_{p1}\ L_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Substituting Equation (28) in Equation (29) and finding the derivative of E_p with respect to D:

$$\frac{dE_p}{dD} = 128.282\ t\ a\ r\ L_p\ (1 + i) = B' \ldots\ldots\ldots\ldots\ldots (30)$$

in which B' is a substitution factor. In Equation (22), substitute Equation (30) for the term, $\dfrac{668\ H\ D\ a\ r\ L_p\,(1 + i)}{s_g\ e_j}$, so that B in Equation (24) becomes $\dfrac{B'}{D}$, and,

$$B' = \frac{G\,J}{D^{x-1}} + \frac{M\,J}{D^{y-1}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

If $x = y = 6.9$ as before, and if the fundamental factors for J, G, M, and B' are substituted logically, Equation (31) may be solved for D in a manner similar to Equation (26):

$$D = \sqrt[6.9]{\frac{J\,(G + M)}{B'}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

approximately, or, in terms of all the fundamental factors involved:

$$D = \sqrt[6.9]{\frac{e\,f\,b\,(5.76392\,K_s\,Q^{2.9}\,L_f + 74.82876\,K_T\,Q^3)}{128.282\,t\,a\,r\,L_p\,(1 + i)}} \quad \ldots\ldots\ldots (33)$$

In order to apply Equation (32) to a practical design, compute the constants, J, G, and M, and determine the economical diameter for different values of t in B' (see Equation (30)). With t and D known the permissible head may be computed from Equation (6) and this value determines the economic diameter at each station on the line where the head corresponds to the values of t and D. As in the case of Equation (24), Equation (31) (with $x = 6.9$ and $y = 6.0$) can be solved for D with greater accuracy by trial and error.

ACCURACY OF EQUATIONS (26) AND (32)

Computations show that the errors in the values of D computed from Equations (26) and (32), for $M = 0.5\ G$ and $M = G$, will not exceed -0.528% and -0.973%, respectively, compared to the accurate solution of those equations. For $M = 0$ corresponding to $K_T = 0$ and with $L = 1$ ft, the equations as given are accurate.

RELATIVE VALUES OF ECONOMIC DIAMETER

Table 1 illustrates the effect of varying the values of the expressions under the radical signs of Equations (11), (26), and (32). It shows that a considerable change in the individual values of the constants in the equations is necessary in order to affect the diameter appreciably. A change in the discharge is much more noticeable. Hence, extreme accuracy is not essential in any particular study, except that the discharge should be estimated as closely as possible.

TABLE 1.—RELATIVE VALUES OF ECONOMIC DIAMETER

Relative value of entire expression under radical sign, Equations (11), (26), or (32) (1)	RELATIVE ECONOMIC DIAMETER, D		Relative value of entire expression under radical sign, Equations (11), (26), or (32) (1)	RELATIVE ECONOMIC DIAMETER, D	
	Computed by Equations (11) and (26) $x = 6.9$ (2)	Computed by Equation (32) $x = 5.9$ (3)		Computed by Equations (11) and (26) $x = 6.9$ (2)	Computed by Equation (32) $x = 5.9$ (3)
0	0	0	1.25	1.033	1.039
0.25	0.818	0.791	1.50	1.061	1.071
0.50	0.904	0.889	1.75	1.085	1.100
0.75	0.959	0.952	2.00	1.106	1.125
1.00	1.000	1.000	5.00	1.263	1.314

COMPARISON OF RESULTS

In 1934, detailed studies were made of a large power development to determine the economic diameter of the penstocks. Subsequently, formulas were developed as outlined herein, using the same general principles as in the detailed studies. For convenience of comparison, the methods used in determining the economic diameter for this development are designated as A, B, and C. The same basic data were used in all three methods.

In Method A the annual cost of friction losses and of the penstock steel was considered. The annual cost of the entire length of penstock, and bend and entrance losses, in addition to the friction losses, was considered in

Method *B*; and, in Method *C*, a detailed study was made of the economic diameter of the penstock. Included in the latter were the annual costs of the penstock, friction, bend, and entrance losses as in Method *B* and, in addition, annual costs due to rack, valve, and cone losses. Water-hammer heads were computed separately for each assumed size of penstock. The economic size of the penstock, based on the minimum annual cost read from the Diameter-Total Annual Cost Curve, was 19.67 ft. The results of the computations by the three methods are summarized in Table 2.

TABLE 2.—COMPARISON OF METHODS TO DETERMINE ECONOMIC DIAMETER OF PENSTOCK

Method (1)	Equation used (2)	Diameter, D, in feet (3)	PERCENTAGE ERROR COMPARED WITH METHOD C: Diameter (4)	PERCENTAGE ERROR COMPARED WITH METHOD C: Total annual cost (5)
A	(11)	18.29	−7.02	+2.22
B	(26)	19.03	−3.25	+0.65
C	Detailed study	19.67	0	0

For this particular development, the bend losses formed a large proportion of the total losses. Hence, since bend losses are neglected entirely in Method *A*, it can be readily seen that the value of D, computed by Method *B*, should be larger than when computed by Method *A*. In the same way, Method *C* includes other losses in addition to those of Method *B*, and hence D, by Method *C*, is larger than by Method *B*. Another factor that must be considered is that Equation (26), for Method *B*, is approximate, and a more accurate solution would give a slightly higher value for D. Column (5), Table 2, shows the percentage errors in total annual cost of Methods *A* and *B* compared to Method *C*. The values of total annual cost, used to determine the percentage errors, were read from the Diameter-Total Annual Cost Curve for diameters corresponding to the three methods.

Table 2 shows that consideration of additional hydraulic losses and the resulting cost of lost power justifies additional capital expenditure for a larger penstock.

Use of Formulas.—The data for the illustrative example are, as follows: $Q = 400$ cu ft per sec; $e = 0.75$; $K_s = 0.401$; $K_e = 0.05$; $k_b = 0.25$; $\Delta = 25^\circ.9$; $K_T = K_e + K_b = 0.05 + 0.1342 = 0.1842$; $L_p = 100$ ft; $L_f = 130$ ft; $H = 130$ ft; $a = \$0.10$ per lb; $r = 0.17$; $s_g = 12\,000$ lb per sq in.; $e_j = 0.90$; $i = 0.20$; $f = 0.25$; and $b = \$0.007$ per kw-hr.

The entire length of the penstock, and the bend and entrance losses, in addition to the friction losses, will be considered in the example. Equation (26) is used to determine the economic diameter, D; but, first, it is necessary to solve for the several constants.

Substituting the foregoing values in Equations (23), and simplifying, $J = 13.14 \times 10^{-4}$; $G = 1\,058 \times 10^7$; $M = 88.3 \times 10^7$; and, $B = 16.4$. Then, from Equation (26), $D = 7.315$ ft; say, 7 ft 4 in.

Choice of Formula for Friction Loss

The Scobey formula (Equation (1)) was selected because it was the most authoritative and the newest available. Mr. Scobey deduced it after an extensive series of tests and a complete review and analysis of previous investigations. Among other factors, he allowed for the kinematic viscosity of water at 15° C (59° F), and for Reynolds' number. He also recommended proper coefficients for use in his formula, based on various classes of pipes and joints and ages of pipe, the coefficients being constant throughout the usual ranges of pipe diameters and water velocities.

Other Formulas for Economic Diameter of Steel Penstocks

Other formulas have been introduced by M. L. Enger[6], and H. K. Barrows[7], Members, Am. Soc. C. E., G. E. Lyon[8], Assoc. M. Am. Soc. C. E., and by Messrs. R. L. Daugherty[9], R. M. Peabody[9], B. F. Jakobsen[10], and W. F. Durand[11].

The Enger formula, as modified somewhat by W. P. Creager and J. D. Justin[12], Members, Am. Soc. C. E., is quite similar to Equation (33) except that they used the Chezy formula for friction loss. Professor Barrows gives a formula similar to Equation (11) except that he used the Weisbach formula for friction and assumed a value of efficiency, e, equal to 1.0. H. L. Doolittle, M. Am. Soc. C. E., outlines a method for the economic design of steel penstocks[13] and in their discussion[9] of that paper, Messrs. Daugherty and Peabody develop analytical solutions for the economic diameter.

Of the foregoing writers, none includes hydraulic losses other than those due to friction, and only Messrs. Creager and Justin attempt to give any specific data on the variation in yearly losses with the annual load factor.

Economic Velocity

Table 3 summarizes computations made to determine the most economic maximum velocity, V, for maximum penstock discharges, Q, of 400, 2 000 and 5 000 cu ft per sec, and heads for shell thickness, H, of 100 and 400 ft for an assumed set of conditions. The assumptions made for the computations are as follows: $K_s = 0.400$; $e = 0.85$; $f = 0.20$ corresponding approximately to a load factor, F, of about 50%; $b = \$0.003$ per kw-hr; $s_g = 15\,000$ lb per sq in.; $e_j = 0.80$; $a = \$0.10$ per lb; $r = 0.07$; $i = 0.10$; and $t =$ minimum shell thickness $= \frac{5}{16}$ in. A 1-ft length of penstock was assumed. Frictional losses only were considered. Except for Item No. 1, Table 3, required shell thicknesses as computed from Equation (6) were in excess of the $\frac{5}{16}$-in. minimum.

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[6] "Economic Design of Penstocks", by M. L. Enger, *Engineering Record*, September 12, 1914, p. 300; also, Editorial in same issue indicating application to concrete pipes; also, inquiry by Mr. C. R. Steiner about article in *Engineering Record*, October 31, 1914, p. 495, and reply by Dean Enger in the same issue, pp. 495–496.

[7] "Water Power Engineering", by H. K. Barrows, 1927, p. 344.

[8] "Economic Size of Pipe for Power Purposes", by G. E. Lyon, *Engineering and Contracting*, May 10, 1922, pp. 436–437.

[9] *Transactions*, A. S. M. E., Vol. 46, 1924 (Paper No. 1946), pp. 1178–1203.

[10] "Economic Pressure Pipe on Penstock Pipe Design", by B. F. Jakobsen, *Engineering and Contracting*, December 11, 1918, pp. 554–556.

[11] "Hydraulics of Pipe Lines", by W. F. Durand, D. Van Nostrand Co., 1921.

[12] Hydro-Electric Handbook, by W. P. Creager and J. D. Justin, John Wiley & Sons, 1927, p. 414.

[13] "A Method for the Economic Design of Penstocks", by H. L. Doolittle, *Transactions*, A. S. M. E., Vol. 46, 1924 (Paper No. 1946), pp. 1165–1178.

TABLE 3.—SUMMARY OF COMPUTATIONS FOR ECONOMIC VELOCITY

Item No. (1)	Maximum discharge, Q, in cubic feet per second (2)	Equation used to compute D (3)	Minimum shell thickness, t, in inches (4)	Economic diameter, D, in feet (5)	Economic velocity, V, in feet per second (6)	Item No. (1)	Maximum discharge, Q, in cubic feet per second (2)	Equation used to compute D (3)	Minimum shell thickness, t, in inches (4)	Economic diameter, D, in feet (5)	Economic velocity, V, in feet per second (6)
H = 100 FEET FOR SHELL THICKNESS						H = 400 FEET FOR SHELL THICKNESS					
1	400	(33)	$\frac{5}{16}$	7.40	9.31	4	400	(11)	Not required	6.03	14.03
2	2 000	(11)	Not required	14.49	12.13	5	2 000	(11)	Not required	11.85	18.13
3	5 000	(11)	Not required	21.30	14.04	6	5 000	(11)	Not required	17.42	20.98

Table 3 shows that V increases with Q and H. Inspection of the velocities in Column (6) indicate that they are fairly representative of existing practice. Economic velocities for conditions other than those assumed may be found in a similar manner, or proportioned from the results given in the table (see Equations (11) and (33)).

ANNUAL POWER LOSS, LOAD FACTOR, AND LOSS FACTOR

The average annual power loss is designated herein as P_a, and the maximum annual power loss for load factor, $F = 1.0$, is designated as P_m. Corresponding discharges are Q_a and $Q_m = Q$. Since discharge is assumed as proportional to power generated,

$$F = \frac{Q_a}{Q_m} = \frac{Q_a}{Q} \quad \dots\dots\dots\dots\dots\dots\dots\dots (34)$$

For a penstock of a given diameter, the head lost varies as $V^{1.9}$ and $Q^{1.9}$. The power lost varies as the product of head lost times discharge. Hence, the power lost varies as $Q^{2.9}$ for friction; and as Q^3 for bend, entrance, and other hydraulic losses where h_h varies as V^2 and Q^2. The equivalent annual discharge which will give a power loss, P_a, corresponding to the load factor, F, and average discharge, Q_a, is:

$$Q_e = \sqrt[2.9]{\frac{q^{2.9}{}_1 + q^{2.9}{}_2 + q^{2.9}{}_3 + \dots q^{2.9}{}_n}{N_1 + N_2 + N_3 + \dots N_n}} \quad \dots\dots\dots\dots\dots (35)$$

in which q_1, q_2, etc., are the discharges of the penstock, in cubic feet per second, for N_1, N_2, etc., hr, for the given period and load curve. For simplicity the exponent and root in Equations (35) and (36) may also be taken as 3 instead of 2.9 if desired. The relations between power losses, discharges, and loss factor is expressed by:

$$f = \frac{P_a}{P_m} = \frac{j\,Q^{2.9}{}_e}{j\,Q^{2.9}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (36)$$

in which j is a constant. For a 1-ft length of penstock, a given diameter, D, and $f = 1.0$ for $F = 1.0$, the value of the constant, j, in Equation (36) is determined from Equation (4) to be:

$$j = 1.17631\ K_s\, e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(37)$$

Messrs. Creager and Justin present data[14] and formulas which, reduced to the symbols of this paper, are as follows:

$$Q^3_a\, C_r = Q^3_e = Q^3_f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(38)$$

Their coefficient, C_r, varies with the load factor, F, and is given in their Fig. 43, which is used in their formula for the economic diameter of a steel penstock.[15] Assuming that $F = \frac{Q_a}{Q}$, a loss factor was computed and plotted as Curve D, in Fig. 1, from the relation:

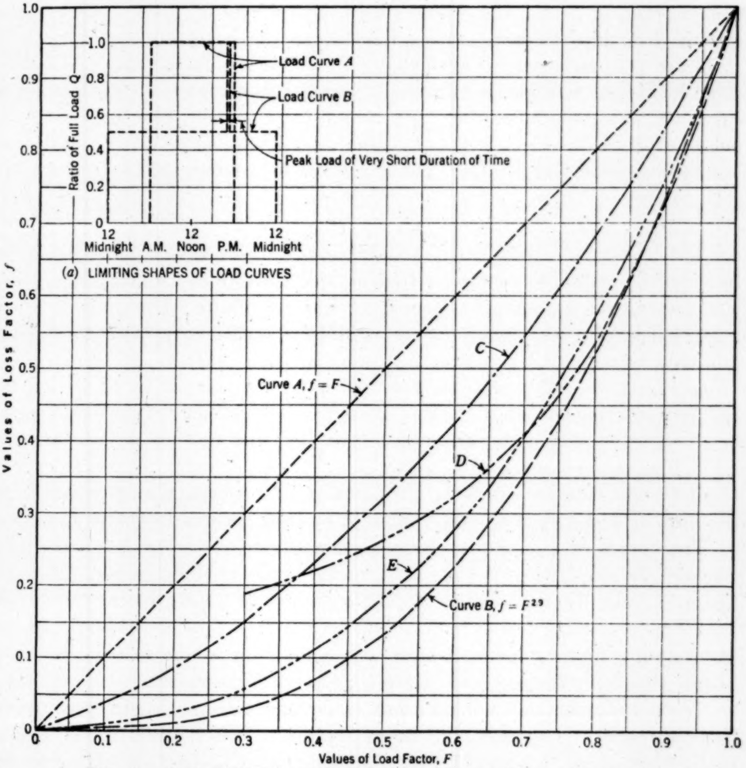

FIG. 1.—LOAD FACTOR-LOSS FACTOR CURVES FOR WATER CONDUITS

$$f = F^3\, C_r \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39)$$

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[14] Hydro-Electric Handbook, pp. 82 and 83, Equation 19, and Fig. 43.

[15] *Loc. cit.*, p. 414.

Loss Factor Data and Curves

Fig. 1 establishes the limiting loss factor curves, A and B, for steel penstocks corresponding to the given limiting shapes of Load Curves A and B in Fig. 1(a). Load Curve A is of rectangular shape and of constant peak load, whereas, Load Curve B is of rectangular shape with a peak load of uniform and very short duration, extending above the rectangular part. The equations of Curves A and B are, respectively, $f = F$ and $f = F^{2.9}$. Messrs F. H. Buller and C. A. Woodrow established similar limiting loss factor curves for transmission lines.[16] Curve C is taken from their average experience loss factor curve for transmission lines, the basic plotted points for which were computed from a series of actual and representative load curves of public utility plants serving diversified loads.

Curve D is an attempt to establish an average loss factor curve from data given by Messrs. Creager and Justin.[14] Curve E is a representative loss factor curve for water conduits, such as penstocks and tunnels, carrying water for public utility hydro-electric power plants serving typical diversified loads. It has been computed as follows: For any given load factor, F:

$$f_E = f^{1.5}{}_C \dots\dots\dots\dots\dots\dots\dots\dots (40)$$

in which f_C = the loss factor for transmission lines from Curve C for a given load factor, and f_E = the loss factor for water conduits. For cases in which a fairly accurate load curve can be established, the longer method of determining a value of Q_e should be used (see Equation (35)).

In Fig. 1, the value of Q for full-load water use is assumed to be that for full gate-opening, or for the rated horse-power of a turbine under its rated head. The friction head varies as $V^{1.9}$ or $Q^{1.9}$, and the power loss varies as $Q^{2.9}$. The power generated is assumed to be proportional to the water use. Until further data are available, the value of f for a typical load curve can be assumed to lie between Curves C and E, probably nearer to Curve E.

The curves of Fig. 1 may be used to estimate values of f for all water conduits in which the friction head varies approximately as V^2. The load factor, F, may be on a daily, weekly, monthly, or yearly basis. Mr. J. G. Tarboux uses a loss factor in formulas he develops for the economic size of transmission line conductors[17], in a manner similar to the method described in this paper, for the economic diameter of a steel penstock.

Table 4 summarizes computations made to check the accuracy of Equation (40) and of Curve E, Fig. 1, as applied to water conduits. A typical load curve[18] of the Alabama Power Company's system (mainly hydro-electric generation) was used as a basis. Its load factor was 0.795. Load curves for load factors of 0.321 and 0.550 were also drawn with shapes similar to that for a load factor of 0.795, except for the peak load. The loss factors for transmission lines in Column (3), Table 4, were computed on the assumption that the power loss varies as the square of the hourly ordinates (or load) of the load curve. The loss factors for water conduits in Column (5) were computed on the assumption that the power loss varies as the cube of the hourly ordinates (or load). Inspection of Table 4 shows that Equation (40) and Curve *E*, Fig. 1, are applicable in all ordinary cases.

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[16] "Load Factor—Equivalent Hour Values Compared", by F. H. Buller and C. A. Woodrow, *Electrical World*, July 14, 1928, p. 59; also, *Electrical World*, August 18, 1928, p. 320.

[17] "Most Economical Conductor", by J. G. Tarboux, *Electrical World*, March 23, 1929.

[18] "Determining Transmission Line Losses" by S. Murray Jones and Joel Tompkins, *Electrical World*, November 17, 1928, p. 993.

TABLE 4.—COMPARISON OF LOSS FACTORS FOR LOAD CURVES

Item No.	Load factor, *F*	Loss Factor *f*			
		Transmission Lines		Water Conduits	
		Computed from load curve	Read from Fig. 1, Curve *C*	Computed from load curve	Read from Fig. 1, Curve *E*
(1)	(2)	(3)	(4)	(5)	(6)
1	0.321	0.144	0.166	0.082	0.067
2	0.550	0.337	0.373	0.225	0.228
3	0.795	0.655	0.677	0.561	0.563

ECONOMIC DIAMETER OF PENSTOCK FROM TOTAL ANNUAL COST CURVE

When it is desired to include costs other than those of, and in addition to, the frictional and hydraulic losses, and penstock steel, as given in the formulas heretofore developed, the procedure will be to assume several different penstock diameters and determine the annual cost for each diameter, including all the factors and elements of cost that vary with the diameter. These additional costs which are affected by the penstock diameter include: (*a*) Intake cost; (*b*) penstock valve costs; (*c*) annual saving in cost of concrete in the dam by using a larger penstock diameter; (*d*) cost of required generator flywheel effect for machine of desired rating, if more than that usually provided in a generator of normal design; (*e*) cost of surge tank, if any; (*f*) governor cost; and (*g*) cost of penstocks supports, etc. Referring to Item (*d*), the flywheel effect, is usually called $W R^2$, which equals the product of the rotating weight of generator rotor, in pounds, and the square of the radius of gyration, in feet.

The totals of the annual costs for all items for each assumed diameter are plotted as ordinates, with penstock diameters as abscissas. The minimum annual cost is found from the Diameter-Total Annual Cost Curve. For a study of this type, Equations (18), (19) and (20) will be found convenient to determine the annual costs of the frictional and hydraulic losses, and penstock steel, respectively.

ECONOMIC DIAMETER OF PENSTOCKS FOR PUMPING PLANTS

In general, the formulas developed in this paper for the economic diameter of steel penstocks for hydro-electric power plants apply to electrically operated pumping plants. In such a case, the annual cost of the power consumed in friction and other hydraulic losses, varies inversely as the over-all efficiency of the plant, *e*, since for a lower efficiency more power must be used to pump a given quantity of water against a given total head. On the other hand for a power plant, the annual cost of the power lost in friction and other hydraulic losses varies directly as the over-all efficiency, *e*, as shown by Equation (5). In Equation (11) for the economic diameter, *e* appears in the numerator of the expression under the radical sign, and in Equations (26) and (32), *e* as one factor in *J* appears in the numerator of the expression under the radical sign.

It follows, therefore, that in using the foregoing formulas to determine the economic diameter of penstock for a pumping plant, the efficiency, *e*, should be placed in the denominator of Equations (11), (26), and (32). In these formulas it is assumed that the head consumed in friction and other hydraulic losses forms only a small part of the static pumping head, and, therefore, that the size of the pumping motors and plant remains constant for a reasonable variation in the size of the penstock.

For a pumping plant with a very long penstock, and relatively low static head, it would be better to use the method outlined under the heading "Economic Diameter of Penstock from Total Annual Cost Curve", and include the annual cost of the pumping plant itself, and other variables, for each assumed diameter or plate thickness.

CONCLUSIONS

The formulas developed in this paper are general, but they are intended to apply particularly to the determination of the economic size of a penstock of uniform diameter for a hydro-electric power plant, particularly where the power-house is situated immediately down stream from the penstock intake section of the dam. Equation (11) or Equation (26) is to be used, depending on the accuracy desired, or the amount of data available.

For a long penstock, the diameter generally decreases toward the lower end, and the penstock is usually made in sections of uniform diameter and the same plate thickness, with transitions connecting the sections of different diameters. Equation (32) applies to this case.

ACKNOWLEDGMENTS

The subject of, and formulas developed in, this paper were suggested by studies made by the writers for a large power development. The senior writer, the late Mr. Charles Voetsch, passed away before he could review the paper entirely. The junior writer wishes to express herein his appreciation of the fine and unfailing spirit of co-operation and friendliness with which Mr. Voetsch drew on his wealth of experience and his valuable engineering library to assist or furnish data to all who desired aid in special problems. I. A. Winter, Assoc. M. Am. Soc. C. E., Senior Engineer, U. S. Bureau of Reclamation, made several valuable suggestions which have been incorporated in the paper. Power plant studies and designs of the U. S. Bureau of Reclamation, including those upon which this paper is based, are under the direction of L. N. McClellan, Chief Electrical Engineer. All studies and designs are under the general direction of J. L. Savage, M. Am.

Soc. C. E., Chief Designing Engineer; all engineering and construction work is under the direction of R. F. Walter, M. Am. Soc. C. E., Chief Engineer, with headquarters at Denver, Colo.; and all activities of the Bureau at the time this paper was prepared were under the general charge of the late Elwood Mead, M. Am. Soc. C. E., Commissioner of Reclamation. John C. Page, M. Am. Soc. C. E., Commissioner of Reclamation, with headquarters in Washington, D. C., is in general charge of Bureau activities.

APPENDIX

NOTATION

The symbols introduced in the paper are defined as follows:

a = the cost of steel in the penstock, in dollars per pound.

A = the cross-sectional area in a penstock, in square feet.

b = the value of power loss because of friction and other hydraulic factors, in dollars per kilowatt-hour.

B = a constant in Equation (26), defined by Equation (23d); B' = a constant in Equation (32), defined by Equation (30).

C_r = the coefficient used by Messrs. Creager and Justin, defined by Equation (38).

D = the economic inside diameter of a penstock, in feet.

e = the over-all efficiency of a plant to: (1) The point where power is sold, for a power plant; and (2) the point where power is purchased, for a pumping plant; e_j = the efficiency of the joints of a penstock, expressed as a decimal.

E = the annual cost (expense) of a penstock of length, L, in dollars; E_f = the annual cost of the power lost because of friction, in a penstock of length, L_f; E_{f_1} = the annual cost of the power lost because of friction in a penstock, 1 ft long; E_h = the annual cost of the power lost because of hydraulic factors other than friction (such as entrance losses, bend losses, etc.); E_p = the annual cost of the steel in a penstock of length, L_p; E_{p_1} = the annual cost of the steel in a penstock, 1 ft long; E_t = the total annual cost of a penstock of length, L; E_{t_1} = the total annual cost of a penstock, 1 ft long.

f = a loss factor, expressed as a decimal, corresponding to the load factor, F, and defined by Equations (36) and (38).

F = the load factor under which the plant operates (= average load ÷ maximum load); for a discharge proportional to the power generated, $F = Q_a \div Q$.

g = the acceleration due to gravity (= 32.16).

G = an algebraic substitution factor in Equations (26) and (32), as defined by Equation (23a).

h = loss of head, in feet; h_f = the head lost in friction, in a penstock of length, L_f; h_{f_1} = the head lost in friction, in a penstock 1 ft long; h_h = the head lost because of hydraulic factors other than friction; h_t = the total head lost because of friction and hydraulic factors; h_b = the additional head lost, due to the introduction of a bend in an otherwise straight penstock; h_e = the head lost at the entrance to a penstock or intake.

H = the average head, on the penstock, including water-hammer effect, in feet (weighted average to be used if necessary).

i = percentage of overweight of steel in a penstock, due to laps, cover-plates, rivets, welds, etc., expressed as a decimal.

j = a constant in Equation (36) defined by Equation (37).

J = an algebraic substitution factor in Equations (26) and (32), as defined by Equation (23c).

k_b = a constant in Equation (14) defined by $k_b = K_b \div \sqrt{\frac{\Delta}{90}}$; usual approximate value = 0.25.

K = a coefficient; K_b = a constant that modifies the expression for head loss when bends are introduced in an otherwise straight pipe (see Equation (14)); K_e = a constant that modifies the expression for head loss at the entrance to a penstock (see Equation (15)); K_s = a general coefficient in the Scobey formula (Equation (1)) which determines the head lost in friction; K_t = a constant in Equation (13) to account for hydraulic effects other than bend losses and entrance losses; K_T = the sum of all constants for hydraulic losses, as defined by Equation (13).

L = length, in feet: L_f = length of penstock used to compute friction loss when it differs from L_p; L_p = length of penstock used to compute the weight of steel when it differs from L_f;

M = an algebraic substitution factor in Equations (26) and (32), as defined by Equation (23b).

N = time, in hours, for a given period and load curve, during which the penstock discharge is q (q_1, q_2, etc., correspond with N_1, N_2, etc.).

P = annual power lost, in kilowatt-hours: P_a = average loss due to friction and hydraulic factors, at the load factor, F; P_f = loss due to friction in a penstock of length, L_f; P_{f_1} = loss due to friction in a penstock 1 ft long; P_h = loss due to hydraulic factors other than friction; P_m = loss due to friction and other hydraulic factors, at full or maximum load, corresponding to the discharge, Q, and the load factor, $F = 1.0$.

q = discharge of a penstock, in cubic feet per second, during a stated time, N, and for a given period and load curve; q_1, q_2, etc., correspond to N_1, N_2, etc.

Q = the rated discharge of a penstock, in cubic feet per second, corresponding to full gate-opening, or the rated horse-power of the connected turbine or turbines (or as limited by the generator capacity), when operating under rated or normal head (in this paper, penstock discharge is assumed to be proportional to the power generated, and Q is assumed equal to Q_m); Q_a = average discharge corresponding to the average power generated during the period and for the load curve considered; Q_e = the equivalent discharge, for the period and load curve considered, to give a power loss for the period equivalent to the actual power loss for the period and the load curve considered (see Equation (35)); Q_m = the maximum discharge for the load factor, $F = 1.0$ (Q_m is assumed equal to Q).

r = the ratio of the annual fixed, operating, and maintenance charges to the construction cost of the penstock.

s = unit stress, in pounds per square inch; s_g = the gross allowable tension in the penstock steel.
t = the thickness of the steel in the penstock, in inches.
V = the velocity of the water in the penstock, in feet per second, corresponding to the discharge, Q.
w = weight of steel, in pounds per cubic foot (= 490).
W = weight of steel, in pounds, in a section of a penstock of length, L; W_1 = weight of steel in a 1-ft section of a penstock.
x = a variable exponent of diameter, D (see Equations (24), (25), and (31)).
y = a variable exponent of diameter, D (see Equations (24) and (31)).
Δ = the angle of bend in a penstock, in degrees.

DISCUSSION

R. A. MONROE,[19] M. AM. SOC. C. E. (by letter).—As noted by the authors, several formulas for computing the economic diameter of steel penstocks have previously been presented. All these formulas are based on the same fundamental principle, namely, that the most economic size of penstock is that for which its annual cost, plus the value of the power lost in friction, is a minimum. For any given set of conditions all will give practically identical results.

The authors have used the Scobey formula for flow in steel pipe lines for the stated reason that: "It is the most authoritative and the newest available." However, the use of the Weisbach formula,

$$Q = \frac{f\,L\,V^2}{2\,g\,D} \quad \text{.......................} \quad (41)$$

(in which f is a friction factor) leads to a considerably simpler mathematical expression for the economic diameter with no appreciable sacrifice in accuracy, providing the factor, f, is properly chosen for the range of penstock diameters being considered. This simplification is particularly desirable when attempting to develop an expression which will include the hydraulic losses, h_h, other than those of friction, all of which are assumed to vary as the square of the velocity head.

The introduction of the load-factor loss curves to enable a quick approximation of the flow for which the economic diameter should be determined is a novel idea. It should be noted, however, that a hydro-electric plant which is tied into a large interconnecting system is seldom operated to produce an output that is proportional to the system load. When ample water is available the hydro-electric plant is usually operated at a high load factor at or near its point of best efficiency for most of the day, dropping off for only a short period when the system load is at a minimum; the peak load of the system is carried by steam plants operating on a very low load factor. During low-flow periods, this method of operation is reversed, particularly where facilities exist for forebay pondage, the hydro-electric plants being operated on the system peak whereas the base load is carried by steam.

In a hydro-electric plant having several units a reduction in output is usually effected by shutting down some of the units and continuing operation of the units remaining in service at the point of best efficiency. In this case the penstock loss is not substantially reduced providing the units are served by separate penstocks. A further point to consider is that peak power is always worth considerably more per kilowatt-hour than the average output of the system, as is indicated by the demand charge which is usually embodied in electric-rate schedules. Consideration of these facts will lead to the selection of a somewhat larger flow on which to base an economic design than would be chosen if the criterion selected was that flow for which the total friction loss is equal to the friction of the actual variable flow. The engineer can usually determine the proper basis of design for any particular case, but as a general proposition it appears that the economic design of a penstock should be based on a flow approaching that which will be utilized by the turbines at the point of best efficiency.

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[19] Head Water Control Engr., TVA, Knoxville, Tenn.

The value of power which should be used in the formula for economic penstock design cannot be determined readily but its exact value is not important since results are affected only in proportion to the seventh root of this factor. The same consideration applies to the ratio of fixed charges to total cost.

An easily applied formula for determining the economic diameter of a steel penstock is very desirable and convenient, particularly for preliminary studies. For the final design of a pipe line in which the various factors become definitely established, it seems quite desirable to make a detailed calculation of the cost and of the hydraulic losses for at least three sizes of penstock. Such a computation will permit a curve to be drawn showing the variation in cost of output in terms of diameter of penstock, which will afford a definite basis for the selection of the proper size. It also makes it possible to take into account many factors which do not lend themselves to formalization. It may perhaps be noted that since the cost curve for varying diameters is usually comparatively flat, the very real advantage of keeping initial costs to a minimum will often cause the selection of a pipe diameter considerably smaller than that shown theoretically as the most economical on the basis of estimates of the future value of power.

WILLIAM E. RUDOLPH,[20] M. AM. SOC. C. E. (by letter).—The application of one of the equations in this paper can be demonstrated with reference to a problem that often arises in penstock design—that of the single pipe line as compared to multiple pipe lines. Especially is this important in hydro-electric development in South America, where it is quite common, first, to install machines for utilizing only a portion of the potential power at a site, and, later, to install other machines to reach the full capacity of the normal water flow. Where there is to be a considerable lapse of years between the first and the later construction stages of a hydro-electric project, the penstock is built of sufficient capacity for the initially installed units only. However, there are instances in which the second stage of development follows within a few years of the first, as, for example, in the development of a mine where the tonnage of ore to be treated increases rapidly. In such cases it is sometimes advantageous to build the penstock for the final project during the first stage of the development, because of the saving in cost of a single pipe line as compared to that of multiple pipe lines.

The writer has been trying to apply the formulas of this paper to a particular problem under consideration. He began by determining the economic diameter of penstock for the amount of power to be installed at once, and then for a single pipe line for the ultimate capacity (which in this case was exactly double the initial installation). The difference in ultimate capital investment between a single large pipe line and (in this case) two smaller pipe lines, was thus estimated. Such definite saving *n* yr hence, was converted back to its present value, ϕ, using the rate of interest applicable. Then, for the pipe line of the capacity of the present installation only, the annual cost of the penstock is defined by Equation (21). For the pipe line of a capacity of present, plus future, installation, the annual cost of the penstock (accurately enough for practical purposes), is:

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[20] Shoreham, N. Y.

$$E_t = E_f + E_h + E_p - \frac{\phi}{n} \quad\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

The smaller annual cost determines the more advantageous installation. In the case under consideration, n being only 2.5 yr, the greater capital expenditure for the full-sized penstock, with the initial installation, appeared to be justified.

PETER BIER,[21] ESQ. (by letter).—Within certain limitations, the economic diameter of steel penstocks and pump discharge lines can be determined rationally by means of this interesting paper. Equation (11) has been derived by using Equation (6) for the calculation of the plate thickness corresponding to a given head. For low-head developments where the plate thickness is not a function of the head, but an arbitrary minimum established to satisfy the requirements of rigidity, fabrication standards, or long life, Equation (11) will not give the proper answer.

An example of a welded-steel discharge pipe for a pumping plant will illustrate this point. Let $Q = 150$; $K_s = 0.36$; $e = 0.71$; $F = 0.425$; $f = 0.42$; $b = \$0.0026$; $s_g = 13\,500$ lb per sq in.; $e_j = 0.90$; $a = \$0.09$; $H = 150$ ft; $r = 0.07$; $i = 0.05$; $L = 1\,920$ ft; and, $t = 0.25$ in. (minimum).

From Equation (11) modified for pump lines:

$$D = 0.50218 \sqrt[6.9]{\frac{0.36 \times 2\,044\,800 \times 0.42 \times 0.0026 \times 13\,500 \times 0.9}{0.09 \times 150 \times 0.07 \times 1.05 \times 0.71}} = 5.44 \text{ ft}$$

With the same data a detailed analysis, based on the total annual cost per foot of pipe, including annual fixed, operating, and maintenance charges, gives the following values, for the various diameters considered:

Diameter of pipe, in feet	Total annual cost, in dollars per foot of pipe
4.75	1.65
5.25	1.50
5.75	1.47
6.25	1.49
6.75	1.56

According to the foregoing data, the economic diameter would be 5.75 ft as compared to 5.44 ft obtained from Equation (11). The discrepancy is due to the fact that in the detailed analysis the diameter derived was not a function of the head and the allowable unit stress as in Equation (11), but resulted from using a minimum plate thickness of 0.25 in. for practical reasons.

In the foregoing example, Equation (6) would yield different plate thicknesses for the varying heads along the profile of the pipe line, with a maximum value of $\frac{3}{16}$ in., which, however, falls short of the practical requirements. Since the values included in Equation (6) enter into Equation (11), this cannot be used for pipes under low heads, where values of t will fall below the minimum plate thicknesses established for reasons other than the requirements of head or the allowable tension in the plate.

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[21] Engr., U. S. Bureau of Reclamation, Denver, Colo. Approved for publication by the U. S. Bureau of Reclamation.

For low-head plants the economic diameter should be determined from Equation (33) by substituting the desired plate thickness into the equation. For the pumping line used in the foregoing example where only pipe friction losses are considered, Equation (33) may be written as follows:

$$D = \sqrt[5.9]{\frac{0.42 \times 0.0026 \times 5.76392 \times 0.36 \times 150^{2.9}}{0.71 \times 128.282 \times 0.25 \times 0.09 \times 0.07 \times 1.05}} = 5.76 \text{ ft}$$

This closely checks the 5.75-ft diameter obtained from the detailed analysis.

Theoretically, the 0.25-in. plate thickness substituted in Equation (6) would give, for $D = 5.75$ ft and $s_g = 13\,500$:

$$H = \frac{0.25 \times 2 \times 13\,500 \times 0.9}{0.4335 \times 5.75 \times 12} = 203.4 \text{ ft}$$

which is the head to which the 0.25-in. shell may ordinarily be subjected if no other considerations enter into the problem.

For pipe lines supported on piers or columns with long spans, a considerable error may be introduced in these equations by using the value, i, which consists of additions to the weight of pipe due to laps, butt straps, stiffener rings, etc. Values of i may vary from 0.05 to 0.25, depending on the span or type of support used. The trend in modern steel-pipe design is to use long spans between supports. This necessitates not only heavy stiffener rings and supports, but also a pipe shell of increasing thickness toward the point of support to reduce the stresses in the shell adjacent to the supporting rings. In such cases it is difficult to formulate the plate thickness or unit cost of the pipe for introduction into the equations unless some preliminary computations or designs are first made for a typical span, and the average plate thickness or cost determined therefrom.

The method presented by the authors should be more economical to use than the detailed analysis after one becomes familiar with its application. The lengthy equations, however, will greatly increase the chance of error. On important and costly installations which justify a more extended analysis it may be advisable to determine the economic diameter first with the authors' method and then check the result with a detailed analysis. By thus limiting the necessary computations to a smaller number of diameters in the detailed analysis, some economy in time may be effected and the result would be more reliable.

Great accuracy on the other hand is not important since certain factors greatly influencing the determination of the economic diameter, such as the numerical relation between the value of power lost in friction and the cost of the pipe line, are necessarily based on more or less accurate estimates, which are subject to change.

Adolpho Santos, Jr.,[22] **Assoc. M. Am. Soc. C. E.** (by letter).—The subject of this paper belongs to that class of engineering problems which, for their proper solution, require the exercise of judgment. Affected as they are by matters of opinion and by a multiplicity of varying, complex, and, sometimes, conflicting factors, such problems cannot be solved with any high degree of precision no matter how dexterous one may be in the use of mathematical tools. Attempts to increase the precision beyond the limits imposed by the nature of the problem are illusory and only add to the complexity. The principal merit of mathematical analyses of such problems is as an aid in understanding them and in reaching sound judgment.

The authors state that, as computed by their formulas, the economic diameter of a penstock for a certain development shows a difference of only 3.25%, as compared to the diameter determined from detailed studies. To the writer this only proves that, starting from identical, basic, and assumed data, the formulas give results which agree with those obtained from detailed studies. It does not prove that the assumptions were correct. Certain of these assumptions, and the haze of uncertainty surrounding them, will be discussed briefly.

The selection of Scobey's formula for estimating the penstock friction loss is justified. It is indeed authoritative, but any estimate of friction-head loss based on it, intended to serve as the basis of economic studies, is nothing more than a forecast of future conditions and, as such, it is not necessarily accurate. The coefficient, K_s, depends on the class of pipes and joints but varies considerably for different penstocks of the same class, and, in many cases, the class itself is only definitely known after the final award of the contract, as joints and other details differ between different manufacturers. It is also known that K_s increases with the age of the penstock, the rate of increase depending on the size of the penstock, the type of protective cover of the internal surface, the quality of the water, climate, etc. The writer knows of cases in which the rate of increase of K_s is 10% each year, and of an extreme case in which it is more than 20% per year. The question arises, therefore, as to what rate of increase and what age of the penstock should be assumed for purposes of economic studies.

The assumption that the friction loss in the penstock varies as the 1.9 power while the other hydraulic losses vary as the square of the velocity seems to be an unwarranted refinement in economic studies because: (*a*) K_s cannot be predicted accurately; (*b*) "* * * it must not be overlooked that the individual values of z vary all the way from 1.73 to 2.14", as Scobey has stated;[23] (*c*) the refinement adds greatly to the complexity of the problem, as will be seen by comparing Equation (11) with Equations (24) to (27); and (*d*), the gain in precision, if any, is negligible.

It would be preferable to select a value of K_s high enough to cover the total losses, particularly in the more common developments where the bend losses are only a small proportion of the total. In the relatively few cases where the bend losses form a great proportion of the total it might be better, in order to avoid unnecessary complexity, to adopt some formula other than that of Scobey in which the friction loss is made to vary as the square of the velocity.

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[22] Asst. Hydr. Engr., The São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[23] "The Flow of Water in Riveted Steel and Analogous Pipes", by Fred C. Scobey, M. Am. Soc. C. E., *Technical Bulletin No. 150*, U. S. Dept. of Agriculture, January, 1930, p. 81.

There is also the question of the discharge which, as the authors state, "should be estimated as closely as possible". For the purpose of economic studies, should the maximum overload, the rated, the most frequent, or some other value of discharge, be assumed? This question is answered partly by the introduction of a factor to take care of the effect of the load factor. The question is not precisely answered, however, because to apply this factor it is necessary to make assumptions regarding the future loading of the unit, and this, again, requires the introduction of the element of judgment. In any case, no matter what value of discharge is selected, the fact remains that the horse-power and efficiency guaranties given by the turbine manufacturers are subject to a certain tolerance, so that at the time of making the economic studies the discharge is not known with a precision greater than 1 to 2 per cent.

In order to determine the thickness of the steel pipes it is necessary to assume a value for the head. This is not quite as simple as it appears. For reasons given elsewhere[24] it is considered that high-head penstocks, at least, should be designed to resist, without appreciable deformation, the stresses caused by the extreme pressures resulting from accidental conditions. These extreme pressures may arise from a variety of causes, some of which depend on the maximum discharge whereas some do not. The probability of any one assumed accidental condition arising during the life of the penstock is a matter for speculation so that weighting is difficult. Then there are the questions of maximum allowable stress intensity under static pressure in the lower part of the penstock, and of minimum thickness to prevent collapse due to vacuum, and as protection against corrosion, in the upper part. As a result of these and other factors it is a nice problem for the judgment and experience of the engineer to determine just which head to use.

All other factors entering into the problem, such as, over-all efficiency of the plant; loss factor corresponding to the load factor under which a plant operates (what should one assume as the period of the average load and as the interval of the maximum load?); value of power lost; unit cost of steel (which varies with the diameter of the penstock not only in purchase price, but also as affected by erection costs); percentage of over-weight; etc., are, each and all, subject to varying degrees of uncertainty. This being the case it follows that the calculated economic diameter is also uncertain and can serve only as a guide to judgment. The probable error or degree of uncertainty of the computed value can be estimated by the well known law of the propagation of errors.

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[24] "High-Head Penstock Design", by Messrs. Billings, Dodkin, Knapp, and Santos; Symposium on Water-Hammer, A. S. M. E., Hydraulic Div., and Am. Soc. C. E., Power Div., p. 29 *et seq.*

For purposes of illustration the writer considers the data given by the authors and assumes certain values for the probable errors of some of the factors: $Q = 400 \pm 4$ cu ft per sec; $e = 0.75 \pm 0.015$; $K_s = 0.401 \pm 0.020$; $K_e = 0.05 \pm 0.005$; $k_b = 0.25 \pm 0.025$; $H = 130 \pm 10$ ft; $i = 0.20 \pm 0.05$; and, $f = 0.25 \pm 0.025$. It will be assumed that the remaining factors: Δ, L_p, L_f, a, r, s_g, e_j, and b are known precisely (which is not the case), and are as given by the authors.

Applying to these data, the law of the propagation of errors, the writer finds from Equation (2) that the head lost due to friction is 0.421 ± 0.022 ft. Similarly, from Equations (2) and (16), it is found that the total head loss is 0.680 ± 0.030 ft. In other words, the probable error of the friction loss is about ± 5.2% and that of the total loss, ± 4.4%, thus indicating that the two methods of arriving at the head loss are of about the same degree of precision. The reason for the large difference in the actual loss as computed by the two methods lies in the fact that the bend losses form a high proportion of the total. This difference could have been made to disappear by increasing K_s proportionately.

Keeping the same number of significant places as the authors, it is found, on the basis of Equations (23) and (26), that $G = 1\,058 \times 10^7 \pm 61 \times 10^7$; $M = 88.3 \times 10^7 \pm 7.3 \times 10^7$; $J = 13.14 \times 10^{-4} \pm 1.34 \times 10^{-4}$; $B = 16.4 \pm 1.4$; and $D = 7.315 \pm 0.154$ ft. This shows that the probable error of the computed value of the economic diameter is about ± 2 in., even when it is optimistically assumed that one-half the factors affecting the result are known exactly. If the retardation coefficient is increased by 15% (and there is reason for doing this when the bend losses are known to be high), thus making $K_s = 0.461$, the diameter computed by the simpler Equation (11) is 7.10 ft, with about the same probable error as the value previously determined.

While on the subject of precision of computations the writer would remark that in deducing Equation (4) from Equation (3), there is no justification for increasing the number of significant places in the constants from three (62.5 and 0.746) to six (1.17631). Similarly, in going from Equation (16) to Equation (17) the increase in the number of significant figures from four to seven is not justified. It is an old rule that "one can never acquire in the product more accurate places than there are in the poorer of the two factors". Division comes under the same rule.

The writer has tried to emphasize the limitations of all formulas purporting to give, directly, the economic diameter of steel penstocks. A further limitation of any such formulas results from the fact that they give one, and only one, value for the diameter, whereas, at least in the case of high-head penstocks, it will be found more economical to vary the diameter with the head. In such cases, it will also be found expedient to proportion the diameters and the length of each reach of different diameter, aside from economical considerations, in such a manner as to reduce to a minimum the pressures resulting from the water-hammers produced during accidents.

JOSEPH D. LEWIN,[25] JUN. AM. SOC. C. E. (by letter).—In that it permits taking into consideration such new factors as bend losses, entrance losses, etc., the development of penstock analysis in this paper is a very interesting one. In the form in which the authors have presented their analysis its application is limited to uniform (that is, horizontal) penstocks. A few changes in the interpretation of the present analysis are sufficient to extend its application to all horizontal and inclined penstocks.

Friction Loss and Annual Cost of Penstock.—In 1914, Professor A. Ludin published his formula[26] which, in the notation of the paper, may be written:

$$D = \frac{\sqrt[7]{101.3\, e_T\, e_G\, e_j\, s_z\, b_w\, T}\ \sqrt[7]{\dfrac{(Q')^3}{H}}}{(1+i)\ \gamma\ C^2_z\ a_s} \ldots\ldots\ldots\ldots\ldots (43)$$

in which e_T = the efficiency of the turbines, in decimals; e_G = the efficiency of the generators, in decimals; e_j = the efficiency of the joints, rivets, etc., in decimals; s_z = allowable stress, in kilograms per square centimeter; b_w = the value of the power at the generators, in marks per kilowatt-hour; T = hours of use; γ = the specific weight of steel (= 7.9); C_z = the friction coefficient in the Chezy formula; a_s = the annual cost of a ton of steel, in marks per ton; and Q' = equivalent discharge, in cubic meters per second. Converted into the units of this paper, Equation (43) becomes:

$$D = 0.56682\ \sqrt[7]{\frac{e\, b\, s_g\, e_j\, T}{a\, r\, C^2_z\, (1+i)}}\ \sqrt[7]{\frac{(Q')^3}{H}} \ldots\ldots\ldots\ldots (44)$$

The differences in the root, in the constant, and in the friction coefficient are due to the use of different equations for determining the friction losses.

The authors used the Scobey formula, Equation (1), and Professor Ludin, the Chezy formula,

$$h_f = \frac{4\ V^2}{C^2_z\ D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

For turbulent flow with rough walls, the exponent, 2, for the velocity seems preferable.

For the penstock under consideration the economic conditions of location, elevation, transportation, etc., determine the first radical in Equation (44) and the corresponding quantity in Equation (11), so that they become constant, which may be expressed, respectively, by the symbols, K'_r and K_r. The economic diameter can be expressed as:

$$D = K_r \sqrt[6.9]{\frac{Q^{2.9}}{H}} = K'_r \sqrt[7]{\frac{Q^3}{H}} \ldots\ldots\ldots\ldots\ldots (46)$$

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[25] Hydr. Designer, Phœnix Eng. Corp., New York, N. Y.

[26] "Die Wirtschaftliche Bemessung von Triebwasserleitungen" von A. Ludin, *Zeitschrift für das gesammte Turbinenwesen*, 1914, No. 13; also, "Wasserkraftanlagen", von A. Ludin, Berlin, 1934, pp. 194–197.

For pressures greater than 300 ft, and for the economic conditions prevailing in Central Europe, Professor Ludin[26] developed the formula:

$$D_m = \sqrt[7]{\frac{5.2\,Q^3}{H}} = 1.26557\ \sqrt[7]{\frac{Q^3}{H}} \quad \text{.................} (47)$$

in which D_m = the economic inside diameter, in meters. The solution of Equation (47) is greatly facilitated by the preparation of a nomogram such as Fig. 2. Equation (11) may seem very complicated, and it will be quite difficult to obtain correct data for the value of the power and costs of the penstock. The value of K_r or K'_r, however, remains nearly unchanged. If the steel costs, a, are greater, the price of power will increase accordingly; or, if the load factor, F, is small (peak power plant), the price of the peak energy, b, will be high too.

This is the reason why Equation (47) is correct for most European penstocks. For all cases with pressures less than 300 ft (100 m) Dr. Bundschu[27] advises heads of 300 ft. Using the values given in the paper under the heading, "Use of Formulas," as prevailing in the United States, Equation (11) becomes:

$$D = 1.33822639\ \sqrt[6.9]{\frac{f\,Q^{2.9}}{H}} \quad \text{.....................} (48)$$

The corresponding diameters can be obtained from Fig. 2 which must be changed accordingly. In Fig. 2, there are two scales for diameters, the left for the European conditions (Equation (47)), and the right for the American conditions (Equation (48)). In the computation the writer prefers to use "the equivalent discharge," Q'_e (discussed subsequently), instead of the authors' quantity, $f\,Q^{2.9}$; but for the purpose of checking the values given in Table 3, with $f = 0.25$, a scale corresponding to,

$$D = 1.38823\ \sqrt[6.9]{\frac{0.25\,Q^{2.9}}{H}} = 1.13555\ \sqrt[6.9]{\frac{Q^{2.9}}{H}} \quad \text{...........} (49)$$

is drawn on the right in Fig. 2. The scales for Equations (48) and (49) differ only by the difference in the constant. Table 5 demonstrates how very closely the computed values coincide with those found nomographically.

TABLE 5.—COMPARISON OF METHODS

Item No.	Maximum discharge, Q, in cubic feet per second (1)	ECONOMIC INSIDE DIAMETER, D, IN FEET: By Equation (48) (2)	ECONOMIC INSIDE DIAMETER, D, IN FEET: By Fig. 2 (3)	ERROR: In feet (4)	ERROR: In percentages* (5)
(a) AVERAGE HEAD, H, EQUALS 100 FEET					
1	400	7.40	7.35	0.05	0.68
2	2 000	14.49	14.41	0.08	0.555
3	5 000	21.30	21.20	0.10	0.47
(b) AVERAGE HEAD, H, EQUALS 400 FEET					
4	400	6.03	6.02	0.01	0.166
5	2 000	11.85	11.90	0.05	0.42
6	5 000	17.42	17.40	0.02	0.115

* All less than 1 per cent.

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[27] "Wirtschaflicher Entwurf von Turbinenrohrleitungen", von Bundschu, *Wasserkraft und Wasserwirtschaft*, 1931, pp. 56–58.

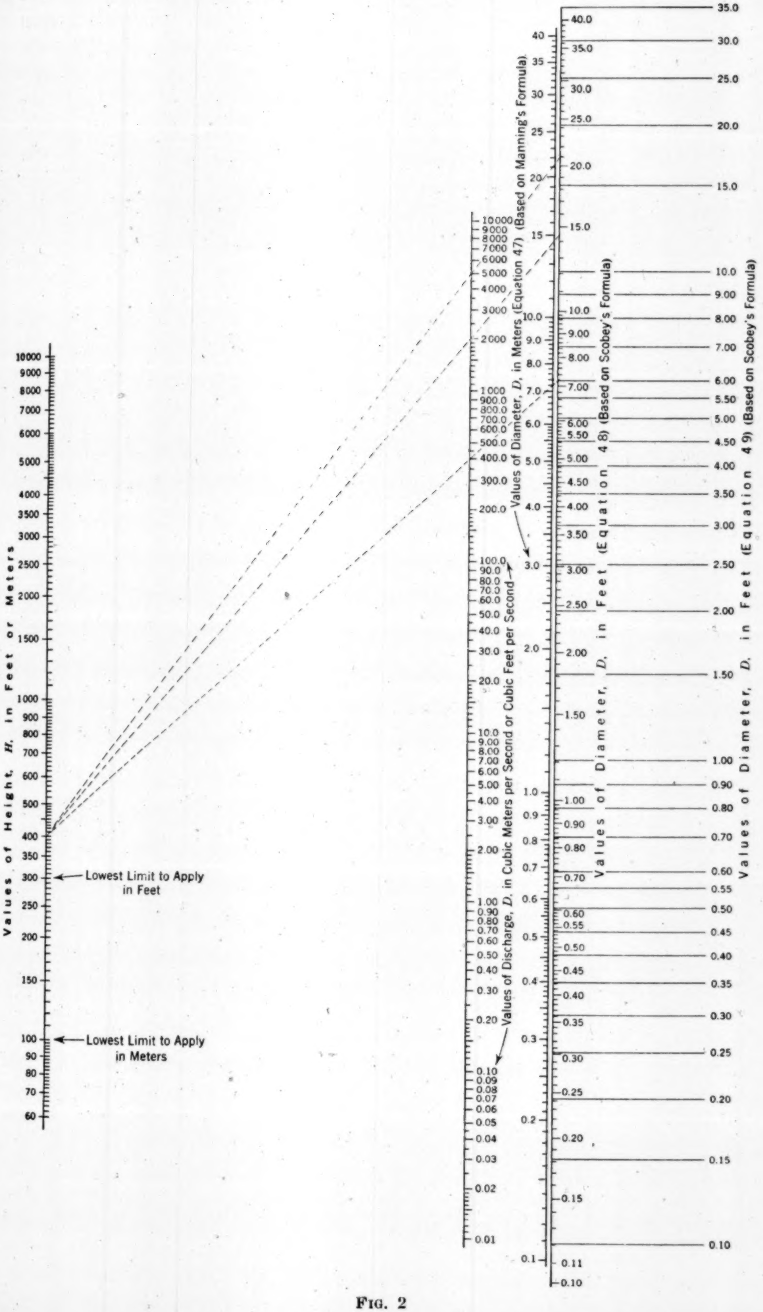
Values of Height, H, in Feet or Meters
10000
9000
8000
7000
6000
5000
4000
3500
3000
2500
2000
1500
1000
900
800
700
600
500
400
350
300
250
200
150
100
90
80
70
60
Lowest Limit to Apply in Feet
Lowest Limit to Apply in Meters
Values of Discharge, D, in Cubic Meters per Second or Cubic Feet per Second
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
900.0
800.0
700.0
600.0
500.0
400.0
300.0
200.0
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
Values of Diameter, D, in Meters (Equation 47) (Based on Manning's Formula)
40
35
30
25
20
15
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Values of Diameter, D, in Feet (Equation 48) (Based on Scobey's Formula)
40.0
35.0
30.0
25.0
20.0
15.0
10.0
9.00
8.00
7.00
6.00
5.50
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.90
0.80
0.70
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.11
0.10
Values of Diameter, D, in Feet (Equation 49) (Based on Scobey's Formula)
35.0
30.0
25.0
20.0
15.0
10.0
9.00
8.00
7.00
6.00
5.50
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.90
0.80
0.70
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10

FIG. 2

Fig. 2 will be the correct dimensions of a penstock under normal economical conditions in Europe, or in the United States, respectively. Should the location of the penstock under consideration cause changes in Equations (47) or (48) a new scale of diameter can be found very easily, in 25 to 30 min, as follows: Substitute the new data in Equation (11) or in Equation (44), depending on whether the Scobey or the Manning formula is preferred. The solution will have the form of Equation (46). The discharge, Q, will be:

$$Q = \sqrt[2.9]{H \left(\frac{D}{K_r}\right)^{6.9}} = \sqrt[3]{H \left(\frac{D}{K'_r}\right)^{7}} \quad \ldots\ldots\ldots\ldots\ldots (50)$$

Next, applying Equation (50), find the discharges for values of $D = 0.1$, 1.0, and 10.0; and $H = 100$, 1 000, and 10 000. The lines connecting the corresponding points, H and Q, in the nomogram will intersect in $D = 0.1$, 1.0, and 10.0. This gives the place of the new D-scales and the size of the unit. The method thus described has the advantage that every D-point is found by intersecting three lines, and thus a double check is possible.

Losses Other Than Those Due to Friction.—The authors limit their analysis to penstocks of uniform diameter, or to a horizontal penstock only. Considering Equation (46) for a constant discharge, Q, the diameters at any elevation, u or v, may be determined by the formulas:

$$D_u = K_r \sqrt[6.9]{\frac{Q^{2.9}}{H_u}} = K \sqrt[6.9]{\frac{1}{H_u}} \quad \ldots\ldots\ldots\ldots\ldots (51a)$$

$\left(\text{or } K'_r \sqrt[7]{\frac{Q^3}{H_u}} = K \sqrt{\frac{1}{H_u}}\right)$; and, similarly,

$$D_v = K \sqrt[6.9]{\frac{1}{H_v}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (51b)$$

$\left(\text{or } K \sqrt[7]{\frac{1}{H_v}}\right)$ in which $K = K_r \sqrt[6.9]{Q^{2.9}}$ (or $K'_r \sqrt[7]{Q^3}$). The relation between two diameters at the different elevations may be expressed as:

$$D_v = D_u \sqrt[6.9]{\frac{H_u}{H_v}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

$\left(\text{or } D_v \sqrt[7.0]{\frac{H_u}{H_v}}\right)$.

In an inclined penstock, therefore, the different diameters, D_v, are determined by Equation (52) after one diameter, D_u, has been computed. Consequently, Equations (27) and (33) of the paper can be used without applying the limitations fixed by the authors. This general application of bend-loss formulas is the new idea introduced by the authors in the analysis of penstocks.

Example.—It is intended to design a penstock, a part of which is horizontal and a part inclined. The angles of bends, and their elevations, are known. Applying Equation (52), reduce the different diameters to a single diameter, D_u, at a basic plane of reference, Elevation u, the result is a series of formulas for annual friction loss, as follows (see Equation (19)):

$$E_{f1} = \frac{1.17631\ K_s Q^{2.9} e f b L_1}{\left(D_u \sqrt[6.9]{\frac{H_u}{H_1}}\right)^{4.9}} = \frac{1.17631\ K_s\ Q^{2.9}\ e f b}{\sqrt[6.9]{H^{4.9}{}_u}}\ \frac{L_1 \sqrt[6.9]{H^{4.9}{}_1}}{D^{4.9}{}_u} \quad \ldots (53)$$

or, substituting $\kappa_f = \dfrac{1.17631\ K_s\ Q^{2.9}\ e f b}{\sqrt[1.41]{H_u}}$:

$$E_{f1} = \kappa_f\ L_1 \frac{\sqrt[1.41]{H_1}}{D^{4.9}{}_u} \quad \ldots\ldots (54)$$

For any inclined part of the penstock the annual cost of friction loss will be:

$$E_{fn} = \frac{\kappa_f\ L_n \sqrt[1.41]{H_n}}{D^{4.9}{}_u} \quad \ldots\ldots (55)$$

for any bend loss, at any elevation:

$$E_{b1} = \frac{18.70719\ Q^3\ e f b}{\left(\sqrt[6.9]{\frac{H_u}{H_1}}\ D_u\right)^4}\ K_{b1}$$

or substituting $\kappa_b = \dfrac{18.70719\ Q^3\ e f b}{\sqrt[1.72]{H_u}}$:

$$E_{b1} = \frac{\kappa_b\ K_{b1} \sqrt[1.72]{H_1}}{D^4{}_u} \quad \ldots\ldots (56)$$

In a penstock consisting of the three sections shown in Fig. 3, the total annual cost of the entire penstock will be the sum of a series of formulas similar to Equations (55) and (56), thus:

$$E_{\text{Total}} = E_{f1} + E_{f2} + E_{f3} + E_{b1} + E_{b2} + E_p + E_e \quad \ldots\ldots (57)$$

The economic diameter is determined by the first differentiation of Equation (57); thus:

$$\frac{dE_t}{dD_u} = -\frac{\kappa_f\, 4.9}{D^{5.9}{}_u}\left[\sqrt[1.41]{H_1}\, L_1 + \sqrt[1.41]{H_2}\, L_2 + \sqrt[1.41]{H_3}\, L_3\right] - \frac{\kappa_b\, 4}{D^5{}_u}\left[\sqrt[1.72]{H_1}\ K_{b1} + \sqrt[1.72]{H_2}\ K_{b2}\right] + B = 0 \quad \ldots\ldots (58)$$

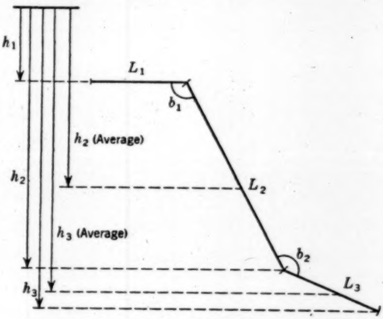

FIG. 3.—SCHEMATIC LAYOUT OF THE PENSTOCK

in which B is defined by Equation (23d). Substituting,

$$U = \frac{4.9\,\kappa_f}{D^{5.9}_u}\left[\sqrt[1.41]{H_1}\,L_1 + \sqrt[1.41]{H_2}\,L_2 + \sqrt[1.41]{H_3}\,L_3\right] \quad \ldots\ldots\ldots (59a)$$

and,

$$V = \frac{4\,\kappa_b}{D_u^5}\left[\sqrt[1.72]{H_1}\,K_{b1} + \sqrt[1.72]{H_2}\,K_{b2}\right] \quad \ldots\ldots\ldots\ldots (59b)$$

in Equation (58) and solving for B,

$$B = \frac{U}{D^x} + \frac{V}{D^y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

which corresponds to Equation (24) and can be solved in a similar manner to yield:

$$D_u = \sqrt[6.9]{\frac{U + V}{B}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Since κ_f, H, L, K_s, and K_b, corresponding to Equation (27), are definite constant values, Equation (61) can be solved. It is also advisable for comparison to compute Equation (61) changing the root from 6.9 to 6.0. so that the limits of the error may be judged.

The foregoing development of the basic formula, Equation (27), extends its application to all penstocks in which H is greater than 300 ft.

Discussion of Fig. 1.—In regard to the Q-value to be used in Equation (43) Professor Ludin prescribes the determination of the equivalent discharge:

$$Q' = Q_{\text{Equivalent}} = \sqrt[3]{\frac{\Sigma\, q^3}{\Sigma\, t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

in which q = partial discharge, in cubic feet per second, during a time interval,[28] t, for every different case.

The authors' computation of different loss-factor values in Fig. 1 is very interesting. It shows that for the usual values of load factor, from $F = 0.4$ to $F = 0.8$, the loss factor ranges from $f = 0.11$ to $f = 0.235$, and, from $f = 0.54$ to $f = 0.685$, respectively, if the curves, E and C, are assumed as limits; or from $f = 0.065$ to $f = 0.5$, and from $f = 0.52$ to $f = 0.8$, respectively, if Curves A and B are assumed as limits. This proves that the error in determining the penstock diameter for small load factors can be very high. For $f = 0.4$, for example, the error in Curves C and E equals $\sqrt[6.9]{\frac{0.235}{0.110}} - 1 = 0.116$; and in Curves A and B, $\sqrt[6.9]{\frac{0.40}{0.065}} - 1 = 0.3013$. The error is between 11.6% and 30.13%, thus nullifying the value of the meticulous and laborious determination of the economic diameter.

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[28] Manuals of Engineering Practice, Am. Soc. C. E., No. 11, p. 6.

The writer considers that it is impossible to give any general factors, such as a loss factor, or any limiting values for the exact computation of penstocks. Every region or district has its own characteristics which must be considered in each case separately. This can be determined, correctly and quickly, in the following manner.

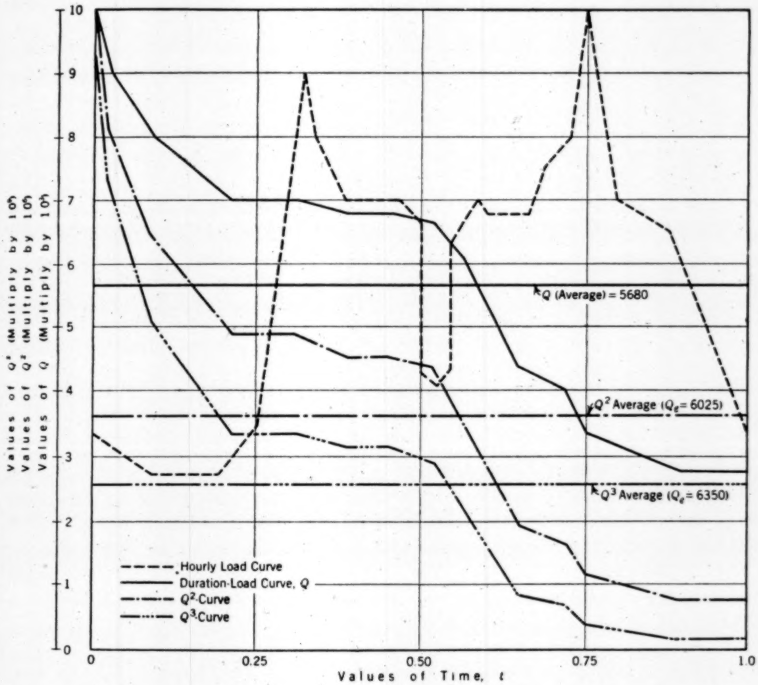

FIG. 4.—COMPUTATION OF EQUIVALENT DISCHARGES

A load-duration curve is usually computed in any case. For different points on such a curve compute the values of Q^2 and Q^3, as shown in Fig. 4. The second and third roots, respectively, the averages of these values, in Equation (62), will give the correct equivalent values of Q_e for the computation of the economical diameter of the penstock. These equivalent discharges are the correct values for the consumption region. This simple computation eliminates any inexactness resulting from values taken from the curves in Fig. 1.

Limitation of Discussion.—It should be noted that this discussion is limited to penstocks, in which the water pressure is great enough, because Equation (6) is correct only if the pressure in the top and in the bottom of the ring can be assumed to be equal (see Fig. 5(*a*)). For penstocks under low and medium pressure ($H < 200$ ft), the ring formula cannot be applied (see Fig. 5(*b*)). Therefore, Item No. 2, in Table 3, has been computed incorrectly. This is also true in the case of Item No. 3.

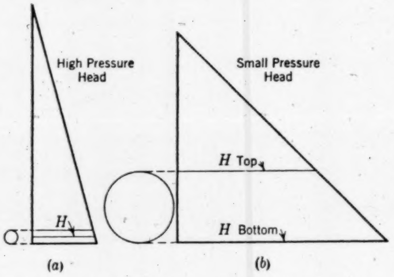

FIG. 5.—PRESSURE DISTRIBUTION IN PIPE

Furthermore, the present analysis is based only on hydrostatic pressure. In most cases the stresses due to the bending of the pipe between the supports must be considered and Equations (6), (7), (9), (10), (11), (20), (22), (23*d*), (27), (28), (30), and (33) must be changed accordingly.

Validity of Equation (6).—In the present analysis Equation (6) is valid only for pipes in which the relation of the thickness of steel to the diameter is very small, $\frac{t}{D} < \frac{1}{20}$. In all other cases the equations must be changed accordingly. Professor Ludin proposes using the outside diameter instead of the inside diameter in Equations (11) and (44).

Other Limiting Factors.—Finally, the diameter is limited due to the fact that, in cases of large discharges, two or more penstock lines are used. This effect can be considered by reducing the equivalent discharge by the number of pipes. Diameters of 21.30 ft and 17.42 ft (Items Nos. 3 and 6, Table 3) seem to be too large to be advantageous in actual operation. Fig. 2 demonstrates that for small heads and large discharges, the values of the penstock diameters do not change very much. For example, when $H = 100$ ft and $Q = 5\,000$ cu ft per sec, $D = 25.50$ ft; and for the same head and $Q = 6\,000$ cu ft per sec, $D = 26.60$ ft. In other words, a change of 20% in the rate of flow, changes the diameter only 7.6 per cent. This is clear because of the unimportance of friction losses in the computation of large diameters. Furthermore, the factors discussed herein under the heading, "Limitation of Discussion," should be considered as well. Thus, the values given in Items Nos. 3 and 6, Table 3, should be considered as being not exactly correct. The present analysis, therefore, is limited to penstocks of small and medium diameters (not greater than 12.0 ft), and great heights ($H > 300$ ft).

It seems to be very questionable whether the diameters of penstocks situated immediately down stream from the dam can be found by the analysis suggested in this paper. Furthermore, the construction of a nomogram has two advantages: (1) It permits finding the maximum heads for given discharges and given diameters, for the standard sizes; and (2), once it has been computed no further calculation is required. Fig. 2 can replace all the calculations in ordinary cases of penstocks.

For assistance in reading this discussion and encouraging the writer in his efforts, acknowledgment is freely given to R. W. Powell, M. Am. Soc. C. E., and Mr. D. C. Williams, of Ohio State University.

F. KNAPP,[29] ESQ. (by letter).—It is a well known fact that, in most cases, penstocks for high-head plants have diameters decreasing toward the power house. The authors do not state the range of applicability of their own formula, nor do they mention the limit of application of the usual orthodox design with varying diameters.

The analysis submitted by the authors, resulting in a penstock with a constant diameter, is applicable as long as: (1) The cost of the penstock per ton is constant irrespective of the variation of the diameters; and (2) the top section of the penstock does not require a minimum thickness, imposed either for practical reasons or in virtue of danger of collapse.

Let ΔL be a short length of the pipe, with an internal diameter, D, under the total head, H. With a circumferential stress, s, the thickness of the shell (using the simple cylinder formula) becomes:

$$t = \frac{0.434\, H\, D \times 12}{2\, s} \qquad \text{.....................} (63)$$

Now, assume that the pipe forms a cone with the small angle, $2\,\alpha$, at the summit. Using the symbol, D_a, to denote average diameter, the weight of the pipe with the length, ΔL, is:

$$W = w\, D_a\, \pi \frac{\Delta L\, t}{\cos \alpha \times 12} = \frac{w\, D^2_a\, \pi\, H \times \Delta L \times 0.434 \times 12}{2\, s \cos \alpha \times 12} = \frac{\text{constant}}{\cos \alpha} \quad .. (64)$$

in which w = the weight of steel, in pounds per cubic foot (= 490).

The loss of head, due to friction, is given by:

$$h_f \doteq K \frac{\Delta L}{D_a} \frac{V^2}{2\, g} = K_1 \frac{Q^2}{D^5_a} \Delta L \qquad \text{...............} (65)$$

Angle α does not appear in Equation (65) and this means that the loss of head in the two cases (constant diameter and decreasing diameter) is the same. However, the weight of the pipe, according to Equation (64), becomes a minimum when $\cos \alpha = 1$. The diameter, therefore, should be constant.

The foregoing two assumptions are the basic requirements for the applicability of the authors' formula. However, in cases where the thickness of the top section of the penstock is not determined by stress conditions, or when a special type of pipe is used in the bottom section (such as forged or banded pipe) which has the effect of increasing unit cost sharply, then the diameters of the penstock, to be the most economical ones, should be stepped.

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[29] Licensed Engr., São Paulo, Brazil.

Certain high-head penstocks are designed on the basis of accidental surge conditions. In such cases, the determination of the design head, including water-hammer, needs to include the influence of variations of diameter and thickness.[30]

A paper on high-head penstock design presented in 1933 contains a proposal to increase the diameters toward the power house,[31] in order to avoid the considerably increased surge in the second positive period, which is sometimes as great as 200% of the primary surge, due to reflections at changes of diameters and thicknesses. This proposal, fortunately, has not been adopted, as it is based on an incomplete study. For penstocks with decreasing diameters and which are designed to take extreme surge conditions into account, the writer has shown that the "reflection-times" of the various diameter sections should be exactly equal and, furthermore, that the "transmission-factors" for the surge wave should be nearly equal.[32] These two requirements (actually incorporated in the design of three important high-head penstocks), together with the consideration of the influence of head losses, have the economic advantage of reducing the absolute maximum surge to values either equal to, or only slightly greater than, the primary surge.

For such a penstock design with limitations, imposed by surge conditions, it becomes almost impossible to determine the best diameters by analytic methods. For the aforementioned penstocks, the writer developed a direct graphical method without the use of "cut and try" methods, which permits investigating all influencing factors with a negligible amount of labor.

The authors based their analysis on minimum total annual cost. This assumption corresponds to a situation in which funds for investment in the project are limited only by the necessity of making the project as attractive as possible. From an economic point of view, it would be justifiable to go beyond the point of minimum total yearly cost to a point corresponding to the case where funds for investment are unlimited, provided the return on each dollar invested will equal or exceed a fixed minimum.

Most penstocks are designed on the basis of "output." In certain cases, it becomes necessary to base the analysis on "capacity" and according to the prevailing conditions, both output and capacity must be taken into account.

It appears from the foregoing that too much refinement of analytical methods is not warranted, and that all such computations should be used, combined with sound judgment, to arrive at the best solution.

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[30] "High Head Penstock Design," by A. W. K. Billings, M. Am. Soc. C. E., O. H. Dodkin, F. Knapp, and A. Santos, Jr., Assoc. M. Am. Soc. C. E., "Symposium on Water-Hammer," Hydraulic Division, A. S. M. E., Power Division, Am. Soc. C. E., 1933.

[31] *Loc. cit.*, p. 44.

[32] "Simple Graphical Solution for Pressure Rise in Pipes and Pump Discharge Lines," by R. W. Angus, *Proceedings*, The Eng. Inst. of Canada, 1935, Vol. 18, p. 72; discussion by F. Knapp, p. 267.

Ralph W. Powell,[33] M. Am. Soc. C. E. (by letter).—In their "Synopsis" the authors state that their formulas differ from those heretofore published in three particulars. The writer believes that Items (*b*) and (*c*) constitute worth while improvements, although in the case of Item (*c*) it may be noted that where a hydro-electric plant is operated at low load factor, it is usually to supply "peak power" which would be more valuable, so that the product, $f\,b$, in Equation (11), may be fairly constant after all. However the authors' form would be useful if it were adapted (as it easily could be) to the case of the economic diameter of a pipe through which water is to be pumped. If the use were not continuous, the economic diameter would certainly be somewhat less than for continuous operation at capacity.

H. L. Doolittle,[18] M. Am. Soc. C. E., as well as William P. Creager and Joel D. Justin, Members, Am. Soc. C. E. (who were cited by the authors), brought the load factor into the problem. Mr. Doolittle stated very clearly that the annual cost of the lost power will vary as the average of the cubes of the various flows; he drew a duration curve and another curve obtained by cubing the ordinates of the duration curve; and showed that f is the cube root of the ratio of the average ordinate of the latter curve to its maximum ordinate. It seems that this is the only satisfactory way to obtain the value, f, and that it cannot be obtained from F alone. There are many possible duration curves, all giving the same power factor, which will yield quite different values of f. For instance, suppose a straight line varying from maximum to zero, and a curve the ordinates of which vary as one plus the cosine of an angle from 0 to 180 degrees. Both give $F = 0.500$, but the first gives $f = 0.250$ and the second, $f = 0.125$ (if 3 is used as the exponent of Q. If 2.9 is used, as the authors have done, $f = 0.256$ and 0.134, respectively. These correspond to Curves *D* and *B* of Fig. 1).

Returning now to the consideration of Item (*a*), the writer questions whether the Scobey formula is the best on which to base the theory of economic design, and whether it would not be better to assume that friction losses vary as the square of the velocity. This statement represents a change in opinion, because in 1927 the writer proposed a formula[34] based on the Hazen-Williams formula for pipe friction. This gave an exponent of 0.146 which is equivalent to the 6.85 root in Equation (11) as compared to 6.9 given by the authors, and 7 by the simpler theory now proposed. This was before Mr. Scobey published his formula. Four or five years later, the writer would undoubtedly have followed Mr. Scobey, as the authors have done; but in the last few years additional light has been thrown on frictional losses at high Reynolds' numbers by J. Nikuradse,[35]

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[33] Assoc. Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[34] "Economic Diameter of Pipe Lines", *Engineering News-Record*, Vol. 98, March 24, 1927, p. 499.

[35] "Strömungsgesetze in rauhen Röhren", von J. Nikuradse, Verein Deutscher Ingenieur, *Forschungsheft*, No. 361, 1933.

V. L. Streeter, Jun. Am. Soc. C. E.,[36] H. Schlichting,[37] and others. These data have been summarized in English by Mr. Streeter and Hunter Rouse, Assoc. M. Am. Soc. C. E.[38] Fig. 2 in Mr. Streeter's paper, and Fig. 10 in that of Mr. Rouse, give Nikuradse's results. The lowest curve, representing a roughness of $\frac{1}{507}$ of the radius of the pipe, would seem to be suitable for most steel penstocks. At any rate plotting Weisbach's f from Mr. Scobey's original data against the Reynolds' number, it seems to agree with this curve as well as with his own equation. Attention is especially called to Runs 65, 66 and 312 of his paper[4] which show a definite tendency of Weisbach's f to increase in the region at a Reynolds' number of approximately 1 000 000, just as it does in Nikuradse's lower curve. (Of course, various types of riveting, age of pipe, etc., will give a series of curves, some lower and some higher, with their minimum points at somewhat different Reynolds' numbers.)

If Nikuradse is correct (and Streeter and Schlichting have confirmed his work), the correct exponent of the velocity to give the friction head in a typical large steel pipe would vary from 1.75 for Reynolds' numbers between about 4 000 and 50 000; then it would increase to 2.00 at about 200 000, and to 2.10 at about 300 000; decrease to 2.00 at about 1 000 000; and remain constant for higher values. It would seem, therefore, that for the Reynolds' numbers usually encountered in a penstock, 2 would be as good a value to use as any. This would change Equation (11) to the seventh root, Equations (32) and (33) to the sixth root, and would greatly simplify the intermediate equations.

It should perhaps be re-emphasized that Equations (11) to (27) apply to high heads where the necessary thickness varies directly as the product of the head and diameter; and Equations (32) and (33) to the almost equally important case of low head in which the thickness of the shell is independent of both the head and the diameter. The writer must confess that he has never studied thoroughly the design of large pipe lines,[39] but he raises the question as to whether there is not a third case to be considered—that in which the maximum shear in the pipe shell, considered as a beam, will govern the thickness so that it will be independent of the head but dependent upon the diameter.

The writer does not wish these remarks to be taken as adverse criticism of the paper as a whole, because he considers it a valuable piece of work done in a careful and thorough manner. He is particularly sorry that the senior author cannot participate in the discussion of the points herein raised.[1] Many years ago he had the privilege of working for some weeks in the same room as Mr. Voetsch and under his immediate supervision. No young engineer could ask for a more kindly chief, or one with wider experience or sounder engineering judgment.

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[36] "Frictional Resistance in Artificially Roughened Pipe", by Victor L. Streeter, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 681–713.

[37] "Experimentelle Untersuchungen zum Rauhigkeitsproblem", von H. Schlichting, *Ingenieur-Archiv*, Band VII, Heft 1, 1936.

[38] "Modern Conceptions of the Mechanics of Fluid Turbulence", by Hunter Rouse, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 463.

[39] "Design of Large Pipe Lines", by Herman Schorer, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), pp. 101–191; and "Line Load Action on Thin Cylindrical Shells", by Herman Schorer, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 767–810.

H. K. Barrows,[40] M. Am. Soc. C. E. (by letter).—The formulas developed by the authors are of interest, and are novel in certain respects. The use of the Scobey formula for friction head loss, involving, as it does, special exponential values for V and D other than the square and first powers, respectively, causes Equation (11) to make the economic diameter vary as the $\sqrt[6.9]{\ }$, whereas if the simpler common formula for friction head losses,

$$h_f = \frac{K'}{D} \times \frac{V}{2\,g} \quad \text{.........................(66)}$$

is used, this becomes $\sqrt[7]{\ }$. It seems questionable whether the use of the Scobey formula is preferable under the circumstances.

Furthermore, the introduction of entrance losses, bend losses, etc., commonly of minor effect, could well be avoided, for the sake of simplicity, without appreciable effect upon results. The use of a loss factor to take account of load factor is logical and warrants further study and determinations of such factors. The writer has found the formula,[41]

$$D = 0.215 \sqrt[7]{\frac{K'\, b'\, Q^3_a\, s}{a\, r\, H}} \quad \text{..................(67)}$$

for steel pipes, convenient for obtaining a quick approximation of the best penstock size as a basis for further detailed study made by assuming different sizes, determining annual cost (of pipe plus value of power lost), and plotting total yearly cost against diameter.

In Equation (67), in addition to the notation of the paper: b' = the value of power, in dollars per theoretical horse-power per year; Q_a = average discharge, or maximum discharge, times a capacity factor; s = unit stress value of steel; and K' = friction factor in the formula,

$$h_f = \frac{K'\, L\, V^2}{2\, D\, g} \quad \text{.........................(68)}$$

The use of the average discharge applies fairly closely where individual penstock lines are considered, if wheel units are operated by number and gate for best results. For a long single penstock the correct discharge would be more nearly the maximum discharge times the square of the capacity factor.

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[40] Prof., Hydr. Eng., Mass. Inst. Tech., and Cons. Engr., Boston, Mass.

[41] "Water Power Engineering", by H. K. Barrows, 1934 Edition, p. 357.

Similar formulas have been deduced for concrete and wood-stave penstocks, the one for concrete being as follows:[42]

$$D = 0.215 \sqrt[7]{\frac{K'\, b'\, Q^3{}_a}{r\left(0.0000290\, a_c + 0.384\, p\, a_s + \frac{a_s\, H}{s_s}\right)}} \quad \ldots\ldots (69)$$

in which, in addition to the notation of the paper: a_c = cost of concrete, in dollars per cubic yard; p = ratio of cross-sectional steel area of longitudinal steel to that of concrete in the same plane; a_s = cost of steel reinforcement, in dollars per pound; and, s_s = allowable stress in hoop steel, in pounds per square inch. The quantity in parenthesis includes the effect of concrete cost, longitudinal steel, and hoop steel, respectively.

For wood-stave penstocks:

$$D = 0.215 \sqrt[7]{\frac{K'\, b'\, Q^3{}_a\, s}{r\, H\, (a_b + 0.000016\, a_w\, s)}} \quad \ldots\ldots\ldots\ldots (70)$$

in which, a_b = cost of steel bands, in dollars per pound; and a_w = cost of wood staves, in dollars per board-foot.

Equation (67) for steel pipes is simple enough to be useful; but Equations (69) and (70) are somewhat long and involved in use, and the writer prefers, in general, to determine the best size of penstock by curves rather than by formula, as this enables all special items of cost, other other than of pipe alone, to be included.

ROBERT W. ANGUS,[43] ESQ. (by letter).—The method of determining the economic diameter of penstocks has been presented in an interesting and valuable manner in this paper, and the authors have expressed the results in convenient formulas.

In the investigation the head, H, is defined as "the average head, on the penstock, including water-hammer effect, in feet", but the water-hammer itself may have an important influence on the size of the pipe, and the pressure rise due to this effect may be so large as to make Equation (6) inaccurate. A few years ago the writer was making some studies with a model pipe line having a glass surge tank. The top of this tank was open, was above the forebay level, and was tested and found to be amply strong for pressures produced by the static head in it. On several occasions, however, the glass surge tank was broken by a quick closure of the turbine gate, although no water came over the top of the tank, and a carefully made indicator showed pressures at its base much in excess of those due to the height of the water column.

Of course, it is well known that under favorable conditions, the water-hammer pressure rise, in feet, may be about 100 times the velocity dissipated, and also that these bad conditions may readily occur in a long penstock or pipe. For example, if a pipe is 6 000 ft long, the foregoing pressure rise will occur for valve closure requiring a period of 4 sec, or less, a length of time easily within the operation range of a governor; and if the pipe is under a low head of perhaps 100 ft and has its velocity of, say, 4 ft per sec dissipated, the pressure from the resulting water-hammer will be five times the normal pressure.

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[42] "Water Power Engineering", by H. K. Barrows, 1934 Edition, p. 358.

[43] Prof. of Mech. Eng., Univ. of Toronto, Toronto, Ont., Canada.

Recently, a case came to the writer's attention, in which two pipes ran parallel to each other from the side of a reservoir and were joined at their lower ends by a **U**-bend. Each pipe had a valve near the reservoir and the valves were under a static head of 66 ft, one valve being open and the other closed at the time. Each pipe also had a turbine connected near its lower end, one of which was shut down, and the other running. The operating turbine was then stopped by a quick, not "sudden", closure of the gates (that is, the closing time exceeded the total time of the pressure wave), but the resulting pressure rise was sufficient to break the closed valve that was near the reservoir. An examination suggested that the resulting pressure was more than twice the static head of 66 ft.

It would appear desirable and necessary, therefore, to include the water-hammer effect in a much more exact manner than by merely taking its average value. In the Symposium on Water Hammer sponsored by the Society, among others,[24] A. W. K. Billings, M. Am. Soc. C. E., has shown that the study of water-hammer modifies the design of the pipe line greatly. In view of the fact that the pressure rise due to this cause may be so easily arrived at graphically,[44] even in fairly complicated systems, it would seem that it should be included in any economic study, although the mathematical expressions for it would be difficult to include in ordinary equations.

M. H. Fresen,[45] Assoc. M. Am. Soc. C. E. (by letter).—The discussions which have been presented, contain much valuable information and constructive criticism.

Economic Diameter Using Weisbach Formula for Friction Loss.—Mr. Monroe and Professors Powell and Barrows prefer to use the Weisbach (also sometimes called Darcy or Chezy) formula for head lost in friction in conduits. Since the paper was prepared, friction factors (Table 6) for use in the Weisbach formula have been made available. For this reason Equations (75) and (76) have been derived.

Using the Weisbach formula for friction head loss (Equation (68) quoted by Professor Barrows), the annual cost of the lost power in a penstock of diameter, D, may be expressed similarly to Equation (5) as:

$$E_f = \frac{18.71\,K'\,L_f\,Q^3\,e\,f\,b}{D^5} \qquad \text{.................... (71)}$$

in which K' equals the friction loss factor, f, in the Weisbach formula.

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[44] "Simple Graphical Solution for Pressure Rise in Pipes and Pump Discharge Lines", by Robert W. Angus, *Journal*, Eng. Inst. of Canada, February, 1935; also, "Water-Hammer in Pipes, Including Those Supplied by Centrifugal Pumps; Graphical Treatment", by Robert W. Angus, *Proceedings*, Inst. of Mech. Engrs., 1937.

[45] Associate Engr., U. S. Bureau of Reclamation, Denver, Colo.

The first derivative of Equation (71) with respect to D is:

$$\frac{dE_f}{dD} = -\frac{93.54\ K'\ L_f\ Q^3\ e\ f\ b}{D^6} \qquad (72)$$

In Equation (72) let $93.54\ K'\ L_f\ Q^3 = G'$, which substituted for G in Equation (24) yields,

$$B = \frac{G'\ J}{D^x} + \frac{M\ J}{D^y} \qquad (73)$$

in which M, J, and B have the values given by Equations (23*b*), (23*c*), and (23*d*), respectively. Letting $x = y = 7$, Equation (73) becomes:

$$D = \sqrt[7]{\frac{J\ (G' + M)}{B}} \qquad (74)$$

approximately, or in terms of the fundamental factors involved, the economic diameter considering the head, H, is:

$$D = \sqrt[7]{\frac{e\ f\ b\ (93.54\ K'\ L_f + 74.83\ K_T)\ Q^3\ s_g\ e_j}{668\ H\ a\ r\ L_p\ (1 + i)}} \qquad (75)$$

which corresponds to Professor Barrow's Equation (67), except that the notation of the paper is used. For greater accuracy, Equation (75) (with $x = 7$ and $y = 6$) can be solved for D by a trial-and-error method.

Considering the shell thickness, t, the following equation is derived similarly to Equation (75):

$$D = \sqrt[6]{\frac{e\ f\ b\ (93.54\ K'\ L_f + 74.83\ K_T)\ Q^3}{128.3\ t\ a\ r\ L_p\ (1 + i)}} \qquad (76)$$

approximately. Equation (76) corresponds to Equation (33) of the paper with the same notation, and is similar to one given by Professor Barrows.[41] For greater accuracy, Equation (31) (with G' substituted for G, and $x = 7$ and $y = 6$) can be solved for D by a trial-and-error method.

TABLE 6.—FRICTION FACTOR, K', FOR USE IN WEISBACH FORMULA, EQUATION (68)

Item No. (1)	Reynolds' number, $R_n = \frac{V\ D}{\nu}$ (2)	Full-riveted pipe (3)	Continuous interior pipe (4)	Item No. (1)	Reynolds' number, $R_n = \frac{V\ D}{\nu}$ (2)	Full-riveted pipe (3)	Continuous interior pipe (4)
1	10×10^3	0.0392	0.0340	6	$5\ 000 \times 10^3$	0.0169	0.0115
2	50×10^3	0.0314	0.0256	7	$10\ 000 \times 10^3$	0.0154	0.0104
3	100×10^3	0.0285	0.0225	8	$50\ 000 \times 10^3$	0.0130	0.0086
4	500×10^3	0.0228	0.0170	9	$100\ 000 \times 10^3$	0.0122	0.0082
5	$1\ 000 \times 10^3$	0.0207	0.0149	..	....	....	....

Based on a study reported in 1936,[46] selected average friction factors (as read from a chart) for use in the Weisbach formula, Equation (68), are given in Table 6, Columns (3) and (4), corresponding to the recommended equations:

---

[46] "Friction Coefficients of Continuous Interior and Riveted Steel Pipes," by M. B. Karelitz and F. M. Watkins, *Technical Memorandum No. 521*, U. S. Bureau of Reclamation, Denver, Colo., May 14, 1936.

$$K' = 0.0043 + \frac{0.160}{R_n^{0.165}} \qquad (77)$$

for full-riveted pipe; and,

$$K' = 0.0043 + \frac{0.254}{R_n^{0.23}} \qquad (78)$$

for continuous interior pipe. The values in Column (2), Table 6, are based on Reynolds' number,

$$R_n = \frac{V\,D}{\nu} \qquad (79)$$

in which ν = the kinematic viscosity of the fluid. For water at 15° C (59°F), $\nu = 1.23 \times 10^{-5}$ sq ft per sec.

Nomogram for Solution of Economic Diameter.—The nomogram, Fig. 2, presented by Mr. Lewin for solving Equations (47), (48), or (49) for the economic diameter, D, directly, constitutes a valuable addition to the paper. To use Fig. 2, the maximum discharge, Q, and the head, H, for shell thickness, must be known. Since it is probable that for different projects the load factor, F, the shape of the load curve, and the corresponding value of the loss factor, f, will vary more widely than the other factors in Equations (11) or (27), the writer prefers to use Equation (48), Fig. 2, which includes the loss factor. The scale for Equation (48), Fig. 2, is such that the diameter read from it is that for unity load factor and unity loss factor. For a smaller load factor and loss factor, the diameter may be read from Fig. 2 (scale for Equation (48)), and corrected by the ratios given in Table 1, Columns (1) and (2). Similarly, Fig. 2 and Table 1 (preferably plotted in logarithmic curve form) may be used to estimate the diameters for assumptions other than those on which Equation (48) is based. It should be noted that Fig. 2 cannot be used to determine the diameter, D, when the head, H, and allowable stress, s_g, give a smaller penstock shell thickness than the minimum thickness permitted by the specifications being followed in the design. For such cases, Equation (33) should be used.

Economic Diameters, in Terms of Head, and Shell Thickness.—For exactly corresponding assumptions, the economic diameter of a steel penstock in terms of the head, H, is about 11% less than the economic diameter obtained when the shell thickness, t, corresponding to H from Equation (6) is used. At first thought this may appear to be inconsistent and incorrect. Fig. 6 has been prepared to show that such results are correct, and will be obtained whether the economic diameters are found from diameter-total annual cost curves similar to Fig. 6, or from solving Equations (27) and (33).

Fig. 6 is based on the following assumptions: $Q = 1\,000$ cu ft per sec; $K_s = 0.34$; $e = 0.85$; $f = 0.20$ (corresponding approximately to a load factor, F, of about 50%); $b = \$0.003$ per kw-hr; $s_g = 15\,000$ lb per sq in.; $e_j = 0.80$; $a = \$0.11$ per lb; $r = 0.07$; and $i = 0.15$. In this example, the following results are obtained: (*a*) From Fig. 6, Curve E, for a head, H, of 100 ft, the economic diameter is found to be 10.36 ft; (*b*) from Equation (6), the thickness of shell corresponding to a head of 100 ft is $t = 0.225$ in.; (*c*) from Fig. 6, Curve D, for a thickness, t, of 0.225 in., the economic diameter is found to be 11.66 ft; (*d*) substituting the foregoing assumed values in Equation (27), and solving, $D = 10.36$ ft for a head of 100 ft, which agrees with Item (*a*); (*e*) similarly, solving Equation (33), $D = 11.66$ ft for a shell thickness of 0.225 in., which agrees with Item (*c*); (*f*) the ratio of $\frac{\text{Item } (a)}{\text{Item } (c)} = \frac{10.36}{11.66} = 0.889$ (or, in general, the economic diameter of a penstock in terms of head is about 11% less than the economic diameter obtained when the corresponding shell thickness is used); and (*g*) the result of Item (*f*) may also be verified algebraically by a study of Equations (6), (27), and (33), and reference to Table 1.

For corresponding assumptions the results obtained, using the equations, are identical with those determined from a detailed analysis. For all ordinary problems the writer believes an appreciable saving of time will result from using the equations of the paper, compared to a detailed analysis.

Miscellaneous.—Mr. Lewin states that the analysis of the paper is "limited to uniform (that is, horizontal) penstocks." He also questions "whether the diameters of penstocks situated immediately down stream from the dam can be found by the analyses suggested in the paper." The writer assumes that Mr. Lewin refers to Equations (11) and (27) for computing D in terms of the head, for a penstock line of constant diameter. Neither of Mr. Lewin's statements applies as long as the following conditions are fulfilled: (*a*) The cost of the penstock per pound is constant irrespective of the variation of the diameters; (*b*) the top section of the penstock does not require a minimum thickness, imposed either for practical reasons or in virtue of danger of collapse; and (*c*) the average weighted head used in the equations is a true numerical function of the weight of the steel (and, therefore, of the cost) for the penstock, profile, and water-hammer conditions under consideration. Conditions (*a*) and (*b*) are limitations explained by Mr. Knapp.

Equations (51*a*) to (61), inclusive, given by Mr. Lewin, are interesting, but the writer doubts whether the average engineer would have the patience to attempt to apply them. The writer believes it would be more practicable to use Equation (33), applying the method suggested in the paragraph immediately following Equation (33).

Messrs. Santos and Knapp, and Professor Angus discuss water-hammer as affecting penstock design. In addition to the Footnote References (30, 44) given by them, several recent papers[47, 48] on water-hammer may be of interest to those who wish to pursue this phase of the problem further.

Messrs. Santos and Knapp very properly point out that too much refinement of analytical methods is not warranted, and that all such computations should be used, combined with sound judgment, to arrive at the best solution. The number of significant figures in the numerical constants of the equations in the paper should be reduced to four at the most, probably three being all that are justified.

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[47] Papers relating to water-hammer phenomena in pipe lines, by Lorenzo Allievi, R. W. Angus, F. Knapp, R. T. Knapp, J. N. Le Conte, O. Schnyder, E. B. Strowger, and F. M. Wood, *Transactions*, Am. Soc. M. E., November, 1937, Vol. 59, No. 8.

[48] "Water-Hammer Pressures in Compound and Branched Pipes," by R. W. Angus, *Proceedings*, Am. Soc. C. E., January, 1938, pp. 133–169.

Mr. Rudolph mentions an interesting application of the formulas of the paper to a problem involving the present value of a saving in the future. This is a phase of the problem in economics which should receive adequate attention.

As mentioned by Messrs. Bier and Lewin, and by Professor Powell, it should be emphasized that Equations (11) and (27) apply to high heads where the necessary shell thickness varies directly as the product of the head and diameter; and Equation (33) to the equally important case of low head in which the thickness of the shell is independent of the head, and for small penstocks, independent of the diameter.[49]

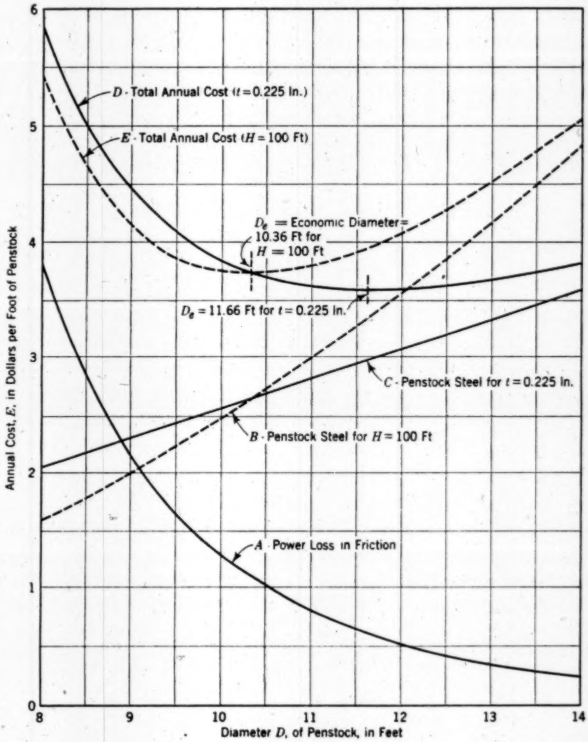

FIG. 6.—ECONOMIC DIAMETER OF STEEL PENSTOCK IN TERMS OF HEAD AND PLATE THICKNESS

Fig. 6 and the example given by Mr. Bier show that the equations of the paper and a detailed analysis give the same results if the same basic data are used throughout.

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[49] "Penstocks—A Report of the Hydraulic Power Committee," Edison Electric Inst., *Publication No. D15*, December, 1936. This recent report gives much valuable data on design and construction of penstocks. Minimum thickness of steel plates is discussed on p. 5.

Mr. Lewin and Professor Powell comment on the wide range in values of loss factor from Fig. 1 for particular values of load factor. Since no one can predict positively in advance the exact shape and magnitude of a load curve for a projected plant, it is probable that values of loss factor estimated from Fig. 1 are as reliable as those computed by the more accurate method given in the paper, and similar methods mentioned by Mr. Lewin and Professor Powell.

In conclusion, the writer wishes to thank those who have contributed discussions, and hence have added considerably to the value of the paper. The writer appreciates in particular the tribute to Mr. Voetsch[1] expressed by Professor Powell, and shares in his sorrow that the senior author cannot participate in this closing discussion, and enrich it from the wide experience he possessed.

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STRESSES AROUND CIRCULAR HOLES IN DAMS AND BUTTRESSES

By I. K. Silverman[1], Jun. Am. Soc. C. E.

With Discussion by Messrs. R. D. Mindlin, Chesley J. Posey, J. H. A. Brahtz, V. L. Fedorov, F. W. Hanna, C. P. Vetter, Fred L. Plummer, and I. K. Silverman.

Synopsis

Cracks that have been observed in dams have sometimes been attributed to the presence of drainage or inspection galleries which are parallel or transverse to the axis of the dam. Some observers have stated that these cracks generally start from the gallery and in some cases travel to both down-stream and up-stream faces, thus dividing the structure into two or more parts. In other cases these cracks are purely local and extend only a short distance from the gallery into the main part of the dam. Both these types of cracks can be observed on the faces of buttresses.

The stresses which occur around these openings are due to volumetric changes arising from shrinkage, temperature, etc., and to water and mass loads. An attempt is made in this paper to analyze the stresses around a circular opening caused by water and mass loads.

Notation.—In the notation introduced herein (see Appendix I) an effort has been made to conform to the American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[2], compiled by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

General Formulas

Both theory and experiment[3] have shown that, in a large body, the effect of a relatively small opening is a purely local one. The stresses at points that are removed a sufficient distance from the discontinuity are essentially the same as if there were no hole. A case that has been investigated to some

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Note.—This paper was published in November, 1936, *Proceedings*.

[1] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[2] A. S. A.—*Z* 10*a*—1932.

[3] "Photo-Elasticity", by E. G. Coker and L. N. G. Filon, pp. 481 *et seq.*, Cambridge Press, 1931.

extent is that of a circular hole in a bar subjected to uniform tension. The normal stress, s_{nx}, on Section AA of Fig. 1[4] is given by,

$$s_{nx} = \frac{s}{2}\left(2 + \frac{r^2_h}{\rho^2} + 3\,\frac{r^4_h}{\rho^4}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

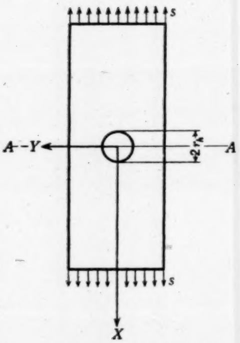


FIG. 1

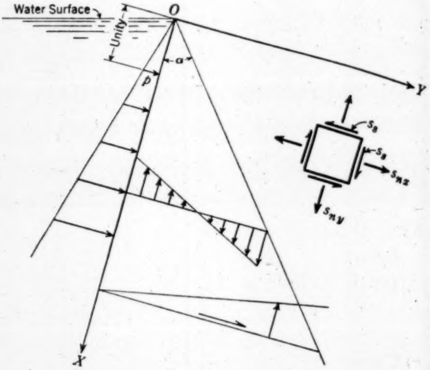


FIG. 2

in which s = the total load divided by the gross area of the bar; r_h = the radius of the hole in the bar; and ρ = radial distance from center of hole. As ρ increases s_{nx} approaches s. In the immediate vicinity of the hole, however, the stresses are larger than those calculated as if no hole existed; and $s_{nx} = 3\,s$ in Equation (1) if $\rho = r_h$.

STRESSES DUE TO WATER LOAD

Although openings in dams or buttresses are not always circular the state of stress around such openings may be examined on the assumption that they are circular. In the analysis which follows the usual assumptions of homogeneity, elastic behavior, and the applicability of Hooke's law, are made. Triangular dams and buttresses subjected to hydrostatic loadings are usually designed on the assumption that the normal stresses, s_n, on a section are distributed linearly (see Fig. 2) and are given by,

$$s_n = \frac{p\,x}{K^2} - 2\,\frac{p\,y}{K^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

in which p = pressure at a unit distance measured from O along the up-stream face (Fig. 2); K = a substitution factor = tan α; α = the angle between the battered faces of a dam; and x and y are variable distances measured in the X- and Y-directions, respectively. The shear stresses, s_s (see Fig. 2), are given by,

$$s_s = -\,\frac{p\,y}{K^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

---

[4] "Theory of Elasticity", by S. Timoshenko, p. 78.

Considered as a two-dimensional problem, the equations of equilibrium of a rectangular particle are:

$$\frac{\partial s_x}{\partial x} + \frac{\partial s_s}{\partial y} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and,

$$\frac{\partial s_y}{\partial y} + \frac{\partial s_s}{\partial x} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

To preserve geometrical continuity under Hooke's law and to satisfy the equations of equilibrium, the stresses, s_x, s_y, and s_s, are defined as follows:

$$s_x = \frac{\partial^2 F}{\partial y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6a)$$

$$s_y = \frac{\partial^2 F}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

and,

$$s_s = - \frac{\partial^2 F}{\partial x \, \partial y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6c)$$

in which, F, the Airy stress function, is given by the following:

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \, \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \quad \ldots\ldots\ldots\ldots\ldots (7)$$

A solution of Equation (7) that satisfies all boundary conditions for a triangular dam section loaded as shown in Fig. 2 is:

$$F = \frac{p \, x \, y^2}{2 \, K^2} - \frac{p \, y^3}{3 \, K^3} - \frac{p \, x^3}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

from which the stresses by Equation (6) are found to be:

$$s_x = \frac{\partial^2 F}{\partial y^2} = \frac{p \, x}{K^2} - \frac{2 \, p \, y}{K^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

$$s_y = \frac{\partial^2 F}{\partial x^2} = - p \, x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

and,

$$s_s = - \frac{\partial^2 F}{\partial x \, \partial y} = - \frac{p \, y}{K^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9c)$$

Transferring the origin of co-ordinates to x_0, y_0 (Fig. 3(a)), and using polar co-ordinates, the stresses acting on a particle bounded by the arcs of two concentric circles and by two radial lines (Fig. 3(b)) are given by:

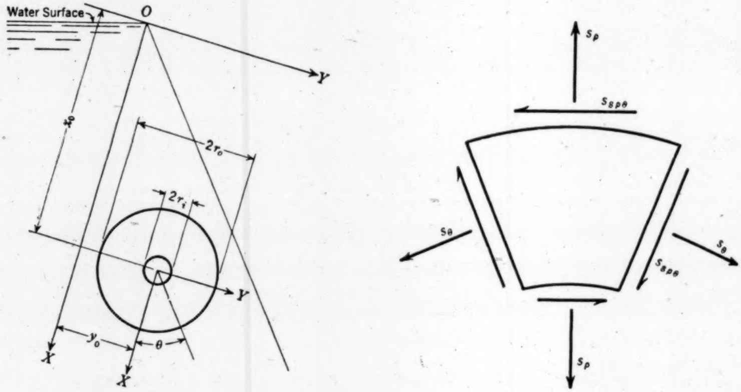
$$\begin{aligned} s_\rho = - \frac{p}{6 \, K^3} \Bigg[& 3 \, \rho \sin \theta + 6 \, K \, y_0 \sin 2 \, \theta + 3 \, \rho \sin 3 \, \theta \\ & + \left(\frac{3 \, \rho K^3}{2} - \frac{3 \, K \, \rho}{2} \right) \cos \theta + \left(6 \, y_0 - 3 \, K \, x_0 - 3 \, K^3 \, x_0 \right) \cos 2 \, \theta \\ & - \left(\frac{3 \, \rho}{2} K^3 + \frac{9 \, \rho}{2} K \right) \cos 3 \, \theta + 3 \, K^3 \, x_0 + 6 \, y_0 - 3 \, K \, x_0 \Bigg] \ldots (10) \end{aligned}$$

$$s_s = \frac{p}{6\,K^3}\left[\left(\frac{3\,\rho}{2}\;K - \frac{3\,\rho}{2}\;K^3\right)\sin\theta + \left(6\,y_0 - 3\,K^3\,x_0 - 3\,K\,x_0\right)\sin 2\,\theta\right.$$
$$\left. - \left(\frac{9\,K\;\rho}{2} + \frac{3\,K^3\,\rho}{2}\right)\sin 3\,\theta + 3\,\rho\cos\theta - 6\,K\,y_0\cos 2\,\theta - 3\,\rho\cos 3\,\theta\right]. \quad (11)$$

and,

$$s_\theta = -\,\frac{p}{6\,K^3}\left[6\,K^3\left(\rho\cos^3\theta + x_0\cos^2\theta\right) + 12\left(\rho\sin^3\theta + y_0\sin^2\theta\right)\right.$$
$$\left. - \,6\,K\left(3\,\rho\sin^2\theta\cos\theta + 2\,y_0\sin\theta\cos\theta + x_0\sin^2\theta\right)\right] \ldots\ldots (12)$$

in which ρ and θ are the polar co-ordinates.



(a) (b)

FIG. 3

If a solid disk of radius, $\rho = r_o$ (outer radius), is considered to be cut from the triangular section, the stresses acting on its boundary are obtained by inserting $\rho = r_o$ in Equations (10) and (11). If, now, $\rho = r_i$ (inner radius) is the radius of the circular opening, then in accordance with the assumption that the stresses on the boundary, $\rho = r_o$, are the same as if no hole existed, the problem reduces to the study of an annular closed ring subjected to the following stresses:

$$s^{\rho o} = A_0 + \sum_{n=1}^{n=3} A_n \cos n\,\theta + \sum_{n=1}^{n=3} B_n \sin n\,\theta \;\ldots\ldots\ldots (13a)$$

$$s_{\rho i} = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

$$s_{so} = C_0 + \sum_{n=1}^{n=3} C_n \cos n\,\theta + \sum_{n=1}^{n=3} D_n \sin n\,\theta \ldots\ldots\ldots (13c)$$

and,

$$s_{si} = 0 \qquad \text{(13}d\text{)}$$

in which $s_{\rho o}$ = the radial stress for a radius, r_o; $s_{\rho i}$ = the radial stress for a radius, r_i; s_{so} = the shear stress for a radius, r_o; s_{si} = the shear stress for a radius, r_i; and A_o, C_o, A_n, B_n, C_n, and D_n are defined as:

$$A_o = -\frac{p}{2\,K^3}\,(K^3\,x_o + 2\,y_o - K\,x_o) \qquad \text{(14}a\text{)}$$

$$A_1 = -\frac{p\,r_o}{4\,K^3}\,(K^3 - K) = D_1 \qquad \text{(14}b\text{)}$$

$$A_2 = -\frac{p}{2\,K^3}\,(2\,y_o - K\,x_o - K^3\,x_o) = -D_2 \qquad \text{(14}c\text{)}$$

$$A_3 = \frac{p\,r_o}{4\,K^3}\,(K^3 + 3\,K) = -D_3 \qquad \text{(14}d\text{)}$$

$$B_1 = -\frac{p\,r_o}{2\,K^3} = B_3 = C_3 = -C_1 \qquad \text{(14}e\text{)}$$

$$B_2 = -\frac{p\,y_o}{K^3} = C_2 \qquad \text{(14}f\text{)}$$

and,

$$C_o = 0 \qquad \text{(14}g\text{)}$$

The analysis of this annular ring, given in Appendix II, results in the following formulas for the radial, circumferential, and shear stresses in the proximity of the opening:

$$\begin{aligned} s_\rho = {} & A_o\left(1 - \frac{r^2_h}{\rho^2}\right) + r_h\left(M_1\cos\theta + M_2\sin\theta\right)\left(\frac{\rho}{r_h} - \frac{r^3_h}{\rho^3}\right) \\ & - \left(A_2\cos 2\,\theta + B_2\sin 2\,\theta\right)\left(4\,\frac{r^2_h}{\rho^2} - 1 - 3\,\frac{r^4_h}{\rho^4}\right) \\ & - r_h\left(M_3\cos 3\,\theta + M_2\sin 3\,\theta\right)\left(-\frac{\rho}{r_h} + 5\,\frac{r^3_h}{\rho^3} - 4\,\frac{r^5_h}{\rho^5}\right) \qquad \text{(15)} \end{aligned}$$

$$\begin{aligned} s_\theta = {} & A_o\left(1 + \frac{r^2_h}{\rho^2}\right) + r_h\left(M_1\cos\theta + M_2\sin\theta\right)\left(3\,\frac{\rho}{r_h} + \frac{r^3_h}{\rho^3}\right) \\ & - \left(A_2\cos 2\,\theta + B_2\sin 2\,\theta\right)\left(3\,\frac{r^4_h}{\rho^4} + 1\right) \\ & + r_h\left(M_3\cos 3\,\theta + M_2\sin 3\,\theta\right)\left(-\frac{\rho}{r_h} + \frac{r^3_h}{\rho^3} - 4\,\frac{r^5_h}{\rho^5}\right) \qquad \text{(16)} \end{aligned}$$

and,

$$s_s = r_h \left(\frac{\rho}{r_h} - \frac{r^3{}_h}{\rho^3}\right)\left(M_1 \sin\theta - M_2 \cos\theta\right) + \left(A_2 \sin 2\,\theta - B_2 \cos 2\,\theta\right)$$

$$\left(3\,\frac{r^4{}_h}{\rho^4} - 1 - 2\,\frac{r^2{}_h}{\rho^2}\right) + r_h\left(4\,\frac{r^5{}_h}{\rho^5} - \frac{\rho}{r_h} - 3\,\frac{r^3{}_h}{\rho^3}\right)\left(M_3 \sin 3\,\theta - M_2 \cos 3\,\theta\right) . \text{(17)}$$

in which $M_1 = -\frac{p}{4\,K^3}\left(K^3 - K\right)$; $M_2 = -\frac{p}{2\,K^3}$; and, $M_3 = \frac{p}{4\,K^3}\left(K^3 + 3\,K\right)$.

Equations (15), (16), and (17) suffice for the determination of the principal stresses near the opening. The hoop stresses may be determined from Equation (16) using $\rho = r_h$ and results in,

$$s_\theta = 2\,A_0 + 4\,r_h\,(M_1 \cos\theta + M_2 \sin\theta) - 4\,(A_2 \cos 2\,\theta + B_2 \sin 2\,\theta)$$

$$- 4\,r_h\,(M_3 \cos 3\,\theta + M_2 \sin 3\,\theta) \ldots\ldots\ldots\ldots\ldots \text{(18)}$$

STRESSES DUE TO MASS

The equations of equilibrium of a particle (Fig. 3(*b*)) under the action of gravity forces are as follows:

$$\frac{\partial s_\rho}{\partial \rho} + \frac{\partial s_s}{\rho\,\partial\theta} + \frac{s_\rho - s_\theta}{\rho} + c\cos(\theta - \beta) = 0 \ldots\ldots\ldots\ldots \text{(19a)}$$

and,

$$\frac{\partial s_\theta}{\rho\,\partial\theta} + \frac{\partial s_s}{\partial\rho} + \frac{2\,s_s}{\rho} - c\sin(\theta - \beta) = 0 \ldots\ldots\ldots\ldots \text{(19b)}$$

in which c = the weight per unit volume; and β = the angle between the direction of gravity and the X-axis.

If the stresses are now defined by,

$$s_\rho = \frac{\partial^2 F}{\rho^2\,\partial\,\theta^2} + \frac{1}{\rho}\,\frac{\partial F}{\partial\rho} - c\,\rho\cos(\theta - \beta) \ldots\ldots\ldots\ldots \text{(20a)}$$

$$s_\theta = \frac{\partial^2 F}{\partial\rho^2} - c\,\rho\cos(\theta - \beta) \ldots\ldots\ldots\ldots\ldots\ldots \text{(20b)}$$

and,

$$s_s = -\frac{\partial}{\partial\rho}\left(\frac{\partial F}{\rho\,\partial\,\theta}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(20c)}$$

the equations of equlibrium and the compatibility resulting from Hooke's law will be satisfied if F is a solution of Equation (21):

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\,\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\,\frac{\partial^2}{\partial\theta^2}\right)\left(\frac{\partial^2 F}{\partial\rho^2} + \frac{1}{\rho}\,\frac{\partial F}{\partial\rho} + \frac{1}{\rho^2}\,\frac{\partial^2 F}{\partial\theta^2}\right) = 0 \ldots \text{(21)}$$

It will be found that the Airy function, F, corresponding to mass and satisfying the boundary conditions for a triangular section is given by,

$$F_g = \frac{1}{6}\,[c_1\,x^3 + (c_1\,k_2 + c_2 - 2\,c_2\,k^2_2)\,y^3 + 3\,c_2\,k_2\,x\,y^2] \ldots\ldots \text{(22)}$$

in which, $c_1 = c\cos\beta$; $c_2 = c\sin\beta$; $k_2 = \cot\alpha$ (see Fig. 2). Transferring the origin to x_o, y_o, and using the definitions stated by Equations (20), the stresses are the following:

$$\begin{aligned} s_\rho = \frac{1}{6}\Bigg[&\left\{\frac{3\,\rho\,c_1}{2} + \frac{3\,\rho}{2}\,c_2\,k_2 - 6\,c_1\,\rho\right\}\cos\theta \\ &+\Big\{3\,c_2\,k_2\,x_o + 3\,y_o\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2) - 3\,c_1\,x_o\Big\}\cos 2\,\theta \\ &+\left\{\frac{9\,\rho}{2}\,c_2\,k_2 - \frac{3\,\rho}{2}\,c_1\right\}\cos 3\,\theta + \left\{\frac{3\,\rho}{2}\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2) - 6\,c_2\,\rho\right\}\sin\theta \\ &- 6\,c_2\,k_2\,y_o\sin 2\,\theta + \frac{3\,\rho}{2}\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2)\sin 3\,\theta \\ &+\Big\{3\,y_o\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2) + 3\,c_2\,k_2\,x_o - 3\,c_1\,x_o - 6\,y_o\,c_2\Big\}\Bigg] \ldots \text{(23)} \end{aligned}$$

$$\begin{aligned} s_\theta = \frac{1}{6}\Bigg[&\left\{\frac{9\,\rho}{2}\,c_1 + \frac{9\,\rho}{2}\,c_2\,k_2 - 6\,c_1\,\rho\right\}\cos\theta \\ &+\Big\{3\,c_1\,x_o - 3\,c_2\,k_2\,x_o - 3\,y_o\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2)\Big\}\cos 2\,\theta \\ &+\left\{\frac{3\,\rho}{2}\,c_1 - \frac{9}{2}\,c_2\,k_2\,\rho\right\}\cos 3\,\theta + \left\{\frac{9\,\rho}{2}\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2) - 6\,c_2\,\rho\right\}\sin\theta \\ &+ 6\,c_2\,k_2\,y_o\sin 2\,\theta - \frac{3\,\rho}{2}\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2)\sin 3\,\theta \\ &+\Big\{3\,y_o\,(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2) + 3\,c_2\,k_2\,x_o - 3\,c_1\,x_o - 6\,y_o\,c_2\Big\}\Bigg] \ldots \text{(24)} \end{aligned}$$

and,

$$\begin{aligned} s_s = -\frac{1}{6}\Bigg[&\frac{3\,\rho}{2}\Big(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2\Big)\cos\theta + 6\,y_o\,c_2\,k_2\cos 2\,\theta \\ &-\left\{\frac{3\,\rho}{2}\Big(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2\Big)\right\}\cos 3\,\theta + \left\{\frac{9\,\rho\,c_1}{2} - \frac{3}{2}\,k_2\,c_2\,\rho - 6\,\rho\,c_1\right\}\sin\theta \\ &+\Big\{3\,c_2\,k_2\,x_o + \Big(c_1\,k_2 + c_2 - 2\,c_2\,k^2_2\Big)3\,y_o - 3\,x_o\,c_1\Big\}\sin 2\,\theta \\ &+\left\{\frac{9\,\rho}{2}\,c_2\,k_2 - \frac{3\,\rho}{2}\,c_1\right\}\sin 3\,\theta\Bigg] \ldots\ldots\ldots\ldots\ldots\ldots \text{(25)} \end{aligned}$$

The stresses given in Equations (23), (24), and (25) may be broken into two separate stress systems each of which are compatible and satisfy Equation (21). Let $s_\rho = (s_\rho)_{\mathrm{I}} + (s_\rho)_{\mathrm{II}}$; $s_\theta = (s_\theta)_{\mathrm{I}} + (s_\theta)_{\mathrm{II}}$; and $s_s = (s_s)_{\mathrm{I}} + (s_s)_{\mathrm{II}}$. The stresses defined by the subscript, II, or the stresses in System II, are given by:

$$(s_\rho)_{\mathrm{II}} = - c_1 \rho \cos \theta - c_2 \rho \sin \theta \ldots\ldots\ldots\ldots \tag{26a}$$

$$(s_\theta)_{\mathrm{II}} = - c_1 \rho \cos \theta - c_2 \rho \sin \theta \ldots\ldots\ldots\ldots \tag{26b}$$

and,

$$(s_s)_{\mathrm{II}} = 0 \ldots\ldots\ldots\ldots \tag{26c}$$

The stresses of System I are then determined by: $(s_\rho)_{\mathrm{I}} = s_\rho - (s_\rho)_{\mathrm{II}}$; $(s_\theta)_{\mathrm{I}} = s_\theta - (s_\theta)_{\mathrm{II}}$; and, $(s_s)_{\mathrm{I}} = s_s$.

Stresses Due to System I.—From this system,

$$A_0 = \frac{y_0}{2}\left(c_1 k_2 - 2 c_2 k_2^2 - c_2\right) + \frac{x_0}{2}\left(c_2 k_2 - c_1\right) \ldots\ldots\ldots\ldots \tag{27a}$$

$$A_1 = D_1 = \frac{r_0}{4}\left(c_1 + c_2 k_2\right) \ldots\ldots\ldots\ldots \tag{27b}$$

$$A_2 = - D_2 = \frac{1}{2}\left\{x_0\left(c_2 k_2 - c_1\right) + y_0\left(c_1 k_2 + c_2 - 2 c_2 k_2^2\right)\right\} . \tag{27c}$$

$$A_3 = - D_3 = \frac{r_0}{4}\left(3 c_2 k_2 - c_1\right) \ldots\ldots\ldots\ldots \tag{27d}$$

$$B_1 = - C_1 = \frac{r_0}{4}\left(c_1 k_2 + c_2 - 2 c_2 k_2^2\right) = B_3 = C_3 \ldots\ldots\ldots\ldots \tag{27e}$$

$$B_2 = C_2 = - c_2 k_2 y_0 \ldots\ldots\ldots\ldots \tag{27f}$$

and,

$$C_0 = 0 \ldots\ldots\ldots\ldots \tag{27g}$$

Using Equation (43), Appendix II, the state of stress around the opening due to System I is described by the following:

$$\begin{aligned} s_\rho = {} & A_0\left(1 - \frac{r_h^2}{\rho^2}\right) + \frac{r_h}{4}\left[\left(c_1 + c\, k_2\right)\cos\theta + \left(c_1 k_2 + c_2 - 2 c_2 k_2^2\right)\sin\theta\right] \\ & \times\left(\frac{\rho}{r_h} - \frac{r_h^3}{\rho^3}\right) - \left(A_2 \cos 2\theta + B_2 \sin 2\theta\right)\left(4\frac{r_h^2}{\rho^2} - 1 - 3\frac{r_h^4}{\rho^4}\right) \\ & - \frac{r_h}{4}\left[\left(3 c_2 k_2 - c_1\right)\cos 3\theta + \left(c_1 k_2 + c_2 - 2 c_2 k_2^2\right)\sin 3\theta\right] \\ & \times\left[-\frac{\rho}{r_h} + 5\frac{r_h^3}{\rho^3} - 4\frac{r_h^5}{\rho^5}\right] \ldots\ldots\ldots\ldots \end{aligned} \tag{28}$$

$$s_\theta = A_0\left(1 + \frac{r_h^2}{\rho^2}\right) + \frac{r_h}{4}\left[\left(c_1 + c_2 k_2\right)\cos\theta + \left(c_1 k_2 + c_2 - 2\,c_2 k_2^2\right)\sin\theta\right]$$
$$\times\left(3\,\frac{\rho}{r_h} + \frac{r_h^3}{\rho^3}\right) - \left(A_2\cos 2\,\theta + B_2\sin 2\,\theta\right)\left(3\,\frac{r_h^4}{\rho^4} + 1\right)$$
$$+ \frac{r_h}{4}\left[\left(3\,c_2 k_2 - c_1\right)\cos 3\,\theta + \left(c_1 k_2 + c_2 - 2\,c_2 k_2^2\right)\sin 3\,\theta\right]$$
$$\times\left(-\frac{\rho}{r_h} + \frac{r_h^3}{\rho^3} - 4\,\frac{r_h^5}{\rho^5}\right) \ldots\ldots\ldots\ldots(29)$$

and,

$$s_s = \frac{r_h}{4}\left[\left(c_1 + c_2 k_2\right)\sin\theta + \left(c_1 k_2 + c_2 - 2\,c_2 k_2^2\right)\cos\theta\right]\left(\frac{\rho}{r_h} - \frac{r_h^3}{\rho^3}\right)$$
$$+ \left(A_2\sin 2\,\theta - B_2\cos 2\,\theta\right)\left(3\,\frac{r_h^4}{\rho^4} - 1 - 2\,\frac{r_h^2}{\rho^2}\right)$$
$$+ \frac{r_h}{4}\left[\left(3\,c_2 k_2 - c_1\right)\sin 3\,\theta - \left(c_1 k_2 + c_2 - 2\,c_2 k_2^2\right)\cos 3\,\theta\right]$$
$$\times\left[4\,\frac{r_h^5}{\rho^5} - \frac{\rho}{r_h} - 3\,\frac{r_h^3}{\rho^3}\right] \ldots\ldots\ldots\ldots(30)$$

Substituting $\rho = r_h$ in Equation (29) the hoop stresses are:

$$s_\theta = 2\,A_0 + r_h\left[\left(c_1 + c_2 k_2\right)\cos\theta + \left(c_1 k_2 + c_2 - 2\,c_2 k_2^2\right)\sin\theta\right]$$
$$- 4\left(A_2\cos 2\,\theta + B_2\sin 2\,\theta\right)$$
$$- r_h\left[\left(3\,c_2 k_2 - c_1\right)\cos 3\,\theta + \left(c_1 k_2 + c_2 - 2\,c_2 k_2^2\right)\sin 3\,\theta\right] \ldots.(31)$$

Hoop Stresses Due to System II.—An investigation of System II shows that these stresses hold the weight of a solid disk of radius r_o, in equilibrium, whereas the stresses of System I give a zero resultant on the circumference, $\rho = r_o$. Consider an annular disk of the outer radius, r_o, and the inner radius, r_i, subjected to gravity forces. If all the stresses on the radius, r_i are zero, the resultant of all the forces on the radius, r_o, must be equal to the weight of the disk. If System II is to be applied to an annular disk, c must be made to equal to:

$$c = -\,c\left(\frac{r_i^2}{r_o^2} - 1\right) \ldots\ldots\ldots\ldots(32)$$

resulting in a modified System II given by $(s_r)_{\text{II}} = c\left(\frac{r_i^2}{r_o^2} - 1\right)\rho\cos\theta'$; $(s_\theta)_{\text{II}} = c\left(\frac{r_i^2}{r_o^2} - 1\right)\rho\cos\theta'$; $(s_s)_{\text{II}} = 0$; and $\theta' = \theta - \beta$.

An Airy function, F, may be written for this system:

$$F = \tfrac{1}{2}\, c\, r_i^2\, \rho\, \theta \sin \theta \ldots\ldots (33)$$

From Equation (20) it follows that,

$$s_\rho = c \left(\frac{r_i^2}{\rho} - \rho \right) \cos \theta \ldots\ldots (34a)$$

$$s_\theta = -\, c\, \rho \cos \theta \ldots\ldots (34b)$$

and,

$$s_s = 0 \ldots\ldots (34c)$$

For $\rho = r_i$, $s_\rho = s_s = 0$; and, for $\rho = r_o$, $s_\rho = c \left(\frac{r_i^2}{r_o} - r_o \right) \cos \theta$; and, $s_s = 0$.

The hoop stresses due to System II are $s_\theta = -\, c\, r_h \cos \theta$. These stresses must be added to those of System I to describe the complete state of stress due to mass.

Application.—The foregoing equations will be applied to the determination of the stresses around a circular hole in a buttress of a hollow type dam with the following design characteristics:

Clear spacing between buttresses, in feet	55
Average thickness of buttress, in feet	5
Up-stream slope of buttress	0.9
Down-stream slope of buttress	0.36
Height of dam, h, in feet	200
Radius of circular opening, $0.02\,h$, in feet	4
Weight of water, in pounds per cubic foot	62.5
Weight of concrete, c, in pounds per cubic foot	150

The location of the center of the opening is given by $x_0 = 0.5h$, and $y_0 = 0.2h$. From the spacing, thickness, and up-stream slope of the buttress, $p = \frac{(62.5)\,(55 + 5)\,(\sin 48^\circ)}{5} = 556$ lb per sq. ft.; and $c = p \times \frac{150}{556} = 0.27\, p$. It follows that: $K = \tan \alpha = 1.864$; $K_2 = \cot \alpha = 0.5365$; and, $\beta = 42$ degrees.

Stresses Due to Water Load.—Using Equations (14):

$$A_0 = -\, 0.2089\, p\, h \qquad M_1 = -\, 0.1780\, p$$
$$A_2 = +\, 0.29\, p\, h \qquad M_2 = -\, 0.07721\, p$$
$$B_2 = -\, 0.05757\, p\, h \qquad M_3 = +\, 0.4657\, p$$

Substituting in Equation (18) the "hoop stresses" due to water load are:

$$s_\theta = p\, h\, [-\, 0.4178 - 0.01424 \cos \theta - 1.16 \cos 2\theta - 0.03726 \cos 3\theta$$
$$-\, 0.006177 \sin \theta + 0.2303 \sin 2\theta + 0.006177 \sin 3\theta] \ldots\ldots (35)$$

Stresses Due to Mass.—Let $c_1 = c \cos \beta = 0.7431\, c$; and, $c_2 = c \sin \beta = 0.6691\, c$. From Equations (27): $A_0 = -\, 0.1616\, c\, h$; $A_2 = -\, 0.02780\, c\, h$; and, $B_2 = -\, 0.07180\, c\, h$.

The "hoop stresses" due to mass are combined with System II:

$$s_\theta = c\, h \Big[-0.3232 + 0.02204 \cos \theta + 0.1112 \cos 2\theta - 0.006678 \cos 3\theta + 0.01365 \sin \theta + 0.2872 \sin 2\theta - 0.01365 \sin 3\theta \Big] \dots\dots\dots (36)$$

Combining Equations (35) and (36) the "hoop stresses" due to water and mass are, when $c = 0.27\, p$, as follows:

$$s_\theta = p\, h \Big[-0.5051 - 0.00829 \cos \theta - 1.13 \cos 2\theta - 0.03906 \cos 3\theta - 0.00249 \sin \theta + 0.3078 \sin 2\theta + 0.00249 \sin 3\theta \Big] \dots\dots\dots (37)$$

TABLE 1.—Hoop Stresses Due to Combined Mass and Water Loads ($p\, h = 771$ lb per in.^{2})

− = Compression			+ = Tension		
θ	s_θ	Unit stress, in pounds per square inch	θ	s_θ	Unit stress, in pounds per square inch
0	$-1.6825\, p h$	−1300	180	$-1.5877\, p h$	−1221
30	$-0.8094\, p h$	−623	210	$-0.7976\, p h$	−613
60	$+0.3583\, p h$	+276	240	$+0.2937\, p h$	+226
90	$+0.6200\, p h$	+478	270	$+0.6300\, p h$	+485
120	$-0.2438\, p h$	−188	300	$-0.1696\, p h$	−131
150	$-1.3282\, p h$	−1022	330	$-1.3451\, p h$	−1035

These stresses are plotted in Fig. 4 and the unit stresses are given in Table 1. Maximum tensile stresses exist in the neighborhood of $\theta = 90°$ and $\theta = 270$ degrees. For $\theta = 90°$, the circumferential stress, s_θ, as a function of $\left(\frac{\rho}{r_h}\right)$ is given by;

$$s_\theta = p\, h \Big[-0.2525 \left(1 + \frac{r^2_h}{\rho^2}\right) - 0.000622 \left(3\, \frac{\rho}{h} + \frac{r^3_h}{\rho^3}\right) + 0.2825 \left(3 \frac{r^4_h}{\rho^4} + 1\right) \times 0.000622 \left(-\frac{\rho}{r_h} + \frac{r^3_h}{\rho^3} - 4\, \frac{r^5_h}{\rho^5}\right) \Big] \dots\dots\dots (38)$$

These stresses are plotted in Fig. 5.

To determine the total tension which must be taken by the reinforcing steel the area under the curve of Fig. 5 may be determined, or the following integral evaluated:

$$\text{Total tension} = \int_{r_h}^{2.5 r_h} s_\theta \, d\rho \dots\dots\dots (39)$$

in which s_θ as given by Equation (38) is to be inserted in Equation (39). The upper limit is determined by an examination of Fig. 5. Performing the integration: Total tension $= ph\, r_h\, [-0.2525\ (2.1) - 0.000622\ (8.295) + 0.2825 \times 2.436 + 0.000622\ (-\ 3.1794)] = 68\,988$ lb. Using an average stress of 18 000 lb per sq in., the area of steel required is $\frac{68\,988}{18\,000} = 3.83$ in.^{2} The spacing of this steel is easily determined from a consideration of Fig. 5. The percentage of steel is $\frac{3.83}{(1.5\ r_h)\ (144)} = 0.444$ per cent. These values are for a unit thickness of buttress.

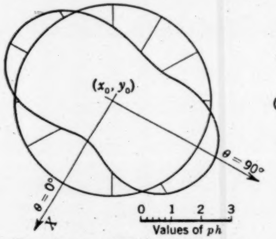


FIG. 4.—s_θ STRESSES DUE TO MASS AND WATER

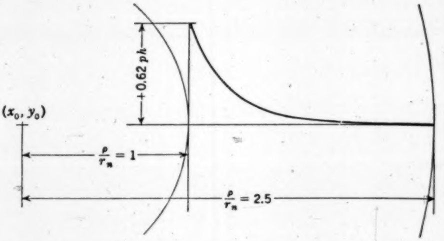


FIG. 5.—VARIATION OF s_θ STRESS ALONG SECTION $\theta = 90$ DEGREES

CONCLUSIONS

The formulas derived in this paper enable the designer to determine the regions around openings in dams and buttresses where tensile stresses may exist. Stress trajectories may also be determined by use of these equations. Furthermore, an estimate may be obtained of the reinforcement needed to take care of such tensile stress. In the case of gravity dams these equations are limited to longitudinal galleries.

APPENDIX I

NOTATION

In the following notation, presented for convenience of reference, an effort has been made to conform as nearly as practicable with "Symbols for Mechanics, Structural Engineering, and "Testing Materials", advanced by the American Standards Association.[2] Symbols used simply as general mathematical coefficients are not included in the list, their definitions being given clearly in the text. Where such coefficients conflict with symbols also used in the paper (such as c = mass), the distinction is explained at each point.

C = a constant;
c = weight per unit volume;
F = the Airy stress function;

f = a substitution factor defined by Equations (47*d*) and (47*h*), Appendix II;

G = a substitution factor defined by Equation (46*b*), Appendix II;

g = the gravity constant; also, where so indicated, g_n = a substitution factor defined by Equations (47*e*) and (47*i*), Appendix II;

h = height; also, where so indicated, h_n = a substitution factor defined by Equations (47*f*) and (47*j*), Appendix II;

J = a substitution factor defined by Equation (47*c*), Appendix II;

K = $\tan \alpha$ = a substitution factor;

k_n = a substitution factor defined by Equations (47*g*) and (47*k*), Appendix II; $k_2 = \cot \alpha$;

$\left.\begin{matrix} M_1 \\ M_2 \\ M_3 \end{matrix}\right\}$ = (see Equations (48), (49), and (50), Appendix II);

n = an abstract number;

p = unit, hydrostatic water pressure;

q = the ratio, $\dfrac{r_i}{r_o}$;

r = radius of a circle, as distinct from the polar co-ordinate distance, ρ; r_h = radius of a circular hole; r_i = the inner radius of an annular ring; r_o = outer radius of an annular ring;

s = unit stress in general; in particular, s = total axial load divided by gross area; s_n = normal stress; s_s = shear stress; s_x = stress in an X-direction; s_y = stress in a Y-direction; s_ρ = stress in a radial plane; s_θ = stress perpendicular to a radial plane at any angle, θ; s_I = stresses defined as belonging to System I; s_{II} = stresses defined as belonging to System II;

x = distance measured in an X-direction; x_0 = the x-co-ordinate of center of hole;

y = distance measured in a Y-direction; y_0 = the y-co-ordinate of center of hole;

α = angle between the battered faces of a dam.

θ = angular distance; a polar co-ordinate;

ρ = a radial distance; polar co-ordinate;

APPENDIX II

ANALYSIS OF ANNULAR RING

An annular ring of outer radius, r_o, inner radius, r_i subjected to the boundary stresses as given by Equations (13) can best be studied by use of Airy's function in polar co-ordinates. Corresponding to Equation (7) in cartesian co-ordinates, there is a similar equation in polar co-ordinates:

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 \phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \phi}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \phi}{\partial \theta^2}\right) = 0 \quad \ldots (40)$$

with stresses defined as follows:

$$\left.\begin{aligned} s_\rho &= \frac{1}{\rho}\frac{\partial\phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\rho^2} \\ s_\theta &= \frac{\partial^2\phi}{\partial\theta^2} \\ s_s &= -\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial\phi}{\partial\theta}\right) \end{aligned}\right\} \qquad (41)$$

and,

The general solution[5] of Equation (40) is:

$$\begin{aligned} \phi &= a_0 \log\rho + b_0\rho^2 + c_0\rho^2\log\rho + d_0\rho^2\theta + a'_0\theta \\ &+ \frac{a_1}{2}\rho\,\theta\sin\theta + \left(b_1\rho^3 + a'_1\rho^{-1} + b'_1\rho\log\rho\right)\cos\theta \\ &- \frac{c_1}{2}\rho\,\theta\cos\theta + \left(d_1\rho^3 + c'_1\rho^{-1} + d'_1\rho\log\rho\right)\sin\theta \\ &+ \sum_{n=2}^{n=\infty}\left(a_n\rho^n + b_n\rho^{n+2} + a'_n\rho^{-n} + b'_n\rho^{-n+2}\right)\cos n\theta \\ &+ \sum_{n=2}^{n=\infty}\left(c_n\rho^n + d_n\rho^{n+2} + c'_n\rho^{-n} + d'_n\rho^{-n+2}\right)\sin n\theta \qquad (42) \end{aligned}$$

in which a_n, b_n, c_n, and d_n are mathematical constants. In order that the stress distribution and displacements may be single valued, $c_0 = d_0 = 0$; and in order to satisfy the boundary conditions imposed by Equations (13):

$$a_0 = -\frac{r_o^2\,q^2}{1-q^2}A_0 \qquad (43a)$$

and,

$$b_0 = \frac{1}{2}\frac{A_0}{1-q^2} \qquad (43b)$$

From the fact that $A_1 = D_1$ and $B_1 = -C_1$, it follows that $d'_1 = b'_1 = 0$. In their complete study of the circular ring, Messrs. Coker and Filon[6], have shown that the remaining constants are given by,

$$b_1 r_o = \frac{A_1}{2(1-q^4)} \qquad (44a)$$

$$a_1 = \frac{A_1 r_o^3 q^4}{2(1-q^4)} \qquad (44b)$$

$$d_1 r_o = \frac{B_1}{2(1-q^4)} \qquad (44c)$$

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[5] "Theory of Elasticity", S. Timoshenko, p. 114.

[6] "Photo-Elasticity", by E. G. Coker and L. N. G. Filon, 1931, pp. 374 *et seq.*

and,

$$c'_1 = \frac{B_1 r^3_o q^4}{2(1 - q^4)} \tag{44d}$$

When n is equal to, or greater than, 2:

$$b_n r_o^n = Q_n f_n - G_n g_n \tag{45a}$$

$$a_n r_o^{n-2} = Q_n g_n - J_n f_n \tag{45b}$$

$$a'_n r_o^{-n} b^{-2} = Q_n h_n - G_n k_n \tag{45c}$$

$$b'_n r_o^{-n} = Q_n k_n - J_n h_n \tag{45d}$$

$$\mathrm{d}_n \mathrm{r}_o^n = Q_n f'_n - G_n g'_n \tag{45e}$$

$$c_n r_o^{n-2} = Q_n g'_n - J_n f'_n \tag{45f}$$

$$c'_n r_o^{-n} b^{-2} = Q_n h'_n - G_n k'_n \tag{45g}$$

and,

$$d'_n r_o^{-n} = Q_n k'_n - J_n h'_n \tag{45h}$$

in which,

$$Q_n = \frac{1 - q^{2n}}{2n\,[(1 - q^{2n})^2 - n^2 q^{2n-2} (1 - q^2)^2]} \tag{46a}$$

$$G_n = \frac{1 - q^{2n-2}}{2(n + 1)\,[(1 - q^{2n})^2 - n^2 q^{2n-2} (1 - q^2)^2]} \tag{46b}$$

$$J_n = \frac{1 - q^{2n+2}}{2(n - 1)\,[(1 - q^{2n})^2 - n^2 q^{2n-2} (1 - q^2)^2]} \tag{46c}$$

$$f_n = n(A_n - D_n) \tag{46d}$$

$$g_n = n A_n - (n + 2) D_n \tag{46e}$$

$$h_n = -q^{2n-2} (n A_n + (n - 2) D_n) \tag{46f}$$

$$k_n = -n q^{2n} (A_n + D_n) \tag{46g}$$

$$f'_n = n(B_n + C_n) \tag{46h}$$

$$g'_n = n B_n + (n + 2) C_n \tag{46i}$$

$$h'_n = -q^{2n-2} \{ n B_n - (n - 2) C_n \} \tag{46j}$$

and,

$$k'_n = -n q^{2n} \{ B_n - C_n \} \tag{46k}$$

Substituting values of A_n in Equations (14) into Equations (46), the factors are determined to be as listed in Table 2. These values can be used

TABLE 2.—LIST OF FACTORS IN EQUATIONS (46), IN TERMS OF A_n AND B_n

Values of n	f	f'	g	g'	h	h'	k	k'
2	$4A_2$	$4B_2$	$6A_2$	$6B_2$	$-2q^2A_2$	$-2q^2B_2$	0	0
3	$6A_3$	$6B_3$	$8A_3$	$8B_3$	$-2q^4A_3$	$-2q^4B_3$	0	0

for the solution of Equations (45). Assuming that a plate is infinite so that $q = \frac{r_1}{r_o} = 0$, the stresses, s_O, s_θ, and s_s, can be obtained by the use of Equations (41) to (45), inclusive, the result being as follows:

$$\begin{aligned} s_\rho = {} & A_o\left(1 - \frac{r^2_h}{\rho^2}\right) + r_h\left(M_1 \cos\theta + M_2 \sin\theta\right)\left(\frac{\rho}{r_h} - \frac{r^3_h}{\rho^3}\right) \\ & - \left(A_2 \cos 2\,\theta + B_2 \sin 2\,\theta\right)\left(4\,\frac{r^2_h}{\rho^2} - 1 - 3\,\frac{r^4_h}{\rho^4}\right) \\ & - r_h\left(M_3 \cos 3\,\theta + M_2 \sin 3\,\theta\right)\left(-\frac{\rho}{r_h} + 5\,\frac{r^3_h}{\rho^3} - 4\,\frac{r^3_h}{\rho^3}\right) \qquad \text{......(47)} \end{aligned}$$

$$\begin{aligned} s_\theta = {} & A_o\left(1 + \frac{r^2_h}{\rho^2}\right) + r_h\left(M_1 \cos\theta + M_2 \sin\theta\right)\left(3\,\frac{\rho}{r_h} + \frac{r^5_h}{\rho^5}\right) \\ & - \left(A_2 \cos 2\,\theta + B_2 \sin 2\,\theta\right)\left(3\,\frac{r^4_h}{\rho^4} + 1\right) \\ & + r_h\left(M_3 \cos 3\,\theta + M_2 \sin 3\,\theta\right)\left(-\frac{\rho}{r_h} + \frac{r^3_h}{\rho^3} - 4\,\frac{r^5_h}{\rho^5}\right) \qquad \text{.......(48)} \end{aligned}$$

and,

$$\begin{aligned} s_s = {} & r_h\left(\frac{\rho}{r_h} - \frac{r^3_h}{\rho^3}\right)\left(M_1 \sin\theta - M_2 \cos\theta\right) \\ & + \left(3\,\frac{r^4_h}{\rho^4} - 1 - 2\,\frac{r^2_h}{\rho^2}\right)\left(A_2 \sin 2\,\theta - B_2 \cos 2\,\theta\right) \\ & + r_h\left(4\,\frac{r^5_h}{\rho^5} - \frac{\rho}{r_h} - 3\,\frac{r^3_h}{\rho^3}\right)\left(M_3 \sin 3\,\theta - M_2 \cos 3\,\theta\right) \qquad \text{.......(49)} \end{aligned}$$

in which $M_1 = -\frac{p}{4\,K^3}\left(K^3 - K\right)$; $M_2 = -\frac{p}{2\,K^3}$; and $M_3 = \frac{p}{4\,K^3}\left(K^3 + 3\,K\right)$

DISCUSSION

R. D. MINDLIN,[7] JUN. AM. SOC. C. E. (by letter).—In examining Equations (28), (29), (30), and (34) for the stresses around a circular hole resulting from the gravitational action of a dam, it is observed that the elastic constants of the material are not involved. This is surprising in the light of a recent extension by M. A. Biot[8] of a theorem by J. H. Michell[9].

Michell's theorem applies to a two-dimensional stress system in a region pierced by holes, no body forces being present. He showed that in such a system the stresses are independent of the elastic constants only if the resultant force over each hole vanishes. This conclusion is found necessary in order to assure single-valuedness of the displacements.

Biot has extended Michell's theorem to the case of gravity stresses. He has shown[8] that "in a solid two-dimensional homogeneous body the stresses due to gravity do not depend on the elasticity constants of the material; they do, however, in general for a body with holes." Therefore, a more detailed examination of the stresses due to mass is indicated.

The author has conveniently divided the stresses in the solid dam into two systems. In System I the stresses on any closed boundary produce zero resultant force. Hence, the additional stresses required to remove the normal and shearing stresses on the periphery of a circular hole also involve zero resultant force on the hole. No difficulty with many valued displacements is encountered, therefore, in dealing with System I, and the author has found the required stress system by a well known method.

In dealing with System II, Mr. Silverman has observed that these stresses produce a resultant force on an internal circular boundary equal to the weight of the enclosed disk. In order to remove the stresses from the periphery of the hole, an additional stress system must be superposed which yields equal and opposite stresses on the circular boundary. The latter system, then, must involve a resultant stress on the hole equal and opposite to the weight of the enclosed disk of radius, r_h. Therefore, according to Michell's theorem, the superimposed system should involve the elastic constants. This was to be expected from Biot's theorem. In fact, the foregoing explanation is simply a derivation of Biot's theorem for the present case.

If the elastic constants do not appear in a case where the resultant force on an internal boundary does not vanish, then, by Michell's theorem, the displacements resulting from the stresses should be many valued. Furthermore, when the internal boundary is closed, as in this case, many-valued functions in the displacements are not permissible, and a solution containing them is not valid.

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[7] Instr., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[8] "Distributed Gravity and Temperature Loading in Two-Dimensional Elasticity Replaced by Boundary Pressures and Dislocations", by M. A. Biot, *Journal of Applied Mechanics*, Vol. 2, No. 2 (June, 1935), p. 41.

[9] "On the Direct Determination of Stress in an Elastic Solid, with Application to the Theory of Plates", by J. H. Michell, *Proceedings*, London Math. Soc. Vol. XXXI (1899), p. 100.

It is necessary, therefore, to proceed to examine the stress function (Equation (33)) which was used to annul the stresses on the boundary, $\rho = r_i$. Writing $c\, r^2_i = a_1$, Equation (33) becomes:

$$F_1 = \frac{a_1}{2}\, \rho\, \theta \sin \theta \qquad (50)$$

The stress function given in Equation (50) is known to produce many-valued functions in the displacements of the form[10], $\frac{a_1}{2}\,\frac{(1-\nu)}{E}\,\theta \sin \theta$; and $\frac{a_1}{2}\,\frac{(1-\nu)}{E}\,\theta \cos \theta$, respectively, in which ν = Poisson's ratio. This was to be expected from the preceding analysis, and it may be concluded that the stress system represented by Equations (34) is not possible in a continuous elastic medium.

To obtain the correct system of stress required to remove the stresses on the periphery of the hole, a bi-harmonic stress function must be found which will satisfy the following conditions:

(1) The displacements introduced by the stress function must be single-valued;

(2) The normal and shearing stresses on the circular boundary, $\rho = r_i$, must be equal and opposite to those of System II; and,

(3) All the stresses introduced by the new function must approach zero as ρ approaches infinity.

In order to satisfy Condition (1) one must combine with Equation (50) a stress function[10],

$$F_2 = b'_1\, \rho \log \rho \cos \theta \qquad (51)$$

in which,

$$b'_1 = -\,\frac{a_1\,(1-\nu)}{4} \qquad (52)$$

The combined functions, F_1 and F_2, satisfy Conditions (1) and (3), but not Condition (2). The addition of F_2 to the system has served to introduce shearing stresses on the circular boundary (proportional to $\sin \theta$), which are not present in System II or in F_1. Therefore, another function, F_3, must be added, which will remove these shearing stresses without introducing normal stresses on the circular boundary other than those proportional to $\cos \theta$. Furthermore, F_3 must satisfy Conditions (1) and (3). Such a function is given by:

$$F_3 = a'_1\, \rho^{-1} \cos \theta \qquad (53)$$

The complete stress function is then given by:

$$F = F_1 + F_2 + F_3 = \frac{a_1}{2}\, \rho\, \theta \sin \theta + b'_1\, \rho \log \rho \cos \theta + a'_1\, \rho^{-1} \cos \theta \qquad (54)$$

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[10] See "Theory of Elasticity", by S. Timoshenko, p. 116, McGraw-Hill, 1934.

To determine the values of the constants it is necessary first to calculate the stresses. Using Equations (20) of the paper, these are found to be:

$$s_\rho = \left(\frac{a_1}{\rho} + \frac{b'_1}{\rho} - \frac{2\,a'_1}{\rho^3} - c\,\rho\right)\cos\theta \;\dots\dots\dots\dots\;(55a)$$

$$s_\theta = \left(\frac{b'_1}{\rho} + \frac{2\,a'_1}{\rho^3} - c\,\rho\right)\cos\theta \;\dots\dots\dots\dots\;(55b)$$

and,

$$s_s = \left(\frac{b'_1}{\rho} - \frac{2\,a'_1}{\rho^3}\right)\sin\theta \;\dots\dots\dots\dots\;(55c)$$

Condition (2) is expressed by the equation:

$$[s_\rho]_{\rho=r_i} = [s_s]_{\rho=r_i} = 0 \;\dots\dots\dots\dots\;(56)$$

and, hence:

$$\frac{a_1}{r_i} + \frac{b'_1}{r_i} - \frac{2\,a'_1}{r^3_i} - c\,r_i = 0 \;\dots\dots\dots\dots\;(57a)$$

and

$$\frac{b'_1}{r_i} - \frac{2\,a'_1}{r^3_i} = 0 \;\dots\dots\dots\dots\;(57b)$$

Condition (1) is expressed by Equation (52). Solving for the three unknowns in Equations (52) and (57):

$$a_1 = c\,r^2_i \;\dots\dots\dots\dots\;(58a)$$

$$b'_1 = -\frac{1-\nu}{4}\,c\,r^2_i \;\dots\dots\dots\dots\;(58b)$$

and,

$$a'_1 = -\frac{1-\nu}{8}\,c\,r^4_i \;\dots\dots\dots\dots\;(58c)$$

Substituting these values in Equation (54) the final stress function becomes:

$$F = \frac{1}{2}\,c\,r^2_i\,\rho\,\theta\sin\theta - \frac{1-\nu}{4}\,c\,r^2_i\,\rho\log\rho\cos\theta - \frac{1-\nu}{8}\,c\,r^4_i\,\rho^{-1}\cos\theta \;\dots\;(59)$$

The corresponding stresses are, from Equations (20):

$$s_\rho = c\left(\frac{3+\nu}{4}\,\frac{r^2_i}{\rho} + \frac{1-\nu}{4}\,\frac{r^4_i}{\rho^3} - \rho\right)\cos\theta \;\dots\dots\dots\;(60a)$$

$$s_\theta = -\,c\left(\frac{1-\nu}{4}\,\frac{r^2_i}{\rho} + \frac{1-\nu}{4}\,\frac{r^4_i}{\rho^3} + \rho\right)\cos\theta \;\dots\dots\dots\;(60b)$$

and,

$$s_s = c\left(-\,\frac{1-\nu}{4}\,\frac{r^2_i}{\rho} + \frac{1-\nu}{8}\,\frac{r^4_i}{\rho^3}\right)\sin\theta \;\dots\dots\dots\;(60c)$$

Equations (60) express the correct stresses for System II. By virtue of Equation (52), Condition (1) is satisfied; by virtue of Equations (57*a*) and (57*b*), Condition (2) is satisfied; and an examination of Equations (60) shows that Condition (3) is satisfied. The hoop stresses at the circular boundary are found by substituting $\rho = r_i$ in Equation (60*b*):

$$[s_\theta]_{\rho=r_i} = -\left(\frac{3-\nu}{2}\right) c\, r_i \cos\theta \quad \text{.................} (61)$$

The foregoing analysis has been made for the case of plane stress. It is approximately applicable to a buttress if the diameter of the hole is small in comparison with its distance from the nearest edge of the buttress, and if the thickness of the buttress is small in comparison with the diameter of the hole. For the case of a longitudinal hole in a gravity section the stresses should be transformed to the corresponding system for plane strain.

Additional references should be given to the results of previous stress analyses for the wedge section. The stress systems represented by Equations (8) to (12) and (22) to (25), inclusive, are due to M. Levy[11] and may be obtained from equations given by Love[12]. This stress system applies only to portions of a dam remote from the base. Hence, in addition to the requirement that the small hole be far from the faces of the dam, it must also be far from the base.

Equations (8) and (22) are seen to be of the same form, differing only in the constants associated with the terms of the third degree polynomial. Equation (22) represents the stress which the author has designated as System I. Hence, the additional stress functions, required to annul the stresses due to System I on a circular boundary, will be the same as the stress functions required to annul the stresses on the circular boundary resulting from the water loading, except for the constants. Therefore, a solution of the problem of the stresses due to mass really includes that of the problem of stresses due to water load.

The author is undoubtedly unaware of a previous solution of this problem by G. Nishimura and T. Takayama[13]. Of the two solutions, the writer prefers that of Mr. Silverman as being far less cumbersome. Very skilfully the author has applied what mathematicians would call an elegant method and, independently, has obtained the solution of an interesting problem.

CHESLEY J. POSEY,[14] JUN. AM. SOC. C. E. (by letter).—The author is to be commended for presenting an analytical proof of the existence of tensile stresses near longitudinal circular holes in concrete gravity dams. The question of the proper reinforcement around holes is of importance in many types of concrete design. Although most designers would provide extra reinforcement around a hole in a girder web, many would not admit that any was necessary for a long, relatively small hole in mass concrete. Mr. Silverman's analysis, made on the basis of reasonable assumptions, shows that such reinforcement is necessary for longitudinal holes in gravity dams, to resist stresses due to water and mass loads. In some cases, stresses around the openings, due to volumetric changes from shrinkage and temperature, may call for even more reinforcement.

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[11] "Sur l'équilibre elastic d'un barrage en maçonnerie à section triangulaire," by M. Lévy, *Comptes Rendus*, Paris, Vol. 127 (1898), p. 10.

[12] "Theory of Elasticity", by A. E. H. Love, Fourth Edition, 1927, Equations (34*A*) and (34*B*), pp. 212 and 213.

[13] "On the Stress Distribution in the Vicinity of a Horizontal Circular Hole in a Gravitating Wedge-Shaped Elastic Solid", by G. Nishimura and T. Takayama, *Bulletin*, Earthquake Research Inst., Tokyo, Vol. 10 (1932), p. 723.

[14] Asst. Prof., Hydraulics and Structural Eng., State Univ. of Iowa, and Assoc. Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

The proper placing of steel bars around holes is of great importance. The steel should never follow around the inside of a hole, for if it is stressed in tension it will tend to straighten, causing tension in the concrete. As soon as this concrete fails, the steel is useless, and cracking is perhaps further advanced than if no steel had been used. Fig. 6(*a*) is an example of incorrect, and Figs. 6(*b*), 6(*c*), and 6(*d*) of correct, placing of the reinforcement around holes. The placing shown in Fig. 6(*d*) is preferable to that shown in Fig. 6(*c*), however. Diagonal bars should be provided at the corners of rectangular holes, for cracks tend to form at the corners, where vertical and horizontal bars are relatively less effective than diagonal bars. If it is certain that tensile stresses can never occur across one of the diagonal sections, the bars crossing that section may be omitted.

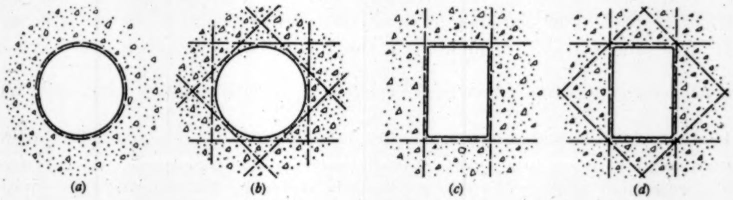

FIG. 6.—METHODS OF PLACING STEEL REINFORCEMENT AROUND HOLES IN MASS CONCRETE

Referring to Fig. 5 of the paper, it is seen that the tensile stresses decrease very rapidly away from the surface of the hole. The steel bars, therefore, should be as close to the surface of the hole as is practicable. The length of bar to be used cannot be computed by any method known at present, and, in the absence of test data, it must be arbitrarily assumed, say, equal to twice the diameter of the hole. The size of bar should then be selected so that the total bond on one-half its length is equal to or greater than its ultimate strength. To prevent cracks, the bar should have as high a ratio of bond stress to slip as is possible. Tests[15] by the writer show that the most favorable ratio is obtained if the bars are straight, and if their surface is rough.

J. H. A. BRAHTZ,[16] ESQ. (by letter).—A new approach is presented in this paper, in the analysis of the stresses in the vicinity of a cylindrical opening in a triangular gravity dam when the axis of the opening is parallel to the face of the dam. The main assumptions are that: (1) The water surface is at the theoretical apex of the dam; (2) the opening is far removed from the faces of the dam; (3) the material is perfectly elastic; and (4) the tensile stresses are so small that the concrete does not crack and, if it does crack, that sufficient steel reinforcement will make the dam act as a homogeneous elastic material.

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[15] "Tests of Anchorages for Reinforcing Bars", *Bulletin 3*, Univ. of Iowa Eng. Studies.

[16] Director, Photoelastic Laboratory, U. S. Bureau of Reclamation, Denver, Colo.

The method is also applicable to a triangular buttress of a hollow dam if the elastic interaction between the up-stream slab or arches and the buttress is ignored. Even if the results obtained are not exact (and, in one instance, incorrect in principle, as will be shown subsequently), it is believed that stresses computed by the author's method will agree quite closely with those obtained by the approximate methods used by the U. S. Bureau of Reclamation. One of these methods, also based on Assumptions (2) to (4), uses the exact solution by Kirsch for a circular hole in an infinite plate acted upon by uniform boundary forces equal to the principal stresses computed in the structure at the center of the opening as if the opening did not exist.[17] This method is far simpler in application than that developed by Mr. Silverman and is useful not only for the specific type of structure and loading treated by him, but for any type of structure and loading in which the principal stresses can be computed at the center of the opening.

Both methods are approximate and are limited to openings at least three times the radius of the opening from the nearest boundary of the structure. In fact, the author's formulas may be restricted to even greater distances since he assumes that $\frac{r_i}{r_o} = 0$ in arriving at Equations (47), (48), and (49). It is evident that the circle, $\rho = r_o$ must lie within the structure, which implies that the opening must be many times r_i removed from the boundaries. The region of validity should be definitely stated by the author. An approximate theory is of little value in practice without a stipulation of the limits of validity.

If the opening is dangerously close to the boundaries of the structure, or if its shape deviates too much from a circle, it is customary at the Bureau of Reclamation to check the approximate analysis by photo-elastic experimentation.[18]

It has been stated that the author's approach is somewhat more laborious than the methods now in vogue, but even so, it has great merit if it is used only as a check on these other methods. It would certainly be of great interest if Mr. Silverman would compare numerical results obtained by each method under identical conditions. In doing this the tensile stresses and the necessary steel reinforcement should be computed over the maximum line, which, in the author's example, is seen by Fig. 4 to occur at about $\theta = +83°$ and $\theta = -103°$, and not at $\theta = \pm 90$ degrees.

A source of uncertainty in the results obtained by both methods of approach is the change in behavior of the structure caused by the reinforcement steel (if such is needed). This question has been discussed by the writer,[17] including the effect of temperature changes and a uniform pressure acting in the circular opening.

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[17] *Technical Memorandum No. 457*, U. S. Bureau of Reclamation.

[18] *Loc. cit., No. 451*, U. S. Bureau of Reclamation.

In publishing solutions of elastic two-dimensional problems many writers consider it necessary to repeat the general theory of the Airy-Maxwell stress function, including the equilibrium equations, stress definitions, and the differential equation which expresses compatibility in accordance with Hooke's generalized stress-strain relations. This seems a needless waste of space, inasmuch as many good references are available. On the other hand, important steps are often omitted which would clarify the situation for the inexperienced reader and facilitate checks on the mathematical procedure.

In the present case the process of transforming the stress functions to the new origin ($x_0\, y_0$) should be given in some detail, because it is of fundamental importance and of general use, and no reference is readily available in case body forces are involved. In manipulating stress functions which represent body forces (weight, inertia, percolation), one soon discovers that it is "tricky" and full of pitfalls.

Change of Co-Ordinates.—The author does not state the definitions of stresses in rectangular components, when body forces, c_1 and c_2, are present. It is here to be noted that more than one stress definition is possible, but if the stress function shall have the same form in rectangular and polar co-ordinates, the writer has shown[19] that the stresses must be defined as follows in rectangular components:

$$s_x = \frac{\partial^2 F}{\partial y^2} - c_1\, x - c_2\, y \quad \text{.................. (62a)}$$

$$s_y = \frac{\partial^2 F}{\partial x^2} - c_1\, x - c_2\, y \quad \text{.................. (62b)}$$

and,

$$s_{xy} = -\frac{\partial^2 F}{\partial x\ \partial y} \quad \text{........................... (62c)}$$

By comparing the author's function, Equation (22), with the function given by the writer[20] for the same case, it may be inferred that the author's stress definitions in rectangular components, are as in Equations (62).

If the origin is now chosen at a point ($x_0\, y_0$) and if the new axes ($\xi\, \eta$) are parallel with the original axes ($x\, y$), then:

$$x = \xi + x_0 \quad \text{........................ (63a)}$$

and

$$y = \eta + y_0 \quad \text{........................ (63b)}$$

Substituting these values in Equations (62), the stress definitions in the new co-ordinate system become:

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[19] *Bulletin*, Am. Math. Soc., June, 1934, p. 427.

[20] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equation (21), p. 1248.

$$s_\xi = \frac{\partial^2 F'_g}{\partial \eta^2} - c_1 (\xi + x_0) - c_2 (\eta + y_0) \ldots\ldots\ldots\ldots (64a)$$

$$s_\eta = \frac{\partial^2 F'_g}{\partial \xi^2} - c_1 (\xi + x_0) - c_2 (\eta + y_0) \ldots\ldots\ldots\ldots (64b)$$

and,

$$s_{\xi\eta} = - \frac{\partial^2 F'_g}{\partial \xi \; \partial \eta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (64c)$$

in which F'_g is obtained simply by substituting Equations (63) into the expression for F_g. In the present case, F_g is given by Equation (22), so that:

$$F'_g = \frac{1}{6} \left[c_1 (\xi + x_0)^3 + (c_1 k + c_2 - 2 c_2 k^2_2 (\eta + y_0)^3 \right]$$
$$+ 3 c_2 k_2 (\xi + x_0) (\eta + y_0)^2 \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

The stresses are now obtained by Equations (64). If, in general, F' is defined as:

$$F' = F_g' - \frac{1}{2} (c_1 x_0 + c_2 y_0) (\xi^2 + \eta^2) \ldots\ldots\ldots\ldots (66)$$

then the stress definitions in the new co-ordinate system may again be expressed in the same general form as Equations (62), namely:

$$s_\xi = \frac{\partial^2 F'}{\partial \eta^2} - c_1 \xi - c_2 \eta \ldots\ldots\ldots\ldots\ldots\ldots (67a)$$

$$s_\eta = \frac{\partial^2 F'}{\partial \xi^2} - c_1 \xi - c_2 \eta \ldots\ldots\ldots\ldots\ldots\ldots (67b)$$

and.

$$s_{\xi\eta} = - \frac{\partial^2 F'}{\partial \xi \; \partial \eta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (67c)$$

in which F' is given by Equation (66). The polar stress components are given by Equations (20). It is important to note that in order to retain the general stress definitions, Equations (62) or Equations (67), it is necessary to subtract the term, 0.5 $(c_1 x_0 + c_2 y_0)$ $(\xi^2 + y^2)$, from the transformed function, F'_g. When no body forces are involved (that is when $c_1 = c_2 = 0$), the transformation is direct.

If the co-ordinate system is to be rotated through the angle, α, about the origin merely replace θ by $\theta + \alpha$ in Equations (20).

Compatibility.—Mr. Silverman makes use of St. Venant's principle when he states that the stresses in the original structure acting on the circle; $\rho = r_0$, are unaffected by the existence of the opening, if r_0 is large compared with r_i.

This principle is somewhat vague, but has been found to hold in many exact solutions and experiments, if applied properly. If r_o falls inside the structure, it is believed that the principle applies in the present case. This then enables the author to ignore compatibility between the regions inside and outside the circle, $\rho = r_o$, so that he only needs to equate the stresses coming from the inside and the outside regions. However, in setting up Stress System II (Equations (34)), in the annular ring, $\rho = r_o \rightarrow r_i$, due to body forces, he makes use of a stress function of the type, $\theta\ \rho \sin \theta$, which gives multi-valued displacements if the origin is entirely surrounded by elastic material. This is not permissible, of course, because it implies that the annular ring has a slit connecting the inside with the outside region. The edges of this slit will be dislocated and must be brought together again by another stress function, giving rise to what is generally known as "dislocation stresses." These are functions of the elastic properties of the material.[21] The complete function which must be used for the author's Stress System II has been presented by the writer.[20]

If no pressure acts on the inside of the hole, this function may be written, in the author's notation:

$$A \left\{ \frac{1}{\rho} + \frac{2\,\rho \log_e \rho}{r_i^2} \right\} \cos(\theta - \beta) + \frac{r_i^2}{2}\, c\, \rho\, \theta \sin(\theta - \beta) \ldots\ldots (68)$$

The stresses are defined by Equations (20). The constant, A, in Equation (68) must be determined so that displacements are single valued; that is, it must be unchanged if θ is replaced by $\theta + 2\,n\,\pi$, in which n is an arbitrary integer. It will be found that,

$$A = -\frac{1}{8}\, r_i^4\, c\, (1 - \mu) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (69)$$

will make the displacements single valued.[22] It is to be noticed that, in Equation (69) in which μ is Poisson's ratio, A is a function of this elastic property, μ.

The Stress System II is now found by applying Definitions (20) to Function (68):

$$s_\rho = -\frac{r_i}{4}\, c\, (1 - \mu) \left(\frac{r_i}{\rho} - \frac{r_i^3}{\rho^3} \right) \cos(\theta - \beta)$$

$$+ \frac{r_i^2}{\rho}\, c \cos(\theta - \beta) - \rho\, c \cos(\theta - \beta) \ldots\ldots\ldots\ldots (70a)$$

$$s_\theta = -\frac{r_i}{4}\, c\, (1 - \mu) \left(\frac{r_i}{\rho} + \frac{r_i^3}{\rho^3} \right) \cos(\theta - \beta) - \rho\, c \cos(\theta - \beta) \ldots (70b)$$

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[21] *Proceedings*, Math. Soc., London, Vol. 30, 1900, p. 103; and, also, "The Mathematical Theory of Elasticity", by A. E. H. Love, 1927, p. 221.

[22] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1246; also, "Photo-Elasticity", by Coker and Filon, 1931, p. 327.

and,

$$s_{\rho\theta} = -\frac{r_i}{4}\,c\,(1-\mu)\left(\frac{r_i}{\rho} - \frac{r^3_i}{\rho^3}\right)\sin(\theta-\beta) \;\ldots\ldots\ldots\ldots (70c)$$

It will be seen that s_ρ and $s_{\rho\theta}$ vanish over the circle, $\rho = r_i$; and, that s_ρ and s_θ become equal to $-\, r_o\, c \cos(\theta-\beta)$ and $s_{\rho\theta} = 0$ over the circle, $\rho = r_o$, if $\frac{r_i}{r_o}$ approaches zero, thus satisfying the author's conditions.

Attention should be called to the fact that in the writer's use of Function (68) for the determination of stresses in a fillet of a dam, the constant, A, is arbitrary because the origin is not completely surrounded by elastic material in the case of a fillet, that is, is always less than 2π. (In this connection compare the methods of finding displacements given elsewhere by the writer[22].)

The magnitude of Stress System II is usually negligible in the regions of validity of the author's formulas so that it could safely be ignored entirely. The reason for bringing to light the correction in System II is that an important principle is involved in the general application of stress functions giving multiple-valued displacements for the determination of stresses due to temperature variations and body forces in structures having doubly connected regions.[23]

V. L. FEDOROV,[24] ESQ. (by letter).—The study of the stress distribution near the holes in dams and buttresses is interesting because it may help to discover the cause of cracks, that often originate in inspection galleries. The solution offered by Mr. Silverman is correct only in the case of water load; for the case of the action of mass load it should be modified as outlined in this discussion.[25]

When a circular ring is cut from a massif subjected to internal strain due to its own weight, the resultant stress on the outer face of the hole is not zero, but equal to the weight of the ring. In this case the stress distribution depends on the elastic constants of the material, which fact was discovered by Michell.

Referring to Fig. 7, assume a ring in which r_o is so much greater than r_i that $\frac{r_i}{r_o}$ may be assumed equal to zero for many practical purposes. The stresses are then expressed by:

$$s_{\rho o} = c\Big[A_o + (A_1 - 1)\, r_o \cos\theta + B_1\, r_o \sin\theta + A_2 \cos 2\,\theta + B_2 \sin 2\,\theta$$
$$+ A_3\, r_o \cos 3\,\theta + B_3\, r_o \sin 3\,\theta\Big] \;\ldots\ldots\ldots\ldots\ldots (71a)$$

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[23] *Journal of Applied Mechanics*, Biot, A. S. M. E., June, 1935.

[24] Dozent, Industrial Inst.; Senior Scientific Worker, All-Union Scientific Research Inst. of Hydrotecknik, Leningrad, Union of Socialistic Soviet Republics.

[25] Based upon a thesis (in Russian) entitled "The Influence of the Round Hole on the Distribution of Stresses in a Dam"; submitted by the writer in 1935 to the State Commission of the Industrial Inst., Leningrad, U. S. S. R., in partial fulfillment of the requirements for the degree of Candidate of Technical Sciences.

and,

$$s_{30} = c\Big[A_1\, r_o \sin\theta - B_1\, r_o \cos\theta - A_2 \sin 2\,\theta + B_2 \cos 2\,\theta$$
$$- A_3\, r_o \sin 3\,\theta + B_3\, r_o \cos 3\,\theta\Big] \quad\ldots\ldots\ldots\ldots\ldots\ldots (71b)$$

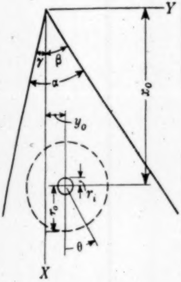

FIG. 7.

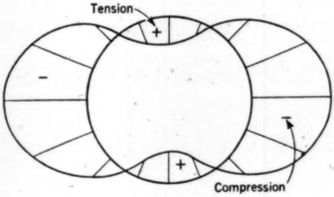

FIG. 8.

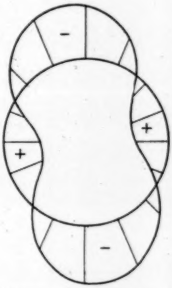

FIG. 9

in which c = weight per unit volume,

$$A_0 = \frac{1}{2}\left[\frac{\sin(\beta-\gamma)\cos\alpha}{\sin^2\alpha}\, y_0 - \left(1 - \frac{2\sin\gamma\sin\beta\cos\alpha}{\sin^2\alpha}\right)x_0\right] \ldots (72a)$$

$$A_1 = \frac{1}{4}\left(1 + \frac{2\sin\gamma\sin\beta\cos\alpha}{\sin^2\alpha}\right) \ldots\ldots\ldots\ldots (72b)$$

$$B_1 = \frac{\sin(\beta-\gamma)\cos\alpha}{4\sin^2\alpha} \ldots\ldots\ldots\ldots\ldots\ldots (72c)$$

$$A_2 = \frac{1}{2}\left[\frac{\sin 2(\beta-\gamma)}{2\sin^2\alpha}\, y_0 - \left(1 - \frac{2\sin\gamma\sin\beta\cos(\beta-\gamma)}{\sin^2\alpha}\right)x_0\right] \ldots (72d)$$

$$B_2 = \frac{\sin\gamma\sin\beta}{\sin^2\alpha}\Big[x_0 \sin(\beta-\gamma) - 2\, y_0 \cos\gamma\cos\beta\Big] \ldots\ldots (72e)$$

$$A_3 = \frac{1}{4}\left[\frac{2\sin\gamma\sin\beta}{\sin^2\alpha}(\sin\gamma\sin\beta + 3\cos\gamma\cos\beta) - 1\right] \ldots (72f)$$

and,

$$B_3 = \frac{\sin(\beta-\gamma)}{4\sin^2\alpha}(\cos\gamma\cos\beta + 3\sin\gamma\sin\beta) \ldots\ldots\ldots (72g)$$

Thus, the stresses in the ring, subjected to load, as determined by Equations (71), are distributed along the outer face (r_o in Fig. 7); there is no load along the inner face, r_i. The stress distribution in this ring may be solved by means of the stress function:

$$\begin{aligned}\phi = c\Big[\, & a_0 \log \rho + l_0 \rho^2 \\ & + \frac{a_1}{2} \rho\, \theta \sin \theta + (l_1 \rho^3 + a'_1 \rho^{-1} + l'_1 \rho \log \rho) \cos \theta \\ & - \frac{c_1}{2} \rho\, \theta \cos \theta + (d_1 \rho^3 + c'_1 \rho^{-1} + d'_1 \rho \log \rho) \sin \theta \\ & + (a_2 \rho^2 + l_2 \rho^4 + a'_2 \rho^{-2} + l'_2) \cos 2\,\theta \\ & + (c_2 \rho^2 + d_2 \rho^4 + c'_2 \rho^{-2} + d'_2) \sin 2\,\theta \\ & + (a_3 \rho^3 + l_3 \rho^5 + a'_3 \rho^{-3} + l'_3 \rho^{-1}) \cos 3\,\theta \\ & + (c_3 \rho^3 + d_3 \rho^5 + c'_3 \rho^{-3} + d'_3 \rho^{-1}) \sin 3\,\theta \Big] \quad \ldots\ldots\ldots\ldots\ldots (73)\end{aligned}$$

The stresses are:

$$\begin{aligned} s_\rho = \frac{1}{\rho}\frac{\partial \phi}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \phi}{\partial \theta^2} - c\,\rho \cos\theta = c\Big[& \frac{a_0}{\rho^2} + 2\, l_0 \\ & + \left(\frac{a_1 + l'_1}{\rho} + 2\, l_1 \rho - \frac{2\, a'_1}{\rho^3} - \rho\right)\cos\theta + \left(\frac{c_1 + d'_1}{\rho} + 2\, d_1 \rho - \frac{2\, c'_1}{\rho^3}\right)\sin\theta \\ & - \left(2\, a_2 + \frac{6\, a'_2}{\rho^4} + \frac{4\, l'_2}{\rho^2}\right)\cos 2\,\theta - \left(2\, c_2 + \frac{6\, c'_2}{\rho^4} + \frac{4 d'_2}{\rho^2}\right)\sin 2\,\theta \\ & - \left(6\, a_3 \rho + 4\, l_3 \rho^3 + \frac{12\, a'_3}{\rho^5} + \frac{10\, l'_3}{\rho^3}\right)\cos 3\,\theta \\ & - \left(6\, c_3 \rho + 4\, d_3 \rho^3 + \frac{12\, c'_3}{\rho^5} + \frac{10\, d'_3}{\rho^3}\right)\sin 3\,\theta \Big] \quad \ldots\ldots\ldots (74a)\end{aligned}$$

$$\begin{aligned} s_\vartheta = -\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial \phi}{\partial \theta}\right) = c\Big[& \left(\frac{l'_1}{\rho} + 2\, l_1 \rho - \frac{2\, a'_1}{\rho^3}\right)\sin\theta - \left(2\, d_1 \rho - \frac{2\, c'_1}{\rho^3} + \frac{d'_1}{\rho}\right)\cos\theta \\ & + \left(2\, a_2 + 6\, l_2 \rho^2 - \frac{6\, a'_2}{\rho^4} - \frac{2\, l'_2}{\rho^2}\right)\sin 2\,\theta - \left(2\, c_2 + 6\, d_2 \rho^2 - \frac{6\, c'_2}{\rho^4} - \frac{2\, d'_2}{\rho^2}\right)\cos 2\,\theta \\ & + \left(6\, a_3 \rho + 12\, l_3 \rho^3 - \frac{12\, a'_3}{\rho^5} - \frac{6\, l'_3}{\rho^3}\right)\sin 3\,\theta \\ & - \left(6\, c_3 \rho + 12\, d_3 \rho^3 - \frac{12\, c'_3}{\rho^5} - \frac{6\, d'_3}{\rho^3}\right)\cos 3\,\theta \Big] \quad \ldots\ldots (74b)\end{aligned}$$

and,

$$s_\theta = \frac{\partial^2 \phi}{\partial \rho^2} - c\,\rho \cos\theta = c\left[-\frac{a_0}{\rho^2} + 2\,l_o \right.$$

$$+ \left(6\,l_1\,\rho + \frac{2\,a'_1}{\rho^3} + \frac{l'_1}{\rho} - \rho\right)\cos\theta + \left(6\,d_1\,\rho + \frac{2\,c'_1}{\rho^3} + \frac{d'_1}{\rho}\right)\sin\theta$$

$$+ \left(2\,a_2 + 12\,l_2\,\rho^2 + \frac{6\,a'_2}{\rho^4}\right)\cos 2\,\theta + \left(2\,c_2 + 12\,d_2\,\rho^2 + \frac{6\,c'_2}{\rho^4}\right)\sin 2\,\theta$$

$$+ \left(6\,a_3\,\rho + 20\,l_3\,\rho^3 + \frac{12\,a'_3}{\rho^5} + \frac{2\,l'_3}{\rho^3}\right)\cos 3\,\theta + \left(6\,c_3\,\rho + 20\,d_3\,\rho^3\right.$$

$$\left.\left. + \frac{12\,c'_3}{\rho^5} + \frac{2\,d'_3}{\rho^3}\right)\sin 3\,\theta \right] \ldots\ldots\ldots\ldots\ldots\ldots (74c)$$

The constant coefficients must be found from the boundary conditions: $\rho = r_i$; $s_\rho = 0$; and, $s_\theta = 0$. For $r = r_o$, when $\frac{r_i}{r_o} = 0$, Equations (74) become the same as Equations (71).

For terms containing $\sin 2\,\theta$, $\cos 2\,\theta$, $\sin 3\,\theta$, and $\cos 3\,\theta$, and for terms independent of θ, all coefficients may be found from the foregoing conditions, as follows: $a_0 = -A_o\,r^2_i$; $l_o = \frac{A_o}{2}$; $a_2 = -\frac{A_2}{2}$; $l_2 = 0$; $a'_2 = -\frac{A_2\,r^4_i}{2}$; $l'_2 = A_2\,r^2_i$; $c_2 = -\frac{B_2}{2}$; $d_2 = 0$; $c'_2 = -\frac{B_2}{2}\,r^4_i$; $d'_2 = B_2\,r^2_i$; $a_3 = -\frac{A_3}{6}$; $l_3 = 0$; $a'_3 = -\frac{A_3\,r^6_i}{3}$; $l'_3 = \frac{A_3\,r^4_i}{2}$; $c_3 = -\frac{B_3}{6}$; $d_3 = 0$; $c'_3 = -\frac{B_3}{3}\,r^6_i$; and, $d'_3 = \frac{B_3\,r^4_i}{2}$.

For terms containing $\cos\theta$ and $\sin\theta$:

$$\frac{a_1 + l'_1}{r_i} + 2\,l_1\,r_i - \frac{2\,a'_1}{r^3_i} - r_i = 0 \ldots\ldots\ldots\ldots\ldots (75a)$$

$$\frac{l'_1}{r_i} + 2\,l_1\,r_i - \frac{2\,a'_1}{r^3_i} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (75b)$$

$$\frac{a_1 + l'_1}{r_o} + 2\,l_1\,r_o - \frac{2\,a'_1}{r^3_o} - r_o = (A_1 - 1)\,r_o \ldots\ldots\ldots (75c)$$

$$\frac{l'_1}{r_o} + 2\,l_1\,r_o - \frac{2\,a'_1}{r^3_o} = A_1\,r_o \ldots\ldots\ldots\ldots\ldots (75d)$$

$$\frac{c_1 + d'_1}{r_i} + 2\,d_1\,r_i - \frac{2\,c'_1}{r^3_i} = 0 \ldots\ldots\ldots\ldots\ldots (75e)$$

$$\frac{d'_1}{r_i} + 2\,d_1\,r_i - \frac{2\,c'_1}{r^3_1} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (75f)$$

$$\frac{c_1 + d'_1}{r_o} + 2\,d_1\,r_o - \frac{2\,c_1'}{r^3_o} = B_1\,r_o \ldots\ldots\ldots\ldots\ldots (75g)$$

and,

$$\frac{d'_1}{r_o} + 2\, d_1\, r_o - \frac{2\, c'_1}{r^3_o} = B_1\, r_o \quad \ldots\ldots\ldots\ldots\ldots\ldots (75h)$$

From Equations (75), it is possible to solve for $a_1 = r^2_i$ and $c_1 = 0$, leaving four equations (Equations (75*b*), (75*d*), (75*f*), and 75*h*)), with six unknowns for the solution of the remaining factors. In order that the displacements in the ring shall be single-valued in the case of plane strain, it is necessary to assume[26] that,

$$l'_1 = -\frac{1 - 2\,\mu}{4\,(1 - \mu)}\, a_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (76a)$$

and,

$$d'_1 = -\frac{1 - 2\,\mu}{4\,(1 - \mu)}\, c_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (76b)$$

in which μ is Poisson's ratio for the material. Then: $l'_1 = -\dfrac{1 - 2\,\mu}{4\,(1 - \mu)}\, r^2_i$; $a'_1 = \dfrac{1}{2}\left[A_1 - \dfrac{1 - 2\,\mu}{4\,(1 - \mu)}\right] r^4_i$; $l_1 = \dfrac{A_1}{2}$; $d'_1 = 0$; $c'_1 = \dfrac{B_1}{2}\, r^4_i$; and $d_1 = \dfrac{B_1}{2}$.

Finally, the stresses are:

$$\begin{aligned} s_\rho = c \Bigg\{ & A_0 \left(1 - \frac{r^2_i}{\rho^2}\right) + (A_1 \cos\theta + B_1 \sin\theta) \left(\rho - \frac{r^4_i}{\rho^3}\right) \\ & + \left[\frac{1 - 2\,\mu}{4\,(1 - \mu)} \left(\frac{r^4_i}{\rho^3} - \frac{r^2_i}{\rho}\right) + \frac{r^2_i}{\rho} - \rho\right] \cos\theta \\ & + (A_2 \cos 2\,\theta + B_2 \sin 2\,\theta) \left(1 + \frac{3\, r^4_i}{\rho^4} - \frac{4\, r^2_i}{\rho^2}\right) \\ & + (A_3 \cos 3\,\theta + B_3 \sin 3\,\theta) \left(\rho + \frac{4\, r^6_i}{\rho^5} - \frac{5\, r^4_i}{\rho^3}\right) \Bigg\} \quad \ldots\ldots (77a) \end{aligned}$$

$$\begin{aligned} s_s = c \Bigg\{ & (A_1 \sin\theta - B_1 \cos\theta) \left(\rho - \frac{r^4_i}{\rho^3}\right) + \frac{1 - 2\,\mu}{4\,(1 - \mu)} \left(\frac{r^4_i}{\rho^3} - \frac{r^2_i}{\rho}\right) \sin\theta \\ & - (A_2 \sin 2\,\theta - B_2 \cos 2\,\theta) \left(1 - \frac{3\, r^4_i}{\rho^4} + \frac{2\, r^2_i}{\rho^2}\right) \\ & - (A_3 \sin 3\,\theta - B_3 \cos 3\,\theta) \left(\rho - \frac{4\, r^6_i}{\rho^5} + \frac{3\, r^4_i}{\rho^3}\right) \Bigg\} \quad \ldots\ldots (77b) \end{aligned}$$

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[26] "Probleme Spannungsferteilung in ebenen Systemen", von Timpe, *Zeitschrift für Math. und Physik*, Bd. 52, 1905, p. 374.

and,

$$
\begin{aligned}
s_\theta = c \Bigg\{ & A_0 \left(1 + \frac{r^2_i}{\rho^2}\right) + (A_1 \cos \theta + B_1 \sin \theta) \left(3\,\rho + \frac{r^4_i}{\rho^3}\right) \\
& - \left[\frac{1 - 2\,\mu}{4\,(1 - \mu)} \left(\frac{r^2_i}{\rho} + \frac{r^4_i}{\rho^3}\right) + \rho\right] \cos \theta \\
& - (A_2 \cos 2\,\theta + B_2 \sin 2\,\theta) \left(1 + \frac{3\,r^4_i}{\rho^4}\right) \\
& - (A_3 \cos 3\,\theta + B_3 \sin 3\,\theta) \left(\rho + \frac{4\,r^6_i}{\rho^5} - \frac{r^4_i}{\rho^3}\right) \Bigg\} \quad \ldots\ldots\ldots (77c)
\end{aligned}
$$

The foregoing solution may be applied to gravity dams. In the case of hollow dams, the buttresses are considered as thin plates, instead of $\dfrac{1 - 2\,\mu}{4\,(1 - \mu)}$ (see Equation (76*a*), one must substitute $\dfrac{1 - \mu}{4}$.

To demonstrate the characteristics of the results, the hoop stresses around a hole due to mass load in a gravity dam are represented in Fig. 8. There is tension along the upper and lower regions, and compression along the sides. The water load for this dam yields the hoop stress distribution shown in Fig. 9.

F. W. Hanna,[27] M. Am. Soc. C. E. (by letter).—A subject that is very important in the design of dams, is described in this paper. That both longitudinal and transverse cracks appear in the inspection and drainage galleries of most dams is to be expected; and, owing to paucity of data on the subject, the designers of dams are in need of technical information on their cause and prevention. Such technical data are provided, in a large measure, by the author. He is to be congratulated therefore, not only on having introduced the subject, but on having presented a careful and laborious analysis of the particular phase of it involved in buttress dams.

The solutions presented are long and the resulting formulas involved and cumbersome. Some of these troubles could have been avoided by taking the origin of co-ordinates at the center of the circular opening initially and using the computed pressures at this point as constant for limited areas around it, the computed pressures being predicated on the absence of the hole in the buttress. This would have involved some approximation, but not sufficient, in a high dam with the opening near the base, to vitiate the results.

Approximate Formulas for Circular Holes.—The writer suggests the use of an approximate solution of the problem of stresses caused by holes in a dam. Let Fig 10(*a*) represent a section of the theoretical profile of a gravity dam with a circular hole through it near the base to serve as a drainage and inspection gallery. Let p be the computed vertical mass pressure per square foot at the center of the hole calculated on the basis of a solid dam. Consider the section, *aa′ b′b*, as a plate, 1 ft thick, perpendicular to the paper, acted on by a uniform unit vertical pressure of p in the vicinity of the hole as indicated in Fig. 10(*b*). The vertical pressure, p, will be a maximum for reservoir empty, as shown by a comparison of Fig. 10(*b*), with Fig. 10(*c*) showing the vertical pressure, p, for reservoir full.

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[27] **Cons. Engr., Ankeny, Iowa.**

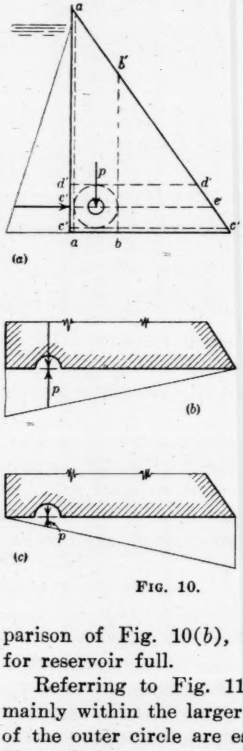

FIG. 10.

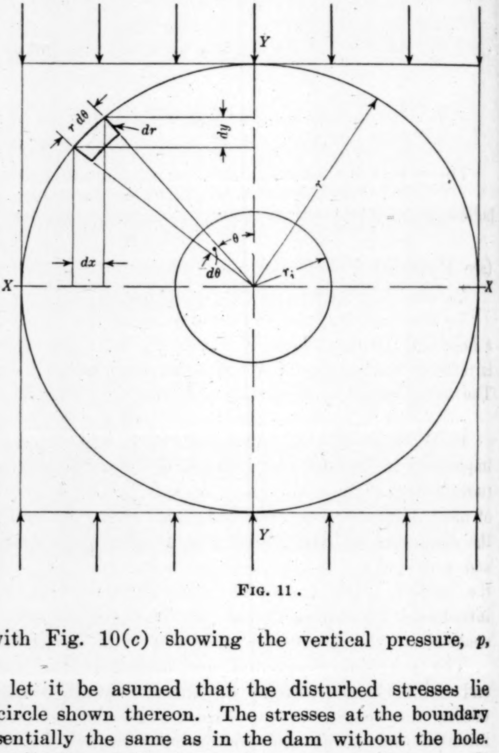

FIG. 11 .

Referring to Fig. 11, let it be asumed that the disturbed stresses lie mainly within the larger circle shown thereon. The stresses at the boundary of the outer circle are essentially the same as in the dam without the hole. The radial stress, s_r, perpendicular to the elementary volume will be $s_r\, r\, d\theta$ and will be balanced by the vertical force, $p\, dx$, resolved on to the opposite side of this volume; therefore, $s_r\, r\, d\theta = p\, dx \cos\theta$, whence $s_r = p \cos\theta\, dx \div r\, d\theta$; therefore,

$$s_r = p \cos^2\theta = \frac{p}{2} + \frac{p}{2}\cos 2\,\theta \qquad (78)$$

In like manner the shearing stress, s_s, would be opposed by the force,

$$s_s = -\,0.5\, p \sin 2\,\theta \qquad (79)$$

and the tangential stress is,

$$s_\theta = p \sin^2 \theta = \frac{p}{2} - \frac{p}{2} \cos 2\,\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

The stress distribution may now be considered as consisting of two parts, one the constant components, $0.5\,p$, and the other the components, $0.5\,p \cos 2\,\theta$, and the shearing force, $0.5\,p \sin 2\,\theta$, both containing functions of θ. The stresses due to these two groups of forces can best be determined separately and their sum taken for their total effect.

The first group of forces act symmetrically around the center of the hole in the dam and is represented by the stress distribution in a thick hollow cylinder subjected to external normal pressure. By adding Equation (78) and Equation (80),

$$s'_r + s'_\theta = p \text{ (a constant)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (81)$$

Now, from Fig. 11, $(s'_r + ds'_r)\,(r + dr) - s'_r = s'_\theta\,dr$, by equilibrating the acting pressure and the resisting stress. Dropping differentials of higher orders,

$$r\,ds'_r + s'_r\,dr = s'_\theta\,dr \quad \ldots\ldots\ldots\ldots\ldots\ldots (82)$$

Solving for s_θ in Equation (81), and substituting the resulting value in Equation (82),

$$r\,ds'_r + s'_r\,dr = (p - s'_r)\,dr \quad \ldots\ldots\ldots\ldots\ldots\ldots (83)$$

Re-arranging,

$$\frac{r\,ds'_r}{s'_r - \frac{p}{2}} = -\frac{2\ dr}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

Integrating,

$$\log\left(s'_r - \frac{p}{2}\right) = \log \frac{C}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (85)$$

Finding the anti-logs of both sides of Equation (85),

$$s'_r - \frac{p}{2} = \frac{C}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (86)$$

When $r = r_1$ in Equation (86), $s'_r = 0$ and, therefore,

$$-\frac{p}{2} = \frac{C}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (87)$$

Solving for C in Equation (87),

$$C = -\frac{p\,r^2_1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (88)$$

Substituting the value of C just found,

$$s'_r - \frac{p}{2} = - \frac{p\, r_1^2}{2\, r^2} \tag{89}$$

Solving for s'_r in Equation (89), the following important equation is derived,

$$s'_r = \frac{p}{2}\left(1 - \frac{r_1^2}{r^2}\right) \tag{90}$$

Eliminating s'_r between Equation (89) and Equation (81) and solving for s'_θ, the companion important equation is found,

$$s'_\theta = \frac{p}{2}\left(1 + \frac{r_1^2}{r^2}\right) \tag{91}$$

Since functions of θ are not involved in this group of forces, a third important result is obtained,

$$s'_s = 0 \tag{92}$$

Addressing the computations to the solution for the stresses produced by the second group of forces, involving functions of θ, let the Airy stress function, F, be taken as,

$$F = f\,(r) \cos 2\,\theta \tag{93}$$

since F is a function of r and θ. Differentiating Equation (93) twice with respect to θ and noting that the result must hold for $\theta = 0$ as well as for all other values around the center of the hole, there results the equation,

$$\frac{d^2 F}{r^2\, d\,\theta^2} = - \frac{4\, f\,(r)}{r^2} \tag{94}$$

Substituting this value in Equation (21), of the paper, the ordinary differential equation becomes,

$$\left(\frac{d^2}{dr^2} + \frac{d}{r\, dr} - \frac{4}{r^2}\right)\left(\frac{d^2 f}{dr^2} + \frac{df}{r\ dr} - \frac{4\, f}{r^2}\right) = 0 \tag{95}$$

The general solution of this equation, as can be determined by checking back on Equation (94), is,

$$f\,(r) = C_1\, r^2 + C_2\, r^4 + \frac{C_3}{r^2} + C_4 \tag{96}$$

Substituting the value of $f\ (r)$ in Equation (93) there results the following stress function equation,

$$F = \left(C_1\, r^2 + C_2\, r^4 + \frac{C_3}{r^2} + C_4\right) \cos 2\,\theta \tag{97}$$

The last term in each of the author's Equations (20) may be dropped due to assumed constancy of weight in the vicinity of the hole. Then these equations become, respectively,

$$s''_r = \frac{d^2 F}{r^2\, d\,\theta^2} + \frac{dF}{r\, dr} \tag{98a}$$

$$s''_\theta = \frac{d^2\, F}{dr^2} \tag{98b}$$

and,

$$s''_s = \frac{d}{dr}\left(\frac{dF}{r\, d\,\theta}\right) \tag{98c}$$

These are the stress components to be found from the use of Equation (97) and the boundary conditions of the problem. Performing the differentiations on Equation (97) as required by Equations (98) and substituting the results in these equations:

$$s''_r = -\left(2\, C_1 + \frac{6\, C_3}{r^4} + \frac{4\, C_4}{r^2}\right) \cos 2\,\theta \tag{99a}$$

$$s''_\theta = \left(2\, C_1 + 12\, C_2\, r^2 + \frac{6\, C_3}{r^4}\right) \cos 2\,\theta \tag{99b}$$

and,

$$s''_s = \left(2\, C_1 + 6\, C_2\, r^2 - \frac{6\, C_3}{r^4} - \frac{2\, C_4}{r^2}\right) \sin 2\,\theta \tag{99c}$$

The constants are to be determined from the external boundary conditions at a distance as required by Equations (78), (79), and (80), from the fact that the stresses are zero at the edge of the hole, and from the fact that $\frac{r_1}{r} = 0$, approximately, for distant points. Applying these principles and solving the constants it will be found that: $C_1 = -\frac{p}{4}$; $C_2 = 0$; $C_3 = -\frac{p\, r_1^4}{4}$; and $C_4 = \frac{p\, r_1^2}{2}$. Substituting these values in Equations (90):

$$s''_r = \frac{p}{2}\left(1 + \frac{3\, r_1^4}{r^4} - \frac{4\, r_1^2}{r^2}\right) \cos 2\,\theta \tag{100a}$$

$$s''_\theta = -\frac{p}{2}\left(1 + \frac{3\, r_1^4}{r^4}\right) \cos 2\,\theta \tag{100b}$$

and,

$$s''_s = -\frac{p}{2}\left(1 - \frac{3\, r_1^4}{r^4} + \frac{2\, r_1^2}{r^2}\right) \sin 2\,\theta \tag{100c}$$

Now, adding Equations (100) to Equations (90), (91), and (92), respectively, and noting that $s'_r + s''_r = s_r$, $s'_\theta + s''_\theta = s_\theta$, and, $s'_s + s''_s = s_s$, the following final important equations are obtained,

$$s_r = \frac{p}{2}\left[1 - \frac{r^2_1}{r^2} + \left(1 - \frac{4\ r^2_1}{r^2} + \frac{3\ r^4_1}{r^4}\right)\cos 2\,\theta\right] \dots\dots \quad (101a)$$

$$s_\theta = \frac{p}{2}\left[1 + \frac{r^2_1}{r^2} - \left(1 + \frac{3\ r^4_1}{r^4}\right)\cos 2\,\theta\right] \dots\dots\dots (101b)$$

and,

$$s_s = -\frac{p}{2}\left(1 + \frac{2\ r^2_1}{r^2} - \frac{3\ r^4_1}{r^4}\right)\sin 2\,\theta \dots\dots\dots\dots (101c)$$

Equations (101) give, respectively, the radial compression, the tangential tension, and the radial shear in the vicinity of the hole. If the value of r is very large, s_r, s_s, and s_θ approach the values given in Equations (78), (79), and (80). If $r = r_i$, $s_r = s_s = 0$, and $s_\theta = p - 2\,p \cos 2\,\theta$. For $\theta = 90°$ and $270°$, $s_\theta = 3\,p$, the maximum compression at the ends of the horizontal diameter. For $\theta = 0°$ and $180°$, $s_\theta = -\,p$, the maximum tension at the end of the vertical diameter. In general, the compressive stresses are not serious in a dam although they may be quite large, but the tensional stresses at the top and bottom of the hole should be provided for with appropriate steel reinforcement.

Formulas similar to Equations (101) have been developed by several investigators[28] for tension in thin plates. In this discussion they have been developed for compression in thin plates in order to make them apply to dams. Such an application has been made previously.[29]

Approximate Formulas for Elliptical Holes.—Drainage and inspection galleries are not often circular in shape, but generally may be made to approach ellipses. By making the same assumptions as to pressures and as to the use of vertical and horizontal strips of the dam as plates, stress formulas may be developed for elliptical holes.

The development of formulas for the determination of stresses caused by elliptical holes in dams involves the use of curvilinear co-ordinates and hyperbolic functions. Formulas have been developed by Inglis for elliptical plates[30] and they may be applied to holes in dams by making the same assumptions as previously made herein for circular holes. The resultant Inglis formulas applicable to the case are not included herein as the reader can find their development in available engineering literature.

Square Holes.—In dams square holes should be avoided, especially with sharp corners. Rounding the corners with liberal radius helps the stress conditions. The injunction against the use of square holes applies to the customary flat bottoms in otherwise well designed drainage and inspection galleries.

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[28] *Zeitschrift* des Vereines Deutscher Ingenieure, Vol. 42, 1898: "Stresses in Plates Having Discontinuities and Some Problems Connected with Them", by K. Suyehiro, *Engineering*, September 1, 1911; "Drang and Zwang", Vol. 1, pp. 314–319, and "Vorlesungen über Technische Mechaniks", by A. Föppel, Vol. V, p. 352; and by "Theory of Elasticity", by S. Timoshenko, pp. 75–78.

[29] "The Design of Dams", by Hanna and Kennedy, pp. 160–162.

[30] "Stresses in a Plate Due to the Presence of Cracks and Sharp Corners", by C. E. Inglis, *Transactions*, Inst. of Naval Archts., 1913, Pt. I, pp. 219–230.

Limitation of Approximate Formulas.—It is appreciated that the foregoing approximate formulas are not applicable where there is too much variation in the pressure in the vicinity of the hole in the dam, which is the case in a low dam. However, inspection and drainage galleries are not often needed in such structures. In high dams where such openings are needed, the pressures are fairly uniform and only slight error is involved in the assumption that uniformity of pressure exists in the vicinity of the hole both for mass and water pressure. It should be observed that mass pressure is predominant and is at a maximum with the absence of water pressure. When the reservoir is full, nearly all the mass pressure disappears and the horizontal water pressure predominates.

Little valid objection can be made to the assumption that a narrow vertical slice of a gravity dam can be considered as a plate. Gravity dam sections are designed on the basis of the existence of such solitary plates, and this in spite of the effects of Poisson's ratio on their sides. Corrections can be made for the effects of this ratio on the sides of the slice by making a separate computation for a stress equal to 25% of the major force. The effect of Poisson's ratio is to lessen that of the major force.

Cracks in Holes in Dams.—As far as the writer is aware, the Pardee Dam was the first in which reinforcement steel was placed around the drainage and inspection gallery to provide against tension. This gallery is 5 ft wide and 10 ft high, circular at the top and flat at the bottom. The floor of the main gallery is 300 ft below the top of the dam, the average mass pressure, p, being equal to 43 200 lb per sq ft at the center of the gallery.

The equation for the total horizontal tensile force at the top of the gallery, on the assumption that it is circular, is given by[31]:

$$F = -\frac{p r_1}{3\sqrt{3}} \qquad \text{(102)}$$

From Equation (102), based on a radius of 2½ ft, the total horizontal tensile stress at the top of gallery of the Pardee Dam is 20 800 lb. The reinforcement consists of 1-in. square bars placed 1 ft apart, extending entirely around the opening. Nevertheless, there is a hair-crack along the top of the gallery for the entire length. No visible cracks exist at the corners of the flat floor, where they might be expected to be found.

The vertical compressive stress at the ends of the horizontal diameter, as given by Equation (101*b*), is $3\,p = 3 \times 43\,200 = 129\,600$ lb per sq ft, or about 900 lb per sq in., which is not excessive.

If the gallery in the Pardee Dam is assumed to be elliptical with the major axis vertical, and equal to twice the minor axis, the opening in the dam will be more closely approximated. The unit tangential compressive stress at the ends of the horizontal diameter due to mass load is given by the formula,

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[31] "The Design of Dams", by Hanna and Kennedy, p. 163.

$$s_\theta = p\left(1 + \frac{2\,b}{a}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (103a)$$

and the unit tensile stress at the ends of the vertical diameter is given by the formula.

$$s_\theta = -\,p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (103b)$$

C. P. Vetter,[32] M. Am. Soc. C. E. (by letter).—The analysis of stresses around a circular gallery in a dam, presented in this paper, follows a method of approach somewhat different from that of previous investigations of the same subject. Such further investigations are of considerable interest since they tend to clarify the problem in the minds of students and designers. The main value, however, lies in comparing results with that of previous work, in emphasizing differences and similarities, in indicating improvement, if any, and in comparing the differences in numerical values obtained by the different methods Any such comparison necessitates giving references to previous work, and such references are often the most valuable part of investigations of this nature. The author has given few such references and has made no such comparisons. The subject has been dealt with in detail by Mr. J. H. A. Brahtz,[33] and it is, therefore, worth investigating to what extent Mr. Silverman has added to present knowledge of the subject.

The paper (see Fig. 1) traces the stresses in a section through a circular hole in a bar subject to uniform tension and refers to the work of Professor S. Timoshenko.[4] In the same book Professor Timoshenko gives[34] the complete solution for stresses at any point of the bar outside the circular opening as well as the proper reference to this well-known Kirsch solution.[35] In the author's notation:

$$s_\rho = \frac{s_x}{2}\left(1 - \frac{r^2_h}{\rho^2}\right) + \frac{s_x}{2}\left(1 + \frac{3\,r^4_h}{\rho^4} - \frac{4\,r^2_h}{\rho^2}\right)\cos 2\,\theta \ \ldots . (104a)$$

$$s_\theta = \frac{s_x}{2}\left(1 + \frac{r^2_h}{\rho^2}\right) - \frac{s_x}{2}\left(1 + \frac{3\,r^4_h}{\rho^4}\right)\cos 2\,\theta \ \ldots\ldots (104b)$$

and,

$$s_s = -\,\frac{s_x}{2}\left(1 - \frac{3\,r^4_h}{\rho^4} + \frac{2\,r^2_h}{\rho^2}\right)\sin 2\,\theta \ \ldots\ldots\ldots (104c)$$

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[32] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[33] "Stresses and Reinforced Steel Around Circular Openings in Concrete Dams," etc., *Technical Memorandum No. 457*, U. S. Bureau of Reclamation, and other memoranda of that Bureau.

[34] "Theory of Elasticity," by S. Timoshenko, p. 77.

[35] *Zeitschrift*, V. D. I., Vol. 42, 1898.

The tension, s_x, in the bar is herein assumed as acting along the X-axis and θ is assumed as positive in the usual way from the X-axis toward the Y-axis. If, in addition, the bar is subjected to a tension, s_y, along the Y-axis, the total stress in any point may be obtained by substituting $\theta + \frac{\pi}{2}$ for θ in Equations (104) and adding; thus:

$$s_\rho = \frac{s_x + s_y}{2}\left(1 - \frac{r^2_h}{\rho^2}\right) + \frac{s_x - s_y}{2}\left(1 + \frac{3\ r^4_h}{\rho^4} - \frac{4\ r^2_h}{\rho^2}\right) \cos 2\ \theta \quad ..(105a)$$

$$s_\theta = \frac{s_x + s_y}{2}\left(1 + \frac{r^2_h}{\rho^2}\right) - \frac{s_x - s_y}{2}\left(1 + \frac{3\ r^4_h}{\rho^4}\right) \cos 2\ \theta \quad ....(105b)$$

and,

$$s_s = - \frac{s_x - s_y}{2}\left(1 - \frac{3\ r^4_h}{\rho^4} + \frac{2\ r^2_h}{\rho^2}\right) \sin 2\ \theta \quad ....\ (105c)$$

This solution has previously been given by Mr. Brahtz.

If the co-ordinate axes are chosen along the lines of principal stresses, s_1 and s_2, at the location of the center of the hole, Equations (105) will give the stresses at any point around the hole. Along the periphery ($\rho = r_h$), for example:

$$s_\rho = 0 \quad ....(106a)$$

$$s_\theta = s_1 + s_2 - 2\ (s_1 - s_2) \cos 2\ \theta_1 \quad ....(106b)$$

and,

$$s_s = 0 \quad ....(106c)$$

in which θ_1 now is measured from s_1. The symbols, s_1 and s_2, denote the principal stresses in a solid dam at a point corresponding to the center of the hole in the structure under consideration. The assumptions on which Equations (105) and (106) are based are obviously that the hole is very small, as compared with the other dimensions, and is sufficiently far removed from the boundaries of the dam.

The author seems to have avoided the necessity for the former assumption and his Equations (42) to (46), inclusive, apparently only assume that the hole is far away from the boundaries of the dam, but not necessarily that it is very small. However, in order to bring his equations to a form which may be of practical use, the author assumes (see following Table 2) that $q = \frac{r_1}{r_0} = 0$. It should be noted that in the main body of his paper, Mr. Silverman does not state that this assumption lies at the foundation of Equations (15) to (17), inclusive. The assumption that $\frac{r_1}{r_0} = 0$ obviously puts the author's equations on par with Equations (105) and (106), obtained from Kirsch's solution, in that they are all based on the same assumptions. Any discussion of the effect of mass, aside from the effect on the principal stresses in the dam, would appear to be a wasted effort since the weight of the concrete in the very small hole cannot possibly affect the stresses to any marked degree.

That the author has gained very little may, perhaps, best be shown by comparing the numerical values obtained by him with those obtained by using Equations (106). At the point on a solid buttress corresponding to the center of the hole in the author's example, the principal stresses are easily obtained[36] from Equations (8) and (22):

$$s_1 = +\,0.0427\ p\ h \ldots\ldots\ldots\ldots\ldots\ldots\ldots (107a)$$

and,

$$s_2 = -\,0.5481\ p\ h \ldots\ldots\ldots\ldots\ldots\ldots\ldots (107b)$$

The angle of the maximum principal stress with the author's X-axis is: $\epsilon = -\,7°\ 38'$; hence; $\theta_1 = \theta - \epsilon = \theta + 7°\ 38'$. From Equations (106) the values given in Table 3 are then obtained and compared with author's values. It will be noted that by the use of the author's extremely unwieldy equations, numerical values are obtained which differ only little from those obtained from Equations (106), and the writer is inclined to believe that the latter are just as "exact" as the formulas given by Mr. Silverman; they certainly are "exact enough" and very much simpler.

TABLE 3.—COMPARATIVE VALUES OBTAINED FROM EQUATIONS (106)
(In all cases, values to be multiplied by $p\,h$)

θ	s_θ by author	s_θ by Equations (106)	θ	s_θ by author	s_θ by Equations (106)
0°	—1.6825	—1.6453	180°	—1.5877	—1.6453
30°	—0.8094	—0.8059	210°	—0.7976	—0.8059
60°	+0.3583	+0.3340	240°	+0.2937	+0.3340
82°22′	+0.6753	+0.6762*	262°22	+0.6567	+0.6762*
90°	+0.6200	+0.6345	270°	+0.6300	+0.6345
120°	—0.2438	—0.2049	300°	—0.1696	—0.2049
150°	—1.3282	—1.3448	330°	—1.3451	—1.3448
172°22′	—1.6319	—1.6870†	352°22′	—1.7221	—1.6870†

* Maximum † Minimum.

FRED L. PLUMMER,[37] M. AM. SOC. C. E. (by letter).—Based on the assumptions defined by Equations (9) the author has developed formulas giving the stresses around circular openings in mass structures. The derivation is presented in a clear and straightforward manner, and illustrates a very skillful use of mathematical theorems. The writer wishes to call attention to the fact that the aforementioned assumptions cannot always be justified, and to question whether, in any case, the designing engineer is warranted in using such complicated formulas when they are developed from assumptions which, at best, can give only a crude approximation of the stresses that will exist in the actual structure.

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[36] See, also, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equations (17) and (21), pp. 1247–1248.

[37] Cons. Engr.; Associate Prof., Structural Eng., Case School of Applied Science, Cleveland, Ohio.

Fig. 12(*a*) shows a buttress similar to the one used by the author in his illustrative example. Obviously, any plane section cut perpendicular to the up-stream face of the buttress, except for the short section, *a-b*, would extend indefinitely into the foundation. The assumptions that the normal stresses on such planes (*c*) are distributed linearly in accordance with Equation (9*a*), and that the shear stresses along such planes (*d*) increase uniformly with the distance from the loaded face in accordance with Equation (9*c*), are obviously far from even a reasonable approximation of the probable stress distribution. On the other hand, the assumption that the direct compression parallel to such planes is constant in accordance with Equation (9*b*) may be more true for planes through *c* and *d*, Fig. 12, than for a plane through *f*.

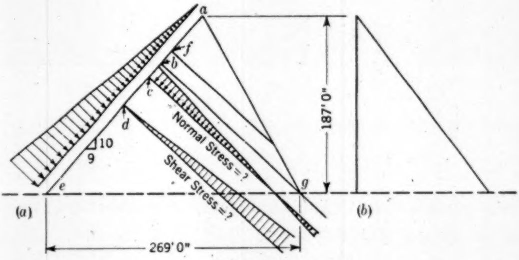
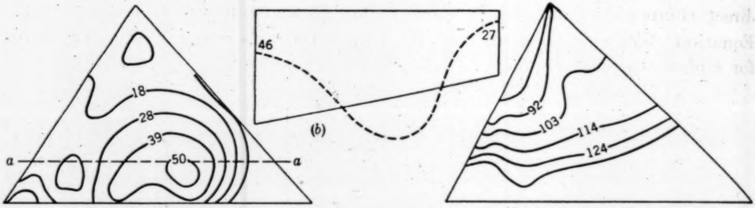


FIG. 12.

It would seem, therefore, that no formula based on these assumptions could be of any value in studying stresses at any point in the section, *b-e-g*, of this buttress. Since stress distributions in Nature cannot be altered abruptly, it is probable that the stresses along a plane through *f* are very similar to those on a plane through *b*. In consequence, it does not seem likely that the stress distribution assumptions defined by Equations (9), nor any formulas based upon these assumptions, could be properly applied to any part of the buttress shown in Fig. 12(*a*), except possibly a small segment at the top of the buttress.

As the slope of the up-stream face of the section is made steeper and the section more nearly approaches that shown in Fig. 12(*b*), the aforementioned assumptions seem to lead to closer approximations of the actual stress distributions. Even in the case shown in Fig. 12(*b*) however, the actual stress distribution may vary quite materially from that indicated by these assumptions. Such variation may be illustrated by considering the very simple case shown in Fig. 13(*a*), which is drawn from a photoelastic picture showing the variation of maximum shearing stresses in the buttress of Fig. 12(*a*) due to the weight of the buttress itself. The usual assumptions, analogous to those based on Equations (9), would indicate maximum shearing stresses along the plane, *a-a*, varying uniformly from 27 lb per sq in. at the up-stream face to 46 lb per sq in. at the down-stream face as shown by Fig. 13(*b*). The distribution of shearing stresses given by the model is shown by the dotted line. This would indicate that the direct compression stresses caused by the triangular mass of concrete above this section, are not distributed uniformly, but that the intensities are much greater near the middle of the section. This effect has been recognized in many other types of problems.

Fig. 13(*c*) shows the variation of shearing stresses in the same buttress due to the weight of the buttress and, in addition, a water load applied to an area on the up-stream face twelve times as wide as the buttress. In each case, the lines represent contours joining those points where the maximum shearing stresses in the analogous concrete buttress equal the values indicated on the diagrams, the unit being pounds per square inch. In neither case, nor along any plane section, does the stress distribution agree closely with that given by the assumptions defined by Equations (9). On the other hand, Fig. 14 shows a section for which the stress distribution given by the model and that based on these same assumptions agree rather closely on several typical sections and for both dead load, and combined dead and water loads.



(*a*) WEIGHT ONLY (*c*) WEIGHT AND WATER LOAD

FIG. 13.

The writer wishes to emphasize the fact that the preceding statements imply no criticism of the author's mathematical derivation, nor of the resulting formulas. Granting the underlying assumptions, the formulas correctly represent the stress conditions around a circular opening. Unfortunately, many designers of engineering structures have been trained to rely too implicitly on printed formulas and, in consequence, often use such aids without understanding the underlying assumptions and, more important, the resulting limitations of every such formula. The formulas under discussion were originally called to the writer's attention by a young engineer who was applying them to a structure for which they could not possibly give results of any value.

It is important that engineers recognize the fact that openings in massive structures in which the normal principal stresses are all compressive may result in tangential tension stresses near the boundaries of such openings. Obviously, in concrete structures, it is wise to provide steel reinforcement in sufficient quantity, placed so as to resist such stresses properly. Equation (1) indicates how much the tension stress in a bar may be increased due to the presence of a small circular hole. Fig. 15 shows the variation of stresses near a circular opening in an area, the least dimension of which is at least four times the diameter of the hole. In Fig. 15 a uniform compressive stress is assumed to act in the direction of the Y-Y axis. Due to the presence of the hole, tension stresses are created, and the compression stresses are increased as indicated. Formulas developed by A. and L. Föppl[38] give the radial, tangential, and maximum shearing stress at any point, *A*, as:

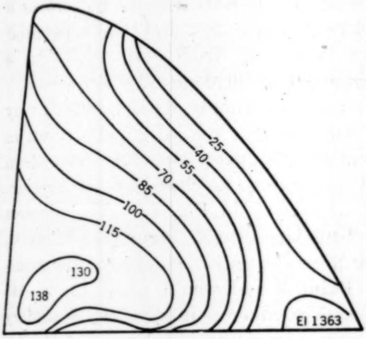


FIG. 14.

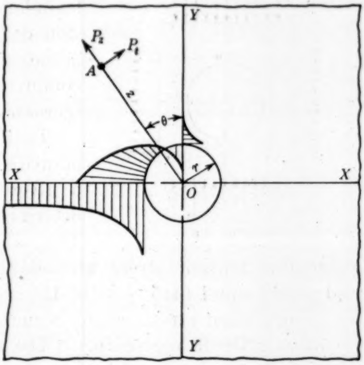


FIG. 15.

$$s_z = \frac{s}{2}\left[1 = \frac{r^2}{z^2} + \left(1 - \frac{4\,r^2}{z^2} + \frac{3\,r^4}{z^4}\right)\cos 2\,\theta\right] \ldots\ldots (108a)$$

$$s_t = \frac{s}{2}\left[1 + \frac{r^2}{z^2} - \left(1 + \frac{3\,r^4}{z^4}\right)\cos 2\,\theta\right] \ldots\ldots\ldots\ldots (108b)$$

and,

$$s_Q = \frac{s}{2}\left[-1 - \frac{2\,r^2}{z^2} + \frac{3\,r^4}{z^4}\right]\sin 2\,\theta \ldots\ldots\ldots\ldots (108c)$$

in which s = the principal stress at *A* if no hole is present (s can act in any direction); r = the radius of the hole; z = the distance from the center of the hole to Point *A*; s_Q = maximum unit shearing stress at Point *A*; and, θ = the angle as shown referred to the direction in which s acts.

If θ is made equal to 90°, Equation (108*a*) reduces to Equation (1) giving the stress distribution along the *X-X* axis. With θ equal to zero, the tangential tension stresses along the *Y-Y* axis may be found. The greatest tension stress occurs at the boundary of the hole and equals in intensity the applied compression stress, *s*. This tension stress decreased along the axis, *Y-Y*, to zero at a point, $r\sqrt{3}$, from the center of the hole.

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[38] "Drang und Zwang," p. 314.

By integration the total tension for unit width may be found to equal $\frac{-s r}{3\sqrt{3}}$, with this resultant force acting at a distance approximately equal to 1.173 r from the center of the hole. If z is made equal to r the resulting formulas define the variation of stress around the boundary of the hole, indicating tangential tension decreasing in intensity from s to zero as θ is increased from zero to 30°, and then tangential compression increasing from zero to $3s$ as θ increases from 30 to 90 degrees.

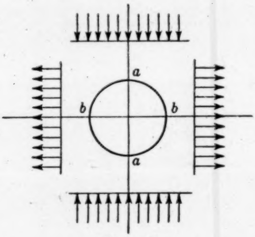


FIG. 16.

To illustrate how simply these relations may be utilized consider that the principal stresses in a given region of a mass structure have been determined, assuming no hole to be present. The values are shown in Fig. 16. The greatest tangential tension stress around the boundary would occur at Point a, and would equal ($445 + 3 \times 15 = 490$ lb per sq in.). Similarly, the greatest compression stress would occur at Point b and would equal ($3 \times 445 + 15 = 1\,350$ lb per sq in.). The stresses at other points on the boundary of the hole can be found in a similar manner by the use of Equation (108b), with $z = r$, the effects of the two principal stresses being computed separately and then combined algebraically. The stress distribution used in this illustration duplicates approximately that which the author found to occur around the hole in the buttress analyzed in his illustrative example.

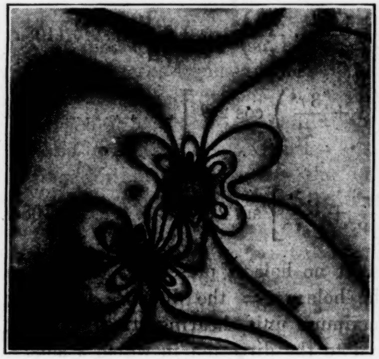


FIG. 17.

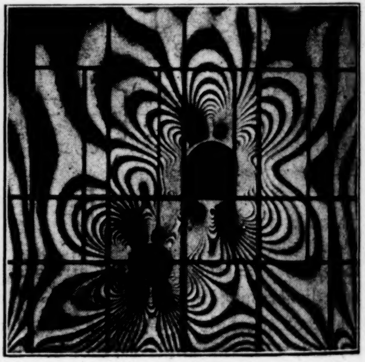
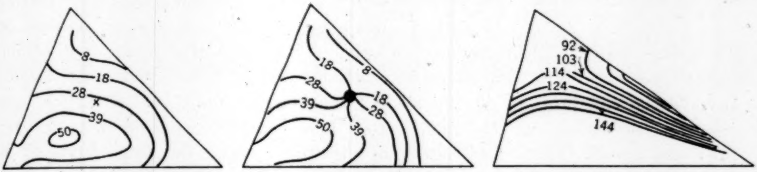


FIG. 18.

Using the data of the paper, the writer has computed the principal stresses that would act in a solid buttress at a point corresponding to the center of the hole specified by the author. Duplicate computations based on three separate sets of generally accepted assumptions result in values of the major principal stress varying in compression from 300 to 570 lb per sq. in. As indicated previously the value obtained by the author was about 445 lb per sq. in. in compression. In the writer's opinion it is this part of the problem which needs serious study. Moreover, it would seem that designers must look to experimental, rather than analytical, methods.

Figs. 17 and 18 show photo-elastic studies of stress distribution in the region of two openings in both a gelatin and a marblette model of a section of a concrete dam. The writer has found this method of study to be very useful and, in general, more reliable than any purely analytical method which of necessity must be based on simplifying assumptions. Obviously, it would be impractical to derive formulas which would be applicable to the complicated situation illustrated by Figs. 17 and 18. Fig. 19 shows a model study of the buttress analyzed by the author. The scale of the model is too small to determine accurately the stress variation around the circular openings. A large scale model of a part of the buttress would be used for this purpose. However, the models show clearly that the stresses in the region of the hole are quite different from those given by computations based on the assumptions used by the author.



(a) WEIGHT ONLY (b) WEIGHT ONLY (c) WEIGHT AND WATER LOAD

FIG. 19.

The curves in Figs. 14 and 17 and the photo-elastic views in Fig. 18, are taken from studies made in co-operation with the office of the U. S. Engineer Corps, at Huntington, W. Va., and under the direction of Lt. Col. John F. Conklin, District Engineer, and A. L. Alin, M. Am. Soc. C. E., Chief of Design on the Bluestone Project.

I. K. SILVERMAN,[39] JUN. AM. SOC. C. E. (by letter).—In a very constructive discussion, Mr. Mindlin shows that the stresses due to gravity in a solid two-dimensional homogeneous body containing holes involve the elastic constants of the material. His analysis yields Equations (60) which are to be compared with the Equations (34) of the paper. The appearance of the elastic constants in Equations (60) is analogous to their appearance in problems dealing with statically indeterminate systems in the theory of structures. Thus, Equations (34) may be considered to be a solution of a statically indeterminate system by methods applicable only to a determinate system. The maximum difference involved in these formulas from the point of view of pounds per square inch of stress can be determined by using $\nu = 0.18$ for Poisson's ratio. The maximum absolute value of this difference is, $c\, r_h \left[1 - \frac{3 - \nu}{2} \right] = 1.71$ lb per sq in. Practically, System II can be neglected entirely, the effect of mass forces being given by Equations (31). This fact suggests that in making photo-elastic tests for mass forces boundary stresses to be applied at the surfaces of the model could be computed by means of Equations (23), (24), and (25), after the stresses defined by Equations (26) are subtracted. Equations (60) are checked both by Equations (70) in the discussion by Mr. Brahtz and by Equations (77) in Mr. Fedorov's contribution.

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[39] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

Mr. Vetter presents Equation (106*b*) to be compared with Equations (18) and (31) of the paper, and shows that the numerical results of Table 3 coincide very closely with those given in Table 1. His reference to the "unwieldy" equations of the paper is not clear. Apparently, he is referring to Equation (18) and Equation (31) which give the hoop stresses at $\rho = r_h$ for water and mass, respectively. These equations are of the simplest type to apply. They contain trigonometric terms and terms which involve the constants of the structure; that is, the angle of batter, the location of the center of the hole, etc. They can be applied without any previous knowledge of the theory of elasticity or of the steps involved in their derivation.

Equation (106*b*), which gives the corresponding hoop stresses, involves the following steps: (1) Determination of the state of stress at the center of the hole; (2) determination of the direction of the principal stresses at the same point; (3) determination of the principal stresses, s_1 and s_2; and, (4) substitution of the various values of the argument, θ_1, in Equation (106*b*). Step (4) involves the use of a new set of co-ordinate axes from which the values of θ_1 are to be measured. In the application illustrated in the paper this step involved a rotation of — 7° 38′ with respect to the natural set of axes taken along the sloping face.

For the sake of completeness the equations required for these operations are given for the case of water loading:

$$s_x = \frac{p\,x}{K^2} - \frac{2\,p\,y}{K^3} \qquad (109a)$$

$$s_y = -\,p\,x \qquad (109b)$$

and,

$$s_s = -\,\frac{p\,y}{K^2} \qquad (109c)$$

$$\tan 2\,\epsilon = \frac{2 s_s}{s_x - s_y} \qquad (110)$$

$$s_1 = \frac{s_x + s_y}{2} + \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + s_s^2} \qquad (111a)$$

and,

$$s_2 = \frac{s_x + s_y}{2} - \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + s_s^2} \qquad (111b)$$

It is seen that an intimate knowledge of the theory of elasticity is required for the correct application of Equation (106*b*) which is simple in form but complex in use.

Mr. Vetter states that the writer has avoided the necessity for the assumption that the hole is very small as compared with the other dimensions of the body in using Equations (42) to (46). The point of the solution, however, is such that this assumption is not needed. The following fact is responsible for the mathematical solution of the problem; Equation (42), the general solution for plane stress problems in polar co-ordinates, yields stresses according to definitions, Equation (41), which are of the identical form of Equations (10) to (12), inclusive. In applying the method of Filon to an annular disk, the ratio of the outer diameter to the inner diameter is involved. For practical purposes, and in agreement with St. Venant's principle, this ratio is taken as zero, and the assumption is made at the proper point. (See text following Table 2.) That this assumption did not appear in the main body of the paper, as Mr. Vetter points out, is due to editorial exigencies, which required that the greater portion of the mathematics involved be placed in an Appendix. In a general way this assumption is implied in non-mathematical language in the first sentence of the opening paragraph of the section entitled "General Formulas."

Mr. Vetter refers to the paucity of references. Footnote references (3) and (4) were deemed to be sufficient cross-references to all work involving stresses around discontinuities.

Professor Plummer questions the assumption of linear stress distribution as given by the Lévy solution, Equations (8) and (22). Both[40] experiment and theory have shown that this departure from linear behavior is confined to the lower third portion of a triangular dam. The maximum possible discrepancy occurs at the boundaries and has very little effect on the principal stresses involved in the region of the hole. The writer believes that the ordinary engineering theory of straight-line variation is sufficiently accurate for all practical purposes, especially when such complex questions of temperature, shrinkage, etc., are equally involved.

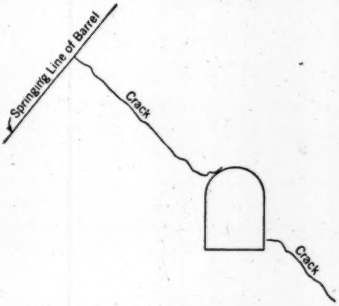

FIG. 20.

Professor Plummer states that his experimental results are quite different from those obtained by using the writer's theoretical equations. Mr. Vetter has shown that comparable results can be obtained by determining the principal stresses at the center of the hole and applying Equation (106*b*). Experiments[41] made at the U. S. Bureau of Reclamation show a very close agreement between experimental and theoretical results. It is not to be expected that the models used by Professor Plummer will give any conclusive experimental results. Probably the maximum size of model possible in a photo-elastic testing apparatus is of the magnitude of 6 in. The diameter of the hole representing a gallery of two one-hundredths of the height would be approximately 0.12 in. The difficulties in measuring stresses around holes of this size are apparent.

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[40] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 489-591; and Vol. 101 (1936), pp. 1258-1259.

The type of model used at the U. S. Bureau of Reclamation consists[42] "of a square slab of plaster celite about seven times the width of the gallery opening. * * * The sides of the square are parallel to the direction of the principal stresses occurring at the center of the area." The slab was roughly 3 ft square.

Mr. Hanna summarizes the formulas in use by designing engineers and describes cracks in dams which he has observed. In a memorandum to J. L. Savage, M. Am. Soc. C. E., Chief Designing Engineer, U. S. Bureau of Reclamation, H. M. Westergaard, M. Am. Soc. C. E., describes cracks observed in dams and mentions a type of crack in buttresses of hollow dams (Fig. 20) which can be explained by the theory as given in this paper. The cracks occur in regions of tensile stress as predicted by Equation (18). In these regions the reinforcement has pulled out, which supports Professor Posey's contention that "the proper placing of steel bars around holes is of great importance."

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[41] Memorandum to Chf. Designing Engr.; "Design Data for Gallery Stresses as Determined by Plaster-Celite Models," by F. M. Russell, Jun. Am. Soc. C. E.; and "Gallery Stresses and Steel Reinforcement," by C. N. Zanger and J. H. A. Brahtz, *Technical Memorandum No. 451.*

[42] "Gallery Stresses as Determined by Means of Plaster Celite Models," by F. M. Russell, Jun. Am. Soc. C. E.

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RECLAMATION AS AN AID TO INDUSTRIAL AND AGRICULTURAL BALANCE

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WITH DISCUSSION BY MESSRS. JOSEPH JACOBS, M. E. MCIVER, C. S. JARVIS, JOHN P. FERRIS, L. C. GRAY, CHARLES P. WILLIAMS, AND ERNEST P. GOODRICH AND CALVIN V. DAVIS.

SYNOPSIS

The economic pattern that has evolved from the troublesome years of 1929 to 1936 indicates that the future industrial structure will be related intimately to agriculture. Reclamation projects scattered over the Western States can have a profound influence in strengthening this link between the factory and the soil because many of these districts are ideally suited for development as complete economic units rather than as agricultural projects alone. A review of the benefits that may be obtained by co-ordinating industrial and agricultural activities on reclamation projects is presented herein.

The writers have studied this problem from three angles: First, the decentralization of industry; second, the diversification of labor; and third, the co-ordination of industry and agriculture. Part I is devoted to a review of the basic principles of each of these factors. In Part II these principles are applied broadly to an actual project to illustrate the industrial and social growth that could follow the construction of hydraulic works in a given area. These potential benefits are illustrated further in Part III by a more detailed analysis which demonstrates the possibilities of co-ordinating industry and agriculture within one of the irrigation districts that will benefit from the projects. Although the conclusions are based on a specific investigation, they would also undoubtedly apply to many of the new irrigated developments that have been made available by Boulder Dam, and other great water conservation projects.

NOTE.—Published in November, 1936, *Proceedings*.

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PART I.—BASIC PRINCIPLES

THE DECENTRALIZATION OF INDUSTRY

Historical.—Industrial congestion in thickly populated cities began with the domestic system long before the industrial revolution. In the early days employers furnished tools, equipment, and housing to workers, who conducted most of the manufacturing operations within their own homes.

The domestic system led to the establishment of the manufacturing towns and cities and these, later, furnished the labor supply for the factories which sprang up during the industrial revolution. The first commercial steam engine manufactured by James Watt in 1776—the engine that was really to begin the industrial revolution—started factories on the road to overgrowth. Steam power had to be used at its source; therefore, great size and economy of operation went hand in hand.

A continued increase in the population densities of industrial cities accompanied the overgrown factories that followed the industrial revolution. Many of the wretched social conditions, that the nation is now striving so desperately to correct, have been inherited from this particular stage of economic evolution. The industrial worker of the city, with his abilities narrowed by mass-production methods, has no alternate means of livelihood during "shut-downs."

The mobility of electric power stands in sharp contrast to the fixed nature of steam power. Electric power, therefore, will be an important factor in the breaking down of great industrial units into smaller and more efficient production plants. Long-distance electrical transmission can make it possible to distribute these smaller plants over rural areas in locations where co-ordination with agriculture may be effected.

Principles and Trends.—Decentralization means the transfer of industry from the great factories of the industrial city to smaller plants in suburban or rural areas. In some cases both workers and plants are being relocated; in others, branch plants are being established.

The purposes of decentralization are: (1) To build partly self-sustaining areas; (2) through the establishment of controls, to balance production and consumption within these areas; (3) to lower greatly the cost of distribution; (4) to increase living standards by decreasing costs and by increasing both production and consumption; (5) to eliminate the evil effects of mass-production methods on industrial workers by the diversification of labor; and (6) to create better social conditions by moving workers and their families away from congested industrial districts and giving them the many advantages incidental to suburban or country life.

Industry is moving away from the congested industrial centers of the East. Evidence of this fact may be found in the "Industrial Census" which shows that the center of gravity of manufactures moved 329 miles westward between 1849 and 1919. During the thirty years between 1899 and 1929, in the areas of "primary concentration" (the big industrial cities), the number of wage jobs per 1 000 population fell from 124 to 106. In the areas of "secondary concentration" (the outlying fringes of the great manufacturing centers and the smaller industrial towns), the corresponding decrease was less—from 106 to 105. Throughout the remainder of the United States, or the rural areas, however, the number actually increased from 34 to 45.

Other evidence of the trend of industry to locate in suburban or rural areas may be found in the statistics of plant movements between 1927 and 1929. During that period, 287 industrial plants, affording 18 599 wage jobs, were moved. Of this number 5 654 employees were moved away from the big cities, 1 933 going to the smaller cities and 3 721 going into the country or to small rural towns.

Future rural relocations will generally be accompanied by decreases in the sizes of production units. Recent studies of the economic sizes of manufacturing plants have shown conclusively that relatively small production units give the most satisfactory results. L. W. W. Morrow, Editor of the *Electrical World*, presented the following significant facts[3] relating to this subject.

An investigation of the machine tool industry made in 1932 shows that the economic size of a plant is one that uses 80 000 to 100 000 man-hr per yr; yet 96½% of the plants in this industry have a greater productive capacity. Studies of three other industries show the same facts—that is, a relatively small production unit is most economical; but at present the greatest productive capacity is found in the larger plants.

In 1933 a certain manufacturer supplied the national market from three plants and manufactured eight products. To-day, this manufacturer has forty plants, one in each market area, and sells sixty-five products through one sales organization. Present earnings are highly satisfactory.

As a result of his studies, Mr. Morrow concluded that:

> "An analysis of the growth of manufacturers shows that small plants or branch plants are being built in each market area and, on the whole, these are more prosperous than the large plants supplying the national market because they restrict their activities to sales in these local areas. The manufacturers who make one product for the national market with a single production plant in one location are suffering the greatest losses."

Mr. Henry Ford, a pioneer in the decentralization movement in the United States, has long advocated both the diversification of labor and a return to the small factory. Extensive experiments in industrial decentralization have been conducted in the vicinity of Dearborn, Mich., with the results summarized in the following statement by Mr. Ford[4]:

> "Ten years ago we started seven village industries on small water power sites, all within twenty miles of Dearborn, our purpose being to combine the advantages of city wages with country living. The experiment has been a continuous success. Overhead cost has been less than in the big factory, and the workers would not hear of going back to the city shops."

Mr. Ford began to acquire land for this experiment several years ago. To-day (1936), his holdings for this purpose embrace 40 000 acres. Farms grouped in large tracts—the largest is 10 000 acres—are tilled on the co-operative plan. Many Ford parts, including starters, lamps, gages, and drills are produced in fourteen small plants located in rural areas within fifty miles of Detroit, Mich. In 1934, these plants produced $8 000 000 worth of parts and tools, employed 2 500 workers, and paid $1 500 000 in wages to these small town workers.[5]

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[3] "Balancing Technocracy" *Electrical World*, January 14, 1933.

[4] "Farm and Factory," *New York Times*, June 3, 1932.

From the foregoing it is evident that decentralization is no longer an academic question for industry; experience has proved this to be a beneficial and constructive measure. No clearer picture of the possibilities of decentralization could be obtained than that given by the Secretary of Agriculture[6]:

> "The ten million unemployed plus the five million living on land which can never be farmed are a continuing menace to the established industry and agriculture of the United States. To solve it means decentralized industrial planning relative to land. If the heads of our two hundred leading corporations were to take into account the full significance of paved roads, autos, trucks, high line electricity, and the increased happiness of human beings close to the land, might they not enthusiastically start a decentralized, industrial, self subsistence homestead program on a scale which would jerk us out of the depression for years to come"?

THE DIVERSIFICATION OF LABOR

Need of Diversification.—The rate of increase of unemployment has kept pace with technological progress. Each year occupational obsolescence has been sending increasing numbers of men to the "scrap heap." Consider, for example, the following unemployment statistics for the relatively prosperous years of 1923 to 1929, inclusive:

Year:	Unemployed:
1923	1 500 000
1925	1 775 000
1927	2 000 000
1929	3 000 000 to 5 000 000

The reasons for this steady increase in unemployment are suggested by the following tabulation[7], which shows, for certain commodities, the increases in production and the decreases in labor force for the years 1923 to 1927, inclusive:

Industry	Increase in production	Decrease in labor force
Oil refineries	84%	5%
Tobacco	53%	13%
Meat	20%	19%
Railroads	30%	1%
Building (Ohio only)	11%	15%
Coal	4%	15%
Steel	8%	9%
Cotton	3%	13%
Lumber	6%	21%

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[5] *New York Times*, September 8, 1935.

[6] "New Frontiers," by the Hon. Henry A. Wallace.

[7] From "An Introduction to Problems of American Culture," by Professor Rugg, pp. 182 and 194.

Instances of unemployment due to technological improvements are legion and space permits only typical examples to be included herein. Improved mining methods have greatly increased the output per worker and, consequently, decreased the number of workers. During the past thirty years the output per worker in soft coal mining increased 60%, in copper mining, 100%, and in the mining of ore, 200 per cent. These increases in efficiency resulted in the permanent unemployment of 64 000 men, engaged in the extractive mineral industries, in the boom year of 1929[8].

The foregoing data present a better picture of occupational obsolescence than could be obtained from similar statistics for the depression years of 1930 to 1935. The abnormal situation in these years was influenced by so many factors that it would be impossible to segregate the effect of any single cause of unemployment.

Probably the most startling development in occupational obsolescence since 1929 was revealed in 1933 by the report of the Henderson Committee[9] in the automobile industry. It was stated therein that keen competition has speeded this industry to a pace considered too fast for men of forty and that, in some instances, 19 men do the work of 250 by contrast with 1929.

Other findings of the Henderson report were of an equally surprising nature; for example, in one plant a door is now made in two stamping operations. In 1929, a door manufactured by this same plant had twenty-six different parts and required several times as many men to make it. Body framing which cost $3.00 in 1929, now (1936) costs $0.30. It cost $0.60 to hang one door in 1929; it cost $0.9 to hang four doors in 1935. Body trimming cost $12.00 in 1929; it costs $4.00 to-day.

The Henderson Committee report stated:

> "Less than five years ago a well known auto manufacturer finished 100 eight cylinder motor blocks on a given line-up with 250 men. To-day the same line-up finishes 250 motor blocks, with 20% more operations with only 19 men. Men were paid $13.20 per hundred blocks five years ago and through the use of tungsten carbide tool tips men now do the same work for $5.20."

It was shown that single machines introduced in factories are automatically removing from 100 to 250 employees.

Many other instances could be cited; the foregoing, however, will serve the purpose of this paper. Without turning to other illustrations of the present trend, it is obvious that two conclusions may be drawn regarding the effects of mass-production methods on a worker: First, his calling may be swept away almost without notice by technological changes, leaving him without alternate means of support; and second, mass-production methods are narrowing to such a degree that little or no mental labor is required. New industries, to absorb the workers thus eliminated from the progressive industries, will doubtless eventually be evolved, but during their development the men and their families suffer greatly.

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[8] "America's Capacity to Produce," p. 155.

[9] *New York Herald-Tribune*, February 7, 1935

The best quick remedy for this situation lies in the diversification of labor. Productive employment in work supplementary to that of the factory must be planned to fill the extended periods of idleness that have accompanied each depression. Although this need has been recognized for many years, it has been only recently that a few industrialists have taken steps to provide some alternate employment during the idle time.

Experiments in the Diversification of Labor.—Obviously, many kinds of work could be planned on a part-time basis. A return to the soil, however, offers the easiest and the most direct method of capitalizing idle hours. This fact was recognized by a group of progressive manufacturers who were trying every available means of coping with the unemployment problem during the depression. With a view, at least, toward keeping workers supplied with food, suitable tracts of land adjacent to the plants were purchased or leased and prepared for planting. Part-time and idle employees then cultivated these tracts and received in return for their labor either the produce of their individual plots or a proportionate share of the yield if the tracts were operated on the co-operative plan.

The experiences gained in administering these agricultural projects as supplements to industry bear a direct relation to this investigation. These experiences enabled the writers to base their analysis on actual operating data rather than on assumptions.

In giving consideration to agriculture as an alternate occupation for industrial employees, many questions arise at once. For example, what classes of labor (skilled or common) adapt themselves most readily to this plan? Would skilled workers care to divide their time between the farm and the factory? Assuming that these workers are inclined to try the experiment, would they be qualified by temperament to grasp an entirely new occupation? What is the cash outlay required of an industry to put such a plan in operation, and what are the returns? Is the worker compensated adequately for his hours of agricultural work? What time must be devoted to such work and how much land is required per dependent? What is the best plan of management? What educational measures are necessary?

In order to obtain answers to these questions the writers' studied in detail the results obtained in eight experiments in industrial co-operative gardening. Space limitations permit only two of these to be described herein.

EXAMPLE 1

General Plan.—During 1931 and 1932 more than 500 former workers of a certain company were unemployed and many others were working only part-time due to an unavoidable curtailment of production schedules. In order to minimize the monetary loss and the physical suffering caused by extensive lay-offs or part-time employment, the Company aligned itself with the municipal government and the civic relief organization. A co-operative farm plan was selected as the most practical solution of the problem.

Consideration was given to both the individual-plot and the mass-production methods of operation. Although no precedent was available for the latter, the management decided that mass-production methods were equally applicable to gardening and manufacturing. The satisfactory results that followed the adoption of this policy are particularly significant.

Early in May, 1932, leases were secured on three tracts aggregating 275 acres in a river valley five miles from the factories. Of this land, 200 acres were tillable. The Employment Manager, was made Director of the project and an engineer with extensive agricultural and industrial experience, was engaged to prepare the detailed plans and to supervise the actual field operations. Plowing was begun as rapidly as the land could be cleared of brush and débris and made ready for cultivation. Drainage ditches, culverts, a small headquarters building, and sanitary facilities were then provided. By May 12, 2 000 early cabbage plants, 2 000 tomato plants, and several bushels of onions had been set out and these were followed immediately by the planting of other vegetables.

The labor-rotation plan, based on the individual working one 8-hr day per week, was followed. Schedules were based on an anticipated total of 150 000 man-hr, embracing 750 men over the 25-week period. Actually, a total of 936 men took an active part in the gardening. When all planting was completed the gardens had the acreage in crops shown by Table 1.

TABLE 1.—ACREAGE OF INDUSTRIAL CO-OPERATIVE GARDEN PROJECT, EXAMPLE 1

Crop (1)	Area, in acres (2)	Crop (1)	Area, in acres (2)	Crop (1)	Area, in acres (2)	Crop (1)	Area, in acres (2)
Potatoes	63.0	Bush and pole beans	8.1	Peppers	2.0	Onions	0.6
Navy beans	40.0	Turnips	7.6	Carrots	1.2	Swiss chard	0.5
Field corn	25.5	Beets	3.8	Kale	1.1	Radishes	0.4
Sweet corn	11.0	Pumpkins	3.0	Summer squash	1.0	Mustard	0.3
Tomatoes	9.5	Peas	3.0	Spinach	1.0	Lettuce	0.2
Cabbage	8.5						

Before planting the potatoes, consultations were held with the staff of the Experiment Station at the State University and with the champion potato grower of the State, a down-State farmer, and 1 300 bushels of certified seed potatoes were obtained from a reliable source.

Less than 25% of the workers had a true farm background. Although this general lack of gardening experience caused the work to progress rather slowly at first, the workers familiarized themselves with their new occupation rapidly. The instructions of the supervisor were grasped quickly and in a few weeks a fair degree of skill in farm work had been developed.

No records were kept of the percentages of the classes of labor (that is, skilled or unskilled), or of the relative performance of each class. It was found, however, that both classes readily adapted themselves to the program. The results of this experiment show conclusively that industrial workers can readily diversify their labor. They represented practically every production division of the factory, and although, their normal occupations had been limited to a highly developed system of mass production, they fitted into the garden plan admirably.

Those who enrolled in the garden force realized the importance of the opportunity provided them in the farming project and entered into the spirit of the work at hand as if they were compensated in money rather than in produce. Former employees, whose individual factory records were above the average, in the majority of cases also proved to be the best "farmers," substantiating the belief that mass-production methods and training can be applied successfully to an agricultural enterprise.

Costs.—The total average investment was about $50 per acre cultivated, as developed from the following itemized summary:

Item	Cost
Plowing, harrowing, planting, including labor, tractor rental, team hire, fuel, mechanical sewer, planter operation, and replacement parts	$1 900
Transportation of farm workers and materials, including use of trucks and passenger bus	1 550
Supervision (Farm Superintendent, part-time truck driver, and incidental specialized labor)	1 400
Spraying materials and equipment	1 100
Seeds, including seed potatoes	1 000
Fertilizer	800
Leases for land	750
Culverts, drain-tile, lumber for headquarters and distribution building, rental of storage trailers, road maintenance	250
Small equipment—tools	150
Incorporation and workmen's compensation	125
Miscellaneous	225
Total	$9 250
Interest, 6 months at 5%	231
Total investment	$9 481

$$\text{Investment per acre} = \frac{\$9\,481}{191.3 \text{ acres}} \doteq, \text{ say, } \$50$$

Return on Investment.—The man who worked 1 day per week on the gardens during the season, without extra work or absence, received the items listed in Table 2, totaling in weight more than 1 ton. This quantity of vegetable food is sufficient to take care of a family of five for nine months. The total value of the produce was easily double the amount invested.

TABLE 2.—GARDEN OUTPUT PER MAN, EXAMPLE 1

Produce	Unit	Quantity	Produce	Unit	Quantity	Produce	Unit	Quantity
Potatoes	Bushels	22	Summer squash	Pounds	31.5	Cayenne peppers	Dozen	16
Cabbage	Pounds	127	Rolled oats	Pounds	25.0	Sweet corn	Dozen	10
Tomatoes	Pounds	98	Greens	Pounds	23.0	Green onions	Dozen	7
Corn meal	Pounds	95	Pumpkins	Pounds	10.0	Hot peppers	Dozen	7
Turnips	Pounds	96	Peas	Pounds	8.0	Carrots	Dozen	4
Navy beans	Pounds	75	Kale	Pounds	5.0	Beets	Dozen	3
Green beans	Pounds	69	Dry onions	Pounds	1.0	Sweet peppers	Dozen	2

The efficiency of operation rapidly increased during the succeeding years. Although employment during 1934 had improved to such an extent that a continuance of the gardens was not necessary, the management felt that the experience was of sufficient value to continue with the work. The area cultivated was increased to 300 acres. The investment amounted to about $20 000, or $66 per acre. The value of the crop harvested and distributed, however, was $60 000, or approximately double the amount per acre of the 1932 harvest.

EXAMPLE 2

Another successful industrial gardening project, operated during 1932, 1933, and 1934 may be discussed as Example 2. About 90 acres of land were rented near the plant for gardening purposes and, in addition, employees were encouraged to cultivate individual gardens.

An agricultural expert was placed in charge of the entire project. Aside from the land and supervision, the Company furnished fertilizers, plowing and harrowing, all seeds and plants, and spraying and spray materials. Seeds and plants only were furnished to those with home gardens. The plant gardens were laid out in plots of 50 by 100 ft, with suitable spaces between lots for paths and driveways. This plan gave centralized operation, combined with the advantages incidental to the cultivation of individual plots. Planting schedules for each plot were planned with a view to providing good substantial food and a well-balanced diet.

Each plot holder was required to sign an agreement which set forth the obligations of both the Company and the gardener. The Company agreed to loan the plot, prepare the soil for planting, furnish the seed, and supervise the planting. The gardener agreed to cultivate the land and keep it free from weeds to the best of his ability. In return for this service the vegetables grown on the land became the sole property of the gardener, except in cases where willful negligence was evident. In this event, the Company reserved the right to revoke the agreement.

The employees on a part-time basis took a keen interest in the gardening work and readily grasped the opportunity to take part in the enterprise, once its advantages became evident. The number of plant gardens and home gardens for each year of operation were as follows:

Year	Plant gardens	Home gardens
1932	345	655
1933	587	813
1934	588	833

About 60% of the workers operating these gardens could be classed as skilled and the remainder as unskilled. Both classes were surprisingly proficient in gardening after the brief instruction period. As in the case of Example 1, the industrial employees who were most proficient in the plant obtained the best results with their gardens.

Accurate cost records were kept during 1933 and 1934 in order to determine the return on the investment. Actually, the gardeners, during these years, received vegetable produce equal in value to between five and six times the total cash outlay. Aside from the monetary value of the product, other benefits were reflected in the improved health of the workers and the close relation that existed between the Company and the employees. The social value of this work in keeping idle hands and minds occupied and in supplying wholesome foods cannot be over-estimated.

Education as well as strict supervision is essential to the success of such a venture. In the beginning the Company had talks about gardening by a County farm adviser and agents of the State Department of Agriculture. Canning demonstrations and individual instruction in the care of produce for winter saving were given. Educational measures were directed toward developing a love of the work concurrently with gardening skill. Efforts in this direction were highly successful.

SUMMARY OF EXPERIENCES WITH THE DIVERSIFICATION OF LABOR

The foregoing, together with the other experiments studied by the writers, lead to the following conclusions:

(1) The occupations of both common and skilled labor may be diversified successfully.

(2) Without exception, agriculture has been selected as the most satisfactory alternate occupation to industrial work.

(3) The most efficient factory employees generally obtain the best results from their garden plots.

(4) With a relatively high yield about 0.25 acre is necessary to provide a family of five with vegetable produce for the greatest part of a year. More land per worker is necessary if the soil is unsuited to intensive farming.

(5) The co-operative plan (that is, all the gardens centralized in one large plot) lends itself to the most efficient management. Mass-production methods may be applied to gardening as well as to manufacturing if this plan is adopted. The costs of plowing, harrowing, planting, and harvesting are reduced to a minimum if the gardening activities are conducted at one place. The principal disadvantage of operating one large plot on a labor-rotation basis is that workers may not have the pride or interest in their work that they would if they were responsible for the sole care of an individual garden. This may be overcome by adopting the plan of dividing the central plot into sections and assigning each section to individual workers (see Example 2).

The cultivation of small gardens adjacent to employees' homes is to be discouraged unless the workers or members of their families are skilled gardeners. This plan has produced only indifferent results as the costs of company operation and management are necessarily much higher than those for centralized plots.

(6) The service of a skilled agriculturist is absolutely essential. Few workers have the gardening experience that is required to insure them an adequate return on their efforts, therefore, additional educational measures are necessary. Lectures, educational bulletins, and planting schedules must be supplemented with individual instruction if the project is to be successful.

(7) Centralized storage facilities must be provided and arrangements made for distributing the produce.

The experiments outlined in the foregoing items do not indicate completely the possibilities of labor diversification. Most of these projects were emergency measures and had to be planned on a subsistence basis only. Regardless of this fact important lessons were learned and there now exists sufficient actual experience to state with certainty that labor diversification in connection with manufacturing operations could be both practical and profitable.

Tangible evidence has been presented to show the benefits that could be derived from both industrial decentralization and the diversification of labor. Nothing has been introduced, thus far, to show the improvements that could result from co-ordinating the operations of adjacent agricultural and industrial developments. The following two specific examples relating to irrigated areas show conclusively that economic conditions improved definitely after the introduction of industrial plants.

The Co-Ordination of Industry and Agriculture within Irrigation Districts

Yakima Valley, Washington.—Industries engaged in the processing of agricultural products followed irrigation development in the Yakima Valley. These industries consist of flour and feed mills, alfalfa mills, fruit dehydrator plants, fruit canning plants, meat packing plants, and various other types of plants for the packing, storing, handling, and distribution of crops. The canning plants alone employ approximately 3 000 annually for about 4 months and the dehydrating plants employ 800 to 1 000 for approximately 6 months.

The Federal Government has expended mostly in Yakima County or in irrigation projects approximately $33 000 000. As a result of these projects and their subsequent industrial development, the population of Yakima County increased from 13 462 in 1900 to 77 402 in 1930. During this same time the population of Yakima City increased from 3 154 to 22 101. The assessed valuation of the County has increased from $25 004 000 in 1900 to $233 041 000 in 1930, or almost tenfold.

Obviously, these benefits have been realized through agriculture, as a population base, being followed by factories for the processing of farm products.

Sugar Production in Sidney, Mont.—An outstanding case illustrating the improvement in the economic status of an irrigation district that follows the introduction of industrial plants may be found at Sidney, Mont. The Lower Yellowstone Irrigation Project was not successful as an agricultural venture alone. For a period of ten years the farmers of this District were faced with severe economic difficulties, with the result that water charges were defaulted.

In 1925 a sugar factory was constructed at Sidney which is centrally located in the Valley. At this time, few of the farmers understood the raising of sugar-beets and, in consequence, only about 50 000 tons of beets were produced annually within the District. There has been a gradual growth of the industry, however, and, in 1935, the growers tributary to the Sidney factory produced approximately 200 000 tons, yielding a return to the farmers of approximately $1 500 000. As a result of linking the manufacture of beet sugar to the agricultural production of the Lower Yellowstone Irrigation Project the farmers of this District are now in good financial condition; many, in fact, are increasing their land holdings.

In the connection with the growth of sugar-beets the District also maintains a substantial live-stock development. During 1935, approximately 150 000 head (including lambs, sheep, and cattle) were fed for the market. This large feeding program is made possible by the by-products from both the factory and the farm. Beet tops and beet pulp and molasses make excellent foods in addition to hay.

The successful growing of sugar-beets calls for the rotation of crops on the farms. Alfalfa, the principal rotation crop, provides an ample quantity of hay for feeding purposes. Combining a sugar-beet and live-stock program has produced increased yields of all other crops grown in rotation with sugar-beets. Feeding of livestock is producing an abundance of manure which, through return to the soil, is gradually building up the fertility and yield of the farms.

PART II.—POTENTIAL INDUSTRIAL GROWTH OF CALIFORNIA RELATED TO CENTRAL VALLEY PROJECT

IRRIGATION AS A BASE FOR WESTERN INDUSTRIAL DEVELOPMENT

Much of the land that will be made irrigable by the construction of hydraulic works in the Western States is adjacent to extensive markets for manufactured products. As these markets are now (1936) only partly supplied by local manufacturing establishments, opportunities exist for the profitable expansion of many industries in the Western States. This situation is especially favorable for decentralized industrial planning in relation to the land.

The benefits that would result from such an arrangement are obvious. Irrigation districts would benefit by having many of their products utilized by adjacent manufacturing plants. Hides, cotton, wool, cereals, sugar-beets, and fruits and vegetables are only a few of the agricultural products that could be processed by adjacent industries. In addition, these irrigation districts would benefit by having a substantial portion of their lands operated by industrial employees on a co-operative basis. This would tend to check the over-production of agricultural products on irrigated areas as the products of the co-operative gardens would be consumed principally by industrial employees. A third benefit would be the tremendous increase in population that could be supported by the land.

Industry would be likewise benefited by co-ordination with agriculture. The labor supply would be more stable and more efficient than in crowded urban locations. Industrial employees would have a "back-log" of agricultural employment and, therefore, would not face serious hardships in the event of a factory shut-down. An abundant supply of cheap power is available in most irrigation districts. These advantages, together with nearness to markets and sources of raw material supplies should offer an incentive to many industries either to relocate their factories or to establish branch plants on projects that are adaptable to co-ordinated industrial and agricultural operations.

The large potential markets that now exist adjacent to recently developed irrigation areas would absorb most of the manufactured products at first. As the projects were settled additional markets would be developed in the immediate vicinity of the manufacturing plants. This would give opportunity for an industrial growth in proportion to the increase in population. This process could continue until the land had been fully developed.

Not only would agriculture and industry in the immediate vicinity of the irrigated areas be benefited by co-ordination, industry in all sections of the country would be helped by the extensive construction program and the additional manufacturing equipment that would be required by industrial decentralization.

The truth of the foregoing statements may best be illustrated by a detailed analysis of a selected irrigation project, the Madera Irrigation District in California, which is suitable for combined industrial and agricultural operations. It is impossible, however, to confine the investigation to the Madera District alone; it is first necessary to consider some of the industrial and economic aspects of both the State of California and the Central Valley Project.

For the purposes of this investigation the State of California was considered as a single-market area. An analysis was then made to determine the opportunities for industrial expansion that existed within the boundaries of this area in 1929. After this information was compiled, the capacity of the Central Valley Project to absorb a program of decentralized industrial development was studied broadly. A proportional part of the potential industrial growth for the entire State was then allocated to the district investigated. This consisted of a selected group of industries for which the location factors in the vicinity of Madera were favorable. It is not meant to imply that the Madera District offers the most favorable location in California for the industries selected. It was beyond the scope of this investigation to do more than to apply principles to a specific area. Similar analyses for other districts should indicate whether they too are suitable for combined industrial and agricultural development.

POPULATION GROWTH RELATED TO LAND DEVELOPMENT

The rate of industrial development has been found to bear a definite relation to population growth.[10] Population studies, therefore, are of primary importance in this investigation. In predicting California's future population sight must not be lost of the fact that a substantial part of the high rate of increase during past years has been due to immigration from other States and from abroad. More than 46% of the people now living in California came there during the past decade. The relationship between that part of the population born in California and the total for the State is shown by

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[10] "Amended Application to Federal Emergency Administration of Public Works," p. 87.

Fig. 1(*a*), which also shows the forecast of population growth from 1932 to 1970, used by the Water Project Authority of the State of California as a basis for the growth curve of future increases in the area of irrigated lands Fig. 1(*b*).

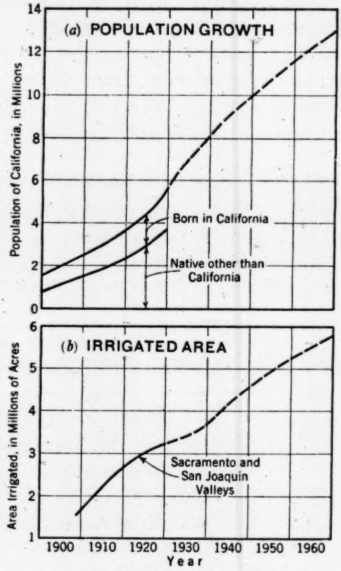

Fig. 1.—Past and Anticipated Growth of California

Agricultural Development

The total cropped area of the State is 8 390 000 acres.[11] Of this area, 4 746 000 acres, or about 57%, is irrigated. The irrigated lands of California, comprising 24% of the entire irrigated area of the United States, are among the most productive in the country. In 1929, California ranked eighteenth in the States of the Union in cropped area, second only to Texas in gross income from crop production. The per capita income of farm population, however, is greater for California than that for any other State.

Industrial Pattern of California

Industry has grown rapidly in California, responding to the westward movement of the center of population. From 1914 to 1929, the value of manufactured products increased 330% and the value added by manufacture, 410 per cent. In each case, this rate of increase was nearly twice that for the United States as a whole.

The 1929 industrial census for California does not present the distorted values that would be given by statistics compiled during the depression years. This investigation, therefore, presents both a cross-section of California industry in 1929 and a reconstruction of the opportunities for industrial expansion that existed in that year. As industrial development has not kept pace with population growth during the depression years there is, in 1936, an even greater opportunity for new industrial development in California than that indicated by the 1929 statistics.

The average value of industrial products per capita is nearly the same for the State of California as it is for the entire country. The following comparison of these values for the years 1929 and 1931 indicates the abnormal situation that existed in 1931 after the depression was well under way:

Description	1929	1931
Total Value of Industrial Products:		
For the entire United States	$70 435 000 000	$41 350 000 000
For California	3 103 000 000	1 985 000 000

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[11] "Permissible Economic Rates of Irrigation Development in California," *Bulletin*, Univ. of California, p. 38.

Description	1929	1931
Value of Industrial Products per Capita:		
For the entire United States	$577	$336
For California	565	330

A superficial examination of the foregoing statistics would seem to show that California was producing at least its proportionate share of the nation's manufactured goods and that little opportunity existed for profitable industrial expansion in 1929. A detailed study of California's manufactured products, however, shows that the picture is distorted greatly by several industries that serve a much larger population than that of the State. For example, in 1929, the industries in California engaged in the canning and preserving of fruits and vegetables served nearly 36 000 000 people on a proportionate basis; the motion picture industry served 85 500 000 people, and the petroleum refineries served 22 500 000 people.

The total value of the products of these three industries was $834 612 000 for the State of California and $3 574 000 000 for the entire United States. If these sums are deducted from the total value of all industrial products shown in the foregoing list it is found that the values of the remaining products per capita were $407 for California and $547 for the entire United States. These averages show conclusively that the majority of the industries in the State were not sufficiently developed to supply their adjacent markets.

The principal industrial areas in California are the Los Angeles District and the San Francisco-Oakland District. Fig. 2 shows the extent of each district and its relation to the Central Valley Area. Table 3 which gives the industrial characteristics of each of the principal districts, and of the remainder of the State, reveals that industry in California has followed the usual course; the greatest industrial concentration is found in the areas of greatest population density. This tendency of industry to follow its markets would not be altered by the adoption of an industrial decentralization program in connection with the Central Valley Project. Actually, decentralized planning would result in a broadening of this same policy. Many of the irrigated dis-

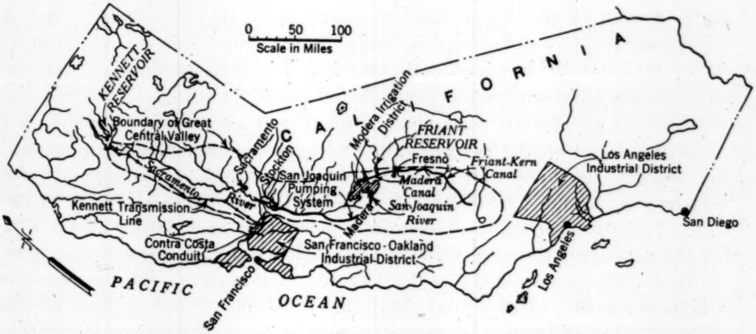
0 50 100
Scale in Miles
CALIFORNIA
KENNETT RESERVOIR
Boundary of Great Central Valley
Sacramento
Stockton
Modera Irrigation District
FRIANT RESERVOIR
Fresno
Friant-Kern Canal
Los Angeles Industrial District
Sacramento River
San Joaquin Pumping System
Madera Canal
San Joaquin River
Madera
Kennett Transmission Line
Contra Costa Conduit
San Francisco-Oakland Industrial District
San Diego
Los Angeles
San Francisco
PACIFIC
OCEAN

FIG. 2.—RELATION OF PRINCIPAL INDUSTRIAL DISTRICTS TO CENTRAL VALLEY AREA

tricts of the Sacramento and San Joaquin Valleys are sufficiently close to the thickly populated areas to be satisfactory for both industrial and agricultural development. Furthermore, a co-ordinated industrial and agricultural program would enable these valleys to support a much greater population density than that which could derive a living from agriculture alone. The distribution of industries throughout the Central Valley therefore, would have a centripetal effect on their respective market areas. Not only would these industries be situated favorably, as far as the existing potential markets are concerned, they would also have the ultimate result of creating important new markets which would develop inwardly toward the factories as focal points.

TABLE 3.—CHARACTERISTICS OF PRINCIPAL INDUSTRIAL DISTRICTS IN CALIFORNIA

Industrial district	Population in 1930	Area, in square miles	Population per square mile, in 1930	Persons engaged in industry in 1929	Total value of products in 1930
Los Angeles	2 209 000	4 115	537	146 000	$1 319 000 000
San Francisco–Oakland	1 307 000	2 464	530	114 000	1 166 000 000
Remainder of State	2 161 000	149 073	14.5	102 000	618 000 000
Total	5 677 000	155 652	36.4	362 000	$3 103 000 000

INDUSTRIAL SURVEY FOR CALIFORNIA

In order to get a true concept of the deficiency of capacity of the California manufacturing concerns in 1929, it is necessary to make a detailed study of each of the 327 industries classified by the industrial census. If the population served by each industry in the State is computed from the census statistics it is possible to establish for each product the potential markets that exist within the State boundaries.

The most satisfactory method of estimating the population served by each industry is based on the average values of manufactured products per capita; for example, the total value of the products in 1929 of the boot and shoe industry was about $966 000 000 for the entire United States; the average value of product per capita, based on a population of 122 000 000, was $7.92. For the same year the total value of the products of the boot and shoe industry in California was $3 095 000. Dividing this sum by the average value of $7.92 per person gives a total population of 391 000 served by the California factories. The difference between the population of the State (5 500 000 in 1929) and the population served by the industry is 5 109 000 persons who purchased shoes that were manufactured outside the State. Further analysis, including estimates of the population per establishment and the employees per establishment, indicates that 56 additional average shoe-manufacturing establishments, employing a total of 9 400 persons, could have been established in California in 1929. Table 4[12] shows the results of an analysis, similar to the foregoing, for each industry that could have been expanded in California in 1929.

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[12] Computed from statistics in United States Industrial Census, 1929.

TABLE 4.—DETAILED STUDY OF CALIFORNIA INDUSTRIES

Item	United States, population per plant	Present number of plants in California	Potential additional plants in California	Potential additional number of industrial employees in California	Potential additional value of industrial products in California
Total for all industries	......	......	1 742	164 273	$940 201 000
Agricultural implements	415 000	18	5	815	$4 600 000
Aluminum manufacturers	812 000	14	6	920	5 820 000
Ammunition and related products	5 800 000	......	1	379	1 965 000
Artificial leather	7 170 000	......	1	142	1 500 000
Artists' materials	1 790 000	4	3	62	383 000
Asbestos products other than steam packing and pipe and boiler covering	1 820 000	......	3	420	2 520 000
Asphalted felt base floor covering	8 700 000	......	1	124	1 440 000
Bags, paper, exclusive of those made in paper mills	1 285 000	......	4	357	2 920 000
Baking powders, yeast, and other leavening compounds	2 560 000	......	2	164	2 370 000
Baskets and rattan and willow ware, not including furniture	447 000	......	12	460	1 030 000
Belting, leather	590 000	10	4	74	745 000
Blacking stains and dressings	717 000	12	4	61	620 000
Blueing	6 100 000	......	1	7	62 000
Bolts, nuts, washers, and rivets not made in plants operated in connection with rolling mills	1 045 000	5	4	580	3 270 000
Bone black, carbon black, and lamp black	1 570 000	......	4	97	910 000
Boot and shoe cut stock not made in boot and shoe factories	580 000	......	10	480	6 050 000
Boot and shoe findings not made in boot and shoe factories	320 000	......	17	560	2 640 000
Boots and shoes, other than rubber	90 700	12	56	9 400	41 000 000
Boxes, paper, not elsewhere classified	98 000	38	13	670	3 040 000
Brooms	300 000	19	4	65	180 000
Brushes other than rubber	400 000	8	9	270	1 340 000
Buttons	508 000	......	11	460	1 305 000
Candles	6 100 000	......	1	47	300 000
Carbon paper and inked ribbons	2 140 000	4	2	62	595 000
Card cutting and designing	1 740 000	......	3	130	887 000
Carpets and rugs — wool, other than rag	1 820 000	......	3	1 569	8 000 000
Carriages, children's	1 540 000	......	4	360	1 305 000
Cars, electric and steam railroad, not built in railroad repair shops	828 000	......	7	2 100	14 800 000
Cash registers and adding, calculating and card tabulating machines	2 950 000	......	2	820	4 950 000
Cast-iron pipe	1 700 000	......	3	942	3 610 000
Cereal preparations	1 010 000	9	4	288	6 100 000
Cheese	44 300	37	21	43	862 000
Chemicals, not elsewhere classified	221 000	48	7	900	8 960 000
Chewing gum	3 300 000	......	2	130	2 710 000
Cleaning and polishing preparations	285 000	34	11	120	1 320 000
Clocks, clock movements, time recording devices, and time stamps	2 170 000	......	3	550	1 900 000
Clothing (except work), men's, youths' and boys' (not elsewhere classified)	33 000	88	130	6 000	31 600 000
Clothing, women's, not elsewhere classified	15 100	305	182	5 000	38 600 000
Combs and hairpins not made from metal or rubber	5 100 000	......	1	32	112 000
Corn syrup, corn sugar, corn oil and starch	3 500 000	......	2	370	7 500 000
Corsets and allied garments	572 000	......	10	720	3 470 000
Cotton goods	95 000	8	53	18 000	62 700 000
Cotton, small wares	605 000	3	9	730	2 840 000
Cutlery (not including silver and plated ware), and edge tools	510 000	7	11	755	3 440 000
Dairymen's supplies, creamery, cheese factory, and butter factory, equipment; poultrymen's and apiarists' supplies	710 000	15	5	214	1 250 000
Dyeing and finishing textiles	167 000	12	30	3 690	19 300 000
Electrical machinery, apparatus, and supplies	67 500	138	48	10 750	61 400 000
Embroideries	150 000	14	27	270	830 000
Emery wheels and other abrasive and polishing appliances	1 800 000	......	3	227	1 415 000
Engraving (other than steel, copper-plate, or wood), chasing, etching, and die sinking	650 000	11	4	56	215 000
Engraving, steel and copper-plate, and plate printing	267 000	4	8	224	850 000
Envelopes	712 000	9	1	73	352 000
Excelsior	1 850 000	......	3	63	225 000
Explosives	1 280 000	......	4	294	3 260 000
Fancy and miscellaneous articles not elsewhere classified	125 000	55	13	350	1 150 000

TABLE 4.—(*Continued*)

Item	United States, population per plant	Pesent number of plants in California	Potential additional plants in California	Potential additional number of industrial employees in California	Potential additional value of industrial products in California
Felt goods, wool, hair, or jute	2 230 000	......	3	285	$2 050 000
Fertilizers	191 000	21	14	550	4 950 000
Files	3 700 000	......	2	194	640 000
Firearms	5 800 000	......	1	350	1 000 000
Fire extinguishers, chemical	4 350 000	......	1	50	385 000
Fireworks	2 440 000	......	2	90	297 000
Flour and other grain - mill products	30 400	54	57	550	14 900 000
Forgings, iron and steel, not made in plants operated in connection with rolling mills	557 000	8	6	635	4 170 000
Foundry supplies	2 080 000	......	3	52	472 000
Furnishing goods, men's, not elsewhere classified	204 000	15	9	450	2 300 000
Gas machines, gas meters, and water and other liquid meters	1 690 000	6	1	105	523 000
Glass	464 000	6	3	920	3 790 000
Glass products (except mirrors) made from purchased glass	283 000	25	12	200	900 000
Gloves and mittens, leather	473 000	17	5	190	720 000
Glue and gelatine	1 645 000	5	2	70	650 000
Gold-leaf and foil	1 625 000	6	2	40	115 000
Handkerchiefs	....	......	5	294	1 330 000
Hardware, not elsewhere classified	252 000	27	18	2 200	8 200 000
Hat and cap materials, men's	1 090 000	......	5	133	1 135 000
Hats and caps, except felt and straw, men's	212 000	32	6	78	383 000
Hats, fur-felt	767 000	6	6	720	3 960 000
Hats, straw, men's	2 440 000	......	2	172	840 000
House-furnishing goods not elsewhere classified	127 000	89	8	166	1 110 000
Ink, printing	890 000	6	2	36	465 000
Ink, writing	4 200 000	......	1	35	207 000
Instruments, professional and scientific	442 000	14	12	885	3 750 000
Iron and steel — steel works and rolling mills	251 000	19	16	14 500	113 000 000
Jewelry	79 200	111	33	732	3 720 000
Jewelry and instrument cases	1 080 000	4	4	94	318 000
Jute goods	6 100 000	......	1	267	987 000
Knit goods	64 500	48	64	7 500	30 200 000
Lace goods	2 900 000	......	2	348	1 320 000
Lasts and related products	2 100 000	......	3	97	346 000
Leather goods, not elsewhere classified	310 000	26	9	163	810 000
Leather, tanned, curried, and finished	258 000	16	17	1 950	17 300 000
Lubricating oils and greases not made in refineries	687 000	13	6	128	2 160 000
Machine tool accessories and small metal-working tools not elsewhere classified	169 000	25	27	1 185	5 480 000
Machine tools	435 000	......	13	2 520	11 100 000
Motor vehicle bodies and motor vehicle parts	106 000	94	39	8 200	52 500 000
Motorcycles, bicycles, and parts	5 800 000	......	1	242	1 070 000
Musical instrument parts and materials, piano and organ	1 820 000	9	2	79	260 000
Musical instruments and parts and materials not elsewhere classified	1 150 000	......	5	190	650 000
Nails, spikes, etc., not made in wire mills, or in plants operated in connection with rolling mills	2 210 000	......	3	110	578 000
Needles, pins, hooks and eyes, and snap fasteners	2 840 000	......	2	316	1 000 000
Oil, cake, and meal, cottonseed	220 000	8	15	530	8 320 000
Paper	177 500	12	19	3 200	26 500 00
Pulp (wood and other fiber)	615 000	......	9	1 220	10 750 000
Paper goods, not elsewhere classified	280 000	14	13	720	5 300 000
Patent or proprietary medicines and compounds	80 000	117	20	340	4 200 000
Paving materials: Asphalt, tar, crushed slag, and mixtures	965 000	5	3	52	552 000
Pencils, lead, including mechanical	320 000	......	2	314	1 260 000
Pens, fountain and stylographic; pen points, gold, steel and brass	1 600 000	......	3	244	1 550 000
Perfumes, cosmetics and other toilet preparations	149 000	55	24	576	5 650 000
Pocketbooks, purses, and card cases	420 000	11	11	451	2 590 000
Printing materials not including type or ink	1 450 000	4	3	38	247 000
Pulp goods	2 710 000	......	2	175	1 230 000
Rayon and allied products	4 200 000	......	2	1 870	6 780 000
Refrigerators, mechanical	3 580 000	......	2	872	7 300 000
Regalia, badges and emblems	1 580 000	......	4	98	358 000
Rubber goods other than tires, inner tubes, and boots and shoes	295 000	28	6	678	3 560 000
Saddlery and harness	468 000	7	7	107	587 000
Safes and vaults	4 500 000	......	1	171	875 000

TABLE 4.—(*Continued*)

Item	United States, population per plant	Present number of plants in California	Potential additional plants in California	Potential additional number of industrial employees in California	Potential additional value of industrial products in California
Saws	1 510 000	......	4	230	1 025 000
Scales and balances	2 070 000	......	3	235	1 375 000
Screw-machine products and wood screws	447 000	9	11	945	4 450 000
Sewing machines and attachments	3 130 000	......	2	550	2 040 000
Shirts	141 000	18	26	1 920	6 850 000
Shortenings, not including lard, and vegetable cooking oils	3 050 000	......	2	158	6 980 000
Silk and rayon manufactures	81 500	......	67	6 250	33 000 000
Silversmithing and silverware	1 465 000	6	3	240	1 180 000
Soda-water apparatus	2 250 000	3	2	134	907 000
Sporting and athletic goods not including firearms and ammunition	503 000	12	10	527	2 340 000
Springs, steel, except wire, not including plants operated in connection with rolling mills	1 415 000	8	2	165	1 160 000
Stamped ware, enamel ware, and metal stamping, enameling, japanning and lacquering	210 000	23	17	1 370	6 100 000
Stationery goods not elsewhere classified	600 000	9	7	462	2 520 000
Steam and other packing pipe and boiler covering and gaskets, not elsewhere classified	712 000	13	2	115	595 000
Stereotyping and electrotyping not done in printing establishments	530 000	7	5	1 850	830,000
Structural and ornamental iron and steel work not made in rolling mills	82 000	136	24	1 120	7 700 000
Surgical and orthopedic appliances	336 000	29	12	350	2 350 000
Suspenders, garters, elastic goods made from purchased webbing	1 340 000	......	4	230	1 370 000
Tanning materials, natural dyestuffs, mordants and assistants, and sizes	967 000	......	6	143	1 800 000
Textile machinery and parts	324 000	......	17	1 425	5 500 000
Tin cans and other tinware not elsewhere classified	525 000	......	11	1 600	13 350 000
Tools, not including edge tools, machine tools, files, or saws	228 000	20	19	830	3 500 000
Toys (not including wagons and sleds), games and playground equipment	256 000	16	17	710	2 700 000
Trimmings (not made in textile mills) and stamped art for embroidery	191 000	24	19	292	1 350 000
Typewriters and parts	....	......	1	832	2 790 000
Umbrellas, parasols, and canes	863 000	......	6	156	902 000
Washing machines, wringers, dryers, and ironing machines for household use	1 880 000	5	3	410	3 400 000
Waste	895 000	6	4	140	1 100 000
Wire drawn from purchased bars or rods	1 500 000	......	4	1 140	9 600 000
Wood, turned and shaped, and other wooden goods not elsewhere classified	137 000	32	25	610	1 970 000
Woolen goods	265 000	5	18	2 500	11 600 000
Wool pulling	6 770 000	......	1	38	612 000
Wool scouring	5 800 000	3	1	71	94 000
Wool shoddy	2 440 000	......	2	82	693 000
Worsted goods	462 000	......	12	4 270	24 300 000

A summation of the deficiencies computed in Table 4 shows that in 1929 the value of products manufactured within the boundaries of California could have been increased by about one-third, or approximately $1 000 000 000, if the needs of the population within the State had been served by local manufacturing establishments. If California industries had taken advantage of this opportunity 1 742 new plants, employing more than 164 000 persons could have been established. Perhaps a clearer way of visualizing these deficiencies is to state that an additional industrial development, nearly equivalent in size to that in either Los Angeles County or in the San Francisco-Oakland District, could have found profitable markets within the State of California in 1929.

THE CENTRAL VALLEY PROJECT

The industrial deficiencies summarized in Table 4 are related intimately to the inadequate water supplies of the southern part of the State. This condition will be corrected by the Central Valley Project.

The principal construction features of this project (see Fig. 2) are the Kennett Dam and power plant, the Kennett transmission line and sub-station, the Keswick Afterbay Dam and Reservoir, the Keswick power plant, the Sacramento-San Joaquin Delta cross-channel, the Contra Costa conduit, the Friant Dam and power plant, the Madera Canal, the Friant Kern Canal, and the San Joaquin pumping system.

LOCATION FACTORS

In order to select a suitable location for each of the industries listed in Table 4, consideration must be given to the following factors: (1) Water supply; (2) proximity to supply and quality of raw materials; (3) location of markets for the finished product; (4) transportation costs; (5) State and local taxes; (6) power costs—hydro-electric, oil, and coal; (7) sources of labor supply, labor rates and attitude of those in control of State and local politics in controversies between labor and capital; (8) living costs; (9) climatic conditions and general health; (10) social conditions; and, (11) availability of industrial sites.

The relative weight of each of the foregoing factors will vary greatly with the particular industry; for instance, industries employing large numbers of men are certain to rate labor factors high. Plants producing heavy or bulky products will place special emphasis on transportation costs and proximity to markets. The development of such industries within certain sections of the Central Valley will be made economically possible by the low-cost water transportation that will follow the channel improvement of the San Joaquin and Sacramento Rivers. Water supply is a particularly important factor in determining the location of breweries, creameries, and packing plants.

It cannot be overlooked that many industries have not followed closely either cheap power and raw materials or favorable living conditions; for example, iron and steel mills are not located in Northern Minnesota and many textile mills are not near cotton or wool-producing sections. Markets, labor supply, and transportation facilities are generally given greater weight than the other factors in locating all industries except those producing foodstuffs. As a typical example, the iron and steel industry has been satisfactorily located on the southern shore of Lake Michigan, because at that place iron ore from Duluth, Minn., and cheap coal from Southern Illinois meet at a point adjacent to the markets and an adequate dependable labor supply.

It would not be possible, of course, to expand all the industries listed in Table 4 within a short period of time. New ones could be developed concurrently with the irrigated lands that will be made available by the Central Valley Project. These lands will form an agricultural base which will be capable of supporting a rapid population growth and, if industrial expansion is correlated with agricultural growth, the population density will increase at even a greater rate.

CORRELATION OF INDUSTRY AND AGRICULTURE

For example, assume that much of the industrial deficiency shown by Table 4 will be made up in the period between 1935 and 1945 through a co-ordinated industrial and agricultural development. Fig. 1(*b*) indicates that the increase in the area of irrigated lands of the State during that period will be about 800 000 acres and Fig. 1(*a*) that the population increase for the State will be about 2 000 000.

It is reasonable to assume that between 1935 and 1945 200 000 industrial employees, in addition to those employed in California in 1929, can be settled permanently in rural districts and can divide their working time between agriculture and industry. If 1 acre of land were allotted to each employee for gardening purposes, only one-fourth the increase in the area of irrigated land would be devoted to co-ordinated agricultural and manufacturing operations.

To provide for the continued future development of irrigated lands, as indicated by Fig. 1(*b*), more water will be required than that which will be furnished by the present Central Valley construction program. These additional supplies will be secured from other reservoirs contemplated in the State's comprehensive plan of water development.

The foregoing examples indicate broadly the opportunities for industrial expansion that exist within the State and also show that future industrial growth could be absorbed by the Central Valley without changing materially the program of agricultural development now planned for California. Although this exposition demonstrates the relation of the entire Central Valley to potential industrial development, it does not show in detail the economic benefits that could be obtained by co-ordinating agriculture and industry. A more comprehensive picture of the possibilities of decentralized industrial planning is needed, and this may best be obtained by a detailed study of a specific project.

PART III.—POSSIBILITIES OF MADERA IRRIGATION DISTRICT AS LOCATION FOR CO-ORDINATED INDUSTRIAL AND AGRICULTURAL PROJECTS

GENERAL DESCRIPTION

The Madera Irrigation District in California is suitable for combined industrial and agricultural development owing to its proximity to important markets, its climate, its large undeveloped areas of fertile soils, and its available labor supply. Fig. 2 shows its relative location. This District is situated in the heart of the San Joaquin Valley, about 150 miles southeast of San Francisco, 150 miles southwest of Stockton, and 22 miles northwest of Fresno, the latter being the largest city in the San Joaquin Valley. The San Joaquin River comprises the southern boundary of the District and the Chowchilla River, the northern boundary. The San Joaquin River has an annual run-off of about 1 750 000 acre-ft per yr, most of which comes down in the flood season of May and June. The Chowchilla River is a flood-water stream, with an average run-off of about 50 000 acre-ft per yr. The District is intersected about midway by the Fresno River, which is also a flood-water stream, with an average run-off practically equal to that of the Chowchilla River.

The total area of the District is 173 179 acres, of which 32 719 acres are rough hardpan, of doubtful value for irrigated land under present economic conditions, and 13 872 acres are irrigable, but not as satisfactory for cultivation as the better class of land. The remainder of the District, or 126 588 acres, is first-class, flat, irrigable land, of which 75 000 acres are now irrigated and cultivated and 51 588 acres are not irrigated. The latter area, which is now (1936) partly used for raising grain, is available for either industrial co-operative gardening or for independent agricultural development. For the purpose of this investigation, it has been assumed that about one-half the first-class available land, or 25 000 acres, will be taken up by combined industrial and agricultural developments.

Population and Labor Supply

The Madera Irrigation District includes two cities within its limits, namely, Madera, with a population of about 6 000, and Chowchilla, with a population of 1 000. The rural population is estimated to be about 7 000, making a total population for the District of 14 000. Aside from these permanent residents, there is a large transient laboring element which follows the cotton and fruit seasons.

Most of the labor available is unskilled. If permanent employment were offered, a large portion of the transient labor would settle permanently in the District and skilled labor would also be attracted. A number of workers are available in winter when they are not engaged in agriculture, and many men who work in the granite quarry and logging camps have extensive "off" seasons. As living conditions within the District are excellent, no difficulties would be experienced in attracting an ample supply of both skilled and unskilled labor.

Climate and Rainfall

The average daily maximum temperature for the year is 75 degrees. The temperature varies from a minimum of 36° in January to a maximum of 99° in July. The climate is dry and exceptionally healthful. Regardless of the high maximum temperatures that are attained during the summer, personal discomfort is seldom experienced owing to the low humidity.

The average annual rainfall is 9.56 in., which is typical for the arid lands of the San Joaquin Valley requiring irrigation. About 85% of the seasonal rainfall comes in the six months between November and April, with a yearly average of about 45 days of rain. There is a peculiar tendency, especially in the spring and autumn, for the rain to occur at night; so there is little interference with work. The climatic conditions of the San Joaquin Valley are especially favorable to industrial development.

Water Requirements and Water Supply

For the purposes of this investigation an analysis has been prepared on the premise that the District was fully developed in 1929. In the following analysis, it has been assumed that 25 000 acres of the first-class land has been utilized by industry for co-operative gardening purposes and that the remainder is developed by individual farmers.

Probably not more than 85% of the available land, or 119 000 acres, will be irrigated in any single year. The remaining 15% may be dry farmed, temporarily, or allowed to lie fallow. The quantity of water that will be required by the project, therefore, will depend upon the relative acreages of the respective crops that are grown upon this land. Table 5 shows the land use assumed in making this investigation and also the water requirements for each crop.

TABLE 5.—Land Use Assumed in Investigation

Crop	Area, in acres	Duty, in acre-feet per acre	Total consumption, in acre-feet
Alfalfa	28 200*	3.0	84 600
Cotton	18 800*	2.5	47 000
Trees and vines	23 500*	2.0	47 000
Grain	9 400*	1.5	14 100
Miscellaneous field crops	9 400*	3.0	28 200
Truck	4 700*	3.0	14 100
Industrial gardens	25 000	3.0	75 000
Total for irrigation	119 000	2.6	310 000
Domestic and industrial use based on future population of 150 000	....	....	17 000
Total annual consumption	....	....	327 000

* Based on percentages used in forecasting demands for Central Valley Project.

A large part of this required supply may be drawn from the San Joaquin and Fresno Rivers after the Central Valley Projects have been constructed. Experience in this District has shown that 25% of the gravity water diverted from the San Joaquin River will be lost by lateral seepage and of this quantity 15% will be recovered by pumping, the other 10% being completely lost. The quantity of water, in acre-feet, to be taken from each source of supply, therefore, is estimated as follows:

From Fresno River	20 000
From San Joaquin River, 258 000 acre-ft, of which three-fourths reaches the land	193 000
Pumped from natural underground recharge	75 000
Pumped from artificial recharge, 15% of 258 000 acre-ft	39 000
Total delivered to land and industries	327 000

This quantity is only about 10% in excess of that which would be required if the District were developed as an agricultural project alone.

Cost of Water Supply

It has been estimated that the cost of water from the San Joaquin and Fresno Rivers will be $3.00 per acre-ft after the construction of the Central

Valley Projects. The average cost of pumping in the Madera District is $0.05 per acre-ft for each foot of lift. For the average lift of 65 ft which exists at present, the average total cost of pumping is $3.25 per acre-ft. This value will apply only after the Central Valley projects are constructed. As the groundwater is now (1936) lowering rapidly, pumping charges will be increased if new sources of supply are not obtained. On the basis of the foregoing unit costs, the total cost of water may be estimated as follows:

Gravity supply, 278 000 acre-ft @ $3.00......	=	$834 000
Pumped supply, 114 000 acre-ft @ 3.25......	=	370 000
Operation and maintenance of gravity system	=	120 000
Total cost of water per year............	=	$1 324 000

$\frac{\$1\,324\,000}{327\,000\text{ acre-ft}}$ = $4.05 per acre-ft, or only 1¼ cents per 1 000 gal, at the land or factory

The proportion of the total annual cost required for each use would be:

Industrial gardens,	75 000 acre-ft @ $4.05	=	$303 750
Industrial and domestic use,	17 000 acre-ft @ 4.05	=	68 500
Independent agricultural development,	235 000 acre-ft @ 4.05	=	951 750
Total	327 000 acre-ft	=	$1 324 000

The construction of the Central Valley Projects will make available an adequate and low-cost supply of water for both agricultural and industrial purposes.

POWER

An ample quantity of power is available for both industrial and agricultural needs. Power charges vary from a minimum of 55 cents per kw-hr for a monthly consumption of more than 5 000 kw-hr, to a maximum of 4.0 cents per kw-hr for general power service.

SOILS

The soils of the Madera District consist principally of sandy loams of excellent fertility, suitable for raising alfalfa, grains, trees and vines, truck, and other field crops.

LAND OWNERSHIP AND VALUE

All the land in the District is privately owned although a large part of it is in the hands of banks and mortgage companies. This situation, which is not uncommon to irrigation districts, as well as other farming areas, points to the need for a re-adjustment in irrigation planning. Obviously, a greater population density is needed, and this may be obtained most readily by industrial developments related to the land as a population base.

Holdings in the District range in size from about 20 acres to 320 acres, with an average of about 80 acres. Values of agricultural land depend at present upon the relative fertility of the soil and the comparative ease with which underground water may be developed. Grain lands, considered non-irrigable under present conditions, are worth from $18 to $25 per acre. Land adapted to the growing of cotton, vines, deciduous fruit, etc., is valued at from $75 to $125 per acre.

TRANSPORTATION

The Madera District is traversed by the Southern Pacific and the Atchison, Topeka, and Santa Fé Railroads which connect it with the most important market areas. Many of the products of the District are transported by truck to Fresno where they are taken over by jobbers and packers for refrigeration and shipment. The average length of haul to Fresno is about 25 miles, and the hauling rates vary from 8 to 12 cents per ton-mile.

VALUE OF CROPS

An estimate of the annual gross value of the crops, if the District were completely developed, is given in Table 6. The prices upon which this estimate is based are the average f.o.b. values per acre for the seasons of 1929 to 1934, inclusive.

TABLE 6.—VALUE OF CROPS

Crop	Area, in acres	Value per acre	Total gross value
Industrial gardens	25 000	$100,00	$2 500 000
Alfalfa	28 200	66,00	1 870 000
Cotton	18 800	55,00	1 040 000
Trees and vines	23 500	94,00*	2 210 000
Grain	9 400	19,00	180 000
Miscellaneous field crops	9 400	50,00	470 000
Truck	4 700	100,00	470 000
Total	119 000	$73.44	$8 740 000

* Based on average value for San Joaquin Valley; derived from data given in "Permissible Economic Rate of Irrigation Development."

The average gross value of crops per acre of crops for the years 1929 to 1934 is about two-thirds of the value for 1929[13]. On this basis the average gross value per acre for the Madera District in 1929 would have beeen $110 per acre, and the total gross value of crops for the District would have been $13 090 000.

The gross value per acre of crops for the Madera District, on the 1929 basis, compared favorably with those of the most productive Federal Reclamation Projects, as follows:

Project	Gross average value of crops per acre (1929 basis)[14]
Okanogan, Washington	$250
Yakima, Washington	120
Madera, California (estimated)	110
Yuma, Arizona	90
Carlsbad, New Mexico	75

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[13] Rept. on Federal Reclamation, by J. W. Haw and F. E. Schmitt, M. Am. Soc. C. E., p. 63.

[14] *Loc. cit.*, pp. 34 to 58, inclusive.

EXISTING INDUSTRIAL PLANTS

There are seven cotton gins scattered throughout the Madera District from Chowchilla to a region eight miles south of Madera. There is also a cottonseed mill and a creamery at Chowchilla. There are three wineries within the project, with a combined tankage of 2 757 000 gal, and also a lumber mill with a capacity of 5 000 000 fbm per season.

FUTURE INDUSTRIAL DEVELOPMENT

It is impossible, of course, to forecast accurately the kinds of industries that may be attracted by a given situation. All that may be accomplished in this investigation is to demonstrate the possibilities of co-ordinating industry and agriculture within the District by studying the characteristics of the area and of a selected group of manufacturing plants. Table 7 gives a list of industries that could be located within the Madera District and also certain average relationships which have beeen computed from the statistics for these industries in the Industrial Census of 1929.

TABLE 7.—INDUSTRIES THAT COULD BE CO-ORDINATED WITH AGRICULTURE ON MADERA IRRIGATION DISTRICT

Industries	Persons in industry	Number of establishments	Salaries	Wages	Cost of materials	Value of products	Value added by manufacture
(*a*) INDUSTRIES ENGAGED IN PROCESSING AGRICULTURAL PRODUCTS							
Leather, tanned, curried and finished..........	2 000	17	$576 000	$2 340 000	$12 400 000	$17 700 000	$5 300 000
Boots and shoes........	5 000	30	1 000 000	4 950 000	11 400 000	21 400 000	10 000 000
Pocketbooks, purses, and card cases...........	500	11	193 000	635 000	1 470 000	2 840 000	1 370 000
Cotton goods..........	5 000	15	470 000	3 720 000	10 400 000	17 400 000	7 000 000
Woolen goods..........	1 000	7	222 000	1 060 000	2 650 000	4 650 000	2 000 000
Knit goods............	3 200	27	615 000	3 060 000	6 500 000	12 900 000	6 400 000
Wood, turned and shaped, other wooden goods...	500	20	130 000	425 000	660 000	1 620 000	960 000
Sub-Total.........	17 200	127	$3 206 000	$16 190 000	$45 480 000	$78 510 000	$33 030 000
(*b*) INDUSTRIES ENGAGED IN MANUFACTURING MECHANICAL AND ELECTRICAL EQUIPMENT							
Agricultural implements.	800	5	$225 000	$990 000	$1 910 000	$4 600 000	$2 690 000
Electrical machinery....	5 000	22	2 110 000	5 650 000	12 000 000	28 500 000	16 500 000
Refrigerators, mechanical	1 000	2	362 000	1 510 000	3 740 000	8 350 000	4 610 000
Hardware manufacturing	1 000	8	328 000	1 120 000	1 270 000	3 810 000	2 540 000
Sub-Total.........	7 800	37	$3 025 000	$9 270 000	$18 920 000	$45 260 000	$26 340 000
Totals............	25 000	164	$6 231 000	$25 460 000	$64 400 000	$123 770 000	$59 370 000

In listing this group, it was assumed that about one-eighth the total industrial deficiency for the State (see Table 4) would be developed within the Madera District. To have concentrated more industries in this area (on the 1929 basis) would have defeated the purposes of an industrial decentralization program. The plants were selected, therefore, with a view to obtaining a well diversified group producing a total annual value of product equal in amount to about $124 000 000.

The selection of the industries shown by Table 7 was based on the following principles:

(1) Industries were selected that would not compete with existing industries within the Madera District.

(2) The location factors, outlined previously under the heading "Possibilities of the Madera Irrigation District as a Location for Co-Ordinate Industrial and Agricultural Projects", are favorable to each of these industries.

(3) All manufacturing plants listed in Table 7 could have their operations co-ordinated readily with agricultural work, that is, workers could divide their time between the shop and their gardens without interfering with industrial operations.

(4) Two groups of industries were selected: The first relates to the processing of agricultural products, and the second consists of certain industries using mass-production methods in their manufacturing processes. Both groups are compatible with a program of decentralization. Industries engaged in the processing of agricultural products that could be raised within the District would effect large savings in transportation costs and offer ready markets for the raw materials produced by adjacent farms. Workers engaged in mass-production operations, as previously emphasized in this paper, need greatly both the physical diversification and the "back-log" of employment that may be obtained through part-time gardening.

(5) The annual value of the products of each of these plants is either equal to or less than the deficiency of this industry within California in 1929.

(6) Raw materials for each industry may either be produced within the District or brought in by rail or water transportation from adjacent sources of supply.

Approximately 25 000 industrial workers would be required to operate the plants shown by Table 7. As each of these would represent a family of about four, it is reasonable to assume that the total increase in population due to this industrial development would be 100 000. Allowing 1 acre of land per worker for gardening purposes, a total of 25 000 acres would be required for decentralized industrial developments as assumed previously herein.

Markets and Competition

The existing plants in California, manufacturing the products listed in Table 7, would offer little competition to similar industries located in the vicinity of Madera. This fact is shown clearly by Table 8 which gives an analysis of the existing market served by each industry shown in Table 7.

The three industrial areas analyzed in Table 8 are outlined in Fig. 2. Values of products and the population served for each industry were segregated for these three Districts. It will be observed from Table 8 that the population served by most of the industries within each District is now appreciably less than the population of the District. This means that the existing industries are not even meeting the needs of their adjacent areas and, therefore, would not offer much competition outside these areas.

Consider, for example, the products of the plants manufacturing boots and shoes. In the Los Angeles industrial area, the shoe factories served only 203 000 persons, leaving a market of more than 2 000 000 persons in this area who must purchase shoes that are manufactured elsewhere. Table 8 shows that the plants manufacturing agricultural implements serve more persons than the population listed under "Remainder of State." A deficiency for this product however, exists within the State, and this justifies the inclusion of such plants in Table 7.

TABLE 8.—ANALYSIS OF MARKETS AND COMPETITION

Industry	STATE OF CALIFORNIA		LOS ANGELES INDUSTRIAL AREA		SAN FRANCISCO OAKLAND INDUSTRIAL AREA		REMAINDER OF STATE	
	Value of product, in thousands of dollars	Population served by industry, in thousands	Value of product, in thousands of dollars	Population served by industry, in thousands	Value of product, in thousands of dollars	Population served by industry, in thousands	Value of product, in thousands of dollars	Population served by industry, in thousands
Population,* 1930 census, in thousands	......	5 677	......	2 209	......	1 307	......	2 161
Leather, tanned, curried and finished	4 410	1 120	....	....	2 946	750	1 464	370
Boots and shoes	3 095	391	1 604	203	....	....	1 491	188
Pocketbooks, purses, and card cases	492	882	298	532	194	350	....	....
Cotton goods	6 406	513	....	....	....	....	6 406	513
Woolen goods	1 561	655	....	....	....	....	1 561	655
Knit goods	10 303	1 400	6 341	862	....	....	3 962	538
Wood, turned and shaped; other wooden goods	1 159	2 040	645	1 130	410	725	104	185
Agricultural implements	7 924	3 480	....	....	....	....	7 924	3 480
Electrical machinery	42 131	2 240	11 009	588	26 865	1 425	4 257	227
Refrigerators, mechanical	....	....	....	....	....	....	....	....
Hardware manufacturing	1 969	1 051	897	480	1 023	545	49	26

* Population served is equal to or greater than that of industrial area.

ECONOMIC BENEFITS RESULTING FROM CO-ORDINATED AGRICULTURAL AND INDUSTRIAL DEVELOPMENTS

The best way to visualize the financial improvement that would result from developing within the District the industries shown by Table 7 is through comparative statements showing the possible results of operating the District, first as an agricultural project alone, and then as a combined industrial and agricultural project. In preparing such statements, the gross annual value of the crops may be compared with the total value of the products given by Table 7. The gross values of the crops represent the prices at the land, before transportation and other charges have been added. The values of products likewise represent the prices at the factory. Both these values are measures of benefit to the agricultural and industrial workers as their individual net incomes should vary in proportion to gross values. It would be better, of course, to base the comparison on the net income per capita before and after the introduction of industrial plants. Unfortunately, such an investigation would involve lengthy studies of agricultural and industrial production costs which vary so widely that generalizations would have little or no value.

In Table 9, the District is assumed to be fully developed. It must not be understood from this comparison that industrial workers would have nearly twice the average net income earned by agricultural workers. It would be reasonable to assume that about one-half the gross agricultural income per capita would be represented by production costs leaving a net average income per capita of $260. This would correspond to the average wage per capita of the industrial population which is estimated from Table 7 to be, $\frac{\$31\,691\,000}{100\,000}$, or $317.

TABLE 9.—COMPARATIVE ESTIMATED STATEMENTS OF ECONOMIC CHARACTERISTICS, 1929 BASIS

Description	Agricultural development	Industrial development	Total co-ordinated agricultural and industrial development
Population	25 000	100 000	125 000
Value of product	$13 000 000	$124 000 000	$137 000 000
Value of product per capita	$520	$1 240	$1 100

With the improvement in agricultural prices that will follow recovery, there is every possibility that the incomes of agricultural workers and industrial workers would be about equal. It is certain that the industrial worker would have nothing to lose by devoting part of his time to gardening as the land utilized for raising truck yields a rate of return that would probably exceed industrial wage rates. On the other hand the worker has everything to gain by having an alternate source of employment during lay-offs and slack periods.

As indicated in Table 9 the population with combined industrial and agricultural developments would be at least five times that of the District if it were devoted to irrigation farming alone. This does not include the great number of additional people that would be necessary for professional and commercial occupations. Although this part of the population is difficult to estimate, it is certain that the increase would be substantial. Tables 1 to 9, therefore, do not include all the benefits of decentralization, industrial construction, highways, schools, churches, and buildings, because social institutions would all help to develop the population and would bring tremendous benefits to the District as a whole.

OPERATING PLAN FOR DECENTRALIZED INDUSTRIAL DEVELOPMENT

The assumption that 1 acre would be cultivated by each industrial worker is rather liberal for the needs of the average family and is greatly in excess of the actual requirements for a subsistence basis. The writers, however, advocate an abundant rather than a subsistence level of living. The care of 1 acre of ground would require about 1.5 days per week per worker if the plots were operated according to the centralized plan as developed in Examples 1 and 2. As a matter of fact, industrial gardening on irrigation projects could be operated under no other plan, as it would be impractical to water and care for widely scattered projects.

Experience has shown that clearing and preparing the industrial gardens for planting could best be handled by the several companies within the District. The land adjacent to the plants could then be laid out in plots which would be assigned to the individual workers. The rotation plan of operation is not recommended as this removes the incentive derived from individual accomplishment.

Educational measures and strict supervision are absolutely essential. It would not be necessary for each plant to employ a trained agriculturist as one organization could handle the agricultural operations of several concerns. In addition to direction and education, the supervisor and his assistants would purchase and distribute seeds and also would manage the storage, distribution, and marketing of the agricultural products. Central storage facilities and distribution systems would be provided by each company. This measure is necessary to insure both the proper care and handling of the produce and a minimum of spoilage.

Conclusions

The foregoing detailed study indicates clearly that economic improvement would follow the co-operation of industry and agriculture on irrigated areas. It is not within the scope of a single paper, however, to present a quantitative analysis that even begins to show the potential benefits made possible by the great water conservation projects now (1936) being constructed.

New thinking is necessary if these results are to be achieved. One cannot view the areas affected by Boulder Dam, or by the works now (1936) being constructed in the Central Valley, the Columbia Basin, or the Tennessee Valley, in terms of flood control, power, or irrigation alone. Larger objectives will be attained if the new lands made available by each project are considered as a population base for a potential industrial development. It is entirely possible that many future industrial areas will develop about water conservation works rather than about congested cities.

These new industrial areas will not be limited by either State lines or other political boundaries; markets and transportation costs will determine the competitive limits within which the complementary industrial and agricultural developments of a water conservation project may serve economically. For example, the area which the future industries of the Central Valley project could serve may be much greater than that of the State of California as assumed in this investigation. To determine accurately the total potential industrial growth of the Central Valley, investigations should be made for the purpose of establishing the limits of the competing areas that will be adjacent to the Columbia Basin projects, to Boulder Dam, and to other projects suitable for industrial development. Obviously, such investigations must have an important place in future national planning.

Widespread industrial decentralization correlated with water conservation can be brought about only by the joint efforts of business and the Federal Government. Manufacturers are turning to industrial decentralization principally because it is profitable. The opportunities for profitable industrial expansion will follow the extensive water-shed development programs that have been sponsored by the Government.

Industrial relocation in rural areas and subsequent co-ordination with agriculture will be beneficial to the entire country. The markets for both agricultural and industrial products will be brought closer together; therefore, decentralization will aid materially in the solution of the distribution problem. Existing industrial areas will not be affected adversely by such a movement; as a matter of fact, relief from over-congestion, due to the abandonment of many obsolete plants, accompanied by the migration of both the industry and its employees, would prove a positive benefit to many cities. Modern production units in healthful locations would replace many of the huge antiquated plants that are now found so frequently in industrial centers.

The stabilization of income through occupational diversification would result in a tremendous increase in demand for all products, and lower production costs would put these products within reach of the workers. It is safe to say, therefore, that the plants moving away from industrial cities would eventually create about them their own markets and would not injure or compete with the plants within existing industrial areas.

It is recognized generally that the present demand is not near the saturation point. What average person, for example, would not be glad to double his (or her) wardrobe if economic conditions permitted? A little thought will show that this situation extends into every department of modern life; a tremendous potential demand is thwarted by putting the products of industry just out of the wage earners' reach. Once these obstacles are overcome, it is safe to say that the total industrial production capacity of the nation will be found to be entirely inadequate.

Actual experience has proved that labor is ready and willing to accept both diversification and decentralization. This transition cannot be accomplished over night; considerable training and re-adjustment will be necessary before the benefits of decentralization may be realized. The end results, however, will more than justify the cost.

The diversification of labor would curtail the over-expansion of farm lands by eliminating from competitive use the areas devoted to industrial gardening. These gardens would not take away part of the existing market for agricultural produce; actually, they would help to lift the burden of relief which is not only robbing the farmer of his market, but increasing his tax load as well. The unemployed man cannot buy produce; it must be given to him, and the farmer now helps with the giving. It is safe to say that the greatest losses of the depression have been the production losses due to idle time. Any system that will help to capitalize idle time, will also reduce the relief load, and thereby improve the economic condition of the entire country. Such improvement results inevitably in better markets for the products of both agriculture and industry.

To summarize, water conservation projects correlated with combined manufacturing and farming developments would assist in bringing about the balance between industrial and agricultural production that is necessary to economic stabilization.

ACKNOWLEDGMENT

This paper offers no brief for any formalized system of government, or controversial class philosophy. The suggestions herein contained are as applicable to the present industrial system and democratic form of government as to any other form. The paper has been kept as free from the mention of specific commercial names as possible in order to emphasize the purpose of the various examples as representative of their kind rather than as an effort to promote a certain isolated case.

Because of the importance of their contribution, however, free acknowledgment should be given to: Edward Hyatt, and Harry Barnes, Members, Am. Soc. C. E.; T. G. Graham, Vice-President of the B. F. Goodrich Company, of Akron, Ohio; A. E. Peters, General Superintendent of the Ingersoll-Rand Company, of Phillipsburg, N. J.; F. O. Hagie, Managing Secretary, Chamber of Commerce, Yakima, Wash.; Mr. F. Stewart Fitzpatrick, Manager, Construction and Civic Development Department, Chamber of Commerce, Washington, D. C.; Mr. Charles F. De Bardeleben, President, Alabama Fuel and Iron Company; Mr. John W. Haw, Director, Agricultural Development Department, Northern Pacific Railway Company, St. Paul, Minn.; Miss Mae A. Schnurr, Assistant to the Commissioner, U. S. Bureau of Reclamation, Washington, D. C.; Mr. John P. Ferris, Tennessee Valley Authority, Norris, Tenn.; C. D. Adams, General Superintendent, Holly Sugar Corporation, Sidney, Mont.; and J. J. Haly, Jr., Administrative Assistant, Department of Public Works, State of California, Sacramento, Calif. Fig. 1 of the paper was prepared from data supplied by research at the University of California. Unpublished data from the private files of the foregoing organizations, in large measure, form the basis for the paper.

DISCUSSION

JOSEPH JACOBS,[15] M. AM. SOC. C. E. (by letter).—One reads with admiration and, in the main, with approval, the forceful presentation made by the authors, of a phase of economics and sociology not often approached from the engineering angle. It is, in other words, the application of engineering analysis to certain economic and sociological problems which all too frequently in the past have measurably lacked that desirable form of analysis. It deals specifically with a phase of national development that is likely to become, if it is not already, an active political issue. The West understands and appreciates the benefits of reclamation, believes it to be a national asset of great importance, and if its value can be materially enhanced in the manner indicated by the authors, it is a matter of national economy and of national well-being in which every one has a vital stake—a consummation to be desired and to be striven for.

Some, no doubt, will feel that the authors are perhaps too optimistic as to the extent that major industries may advantageously be decentralized and transplanted to newly developed, or long existent, reclamation areas, and that the doctrine of self-sufficiency may be over-emphasized. That there are, however, distinct possibilities in that direction, and that there are some present tendencies toward industrial decentralization, there can be no question. Without intent to take issue with any of the authors' contentions, the writer desires to submit a few brief comments on their main thesis, as follows:

(*a*) A well balanced distribution of industry and agriculture is desirable and, unquestionably, diversification of occupation and decentralization of industry afford social advantages as pointed out by the authors. However, with the nation's splendid and steadily improving transportation and communication systems, complete balance for individual areas is not absolutely necessary and, in many cases no doubt, not entirely desirable or possible from economic considerations. In general, both industrial and agricultural production should be fostered and expanded only in those localities which are best adapted to the particular industry and the particular agriculture in question. Development should not be unduly forced on a fetich of self-containment.

(*b*) If California is not entirely self-sustaining as to some industries; that is, if it is not a completely balanced economic unit, it is more than self-sustaining as to other industries for which it has a nation-wide and, to some extent, a world-wide market. Certainly, it would not desire to surrender these markets. Other States and other localities are similarly situated and that is, by no means necessarily, an unwholesome situation.

(*c*) More can be said for the need of national self-sufficiency than for State or local self-sufficiency. However, with a broadening of understandings and of amicable agreements between nations, and again with the steadily improving transportation and communication systems in the United States, even complete national self-sufficiency, in the aforementioned sense, is far from being a present-day imperative necessity. An active interchange of world trade is highly important to national prosperity and, similarly, an active interstate commerce is of high economic value. These desirable ends would be largely sacrificed if the doctrine of self-sufficiency is carried too far in practice.

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[15] Cons. Engr., Seattle, Wash.

(*d*) Important developments in the direction indicated by the authors are no doubt possible and desirable, but there is always economic danger in broad generalizations, and caution is essential in the application of new economic theories and doctrines. The fact is that each particular industry, and each particular locality, demands a detailed economic analysis by experts in the particular lines involved before a safe program in respect thereto can be outlined. That the authors recognize the essential correctness of this statement is evidenced in their discussion of "Location Factors."

It is noted (see heading "Cost of Water Supply"), that a uniform price of $4.05 per acre-ft is designated for all classes of water use. If the water charge for industrial and domestic use were increased three or fourfold (a charge that could readily be borne for those uses), it would permit a material decrease in the charge for irrigation water. Industrial gardens can stand a heavier water charge than general agriculture but, for either, a charge of $4.05 per acre-ft, which amounts to from $10.00 to $12.00 per acre per season, is rather heavy—much heavier than is customary on most irrigation projects. A variation of rates for the different uses would seem to be desirable.

M. E. McIver[16] (by letter).—It is re-assuring to find engineers supporting remedial measures, such as those suggested in this paper. The writer believes that the future may be considered safe when engineers, rather than sociological and political theorists, prescribe for the ills of society.

The paper contains the kind of "medicine" for social and economic ills that the late Henry Arnstein, noted Consulting Chemist of Philadelphia, Pa., urged in the pages of every publication available to him. "Get the most out of natural resources", he pleaded, and his doctrine was more widely accepted in the thirty foreign countries to which he was special adviser in conservation policies, than in his own wealthy and profligate country.

The authors should add to their treatise an appeal for the development in the areas described, not only of branches of old established industries, but of new manufacturing agencies to produce—from surpluses, and from waste material—synthetics suitable as substitutes for material now imported. That was the final treatment in Dr. Arnstein's formula for changing a wasteful nation into an independent and stable power.

Much of the appalling extravagance of the last few years will be compensated and the future tax burden not only justified but "written off", if the policies outlined in this paper can be put into practice; and great social values can thus be developed from mere (but inconceivably expensive) power projects. Engineers should teach the politicians to go on to the ultimate Arnstein project and demand that, in addition to reforestation, for example, they learn all the synthetic possibilities of every splinter of every tree that is cut; that instead of merely making more sugar to compete with "reciprocity" sugar, the new industries be encouraged to develop fuel alcohol; and, that in some of these areas, the dyestuffs and fertilizers and hundreds of other products now imported, may be produced by engineering research magic from surplus and waste. Engineers will have occasion for greater pride in their leadership in the field of conservation than in the construction of the tallest buildings or the biggest bridges, which seem to be the largest scale production now attributable to engineering knowledge and skill. The latter utilities are evidence of genius, but conservation progress is genius plus patriotism.

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[16] Secretary, Am. Assoc. of Engrs., Chicago, Ill.

C. S. JARVIS,[17] M. AM. SOC. C. E. (by letter).—The clearly expressed views presented in this paper are both impressive and convincing because of the sound logic used, the practical examples cited, and the reasonable conclusions reached. Such examples and principles must be relied upon largely for future guidance in the decentralization of industry, orderly diversification of labor, and co-ordination of agriculture with other essential and basic industries.

Pioneers and frontiersmen generally have had to develop ingenuity in shifting for themselves, to become both self-reliant and reasonably self-sustaining, and thus to become largely immune to the hardships ordinarily attending isolation. Over-specialization of industry in crowded centers has had the opposite trend and effect; any reasonable means for bringing about decentralization of industry and diversification of labor, especially among rural surroundings and in more or less closely related branches of agriculture, should be encouraged.

Too often such motives have been announced, the colonies and industries established, the land subdivided and disposed of at rising prices, only to suffer collapse when it has been demonstrated that the proposed industries cannot thrive under local conditions as to raw materials, power, labor, water supply, or market. In other instances, smelters and mills for the reduction of ore have been established in rural communities, only to be shut down and removed after conclusive demonstration that the by-products of flues and the odors or dust attending the processes, and the effluents from drains are objectionable and harmful to various forms of life. Many communities have been brought to the realization that they have paid too high a price for industrial activity and temporary prosperity, when they discovered the deleterious effects of pollution in atmosphere, streams, pastures, fields, orchards, and in the soil itself. Other objectionable effects have attended the establishment of industries with characteristically unpleasant and noxious odors.

If the well-matured studies and procedure associated with community planning are utilized in connection with both reclamation and the various industries that are considered in an effort to promote a stabilized industrial activity, there is reason to expect attainment of the desired objectives, with avoidance of the costly errors so often experienced heretofore. Long-range planning implies the consideration of all lines of activity and growth that may be reasonably foreseen or imagined, with adequate provision for each essential element of the industrial system along with agriculture and its ramifications, and finally, adjustment and reconciliation of their divergent requirements.

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[17] Cons. and Hydr. Engr., SCS, Washington, D. C.

The citizens of the country must recognize and adjust themselves to the inevitable trend away from the smallest industrial units—the "single run-of-stone" grist-mill, now generally supplanted by roller mills; the water-driven, seasonally operated saw-mills and planing mills or wood-turning shops, ordinarily giving way to electrically operated all-year plants; the small textile factories unable to compete with the larger units or associations, and, therefore, readily absorbed during dull periods at bargain prices; the small tanneries, cement works, lime-kilns, foundries, glass works, canneries, dehydration plants for fruit and vegetables; the small creamery, ice-plant, storage warehouse, laundry, and indeed nearly all the ordinary commercial and manufacturing enterprises. The increasing complexities of accounting, inspection, chemical analysis and testing, research laboratories, water supply, fire protection, storage, buying of raw materials at favorable prices, marketing of the finished product, smoke abatement, industrial and household waste disposal, and maintaining good-will of customers and of public officials—all these have seemingly grown beyond the capacity of an individual or of a small group. They demand a technical staff with background and experience as broad as the problems they will be likely to encounter. The dependence of a highly trained staff, the operators, and their families, upon the stability of the enterprise calls for the maintenance of a substantial reserve of both raw materials and finished products, as well as substantial credit and liquid assets, to bridge over periods of depression.

For those occasional instances where a small industry can thrive, such as for corn-meal or other feed-grinding units at a centrally located farm, convenient to dependable and low-cost power, there should be an attractive future as long as the quality of the product is conscientiously maintained according to acceptable standards. Service to certain farms with heavy tonnages may be rendered to best advantage by means of portable grinding units. As such work is usually seasonal in character, and follows the main harvests of grain, it could well be one of the auxiliary activities maintained in connection with a well-equipped farm.

Other services, trades, and specialized work are normally in demand wherever a community establishes itself and has a reasonable assurance of continued growth and prosperity; but none of them has so nearly a guaranteed supply of raw materials nor so wide a market demand as have those branches of agriculture which furnish foodstuffs within ready access of population centers. The important part of the unemployed families' living supplied by the industrial co-operative garden projects, as described so interestingly by the authors, vividly illustrates the nearly independent status of rural home owners with small acreages and skill, industrious habits, and enthusiasm for agricultural enterprises appropriate to the region. To such a citizen, trained to thrifty ways and inclined to make some provision for the future, occasional interruptions of industrial employment mean merely a better opportunity to concentrate on his gardening or such other activities as he has established, more rigorous reduction of expenditures, depletion of savings, or a combination of such measures. He is not facing unemployment, but rather the problem of adaptation to available employment.

It must be recognized that many individuals and families are quite unfitted by natural inclination, training, and habits of life to become contented and dependable members of rural communities relying mainly upon agricultural pursuits. Early Colonial history and subsequent frontier life in this country afford many examples of failure because of unwillingness or inability to forego the use of seed-grain and vegetables during periods of privation, when such temporary appeasement of hunger plainly meant another season of famine or abandonment of the venture, admitting defeat at their own hands and under the stress of circumstances.

Granting that improved means of communication and the recognition of society's responsibility to aid individual members during emergencies have radically changed the outlook for struggling colonists and "back-to-the-soil" devotees, some vestige of that self-restraint, control over appetites, and long-range planning for economic stability must be maintained if resettlement projects are to prosper in an approach to their main objectives. Among these are decentralization of industry, diversification of labor, co-ordination of agriculture with other basic industries, thereby establishing a positive defense against the recurrent depressions, temporary collapse of property and security values, disorganization, distress, discouragement, and attendant ills.

On many reclamation projects where irrigation is the main dependence for crop production, snow surveys afford a satisfactory index to the season's water supply. If reservoirs provide considerable carry-over from years of surplus, the inclination is to draw a minor portion of such reserve during any one season. During years of extreme drought, and before adequate reservoirs are provided, it occasionally happens that the limited irrigation supply is required mainly within orchard and small garden tracts. The main recourse for the growing of forage and grain crops, under such circumstances, has been dry-farming, resulting in the improvement of methods and the pronounced extension of areas within which it has proved moderately successful, year after year. Even such persistent weeds as the Russian thistle provide excellent, succulent pasturage as long as they are kept closely grazed; and the mature plants have much nutriment available for livestock if included with ensilage, or if moistened thoroughly to soften the thorny spikes.

Unfortunately, mounting tax rates have become more and more burdensome to the land-holder and home owner, and this item is not subject to curtailment during periods of depression and retrenchment. If the assessed valuation of a county is reduced, the requirements for revenue result in a proportionate increase of tax levies, thus leading to delinquencies and ultimate forfeiture unless normal market demands and prices for agricultural products are restored without undue delay.

Either depression or prosperity incident to agriculture seems to be reflected throughout the national economic structure sufficiently to make the co-ordination and stabilization of its various branches, along with other basic industries, one of the foremost national problems of to-day.

JOHN P. FERRIS,[18] ESQ. (by letter).—This paper states the case for an attempt to create a new industrial region to accompany a new agricultural region, which latter would normally be expected to come into existence as a result of a major reclamation development.

The National Industrial Conference Board reports[19] a startling displacement of employment in agriculture relative to other types of employment. In 1930, about 88 persons per thousand of the population of the United States were employed in agriculture, whereas two generations earlier (1870) there were about 180 persons per thousand thus employed. A variety of causes accounts for this astonishing shift, some of which the writer has discussed elsewhere.[20] As this displacement has occurred in existing agricultural regions it is natural that it has produced a large surplus rural population. In many counties in the rural Southeast this surplus appears to be from 400 to 1 800 workers.

The nation, therefore, faces a parallel problem to that discussed by the authors—the problem of the region that is being subjected to excessive drainage of both human and material resources. In such areas the outgo exceeds income, and the "books" are balanced by a continual depletion of basic resources and by increasing debt.

Migration is beset by great practical difficulties. A logical starting point for the diffusion of industrial employment into rural areas seems to be in some of the regions of increasing surplus rural population, where, at the same time, the new "framework" of roads and power can be made available. Of course, a number of the new water-shed projects are situated in such areas.

Surplus rural population results both from the large technological displacement in agriculture generally, and from the high birth rate in many rural areas; for instance, the farm population of the South, in the opinion of O. E. Baker,[21] Senior Agricultural Economist, Bureau of Agricultural Economics, U. S. Department of Agriculture, is multiplying at a rate which may result in its contributing approximately ten times as many people to the national total at the end of a century as an equal number of adults in the large cities.

In connection with this matter of population, the data of the California Water Project Authority seem to be open to question. Messrs. Warren S.

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[18] Norris, Tenn.

[19] "Gainful Workers per 1000 Population in the United States, 1870–1930", *Bulletin*, National Industrial Conference Board, Inc., August, 1932, Chart No. 283, Road Maps of Industry Series.

[20] "Industry, Agriculture, and Population Readjustment", *Mechanical Engineering*, 1936.

[21] Opinion expressed in 1936 in a letter to the writer.

Thompson and P. K. Whelpton, of the Scripps Foundation for Research in Population Problems,[22] estimate that in 1960 the population of California will be about 7 940 000, instead of 11 600 000 estimated by the Water Project Authority.

The writer is in hearty agreement with the authors concerning the desirability of diversification of employment. Many of the benefits, however, are achieved provided there is diversification within a small area; it is not necessarily most efficient for an individual to work both on the farm and in industry. Much can be said for an arrangement under which some member of a farm family spends his full time in farming, with industry and related rural community services absorbing the surplus workers from farms. This preserves the family farm system, which has many advantages, both in the matter of incentives and in the matter of security. Although many of the part-time farming or gardening projects to which Messrs. Goodrich and Davis refer, have been measurably successful, the fact that the land is worked by non-owners with no permanent security has caused many of these enterprises to be regarded by the participants as emergency relief, rather than as a way of living.

A third form of decentralization which has occurred in many parts of the country, results from highly developed transportation facilities and widespread availability of electric power in hundreds of the smaller towns. An increasing number of city workers with cash incomes are actually living on farms near industrial centers and "commuting" back and forth, the cost being divided among the several workers who ride in each car. Such families are supplementing their income from industry with some "live-at-home" income derived from small tracts of their own land.

Such programs as the foregoing, as well as the type of development proposed by Messrs. Goodrich and Davis, assume a new relation of industrial location to the national welfare. In the first stages of America's economic development, the basic soil economies shaped manufacturing developments. McCormick manufactured his reaper at Chicago, Ill., because this city was a commercial outpost in communication with the great wheat area, and because coal and steel could be shipped cheaply from Pennsylvania on the Great Lakes waterway, and, later, on the east-west railroads. Settlement of the industrially skilled immigrant populations from Europe naturally followed these same transportation routes.

Now, however, the country seems to be entering a new stage, one in which industry can exert a powerful guiding force in shaping the basic economies under which the people live. To-day, there are "network" systems of roads and electric power; many industries no longer find it necessary, as they did in the past, to huddle in large cities along the few old transportation "arteries." Industry can, to a much greater extent than ever before, now establish the location of its demand for the materials which it buys and thus influence the location of their production.

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[22] "Estimates of Future Population by States, 1935 to 1960", Oxford, Ohio.

L. C. Gray,[23] Esq. (by letter).—According to the authors, advantages would accrue if industry were decentralized and small plants co-ordinated with agriculture in irrigated areas. Under this plan industrial workers in small factories would cultivate small tracts of land, either co-operatively or individually, and would produce at least part of their vegetable food requirements. Undoubtedly, certain advantages can be gained by proper co-ordination of agriculture and industry. However, the number and extent of benefits to be derived appear to be somewhat over-emphasized in this paper.

Both social and economic factors should receive consideration in judging the desirability of such a program. Social aspects are less tangible and less subject to measurement than economic factors, and the relative weight that should be given to them cannot be determined with precision. The value placed on certain social aspects by different individuals varies considerably. What one would consider highly desirable, another might consider undesirable. Because of this fact, the social advantages and disadvantages of decentralizing industry and integrating it with agriculture, have been omitted from this discussion.

The degree of economic success that can be attained by such integration will depend largely upon certain factors influencing the industrial enterprises and agricultural production. Sufficient quantities of raw products should be available under conditions which would allow successful competition with other manufacturers of similar products. In some of the irrigation projects of the West, a large amount of hydro-electric power is available at low rates. Such areas often offer an advantage to manufacturing industries which require large quantities of power, and in which the cost of power represents a large part of the total production cost. Under such conditions the manufacturing of non-agricultural products may assume considerable proportions. A local product can seldom, however, be expected to displace all similar products in near-by markets. If plants have a capacity greater than can be disposed of satisfactorily near-by, competition with more distant plants in other areas at higher transportation costs should be expected.

Conditions should also be favorable to agriculture in the area, both for the production of any raw materials for manufacture and for garden crops. The cost of water in relation to crop values should be moderate. Soil should be fertile and of suitable texture for irrigation. Topography should be fairly level, and other agricultural and water factors should be favorable.

If agriculture and industry are to be co-ordinated according to some such plan on newly constructed projects, the development of the irrigation projects and industrial plants should take place simultaneously. Workers will depend upon industry for the major part of their income from the time they arrive on the project, and farmers will depend on it for marketing their crops. This situation may be difficult to bring about. Irrigation projects usually require heavy expenditures and large units become available for use at one time. If the facilities, both land and power, are not used immediately, losses result. Close co-ordination of the building of the project, the location and building of industry, and colonization is highly important.

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[23] Asst. Administrator, Resettlement Administration, U. S. Dept. of Agriculture, Washington, D. C.

The extent of integration, and the balance between agriculture and industry which is most desirable, will vary between areas. In some areas large supplies of raw products, power, and adjacent markets may make a high degree of co-ordination desirable, whereas in other areas conditions are such that little if any integration is practicable. A policy concerning the integration of agriculture and industry in irrigation areas, therefore, should be very general in nature. Whether such a plan is applicable to an individual area, and the degree of integration which is most desirable, should be determined for each proposed area after a consideration of the particular circumstances.

It is particularly to the disadvantage of Western agriculture to stress area self-sufficiency since a large part of the production of several of its most important crops depends primarily upon wide distribution. Some past attempts at area self-sufficiency made elsewhere in the United States have worked to the direct disadvantage of certain types of Western agriculture. Rather than to aim at the highest possible degree of area self-sufficiency, as the authors recommend, efforts should be concentrated upon making agricultural and manufactured products which can be produced to greatest comparative advantages in the area. For instance, parts of California have a decided advantage over most of the United States in producing oranges, but have a distinct climatic disadvantage in manufacturing cotton goods. For certain types of metal goods, any marketing advantage may be entirely outweighed by a freight disadvantage in obtaining heavy raw materials.

The authors stress the production of garden products for home use by industrial workers. In addition, some part-time farming could probably be developed to co-ordinate with industry. In present irrigation projects some farmers find periodic work in industries processing agricultural products. Some co-operative plants hire mostly farmers. Successful part-time farming depends upon considerable income from sources outside the farm. Outside work should be available at times not seriously conflicting with important farm work. In addition to farmers finding work in industry through close co-ordination, industrial workers might be used on farms during periods of peak employment, such as at the harvesting of specialty crops.

Where the production of large quantities of specialty crops on new projects is contemplated, market outlets should be carefully determined before the project is set up. A delicate balance often exists between production and consumption of such crops, which a decided increase in production would destroy. Disturbance of the equilibrium usually causes wide price changes. A large increase in production would probably cause great difficulties both for the new farmers as well as for other farmers already producing that crop.

Although there are large numbers of unemployed industrial workers, an already over-burdened agriculture should not be expected to assimilate many of them. Rather than for the Government to encourage "back to the land" movements which tend to increase the supply and decrease effective demand for agricultural products, it would seem more logical to encourage existing industry, and to foster, particularly, the establishment of new industries to produce goods and services for which there would be a large public demand. The development of new industries may offer a more important method for the solution of the unemployment problem than industrial decentralization, and appears to be worthy of much consideration.

To summarize, integration of agriculture and industry should offer certain benefits. Irrigated projects often present conditions favorable to such integration. As conditions vary widely between irrigated areas, however, the adaptability of each area to such a plan should be judged individually on the basis of both economic and social values.

CHARLES P. WILLIAMS,[24] M. AM. SOC. C. E. (by letter).—A clear and convincing argument regarding what may be accomplished by the co-ordination of agriculture and industrial development, is contained in this paper. In the development of a large irrigation project the economic phases constitute a difficult and paramount problem. Of the irrigation projects built by the U. S. Bureau of Reclamation none can be considered, in itself, a commercially feasible investment. A project that cannot return the construction cost, without interest, within twenty-five years or more, cannot be considered commercially feasible. By no means does it follow that the investments were not justified. Incidental to the agricultural project, towns developed within the irrigated areas, with small mercantile stores of all kinds. There were established sugar factories, alfalfa meal mills, flour mills, creameries, cheese factories, canning factories, packing houses, and other small industries of various kinds. The benefits of the project were not confined to the irrigated area. A market was created for materials and supplies manufactured outside the project. The prosperity of such projects undoubtedly would be greatly increased by the introduction of other industries for supplying not only existing local needs, but also the increased demand created by the increased population.

In the paper, attention is called to the tendency toward the decentralization of industry. Such decentralization has been due, no doubt, not only to the greater economy of relatively small production units, to which the authors call attention, but also to the very high value of land in, or immediately adjacent to, large cities, and the consequent high value of plant sites, high taxes, and necessarily higher wages, due to the increased cost of living, for employees. Such decentralization, of course, has been on the initiative of the plant owners and management. Incidentally, labor has benefited by a lowered cost of living and more agreeable and healthful living conditions.

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[24] Project Engr., Ambursen Eng. Corporation, Possum Kingdom Dam, Mineral Wells, Tex.

The marked increase in unemployment during the relatively prosperous years 1923 to 1929, inclusive, is discussed by the authors, as well as the percentage of decrease in labor forces and the concurrent percentage of increase in production, in each of nine important industries, during the period, 1923 to 1927, inclusive. Various examples have been cited of the remarkable increase in efficiency in various industries by the introduction of machines and more efficient methods, thus reducing the labor necessary, and adding greatly to the army of the unemployed.

The authors are of the opinion that the plan proposed, if adopted generally, would be very effective in reducing unemployment. The writer doubts the validity of this conclusion unless other measures, not mentioned by the authors, are taken. The economic structure of the United States is a very intricate and complicated system, and a complete and comprehensive discussion of the causes of depressions and recoveries is obviously far beyond the scope of this discussion. However, there are certain basic principles affecting the system which, to the writer, appear to be evident.

Undoubtedly, there are many factors which contribute to unemployment. One very important factor, if not the most important, is the introduction of machines and more efficient methods, thus reducing greatly the number of men necessary for the accomplishment of any industrial work. However, no sane man would advocate the discard of machines, or the return to less efficient methods. Even greater efficiency is desirable wherever possible. The problem is to determine how efficiency can be retained, and even increased, and, at the same time, how to eliminate unemployment, or at least, to reduce it greatly.

Assume a country in which there is such industrial and vocational balance that all men capable of working are employed and that all are receiving such just compensation as enables each to satisfy his needs of goods produced and services rendered. Now, let machines and more efficient methods be introduced to such extent that the total number of man-hours of work, in the country as a whole, necessary for the production of those things which before had been produced, and the rendering of the services before rendered, are reduced, say, 10 per cent. If then the industrial and vocational balance is to be maintained, one of the following two plans, or a combination of the two, must be adopted:

(*A*) The total number of hours of work of those engaged in production or service may be reduced 10%, each individual receiving such compensation as will enable him to continue to satisfy his needs of goods produced and services rendered; or,

(*B*) The number of hours of work per individual and the wages or salaries per individual, in the industries and vocations in which machines and more efficient methods have been introduced, may be retained as before, and the number employed may be reduced to that necessary for the same amount of production and service as before. Those not retained in these industries and vocations may be employed in new industries or services, with appropriate hours of labor and appropriate compensation, or in old industries or services in which the forces employed therein are insufficient to supply the demand for the products or services.

Under Plan (*A*), the cost of production and services, prices, ability to purchase, and profits will remain the same as before. The only advantage will be that: (1) All are employed as before; (2) every one will be able to enjoy the same comforts as before; and (3) the workers will have more leisure for recreation, mental improvement, social activities, and similar avocations.

Under Plan (*B*), each of the workers retained in the former industries and vocations will receive the same wage or salary as before. However, the cost of production and service will be reduced, prices can be reduced, and the worker can buy the products and services before purchased for less than the sum originally expended therefor, leaving a part of his earnings for the purchase of goods produced in the new industries and of the new services introduced. Likewise, those employed in the new industries and vocations will be able to purchase their proportionate parts of the goods originally produced and the services originally rendered, and also the products of the new industries and the services of those in the new vocations.

Plan (*B*) evidently is preferable to Plan (*A*). Under Plan (*B*), a higher standard of living will result. Owners of industries may receive the same profits as before, but at a higher percentage, or they may receive even greater profits if not all the decreased cost of production is applied to a decrease in prices.

Due to the very great reduction in labor needed in industry in recent years, it probably would not be possible to provide sufficient work in new industries and new vocations to reduce unemployment greatly. Such great reduction could probably be accomplished only by the combination of Plans (*A*) and (*B*).

In the plan of combined agricultural and industrial development, suggested by the authors, it is proposed that the workers be employed part-time in the industrial plants and part-time in agriculture. Agriculture is not a new industry; neither is agricultural production insufficient to supply the demand for agricultural products. The decrease in the hours of labor per man in industry would increase the number of men employed in industry, but if the reduction in hours of labor per man in industry is applied to agricultural production, to the same extent would farm labor by those engaged primarily in agricultural pursuits be reduced.

The writer does not wish to be understood as being of the opinion that the Madera District would not be benefited by the plan proposed by the authors. He believes that it would be. More particularly would the industrial workers be benefited by being occupied part-time in healthful, outdoor work, as well as by obtaining increased compensation. As a means of reducing unemployment, however, he believes that it would have little, if any, value.

The writer is of the opinion that, although the co-operative plan of gardening might result in greater efficiency, the allotting of individual tracts to workers would be more beneficial for the reasons stated by the authors. In this case, it is believed that efficiency is of less importance than the psychological effect. It would be desirable to have a demonstration farm where the men could work for a time until they become acquainted with effective agricultural methods. At first, this farm could be of large area (embracing the entire 25 000 acres, in the case of the Madera District). Individual allotments could then be made therefrom as the men become competent to conduct their own operations efficiently. The stabilizing influence, to which the authors refer, would be much greater if each man had a proprietary interest in his own individual tract than if his work were on a co-operative basis. As far as conditions would permit, each could live on his tract and could establish a permanent home on it. It is true that a centralized plan would be necessary, but it need not be co-operative, except for single men and those lacking agricultural training.

The authors propose that the managing, and presumably the financing of the industrial gardens be by the several industrial companies. The writer believes this to be inadvisable. Some industrial owners may be philanthropic and altruistic, but, in general, their chief aim naturally is to secure a maximum profit. Company farms, like company stores, and large tenant plantations might become a force for economic slavery. The agricultural unit, including houses for workers, could easily be made a State, or National Government, self-liquidating project, financed in a manner similar to the operation of the Federal Housing Administration, the manager, agriculturist, and all assistants being State employees.

ERNEST P. GOODRICH[25] AND CALVIN V. DAVIS,[26] MEMBERS, AM. SOC. C. E. (by letter).—In closing, the writers re-emphasize the existing needs for the economic re-adjustments outlined in this paper: (1) Technological advances are continuing to increase unemployment; (2) the workers thus displaced have, in most cases, no secondary means of support; (3) the overcrowded and restricted conditions in many industrial cities prevent any constructive use of idle time; and (4) the accumulated national loss resulting from this idle time is such a staggering value that almost any means, no matter how drastic, would be justified to prevent it. Consider, for example, a fair measure of this loss. At the nominal unit rate of $1 000 per worker the annual loss to the country, with 10 000 000 persons idle, would be $10 000 000 000. From a banker's viewpoint the prevention of this loss would be worth an investment of $100 000 000 000, or approximately three times the national debt. This is a challenge to engineers.

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[25] Cons. Engr., New York, N. Y.

[26] Formerly, Chf. Engr., Ambursen Dam Co., Inc., New York, N. Y.

There are indications that the Medical Profession is already aware of the need for the social changes recommended by the writers. For example, Dr. Alexis Carrel has written:[27]

> "There have been in the past industrial organizations which enabled the workmen to own a house and land, to work at home when and as they willed, to use their intelligence, to manufacture entire objects, to have the joy of creation. At the present time this form of industry could be resumed. Electrical power and modern machinery make it possible for the light industries to free themselves from the curse of the factory. Could not the heavy industries also be decentralized? * * * Men would live in small communities instead of immense droves. Each would preserve his human value within his group. Instead of being merely a piece of machinery he would become a person."

The writers are in accord with Mr. Jacobs and Mr. McIver that this entire matter should be approached from an engineering angle. Engineers should be willing to assume a large measure of responsibility during the transition period.

The suggestions and analyses presented by the writers were not submitted as a possible cure-all for the problems outlined in the foregoing; these were intended to show by one specific example the possible benefits that could be realized from correlating the conservation of natural resources with industrial and economic adjustments. Conservation programs, such as are now (1938) under way in the arid regions of the West, could have as their larger objective the ending of unemployment. The investment in each project, in a sense, could be considered as an increment of the total that would be required to capitalize the time now being lost during depressions. Mr. Williams makes a point in this respect by indicating the over-all economic effects of the projects built by the U. S. Bureau of Reclamation even if many of these projects would not be commercially feasible investments when considered individually. Add to these existing benefits the partial elimination of unemployment, or, as Mr. Williams prefers to call it, furnishing a supplementary means of income during slack periods, and the accumulated benefit values will be many times the project costs.

Improvement, of course, cannot be brought about quickly; it will be rather a matter of slow growth. A long period of preparation, co-ordination, and training will be required, as suggested by Mr. Jarvis. It is unlikely that industrial growth will parallel colonization and agricultural development, as suggested by Mr. Gray; the writers are of the opinion that factories will follow only after the country is fairly well developed.

The writers intended to emphasize the need of a better balance between agriculture and industry in a given area rather than complete self-sufficiency as inferred by Mr. Gray and Mr. Jacobs. As Mr. Ferris so ably stated, there has been a mining process in some areas which has drained them of both human and material resources. In some cases, decentralization will help to restore, and in others establish, a more balanced economy.

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[27] "Man the Unknown," by Alexis Carrel, p. 315.

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CONSTRUCTION AND TESTING OF HYDRAULIC MODELS, MUSKINGUM WATER-SHED PROJECT

BY GEORGE E. BARNES[1], M. AM. SOC. C. E., AND
J. G. JOBES[2], JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. G. W. HOWARD, F. W. EDWARDS, T. T. KNAPPEN, RALPH W. POWELL, AND GEORGE E. BARNES, AND J. G. JOBES.

SYNOPSIS

Hydraulic model studies were conducted on eleven of the dams for the Muskingum Water-Shed Project, north of Zanesville, Ohio. This paper describes the laboratory facilities used in the tests, and the more important features of model fabrication, operation, and test procedure. The time and personnel required, and the cost of doing the work are given. An effort is made to present a picture of the physical equipment and organization required, and also the cost of securing results on a well defined, although not necessarily typical, program of model studies. It is believed that such material, as distinct from technical findings alone, has not formerly been made available to the extent desired by the profession. Thus, more discussion is devoted to the manner in which the work was done, than to a detailed analysis of the test measurements. However, certain selected test observations of special importance, which are considered to be of general interest as well, are presented herein.

METHOD OF ATTACK

In each of the eleven studies, a working model simulating the outlet works, spillway, or other features governing hydraulic performance, was constructed from preliminary drawings of the dam. The general purpose of all tests was: (1) To observe and record the behavior of the model, built in

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NOTE.—Published in December, 1936, *Proceedings*.

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[2] Engr., U. S. Engr. Office, Los Angeles, Calif.

accordance with the preliminary design; (2) to determine revisions necessary or desirable for optimum performance; and (3) to revise the model to complete or substantial agreement with the final design and to make complete record tests under the full range of flow conditions anticipated for the prototype.

The laboratory personnel was given the relation between tail-water stage and discharge at each site, based upon channel studies. In certain cases, where considerable variation in stage might be expected at a given discharge, the best performance was demanded only for most frequent operating conditions, good performance for less frequent circumstances, and safe operation under circumstances likely to occur only at rare intervals. Adequate discharge capacity and safety were of first importance.

The models were constructed with surfaces in contact with the water (except non-essential detail) similar to the proposed structure. The models were not distorted, and were built to the same vertical and horizontal scale, varying from 1:20 to 1:40. Each model included some or all of the following elements: (*a*) Reservoir approach topography at the control works; (*b*) intake tower and gates; (*c*) outlet conduits or tunnels; (*d*) stilling-basin; (*e*) outlet channel; and (*f*) spillway.

The laboratory studies extended from August 1, 1934, to June 20, 1935. They began immediately after the preparation of the official "Plan of Flood Control and Water Conservation Reservoirs" by the United States Engineer Office of the Zanesville District, and were continued throughout the design period. As the basic data for design were supplemented by field measurements and revised by analysis, and as laboratory test data accumulated, there was a constant rotation of data between the office and the laboratory during this interval. When acceptable performance for stipulated operating conditions was secured in the model, record drawings of the final model were put in the hands of the designers. These drawings were supplemented by a final report giving a complete record of all tests for that model.

LABORATORY FACILITIES

The model studies were made in the Warner Hydraulic Laboratory of the Department of Civil Engineering, Case School of Applied Science, in Cleveland, Ohio. This laboratory includes two flumes, both of which were used in the work, and the larger one of which was added specifically for these tests. Fig. 1 is a longitudinal elevation of the circulating system, showing the metering devices and the flumes with essential accessories.

The laboratory occupies a total of 6 000 sq ft of floor space on two floors of a modern building, with a trucking entrance on the ground floor and a 3-ton traveling crane for equipment and material handling. From a 50 000-gal recirculating reservoir under the ground floor, a battery of three centrifugal pumps, operating in parallel, with a total capacity of about 7.5 cu ft per sec, deliver through a 10-in. common header to a constant-level tank on the second floor, or to the supply mains direct, if desired. Flow from the constant-level tank is through a 12-in. or a 4-in. supply line, each equipped with a Venturi tube, the flow being registered with indicating, recording, and totalizing meters. The metered supply reaches the River Flume on the ground floor through a 10-in. riser, the discharge from the flume being returned to the recirculating reservoir. On the second floor, the Experimental Flume is fed by a 10-in. take-off from the riser, or from a 6-in. take-off and weir box.

Discharge from the experimental flume on the second floor reaches the reservoir through a 10-in. return, or may be passed through any one of six volumetric measuring tanks, each 6.0 ft in diameter and 10.0 ft deep, arranged in pairs, with flipper devices for continuous running to calibrate the meters or measure the discharge, and each equipped with telescopic level readers and quick-opening valves through which water is returned to the reservoir.

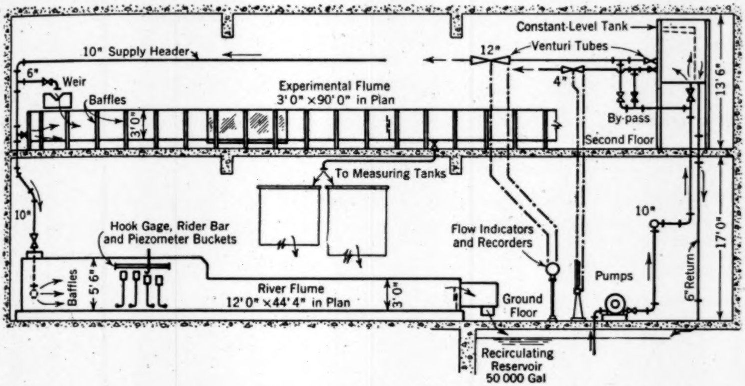
Constant-Level Tank
10" Supply Header
12"
Venturi Tubes
6"
Weir
4"
Experimental Flume
3' 0" ×90' 0" in Plan
Baffles
3' 0"
By-pass
Second Floor
13' 6"
To Measuring Tanks
10"
10"
Hook Gage, Rider Bar
and Piezometer Buckets
Flow Indicators
and Recorders
17' 0"
Baffles
5' 6"
Pumps
6" Return
River Flume
12' 0" ×44' 4" in Plan
3' 0"
Ground
Floor
Recirculating
Reservoir
50 000 Gal

FIG. 1.—LONGITUDINAL ELEVATION OF HYDRAULIC LABORATORY (NOT TO SCALE)

Both the second-floor experimental flume, with glass section, and the river flume on the ground floor, are equipped with adjustable tail-gates and with numerous piezometer connections. Each separate model requires a newly built piezometer assembly, leading in each case to a battery of open buckets in which levels are read by a hook-gage traveling on a carefully leveled rider bar. Each separate model, likewise, requires newly set rails supporting point-gage bridges, or sounding devices for measuring scour.

Required flows are set by valve manipulation and indicated by either the 4-in. or the 12-in. Venturi tubes. At low flows, a U-tube manometer containing carbon tetrachloride is used in tests as it gives much greater deflections and greater precision in reading. To insure minimum error, the 4-in. and the 12-in. meters are rated by volumetric measurement. The models for the Wills Creek, Charles Mill, Mohawk, Dover, Mohicanville, and Pleasant Hill Dams (see Fig. 2) were tested on the ground floor and the models for the Tappan, Clendening, Piedmont, Senecaville, and Bolivar Dams were tested on the second floor.

Any description of laboratory facilities needed for model studies would be decidedly incomplete without mention of the shop facilities, tools for maintenance work, and the fabrication of special instruments, or other devices required for a wide range of purposes. In a large city such as Cleveland, the availability of industrial shops is an added asset. Furthermore, room is needed for the preparation of drawings, filing of records, and reference material

TYPES OF MODELS

The characteristics of the various dams have been described by E. L Winslow, Jr., Assoc. M. Am. Soc. C. E.[3] The following description gives only the distinguishing features of the several models studied. Aside from site

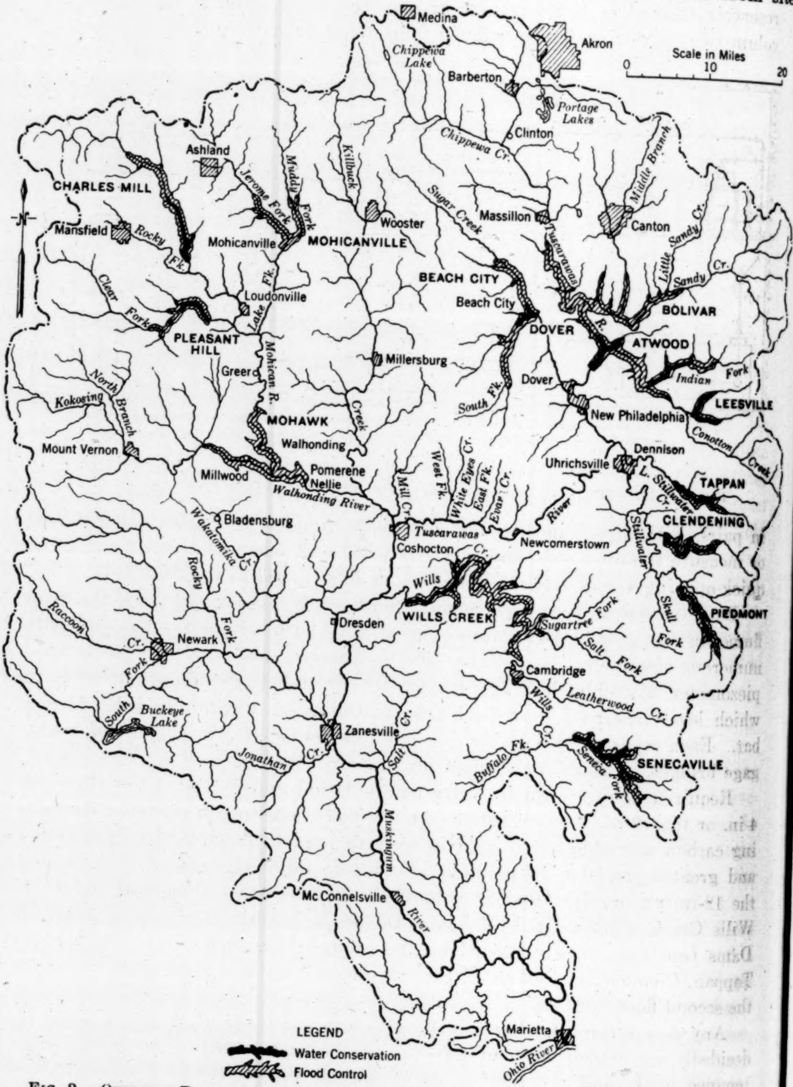
Medina
Chippewa Lake
Akron
Barberton
Scale in Miles
0
10
20
Portage Lakes
Clinton
Chippewa Cr.
Middle Branch
Ashland
Jerome Fork
Muddy Fork
Killbuck
CHARLES MILL
N
Mansfield
Rocky Fk.
Mohicanville
MOHICANVILLE
Wooster
Sugar Creek
Massillon
Tuscarawas
Canton
Little Sandy Cr.
Sandy Cr.
BEACH CITY
Beach City
Clear Fork
Loudonville
Lake Fk.
PLEASANT HILL
BOLIVAR
DOVER
R.
ATWOOD
Mohican R.
Millersburg
Indian Fork
Greer
Dover
Kokosing
North Branch
South Fk.
LEESVILLE
New Philadelphia
MOHAWK
Creek
Conotton Creek
Walhonding
Mount Vernon
West Fk.
White Eyes Cr.
East Fk.
Evans Cr.
Dennison
Uhrichsville
Pomerene
Nellie
Millwood
Walhonding River
Mill Cr.
River
L. Stillwater Cr.
TAPPAN
CLENDENING
Bladensburg
Wakatomika Cr.
Tuscarawas
Coshocton
Cr.
Newcomerstown
Stillwater
Rocky
Wills
WILLS CREEK
Skull Fork
PIEDMONT
Raccoon
Dresden
Sugartree Fork
Fork
Cr.
Newark
Salt Fork
Fork
Cambridge
Wills
Leatherwood Cr.
South
Buckeye Lake
Cr.
Zanesville
Salt Cr.
Jonathan Cr.
Buffalo Fk.
Seneca Fork
SENECAVILLE
Muskingum
Mc Connelsville
River
Marietta
Ohio River
LEGEND
Water Conservation
Flood Control

FIG. 2.—OFFICIAL PLAN OF RESERVOIRS AND DAMS IN THE MUSKINGUM WATER-SHED IN OHIO.

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[3] *Civil Engineering*, January, 1936, p. 1.

requirements, dissimilarity between structures is occasioned partly by the fact that operation of the several works was planned so as to be interrelated, and the performance of one outlet, in certain cases, affects another. These circumstances result in a great variety of hydraulic problems to be solved in design, and the model studies, therefore, vary in character to a commensurate extent.

For each dam, the outlet works terminates in a stilling-basin in which an hydraulic jump is formed, to dissipate the energy of conduit discharge. Since, in any case, spillways on these dams discharge under emergency conditions only to safeguard embankments, model spillways were studied in the laboratory only for those few dams where the confluence of spillway and outlet discharge occurs in such a manner as to affect, appreciably, the design of the spillway, the outlet works, or both.

The Wills Creek, Mohawk, and Bolivar models each include an intake tower with six control gates discharging through twin tunnels or conduits of horseshoe section, into a stilling-basin. The dimensions, heads, capacities, and tail-water conditions differ widely for the three dams.

The Clendening, Tappan, and Piedmont models each discharge through a single horseshoe conduit into a different type of stilling-basin. The Clendening, Tappan, and Piedmont Basins are similar to each other, but unlike

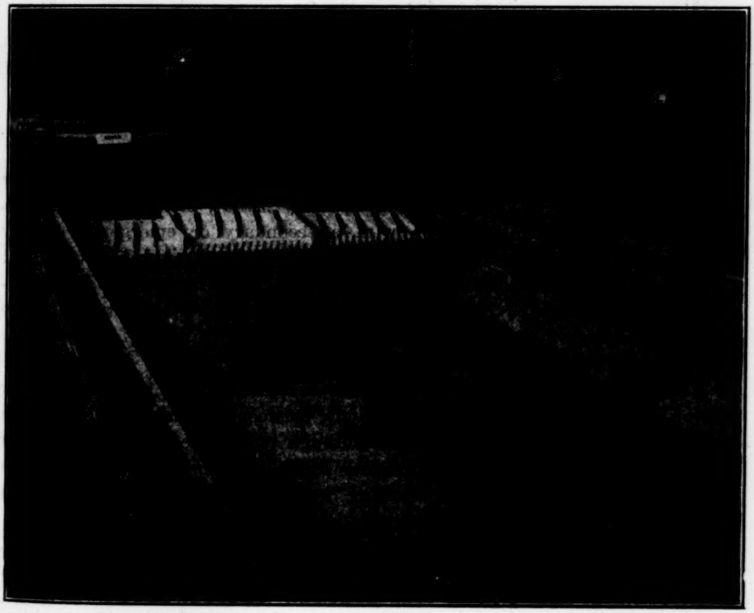


FIG. 3.—MODEL OF SPILLWAY AND OUTLET WORKS, DOVER DAM

those developed for other dams of the project. The distinctive feature is the rise of the curved apron below the portal to an elevation higher than the invert of the tunnel, followed by a parabolic drop to the floor of the stilling-basin. The formation of an hydraulic jump with relatively high tail-water and low tunnel elevation and discharge is thereby obtained.

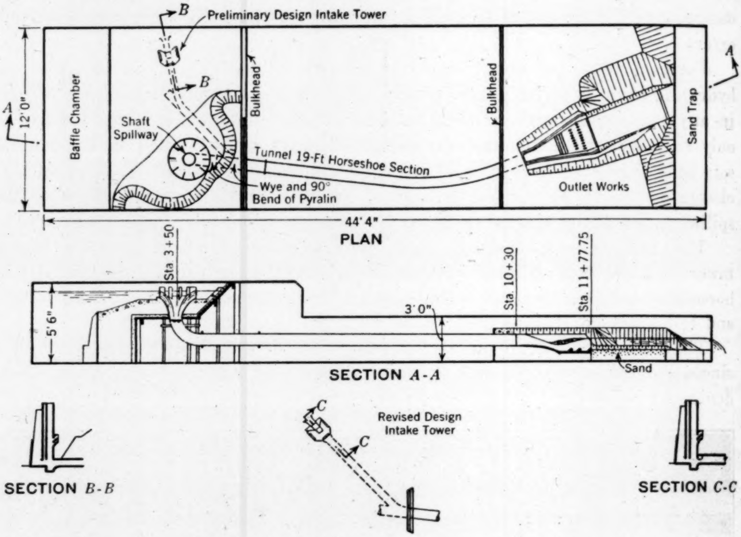

B
Preliminary Design Intake Tower
B
A
12'0"
Baffle Chamber
Shaft Spillway
Bulkhead
Tunnel 19-Ft Horseshoe Section
Wye and 90° Bend of Pyralin
Bulkhead
Outlet Works
Sand Trap
A
44'4"
PLAN
Sta 3+50
Sta. 10+30
Sta. 11+77.75
5'6"
3'0"
SECTION A-A
Sand
SECTION B-B
C
Revised Design Intake Tower
C
SECTION C-C

FIG. 4.—MODEL LAYOUT, PLEASANT HILL DAM

The Charles Mill and Mohicanville models, as finally crystallized after substantial changes to the preliminary spillway layout, include short conduits through a solid masonry section with a secondary or emergency spillway immediately adjacent. The Senecaville model discharges through twin short conduits set in a gate-tower centered between adjacent twin over-flow sections, each surmounted by a Taintor gate. For Senecaville, the discharge from all outlets combines in a long paved channel with a steep grade leading to a stilling-basin, 400 ft (20 ft in the model) down stream.

The model of Dover Dam (see Fig. 3), simulates a solid masonry, overflow section, 338 ft long (9.66 ft in the model), between abutments, with eighteen gate-controlled, short, outlet conduits through it. These conduits are set in three batteries of six units each, at three levels. The outlet portals discharge over a curved apron between training walls into a stilling-pool which is required to be very shallow because of foundation conditions at the site.

The model of Pleasant Hill Dam (see Fig. 4), includes an intake tower, enclosing a vertical shaft, into which flood waters are discharged through six orifices, without control gates. The orifices are set at two different elevations, and are dimensioned to secure capacity which will predetermine the reservoir rise automatically. The model shaft discharges at the base through a constricted throat (for the purpose of maintaining a plunge pool in the shaft) into a 6-in. (15-ft on a 1:30 scale), circular tunnel, and blow-off gates are provided at the tower base, as well as temporary openings for diverting water during the construction period. The spillway is of the shaft type with a circular crest, 2 ft 8 in. (80 ft) in diameter, surmounted by piers to eliminate vortex action, and with its ogee surface converging to an 8-in (20-ft) circular shaft and 90° bend to the horizontal. The model tower and spillway tunnels meet in a wye junction to form a tunnel (horseshoe section) 8 in. (20 ft) in height and width, leading to a stilling-basin below the dam.

Model Fabrication

From the dimensions on the design drawings the Laboratory Staff prepared detailed model drawings for the fabrication of the several elements, and also layout sheets to guide assembly and erection. In making the models, skilled labor was employed in the laboratory and shop. Certain units were made for a price by commercial organizations in Cleveland. In the special instance of the wye-tunnel junction for Pleasant Hill (required to be made of a transparent material so that flow inside the junction could be observed and photographed), the piece was fabricated at the University of Iowa, at Iowa City, Iowa, from drawings prepared at Case School, and then shipped to Cleveland for erection. Speed was essential and all available facilities were needed. Where complex details had to be reduced to model dimensions, as, for example, in the gate-towers, and tunnel and portal sections, the model drawings were made to full model scale. Topography in the model reservoir and in the channel below the dam was reproduced in the model by inundated sand fill, topped with a 1½-in. mortar coat, placed between sheet metal templets cut to the proper transverse profile and set at several stations on the center line of the works. A tinsmith cut these templets from full-sized paper patterns. The original pencil fabrication and layout drawings were usually handled by the craftsmen working on the model, and due to the different scales required for the drawings, it was not always feasible to establish a uniform size for the sheets. Experience indicates that it might be preferable to maintain a uniform size for filing and to work from blue-prints, since the drawings suffered considerably in the handling.

To secure the complicated shapes with reasonable accuracy and speed, it was found best to work in sheet steel. The model tunnels and gate-towers, therefore, were made from welded 16-gage material. The Morning Glory spillway was spun from sheet aluminum, on a laminated solid wooden core, accurately turned to shape. The curvature in the tunnels was made as a series of short tangents. The joints between tunnel sections were male and female with bolted flanges. In forming the multiple short conduits—as for the Dover, Charles Mill, and Mohicanville Dams—a box-like steel shell formed the floor, ceiling, and the side walls of the outermost conduits. Individual conduits were then formed by wood-block inserts or piers fashioned by a pattern-maker with snug fit between floor and ceiling. The gates for all dams were vented immediately down stream. These vents were reproduced accurately in the models with welded steel, with openings carried through risers, headers, or stacks as in the prototype, either to scale or slightly larger. The stilling-basins were usually formed with wood floor, walls, baffle-piers, and terminal sill. Where the side walls were warped, it was easier to secure proper dimensions with 22-gage galvanized sheet metal, or some comparable commercial product. Where changes were not anticipated the walls and floor of the stilling-basin were formed in mortar.

Numerous piezometer connections were installed along the floors of the conduits and at other points where pressure readings were desired. Unless piezometer openings are mechanically true, and unless the surfaces in which they are set are absolutely smooth and undistorted, false readings will be secured. For these models the best piezometer connection was found to be made by brazing a solid lug about 1 in. long to the under side of the steel floors, and drilling either a single $\frac{1}{8}$-in. hole through it in a direction normal to the surface, or three of such holes, and dressing the opening at the surface with a slight radius. The opposite end of the lug was tapped for the insertion of a $\frac{1}{4}$-in. or a $\frac{3}{8}$-in. pipe leading to the piezometer buckets. On the surface of the Morning Glory spillway of the Pleasant Hill model seven pairs of piezometer openings, diametrically opposed, were connected through similar lugs to copper tubing leading to a piezometer panel on which was mounted 0.5-in. transparent tube through which the pressure gradient was read directly against paper scales. It should be remembered that piezometers record the

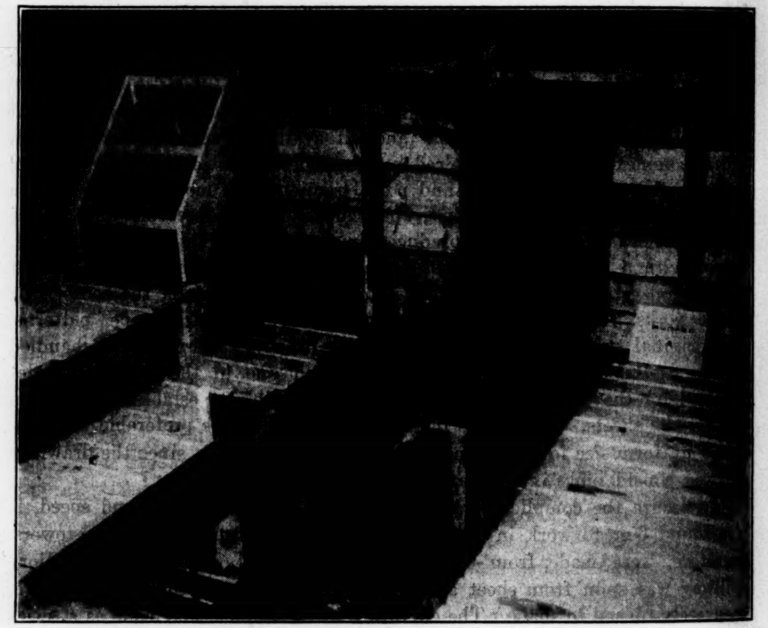
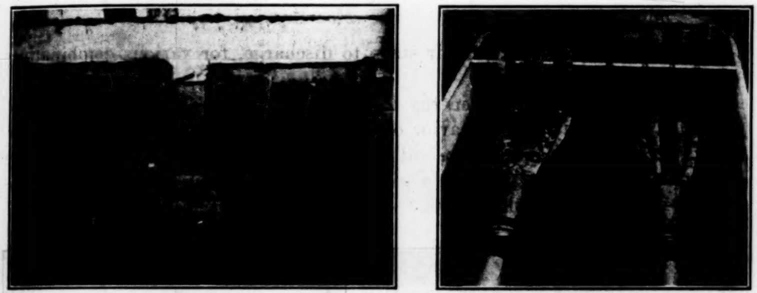


FIG. 5.—MODEL OF INTAKE AND TWIN TUNNELS, MOHAWK DAM (ORIGINAL DESIGN BEFORE REVISIONS)

pressure and not the water depth where curvilinear flow exists in a vertical plane. Of all the measurements taken, piezometer readings were most difficult and unsatisfactory.

Fig. 5 shows a model of sheet steel with the intake ceiling elliptical in profile and of transparent material. The gates are steel flats, sliding in grooves in the wood division piers, and vented through rectangular slots and stacks. The tunnels are 20 ft high (model scale, 1:40). The complete model for the Dover Dam to a scale of 1:35 is shown in Fig. 3. Typical model construction is shown in Fig. 6[4]. The Bolivar tunnels are 16 ft high to a scale of 1:40. Typical of the stilling-basins for Clendening, Piedmont, and Tappan, is the model shown in Fig. 7.



(a) TWIN INTAKES (b) GATE STRUCTURES

FIG. 6.—TYPICAL MODEL CONSTRUCTION, BOLIVAR DAM

FIG. 7.—MODEL OF STILLING-BASIN, CLENDENING DAM

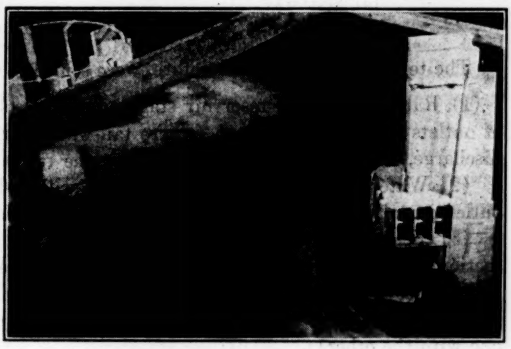


FIG. 8.—MODEL OF INTAKE TOWER, PLEASANT HILL DAM

Some of the most complex model fabrication was required in the case of the Pleasant Hill Dam. Fig. 8 shows the model intake tower of welded steel sheet. The Morning Glory spillway of spun aluminum with piezometer connections is shown in Fig. 9(*a*). The wye transparent junction and steel horseshoe tunnel are exposed for their full length to provide access to piezometers. The wye is shown in Fig. 9(*b*).

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[4] See, also, *Civil Engineering*, January, 1936, p. 16.

MEASUREMENTS

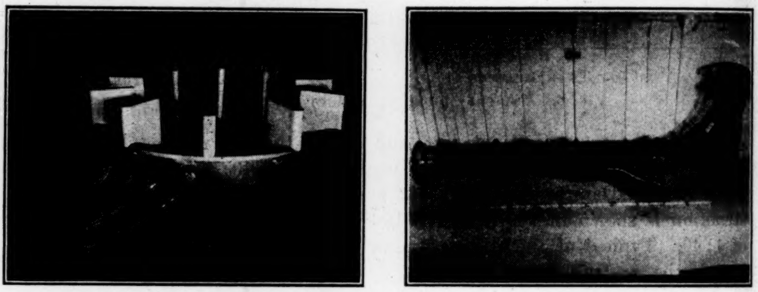
For each of the eleven dams certain features of operation and performance were studied, including:

(*a*) The relation of reservoir stage to discharge, for various combinations of outlet openings;

(*b*) The distribution of energy losses;

(*c*) The capacity and behavior of the spillways; and,

(*d*) The performance of the stilling-basin, with particular reference to the variation in tail-water level at a given discharge, or the unsymmetrical discharge from the multiple outlets.



(*a*) SPILLWAY (*b*) TUNNEL JUNCTION

FIG. 9.—TYPICAL MODEL CONSTRUCTION, PLEASANT HILL DAM

The test measurements, therefore, were aimed to secure the following data:

(1) Rating curves of reservoir stage against discharge for all combinations of outlets open or partly open (where tail-water variation affected the rate of discharge, rating curves were secured for the range of tail-water levels);

(2) Water surface profiles through the stilling-basin, extending from the outlet portal through the hydraulic jump to the channel below;

(3) Velocity traverses showing the distribution of velocities across various sections in the stilling-basin, and in the channel below;

(4) Scour (with the sand bed of the outlet channel screeded level before a test run, cumulative scour over an hour's time was measured to compare tests, one with the other);

(5) Flow conditions, recorded by photographs and motion pictures;

(6) Observations without measurement (the effect of rising tail-water on the hydraulic-jump performance was recorded as observed by eye for small increments of rise);

(7) Check surveys to determine the actual measurements of the models at various times as against the measurements to which they were set (in some

cases swelling wood or settlement of supports made it necessary to re-set the model; the actual dimensions of the model tunnels and outlets were recorded); and, finally,

(8) Record tests.

The latter (Item (8)) were so-called formal tests conducted with all the measurements required to determine the behavior under stipulated operating conditions. They included hydraulic gradients through the structure, the upper pool elevation and discharge, the water-surface profile of the hydraulic jump, the distribution of velocities in the stilling-basin, and the topography of scour at the end of an hour's run.

Model Operation

For taking measurements it was found convenient to furnish the laboratory operators with mimeographed sheets, there being a separate form for the: Rating curve; water-surface profile; velocity traverse; hydraulic gradients; scour sheets; and daily diary.

In operation, two men could usually handle the model on an 8-hr shift, one acting as reader and the other as recorder. When the model was being built or dismantled, skilled and unskilled labor was employed in addition. The laboratory operated continuously on a 24-hr, three-shift basis for about seven months. This continuous operation and accumulation of data kept about ten draftsmen busy, on an 8-hr basis, plotting, computing, and recording all measurements and model details, and preparing fabrication drawings for new models.

For a record test, the sand bed below the stilling-basin was first screeded level to the desired contour. From a 2-in. flood line, water was admitted slowly at the tail-gate to approximate tail-water depth for the intended discharge, so that the initial scour during the run would be slight. The proper flow was set by the meters, and the reservoir level and tail-water level were brought to equilibrium. All piezometers were blown free of air and stabilized. The hydraulic gradients were then recorded, water-surface profiles were taken in the stilling-basin with a point-gage and the distribution of velocities was recorded by Pitot tube or Bentzel tube in the stilling-basin and below the terminal sill in the outlet channel. At the end of an hour's run, the flow was stopped, the reservoir was allowed to empty, and the lower pool was drained to expose the sand bed, which was then measured with a sounding rod to record the depth and location of scour.

Scour measurements were taken as an index by which performance between tests could be compared roughly, since it denoted concentration of high velocities or persistent eddies and whorls along the bottom of the channel. Excessive or irregular scour in any case showed poor distribution of terminal velocities. From velocities at two-tenths and eight-tenths of the depth, the discharge was computed by ordinary stream-gaging methods to check by agreement with Venturi meter readings.

For Wills Creek, a preliminary model to a scale of 1:40 was superseded by a final model to a scale of 1:25. For Charles Mill, a large scale (1:20) model was used for conduit and stilling-basin studies only, followed by a 1:30 scale model complete with spillways.

Time, Personnel, and Cost Requirements

Fig. 10, "Laboratory Schedule", shows the dates between which model erection and model testing occurred. The interval shown for erection and testing does not include time for preparing drawings, for the fabrication of model parts, or for handling and digesting test results. As a rule, no time elapsed between clearing the flumes of one model and erecting another. The vertical lines in Fig. 10 indicate the man-hours for that week's payroll, including all operations. Rates for engineering assistants (draftsmen, computers, model builders, and model operators) ran from $0.60 to $1.50 per hr, limited to a 40-hr week. Rates for skilled labor (machinists, carpenters, cement finishers, pattern-makers) ran at the prevailing local scale, from $1.00 to $1.25 per hr, limited to a 30-hr week.

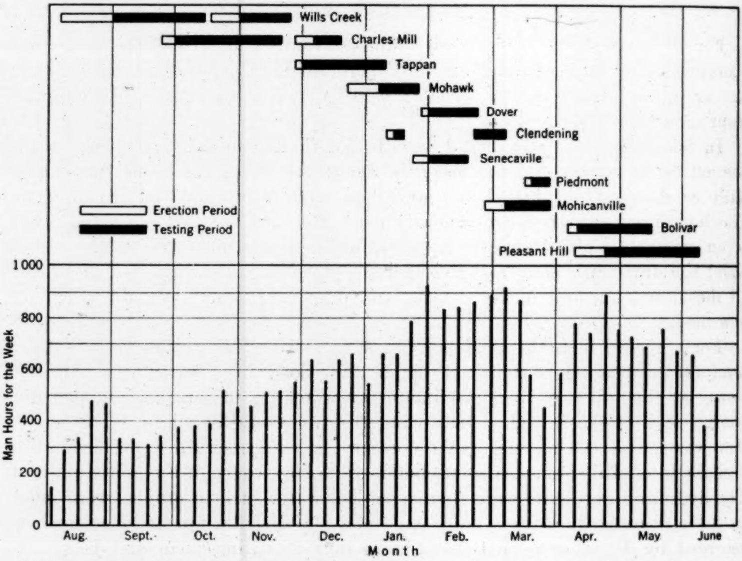


Fig. 10.—Laboratory Schedule, Showing Dates Between Which Model Erection and Model Testing Occurred

The direct cost of materials and labor for the several models is shown in Table 1. It includes a total of about $1 800 for blueprinting required for the several reports and $1 100 for photographic prints used in the reports. The power consumed in pumping totaled about 20 000 kw-hr. To these data should be added approximately 40% to cover rentals, supervision, miscellaneous services and supplies, travel and consulting expenses, and the salaries of three engineers from the Zanesville U. S. Engineer Office stationed at the laboratory. Included in the 40% is the cost of the river flume ($1 250), which was installed specifically for these tests and transferred to Case School as part compensation for the work.

TABLE 1.—Direct Labor and Materials Involved in Model Testing

Structure (see Fig. 2)	Scale of model	Units tested	Costs		
			Labor	Materials	Total
Wills Creek	1:40 1:25	Intake, twin tunnels, stilling-basin Intake, twin tunnels, stilling-basin	$4 650.59	$1 382.48	$6 033.07
Charles Mill	1:20 1:30	Outlet conduits, stilling-basin Outlet conduits, overflow section, stilling-basin, spillway	3 010.00	977.81	3 987.81
Tappan	1:20	Tunnel, stilling-basin	935.94	277.87	1 213.81
Clendening	1:20	Tunnel, stilling-basin	972.31	262.87	1 235.18
Piedmont	1:20	Tunnel, stilling-basin	398.03	131.34	529.37
Mohawk	1:40	Intake, twin tunnels, stilling-basin.	2 468.06	1 106.39	3 574.45
Senecaville	1:20	Outlet conduits, Taintor gate sections, outlet channel, stilling-basin	1 106.05	236.46	1 342.51
Dover	1:35	Spillway section, outlet conduits, stilling-basin	2 038.85	960.52	2 999.37
Bolivar	1:40	Twin intakes, tunnels and stilling-basin	1 231.50	580.02	1 811.52
Mohicanville	1:25	Outlet conduits, outlet channel, stilling-basin, spillways	1 546.56	499.35	2 045.91
Pleasant Hill	1:30	Intake tower, spillway tunnels, stilling-basin	3 167.83	1 574.40	4 742.23
Beach City	....	Preliminary drawings for model	33.00	....	33.00
Cost	......	....	$21 558.72	$7 989.51	$29 548.23

Exclusive of the building, a 50 000-gal. reservoir, the general plant, shop, and tools, the permanent investment, installed, for the principal items of the fixed equipment used for the tests is:

Three motor-driven pumps and controllers	$ 3 400
Major piping and valves	1 750
Constant level tank	300
Venturi tubes and meters	600
Experimental flume (3 ft 0 in. by 3 ft 0 in. by 90 ft 0 in.) with drains	1 300
River flume (5 ft 6 in. by 12 ft 0 in. by 44 ft 4 in.)	1 250
Measuring tanks	2 400
Total	$11 000

Caution must be used in comparing the cost of model studies, one with the other, because they are likely to be unrelated in character. The costs shown are those for a concentrated extensive program of studies generally similar and related, under a carefully planned and rigorous schedule. Furthermore, for these studies, work ceased the moment major decisions on design could be made, and the models were built in an economical manner for this immediate purpose only.

Operating Difficulties

Mechanical imperfection of the models themselves, together with the physical difficulty of taking precise measurements in all laboratory work, may seriously affect the results of an hydraulic model study, if these factors are under-rated or their significance ignored. Some of these factors deserving special consideration are: Discharges, piezometer readings, points of filling, orifice coefficients, joints and connections, warping, settlement, water-surface elevations, velocity measurements, and entrained air.

Discharges.—The small geometric scales necessary, require a large ratio of prototype flow to model flow, necessitating the accurate measurement of discharge. Where a weir is used, calibration should be directly by volume or weight if possible. Otherwise, proper precautions must be followed to insure that the weir corresponds in setting to the original by which formulas have been derived. With the Venturi meter, direct reading may be obtained at about full capacity with ordinary differential manometers. As previously noted, it is much more satisfactory with lower flows to magnify the deflections of the manometer by the use of a lighter liquid, such as, carbon tetrachloride. The specific gravity of the carbon tetrachloride must be determined carefully and frequently, and the glass tube should be at least 0.5 in. in diameter for easy cleaning and to avoid the effects of surface tension on the meniscus.

Piezometer Readings.—Where high velocities occur, piezometer readings are the most difficult measurements to obtain most accurately. A piezometer orifice with $\frac{1}{4}$-in. hole and $\frac{1}{16}$-in. radius of rounding is recommended for standard practice by C. M. Allen, M. Am. Soc. C. E. For smaller pipe sizes, he suggests a $\frac{1}{8}$-in. hole with $\frac{1}{32}$-in. radius.[5] The latter suggestion was adopted by the writers for model tunnels about 4 in. to 8 in. in diameter. The relation between discharge in the prototype and that in the model depends on the ratio, *n*. Since *n* must be determined by piezometer readings their importance and the care to be exercised in the fabrication of piezometer connections cannot be over-emphasized.

Points of Filling.—That point at which the model tunnel ceases to flow partly full and begins to flow completely filled, is difficult to determine. Even where this point is determined accurately in the model there may be some question as to whether or not the prototype will behave in exactly the same manner.

Orifice Coefficients.—The use of models in which orifices are involved raises the question as to how the coefficients are affected by actual size. Tests covering small orifices show a variation in the coefficient with the actual size of opening.

Joints and Connections.—As few joints as possible in a model appears to be desirable, inasmuch as these joints affect flow conditions both up stream and down stream with consequent breaks in the hydraulic gradient, as shown in the piezometer readings. A smooth hydraulic gradient is desirable in order to obtain the value of *n* for the model.

Warping.—It is extremely difficult to prevent warping in wooden models. Slight warping often produces incorrect flow conditions, which affect the results seriously. Many methods of water-proofing have been tried by the different hydraulic laboratories. The use of celluloid dissolved in banana oil, and the use of aluminum foil have been tried.

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[5] "Piezometer Investigation", by C. M. Allen, *Transactions*, A.S.M.E., May 15, 1932.

Settlement.—Settlement of the sand base under mortar surfaces leaves the templets exposed and elevations in error so that often it is necessary to rebuild the model after it has been in use. If the sand base is thoroughly inundated prior to placing the mortar surfaces, and not permitted to dry out, this difficulty is kept at a minimum.

Water-Surface Elevations.—Water-surface elevations are difficult to obtain, and care in leveling the base rail and in checking of the gage zeros, must be maintained at all times.

Velocity Measurements.—Velocity measurements in a model are difficult to obtain inasmuch as they usually cover a wide range, from considerably less than 1 ft per sec to greater than 10 ft per sec. For the low velocities the Bentzel meter, specially constructed Pitot tubes, and small current meters have all been used by the various laboratories.

Entrained Air.—Most hydraulic formulas are based on the assumption that water weighs 62.5 lb per cu ft. However, they do not hold exactly where the hydraulic jump, or where flow conditions in which air is entrained occur. Thus, any model that includes these flow conditions should have a factor of safety added greater than the minimum dimensions shown by the model tests.

OBSERVATIONS AND FINDINGS

In model studies, preliminary analysis is essential in order that time and money spent on laboratory manipulation may be minimized. The test measurements themselves are worthy of equally careful analysis and interpretation. Analysis of the detailed test observations, however, is beyond the intended scope of this paper, and the following discussion includes certain findings which are stated in general terms only, to give a brief picture of certain selected results.

The outstanding single result of tests was an improvement to all stilling-basins. In the case of single or twin outlet conduits, the typical features of the model basins are illustrated in Figs. 4 and 7. With the jet issuing from the portals between flaring side-walls over a curved apron, the models demonstrated, for these outlets, that the jump could be held with slight change in location on the apron, for all rates of discharge and corresponding tail-water levels, by block-shaped baffle-piers with stepped up-stream faces, anchored to the basin floor. For a given discharge, the jump appeared to be slightly more stable when the apron itself was stepped.

With excessive flare and batter to the side-walls, lateral distribution of the shooting flow on the apron was poor. In such cases the water failed to follow the flare, and lack of forward momentum at the side-wall permitted back rollers from the tail-water in the lower end of the basin, which skewed the jump to one side. The result was persistence of whorls, eddies, and concentrated high velocities at the exit from the basin. Similar behavior accompanied the presence of excessively high tail-water in any case. A low value of average velocity computed on the basis of high tail-water proved to be a deceptive index to flow conditions, if the jump itself was neither positive nor balanced.

In the models, good results were secured when the side-walls up stream from the jump were not flared more than 1 on 4 (preferably 1 on 5 or 1 on 6), and battered not more than 1 on 10. The parallel side-walls of the basin, below the jump, were battered as suitable for their construction as paved slopes or retaining walls, as the case might be. With tranquil flow instead of shooting flow, the slopes could be flatter without serious effect on performance, although slopes as flat as 4 on 1 produced eddies in some cases.

For structural or other reasons, the outlet tunnels for several of the dams were originally set lower than proved suitable for good performance with the coincident tail-water conditions corresponding to the several rates of discharge. In some cases, the tunnel was raised as a result of the tests; in others, where this was not feasible, a unique expedient solved the difficulty.

The Clendening stilling-basin has already been specially mentioned. In the original design, the tunnel invert was placed low, and high tail-water in the model drowned out the jump. To secure a good hydraulic jump without raising the tunnel, proper relation between momentum of outflow and tail-water depth was secured by passing the water over a humped apron (as described under the heading, "Types of Models"), through which a narrow gutter was cut for drainage. The hump was first built crudely with wooden flats in the model, and then carefully shaped for easy transition. This feature was also adopted, after trial, in the models for the Tappan and Piedmont Dams as well.

To reduce concentration of scouring velocities along the bed of the outlet channel, a low terminal sill with the up-stream face stepped, was placed at the end of the stilling-basin and was found to be the simplest and most effective, although other types were studied.

One expedient to reduce the length of stilling-basins was to carry the horizontal flare into the tunnel portals. In the first Mohawk model, with portal and tunnel height equal, the portal flare and increase in area resulted in the hydraulic control being shifted from the tunnel section at low flows, to the portal section at maximum flows. With the large portal flowing full at atmospheric pressure, the differential between the portal velocity head and the tunnel velocity head was roughly 20 ft. Consequently, negative head occurred in like amount in the tunnel. When the flare was omitted from the tunnels, and stilling-basin side-walls were adjusted to correspond with increased warp and flare, the hydraulic jump was unbalanced. The flare was restored, therefore, to the portal section, but the portal area (and negative tunnel head) was reduced by dropping the portal crown. Piezometer readings in the model disclosed the shift in the pressure gradient.

Finally, for all stilling-basins, tail-water stages were such that the depth in the basin much below the stream bed was, in general, found to be needless, and the model studies resulted usually in reducing the length or width of the basins, or both. Liberal size alone was found to be insufficient guaranty of adequate performance.

The true head between reservoirs and outlets is difficult to discover. For preliminary designs, outflow was predicated on head measured as reservoir elevation, minus portal invert elevation, minus the average depth at the portal. Actually, the head will be greater, since the jet from the portal is accelerated downward by gravity, and the pressure head on the apron is less than that caused by an equal depth of still water. The discrepancy between assumption and fact will depend on the sharpness of apron curvature and jet velocity. In the case of the Pleasant Hill model, the true head by piezometers appeared to have a value closer to reservoir elevation, minus portal invert elevation, minus one-half the depth. The model discharge was greater than anticipated and, in fact, substantially exceeded the required capacity, so that the original 20-ft tunnel and shaft was reduced to 19 ft, with incidental changes resulting in reduced cost of construction.

Another feature of the Pleasant Hill model was the transparent wye-junction through which flow conditions, much under discussion during staff conferences, could be observed and evaluated. With certain rates of flow (to occur in Nature at rare intervals), the model showed that discharge from the gate tower would enter the junction in such fashion as to climb the opposite wall and spiral completely around the interior face of the main tunnel.

The Bolivar model was operated with its portal submerged by the tail-water, as would be required for this particular dam at certain outflows, and when the Dover Reservoir, located down stream, is filled to crest level. Quick closure of the model gates caused a sudden inrush of air through the vents, the slug of air traveling down the tunnel to be released at the portal. Release of air was followed by a sudden rush of tail-water into the portal, with severe slap and shock. Disturbance also occurred with the gates stationary in a throttled position, provided the portal was submerged. The gates in the model, of course, can be closed almost instantaneously, whereas the mechanism in the actual structure will not permit any such rapid gate manipulation. Nevertheless, the model showed that it might be important to release the water through a few gates fully opened rather than through several in a throttled position, and to restrict the speed of gate closure.

With multiple short conduits through a masonry section, as for Charles Mill and Dover Dams, increase in capacity or improved lateral spread on the apron was attempted with conduits having a curved or offset ceiling profile, or horizontal flare, or both. The benefits, if any, indicated in the models were not considered commensurate with the probable cost of the more complex shapes and, in each case, the final models crystallized into conduits of simpler uniform rectangular section and liberal elliptical easement at the inlet, which appeared to be entirely suitable.

The size and position of training walls, and the shape of rock-cut benches by which spillway discharge could be directed with a minimum disturbance into the outlet channels, were studied in the Charles Mill and Senecaville models. Studies of alternate types of spillways were made for Mohicanville. The approach cuts and wing-walls at the inlet were improved in the models for Wills Creek, Charles Mill, and other dams.

Relation Between Capacity of Prototype and Model

To check the stipulated capacity of the outlet works at a given reservoir stage, it is necessary to convert the model discharge into corresponding flow in the prototype. Where gravity is the predominating factor in inducing flow, with comparatively slight retardation from friction, the flow in the model is directly converted to flow in the prototype by the $\frac{5}{2}$ power of the scale ratio. Such a conversion factor would be appropriate for overflow spillways, stilling-basins and (for simplicity, and on the safe side, for capacity), for short conduits. In long tunnels where frictional retardation has a great influence on discharge, it was difficult or impossible with these model scales and structures to secure an interior roughness factor, or Kutter's n, low enough to give the foregoing conversion factor where n in the prototype is to be 0.011 or 0.013. Consequently, it was necessary to determine the value of n for the model by observing, carefully, the hydraulic gradients in a straight section where uniform flow obtained. For example, the value of n for the model of the Mohawk Tunnels (1:40 scale) was found to be 0.0091, which indicates that the models, however smooth mechanically, are nevertheless too rough to simulate conditions in the prototype. However, knowing the value of n for the model, the corresponding flow in the prototype can be computed by Manning's formula, or by other accepted formulas. For these models too much confidence is not warranted in the experimental values of n, because the model tunnels are believed to be too short to establish uniformly good flow conditions.

Summary and Conclusions

The fact that hydraulic models cannot be made to reproduce, in exact miniature, all the phenomena that will occur in the prototype does not detract from their genuine utility. The performance of hydraulic models may be interpreted to great advantage by a designing engineer. It is obvious that certain of the foregoing findings could not be determined over the drafting-board, or by analysis alone, and that other measurements furnish the data for needed analysis. The model studies described in this paper were in the nature of check tests for the sole purpose of expediting the production schedule demanded of the Designing Staff for this project, by contributing a valuable supplementary method for attacking complex problems of hydraulic design. The tests more than saved their own cost by actually reducing construction expenditure, and, in addition, gave invaluable assurance as to the probable behavior of the finished structures of the Muskingum Water-Shed Project. It is hoped that the data presented herein may be helpful in planning other model studies of somewhat related character.

Acknowledgments

The model studies were conducted under the direction of the senior writer, with the junior writer assigned as Assistant in active charge of laboratory operation. W. A. Snyder, Junior Engineer, and L. G. Leach, Jun. Am. Soc. C. E., Junior Computer, were also stationed by the Zanesville Engineer Office at the Laboratory. The Laboratory Staff included G. Brooks Earnest, Assoc. M. Am. Soc. C. E., W. J. Hopkins, Jun. Am. Soc. C. E., and fifteen to twenty Engineering Assistants at the Case School of Applied Science. The co-operation of the Designing Staff throughout the work was most helpful.

DISCUSSION

G. W. HOWARD,[6] JUN. AM. SOC. C. E. (by letter).—In view of the increasing use of model studies and the corresponding increase in difficulties of construction and operation of models, this paper is extremely helpful in its presentation of the author's experiences in conducting the studies made in connection with the Muskingum Water-Shed Project.

It is doubtful whether the troubles that are encountered with piezometers can be over-emphasized. The authors state that, of all measurements taken, the piezometer readings were the most difficult and unsatisfactory. This is generally true in almost every study of the type discussed, where much use is made of piezometer readings. The general difficulty encountered is in fluctuations in the manometer tubes, caused by entrained air. To care for this factor, it is frequently possible to install bleeder valves at certain points in the line to permit correction of this difficulty and also to flush the lead line.

When there are excessive fluctuations in the tube, it is also possible to use several arrangements for dampening these fluctuations in an effort to obtain an average reading. One method is to choke the line, either by using a clamp or by a reduction in size of the line. Another method is to have a stilling-well, say, 9 in. by 9 in. by 15 in., into which the line is led before it is taken to the manometer tube or gage bucket. This second method has been found by the writer to be preferable to choking the line. However, any method for dampening fluctuations to obtain an average reading must be used with caution. Each gage should be investigated carefully to determine the time required for equilibrium to be reached, because inconsistencies in results can quite often be traced to a manometer observation made prior to equilibrium.

F. W. EDWARDS,[7] ASSOC. M. AM. SOC. C. E. (by letter).—Former papers on hydraulic investigations utilizing models have described briefly the laboratory equipment and the physical features of the models as incidental to the more detailed presentation of the test results. In this paper, the opposite is true. Two objectives are reached by this change: First, to other research workers, many hints on model construction and operation are given; and, second, to the designers who put into practical use the results of tests, some of the problems to be met and some of the limitations to be expected in model studies are presented, thus promoting better understanding between the research and the design groups.

It is proposed herein to add a few remarks on the economics of model investigations, to elaborate somewhat upon the operation difficulties listed by the authors, and to discuss the relation between the capacity of the prototype and the model as mentioned in the paper.

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Economics of Model Investigations.—The cost of a model investigation is difficult to determine in advance. As emphasized by the authors, caution should be used in comparing the costs of different models. The construction cost for some types of models is relatively large whereas the operation cost of others is the major item. The cost of the construction of the first test set-up and of the operation of the first series of proposed tests may be estimated readily, but the results of the first few tests may change the nature of the entire future program. Before the completion of the study, it is often necessary to develop new instruments to obtain the results desired. Furthermore, models similar in character studied at one place during an extensive program (as were the Muskingum studies), will cost less on the average than a single model.

Offsetting the uncertainty of cost of a proposed investigation is the fact that model studies usually point to economies either by indicating inadequate designs, by directly reducing construction costs, or by both. Savings created by the elimination of unsafe designs are indirect and often cannot be evaluated accurately. However, there are many examples of reductions in construction cost, subject to accurate estimation, which were created entirely by model investigations.

An example of a model study which eliminated an unsafe design and, at the same time, reduced the estimated cost of the structure is that of the stilling-basin for the Conchas Dam which was completed at the U. S. Waterways Experiment Station, at Vicksburg, Miss. This study exposed a dangerous flow condition in the basin for the preliminary plan and led to the adoption of an alternate plan having a reported cost[8] of approximately $625 000 less than the original design. The cost of the model study was less than $10 000.

The estimated construction cost reductions which were made possible by the tests for the Muskingum Water-Shed Project are not known to the writer, but the general statement made by the authors, that "* * * the model studies resulted usually in reducing the length or width of the basins, or both," indicates that the money saved made the cost of the investigation appear insignificant. There are many other examples of great economies resulting from model experimentation.

Operation Difficulties.—Operation difficulties place definite limitations upon model studies. The recognition of these limitations is essential to proper analysis of the test results. The authors state that piezometer readings are the most difficult to obtain accurately. Of course, the importance of the various measurements depends upon the particular problem being studied, but the writer believes that, ordinarily, velocity measurements in the type of model dealt with in the paper are the most difficult and unsatisfactory. Pitot tubes, Bentzel tubes, and small current meters all have their shortcomings particularly in exceptionally turbulent flow similar to that encountered in a stilling-basin. Sometimes it is possible to obtain relative values, but in some cases it is practically impossible to obtain velocity measurements that have any value whatever. New instruments based on different principles are needed for these measurements.

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[8] "Hydraulic Laboratory Projects of the Corps of Engineers, U. S. Army," by F. H. Falkner, Jun. Am. Soc. C. E., *Transactions*, A. S. M. E., Vol. 58, No. 7, p. 561.

Entrained air, referred to in the paper, is not only a factor in scale conversions, but often becomes a source of trouble in piezometer and Pitot tube connections. These connections, from orifice opening to manometer, should be as short as possible. In addition to avoiding sharp bends, they should slope upward throughout their length to eliminate sags where air often becomes entrapped. Mechanical devices for drawing the air out of connections are desirable if not essential. Vacuum pumps or air ejectors are most satisfactory for this purpose.

Capacity of Prototype and Model.—The authors mention the impossibility of reproducing the frictional factor to scale in models of the size of those discussed. They suggest measuring the value of n experimentally and then using it to determine the discharge conversion factor. In a model in which the roughness is significant, this method must be used with caution because the corresponding stages may be considerably in error and the location of the hydraulic jump affected accordingly. For example, consider a model that has too great a value for n. For normal head-water elevation, the conduit discharge would be too small. Under this circumstance a tail-water elevation in the model corresponding to the correct prototype discharge would cause the hydraulic jump to form up stream from its correct position. Particularly in a basin having warped sides and sloping floor, this condition would be difficult to analyze. However, by varying the tail-water elevation for a given discharge, sufficient data may be collected to permit proper analysis of the basin action when the head-water elevation, conduit discharge, and tail-water elevation are correlated.

One solution for the problem arising from the inability to reduce the friction factor below a certain limit is to use a model that is large enough to eliminate the scale effect. This often would require space and discharge capacity beyond the laboratory facilities.

Another solution, which would eliminate the conduit section and provide for studying the intake and stilling-basin in separate models, should be considered for each specific problem. The friction loss in the conduit section would be left entirely to computation.

T. T. KNAPPEN,[9] M. AM. SOC. C. E. (by letter).—The general plan for the Muskingum project, referred to by the authors as the "Official Plan", was prepared by the U. S. Engineer Office, at Zanesville, Ohio, in co-operation with the Muskingum Watershed Conservancy District and was completed about August, 1934. This project then was constructed by the U. S. Engineer Department under the co-operative agreement between the Muskingum Watershed Conservancy District and the U. S. Engineers, representing the Public Works Administration. Since the work was to be constructed with P.W.A. funds and since the law at that time required that all funds be obligated by July 1, 1935, the Designing Staff was faced with the problem of completing the plans and specifications for the entire project by the following May, in order to allow a sufficient period for advertising the work and awarding the contracts. In analyzing the design, it was apparent that numerous difficult hydraulic problems would develop and that the time for their solution would be limited. Negotiations were started, therefore, early in the summer of 1934 with the object of devising an arrangement for testing the proposed hydraulic designs in a suitably equipped hydraulic laboratory. An agreement was finally made with Professor Barnes to have this work done at the Case School of Applied Science, in Cleveland, Ohio, and Mr. Jobes was assigned as representative of the U. S. Engineer Office to assist Professor Barnes in performing the experiments and to serve as a "contact man" between the two organizations.

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[9] Associate, Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

At the time that these negotiations were going forward, a section was being organized in the Zanesville Office to design the hydraulic structures. Mr. Paul H. Jaenichen was placed in charge of this section. The preliminary hydraulic designs were developed under his direction, and laboratory tests were then run to check and improve the tentative plans. A close relation was maintained at all times between the design section and the laboratory. The hydraulic model experiments were considered as being a part of the actual design. Throughout the course of the experiments, this policy was strictly maintained. Under ordinary circumstances one might have been warranted in conducting these experiments to extend the range of test data beyond the immediate needs of a specific design problem and to obtain information of more general applicability; but the severe time limitation made it necessary to dismantle each model and replace it with another one as soon as the particular problem was sufficiently clarified. Exceptions were made in cases where existing models could be used with little or no alteration for preliminary investigations of similar features on other dams in the project.

Table 1, showing the cost of labor and materials involved in the model testing, is known to be accurate, as a strict system of accounting was established as a basis of approving bills for the work. The over-head charges of 40% are reasonably representative of the actual over-head costs. The total cost of the hydraulic model work, including over-head, was approximately \$42 000, or an average cost of about \$4 000 per model. The construction cost of the fourteen reservoirs of the Muskingum project was approximately \$23 000 000, so that the cost of the hydraulic model testing was slightly less than 0.2% of the project cost. Although hydraulic experiments were made for only eleven of the dams, the results were used on all fourteen. The writer feels that the expenditure was more than justified by the results obtained.

At the beginning of the hydraulic design work, little information was available about hydraulic-jump action in sloping and flaring channels. Considerable difficulties were encountered in the first designs, and many changes were necessary in the first models, in order to arrive at a good stilling-basin arrangement; but as the hydraulic experimental work progressed and test results from the first models were analyzed, definite design rules were developed and the art of designing stilling-basins was gradually mastered. Two models of the outlet structure of the Wills Creek twin conduit were built before a satisfactory solution was found; but the largest part of this cost should be distributed among the other dams, since all the later designs benefited by these first experiments. The designs of the Mohawk and Bolivar outlet works, also twin-conduit structures, gave such surprisingly good check results during the model tests that it was found unnecessary to perform experiments on the Beach City structure, which is in principle the same as the Wills Creek layout. The cost of the Mohawk tests was relatively high, due to the completeness with which the intake structure and the conduit system were reproduced in the model.

The experience with the small single-conduit outlets was similar to that with the large twin-conduit structures. The Tappen and Clendening outlets were treated very completely in the experimental work, and numerous changes were made in these models. The stilling-basin designs for Piedmont and Pleasant Hill were based on these experimental findings, and when tested in the models gave such close check on the original designs that it was felt that the Leesville and Atwood structures could safely be eliminated from the experimental program. The relatively high cost figures for the hydraulic experimental work for the Charles Mill, Dover, Mohicanville, and Pleasant Hill structures are due to the fact that the spillways were included in these models and that the design problems were of an intricate nature.

The use of the hump in the stilling-basin apron at the ends of the tunnels for Tappan, Clendening, and Piedmont, first developed by Mr. R. G. Schneider while assisting in the design of these structures, proved to be an interesting feature. In these cases the conditions for tunnel driving were most favorable at the elevations selected. At Tappan, for instance, if the tunnel had been raised to the required invert elevation to avoid the use of the hump, the crown would have broken through a hard sandstone into soft shale which would have required timbering the tunnel at an increased cost of about $10 000.

It is worthy of note, that the hydraulic jump at Clendening is created by the hump in the stilling-basin apron; without it, unsatisfactory stilling performance would have to be expected under the prevailing conditions of high tail-water. In the case of the work at Tappan, plans and specifications were issued, and the construction was started in advance of the completion of the hydraulic experiments. Because of this, it was not feasible to make all the modifications indicated by these studies. The model experiments showed that an eddy could be expected in the stilling-basin under certain conditions of discharge and tail-water, because of the unnecessary basin width, whereas under more favorable conditions this eddy would not occur. The writer was fortunate enough to see both these conditions reproduced in the prototype in the spring of 1936, after having witnessed the condition in the model. A very close agreement between behavior of model and actual structure was observed.

During the 1937 flood, the Muskingum Conservancy District was unable, because of legal difficulties, to close any of the gates for regulation, so that even with considerable outflows, discharge heads were relatively small and tail-water levels were higher than normal, due to the small controlling effect of the reservoirs. Thus, stilling-basins during this flood were subject to an extreme and very unusual test. In all cases observed, the hydraulic jump developed very close to the exit portal. The condition of outflow with low kinetic energy and high tail-water is difficult of proper treatment in the design, due to the proximity of unbalanced hydraulic-jump conditions, but the structures acted remarkably well under this test. In comparison with the hydraulic model work, it may be stated herein that the appearance of the stilling performance in the actual structures was far better than that found under similar conditions in the model tests. Various small and rather unimportant flow details that had been observed on the models were reproduced in the actual structures in striking similarity.

An interesting point developed by the model tests was that the flare of the side-walls and the width of the stilling-basin should be limited to obtain the best hydraulic-jump action. This was borne out by a comparison of the action of the Wills Creek and Mohawk Dams during the 1937 flood (Figs. 11 and 12). In their general layout these outlet structures have much in common, both having large twin conduits. The length of the Wills Creek Basin was limited by the dropping off of the rock formation under the far end of the right-basin wall. Its sloping apron is slightly too steep, and the basin width required to reduce the exit velocity to safe limits is somewhat too large for proper jump action. Fig. 11 shows the Wills Creek Dam discharging about 15 000 cu ft per sec, or 7 500 cu ft per sec from each conduit, during the January, 1937, flood. The back rolls along each side-wall, induced by the departure from ideal conditions described previously, are plainly visible. Fig. 12 shows the Mohawk Dam during the same flood, discharging about 16 000 cu ft per sec under similar tail-water conditions. The attainment of close to an ideal design is indicated by the absence of back rolls.

The writer would like to emphasize the effect of baffle-blocks as described by the authors. The model studies showed repeatedly that properly located baffle-blocks stabilize the jump action, permit the use of somewhat shorter stilling-basins, and provide a factor of safety against unforeseen low tail-water conditions.

A controlling influence in the design of outlet structures is the correct determination of the tail-water conditions. The writer was impressed, by the Muskingum model studies, with the importance of securing accurate survey data from which to compute rating curves below dam sites.

The authors are to be congratulated on presenting this excellent paper, and particularly for setting forth cost data and describing in considerable detail the method of model construction. As they suggest, the cost data must be used with caution, as they are representative of a project involving a large number of structures. Had the model works not been grouped in a continuous program, the costs would have been probably 25% to 50% greater. The writer feels that the expenditure was more than justified

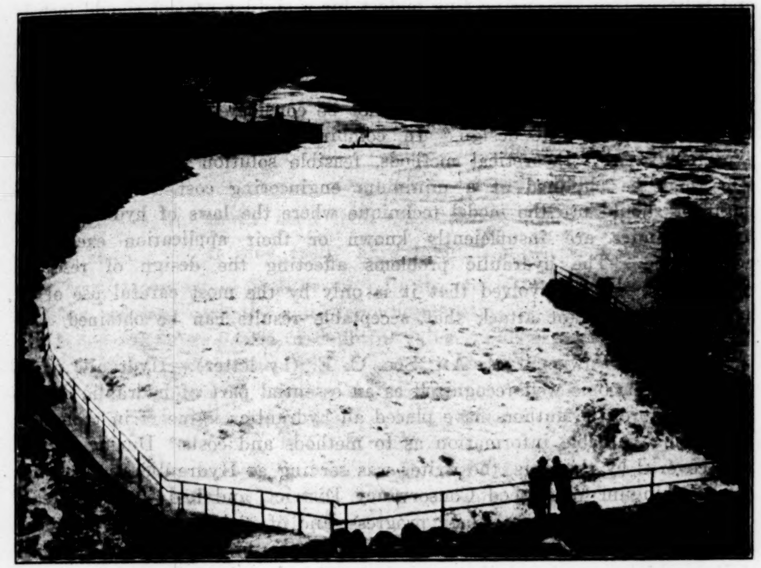

FIG. 11.—WILLS CREEK DAM, OHIO, 1937: STILLING-BASIN FROM TOP OF DAM (DISCHARGE, 15 000 CUBIC FEET PER SECOND)

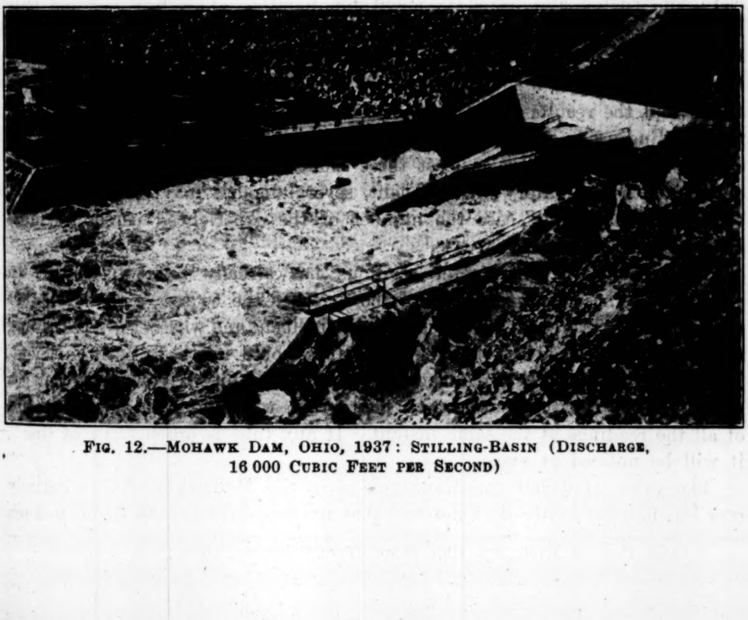

FIG. 12.—MOHAWK DAM, OHIO, 1937: STILLING-BASIN (DISCHARGE, 16 000 CUBIC FEET PER SECOND)

and believes that the engineering budget for a similar project should include an allowance for model tests as a part of the necessary and warranted design cost.

The writer concludes from his experience on the Muskingum project that model tests of this type should not be considered as a full substitute for careful analytical design. In combining, properly, hydraulic model experiments with theoretical methods, feasible solution of hydraulic problems may be obtained at a minimum engineering cost. The hydraulic engineer should use the model technique where the laws of hydraulics and hydrodynamics are insufficiently known or their application excessively complicated. The hydraulic problems affecting the design of reservoir outlet work are so involved that it is only by the most careful use of all available methods of attack that acceptable results can be obtained.

RALPH W. POWELL,[10] M. AM. SOC. C. E. (by letter).—Hydraulic model testing has become well recognized as an essential part of hydraulic design, and, therefore, the authors have placed all hydraulic engineers in their debt by sharing valuable information as to methods and costs. During the period covered by the tests, the writer was serving as Hydraulic Engineer for the Muskingum Watershed Conservancy District, and had the pleasure of observing several of the tests in progress, and of making a fairly thorough study of the complete reports which were prepared. This contact gave the writer a high regard for what was accomplished under very definite limitations.

As far as the writer is aware, no equal amount of model testing has ever been done in one laboratory in an equal length of time. Thirteen models were constructed and tested in about ten months. This was possible only by working the models on three 8-hr shifts, and meant 13 or 14-hr days for the Supervisory Staff. Although the authors are to be congratulated on the work done under such conditions, they would probably be the first to admit that the results might have been better if more time could have been taken. Engineers who wish model tests comparable to those described should, if possible, allow at least three months for each model. If the facilities of the laboratory are sufficient, several models may be tested simultaneously, but in this case the number of the experienced personnel must be increased accordingly.

The problem of piezometer readings has already been discussed. In view of the uncertainty of many of the readings the writer wonders whether the hook-gages were not an unnecessary refinement and whether direct reading manometers arranged in "gangs" on boards would not have been sufficient. This arrangement has the advantage that the prototype elevations can be marked plainly on the board, and a photograph will give a permanent record of all the readings at the same instant. If any tube is much "out of line", it will be noticed at once.

The value of 0.0091 for Manning's n in the Mohawk model is entirely possible, but the writer does not feel that much weight can be given to such measurements. Prandtl and Tietjens[11] state that in the case of turbulent flow, the kinetic energy head increases from $\frac{V^2}{2g}$ to $1.09\,\frac{V^2}{2g}$ in the first 40 diameters of the length of the pipe. In a complicated structure like the outlet works of the Mohawk Dam, it is somewhat of a question what point to consider as the entrance, but using the gate slots, the points at which the pressure was measured were approximately 11 and 21 tunnel diameters down stream from the entrance, respectively. How much of the $0.09\,\frac{V^2}{2g}$ increase in kinetic energy head took place between these two sections is difficult to say, but it may easily have been as much as $0.02\,\frac{V^2}{2g}$. This would be of the order of 10% or more of the entire pressure drop observed. This fact, coupled with the admitted uncertainties in the piezometer readings, makes the figure 0.0091 of doubtful value. This is entirely apart from the doubt which grows constantly in the writer's mind as to whether Manning's formula should be used in model work at all. A better means of expressing fluid friction in conduits is badly needed.

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George E. Barnes,[12] M. Am. Soc. C. E., and J. G. Jobes,[13] Jun. Am. Soc. C. E. (by letter).—The discussion shows corollary experience in other laboratories, with the inherent difficulty of securing desired precision in piezometer and velocity readings. The value of model studies, now established beyond question, will be much enhanced, of course, by any improvements in laboratory technique.

Messrs. Howard and Edwards both submit practical suggestions on the piezometer connections, piezometer tubing, and indicating devices. With these, and with Mr. Edwards' comments on velocity meters, the writers concur. In the laboratory at Case School of Applied Science, hook-gage buckets for gangs of piezometers have been found less practical, in most cases, than a bank of piezometer-indicating tubes joined at the top by a header, and connected to pressure and suction lines by suitable valves. It all depends, however, upon the precision and convenience desired, and no generalization is offered.

As the writers stated, too much confidence is not warranted in the experimental values of n determined for these short model tunnels. Professor Powell amplifies this observation by reference to the probable influence of the intake structure on velocity distribution. In a thesis study,[14] it was demonstrated conclusively that the apparent value of n in the model tunnel of Pleasant Hill Dam could be changed by baffling the entrance in a manner that would influence velocity distribution. Furthermore, the measurements indicated that the value of n may change with the rate of discharge; and, as implied by Mr. Edwards, a value of n which is on the safe side for the capacity of the tunnels, is on the unsafe side for the performance of the stilling-basin. It is necessary to determine the value of n in order to calibrate the delivery capacity of the outlet works. In operating the stilling-basin, however, it is the quantity of discharge that is important. For a given discharge, excess head may be needed.

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[11] "Applied Hydro- and Aero-Mechanics", by Prandtl and Tietjens, McGraw Hill Book Co., 1934, pp. 49–51.

[12] Head, Dept., of Civ. Eng., and Prof., Hydr. and San. Eng., Case School of Applied Science, Cleveland, Ohio.

[13] Engr., U. S. Engr. Office, Los Angeles, Calif.

[14] "Determination of Kutter's *N* for the Tunnel of the Pleasant Hill Dam Model," by Edward R. Van Driest, Harold E. Orford, and Charles M. Gard. Thesis submitted for the degree of Bachelor of Science in Civil Engineering, Case School of Applied Science, 1936.

All observers agree that the design must provide for probable deviations in tail-water level from those predicted. The extent of this deviation is a matter of judgment based on the probable strength of the data from which the tail-water rating curve is derived.

Mr. Knappen's discussion is most generous in scope. As the then Chief of the Engineering Division of the U. S. Engineer Office, at Zanesville, Ohio, his viewpoint is most valuable. Perhaps too much weight is given the dollars and cents savings made possible by model studies and too little weight is given the fact that such studies, in many cases, are worth while simply as a guaranty of good performance. Certainly, the art of stilling-basin design alone was greatly advanced in this work, and as Mr. Knappen indicates, it became more and more possible to be right the first time as the studies progressed. This was despite the wide variety of designs for the several dams.

The design of the Clendening Dam, discussed by Mr. Knappen, was mentioned by the writers as an interesting departure from orthodox design. It also illustrates how an idea for design will germinate during the manipulation of a model. In previous and incompleted tests on Tappan Dam, stilling-basin performance with the tunnel at various elevations was studied for low discharges by merely inserting filler pieces or "humps" on the floor of the tunnel portal, as described. This was quicker and more convenient than continually changing the model around and, in a rough and tentative manner, it gave the approximate effect of raising or lowering the tunnel. Here was born the idea of a low tunnel, with a portal or apron of greater lateral width and high invert, although a simpler design was preferred if field conditions would permit raising the tunnel throughout its length. However, the so-called "hump" proved well adapted to field conditions at Tappan, Clendening, and Piedmont Dams, and the first design for Clendening was so drawn up, although some changes were made after test.

Professor Powell dwells on the expedition with which the work was handled, and the limitations which this imposed on the scope of the tests. Expedition marked the handling of the Muskingum Water-Shed Project as a whole, and was, in fact, a condition precedent to the approval of the project. It might be stated, therefore, that the model studies would have failed in their purpose, had each individual study been extended beyond the time allowed. Unquestionably, this program limited the findings secured.

The writers did not intend to elaborate upon technical findings and design, but rather on matters affecting the programming of model studies. The following remarks are offered in closure: As part of the design procedure, model studies for a given project may be simple or elaborate depending on objectives. These objectives may be limited to taking observations on matters of only immediate or primary concern in the design, or they may be extended in detail or expanded to general research to facilitate the planning of a series of integrated developments with other projects in view. Model studies ordinarily should start with preliminary studies and should be carried along concurrently with design. The writers believe it undesirable to issue contract drawings for bidding purposes before the model studies are completed, on the assumption that changes will be minor in character, since under these circumstances they are likely to be in the nature of a compromise and the design does not receive the full benefit of the model studies. In the case of the Muskingum Water-Shed Project, model studies commenced at the very beginnings of design, which is the best possible arrangement, and most productive for all concerned.

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ANALYSIS OF STRESSES IN SUBAQUEOUS TUNNEL TUBES

By A. A. Eremin,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. David J. Peery, Orrin L. Brodie, and A. A. Eremin.

Synopsis

Stresses in a concrete tunnel tube with flexible horizontal ties, are analyzed in this paper. The equations for computing stresses in the tube and ties are based on the Maxwell theorem of reciprocal displacements. Stresses in a tube may also be determined by the Method of Least Work, although the resulting equations are lengthy and require considerable computation. The dimensions of a tube and its tie-rods are generally proportioned by trial and error and, therefore, it is essential to save time in the analysis of the stresses. Changes in the dimensions of the tube and ties require the computation of fewer terms in the equations based on the Maxwell theorem than in the Method of Least Work.

Introduction

The elastic tube lining with flexible horizontal ties has found wide application in the construction of subaqueous tunnels. In Fig. 1, for example, the roadway is placed at the center of the section. The upper horizontal ties are enclosed in the ceiling slab, the space above being used for an exhaust ventilating duct. The lower horizontal ties are placed in the roadway slab, the space beneath it being occupied by the fresh-air ventilating duct. These rods cause a more even distribution of the stresses in the ring.

The tunnel lining may be either a reinforced concrete tube or a series of cast-iron rings bolted at the flanges. Each type has been found advantageous from a construction or economic standpoint, depending on the site and the soil to be penetrated. In the Posey Tube under the estuary channel between Oakland and Alameda, Calif., reinforced concrete pre-cast sections were used.

Note.—Published in December, 1936, *Proceedings*.

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In this case the horizontal steel tie-rods were placed in the ceiling, the roadway slab being designed to act as a strut to resist external soil pressure.[2] In the Holland Vehicular Tunnel under the Hudson River between New Jersey and New York, the tube was made of bolted cast-iron rings. The high tensile steel tie-rods were placed in the ceiling and roadway slabs.[3]

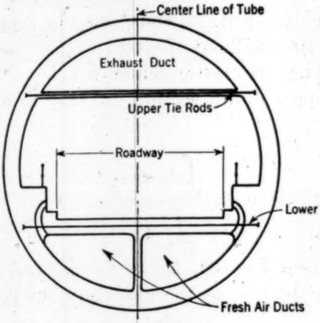

FIG. 1.—TYPICAL SECTION OF SUBAQUEOUS TUNNEL TUBE.

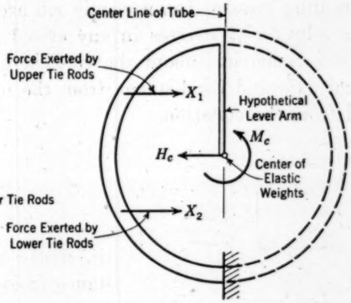
Center Line of Tube
Force Exerted by Upper Tie Rods
X_1
Hypothetical Lever Arm
M_c
H_c
Center of Elastic Weights
X_2
Force Exerted by Lower Tie Rods

FIG. 2.—DIAGRAM OF BASIC TUBE.

The equations developed in this paper refer to the analysis of stress in a reinforced concrete tube. However, they may also be used for the analysis of stresses in a cast-iron tube by substituting the proper values for the modulus of elasticity. The basic assumptions made for the elastic materials generally used in the theory of elasticity were adopted for this paper. However, the possible effect of plastic flow in concrete was not considered.

Notation.—The letter symbols in this paper are introduced in the text as they occur and are summarized for reference in the Appendix. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials"[4] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

DERIVATIONS

The loading sustained by a tunnel tube consists of external earth and water loads, the weight of the structure, the live loads on the roadway deck, and the force of buoyancy. The ring and the loads acting on it are generally symmetrical about a vertical axis through the crown section, and, consequently, the stresses involved are also symmetrical about the vertical axis. The shearing stresses at the upper and lower crown sections are equal to zero. In developing the equations the half-ring shown in Fig. 2 is analyzed, representing a 1-ft length of tube, fixed at the bottom, with the right half removed. The tie-rods are replaced by tensile forces, and the reactions from the right half are expressed by the horizontal force and bending moment applied at the elastic center of the tube.

An elastic tube with two horizontal flexible tie-rods is statically indeterminate in the fourth degree; that is, there is a deficiency of four equations for solution as a statically determinate problem. The following are the conditions to be met: (1) From symmetry, the vertical diameter in Fig. 2 may be assumed as fixed; (2) the horizontal displacement and the rotation of the crown sections may be equated to zero; and (3) the horizontal displacements of the ring, at the points where the tie-rods are connected are equal to the strains in the corresponding ties.

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[2] *Engineering News-Record*, October, 1924, p. 720.
[3] *Loc. cit.*, December 3, 1925, p. 902.
[4] A.S.A.—*Z10a*—1932.

Case 1.—Tunnel Section Without Tie-Rods.—Equations developed for determining stresses in an elastic rib arch fixed at the ends,[5] may be adapted to the solution of stresses in any closed ring without tie-rods. Since such a ring is symmetrical about the vertical axis, its elastic center will be on the vertical axis and its distance from the upper crown section, y_c, may be computed from the equation,

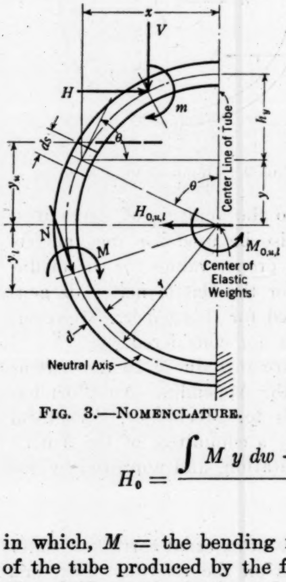

FIG. 3.—NOMENCLATURE.

$$y_c = \frac{\int h_y\, dw}{\int dw} \quad \ldots\ldots\ldots\ldots (1)$$

in which (see Fig. 3), h_y = the vertical distance from the axis at the crown, to any point, x, y, on the ring; $dw = \frac{ds}{E_c I}$, the elastic weight of the arch element; ds = the length of an element measured along the neutral axis; E_c = the modulus of elasticity of the concrete in compression; and I = the moment of inertia of the section of the tube.

The horizontal force, H_0, that expresses the effect of the right-half ring (transmitted through the hypothetical arm shown in Fig. 2) is determined from:

$$H_0 = \frac{\int M\, y\, dw + \int H \cos^2\theta\, dv + \int V \sin\theta \cos\theta\, dv}{\int y^2\, dw + \int \cos^2\theta\, dv} \quad \ldots\ldots (2)$$

in which, M = the bending moment at some point, x, y, on the neutral axis of the tube produced by the forces applied between that point and the crown; H and V are the horizontal and vertical components of the sum of all the forces between Point x, y, and the crown; $dv = \frac{ds}{E_c A}$; A = the area of the section cut by a radial plane, per linear foot of tunnel; and θ = the angular distance of Point x, y, measured counter-clockwise. The moment, M_0, at the elastic center, introduced to allow for the removal of one-half the tunnel tube, is:

$$M_0 = \frac{\int M\, dw}{\int dw} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

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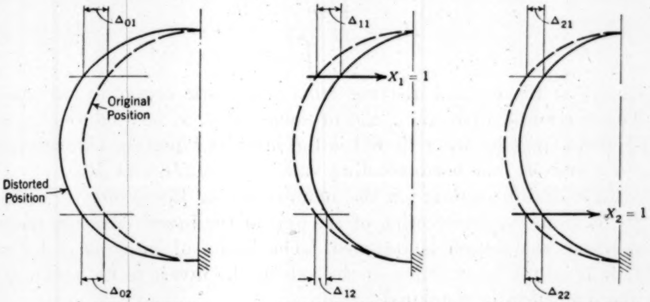
[5] "Movable and Long-Span Steel Bridges", by G. A. Hool and W. S. Kinne, p. 440.

The horizontal distortion at Point x, y, (see Fig. 4) may be determined by:

$$\Delta_{0u} = \int M\, y\, dw + \int H \cos^2 \theta\, dv + \int V \sin \theta \cos \theta\, dv$$
$$- H_0 \left(\int y^2\, dw + \int \cos^2 \theta\, dv \right) - M_0 \int y\, dw \; \ldots \ldots \ldots \ldots (4)$$

the integration being taken from the fixed support to Point x, y. When the external load is a unit horizontal force applied at the upper tie-rod joint (Fig. 4(*b*)), the horizontal and vertical components are $H = 1$, and $V = 0$,



(*a*) DUE TO EXTERNAL LOADING (*b*) DUE TO UNIT FORCE AT UPPER TIE ROD (*c*) DUE TO UNIT FORCE AT LOWER TIE ROD

FIG. 4.—DIAGRAMS OF HORIZONTAL DISTORTION.

and the bending moment at Point x, y (Fig. 3), on the neutral axis below the upper tie-rod joint is,

$$M = y_u - y \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

in which, y_u = the vertical distance of the upper tie-rod joint from the elastic center. Therefore, from Equation (2), the horizontal reaction at the elastic center, produced by a unit force at the upper tie-rod joint is:

$$H_u = \frac{\int y\,(y_u - y)\, dw + \int \cos^2 \theta\, dv}{\int y^2\, dw + \int \cos^2 \theta\, dv} \ldots \ldots \ldots \ldots (6)$$

and the reaction bending moment, from Equation (3) is,

$$M_u = \frac{\int (y_u - y)\, dw}{\int dw} \ldots \ldots \ldots \ldots \ldots \ldots (7)$$

From Equation (5) the horizontal distortion at some point, x, y, on the neutral axis, produced by a unit force acting along the upper tie-rod is:

$$\Delta_{1u} = \int y\,(y_u - y)\, dw + \int \cos^2 \theta\, dv$$
$$- H_u \left(\int y^2\, dw + \int \cos^2 \theta\, dv \right) - M_u \int y\, dw \ldots \ldots \ldots (8)$$

Likewise, the horizontal thrust, H_l, and the bending moment, M_l, at the elastic center of the ring produced by a unit force applied along the lower tie-rod, are:

$$H_l = \frac{\int y\,(y_l - y)\,dw + \int \cos^2\theta\,dv}{\int y^2\,dw + \int \cos^2\theta\,dv} \quad \dots\dots\dots\dots\dots (9)$$

and,

$$M_l = \frac{\int (y_l - y)\,dw}{\int dw} \quad \dots\dots\dots\dots\dots\dots\dots (10)$$

in which, y_l = the vertical distance from the elastic center to the lower tie-rod. The horizontal distortion, Δ_{1l}, of some point, x, y, produced by a unit force applied along the lower tie-rod is the same as Equation (8), substituting for y_u, H_u, and M_u, the corresponding values of y_l, H_l, and M_l.

The limit of integration for the nominators in Equations (6), (7), (9), and (10), is from the intersection of the axis at the invert crown to the point, x, y, at which the tie-rod is fastened. The limits of integration for Equation (8) is from the intersection of the axis at the invert to the point, x, y, at which the distortion is computed.

Case 2.—Tunnel Section with Horizontal Tie-Rods.—Whereas, a circular, elastic tube with two horizontal tie-rods is statically indeterminate in the fourth degree, without the tie-rods the section is statically indeterminate in the second degree, and in the latter case, the reaction forces may be computed by Equations (2) and (3). Stresses in the tie-rods may be determined from the Maxwell theorem of reciprocal displacements.

The horizontal distortions at the level of the upper and lower tie-rods are, respectively, Δ_{ou} and Δ_{ol}. In the tube with ties these distortions are corrected by tensile forces in the tie-rods. The final force in the upper tie-rod, assisting the tube to carry the external loads, is indicated by X_u and at the lower rod by X_l. The horizontal distortion at the upper tie-rod joint produced by the upper tie-rod stress is $X_u \Delta_{1u}$, and the distortion produced by the lower tie-rod stress is $X_l\,\Delta_{1l}$. The total distortion of the ring at the level of the upper tie-rod, from its original position, is equal to one-half the strain in that tie-rod, or

$$\Delta_1 = \frac{X_u\,L_u}{2\,E_s\,A_u} \quad \dots\dots\dots\dots\dots\dots\dots\dots (11)$$

(assuming no initial tension in the tie-rod). Therefore, from the Maxwell theorem the equation may be written:

$$\frac{X_u\,L_u}{2\,E_s\,A_u} = X_u\,\Delta_{1u} + X_l\,\Delta_{1l} + \Delta_{ou} \quad \dots\dots\dots\dots\dots (12)$$

Likewise, for the lower tie-rod an equation of distortion may be written:

$$\frac{X_l\,L_l}{2\,E_s\,A_l} = X_l\,\Delta_{2l} + X_u\,\Delta_{2u} + \Delta_{ol} \quad \dots\dots\dots\dots\dots (13)$$

in which, A_u and A_l are the sectional areas of the upper and lower tie-rods, respectively. From Maxwell's theorem $\Delta_{1l} = \Delta_{2u}$. Solving, simultaneously, Equations (12) and (13) the reactions, X_u and X_l, in the tie-rods may be determined.

The horizontal thrust, H_c, and the bending moment, M_c, at the elastic center of the tube, with tie-rods, may be computed from:

$$H_c = H_0 + X_u H_u + X_l H_l \text{ ....} (14)$$

and,

$$M_c = M_0 + X_u M_u + X_l M_l \text{ ....} (15)$$

The bending moment, M, and the normal thrust, N, at any radial section may be computed from statics; thus, for points on the axis of the ring between the upper tie-rod and the horizontal diameter of tube:

$$M_q = - M_c + H_c\, y + M + X_u\, (y_u - y) \text{ ....} (16)$$

and,

$$N = H_0 \cos\theta - H \cos\theta + V \sin\theta - X_u \cos\theta \text{ ....} (17)$$

Case 3.—Temperature Stresses.—Assuming the tie-rod removed, the horizontal thrust at the elastic center, produced by a change (t) in the temperature of the atmosphere is expressed by:

$$H_t = \frac{c\, t\, r\, E_c}{\int y^2\, dw + \int \cos^2\theta\, dv} \text{ ....} (18)$$

in which, c = a coefficient of temperature expansion or contraction of concrete for 1° variation in temperature; and r = the radius of the neutral axis of the tube.

The horizontal distortion of some point, x, y, on the neutral axis of the tube for a change of temperature of $t°$ is expressed by:

$$\Delta_t = c\, t\, r\, E_c - H_t \left(\int y^2\, dw + \int \cos^2\theta\, dv \right) \text{ ....} (19)$$

Tension or slack in the upper and lower tie-rods produced by temperature changes in the atmosphere may be computed from Equations (12) and (13), except that Δ_{01} and Δ_{02} must be changed to the temperature distortions, Δ_{tu} and Δ_{tl}, in the upper and lower rods. Temperature stresses may be computed also for differences in temperature between rods and ring, with appropriate distortions, Δ_{tu} and Δ_{tl}. It should be noted that the earth pressure loads may also change with changes in temperature.

Case 4.—Circular Tunnel Tube of Uniform Thickness.—The equations for computing stresses in a circular ring of uniform thickness may be simplified considerably. The elastic center of the tube will be in the geometric center of the ring. The co-ordinates of a radial section may be expressed by angular functions, thus,

$$x = r \sin \theta \quad \text{.........................} \quad (20)$$

$$y = r \cos \theta \quad \text{.........................} \quad (21)$$

$$ds = r \; d\theta \quad \text{.........................} \quad (22)$$

$$dw = K_1 \; r \; d\theta \quad \text{.........................} \quad (23)$$

and,

$$dv = K_2 \; r \; d\theta \quad \text{.........................} \quad (24)$$

in which, $K_1 = \dfrac{1}{E_c \, I}$; and, $K_2 = \dfrac{1}{E_c A}$. Substituting Equations (20) to (24) in Equations (2), (3), and (4):

$$H_0 = \frac{r \; K_1 \int M \cos \theta \; d\theta + K_2 \int H \cos^2 \theta \; d\theta + K_2 \int V \sin \theta \cos \theta \; d\theta}{(r^2 \, K_1 + K_2) \int \cos^2 \theta \; d\theta} \quad \text{....} \quad (25)$$

$$M_0 = \frac{\int M \; d\theta}{\int d\theta} \quad \text{.........................} \quad (26)$$

and,

$$\Delta_0 = r \, K_1 \int M \cos \theta \; d\theta + K_2 \int H \cos^2 \theta \; d\theta + K_2 \int V \sin \theta \cos \theta \; d\theta$$

$$- \; (K_1 \, r^2 + K_2) \int \cos^2 \theta \; d\theta - r \, K_1 \, M_0 \int \cos \theta \; d\theta \quad \text{.........} \quad (27)$$

EXAMPLE 1

As an illustration of the practical application of the equations developed, the stresses in the elastic tunnel tube with the tie-rods in the ceiling were computed. Fig. 5 shows the dimensions of the tube and the external loads acting on it. In addition, the sectional area of the tie-rods is $A_u = 1.3$ sq in. per lin ft of tunnel; the length of the tie-rods is $L_u = 29.67$ ft; and the vertical distance to the tie-rod from the center of the tube is $y_u = 8.80$ ft.

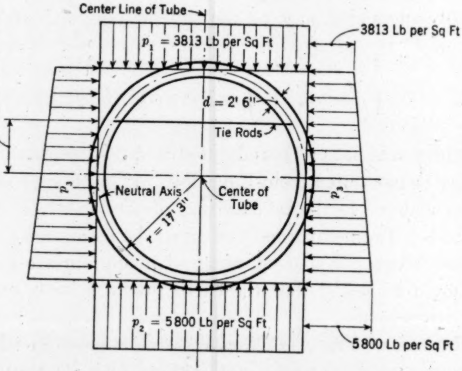

Center Line of Tube
p_1 = 3813 Lb per Sq Ft
3813 Lb per Sq Ft
y_u = 8.80 Ft
d = 2' 6"
Tie Rods
Neutral Axis
Center of Tube
r = 17' 3"
p_3
p_3
p_2 = 5800 Lb per Sq Ft
5800 Lb per Sq Ft

FIG. 5.—TUBE SECTION AND LOADING USED IN EXAMPLE.

From symmetry it follows that the elastic center of the tube corresponds to the geometric center of the ring. The horizontal thrust, H_0, and the bending moment, M_0, produced by the external loading, as computed by Equations (25) and (26), are $H_0 = 81\,500$ lb and $M_0 = 1\,450\,000$ ft-lb. The horizontal distortion, Δ_{01}, at the point, y_u, (with the tie-rods removed), due to the external loading, is computed by Equation (27): $\Delta_{0u} = 0.248$ ft. The horizontal thrust, H_u, and bending moment, M_u, at the elastic center produced by a horizontal load of 1 000 lb at the point, y_u, as computed by Equations (6) and (7) are: $H_u = 668$ lb, and $M_u = 10\,600$ ft-lb. The horizontal distortion of the tie-rod joint of the tube produced by a horizontal force of 1 000 lb at y_u, as computed by Equation (8) is, $\Delta_{1u} = 0.011$ ft. The strain in the tie-rod produced by a tensile force of 1 000 lb, is: $\Delta_u = \dfrac{1\,000 \times L_u}{2\,E_s\,A_u} = 0.00038$ ft.

Substituting the values of the distortions in Equation (12) and solving, the unknown reaction in the tie-rods is found to be $X_u = 22\,877$ lb. In Equation (12), for the present example, $X_l = 0$. The unit stress in the tie-rods is: $f_s = \dfrac{X_u}{A_u} = \dfrac{22\,877}{1.3} = 17\,598$ lb per sq in. The effect of the direct stresses on the reactions at the elastic center and the horizontal distortions of the tube have not been considered in this example.

SUMMARY

The advantages of the equations based on the Maxwell theorem in computing the stresses in a tunnel tube may be summarized, as follows:

(1) The stresses in the shell and tie-rods of a tunnel may be computed by an adaptation of Maxwell's theorem with the same precision as by any other refined method based on the theorem of least work.

(2) The significance of the various terms in the equations for computing the stresses may easily be visualized. Therefore, the computation of stresses may be simplified by reducing them for negligibly small terms. Furthermore, the simplicity of interpreting the signs of the expressions for distortion at tie-rods is self-evident.

(3) The "trial-and-error" changes in the dimensions of a tunnel tube and the tie-rods in the method based on Maxwell's theorem will affect a smaller number of the terms than is the case with the method of least work.

(4) Further simplifications may be introduced by preparing tables and diagrams for the geometrical constants involved.

(5) The method of computing stresses in the tube based on Maxwell's theorem has a wide application. It may readily be used in computing stresses in any other type of statically indeterminate structures.

The necessity for a shorter and more accurate method of computing stresses in a tunnel tube is unquestionable. In this respect, the method presented herein, based on Maxwell's theorem, will be of interest to the designers of tunnels.

APPENDIX

NOTATION

The following symbols, defined where first introduced in the paper, are re-arranged herein for convenience of reference:

A = area of a radial section of a 1-ft length of tunnel; also, with subscripts, A = cross-sectional area of a tie-rod per linear foot of tunnel, A_u referring to the upper of two rods, and A_l to the lower rod;

c = coefficient of expansion or contraction of concrete for a change in atmosphere of 1° F;

d = thickness of the tunnel ring;

E = modulus of elasticity: E_c, for concrete in compression; and E_s, for steel in tension;

f = unit stress, f_s, referring to unit steel stress;

H = horizontal thrust or reaction; horizontal component of force at any point, x, y, produced by forces on the ring between that point and the crown; H_c = horizontal component of force at the crown section, transferred to the center of the elastic weights; H_0 = horizontal component of force applied at the center of the elastic weights to allow for the removal of one-half the tunnel section; H_u = a horizontal force at the center of the elastic weights produced by a unit force applied along the upper tie-rod; H_l = a horizontal force at the center of the elastic weights, applied along the lower tie-rod; H_t = a horizontal thrust at the center of the elastic weights, produced by a change, t, in the temperature of the atmosphere;

h = height; h_y = the vertical distance from the crown to any point, x, y, on the axis of the tunnel ring;

I = moment of inertia of a radial section cut from a 1-ft length of tunnel;

K = a constant; $K_1 = \dfrac{1}{E_c\,I}$; and $K_2 = \dfrac{1}{E_c A}$;

L = length of a tie-rod between its intersection with the axis of the tunnel ring, L_u referring to the upper of two rods and L_l to the lower rod;

M = moment; the bending moment at any point, x, y, produced by the forces on the ring between that point and the crown; M_c = moment at the crown, transferred to the center of the elastic weights; M_0 = moment applied at the center of the elastic weights to compensate for the removal of one-half the ring; M_u and M_l = moment at the center of the elastic weights produced by a unit rotation in the tunnel ring at y_u and y_l, respectively; and M_t = moment at the center of the elastic weights produced by a change, t, in the temperature of the atmosphere; M_q = the bending moment at any point of the tunnel tube with tie-rods, Equation (16).

N = component of thrust normal to a radial plane through any point of the tunnel tube;

p = pressure per unit area; on the 1-ft length of tunnel, p_1, p_2, and p_3, = the uniformly distributed loads on the top, bottom, and sides, respectively, of the tunnel ring (see Fig. 5);

r = radius of a circle described from the center of elastic weights; the radius of the geometric center of the neutral axis of a circular tunnel section;

s = linear distance, measured along the axis of a tunnel section; ds = an element of s;

t = change in the temperature of the atmosphere, in degrees Fahrenheit;

V = vertical component of force at any point, x, y, due to forces on the ring between that point and the crown;

dv = elastic weight with normal stress $= \frac{ds}{E_c A}$;

w = weight; dw = elastic weight $= \frac{ds}{E_c\, I}$;

X = a tie-rod force, or reaction, X_u, referring to the upper tie-rod and X_l to the lower one;

x = a horizontal distance measured from the center of elastic weights;

y = a vertical distance measured from the center of elastic weights; y_c = the distance to the neutral axis of the tunnel ring at the crown; y_u = the distance to the upper tie-rod; y_l = the distance to the lower tie-rod;

Δ = horizontal distortion or displacement; Δ_0 = the distortion at any point x, y, due to external loading; Δ_u = the distortion in the tunnel ring, at Point y_u; Δ_1 = the distortion at any point, x, y, due to a unit load applied along the upper tie-rod; Δ_2 = the distortion at any point, x, y, due to a unit load applied along the lower tie-rod; Δ_l = the distortion in the tunnel ring at y_l; Δ_t = distortion at any point, x, y, due to a change in the temperature of the atmosphere; Δ_{01} = the distortion in the tunnel ring at Point y_u due to strain in the upper tie-rod $= \frac{X_u\, L_u}{2\, E_s\, A_u}$; Δ_{02} = the distortion in the tunnel ring at Point y_l due to strain in the lower tie-rod $= \frac{X_l\, L_l}{2\, E_s\, A_l}$;

θ = angular distance; the angle between a horizontal line through any point, x, y, and the tangent to the neutral axis at that point; in a circular tunnel section the angular distance (central angle) to Point x, y, measured counter-clockwise.

DISCUSSION

DAVID J. PEERY,[6] JUN. AM. SOC. C. E. (by letter).—The method described in this paper is quite convenient for purposes of computation and design, being a very simple and ingenious method for analyzing a structure which is statically indeterminate in the fourth degree. Equation (4), however, represents the horizontal deflection of the elastic center, rather than the deflection of a point on the ring. In order to avoid ambiguity in nomenclature, assume that the specific point at which the deflection is desired has the co-ordinates, x', y'. Other nomenclature will be as shown in Fig. 3, the variables, x and y, representing the co-ordinates to the curve of the neutral axis of the ring. The horizontal distortion at Point x', y', will be.

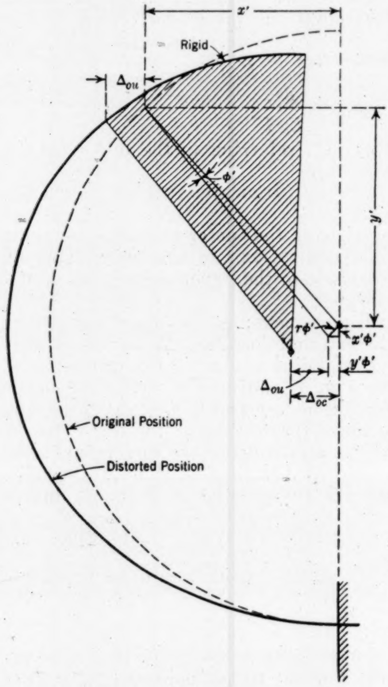


FIG. 6

$$\Delta_{ou} = \int M (y - y')\, dw + \int H \cos^2 \theta\, dv + \int V \sin \theta \cos \theta\, dv - H_0 \left[\int y (y - y')\, dw + \int \cos^2 \theta\, dv \right] - M_0 \int (y - y')\, dw \quad \ldots\ldots\ldots\ldots (28)$$

the limits of integration being taken from the fixed support to Point x', y'. If the limits of integration are taken from the fixed support to the Point x', y', Equation (4) represents the horizontal deflection of the elastic center when the part of the ring between the fixed support and the Point x', y', is considered flexible and the other part of the ring is considered rigid. If this deflection is Δ_{oc}, Equation (4) and Equation (28) yield:

$$\Delta_{ou} = \Delta_{oc} - y' \int (M - H_0\, y - M_0)\, dw \quad \ldots\ldots\ldots\ldots (29)$$

The term under the integral sign represents the angular rotation of the tangent at Point x', y'. Equation (4) will represent the horizontal deflection of Point x', y', only when there is no rotation of the tangent at the point, x', y'. This will not be true in a general case. If this rotation at Point x', y', is called ϕ', Equation (29) becomes:

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$$\Delta_{ou} = \Delta_{oc} - y' \phi' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The geometric significance of the last term in Equations (29) and (30) is evident from Fig. 6. Equations (8) and (27) of the paper will also have to be adjusted in the same manner.

ORRIN L. BRODIE,[7] M. AM. SOC. C. E. (by letter).—Simplification of analyses for obtaining the most probable stresses to be expected in subaqueous tunnel tubes is always welcome and the author has presented his method of analysis clearly and briefly. His paper should prove serviceable to all engaged in this kind of work.

The application of the Maxwell theorem of reciprocal displacements, while not novel, is extremely apt in connection with the feature of the restraining effect of horizontal tie-rods. Mr. Eremin is correct in his conclusion that the use of this theorem, in this respect, affords a simplification over that of the method of least work; but the writer has applied the latter to tunnel stress analysis, by means of a simple graphic adaptation of the method, which served as a satisfactory check to the more precise method by deflections. The use of the elastic center for the origin is also simpler, in the deflection method, than placing the origin at the invert, as has been done in studies of this kind. Tabular methods of computation always should be used; but after the basis of computation has been established by the theory, it is doubtful whether or not, in general, the subsequent labor of computing varies much. As the author has stated, the mechanical part is a matter of preparing tables and diagrams.

Although it is obvious that the theoretical derivations, such as those outlined in the paper, are independent of the physical conditions surrounding the structure, nevertheless the paper arouses curiosity along the lines of the assumed loadings for the tunnel considered. Some developments of tunnel stress theory have incorporated the "external loading" in the integrations. The author very properly has omitted these, as they only encumber the analysis unnecessarily, especially as the use of unit forces and moments suffices in this respect; but as the main objective of any analysis is to make practical application toward evaluating the stresses in the structure for proper design, it must be admitted that some appraisal of the external forces forms the other half of the "picture."

No doubt it is true that each tunnel project presents conditions in this respect peculiar to itself; but the writer is out of sympathy with the thought that because the determination of external forces is often more or less uncertain, in an absolute sense, theoretical studies, such as the author has presented, are of no practical value. In each undertaking more is learned, and uncertainties of yesterday become cleared up to-day. The value of theoretical investigation is that by its application in a comparative manner, the experience gained by the construction of works successfully accomplished may be carried over to present problems of projected work. The theory provides the vehicle. In other words, theoretical analyses permit of comparing tunnels of different sizes and types under approximately like conditions. In order to emphasize this the writer ventures to review, as briefly as possible, the main results of the Holland Tunnel studies, after which he will illustrate one phase of this work by considering the example shown by the author in Fig. 5.

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The comparative studies were made mainly on the Pennsylvania Tunnel tube, as being most similar in kind of material, along with the tunnel tube considered, with others suggested. Extensive stress analyses were made by means of the method of least work and the method of deflections. Studies of the local stresses at the joints, both end and side, were based upon the results of these analyses. The cast-iron ring containing the entire internal structure and the cast-iron ring alone (as it would be during construction), with and without compressed air, were subject to the investigation. The depths over the top of the tunnel ring which gave the probable, greatest loadings, were 31 ft of silt beneath 38 ft of water, and the weights of silt and water were taken, respectively, at 104 and 64 lb per cu ft.

Due to the wide variations in the behavior of the silt of the Hudson River, as had been observed by different people at different times and under varying conditions, diverse opinions existed as to the actual forces that might be expected to be active externally. This is explainable if one considers the noticeable variation in the consistency, composition, water content, and weight of silt, after the violent disturbance of the shoving of the shield and then after a considerable time during which it has become quiescent around the tube. To include material of varying consistency, Rankine's theory of earth pressure was applied, through varying "angles of repose," although it was well recognized that that theory did not include the tenacity, or cohesion inherent to a more or less determinable degree, in the river silt. (Subsequent experiments disclosed a value reaching 0.4 lb per sq in., with a water content of 55% by volume. This value was indicated by a computation to contribute toward offsetting the tendency to rise because, by the Rankine theory, the result was that in some cases the weights of the tunnel ring alone, with the overlying material bearing upon it, were less than the upward, active pressures of the silt.)

Where "floatation" was indicated, due compensation was made by the distribution of a uniform load either at the top or bottom, as required, before proceeding with an analysis.

During construction, silt pressures were observed around the tube over a considerable period of time, by specially designed pressure gages, the tubes of which were let through the grout holes of the ring castings. Concurrently with these observations weight and water content determinations were made. The silt weight ranged from 93 to 114 lb per cu ft and the water content, from 52% to 64% by volume. Laboratory tests followed on transmission of conjugate pressures, whence as an index, angles of repose were computed. These angles varied between 1° 45′ and 2° 40′. The pressure gages in the tunnel disclosed that these angles were less at the top than at the bottom of the ring. The recent advance in the realm of "soil physics" gives promise, along these lines, of clarifying much that, in the initial stages of the Holland Tunnel construction, was more or less speculative and rendered the minds of some engineers skeptical of theoretical analyses.

In mentioning the Holland Tunnel, the author referred to high tensile steel in the tie-rods. It is believed that this tunnel was the first to have the tie-rods introduced. The tunnel ring bolts were also of the same material. It might be of interest to note in this connection that the idea for the use of high tensile steel in the Holland Tunnel, for this purpose, was proposed originally in December, 1920, by Ralph Smillie, M. Am. Soc. C. E., then Designing Engineer on the Tunnel Staff. There were several other features of design besides the bolts and tie-rods which were developed by the comparative studies made possible by the analyses. The segments of cast iron and steel were thickened at the corners of their flanges, which insured better castings and reduced local stresses in the flanges. It had been demonstrated that changing the location of the bolts connecting the end flanges of the ring segments, by moving them nearer to the inner sides of the web and side flanges of the segment, resulted in a more rigid joint and, consequently, a stiffer tunnel ring. Furthermore, it was proved that the lower middle bolt, then customary in end-flange design, was entirely inutile; but only with the introduction of the high tensile steel and corresponding decrease in bolt diameter were these innovations made possible. The bolts were tightened to an initial stress of 25 000 lb per sq in. and both the saving in bolt and tie-rod metal, together with a great reduction in the necessity of bolt tightening after each shove of the shield, amply justified the modifications in design. The effect of the tie-rods was approximated by supplemental calculations, which indicated a reduction in flexure in the ring by about one-half to nearly two-thirds, from that without their restraining effect.

The interest in the "loading" applied by the author was aroused by Fig. 5, in his exemplification of the application of the equations, illustrating the writer's contention of the affinity of the conditions surrounding a tunnel to stress analyses. Fig. 5, however, indicates that the example loading is fluid in nature, or that the Rankine angle of repose is zero. Furthermore, it is presumable that the dead weight of the tube is equal to the difference between the upward and downward vertical forces. For this reason the writer would be interested to learn of the actual conditions of loading assumed by the author, if those given do not rightly reflect them.

Table 1 serves to compare the moments in the Posey Tube between Oakland and Alameda, Calif., with one of cast iron, of the same outside diameter (37 ft), first under the loading inferred from Fig. 5 and then under the weight of material surrounding the Holland Tube as assumed for that tunnel, and with an angle of repose of 3 degrees. As a measure

TABLE 1.—Comparison of Maximum Moments in Tunnel Rings

Tunnel	For a Load, W_1, in Pounds per Cubic Foot, of:	
Angle of repose, ϕ	53.7	104
	0°	3°
Posey	24 500	20 000
Cast iron	1 100	36 000

for comparison, the maximum bending moments for each have been computed, without tie-rod restraint, by the following equation:

$$M = w{\cdot}D\,a + \left(1 - k - \frac{D}{H_s}b\right)\frac{W_1\,H_s\,D^2}{c}\;\ldots\ldots\ldots\ldots(31)$$

in which: M = maximum bending moment (in foot-pounds) in a tunnel ring 1 ft long; w = weight of tunnel, in pounds per linear foot; D = external diameter of tunnel ring, in feet; H_s = depth, in feet, of the center of the tunnel ring below the surface of the silt (the depth of water above is reduced to an equivalent depth of silt, which is added to the actual depth of silt); W_1 = weight of 1 cu ft of silt or material, in pounds; a, b, and c are constants; and:

$$k = \frac{1 - \sin\phi}{1 + \sin\phi}\;\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

in which ϕ = the angle of repose of the material surrounding the ring. Values of the constants, a, b, and c, for use in Equation (31) are given in Table 2. It should be borne in mind that Equation (31) yields only the value (not the location) of the maximum bending moment. A negative moment in an answer signifies only a reversal from the moments that have a plus sign.

TABLE 2.—Value of Constants for Use in Equation (31)

Angle of repose, ϕ	a	b	c	$1 - k$	Angle of repose, ϕ	a	b	c	$1 - k$
0°-0′	+0.0361	0.1084	4.00	0.000	7°-30′	0.0343	0.4706	17.20	0.230
2°-0′	−0.0236	0.0974	5.41	0.067	8°-0′	0.0363	0.4875	17.42	0.244
3°-0′	−0.0247	0.1295	6.94	0.101	8°-30′	0.0377	0.4995	17.50	0.258
3°-30′	−0.0120	0.1758	8.65	0.117	9°-0′	0.0387	0.5080	17.50	0.270
4°-0′	−0.0078	0.2620	11.26	0.130	9°-30′	0.0394	0.5145	17.41	0.283
4°-30′	0.0102	0.2824	12.00	0.144	10°-0′	0.0372	0.5030	17.15	0.295
5°-0′	0.0232	0.3740	15.03	0.160	10°-30′	0.0349	0.4920	16.75	0.309
5°-30′	0.0275	0.4125	16.20	0.176	11°-0′	0.0326	0.4805	15.97	0.321
5°-45′	0.0263	0.4040	15.96	0.182	12°-0′	0.0327	0.4862	15.83	0.344
6°-0′	0.0411	0.5305	18.90	0.189	13°-0′	0.0328	0.4919	15.70	0.368
6°-30′	0.0359	0.4835	17.85	0.204	14°-0′	0.0329	0.4975	15.57	0.390
7°-0′	0.0307	0.4440	16.86	0.218	15°-0′	0.0330	0.5030	15.43	0.411

Fig. 5 indicates that the specific gravity of the material appears to be only 0.86. The head on the center of the 37-ft tube is 89.5 ft; its weight is 73 500 lb per lin ft, the weight of material around it, 53.7 lb per cu ft, and, the angle of repose, first 0° and second, 3 degrees.

The weight of a cast-iron tube, correspondingly, should be about 56 000 lb per lin ft. The results in Table 1 indicate that the Posey Tube sustains less flexure in the heavier cohesive material, whereas the cast-iron tube fares better in the liquid. No doubt it would float; but this has not been considered in the example.

Equation (31) was derived and constants computed as an epitome, as it were, of the studies alluded to by the writer. It may be used directly for fairly close approximations of values for tunnels of different sizes and weights and depths of submergence. Curves in terms of angles of repose and the moments as arguments may then be plotted and comparisons made.

The maximum bending moment usually occurs at the crown or invert although it has been found at the haunches, depending upon conditions, and occasionally between the crown and the haunch. Moments at the haunches are of opposite sign to those at the crown and invert, and the usual assumption is to consider moments to be positive when producing compression at the "extrados" of the ring, and negative if they produce tension.

A. A. EREMIN,[8] ASSOC. M. AM. SOC. C. E. (by letter).—The favorable response to the method of analysis presented in this paper is gratifying. The discussion contains much valuable information and constructive criticism.

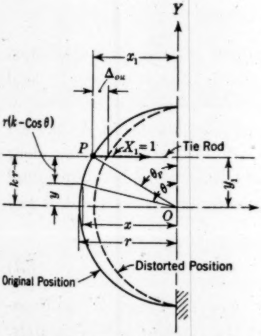
Y
x_1
Δ_{ou}
r(k - Cos θ)
P
$X_1 = 1$
Tie Rod
kr
y
θ_P
θ
y_1
X
O
x
r
Distorted Position
Original Position

FIG. 7.—DIAGRAM OF HORIZONTAL DISTORTION

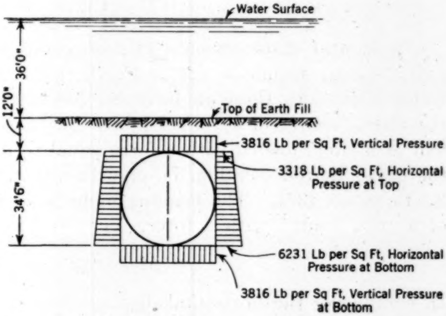
Water Surface
36'0"
12'0"
Top of Earth Fill
3816 Lb per Sq Ft, Vertical Pressure
3318 Lb per Sq Ft, Horizontal Pressure at Top
34'6"
6231 Lb per Sq Ft, Horizontal Pressure at Bottom
3816 Lb per Sq Ft, Vertical Pressure at Bottom

FIG. 8.—TYPICAL TUBE LOADING

Equations (28) to (30) for horizontal displacement at Point x, y (Fig. 6), as presented by Mr. Peery, are correct. However, the computations of the horizontal displacements of the tube may be greatly simplified. For convenience of reference the left half of the tube, with Point P defined by the co-ordinates, x_1 and y_1, is shown in Fig. 7. The elastic center of the tube and the origin of the co-ordinates, x and y, are at the center. The vertical distance of the point, P, to the horizontal diameter of the tube, y_1, expressed as a proportion of the radius, is:

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[8] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

$$y_1 = k\,r \qquad (33)$$

Therefore, the vertical distance from some point, x, y, on the shell to Point P, Fig. 7, is,

$$(y_1 - y) = r\,(k - \cos\theta) \qquad (34)$$

and the horizontal displacement at Point P on the left half of the tube fixed at the invert, Fig. 7, is,

$$\Delta_{0u} = r\int M\,(k - \cos\theta)\,dw + \int H\cos^2\theta\,dv + \int V\sin\theta\cos\theta\,dv$$

$$- H_0\left[r^2\int (k - \cos\theta)\cos\theta\,dw + \int \cos^2\theta\,dv\right] - M_0\,r\int (k - \cos\theta)\,dw. \qquad (35)$$

For practical purposes in computing horizontal displacements the influence of direct and shear stresses may be neglected without serious error. Therefore, Equation (35) may be written,

$$\Delta_{0u} = r\int M\,(k - \cos\theta)\,dw - H_0\,r^2\int (k - \cos\theta)\cos\theta\,dw$$

$$- M_0\,r\int (k - \cos\theta)\,dw \qquad (36)$$

Horizontal displacements can be computed more quickly by Equation (36) than by Equation (29). Furthermore Equation (36) involves integral terms similar to those in formulas for computing the redundant forces at the elastic center of a tunnel tube. The horizontal displacement of the left half of a tube, loaded with a force equal to unity ($X_1 = 1$) and acting along the upper tie-rod (see Fig. 7), or with any other loading, may be determined by Equation (36). The bending moment at any point, x, y, on the left half of a tube loaded with a force, $X_1 = 1$ (Fig. 7), is,

$$M = r\,(k - \cos\theta) \qquad (37)$$

In computing the horizontal displacement of the tube at Point P by Equation (36), each term containing an integral may be computed separately. Substituting the bending moment from Equation (37) in Equation (36), the first term in the latter is,

$$B_1 = r^2\int_{\pi}^{\theta_p} (k - \cos\theta)^2\,dw \qquad (38)$$

Integrating and simplifying,

$$B_1 = \frac{r^2}{E\,I}\left[k\,\pi + \frac{\pi}{2} - k^2\,\theta_p + 2\,k\sin\theta_p - \frac{1}{2}\,(\theta_p + \sin\theta_p\cos\theta_p)\right] \qquad (39)$$

Integrating and simplifying the second term of Equation (36), B_2 is:

$$B_2 = H_0 r^2 \int_{\pi}^{\theta_p} (k - \cos\theta) \cos\theta \, dw$$

$$= \frac{H_0 r^3}{E I} \left[- \frac{\pi}{2} - k \sin\theta + \frac{1}{2} (\theta_p + \sin\theta_p \cos\theta_p) \right] \ldots\ldots\ldots (40)$$

Integrating and simplifying the third term of Equation (36), B_3 is:

$$B_3 = M_0 r \int_{\pi}^{\theta_p} (k - \cos\theta) \, dw = \frac{M r^2}{E I} (k \pi - k \theta_p + \sin\theta_p) \ldots. (41)$$

Therefore, the horizontal displacement at Point P, Fig. 7, as computed by Equations (36) to (41) is,

$$\Delta_{0u} = B_1 + B_2 + B_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Information concerning mud pressures on subaqueous tunnels, as given by Mr. Brodie, is most interesting. It is true, as Mr. Brodie states, that the method of considering the distribution of external pressures in subaqueous tunnels "forms the other half of the 'picture'."

In Example 1, Fig. 5, the loadings are assumed. It may be interesting to compare them with the specified external loading used in the design of the Posey Tunnel. Water pressure on that structure was considered as acting (independent of earth pressure) radially over the entire circumference. The vertical pressure on the top was considered equal to the total weight of water, at 64 lb per cu ft, plus the weight of the submerged earth at 62 lb per cu ft. The intensity of the lateral pressure at any section was assumed equal to total vertical pressure, excluding water pressure, multiplied by 33%, plus the intensity of water pressure. This also agrees with lateral pressure computed with Rankine's formula for wet sand with an angle of repose of 30° recommended by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E.[9]

Assume the tunnel in Example 1 to be placed 48 ft below the water surface, as shown in Fig. 8. The top of the earth fill is 12 ft above the top of the tunnel. Therefore, the vertical pressure on top of the tube is $64 \times 48 + 62 \times 12 = 3\,816$ lb per sq ft. The side pressure at the top of the tube is $64 \times 48 + 62 \times 12 \times 0.33 = 3\,318$ lb per sq ft. The side pressure at the bottom is $64 \times 82.5 + 62 \times 46.5 \times 0.33 = 6\,231$ lb per sq ft.

Assuming the weight of tube to be equal to the buoyant force of the submerged tube, the vertical pressure at the invert will be the same as that at the top. If the tube is filled with water, and if the earth fill is placed directly on top of the tube, the vertical pressure on the invert will be greater than that on top by an amount equal to the weight of the submerged tube. To simplify computations in Example 1 the loading on the tube was as shown in Fig. 5. Evidently, this loading gives greater stress in the tie-rods than that in Fig. 8. The exact loading on subaqueous tunnels varies with the method of construction, the supporting foundation beneath the finished tunnel tube in place, the variation of the water level, and the plastic flow in the earth and the mud fill on top of the tube. Interesting effects of the plastic flow of mud on the external pressures and deformations of the Hudson and Manhattan Railroad tubes were observed during the construction of the cast-steel rings. Perfectly circular steel rings, when shoved forward under pressure by means of jacks, were changed in form by the lengthening of their vertical diameters and the shortening of their horizontal diameters. After the plastic flow of mud had occurred the shape was reversed; the vertical diameter decreased, and the horizontal diameter increased. By tightening the steel tie-rods the tube rings may be returned to their original form. Equation (31) gives the value of the maximum bending moment in the circular tunnel tube without tie-rods. This bending moment may occur at the crown, at the invert, or at the haunch section as stated by Mr. Brodie. It is to be regretted that Mr. Brodie did not give sufficient information to permit one to check Equation (31). The need for a more determinate method of computing bending moments at desired sections is self-evident. The redundant force, H_0, and the bending moment, M_0, at the elastic center of the tunnel tube without tie-rods may be computed by Equations (25) and (26), respectively. The bending moment, M_1, at any point on the tube without tie-rods, by Equation (16), is:

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[9] "The Design of Walls, Bins, and Grain Elevators," by Milo S. Ketchum, Third Edition, p. 73.

$$M_1 = - M + H_0 y + M_0 \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

The maximum bending moment may be determined algebraically by Equation (43), with the external loading expressed by a continuous function or by plotting a force polygon. In practice, however, with a few trial computations, the maximum bending moment in a tube may be determined without much effort.

A method of computing stresses in subaqueous tunnel tubes without tie-rods,[10] has recently been advanced by M. Alfano, who assumes the origin of the co-ordinates at the invert instead of that at the center of the tube. However, he failed to show any advantages in computing stresses with the origin of co-ordinates selected in this manner.

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[10] *Annales des Ponts et Chaussees,* February, 1937, p. 197.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 1987

DEFLECTIONS BY GEOMETRY

BY DAVID B. HALL[1], ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. A. J. MCGAW, L. E. GRINTER, RALPH W. STEWART, WILLIAM BERTWELL, AND DAVID B. HALL.

SYNOPSIS

The fundamental method for finding deflections is by geometry, rather than by the law of conservation of energy or the principle of least work. The few simple rules that are necessary in a geometrical analysis are given in this paper, and are illustrated by developing methods for calculating influence lines for such structures as arches, and deflections in trusses.

GENERAL THEORY

For the engineer who is constantly dealing with stresses, but meets only occasionally with deflections or statically indeterminate structures, the methods of calculating deflections in terms of imaginary forces are of great assistance. They enable him to make computations with a minimum knowledge of the subject of deflections. For this same reason, however, such indirect methods as that of least work, or of elastic weights, tend to stand in the way of acquiring real facility in the subject.

A simple illustration will show the real inward nature of the problem. Consider the four following problems.

Problem 1.—Determine the Force X, in Fig. 1, by the Principles of Statics.—The solution is evidently to take moments about the support; whence $X = 2$ lb.

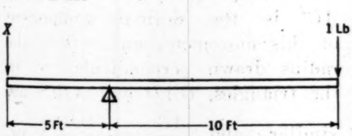

FIG. 1.—MECHANICS PROBLEM.

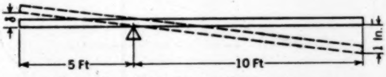

FIG. 2.—GEOMETRY PROBLEM.

Problem 2.—Determine the Distance, δ_x, in Fig. 2, by Geometry.—From similar triangles it is evident that $\delta = 0.5$ in.

NOTE.—Published in December, 1936, *Proceedings*.

[1] Detailer, Ash-Howard-Needles & Tammen, New York, N. Y.

Problem 3.—Determine the Force, X, Fig. 1, in Some Manner Other Than by Statics.—If the force of 1 lb is balanced by the unknown force, X, and the lever is rotated slightly, one side will gain, and the other side will lose, energy. According to the law of conservation of energy, these two changes must be equal. Therefore, continuing from the solution of Problem 2, $X \times 0.5$ in. $= 1$ lb $\times$ 1 in.; and, $X = 2$ lb.

Problem 4.—Determine the Deflection δ, Fig. 2, in Some Manner Other Than by Geometry.—Once more using the law of conservation of energy, apply a force on the left end and find by statics the force on the right end to balance it; then find how far the first force must move to balance the energy gained or lost by the second force in moving 1 in. This distance will be 0.5 in.

Comment.—In the foregoing group of problems it is quite obvious that the simplest solution of the mechanics problem is by mechanics and the simplest solution of the geometrical problem is by geometry. The solution presented in Problem 4, however, is often referred to as the fundamental method of solving deflection problems. It is none other than the method of work. By the more usual method of presentation, the force corresponding to that on the right side of the lever would be furnished by elastic members, and, therefore, both it and the force on the left would be functions of the deflections and the energy would be determined by integral calculus, although these complications seem to be irrelevant.

The group of problems suggests one constructive idea, however: When new methods for applying the geometry of deflections are being sought, there will usually be similar familiar methods for solving corresponding mechanics problems to serve, not as substitutes, but as guides.

Probably the most important theorem of the geometry of deflections is one which bears a very close resemblance to the principle of moments:

Theorem 1.—If a body rotates about a point through an infinitesimal angle (in ordinary structures all deflections are treated as infinitesimal quantities), the component in any direction of the movement of any point in the body is proportional to the perpendicular distance from the line of that component to the center of rotation.

The proof of Theorem 1 is quite simple. In Fig. 3 it will be seen, in the first place, that the actual motion of a point such as A is at right angles to the radius, OA, and proportional to its length. (The motion is numerically equal to $\overline{OA} \times \alpha$.) If AC is the desired component of this movement, and OD is the radius drawn perpendicular to it, the triangles, OAD and ABC, are similar, and $\frac{\overline{AC}}{\overline{AB}} = \frac{\overline{OD}}{\overline{OA}}$; or,

$$\overline{AC} = \frac{\overline{OA} \times \alpha \times \overline{OD}}{\overline{OA}} = OD \times \alpha.$$

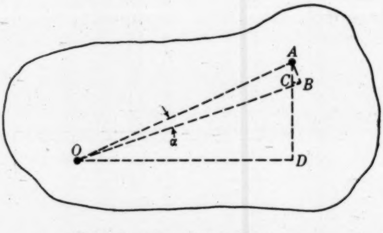

FIG. 3.—ROTATION THEOREM

When dealing with deflections in terms of vertical and horizontal components, the foregoing theorem can be stated more conveniently as follows:

Theorem 2.—When a body rotates about a point, the vertical displacement of any point is proportional to its horizontal distance from the center of rotation, and the horizontal displacement is proportional to its vertical distance.

DEFLECTIONS OF BEAMS

The simplest type of beam to investigate is a cantilever. Consider the beam in Fig. 4. Suppose that the moment in the shaded area causes this portion of the beam to bend through an angle, α. (This angle can be calculated by the well known formula, $\alpha = \frac{M\,dL}{E\,I}$, in which dL is the length of the portion considered; M is the moment; E, the modulus of elasticity; and I, the moment of inertia.) Then Theorem 2 can be applied directly to determine the deflection at the end contributed by this part of the beam. The vertical deflection is $x\,\alpha$, or $\frac{M\,dL}{E\,I}\,x$, and the horizontal deflection is $\frac{M\,dL}{E\,I}\,y$. To find the complete deflection, it is necessary to add the contributions from each part of the beam, integrating by calculus if possible, or by dividing the beam into a number of sections and treating the average properties of each section.

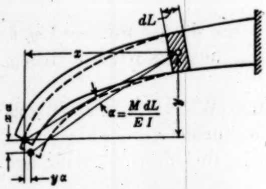

FIG. 4.—DEFLECTION OF A CANTILEVER.

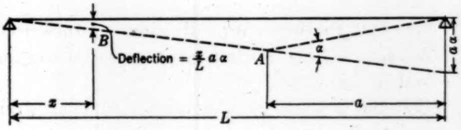

FIG. 5.—DEFLDCTION OF A SIMPLE BEAM.

To find the moments in a cantilever it is only necessary to multiply the loads and their lever arms. To find the moments in a simple beam an additional step is required: (1) Take moments about one support and find one reaction; and (2) find the moment caused by this reaction. Similarly, an extra step is required in finding deflections in a simple beam. The process is illustrated in Fig. 5. Bending through an angle, $\alpha \left(= \frac{M\,dL}{E\,I}\right)$, at Point A would produce an upward deflection, $a\,\alpha$, at the right end if the left part were held fast. If the beam were then rotated as a whole until the right end rested on its support, the beam would assume the position which it actually does occupy, and the proportionate deflection at Point B would be $\frac{x}{L}\,a\,\alpha$.

In both of the foregoing examples it will be discovered that the coefficient of $\frac{M\,dL}{E\,I}$ is identical to the moment, m, which a unit load applied where the deflection is measured would produce at the point that is bending. This again illustrates the connection between the methods of Problems 2 and 4, and forms the basis of the dummy load formula:

$$\delta = \int \frac{M\,m\,dL}{E\,I} \quad \text{......................} (1)$$

There are two general methods of determining deflections: The foregoing, which might be referred to as single integration, and the method of double integration. The second method is particularly adapted to determining deflections at all the division points of an irregular beam such as an arch. In order to have something definite and clear cut upon which to perform the various operations that are involved, the beam should be visualized in the following manner:

1.—Cut the beam into divisions as indicated by the dotted lines in Fig. 6. (The dotted lines are necessary for the explanation but are not essential in actual design.)

2.—Set a point in the middle of each division. When the beam is subjected to bending the faces of Division 4, say, are turned through a certain angle relative to each other, the part of the axis in this division being bent in a curve. This angle is $\frac{M\ dL}{E\ I}$, in which dL is the length of the division. The same happens in each of the other divisions.

3.—Instead of considering the divisions curved by the moments, the beam is assumed to consist of a series of straight lines from point to point and the bending to consist of a change in the angles at the joints in these lines. With the beam thus articulated it is a simple matter to visualize any operation that has to be performed.

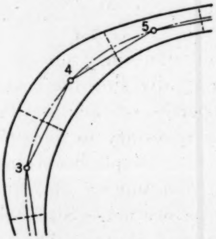

FIG. 6.

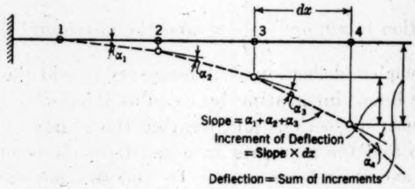
dx
1
2
3
4
α_1
α_2
α_3
α_4
Slope = $\alpha_1+\alpha_2+\alpha_3$
Increment of Deflection = Slope × dx
Deflection = Sum of Increments

FIG. 7.—DEFLECTION BY DOUBLE INTEGRATION.

The deflection of a cantilever by double integration is illustrated in Fig. 7. The process is exactly the same for a curved beam. A straight beam was illustrated to avoid confusing the reader with two sets of angles. For any other beam than a cantilever the best method is to determine the rotation at some point first and then proceed from there to sum up the angles at the joints. For a symmetrical simple beam, symmetrically loaded, this is accomplished by starting at the center where the slope is zero. It may be noted that the point of initial slope and the point of zero deflection do not need to be the same; the first summation can proceed in one direction, and the second in the opposite direction.

Influence Lines for Arches.—Calculation of the influence line for the thrust in a symmetrical two-hinged arch (see Fig. 8) is a good illustration of the foregoing method. Instead of placing a unit load at successive points to find the displacement of the hinges, the vertical deflections of points along the arch due to a unit thrust at the hinges may be calculated, since by the theorem of reciprocal displacements both are alike. In Table 1 it will be noted that the lines are double-spaced. Entries opposite the points refer in some manner to the individual points (co-ordinates, angles, etc.), whereas entries between points refer to the lines joining them (slopes, increments of deflection, etc.). For divisions of equal length, $\frac{dL}{EI}$ is proportional to $\frac{1}{t^3}$, in which t is the thickness of the section. For a unit thrust, $M = y$, and

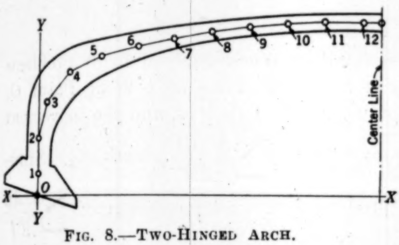

FIG. 8.—TWO-HINGED ARCH.

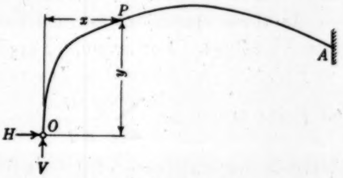

FIG. 9.—MATHEMATICAL EQUIVALENT OF THE BEGGS METHOD.

TABLE 1.—INFLUENCE LINE FOR TWO-HINGED ARCH (SEE FIG. 8)

Point	Abscissa, x (1)	Element, dx (2)	Thickness of arch, t (3)	t^3 (4)	Ordinate, y (5)	Horizontal deflection, $\frac{y^2}{t^3}$ (6)	Angle, $\frac{y}{t^3}$ (7)	Slope, $\Sigma \frac{y}{t^3}$ (from crown) (8)	Deflection increment, Column (8) × Column (2) (9)	Vertical deflection, Σ Column (9) (from hinge) (10)	H influence factors, $\frac{\text{Column (10)}}{627.2}$ (11)
1......	0		4±	100±	2.2	0.0	0.02			0	0
		0.0						16.32	0.0		
2......	0		3.70	50.7	6.6	0.9	0.13			0	0
		1.1						16.19	17.8		
3......	1.1		4.90	117.6	10.9	1.0	0.09			17.8	0.028
		3.0						16.10	48.3		
4......	4.1		5.12	134.2	14.1	1.5	0.10			66.1	0.105
		4.1						16.00	65.5		
5......	8.2		3.96	62.1	15.9	4.1	0.26			131.6	0.210
		4.3						15.74	67.7		
6......	12.5		3.00	27.0	17.1	10.8	0.63			199.3	0.318
		4.3						15.11	65.0		
7......	16.8		2.46	14.9	18.0	21.8	1.21			264.3	0.421
		4.4						13.90	61.1		
8......	21.2		2.13	9.66	18.75	36.4	1.94			325.4	0.519
		4.5						11.96	53.8		
9......	25.7		1.96	7.53	19.3	49.5	2.56			379.2	0.604
		4.4						9.40	41.4		
10......	30.1		1.88	6.64	19.7	58.5	2.96			420.6	0.670
		4.4						6.44	28.4		
11......	34.5		1.85	6.33	20.0	63.2	3.16			449.0	0.715
		4.5						3.28	14.8		
12......	39.0		1.83	6.13	20.1	65.9	3.28			463.8	0.739
		2.2						0	0		
Crown...	41.2		....	...	20.1	...	..			463.8	0.739
$\Sigma \frac{y^2}{t^3}$	....	...	....	...	...	313.6	..	....	......	....	....
$2\Sigma \frac{y^2}{t^3}$	....	...	....	...	...	627.2	..	.. ...	......	....	....

$\frac{M\ dL}{E\ I}$ is equivalent to $\frac{y}{t^3}$. The horizontal displacement between the hinges is equal to $\sum \frac{y^2}{t^3}$, and the influence line is obtained by dividing the deflections for unit loads by this displacement.

As these operations become more familiar, increasingly difficult problems can be solved. For example, apply a horizontal force of 1 lb at Point O, in Fig. 9. The moment at any point, P, will be 1 lb × y, and the movement at Point O will be, $\sum \frac{y^2\,dL}{E\,I}$, horizontally, and $\sum \frac{x\,y\,dL}{E\,I}$, vertically. Similarly, a vertical force of 1 lb will produce a vertical movement of $\sum \frac{x^2\,dL}{E\,I}$ and a horizontal movement of $\sum \frac{x\,y\,dL}{E\,I}$.

When these deflections have been determined it is a simple matter to compute a combination of vertical and horizontal forces at Point O which will cause a horizontal deflection of one unit and no vertical deflection. The formulas for this condition, and for a unit vertical deflection, are as follows:

For a unit horizontal deflection:

$$H = \frac{\Sigma\ (x^2\,G)}{B} \quad \text{..........................} (2)$$

and,

$$V = \frac{\Sigma\ (x\,y\,G)}{B} \quad \text{..........................} (3)$$

and, for a vertical deflection:

$$H = \frac{\Sigma\ (x\,y\,G)}{B} \quad \text{..........................} (4)$$

and,

$$V = \frac{\Sigma\ (y^2\,G)}{B} \quad \text{..........................} (5)$$

in which $G = \frac{dL}{E\,I}$; and.

$$B = \Sigma\ (x^2\,G)\ \Sigma\ (y^2\,G) - \left[\Sigma\ (x\,y\,G)\right]^2 \quad \text{................} (6)$$

Furthermore, it is not difficult to compute the moments and, in turn, the deflections at a succession of points along the structure due to these forces at Point O. These deflections constitute the influence line for the horizontal reaction at Point O in a structure fixed at Point A (Fig. 9) and hinged at Point O, because the foregoing process is the mathematical equivalent of the Beggs mechanical method for obtaining influence lines.

This principle will be illustrated by the calculation of the constants for unsymmetrical beams of variable section. If the right end of Beam AB

(Fig. 10(*a*)) is fixed and the left end is rotated through a unit angle, the resulting end moments can be called stiffness factors. The moment at End *A* will be denoted as K_A; the moment at End *B* will be denoted as K_{AB} (or K_{BA},

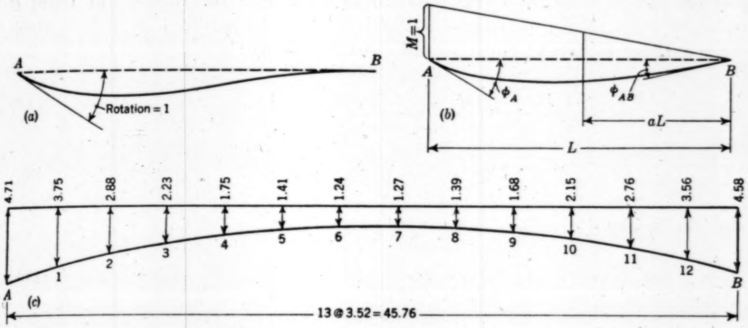

FIG. 10.

since it will be found that the fixed-end moment for either end, when the opposite end is rotated, is the same). Similarly, if End *A* is fixed and End *B* is given a unit rotation, the moment at End *B* will be K_B.

For the moment distribution method, K_A and K_B are the usual stiffness factors, and $\frac{K_{AB}}{K_A}$ and $\frac{K_{AB}}{K_B}$ are the carry-over factors. For the slope deflection method,

$$M_{AB} = K_A(\theta_A - R) + K_{AB}(\theta_B - R) + C_{AB} \dots\dots\dots\dots (7)$$

and,

$$M_{BA} = K_B(\theta_B - R) + K_{AB}(\theta_A - R) + C_{BA} \dots\dots\dots\dots (8)$$

in which C_{AB} and C_{BA} are fixed beam moments, the signs being algebraic.

The method of obtaining the foregoing *K*-factors is as follows: The clockwise rotation of End *A* in the simple beam in Fig. 10(*b*) is equal to the deflection of End *B* from the tangent at End *A*, divided by *L*, and can readily be shown to be:

$$\phi_A = \int \frac{a^2\, dL}{E\, I} \dots\dots\dots\dots (9)$$

or for finite divisions,

$$\phi_A = \frac{\Sigma\, a^2\, dL}{E\, I} \dots\dots\dots\dots (10)$$

This rotation will be referred to as ϕ_A. The counterclockwise rotation at Point *B*, is equal to:

$$\phi_{AB} = \frac{\Sigma\, a\,(1 - a)\, dL}{E\, I} \dots\dots\dots\dots (11)$$

For a unit moment at Point B, $\phi_B = \sum \frac{(1-a)^2\, dL}{E\, I}$, and ϕ_{BA} is the same as ϕ_{AB}. To obtain the stiffness factors, K_A and K_{AB}, it is simply necessary to make the rotation at Point A equal to unity and the rotation at Point B equal to zero, thus producing the condition illustrated in Fig. 10(a), which defined these factors. The necessary equations are:

$$M_A\, \phi_A - M_{AB}\, \phi_{AB} = 1 \quad \text{.....................(12)}$$

and,

$$M_A\, \phi_{AB} - M_{AB}\, \phi_B = 0 \quad \text{.....................(13)}$$

and the solution gives,

$$M_A = K_A = \frac{\phi_B}{\phi_A\, \phi_B - (\phi_{AB})^2} = \frac{\phi_B}{D} \quad \text{..............(14)}$$

and,

$$M_{AB} = K_{AB} = \frac{\phi_{AB}}{D} \quad \text{.....................(15)}$$

Similarly,

$$K_B = \frac{\phi_A}{D} \quad \text{.........................(16)}$$

A typical beam is shown in Fig. 10(c), and the evaluation of these constants for this beam is contained in Table 2. It will be noticed that multiplying or dividing at strategic points by the number of divisions (13) facilitates the numerical work. Referring to the summation of Table 2:

TABLE 2.—EVALUATION OF CONSTANTS FOR AN UNSYMMETRICAL BEAM (SEE FIG. 10(c))

Point	Elastic factor, $\frac{dL}{I^3}$ (1)	Values of $(13a)^2$ (2)	Values of $13a$ $(13 - 13a)$ (3)	Values of $(13 - 13a)^2$ (4)	Column (1) × Column (2) $= 13^2\, \phi_A$ (5)	Column (1) × Column (3) $= 13^2\, \phi_{AB}$ (6)	Column (1) × Column (4) $= 13^2\, \phi_B$ (7)
A*	0.017	169	0	0	2.87	0.00	0.00
1	0.067	144	12	1	9.65	0.80	0.07
2	0.147	121	22	4	17.79	3.23	0.59
3	0.317	100	30	9	31.70	9.51	2.85
4	0.657	81	36	16	53.22	23.65	10.51
5	1.256	64	40	25	80.38	50.24	31.40
6	1.846	49	42	36	90.45	77.53	66.46
7	1.718	36	42	49	61.85	72.16	84.18
8	1.311	25	40	64	32.78	52.44	83.90
9	0.742	16	36	81	11.87	26.71	60.10
10	0.354	9	30	100	3.19	10.62	35.40
11	0.167	4	22	121	0.67	3.67	20.21
12	0.078	1	12	144	0.08	0.94	11.23
B*	0.018	0	0	169	0.00	0.00	3.04
Summation...	.....	...	..	...	396.50	331.50	409.94

* One-half section.

$$13^4\,\phi_A\,\phi_B = \text{product of Columns (5) and (7)} = 162\,541$$

$$13^4\,\phi_{AB}^2 = \text{Column (6) squared} = 109\,892$$

$$13^4\,D = 13^4\,(\phi_A\,\phi_B - \phi_{AB}^2) = 52\,649$$

Finally, the stiffness factors are:

$$K_A = 409.94 \times \frac{13^3}{52\,649} = 1.31588$$

$$K_{AB} = 331.50 \times \frac{13^3}{52\,649} = 1.06409$$

and,

$$K_B = 396.50 \times \frac{13^3}{52\,649} = 1.27274$$

The stiffness factors having been calculated, it is now a simple matter to apply the equivalent of the Beggs method for finding the fixed-beam end moments. To obtain actual numerical values for the moment at Point A, the deflections must be determined while End A is rotated through a unit angle and End B is held fixed, exactly the same as in a mechanical model. Referring to Fig. 10(a), it is evident that this condition can be realized by applying at End A the moment, K_A (= 1.31588), and at End B the moment, K_{AB} (= 1.06409). Since one of the operations will consist of multiplying

TABLE 3.—Computation of Fixed-Beam Moments

Point	C_{AB}					C_{BA}			
	Elastic factor, $\frac{dI}{I^3}$ (1)	Moment (2)	Values of $\frac{M\,dL}{I^3}$ (3)	Values of Σ (from Point B) (4)	Values of $\Sigma\Sigma = C_{AB}$ (from Point A) (5)	Moment (6)	Values of $\frac{M\,dL}{I^3}$ (7)	Values of Σ (from Point A) (8)	Values of $\Sigma\Sigma = C_{BA}$ (from Point A) (9)
				3.5196					
A	0.017	4.6319	0.0787		0.000	3.7456	0.0637		0.000
				3.4409				0.0637	
1	0.067	3.9875	0.2672		3.441	3.1129	0.2086		0.064
				3.1737				0.2723	
2	0.147	3.3431	0.4914		6.615	2.4801	0.3646		0.336
				2.6823				0.6369	
3	0.317	2.6986	0.8555		9.297	1.8474	0.5856		0.973
				1.8268				1.2225	
4	0.657	2.0542	1.3496		11.124	1.2146	0.7980		2.195
				0.4772				2.0205	
5	1.256	1.4098	1.7707		11.601	0.5819	0.7309		4.216
				−1.2935				2.7514	
6	1.846	0.7654	1.4129		10.307	−0.0508	−0.0938		6.967
				−2.7064				2.6576	
7	1.718	0.1209	0.2077		7.601	−0.6836	−1.1744		9.625
				−2.9141				1.4832	
8	1.311	−0.5235	−0.6863		4.687	−1.3163	−1.7257		11.108
				−2.2278				−0.2425	
9	0.742	−1.1679	−0.8666		2.459	−1.9490	−1.4462		10.866
				−1.3612				−1.6887	
10	0.354	−1.8123	−0.6416		1.098	−2.5818	−0.9140		9.177
				−0.7196				−2.6027	
11	0.167	−2.4568	−0.4103		0.378	−3.2145	−0.5368		6.574
				−0.3093				−3.1395	
12	0.078	−3.1012	−0.2419		0.069	−3.8473	−0.3001		3.435
				−0.0674				−3.4396	
B	0.018	−3.7456	−0.0674		0.0016 (Check)	−4.4800	−0.0806		−0.0049 (Check)
								−3.5202	

the various slopes by dL (see Fig. 7), it will save work to do this in advance, by using the moments, $K_A \times 3.52 = 4.6319$, and $K_{AB} \times 3.52 = 3.7456$. The moments along the beam vary between these two values by thirteen equal increments, so that they are very easily tabulated (see Column (2), Table 3). Multiplying by $\dfrac{dL}{EI}\left(\text{or, } \dfrac{dL}{t^3}\right)$ gives the angle at each joint; one summation gives the slope of each section (note that the slope at End A equals 3.52 as a check); and the second summation gives the ordinates of the influence line. A similar operation gives the same data for End B (Columns (6) to (9), Table 3).

The values of the moment increments are:

$$
\begin{array}{lcl}
\text{For Column (2):} & & \text{For Column (6):}\\
3.52\,K_A = 4.6319 & & 3.52\,K_B = 4.4800\\
3.52\,K_{AB} = 3.7456 & & 3.7456\\
\hline
\phantom{3.52\,K_{AB} =}\ 8.3775 & & 8.2256
\end{array}
$$

$$
\text{Increment} = \frac{8.3775}{13} = 0.644423 \qquad \frac{8.2256}{13} = 0.632738
$$

This same method is well adapted to calculating influence lines for fixed arches, especially if the redundants are taken at the elastic center of the arch. It has been used successfully on unsymmetrical two-span arches, such as those illustrated in Fig. 11(a). In this case the statically determinate base system and the redundant forces, X, were assumed to be as shown in Fig. 11(b). The three deflections due to each of the three redundants were calculated; then, for each influence line, three simultaneous equations were solved to make the corresponding deflection unity and the other two deflections, zero; and through each of these stages a record was kept of the rotation of the top of the center pier, to serve as a starting point for summing up the changes in direction from point to point.

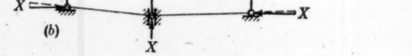

FIG. 11.—A TWO-SPAN FRAME.

TRUSS DEFLECTIONS

The geometry of trusses, if anything, is even simpler than the geometry of beams, since a few definite members take the place of continually changing infinitesimal elements. Most of the methods of stress calculation, both graphic and analytic, have some kind of a counterpart in deflection calculation. There is a distinction, however, in the case of resolution of forces or displacements at a joint. From the law of conservation of energy, it will be observed that the relation between displacements is the reciprocal of the relation between forces, since the product of forces and displacements must be constant. The relation of forces at a joint can be represented by a force triangle or polygon, and the relation between displacements, therefore, automatically cannot. This is the reason that Williot diagrams are inherently more complicated than stress diagrams. For this same reason the dummy load method expedites deflection calculations for single joints or panels.

For complete truss deflections it is preferable to deal with rotations, just as a majority of stress calculations are made by moments. Most of the conditions that can arise, are illustrated by Fig. 12. Consider the deflection at Point *A* due to a change in length of Member *CD*. The part of the truss bounded by Points *d, D, F,* and *f* will remain fixed in place. The part bounded by Points *d, C, A,* and *b* will rotate as a unit about Point *d.* The extent of this rotation is the stretch of Member *CD* divided by the perpendicular distance from *CD* to Point *d,* and the vertical movement of Point *A* (or any other point on the moving part of the truss) is equal to this rotation times the horizontal distance to Point *d.* If the member, *dE,* stretches, the polygon, *dDAb,* will rotate about Point *O.* This motion can also be treated as a combined translation and rotation about Point *D* or Point *E,* expressed entirely in terms of the dimensions of Panel *dDEe*[2].

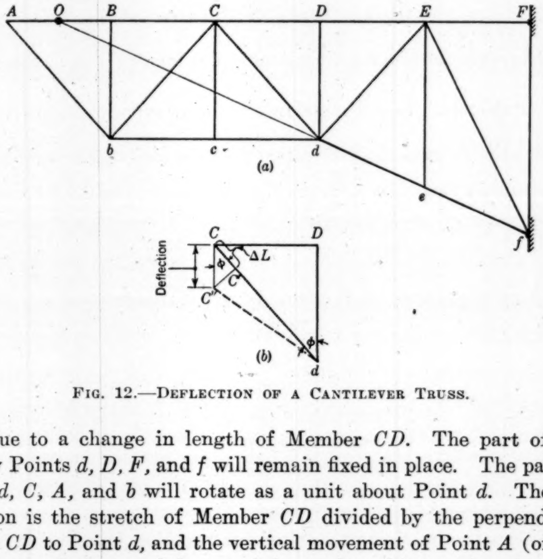

FIG. 12.—DEFLECTION OF A CANTILEVER TRUSS.

For a change in length of Member *Cd,* the rectangle, *dDCc,* is distorted into a parallelogram, and the remainder of the truss is displaced without rotating. The movement is equal to the change of length of Member *Cd* (ΔL)

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[2] *Engineering News-Record,* November 1, 1934, p. 565; and *Transactions,* Am. Soc. C. E., Vol. 102 (1937), p. 206.

times $\frac{\overline{d\,C}}{\overline{d\,D}}$ as shown in Fig. 12(*b*), because the triangle, *CC′ C″*, is similar to Triangle *dDC*, and $\frac{\overline{C\,C''}}{\overline{C\,C'}} = \frac{\overline{d\,C}}{\overline{d\,D}}$.

The actual systematic computation of the deflection of a simple truss is shown in Fig. 13 with Table 4. In Fig. 13, the upper values denote changes of length, ΔL; and the lower values denote the factor, $\frac{40}{50}\,\Delta\,L$, for chords and $\frac{64}{50}\,\Delta\,L$ for diagonals. In a simple truss of this type no algebraic signs are required since the deformations of all members contribute to downward deflection. An alternative method of solving this problem, based on single, instead of double, integration has been presented elsewhere[3] by the writer.

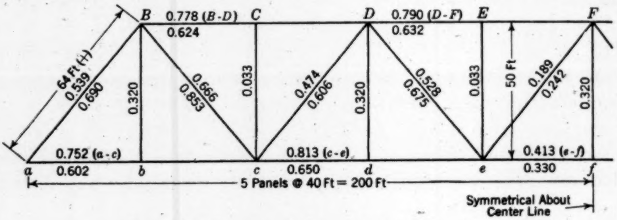
B
0.778 (B-D)
0.624
C
D
0.790 (D-F)
0.632
E
F
64 Ft (±)
0.539
0.690
0.320
0.666
0.853
0.033
0.474
0.606
0.320
0.528
0.675
0.033
50 Ft
0.189
0.242
0.320
0.752 (a-c)
0.602
0.813 (c-e)
0.650
0.413 (e-f)
0.330
a
b
c
d
e
f
5 Panels @ 40 Ft = 200 Ft
Symmetrical About Center Line

FIG. 13.—DEFLECTION OF A SIMPLE TRUSS.

TABLE 4.—DEFLECTIONS OF A SIMPLE TRUSS

Panel Points (see Fig. 13)	CHORDS		Diagonals (3)	Main panel points Column (2) + Column (3) (4)	Verticals (5)	Secondary panel points (6)
	Rotation (1)	Slope (2)				
a		2.838	0.690	0		
b	0.602	2.236	0.853	(*B*) 3.528	0.320	(*b*) 3.848
c	0.624	1.612	0.606	(*c*) 6.617	0.033	(*C*) 6.650
d	0.650	0.962	0.675	(*D*) 8.835	0.320	(*d*) 9.155
e	0.632	0.330	0.242	(*e*) 10.472	0.033	(*E*) 10.505
f	0.330			(*F*) 11.044	0.320	(*f*) 11.364

SCOPE OF THE GEOMETRICAL METHOD

Only a few specific applications of the theory of determining deflections by geometry have been presented herein. However, what the writer particularly wishes to emphasize is that a familiarity with the basic rules, comparable with the average engineer's familiarity with forces and moments, will make it possible to improvise methods for solving problems which formerly appeared to be quite difficult.

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[3] *Engineering News-Record,* June 7, 1934, p. 746; and *Transactions,* Am. Soc. C. E., Vol. 102 (1937), p. 202.

For any one who is interested in acquiring proficiency in this method, a few suggested exercises are as follows:

(1) In a three-hinged arch, write the formula for the deflection at some point on one side due to an angle at some point on the other side. This involves one more operation than a simple beam, just as the simple beam requires one more operation than a cantilever.

(2) Devise a method for calculating the dead load moments in a two-hinged arch such as in Fig. 8. Then, find what the dead load moments will be if the base is fixed, not by beginning over again, but simply by finding the rotation of the base of the two-hinged arch and then eliminating it, using data on hand as much as possible.

DISCUSSION

A. J. McGaw,[4] Jun. Am. Soc. C. E. (by letter).—The author has focused much needed attention upon the principles of geometry as they affect structural action. Although equally accurate results may be obtained by the methods embracing the law of conservation of energy and the principles of least work, and although these methods are in many instances desirable as analytical procedures, they do side-track the geometry involved, which is so essential to complete understanding of the subject of indeterminacy. As a teaching tool, at least, the geometric method has much in its favor. The physical conception of structural deformation and the part played by geometry must, of necessity, go hand in hand. The formula-minded analyst, in many cases, fails to see his structure act. It is probably safe to state that analyses made under such conditions are dangerously likely to contain error. There is little to fear from the designer who can sketch the structure at hand in its deformed shape. It follows that an understanding of the principles of geometry as they affect structural action is highly desirable.

Along this same line of thought, it seems that the development of fundamental theory, through the utilization of geometric principles, may be extended advantageously to include Maxwell's law of reciprocal deflections. Surely, any haziness in one's mind concerning the truth and logic of this fundamental and much used tool will prevent, to a considerable extent, real mastery of the subject of indeterminacy. With this in mind, the writer has adapted the reasoning presented by the author to evolve a proof of Maxwell's law.

As in the paper, the beam, *AB*, in Fig. 14(*a*) has been assumed rigid, except at Point *G*. Due to the unit load at Point *D*, the moment, M_G, at Point *G*, has produced a rotation at that point through the angle, α_D, equal to:

$$M_G = \frac{x\,a}{L} \qquad \text{.........................(17)}$$

and since α_D is proportional to M_G,

$$M_G\,K = \alpha_D = \frac{x\,a\,K}{L} \qquad \text{....................(18)}$$

Furthermore, the ordinate, *BF*, in Fig. 14, equals,

$$\alpha_D\,a = \frac{x\,a^2 K}{L} \qquad \text{....................(19)}$$

and the deflection at Point *E* equals:

$$\delta_E = \frac{\overline{BF}\,x_E}{L} = \frac{x\,a^2\,x_E\,K}{L^2} \qquad \text{..................(20)}$$

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[4] Instructor of Civ. Eng., Coll of Eng., Univ. of Wyoming, Laramie, Wyo.

Similarly, the deflection at Point D (Fig. 14(b)), with a unit load at Point E, may be obtained as follows:

$$\alpha_E = M_G\,K = \frac{x_E\,a\,K}{L} \qquad \text{(21)}$$

and, the ordinate, BF, is equal to:

$$\alpha_E\,a = \frac{x_E\,a^2 K}{L} \qquad \text{(22)}$$

and,

$$\delta_D = \frac{\overline{BF}\;x}{L} = \frac{x\,a^2\,K\,x_E}{L^2} \qquad \text{(23)}$$

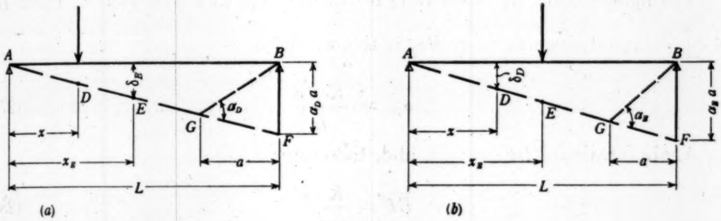
A
B
D
E
G
F
δ_E
α_D
$\alpha_D\,a$
x
x_E
a
L
(a)
A
B
D
E
G
F
δ_D
α_E
$\alpha_E\,a$
x
x_E
a
L
(b)

FIG. 14

Comparing Equations (20) and (23), $\delta_E = \delta_D$, which shows that the deflection at E due to a unit load at Point D equals the deflection at Point D due to a unit load at Point E. The flexure effects for all points along the beam must be summed up to obtain the total deflections. This is not necessary to the proof, however, since, obviously, the sums of equals must be equal.

That the vertical deflection at Point D caused by a unit moment at Point A (Fig. 15(a)) is equal to the rotation at Point A caused by a unit load at Point D (Fig. 15(b)) may be shown in a similar manner. A unit

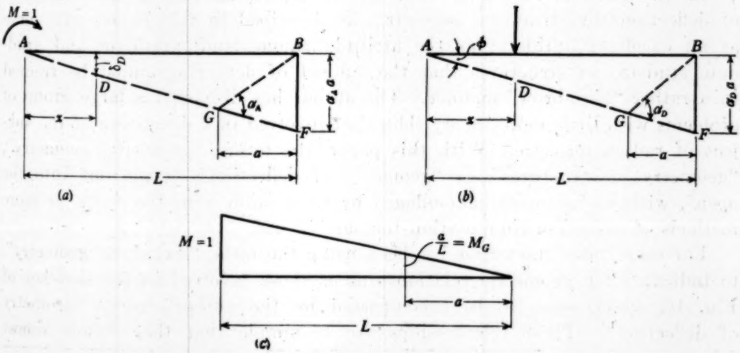
M = 1
A
B
D
G
F
δ_D
α_A
$\alpha_A\,a$
x
a
L
(a)
A
ϕ
B
D
G
F
α_D
$\alpha_D\,a$
x
a
L
(b)
M = 1
$\frac{a}{L} = M_G$
a
L
(c)

FIG. 15

moment is applied at Point *A*, Fig. 15(*a*), and the corresponding moment diagram for the beam is shown in Fig. 15(*c*). Then:

$$\alpha_A = \frac{K\,a}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

and since Ordinate $BF = \alpha_A\,a$,

$$\overline{BF} = \frac{K\,a^2}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The vertical deflection at Point *D*, Fig. 15(*a*), is then:

$$\delta_D = \frac{\overline{BF}\,x}{L} = \frac{K\,a^2\,x}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

The moment, M_G, at Point *G* (Fig. 15(*b*)), due to a unit load at Point *D*, is $\dfrac{x\,a}{L}$, and since $\alpha_D = K\,M_G$, it follows that:

$$\alpha_D = \frac{K\,x\,a}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Again, Ordinate $BF = \alpha_D\,a$, and, therefore:

$$\overline{BF} = \frac{K\,x\,a^2}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

and Angle ϕ equals $\dfrac{\overline{BF}}{L}$, so that,

$$\phi = \frac{K\,x\,a^2}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Comparing Equations (26) and (29), $\phi = \delta_D$, which shows that a rotation at Point *A* caused by a unit load at Point *D* equals the deflection at Point *D* due to a unit rotation at Point *A*.

L. E. Grinter,[5] M. Am. Soc. C. E. (by letter).—The determination of deflections by structural geometry, as described in this paper, will serve as an excellent antidote for the attitude among undergraduate and graduate students of structures that the subject of deflections must be treated in a rather "high-brow" manner. The author has discussed a large group of problems with little more theory than that involved in a discussion of the subject of radian measure. With this paper, the terms, "structural geometry", "geometry of structures", or "geometry of deflection", "come out into the open", without becoming camouflaged by association with the dozen or more methods of deflection computation that are in use.

For some years the writer has been using the term, "structural geometry", to indicate such geometric relationships as those involved in the sketches of Fig. 16, which seem to be encompassed by the author's term, "geometry of deflection." These relationships are so simple that they follow almost without proof and for this reason they form a proper introduction to the study of deflections. Furthermore, they involve no abstruse conceptions, such as the use of a "dummy" load, or the imaginary application of an $\frac{M}{EI}$ - loading. For the undergraduate student, or the sub-professional engineering employee, these geometric relationships are the proper starting point in a study of deflections. This, it seems, is also the author's belief. Textbooks on the strength of materials should emphasize such applications of structural geometry more decidedly than the common procedure of double integration. That authors of standard textbooks have always reversed this emphasis is one reason why undergraduate students, and even engineering graduates, often face the subject or the practice of indeterminate structural analysis and design with a complete lack of confidence.

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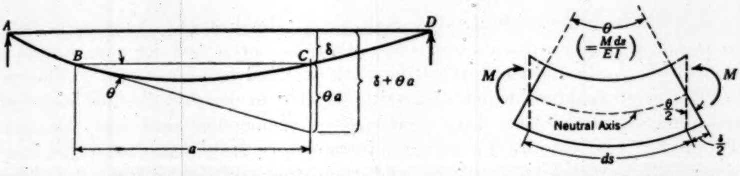
[5] Director of Civ. Eng., and Dean, Graduate Div., Armour Inst. of Technology, Chicago, Ill.



(a) RELATIVE DEFLECTIONS (b) INTERNAL DISTORTION

FIG. 16.—STRUCTURAL GEOMETRY.

On the other hand, there is much to be said for the methods of virtual work, area moments, the conjugate beam, elastic weights for trusses, and the Williot-Mohr diagram. Each of these methods, in certain of its forms, offers a semi-automatic procedure of deflection computation, a very desirable feature where the process is to be repeated over and over as in the analysis of certain indeterminate structures. Furthermore, the method of virtual work, for example, can be used not only to find a deflection caused by flexure, but one caused by shear, torsion, or direct stress. It is also quite as applicable to the determination of rotations caused by any internal distortion. The author's conceptions of "geometry" are scarcely so extensive. For graduate study, then, assuming that the conceptions of structural geometry have been fixed in the student's mind in his undergraduate courses, the writer feels that the student should become familiar with each of the aforementioned methods. He should also learn that Castigliano's theorem is simply a reversed statement of virtual work (that is, a reversed order of the differentiation and integration), and he should be made to realize that such terms as elastic weights, moment weights, angle loads, etc., are all simply names for the same conception of an $\frac{M}{EI}$ - loading revised for application to trusses.

In other words, the most useful tool of deflection computation for the beginner is that method which is most nearly physical or visual; but the method that will be most useful to the experienced engineer may well be that which requires the least visualization and, therefore, tends to become most nearly automatic in its application. Here, therefore, is a conflict that will always appear. It explains why such intangible methods as virtual work and the conjugate beam are so widely useful and so widely used. Such methods are difficult to learn but easy to apply. When properly presented, the Williot-Mohr diagram can be shown to be both visual and semi-automatic. The author has presented the geometrical method as a very simple visual procedure. To interest the practical engineer, he should also emphasize its possibilities for rapid semi-automatic computations, such as those presented in Table 1.

Ralph W. Stewart,[6] M. Am. Soc. C. E. (by letter).—Adhering closely to sound conventional methods, the author has presented an able introduction to his theme and has carried the work outlined to a successful conclusion.

The only weakness which the writer is able to discover is that a part of the analysis could have been made more geometrical and less laborious. To illustrate this point the author's introductory statements regarding rotations will be slightly expanded and then the stiffness factors and other constants for the beam of variable and unsymmetrical section, represented by Fig. 10, will be developed in a more purely geometrical manner, which will substantially reduce the arithmetical work.

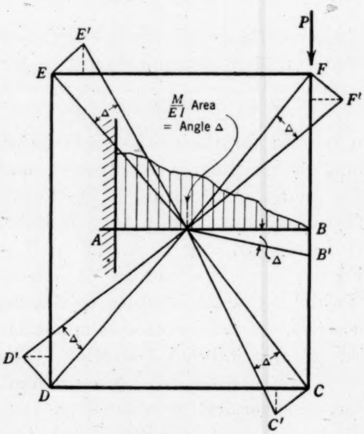


Fig. 17

Fig. 17 represents the application of the author's rotation Theorems 1 and 2, illustrated by Fig. 3, with reference to a specific cantilever beam and some connecting members, rather than with reference to an abstract point, *O*, which is not tied in to any structure. The rectangular frame, *CDEF*, is free except for its integral connection with the beam at Point *B*. It may be conceived that Sides *EF* and *DC* pass through slots in the wall which fixes the beam at Point *A*.

If now, a load, *P*, is applied either over Point *B*, or at any point on the beam, a cantilever moment will be created. Regardless as to whether the beam section and its modulus of elasticity are constant or variable, the computer can lay out the $\frac{M}{EI}$-diagram due to this cantilever moment, if the make-up of the beam is known. The point where the center of gravity of this $\frac{M}{EI}$-diagram is projected upon the unsprung axis of the beam will now be the center of rotation about which the deflection of Point *B* and all points of the tributary frame occur. This center of rotation is a fixed point independent of the magnitude of the load. Its location, for any beam, can be determined by computing the $\frac{M}{EI}$-diagram for a trial load or a trial moment when the real load or the real moment is known as to position, but not as to amount. The rotation, expressed as an angle, is equal numerically to the total area of the $\frac{M}{EI}$-diagram.

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[6] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

In Fig. 17 the rotation is represented by the angle, Δ, and the movement of each lettered point on the frame is indicated by the primed letters. The formulas for the movement of each point are correctly stated by the author. A beam having two end moments will have two such centers of rotation, which, if the beam has end slopes, will be projected on the end slope lines instead of the unsprung axis.

A paper explaining the geometrical methods of analyzing the flexure of continuous structures by means of a series of lines connecting these centers of rotation, has been published.[7]

TABLE 5.—COMPUTATION OF THE CONSTANT FOR THE BEAM IN FIG. 10

Point * (1)	Moment, M (2)	Values of $\frac{M}{t^3}$ (3)	Moment of Column (3) (4)	Point * (1)	Moment, M (2)	Values of $\frac{M}{t^3}$ (3)	Moment of Column (3) (4)	Point * (1)	Moment, M (2)	Values of $\frac{M}{t^3}$ (3)	Moment of Column (3) (4)
A	13	0.124	0.788†	5	8	2.856	22.848	10	3	0.302	0.906
1	12	0.228	2.736	6	7	3.668	25.676	11	2	0.095	0.190
2	11	0.460	5.060	7	6	2.928	17.568	12	1	0.022	0.022
3	10	0.903	9.030	8	5	1.900	9.500	*B*	0	0	0
4	9	1.678	15.102	9	4	0.844	3.376	Summation....		15.945	112.802

* See Fig. 10. † $\frac{0.124}{2} \times 12\frac{2}{3} = 0.788$.

In an analysis such as that of the variable beam (Fig. 10), it is better to find these centers of rotation and the ratio between an end moment and its corresponding angle of rotation and subsequently to treat the flexure in the span due to each end moment as a single item rather than as a series of small items each resulting from the curvature of a separate increment, *dl*. This permits the mechanics of the problem to be transformed into a purer form of geometry and also reduces the number of arithmetical operations.

To compute the constant for the beam in Fig. 10 apply a trial end moment of 13 at End *A* with End *B* assumed to be hinged. In Table 5, Columns (1) represent the moment at each division point of the beam and Columns (2), the $\frac{M}{t^3}$-value at each division point. The $\frac{M}{t^3}$-area, taking each division length as unity instead of 3.52, is the summation of

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[7] "The Analysis of Continuous Structures by Traversing the Elastic Curves", by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

Columns (2), except that before adding, the top and bottom items are divided by 2 as they apply to only one-half as much length as the others.

To get the position of the center of gravity of the $\frac{M}{t^3}$ - area the moment of each elementary area about Point *B* is computed as shown in Columns (3) still taking the *dl* lengths as unity instead of their real lengths.

The relative distance to the center of gravity of the $\frac{M}{t^3}$ - area is now the summation, Columns (3) divided by the summation of Columns (2); thus (see Table 5): $\frac{112.802}{15.945} = 7.06$. If this actual distance is desired, multiply the relative distance by 3.52, the division length. A similar computation is made for a trial moment of 13 at Point *B* with Point *A* hinged, or by utilizing Maxwell's theorem of reciprocal deflections, the computation for the center of rotation for a moment at *B* can be made with one less column.

It is found that a trial-end moment of 13 at Point *A* gives an $\frac{M}{t^3}$ - area of 15.94 with its center of gravity 7.08 division lengths from Point *B*, and a trial-end moment of 13 at Point *B* gives an $\frac{M}{t^3}$ - area of 16.27 with its center of gravity 7.19 division lengths from Point *A*.

All constants for the beam can now be taken by simple ratio and proportion from the geometry of the traverse diagrams, Fig. 18. These diagrams are similar to the author's Fig. 10 (*a*), except that they are completed to show the centers of rotation which correspond to Point *O* in Fig. 3, or Δ in Fig. 17. Fig. 18 shows a rotation of unity at Point *A*. The other angles are evaluated from the fact that in the traverse triangle the angles are proportional to the opposite sides. The moments appurtenant to the two Δ-angles bear the ratio to the trial moment of 13 that their Δ-angles bear to the $\frac{M}{t^3}$ - area for the trial moment of 13. It is now seen at once that a unit rotation at Point *A* with Point *B* fixed gives

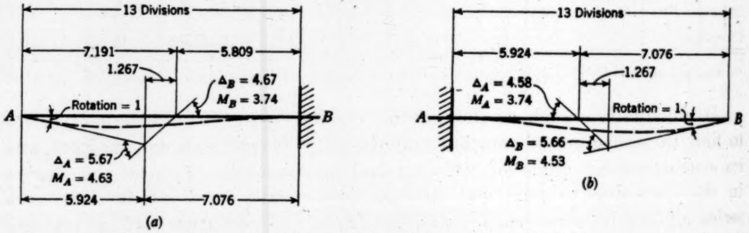

FIG. 18

$M_A = 3.74$, which is the stiffness factor as defined by the author; and a unit rotation at Point B with Point A fixed gives $M_B = 4.53$. These agree with the stiffness factors computed by the author (see, following Table 3). The carry-over factors are also taken off these traverse diagrams by dividing the moment at the fixed end by that at the rotated end.

The saving in computation as compared with Table 2 is more than appears at first glance, because when a slide-rule is set for an $\frac{M}{t^3}$ - value in Table 5[*] it is only necessary to move the runner to the square scale, to extend the computation to Columns (3), since the selection of 13 for the trial moment makes $\frac{M\,x}{t^3} = \frac{M^2}{t^3}$. Taking stiffness and carry-over factors from these traverse diagrams is recommended as being superior to the algebraic solution by means of the Equations (7) to (16), inclusive.

At the end of the paper the author suggests some exercises for any one wishing to acquire proficiency in the geometric treatment of deflections. The last exercise suggested is to devise a method of computing the dead load moments in a two-hinged arch and then find what the dead load moments will be if the structure is transformed into a fixed arch by eliminating the rotation at the base of the two-hinged arch. This problem can be made short and its solution conveniently given in full by selecting, for the arch, a portal frame with level deck and vertical abutments, with uniform moment of inertia throughout, and height = length = L.

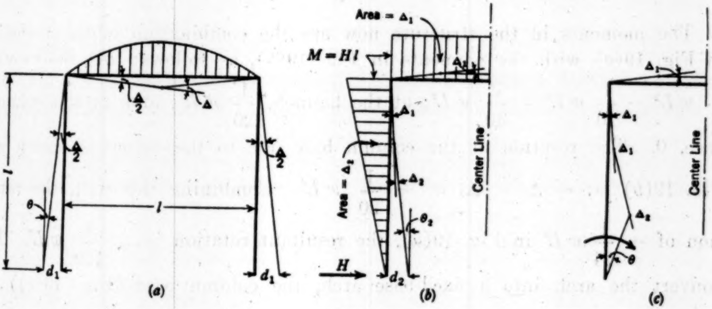

FIG. 19.

In the following solution the Δ-angle points represent centers of geometrical rotation which result from appurtenant sections of the $\frac{M}{EI}$ - diagram. Since E and I (or t^3) are constant throughout, they may be omitted from the computations. For Condition No. 1 (Fig. 19 (*a*)), the arch is taken as a simple structure with no restraint, subjected to a uniform load, w, per unit of length.

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[*] A complete explanation of the technique of Table 5 may be found in *Bulletin No. 66*, Ohio State Univ., Columbus, Ohio.

The moments are: At the crown, $\frac{1}{8}\ wL^2$; at the haunch, 0; and at the column base, 0. The area of the moment diagram equals,

$$\Sigma M\ dL = \frac{1}{12}\ w\,L^3 = \Delta \ \ldots\ldots\ldots\ldots\ldots\ldots \ (30)$$

The spread at the bottom of the columns equals,

$$2\ d_1 = \Sigma M\ dL\ y = \Delta\ L = \frac{1}{12}\ w\,L^4 \ \ldots\ldots\ldots\ldots\ldots \ (31)$$

and,

$$\theta = \frac{\Delta}{2} = \frac{1}{24}\ w\,L^3 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (32)$$

For Condition No. 2 (Fig. 19(*b*)), the structure is converted into a two-hinged arch by applying a thrust, H, sufficient to bring the column bases to their original positions: $d = \frac{1}{24}\,w\,L^4 = \Sigma\ M\ dL\ y = \Delta_1\ L + \Delta_2 \left(\frac{2}{3}\ L\right) = \frac{H\,L^3}{3} + \frac{H\,L^3}{3} = \frac{5}{6}\ H\,L^3$. Therefore,

$$H = \frac{1}{20}\ w\ L \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (33)$$

The moments in the structure now are the combination of the moment in Fig. 19(*a*) with the moments in Fig. 19(*b*), as follows: At the crown, $\frac{1}{8}\ w\,L^2 - \frac{1}{20}\ w\,L^2 = \frac{3}{40}\ w\,L^2$; at the haunch, $\frac{1}{20}\ w\,L^2$; and, at the column base, 0. The rotation at the column base due to the moments shown on Fig. 19(*b*) is: $-\ \Delta_1 - \Delta_2 = -\ \frac{1}{20}\ w\,L^3$. Combining this with the rotation of $+\ \frac{1}{24}\ w\,L^3$ in Fig. 19(*a*), the resultant rotation is $-\ \frac{1}{120}\,w\,L^3$. To convert the arch into a fixed-base arch, the column base (Fig. 19(*c*)) is made vertical by rotating it through the angle, $\theta = \frac{1}{120}\ w\,L^3$, in a clockwise direction and the resulting moments are determined and combined with those of Fig. 19(*a*) and Fig. 19(*b*). To solve Fig. 19(*c*), a traverse computation is made from the top to the bottom of the column, calling clockwise rotations positive: $-\ 3\Delta_1 - 2\Delta_1 + \Delta_2 = 0$. Therefore, $\Delta_2 = 5\Delta_1$. Summing the angles: $3\Delta_1 = \frac{1}{120}\ w\,L^3$; M (at the haunch) $= \frac{2\ \Delta_1}{L} = \frac{1}{180}\ w\,L^2$; M_2 (at the column base) $= 5\,M_1 = \frac{1}{36}\ w\,L^2$; and the crown moment $=$ the haunch moment $= \frac{1}{180}\ w\,L^2$.

Combining these quantities with the moments in Fig. 19(*a*) and Fig. 19(*b*), the respective moments for the fixed arch are as follows: Crown moment, $\frac{1}{8}\,w\,L^2 - \frac{1}{20}\,w\,L^2 - \frac{1}{180}\,w\,L^2 = \frac{5}{72}\,w\,L^2$; haunch moment, $\frac{1}{20}\,w\,L^2 + \frac{1}{180}\,w\,L^2 = \frac{1}{18}\,w\,L^2$; and, moment at the column base, $\frac{5}{180}\,w\,L^2$.

The foregoing analysis carries the structure progressively, by a geometric treatment of its deflections, through the three conditions of free base, hinged base, and fixed base.

Except for the parts which fail to utilize fully the geometrical properties of flexure (notably the part relating to the constants for a variable section beam), the paper merits attention.

WILLIAM BERTWELL,[9] ASSOC. M. AM. SOC. C. E. (by letter).—Being simple and explicit, this valuable paper is easy to follow. An actual trial of the geometric method should convince most engineers that the usual textbook treatment of many problems (particularly arch analysis) is cumbersome and difficult to remember. The author is to be congratulated for his careful development of the fundamental conceptions as well as for his choice of illustrations.

The virtues of the geometric attack are especially apparent in dealing with trusses, as Mr. Hall shows. Although it is equally convenient to compute the deflections of flexural sections by geometry, in this case the geometric method is also, essentially, the method of elastic weights, since the elastic weight of an element of an elastic rib may be defined as the angular distortion caused by the application of a unit moment couple to that element. Unfortunately, however, the method of elastic weights has come to be completely identified with the conjugate beam convention in the minds of many engineers, and, hence, it may be that the adoption of such a term as "geometric method" is desirable.

For some time the writer has been using the double summation procedure for the calculation of influence lines for the redundants of a fixed arch, and has found this process to be simpler and shorter than any other, unless one uses a definite mathematical arch such as that developed by Charles S. Whitney, M. Am. Soc. C. E.[10] For designers who take rib-shortening into account in arch design, it might be explained that this effect may be included quite simply in the geometric method. For example, in Table 1, the elastic shortenings of the elements are to be computed and the components of these shortenings in the directions of X-X and Y-Y recorded. Then a single summation of the vertical components will provide the corrections to the vertical deflections at the division points, and the total of the horizontal components gives the correction to the horizontal movement of the hinge point.

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[9] Regional Office, U. S. Bureau of Public Roads, San Francisco, Calif.

[10] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 931.

The author's analysis of the unsymmetrical beam of Fig. 10(*c*) is brief, but the results may be reached with equal facility by adhering to the general method—the mathematical equivalent of the mechanical method. In this case, the analysis may be made by removing one support, fixing the other, and then applying a unit moment and a vertical force at the free end such that the total deflection of that end is zero. The problem is easily visualized in this manner, and one needs no formulas for the evaluation of the constants of the moment-distribution method; it is only necessary to remember their definitions. This process is exemplified in Table 6 by an analysis for one end of the unsymmetrical beam used by the author. (In Table 6, in addition to the notation of the paper, $\Delta = \frac{3.52}{t^3}$; and y = deflection.)

The author's division of the beam into twelve full sections and two half-sections draws attention to a common inaccuracy which can be easily eliminated. Briefly, the correct influence values, found by any process of finite summation, occur only at the divisions between the finite elements—not at their centers. The writer realizes that it is unnecessary to secure extremely accurate results because of the designer's many assumptions, such as the magnitude of the live load and impact forces, the degree of elasticity of reinforced concrete members, and the effect of reinforcing steel in such members. Nevertheless, it is apparent that the results sought are the angular distortion of the element and the deflection of one end from the tangent at the other. The first is the product of the moment and the elastic weight, and the second is the product of the angular distortion and the distance from the instantaneous center of rotation to the face of the element. Hence, one can calculate true deflections only at the division points between the elements. In the graphical solution this means that the true influence curve is tangent to the equilibrium polygon at the division points between the beam elements. Column (11) of Table 6 takes account of this, and it is apparent that the extra labor is slight.

In Column (13), Table 6, are values obtained from an analytical solution of the same beam, taking into account all refinements except shear distortion and the curvature of the top and bottom of the beam between division points. If these values are considered comparatively exact, the error of the writer's approximate solution is only about one-half as great as that which results from using the points at the centers of the elements. The error of the author's solution is small, however, and this may be attributed to his having used rather short divisions of the beam.

It may be well to explain the refinements used in obtaining the "exact" values in Column (13), Table 6. Consider the fundamental elastic properties of a trapezoidal element. It may be shown that the center of gravity of infinitesimal elastic weights is at the intersection of the diagonals, rather than at the mid-point between the parallel bases. Furthermore, the

TABLE 6—INFLUENCE LINE FOR FIXED-BEAM MOMENTS AT END A

Division point	Depth of beam, t	$\frac{3.52}{t^3} = \Delta$	UNIT MOMENT AT A		UNIT VERTICAL FORCE AT A				(3.52 V = 0.138907)	$\left(M = 1; \text{and } V = \frac{0.138907}{3.52}\right)$		Influence ordinates $= \frac{y}{0.7388}$	True values
			$\Sigma \Delta = \theta$	$\Sigma \theta = \frac{y}{3.52}$	$\frac{x}{3.52}$	$\frac{x \Delta}{3.52}$	$\frac{\Sigma (x \Delta)}{3.52} = \frac{\theta}{3.52}$	$\Sigma \frac{\theta}{3.52} = \frac{y}{3.52^2}$	$\frac{y}{3.52}$	$\frac{y}{3.52}$	$\frac{y}{3.52}$ = mean values from Column (10)		
		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
A				54.8603				394.9423	54.8603	0		0	0
	4.23	0.0465	8.3593	50.6807	0.5	0.0232	54.8602	367.5122	51.0500	0.3693			
1											0.7171	3.416	3.425
	3.315	0.0965	8.3128	42.3679	1.5	0.1447	54.8370	312.6752	43.4328	1.0649			
2											1.3745	6.548	6.572
	2.555	0.2110	8.2163	34.1516	2.5	0.5275	54.6923	257.9829	35.8356	1.6840			
3											1.9248	9.170	9.214
	1.99	0.4467	8.0053	26.1463	3.5	1.5634	54.1648	203.8181	28.3118	2.1655			
4											2.2915	10.917	10.988
	1.58	0.8924	7.5586	18.5877	4.5	4.0158	52.6014	151.2167	21.0051	2.4174			
5											2.3761	11.320	11.417
	1.325	1.5132	6.6662	11.9215	5.5	8.3226	48.5856	102.6311	14.2562	2.3347			
6											2.1148	10.075	10.156
	1.255	1.7808	5.1530	6.7685	6.5	11.5752	40.2630	62.3681	8.6634	1.8949			
7											1.5885	7.568	7.581
	1.33	1.4962	3.3722	3.3963	7.5	11.2215	28.6878	33.6803	4.6784	1.2821			
8											1.0070	4.798	4.762
	1.535	0.9732	1.8760	1.5203	8.5	8.2722	17.4663	16.2140	2.2522	0.7319			
9											0.5448	2.596	2.552
	1.915	0.5012	0.9028	0.6175	9.5	4.7614	9.1941	7.0199	0.9751	0.3576			
10											0.2506	1.194	1.163
	2.455	0.2379	0.4016	0.2159	10.5	2.4980	4.4327	2.5872	0.3594	0.1435			
11											0.0910	0.434	0.416
	3.16	0.1115	0.1637	0.0522	11.5	1.2822	1.9347	0.6525	0.0906	0.0384			
12											0.0192	0.091	0.084
	4.07	0.0522	0.0522	0	12.5	0.6525	0.6525	0	0	0			
B												0	0
$3.52\ V = \frac{54.8603}{394.9423} = 0.138907$; or V =												0.039462	0.039482
Total angular rotation at A = 8.3593 − 0.138907 × 54.8602 =												0.7388	0.7458
K_A = reciprocal of total angular rotation at A =												1.3535	1.3408
Carry-over factor = 1 − 13 × 0.138907 =												0.80579	0.80670
K_{AB} = 0.80579 × 1.3535 =												1.0906	1.0817

true elastic weight of the element is measured by the sum of the two parallel bases divided by the square of their product, rather than by the reciprocal of the cube of the depth at the mid-point. For example, if the larger base is 1.25 times the smaller, and the computations are made in the ordinary manner, the error in the location of the elastic center of the element is 5.55% of the length of the element, and the error in the value of the elastic weight is 2.46 per cent. The ratio of 1.25 between the bases is not uncommon, and is slightly exceeded in the author's example. The problem is still further complicated because the instantaneous center of rotation developed in each element by the vertical force at the end is not at the center of gravity of the infinitesimal elastic weights, but at the center of gravity of the infinitesimal angle changes caused by the force. This shift in the center of rotation, when a shear causes the moment in the element (that is, when the moment is not constant throughout the element), is not usually large except for the one or two elements adjacent to the point where the vertical force is applied. In the writer's solution of the author's example, this shift is 0.51 ft for Element *A*-1 and 0.18 ft for Element 1-2. It is a simple matter to use the correct elastic weights, but the variations in their location would indicate that a graphical solution, in which the equilibrium polygon corresponds to the elastic line of the beam, would provide greater accuracy with less labor than the analytical solution. However, an analytical solution taking all these refinements into account is not too lengthy if a computing machine is available. In fact, in this instance only two more operations would have been necessary to include the effect of the shearing distortion.

David B. Hall,[11] Assoc. M. Am. Soc. C. E. (by letter).—In a method of solving problems for which other solutions already exist, it is natural to inquire into the relative advantages of the proposed method. The writer is naturally pleased to observe that those who have discussed the paper have expressed emphatic, favorable views on this question, finding that the proposed method makes the subject of deflections more clearly and easily understood. Mr. McGaw and Professor Grinter, who are both teachers, have especially stressed its value for beginners on this account.

There appears to be some skepticism, however, as to the possibility of this geometrical method taking its place beside some of the older methods for ordinary routine computations, and also for solving difficult problems. It is the writer's feeling that such advantages as the older methods may have are not inherent, but must represent developments with which the newer method has not "caught up." After all, any substitute method depends for its existence upon some mathematical relationship, and however varied its physical interpretations may be, its mathematical operations must be substantially the same. One tool that Professor Grinter mentions, the Williot diagram, in reality, should be classed as one of the most legitimate of the methods of the geometry of deflections.

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[11] Detailer, Ash-Howard-Needles & Tammen, New York, N. Y.

To make the geometrical method comparable with the older methods as a practical working tool, the greatest need is evidently an accumulation of working details. Several valuable contributions are to be found in the discussions.

Mr. McGaw has begun this filling in by translating the law of reciprocal deflections into geometric terms; and in doing so he has probably made this law more intelligible to many readers. Perhaps it should be noted that one additional step is necessary to develop, fully, a formal proof along this line, because the law of reciprocal deflections applies to any structure in which the materials follow Hooke's law, whereas the foregoing proof is based on relations that exist only in a statically determinate structure. The additional step is merely to trace the effect of adding a redundant support or member to a simple structure.

Specifically, consider three points, *A*, *B*, and *C*, on a simple beam. A redundant support is to be added at Point *B*. When a load is applied at Point *A*, the beam will deflect at Point *B*. If a support is added, the reaction will be proportional to this deflection. Finally, this reaction will produce a deflection at Point *C*. Since, in the simple beam, the reciprocal relation is known to exist between Points *A* and *B*, *B* and *C*, and *A* and *C*, all the factors, multiplied together to find the deflection at Point *C*, due to a load at Point *A*, appear in finding the deflection at Point *A* due to a load at Point *C*. Finally, if the principle applies for a structure with n redundants, the foregoing proof shows it to apply for $(n+1)$ redundants, thus completing the proof by mathematical induction.

A useful geometrical conception, essential to any comprehensive treatment of the subject, is Mr. Stewart's method of traversing the elastic curve. Although it may take a little time to grasp this method fully, the speed and simplicity with which it may be applied to a wide variety of problems make it well worth mastering.

In presenting two alternative solutions for the unsymmetrical beam of Fig. 10(*c*), Mr. Stewart and Mr. Bertwell have illustrated, effectively, the readiness with which solutions may be improvised by these methods—in contrast, for instance, to the necessity for following, faithfully and blindly, some of the older standardized forms for arch computations.

In the second half of his discussion Mr. Bertwell has made a number of observations pertaining to an intriguing branch of mathematics—approximate integration. In its elementary form this is the problem of constructing a polygon to take the place of a curve (as in Fig. 7, for example), and determining the location of the polygon with respect to the curve. In Fig. 7, this polygon consists of a series of tangents to the curve. In such a case Mr. Bertwell's proposal to use interpolated values, which are at the points of tangency of the polygon and curve, is sound. An alternative refinement, used at one time by the writer, was to make corrections at the vertices of the polygon in the same manner that the middle offset of a vertical curve is computed. This correction is $\frac{1}{8}\ \alpha\ dx$. Thus, in Table 1, the correction at Point 10 would be $\frac{1}{8} \times 2.96 \times 4.4 = 1.6$; and the correct deflection would be, $420.6 - 1.6 = 419.0$.

The arrangement in Fig. 10(*c*) and in Tables 2 and 3, however, presents the following entirely different approach, which Mr. Bertwell failed to recognize. If a curve is drawn and marked off at equal intervals, and a polygon is then drawn consisting of a series of chords joining these points, then the angle at any vertex of the polygon will be the product of the curvature in its vicinity and the length of one interval; and the angle between the last chord and the end tangent will be one-half the product of the curvature there and the length of an interval. In this manner the desired results were obtained directly with no extra operations.

It is believed that further refinements, when they are warranted, may be developed more easily in the direction of integration by Simpson's rule, than along the lines described by Mr. Bertwell.

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GRAPHICAL DISTRIBUTION OF VERTICAL PRESSURE BENEATH FOUNDATIONS

By DONALD M. BURMISTER,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. WILLIAM P. KIMBALL, I. M. NELIDOV, GEORGE PAASWELL, JACOB FELD, NATHAN M. NEWMARK, A. E. CUMMINGS, D. P. KRYNINE, A. A. EREMIN, AND DONALD M. BURMISTER.

SYNOPSIS

The most important part of the analysis of foundations deals with the settlement of structures due to the consolidation of compressible strata of clay and fine saturated silt located at some depth beneath the structure. This involves the practical application of the theory of elasticity in the determination of the state of stress in the underground. In the present state of knowledge the Boussinesq equation for the pressure at a point within the soil mass, due to a point load concentrated at the surface, is the logical basis for this determination. The practical side of the solution of such problems requires a simplification of procedure, in order to make the valuable method of analysis more available. A method is suggested which makes use of the ideas of the influence line and of graphical integration.

A study of the discussions of foundation problems appearing in recent years makes it seem reasonable to divide them into two general classes. The first class deals with near-surface phenomena, involving the behavior of the soil in the "disturbed zone" close to the individual footings. Bearing capacity is developed by an almost immediate compacting and settling of the soil to a stable condition with some lateral displacement, provided that a certain quite definite loading is not exceeded. The soil may have any moisture content from the dry state to saturation. The second class deals especially with the settlement, over a long period of time, of a structure underlaid by highly compressible saturated layers. The settlement due to the surface phenomena may be negligible compared with the latter.

The essential requisites for such analyses are the determination of the physical properties of the different soil strata and of their behavior under

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load by means of physical tests, and the determination of a fairly complete picture of the distribution of pressure in the underground due to foundation loads. The theoretical aspects of the stress conditions for the surface problems, the points of agreement and disagreement, and the results of investigations have been discussed very completely by Messrs. J. A. Moyer, M. L. Enger, A. T. Goldbeck, W. S. Housel, J. H. Griffith, and A. E. Cummings, Members, Am. Soc. C. E., Dr. F. Kögler, Dr. A. Scheidig, Dr. O. K. Froehlich, and others. Experience has shown that the settlement of full-scale footings is often considerably less and sometimes more than is indicated by extrapolation of small-scale test curves for granular soils. Hence, the important load-area-settlement relations have yet to be completely defined; likewise, the effects of lateral displacement of the soil and of differences of physical properties for full-scale footings are subject to further clarification.

It is well to consider certain questions in order to justify the use of the Boussinesq equation, which represents the only means available for the solution of the second class of foundation problems. The major settlement of structures in this class is due to consolidation of saturated compressible layers, which may be at considerable depth. At a comparatively shallow depth beneath a footing, two to three times its greatest width, the surface loading, due to individual footings, loses its isolated character and merges into a single pressure distribution for the structure as a whole. This occurs below the "disturbed zone" as defined by Kögler and Scheidig. Furthermore, the distribution of the contact pressure, which varies with both the elastic properties of the footing and the soil, becomes unimportant[2].

It is suggested by D. P. Krynine[3], M. Am. Soc. C. E., that the concentration factor, n, in the modified Boussinesq equation[2],

$$p_z = \frac{n\,p}{2\,\pi\,R^2}\left(\frac{z}{R}\right)^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

is a function of depth, tending toward a value of 3 as depth increases, and that the differences between theory and the results of investigations are merely surface phenomena confined to the disturbed zone. Experience, thus far, seems to indicate that for finer grained soils possessing considerable cohesion and situated at depths at least greater than two to three times the greatest width of the footing, the soil may behave in accordance with the theory of elasticity; and the Boussinesq equation, for which $n = 3$, should give pressures in reasonably close agreement with the actual, if they could be determined in the natural soil.

It would follow from this fact that the principle of superposition of loads may be applied in order to obtain the effect of distributed loads. Glennon Gilboy, Assoc. M. Am. Soc. C. E., has investigated[4] the proper degrees for subdividing a footing into units of such size that the loads on each unit may be considered a point load as assumed in the Boussinesq equation, and has indicated the accuracy that may be attained by such subdivision and superposition of point loads.

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[2] "Distribution of Stresses Under a Foundation", by A. E. Cummings, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1072.

[3] *Loc. cit.*, p. 1089.

[4] Progress Rept., Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 789.

The detailed analysis of the pressure distribution under a large structure becomes exceedingly long and tedious and involves the determination of a complete picture of the state of stress with respect to the compressible layer. The table of Boussinesq coefficients, K, corresponding to values[3] of $\frac{r}{z}$, for determining the vertical intensity of pressure at a point in the underground due to a point load concentrated at the surface in the equation,

$$p_z = \frac{K\,p}{z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

has placed a useful and valuable tool in the hands of the foundation engineer.

The following method is suggested as a practical simplification of the procedure for determining the stresses beneath a structure. The distribution of vertical pressure on a horizontal plane, mn, at some depth, z, due to a unit point load, p_0, at the surface at Point O, is shown by the typical curve in Fig. 1. The intensity of vertical pressure, p_z, at any point (B, Fig. 1) is given by the ordinate, ab, to the curve at that point. This distribution curve may be used conveniently as an influence line, as suggested by S. Timoshenko[5], for a unit load, p_0 of 1 ton. Then, for a load, P_1, at some point, O_1, the vertical pressure at Point A is given by the product of the ordinate, cd, due to a unit point load directly over A, by the load, P_1. It follows that the pressure, p_z, at Point A for a series of point loads, is given by the sum of the ordinates directly under the point loads, multiplied by p.

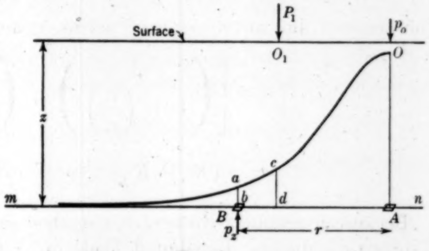

FIG. 1

This idea may be extended to a uniformly distributed line load of p tons per ft, covering all, or any part, of the foundation. The pressure, p_z, at Point A is then given by the area of the distribution curve directly under the loading, multiplied by the uniform loading of p tons per ft.

If this pressure-distribution curve is now revolved about its center at Point A, at which the pressure is to be computed, a bell-shaped surface is formed. Using this area as an influence surface, the pressure, p_z, at Point A is now given by the volume directly under the uniformly loaded area. As the distribution curve is revolved it sweeps out circular segments on the footing area, which are contours of equal pressure. By summing the products of the lengths of each arc by its average pressure intensity and multiplying by the footing load (p tons per sq ft), the total pressure at Point A is obtained. This is simply a graphical method of integration (see Fig. 2), and is the clue to the method proposed herein. The total pressure at Point A due to each

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[5] "Theory of Elasticity", by S. Timoshenko, Engineering Societies Monographs, p. 88.

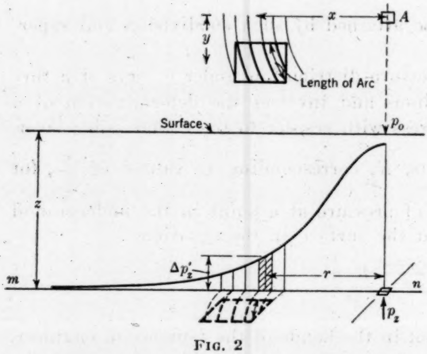

FIG. 2

ring of 1 ft width, is equal to the product of the pressure represented by the shaded area of the distribution curve, having an average ordinate, $\Delta p'_z$, multiplied by $2\pi r$, in which r is measured to the center of gravity of the shaded area. The pressure due to each of these rings may be determined in a very simple manner, because the Boussinesq equation can be readily integrated for circular areas. The pressure at a depth, z, directly under the center of a uniformly loaded circular area of radius, r, is given by,

$$p_z = p\left[1 - \left(\frac{1}{1 + \left(\frac{r}{z}\right)^2}\right)^{\frac{3}{2}}\right] \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The pressure due to any ring of radius, r_1 and r_2, is then,

$$p_z = p\left[\left(\frac{1}{1 + \left(\frac{r_2}{z_1}\right)^2}\right)^{\frac{3}{2}} - \left(\frac{1}{1 + \left(\frac{r_1}{z_1}\right)^2}\right)^{\frac{3}{2}}\right]$$

$$= p\,(K'_2 - K'_1) = p \times \Delta K' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Because a graphical integration method greatly simplifies the computation of pressures due to distributed loads on a foundation, the writer has constructed charts for depths of z = 10, 20, 40, and 80 ft. A sample tabulation

TABLE 1.—SAMPLE TABULATION OF VALUES OF $\Delta K'$ AND THE DEGREES OF ARC FOR p_z = 0.001 FOR CONSTRUCTING PRESSURE CHARTS

Ring No.	DEPTH, z, IN FEET							
	10		20		40		80	
	$\Delta K'$	Degrees of arc	$\Delta K'$	Degrees of arc	$\Delta K'$	Degrees of arc	$\Delta K'$	Degrees of arc
1	0,01418	24.3	0.0037	96.0	0.0009		0,0002	.....
2	0.0423	8.51	0.0110	32.58	0.0023	156.7	0.0007	.....
3	0.0642	5.61	0.0180	30.00	0.0046	81.8	0.0012	300
4	0.0783	4.60	0.0243	14.81	0.0064	56.25	0.0016	225
5	0.0848	4.24	0.0298	13.08	0.0082	43.9	0.0021	171.5
6	0.0851	4.25	0.0344	10.46	0.0098	36.7	0.0025	.....
7	0.0807	4.46	0.0378	9.52	0.0114	31.6	0.0030	.....
8	0.0737	4.88	0.0405	8.89	0.0129	27.9	0.0035	.....
9	0.0654	5.51	0.0420	8.57	0.0142	25.3	0.0039	.....

of values of $\Delta K'$, which were computed for the purpose of constructing these charts, is given in Table 1. It is seen from the foregoing discussion that the pressure, Δp_z, at Point A is directly proportional to the percentage of each ring swept out in Fig. 2. Each ring, therefore, is subdivided for interpolation into such unit lengths of arc that the pressure represented is some convenient value as 0.01, 0.02, 0.03 tons per sq ft, etc., and multiples of 10 thereof. The construction of a chart of equal vertical pressure intensities is now very simple. For example, $\Delta K'$ for $z = 10$ ft and a ring of radii of 6 ft and 7 ft, that is, Ring No. 7, is equal to 0.0807 ton per sq ft. The degrees of arc for a pressure intensity of 0.01 ton per sq ft would be, $\frac{360° \times 0.01}{0.0807} = 44.6$. Interpolation is simply in direct proportion to the length of arc. The degree of arc for the pressure intensity of 0.01 for all other rings is obtained from the general equation.

$$\alpha = \frac{360° \times 0.01}{\Delta K'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

A smooth curve drawn through these points defines the curve for $p_z = 0.01$ ton per sq ft. The curve for $p_z = 0.01$ is one-tenth of the arc for the 0.01 value on each ring. Hence, having constructed the curves for 0.01, 0.001, 0.0001, etc., any degree of subdivision may be obtained by a pair of dividers. The arcs are laid out at the center of each ring which, for all practical purposes, is the center of gravity of the pressure on the ring. Such a chart[6] for $z = 10$ ft and a unit uniform load, p, of 1 ton per sq ft on each ring, is shown in Fig. 3.

The total pressure at Point A due to a uniformly distributed load of 1 ton per sq ft on a footing of any shape, located anywhere within the range of the chart, is now given by the sum of the arc intercepts enclosed on the chart by the footing. The graphical integration is actually accomplished by revolving the footing layout about Point A, so that the reference sides of the footing, *ab* and *cd* (Fig. 3), are brought successively for each ring to the base line, AB, where the arc intercepts can then be readily interpolated. The sum of these intercepts, multiplied by the actual loading in tons per square foot, gives the total pressure due to the footing. The interpolation between pressure curves on the chart is directly proportional to the length of arc intercepted, and is readily made by inspection.

If the point, A, falls within the loaded area, a certain number of complete rings may be contained within the area. The pressure due to these rings is tabulated below the ring numbers. The pressure due to the remaining area is found in this case by interpolating the intercepts as before. The total pressure at Point A due to all the footings within the limits of the chart is obtained by repeating the process for each and summing the effects. This is a simple graphical process and a simple adding machine computation.

The foundation plan is made on tracing paper to the same scale as the horizontal distances on the chart. Each successive point at which the pressure is to be determined in the soil, is pinned down at Point A on the proper chart for depth. If the foundation plan is regular, these points would be most conveniently placed on the center line beneath each footing. The procedure for

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[6] A full set of charts, for $z = 10$, 20, 40, and 80 ft., is included with the closing discussion.

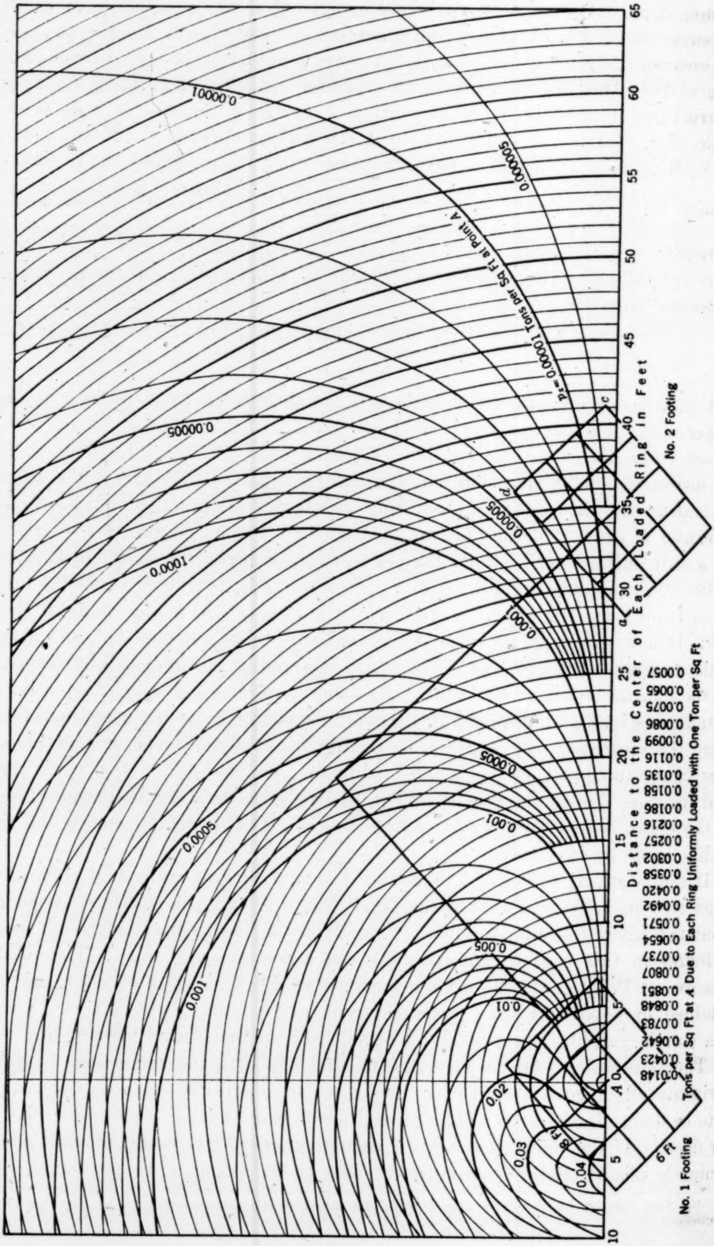
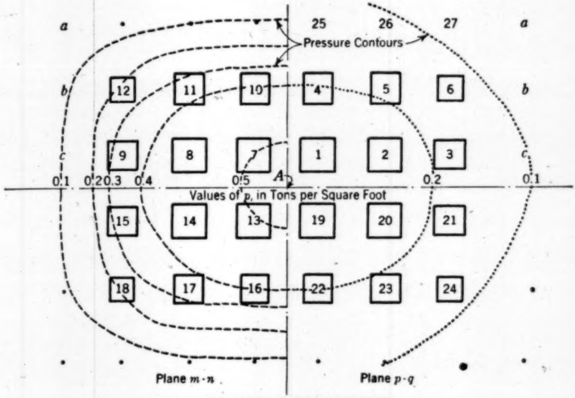
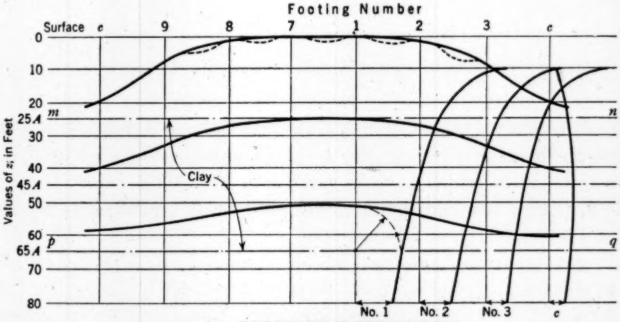


FIG. 3.—PRESSURE CHART FOR z = 10 FEET

the analysis of foundation pressures is indicated in Fig. 4. The pressure at Point *A* due to all footings having an appreciable influence may now be obtained by revolving the foundation plan about Point *A* and summing the effects for each footing, successively. An orderly and logical arrangement of the work will materially facilitate the computation. Most layouts have many


a
25
26
27
a
Pressure Contours
b
12
11
10
4
5
6
b
c
9
8
7
1
2
3
c
0.1
0.2
0.3
0.4
0.5
A
0.2
0.1
Values of p, in Tons per Square Foot
15
14
13
19
20
21
18
17
16
22
23
24
Plane m-n
Plane p-q

(*a*) FOUNDATION LAYOUT



(*b*) PRESSURE DISTRIBUTION
VERTICAL SECTION THROUGH *c-c*

FIG. 4.—PROCEDURE FOR ANALYZING FOUNDATION PRESSURES. HORIZONTAL LINEAR SCALE, 1 INCH = 50 FEET; AND PRESSURE SCALE, 1 INCH = 1.0 TON PER SQUARE FOOT.

symmetrical arrangements of footings as to location, size, and loading. If these similarities are noted, duplication is avoided and the work is decreased. A sample arrangement of some of the duplicate values to illustrate this problem is given in Table 2. For example, the value obtained for the pressure beneath Footing No. 1 due to pressures exerted by Footing No. 2, when multiplied by 3, takes care of the influence of Footings No. 2, 7, and 19 in the third column. Thus, a set of values indicated by the first and second columns, when completed, serves to fill out the remaining four columns, thus giving the data necessary for determining the distribution of pressure for Section *c-c* at any given elevation, *z*. The sum of all the values found for Footing No. 1 (that is, in the third column complete of Table 2) gives the total pressure beneath Footing No. 1. This value, multiplied by 3 tons per sq ft gives the value which is to be plotted in Fig. 4(*b*). Most of the values in Table 2, with a few additional values, will serve for Sections *a-a* and *b-b*, by noting the symmetry of arrangement.

TABLE 2.—Tabulation of Duplicate Values for Any Depth Along Section *c-c* (Fig. 4).

The pressure beneath Footing No.	Due to pressures exerted by Footing No.	Is the Same as the Pressure Under Footings Nos.			
		1	2	3	*c*
		Due to Pressures Exerted by Footings Nos.			
1	1	1	2	3	....
1	2	2, 7, and 19	1 and 20	2	....
1	3	3 and 22	23	....	....
1	4	4	3 and 5	21	3
1	5	5 and 10	4 and 21	5	21
1	6	6	24	....	24
7	2	8	7	1	2
7	3	9	....	7	....
13	1	13 and 20	19	20	....
13	2	14	13	19	20 and 5
13	3	15	11	13	4
13	5	17	16	22	23
13	6	18	....	....	....
19	3	11, 16, 23, and 21	22 and 10	4 and 23	....
19	6	24	....	....	....
2	6	....	6	....	6
8	2	....	8	....	1
8	5	....	....	10	....
14	2	....	14	....	19
14	5	....	17	16	22
3	6	....	....	6	....
3	3	....	....	3	....
4	6	....	....	24	....

It can also be readily observed which footings have a negligible effect by noting in what zone it lies on each chart with respect to the center point, *A*, For a depth of $z = 10$ ft, a footing more than 50 ft away produces a negligible pressure because the maximum pressure contours covered equal only 0.00001 ton per sq ft, whereas a footing at Point *A* includes four and one-half full rings of 0.08 maximum pressure. For the example shown footings that are at a greater distance from the one considered (Point *A*), than the following values will not affect materially the third significant figure, which is about the limit of plotting:

Depth, in feet, below the ground surface	Distance, in feet, from any given point
10	50
20	60
40	70
80	80

The diagonal limit of Fig. 3 is at 70 ft.

Examples of the measurement of pressure at a depth, $z = 10$ ft, and $p = 3$ tons per sq ft, are given in Table 3.

TABLE 3.—EXAMPLE OF THE MEASUREMENT OF PRESSURE AT A DEPTH, $z = 10$ FEET, AND $p = 3$ TONS PER SQUARE FOOT (SEE FIG. 3).

MEASUREMENT OF PRESSURE BENEATH FOOTING NO. 1, DUE TO PRESSURE EXERTED BY:

Footing No. 1		Footing No. 2		Footing No. 24	
Remarks	"Influence" pressures	Remarks	"Influence" pressures	Remarks	"Influence" pressures
Four full rings.............	0.0142 0.0423 0.0642 0.0783	Eight segments.	0.00160 0.00130 0.00105 0.00085 0.00070 0.00058 0.00048 0.00039	Eight segments.	0.000002 0.000005 0.000007 0.000007 0.000006 0.000004 0.000002 0.000001
Approximately one-half a ring.....................	0.0424				
Four segments @ 0.0045 =	0.0180				
		Three-fourths of 0.00033.....	0.00025		
Total "influence" pressures.	0.2594	....	0.00710	....	0.000038
Total pressure, $p = 3$ tons per sq. ft. × the total "influence" pressure.....	0.7782	....	0.02130	....	0.000112*

* Negligible.

The pressure distribution on a vertical section through Section *c-c* along the center lines of Footings Nos. 1, 2, and 3, is shown in Fig. 4(*b*). It is a simple matter to lay out, by dividers, the distribution of pressure on the horizontal lines, *m-n* and *p-q*, which define the limits of the clay layers. In fact, the pressure at such depths only as are necessary to determine the distribution of pressure within the clay layer, defined by the planed *m-n* and *p-q*, need be determined, as, for example, for $z = 20$, 40, and 80 ft. Similar diagrams may be drawn for Sections *a-a* and *b-b*. The distribution of the pressure on any horizontal plane, such as *m-n* and *p-q*, may then be easily drawn by interpolating contours of equal pressure from curves, such as Fig. 4(*b*), for Sections *a-a*, *b-b*, and *c-c*. These contours have been sketched in in Fig, 4(*a*). The pressures are given in tons per square foot. When the relations between pressure and settlement are obtained from consolidation tests, the pressure contours will define curves of settlement.

Table 4 contains a comparison of values obtained by pressure charts, such as Fig. 3, by the subdivided area method, and by the single, concentrated point load method. The pressures are computed for each footing separately at depths of 10, 20, 40, and 80 ft, along the center line of Footing No. 1 (see Fig. 5).

CONCLUSION

These results show that the point load method is considerably in error, especially close to the footing, whereas the differences between the chart

TABLE 4.—Comparison of Pressures Under Footing No. 1, by the Subdivided Area Method, and by the Point-Load Method Obtained by Charts Such as Fig. 3 (See Fig. 5)

Depth, in feet	Due to Footing No. 1			Due to Footing No. 2		
	By pressure charts	Load applied at twelve points*	Load applied at a single point	By pressure charts	Load applied at twelve points*	Load applied at a single point
10	0.1853	0.1942	0.2290	0.00096	0.00097	0.00091
20	0.0544	0.0549	0.0572	0.00369	0.00373	0.00367
40	0.0133	0.0142	0.0143	0.00499	0.00518	0.00521
80	0.0034	0.0036	0.0036	0.00265	0.00264	0.00263

* Subdivided area method.

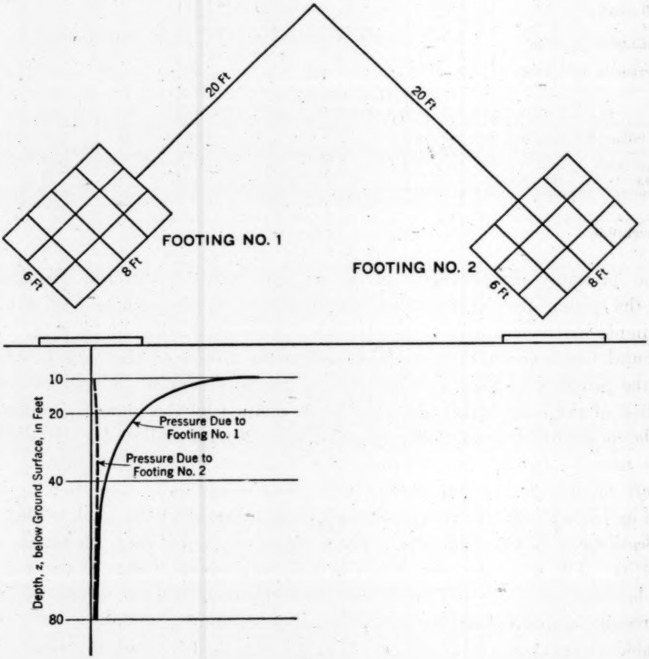

20 Ft
20 Ft
FOOTING NO. 1
6 Ft
8 Ft
FOOTING NO. 2
6 Ft
8 Ft
10
20
40
80
Depth, z, below Ground Surface, in Feet
Pressure Due to Footing No. 1
Pressure Due to Footing No. 2

Fig. 5.

method and the subdivided area method in general, are small. The saving in time of computation, however, is the important factor. The recent trend toward the use of such labor-saving computing charts in all branches of engineering is indicated by the number of such charts which have appeared from time to time in engineering literature. It is hoped that these charts will prove to be useful and by saving time and labor will make possible a more complete stress analysis in the soil beneath foundations. This would mean that the field of uncertainties would be narrowed down to other considerations and the correlation between the behavior of structures and theoretical and experimental soil mechanics would indicate the necessary modifications and re-adjustments.

The influence-line, graphical integration method may be extended to other problems for which the stress distribution is essential wherever the equation may be integrated for circular areas or by using the method outlined in Fig. 2 and the discussion immediately following.

DISCUSSION

William P. Kimball,[7] Assoc. M. Am. Soc. C. E. (by letter).—As long as the Boussinesq equation is used for the determination of vertical stresses in soil underlying foundations, accurate short-cut methods of applying the equation to actual problems will be of value to the soil engineer. In his paper the author has provided such a tool. The writer has had many occasions to compute, by the Boussinesq equation, the stresses under foundations of various types, and after experimenting with the author's graphical method believes that it represents one of the most efficient time-saving devices available for the solution of most stress-distribution problems.

To illustrate the application of the method the author has shown the analysis of stresses under the centers of individual spread footings supporting a building. A different type of foundation to which the method is equally applicable is a single large area, such as the base of a bridge pier. In this case it is not enough to find the stresses at various elevations under only the center of the area. The net settlement of a bridge pier due to the consolidation of the underlying soil must be caused by the average stresses under the loaded area rather than by the maximum stresses which occur, at least at a distance below the loaded area, under the center of the area. The determination of the average stresses at various elevations under the loaded area

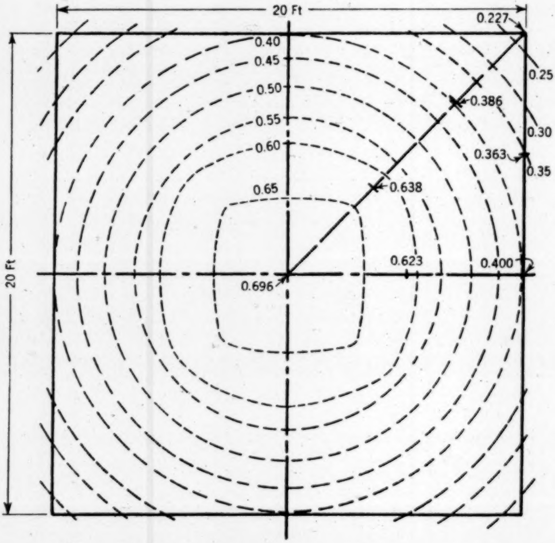


Fig. 6.—Vertical Pressures at $z=10$ Feet Beneath a 20-Foot by 20-Foot Loaded Area

becomes necessary. Using the pressure chart for $z = 10$ ft (Fig. 3) the writer has determined the stresses under various points of a footing, 20 by 20 ft in area, uniformly loaded with 1 ton per sq ft. These stresses, together with the resulting stress contours, are shown in Fig. 6. From these contours it is a simple matter to compute, with satisfactory accuracy, the average stress over the entire area. In this particular instance the average stress was computed to be 0.500 ton per sq ft, as compared with the maximum center stress of 0.696 ton per sq ft. The difference between the stress under the center and the average stress decreases, of course, as the distance below the loaded area increases, and the limit beyond which it is not necessary to compute the average stress can be determined readily by practice.

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[7] Asst. Prof. of Civ. Eng., Thayer School of Civ. Eng., Dartmouth Coll., Hanover, N. H.

An additional short-cut which may be used in certain cases has been previously suggested[8] by the writer, namely, to compute the stress under the center as under the center of a uniformly loaded equivalent circular area. An equivalent circular area to the 20 by 20-ft square has a diameter of 22.5 ft. Substituting the proper values in Equation (3) of the paper a center stress of 0.707 ton per sq ft is obtained for $z = 10$ ft. This compares favorably with the 0.696 ton per sq ft obtained by the graphical method, and results in a substantial saving of time. The accuracy of this approximation increases, quite apparently, as the distance below the loaded area increases, but decreases as the shape of the loaded area digresses from the circular form. Whether this approximation is sufficiently accurate in any particular case can only be determined from experience.

I. M. Nelidov,[9] Assoc. M. Am. Soc. C. E. (by letter).—Without raising questions as to the distribution of pressure in the upper strata of foundations, the value of the concentration factor, n, and other factors pertinent to the problem of the behavior of soils under load, the writer wishes to apply the principle proposed by the author to the case in which the pressure is not uniform, but varies linearly. In this case the pressure at Point A (Fig. 7), due to a loaded foundation area some distance away, will be:

$$p_z = \sum \left\{ \frac{\Delta K'}{2\pi} \left[p_A \int_{\phi=\alpha_1}^{\phi=\alpha_2} d\phi + r \tan i \left(\int_{\phi=0}^{\phi=\alpha_2} \sin\phi \, d\phi - \int_{\phi=0}^{\phi=\alpha_1} \sin\phi \, d\phi \right) \right] \right\}$$

$$= \sum \left[p_A \frac{\Delta K'}{2\pi} (\alpha_2 - \alpha_1) + \tan i \, \frac{\Delta K'}{2\pi} \left(l_{x2} - l_{x1} \right) \right] = p_{z1} + p_{z2} \ldots (6)$$

in which, in addition to the notation of the paper: p_A is the uniform pressure at Point A, in tons per square foot; p_{z1} and p_{z2} are correspondingly the pressure due to uniform and non-uniform loading; tan i is the slope of the linear variation of the stress taken normally to the line of zero stress due to bending only, in tons per cubic foot (see Fig. 7); and l_{x1} and l_{x2} are the projections of the arc of radius, r, intercepted by the loaded area, on the line, O-X.

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[8] *Proceedings* Am. Soc. C. E., August, 1933, p. 1060.

[9] Senior Engr. of Hydr. Structure Design, State Dept. of Public Works, Sacramento Calif.

The derivations for obtaining the pressure, p_{z1}, are given by the author. To determine the pressure, p_{z2}, Fig. 8 is constructed similarly to Fig. 3. This is done by taking as abscissas the radii, r, and laying out the distances, l_x, to the left of the point of intersection of the arc corresponding to r, with the line of abscissas, and erecting perpendiculars to the intersection with the arc. The distances, l_x, are selected so as to give even intervals of the load, p_{z2}, with $\tan i = 1$. In this manner,

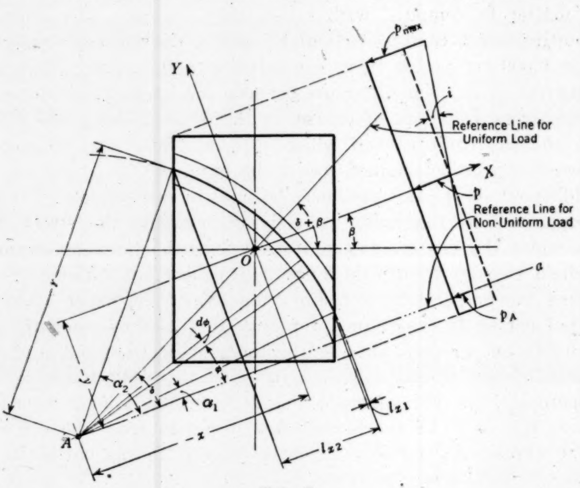

FIG. 7.

$$l_x = p_{z2} \frac{2\pi}{\Delta K'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

in which p_{z2} is the load, in tons per square foot. For small values of the loads the intersection was determined analytically from the expression, $\cos \Delta = 1 - \frac{l_x}{r}$ in which Δ is the angle between r and the reference line. In order to compute the pressure, p_{z2}, the loaded area is placed on the chart (Fig. 8) so that Line *A-a*, Fig. 7, coincides with the reference line of the chart, and Point *O* is located by the co-ordinates, x and y. Then, positive pressures are read along the arcs intercepted by the periphery of the loaded area. In order to read negative pressures, the footing is rotated 180° about the reference line of the chart, and pressures are read again. These pressures must then be multiplied by $\tan i$ in order to obtain true values of p_{z2}.

For example, consider a rectangular footing 7 ft by 10 ft in area. The uniform pressure is 2.5 tons per sq ft; $\tan i = 0.15$; and the line of zero bending stress is inclined to the axis of symmetry of the footing at an angle, $\beta = 20$ degrees. It is required to find the pressures, p_z, at a point, A, with the co-ordinates, $x = -10$ ft and $y = -5$ ft in the co-ordinate system, XOY (Fig. 7).

The intensity of the uniform pressure at Point A is: $p = 2.5 \times 0.15 \times 5 = 1.75$ tons per sq ft; the total pressure at Point A due to this uniform pressure is obtained with the aid of Fig. 3; and, the angle of inclination of the axis of symmetry of the footing to the base line of the chart, $\beta + \delta$ equals $20 + 27 = 47$ degrees. Consequently, by Equation (6), $p_{z1} = 1.75\ (0.0015 + 0.0025 + 0.0028 + 0.0030 + 0.0029 + 0.0027 + 0.0024 + 0.0022 + 0.0012 + 0.0006 + 0.0003) + (0.0033 + 0.0045 + 0.0048 + 0.0047 + 0.0044 + 0.0041 + 0.0033 + 0.0016 + 0.0008 + 0.0004 + 0.0001) = 1.75 \times 0.0541 = 0.0946$ ton per sq ft.

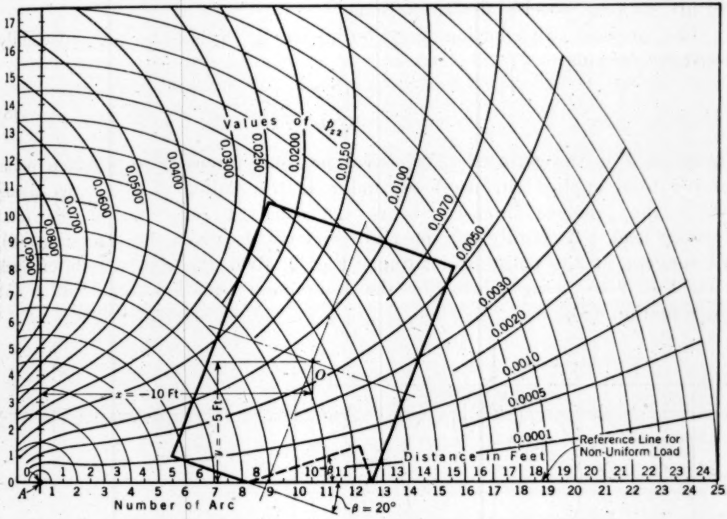

FIG. 8.—CHART FOR DETERMINATION OF VALUES OF p_{z2} WITH TAN $\phi = 1$

The compressed part of the footing is shown in Fig. 8 in full lines, and the tension part is shown in dotted lines. The angle of inclination of the axis of symmetry with the base line of the chart is $\beta = 20$ degrees. The pressure due to non-uniform stresses will be, similarly to p_{z1}: $p_{z2} = 0.15\ (0.0040 - 0.0010) + (0.0100 - 0.0002) + 0.0160 + 0.0205 + 0.0230 + 0.0250 + 0.0250 + (0.0250 - 0.0002) + (0.0230 - 0.0015) + (0.0145 - 0.0029) + (0.0095 - 0.0042) + (0.0066 - 0.0050) - 0.0001 - 0.0001 - 0.0004 - 0.0005 = 0.15\ (0.1741 - 0.0011) = 0.0260$ ton per sq ft. The total pressure will be: $p_z = p_{z1} + p_{z2} = 0.0946 + 0.0260 = 0.1206$ ton per sq ft. If only a uniform pressure of 2.5 tons per sq ft were considered, the pressure at Point A would be: $p_z = p_{z1} = 2.5 \times 0.0541 = 0.1350$ ton per sq ft.

This method may be applied to the computation of pressures caused by continuous non-uniform loadings, such as earth-fill or rock-fill dams, gravity dams, etc. The principle of graphical computation of pressure proposed by the author is practical and labor-saving.

GEORGE PAASWELL,[10] M. AM. SOC. C. E. (by letter).—The recognized importance of determining pressures and their manner of distribution beneath foundations makes it necessary that the formulas covering the determination of such distributions be of ready application. Mr. Burmister's paper is "a step in the right direction." It is realized that the exact determination of the distribution is not necessary; as a matter of fact, present knowledge as to the nature of foundation materials does not yet permit making precise determinations.

The pressure distribution may be given by a simple yet sufficiently accurate formula,

$$r = \frac{0.32}{\sqrt{k}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which k is the ratio of the vertical pressure intensity at a given point to the total applied surface load, and r is the radius of the circle along which the pressure intensity bears the constant ratio, k, to the total surface load. A family of spherical surfaces is given by Equation (8), all tangent at the point of loading. Fig. 9 gives the comparison of such loci, Fig. 9(a) being plotted by the Boussinesq equation, and Fig. 9 (b) by Equation (8).

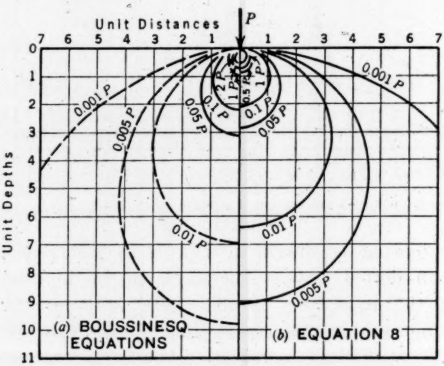

FIG. 9.—LOCI OF CONSTANT VERTICAL PRESSURE INTENSITIES.

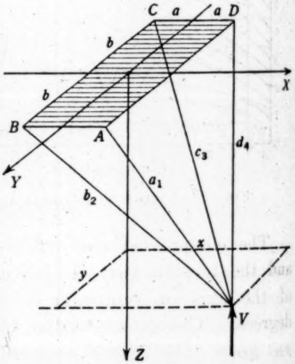

FIG. 10.—DISTRIBUTION OF VERTICAL PRESSURES UNDER UNIFORMLY LOADED RECTANGLE.

The question of the partition of a concentrated load into sufficiently small areas to give correct distribution becomes important at points close to the load. Professor A. E. H. Love has given a solution[11] for the distribution of pressures beneath a rectangle, Fig. 10. The vertical intensity of pressure at a point, x, y, z, with respect to the center of the rectangle as the origin is,

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[10] Secy. and Treas., Spencer & Ross, Inc., Detroit, Mich.

[11] *Philosophical Transactions* Royal Soc., Series A, 1929.

$$V = -\frac{A - z\,B}{2\,\pi}\,p \quad \text{(9)}$$

in which p is the surface load per unit of area. The other terms are as follows (a negative value of V denotes a compressive stress):

$$A = -[2\,\pi - (\phi_1 + \phi_2 + \phi_3 + \phi_4)] \quad \text{(10a)}$$

$$B = (a - x)\,s_1 + (a + x)\,s_2 + (b - y)\,s_3 + (b + y)\,s_4 \quad \text{(10b)}$$

$$\cos\phi_1 = \frac{(a - x)\,(b - y)}{k_1\,k_3} \quad \text{(11a)}$$

$$\cos\phi_2 = \frac{(a - x)\,(b + y)}{k_1\,k_4} \quad \text{(11b)}$$

$$\cos\phi_3 = \frac{(a + x)\,(b - y)}{k_2\,k_3} \quad \text{(11c)}$$

$$\cos\phi_4 = \frac{(a + x)\,(b + y)}{k_2\,k_4} \quad \text{(11d)}$$

$$k^2_1 = (a - x)^2 + z^2 \quad \text{(12a)}$$

$$k^2_3 = (b - y)^2 + z^2 \quad \text{(12b)}$$

$$k^2_2 = (a + x)^2 + z^2 \quad \text{(12c)}$$

$$k^2_4 = (b + y)^2 + z^2 \quad \text{(12d)}$$

$$s_1 = \frac{1}{k^2_1}\left(\frac{b - y}{a_1} + \frac{b + y}{d_4}\right) \quad \text{(13a)}$$

$$s_2 = \frac{1}{k^2_2}\left(\frac{b - y}{b_2} + \frac{b + y}{c_3}\right) \quad \text{(13b)}$$

$$s_3 = \frac{1}{k^2_3}\left(\frac{a - x}{a_1} + \frac{a + x}{b_2}\right) \quad \text{(13c)}$$

$$s_4 = \frac{1}{k^2_4}\left(\frac{a - x}{d_4} + \frac{a + x}{c_3}\right) \quad \text{(13d)}$$

The angles are given in circular measure and may vary from 0 to π. If the fractions are negative (that is, x or y greater than a or b), take the value of the cosine corresponding to the positive value of the fraction and subtract the angle thus found from π. Symbols a_1, b_2, c_3, and d_4 denote the distances from the point to the four corners of the rectangle (see Fig. 10). Equations (9) to (13) fail for points directly under the edges of the rectangle. For such points select points just outside the projections of the rectangle.

It may be interesting to observe how many subdivisions are required, using the Boussinesq formula, to approximate the values as obtained from Equation (9). Select a series of points directly below the center of the rectangle, $x = y = 0$, and assume that $a = 5$; $b = 3$; and $p = 1$. The area of the rectangle is 60 units, and the total load, P, in the Boussinesq formula, is 60.

TABLE 5.—COMPARISON OF VERTICAL PRESSURE INTENSITIES

Values of z	Love	BOUSSINESQ		
		One subdivision	Four subdivisions	Eight subdivisions
1	0.98	28.6	0.01	0.29
2	0.91	7.2	0.42	0.73
3	0.79	3.2	0.61	0.78
4	0.66	1.8	0.62	0.68
5	0.54	1.1	0.60	0.56
10	0.23	0.3	0.23	0.23
20	0.06	0.07	0.07	0.07

Table 5 shows the vertical intensity of the pressure at depths from 1 to 20 as given by Love and by Boussinesq when the rectangle is subdivided into one, four, and eight parts It indicates that a single concentration may be assumed for the rectangular distribution when the point in question is at least 10 ft below the surface.

The writer feels that the method of plane sections[4] as covered in the report of the Committee on Earths and Foundations, is an extremely useful one in considering the effect of a group of pier loads on the deeper strata. A uniform distribution of load may be taken as the equivalent of the various piers and the simple formulas given in the report may be used to determine the vertical loadings at the depths required. The prediction of settlement from an analysis made in this manner will be exact enough in every sense, to make a complete study of the foundation problem.

JACOB FELD,[12] M. AM. SOC. C. E. (by letter).—The graphical interpretation, by Professor Burmister, of the integration of pressure distributions over volumes below the loaded areas is a time-saving tool, which does not reduce the accuracy of the method. However, it is important that one who uses it knows the limitations of the theory and does not expect a result more accurate than can be expected from a tedious analytical computation based on that theory.

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[12] Cons. Engr., New York, N. Y.

Very properly, the author distinguishes between the problems of "disturbed zone" and deeper layers as an explanation of settlements. The Boussinesq formula, with or without modifications, is only applicable in isotropic bodies. The soils in a "disturbed zone" directly below a foundation are not isotropic. The soils in deeper layers, under initial compression (before the additional loading) are often under such low stresses, that isotropy is closely simulated. Hence, the formula and the author's method are not applicable to the determination of settlements in the "disturbed zone," but can be used for such determinations at greater depths.

The careful reader will notice that the theoretical load distribution is not dependent upon the soil characteristics. This is neither logical nor true. Under low stresses, the error introduced by the assumption that all soils are equal, is so quantitatively small, because the numerical values are small, that it can be disregarded.

Especially in the "disturbed zone" any theory that is based on the assumption that, under equal loading the same distribution will result in mud and in rock, must require modification.

Referring to the numerical example, especially Table 4, and from a general study of the problem of settlements under foundations, one is quite safe in drawing the following conclusions:

(*a*) At depths of 20 ft, or more, the assumption of a single point loading for the usual column footings gives pressure distributions (computed either numerically or graphically) which are as accurate as the computed loadings on such footings.

(*b*) At lesser depths, if the soil is of such nature that isotropic behavior cannot be expected, settlements will occur which cannot be permitted. In this case, loads are transmitted to lower and better layers of soil, by means of piles or caissons.

(*c*) At lesser depths, if the soil is of such nature that isotropic behavior can be expected, settlements are so small that there is no problem.

In all these studies for the estimation of expected settlements, in addition to the nature of the soil, the designer often disregards the effect of the shape of the footings and edge resistance, as well as relative settlements of unequal areas loaded with the same load per unit area. These effects are most important in soils that cannot be assumed to fall in the isotropic group.

NATHAN M. NEWMARK,[13] ASSOC. M. AM. SOC. C. E. (by letter).—The method presented by Mr. Burmister for finding vertical pressures beneath foundations graphically is interesting and simple to use. The author's procedure is more rapid than the method of dividing the loaded area into elements of load which may be replaced by concentrated forces. The writer wishes to supplement Mr. Burmister's method by describing for comparison two other procedures, one analytical and one graphical, which seem to have similar advantages of speed and simplicity.

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[13] Research Asst. Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

An analytical procedure for computing the intensity of vertical stress beneath a loaded area was developed by the writer at the University of Illinois.[14] This procedure makes use of a table of values of vertical pressure at unit depth beneath the corner of a uniformly loaded rectangle of dimensions, *m* by *n*. The table is applicable to all depths and to all dimensions of loaded area. The formula by which the numerical values were computed was obtained by integration over a rectangular area of Boussinesq's formula for the pressure due to a concentrated load. The tabulated coefficients of pressure are given in terms of the vertical load per unit of area on the rectangular area and are dimensionless.

The pressure at points other than under the corner of a rectangular area is found[14] by combining upward and downward loads on not more than four rectangles. For example, the vertical pressure at a depth of 10 ft beneath the center of the footing of dimensions, 6 ft by 8 ft, shown in Fig. 5, is found as the sum of the vertical pressures 10 ft beneath the common corner of four equal rectangles, 3 ft by 4 ft. The "relative dimensions" (*m* and *n*) for these rectangles are 0.3 by 0.4. With these values of *m* and *n*, the table[14] referred to lists a value of influence pressure of 0.04742. Then the influence pressure for the entire footing is four times this value, or 0.18968.

TABLE 6.—INFLUENCE PRESSURES

Depth under center of Footing No. 1, in feet	INFLUENCE PRESSURE DUE TO:	
	Footing No. 1	Footing No. 2
10	0.1897	0.00097
20	0.0542	0.00372
40	0.0141	0.00520
80	0.0035	0.00265

For the problem shown in Fig. 5 of the paper the results given in Table 6 were obtained by this procedure. The writer was unable to make comparisons with the results obtained for the problem of Fig. 4, since no dimensions are given for that problem. Except for pressures near the surface, such a foundation plan can be considered to be replaced by a rectangular area uniformly loaded with the same total load as is applied to the footings. In view of the fact that the computation of pressure in soil is based on assumptions that are only approximately true, such a replacement is permissible. Near the surface, the pressures are affected by the distribution of contact pressure between the footings and the soil, and this distribution may vary between rather wide limits.

It seems that the author's graphical procedure is limited to computations at depths for which charts are available. Only one chart is necessary for all depths if the scale of the foundation plan is changed properly with the depth. For example, Fig. 3, the author's chart for a 10-ft depth, may be used for, say, a 20-ft depth if the scale of distance to the center of each loaded ring is multiplied by 2, so that the rings become 2 ft wide.

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[14] "Simplified Computation of Vertical Pressures in Elastic Foundations", by Nathan M. Newmark, Univ. of Illinois, Eng. Experiment Station, *Circular No. 24* (1935).

If a graphical procedure is to be used, the writer prefers a pressure chart such as Fig. 11 in which each loaded ring accounts for the same influence pressure. Subdivisions of the rings may then be made by radial lines. The

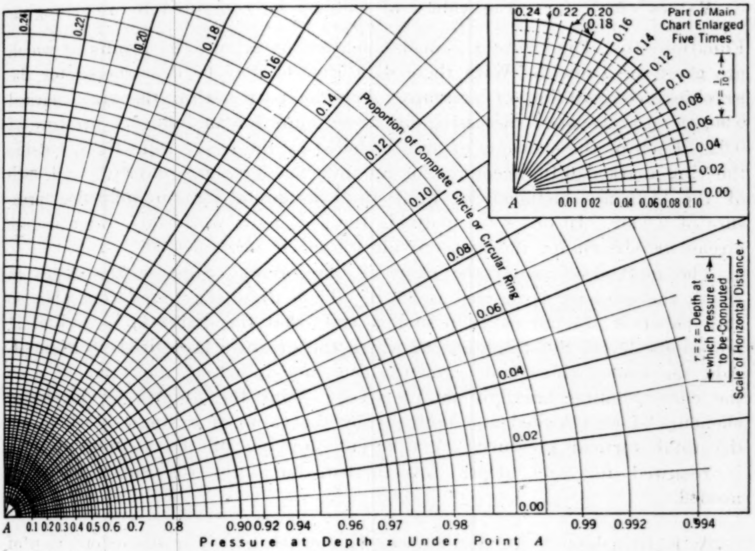

FIG. 11.—FOR UNIFORM LOAD OF UNIT INTENSITY ON AREA OF COMPLETE CIRCLE BOUNDED BY RING DESIGNATED.

TABLE 7.—VALUES OF RELATIVE RADIUS OF CIRCLE, $\frac{r}{z}$, FOR GIVEN VALUES OF $\frac{p_z}{p}$

$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)	$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)	$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)	$\frac{p_z}{p}$ (1)	$\frac{r}{z}$ (2)
0	0	0.36	0.5887	0.70	1.1097	0.970	3.0590
0.02	0.1164	0.38	0.6126	0.72	1.1561	0.972	3.1377
0.04	0.1661			0.74	1.2062	0.974	3.2240
0.06	0.2052	0.40	0.6370	0.76	1.2607		
0.08	0.2391	0.42	0.6617	0.78	1.3206	0.976	3.3194
		0.44	0.6869			0.978	3.4259
0.10	0.2698	0.46	0.7127	0.80	1.3871	0.980	3.5457
0.12	0.2983	0.48	0.7392	0.82	1.4618	0.982	3.6823
0.14	0.3252			0.84	1.5469	0.984	3.8404
0.16	0.3511	0.50	0.7664	0.86	1.6459		
0.18	0.3761	0.52	0.7945	0.88	1.7636	0.986	4.0268
		0.54	0.8235			0.988	4.2519
0.20	0.4005	0.56	0.8536	0.90	1.9083		
0.22	0.4244	0.58	0.8849	0.91	1.9948	0.990	4.5326
0.24	0.4481			0.92	2.0943	0.992	4.8990
0.26	0.4715	0.60	0.9176	0.93	2.2108	0.994	5.4116
0.28	0.4948	0.62	0.9519	0.94	2.3505	0.996	6.2197
		0.64	0.9880	0.95	2.5235	0.998	7.8738
0.30	0.5181	0.66	1.0261				
0.32	0.5415	0.68	1.0665	0.96	2.7477	1.0	∞
0.34	0.5650						

chart can be made independent of depth by drawing the foundation plan to scale such that the depth at which the pressure is to be computed is taken as the unit of length.

Values of $\frac{r}{z}$ for given values of $\frac{p_z}{p}$ may be computed by re-arranging Equation (3). The writer has made such computations, the results of which are given in Table 7. With these data Fig. 11 may be constructed for the graphical computation of pressure. For the part within the ring marked 0.90, the area between successive circles accounts for an influence pressure of 0.02. Each of the elements of area bounded by the heavy solid lines, outside the ring marked 0.10, accounts for an influence pressure of 0.002, and each of the elements bounded by light lines accounts for an influence pressure of 0.0002. In order to subdivide the area more uniformly, intermediate circles are drawn for the area outside the circle marked 0.90.

The method of using the chart is, as follows: Suppose the depth at which the pressure is desired is 35 ft. Then draw the foundation plan on thin paper or tracing cloth to such a scale that the distance, z, in Fig. 11 represents 35 ft. Set the point at which the pressure is desired at Point A. Add the loaded rings and segments, each multiplied by the value of p on the corresponding area, to obtain p_z; or count the number of blocks or elements of area loaded and multiply by the value of a single block to get the total vertical pressure. A different drawing of the foundation plan is required for each depth investigated, but only the single chart is needed.

A. E. CUMMINGS,[15] M. AM. SOC. C. E. (by letter).—For the calculation of the stress distribution in the ground under a foundation, the author recommends the use of a simple algebraic expression which is often referred to as "the Boussinesq equation." According to Professor Burmister, this equation represents the only means available for the solution of such problems. The writer is of the opinion that the derivation of the equation is based on assumptions which are rarely fulfilled by an actual foundation problem. Accordingly, he proposes to outline briefly the original derivation[16] of the equation and to discuss the assumptions on which it is based.

Boussinesq assumes a rectangular co-ordinate system (Fig. 12), with the XY-plane as the plane horizontal boundary of the semi-infinite solid and with the positive Z-axis directed downward into the solid. He begins with the general differential equations of elastic equilibrium in the form commonly known as the body-shift equations. These are:

$$(\lambda + \mu)\frac{\partial \Delta}{\partial x} + \mu \nabla^2 u = 0 \qquad \text{(14a)}$$

$$(\lambda + \mu)\frac{\partial \Delta}{\partial y} + \mu \nabla^2 v = 0 \qquad \text{(14b)}$$

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[15] Dist. Mgr., Raymond Concrete Pile Co., Chicago, Ill.

[16] "Application des Potentiels a l'Etude de l'Equilibre et du Mouvement des Solides Elastiques", by J. Boussinesq, Gautier-Villars, Paris, 1885.

and,

$$(\lambda + \mu)\,\frac{\partial \Delta}{\partial z} + \mu \nabla^2 w = 0 \ldots\ldots\ldots\ldots\ldots\ldots (14c)$$

In Equations (14), u, v, and w are the displacements in the X, Y, and Z directions, respectively; and λ and μ are the elastic constants of Lamé, μ being the shear modulus. The cubical dilatation is:

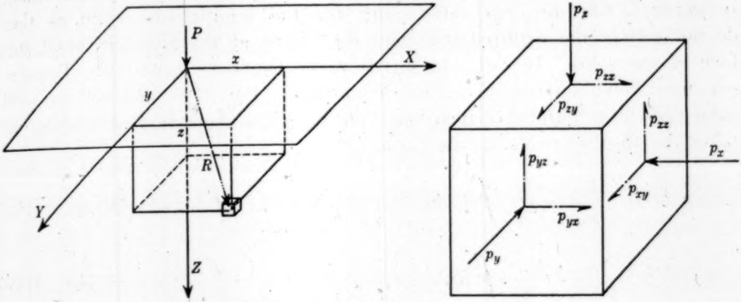

FIG. 12.

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

and ∇^2 is the Laplacian operator:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

It is necessary to find expressions for u, v, and w that will satisfy the differential Equations (14) and for this purpose Boussinesq uses certain potential functions. He expresses the displacements in terms of the following:

The inverse or Newtonian potential is:

$$U = \int \frac{dm}{R} \ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

the direct potential is:

$$V = \int R\, dm \ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

the first logarithmic potential of three variables is:

$$\psi = \int \log\,(z + R)\, dm \ldots\ldots\ldots\ldots\ldots\ldots (17c)$$

and the second logarithmic potential of three variables is:

$$\Psi = \int [-R + z \log\,(z + R)]\, dm \ldots\ldots\ldots\ldots\ldots\ldots (17d)$$

In Equations (17), dm is the infinitesimal element of the mass producing the field of force and $R^2 = x^2 + y^2 + z^2$. The integration extends over the mass, m.

By expressing the displacements as the space derivatives of one or the other of these potential functions, Boussinesq finds solutions of the differential Equations (14). These solutions he calls "simple integrals" but, in general, they are not satisfactory for the semi-infinite solid as they do not satisfy the required boundary conditions at the free horizontal surface of the solid. In order to satisfy these boundary conditions, Boussinesq uses combinations of several potentials. For the semi-infinite solid subjected to a purely vertical load on its plane horizontal boundary, he takes the displacements in the form:

$$u = -\frac{1}{4\,\pi\,\mu}\,\frac{\partial^2}{\partial x\,\partial z}\int R\,dm - \frac{1}{4\,\pi\,(\lambda + \mu)}\,\frac{\partial}{\partial x}\int \log\,(z + R)\,dm \quad \text{..(18a)}$$

$$v = -\frac{1}{4\,\pi\,\mu}\,\frac{\partial^2}{\partial y\,\partial z}\int R\,dm - \frac{1}{4\,\pi\,(\lambda + \mu)}\,\frac{\partial}{\partial y}\int \log\,(z + R)\,dm \quad \text{..(18b)}$$

and,

$$w = -\frac{1}{4\,\pi\,\mu}\,\frac{\partial^2}{\partial z^2}\int R\,dm + \frac{2\,\lambda + 3\,\mu}{4\,\pi\,\mu\,(\lambda + \mu)}\,\frac{\partial}{\partial z}\int \log\,(z + R)\,dm \quad \text{..(18c)}$$

Equations (18) are seen to be combinations of the direct potential and the first logarithmic potential, and it is easily shown by differentiation that they are solutions of the differential Equations (14).

To obtain the stresses from the displacements, Boussinesq makes use of the equations of the mathematical theory of elasticity in which the stresses are expressed in terms of the derivatives of the displacements. These are:

$$p_x = -\lambda\,\Delta - 2\,\mu\,\frac{\partial u}{\partial x} \quad \text{........................(19a)}$$

$$p_y = -\lambda\,\Delta - 2\,\mu\,\frac{\partial v}{\partial y} \quad \text{........................(19b)}$$

$$p_z = -\lambda\,\Delta - 2\,\mu\,\frac{\partial w}{\partial z} \quad \text{........................(19c)}$$

$$p_{zx} = p_{xz} = -\mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \quad \text{..................(19d)}$$

$$p_{zy} = p_{yz} = -\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \quad \text{..................(19e)}$$

and,

$$p_{xy} = p_{yx} = -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad \text{..................(19f)}$$

In Equations (19), the values of p with the single subscript are the normal stresses and those with the double subscripts are the shears.

If it is assumed that a point load, P, is applied normally to the surface of the solid so that $\int dm = P$, Equations (18) and (19) give the stresses at any point within the solid, as follows:

$$p_x = \frac{P}{2\,\pi}\left\{\frac{3\,z\,x^2}{R^5} + \frac{\mu}{\lambda + \mu}\left[\frac{R^2\,(z + R) - x^2\,(z + 2\,R)}{R^3\,(z + R)^2} - \frac{z}{R^3}\right]\right\} \quad \text{..(20a)}$$

$$p_y = \frac{P}{2\,\pi}\left\{\frac{3\,z\,y^2}{R^5} + \frac{\mu}{\lambda + \mu}\left[\frac{R^2\,(z + R) - y^2\,(z + 2\,R)}{R^3\,(z + R)^2} - \frac{z}{R^3}\right]\right\} \quad \text{..(20b)}$$

$$p_z = \frac{3\,P\,z^3}{2\,\pi\,R^5} \quad \text{......(20c)}$$

$$p_{zx} = \frac{3\,P\,z^2\,x}{2\,\pi\,R^5} \quad \text{......(20d)}$$

$$p_{zy} = \frac{3\,P\,z^2\,y}{2\,\pi\,R^5} \quad \text{......(20e)}$$

and,

$$p_{xy} = \frac{P}{2\,\pi}\left[\frac{3\,x\,y\,z}{R^5} - \frac{\mu}{\lambda + \mu}\,\frac{x\,y\,(z + 2\,R)}{R^3\,(z + R)^2}\right] \quad \text{......(20f)}$$

In these stress equations the plus sign represents compression and the minus sign, tension. Equation (20*c*) is easily recognized as "the Boussinesq equation."

Whether or not Equation (20*c*) is applicable to a given foundation problem depends on how well the foundation problem conforms with the assumptions on which the derivation of the equation is based. As an illustrative example, the author has used a structure built on isolated spread-footings (see Fig. 4). However, there is nothing in the paper to indicate that the charts should not be used for large mat foundations. The assumptions on which Equation(20*c*) is based are more readily understood by consideration of a mat foundation. Furthermore, the author has made several statements which are of considerable interest when applied to a large mat. The writer proposes, therefore, to consider not only Fig. 4, but also the foundation problem illustrated in Fig. 13. This is not an imaginary soil condition. It is more or less typical of certain parts of Chicago, Ill., Detroit, Mich., Toledo, Ohio, and other cities on the Great Lakes.

One important assumption on which Equation (20*c*) is based is that the solid must be homogeneous to an infinite depth. This is required by Equations (18) since the space derivatives of the potential functions in these equations are continuous functions and do not vanish except at infinity. Discontinuities in the soil structure are not compatible with

Equations (18). In Fig. 13, the surface of the bed-rock is at a depth of 90 ft below the ground surface. This rock may be considered as practically rigid so that the vertical displacements must vanish at the plane of the rock surface. The horizontal displacements may or may not vanish, depending on the friction between the rock and the soil stratum just above it. Equation (20*c*) does not apply to this case.

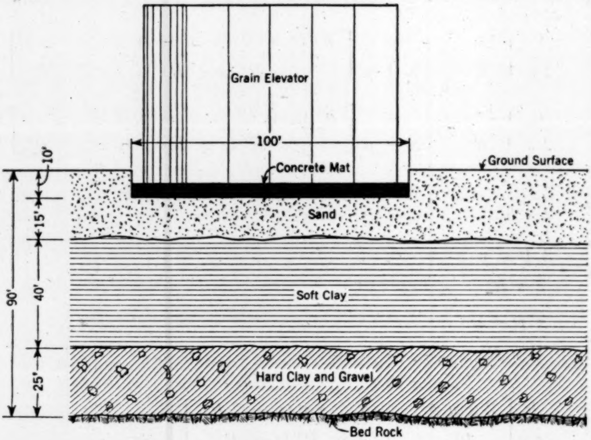

FIG. 13.

A method of taking into account the discontinuity at the rock surface has been published by Professor M. A. Biot.[17] According to Professor Biot's analysis, the vertical normal stress on the center line of the load at the rock surface is considerably greater than the stress calculated with Equation (20*c*). The plane of contact between the sand and the soft clay represents another type of discontinuity. Arthur Casagrande, Assoc. M. Am. Soc. C. E., has studied this condition[18] and has found that a dense sand bed apparently spreads the load so that the vertical normal stresses in the clay are much less than the values given by Equation (20*c*). This question of discontinuities in the soil structure applies to Fig. 4 as well as to Fig. 13. In Fig. 4, it is the width of the entire building that must be considered and not the diameter of the individual footings. Whenever there is bed-rock or some other abrupt change in soil stratification at a depth no greater than one or two times the diameter of the building, Equation (20*c*) does not provide the proper answer.

A second assumption on which Equation (20*c*) is based is that the load must be at the surface of the solid and not below the surface. This requirement is also due to Equations (18). Fig. 13 shows the bottom of the mat at a depth of 10 ft below the surface. This is not an uncommon condition and, in some cases, buildings have several sub-basements so that their foundations are 25 or 30 ft below the surface. Equation (20*c*) is not suitable for such conditions, and it is necessary to use an analysis of the type published in 1936[19] by R. D. Mindlin, Jun. Am. Soc. C. E. Stresses obtained from Mr. Mindlin's equations may vary considerably from those given by Equation (20*c*), depending on the size of the loaded area, its depth below the surface, and the depth of the plane for which the stresses are being calculated.

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[17] "Effect of Certain Discontinuities on the Pressure Distribution in a Loaded Soil", *Physics*, December, 1935, Vol. 6, p. 367.

[18] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1122.

A third assumption involved in the derivation of Equation (20*c*) is that the load must be normal to the surface of the solid. The displacements represented by Equations (18) are for normal loads only. When the point load, P, is changed to a distributed load, $p\,dx\,dy$, which is to be integrated over a finite area, this normal load requirement does not change, and the load on the finite area must also be purely normal to the surface of the solid. In some cases there is a tendency for the soil to attempt to escape laterally from under the foundation. The condition is similar to that of a compression test on a concrete cylinder in which there are tangential friction forces in the planes of contact between the test blocks and the ends of the cylinder. If the soil attempts to move outward, the friction forces on the soil surface are directed radially inward.

The resultant of the normal load and the friction force at any point makes an angle with the normal, the magnitude of the angle depending on the amount of friction. This oblique resultant force represents a more general case than the purely normal force. For the oblique force the stress distribution within the solid is quite different from that produced by the normal force alone. Boussinesq[20] has solved the problem for the oblique force with the aid of the second logarithmic potential given by Equation (17*d*). Professor O. K. Froehlich[21] has derived equations which show the effect of tangential forces in the contact plane. The existence of this phenomenon has been noticed and discussed[22] by M. L. Enger, M. Am. Soc. C. E. When there are tangential friction forces in the contact plane, the stress distribution in the soil is not that of Equations (20).

The fourth assumption on which Equation (20*c*) is based is that the solid must satisfy Hooke's law and must be a truly elastic isotropic solid in the sense required by the mathematical theory of elasticity. The fact that no elastic constants appear in Equation (20*c*) does not nullify this requirement. The derivation of this formula is based on the differential Equations (14) and these apply only to elastic isotropic solids which satisfy Hooke's law. In order to fulfill these requirements the soil would have to be capable of resisting all tensile, compressive, and shearing stresses represented by Equations (20). Granular soils, for instance, lack the capability of resisting tensile forces except in so far as they may already be under compression due to body forces. At a point on the X-axis at the surface of the solid (in Fig. 12, $z = 0$; $y = 0$; and $x = R$), the shears, p_{zx}, p_{zy}, p_{xy}, and the normal stress, p_z, all vanish. The normal stress p_x, is a radial tension and p_y is a tangential compression. Concentric circles drawn on the surface of the solid, with the origin as a common center, are contracted.

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[19] "Force at a Point in the Interior of a Semi-Infinite Solid", *Physics*, May, 1936, Vol. 7, p. 195.

[20] "Application des Potentiels", p. 182.

[21] "Druckverteilung im Baugrunde", Julius Springer, Vienna, 1934, p. 55.

[22] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1580.

If the physical properties of the soil are such that it cannot resist this tensile stress, p_x, the stress field cannot be represented by Equations (20). Isotropy requires that the elastic properties of the soil would have to be the same in all directions. Stratified silts and varved clays are not isotropic in this sense. Hooke's law requires a linear relationship between stress and strain both for loading and for unloading. In some soils the stress-strain loading curve is a straight line for small strains but in many soils it is not and, in most soils, the process is not reversible. The result is that, in general, it is not valid to consider the soil under a foundation as an elastic isotropic solid. A method of determining the stress distribution in certain types of anisotropic solids has been published by K. Wolf.[33]

The author mentions the "disturbed zone" defined by Kögler and Scheidig and remarks that this region of disturbance extends to a depth of two or three times the greatest width of the footing. Apparently, the author is thinking only of the relatively small spread-footings shown in Fig. 4. The question arises as to how this idea may be applied to Fig. 13. Two or three times the width of this footing means 200 or 300 ft in the ground. It seems scarcely likely that a region of disturbance would extend to any such depth as this. In their original description[34] of this phenomenon, Kögler and Scheidig did not publish any mathematical analysis by means of which the extent of this region could be calculated. However, they called attention to the fact that the depth of the disturbance depended on the physical properties of the soil and the intensity of the surface load. Actually this region of disturbance is a region of plastic flow.

Professor O. K. Froehlich has published[35] equations which define the boundaries between the elastic and the plastic regions. Froehlich's analysis is based on Mohr's theory of plastic flow and his equations include the size of the loaded area, the intensity of the surface load, and the physical characteristics of the soil. In either Fig. 4, or Fig. 13, the extent of this region of plastic flow would depend on these various factors. The region is not merely a function of the diameter of the footing. In addition to this "region of disturbance," Kögler and Scheidig described a "region of no stress" which lies near the surface and surrounds the loaded area. It is in this "region of no stress" that the theory of elasticity, as represented by Equations (20), includes a tensile stress. The inability of sand to resist this tensile stress is part of the cause of the existence of the "region of disturbance."

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[33] "Ausbreitung der Kraft in der Halbebene und im Halbraum bei Anisotropen Material", *Zeitschrift für ang. Math. und Mech.*, 1935, Vol. 15, No. 5, p. 249.

[34] *Die Bautechnik,* April 6, 1928, p. 206.

[35] "Druckverteilung im Baugrunde", p. 72.

The author also discusses the distribution of pressure in the contact plane and states that this becomes unimportant at depths several times the width of the footing. Again, he appears to be thinking only of the small footings of Fig. 4. It is interesting to consider this statement in connection with Fig. 13. As Professor Burmister states, this surface pressure distribution depends on the relative elastic properties of the soil and the structure. It can vary from a distribution which is a maximum at the center and a minimum at the edges to one which is a minimum at the center and a maximum at the edges. In Fig. 13 the entire depth of soil between the bottom of the footing and the surface of the rock is only eight-tenths of the diameter of the footing. It is in this region that the non-uniformity of the surface loading is most important. Even if the rock were not there, and if the soil were a homogeneous, elastic, isotropic solid, the vertical normal stresses on horizontal planes in the region down to an 80-ft depth would be seriously affected by the distribution of pressure in the contact plane. The author's remark about the unimportance of the distribution of contact pressure is apt to be misleading. There are many kinds of foundation problems in which the distribution of pressure in the contact plane cannot be ignored.

The calculation of stress distributions in the ground under a foundation represents the first half of the problem of settlement analysis. The other half is concerned with the laboratory soil tests and the application of the test results to full-sized structures. Many records of observations on the settlements of structures have been published during the past few years. In a few cases there is agreement between prediction and observation, but in many cases there are wide variations. These differences between prediction and observation could result either from the laboratory tests or from the assumed stress distribution in the soil. As far as the stress distribution is concerned, it is the writer's opinion that this so-called "Boussinesq equation" is being somewhat overworked. Foundation problems seldom conform with the assumptions which are the bases of its derivation. Furthermore, the equation is not the only means available for the analysis of stress distributions in the ground under a foundation.

The extent of actual knowledge available at the present time on stress distributions under full-sized structures is extremely limited. The writer, therefore, is inclined to question the desirability of putting into general use such charts as those proposed by the author. These charts are based on an equation taken from the mathematical theory of elasticity, and the equation itself is subject to several definite restrictions. If the assumption is granted that the soil is an elastic, isotropic solid there still remain such questions as: Is the load at the surface or below the surface? Is the load normal to the surface or are there horizontal forces in the contact plane which produce, with the vertical force, an oblique resultant? Is there an unyielding rock stratum or other discontinuity in the soil at a depth no greater than one or two times the diameter of the structure? The author's charts do not take any of these conditions into account.

If it is admitted that the soil is not an elastic, isotropic solid, it becomes necessary to abandon the use of the theory of elasticity altogether, or at least to modify the equations of that theory before they are used to calculate stress distributions in soils. Apparently, it is true that, at considerable depths below the surface, the materials of the earth behave approximately as elastic, isotropic solids. Much of the science of seismology is based on this assumption.[26] However, it is the writer's opinion that most foundation problems are concerned with surface phenomena. In general, the entire depth of soil involved in the problem is of the same order of magnitude as the diameter of the structure, and there are usually a number of other special conditions to be considered. It would seem better, therefore, to leave the door open to a consideration of these special conditions rather than to advocate the use of charts which do not take them into account at all. In any case, the charts and tables should be supplemented by a complete statement of the assumptions on which they are based so that the user may be fully aware of their limitations.

D. P. Krynine,[27] M. Am. Soc. C. E. (by letter).—The object of this discussion is: First, to emphasize the outstanding place which the chart method advanced by Professor Burmister occupies among other methods of determining the vertical pressure within a loaded earth mass; and, second, to suggest some further simplifications of the method in question.

Existing Methods of Determining Vertical Pressure Within a Loaded Earth Mass.—The problem of determining the vertical pressure, p_z, at a point within an earth mass as caused by a structure at the top (horizontal boundary plane) of that mass, has been solved, to date, in four different ways: (1) Subdividing the loaded area at the base of the structure into small rectangles and replacing each of them with an equivalent concentrated load acting at the center of the rectangle in question; (2) subdividing the loaded area by arcs of circles and their radii into small elements, and replacing the latter with equivalent concentrated loads; (3) developing theoretical (so-called "strict") formulas to express the pressure, p_z, caused by a rectangular or circular loaded area; and (4) compiling tables and tracing graphs. In practically all existing methods the validity of the Boussinesq formula is an assumption; in other words, the earth mass is considered to be an elastically isotropic body. Furthermore, only uniform load distribution through the loaded area is considered, with a few exceptions, mentioned subsequently. The writer has also proposed a method of determining the vertical pressure, p_z, within any earth mass and under uniformly or non-uniformly loaded structures of an arbitrary shape.[28]

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[26] "Physics of the Earth", Vol. VI, The National Academy of Sciences, Washington, D. C., 1933.

[27] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[28] *Transactions*, Am. Soc. C. E., Vol. 103 (1938).

For the purposes of this discussion, however, the writer's method will be left entirely out of consideration.

A characteristic representative of the methods corresponding to Item (1) is that proposed by Glennon Gilboy, Assoc. M. Am. Soc. C. E.,[29] in 1933. This method is the one most used by soil mechanics investigators in the United States. It yields quite satisfactory results in the case of uniformly loaded areas composed of one or more rectangles. A semi-graphical method elaborated by Messrs. Kögler and Scheidig,[30] of Germany, corresponds to Item (2). This method may be applied to both uniformly and non-uniformly loaded areas. Owing to certain assumptions the method furnishes approximate results. The group under Item (3) is the most numerous. The stressed condition beneath circular and rectangular uniformly and non-uniformly loaded areas has been studied by various investigators, and pressures have been found mostly at points along the vertical center line of a circle or of a rectangle, as well as under a corner of a rectangle. In this respect, works of Boussinesq, Love,[31] and other theoreticians in the field of elasticity, are to be mentioned (also those of Professor F. Schleicher,[32] of Germany, Dr. O. K. Fröhlich,[33] of Holland, and Professor F. E. Relton,[34] of Egypt). A. E. Cummings,[35] M. Am. Soc. C. E., has studied, in a comprehensive manner, a considerable number of cases of uniformly and non-uniformly loaded circular areas and has developed formulas to compute pressures at points along the vertical center line. Since the methods corresponding to Item (3) are applicable only in the case of regular, circular, or rectangular areas, they have, as a rule, only a restricted value.

So far as the writer is aware, there are two works to be classified under Item (4): That of Nathan M. Newmark, Jun. Am. Soc. C. E.,[36] and that of Dr. Wilhelm Steinbrenner,[37] of Austria. Both these investigators studied the case of uniformly loaded rectangles and computed pressures at the corner; Mr. Newmark has compiled comprehensive tables and Dr. Steinbrenner has traced ingenious graphics which permit the computation not only of the pressure, p_z, but also of the value of the elastic settlement at the corner of a uniformly loaded rectangle. Although the methods of this group refer to rectangular loaded areas only, their particular value consists in their tendency to facilitate the work of the designer, placing in his hands such time-saving devices as tables and graphs.

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[29] Progress Report of the Committee of the Society on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 784.

[30] *Die Bautechnik*, Vol. 6, p. 231 (1928).

[31] *Philosophical Transactions*, Royal Soc., Lond., Series A, Vol. 228, p. 377 (1928–29); also "Theory of Elasticity", by S. Timoshenko, p. 335 (1934); see, also, Bibliography given by Love, *Philosophical Transactions*, Royal Soc., Lond., Series A, Vol. 228, p. 377 (1928–29).

[32] *Der Bauingenieur*, Vol. 7, pp. 931, 949 (1926); also, Vol. 14, p. 242 (1933).

[33] "Druckverteilung im Baugrunde", 1934.

[34] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Vol. I, p. 57 (1936).

[35] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1072.

[36] *Circular 24*, Eng. Experiment Station, Univ. of Illinois. Urbana, Ill., 1935.

[37] *Die Strasse*, October, 1934; also a special publication by that journal entitled, "Bodenmechanik und neuzeitlicher Strassenbau", p. 75 (1936).

Chart Method Discussed.—The work of Professor Burmister belongs to Group (4); but it is more applicable to actual engineering computations than the work of Dr. Steinbrenner which is the closest to it as far as the method of attack is concerned. This is because Professor Burmister's method can be used in the case of a uniformly loaded structure of an arbitrary shape. His graphs permit the taking of readings at every foot, or, if desired, more closely, thus approaching the continuity of load distribution. The writer cannot qualify the work of Professor Burmister otherwise than as a correct step in the right direction; but, at the same time, he wishes to emphasize a few features of this method which should be improved, at least partly.

The fact that, in this method, the earth mass is considered only as an elastically isotropic body, cannot be considered as a weakness of the method since the theory of the concentration factor has not been generally adopted as yet by all soil mechanics investigators; and yet it has been felt that at greater depths the Boussinesq formula holds. The method in question, however, involves some less important disadvantages of a purely practical character; (*a*) The given structure or set of footings must be re-drawn on transparent paper to the scale of the diagram (Fig. 3), and this means a certain loss of time; and (*b*), the pressures can be determined only for the depths, for which charts have been prepared. It is true that for intermediate values the pressures can be determined by interpolation; but to use the interpolation method, at least three pressures at three different depths must be found first, before it is possible to trace a curve. Two pressures would give only a straight line and a rough interpolation.

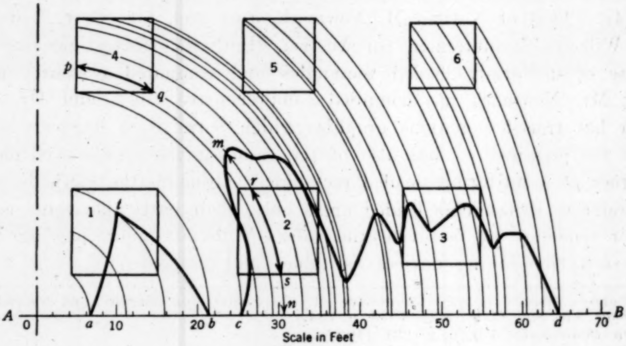

FIG. 14.

A Simplification Suggested.—Furthermore, Professor Burmister suggests that the re-drawn plan of the structure be pinned down at Point *A*, Fig. 3, and that the plan be revolved for making readings. This procedure may be substantially simplified as follows: (*a*) Charts are prepared on tracing cloth; and (*b*), the plan of the structure is drawn on ordinary paper to the scale of the chart. Fig. 14 represents the first quarter of the foundation layout in Fig. 4. If the vertical pressure is to be determined at some depth under Point *A*, all the footings should be referred to an arbitrary line, *AB*, passing through Point *A*, as follows: With Point *A* as a center, arcs are drawn through all breaks in the plans of the footings. Distances of each arc intersecting footings are added, using dividers to measure chords; for this purpose, these distances are plotted along each corresponding arc, starting from the point of intersection of that arc with Line *AB*. For instance, Segments *pq* (Footing No. 4) and *rs* (Footing No. 2), when plotted upward from Line *AB*, Fig. 14, are represented in Segment *mn*, the length of the latter being equal to the sum, $\overline{pq} + \overline{rs}$. Joining all points such as *m*, Areas *atb* and *cmd* would be obtained bounded at the bottom by the straight line, *AB*, and at the top by the irregular curves shown in Fig. 14. The vertical pressure, p_z, produced by the loaded areas, *atb* and *cmd*, at a particular point of the earth mass located at the vertical passing through Point *A*, equals that produced at that particular point by Footings Nos. 1, 2, 3, 4, 5, and 6. To measure this pressure, the transparent chart is simply placed over the layout of the foundation, making Points *A* of both drawings coincide (see Figs. 3 and 14); and Line *AB* of the layout (Fig. 14) is made to coincide with the basic horizontal line at the bottom of the chart (Fig. 3). Then readings are made along the upper boundary curve, of Areas *atb* and *cmd*.

The writer has made a comparative computation using both Professor Burmister's chart method and the suggested simplification. The saving in time obtained in the latter case can be estimated at least at 25 per cent. The more numerous the footings (or the buildings in the case of a built-up area), the more considerable will be the time saved. Such a feature of the chart method as Table 2 should be abandoned if this simplification is used. Moreover, it would be no longer necessary to pin the graph at Point *A* and rotate the plans.

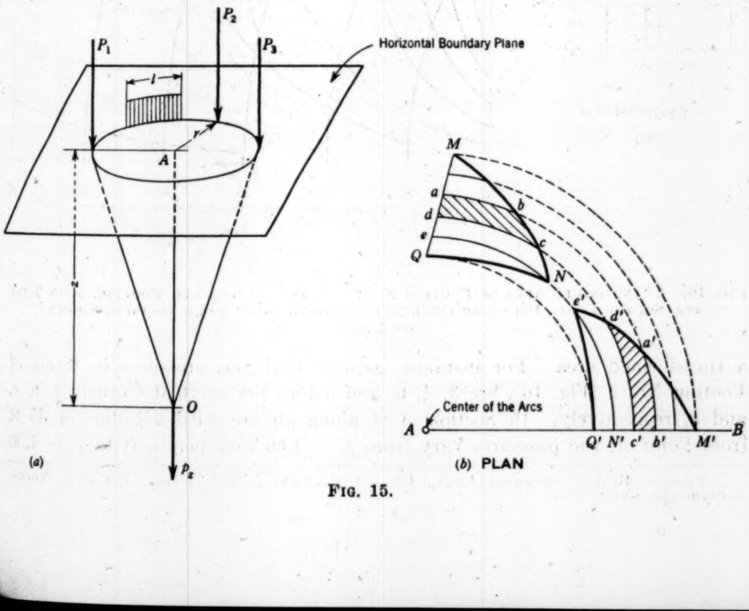

FIG. 15.

Theoretical Basis of the Simplification.—The theory underlying the suggested simplification, may be deduced from Fig. 15. In both Fig. 14 and Fig. 15, Point *A* is the projection on the horizontal boundary plane of Point *O*, at which the vertical pressure, p_z, is to be determined. Loads $P_1 = P_2 = P_3$, acting at different points of a circumference for which Point *A* is the center, produce equal vertical pressures, p_z, at Point *O* (Fig. 15(*a*)). This is because the values, *r* and *z*, influencing the vertical pressure, p_z, at Point *O*, are constant for all points along that circumference. In an analogous manner, a line load of a certain intensity and of a length, *l* (Fig. 15(*a*)), acting along a part of the circumference under consideration, may be moved to any other place on the same circumference without influencing the vertical pressure, p_z, at Point *O*. Furthermore, a loaded strip, *abcd* (Fig. 15(*b*)), may be transferred to Line *AB*, in the form of the strip, *a′b′c′d′*. The entire area, *MNQ*, if referred to Line *AB*, would be represented by the area, *Q′e′M′*. The construction used in Fig. 14 is based on the same idea.

A. A. Eremin,[38] Assoc. M. Am. Soc. C. E. (by letter).—An interesting method of computing foundation stresses by means of the Boussinesq equation, is contained in this paper. Mr. Burmister has applied his method to footings with uniform pressures. However, Fig. 3 may also be used in computing foundation stresses beneath footings with non-uniform pressure distribution. The areas of such footings should be expressed in terms of a transformed area. For instance, assume that the pressures in terms of Footing No. 2 (Fig. 16), are 3, 4, 6, and 5 tons per sq ft at Corners *a*, *b*, *c*, and *d*, respectively. In Section *A–A* along an arc with a radius of 35 ft from Point *A*, the pressures vary from $p_1 = 4.66$ tons per sq ft to $p_2 = 4.50$

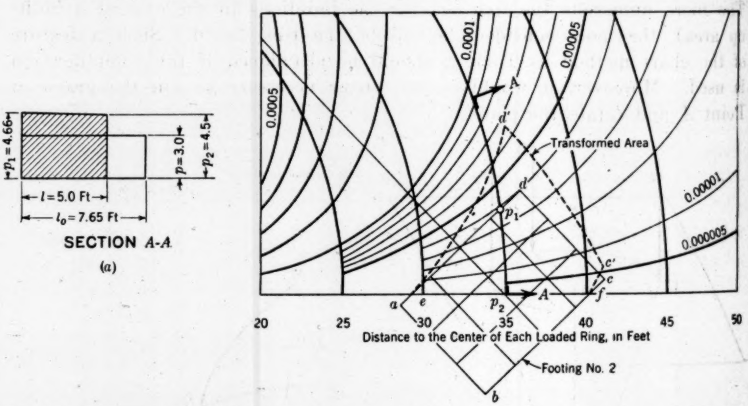

Fig. 16.—Transformed Area of Footing No. 2. Pressure Beneath Footing, *p*, in Tons per Square Foot; Pressure Chart and Footing No. 2 Same as That Shown on Fig. 3.

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[38] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

tons per sq ft (see Fig. 16(*a*)). By changing the length, l, the area of the pressure diagram may be expressed as an equivalent area of uniform pressure. The length, l_0, of the footing in Section *A–A*, Fig. 16(*a*), with a uniform pressure of $p = 3$ tons per sq ft, is,

$$l_0 = \frac{(p_1 + p_2)\, l}{2p} \quad \text{.......................} (21)$$

or, $l_0 = \frac{(4.66 + 4.50)\, 5}{2 \times 3.0} = 7.65$ ft.

The width of the transformed area may be constructed along the arcs at various distances from Point *A*. The computed area of the footing, with a uniform pressure of $p = 3$ tons per sq ft, is shown in Fig. 16 by dotted lines, *e*, *d'*, *c'*, and *f*. Area *abfe* (Fig. 16) is revolved 180° about Line *ef* and is also converted or transformed into an equivalent uniform pressure. Foundation stresses beneath this transformed area may be computed as shown by the author in Fig. 3.

If the pressure curves under the footings are irregular, the area of the pressure diagram along the arcs may be determined by planimeter measurements, or by approximate summation. The width of the transformed footing along an arc, l_0, will be,

$$l_0 = \frac{A_p}{p} \quad \text{.........................} (22)$$

in which A_p is the area of the pressure diagram, and p is the assumed uniform foundation pressure. For convenience of computations the uniform foundation pressure may be assumed to be the same as the average footing pressure.

The method presented by Mr. Burmister saves time in computing foundation stresses beneath multiple footings. Most of the labor involved in his method is that required in preparing the charts for pressures in planes at various depths, and the author, therefore, should be encouraged to present his complete set for practical use in design problems. Without such charts, if the pressures considered are produced by a single footing, foundation stresses could be determined in less time by the graphical method developed by D. P. Krynine, M. Am. Soc. C. E.,[28] in which construction of pressure charts is not required.

DONALD M. BURMISTER,[39] ASSOC. M. AM. SOC. C. E. (by letter).—The constructive nature of the discussions is gratifying, and the writer feels that the value of the paper has been materially increased thereby. The comments have revealed interesting practical uses, and have suggested valuable "short-cut" procedures.

Although it was not the intent of the writer to go into the limitations of the Boussinesq theory, it is of the greatest importance to have a clear understanding of the situation in any given case. Mr. Cummings has given an excellent discussion of how well the foundation problem conforms with the assumptions on which the Boussinesq theory is based, and

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[39] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

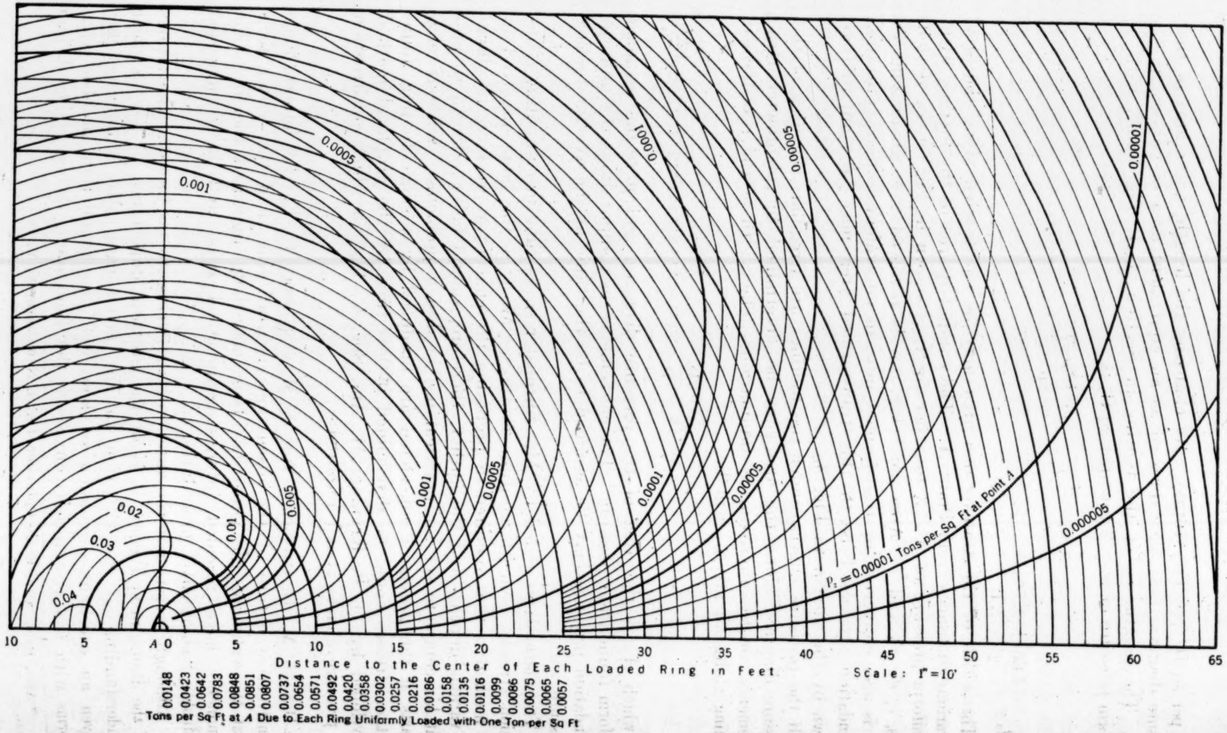

FIG. 17.—PRESSURE CHART FOR $s = 10$ FEET.

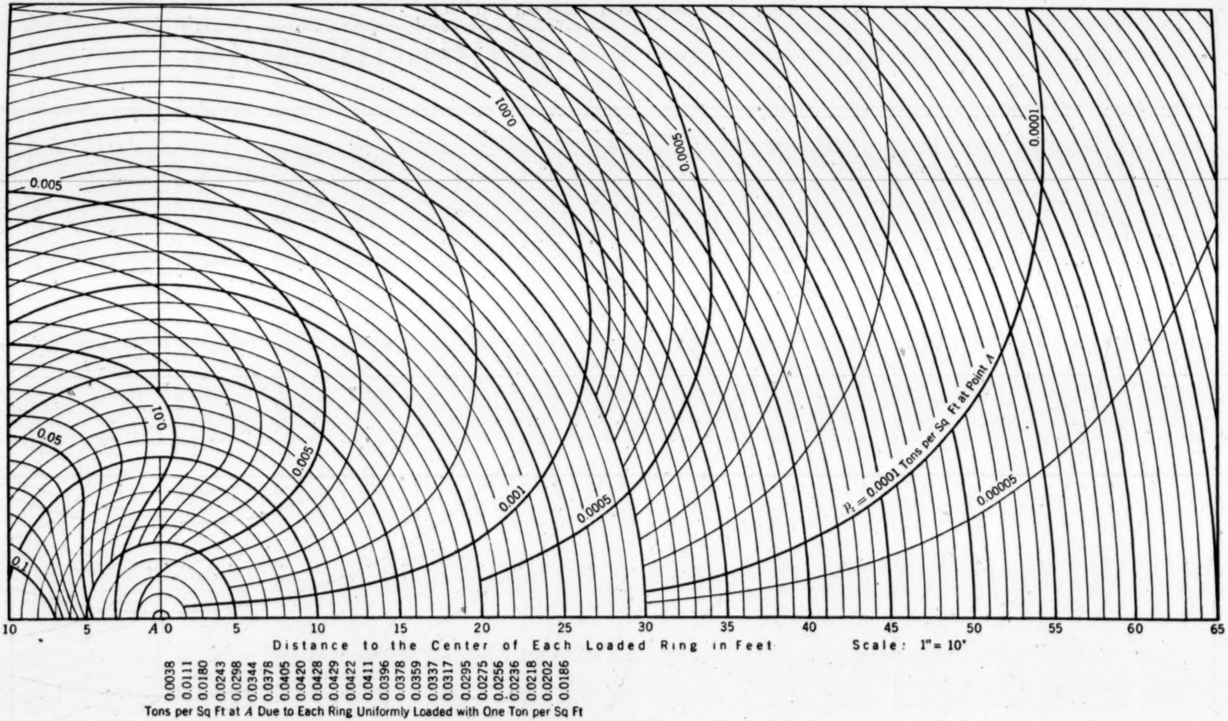

FIG. 18.—PRESSURE CHART FOR $z = 20$ FEET.

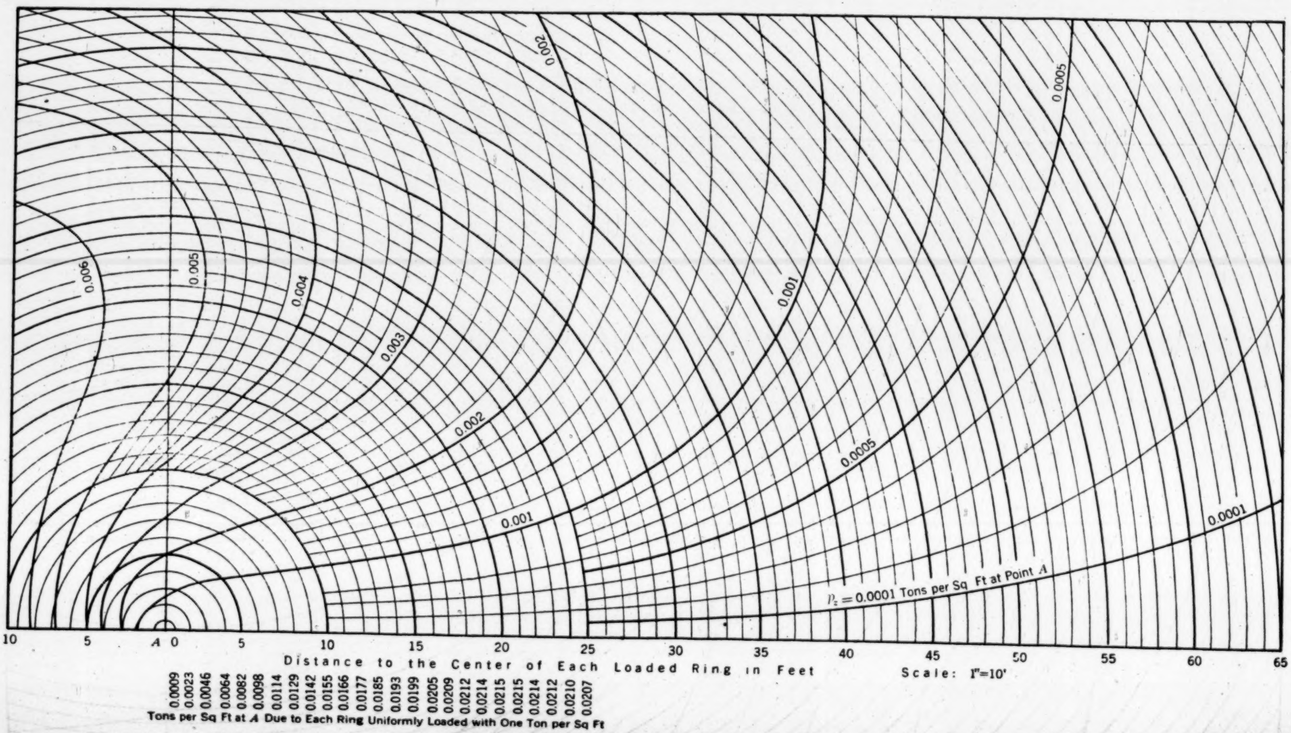

FIG. 19.—PRESSURE CHART FOR $z = 40$ FEET.

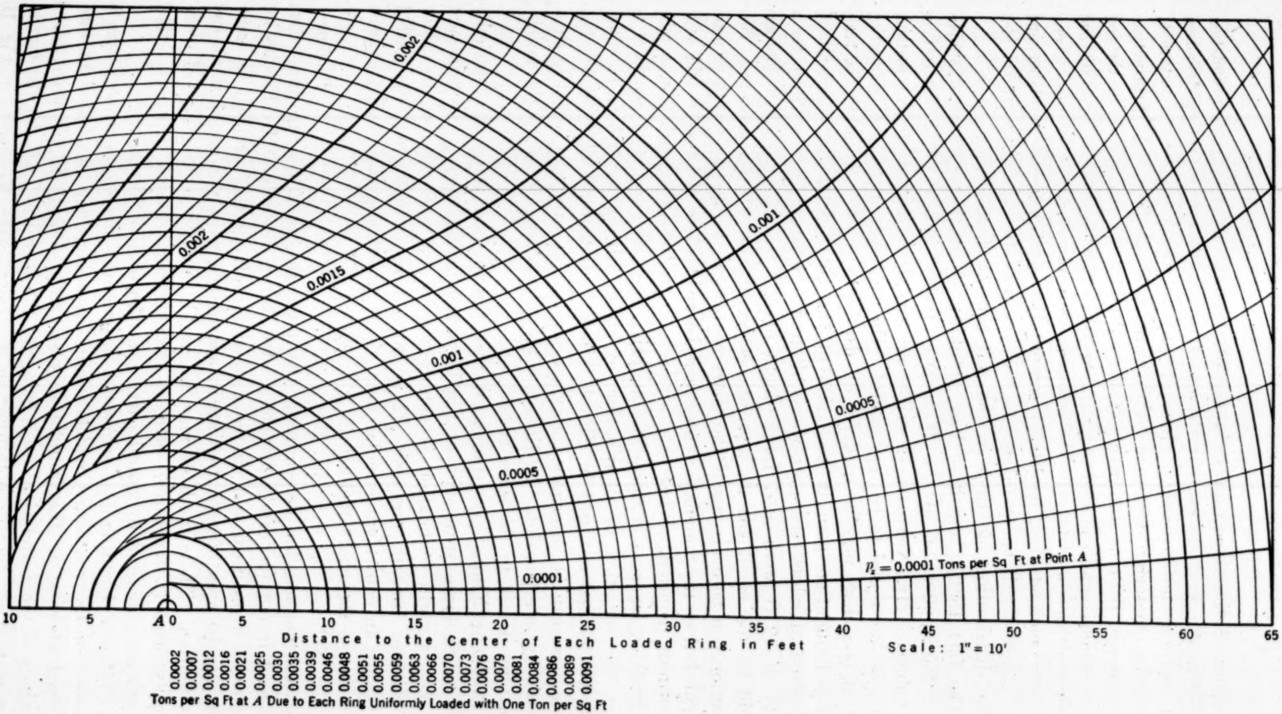

FIG. 20.—PRESSURE CHART FOR $z = 80$ FEET.

has outlined the important influence of discontinuities in the foundation soil. Mr. Paaswell shows that the exact determination of the distribution of pressure is not necessary, since present knowledge as to the nature of foundation material does not yet permit making precise determination. For example, little is known about the actual stress conditions in the soil under various conditions of loading, or the influence of stratification and discontinuities upon the way the stress is transmitted into the soil. Therefore, any method that facilitates computation and reduces the labor and time involved will be useful to the soil engineer in obtaining a good idea of the distribution of stress in the soil beneath foundations.

The fact that others have been thinking about the problem of simplifying the application of the Boussinesq theory indicates the interest in the subject. Mr. Newmark describes an analytical and graphical method, which he has found to be very useful. He has made a valuable suggestion in that one chart may be used for all depths by simply changing the scale of the foundation plan in the ratio of the chart depth to the depth of the plane at which the pressure is desired. However, if the chart for a depth of 10 ft were used for a depth of 100 ft, the individual footings would be too small for an accurate determination of the pressure. This objection can be overcome if the chart nearest to the desired depth is used, provided that the foundation plan can be reduced easily to the odd scale sometimes required. Where the stresses are to be investigated in a relatively thick layer of compressible material, such as clay, it is necessary to know the variation or distribution of stress within the layer, in which case the method outlined in Fig. 4 is most satisfactory. The distribution of pressure can be determined at three levels, obtaining the complete stress picture by interpolation. In this case the charts are used with the foundation plan drawn to the same scale.

Professor Kimball has indicated that the method is equally applicable to a single large area, such as the base of a bridge pier. The settlement of such a rigid foundation is determined by the average pressure acting in the soil beneath the pier. It is interesting to note that the method of taking an equivalent circular area yields practically the same maximum pressure as the chart method.

Mr. Nelidov has extended the usefulness of the charts by developing a practical method for determining the pressure beneath a footing, where the load is not uniformly distributed but varies linearly. Mr. Eremin has suggested a practical and simple method of solving this problem by transforming the area of each footing in accordance with the non-uniform load distribution. These methods may be readily applied to the case of eccentrically loaded structures, such as eccentrically loaded footings, bridge abutments subjected to lateral earth pressure of the fill, etc.

Professor Krynine has suggested a practical improvement and simplification in the procedure, which results in a considerable saving of time. Instead of rotating the foundation plan about Point *A*, the areas of all footings of the foundation are referred to the base line, *AB*, on the foundation plan itself, thereby obtaining an equivalent area, as shown in Fig. 14.

The base line, AB, of the footing plan is made to coincide with that of each chart in turn, and the readings to the boundary curve of the equivalent area are then added to obtain the total pressure of all footings at the point under consideration. Fig. 3, for $z = 10$ ft, was submitted as a specimen to demonstrate the type of chart recommended.

Figs. 17 to 20 are pressure charts corresponding to Fig. 3, designed for use in actual practice. The scale reduction has been adjusted purposely to 1 in. = 10 ft.

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Paper No. 1989

RAINFALL INTENSITIES AND FREQUENCIES

By A. J. Schafmayer[1], M. Am. Soc. C. E., and B. E. Grant[2], Esq.

With Discussion by Messrs. Victor H. Cochrane, LeRoy K. Sherman, J. O. Jones, Charles W. Sherman, Glen N. Cox, Garrett B. Drummond, Eugene L. Grant, Adolph F. Meyer, Clinton L. Bogert, C. H. Eiffert, C. S. Jarvis, and A. J. Schafmayer.

Synopsis

An investigation of the relation of frequency to rainfall intensity, by the statistical method, forms the basis of this paper. All the available records of excessive rainfall published by the United States Weather Bureau for nineteen cities were first examined and those exceeding certain rates were tabulated and plotted on semi-logarithmic paper. The graphs were straight lines of marked regularity in their arrangement. Then, the data from ten cities of the original nineteen were tabulated and plotted, thus using about one-half the quantity of data first used. A similar regularity and consistency in the curves were found. Finally, the data from fourteen rain-gages in the Chicago (Ill.) District were used for finding curves and formulas for the use of the City of Chicago. The formulas for intensity are rectangular hyperbolas and are plotted on hyperbolic paper as straight lines. The study is limited to excessive storms having durations of 120 min, and less.

Introduction

Rainfall intensity is involved in the design of storm sewers, as the rate of run-off is generally a function of the intensity. Adequate knowledge of intensities only becomes available with the accumulation of long-time records of automatic recording rain-gages. As the automatic gage has been in general use for only a few decades, formulas for intensity derived in the early part of the Twentieth Century were based on scanty information and should be revised.

The Board of Local Improvements of Chicago, a department of the Municipal Government which designs and builds most of the City's sewers, has

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Note.—Published in February, 1937, *Proceedings*.

[1] Engr., Board of Local Impvts., City of Chicago, Chicago, Ill.

[2] Div. Engr., Board of Local Impvts., City of Chicago, Chicago, Ill. Mr. Grant died on October 30, 1936.

been using a formula for rainfall intensity of the exponential type which was adopted in 1915. It was based on the rather scant information about the rainfall of Chicago available at that time. (The Sanitary District of Chicago, a separate and independent municipality, builds and maintains intercepting sewers which carry the dry-weather flow of the sewers of Chicago and other cities to treatment plants. Its sewers, not being owned or controlled by the City of Chicago, are not classed herein as city sewers.)

The multiplication of gages in recent years and the corresponding increase in the available data have led to a more complete study of rainfall intensities and frequencies by the Board than had previously been made, and to the search for formulas that would be the basis for more rational and economical design.

The study included the determination of a method for using data from a number of gages and an investigation of the validity of the station-year method for utilizing such data. Some of the results of that study are presented herewith in an abridged form.

The hyperbolic type of formula for intensity of rainfall, originated about forty years ago by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., has been found to fit the data more consistently than the exponential type. This study includes only storms of 2-hr duration and less, and the formulas are not intended for longer durations. However, this time is sufficient for the majority of cases in the design of city sewers.

In order to obtain a sufficient quantity of data on excessive storms, the records of a number of rain-gages in different cities are combined. These records give consistent and characteristic curves which afford a basis for judging the sufficiency of the lesser number of records which are available in a single city.

The rainfall at any given point results from the operation of a number of unknown laws. It is classified, therefore, as a chance event. If rainfalls of certain intensities and frequencies do follow a law within limits, the question arises as to how many data are needed to determine such law. The record from a single rain-gage for 100 yr is much more valuable than the record from the same gage for 25 yr, or even for 50 yr.

The tipping-bucket gage of the U. S. Weather Bureau which gives short-time records of excessive storms has not been in use long enough to give the desired quantity of data, and, therefore, some method must be used which will give reasonable results with scanty data.

The method of approach used in this paper accepts and classifies the available authentic data of a region or an area having somewhat similar rainfall characteristics and thus accumulates a number of observations great enough to indicate a rule or a law.

DEFINITIONS OF TERMS

The following definitions, abbreviations, and letter symbols apply thoroughout this paper:

"Intensity", i, is the rate of rainfall, in inches per hour.

"Duration", t, is the elapsed time, in minutes, for the excessive rainfall.

"Quantity of precipitation", d_a, is the total rainfall, in inches, for the time given.

"Frequency", F, is the average time, in years, between occurrences.

"Station-years" for a number of stations is the sum of the years of record for all stations in the group.

"Excessive rainfalls" are those equal to or greater than the smallest quantities used in the tabulations. These quantities are:

Rainfall, in inches	Duration, in minutes	Rainfall, in inches	Duration, in minutes
0.35	5	1.20	60
0.50	10	1.30	100
0.65	15	1.40	120
0.90	30		

"Chance event", is one in which the determining factors are unknown or are so complex that the result can be predicted only as an average, if at all.

DEFICIENCIES IN DATA

When one attempts to find an expression for intensity of rainfall for any given frequency for Chicago, several difficulties appear:

(*a*) The record of excessive storms is a comparatively short one.

(*b*) The number of gages having an available record is too small.

(*c*) The published records give the quantity of rainfall for successive 5-min intervals, and it is only by chance that these may correspond to maximum quantities for short durations.

(*d*) The type of gage in use does not make a correct record of high intensities of short duration.

(*e*) The published summaries of the U. S. Weather Bureau give excessive rates of rainfall for durations of 2 hr and less, only.

(*f*) A single rain-gage gives a record of the precipitation at one point only, with no indication of the variation in any direction. It merely takes a sample, 1 ft wide, out of a storm that may have a width of 50 000 ft.

(*g*) A large number of storms might pass over a single gage before it would make a record of the maximum intensity of any one storm.

(*h*) It seems that no one has ever installed enough gages to show a reasonably correct picture of an excessive rain storm. In the Chicago District (see Fig. 1 and Table 1) with an area of about 250 sq miles, there are now (1937) fourteen rain-gages of the tipping-bucket type. The distance between them is still too great to give satisfactory storm patterns, although an effort has been made to use the available information.

These deficiencies probably explain inconsistencies in the results of rainfall studies and justify efforts to increase the statistical material applicable to a specific problem so as to reduce the percentage of probable errors. The addition of more gages multiplies the data each year and increases the points of application; for instance, in one Chicago Sewer District, comprising 4 500 acres, three gages were installed in 1933 to furnish data for a special study of the District as the entire run-off passes through a pumping station equipped with recording Venturi meters. It is hoped that some definite relation may be established between rainfall intensity, impervious area, rate of run-off, total run-off, and time.

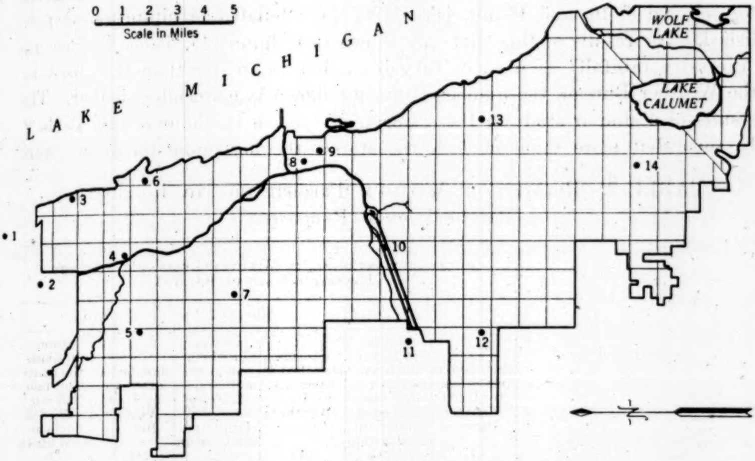

FIG. 1.—LOCATION OF RAIN-GAGES, CHICAGO, ILL. (SEE TABLE 1).

TABLE 1.—LOCATION OF RAIN-GAGES IN THE CHICAGO, ILLINOIS, DISTRICT

Rain-Gage No. (see Fig. 1)	Location	Address
	(a) UNITED STATES WEATHER BUREAU	
13	University of Chicago	Fifty-ninth Street and Ellis Avenue
8	United States Court House	Adams and Clark Streets
12	Municipal Airport	5932 South Cicero Avenue
	(b) CITY OF CHICAGO, ILLINOIS	
5	Mayfair Pumping Station	4850 Wilson Avenue
7	Springfield Avenue Pumping Station	1747 North Springfield Avenue
14	Roseland Pumping Station	351 West One-Hundred and Fourth Street
6	Lake View Pumping Station	742 Montrose Avenue
	(c) SANITARY DISTRICT OF CHICAGO	
1	Evanston Pumping Station	1457 Elmwood Avenue, Evanston
2	North Side Treatment Works	Howard Street and McCormick Road
9	Standard Oil Building	910 South Michigan Avenue
10	Electric Sub-station	Thirty-first Street and Western Avenue
11	West Side Treatment Works	Pershing Road and Fifty-second Avenue
3	Loyola University	Sheridan Road and Loyola Avenue
4	Winnetka (Discontinued September 27, 1933) (Now (1937) North Branch Pumping Station)	560 Orchard Lane, Winnetka Lawrence Avenue and Franciso Avenue

RELATION OF MAXIMUM PRECIPITATION TO TABULATED (PUBLISHED) QUANTITIES FOR 5, 10, AND 15 MINUTES

The tabulation of excessive rainfall published by the U. S. Weather Bureau shows the quantities for successive 5-min intervals, beginning with the excessive rate. The maximum may occur partly in one and partly in another of the 5-min intervals. When this occurs the published records do not show this maximum. An examination was made of all the original rain-gage charts of the excessive rainfall records for the Federal Building, in Chicago, for the 10-yr period, 1919–1928, to find the relation of maximum precipitation for selected 5, 10, and 15-min periods to the tabulated precipitation as published. The results of this study are shown in Column (1), Table 2. Because "excessive rainfall" as used in this discussion is greater than that used by the Weather Bureau, the number of storms shown is materially smaller. The results of a similar study of these storms are shown in Column (2), Table 2. In somewhat more than one-half the storms the maximum quantities were greater than those shown in the official tabulations for the given intervals. The increase averaged from 2 to 3% for that part of the record used for the curves.

TABLE 2.—RELATION OF MAXIMUM PRECIPITATION TO TABULATED PRECIPITATION AS PUBLISHED

Description	SUCCESSIVE INTERVAL, IN MINUTES, BEGINNING WITH THE EXCESSIVE RATE OF RAINFALL					
	5		10		15	
	Storms classified as excessive by U. S. Weather Bureau (1)	Storms in which the maximum rainfall rate exceeded minimum used herein (2)	Storms classified as excessive by U. S. Weather Bureau (1)	Storms in which the maximum rainfall rate exceeded minimum used herein (2)	Storms classified as excessive by U. S. Weather Bureau (1)	Storms in which the maximum rainfall rate exceeded minimum used herein (2)
Number of storms tabulated.....	65	12	62	17	52	18
Number of storms in which the maximum rate exceeded tabulated rates.................	42	6	33	11	29	11
Percentage excess of maximum rainfall over tabulated rainfall.	9.9	3.2	3.3	2.1	2.3	2.0

This record of one gage for ten years may be taken as an indication of the failure of the tabulated records to show maximum rainfall for short durations, but the data are too scanty to give general averages and no correction factor has been applied in this study.

NEW RULE OF THE UNITED STATES WEATHER BUREAU FOR TABULATION OF EXCESSIVE RAINFALL

The U. S. Weather Bureau adopted rules, in March, 1934, relating to the tabulation of excessive precipitation which are expressed by two formulas, one for the Southern States and one for the Northern States. Table 3, con-

TABLE 3.—LIMITS AT WHICH PRECIPITATION MAY BE CONSIDERED EXCESSIVE

Description	DURATION OF STORM, IN MINUTES										
	5	10	15	20	25	30	35	40	45	50	60
Depth of precipitation, in inches...........	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.80

tained in a set of instructions issued by the Weather Bureau, explains these rules It is based upon the formula:

$$d_a = t + 20 \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

in which d_a is the accumulated depth, in hundredths of inches; and t is the time, in minutes. For stations in the Southern States (including North Carolina, South Carolina, Georgia, Florida, Alabama, Mississippi, Tennessee, Arkansas, Louisiana, Texas, and Oklahoma), where brief, heavy showers are comparatively frequent, the excessive rate, for the purpose of tabulation, is based upon the formula:

$$d_a = 2\,t + 30 \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

For a rain lasting 5 min, for example, the total depth in Table 3 would be 0.40 in.; for $t = 30$ min, $d_a = 0.90$ in.; and, for $t = 60$ min, $d_a = 1.50$ in.

The U. S. Weather Bureau rule provides that the accumulated depths are to be tabulated when the fall equals or exceeds the rates determined by Equations (1) and (2) and that all tabulations must show the accumulations for 60, 80, 100, and 120 min, even where the excessive rate does not continue for such periods.

The rule for the Northern District applies to Chicago. However, the storms rated as excessive in this paper are in accordance with the definition previously given. The data published under the new rules are more complete and permit of more accurate analysis inasmuch as the precipitation for excessive storms is given for intervals as long as 120 min, although the excessive rate may not have continued for that length of time. This was not done in records previous to 1933, and any precipitation occurring within the 120-min period, but subsequent to the expiration of the excessive rate, was not shown.

STATION-YEARS AND FREQUENCIES

The data on excessive rainfall from a single station are very scanty because the length of the record giving intensities for short durations extends over only a short term of years, and if storms of long-time frequencies occur in the record, it is difficult, if not impossible, to determine their frequencies. The best way to overcome this defect seems to be to combine the records of several stations and thus to obtain sufficient data to treat them statistically. The idea of combining records of different stations in hydraulic studies is not a new one. It was used by the late W. E. Fuller, M. Am. Soc. C. E.[3], by A. F. Meyer, M. Am. Soc. C. E.[4], and by the late Allen Hazen, M. Am. Soc. C. E.[5] The data in Meyer's compilation of excessive precipitation ended with the year 1914. Since then the total data available have been more than doubled by the publications of the U. S. Weather Bureau.

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[3] *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 564.

[4] "Hydrology", by A. F. Meyer, John Wiley & Sons, 1928.

[5] "Flood Flows", by Allen Hazen, John Wiley & Sons, 1930, p. 90.

If there are 10 stations with a 30-yr record for each station, there are 300 station-yr. A record that occurs three times in the entire 30 yr has a probable frequency of 100 yr for any one station, or the chance of such a storm occurring at a given station in any year is 1 in 100. In the same manner a record that occurs thirty times has an average frequency of 10 yr, and a record that occurs sixty times has a frequency of 5 yr. If the distribution were uniform and if each station showed that a certain record occurred six times in the 30 yr, the 5-yr frequency would be apparent, and the record of one station, in so far as this record was concerned, would be as good as the record of ten stations, but the distribution is not uniform and an average frequency is sought. The use of several rain-gages distributed throughout the area considered is a reasonable method of accumulating the needed data if the record extends over a considerable number of years.

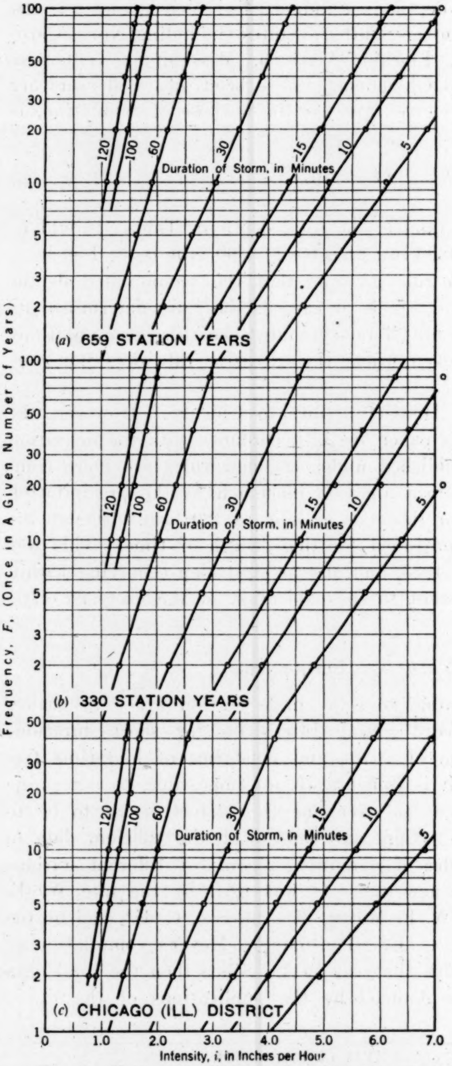

FIG. 2.—FREQUENCY-INTENSITY GRAPHS, FROM EQUATIONS OF THE FORM, $i = \frac{d_a}{t+b}$ (SEE TABLE 5).

INTENSITY CURVES ON SEMI-LOGARITHMIC PAPER

The curves from formulas for intensity of the compound interest law type when plotted on semi-logarithmic paper are straight lines, and, therefore, the intensities for any given duration have a constant difference for frequencies that vary in a geometric ratio, such as 5, 10, 20 and 40 yr.

The lines that depart most from the vertical show the greatest difference in intensities for any given frequencies; hence, in Fig. 2, the 5-min line shows the greatest differences and the 120-min line the least, as would be expected.

In 1930, C. S. Jarvis, M. Am. Soc. C. E., presented[6] a chart of frequency curves on semi-logarithmic paper, which shows the minimum and maximum annual precipitation for four stations, namely: Padua, Italy, 202 yr; Paris, France, 214 yr; Charleston, S. C., 137 yr; and, Boston, Mass., 111 yr. Commenting on "some interesting and useful relations" regarding periodic maximum and frequency curves, Mr. Jarvis calls attention to the fact that the mean monthly or the mean annual precipitation seems to form a simple arithmetical progression, whereas the related time intervals form a geometrical progression. His graph[6] illustrates, for the yearly precipitation, "how nearly the straight-line relation holds for frequency curves when they are thus plotted on semi-logarithmic paper." Similar results are apparent for monthly periods.

STATIONS STUDIED

The records of nineteen U. S. Weather Bureau Stations for all storms occurring in the years, 1915 to 1932 (several thousand in number), that were classified as excessive by the Weather Bureau, were examined for intensities and durations coming within the limits of this study. The records selected were summarized by classes and added to the Meyer summary for the earlier years. No correction factor was used for the precipitation corresponding to any duration.

The nineteen stations selected were: Albany, N. Y.; Asheville, N. C., Boston, Mass., Cairo, Ill., Chicago, Ill., Cincinnati, Ohio, Cleveland, Ohio, Detroit, Mich., Dodge City, Kans., Elkins, W. Va., Grand Haven, Mich., Indianapolis, Ind., Knoxville, Tenn., Madison, Wis., Memphis, Tenn., Moorhead, Minn., Pittsburgh, Pa., St. Paul, Minn., and Yankton, S. Dak. Independent studies and graphs were made for each of these nineteen stations for the 37 yr covered by the 1915–1932 data (18 yr) combined with the Meyer data (19 yr). A considerable degree of similarity was found with only such discrepancies as might be expected from data of such limited nature covering chance occurrences.

Graphs and formulas were derived from these data, covering the quantities, intensities, frequencies, and durations for excessive storms. It was found, by plotting the tabulated statistical results on semi-logarithmic paper for storms of 5, 10, 15, 30, 60, 100, and 120-min duration, that the data were best represented by straight lines, as shown in Fig. 3(*a*). The precipitation, in inches of rainfall, was plotted on the arithmetical scale and the number of storms exceeding the given depth of rainfall on the logarithmic scale. The means of class intervals were used in plotting the curves. For example, on the 30-min graph shown in Fig. 3(*a*), 353 storms were plotted against 1.05 in. of rainfall for the class interval extending from 1 in. to 1.10 in., it being assumed that as many storms in the class interval exceeded the mean as were below the mean. The class intervals vary for storms of different durations, but are the same for each duration in all these studies. With this

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[6] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), Fig. 3, p. 400.

TABLE 4.—SUMMARY OF RECORDS OF STORMS AT TEN STATIONS, 1900 TO 1932

Precipitation, in inches (1)	Knoxville, Tenn. (2)	Memphis, Tenn. (3)	Indianapolis, Ind. (4)	Cincinnati, Ohio (5)	Detroit, Mich. (6)	Chicago, Ill. (7)	St. Paul, Minn. (8)	Cairo, Ill. (9)	Yankton, S. Dak. (10)	Dodge City, Kans. (11)	Total (12)	Summary (13)
(*a*) PRECIPITATION IN 5 MINUTES												
0.35 to 0.39	14	12	20	11	11	14	10	9	10	8	119	227
0.40 to 0.44	4	4	6	3	8	3	4	3	7	4	46	108
0.45 to 0.49	4	1	4	3	4	3	6	1	1	3	30	62
0.50 to 0.54	1	0	3	4	1	3	2	1	2	4	21	32
0.55 to 0.59	0	1	0	0	0	2	2	0	0	1	6	11
0.60 to 0.69	0	1	0	0	0	0	0	1	0	0	2	5
0.70 to 0.79	0	1	0	1	1	0	0	0	0	0	3	3
0.80 to 0.89	0	0	0	0	0	0	0	0	0	0	0	0
Total......	23	20	33	22	25	25	24	15	20	20	227	.....
(*b*) PRECIPITATION IN 10 MINUTES												
0.50 to 0.54	12	17	16	14	13	7	12	22	17	6	136	396
0.55 to 0.59	9	13	13	10	3	10	6	7	12	3	86	260
0.60 to 0.69	9	5	16	8	10	7	7	8	11	8	89	174
0.70 to 0.79	4	3	4	2	7	5	6	5	4	5	45	85
0.80 to 0.89	2	0	4	2	1	4	1	0	1	0	15	40
0.90 to 0.99	2	1	2	2	1	2	2	2	1	4	19	25
1.00 to 1.09	0	1	0	0	0	0	0	0	0	2	3	6
1.10 to 1.19	0	0	0	0	0	0	0	0	0	0	0	3
1.20 to 1.29	0	1	0	0	0	0	0	0	0	0	1	3
1.30 to 1.39	0	0	0	0	1	0	0	0	0	0	1	2
1.40 to 1.49	0	0	0	1	0	0	0	0	0	0	1	1
Total......	38	41	55	39	36	35	34	44	46	28	396	.....
(*c*) PRECIPITATION IN 15 MINUTES												
0.65 to 0.69	5	13	11	5	6	6	5	13	7	4	75	334
0.70 to 0.79	14	14	15	9	8	11	7	14	12	10	114	259
0.80 to 0.89	4	3	14	5	4	6	9	5	7	5	62	145
0.90 to 0.99	3	4	2	2	8	5	1	5	3	5	38	83
1.00 to 1.09	4	1	5	2	1	1	2	1	1	0	18	45
1.10 to 1.19	0	0	2	0	0	3	0	0	1	4	10	27
1.20 to 1.39	3	0	1	2	0	1	2	2	1	2	14	17
1.40 to 1.59	0	0	0	0	0	0	0	0	0	0	0	3
1.60 to 1.79	0	1	0	1	0	0	0	0	0	0	2	3
1.80 to 1.99	0	0	0	0	1	0	0	0	0	0	1	1
Total......	33	36	50	26	28	33	26	40	32	30	334	.....
(*d*) PRECIPITATION IN 30 MINUTES												
0.90 to 0.99	6	13	10	11	7	4	4	9	13	8	85	295
1.00 to 1.09	5	10	8	5	5	8	4	8	6	3	62	210
1.10 to 1.19	2	9	7	1	4	6	4	9	5	5	52	148
1.20 to 1.39	4	2	5	0	6	7	5	6	7	5	47	96
1.40 to 1.59	5	1	2	4	2	2	3	1	2	6	28	49
1.60 to 1.79	1	0	1	1	0	1	0	3	2	0	9	21
1.80 to 1.99	1	0	1	0	0	0	0	0	0	1	3	12
2.00 to 2.19	1	0	0	0	0	1	2	0	0	0	4	9
2.20 to 2.39	0	0	0	1	1	0	0	0	0	1	3	5
2.40 to 2.59	0	0	0	1	0	0	0	0	0	0	1	2
2.60 to 2.79	0	0	0	0	0	0	0	0	0	0	0	1
2.80 to 2.99	0	1	0	0	0	0	0	0	0	0	1	1
Total......	25	36	34	24	25	29	22	36	35	29	295	.....

method of plotting, double points occur on certain lines as on the 10-min line in Fig. 3(*b*). The list of excessive storms shown in Table 4 indicates a total of three storms for each of the intervals, 1.10 to 1.19 in. and 1.20 to 1.29 in. The statistical results for the 330 station-yr study, given in Table 4, is typical. The respective class intervals are shown in Column (1) and the number of storms shown in Column (13) were the values plotted for each class interval. The deviation of individual points from a straight line was a matter of only a few hundredths of an inch of rainfall, except in

TABLE 4.—(*Continued*)

(*e*) PRECIPITATION IN 60 MINUTES

Precipitation, in inches (1)	Knoxville, Tenn. (2)	Memphis, Tenn. (3)	Indianapolis, Ind. (4)	Cincinnati, Ohio (5)	Detroit, Mich. (6)	Chicago, Ill. (7)	St. Paul, Minn. (8)	Cairo, Ill. (9)	Yankton, S. Dak. (10)	Dodge City, Kans. (11)	Total (12)	Summary (13)
1.20 to 1.29	2	6	3	5	3	5	5	6	5	2	42	209
1.30 to 1.39	7	4	2	2	4	6	3	4	3	6	41	167
1.40 to 1.49	3	5	3	2	2	4	3	1	4	3	30	126
1.50 to 1.59	1	3	4	1	0	1	2	6	2	0	20	96
1.60 to 1.79	3	5	3	2	3	2	1	3	6	3	31	76
1.80 to 1.99	2	1	0	1	0	2	0	1	3	2	12	45
2.00 to 2.19	0	0	3	2	1	0	1	2	1	3	13	33
2.20 to 2.39	1	0	0	1	1	2	2	0	1	2	10	20
2.40 to 2.59	2	0	0	0	0	0	1	0	0	0	3	10
2.60 to 2.79	0	0	1	1	0	0	1	0	0	0	3	7
2.80 to 2.99	0	0	0	0	0	0	0	0	0	0	0	4
3.00 to 3.24	0	0	0	0	1	0	0	1	0	0	2	4
3.25 to 3.49	0	1	0	0	0	0	0	0	0	1	2	2
Total......	21	25	19	17	15	22	19	24	25	22	209	.....

(*f*) PRECIPITATION IN 100 MINUTES

Precipitation, in inches (1)	Knoxville, Tenn. (2)	Memphis, Tenn. (3)	Indianapolis, Ind. (4)	Cincinnati, Ohio (5)	Detroit, Mich. (6)	Chicago, Ill. (7)	St. Paul, Minn. (8)	Cairo, Ill. (9)	Yankton, S. Dak. (10)	Dodge City, Kans. (11)	Total (12)	Summary (13)
1.30 to 1.39	7	3	2	1	4	6	3	5	2	8	41	192
1.40 to 1.49	3	2	2	3	2	4	4	3	3	4	30	151
1.50 to 1.59	1	3	5	1	0	2	2	3	2	1	20	121
1.60 to 1.79	2	4	2	4	1	2	2	3	4	3	27	101
1.80 to 1.99	2	5	2	0	3	2	2	3	5	1	25	74
2.00 to 2.19	0	1	2	1	1	1	1	4	2	1	14	49
2.20 to 2.39	0	2	2	2	0	2	1	0	0	3	12	35
2.40 to 2.59	2	1	0	1	0	0	1	1	1	1	8	23
2.60 to 2.79	1	0	0	1	2	0	1	0	0	1	6	15
2.80 to 2.99	1	0	1	0	0	0	1	0	0	0	3	9
3.00 to 3.24	0	0	0	0	0	0	0	1	1	0	2	6
3.25 to 3.49	0	0	0	0	0	0	0	0	0	0	0	4
3.50 to 3.74	0	0	0	0	0	0	0	1	0	0	1	4
3.75 to 3.99	0	0	0	0	1	0	0	0	0	0	1	3
4.00 to 4.49	0	0	0	0	0	0	0	0	0	0	0	2
4.50 to 4.99	0	1	0	0	0	0	0	0	0	1	2	2
Total......	19	22	18	14	14	19	18	24	20	24	192	.....

(*g*) PRECIPITATION IN 120 MINUTES

Precipitation, in inches (1)	Knoxville, Tenn. (2)	Memphis, Tenn. (3)	Indianapolis, Ind. (4)	Cincinnati, Ohio (5)	Detroit, Mich. (6)	Chicago, Ill. (7)	St. Paul, Minn. (8)	Cairo, Ill. (9)	Yankton, S. Dak. (10)	Dodge City, Kans. (11)	Total (12)	Summary (13)
1.40 to 1.49	3	2	2	3	2	4	4	3	3	4	30	152
1.50 to 1.59	1	3	5	1	0	3	2	3	2	1	21	122
1.60 to 1.79	2	2	2	2	1	2	2	3	4	3	23	101
1.80 to 1.99	2	4	2	2	3	2	2	3	5	1	26	78
2.00 to 2.19	0	3	2	1	1	1	1	3	1	1	14	52
2.20 to 2.39	0	2	2	2	0	2	1	1	0	3	13	38
2.40 to 2.59	1	1	0	1	0	0	1	0	1	1	6	25
2.60 to 2.79	1	0	0	1	2	0	1	1	0	0	6	19
2.80 to 2.99	1	1	1	0	0	0	0	0	1	0	4	13
3.00 to 3.24	0	0	0	0	0	0	1	1	1	1	4	9
3.25 to 3.49	1	0	0	0	0	0	0	0	0	0	1	5
3.50 to 3.74	0	0	0	0	0	0	0	1	0	0	1	4
3.75 to 3.99	0	0	0	0	1	0	0	0	0	0	1	3
4.00 to 4.49	0	0	0	0	0	0	0	0	0	0	0	2
4.50 to 4.99	0	1	0	0	0	0	0	0	0	1	2	2
Total......	12	19	16	13	10	14	15	19	18	16	152	.....

certain cases where the indicated frequency exceeded 100 yr, which is to be expected, as the correct frequency of rare storms is always a matter of uncertainty in a short record.

Some frequency lines are also shown on Fig. 3(*a*). The 10-yr frequency is represented by the horizontal line through the point, 65.9, which represents one storm in 10 station-yr. A frequency-quantity graph was derived directly from Fig. 3(*a*) by plotting the values of the depths for the 100-yr frequency line and the depths for the 5-yr frequency line, and drawing straight lines

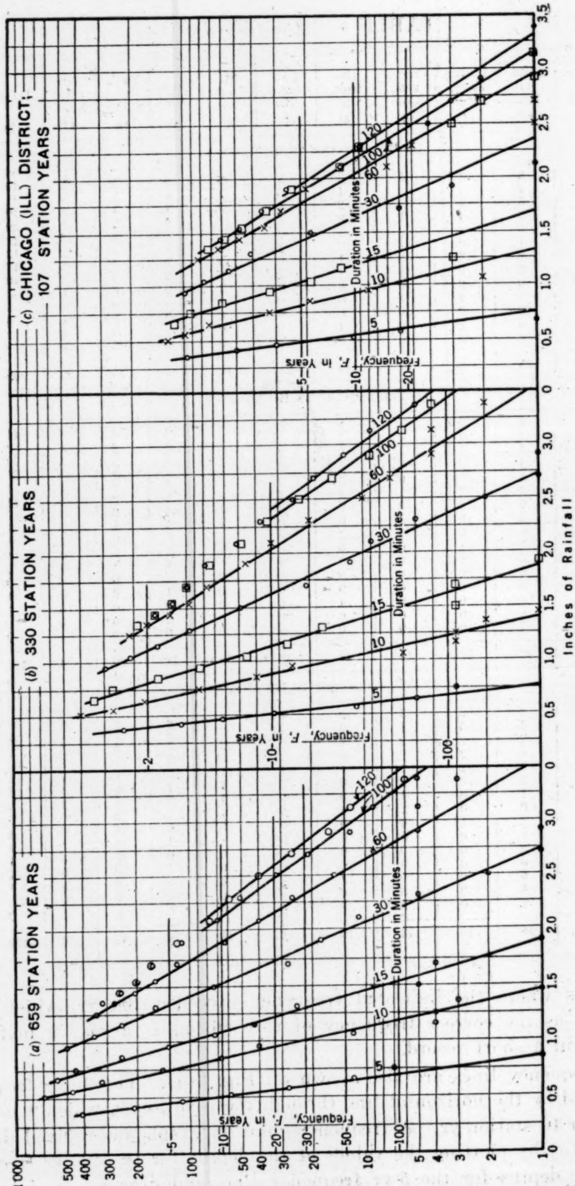

FIG. 3.—GRAPHS OF STATISTICAL DATA.

between the points plotted. In other words, such a graph would be similar to Fig. 3(*a*), except that the curves are inverted, and the logarithmic scale then gives the frequency for any number of years from 1 to 100.

Fig. 4 gives the relation between frequency and intensity for the nineteen stations. These graphs were obtained directly from the aforementioned frequency-quantity graph by converting the quantities for the various durations into intensities, or rates per hour, and by plotting the frequency on the logarithmic scale. This chart shows frequencies up to 100 yr, but as the data cover 659 station-yr, it might be reasonable to extend the graphs to an even higher limit.

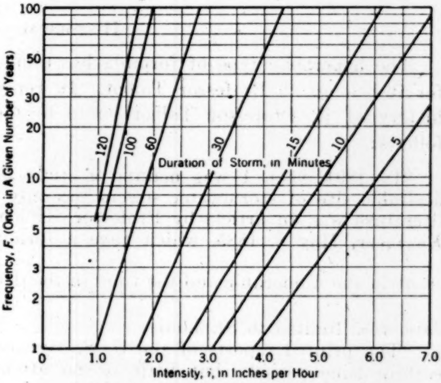

FIG. 4.—FREQUENCY-INTENSITY CURVES OF STATISTICAL DATA; 659 STATION-YEARS.

Although the relation between durations and depths of rainfall for any frequency appears to follow a law as expressed by logarithmic formulas, it was found that the hyperbolic type of formula represented by,

$$i = \frac{A}{t + b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

was the best form for showing the relation between intensity and duration for storms not exceeding 120 min of a given frequency. (In Equation (3), A and b are abstract constants, as explained subsequently under the heading "The Constants in the Formulas for Intensity".) Formulas of this type were derived from the 659-station-yr data. These formulas were then plotted, as shown by dots in Fig. 2(*a*), and furnish a basis of comparison with the graphs as derived from the statistical data as shown on Fig. 4. The points as plotted from these formulas give a straight line for each duration, indicating a remarkable consistency in the formulas. These graphs give values very close to those shown by the statistical graphs, except in the case of the

TABLE 5.—VARIATIONS OF CONSTANTS IN THE FORM OF EQUATION (3)

Item No.	Formulas for intensity of rainfall, *i*, used in diagrams	FRRQUENCY, IN YEARS											
		2		5		10		20		40		80	
		Numerator, *A*	*b*	Numerator, *A*	*b*	Numerator, *A*	*b*	Numerator, *A*	*b*	Numerator, *A*	*b*	Numerator, *A*	*b*
1	2(*a*).........	97	16	126	18	152	20	177	21	202	22	227	23
2	2(*b*) and 5....	102	16	138	19	166	21	193	22	222	24	251	25
3	2(*c*).........	108	17	137	18	157	18	181	19	202	19	.....	..

5-min line, and probably the values given by the formula are more nearly correct for this duration than the values given by the statistics. The curves of Fig. 2(a) are variations of Equation (3), as defined by Item No. 1, Table 5.

HISTORICAL

The hyperbolic type of formula has been used since 1891, and the credit for it belongs to Professor Talbot. In reply to an inquiry concerning the history of it, Professor Talbot wrote to the writer on July 31, 1933, as follows:

> "In 1891 when I was making a study of rates of rainfall as affecting desirable size of stormwater sewers the only reference found in engineering literature was an article by Professor Nipher of St. Louis in the *American Engineer,* May 8, 1885, which gave a formula for intensity having a constant in the numerator and the time in the denominator $\left(i = \frac{360}{t}\right)$. Nipher's data were limited to St. Louis.
>
> "The printed reports of the U. S. Weather Bureau gave the rainfall for certain time periods, but little or no attention was given to the rate or intensity of the rainfall as such. After plotting the rates against time for all the data available in the published records of the U. S. Weather Bureau and for different sections of the Country and for the larger cities, my studies indicate that a formula of the form of a constant in the numerator and the time plus a constant in the denominator would best envelop the plotted points for 'rare rainfalls' and another of the same form those for what were called 'ordinary rainfalls.' The studies further brought out that the maximum rates shown by the accumulation of data were not greatly different for the several sections of the country considered (the maximum rates, not their frequency), a conclusion quite different from the views then commonly held.
>
> "The studies were published as 'Maximum Rates of Rainfall' in *The Technograph,* No. 6, 1891–92, University of Illinois. This article was republished in whole or in part in engineering journals and has been abstracted or referred to through the years in discussions and publications here and abroad. For thirty years hardly a year went by without seeing a reference to it in print.
>
> "So far as the form of the formula is concerned, it was devised by me, and I have seen nothing since that indicated a prior use by another. Mr. Kuichling told me later that he had used a straight-line formula in 1889. Mr. Rudolph Hering once showed me the rainfall plots he had made for New York City, but after reading my article he had not gone further in the study of the subject.
>
> "In the years intervening since the publication of 'Maximum Rates of Rainfall' there has been a great and valuable accumulation of rainfall statistics. It would not be strange if now a new constant or even a new form of formula would better fit the data. Naturally I am pleased to learn that the form of the formula devised by me so many years ago is useful in expressing the intensity of rainfall for Chicago."

Some of the hyperbolic formulas in common use are of the Talbot type. Formulas of this type that are used in other cities have been studied in connection with the frequency-intensity graphs, Figs. 2(a) and 4, and also on hyperbolic paper with varying results. Considerable variation in frequency has also been found for formulas of the exponential type such as,

$$i = \frac{A}{(t + b)^n} \quad \text{.........................(4)}$$

Thus far, it has not been found, in any of these investigations, that a formula of this type would give a constant frequency for all durations between 5 and 120 min. Rectangular hyperbolas fit the data better than any other curves and the Talbot type of formula is the simplest expression found for them for a given frequency.

LIMITATIONS OF FORMULA FOR INTENSITY

Although the hyperbolic type of formula appears to be singularly adapted to express the intensity for a given frequency, for excessive storms of durations not exceeding 120 min, it is, from its fundamental structure, not applicable for storms of long duration, as shown by the following analysis. The formula,

$$i = \frac{222}{t + 24} \quad \text{.........................(5)}$$

gives the intensity or rate per hour for a 40-yr frequency curve. If it is multiplied by $\frac{t}{60}$, the result will be the precipitation, d, for the time, t. Then:

$$d_a = \frac{222}{t + 24} \times \frac{t}{60} = \frac{3.7\,t}{t + 24} \text{.....................(6)}$$

The factor, $\frac{t}{t + 24}$, will always be less than unity for any value of t and, therefore, the value of d_a cannot exceed 3.7 in. for any time. When t becomes large, the change in d_a is small for successive changes in t, as shown by the following tabulation:

Duration, t, in minutes	Precipitation, d_a, in inches
120	3.08
600	3.56
1 440	3.64
2 880	3.67

As the rainfall in 10 hr, or 24 hr (600 min, or 1 440 min) may greatly exceed the corresponding values shown for d_a, Equation (6) is evidently not applicable for the longer durations.

A formula for depth of rainfall was derived as a function of log-time which fits closely the data used in the hyperbolic formula (Equation (3)) from $t = 10$ to $t = 120$ min and does not have the defect of reaching a maximum.

In the case of the 40-yr frequency, it was:

$$d_a = 1.89 \log t - 0.80 \text{.....................(7)}$$

The hyperbolic form is simpler for computation and is the one preferred when the time extends only to 120 min.

CURVES FOR TEN SELECTED STATIONS

After analyzing the data for the entire nineteen stations results were examined to discover the ten stations having characteristics similar to Chicago conditions. This was done for the purpose of securing data that corresponded most closely to the Chicago conditions, in order to amplify limited records available locally. The ten stations were: Cairo, Ill., Chicago, Ill., Cincinnati, Ohio, Detroit, Mich., Dodge City, Kans., Indianapolis, Ind., Knoxville, Tenn., Memphis, Tenn., St. Paul, Minn., and Yankton, S. Dak.

After selecting these stations an independent study of the records of the U. S. Weather Bureau for the 33 yr from 1900 to 1932, inclusive, was made and the excessive storms were tabulated as shown in Table 4. Similar tables, which are not shown, were prepared for the studies for 659 station-yr, and for Chicago. Graphs were made similar to the graphs prepared for the 659 station-yr in the preliminary studies.

It was found that the use of the data for storms of low intensity added little to the value or the accuracy of the graphs. In general, Table 4 includes only storms having intensities equal to or greater than those indicated in the definition of excessive storms. These intensities include all frequencies likely to be of interest to the designer of storm sewers for the reason that sewers should be designed to provide for storms of greater intensity than those likely to occur once a year or oftener.

Fig. 3(*b*) shows the statistical data for ten stations for the years, 1900 to 1932, inclusive, or a total of 330 station-yr plotted on semi-logarithmic paper, with the depth of rainfall shown on the arithmetical scale and the number of storms on the logarithmic scale Fig. 3(*b*) corresponds, for the ten stations, with Fig 3(*a*) showing the nineteen stations, and is similar in the relative position of the graphs. More than 1 800 excessive storms were tabulated and studied to make the chart for ten stations.

Fig. 2(*b*) shows frequency-intensity graphs for 330 station-yr plotted from the hyperbolic formulas which were derived from the statistical data. For the ten stations, Fig. 2(*b*) is comparable with Fig. 2(*a*) for the nineteen stations. With the exception of the 5-min and 10-min lines, the graphs fit the original data so nearly that they are practically coincident. From the formulas (Equation (3), and Items Nos. 1 and 2 in Table 5), the 5-min line is probably more nearly correct than the original data, as it gives larger values. The equation for the 60-min curve on this chart may be written in two forms:

$$i = 1.03 + 1.00 \log F \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

or,

$$F = 0.0933\ e^{2.3i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which *e* is the Naperian base. The curves of Fig. 2(*b*) are variations of Equation (3), as defined by Item No. 2, Table 5.

Derivation of a Formula for the 60-Minute Graph on the Frequency Intensity Chart for 330 Station-Years (See Fig. 2(*b*))

Any curve that appears as a straight line on semi-logarithmic paper has an equation of the form:

$$y = k\ e^{rx} \quad \dots\dots (10)$$

in which k is a constant, and r is the rate of increase. Substituting F, the frequency, and i, the intensity, for y and x, Equation (10) becomes:

$$F = k\ e^{ri} \quad \dots\dots (11)$$

Taking values of i from the curve for the 60-min storm (Fig. 2(*b*)) when $F = 100$ and $F = 1$ and substituting them in Equation (11): $100 = k\ e^{3.03\ r}$; and, $1 = k\ e^{1.03\ r}$. Then, converting to the logarithmic form:

For $F = 100$:

$$\log 100 = \log k + 3.03\ r \log e \quad \dots\dots (12a)$$

for $F = 1$:

$$\log 1 = \log k + 1.03\ r \log e \quad \dots\dots (12b)$$

and, by subtraction,

$$2 = 2.00\ r \log e \quad \dots\dots (13)$$

Finally, $r = \dfrac{1}{0.4343} = 2.30$; and, by substituting this value in Equations (12) and solving simultaneously: $\log k = -1.03 = 8.97 - 10$; $k = 0.0933$; and, substituting in Equation (11):

$$F = 0.0933\ e^{2.3i} \quad \dots\dots (14)$$

Equation (14) may be reduced to the formula for intensity, as follows:

$$\log F = \log 0.0933 + 2.3\ i \log e \quad \dots\dots (15)$$

and since $2.3 \log e = 1$,

$$\log F = \log 0.0933 + i \quad \dots\dots (16)$$

and,

$$i = \log F - \log 0.0933 = \log F + 1.03 \quad \dots\dots (17)$$

which may also be obtained by inspection of Fig. 2(*b*). The equation for any other line on Fig. 2(*b*) can be obtained in a similar manner:

For $t = 120$,

$$i = 0.57 + 0.61 \log F \quad \dots\dots (18a)$$

for $t = 100$,

$$i = 0.67 + 0.70 \log F \quad \dots\dots (18b)$$

for $t = 60$,

$$i = 1.03 + 1.00 \log F \quad \dots\dots (18c)$$

for $t = 30$,

$$i = 1.80 + 1.46 \log F \qquad (18d)$$

for $t = 15$,

$$i = 2.73 + 1.87 \log F \qquad (18e)$$

for $t = 10$,

$$i = 3.32 + 2.03 \log F \qquad (18f)$$

and, for $t = 5$,

$$i = 4.18 + 2.25 \log F \qquad (18g)$$

The constant is the value of the intensity for the 1-yr frequency and the coefficient of log F is the 10-yr intensity minus the constant. As the logarithm of 10 is 1, it is apparent that the value of the 10-yr intensity is the sum of the constant and the coefficient.

THE CHICAGO CURVES

The methods of studying the nineteen stations and, later, the ten stations were then applied to the data of the U. S. Weather Bureau for the Chicago Station. Following the procedure previously described for the nineteen stations, a chart was prepared with graphs showing storms from 5 to 60-min duration, based on the statistical data for a period of 33 yr, which were all the official data available for that station, for excessive storms of those durations.

Other rain-gages in the Chicago District furnished records that might be combined with the data of the Weather Bureau Station, to give charts based on nearly three times the statistics of the single station. Thirteen rain-gages of the automatic type furnished the precipitation data for the City of Chicago used in this study. The location of the gages is shown in Fig. 1 and Table 1. Of these, two are maintained by the U. S. Weather Bureau (three in 1937), four by the City, and seven by the Sanitary District of Chicago (six in 1937). The distribution of the gages, although not ideal, is still fairly good as may be seen by reference to the outline map of Chicago in Fig. 1.

Three gages were established in 1933. Five gages have records beginning in 1926. The records of excessive precipitation from two of the automatic gages of the Federal Government extend back to 1919 and 1900. It is quite possible in combining these records that too much emphasis is given to records of recent years, but there is no obvious method for balancing this defect except by adding new data as they occur. Seven years of observation with the present number of gages will about double the data.

The official station (for observation purposes) of the U. S. Weather Bureau, in Chicago, was on the United States Court House (formerly the Federal Building) previous to 1926, when the official station was moved. Rosenwald Hall and the adjoining campus at the University of Chicago then became the official station (for observation purposes), although it had been in existence as an unofficial station of the Weather Bureau since 1916. The station on the U. S. Court House has been continued, but its records have not been published since 1925.

Curves Applying to the Chicago District

Fig. 3(*c*) shows the statistical data for the combined records of two Weather Bureau gages, and the several gages maintained by the City of Chicago and the Sanitary District of Chicago to the end of 1933, amounting to 107 station-yr. The longest record for one gage is for 34 yr. Fig. 2(*c*) shows frequency-intensity graphs for storms from 5 to 120-min duration, based on formulas derived from the foregoing statistics. The plotting of the formulas in this case shows the same consistency of the points on the lines and the relative position of the graphs that was shown in Figs. 2(*a*) and 2(*b*). The curves of Fig. 2(*c*) are variations of Equation (3), as defined by Item No. 3, Table 5.

Semi-Hyperbolic Paper

The Talbot formula, Equation (3), is a special form of the general equation $xy = k$, representing a rectangular hyperbola. Since the value of x varies as a hyperbola, the locus of the equation should be a straight line when plotted on semi-hyperbolic paper, that is paper graduated in one dimension on an arithmetical scale and in the other dimension on a hyperbolic scale.

As an investigation failed to disclose any hyperbolic paper of suitable size on the market, paper of this type was prepared. For the purpose a form of semi-hyperbolic paper of letter-sheet size was prepared with the hyperbolic scale covering 10 in. and the arithmetical scale, 7 in. The hyperbolic scale is derived from the formula for a rectangular hyperbola. In this case the base line equals 10 in., and the divisions from the right-hand margin are equal to ten times the reciprocals of the numbers from 1 to 10. For example: The line, 5, is ten times $\frac{1}{5} = 2$ in. from the margin; the line, 2, is ten times $\frac{1}{2} = 5$ in. from the margin; the value of the line at the left margin is 1.0; and the value of the line at the right margin is infinity.

Equations of the form, $xy = k$, which includes equations for intensity of rainfall, such as $i = \dfrac{126}{t + 18}$, appear as straight lines on this paper. It has a range for $i = 1$ to $i = 10$; and a range for $t = 0$ to $t = 120$.

Intensity Curves on Semi-Hyperbolic Paper

Fig. 5 shows, on semi-hyperbolic paper, the intensity-duration relationship for storms varying in frequency from 2 to 80 yr as determined by the formulas derived from the studies for the ten stations previously mentioned, including 330 station-yr (see Table 5). The frequencies shown, for $F = 5, 10, 20, 40,$ and 80 yr, are in a geometrical progression; the 2-yr frequency is an exception. The graph for the 10-yr frequency is midway between the graphs for the 5-yr and 20-yr frequencies, and the graph for any term in the geometrical progression lies midway between the graphs of the adjacent terms. The application of this relationship for interpolating intermediate frequencies and for deriving formulas will be discussed subsequently.

The intensity curves on hyperbolic paper are best adapted for use in preparing sewer designs. The range of the charts is sufficient to include all except the larger sewer projects. Curves are given for five or six different frequencies and for durations of as much as $t = 120$ min with 2-min intervals. Graphs for other frequencies than those shown, can easily be added if needed.

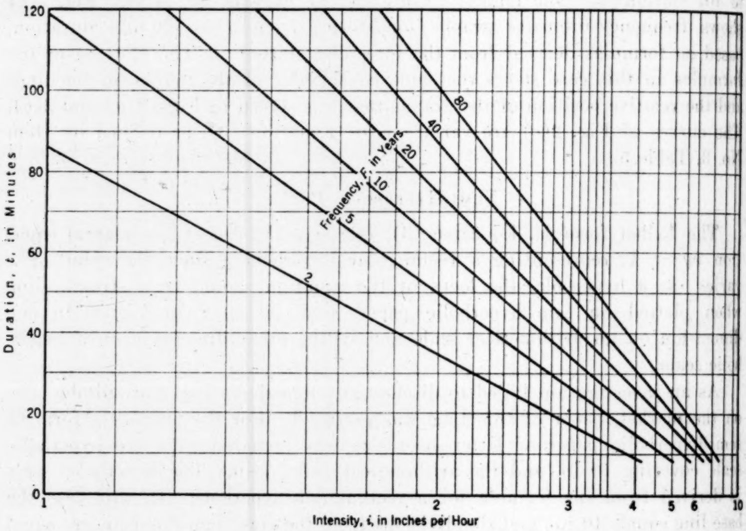

FIG. 5.—INTENSITY-DURATION GRAPHS; 330 STATION-YEARS (SEE TABLE 5, ITEM NO. 2).

A brief examination of these charts reveals the great importance of the time element. A change of 10 min in the estimate of the concentration period, as from 50 to 40 min, may be the equivalent of changing from a 5-yr frequency to a 10-yr frequency; for example, the intensity for a 5-yr frequency is 2 in. for 50 min, and 2.33 in. for 40 min, the latter being the same as the intensity for a 10-yr frequency for 50 min.

THE CONSTANTS IN THE FORMULAS FOR INTENSITY

The constants in the formulas for intensity were found originally by trial to make the formulas fit the values in the statistical data. This is easily done with the slide-rule. They may also be found by solving Equation (3), using the values for i and t that are obtained from the graphs of the statistical data (Fig 3). After the formulas were determined for the various frequencies, the results were verified, and the accuracy and consistency of the formulas and data were examined by plotting the constants in the numerators and denominators on semi-logarithmic paper.

Fig. 6 shows the constants used in the intensity formulas applying to the data for nineteen Weather Bureau stations. The values for the constants in the numerators lie on a straight line with one exception and the values used in the denominators are close to a line with very slight curvature. All the constants in these formulas are whole numbers, as it seemed that the introduction of fractions would be a needless refinement. By the use of the graphs the approximate values of the constants in the formulas for any desired frequency from 2 to 100 yr may be obtained, and formulas for the intensity for any desired frequencies within the range of the chart may be written. This gives a simple method for computing two values and drawing any intermediate frequency line on the hyperbolic chart.

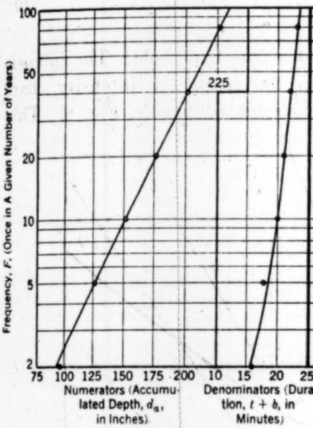

FIG. 6.—CONSTANTS IN INTENSITY FORMULAS, 659 STATION-YEARS.

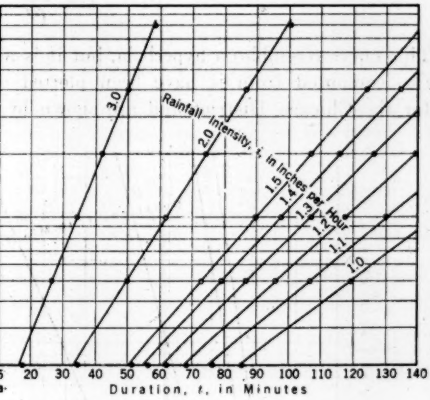

FIG. 7.—FREQUENCY DURATION GRAPHS FOR THE INTERPOLATION OF FREQUENCIES.

GRAPHICAL INTERPOLATION OF FREQUENCY LINES

A graphic method for interpolating frequencies on a hyperbolic chart of rainfall intensities for the ten stations, is shown in Fig. 7, in which intensity lines are drawn with frequency and time as the co-ordinates. In other words, it is a frequency-duration chart for various intensities. The points were found by solving for time in the intensity formulas for the several frequencies with assumed intensities from 1 in. per hr to 3 in. per hr. These points, for frequencies of 5 yr, or more, when plotted on the semi-logarithmic paper give a straight line for each intensity; for example, if it is desired to interpolate on the hyperbolic chart, Fig. 5, a line giving the intensities for a frequency of 15 yr, two values should be found for the time for that frequency, using different intensities. For an intensity, $i = 3$, Fig. 7 gives the time, $t = 39$ min, and for $i = 1.4$, $t = 108$ min. This permits the plotting of two points on the 15-yr line on the hyperbolic chart (Fig. 5) and the drawing of a straight line through these points, which will give the intensities for any time up to 120 min.

The formula adopted in 1914–15 for use in Chicago was of the form of Equation (4):

$$i = \frac{28}{t^{0.7}} \qquad (19)$$

This curve resembles a hyperbola, but it is not a true hyperbola. The intensities computed from it have been plotted on the frequency intensity chart for the Chicago District and are shown by the curved line in Fig. 8. This graph indicates that the frequencies represented by Equation (19) vary from about 3 yr to more than 20 yr. Before these studies were made it was thought that the frequency corresponding to the intensities obtained by Equation (19) was approximately 5 yr, but such is the fact only for durations of 15 and 120 min, and it is not true for storms of other durations.

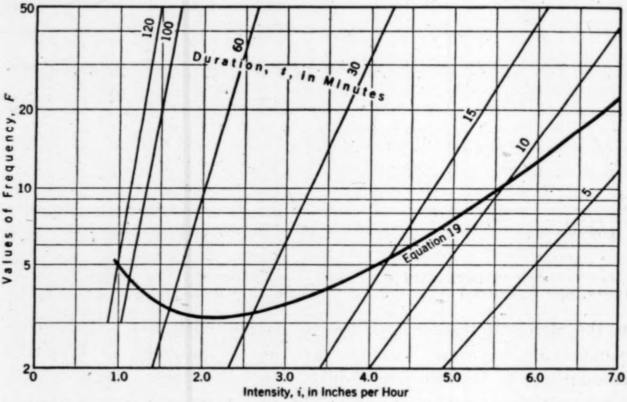

FIG. 8.—CURVES SHOWING VARIABLE FREQUENCY OF OLD FORMULA.

CONSTANT RATIO FOR FREQUENCIES WHEN INTENSITIES INCREASE BY A FIXED SUM

A characteristic of any formula which plots as a straight line on semilogarithmic paper, is that adding a fixed sum to the intensity will multiply the frequency by a fixed sum. In other words, successive additions of a fixed sum to the intensity will correspond to frequencies with a constant ratio. As an example, consider the following quantities picked from the 60-min frequency curve for 330 station-yr:

Intensity, in inches	Frequency, in years	Intensity, in inches	Frequency, in years
1.25	1.6	2.00	8.9
1.50	2.85	2.25	16.0
1.75	5.0	2.50	28.5

In each case, the ratio between successive frequencies is consistently 1.78.

RATIO OF DEPTHS OF RAINFALL (AND INTENSITIES) FROM CURVES FOR 330 STATION-YEARS

Table 6 shows depths of rainfall for storms of 1, 10, and 100-yr frequency for durations of 5, 10, 15, 30, and 60 min, taken from graphs; it also shows the computed ratio of the depth corresponding to a frequency higher than the ratio for the next lower frequency. The precipitations for various durations are shown, but the intensities will have the same ratios.

TABLE 6.—COMPARISON OF FREQUENCY STUDIES

Frequency, F, in years (1)	Duration, t									
	5		10		15		30		60	
	d_a, in inches (2)	Ratio (3)	d_a, in inches (4)	Ratio (5)	d_a, in inches (6)	Ratio (7)	d_a, in inches (8)	Ratio (9)	d_a, in inches (10)	Ratio (11)
1	0.34		0.54		0.67		0.91		1.04	
		1.50		1.63		1.70		1.79		1.96
10	0.51		0.88		1.14		1.63		2.04	
		1.33		1.40		1.42		1.44		1.49
100	0.68		1.23		1.62		2.35		3.04	

An equation such as,

$$i = \frac{C F^n}{(t + 7)^{0.7}} \quad \dots\dots (20)$$

will not fit the data in Table 6 because F, with a constant exponent, indicates a constant ratio of the intensities for all durations when the frequencies are in a geometrical progression. The ratio of intensities actually increases with the duration and decreases with the frequency in Table 6.

INTERPOLATION OF FREQUENCY CURVES BY GEOMETRIC MEAN

Charts on hyperbolic paper show that if the frequency lines are in a geometrical progression they will intercept a given intensity line with equal intervals (time) between them; for example, the 10-yr frequency line in Fig. 5 crosses the 2-in. intensity line half way between the 5-yr and the 20-yr frequency lines. Similarly, the line midway between the 5-yr and 10-yr lines will have a frequency of 7.07 yr, which is the square root of 5 × 10, and the line midway between the 10 and 20-yr lines will have a frequency of 14.14 yr, which is the square root of 10 × 20. A geometrical progression has the form,

$$x = a + ar + ar^2 + ar^3 + ar^4 \dots + ar^{n-1} \quad \dots\dots (21)$$

in which a = the first term; r = the ratio; and n = the number of terms. The ratio of the last term to the first term $\left(\frac{\text{Last term}}{\text{First term}}\right)$ is:

$$\frac{a\,r^{n-1}}{a} = r^{n-1} \quad \dots\dots (22)$$

Then, for the interpolation between $F = 5$ and 10, or $F = 10$ and 20:

$$\log r = \frac{1}{n-1} \log 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

When the first and last terms are given, all the intermediate terms may be found.

Solving a 5-term series for three intermediate frequencies between 5 and 10 yr gives lines for $F = 5.95$, 7.07, and 8.41 yr, equally spaced. These values may be found by taking square roots, but when the exact position is required for an integral frequency, it is generally necessary to find an exponent by logarithms.

The integral numbers, 5, 6, 7, 8, 9, and 10, apparently, are terms in an arithmetical progression, but they are also approximate terms in a geometric progression that has a large number of terms (say, 1 001). Applying this method it is found that the spacing for the frequencies, $F = 6$, 7, 8, and 9 will be, respectively, 26.3%, 48.5%, 67.8%, and 84.8% of the distance between the 5-yr and the 10-yr frequencies. Intermediate integral frequencies between 10 and 20 will have the same proportionate spacing.

Difference in Time When Frequencies Are in Geometrical Progression

If the several formulas for intensity used in Fig. 5 (see Table 5, Item No. 2) that were derived from the chart showing the data for 330 station-yr are solved for the time with the same assumed intensity, it is found that the differences in time or in duration are constants for each intensity when the frequencies are in a geometrical progression. This is also true within narrow limits for the corresponding formulas for intensity for the Chicago District. This characteristic indicates a possible method for drawing additional curves on a chart.

TABLE 7.—Differences in Duration Time, Δt

Frequency, F	Intensity of Rainfall, in Inches per Hour							
	1		2		3		4	
	Time, t, in minutes	Difference, Δt	Time, t, in minutes	Difference, Δt	Time, t, in minutes	Difference, Δt	Time, t, in minutes	Difference, Δt
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5	108		45		24		13¾	
		24		11		6⅔		4¼
10	132		56		30⅔		18	
		24		11½		7⅓		5¼
20	156		67½		38		23¼	
		24		11½		7⅓		5¼
40	180		79		45		28½	
		24		11½		7⅓		5¼
80	204		90½		52⅓		33¾	

Table 7 gives values of Δt from the formulas for 659 station-yr for frequencies of 5, 10, 20, 40, and 80 yr (see Table 5, Item No. 1). From inspection of Table 7 it appears that,

$$\Delta t = \frac{25 - i}{i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

when frequencies form a geometrical progression. If $i = 2$ in. per hr, $\Delta t = \frac{25-2}{2} = \frac{23}{2} = 11\frac{1}{2}$; and, if $i = 4$ in. per hr, $\Delta t = \frac{25-4}{4} = \frac{21}{4} = 5\frac{1}{4}$. The differences are constant for a given intensity when the frequencies vary as in a geometrical progression.

CONCLUSIONS

The following conclusions are indicated by the foregoing arguments:

(1) The statistical method which combines station-years affords a means for studying rainfall data which gives reasonably regular and consistent results.

(2) Semi-logarithmic and hyperbolic papers are effective tools in studies of rainfall.

(3) For the shorter durations, the inlet time is an important factor in the time of concentration, as suggested in the comment on Fig. 5. Studies to find its probable mean value are desirable.

(4) Formulas such as Equation (7) indicate that when the durations are in a geometrical series the differences in rainfall values are a constant. This suggests a method that may be useful in a study of storms exceeding 120 min. Fig. 5 indicates that when the frequencies are in a geometrical series the differences in durations for a given intensity are constant. These relations would seem to be more than a coincidence and may be worthy of more extended examination.

(5) For the 5-yr and the 10-yr frequencies a close degree of similarity is shown for intensity values by the curves for the Chicago District and those for the 330 station-yr, except for 5-min and 10-min durations.

(6) The frequency-intensity curves give the probable intensity of rainfall for certain durations for a single gage. The average intensity over a large area, of course, is less than the maximum intensity. The ratio of this average to the maximum intensity is as yet unknown, and is one of the factors needing further investigation. One of the methods suggested for overcoming this difficulty in sewer design is the use of a frequency which varies in some inverse ratio with the duration or time of concentration. The frequencies that should be used in sewer design, although theoretically a matter of economics, are actually a matter of judgment based on experience.

(7) A study should be made of the relation of average intensities to maximum intensities of rainfall on small areas, such as may be used in sewer design, say, to 10 000 acres. This would require a considerable number of gages spaced in a regular pattern so as to permit the drawing of isohyetals and the computation of average depths of rainfall. It is believed that the frequency graphs give values for areas that are comparable with the size of sewer districts, and that the maximum size of such areas may be between 1 000 and 10 000 acres.

(8) The values obtained from the charts developed for the Chicago District have greater statistical foundation and appear to be more reliable than

any curves previously used by the Board of Local Improvements, City of Chicago. These charts are now (1937) being used by the engineers of the Board.

ACKNOWLEDGMENTS

Co-operation and assistance in the work outlined in this paper have been given in various ways by the following: The Sanitary District of Chicago, through the members of its Engineering Staff (Philip Harrington, Chief Engineer, O. L. Eltinge, M. Am. Soc. C. E., Engineer of Sewer Design, and L. W. Hall, M. Am. Soc. C. E., Hydraulic Engineer); the U. S. Weather Bureau, by W. R. Gregg, Chief of Bureau, Washington, D. C., and C. A. Donnel, Principal Meteorologist in Charge, Chicago, Ill.; Loyola University, Chicago, by its President the Rev. R. M. Kelly, S. J.; the City Engineer of Chicago, the late Myron B. Reynolds, and the Water Purification Division of the City Engineering Department; the late George C. D. Lenth, M. Am. Soc. C. E., Consulting Engineer, Chicago; T. S. Ford, Assistant Chief Engineer, Board of Local Improvements, and A. L. Appelbaum, Junior Engineer; and A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., as previously noted.

DISCUSSION

VICTOR H. COCHRANE,[7] M. AM. SOC. C. E. (by letter).—The remarkably consistent results obtained by the authors and other investigators indicate that the hyperbolic, or Talbot, type of formula is correct for precipitation lasting two hours or less. For some years the writer has made use of a formula giving the intensities of precipitation for various periods in terms of I, the precipitation in 1 hr. If in a given case, I is known or assumed, the proper intensities for other periods from 5 to 120 min may be calculated from the formula,

$$i = \frac{60 + n\,I}{\dfrac{t}{I} + n} = \frac{(60 + n\,I)\,I}{t + n\,I} \quad\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

in which n is a constant depending upon the frequency, F. Its value varies from approximately 8 for $F = 100$ to about 12 for $F = 1$. It may be assumed according to the formula,

$$n = 12 - 2 \log F \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

Combining Equations (25) and 26):

$$i = \frac{60 + (12 - 2 \log F)\,I}{\dfrac{t}{I} + (12 - 2 \log F)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

If $F = 10$, $n = 10$, an average value. In case it is desired to use a value of I for which the frequency is unknown, this average value of n may be used for practical purposes.

TABLE 8.—COMPARATIVE VALUES OF i.

Elapsed time, t, in minutes	FREQUENCY EQUALS ONCE IN EVERY:					
	2 yr.		10 yr.		100 yr.	
	Equation (18)	Equation (27)	Equation (18)	Equation (27)	Equation (18)	Equation (27)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
I=	1.34	1.34	2.03	2.03	3.03	3.03
n=	.....	11.4	.....	10.0	.....	8.0
120.....	0.75	0.75	1.18	1.16	1.79	1.77
100.....	0.88	0.88	1.37	1.36	2.07	2.05
60.....	1.34	1.34	2.03	2.03	3.03	3.03
30.....	2.24	2.23	3.26	3.24	4.72	4.70
15.....	3.29	3.33	4.60	4.62	6.47	6.50
10.....	3.93	3.99	5.35	5.38	7.38	7.45
5.....	4.86	4.98	6.43	6.44	8.68	8.73
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....

Table 8 shows comparative results computed from Equations (18) and (27). The fact that hyperbolic curves fit rainfall data covering short periods suggests that a formula giving flood flows from small water-sheds should likewise be hyperbolic.

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[7] Cons. Engr., Tulsa, Okla.

LeRoy K. Sherman,[8] M. Am. Soc. C. E. (by letter).—Rainfall data are essential for the design of storm sewers and many other works. Analysis and interpretation of the data are of equal importance. The utilization of station-years in statistical methods, where such stations are near isohyetals for the same rainfall durations and frequencies, appears to be sound. Stations with materially different precipitation characteristics, such as Chicago, Ill., and Nashville, Tenn., should not be grouped together. Stations, such as St. Paul, Minn., Chicago, and Central New York State, all near the same isohyetal, may properly be placed in the same group.

The authors give preference to the hyperbola type of curve to express the rainfall relations. Others who have made similar studies prefer the exponential type of rainfall equation. In so far as the mathematical expression closely fits the data, the form of expression is relatively unimportant. The form which fits the widest range of observations is frequently the most desirable.

An examination of a large number of automatic rain-gage records in the District of Columbia[9] showed few instances of even approximate uniform rainfall rates for periods of 30 min or more. A fair average rainfall pattern was triangular in form, with time duration as the base and the peak rate at twice the average rate. These rain irregularities are partly "ironed out" on large areas during the time of run-off. Does the same apply to small urban areas? The authors' diagrams show that 2 in. of rain may fall during 1 hr: 1.3 in. in 20 min, and 0.7 in 40 min.

The rational method is now commonly used to estimate run-off from severed areas. The average hourly rate of rain for the period of concentration is used. It is pertinent to find what, if any, difference in run-off exists under the assumption of the average and the actual rates of rain.

The writer has determined this for a particular case—a severed area of 420 acres in Evanston, Ill., north of Chicago. The run-off characteristics of this area are given in a paper by S. A. Greeley,[10] M. Am. Soc. C. E. The writer has derived the typical percentage distribution of run-off from this area so as to conform to Mr. Greeley's time-run-off data. He has applied his own unit hydrograph method[11] and the same method as improved by Merrill M. Bernard,[12] M. Am. Soc. C. E., and by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt,[13] Assoc. M. Am. Soc. C. E.

A 10-yr storm for 50 min was taken. By Fig. 5, the average intensity of rain per hour for 50 min is 2.34 in., or a total of 1.95 in. The initial and infiltration loss was taken at 0.45 in. Under these conditions, the peak rate of run-off, by the rational method, is 755 cu ft per sec. The peak run-off for this same case, computed by the unit hydrograph method, is practically the same (762 cu ft per sec) as shown in Table 9(*a*). If the rain is continued beyond the time of concentration, the run-off rate (762 cu ft per sec, in Column (8), Table 9), will remain constant.

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[8] Cons. Civ. Engr., Chicago, Ill.

[9] "Elimination of Pollution of Rock Creek, August 1, 1935", by L. K. Sherman and W. W. Horner, Members, Am. Soc. C. E., Gov't. Printing Office, Washington, D. C.

[10] *Journal*, Western Soc. of Engrs., September, 1913; also, "Design of Sewers", by Metcalf and Eddy.

[11] "Runoff from Rainfall by the Unit Graph", *Engineering News-Record*, April 7, 1932.

[12] "An Approach to Determinate Stream Flow", *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 347.

[13] "Relation Between Rainfall and Run-Off from Small Urban Areas", *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

TABLE 9.—COMPUTATION OF RUN-OFF

Elapsed time, in 10-min. intervals	RAINFALL, IN INCHES OF DEPTH			Percentage, distribution of rainfall	DISTRIBUTION OF RUN-OFF, IN INCHES						RUN-OFF	
	Gage reading	Loss	Net rainfall								In inches	In cubic feet per second
(1)	(2)	(3)	(4)	(5)	(6*a*)	(6*b*)	(6*c*)	(6*d*)	(6*e*)	(6*f*)	(7)	(8)
					(*a*) WITH UNIFORM RATES OF RAINFALL							
1.......	0.39	0.09	0.3	8	0.024	.....	.....	.....	.....	....	0.024	61
2.......	0.39	0.09	0.3	19	0.057	0.024	.....	.....	.....	....	0.081	206
3.......	0.39	0.09	0.3	28	0.084	0.057	0.024	.....	.....	....	0.165	419
4.......	0.39	0.09	0.3	23	0.069	0.084	0.057	0.024	.....	....	0.234	595
5.......	0.39	0.09	0.3	15	0.045	0.069	0.084	0.057	0.024	....	0.279	708
....	....	....	....	7	0.021	0.045	0.069	0.084	0.057	0.024	0.300	762
....	....	....	....	....	.....	0.021	0.045	0.069	0.084	0.057	0.276	701
....	....	....	....	....	.....	.....	0.021	0.045	0.069	0.084	0.219	556
....	....	....	....	....	.....	.....	.....	0.021	0.045	0.069	0.135	343
....	....	....	....	....	.....	.....	.....	.....	0.021	0.045	0.066	168
....	....	....	....	....	.....	.....	.....	.....	.....	0.021	0.021	53
....	1.95	0.45	1.50	100	0.30	0.30	0.30	0.30	0.30	0.30	1.8000	....
					(*b*) WITH NON-UNIFORM RATES OF RAINFALL							
1.......	0.22	0.09	0.13	8	0.0104	.....	.....	.....	.....	....	0.0104	25
2.......	0.22	0.09	0.13	19	0.0247	0.0104	.....	.....	.....	....	0.0351	92
3.......	0.40	0.09	0.31	28	0.0364	0.0247	0.0248	.....	.....	....	0.0359	218
4.......	0.90	0.09	0.81	23	0.0299	0.0364	0.0589	0.0648	.....	....	0.1900	482
5.......	0.21	0.09	0.12	15	0.0195	0.0299	0.0868	0.1539	0.0096	....	0.2997	760
....	....	....	....	7	0.0091	0.0195	0.0713	0.2268	0.0228	....	0.3495	890
....	....	....	....	....	.....	0.0091	0.0465	0.1863	0.0336	....	0.2755	700
....	....	....	....	....	.....	.....	0.0217	0.1215	0.0276	....	0.1708	434
....	....	....	....	....	.....	.....	.....	0.0567	0.0180	....	0.0747	190
....	....	....	....	....	.....	.....	.....	.....	0.0084	....	0.0084	21
....	1.95	0.45	1.50	100	0.13	0.13	0.31	0.81	0.12	....	1.5000	....

The peak run-off for this non-uniform rate of rainfall is 890 cu ft per sec (Table 9(*b*)). This is about 17% greater than the run-off due to uniform rainfall. One may conclude that, for a given coefficient or a rate of loss, the rational method will forecast the minimum rate of run-off corresponding to the storm duration and frequency. On large natural areas, with flood-producing storms lasting for several days, and where two-thirds of the total rainfall commonly occurs in one day, the use of average storm rates is inadmissible.

All of the foregoing goes to show the importance of rainfall data segregated into divisions of duration-intensity-frequency, such as the authors have presented.

J. O. JONES,[14] ASSOC. M. AM. SOC. C. E. (by letter).—A method of increasing the size of the sample of excessive precipitation data for storms of short duration (not exceeding 120 min) is described in this paper. The writer agrees that there is no single station record long enough to give what may be regarded as a fair sample of the total "population" of such data. In any statistical study of chance events the methods of statistics assume that an unbiased sample is available. The authors are to be congratulated for a fine contribution to a difficult and important subject.

It may be a debatable question whether it is permissible to combine the records of stations widely separated geographically, unless the rainfall characteristics are similar in other respects. As an extreme example, one would scarcely feel justified in combining the record of a station in Assam, (where the warm southwest monsoons from the Indian Ocean coming in contact with the Himalayas produce precipitation of from 400 in. to 800 in. per yr) with that of Boise, Idaho.

TABLE 10.—RAINFALL CHARACTERISTICS, STATE OF KANSAS

Division	Average coefficient of variation	Average ratio of the minimum to the mean	Average ratio of the maximum to the mean
Eastern	0.21	0.60	1.50
Middle	0.25	0.55	1.63
Western	0.28	0.49	1.72

It does seem reasonable to combine the records from a small area, such as the Sanitary District of Chicago, or even an entire State, where the rainfall characteristics are similar over the State. This the writer has done for the State of Kansas. There is some difference in the rainfall characteristics of Eastern Kansas when compared with the western part of the State. The average annual rainfall varies from about 17½ in. in Longitude 101° to 40 in. in the southeast corner of the State; but the time of occurrence is similar, being greatest in the warm months. As regards annual rainfall, the coefficient of variation and the coefficient of skew are not greatly different for the eastern as compared to the western part of the State. For eighteen stations in the Eastern Division, thirteen in the Middle Division, and eleven in the Western Division, the average coefficient of variation, the average ratio of the minimum to the mean, and the average ratio of the maximum to the mean is as shown in Table 10. Thus, it appears that the annual rainfall is less and more variable in the western part of Kansas. The differences in the statistical parameters, however, are not great. The monthly distribution is similar over the State, although the total depth, month by month, is greatest near the eastern border.

In view of the foregoing, it seems permissible to combine the records of excessive precipitation for the five U. S. Weather Bureau stations within the State, together with Kansas City, Mo., which is on the eastern border. The lengths of the records included in the study are: Concordia, 1908 to

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[14] Prof. of Hydr., Univ. of Kansas, Lawrence, Kans.

TABLE 11.—FREQUENCY OF EXCESSIVE PRECIPITATION

Inches of rainfall	Number of Occurrences						
	Kansas City	Topeka	Iola	Wichita	Dodge City	Concordia	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(*a*) FIVE-MINUTE DURATION							
0.125	21	17	23	18	13	7	99
0.175	54	60	54	60	37	28	293
0.225	59	70	60	50	43	40	322
0.275	47	61	36	38	22	19	223
0.325	26	24	34	23	12	21	140
0.375	14	21	9	10	8	6	68
0.425	4	8	10	7	5	7	41
0.475	4	6	4	3	6	2	25
0.525	5	1	3	2	3	3	17
0.575	2	1	0	3	1	1	8
0.625	1	..	1	..	0	0	2
0.675	..	..	1	..	0	0	1
0.725	..	..	0	..	1	1	2
0.775	..	..	0	..	..	..	0
0.825	..	..	1	..	..	..	1
0.875	..	..	0	..	..	..	0
0.925	..	..	0	..	..	..	0
0.975	..	..	1	..	..	..	1
	..	..	..	..	..	..	1 243
(*b*) TEN-MINUTE DURATION							
0.175	..	1	..	1	1	1	4
0.225	12	13	15	16	7	5	68
0.275	42	28	27	23	15	14	149
0.325	39	59	46	51	37	27	259
0.375	36	47	39	31	22	27	202
0.425	25	31	32	19	24	13	144
0.475	16	19	17	17	12	8	91
0.525	33	30	12	16	5	7	103
0.575	9	10	16	5	4	5	49
0.625	6	7	7	6	3	8	37
0.675	2	7	5	8	8	6	36
0.725	2	6	5	5	3	5	26
0.775	2	3	1	4	2	1	13
0.825	2	3	4	3	1	3	16
0.875	3	2	1	1	0	1	8
0.925	1	..	1	3	2	1	8
0.975	2	..	2	..	3	0	7
1.025	1	..	0	..	2	0	3
1.075	1	..	1	..	..	0	2
1.125	1	..	0	..	..	0	1
1.175	..	..	0	..	..	1	1
1.225	..	..	0	..	..	..	0
1.275	..	..	0	..	..	..	0
1.325	..	..	0	..	..	..	0
1.375	..	..	1	..	..	..	1
	..	..	..	..	..	..	1 228
(*c*) FIFTEEN-MINUTE DURATION							
0.275	9	5	12	4	1	1	32
0.325	17	18	13	15	10	5	78
0.375	34	37	27	34	16	26	174
0.425	37	29	28	30	24	21	169
0.475	16	30	28	17	14	13	118
0.525	23	30	30	18	15	9	125
0.575	13	25	13	16	11	8	86
0.625	20	10	14	10	9	5	68
0.675	19	15	12	8	6	5	65
0.725	13	7	7	10	3	5	45
(*c*) FIFTEEN-MINUTE DURATION (*Continued*)							
0.775	3	10	6	4	6	6	35
0.825	1	4	5	3	5	6	24
0.875	4	3	4	6	2	5	24
0.925	3	5	5	5	2	3	23
0.975	3	4	4	5	4	1	21
1.025	2	5	2	2	2	2	15
1.075	0	1	1	2	0	0	4
1.125	3	1	2	2	2	1	11
1.175	1	..	0	0	2	0	3
1.225	0	..	1	1	0	0	2
1.275	0	..	0	1	1	2	4
1.325	0	..	1	1	1	0	3
1.375	0	..	0	..	..	1	1
1.425	0	..	0	..	..	0	0
1.475	1	..	0	..	..	0	1
1.525	0	..	0	..	..	0	0
1.575	1	..	0	..	..	1	2
1.625	0	..	1	..	..	..	1
1.675	1	..	..	..	..	..	1
	..	..	..	..	..	..	1 136
(*d*) TWENTY-MINUTE DURATION							
0.35	14	8	14	9	6	6	57
0.45	50	42	31	44	19	24	210
0.55	36	46	45	27	27	20	201
0.65	26	33	29	31	16	10	145
0.75	28	23	19	18	12	8	108
0.85	15	11	9	16	10	6	67
0.95	10	6	8	5	6	10	45
1.05	4	9	5	8	4	8	38
1.15	4	4	9	5	5	1	28
1.25	4	4	1	5	3	3	20
1.35	2	0	2	1	1	1	7
1.45	0	1	1	1	0	1	4
1.55	0	..	0	1	1	0	2
1.65	1	..	1	..	1	1	4
1.75	0	..	1	..	..	1	2
1.85	1	..	..	..	..	..	1
1.95	0	..	..	..	..	..	0
2.05	0	..	..	..	..	..	0
2.15	1	..	..	..	..	..	1
	..	..	..	..	..	..	940
(*e*) THIRTY-MINUTE DURATION							
0.30	..	..	..	..	..	1	1
0.40	3	4	2	..	..	0	9
0.50	13	9	13	7	8	4	54
0.60	28	16	15	13	8	12	92
0.70	14	23	20	13	15	7	92
0.80	15	22	18	12	9	6	82
0.90	10	14	19	15	11	6	75
1.00	12	9	7	12	4	2	46
1.10	11	9	4	11	6	6	47
1.20	6	7	3	2	5	8	31
1.30	4	3	7	6	3	2	25

TABLE 11.—(*Continued*)

Inches of rainfall	Number of Occurrences						
(1)	Kansas City (2)	Topeka (3)	Iola (4)	Wichita (5)	Dodge City (6)	Concordia (7)	Total (8)
(*e*) Thirty-Minute Duration (*Continued*)							
1.40	5	1	5	2	4	2	19
1.50	4	3	1	1	4	1	14
1.60	3	0	2	2	2	1	10
1.70	0	1	0	1	0	1	3
1.80	0	1	2	4	0	1	8
1.90	1	2	0	1	0	0	4
2.00	0	..	0	..	1	0	1
2.10	0	..	0	..	..	1	1
2.20	0	..	0	..	..	..	0
2.30	1	..	1	..	..	..	2
2.40	0	..	..	..	..	..	0
2.50	0	..	..	..	..	..	0
2.60	0	..	..	..	..	..	0
2.70	0	..	..	..	..	..	0
2.80	0	..	..	..	..	..	0
2.90	0	..	..	..	..	..	0
3.00	0	..	..	..	..	..	0
3.10	1	..	..	..	..	..	1
	..	..	..	..	..	..	617
(*f*) Forty-Five-Minute Duration							
0.625	9	2	3	4	1	3	22
0.775	16	9	9	5	4	3	46
0.925	14	16	13	3	8	3	57
1.075	7	7	11	6	6	5	42
1.225	5	10	5	6	9	1	36
1.375	1	4	2	4	3	2	16
1.525	9	5	2	4	1	3	24
1.675	8	0	3	2	6	3	22
1.825	1	2	2	1	1	1	8
1.975	0	0	1	0	0	0	1
2.125	4	1	0	4	0	0	9
2.275	0	0	0	3	1	1	5
2.425	0	2	1	1	0	1	5
2.575	0	1	1	..	0	0	2
2.725	1	..	..	..	0	0	1
2.875	0	..	..	..	0	0	0
3.025	0	..	..	..	1	0	1
3.175	0	..	..	..	..	0	0
3.325	0	..	..	..	..	1	1
3.475	0	..	..	..	..	..	0
3.625	0	..	..	..	..	..	0
3.775	0	..	..	..	..	..	0
3.925	0	..	..	..	..	..	0
4.075	1	..	..	..	..	..	1
	..	..	..	..	..	..	299
(*g*) Sixty-Minute Duration							
0.825	3	4	3	1	1	2	14
0.975	12	2	4	5	3	2	28
1.125	8	11	9	0	5	4	37
1.275	2	7	6	1	2	1	19
1.425	1	3	3	3	2	0	12
1.575	4	4	1	3	1	0	13
1.725	6	5	1	4	2	1	19
1.875	5	0	0	1	1	1	8
2.025	3	0	2	1	1	0	7
2.175	1	1	1	1	1	0	5
(*g*) Sixty-Minute Duration (*Continued*)							
2.325	3	0	0	1	1	0	5
2.475	0	0	0	3	1	0	4
2.625	1	1	0	2	0	1	5
2.775	0	1	1	..	0	0	2
2.925	1	1	1	..	0	0	3
3.075	0	..	..	..	0	0	0
3.225	0	..	..	..	0	0	0
3.375	0	..	..	..	0	0	0
3.525	0	..	..	..	1	1	2
3.675	0	..	..	..	..	..	0
3.825	0	..	..	..	..	..	0
3.975	0	..	..	..	..	..	0
4.125	0	..	..	..	..	..	0
4.275	0	..	..	..	..	..	0
4.425	0	..	..	..	..	..	0
4.575	0	..	..	..	..	..	0
4.725	1	..	..	..	..	..	1
	..	..	..	..	..	..	184
(*h*) Eighty-Minute Duration							
0.8	7	..	2	2	2	2	15
1.0	8	1	7	0	4	2	22
1.2	2	3	5	5	1	1	17
1.4	4	2	1	1	0	0	8
1.6	3	3	2	0	1	1	10
1.8	3	1	2	2	2	..	10
2.0	3	1	0	0	0	..	4
2.2	1	1	1	2	1	..	6
2.4	0	1	1	..	0	..	2
2.6	1	..	..	..	0	..	1
2.8	0	..	..	..	0	..	0
3.0	0	..	..	..	0	..	0
3.2	0	..	..	..	1	..	1
3.4	0	..	..	..	..	..	0
3.6	0	..	..	..	..	..	0
3.8	0	..	..	..	..	..	0
4.0	1	..	..	..	..	..	1
	..	..	..	..	..	..	97
(*i*) One-Hundred-Minute Duration							
0.6	1	..	..	..	1	..	2
0.8	2	..	2	..	1	1	6
1.0	4	3	2	..	1	..	10
1.2	1	2	0	1	0	..	4
1.4	1	3	0	2	1	..	7
1.6	3	1	2	1	1	..	8
1.8	2	0	1	1	0	..	4
2.0	0	1	..	..	0	..	1
2.2	2	..	..	..	0	..	2
2.4	0	..	..	..	0	..	0
2.6	0	..	..	..	0	..	0
2.8	0	..	..	..	0	..	0
3.0	0	..	..	..	1	..	1
3.2	0	..	..	..	..	..	0
3.4	1	..	..	..	..	..	1
	..	..	..	..	..	..	46

1929; Dodge City, 1892 to 1929; Iola, 1906 to 1929; Kansas City, 1899 to 1929; Topeka, 1899 to 1929; and Wichita, 1903 to 1929—a total of 167 station-yr. These data include all storms for which the depth of precipitation is equal to or exceeds the quantities given by the authors in Table 3. From the records the frequencies in Table 11 were compiled. The values given in the left-hand column, "Inches of Rainfall," are the central values of groups; for example, for rainfall of 5-min duration, the depth, 0.275 in. includes all values from 0.25 in. to 0.29 in., 0.275 in. being the central value of the group.

From the data in Table 11 the usual statistical parameters were calculated for each station separately, with the result shown in Table 12. The remarkable agreement among the several stations, for the several durations, as regards the mean and the coefficient of variation, indicates that the data for the several stations are comparable. The somewhat greater variation in the coefficient of skew is not particularly significant considering its relatively large probable error when the coefficient is large and the sample is small.

TABLE 12.—EXCESSIVE PRECIPITATION IN KANSAS, U. S. WEATHER BUREAU STATIONS

(i = Mean intensity, in inches per hour; c_v = coefficient of variation; c_s = coefficient of skew.)

City	i	c_v	c_s	i	c_v	c_s	i	c_v	c_s
	5-min.			10-min.			15-min.		
Kansas City	3.06	0.37	1.15	2.54	0.39	1.68	2.24	0.39	1.93
Iola	3.09	0.43	2.31	2.57	0.39	1.86	2.19	0.39	1.55
Topeka	3.04	0.33	0.89	2.51	0.33	1.04	2.16	0.33	1.16
Dodge City	3.07	0.41	1.45	2.53	0.42	1.53	2.29	0.39	1.28
Concordia	3.19	0.37	1.42	2.65	0.39	1.31	2.29	0.42	1.51
Wichita	2.96	0.37	1.23	2.52	0.38	1.18	2.22	0.39	1.24
Average	3.07	0.38	1.41	2.55	0.38	1.43	2.23	0.38	1.44
	20-min.			30-min.			45-min.		
Kansas City	2.00	0.41	1.89	1.78	0.44	1.58	1.57	0.48	2.12
Iola	2.00	0.38	1.47	1.76	0.38	1.40	1.52	0.37	1.54
Topeka	1.94	0.33	1.25	1.72	0.35	1.24	1.55	0.37	1.52
Dodge City	2.09	0.38	1.20	1.84	0.35	0.88	1.67	0.36	1.64
Concordia	2.12	0.41	1.19	1.89	0.38	0.96	1.77	0.46	1.42
Wichita	2.01	0.37	1.12	1.88	0.35	1.02	1.78	0.39	0.55
Average	2.03	0.38	1.35	1.81	0.38	1.18	1.64	0.40	1.46
	60-min.			80-min.			100-min.		
Kansas City	1.58	0.47	1.86	1.39	0.50	2.06	1.45	0.47	1.02
Iola	1.36	0.38	1.65	1.30	0.32	1.06	1.02	0.31	1.32
Topeka	1.42	0.36	1.50	1.32	0.31	0.89	....	....	....
Dodge City	1.55	0.40	1.42	....	....	....	....	....	....
Concordia	1.47	0.52	1.54	....	....	....	....	....	....
Wichita	1.69	0.34	0.19	....	....	....	....	....	....
Average	1.51	0.41	1.36	1.33	0.38	1.34	....	....	....

The coefficient of skew for the Kansas City data is greater than the average of all the stations for all durations except those of 5 min and 100 min. This is the effect of a single storm, August 23, 1906, which was an unusual one.

When the records of the stations are combined the result given in Table 13 is obtained. A typical histogram and frequency curve is shown for the 5-min duration in Fig. 9. The frequency curve is fitted to the data by Pearson's Type III curve.[15]

TABLE 13.—EXCESSIVE PRECIPITATION; SUMMARY OF STATISTICAL PARAMETERS, KANSAS DATA

Total frequency, F (1)	Duration, in minutes, t (2)	Average intensity, in inches per hour, i (3)	Coefficient of variation, c_v (4)	Coefficient of skew, c_s (5)
1 243	5	3.05	0.382	1.53
1 228	10	2.56	0.366	1.69
1 136	15	2.21	0.385	1.50
940	20	2.00	0.377	1.42
617	30	1.80	0.379	1.43
299	45	1.66	0.392	1.68
184	60	1.50	0.407	1.67
97	80	1.40	0.402	1.58
46	100	1.38	0.410	1.43
21	120	1.24	0.338	0.24
Average*	...	....	0.386	1.55

* The average does not include the 120-min. duration, because of the small total frequency.

Using the average intensities, i (Column (3), Table 13), and the average values, $c_v = 0.386$ and $c_s = 1.55$, the ogive curves, Fig. 10, were constructed by the use of Foster's skew factors.[16] The "raw" data for the 5-min duration

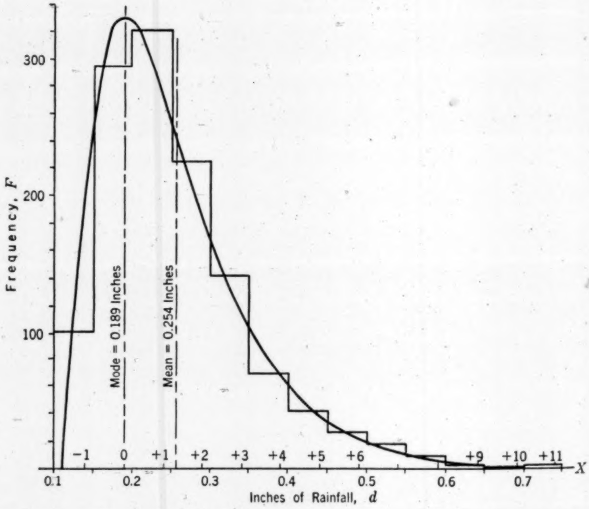

FIG. 9.—EXCESSIVE PRECIPITATION IN KANSAS; INCHES OF RAINFALL IN 5 MINUTES; U. S. WEATHER BUREAU STATIONS.

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[15] *Transactions*, Am Soc. C. E., Vol. LXXXVII (1924), p. 148.
[16] *Loc. cit.*, p. 162.

is compared with the ogive curve obtained by the use of Foster's skew factors, and with the curve obtained by the use of Pearson's Type III frequency curve, in Fig. 11. No comparison was made with the "raw" data for the other durations. The labor of computing the ogive curve by means of Pearson's equation is prodigious. An examination of the method used by Foster[17], in deriving his skew factors for the Type III curve, leaves one with the conviction that the ogive curve, obtained by the application of the skew factors, will agree closely with that obtained from Pearson's equation, as it does for the 5-min duration.

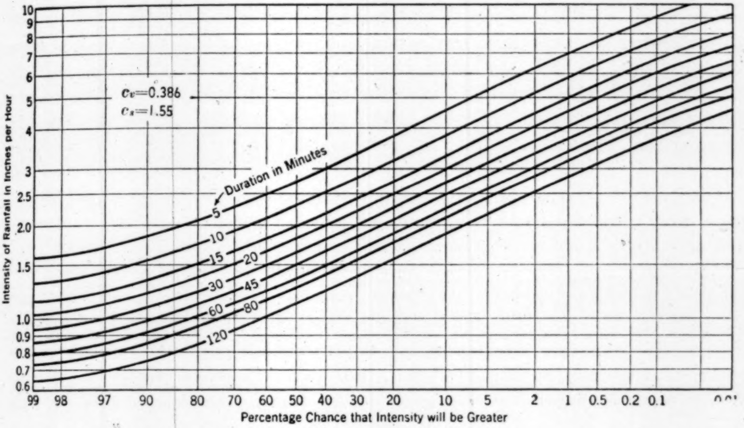

FIG. 10.—EXCESSIVE PRECIPITATION IN KANSAS; U. S. WEATHER BUREAU STATIONS.

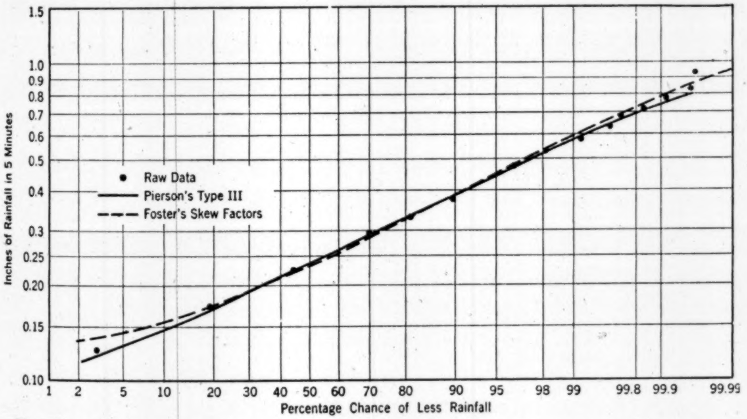

FIG. 11.—COMPARISON OF RAW DATA WITH FOSTER'S SKEW FACTORS AND PEARSON'S TYPE III FREQUENCY CURVE.

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[17] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 159 *et seq.*

Equations 18(*a*) to 18(*g*) yield results which agree closely with Meyer's[18] equation for his Group 3; but a comparison with the results from the ogive curves is difficult. It depends upon what is meant by the term, "frequency". To state that a rainfall has a frequency of "once in 100 yr" does not necessarily mean that it has a 1% chance of occurring. There is some danger in assuming that because a certain maximum intensity has only been recorded once in 100 station-yr it, therefore, has a frequency of once in 100 yr. At Topeka, for the 5-min duration, an intensity of 6.9 per hr was experienced only once prior to 1929. In the interval, 1929 to 1935, inclusive, an intensity of 6.9 in. per hr was experienced once, and an intensity of 8.0 in. per hr was experienced once. Thus, prior to 1929 an intensity of 6.9 in. per hr had a frequency of once in 30 yr, but by the end of 1935 that intensity had a frequency of once in about 12 yr. Of course, by taking a large number of station-years such absurdities are "ironed out".

It would seem to be more logical to apply the methods of statistical analysis to chance events, and some statement involving the term, "percentage chance", would be preferable to the term, "frequency", as commonly used. To illustrate, there are on the average about four excessives per year, of 15-min duration, at Dodge City, whereas, at Topeka, there are on the average about eight. For any one of these excessives there is a 50% chance that it will exceed 2.0 in. per hr; a 10% chance that it will exceed 3.3 in. per hr; a 1% chance of 5.1 in. per hr; and 1 chance in 1 000 of an intensity of 6.7.

The Talbot formula has long been a favorite, because of its simplicity and its successful use in design. Considering the paucity of data at the time it was first proposed, it has given remarkable results in the design of storm sewers. It is not a good 'fit", however, to the record of a single storm. It can be stated in the form,

$$i\,t = A - i\,B \qquad \text{..................} (28)$$

in which $i\,t$ (intensity times duration) is the y-variable and i (intensity) is the x-variable. In this form, it is the equation of a straight line, A being the intercept on the y-axis and B, the slope of the line. In Fig. 12 is shown the intensity duration of the four greatest storms that have been recorded at Kansas City. Only one of these storms, that of September 6 and 7, 1914, is a good approximation to a straight line. The storms of June 8, 1910, and September 15, 1914, approximate a straight line if the 5-min and 10-min durations are omitted.

The result is little, if any, better when the combined data of several stations are considered. In Table 14 is listed the twenty greatest intensities from the record of 167 station-yr for the State of Kansas, arranged in order of magnitude without regard to the location of the station. These data, when plotted as in Fig. 12, are irregularly curved lines until the high orders of magnitude are reached.

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[18] "Hydrology," by Adolph Meyer, M. Am. Soc. C. E.

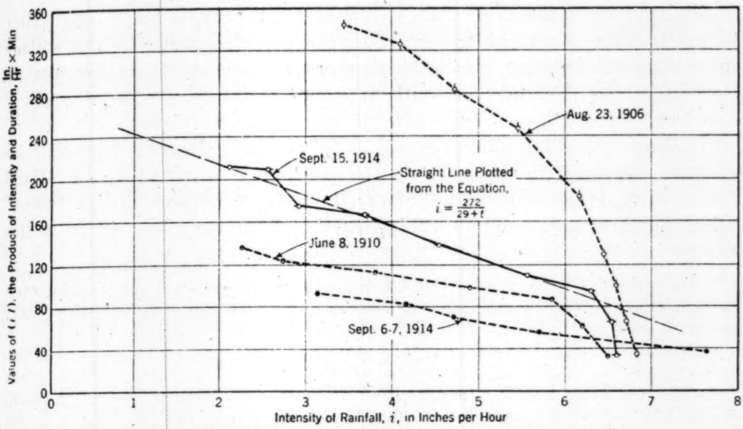

FIG. 12.—EXCESSIVE PRECIPITATION, KANSAS CITY, MO.

The writer is forced to the conclusion that attempts to generalize, in the form of an algebraic equation, are futile as far as conditions in Kansas are concerned. Formulas are convenient, and, in the hands of an expert, may serve a useful purpose, but they appear to be so easy to apply that they are likely to lead to absurd results in the hands of the inexperienced. It would seem that the histogram or the ogive curve is preferable.

TABLE 14.—MAXIMUM INTENSITIES OF RAINFALL IN KANSAS, ARRANGED IN THE ORDER OF MAGNITUDE

	DURATION IN MINUTES									
	5	10	15	20	30	45	60	80	100	120
1	11.64	8.22	6.60	6.45	6.14	5.47	4.74	4.09	3.44	2.26
2	9.60	7.08	6.40	5.55	4.58	4.37	3.52	3.20	2.92	1.75
3	9.12	6.72	6.28	5.22	4.54	3.93	3.47	2.56	2.12	1.60
4	8.40	6.54	6.20	5.10	4.26	3.68	2.96	2.42	2.11	1.58
5	7.80	6.30	5.84	5.07	3.98	3.51	2.93	2.37	1.92	1.56
6	7.68	6.18	5.48	5.04	3.84	3.51	2.92	2.34	1.85	1.50
7	7.20	6.12	5.32	4.89	3.80	3.36	2.81	2.25	1.85	1.46
8	6.96	6.12	5.24	4.80	3.78	3.31	2.73	2.19	1.79	1.44
9	6.84	5.94	5.20	4.65	3.72	3.24	2.65	2.13	1.72	1.42
10	6.84	5.70	5.16	4.62	3.68	3.16	2.65	2.12	1.67	1.26
11	6.84	5.70	5.08	4.41	3.66	3.15	2.64	2.11	1.64	1.23
12	6.72	5.70	5.00	4.41	3.64	3.05	2.56	2.06	1.61	1.22
13	6.72	5.70	5.00	4.29	3.64	3.04	2.55	2.03	1.58	1.15
14	6.60	5.70	4.92	4.29	3.64	2.95	2.50	1.94	1.58	1.02
15	6.60	5.70	4.92	4.14	3.56	2.93	2.44	1.90	1.55	0.97
16	6.48	5.70	4.72	4.08	3.56	2.93	2.43	1.88	1.52	0.97
17	6.48	5.64	4.72	4.08	3.54	2.92	2.40	1.88	1.47	0.92
18	6.48	5.58	4.64	4.05	3.50	2.91	2.38	1.87	1.41	0.79
19	6.48	5.58	4.52	3.96	3.44	2.89	2.37	1.85	1.41	0.68
20	6.48	5.46	4.52	3.96	3.40	2.84	2.30	1,84	1.39	0.66

CHARLES W. SHERMAN,[19] M. AM. SOC. C. E. (by letter).—The writer is particularly impressed by the almost unqualified faith of the authors in the assumption that the total number of station-year records may be used in studies of frequencies without regard to the matter of geographical locations, topographical conditions, or meteorological cycles.

Perhaps the first extensive use of combined records from various stations appeared in Part V of the Technical Reports of the Miami Conservancy District, entitled "Storm Rainfall of Eastern United States," which appeared originally in 1917. Quoting from the 1936 edition of this report:

> "In order to utilize all existing records that possessed any value, even though they differed materially in length, a method was adopted which may be explained as follows:
>
> "Assume a number of rainfall stations, say, five, located within an area possessing uniform rainfall characteristics. At station A complete records have been kept for a period of 70 years; at station B, for 40 years; at station C, for 60 years; at station D, for 80 years; and at station E, for 50 years; the aggregate of the period of records or sum being 300 years. Treating this aggregate as a single record for the area under consideration, it may be said by the above definitions of frequency, that the highest rainfall intensity recorded in the entire period has occurred with a probable frequency of once in 300 years."

The most important words in this description are "located within an area possessing uniform rainfall characteristics." Too many studies have been made in which the writers have totally lost sight of this qualification, and the authors appear to have given little consideration to this requirement when they combined in their studies the records of stations as far apart geographically as Boston, Mass., Dodge City, Kans., Knoxville, Tenn., and Yankton, S. Dak. The topography and climatological conditions affecting quantities and intensities of rainfalls within this wide area vary so much that there is grave doubt whether a combination of the records is of any significance whatever.

Indeed, it may be noted that stations located only a few miles apart may be subject to such different conditions that it would be distinctly improper to combine the records. For instance, data relating to rainfall intensities at the summit of Mt. Washington, in New Hampshire, and at a point perhaps not more than 20 or 30 miles distant at a much lower altitude, certainly should not be combined. It is questionable also how far the records of several stations located within a district measuring not more than 10 to 15 miles in any direction should be combined in any analysis of frequency. Records from such gages are of great importance in studying the extent of the area covered by intense rains[20]; but that the records of ten such gages which have been maintained for 10 yr could be of equal value to a similar record of a single gage for 100 yr in any study of frequency, obviously is impossible.

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[19] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[20] "The Distribution of Intense Rainfall and Some Other Factors in the Design of Storm-Water Drains," by Frank A. Marston, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 535.

Another item which is fully brought out in the foregoing quotation, but perhaps not emphasized as much as it should be, is that the record at each of the stations from which data are proposed to be combined, should be of considerable length. The typical example quoted relates to five stations, with a least period of record of 40 yr. Granted that the topographical conditions are such that rainfalls of like frequency and intensity may occur at each of these several stations, there can be no question that the combined record of the five stations is of much more value than that of a single station. If, however, the combined record totaling 300 yr in the example given, were based on 50 stations having an average length of record of 6 yr per station, and perhaps ranging from 1 yr to 20 yr at the different stations, the combined record would be of little greater value than that of the single station having the longest record.

A paper[21] presented by the late David L. Yarnell, M. Am. Soc. C. E., which was issued in 1935, contains a much more extensive analysis of Weather Bureau records of intense precipitation than that shown by the authors, and appears to be of distinct importance. Mr. Yarnell stated: "The methods followed and the results obtained differ considerably from those of the Miami Conservancy District."

The importance of meteorological cycles must not be forgotten in any study of this kind. In this discussion the term, "cycles," is used without particular reference to its precise applicability. Whether or not changes of meteorological conditions occur with such regularity that they may properly be called cycles, there can be no question that wide differences of climatological and meteorological conditions have occurred within historic times. For the purposes of this discussion it may be considered that what is herein called a "cycle" probably has a duration of not less than 300 yr and may be as long as 1 000 yr. Any combination of the records from a number of stations, which have been obtained in the years during the part of a cycle when the rainfall is either high or low, cannot be accurate when consideration of frequencies of 300, 500, or 1 000 yr is discussed.

With regard to the type of formula which may best represent the relation between intensity of rainfall and its duration, for any given frequency, the authors state that the form, $i = \frac{A}{t + b}$, was found to be best. It might well have been noted that the form,

$$i = \frac{A}{(t + b)^d} \qquad \text{(29)}$$

would be a more satisfactory form and one applicable to all attempts thus far made, of which the writer has knowledge, to express this relation by a formula. If the exponent, d, is unity, Equation (29) is identical with

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[21] "Rainfall Intensity-Frequency Data." *Miscellaneous Publication No. 204*, U. S. Dept. of Agriculture.

Equation (3). If, at the same time, the value of b is 0, Equation (29) conforms to Professor Nipher's formula of 1885 for St. Louis, Mo. If b has a value of 0 and d a value differing from unity, the formula corresponds to that used by the writer[22] in 1905; and if b has a value other than 0 and d a value other than unity, the formula would correspond to that used by the writer[23] in 1931.

Conclusions (6) and (7) of the paper might perhaps have been considerably modified after consideration of the discussion in Mr. Marston's paper.[20] It appears probable that the data studied by the authors might have been analyzed by the methods suggested by Mr. Marston so as to provide information of great value regarding the variations in rainfall intensity over areas of moderate size.

GLEN N. COX,[24] ASSOC. M. AM. SOC. C. E. (by letter).—Many data have been presented very ably in this paper, which should make possible closer predictions as to expected rainfall rates for any mid-western section. Nevertheless, it raises certain questions as follows: The U. S. Weather Bureau standard recording rain-gage is a tilting bucket type. Gages of this type are able to operate at a certain maximum rate due to the inertia of the parts and will not record correctly excessive precipitation. The readings given by the record chart are low. This under-recording will begin when the intensity exceeds about 3.5 in. or 4 in. per hr. Then it becomes necessary to distribute any difference between the stick measurement and the chart value by empirical methods. The last sentence of the first paragraph under the heading "Stations Studied" reads: "No correction factor was used for the precipitation corresponding to any duration." In Table 4 (*a*), three occurrences of precipitation between 0.70 and 0.79 in. are listed. These values correspond to an intensity of about 9 in. per hr which is truly of cloudburst proportions and one which a gage of the tilting type can not record accurately. For this reason, one wonders as to the exact meaning of the foregoing quotation.

Some consideration should have been given to the time of year when certain rates could be expected. After all, the engineer is only interested in precipitation intensities inasmuch as certain run-off rates follow. Many factors other than rainfall intensity affect the run-off rate. For any one water-shed, the time of year, the temperature, extent, and condition of vegetation, the condition of the soil, and previous precipitation are very important; and, in addition, the properties of the water-sheds vary greatly. It is entirely possible for a 1-in. slowly falling rain in early spring to cause a greater run-off than a 3-in. dashing rain of midsummer. After all, 1 000 cu ft per sec per sq mile only corresponds to a rate of 1.55 in. per hr. Run-off rates of this magnitude may follow after previous precipitation has saturated the soil and filled the surface storage.

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[22] *Transactions*, Am. Soc. C. E., Vol. LIV (June, 1905), p. 173.

[23] *Loc. cit.*, Vol. 95 (1931), p. 951.

[24] Prof. of Mechanics and Hydraulics, Louisiana State Univ., University, La.

Since April, 1933, the Louisiana State University, in co-operation with the U. S. Geological Survey, under the writer's direction, has maintained a number of U. S. Weather Bureau standard rain cans and one weighing type of recording gage. During this time, data have been obtained which permit the comparisons of Table 15. The number of times that the Weather Bureau designations were exceeded for excessive rates, for the 5, 10, 15, 30, 60, 100, and 120-min intervals, was 3, 11, 17, 17, 8, 1, and 0, respectively. During this period, in spite of the fact that Louisiana is subject to precipitation of cloudburst intensity, the maximum depth for the 5-min period was 0.42 in., which corresponds to an intensity of 5.04 in. per hr. These data show the relative improbability of obtaining the quantities designated by the authors, or by the Weather Bureau, for the short intervals, and also the improbability of obtaining the Weather Bureau's designation for the longer periods.

TABLE 15.—NUMBER OF TIMES VALUES IN TABLE 6 WERE EXCEEDED

Frequency, in years	Duration, in Minutes						
	5	10	15	30	60	100	120
1	6	8	12	17	17	16	16
10	0	0	0	0	1	1	1
100	0	0	0	0	0	0	0

GARRETT B. DRUMMOND,[25] ESQ. (by letter).—A careful analysis of the method followed by the authors in determining the constants in their

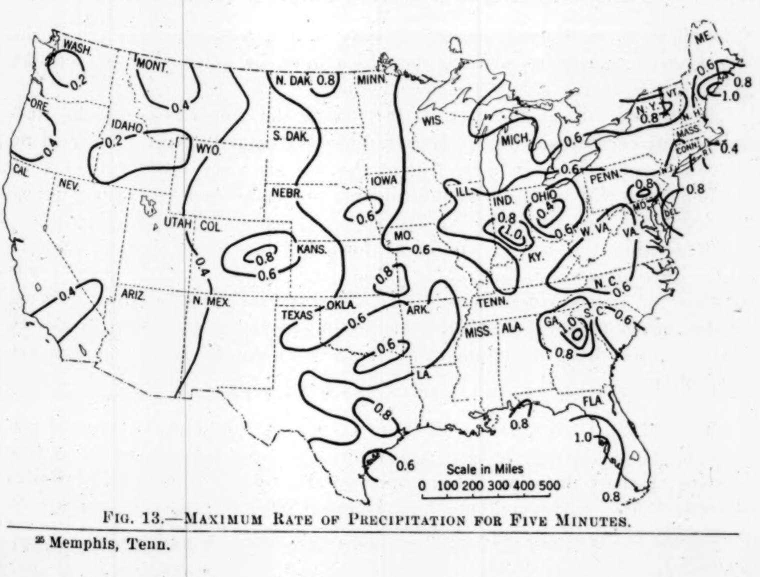
WASH.
MONT.
N. DAK.
MINN.
ME.
ORE.
IDAHO
WYO.
S. DAK.
WIS.
MICH.
N. Y.
VT.
N. H.
MASS.
CONN.
R. I.
CAL.
NEV.
NEBR.
IOWA
ILL.
IND.
OHIO
PENN.
N. J.
MD.
DEL.
UTAH
COL.
KANS.
MO.
W. VA.
VA.
KY.
N. C.
ARIZ.
N. MEX.
OKLA.
TEXAS
ARK.
TENN.
S. C.
MISS.
ALA.
GA.
LA.
FLA.
0.2
0.4
0.6
0.8
1.0
Scale in Miles
0 100 200 300 400 500

FIG. 13.—MAXIMUM RATE OF PRECIPITATION FOR FIVE MINUTES.

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[25] Memphis, Tenn.

various equations raises the question of the actual dependability of empirical formulas. In their application it should be remembered that such formulas are reliable only within the range of data from which they are derived. Especially is this true when the mass of data is segregated arbitrarily into periods of durations, as is done in the case of rainfall intensities. Actually, the "picture" presented by such data is the complete regimen of rainfall. Therefore, by considering separately that falling within certain time limits, there is introduced an artificiality which must be taken into consideration when utilizing the results of such a study.

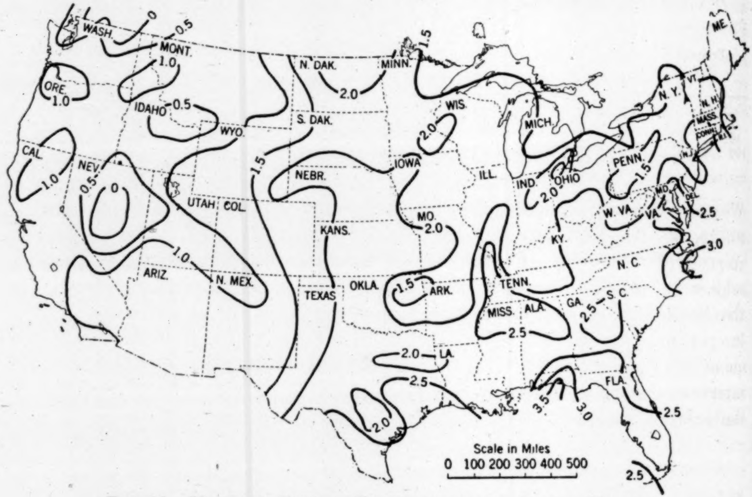
WASH.
MONT.
N. DAK.
MINN.
ME.
ORE.
IDAHO
WYO.
S. DAK.
WIS.
MICH.
N. Y.
VT.
N. H.
MASS.
CONN.
R. I.
CAL.
NEV.
UTAH
COL.
NEBR.
IOWA
ILL.
IND.
OHIO
PENN.
N. J.
MD.
DEL.
W. VA.
VA.
KANS.
MO.
KY.
N. C.
ARIZ.
N. MEX.
TEXAS
OKLA.
ARK.
TENN.
S. C.
MISS.
ALA.
GA.
LA.
FLA.
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Scale in Miles
0 100 200 300 400 500

FIG. 14.—MAXIMUM RATE OF PRECIPITATION FOR THIRTY MINUTES.

However, this should not detract from the importance of the study which has been submitted by Messrs. Schafmayer and Grant. Their efforts, although restricted to Chicago, point the way to other investigations which undoubtedly will add to the information now possessed concerning precipitation.

Regarding the use of empirical formulas for sewer design, especially those involving frequency, it is interesting to compare the results with a study of the maximum precipitation for various time-lengths. This, the writer has done for 5-min intervals to include 30 min. Figs. 13 and 14, for example, show maximum rates of precipitation for 5 min and 30 min, respectively.

EUGENE L. GRANT,[26] M. AM. SOC. C. E. (by letter).—An implied conclusion of this study is that the intensities and frequencies of extreme storms of short duration do not vary greatly over a large area of Central United States. Although the area bounded by a line passing through St.

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[26] Associate Prof. of Civ. Eng., Stanford Univ., Stanford University, Calif.

Paul, Minn., Detroit, Mich., Knoxville, Tenn., Memphis, Tenn., Dodge City, Kans., and Yankton, S. Dak., contains wide variations in annual rainfall and other climatic characteristics, it is apparently homogeneous with respect to the frequencies of the intense storms which were studied. This implied conclusion should interest all students of meteorology and hydrology; it is of immediate practical importance in that the curves in the paper—particularly the very useful Fig. 5—may be applied to the economic design of storm drains throughout this entire area.

Because of the importance of this conclusion, and because the combining of a 33-yr record for ten stations into what amounts to a 330-yr record seems to be justified only on the assumption that the storms at each station are independent events which are subject to the same system of chance causes, it is desirable to check this conclusion in all possible ways. One such check was the observance by the authors of the close correspondence between the (unpublished) graphs for the individual stations. Other checks may be devised which make use of the methods of mathematical statistics, and which give results expressed in terms of probability. Two types of tests of this character which have been made by the writer seem to confirm the conclusion that the observed differences between the ten stations studied in detail are such as might be expected as chance variations in samples of the size observed, if there were no variations in the cause systems affecting the ten stations.

The first of these tests is the so-called χ^2 test of goodness of fit devised by Karl Pearson and elaborated by Mr. R. A. Fisher[27] which has been applied to each of the seven sections of Table 4. A sample calculation relative to the 30-min storms of Table 4(d), is shown in Table 16, in which

TABLE 16.—TEST FOR CONSTANCY OF CAUSE SYSTEM AT TEN RAINFALL STATIONS

City	Number of Storms with the Following Precipitation (in Inches) During 30 Minutes							
	From 0.90 to 0.99		From 1.00 to 1.09		From 1.10 to 1.19		1.20 and more	
(1)	Number observed, f_o (2)	$\frac{(f_o-f_e)^2}{f_e}$ (3)	Number observed, f_o (4)	$\frac{(f_o-f_e)^2}{f_e}$ (5)	Number observed, f_o (6)	$\frac{(f_o-f_e)^2}{f_e}$ (7)	Number observed, f_o (8)	$\frac{(f_o-f_e)^2}{f_e}$ (9)
Knoxville, Tenn.	6	0.74	5	0.23	2	1.97	12	0.60
Memphis, Tenn.	13	2.38	10	2.33	9	2.78	4	3.27
Indianapolis, Ind.	10	0.26	8	0.52	7	0.62	9	0.04
Cincinnati, Ohio	11	0.74	5	0.23	1	3.39	7	0.70
Detroit, Mich.	7	0.26	5	0.23	4	0.28	9	0.04
Chicago, Ill.	4	2.38	8	0.52	6	0.12	11	0.20
St. Paul, Minn.	4	2.38	4	0.78	4	0.28	10	0.02
Cairo, Ill.	9	0.03	8	0.52	9	2.78	10	0.02
Yankton, S. Dak.	13	2.38	6	0.01	5	0.01	11	0.20
Dodge City, Kans.	8	0.03	3	1.65	5	0.01	13	1.20
Number expected, f_e*	8.5	....	6.2	....	5.2	....	9.6	....
Value of χ^2	....	....	....	....	....	....	....	37.13†

* Assuming uniform conditions. † Sum of Columns (3), (5), (7), and (9).

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[27] A general explanation of this test may be found in "Probability and Its Engineering Uses", by T. C. Fry, Articles 102–104. The specific application to a problem of the type presented in testing Table 4 for homogeneity is based on unpublished notes on "Statistical Inference", by Prof. Harold Hotelling, of Columbia Univ., New York, N. Y.

f_o = number of storms of a given magnitude observed at a given station in the 33-yr period; and f_e = number of storms expected, assuming uniform conditions.

The χ^2 test involves the classification of the observed data into groups or "cells"; the number expected in each cell according to the hypothesis to be tested is then computed. In this case the hypothesis is that all the ten stations are alike with respect to the occurrence of intense storms of each magnitude, so that the number expected at each station is the average number of storms observed at the ten stations for each precipitation. The departure from expectancy in each cell is then squared and divided by the number expected in the cell; the sum of these computed values is called χ^2. Probability theory then gives the relation between χ^2, the number of "degrees of freedom" (the total number of cells compared minus a correction for the number of ways in which the numbers expected are made to agree with numbers observed), and the probability, *P*, that if the expected system of causes were really operating, a departure from the expected number as great as, or greater than, that observed would occur as a matter of chance in random sampling. The purpose of the test, of course, is the determination of this value of *P*.

In Table 16 it has been necessary to group together all storms in excess of 1.20 in. at each station; experience with the test indicates that it cannot be expected to give satisfactory results if the expected frequency in any cell is fewer than five. A similar condensation of cells has been made in testing each section of Table 4.

Table 16 gives a χ^2-value of 37.1. There are 36 degrees of freedom, 40 cells minus 4 restrictions imposed by the condition that the total number expected in each precipitation group agree with the total number observed in that group. Entering a table of χ^2-values, such as may be found in many books on statistical methods,[28] these two values give a probability, *P*, of 0.4. This is interpreted to mean that if there were no real difference in the cause systems affecting 30-min excessive storms at the ten stations, a variation from uniform results as great as or greater than, that observed might be expected four times out of ten. In other words, the observed data are not inconsistent with the hypothesis of uniformity. The results obtained from χ^2-tests on the other sections of Table 4, are shown in Table 17; they seem to confirm this conclusion. The best fit to the assumption of uniform causes is found in the storms from 60 to 120 min in duration.

The second statistical test made by the writer involved the checking of an hypothesis concerning the relation between the average number of excessive rainfalls per year of each duration, and the probability of any given number of such excessive rainfalls occurring in any single year. This hypothesis is that this probability may be determined by the Poisson law[9], which expresses the probability of any given number of occurrences per period as a function of the average number of occurrences, m. The Poisson law gives the probability of no occurrences as e^{-m}, of exactly one occurrence as me^{-m}, of exactly two as $\frac{m^2}{2} e^{-m}$, of exactly three as $\frac{m^3}{3 \times 2} e^{-m}$, of exactly four as $\frac{m^4}{4 \times 3 \times 2} e^{-m}$, etc.

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[28] "Statistical Methods for Research Workers", by R. A. Fisher, Table III, pp. 96–97, or Fry's "Probability and Its Engineering Uses", Table VIII.

TABLE 17.—RESULTS OF χ^2-TESTS APPLIED TO EXCESSIVE RAINFALL RECORDS OF TEN STATIONS

Duration of excessive storms, in minutes	TEST OF DATA IN TABLE 4 FOR HOMOGENEITY				Value of P obtained from comparison with the Poisson law
	Cells compared	Degrees of freedom	Calculated χ^2	Value of P	
5	30	27	23.6	0.7	0.08
10	40	36	43.0	0.2	0.97
15	40	36	41.0	0.3	0.7
30	40	36	37.1	0.4	0.3
60	30	27	14.9	0.97	0.5
100	30	27	20.8	0.8	0.3
120	30	27	13.8	0.98	*

* Not determined.

The characteristics of extreme rainstorms would seem on theoretical grounds to agree with the assumptions underlying the Poisson law, at least in so far as the rainstorms at any one station are concerned. In brief, these assumptions are that the quantity measured is the number of occurrences of a particular event, each sample consisting of a very large number of independent trials of the event in which the probability of the event occurring on any individual trial is very small. Extreme storms are presumably independent events; there are a great many short intervals of time during a year which might have such storms; the probability of an extreme storm in any one of them is very small indeed.

In testing the Poisson law hypothesis it was necessary for the writer to examine the original rainfall data assembled by Messrs. Schafmayer and Grant in order to construct frequency distributions which showed in how many station-years there were no excessive storms of a given duration, and in how many there were one, two, three, four, etc. The expected number of years which would have each number of storms was then calculated by the Poisson law, using the observed average. Table 18 gives the data for the storms of 10-min duration, and illustrates the method followed. Dividing the total of Column (3) by the total of Column (2), the average number of storms per year, m is found to be $\frac{396}{330} = 1.20$; $e^{-m} = 0.3012$; $me^{-m} = 1.20\ (0.3012) = 0.3614$; $\frac{m^2}{2} e^{-m} = \frac{1.20}{2}$ $(0.3614) = 0.2168$; $\frac{m^3}{3 \times 2} e^{-m} = \frac{1.20}{3}\ (0.2168) = 0.0867$; etc.

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[9] For mathematical derivations and discussions of the Poisson law, see Fry's "Probability and Its Engineering Uses", Articles 83-88. For examples of a number of other series which are fitted by the Poisson law, see "Applications of Poisson's Probability Summation", by Frances Thorndike, *Bell System Technical Journal*, Vol. 5 (1926), p. 604.

It will be noted that the number of 10-min storms shown in Table 18 for the 330 station-yr is necessarily the total of 396 shown in Table 4(*b*) for the same station-years. The correspondence between the observed frequencies shown in Column (2), and the theoretical frequencies from the Poisson law shown in Column (5), is extraordinarily good. Similar checks between observed and expected values—although not quite as close as this one—were found for the frequency tables for the storms of other durations. The χ^2-test was used to compare these observed and expected frequencies; the values of *P* obtained from these tests are included in Table 17.

TABLE 18.—POISSON LAW FITTED TO NUMBER OF 10-MINUTE EXCESSIVE STORMS PER STATION-YEAR

Number of excessive storms *N*, in year (1)	Observed number of station-years with exactly *N* storms, *f* (2)	Total storms in each group *fN* (3)	Probability of exactly *N* storms by the Poisson law (4)	Expected frequency in 330 station-years by the Poisson law (5)
0..............	102	0	0.3012	99
1..............	114	114	0.3614	119
2..............	74	148	0.2168	72
3..............	28	84	0.0867	29
4..............	10	40	0.0260	9
5..............	2	10	0.0062	2
Totals..............	330	396	......	330

It would seem reasonable that the relationship herein discovered between the average number of storms per year of a given magnitude and duration, and the probability of any given number of such storms occurring in any one year, justifies the use of the Poisson law for similar calculations relative to extreme storms of any frequency. For instance, any storm shown on Fig. 5 with a frequency, *F*, of 5 would correspond to an average of 0.2 storm per yr. The probability of no such storms in any one year may be computed by the Poisson law to be 0.819, and of exactly 1 yr to be 0.164; thus, in only 17 yr out of 1 000 would two or more such storms occur in the same year. Such computations would seem to be more of academic than of practical importance, although they might afford a kind of doubtful comfort to the engineer who has just seen his storm sewers overflowed by one such storm.

The important practical conclusion from the Poisson calculations is that they furnish another check on the hypothesis that the excessive storm characteristics are the same at all of the ten stations studied. It is a mathematical property of the Poisson law that two or more Poisson distributions with different averages do not add up to make another Poisson distribution; thus, the check between the total results for the ten stations and the Poisson law is good evidence of similarity between the stations.

The points of similarity and the fundamental differences between this study of rainfall intensity, and the many interesting studies which hydraulic engineers have made of flood frequencies, are worthy of comment. Although the authors have made use of no "probability curves" such as have been used in many flood studies, the same general philosophy underlies the approach to both problems. The evident fact that the authors' studies of excessive rainfall frequency seem much more conclusive than any published studies of flood frequency, results from certain characteristics of the rainfall problem which are not found in the flood problem. These advantages may be recognized if the implications of a frequency or probability approach to a problem of this type are noted.

The statistical or frequency definition of probability may be stated as follows: Assume that if a large number of trials of an event are made under the same essential conditions, the ratio of the total number of trials in which a certain event happens, to the total number of trials made, will approach a limit as the latter number is increased indefinitely; then the limit thus described is defined as the probability that the event shall happen under these conditions.

In the flood studies such as those made by the late Allen Hazen,[30] M. Am. Soc. C. E., the trial may be thought of as a year; the event which either happens or fails to happen is a flood of a given magnitude. Several assumptions are made, either consciously or by implication, in flood studies leading to probabilities for floods of any given magnitude. Among them are: (1) There really is a natural law which relates to the probability of a flood of a given magnitude at a given point on a given stream at a particular time; if it were possible to secure records for a great many years during which time there was no fundamental change in climate or in the characteristics of the stream's drainage area, the ratio of occurrences of a given flood to total years would actually approach a statistical limit; (2) successive trials are made under the same essential conditions or nearly enough so for practical purposes; this implies among other things that they are independent of one another; that is, the flood magnitude in one trial is not related to a flood magnitude in preceding or succeeding trials; and (3) there is a definite relation between the probability or frequency of a flood of given magnitude and the magnitude of the flood, which relationship would be apparent if one had a sufficiently long record. On the basis of a limited record an attempt is made to fit some kind of probability curve to the available data, extrapolating if necessary in order to obtain desired probabilities. Thus, it is common for investigators using these methods to calculate the probability of the 1-yr-in-100 flood from a 30-yr record—or sometimes even the 1-yr-in-10 000 flood.

The logical weakness of such extrapolation beyond the limits of a poorly defined frequency curve is obvious. Different forms of curves may seem to fit the limited data fairly well, and yet have widely different implications outside its range when they are used as a basis of extrapolation. Thus, the methods developed by different investigators of flood frequencies give very different results when applied to the same problem. One investigator may designate as a 1-yr-in-57 flood what another calls a 1-yr-in-500 flood.[31]

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[30] "Flood Flows", by Allen Hazen, John Wiley & Sons, 1930.

[31] See the discussion by C. S. Jarvis, M. Am. Soc. C. E., in *Proceedings* Am. Soc. C. E., Papers and Discussions, October, 1927, p. 2028, for this particular example. The wide differences in conclusions obtained by different methods of extrapolation are familiar to any student of this subject.

Two attempts have been made by investigators to avoid extrapolation by increasing the data used from a limited number of years of record. One method has been to combine the records for a number of streams; the other has been to take all the flood peaks each year in order to obtain a great many floods from a record of short duration. Neither of these devices seems reasonable with respect to flood probability studies. The water-shed characteristics of the various streams are sufficiently different so that floods on separate streams are not under "the same essential conditions;" the successive floods on a given stream in a given year are certainly not independent events.

In studies of intensity and frequency of excessive rainfall, however, it does seem reasonable to use both these devices. As was evidenced by the application of the χ^2-test to the data in Table 4, the excessive rainfalls of short duration at widely separated stations seem to come from the same system of chance causes. Excessive rainstorms are local in character; the storms at different stations at a considerable distance from one another are obviously independent events. Thus, it is legitimate to combine the 33-yr records of ten stations to secure what in effect is a 330-yr record.

It also seems reasonable in these rainfall studies to make use of all the excessive storms that have occurred in a given year. Although two or more flood peaks in a year on the same stream would seem to be definitely related, two or more excessive rainstorms of short duration in the same year at one station might be expected to be completely independent. This expectation is confirmed by the close fit of the number of storms per year to the Poisson law.

Finally, the investigator of the frequencies of excessive rainfalls who wishes to use his conclusions in the economic design of storm sewers has a great advantage over the investigator of flood frequencies who wishes to use his conclusions in the economic design of structures subject to flood. In theory at least, design for long-run economy of both storm sewers and flood spillways would seem to require consideration of the cost of providing each possible capacity, the expected frequencies with which such capacity would be exceeded, and an estimate of the probable resulting damage. However, the conclusions of economy studies in the two fields, as to the frequencies against which it is economical to design, are likely to be quite different. A typical conclusion might be that a spillway should be designed against at least the 1-yr-in-100 flood, whereas a storm sewer should be designed, at most, against the 1-yr-in-5 excessive rainfall. In the first instance extrapolation to estimate the 1-yr-in-100 flood from a record of 30 yr or less involves much guesswork as to the form of the frequency function. In the second instance, as the authors have demonstrated, a 330-yr record defines its own frequency function so that it is possible to estimate the 1-yr-in-5 rainfall with a high degree of confidence, and there is no necessity for extrapolation.

ADOLPH F. MEYER,[32] M. AM. SOC. C. E. (by letter).—This paper gives encouragement to those who have labored long in favor of the use of station-year records from a relatively large area in preference to the present-day inadequate records of intense precipitation at a single station, for the purpose of predicting rainfall frequency. The fundamental soundness of the method of determining frequencies from station-year records is demonstrated by the fact that, notwithstanding the recent severe droughts, the last eighteen years of rainfall records (1929-37), when added to the earlier nineteen years of record for stations analyzed by the writer elsewhere,[33] do not result in any material change in the conclusions respecting the frequencies of given rates of intense precipitation to be expected in the several time intervals from 5 min to 2 hr.

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[32] Cons. Hydr. Engr.; Engr. (Meyer Governor Co.), Minneapolis, Minn.

A comparison of the frequency of intense precipitation, as determined from the authors' graphs in Fig. 3, for 659 station-yr, with the frequency of the same rates of intense precipitation for the City of Chicago, Ill., taken from charts presented by the late David L. Yarnell, M. Am. Soc. C. E.,[34] is most striking. The values are almost identical, with the exception of the 100-yr, 2-hr rate shown in Fig. 3 of the paper. It is the writer's judgment, however, that curves, and not straight lines, should be used to represent the actual facts shown by the observational data in Fig. 3. If curves are used, the 2-hr precipitation of 100-yr frequency is substantially reduced and comes into much better agreement both with the Yarnell charts and with the writer's formulas, first published twenty years ago.[35]

The writer has found further satisfaction in the authors' use of definite rates of precipitation for the various time intervals as measures of what constitutes "excessive rainfall" rather than the use of percentages of annual rainfall. It has always been the writer's contention that there is no material relationship between intense precipitation during short-time intervals and annual precipitation. Stations with low annual precipitation are quite likely to have very intense precipitation for short-time intervals, although the frequency of such excessive rainfalls is usually greater in regions of moderate and heavy rainfall.

Another point cited by the authors which is worthy of emphasis is the fact that the maximum rainfall determined from the original rainfall recording gage charts for any given time interval is greater than the maximum determined from the published tabular data. The writer has called attention to this fact[35] in analyzing approximately 300 storms for Minneapolis and St. Paul, Minn., with a view to determining the correction which should be applied to the tabular data published by the U. S. Weather Bureau. The data published for Minneapolis and St. Paul give results 9% too small for the 5-min interval, 11% too small for the 2-hr interval, and from 2 to 4.5% too small for the intermediate intervals from 10 min to 60 min. These errors of about 10% for the 5-min and 120-min intervals are far in excess of the differences between conclusions drawn from the writer's 19-yr record,[33] as compared with those drawn from the present 37-yr record.

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[33] "The Elements of Hydrology", by Adolph F. Meyer, 1917, Group 3, p. 198.

[34] *Miscellaneous Publication No. 204*, U. S. Dept. of Agriculture.

[35] "The Elements of Hydrology", 1917, p. 165.

Another fact which bears emphasis, because of a deficiency in the earlier published data of the Weather Bureau and of errors frequently made by those who use such data, is that the accumulated precipitation for 60 or 80 min, for example, must also be considered as having fallen in 100 and 120 min, even if the rain did not continue that long, or was not excessive toward the end and, therefore, was not reported. Where the rain did continue, but the rate was not excessive, the later records of the Weather Bureau now give the necessary information.

Most of the questions to which attention has been directed herein result in increasing the quantity of rain which may be expected in a given time interval. If designs are to be conservative, these several factors must be taken into consideration.

When it is noted that the formula heretofore used in Chicago gave frequencies varying from 3 to 20 yr, which were supposed to represent approximately 5-yr frequencies, it must be apparent that the adoption of new values of frequencies for given rates of intense precipitation will result in better balanced and more economical sewer designs for the City of Chicago.

The writer believes that the Yarnell charts,[34] are by far the best measure of the frequency of given rates of intense precipitation over most of the United States, available to-day.

CLINTON L. BOGERT,[36] M. AM. SOC. C. E. (by letter).—A very complete study of rainfall intensities is reported in this paper. The authors have extended the analytical methods of comparing rainfall intensities and frequencies.

In pursuing statistical methods, engineers should not become so engrossed in the formulas and curves derivable from the data at hand, as to lose sight of the ultimate aim of the study, which is to obtain the run-off from a given area under the worst conditions.

This run-off is dependent on two factors of about equal weight, namely, the rainfall intensity with which this paper is concerned, and the run-off factor. Assuming that the studies are being made to establish the size of drains, the drains will be improperly proportioned if an error in judgment is made in selecting either of the factors. The run-off factor depends upon the slopes and the extent of the built-up and paved areas in the drainage district. Zoning will tend toward making this factor slightly more determinate, but there is no telling when, in the future, Court or legislative action will alter zoning restrictions and automatically put the engineer's assumptions of the future character of the district awry.

Another variable in the problem is the frozen ground in winter which is not considered in arriving at a run-off coefficient. A snow-covered lawn with an ice glazing is equivalent in run-off capacity to a paved area. In most cases of inadequate drains, the flooding will be found to occur in the early spring, due to high run-off from frozen ground and glazed snow.

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[36] Cons. Engr. (Sanborn & Bogert), New York, N. Y.

C. H. EIFFERT,[37] M. AM. SOC. C. E. (by letter).—The writer is very much interested in this paper and is pleased to note that the authors apply the method of combining years of record of a number of stations to be used in the determination of frequencies. This method was used extensively in one of the Technical Reports of the Miami Conservancy District, "Storm Rainfall of the Eastern United States", published in 1917. This report was revised and reprinted in 1936. Elsewhere,[38] the writer has attempted to emphasize some of the possibilities of this method for increasing the usefulness of precipitation records of comparatively short duration. The Miami Conservancy District studies were of great storms lasting from 1 day to 5 days and were made for flood-control purposes, whereas the investigations described by the authors were made in connection with sewer design and refer to excessive precipitation lasting from 5 min to 2 hr. However, the principle of combining the years of record is the same.

It is perhaps pertinent to state that judgment and caution must be used in the application of this method. The area from which the records are combined must have uniform rainfall characteristics; otherwise, the results will not be correct for different points within the area. For instance, an area in which there are great differences in altitude would probably also have great differences in precipitation both as to type and quantity. The years of record for stations in such an area could not be combined satisfactorily.

The records for the individual stations should be long enough to cover a fairly complete rainfall cycle. Such cycles are indefinite at best; yet it is well known that periods of wet and dry years do occur. Therefore, any record so short that it covers a wet or a dry period only would not be suitable for use in this connection.

In the aforementioned work of the Miami Conservancy District, the eastern half of the United States was divided into 2° quadrangles. The records of the stations in each of these quadrangles were added to obtain the total years of record for the quadrangle. Records of less than 5-yr duration were discarded. The size of the quadrangles used is somewhat arbitrary. It should probably be changed in order to obtain the most satisfactory results for certain localities. In the study of the entire eastern half of the United States this refinement could not be attempted on account of the work involved.

There seems to be no good reason for combining the records of stations as far apart and differing as much in climatic character as Boston, Mass., Yankton, S. Dak., and Knoxville, Tenn. The combination of records from stations in the Chicago (Ill.) area should be quite satisfactory. It is true that the records of a number of the stations are much shorter than is desirable so that only part of a cycle is covered and this fact should be borne in mind when the data are being used. Even if the period is short it is probably the best possible way of using these records. It will be necessary to revise the curves from time to time as records accumulate.

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[37] Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

[38] *Engineering News-Record,* December 31, 1936.

A very valuable publication on "Rainfall Intensity-Frequency Data", by the late David L. Yarnell, M. Am. Soc. C. E., was issued by the U. S. Department of Agriculture in August, 1935. This record covers periods from 5 min to 24 hr. The values taken from the authors' curves for Chicago agree very closely with those for the same location taken from Mr. Yarnell's maps.

The writer has seized every opportunity in the past to emphasize the necessity for obtaining more complete records of rainfall and run-off, and cannot refrain from doing so again in this connection. The U. S. Weather Bureau needs many more recording rain-gages. The records should be made continuous and should not be subject to termination on account of lack of appropriations as they are at the present time. Fragmentary records lose a great deal of their value for the aforementioned reason, namely that they cover only parts of cycles. Engineers should emphasize the importance of such records whenever possible because legislators, as a rule, do not realize the necessity for them.

C. S. JARVIS,[39] M. AM. SOC. C. E. (by letter).—The persistency of the straight-line relations as shown in Fig. 2 is so striking as to invite further consideration. It is found that the data included in Meyer's Table 15, Groups 2, 3, and 4,[40] conform moderately well to straight lines for the lower rainfall intensities (Fig. 15). However, for Group 1 the divergence of the data from straight-line trends becomes quite marked, thus requiring the use of irregular curves. Similar divergence from the straight-line relation is apparent likewise in the data furnished for Boston, Mass., by Charles W. Sherman,[41] M. Am. Soc. C. E., and re-plotted on semi-logarithmic paper in Fig. 16(*a*).

When the data from Group 3 of Meyer's Table 15 were plotted similarly on the same scale as that used for Fig. 2 of the paper, their inclination to fall on straight lines was quite pronounced; but when plotted on a much larger scale, the divergences became proportionately more apparent.

Approaching the problem independently, the writer undertook to analyze the excessive precipitation data[42] for Indianapolis, Terre Haute, Evansville, Fort Wayne, and Royal Center, in Indiana; Cleveland, Columbus, Dayton, and Cincinnati, in Ohio; Pittsburgh, Pa.; St. Louis, Mo.; and Chicago, Ill., representing a total of 301 yr of record, or an average of 25 yr per station, the maximum record period being about 34 yr for Indianapolis, with 33 yr for Pittsburgh, Columbus, and Cincinnati. The resulting intensity-frequency graphs are shown in Fig. 16(*b*), on which the data from 301 station-yr of composite record are summarized graphically, all conforming more or less closely to straight-line trends.

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[39] Cons. and Hydr. Engr., SCS, Washington, D. C.

[40] "Elements of Hydrology," by Adolph F. Meyer, 1917, pp. 181–184.

[41] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 956.

[42] *Bulletin W*, U. S. Weather Bureau, 1930.

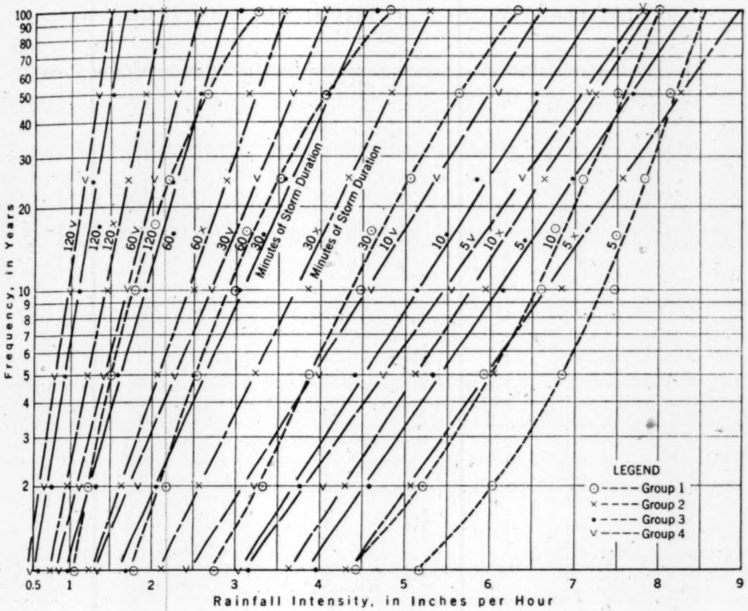

FIG. 15.—COMPARISON OF RAINFALL FREQUENCY-INTENSITY RELATIONS.

In order to afford a comparison of data for Cincinnati and Chicago from record periods terminating in 1914 and 1930, respectively, Tables 19 and 20 were compiled. These tables show generally increased intensities for given frequencies, due to the inclusion of the more recent data. The

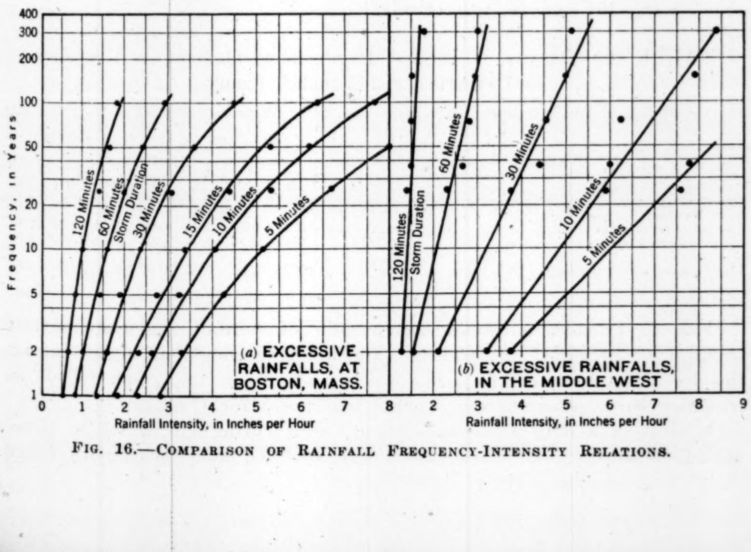

FIG. 16.—COMPARISON OF RAINFALL FREQUENCY-INTENSITY RELATIONS.

TABLE 19.—Comparison of Intensity-Frequency Relations

Frequency years	Precipitation Depths, in Inches, Exceeded Once in Given Number of Years During Designated Time											
	Time, in Minutes											
	5			10			30			60		
	Meyer*	U. S. Weather Bureau†	Authors‡	Meyer*	U. S. Weather Bureau†	Authors‡	Meyer*	U. S. Weather Bureau†	Authors‡	Meyer*	U. S. Weather Bureau†	Authors‡
(a) Chicago, Illinois												
	....	0.37§	....	....	0.61§	....	....	1.18§	....	....	1.49§	....
2	0.36	0.44	0.40	0.64	0.68	0.63	1.05	1.29	1.10	1.28	1.63	1.34
5	0.45	0.51	0.50	0.75	0.77	0.78	1.25	1.45	1.40	1.45	1.87	1.74
10	0.55	0.55	0.55	0.85	0.82	0.89	1.35	1.55	1.62	1.60	2.05	2.05
(b) Cincinnati, Ohio												
	....	0.35§	....	....	0.59§	....	....	1.33§	....	....	....	....
2	0.34	0.44	....	0.56	0.69	....	0.98	1.43	....	1.15	1.85	....
5	0.40	0.51	....	0.65	0.84	....	1.20	1.84	....	1.50	2.26	....
10	0.48	0.55	....	0.78	0.96	....	1.60	2.14	....	2.00	2.46	....

* Data from Table 14, pp. 178–179, Meyer's "Hydrology," 1917.
† Analysis of data from U. S. Weather Bureau *Bulletin W*, 1930, by C. S. Jarvis, as shown in Fig. 17.
‡ Data from Fig. 5, Schafmayer and Grant.
§ Derived from all pertinent data in U. S. Weather Bureau *Bulletin W*, instead of being limited to those higher values defined by the authors as "excessive rainfalls."

quantities in the columns headed, "Authors," were derived from Fig. 5, and show fairly close agreement between results of independent approaches by the authors and the writer toward rainfall frequency-intensity relationships for Chicago, even if the station-years of record considered were unequal, being 107 and 25, respectively.

TABLE 20.—Depths, in Inches, Required to Qualify as Excessive Precipitation for Various Periods

Authority	Duration of Storm, in Minutes						
	5	10	15	30	60	100	120
U. S. Weather Bureau:							
Northern States	0.35	0.50	0.65	0.90	1.20	1.30	1.40
Southern States	0.40	0.50	0.60	0.90	1.50	2.30	2.70
The Authors, under heading, "Definitions of Terms"	0.35	0.50	0.65	0.90	1.20	1.30	1.40

Fig. 17 provides a graphic comparison of rainfall frequency-intensity trends for Cincinnati and Chicago, and indicates somewhat higher intensities for Cincinnati, except for 5-min periods, when the graph follows an irregular curve, crossing the straight-line trend for Chicago.

Whether the data from 12 stations, representing an aggregate record period of 301 yr, may be regarded and analyzed as a continuous record at one station for three centuries has never been settled authoritatively, although the assumption has been widely accepted. Valuable generalizations, trends, and conclusions have been derived from such composite records; but, thus far, no one has claimed and proved their equivalence to continuous records of equal lengths from single stations. Neither has it been established satisfactorily that straight-line relations inherently exist among observed hydrologic or other physical data, no matter what scale or form of chart is used. The natural inequalities and irregularities appearing in practically all basic data must be "ironed out" by some selective grouping, averaging, or other leveling process, or by plotting on small-scale charts, before the inherent deviations from the trend lines or curves are practically eliminated, or reduced to significant, readily understandable quantities, susceptible of logical interpretation.

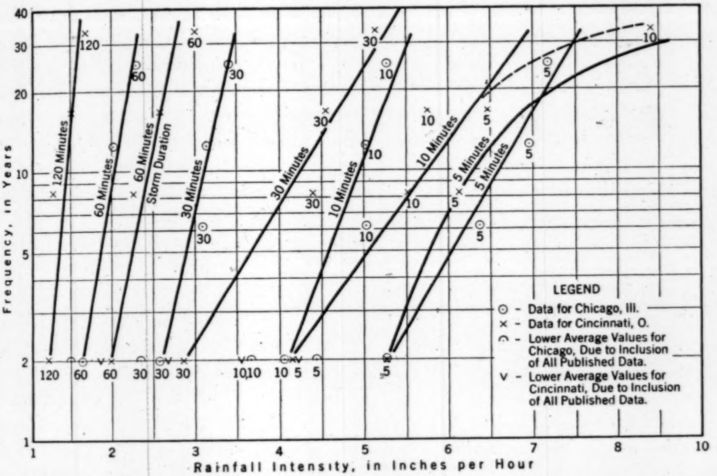

FIG. 17.—COMPARISON OF RAINFALL FREQUENCY-INTENSITY RELATIONS FOR CINCINNATI, OHIO, AND CHICAGO, ILLINOIS.

The writer has been unable to reconcile the various trends of Fig. 2, whereby the rainfall intensities for given frequencies become slightly less severe as the station-years increase, although the added stations are supposed to be fairly comparable. One would naturally expect higher records to be developed as stations are increased in number and as the record periods are extended.

The authors have made important contributions to the subject of their paper, and have apparently established a safe basis for the design of storm sewers in the Chicago District.

A. J. SCHAFMAYER,[43] M. AM. SOC. C. E. (by letter).—The constructive type of discussion submitted brings out some interesting points. For example, Mr. Cochrane's comparison of Equations (18) and (27) in Table 8 shows a very remarkable agreement in intensities.

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[43] Engr., Board of Local Impvts., City of Chicago, Chicago, Ill. Mr. Grant, co-author of this paper, died on October 30, 1936.

Mr. LeRoy K. Sherman questions the grouping of the Tennessee stations with the others included in the Chicago group. As pointed out by Professor Meyer, the writers are in agreement with his deduction that there is no material relationship between the intense precipitation for short periods of time and the quantity of annual rainfall.

As to Mr. Sherman's question: Are irregularities "ironed out" for small urban areas? the writers' study of the data led them to believe that irregularities are "ironed out" for areas requiring a time element of 15 min, or more, for excessive intensities. However, this may not be true for annual quantities, as shown by George W. Pickels,[44] M. Am. Soc. C. E. The writers made no attempt in this study to relate intensity and run-off or to secure simultaneous intensity and run-off data, as was done in the study by Messrs. Horner and Flynt.[13] It is evident, however, as stated by Mr. Sherman, that the intensity rates may be applied to run-off computation by the so-called rational method, or by the Sherman unit hydrograph[11, 12] method, or by empirical graphs, or tables of other types. Table 9, submitted by Mr. Sherman, shows the value of careful study in this part of the design.

In his studies of rainfall for Kansas, Professor Jones presented some interesting data. As he states, however, comparisons, especially with his ogive curves, are difficult. Although their studies led the writers to believe that the quantity of annual rainfall has no relation to the intensity for short durations, it appears that this is true only for the excessive rates with a high minimum intensity used by them, as shown under the heading, "Definitions of Terms." If lower intensities are included and a large number of storms of minor intensity are added to the tabulations, the principle of independence between intensity and annual precipitation is modified, and this independence tends to decrease as the minimum rate is reduced. For example, in Section (*a*) of Table 11, the occurrences listed by Professor Jones with intensities less than that used by the writers, number 1 077 in a total of 1 243 items, or more than 86%; but even with this increase in number of occurrences, the data as shown by the analysis of Professor Jones may be statistically comparable. Although such storms of minor intensity are of great importance from the standpoint of storage for agricultural or hydro-electric purposes, etc., they are of slight importance in the design of storm sewers and were not considered by the writers. The stations used were selected on the basis of the uniformity in the recorded data as shown in Table 4.

In Fig. 12 Professor Jones presents a chart showing the quantity-intensity relationship, since multiplying intensity by time gives the quantity or accumulated depth. The use of minutes in multiplying in this particular case gives a function of the quantity instead of the actual depth, but the ratio is constant. Professor Jones shows clearly the variations of the records of single storms from the Talbot type of formula. The writers also found that records of single storms showed considerable variation from the formula. However, the averages of a great many records of high intensity appear to fit very closely.

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[44] "Run-Off Investigation in Central Illinois," *Bulletin No. 232*, Eng. Experiment Station, Univ. of Illinois, pp. 91–93.

Mr. Charles W. Sherman brings up an important point. Geographical and topographical influences undoubtedly affect rainfall. The stations used, however, showed, by recorded data, a similarity of occurrences of high intensity under the existing geographical and topographical variations. It is as conceivable that several places more or less remote from each other should have similar rainfall intensity and frequency characteristics in the excessive range, while differing greatly in total annual rainfall, as that several such places might have the same altitude, even if hundreds of miles of mountains and valleys separate them geographically. The writers realized that the stations used are not "located within an area possessing uniform rainfall characteristics," as far as total annual rainfall is concerned. They believed, however, that these stations have very similar characteristics in the range of excessive intensities used in their investigations.

Meteorological cycles were considered in this study, but no attempt was made to determine the effects of such cycles except to assume that the data for the 659 station-yr and the 330 station-yr studies include approximately three complete registrations of the common or sun-spot cycle (in so far as it affects the high intensities considered). As to cycles of the order of hundreds of years, the writers agree with Mr. Sherman that the station-year method gives no indication of the effect of such cycles. More years of scientific recording than are now available will be necessary to determine their effect on rates of rainfall.

Conclusions (6) and (7) were based on the studies for the published paper and on unpublished studies of storm patterns which Mr. Grant had been making for some years prior to his death and which were somewhat similar to the studies by Frank A. Marston,[20] M. Am. Soc. C. E., referred to by Mr. Sherman. Because the Chicago gages are spaced irregularly and some are considerable distances apart, only fragmentary storm maps were secured in many cases. Only a comparatively few closed isohyetals near the storm centers could be developed, and most of the storms appeared to be much more irregular in shape and movement than those reported by Mr. Marston. Because these studies were incomplete, it was decided not to include them in the paper as presented. Therefore, no reference was made to the paper[20] by Mr. Marston. However, since this aspect of the general rainfall problem was noted in Conclusions (6) and (7), perhaps such a reference should have been made.

The inaccuracy of the tilting bucket type of record for very high intensities is pertinently noted by Professor Cox. Undoubtedly, the under-recording occurs principally during the periods of very high intensities. The statement quoted that "no correction factor was used for the precipitation corresponding to any duration", means that the writers applied no correction factor in using the published records. Wherever local observers, in preparing rainfall data for publication, may have distributed an excess, as shown by stick measurement over the automatic record, such distribution or correction, of course, was included in the published records, and was used by the writers. No study was made to determine the extent of such correction or distribution and no special consideration was given to it except to mention it in Item (*d*) under the heading, "Deficiencies in Data." A study was made for the Chicago District to determine the time of year when excessive rates occur. A chart showing these occurrences and their dates is included among the unpublished papers, and it can be summarized by stating that all the excessive rates for this District occurred between May 1 and October 21. The comments of Professor Cox as to the conditions affecting run-off, other than the rainfall rate, apply more directly to suburban or rural areas. In metropolitan sewer districts with large proportions of impervious areas those other factors have diminishing influences on the rate of run-off.

Mr. Drummond's charts, shown in Figs. 13 and 14, supply interesting information and should aid an engineer in determining whether the results of these studies can be applied to his local problem.

The statistical analyses of the results of these studies presented by Professor Eugene L. Grant, provide tests of validity of a different type than any considered by the writers. It is gratifying to note that they qualify under such tests. The important implication, as emphasized by Professor Grant, and again by Professor Meyer, is that, within the high intensity brackets used in these studies, stations many hundred miles apart and with dissimilar annual rainfall characteristics have similar frequency-intensity characteristics.

Mr. Bogert emphasizes the run-off factor in the final design of sizes and capacity of drains. The writer agrees with Mr. Bogert that these studies dealt with only one of the elements concerned. However, the determination of run-off is another step that requires a specific consideration of the items enumerated by Mr. Bogert for the particular locality as well as a considerable element of judgment.

The statement by Mr. Eiffert that judgment and caution should be used in applying the station-year method is undoubtedly true. It was the opinion of the writers, however, that the similar characteristics for the high intensities shown in the data assembled in their studies as well as in the studies by Meyer[4] justified using the stations selected. Mr. Eiffert's comments as to cycles as well as those of Mr. Charles W. Sherman are quite correct. Although the 19-station and the 10-station series in the paper include approximately three of the sun-spot cycles, they cannot show the effect of long-term cycles. Some of the older local stations and the two Chicago stations of the U. S. Weather Bureau, in the Chicago District series show more than one complete sun-spot cycle. The newer stations in Chicago are of such recent date that they do not cover even one of these cycles, as noted in the paper. It is very probable that further accumulation of local records will alter the local data somewhat. The writers were in complete agreement with Mr. Eiffert as to the need for more complete rainfall records.

As stated by Mr. Jarvis, the straight-line relationship is more apparent in some cases than in others. The writers noted, however, that the divergence appeared to decrease as the quantity of data increased. The variations from a straight line for the Group 1 and Boston data in Fig. 15 and Fig. 16 (*a*) are interesting and worthy of further study. The writers also found the greatest deviations from a straight line in the records of the 5-min and 10-min storms. Although the writers had observed the apparent arithmetical-geometrical progression relationships between the intensity and the corresponding time intervals, their studies were stimulated when the paper by Mr. Jarvis[6] was published in 1930.

Referring to the last part of Table 20, the Weather Bureau standard for excessive rates shown on the top line agrees with the writers' definition on the bottom line. Unless this is a typographical error, there must have been a new ruling since that of March, 1934, shown by the writers in detail, under the heading "New Rule of the United States Weather Bureau for Tabulation of Excessive Rainfall." The trend mentioned by Mr. Jarvis indicated in Fig. 2 may be partly due to the method followed by the writers. The 19-station series was developed first. Then the Chicago District was studied and tabulated, using the Weather Bureau stations, together with the City and Sanitary District stations. These latter stations included data from some storms of much higher intensities than were recorded on any of the local Weather Bureau records; for instance, the Mayfair gage of the City of Chicago in the storm of July 26, 1932, recorded 3.87 in. in a 2-hr storm, with diminishing intensities on adjacent gages and with only 1.06 in. recorded for a storm of 1.5 hours' duration at the official University Station of the Weather Bureau about 15 miles away. Hence, the averages for the Chicago District were higher than those for the nineteen original stations. Then, the ten stations whose average data were in closest correspondence with the Chicago data, were selected from the nineteen stations for the 330 station-yr series. This resulted in the averages of the ten stations being higher than those for the entire nineteen stations, but not as high as the averages for the Chicago District.

The study was prompted by the desire to develop a method for using the cumulative data from an increasing number of rain-gages and to investigate the validity of the station-year method of using such data. The recent increase in the number of such gages in the Oklahoma[45] and Ohio[46] studies by the Federal Government may develop still further information on these and other points.

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[45] "The Life History of Rainstorms," Progress Report from the Oklahoma Climatic Research Center, by C. Warren Thornthwaite, *The Geographical Review*, Vol. XXVII, No. 1, January, 1937, p. 92.

[46] The Section of Climatology, Branch of Research, Soil Conservation Service, U. S. Dept. of Agriculture, is installing in the Muskingum Conservancy District in Southeastern Ohio 500 recording stations on 4½-mile quadrates over 8 000 sq miles. One district at Coshocton, Ohio, has gages on quadrates of mile over an area of approximately 6 000 acres.

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FLOW CHARACTERISTICS IN ELBOW DRAFT-TUBES[1]

By C. A. Mockmore,[2] M. Am. Soc. C. E.

With Discussions by Messrs. F. T. Mavis, Jerome Fee, R. E. B. Sharp, L. F. Harza, Ellery R. Fosdick, Howard L. Cooper, and C. A. Mockmore.

Synopsis

The purpose of this study was to investigate the flow characteristics in the elbow draft-tube, placing special emphasis on the bent part of the tube. The first part of the study consisted of designing and building several different pipe bends and testing them with a Pitot tube for filamental velocities and pressures. Measurements for loss of head were made for various rates of flow to determine which shape of bend offered the least resistance to the flowing water.

The second part of the study involved the design, construction, and testing of several model draft-tubes, patterned after the type of those at the Bonneville Dam, on the Columbia River, in Oregon. The model draft-tubes, as well as the experimental pipe bends, were made of pyralin, a transparent material through which it was possible to take photographs of the phenomena. Motion pictures were made for all conditions of flow in the pipe bends and the draft-tubes.

The tests indicated that the pipe bend which was flattened in the direction of the plane of the bend offered less resistance to flow than any of the other bends, regardless of cross-sectional area. It also appeared that the distance between the inside and outside walls at the outlet of the bend of an elbow draft-tube should be small compared to the throat diameter of the tube.

Introduction

Function of Draft-Tubes.—The draft-tube of a water turbine is an extension of the wheel passages, made so as to perform two principal functions:

Note.—Published in February, 1937, *Proceedings*.

[1] Based upon a thesis submitted to the State University of Iowa in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The data supporting this paper have been filed complete for reference in the Library of the State University of Iowa, Iowa City, Iowa, and in Engineering Societies Library, in New York, N. Y.

[2] Prof. and Head, Dept. of Civ. Eng., Oregon State Coll., Corvallis, Ore.

(1) To reduce the velocity of the water in an efficient manner as it passes through the tube; and (2) to permit the turbine runner to be placed at a convenient distance above the tail-water.

It would be difficult to state which of these two functions is the more important because their relative values may change from one installation to another, depending upon the physical conditions of the plant. Whatever the relative values may be, the total head should not exceed about 25 ft, and this should vary inversely with the specific speed of the turbine, in order that the danger of cavitation may be lessened.

Historical Development of Draft-Tubes.—For those who may be interested in the historical development of draft-tubes, the Appendix will serve as a valuable check list of reading material[8]. In general, they may be divided into two classes: (*a*) Those having straight axes; and (*b*) those having curved axes. This paper presents in detail a study of the flow characteristics in quarter-turn draft-tubes. Draft-tubes with straight axes, mentioned only incidentally herein, have been studied further at the State University of Iowa by Andreas Luksch (6).

The most recent and most complete study of draft-tube literature the writer has been able to obtain was reported in April, 1935, by the National Bureau of Standards (16), of the Department of Commerce, at the request of the Tennessee Valley Authority. According to the report,

"The purpose of this investigation was to examine and report on all published literature on draft-tubes published in English and foreign languages during the past 20 years, to prepare an annotated bibliography of this material, and to make recommendations as to the need for further experiments on draft-tubes, and as to the direction which such experiments should take."

"The current design of draft-tubes," according to the report, "shows a persistent tendency to depart from the use of the spreading and symmetrical types." This is attributed to the better structural features of the elbow draft-tubes and to the improvement in their hydraulic efficiencies. As an example of the findings in improved efficiency for the elbow draft-tube, reference is made to the Lilla Edet (17) experiments in which a "special curved tube, shallow and very long, proved to be superior to the Moody (11) and White (9) tubes at full load performance."

A listing of the recent installations of elbow draft-tubes, such as that given by J. S. Ball (18), seems to vindicate the pioneer work of the late Gardner S. Williams, M. Am. Soc. C. E., who was among the earliest to claim that the elbow draft-tube may have as high efficiency as any other tube built. However, this is not to be construed to mean that the elbow draft-tube will ever entirely displace all draft-tubes having straight axes. No one type of tube could possibly be superior to all other types under all conditions of operation. It was in this connection that Dr. D. Thoma (19) stated: "It is firmly established today that a given draft-tube may be the best possible form for one runner, but not for another. Consequently, the runner and draft-tube should be investigated as a single system."

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[8] In this paper numerals in parentheses, thus (10), refer to corresponding numbers in the Appendix.

Characteristic Parts of a Draft-Tube.—It appears logical that an elbow-type draft-tube may be considered as composed of three principal parts: (1) The vertical leg; (2) the bend; and (3) the lower leg, sometimes called the horizontal leg.

In the report (16) by the Bureau of Standards, a hypothetical form of draft-tube was assumed as made up of four principal parts for the purposes of discussion. The fourth part was added following the bend, and assumed as consisting of a section of the tube in which there was no increase in cross-sectional area. It appears that this part was added to take care of the suggestion of Viktor Kaplan (13) that in his experiments he found that the addition of such a section had increased the efficiency of the draft-tube. To the writer it appears that this is merely a part of the bend, or should be considered as such. Many designers of elbow draft-tubes believe that, in designing the bent part, the cross-sectional areas from section to section along the axis should increase in diminishing degree until near the end of the bend there is little or no increase in cross-sectional area from section to section. Then, as the lower leg is encountered, the flare may again be resumed.

EXPERIMENTS WITH PIPE BENDS

Notation.—The symbols used throughout this paper are defined where they first appear. An effort has been made to conform as nearly as practicable with "Symbols for Hydraulics", compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929. (A.S.A.—*Z*10*b*—1929.)

Theory of Induced Spirals in Pipe Bends.—The bend of the quarter-turn draft-tube is the part in which the greatest complications arise in the function of velocity reduction. In fact, complications are inherent with the flow of water in any channel where the filaments are forced to undergo a general change of direction. If there were stream-line flow of equal velocities at the entrance to a bend, and if friction could be eliminated, no complications of any great import would be encountered. With a whirl component at the entrance to a pipe bend, even without frictional resistance, complications would arise due to gyral action.

No pipe can be made in which frictional resistance to flow will not exist. Therefore, the filaments of flow near the walls of the pipe will be of less velocity than those near the center, and, as the water enters a bend, a spiral flow is induced automatically.

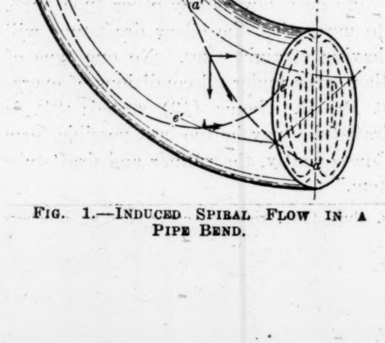


FIG. 1.—INDUCED SPIRAL FLOW IN A PIPE BEND.

As an explanation of the cause of the spiral flow, let a small block of water, b, at a distance, r, from Point O (Fig. 1) have unit length and breadth, and a thickness, dr. Its volume then would be infinitesimally small as represented by the expression: $dQ = dr$; and its mass would be,

$$dm = \frac{w}{g}\, d\,Q = \frac{w}{g}\, dr \text{ ....(1)}$$

in which w = the weight per unit volume; and g = the acceleration of gravity.

From mechanics it is known that the centrifugal force induced by the water moving with a velocity, V, being deflected (20) by a circular bend of radius, r, is:

$$dF = dm\,\frac{V^2}{r} = w\,V^2\,\frac{dr}{r\,g} \text{ ....(2)}$$

The area of the side of the small block is unity, since it is of unit length and breadth, and the intensity of pressure, increased on the left side of the small block caused by the centrifugal force, is:

$$dp = w\,V^2\,\frac{dr}{r\,g} \text{ ....(3)}$$

If the velocities of the filaments between Points a and b are considered equal, and if the inner radius is designated by r_i and the outer radius by r_o, the difference in pressure between these limits integrates into the following:

$$\Delta p = w\,\frac{V^2}{g}\int_{r_i}^{r_o}\frac{dr}{r} = \frac{w\,V^2}{g}\log\left(\frac{r_o}{r_i}\right) \text{ ....(4)}$$

Equation (4) indicates that the difference in pressure caused by the centrifugal force is a function of the square of the velocity. It is similar to the expression for the impulse, or reaction of a jet of water:

$$F = \frac{W\,V}{g} \text{ ....(5)}$$

in which, F = the force, in pounds; V = the velocity of the jet, in feet per second; and W = the flow of water, in pounds per second. The weight, in pounds per second, is then a function of velocity, so that the force, F, is a function of the square of the velocity. Thus, it appears logical that as the bend is encountered the pressure either should be increased at Point e (Fig. 1), or reduced at Point a by some amount, not necessarily exactly equal to that indicated by Equation (4), because the velocities along the traverse between Points a and e are not all of equal magnitude. These velocities will change as the pressures change in the flow around the bend, and perhaps changes will occur even in advance of the bend.

If all the filamental velocities of the water were equal at the entrance to the bend (a condition which could not be obtained easily without a very special apparatus) the pressure at Point d (Fig. 1) would be the same as that at Point *b*; but, due to the lower velocities in the elementary strip, *c-d*, the pressure at Point *d* will be less than that at Point *b*. Therefore, a transfer of water from Point *b* toward Point *d* will tend to take place. Likewise, movement will tend to occur from Point *a* toward Point *b*, and a double spiral is induced. The two principal causes of this condition are: (1) The centrifugal force in the pipe bend; and (2) the existence of friction on the pipe walls so as to give higher velocities near the center of the pipe than near the walls. Under actual conditions the particles of water near Point *e* (Fig. 1), might progress in a path such as *e-e′-e″*, and another particle, such as at Point *a*, might progress in a path such as *a-a′-a″*.

An explanation of the induced spiral probably was offered first by Professor James Thomson (21), in 1876. Recently, tests on the flow of water around bends in pipes were conducted by D. L. Yarnell and the late F. A. Nagler (22), Members, Am. Soc. C. E., at the Iowa Institute of Hydraulic Research. These experiments were undertaken for the purpose of determining the laws governing the changes in pressure and velocity in different parts of the flowing stream, as the moving water undergoes the transition from motion along a straight path to motion around a curve, and then as it undergoes the opposite transition back to final straight-line motion. These experiments are especially valuable because of the numerous Pitot tube measurements of the velocity and pressure at the various sections of the pipe bends. Several different shapes of the pipe bends were used, and tests were made to determine the loss of head due to flow around each bend. The closing statement in the discussion of that paper is of particular interest to the designer of an elbow draft-tube: "Probably the most important finding in the investigation is that it is possible to have such conditions of flow that the loss of head may be very little, or unusually large, in the same bend for identical discharges", depending upon the velocity distribution at the entrance to the bend.

Several of the pipe bends reported in this paper were sent to the Iowa Institute of Hydraulic Research and were tested for flow characteristics. Only part of the results of these tests were described by Messrs. Yarnell and Nagler.

Design of Experimental Pipe Bends.—The general equation of continuity of flow of a fluid is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V_x)}{\partial x} + \frac{\partial (\rho V_y)}{\partial y} + \frac{\partial (\rho V_z)}{\partial z} = 0 \ldots\ldots\ldots\ldots (6)$$

in which ρ = density of the fluid; t = element of time; V_x = component of velocity along the X-axis; V_y = component of velocity along the Y-axis; and V_z = component of velocity along the Z-axis.

The density of the water may be regarded as constant, and the equation of continuity becomes:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 = \text{divergence} \ldots\ldots\ldots\ldots (7)$$

If the motion is irrotational (23) the curl of the velocity vector is zero, and the vector of which V_x, V_y and V_z are components is derivable from a potential.

For irrotational motion, the components of flow in the direction of the axes, X, Y, and Z, are: $V_x = -\frac{\partial \Psi}{\partial x}$; $V_y = -\frac{\partial \Psi}{\partial y}$; and $V_z = -\frac{\partial \Psi}{\partial z}$. The equation of continuity then becomes:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

which is the Laplace equation.

If this is taken as a special case of two-dimensional flow, say, in the X-Y plane, then,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The real part, ψ, of any function, $f(x + iy)$, of a complex variable satisfies the equation for the velocity potential, and, at the same time, the imaginary part gives the stream function. Assume that:

$$\Psi + i\Phi = (x + iy)^2 = x^2 - y^2 + 2i(xy) \ldots\ldots\ldots\ldots (10)$$

Therefore,

$$\Psi = x^2 - y^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and,

$$\Phi = 2xy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

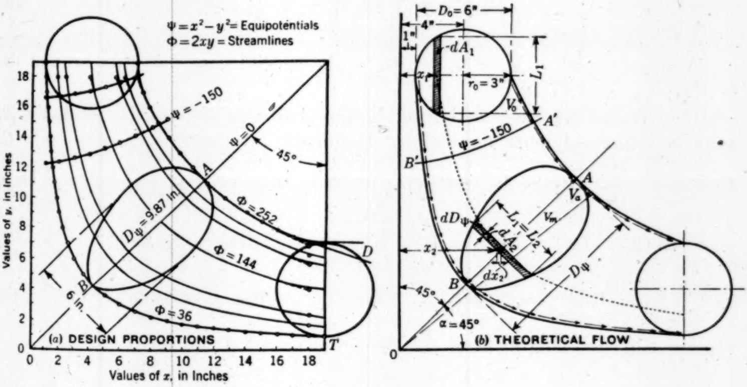

Ψ = x² − y² = Equipotentials
Φ = 2xy = Streamlines
ψ = −150
ψ = 0
45°
A
Φ = 252
Dψ = 9.87 In.
Φ = 144
D
B
6 In.
Φ = 36
T
(a) DESIGN PROPORTIONS
Values of y, in Inches
Values of x, in Inches
0
2
4
6
8
10
12
14
16
18
D₀ = 6"
4"
1"
dA₁
x₁
r₀ = 3"
L₁
V₀
A′
ψ = −150
B′
A
Vₐ
dDψ
L₁ = L₂
Vₘ
dA₂
x₂
dx₂
Dψ
B
45°
α = 45°
0
(b) THEORETICAL FLOW

FIG. 2.—DESIGN OF PIPE BEND NO. 2.

Equations (11) and (12) satisfy the specifications for two-dimensional irrotational flow since the divergence is zero:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = \frac{\partial^2 (x^2 - y^2)}{\partial x^2} + \frac{\partial^2 (x^2 - y^2)}{\partial y^2} = 0 \quad \ldots\ldots\ldots (13)$$

To design a pipe bend from this relation and not to take friction into account, of course, would omit an essential factor in the flow of water in pipes, but it was deemed desirable to try it and observe the results. Accordingly, a bend suitable for a 6-in. pipe was designed, more or less by trial, until that shown in Fig. 2 was obtained. If the co-ordinates, in inches, of the beginning point, *T*, were used as (1, 12) instead of the (1, 18), the angle at Point *D* would become large. To use (0.5, 18) instead of (1, 18) made the width, *A*-*B*, rather large and, at the same time, gave a sharper curve near Point *A*. To avoid an abrupt angle at Point *D*, 1 in. was added to each of the x and y-distances to afford an opportunity to smooth out the hyperbolic curves to meet the parallel filamental flow from the 6-in. pipe at the entrance to the bend. The leg length, *L*, of the pipe thus became 15 in. as shown in Fig. 2.

The distance across the pipe from side-wall to side-wall, measured along the equi-potential lines, is expressed by the equation:

$$D_\Psi = \int \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{0.5} dx \qquad (14)$$

Since, $x^2 - y^2 = C =$ a constant; $2\,x\,dx - 2\,y\,dy = 0$; $\frac{dy}{dx} = \frac{x}{y}$; and, $\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{y^2}$:

$$D_\Psi = \int \left[1 + \left(\frac{x^2}{y^2}\right)^2\right]^{0.5} dx \qquad (15)$$

When $x^2 - y^2 = \Psi = 0$ (which is the special case half-way around the bend), $D_\Psi = 9.87$ in.

The area of the cross-section of the pipe half-way around the bend, Fig. 2, may be expressed mathematically. Assuming, for example, $x\,y = 18 = C$, then $x_1 = \frac{C}{18}$. At the 45° section, $x = y$, so that, $(x_2)^2 = C = 18\,x_1$. Then, $x_2 = 3\,(2)^{0.5}\,(x_1)^{0.5}$; $dx_2 = \left(\frac{3}{2}\right)(2)^{0.5}\,\frac{dx_1}{x_1^{0.5}}$; $dD_\Psi = dx_2\,(2)^{0.5} = 3\,\frac{dx_1}{x_1^{0.5}}$; and, $dA_2 = L\,dD_\Psi = 3\,L\,\frac{dx_1}{x_1^{0.5}}$. Since the radius, r, of the entrance of the pipe is 3 in., the length, L, of the elemental area, dA, is:

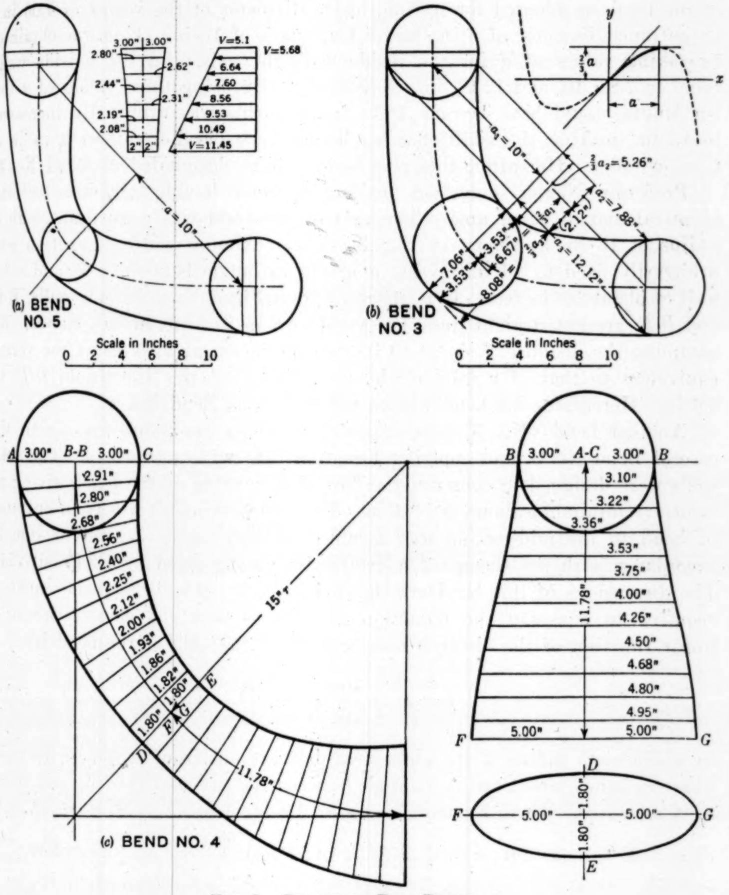
$$L = 2\,[9 - (x_1 - 4)^2]^{0.5} \qquad (16)$$

which is the same as the length of the corresponding elemental area for the section half-way around the bend. Then,

$$d\,A_2 = 3\,L\,\frac{dx_1}{x_1^{0.5}} = 6\int_1^7 \left(8 - x_1 - \frac{7}{x_1}\right)^{0.5} dx_1 \qquad (17)$$

Equation (17) is an elliptic integral, which can be evaluated. By plotting the curve to scale and using a planimeter, the area was found to be 47.0 sq in.

The generating angle of a cone having an equivalent increase in area in the same length as this hyperbolic bend between entrance and vertex was computed and found to be 4° 20′, which is well below the usual allowable value of the angle of flare for draft-tubes. This bend hereafter will be referred to as Bend No. 2. Bend No. 1 is a standard 6-in. pipe bend of 90° having a center-line radius of 8 in.



(a) BEND NO. 5
3.00" 3.00"
2.80"
2.62"
2.44"
2.31"
2.19"
2.08"
2" 2"
V=5.1
V=5.68
6.64
7.60
8.56
9.53
10.49
V=11.45
a=10"
Scale in Inches
0 2 4 6 8 10
(b) BEND NO. 3
y
x
$\frac{2}{3}a$
a
$a_1=10"$
$\frac{2}{3}a_2=5.26"$
$a_2=7.88"$
$(2.12")$
$\frac{2}{3}a_1$
$a_3=12.12"$
3.53"
6.67"
7.06"
3.53"
8.08"
$\frac{2}{3}a_3$
Scale in Inches
0 2 4 6 8 10
(c) BEND NO. 4
A 3.00" B-B 3.00" C
2.91"
2.80"
2.68"
2.56"
2.40"
2.25"
2.12"
2.00"
1.93"
1.86"
1.82"
1.80"
1.80"
E
F G
D
15"
11.78"
B 3.00" A-C 3.00" B
3.10"
3.22"
3.36"
3.53"
3.75"
4.00"
4.26"
4.50"
4.68"
4.80"
4.95"
11.78"
F 5.00" 5.00" G
D
F 5.00" 1.80" 5.00" G
1.80"
E

FIG. 3.—DESIGN OF PIPE BENDS.

S. M. Woodward, M. Am. Soc. C. E., has suggested designing a bend with side-walls defined by the cubic equation,

$$y = x - \frac{x^3}{a^2} \qquad \text{(18)}$$

This curve has a slope of 45° at the origin, and is horizontal at its maximum point where $x = a$, and $y = \frac{2a}{3}$. The radius of curvature is infinity at the origin, and is $0.5\,a$ at the maximum point. The part of the curve between the origin and the maximum point was used to make one-half the bend; and then it was turned over to make the other half of the bend, as shown in Fig. 3.

By assuming values of a in Equation (18), the side-walls of the bend were obtained. This bend would always bulge at the vertex. The final dimensions of the bend, as adopted for testing, had a diameter at the vertex of 7.06 in., an entrance diameter of 6 in., and a leg length of 15 in. This was obtained by taking values of a for the inside wall, the axis, and the outside wall, equal to 7.88, 10, and 12.12, in., respectively. By using these values of a the leg length would have become 14.14 in., but this was arbitrarily increased to 15 in., making the additional leg length have parallel side-walls as in a 6-in. cylinder. Hereafter, this pipe bend will be designated as Bend No. 3.

Professor Nagler suggested making a bend having all cross-sections elliptical and of equal area. The axis of the bend was a quadrant with a radius of 15 in. as shown in Fig. 3. The maximum width of section was arbitrarily fixed at 10 in., giving a maximum angle between axis and side-wall of about 10 degrees. It is apparent from Fig. 3 that the side-walls, *B-G* and *B-F*, are not straight lines, but are drawn in to form smooth curves. By assuming the distance, *F-G*, as 10 in., and maintaining an area at the vertex equivalent to that of a 6-in. circle, the ellipse had the dimension, *D-E*, of 3.6 in. Hereafter, this bend will be referred to as Bend No. 4.

Another bend (No. 5) was designed by using the same axis as in the case of Bend No. 3, and applying Equation (18) with $a = 10$ in. The side-walls were designed to constrict the flow at the vertex of the bend, giving an arbitrary minimum diameter of 4 in. The increase in velocity from entrance of bend to the mid-section was assumed to vary as in a straight line in accordance with the theory of a draft-tube, as suggested by F. Prasil (24). The dimensions of this bend are shown in Fig. 3. The differential equation required to represent the reduction in velocity, and the deceleration as a linear function of the distance measured along the axis of the bend is:

$$\tan \alpha = \frac{dr}{dx} = -\frac{r_0}{2x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which r_0 = radius of the pipe; x = distance out from origin to the point where r_0 is measured; and, α = angle between the axis and the side-wall where r_0 is measured. Then, $2\,x\,dr = -\,r_0\,dx$; $2\,x\,dr_0 + r_0\,dx = 0$; $d\,(r^2_0\,x) = 0$; $r^2_0\,x = K$ = constant; $\pi\,r^2_0\,x = K' = A\,x$; and, $\frac{Q\,x}{V} = K'$. Therefore,

$$V = \left(\frac{Q}{K'}\right) x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

which represents a straight line.

If the velocity is differentiated with respect to time, the result will be deceleration; thus:

$$\frac{dV}{dt} = \left(\frac{Q}{K'}\right)\frac{dx}{dt} = \left(\frac{Q}{K'}\right) V \quad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

This, again, is a linear equation in x, since $\frac{dx}{dt}$ is velocity. The velocity variation from the entrance to the mid-point of this bend is shown in Fig. 3 for a special case in which the rate of flow is 1.0 cu ft per sec.

TABLE 1.—Summary of Pipe-Bend Dimensions

Bend No.	Vertex area, in square inches	Length of bend on axis, in inches	Generating angle of equivalent conical tube, in degrees	Radius of curvature of axis at vertex of bend, in inches
1..............	28.27	12.6	0	8
2..............	47.00	23.0	4° 20′	12
3..............	38.15	26.4	2° 20′	5
4..............	28.27	23.6	0	15
5..............	12.57	26.4	4° 20′	5

Table 1 shows a summary of the dimensions of Bends Nos. 1 to 5, each of which had an entrance diameter of 6 in., and (except Bend No. 1, in which, $L = 8$), a leg length of 15 in.

Construction of Pipe Bends.—All the pipe bends were constructed of transparent pyralin. After some preliminary experiments it was discovered that this material would become plastic when heated to a temperature of about 275° F, without losing its transparency, and when it was pressed into various shapes and cooled, it again became rigid.

The first stage in the construction of the bends was to make the moulds for pressing the material into the desired shapes. Full-sized drawings were made showing the intersections of planes passed through the bend parallel to, and at distances, in inches, of 1, 2, 3, etc., from the axis. The traces marking the intersection of the planes with the barrel of the pipe formed templates for marking the pieces of 1-in. sugar pine lumber so that they could be cut with a band-saw to the proper shapes. These pieces were then glued together and gouged out with a chisel to obtain the various cross-sections according to the different designs. Allowance was made in the construction of the moulds for the $\frac{1}{8}$-in. thickness of the pyralin between the two parts of each mould.

The material was heated in an electric oven at a temperature of about 275° F, and when the sheets became plastic they were placed between the moulds and pressed into shape. The pieces were put together with cement made by dissolving pyralin in acetone.

If the material was heated too much, it would lose its transparency; if not heated enough, it would wrinkle when pressed into warped surfaces; and, if heated too quickly, the surface became "frosty", and impaired the transparency. The "frosty" condition was overcome by rubbing the material with water sand-paper and polishing it with pumice.

Laboratory Experiments with Pipe Bends.—Fig. 4 shows the general arrangement of the apparatus during a test. The water, admitted at Point *K*, flowed through a conical expanding tube, and was deflected by a flat plate, into Tank *A*. A conical bottom, *C*, and guide-vanes in the bottom of the head-water tank directed the water, with or without a whirl component as desired, into a vertical pipe, *D*, 6 in. in diameter and 24 in. long, above the bend. The bend to be tested is indicated by *F*. After leaving the bend, the water flowed through a straight section of 6-in. pipe, 32 in. long, to a second tank, *B*. This tank was open at the top and fitted with a discharge valve at Point *H*.

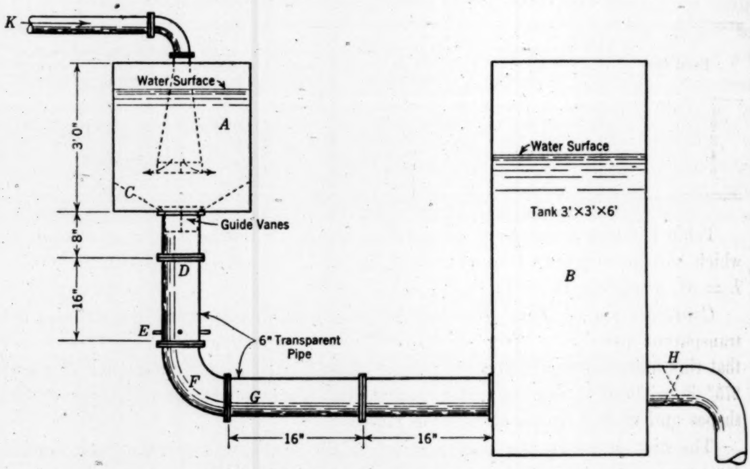
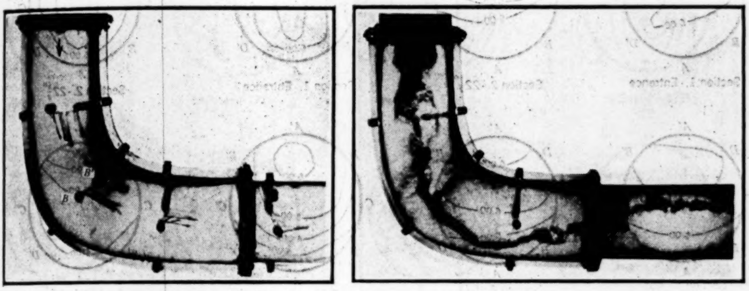


FIG. 4.—DESIGN OF LABORATORY APPARATUS.

The velocities of the water filaments were measured with a Pitot tube which was calibrated at the State University of Iowa, the Colorado Agricultural College, and at the Oregon State College. Its coefficient was found to be 0.86.

The rate of flow was determined by volumetric measurement. The water flowing from the test apparatus was collected in a concrete tank, and by use of a stop-watch and point-gage, readings were taken to determine the time required to raise the water surface to a given level in the tank so that the rate of flow could be established.

The bends were tested with two types of flow: (1) Direct flow without whirl component; and (2) helical flow, or that with a whirl component at the entrance to the bend. It has been stated that when the water approaches the bend with approximately uniform filamental velocities, there will be an induced double spiral as shown in Fig. 1. That this double spiral actually developed to a marked degree is indicated by Fig. 5(*a*), which is a view of the direct flow in Bend No. 3. At the vertex of the bend is shown a string stretched from the Pitot tube connections marked *B-B'*, and attached to this string are several short pieces of yarn. The yarns show that the filaments of flow near the side-walls of the pipe, as at *B* and *B'*, have velocity components toward the inside of the bend, whereas the filaments of flow near the center of the pipe have velocity components toward the outside wall of the bend. A double spiral flow has been induced, as shown in Fig. 1.



(*a*) Direct Flow. (*b*) Helical Flow.

Fig. 5.—Direct and Helical Flow in Bend No. 3.

It is possible for a single spiral to exist in a pipe bend, with the water rotating in one direction or the other, depending on the flow at the entrance to the bend. This was shown in a series of experiments conducted by Messrs. Yarnell and Nagler (22) in 1934, in which the water at one side of the entrance to the bend had higher filamental velocities than on the other side. On the side where the axial velocities were small the water would move toward the inside, and on the side where the axial velocities were large the water would move toward the outside, of the bend, causing a single spiral instead of a double spiral.

Fig. 6 shows the velocity contours at different sections of Bends Nos. 1, 2, 3, and 4. In these diagrams the points marked *A* are at the outside of the bend, and the points marked *A'* are at the inside of the bend. In all cases the view is looking down stream. From these diagrams it is evident that, as the water approached the bend, the filaments of flow along the inside of the bend accelerated until they reached a maximum velocity about half-way around the bend. From that point the thread of maximum velocity moved to the outside of the bend.

Attention should be called to the unsymmetrical velocity contours in Bends Nos. 2 and 4. This was caused by having the cone in the head-water tank, *A* (Fig. 4), slightly off center. When this cone was centered the velocity contours were symmetrical about the center of the pipe, as shown for Bends Nos. 1 and 3 (see Fig. 6). It should be noted that any minor disturbance up stream from the bend was found to be reflected in the velocity distributions throughout the bend.

Curves were drawn for the combined velocity head and pressure head for all bends, as shown in Fig. 7. In the data the pressure has been recorded as p, for brevity, which was the indicated pressure head as given by the pressure

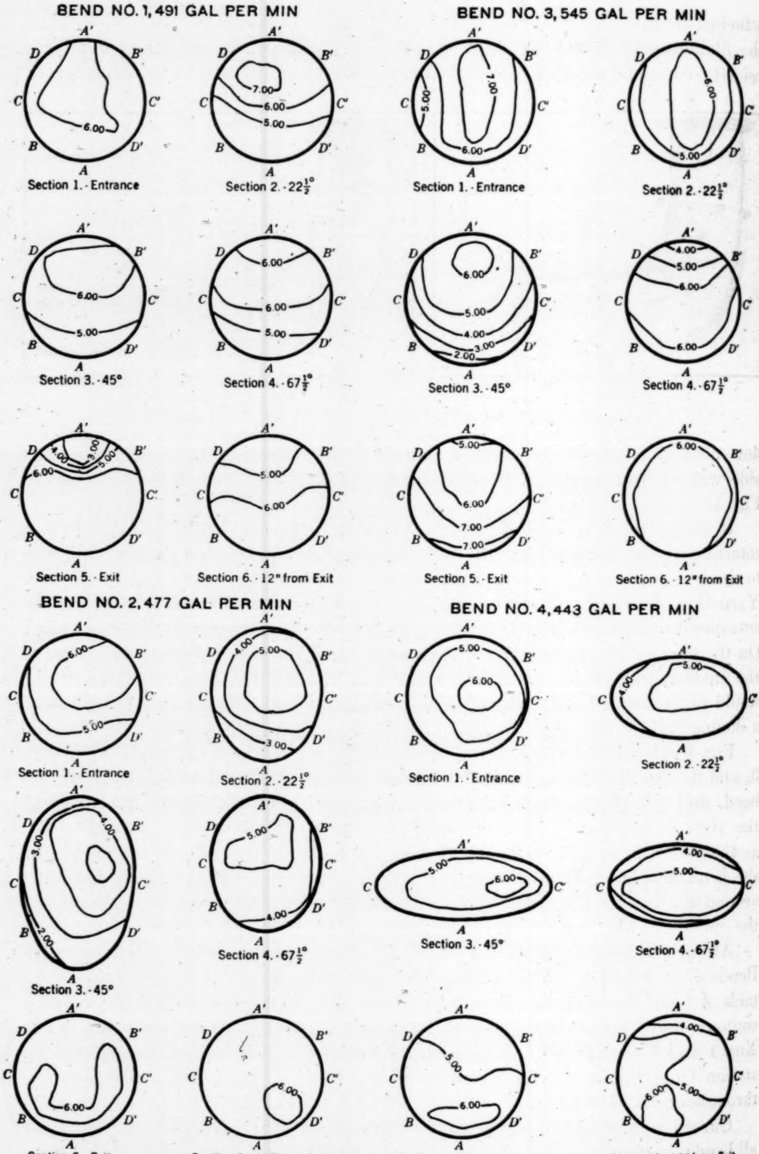
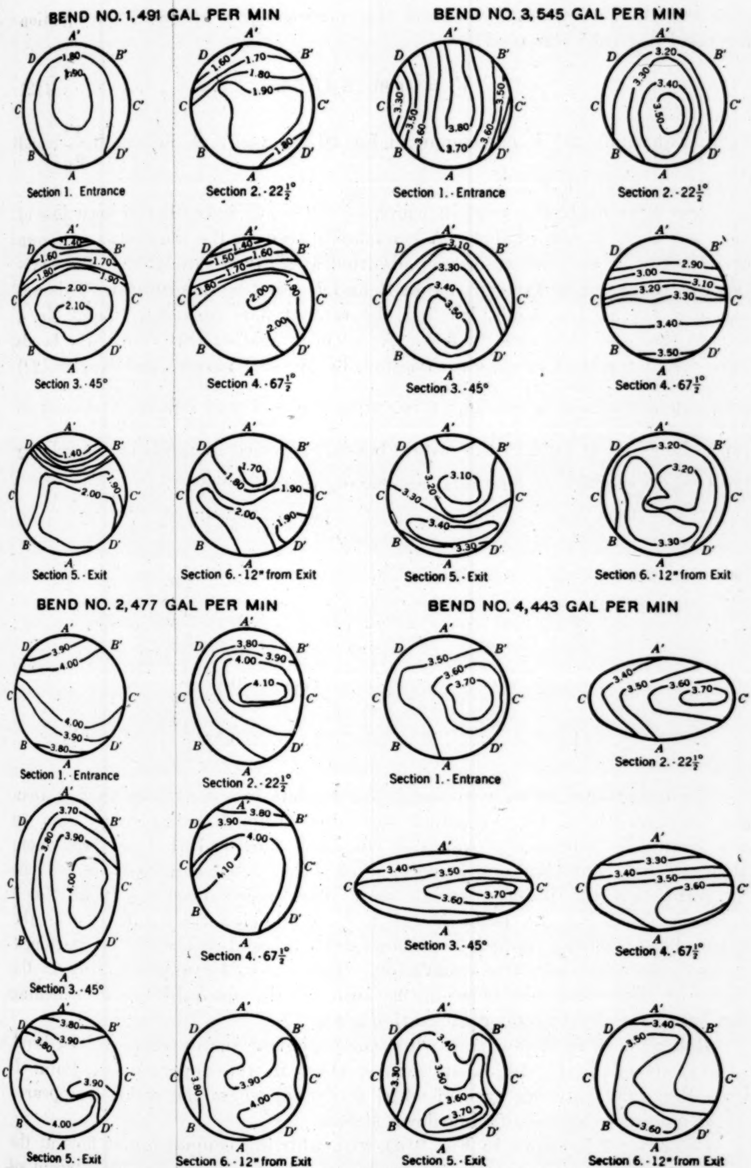


FIG. 6.—VELOCITY CONTOURS AT DIFFERENT SECTIONS OF EXPERIMENTAL BENDS.


BEND NO. 1, 491 GAL PER MIN

Section 1. Entrance
Section 2. - 22½°
Section 3. - 45°
Section 4. - 67½°
Section 5. - Exit
Section 6. - 12" from Exit

BEND NO. 3, 545 GAL PER MIN

Section 1. - Entrance
Section 2. - 22½°
Section 3. - 45°
Section 4. - 67½°
Section 5. - Exit
Section 6. - 12" from Exit

BEND NO. 2, 477 GAL PER MIN

Section 1. Entrance
Section 2. - 22½°
Section 3. - 45°
Section 4. - 67½°
Section 5. - Exit
Section 6. - 12" from Exit

BEND NO. 4, 443 GAL PER MIN

Section 1. - Entrance
Section 2. - 22½°
Section 3. - 45°
Section 4. - 67½°
Section 5. - Exit
Section 6. - 12" from Exit

A'
B'
C'
D'
A
B
C
D
1.80
1.90
1.60
1.70
2.00
2.10
1.40
1.50
3.30
3.50
3.60
3.70
3.80
3.20
3.40
3.10
2.90
3.00
3.90
4.00
4.10

FIG. 7.—COMBINED VELOCITY-HEAD AND PRESSURE-HEAD CONTOURS AT DIFFERENT SECTIONS OF EXPERIMENTAL BENDS.

orifice of the Pitot tube. To obtain the corrected pressure head the equation for the Pitot tube was used:

$$V = 0.86 \ (2 g h)^{0.5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

From Equation (22) h was computed for all the readings, added to $\frac{V^2}{2g}$, and plotted as shown in Fig. 7.

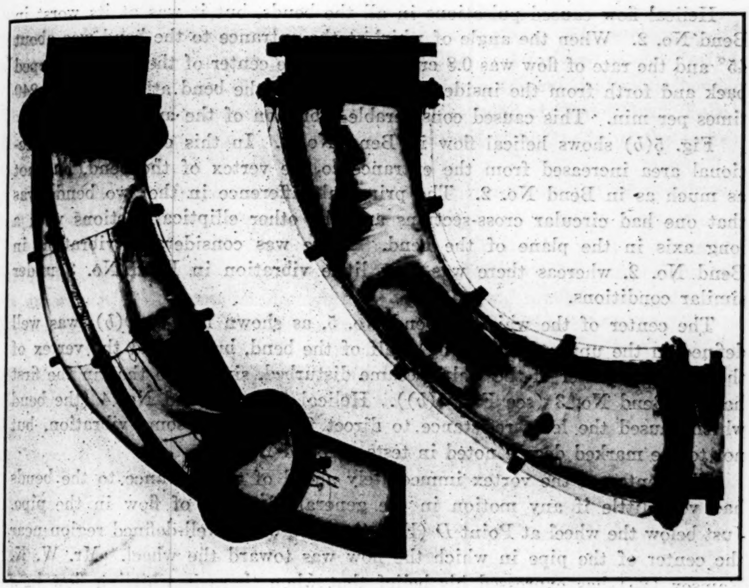
Tests were made to ascertain which of the bends induced the least loss of head for a given rate of flow. A bend was placed in the apparatus as shown at F in Fig. 4, and the water was admitted at a given rate so that the elevations of the water surfaces in Tanks A and B could be determined. This was repeated for all the bends, keeping the rate of flow constant. Bend No. 4 offered the least resistance to flow. As a check against this result, the bends were tested for head loss due to friction, by Messrs. Yarnell and Nagler (22) who found that, in a straight 6-in. pipe, the loss was 0.0302 $\frac{V^2}{2g}$ per ft of pipe. The loss of head in the several bends, expressed in terms of the velocity head, and exclusive of frictional resistance, was as follows:

Bend No.	Loss of head in 90° bend (exclusive of friction)
1	$0.15 \ \frac{V^2}{2g}$
2	$0.15 \ \frac{V^2}{2g}$
3	$0.17 \ \frac{V^2}{2g}$
4	$0.13 \ \frac{V^2}{2g}$

This tabulation shows that Bend No. 4 offers less resistance to flow than any of the others, the advantage over Bend No. 3 being more than 30 per cent. Its advantage over the standard short-radius bend was more than 15 per cent. By an inspection of Figs. 5(a) and 8(a) it will be apparent that the induced spirals were more pronounced in Bend No. 3 than in Bend No. 4. Bend No. 4 has an additional advantage over the other bends when applied to elbow draft-tubes because it is flattened properly to avoid excessive excavation. This was a fortunate finding in the light of draft-tube construction, because of the desirability of widening the tube laterally to reduce excavation costs.

Helical flow, or that with a whirl component at the entrance to the bend, was produced by placing a stationary wheel in the apparatus at Point D (see Fig. 4). Only one type of whirl was used, where the rotation appeared to be counter-clockwise, facing down stream.

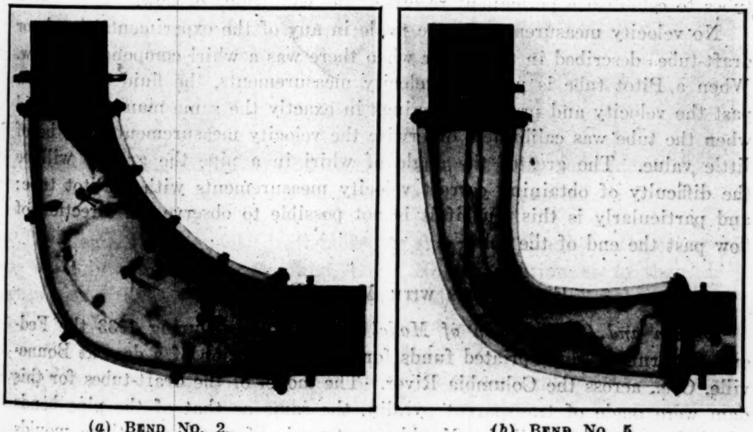
Bend No. 2 is shown in Fig. 9(a) with whirling component of flow at the entrance. The dark areas were made by admitting air in the stream of



(a) DIRECT FLOW. (b) HELICAL FLOW

FIG. 8.—DIRECT AND HELICAL FLOW IN BEND NO. 4.

inflowing water. The air bubbles moved into the region of least pressure, near the center of the whirl, and when a light was placed behind the pipe the air bubbles intercepted the light.



(a) BEND NO. 2. (b) BEND NO. 5.

FIG. 9.—EXAMPLES OF HELICAL FLOW.

Helical flow caused pulsations in all the bends, but it was at its worst in Bend No. 2. When the angle of whirl at the entrance to the bend was about 45° and the rate of flow was 0.8 cu ft per sec, the center of the vertex whipped back and forth from the inside to the outside of the bend at the rate of 240 times per min. This caused considerable vibration of the apparatus.

Fig. 5(*b*) shows helical flow in Bend No. 3. In this case the cross-sectional area increased from the entrance to the vertex of the bend, but not as much as in Bend No. 2. The principal difference in the two bends was that one had circular cross-sections and the other elliptical sections with a long axis in the plane of the bend. There was considerable vibration in Bend No. 2, whereas there was very little vibration in Bend No. 3 under similar conditions.

The center of the whirl in Bend No. 5, as shown in Fig. 9(*b*), was well defined in the upper (converging) half of the bend, but beyond the vertex of the bend the center of the whirl became disturbed, similar to that in the first half of Bend No. 3 (see Fig. 5(*b*)). Helical flow in Bend No. 4 (the bend which caused the least resistance to direct flow) caused some vibration, but not to the marked degree noted in tests of Bend No. 2.

The center of the vortex immediately ahead of the entrance to the bends had very little if any motion in the general direction of flow in the pipe. Just below the wheel at Point *D* (Fig. 4), there was a well-defined region near the center of the pipe in which the flow was toward the wheel. Mr. W. K. Ramsey (25) has expressed his belief that when there is whirling flow in a draft-tube, whether of straight or curved axis, there is a backward movement of water in the center of the whirl extending from the exit of the tube to the turbine runner. In none of the experimental bends or model draft-tubes described in this paper did the region of backward movement of water extend half-way around the bend, and in most cases it did not reach as far as the bend. Motion pictures were made of the flow through all the bends and draft-tubes to establish a permanent record of the conditions of flow.

No velocity measurements were made in any of the experimental bends or draft-tubes described in this paper when there was a whirl component of flow. When a Pitot tube is used for velocity measurements, the fluid should flow past the velocity and pressure openings in exactly the same manner as it did when the tube was calibrated, otherwise the velocity measurements may be of little value. The greater the angle of whirl in a pipe the greater will be the difficulty of obtaining correct velocity measurements with a Pitot tube; and particularly is this true if it is not possible to observe the direction of flow past the end of the tube.

EXPERIMENTS WITH MODEL DRAFT-TUBES

Design and Construction of Model Draft-Tubes.—During 1933 the Federal Government appropriated funds for the construction of a dam at Bonneville, Ore., across the Columbia River. The model of the draft-tubes for this dam were made of transparent pyralin, the same as that of the pipe bends which have been described. Moulds were made of wood, and these moulds were used to press the sheets of transparent material into the desired shapes. Some trouble was encountered in pressing the flat sheets into such warped shapes, but for the most part a fairly workmanlike and finished model draft-tube was obtained. Fig. 10 shows the moulds for a typical draft-tube bend. The mould on the left was for the lower half of the bend of Model Draft-Tube No. 1, and that on the right was for the inner half of the bend of the same tube.

Pyralin is much superior to glass as a transparent material of which to build model draft-tubes because of its durability and ease of fabrication. Prasil (26) used glass in the construction of a draft-tube with a straight center line, but no mention was made of the use of Pitot tube connections in his experiments. Piezometer connections can be made much more easily of pyralin than of glass, and with less trouble due to breaking.

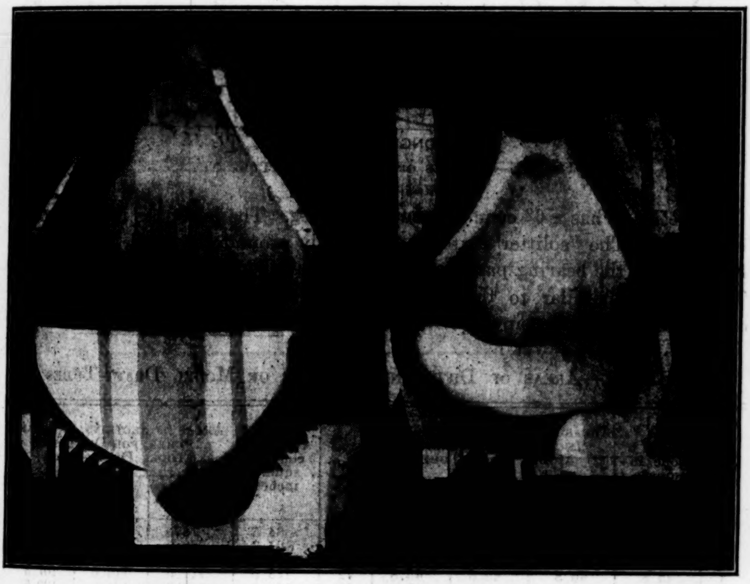


FIG. 10.—MOULDS FOR BEND OF MODEL DRAFT-TUBE NO. 1.

The design of the model draft-tubes for the U. S. Engineer Department was made in the office at Portland, Ore. No information as to their design was available to the writer other than the detail drawings, such as shown in Fig. 11, Draft-Tube No. 1, and the cross-sectional areas at different points along the axis of the tubes, as shown in Table 2. The cross-sections of Tube No. 1 are not circular at any point along its axis except at the entrance. The tube expands at the same rate as a cone having a 6° generating angle. There is no "splitter", and the bearing partition does not enter the bend. The distinctive feature of this bend is the manner in which the walls are "dished" down on the inside of the bend and lower leg.

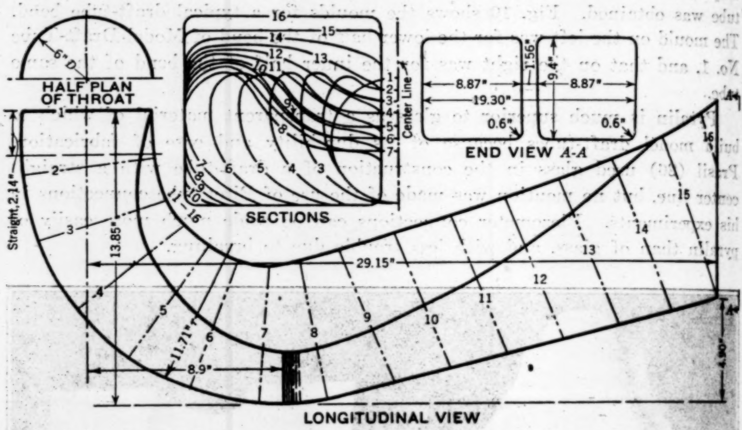


FIG. 11.—DETAILS OF MODEL DRAFT-TUBE NO. 1.

Tube No. 2 has a 6° cone for the vertical leg. The lower leg is rectangular in section. The "splitter" extends completely around the bend, as shown in Fig. 12, and the bearing partition terminates at about the middle of the bend. This tube is similar to the draft-tubes built at Ryburg-Schwörstadt (27) on the Rhine, in Germany.

TABLE 2.—AREAS OF DIFFERENT SECTIONS OF MODEL DRAFT-TUBES

Section No.	Area of 6° cone, in square inches	Areas of Sections, in Square Inches, for Different Model Draft-Tubes			Section No.	Area of 6° cone, in square inches	Areas of Sections, in Square Inches, for Different Model Draft-Tubes		
		Tube No. 1	Tube No. 2	Tube No. 3			Tube No. 1	Tube No. 2	Tube No. 3
1	28.3	28.3	28.3	28.3	9	84.5	76.0	83.7	68.5
2	33.6	33.6	33.6	33.6	10	91.7	88.7	96.5	81.0
3	39.4	39.0	39.4	39.4	11	103.0	100.3	104.5	94.5
4	45.8	46.8	45.6	45.8	12	113.0	111.0	114.0	107.5
5	52.6	53.5	54.3	47.0	13	124.0	122.3	124.0	120.5
6	60.0	61.5	60.0	45.5	14	135.0	132.8	135.5	134.0
7	67.7	60.0	66.4	45.5	15	146.0	143.0	148.5	146.5
8	75.7	65.0	75.0	55.5	16	159.0	153.0	.....	.....

Tube No. 3 is dished on the inside of the bend, but to a less degree than Tube No. 1. The vertical leg widens in a direction normal to the plane of the bend, and there is no "splitter." The areas of the sections at different points along the axis of the tube are about the same as that for a 6° cone as given in Table 2. The bearing partition extends almost completely around the bend, as shown in Fig. 13.

Design of Model Draft-Tube No. 4.—This tube was designed for the same setting as the model draft-tubes for the U. S. Engineer Department. The given data were: (1) Elevation of entrance of draft-tube; (2) entrance diameter; (3) elevation of top edge of outlet; (4) maximum width; and, (5) normal discharge. The vertical leg of the tube was made as long as it could be conveniently, so that the velocity would be reduced before the water entered the bent part of the tube. It was trumpet-shaped, as suggested by Prasil (24). This type of vertical leg appeared to have certain advantages since the deceleration at the entrance of the tube was small and there seemed

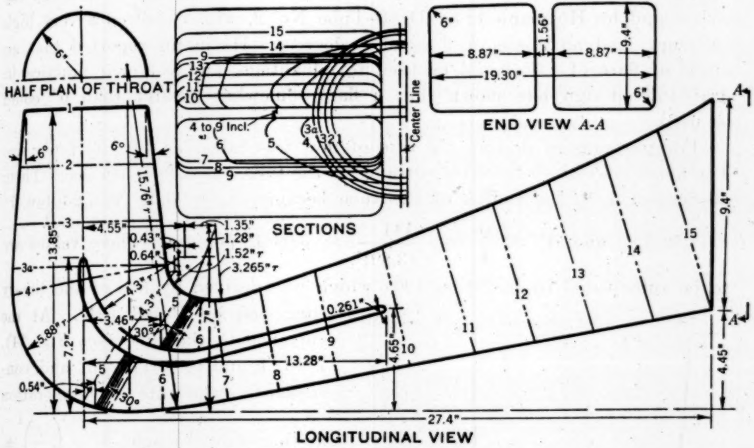

HALF PLAN OF THROAT
6"
SECTIONS
15
14
9
13
12
11
10
4 to 9 Incl.
6
5
3a
4
3 2 1
7
8
9
Center Line
END VIEW A-A
6"
8.87"
1.56"
8.87"
9.4"
19.30"
6"
A
A
1
2
3
3a
4
6°
6°
15.76" r
13.85"
4.55"
0.43"
0.64"
1.35"
1.28"
1.52" r
3.265" r
4.3" r
4.3" r
3.46"
5.88" r
7.2"
30°
30°
7"
5
5
6
6
6
7
8
9
10
11
12
13
14
15
0.261"
4.65"
13.28"
9.4"
4.45"
0.54"
27.4"
LONGITUDINAL VIEW

FIG. 12.—DETAILS OF MODEL DRAFT-TUBE NO. 2.

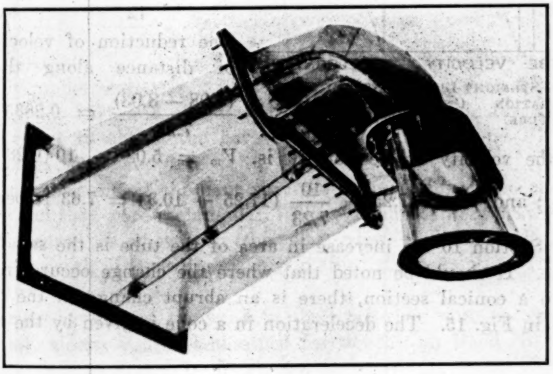


FIG. 13.—MODEL DRAFT-TUBE NO. 3

to be less likelihood of inducing cavitation. The length of the vertical leg used was the same as that of Tube No. 2—7.23 in.—and the entrance was 6 in. in diameter.

The angle of flare of 7° was selected after considering earlier tests by the writer and by Hofmann (8). Draft-Tube No. 2, which showed a very high efficiency, had an angle of flare of 6 degrees. Hofmann reported that an angle of flare of 8.5° could be used to advantage, and common practice in draft-tube design has shown that a flare somewhat greater than 6° could be used.

For purposes of design, the rate of flow was taken as 1.0 cu ft per sec. The mean velocity in the 6-in. entrance to the tube was 5.08 ft per sec. Then, at Section 7.23, the radius of the tube became: $r_0 = 3.00 + 7.23 \tan 7°$ $= 3.89$ in.; and, $V = \frac{Q}{A} = \frac{144}{\pi\,(3.89)^2} = 3.03$ ft per sec. These values are to be substituted in Equation (20) which was derived by use of the theory suggested by Prasil (24). At the entrance to the tube (see Fig. 14), x was found to be 17.94 in. and, consequently, the rate of deceleration at that point becomes $\delta_0 = \left(\frac{Q}{K'}\right)^2 x_0$ $= [(5.08)\ (12) \div 17.94]^2\ (17.94 \div 12) = 17.25$ ft per sec²; and at Section 7.23, $\delta_{7.23} = (3.4)^2 \times \frac{(17.94 - 7.23)}{12} = 10.3$ ft per sec.² The reduction of velocity per inch of distance along the axis is, $\frac{(5.08 - 3.03)}{7.23} = 0.2835$ ft per sec per ft; the velocity at Section 10 is, $V_{10} = 5.08 - 10\,(0.2835) = 2.25$ ft per sec; and, $\delta_{10} = 17.25 - \frac{10}{7.23}\,(17.25 - 10.3) = 7.63$ ft per sec.²

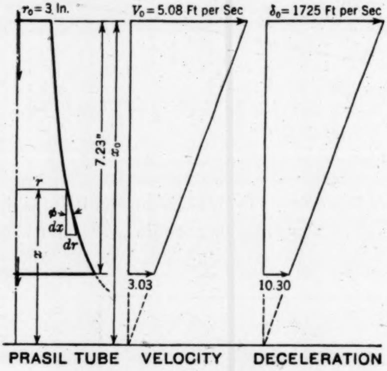


FIG. 14.—STRAIGHT-LINE VELOCITY AND DECELERATION CURVES IN PRASIL DRAFT-TUBE.

From Section 10 the increase in area of the tube is the same as that for a 7° cone. It should be noted that where the change occurs from a Prasil section to a conical section, there is an abrupt change in the deceleration, as shown in Fig. 15. The deceleration in a cone is given by the equation:

$$\delta_x = \frac{2 \tan \phi\,(V_x)^{2.5}}{r_0\ V_0^{0.5}} \qquad (23)$$

in which, r_0 = radius of cone at entrance of tube; V_0 = entrance velocity of water; and, V_x = velocity of water at any other section where deceleration is desired. The radius of a circle having an area of 0.448 sq ft at Section 10, is 4.53 in., which, substituted in Equation (23), yields, $\delta_{10} = \dfrac{2\,(\tan 7^\circ)\,(2.25)^2}{\dfrac{4.53}{12}}$ $= 3.3$ ft per sec^2. Thus, the deceleration drops suddenly from 7.6 ft per sec.^{2} to 3.3 ft per sec.^{2}, as shown in Fig. 15. The deceleration at the entrance of the Prasil tube was computed to be 12.16 ft per sec.^{2}, as compared to 15.3 ft per sec.^{2} at the entrance of a 6° cone.

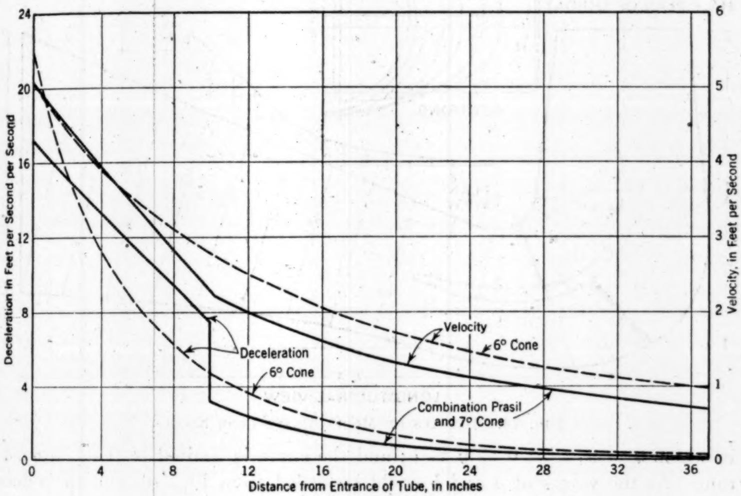


Fig. 15.—Velocity and Deceleration in Model Draft-Tube No. 4

The horizontal leg of the draft-tube, as designed, was rectangular in section. The upper wall was a plane surface, the sides were parallel, and the floor slope was varied in the direction of flow to provide for the increase in area necessary to obtain the flare equivalent to a 7° cone. Fig. 16 shows the details of the model draft-tube.

At the upper end of the bent part the cross-sections were circular and at the lower end they were rectangular. The side curves were drawn in to obtain a smooth curve as shown in the plan view. The outside curves were drawn as smooth lines without any mathematical relation from which to establish them.

The cross-sections of the bend in the draft-tube were patterned after the energy and velocity contours obtained for the test on Bend No. 4 and shown in Sections 4, Figs. 6 and 7. The general plan of the design of the cross-sections of the bend was as follows: At the beginning of the bend, use a circular cross-section; at 22½°, use an elliptical section; at 45°, use an ellipse of greater eccentricity for the inside than for the outside part of the section; and, at the end of the bend, use a rectangular section.

An equation suitable for obtaining such curves as could be used for the sections between the vertex and the end of the bend where the transition must be made from an ellipse to a rectangle, has the general form:

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

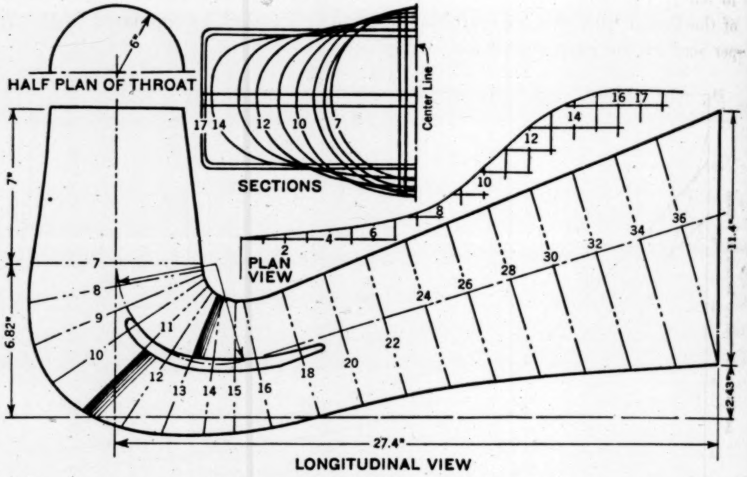

HALF PLAN OF THROAT
6"
SECTIONS
17 14 12 10 7
Center Line
PLAN VIEW
2
4
6
8
10
12
14
16 17
7"
6.82"
7
8
9
10
11
12
13 14 15 16
18
20
22
24
26
28
30
32
34
36
11.4"
2.43"
27.4"
LONGITUDINAL VIEW

FIG. 16.—DETAILS OF MODEL DRAFT-TUBE NO. 4.

in which $0 \leq x \leq a$; $0 \leq y \leq b$; and the curve is limited to the first quadrant. As the values of a and b could be scaled from Fig. 16, and the values of the exponent, n, could be varied to obtain the approximate total cross-section that would give the required area, an exact definition of the cross-sections could be given. However, this refinement was not necessary, and no further mathematical analyses of the sections of the bends were made.

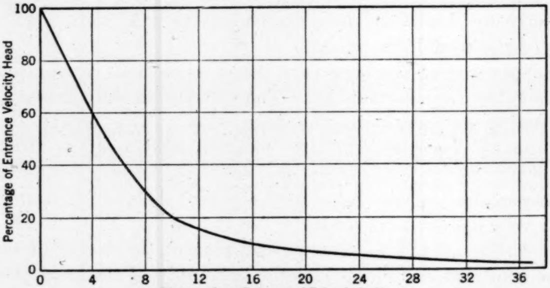


FIG. 17.—PERCENTAGE OF VELOCITY HEAD REMAINING AT VARIOUS SECTIONS OF MODEL DRAFT-TUBE NO. 4.

Fig. 17 shows the percentage of the entrance velocity head that remains in the water at different sections of Model Draft-Tube No. 4, the flow at the entrance being 1.0 cu ft per sec. From this curve it is evident that the major portion of the velocity head should be reclaimed in the vertical leg of the tube.

Laboratory Tests on Model Draft-Tubes.—The tests of the model draft-tubes were made with the same apparatus that was used in testing the experimental pipe bends (see Fig. 4). The only change in this apparatus was to fasten the lower, or outlet, end of the draft-tube to Tank *B*. The throat of the draft-tube was fastened to the same flange to which the entrance of the pipe bend was fastened, as at Point *E*.

The water that flowed through the draft-tube was measured by volume to establish the rate of flow, which was regulated by the valves in the inlet pipe, *K*, and in the outlet pipe, *H*.

Each draft-tube was tested first with a direct flow of the entering water. The pressure at the throat of the tube was obtained through four piezometer connections spaced at equal intervals around it, and the pressure at the exit of the tube was obtained from the elevation of the water surface in the tank, *B*. Pitot tube measurements of velocities in the lower leg of two of the draft-tubes were recorded for several different rates of flow, both with and without the "splitter" in place.

To obtain whirling flow in a draft-tube, a special set of curved vanes was used above the entrance to the tube replacing the guide-vanes shown in Fig. 4. Air was admitted with the water to assist in observing and photographing the position of the center of the whirl. Motion pictures were taken of all the different conditions of flow.

The efficiency of a draft-tube as a regainer of static pressure has been defined in various ways. To the writer it seems logical to define it as the ratio of the head actually recovered to the head available for recovery, or,

$$e = \frac{\text{Regain}}{\text{Throat velocity head}} = \frac{\dfrac{p_2}{w} - \dfrac{p_0}{w}}{\dfrac{V_0^2}{2g}} \quad \ldots\ldots\ldots\ldots\ldots (25)$$

in which p_2 = the pressure head at the exit of the tube; p_0 = the pressure head at the throat of the tube; and V_0 = the velocity at the throat of the tube. The Power Test Code of the American Society of Mechanical Engineers (28) recommends deducting the velocity head at the exit of the tube in computing efficiency. Luksch (6) stated that if the discharge velocity is deducted in computing the efficiency of a draft-tube the result would be indeterminate if applied to a straight cylinder, and he concluded "that it is not justified to account for the discharge energy out of the draft tube, $\frac{V^2}{2g}$, as acting in the sense of an improvement of the efficiency of a draft tube."

No method was devised by the writer for obtaining the efficiency of draft-tubes for conditions of whirling flow. When the water had a whirling motion at the entrance to the tube, the pressure as indicated by the piezometers at the walls of the pipe gave no indication of the mean pressure at that point. Under the conditions of testing as shown in Fig. 4, the pressure at the center of the entrance pipe of the draft-tube was below atmospheric, whereas the pressure near the walls of the entrance pipe was decidedly above atmospheric.

Table 3 shows the efficiencies of the several draft-tubes under conditions of direct flow. Draft-Tube No. 2 showed the highest efficiency (about 65%), indicating that it actually converted into pressure head about 65% of the velocity head at the throat of the tube.

TABLE 3.—SUMMARY OF OBSERVED EFFICIENCIES OF MODEL DRAFT-TUBES

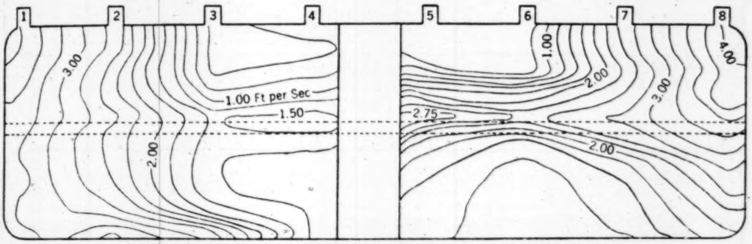
Flow, Q, in cubic feet per second (1)	Throat velocity, V, in feet per second (2)	Velocity head, $\frac{V^2}{2g}$ (3)	PRESSURES		Head regained by tube (6)	Percentage, efficiency (7)
			Entrance (4)	Exit (5)		
(*a*) TUBE NO. 1, COMPLETE						
1.66	8.74	1.185	3.360	3.910	0.550	46.5
1.66	8.74	1.185	3.310	3.870	0.560	47.2
1.65	8.68	1.170	3.310	3.870	0.560	47.8
1.31	6.90	0.740	3.800	4.150	0.350	47.3
Average	.....	.....	.....	.....	.....	47.2
(*b*) TUBE NO. 2, COMPLETE IN PLACE						
0.328	1.72	0.046	3.020	3.050	0.030	65.0
0.725	3.81	0.226	4.070	4.212	0.142	62.8
1.684	8.86	1.220	2.880	3.690	0.810	66.3
1.720	9.05	1.270	2.910	3.740	0.830	65.4
1.250	6.60	0.675	3.424	3.864	0.440	65.1
Average	.....	.....	.....	.....	.....	64.9
(*c*) TUBE NO. 2, WITH HORIZONTAL SPLITTER REMOVED						
1.270	6.65	0.689	3.502	3.937	0.435	....
Average	.....	.....	.....	.....	.....	62.9
(*d*) TUBE NO. 3, COMPLETE IN PLACE						
0.844	4.44	0.381	3.017	3.160	0.153	46.3
0.975	5.13	0.409	2.785	2.980	0.195	47.8
0.995	5.24	0.426	2.785	2.980	0.195	45.7
1.160	6.10	0.578	3.195	3.470	0.275	47.7
1.255	6.62	0.680	3.596	3.920	0.324	47.7
Average	.....	.....	.....	.....	.....	47.0
(*e*) TUBE NO. 4, COMPLETE IN PLACE						
0.650	3.41	0.181	3.663	3.775	0.112	62.0
0.817	4.30	0.288	3.642	3.822	0.180	62.5
0.822	4.32	0.291	3.674	3.855	0.181	62.2
0.828	4.36	0.295	3.640	3.826	0.186	63.0
0.830	4.37	0.297	3.661	3.846	0.185	62.3
1.000	5.25	0.427	3.648	3.922	0.274	64.0
Average	.....	.....	.....	.....	.....	62.7

When the horizontal "splitter" in Draft-Tube No. 2 was removed, the tests, summarized in Table 3, showed that the efficiency was lowered about 2 per cent. Of course, some of the cross-sectional areas of the bend were increased about 10% or 15%, but the writer does not believe this could account entirely for the reduced efficiency.

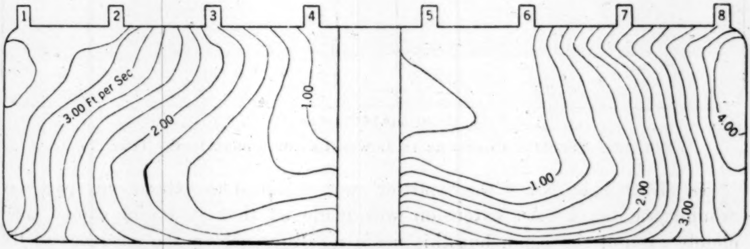
Draft-Tubes Nos. 1 and 3 had an efficiency of about 47 per cent. These two tubes had the central portion of the top surface of the entrance to the lower leg "dished down" very much in the same manner as shown in drawings of a draft-tube developed by G. A. Jessop, Assoc. M. Am. Soc. C. E. (29). Although the lowering of the roof part of the entrance of the lower leg might have been expected to eliminate the dead-water space common in some elbow draft-tubes and thus show good efficiency as a regainer the results were not very good.

There appeared to be no marked difference in the efficiency of the tubes for the different rates of flow. The efficiency was about the same for the lowest rate of flow as for the highest rate; therefore, the fluctuations in efficiency as shown in Table 3 were attributed to experimental error. In tests on draft-tubes having throat diameters of 3 in., Gibson and Labrow (30) concluded that the efficiency was sensibly independent of the velocity for velocities exceeding about 5 ft per sec. In an investigation of the transformation of water velocity into pressure with diffusers having a throat diameter of 75 mm, K. Andres (31) found that the efficiency exhibited by any tube was independent of the velocity over a range of velocities between 30 and 130 ft per sec.

The Pitot tube measurements of the velocities near the entrance (Section 10) of the lower leg of Draft-Tube No. 2, both with and without the "splitter", and facing in the direction of flow, are shown in Fig. 18. From these curves, it is evident that the "splitter" had a material effect on the filamental velocities, but did not eliminate entirely the region of lessened velocities near the inner part of the exit of the bend. From the curves it appears that the velocities were higher near the right side of the tube, looking down stream, than on the left side. This was found to have been caused by the flaring tube in Tank *A* (Fig. 4), being about $\frac{1}{2}$ in. off center. When the flaring tube was adjusted to the center of Tank *A*, its two sides showed approximately symmetrical conditions of flow.



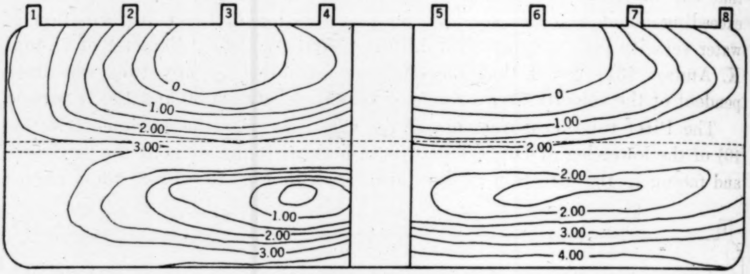
(*a*) With Horizontal Splitter and Vertical Bearing Partition; Avg Q = 1.204 Cu Ft per Sec



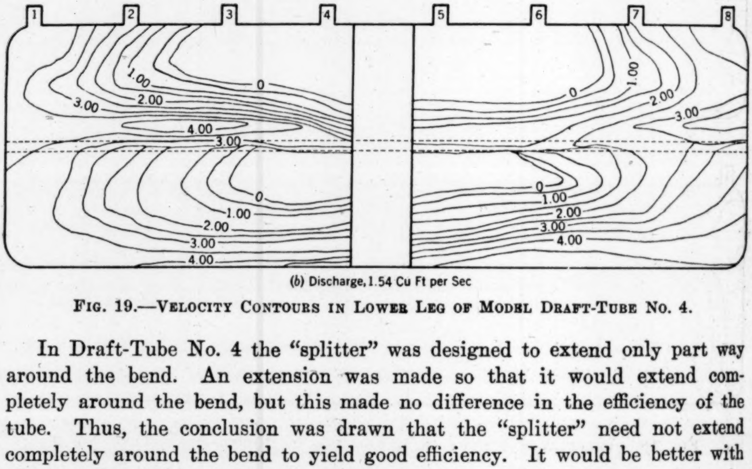
(*b*) With Vertical Bearing Partition; Avg Q = 1.193 Cu Ft per Sec

FIG. 18.—VELOCITY CONTOURS IN LOWER LEG OF MODEL DRAFT-TUBE NO. 2.

Fig. 19 shows velocity contours at a section (Section 20) near the exit of the bend of Draft-Tube No. 4, facing in the direction of flow. It is evident that there was considerable dead water near the inside of the bend, a condition that did not exist in Tube No. 2. The reason for the occurrence of the dead water was attributed to too great a distance between the top and bottom walls of the exit of the bend, a measurement hereafter designated as the Δ-distance of the tube.



(a) Discharge, 1.23 Cu Ft per Sec



(b) Discharge, 1.54 Cu Ft per Sec

FIG. 19.—VELOCITY CONTOURS IN LOWER LEG OF MODEL DRAFT-TUBE NO. 4.

In Draft-Tube No. 4 the "splitter" was designed to extend only part way around the bend. An extension was made so that it would extend completely around the bend, but this made no difference in the efficiency of the tube. Thus, the conclusion was drawn that the "splitter" need not extend completely around the bend to yield good efficiency. It would be better with whirling flow if the "splitter" were limited to a position about as shown in Fig. 16.

With whirling flow there was considerable turbulence throughout the length of all the draft-tubes. This was at its worst with those tubes having the "splitter", such as Tube No. 2, if the "splitter" extended completely around the bend. An attempt was made to correct the turbulence in Tube No. 2 by placing a "twisted fin" on the upper end of the "splitter." The results of this experiment are shown in Table 4.

The loss of head in flow through the tube was somewhat reduced by the "fin", but the difference was not great. It is known that the whirl component of the filamental velocities in draft-tubes may be clockwise at one time and counter-clockwise at another, depending upon the changes in head and gate-opening It is possible that the whirl may be clockwise near the walls of the draft-tube and counter-clockwise near the center of the tube. It is evident, therefore, that the use of any such deflector as described herein for correction of whirl in a draft-tube is not practicable.

TABLE 4.—ELEVATIONS OF WATER SURFACES IN TANKS *A* AND *B* FOR DRAFT-TUBE No. 2, WITH AND WITHOUT SPECIAL FIN FOR CORRECTING WHIRL

WITH FIN IN PLACE (RATE OF FLOW, 0.961 CUBIC FOOT PER SECOND. MEAN VELOCITY AT ENTRANCE OF TUBE, 5.11 FEET PER SECOND)			WITHOUT FIN IN PLACE (RATE OF FLOW, 0.975 CUBIC FOOT PER SECOND. MEAN VELOCITY AT ENTRANCE OF TUBE, 5.13 FEET PER SECOND)		
Water-Surface Elevations		Lost head	Water-Surface Elevations		Lost head
Tank *A*	Tank *B*		Tank *A*	Tank *B*	
4.327	2.034	2.293	4.410	2.020	2.390
4.328	2.035	2.293	4.418	2.027	2.391
4.335	2.037	2.298	4.408	2.012	2.396
4.341	2.024	2.317	4.407	2.027	2.380
4.335	2.035	2.300	4.410	2.012	2.398
4.339	2.044	2.295	4.420	2.025	2.395
4.310	2.015	2.295	4.415	2.024	2.391
4.345	2.025	2.320	.....	.....	.....
Average.....	.....	2.310	Average.....	.....	2.391

RECOMMENDATIONS FOR THE DESIGN OF ELBOW DRAFT-TUBES

The Vertical Leg.—An elbow draft-tube should have a trumpet-shaped vertical leg as long as the suction head and the construction costs will permit. A long vertical leg seems to be justified by the fact that the bent part of an elbow draft-tube induces spiral flow and, consequently, friction and eddy losses which vary approximately as the square of the velocity of the flowing water. If there is a vertical leg in a draft-tube, the induced spiral velocities in the bend and the resultant hydraulic losses should be smaller than in an elbow draft-tube which has no vertical leg.

As indicated in Fig. 17, there is a greater conversion of kinetic energy to potential energy per foot of draft-tube where the mean velocity is high than where it is low. This is due to the fact that the velocity head varies as the square of the velocity. In the case of Model Draft-Tube No. 4, it would be theoretically possible, under ideal conditions, to obtain an efficiency of more than 80% with the tube cut off at Section 10. Of course, it would be impossible to obtain ideal conditions where there would be no friction and eddy losses, but there can be a closer approach to ideal conditions in a straight pipe than in a curved pipe. Friction on pipe walls cannot be eliminated, and centrifugal force is inherent to change in the direction of flow. These two in combination induce spiral flow as shown in Fig. 5(*a*). The induced spirals cause hydraulic losses that would not occur in a straight pipe.

There is a possibility that the induced spirals due to the bend in an elbow draft-tube may approximately offset the undesirable whirl components of flow from a turbine runner. It is very doubtful, however, whether a designer, either by chance or by careful planning, could discover the combination of conditions needed for this delicate balance. Further research along this line may be expected in the future, since rational and experimental evidences have been submitted (32) (33) (34) by many able engineers in support of their conclusions that the form of draft-tube affects the performance of the runner.

The trumpet shape of the vertical leg, such as that suggested by Prasil (24) seems to be preferable to the straight circular cone. The experiments of Hofmann (8) Gibson and Labrow (30) and Andres (31) indicate that the trumpet-shaped tube has as great an efficiency as a straight cone, or greater. The cone-shaped tube has a further disadvantage that at the entrance to the tube the deceleration is comparatively high and is changing rapidly. This occurs in a region where cavitation troubles may arise. Research at the Massachusetts Institute of Technology (35) shows that cavitation is a mechanical phenomenon in which there is a rapid formation and collapse of small cavities in the flowing water. It appears that the rapidly changing decelerating force at the entrance of a conical tube would make a turbine runner more susceptible to cavitation than the lesser decelerating forces in the trumpet-shaped draft-tubes. It seems, therefore, that the vertical leg of a draft-tube should be trumpet shaped.

The Bend.—In the test of the experimental pipe bends it was shown that Bend No. 4, which was widened in the direction normal to the plane of the bend, offered the least resistance to flow. This was a fortunate finding, because of the ease with which this type of bend can be adapted to an elbow draft-tube.

In the report of the Bureau of Standards (16) on the investigation of literature on draft-tubes, the following statement was made:

> "* * *, some conditions for improving the efficiency of elbow type draft tubes will be stated, which probably are correct, since apparently they are all met with in practically all the elbow tubes of the better design. First, the curvature of the inside wall should be sharp, reducing the length of the curve. Secondly, the depth of the tube at the section where the water is deviated should be small. Third, where the inside wall has considerable curvature, the areas of the sections from point to point should be nearly constant."

It is interesting to note that Bend No. 4 essentially fulfills all these conditions, since the radius of curvature of the inside wall is smaller than that of the outside wall, all the cross-sectional areas are alike, and the pipe is flattened in the plane of the bend.

This flattening, arranged so that the depth of the draft-tube at the exit of the bend is small, seems to be of considerable importance. Draft-Tube No. 2 (Fig. 12) has less depth at the exit of the bend than Draft-Tube No. 4 (Fig. 16). Otherwise, the shapes are quite similar Draft-Tube No. 2 had no dead-water space in the lower leg (Fig. 18), whereas Tube No. 4 had considerable dead-water (Fig. 19), with a resultant lowered efficiency.

Luksch (6) has shown that with direct flow in a trumpet-shaped draft-tube, the maximum efficiency occurred when the impact plate was placed so that $\frac{\Delta}{D_0} = 0.27$, in which Δ = the distance between the impact plate and the bottom of the draft-tube; and D_0 = the throat diameter of the draft-tube.

With values of $\frac{\Delta}{D_0}$ greater than 0.27, the water would jump free from the inner face of the tube and permit the formation of a dead-water zone. This same phenomenon probably occurs in an elbow draft-tube, because the bottom part of the bend acts in a manner similar to an impact plate.

It was stated that the radius of curvature of the inside wall of the bend should be smaller than that of the outside wall. Just what relation these radii should have to the throat diameter of the tube is not known, but some notion may be had from the experiments of Hofmann (8) who showed that if an impact plate is used below a conical draft-tube with rounded bottom edges, the most favorable ratio for rounding is, $\frac{r}{D_0} = 0.85$, in which D_0 = the throat diameter of the tube; and r = the radius of the rounded bottom edges of the draft-tube. He showed, furthermore, that $\frac{r}{D_0}$ may vary between 0.67 and 1.00 without much change in efficiency of the tube. This indicates that the elbow draft-tube may have a wide range of values of the radius of curvature for the inner walls of the bend without losing materially in efficiency.

The shape which should be given the curve shown as the plan view, in Fig. 16, seems to be of no great importance, except that preferably, it should be a smooth curve. In all the model draft-tubes tested by the writer the water had comparatively high velocities of flow along the sides of the tubes, as shown in Figs. 18 and 19.

The cross-section at the entrance of the bend should be circular, and, at the exit, it could well approach that of a rectangle. The intermediate sections should be flatter on the inside of the bend than on the outside.

A "splitter" should be used in the bend of an elbow draft-tube because of its tendency to assist in getting the water around the bend in an orderly manner. Tests on Draft-Tube No. 2 gave about 2% higher efficiency with the "splitter" than without it. In the Lilla Edet (17) tests, where the draft-tube had practically no vertical leg, the "splitter" increased the efficiency slightly.

It is undesirable to extend the "splitter" completely around the bend. Tests with Draft-Tube No. 4 were made first with the "splitter" beginning at about one-third the way around the bend (Fig. 16) and then extended so that it began at the very entrance to the bend. No noticeable change in the efficiency was observed. Caflisch (36) tested a turbine of the Francis type for which he showed that the "splitter" reduced the efficiency when extended into the vertical leg of the draft-tube. This reduction was to be expected, as the turbine caused a whirl component of flow at the part gate-opening. At the larger gate-openings the "splitter" seemed to have little effect on the efficiency. Had it been extended only part way around the bend, such as at the Kembs Power Plant (37), on the Rhine, there probably would have been less turbulence at part-gate operation.

Professor Spannhake (38) has mentioned the desirability of whirl in the draft-tube in the positive sense of the rotation of the runner, this whirl allowing the runner blades to be designed for lower relative velocities and, in consequence, for reduced possibility of cavitation. If this should be the practice, then the extension of the "splitter" into the vertical leg is undesirable. However, it does seem desirable to have both the "splitter" and the vertical bearing partition, commonly built in the lower leg of the tube, extend some distance into the bend.

The Lower Leg.—The length, and particularly the width, of the lower leg of an elbow draft-tube greatly influence the design of the power house. Generally, the width of the draft-tube controls the spacing of the turbines, and the type and size of the plant determine whether or not a bearing partition shall be used in the lower leg of the draft-tube as a means of giving stability to the power-house structure.

The proportions that should be given the lower leg of elbow draft-tubes have been the subject of an investigation (39) for the Wheeler Dam, in the Tennessee Valley. Several tubes were tested with the result that the best efficiency in each tube was obtained when the length of the lower leg was the longest tested. In Plate A-6 of the report (15) of the National Electric Light Association on draft-tubes, are given the results of tests on changing the length of the lower leg of an elbow draft-tube by the Newport News Shipbuilding and Drydock Company. Leg lengths of 2.50, 3.00, and 3.71 times the throat diameter of the tube were used, and the efficiency of the tube was increased with each increase in leg length. This does not mean that the length could be increased indefinitely with a continued increase in efficiency. Hofmann's tests (8) on conical tubes led him to conclude that at first an increase of the length of a draft-tube will cause an improvement in efficiency and, finally, that the wall friction becomes too great and causes a decrease in efficiency. Further experiments are required to determine what the optimum length of the lower leg of an elbow draft-tube should be.

SUMMARY AND CONCLUSIONS

Experiments with Pipe Bends.—

(1) The loss of head due to the flow of water through a standard 6-in. pipe bend of 8-in. axial radius was 0.15 $\frac{V^2}{2g}$ as compared to values of 0.13 $\frac{V^2}{2g}$ to 0.17 $\frac{V^2}{2g}$ for the other pipe bends tested. The specimen that offered the least resistance to the flow of water was flattened in the plane of the bend. Its superior performance probably was due to the fact that fewer induced spirals occurred in this form than in the other bends.

(2) The specimen that was widened in the plane of the bend had more vibration than the other bends when tested under conditions where the entering water had a whirl component of velocity.

Experiments with Model Draft-Tubes.—

(3) The efficiency of the model draft-tubes of 6-in. throat diameter as a device for converting velocity head into pressure head varied between 47% and 65% when the efficiency was defined as the ratio of the head actually recovered to the head available for recovery. There was no appreciable change in the efficiency of the model draft-tubes for changes in rate of flow.

(4) When there was considerable whirl component of the velocities of flow in the draft-tubes, there was a central rotating core in the vertical leg of the tube in which there was a return flow of the water toward the entrance of the tube.

(5) The insertion of a deflector in the model draft-tubes for the purpose of correcting the whirl component of flow was found to be impracticable.

Suggestions for the Design of Elbow Draft-Tubes.—

(6) The vertical leg of an elbow draft-tube should be trumpet-shaped. This shape is just as efficient as any other. It has the advantage over the conical tube that, for a given reduction in mean velocity in a given length of tube, the deceleration is less in the regions of higher velocity.

(7) The following suggestions are offered as a basis for designing the cross-sections of the bend of an elbow draft-tube: (*a*) At 0°, beginning of bend, use a circular cross-section; (*b*) at 22½°, use an elliptical section, with the major axis of the ellipse normal to the plane of the bend; (*c*) at 45°, use an ellipse of greater eccentricity for the inside than for the outside part of the section; and, (*d*) at the end of the bend, use a rectangular section.

(8) The radius of curvature of the inside walls of the bend of an elbow draft-tube should be smaller than the radius of curvature of the outside walls of the bend. The centers of the radii of curvature of the inside and outside walls of the bend should be located so that the distance between the walls diminishes from point to point along the axis of the bend.

(8) A comparison of the results of the tests on pipe bends and draft-tubes, as described in this paper, with the performance of the best elbow draft-tubes described in current technical literature, indicates that, similar to a conical or trumpet-shaped draft-tube of given length and flare in which there is an optimum ratio of the distance between a deflector plate and the bottom edge of the tube to the throat diameter, there probably is an optimum ratio of the distance between the top and bottom walls of the exit of the bend of an elbow draft-tube of given length and flare to the throat diameter of the tube. Further experimentation is needed to establish the optimum ratio to be used in design.

(10) When a guide-vane or "splitter" is used in the bend of an elbow draft-tube, it should extend to about the third point of the bend. The extension of a "splitter" completely around the bend effects no increase in efficiency for direct flow, and causes loss in efficiency for whirling flow. When a bearing partition is used in the lower leg of an elbow draft-tube, it should extend about half way around the bend to help divide the flow between the two halves of the tube when there is whirl component of the filamental velocities.

ACKNOWLEDGMENTS

The writer earnestly wishes to express his appreciation to all those who have helped to make this paper possible. Particular mention should be made of the help and advice of S. M. Woodward, F. T. Mavis, B. J. Lambert, and the late Floyd A. Nagler, Members, Am. Soc. C. E.; Professor Lewis F. Moody, of Princeton University, Princeton, N. J.; D. L. Yarnell, M. Am. Soc. C. E., F. Merryfield, Assoc. M. Am. Soc. C. E., Douglas G. Baird, Jun. Am. Soc. C. E., and Mr. Grant Robley, of the Oregon State Agricultural College, Corvallis, Ore. Draft-tubes for this project were designed under the supervision of Lt. Col. Charles F. Williams, Corps of Engineers, U. S. A., District Engineer, U. S. Engineer Department, at Portland, Ore., and models were authorized to be built and tested at the Oregon State College. The tests of the model draft-tubes were conducted under the direction of the writer, and with the understanding that the experimental data might form a part of this thesis. C. I. Grimm, M. Am. Soc. C. E., was Chief Civilian Engineer in charge of the Bonneville Project.

APPENDIX

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(2) **"Lowell Hydraulic Experiments."** By James B. Francis. (p. 2.) (In this book Uriah A. Boyden was given credit for inventing a turbine in which he "introduced several improvements of great value").

(3) **"The Control of Water."** By Philip A' Morley Parker, 1913. (On p. 870 the author states that the type of turbine used by Francis was a very special form for a far larger class termed the Francis Turbine, and was "probably invented by Boyden.")

(4) **"Hydraulics and Its Application."** By A. H. Gibson. (Gives credit to Uriah A. Boyden for inventing the diffuser.)

(5) **"The American Mixed-Flow Turbine and Its Setting."** By Arthur T. Safford and Edward Pierce Hamilton. *Transactions*, Am. Soc. C. E., Vol. 85 (1922), pp. 1337–1356. (The water flowed radially inward and curved gradually so as to have velocity components parallel to the axis of the runner.)

(6) **"An Experimental Study of the Most Favorable Distance of a Water Turbine Draft Tube from the Tailrace."** By Andreas Luksch, 1935. (A thesis presented to the State University of Iowa in partial fulfillment of the requirements for the degree of Doctor of Philosophy.)

(7) **"Water Power Engineering."** By H. K. Barrows. 1927, pp. 230, 397. (Shows mathematically that, as the head on a turbine increases, the wheel must be lowered with reference to the tail-water level.)

(8) **"Die Energieumsetzung in saugrohränlich-erweiten Düzen."** ("The Conversion of Energy in Nozzles Shaped like Draft-Tubes"). By A. Hofmann. *Mitteilungen* des Hydraulischen Institut der Technischen Hochschule, München, Heft 4, 1931, pp. 44–69. (States that it is permissible to let the angle of flume of a draft-tube be increased beyond that in common practice.)

(9) **"The White Hydraucone Regainer."** By W. M. White. 1921, pp. 255–324. (First improvement of the draft-tube after its invention in 1840.)

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DISCUSSION

F. T. Mavis,[4] M. Am. Soc. C. E. (by letter).—The author is to be commended for his painstaking work in constructing the model bends and in performing the tests which have been briefly summarized in this paper. He has demonstrated remarkable skill in making transparent model draft-tubes and in shaping and assembling the pyralin sections to be tested.

A quarter-turn draft-tube is no simple conduit. It may be visualized as a combination of a bend and an expanding section of pipe which changes not only in cross-sectional area but in shape as well. This combination, however, is not one that can be investigated readily by studying the peculiar behavior of each part and superposing the flow conditions which would obtain in each part separately. Each of these elements (the bend and the expanding section) has been studied by investigators over a period of more than 60 yr. There is still much to be learned about the behavior of water as it flows in a curved channel and in an expanding conduit, and the investigation of flow in draft-tubes has little more than made its "debut" in the hydraulic laboratory.

The writer proposes to confine his discussion to certain phases of the flow of water in curved pipes which have substantially the same cross-sectional area at the inlet and at the outlet. He will not attempt to discuss the hydraulics of expanding sections nor to dwell upon the combination of bend and diffuser in a draft-tube of the quarter-bend type.

Professor Mockmore has referred to the comprehensive tests on the flow of water in bends which were performed by Messrs. Yarnell and Nagler (22).[3] With reference to investigations of head loss in bends, mention should be made of the pioneer work of Weisbach[5] in Germany, about the middle of the Nineteenth Century; and of the work of Alexander[6] and Brightmore[7] in England; and Williams, Hubbell, and Fenkell,[8] and Schoder[9] in the United States twenty-five to thirty years ago. More recently, investigations on the flow of water in curved pipes and miter bends have been carried on in Germany by Hofmann,[10] Schubart,[11] and others,[12] working in the Hydraulic

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[4] Prof. and Head, Mechanics and Hydraulics Dept., and Consultant, Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[5] "Lehrbuch der Ingenieur und Maschinen Mechanik", von J. Weisbach, Leipzig, 1845, p. 439.

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Institute, at Munich; in Danzig, by Nippert,[13] and in the United States, by Giesecke,[14] by the staff of the National Hydraulic Laboratory, under the direction of Mr. H. N. Eaton, and by Wallace M. Lansford, Assoc. M. Am. Soc. C. E.,[15] at the University of Illinois. Most of these experiments were conducted on pipes of circular cross-section, although some were made on pipes of square or rectangular section. There is a mass of experimental data which could be collected from many published and unpublished investigations. It would be worth considerable to have these data brought together in a single report, and to have them re-appraised and correlated on the basis of present knowledge of the behavior of water as it flows around a bend.

The problem of the hydraulics of bends has also been investigated in some detail from an analytical, or mathematical, basis. Zur Nedden[16] in a paper on induced currents in fluids published in 1916, presented certain aspects of the problems which merit the mention of his paper in particular. He calls attention to the fact that a bend may perform quite differently depending on whether the flow of the fluid through it is laminar or turbulent. He cites specifically the case of a bend of rectangular cross-section, the inner and outer curves of which were hyperbolic, as causing a lower head loss than a circular bend as long as the flow remained laminar; and that as soon as the flow became turbulent, the hyperbolic bend caused a greater head loss than the circular bend. This behavior, of course, would be apparent if the coefficients of velocity head, representing the excess head loss in the bend, were plotted as a function of Reynolds' number. The particular bend to which Zur Nedden refers was one tested by Grether[17] which in plan was similar to the bend which Professor Mockmore calls Bend No. 2.

To-day, it is generally recognized that a liquid, in flowing around a bend under turbulent flow conditions, moves in a so-called "spiral" or "helical" path, and that the induced currents and attendant impact are largely responsible for excess losses of head in bends. In 1902, the late H. W Brinckerhoff, M. Am. Soc. C. E., discussing[18] a paper by Clarence W. Hubbell George H. Fenkell, and the late Gardner S. Williams, Members, Am. Soc. C. E., explained the fact that a bend offers greater resistance than an equal length of straight pipe on a hypothesis of spiral flow which he described in its essentials precisely as Professor Mockmore and other observers have done. In their closing discussion, the authors of this paper replied to Mr. Brinckerhoff's comments as follows:

> "* * * at one time, one of them [the authors] was disposed to explain certain phenomena on the theory of a spiral motion, but in spite of attempts to locate such motion, no direct experimental evidence of its existence has yet

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[13] Uber den Strömungsverlust in gerkrümmten Kanälen,", von H. Nippert, *Forschungsarbeiten*, Heft 32 (1929).

[14] "Friction of Water in Elbows", by F. E. Giesecke, M. Am. Soc. C. E., *Transactions*, Am. Soc. for Heating and Ventilating Engrs., Vol. 32, pp. 303–314, 1926.

[15] "The Use of an Elbow in a Pipe for Determining the Rate of Flow in the Pipe", by Wallace M. Lansford, *Bulletin No. 289*, Eng. Experiment Station, Univ. of Illinois, 1936.

[16] "Induced Currents in Fluids", by F. Zur Nedden, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 844–913.

[17] "Krümmer die Potentialbewegung zulassen", von H. Grether, *Verhandlung* d. Ver. z. Beförderung d. Gewerbefl., 1909, p. 117 ff.

[18] *Transactions*, Am. Soc. C. E., Vol. XLVII (1902), p. 272.

been obtained, and, though the writers consider that Mr. Brinckerhoff's theory is ingenious and possibly correct, they are not prepared to endorse it."

Transparent model bends, such as those used by Professor Mockmore, permit the observer to see this spiral or helical flow. These transparent models have done much to promote a better understanding and a more intimate knowledge of the phenomena of hydraulics. To-day, this spiral flow in bends can be observed; three decades ago, it was an hypothesis which eminent hydraulic engineers were then " * * * not prepared to endorse * * *."

The intensity of induced currents depends chiefly upon the pressure gradient along a section which is normal to the direction of motion of particles of water in the bends. With slight modification, Equation (4) can be written as follows:

$$p_2 - p_1 = \frac{w}{g} \int_{r_1}^{r_2} V^2 \frac{dr}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which $p_2 - p_1$ = the difference in pressure at Points (2) and (1) on a section normal to the flow of water particles; r_2 and r_1 = the radii of curvature of stream filaments at Points (2) and (1), respectively; V = the velocity of flow along any filament the radius of curvature of which is r (and V is, therefore, a function of r); and, $\frac{w}{g}$ = the specific mass of water.

If the width of bend $(r_2 - r_1)$ is small in proportion to the radius of curvature, $0.5\ (r_2 + r_1)$, and if the depth of the channel is large in a direction normal to the plane of curvature; that is, several times $r_2 - r_1$, the velocity will probably be nearly constant and it may be replaced by the average velocity, V_{av}. Under these conditions it may be justifiable to state as a fair approximation that the difference in pressure on the rigid side-walls which are a distance $(r_2 - r_1)$ apart will be, as given by Equation (4),

$$p_2 - p_1 = \frac{w}{g} V^2_{av} \log_e \left(\frac{r_2}{r_1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Co-axial guide-vanes, in general similar to the horizontal partition in Draft-Tube No. 2, may be expected to reduce the head loss in bends for three reasons: (1) The vanes tend to act as baffles and equalize the velocity distribution; (2) by reducing the ratio, $\frac{r_2}{r_1}$, for the individual channels and making it nearer unity than the corresponding ratio for the bend as a whole, the pressure difference, $p_2 - p_1$, is reduced between any two given vanes; and (3), the unbalanced force exerted on a given guide-vane is distributed over a curved plate supported on the side walls of the bend. The efficacy of guide-vanes in reducing losses due to the flow of air in bends has been demonstrated, and bends fitted with co-axial guides have been used in wind tunnels of the return-flow type. In principle, they should also be effective in water conduits although there are certain apparent obstacles in the way of obtaining structural strength at costs low enough to justify the construction of guide-vanes.

The phenomena of flow of water through bends, expanding conduits, and draft-tubes, deserve intensive study in model and prototype. Professor Mockmore's approach through the medium of transparent models leads the hydraulic engineer a long way toward a better understanding of those phenomena.

Jerome Fee,[19] Assoc. M. Am. Soc. C. E. (by letter).—This important paper is distinguished by the experimental technique displayed and the excellent form in which the results are presented. Apart from the special field of draft-tube design, the study is one of great interest in connection with general problems of hydraulics.

The author refers to spiral flow as an important factor. Because of its significance in these experiments and its bearing on diverse subjects, the writer is prompted to discuss at some length the author's "Theory of Induced Spirals in Pipe Bends." Despite the fact that many attempts have been made to explain the phenomena of spiral flow, the writer feels that, even now, a complete and satisfactory analysis is lacking. The purpose of this discussion is not to propose a new explanation, but to call attention to imperfections in the theories which have been advanced from the time of James Thomson (21),[8] in 1876, to Albert Einstein,[20] in 1933, or L. Prandtl,[21] in 1935. It is astonishing how frequently this subject has been referred to in scientific literature and how many outstanding men of science have given it their attention.

There is a close similarity between Thomson's solution and that of Einstein. The author's theory seems to be based on the same general ideas, but the relationship is not so apparent.

Referring to flow in river channels, Thomson writes (21):

> "But the layer of water along the bottom, being by friction much retarded, has much less centrifugal force in any bar of its particles extending across the river; and consequently it will flow sidewise along the bottom towards the inner bank, and will, part of it at least, rise up * * *."

Einstein describes the circulating motion of water stirred in a cup, and likewise finds that the velocity near the bottom and, hence, the centrifugal force, is reduced, from which he concludes that a circulatory motion will occur in radial planes, that is, normal to the general whirling of the water. The writer wishes to take no undue liberties and would prefer to give Professor Einstein's exact words, but is restrained by the formidable phrasing of the copyright clause. It should be mentioned that Einstein was writing for non-technical readers.

It can scarcely be conceived that a reduction in centrifugal force, by itself, will cause a particle to move radially inward toward the center of a curved path. That would be equivalent to saying, "because of a reduction in the force of gravity the particle started to rise." Nevertheless, this method of explaining spiral flow is quite common. To cite one additional example, Richard Hein[22] writes:

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[19] Asst. Engr., San Francisco Water Dept., San Francisco, Calif.

[20] "The World as I See It", p. 111.

[21] "Aerodynamic Theory", Vol. III, p. 161 (Guggenheim Fund Series, W. F. Durand. Editor in Chief).

> "The layers of water adjacent to the upper and lower sides [actually the top cover and bottom of a closed vortex chamber] experience a breaking effect due to the skin friction, and the resulting reduction in centrifugal force causes flow towards the center of the vortex."

Directing attention next to the explanation given by the author in the paragraphs following Equation (5) where he states that "the pressure at Point *d* will be less than at Point *b*," this opinion appears to be based on the inference that, because of the proximity of the channel walls, the velocity is reduced; consequently, the centrifugal force and, therefore, the pressure are reduced. The writer would expect a high, not a low, pressure at Point *d* in conformity with the usual rule that high pressures go with low velocities. The validity of this rule is proved in an interesting manner by comparing Figs. 6 and 7.

Of the various explanations of spiral flow which the writer has examined, that of L. Prandtl[23] seems the nearest approach to a correct solution. The writer regrets that certain of Prandtl's references are not available, particularly those of which he writes: "The results of experiments recently performed, or about to be performed, should, after some time, permit the definite quantitative determination of the magnitudes involved in the phenomena just described."

Prandtl finds that two distinct cases are involved. One, as the writer understands it, may be illustrated by considering the flow in a curved open channel with frictionless sides, but a rough bottom. In this case, the pressure gradient at the bottom is determined by the pressure gradient of the more freely flowing filaments immediately above. The differential pressure on a bottom particle, not being opposed by as high a centrifugal force as for those above, causes the particle to move radially inward.

In the other case, the curved open channel should be thought of as having a frictionless bottom, but rough curved sides. Prandtl considers that the particles close to the convex wall, by having their velocity reduced, are subject to lower centrifugal forces than adjacent filaments and tend to remain close to the wall. On the other hand, particles at the concave wall, likewise subject to smaller centrifugal forces than adjacent particles, will be pressed away from the wall by such faster moving particles.

Prandtl expresses these ideas with such brevity that the writer feels justified in giving his interpretation, for which he alone is responsible. The writer may have misconstrued or misunderstood the matter, but he is unable to see how spiral flow follows as a result, although the argument for the first case seems the more convincing of the two.

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[22] "Investigation of the Thoma Counter Flow Brake", by Richard Hein, *Transactions*, Munich Technical Univ., *Bulletin 3*, pub. in 1935 by Am. Soc. of Mech. Engrs.

[23] "Aerodynamic Theory", Vol. III, p. 161.

The situation in regard to spiral flow seems to parallel a similar problem in aerodynamics where it is necessary in order to explain "lift" to refer to a "circulation" of air about an airplane. To account theoretically for this circulation has been a matter of great difficulty. In a perfect fluid, circulation about an airplane wing would be impossible. In a real, viscous fluid, to account for circulation by the simple statement that the velocity is reduced in the boundary region is quite inadequate.

The writer is of the opinion that all attempts to explain the circulation of water flowing in bent pipes (which circulation, of course, is the distinguishing component of spiral flow), without reference to vortex filaments and surfaces of discontinuity of flow, will ultimately be rejected. The use here of the word, "vortex," has reference not to the entire phenomena of circulation, but to the localized vortices which sometimes determine the nature of the entire flow.

One very interesting feature of the author's study appears to have been the formation of two distinct types of spiral flow. In one type the motion is in circular paths with the fastest moving particles, neglecting longitudinal flow, farthest from the center, or approximately so. This is the type of circulation, as the writer understands it, which Einstein and Thomson describe, and which may exist as one single circulation or two circulations combined, as shown in Fig. 1. On the other hand, such spiral flow as that illustrated in Bend No. 5, Fig. 9(*b*), of sufficient intensity to cause vibration, must be the better known "free vortex" flow in which the velocities increase, approaching the center.

To revert to the first-mentioned type, the writer has before him the typewritten notes recently used in a college class in hydrodynamics. A description of the flow in pipe bends states that "the circulation corresponds closely to that of the Rankine combined vortex, the tangential velocity component being greatest at a point between the center line and the pipe wall, and decreasing toward the center and wall." This is the only specific reference the writer has seen to the distribution of velocities of circulation in pipe bends. It is clear that the distribution differs from that of the free vortex.

These lecture notes explain the circulation by stating that "since these centrifugal forces diminish as the velocity approaches zero, the pressure in the boundary region at the outside of the bend is less than that just within this boundary layer." The writer believes that this is incorrect, as it implies that pressure may manifest itself only as a consequence of centrifugal force.

No one interested in the subject of hydraulics can fail to appreciate the remarkable value, from both the theoretical and practical standpoint, of Professor Mockmore's study.

R. E. B. SHARP,[24] ESQ. (by letter).—It is noted that the losses for Bend No. 5 were not included in the tabulation following Equation (22). It would be of interest to have this information. A comparison of Bends

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[24] Hydr. Engr., I. P. Morris Div., Baldwin-Southwark Corporation, Philadelphia, Pa.

No. 1 and No. 4 as made and tested by the author does not show the true relation. The latter shows the smaller loss, but to give a true picture of the relative advantage of adopting the shape of Bend No. 4 as compared to the circular shape, the same radius of curvature of axis should have been used for both bends. For Bend No. 4, $\frac{R}{d} = \frac{15}{6} = 2.5$, whereas this value for Bend No. 1, equals 1.333. If the loss with Bend No. 1 is corrected from $\frac{R}{d} = 1.333$ to $\frac{R}{d} = 2.5$, in accordance with tests reported by Dr. Albert Hofmann[25], the following approximate values are derived:

$$(a)\ \text{Loss} = \frac{0.15\ V^2}{2\ g}\ \text{(Mockmore)} \times \frac{0.125}{0.16}\ \text{(Hofmann tests with smooth walls)} = \frac{0.117\ V^2}{2\ g}$$

$$(b)\ \text{Loss} = \frac{0.15\ V^2}{2\ g}\ \text{(Mockmore)} \times \frac{0.14}{0.17}\ \text{(Weisbach)} = \frac{0.1237\ V^2}{2\ g}$$

and,

$$(c)\ \text{Loss} = \frac{0.15\ V^2}{2\ g}\ \text{(Mockmore)} \times \frac{0.09}{0.11}\ \text{(Alexander)} = \frac{0.1225\ V^2}{2\ g}$$

The foregoing corrected values compare favorably, from the standpoint of the circular shape, with those obtained by the author for the elliptical shape of Bend No. 4, in which he found the loss to be $\frac{0.13\ V^2}{2\ g}$.

The tests of the model draft-tubes, as presented by the author, are of interest as such. Tests of this nature may serve as guides for the construction of model draft-tubes for tests with model turbines, but should not be used as the bases for the construction of large tubes in the field. The reason for this, of course, is the impossibility of distributing, properly, the flow at the entrance to model tubes. It is noted that although Model Draft-Tube No. 2 gave the most favorable results, the author did not include data as to its dimensions or the area of its various sections. The lack of these data prevents a comparison and study of this design.

The writer does not think that the author is justified in submitting general recommendations for the designs of elbow draft-tubes on the basis of the tests submitted in his paper. His recommendation that the vertical leg be trumpet-shaped is not borne out by his comparative tests (Bend No. 2 as compared to Bend No. 4). Not only is the efficiency of Bend No. 2, with the conical vertical leg, higher than Bend No. 4, with the trumpet-shaped vertical leg, but, in addition, the outflow loss from Bend No. 2 is decidedly greater than that from Bend No. 4, so that if correction for this fact were made, the efficiency of Bend No. 2 would be relatively increased to a value still greater than Bend No. 4.

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[25] "Loss in 90° Pipe Bends of Constant Circular Cross-Section," by Albert Hofmann, *Transactions*, Munich Hydr. Inst., *Bulletin* (pub. by A.S.M.E.), Fig. 1.

The Company with which the writer is connected abandoned long ago the Prasil principle of uniform deceleration, as it was demonstrated in practice that where the diverging angle of the tube to meet this requirement exceeded allowable values (7° to 8°) the water did not follow the tube walls, and harmful eddies were formed. Any impact plate at the end of the tube alters the conditions, resulting in flow in a radial direction. However, this condition does not apply to the vertical leg of a draft-tube.

Furthermore, the writer cannot agree that the trumpet-shape will reduce the cavitation tendency of the runner. It is true that the throat rings of some turbines of the Kaplan type tend to corrode. This, however, does not extend down into the tube, even with 7° to 8° flare, but appears to be a function of the movable guide-vanes above, as the corrosion is limited to parts in the line of flow from the guide-vanes.

Although a splitter is often beneficial, the author should draw no conclusions on that basis because Draft-Tube No. 2 showed better results with, than without, a splitter. This tube was apparently designed for a splitter, and the areas proportioned accordingly. Removing the splitter greatly increases the areas around the bend of the tube — at the very region where the rate of diffusion should be a minimum.

L. F. Harza,[26] M. Am. Soc. C. E. (by letter).—It seems to be a general principle that water will flow with the least loss of energy if permitted to conform as nearly as possible with its natural tendencies.

The logic of a wide, shallow design, such as Bend No. 4 (see Fig. 3) can be easily demonstrated by projecting a free stream of water tangentially against a curved sheet of metal. The stream will spread out into a thin sheet as deflected around the curve. The writer is surprised, however, at the small reduction in the loss experienced in Bend No. 4, as compared with the conventional elbow, Bend No. 1. It seems possible that considerably more improvement would result from a still shallower and wider section at the bend if not, indeed, a section that is actually flat on top and bottom. The flattening of Bend No. 4 would not seem to be enough to prevent the formation of the double spiral which could hardly exist in a thin wide section.

The saving of head in Bend No. 4 as compared with Bend No. 1 is only the equivalent of the loss occurring in about 8 in. of straight pipe, which would scarcely seem to justify the more expensive construction of the flattened bend in practical pipe manufacture.

In general, the writer is in agreement with the stated draft-tube principles, except perhaps that the vertical cone should be as long as possible, with short-radius elbow and, consequently, a sharp inside curvature. The writer would indeed agree also with this principle were it not for the better results apparently shown by later tests with the long-radius, sweeping elbow which was finally adopted for Bonneville Dam. It is to be regretted that a parallel test of this design, in a 6-in. inlet size, was not incorporated in the author's comparison. Apparently, it was not included because of the development of this design subsequent to the completion of the 6-in. model series.

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[26] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

ELLERY R. FOSDICK,[27] ESQ. (by letter).—The improvements which heretofore have been made in the science of hydraulics as related to power production, have been almost entirely in the field of dynamics and have usually been confined to such equipment as turbines, needle-valves, the velocity-recovering ability of draft-tubes, and intakes to flow lines. Practically no attempts have seemingly been made to improve the design and operating characteristics of such important structures as bends, elbows, and wye branches. The excellent work presented in this paper by Professor Mockmore, therefore, is a valuable addition to the information that is available on the subject of losses in conduit bends.

The losses that result from bends and wye branches in conduits are frequently quite large and seem to justify more consideration than is usually given them. They are of sufficient magnitude in some installations to warrant the expenditure of additional capital for their reduction.

Draft-tubes are normally subjected to very low heads or to a partial vacuum whereas conduit bends and wye branches operate under higher heads. It follows, therefore, that the type of design which is most suitable for a draft-tube may not be practicable for use in conduit bends and wye branches since for this type of structure it must possess mechanical stability under high heads and, at the same time, be justifiable economically.

The improvements that have been made to hydraulic structures, other than bends and wye branches, have reduced the losses occurring in them to such a low point that no material improvements can be expected in the future and, therefore, the only changes which it is likely will be made are refinements that may result in very minor increases in efficiency. On the other hand, the designs of bends and wye branches have not been changed, for the most part, since the inception of conduits, having a circular cross-section, and the losses in these structures are sufficiently large in some instances to justify the expenditure of additional sums of money, in order to improve the designs toward the end of increasing their efficiency and improving their operating characteristics.

General.—In view of the foregoing facts, the writer has conducted an investigation during the past few years similar to that made by Professor Mockmore, to determine the character of the losses that occur in bends and wye branches, whereas the different types of bends investigated by Professor Mockmore were principally suited for use under low heads as contrasted with the types of structure that were studied by the writer for use under high heads.

The passage of a fluid around a bend in a conduit results in a double spiral eddy in which the diversion loss occurs. This eddy is produced as a result of the distortion to the normal distribution of velocity and an abnormal pressure created by the centrifugal thrust of the fluid. These phenomena have been studied by several investigators,[28] and the results of their work have been previously made available to the Engineering Profession through the medium of various technical publications. Consequently, no further discussion of the subject will be given herein.

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[27] Elec. Engr., Spokane, Wash.

It has been known to hydraulic engineers for some time that the losses resulting from a bend may be decreased materially through the use of a conduit flattened in a plane at right angles to the plane or the bend. Such a structure, however, is suitable only for very low heads and could not be made physically stable for high heads without excessive cost.

In view of these facts, the selection of a design for reducing the losses in conduit bends would be more or less limited to a cross-sectional shape that was circular, or some part of a circle, so that it would tend to maintain its normal contour under high pressures. After much investigation of this problem before the beginning of these tests, it was finally decided to study a bend with a semi-circular cross-section with the shortened diameter lying in the plane of the bend. Such a conduit might be easily constructed for use under high pressures from a circular pipe by merely installing a diametrical partition wall with relatively small vent holes in it, which would transmit the fluid pressure to the side of the partition wall away from the moving column of water. In this manner the hydrostatic pressure of the fluid would be exerted upon a physically stable structure with a circular cross-section, while the dynamic flow would take place through a conduit having a semi-circular cross-section and the partition wall would only have to withstand the forces produced by the dynamic flow of the fluid passing around the bend.

The tests that are described in this discussion were conducted in the Hydraulics Laboratory at Washington State College, Pullman, Wash., using a standard 6-in. pipe for the supply and discharge lines from a 90° test bend. A schematic arrangement of the flow line which was used for these tests is shown in Fig. 20. It will be observed that the supply line was a straight section of 6-in. pipe practically 33 ft long, whereas the discharge pipe was a straight section of 6-in. pipe approximately 16 ft long. The 90° test bends were inserted between these two sections of pipe. The water supply was obtained from a centrifugal pump, and the flow was measured over a triangular weir shown in Fig. 21, which had previously been calibrated within very close limits. The flow was regulated by means of a valve on the discharge side of the pump and the pressure on the flow line and test bend was regulated by means of a valve in the discharge pipe.

Ten measuring sections were laid out along the 6-in. pipe, as shown in Fig. 20. Four piezometers were placed at each of these sections, spaced 90° apart, with two of them lying in the plane of the bend and two in the plane at right angles to the plane of the bend. Each of these piezometers consisted of a $\frac{1}{16}$-in. hole which had been drilled radially into the pipe and reamed with a specially built reamer so that the broken bits of metal were removed from the inside of the pipe at the edge of the hole and also so that the inside edge of the hole was slightly rounded. Figs. 22 and 23 show the supply line and a test bend in place in the flow line and the discharge line.

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[28] "Flow of Water Around Bends in Pipe", by the late David L. Yarnell and Floyd A. Nagler, Members, Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 100 (1935) p. 1018; "Modern Conceptions of the Mechanics of Fluid Turbulence", by Hunter Rouse, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 463; "Loss in 90-Degree Pipe Bends of Constant Circular Cross-Section", by Albert Hofmann, *Transactions*, Munich Hydr. Inst., *Bulletin 3*.

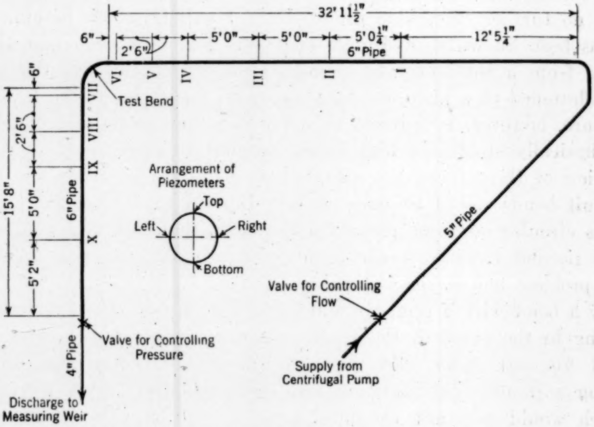
32' 11½"
6"
2' 6"
5' 0"
5' 0"
5' 0¼"
12' 5¼"
6" Pipe
VI
V
IV
III
II
I
VII
VIII
IX
X
Test Bend
6"
2' 6"
15' 8"
5' 0"
5' 2"
6" Pipe
Arrangement of Piezometers
Top
Left
Right
Bottom
5" Pipe
Valve for Controlling Flow
Valve for Controlling Pressure
4" Pipe
Supply from Centrifugal Pump
Discharge to Measuring Weir

FIG. 20.—DIVERSION LOSSES IN PIPE BENDS; ARRANGEMENT OF FLOW LINE USED FOR TESTS.

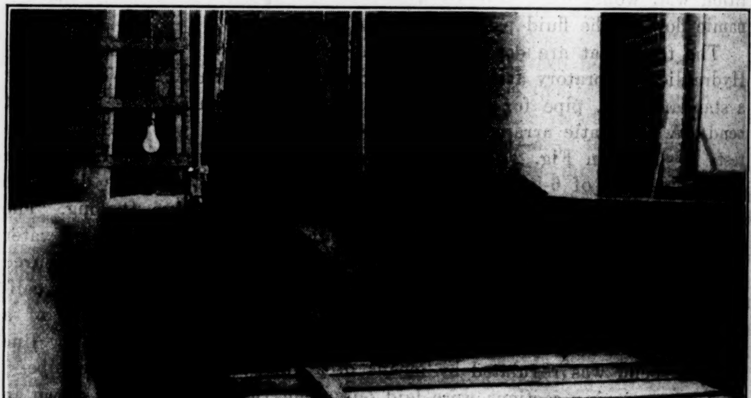

FIG. 21.—CONCRETE FLUME AND TRIANGULAR MEASURING WEIR.

Each of the piezometers was connected to a vertical, glass tube manometer. Throughout the tests of the first two bends, the oscillations of the water column in the manometers were damped by pinching the rubber tubes that connected the manometers to the piezometers. During the tests on the third bend a 6-in. length of capillary tube was placed in series between each manometer and the piezometer for the purpose of damping these oscillations. This latter arrangement was found to be somewhat more satisfactory than the one originally used.

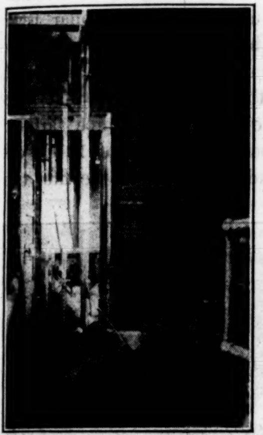

FIG. 22.—SUPPLY LINE AND TEST BEND.

FIG. 23.—DISCHARGE LINE AND CONTROL VALVE.

Three types of 90° bends were tested; the first had a semi-circular cross-section with the same area as a standard 6-in. pipe with the flat side of the semi-circular cross-section at right angles to the plane of the bend

FIG. 24.—FRONT VIEW OF IMPROVED SPECIAL ELBOW NO. 1.

and on the side of the pipe away from the center of curvature. This bend was connected to the pipe having a circular cross-section by means of transition sections. After some preliminary tests had been run on this particular bend another one was constructed, in which the transition and the bend were cast integral in one piece. This bend is shown in Fig. 24.

The next set of tests was run on a standard 6-in. flanged, short-radius elbow. The third set of tests was run when using a semi-circular bend with a cross-sectional shape similar to that in the first set, except that the flattened surface was located on the side of the bend nearest the center of curvature, as shown in Figs. 25 and 26. All the special bends that were tested had the same radius of curvature on the face of the pipe farthest from the center of curvature as the standard bend. A total of 10 564 observations was made of the losses and corresponding flows for the first two of these bends, and 4 496 observations were made of the third one. The results of these observations have been computed and analyzed graphically. A summary of the information which has been obtained follows.

FIG. 25.—END VIEW OF IMPROVED SPECIAL ELBOW NO. 2.

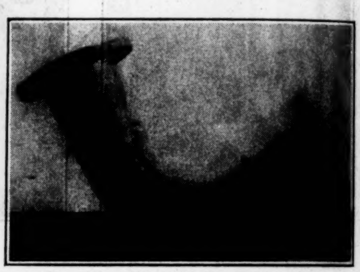

FIG. 26.—SIDE VIEW OF IMPROVED SPECIAL ELBOW NO. 2.

Method of Testing.—Observations of pressure heads were made in the manometers connected only to the right-hand piezometers at the various stations along the flow line, and the equation between the water level in the right-hand and left-hand piezometer at each station was carefully observed throughout the test runs, and recorded at the end of each group of readings. It was found that the equation between the right-hand and left-hand piezometers at any one station remained substantially the same for any given condition of flow, and for this reason it was unnecessary to take readings in both of them.

A group of twenty-five simultaneous readings of the pressure head in the flow line was made for each condition of flow at each station when testing the standard bend and Special Bend No. 1. These readings were made at 10-sec intervals and were found to cover at least one complete cycle of the variations in pressure that were found to be present in the flow line under stable conditions of flow. Consequently, the average of these twenty-five readings for any one flow completed a fairly accurate measure of the normal average pressure head that existed during the particular run.

In making the observations for Special Bend No. 2, the number of consecutive simultaneous readings that was made for any one condition of flow, was reduced to twenty, as this appeared to be adequate for the degree of accuracy desired. The method of reading and the interval of time between readings were the same as for the previous tests.

Due to the limited number of observers it was impossible to read all the piezometers simultaneously and, therefore, it was necessary to take three groups of readings for each test run at each of four adjacent stations. The stations that were read in each group of observations had to overlap in order to determine a continuous hydraulic gradient for the entire flow line; that is, the last, or fourth, station in the first and second groups of readings were the first stations read in the second and third groups of readings.

Observations of the flow were made by taking readings of the head on the weir at 30-sec intervals throughout each test run. The head on the weir was measured with a standard hook-gage and the observations were read to 0.0001 ft. The standard bend was tested under fifteen different conditions of flow with a range of velocity from 2.5 to 7.2 ft per sec. Improved Bend No. 1 was tested with a total of twenty-one different flows, having a range of velocity from 2.4 to 7.1 ft per sec and Improved Bend No. 2 was tested for nine different flows having a range of velocity from 1.7 to 7.5 ft per sec.

Computation of Results.—The diversion loss resulting from the fluid passing around the bend was found by subtracting the total friction loss in the flow line from the total loss in the flow line.

The temperature of the water passing through this flow line during the tests was measured at frequent intervals, and, from this, it was possible to ascertain the kinematic viscosity. With this factor and the mean velocity of the fluid and the diameter of the pipe known, it was possible to compute Reynolds number for the various conditions of flow. This affords a dimensionless parameter for designating the results of these tests so they may be transposed to other hydraulic structures having the same geometric similitude.

It will be observed from Fig. 27, which contains a summary of the information obtained in these tests, that the diversion loss factors for the standard and special bends approach a constant value at about $\mathbf{R} = 250\,000$. This is in accordance with the observations of other investigators and indicates that the loss factors for a Reynolds number of approximately 250 000 may be used with a fair degree of accuracy in computing the losses with a flow having a Reynolds number many times greater.

No observations were made for a Reynolds number much less than 50 000 as this represents a very low mean velocity of flow for a pipe of this size and is a condition that is not ordinarily of importance in the operation of hydraulic structures, therefore, although it may be of theoretical interest, it is not of any practicable importance. It is probable, however, that if these observations had been made, the loss factor curves would have followed approximately the course indicated by the dashed lines on Fig. 27.

Conclusion.—The results of the tests summarized in Fig. 27 indicate clearly that a considerable reduction can be made in the diversion losses that result from the passage of a fluid around a conduit bend by the use of conduits having a semi-circular shape when they are arranged with the flat side at right angles to the plane of the bend. This is in agreement with the conclusions reached by Professor Mockmore and others that the losses resulting from the passage of a fluid around a conduit bend can be reduced by flattening the structure in the plane of the bend. The semicircular shape of the conduit bends that have been used in these tests create a redistribution of the fluid velocity that minimizes the losses by reducing the unbalance of pressure resulting from the centrifugal thrust of the fluid column and by decreasing the resistance to flow around the bend on the side nearest the center of curvature. The types of bends tested by the writer were designed to withstand high heads in addition to reducing the losses whereas the bends that were tested by Professor Mockmore were designed for use under low heads. As a result there is not sufficient similarity between the different bends that were tested to permit a comparison of the results.

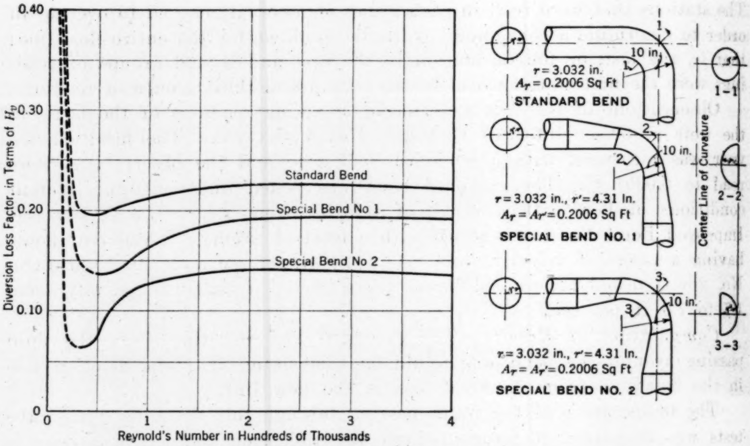

FIG. 27.—COMPARISON OF DIVERSION LOSS FACTORS FOR VARIOUS TYPES OF CAST-IRON FLANGED 90-DEGREE BENDS

The loss factors for Special Bend No. 1 with the flat side radially outermost were found to be materially less than the corresponding loss factors for a standard bend of circular cross-section having the same cross-sectional area and radius of curvature. The loss factors for Special Bend No. 2, which had the flat side radially innermost, were found to be much less than the corresponding loss factors for Special Bend No. 1 and very much smaller than the losses for the standard bend.

The construction of a conduit bend having a semi-circular shaped passage for the moving fluid is feasible for operation under high heads with the shell and partition design. In this type of structure a circular pipe having approximately twice the required cross-sectional area, and designed to withstand the fluid pressure, has a relatively thin longitudinal partition wall that is secured in the pipe in a diametrical position in the plane at right angles to the plane of the bend. Relatively small holes in the partition wall transmit the fluid pressure to both halves of the conduit with the result that the only force acting on the partition wall is the relatively small pressure produced by the dynamic action of the fluid.

Acknowledgment.—The facilities of the Hydraulics Laboratory at Washington State College were made available for these tests by H. V. Carpenter, Dean of the Engineering School, and M. K. Snyder, M. Am. Soc. C. E., Head of the Civil Engineering Department. J. G. Woodburn, Assoc. M. Am. Soc. C. E., contributed much of his time and energy in assisting with the work that was done in the Laboratory, and it was largely through his efforts and co-operation that these tests were successfully completed.

HOWARD L. COOPER,[29] ESQ. (by letter).—The type of experimentation developed by Professor Mockmore will have a marked impression on the future of hydraulic machines and structures. This fact has certainly been demonstrated in the turbine and draft-tube laboratory of the U. S. Army Engineers, at Portland, Ore., in connection with the Bonneville Project. The writer wishes to discuss further some of the statements made by Professor Mockmore and to give a little more information on the draft-tube report, especially pertaining to the tubes that were tested after the work was finished at the Oregon State Laboratory and moved to the U. S. Engineer Laboratory, at Linnton, Ore.

Figs. 28 and 29, are submitted as evidence of the study that was started in an effort to find a relation between diameter of jet, radius of curvature, and angle of flare of a jet of water passing around a curve on a flat plate. Such a jet represents the natural flow as it probably offers less resistance than any other method of changing the direction of flow through 90 degrees. It is very easy to visualize the double spiral if one pictures this fan-shaped jet folded up from each side upon itself until it is confined in a 90° elbow. The major losses are then the energy required to form the double spiral and also the internal friction caused by the different strata of the whirl meeting with different relative velocities with respect to the initial flow.

The writer agrees with Professor Mockmore that Elbow No. 4 should be the most efficient, as the flattened shape conforms more nearly to the fan-shaped jet shown in Fig. 28 and Fig. 29. The double spiral is dissipated to a certain degree while making the turn. Fig. 30 shows a jet of water passing around a standard elbow with the jet smaller than the elbow inlet; it can be clearly seen how one-half the double spiral begins to develop.

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[29] Office of Div. Engr., North Pacific Div., U. S. War Dept., Portland, Ore.

Referring to the elbow studies, the writer is convinced that a slow-motion study of elbows, possibly using additional shapes, would show many interesting phenomena and possibly develop some ideas that would be a real contribution to the study of hydraulics.

FIG. 28.

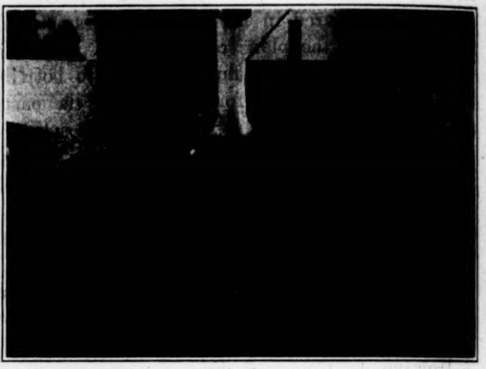

FIG. 29.

Referring to the functions of a draft-tube, the writer wishes to take the liberty of discussing the advantages in a little more detail. The velocity of the water is reduced after leaving the turbine runner, but in doing so it maintains a seal at all times and lowers the absolute pressure directly under the runner and, therefore, increases the total effective head on the unit. A poor draft-tube that does not maintain this seal loses this gain in velocity head and, therefore, reduces the output of the turbine, and also frequently causes serious vibration, which naturally reduces the life and mechanical efficiency of the unit.

FIG. 30.

The location of the center line of the runner above the tail-race is an important feature. Francis wheels can be placed much higher than those of the propeller type. A very careful study must be made of propeller type settings with respect to their elevation above tail-water, as the location depends on the proper combination of the plant, σ, specific speed, ϕ, and the general design of scroll and draft-tube, in order to prevent cavitation at the higher loads and low tail-water conditions. ($\sigma =$ the cavitation coefficient $=$ the ratio of the maximum possible pressure reduction within the runner without vaporization of the water, to the total effective head on the turbine. Plant σ is distinguished from Field σ by a suitable factor of safety. The ratio of the peripheral speed of the runner to $\sqrt{2\,g\,H} = \phi$.) Cavitation not only affects parts of the runner blades and throat liners, but sets up detrimental vibrations in the unit and the structure.

A draft-tube also should have enough velocity at the exit, together with the hydraulics of the tail-race, to keep the velocity high enough to clear the tail-race without the building up of undue head.

In large draft-tubes the writer makes the following recommendations based upon experimental observation. In addition to the horizontal splitter, at least two vertical splitters are suggested; also, the placing of six or more vertical, stream-lined struts in the exit half of the horizontal leg, the spacing to be arranged by experiment. These struts will cause the water to seal the exit with a uniform discharge and, therefore, increase the re-gain within the draft-tube.

TABLE 5.—HEAD, IN FEET, REQUIRED TO FORCE A FLOW OF 0.997 SECOND-FOOT THROUGH DRAFT-TUBE NO. 2

Description	ANGLE OF WHIRLING COMPONENT AT ENTRANCE	
	30°	40°
Without struts	0.4062	1.709
With struts, Arrangement No. 1	0.3971	1.695
With struts, Arrangement No. 2	0.3821	1.683

The information contained in Table 5 is taken from experimental data, Draft-Tube No. 2 being operated with, and without, the vertical struts in the horizontal leg near the exit (see Fig. 31).

Draft-tube passages that do not allow the water to flow smoothly reduce the re-gain in velocity head, and this method of experimentation, together with the use of slow motion pictures, possibly will enable most of the internal cross-currents to be checked and corrected, thereby increasing the re-gain in velocity head. The writer suggests that a material of practically neutral buoyancy be used in the water instead of the injection of air, because the air bubbles will flow toward the upper surfaces and will not give a true pattern of the flow. Besides, the air bubbles change the specific gravity of the liquid used, which change can not be conveniently compensated, and no doubt will cause Pitot tube velocity readings to be incorrect.

After weeks of searching, the writer found a hardwood granular sawdust that would give a true pattern after being soaked in water for about 24 hr; it would neither float nor sink, and being practically the same weight as water, did not affect the specific gravity of the liquid. This material photographs very well in slow motion pictures. The writer also agrees that any attempt to simulate the flow from a water-turbine runner by mechanical means is useless. An homologous model runner should be used in all scroll-case and draft-tube studies. There is no doubt in the writer's mind about the statement that every draft-tube should be designed especially for its own setting and turbine.

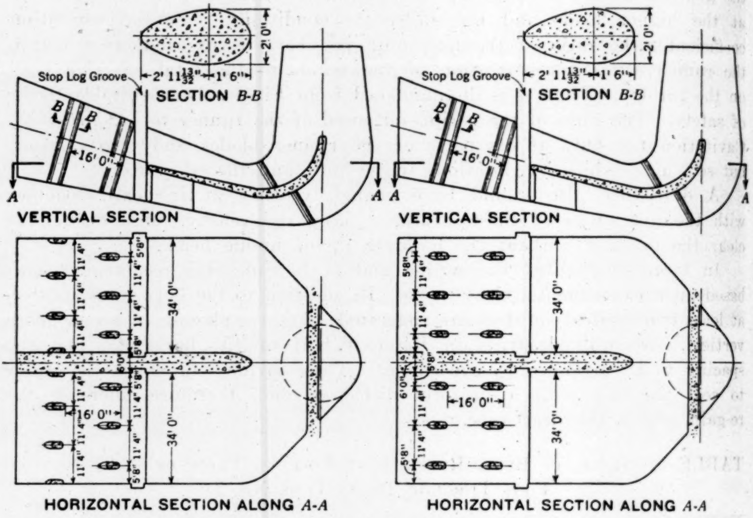
Stop Log Groove
2' 11 13/32"
1' 6"
2' 0"
SECTION B-B
B
B
16' 0"
A
A
VERTICAL SECTION
11' 4"
5' 8"
6' 0"
16' 0"
34' 0"
34' 0"
HORIZONTAL SECTION ALONG A-A
Stop Log Groove
2' 11 13/32"
1' 6"
2' 0"
SECTION B-B
B
B
16' 0"
A
A
VERTICAL SECTION
5' 8"
11' 4"
6' 0"
16' 0"
34' 0"
34' 0"
HORIZONTAL SECTION ALONG A-A

FIG. 31.—ARRANGEMENT OF ELECTRODES IN DRAFT-TUBE TESTS.

The writer would carry the increasing diminishing degree of the area in the elbow a little further and reduce the sections around the bend, giving the flow a slight acceleration. All model tests showed that when the velocity was slightly accelerated, very smooth flow resulted. This can also be observed by inspecting Bend No. 5, in Fig. 9(*b*) of the paper. After the flow begins to decelerate, it moves in various directions, but while it is accelerating, it is noticeably smooth.

A method of measuring the efficiency of elbows and draft-tubes when the water has a whirling component is very important and such measurement could be accomplished in at least three ways: (1) One with a small tube with two holes in the sides located diametrically opposite and then placed into the flow with the holes 90° from the line of flow; (2) another method would be to use the outside piezometers and correct for the centrifugal force of the whirling body of water; and (3) the third suggestion would be to use the method developed by the Reclamation Bureau and used in the War Department laboratory at Portland; that is, find the entrance loss in the entrance tube without the draft-tube in place for the various whirling components, and then operate the entrance tube in conjunction with the elbow tube. Combining results:

$$\text{Draft-tube efficiency} = \frac{\left(\frac{C_2}{C_1}\right)^2 - 1}{(C_2)^2} \dots\dots\dots\dots\dots (28)$$

in which C_1 = the coefficient of discharge of the entrance tube; and C_2 = the coefficient of discharge of the entrance tube, together with the elbow tube. In all experiments the flow should be corrected to one common quantity by using the coefficient of discharge method; that is, find the coefficient of discharge and substitute back in the orifice formula, using a constant rate of discharge, Q, for the final data. This may not indicate large variations because the efficiency varies little with a change in the quantity; however, it does make the data comparable.

There is no doubt that the most satisfactory method of checking the operating advantages of draft-tubes would be to run them on a model turbine that is equipped with an accurate dynamometer. This will give the relative operating efficiency of the various combinations, which is what is required in the final analysis. The following comments are offered on the draft-tube models designed by the U. S. Engineer Department.

Draft-Tube No. 1.—The principle of the fan-shaped jet, as shown in Figs. 28 and 29, was applied to this design within the practical limits of the economical center spacing of units in the power house. The elbow flow was very smooth; however, the high velocity at the extreme sides caused a reversal of flow in the center and near the top of the horizontal leg, and in many places there were no-velocity areas.

Draft-Tube No. 2.—This tube was patterned in general after the Ryberg-Schwerstadt tube, the unit being as near to the Bonneville size and power as any other Kaplan turbine then in operation. As the first tests show, in Table 6, this tube gave fairly good results. The tube was then taken to the U. S. Engineer Laboratory, at Linnton, Ore., and, as a result of slow motion pictures and general observation, several alterations were made: (*a*) The deceleration around the bend was changed to a slight acceleration; (*b*) the top of the vertical leg near the bend was lowered to fill a no-velocity area; and (*c*) the angle of the elbow sides was reduced with respect to the center line. All these changes indicated a marked improvement in the flow conditions and, based on the same method of testing as that done under the direction of Professor Mockmore, showed an average efficiency of 70.0% an increase of about 5 per cent.

The fact that this increase was due to the visual study would indicate that many additional improvements would be possible with added research.

This tube would give very good operating results on a turbine having a small whirling component in either the forward or backward direction. The elbow is too short and the splitter too high in the vertical leg for a wheel having a large whirling component which would set up violent disturbances and reduce the efficiency. All tests definitely showed that wheels giving a large whirling component should have the splitter located above one-third of the way around the board as recommended by Professor Mockmore.

TABLE 6.—LABORATORY TEST DATA AFTER ALTERATIONS (*a*), (*b*), AND (*c*) WERE MADE, DRAFT-TUBE NO. 2; $A = 0.1967$ SQUARE FOOT

Run No.	QUANTITY		TANK		Throat gage	$V = \frac{Q}{A}$	$\frac{V^2}{2g}$	Velocity head reclaimed, Column (5) − Column (6)	Percentage efficiency, Column (9) ÷ Column (8)
	Gage	In second-feet	*A*	*B*					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	0.691	..	3.300	3.084	2.809	..	..	0.275	..
2	0.693	..	2.851	2.630	2.353	..	..	0.277	..
3	0.694	..	2.457	2.239	1.958	..	..	0.281	..
Average	0.693	0.997	..	..	..	5.07	0.399	0.278	69.7
4	0.671	..	3.276	3.086	2.848	..	..	0.238	..
5	0.671	..	2.860	2.663	2.424	..	..	0.239	..
6	0.672	..	2.460	2.266	2.026	..	..	0.240	..
Average	0.671	0.920	..	..	..	4.67	0.339	0.239	70.5
7	0.629	..	3.289	3.148	2.978	..	..	0.170	..
8	0.629	..	2.854	2.711	2.540	..	..	0.171	..
9	0.630	..	2.423	2.279	2.103	..	..	0.176	..
Average	0.629	0.783	..	..	..	3.99	0.247	0.172	69.6
10*	0.602	..	3.297	3.180	3.043	..	..	0.137	..
11*	0.602	..	2.831	2.712	2.578	..	..	0.134	..
12*	0.601	..	2.466	2.348	2.208	..	..	0.140	..
Average	0.602	0.701	..	..	..	3.56	0.197	0.137	69.5
13*	0.562	..	3.338	3.250	3.151	..	..	0.099	..
14*	0.564	..	2.843	2.751	2.651	..	..	0.100	..
15*	0.564	..	2.427	2.338	2.237	..	..	0.101	..
Average	0.563	0.593	..	..	..	3.01	0.141	0.100	70.8
Average	..	..	..	..	..	..	..	..	70.0

* Data, October 17, 1934.

Draft-Tube No. 3.—This tube incorporated the flat jet idea that was used in Tube No. 1, together with the spreading-tube principle as used by some turbine manufacturers. The results were practically identical to those found in Tube No. 1, and the flow conditions within the tube were quite similar.

Draft-Tube No. 4.—This tube was designed and tested by Professor Mockmore. The writer did not see it operate, and, therefore, will make no comment on general operation.

Draft-Tube No. 5.—This draft-tube (not shown in Professor Mockmore's paper) was designed so that the entire whirling component was transferred into velocity head without the use of splitters; only one vertical splitter was used and that was for structural reasons. The one model that was made showed very good efficiency. No alterations were made, due to the lack of time, but the writer is confident that, with several alterations and re-proportioning, the tube could be made into an excellent draft-tube. The cost of construction should not exceed the cost of splitter tubes.

An interesting phenomenon occurs when under operation; that is, no matter whether the entrance whirling component is 1° or 50°, the flow is evenly divided between the two exit channels with no eddies or high velocity areas. The tail-race is exceedingly smooth under all load conditions. The slow motion pictures showed that this double spiral was completely dissipated before passing around the bend.

TABLE 7.—LABORATORY COMPARISON OF STRAIGHT-LINE AND PARABOLIC DECELERATION IN VERTICAL LEG OF DRAFT-TUBE No. 2

Vanes	$H_a - H_b$	$\sqrt{H_a - H_b}$	$H_a - H_b$ corrected for Q difference	h_2	$\sqrt{h_2}$	Q	C_2*
			(*a*) WITH TRUMPET-SHAPED VERTICAL LEG				
Straight	0.155	0.394	0.159	0.151	0.389	0.771	1.270
	0.250	0.500	0.260	0.247	0.497	0.980	1.275
7½°	0.147	0.3835	0.151	0.143	0.378	0.771	1.307
	0.239	0.489	0.247	0.234	0.484	0.983	1.310
15°	0.181	0.426	0.186	0.178	0.422	0.771	1.170
	0.289	0.538	0.297	0.284	0.553	0.986	1.190
30°	0.366	0.605	0.375	0.376	0.606	0.771	0.815
	0.590	0.768	0.610	0.597	0.772	0.983	0.821
45°	1.488	1.220	1.500	1.492	1.221	0.777	0.405
	2.371	1.540	2.440	2.427	1.560	0.986	0.407
			(*b*) WITH STRAIGHT CONICAL VERTICAL LEG				
Straight	0.144	0.379	0.150	0.142	0.377	0.764	1.312
	0.234	0.484	0.241	0.228	0.474	0.986	1.337
7½°	0.139	0.373	0.145	0.137	0.370	0.764	1.336
	0.223	0.472	0.231	0.218	0.467	0.983	1.356
15°	0.173	0.416	0.177	0.169	0.411	0.771	1.203
	0.281	0.530	0.291	0.278	0.527	0.983	1.202
30°	0.348	0.590	0.356	0.348	0.590	0.771	0.838
	0.564	0.751	0.584	0.571	0.756	0.983	0.839
45°	1.386	1.178	1.420	1.412	1.189	0.771	0.416
	2.339	1.528	2.370	2.357	1.535	0.992	0.413

* C_2 = coefficient of discharge.

The writer cannot see why Tube No. 4 which contains many of the author's recommendations for designs of draft-tubes, should show a lower efficiency than the Tube No. 2 which is similar in general shape but does not contain any recommended designs. The trumpet-shaped vertical leg or straight-line deceleration showed less efficiency when installed on Tube No. 2 than the straight cone. The data given in Table 7 are offered as evidence.

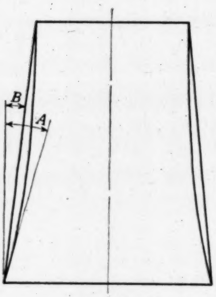

FIG. 32.

The writer agrees that a sudden deceleration near the runner may cause cavitation, but he does not believe that the difference between the straight cone and the trumpet-shaped vertical leg would be large enough to cause any trouble. It would be possible, however, to set up eddy and shock losses at the lower end of the trumpet-shaped vertical leg if the deceleration was relatively large because the angle of flare (Angle *A*, Fig. 32) would exceed the angle that water will follow without leaving the surface, where a smaller angle (Angle *B*, Fig. 32), might be satisfactory. The trumpet-shaped vertical leg has one decided advantage (if Angle *A* is not too large) and that is that this change in shape will naturally conform to the widening required within the bend.

Table 8 is submitted as evidence that for a flow that is either straight or with a small whirling component, the location of the splitter as shown on Tube No. 2 is the most efficient. There is a pos-

TABLE 8.—LABORATORY TEST DATA OF SPLITTER ALTERATIONS, DRAFT-TUBE NO. 2

Flow	Basic Q, in cubic feet per second	Measured Q, in cubic feet per second	$H_a - H_b$	$\sqrt{H_a - H_b}$	$H_a - H_b$ corrected for Q difference	h_2 $H_a - H_b$ corrected for Q difference and exit $\frac{V_2}{2g}$	$\sqrt{h_2}$	C_2*
(*a*) SPLITTER IN ORIGINAL POSITION								
Straight	1.000 0.780	0.986 0.764	0.234 0.144	0.484 0.379	0.241 0.150	0.228 0.142	0.474 0.377	1.337 1.312
(*b*) 2-INCH CUT-OFF FROM TOP OF HORIZONTAL SPLITTER								
Straight	1.000 0.780	0.961 0.743	0.226 0.141	0.475 0.375	0.494 0.394	0.244 0.155	0.481 0.383	1.318 1.291
(*c*) 4-INCH CUT-OFF FROM TOP OF HORIZONTAL SPLITTER								
Straight	1.000 0.780	0.958 0.795	0.237 0.174	0.487 0.417	0.508 0.409	0.258 0.167	0.495 0.399	1.281 1.239
(*d*) HORIZONTAL SPLITTER REMOVED								
Straight	1.000 0.780	0.969 0.783	0.295 0.203	0.543 0.451	0.560 0.449	0.314 0.202	0.549 0.440	1.155 1.124
(*e*) WITH FIN ON TOP OF SPLITTER								
Straight	1.000 0.780	0.969 0.761	0.220 0.139	0.469 0.373	0.234 0.146	0.221 0.138	0.470 0.371	1.349 1.334

* C_2 = Coefficient of discharge.

sibility that with the splitter removed the change in rate of deceleration caused the drop in efficiency. The fin on top of the splitter would no doubt be impractical, according to the author, due to the large variation in whirling component; but no doubt it could be used to good advantage in pipe elbows with a fixed angle of whirl.

The writer agrees that the whirling component changes from forward to backward in the Francis type of turbines depending upon load conditions, but for Kaplan propeller types within the normal range of specific speed this component is always forward in the direction of the propeller rotation. There is belief in the writer's mind that one could capitalize on this phenomenon and the energy within this rotating body of water regained in the form of useful work (Draft-Tube No. 5, for example).

The writer can not subscribe to the belief expressed by Professor Mockmore regarding a central rotating core in the vertical leg moving toward the entrance of the tube. This may have been possible when the core was composed mostly of air, but in all tests at the Portland laboratory where a neutral buoyancy material was used to give the flow pattern, no reversal of flow was observed under any operating condition.

Considerable advance in the art of constructing pyralin models has been made in the past few years. One arrangement that gave very accurate results without wrinkling or discoloring the pyralin consisted of an electric oven, thermostatically controlled, into which the moulds for forming the models were placed. The moulds with the pyralin sheets in place were forced together by a hydraulic ram operated from the outside, one side of the oven containing a mica window for observation. After the moulds were pressed together completing the forming operation, the oven was allowed to cool before they were removed. Motion picture cameras taking 64 frames per sec are fast enough to photograph the flow in models of this size. From experience, it is recommended that they be taken at night under artificial light.

The longer draft-tube should be the most efficient for two important reasons: (1) It effects a greater re-gain in velocity head; and (2) it allows whirling and elbow disturbances a longer passage in which to straighten out and make a better seal at the exit of the tube.

The writer would like to make the following suggestion for design of elbow draft-tubes: Carry the circular and elliptical sections down as far as possible in the vertical leg; in order to make the bend, select the inside and outside radii by experience and model testing; reduce the section areas slightly, allowing the top and bottom to conform to the fixed radii and the sides to the correct areas, the flow lines, of course, to be fair at all times. The flow can be decelerated finally in the horizontal leg. In large units install at least two vertical splitters in the horizontal leg and extend them about one-fourth of the way around the bend, one horizontal splitter which does not extend more than one-third of the way around the elbow, and six or eight vertical stream-lined struts in the exit half of the horizontal leg.

The following list gives an idea of many features that could possibly be improved by visual studies of draft-tubes for various specific speeds and types of wheels over a wide range of operating speeds: The location of the splitter; the shape of the splitter; the number of vertical splitters; the proper relation of deceleration and acceleration; shock and eddy losses; no-velocity areas; high velocity areas; reversals of flow; cavitation; vibration; straight-line or parabolic deceleration; minimum exit velocity; use of stream-lined vertical struts; ratio of total depth to diameter of wheel; and inside and outside radii of elbow.

C. A. Mockmore,[30] M. Am. Soc. C. E. (by letter).—There is ample reason to agree with Professor Mavis that "the investigation of flow in draft-tubes has little more than made its 'debut' in the hydraulic laboratory." The findings made in this study and the suggestions made for the design of draft-tubes are not regarded as final. In this day of research, even the validity of the Newton law of gravitation is subjected to careful scrutiny.

Mr. Fee refers to Fig. 1 and the discussion following Equation (5), and states that he "would expect a high, not a low, pressure at Point *d* in conformity with the usual rule that high pressures go with low velocities." This belief is apparently based on the Bernoulli theorem, and would be correct for a single particle of water, but in Fig. 1 two different particles are under consideration.

Consider a case of two particles of water such as at Points *b* and *c* of Fig. 1 if they are moved up stream a reasonably short distance, say, 1 ft. Would the two particles be subjected to pressures of equal magnitude? Certainly the particle near the center of the pipe would have a greater velocity head than the one near the walls of the pipe. If the two particles possessed the same total energy the pressure near the walls would be greater than at the center of the pipe. This seems questionable. If there is a difference it could not well be detected by the pressure orifice of the Pitot tube used in these experiments. If the pressures are practically uniform across the pipe section and the filamental velocities are highest near the center of the pipe then the highest total energy content would be near the center of the pipe, as indicated for Section 1 of Bend No. 1, Fig. 7.

Now consider particles at Points *b* and *d* of Fig. 1 again, just before they start around the pipe bend. Suppose each particle possesses approximately the same pressure head, but different velocity heads. Then an instant later each particle starts its course around the bend. Particle *b* will have the greater centrifugal force and thus will tend to crowd Particle *d* toward the center of curvature. Two fundamental conditions were necessary to induce the spiral, namely, the presence of a curve in the channel to induce centrifugal force, and a difference between filamental velocities near the center and walls of the conduit.

Mr. Sharp thinks that the writer is not justified in submitting general recommendations for the design of elbow draft-tubes on the basis of the tests submitted in the paper. Of course, the recommendations are not drawn from the tests described only, but from other sources, such as a review of the literature, inspection of turbines in operation and repair, and from visits to various turbine laboratories where scores of model draft-tubes of almost every conceivable shape have been tested. For example, at one commercial laboratory visited in 1932, a test was described in which a movable false bottom was installed in an elbow tube such as one might imagine for Tube No. 2, Fig. 12, between Sections 5 and 10, so that the vertical dimension along Section 7 (termed the Δ-distance in this discussion) might be reduced. In the test referred to, the tube efficiency was increased by reducing the Δ-distance.

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[30] Prof. and Head, Dept. of Civ. Eng., Oregon State Coll., Corvallis, Ore.

The data on the areas at various sections of Draft-Tube No. 2 are given in Table 2, and are compared to a 6° conical tube.

Mr. Sharp states that tests on model draft-tubes should be used as a guide for tests with model turbines " but should not be used as the bases for the construction of large tubes in the field." It would be difficult to convince many engineers interested in model research that such work had no value in relation to the design and construction of large water-turbine developments. Very little experimental data can be had on such large developments as that on the Columbia River. Changes are difficult and expensive to make, and measurement of flow of water through the turbines is especially troublesome and questionable in accuracy.

The trumpet-shaped vertical leg of the draft-tube need not extend far from the exit of the turbine runner, so that the allowable values of flare are not exceeded, somewhat as an easement spiral is placed on a railroad curve. Suppose the efficiency of Draft-Tube No. 2 were higher than that of Tube No. 4, as Mr. Sharp points out, one wonders why he did not show experimental evidence or other proof, that it could have been due to the Prasil vertical leg. Experiments of Hofmann (8),[3] Gibson and Labrow (30),[3] and Andres (31),[3] indicate that the trumpet-shaped tube has as high, if not higher, efficiency as a straight cone. As indicated in Figs. 18 and 19, there was greater dead-water space in Tube No. 4 than in Tube No. 2. Surely this should not be ascribed to the Prasil vertical leg. Would it not more logically be due to too great a Δ-distance? In this same connection Mr. Harza suggests that considerably more improvement would result from a still shallower and wider section of a pipe bend, which would call for additional lessening of the Δ-distance of an elbow tube.

The loss in flow through the experimental pipe bends appears to have interested, not only Mr. Sharp and Mr. Harza, but brought forth the description of an interesting experiment by Mr. Fosdick. His special Bend No. 2 gave the most favorable loss factor, and, in general, seems to verify the writer's suggestion, (7(*c*)), for the design of the bend of an elbow draft-tube.

It is regretted that Mr. Cooper did not submit the design of the Draft-Tube No. 5, which showed such fine characteristics in operation. He states that it " was designed so that the entire whirling component was transferred into velocity head." The test would really be met when this whirling component is transferred into effective head on the turbine.

Mr. Cooper submits Table 8, as evidence that the splitter should extend completely around the bend as in Draft-Tube No. 2, ostensibly to refute Conclusion (10), and then proceeds to give his own suggestions for design to include "one horizontal splitter which does not extend more than one-third of the way around the elbow." No reason is given for the apparent inconsistency.

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Paper No. 1991

A NEW THEORY OF RAIL EXPANSION

By Alfred Africano,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Chester F. Gailor, E. F. Kenney, C. W. Baldridge, George W. Hunt, Frank B. Walker, Arthur N. Talbot, Randon Ferguson, A. N. Reece, G. M. Magee, H. D. Hussey, and Alfred Africano.

Synopsis

The change in length of a rail resulting from a change in temperature is ordinarily taken as the product of the coefficient of linear expansion times the length times the change in temperature. The action of the ties in resisting longitudinal movement of the rail is neglected.

For a 33-ft rail, with ordinary track construction, the resulting error is less than 2%, and, therefore, may be justly neglected; for a 66-ft rail, however, it may be from 10% to 40%, depending upon how securely the rail is fastened to the ties; and for the long continuous welded rails, the error is so great that it is evident that tie resistance must be taken into account.

In the following analysis of the problem, formulas are developed which include the action of the ties, and, in addition, the effect of joint restraint. The resultant theoretical rail expansions are in good agreement with the general observations made on the German railroads, and the careful measurements made on four half-mile length rails in the Delaware and Hudson track, at Mechanicville, N. Y.

Notation.—The letter symbols in this paper are introduced in the text as they occur and are summarized for reference in Appendix I. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials"[2] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

The coefficient of linear expansion, n, for hard steel is equal to 0.0000073 in. per degree Fahrenheit per in. of length[3]. Therefore, the free expansion

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Note.—Published in February, 1937, *Proceedings*.

[1] Asst. Engr., Interborough Rapid Transit Co., New York, N. Y. Elected Assoc. M. Am. Soc. C. E., on October 11, 1937.

[2] A.S.A.—Z10*a*—1932.

[3] Bethlehem Manual of Steel Construction, Catalogue S-47, 1934, p. 299.

of a rail, expressed as a formula, is,

$$\Delta l_f = n \, \Delta t \, l \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which l = half the length of a rail; and Δl_f = change in length, due to a temperature change, Δt, when the rail is free to expand. If this expansion is prevented by fixing the ends of the rail, the resulting uniform stress, s, in the rail is, in pounds per square inch,

$$s = E\delta = E\frac{\Delta l_f}{l} = E \, n \, \Delta t \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which δ = unit strain, in inches per inch; and E = modulus of elasticity. For $E = 30\,000\,000$, $s = 219 \, \Delta t$, or the stress is 219 lb per sq. in. for every degree change in temperature. It should be noted that this stress is independent of the length of the rail.

If $\Delta t = 100°$ F is the rise in temperature over that at which the rail was laid, the stress would be 21 900 lb per sq in.; and if the rail section were a 100-lb A R A-B[4], with a cross-sectional area, $A = 9.85$ sq in., the force, F, required at each end to prevent expansion would be $F = sA = 21\,900 \times 9.85 = 216\,000$ lb. To develop this force in one-half the length, l, of a 33-ft rail, with the tie spacing, c, equal to 22 in., would require the average tie resistance to longitudinal movement, T, to be: $T = \dfrac{216\,000 \times 2 \times 22}{33 \times 12} = \dfrac{216\,000 \text{ lb}}{9 \text{ ties}}$ = 24 000 lb per tie.

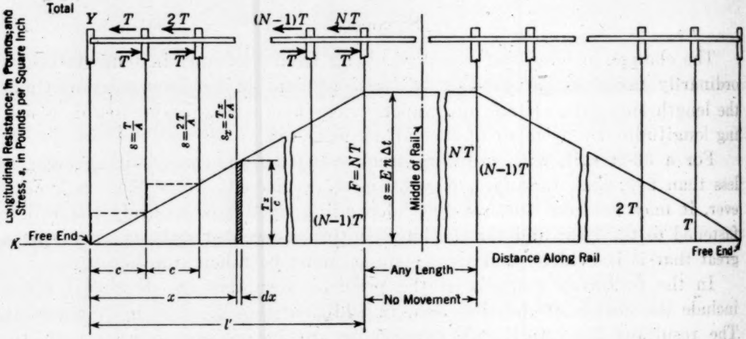

Total
Longitudinal Resistance; in Pounds; and Stress, s, in Pounds per Square Inch
Y
T
2 T
(N−1)T
NT
T
T
T
T
$s = \frac{T}{A}$
$s = \frac{2T}{A}$
$s_x = \frac{Tx}{cA}$
$\frac{Tx}{c}$
F = NT
$s = En\Delta t$
Middle of Rail
NT
(N−1)T
(N−1)T
2 T
T
K
Free End
Free End
Distance Along Rail
c
c
x
dx
l′
Any Length
No Movement

FIG. 1.—TEMPERATURE STRESSES IN A RAIL

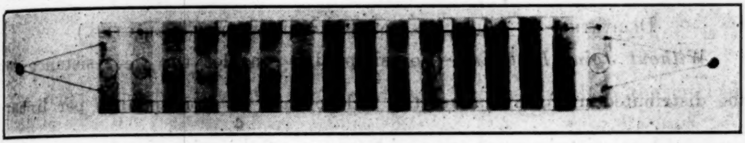
TABLE 1.—AVERAGE TIE RESISTANCE REQUIRED TO DEVELOP 216 000 POUNDS AT THE MIDDLE OF A RAIL

No. of 33-ft rails	Length, L, in feet	Resistance, T, in pounds per tie	No. of 33-ft rails	Length, L, in feet	Resistance, T, in pounds per tie
1	33	24 000	15	495	1 600
2	66	12 000	30	990	800
3	99	8 000	45	1 485	530
5	165	4 800	60	1 980	400

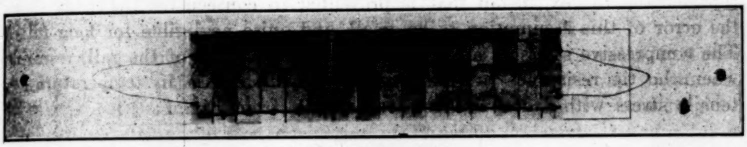
---

[4] Bethlehem Manual of Steel Construction, Catalogue S-47, 1934, p. 78.

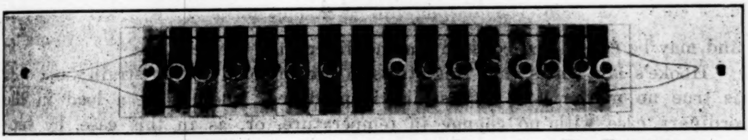
Since it is improbable that one rail-fastening and the tie will develop a resistance of 24 000 lb, unless set in concrete, a 33-ft rail expands quite freely; but when several rails are made continuous the average tie resistance drops to an amount that can be developed reasonably, and the fixed-end condition is approached. Table 1 illustrates how the average tie resistance required to prevent expansion varies inversely as the length of the rail.



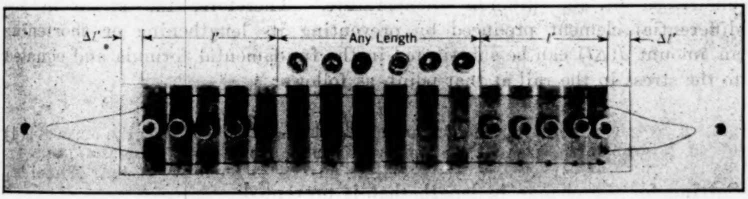
(*a*) ORIGINAL LENGTH OF RAIL



(*b*) FREE CONTRACTION



(*c*) CONTRACTION DECREASED BY LONGITUDINAL RESISTANCE, NOT UNIFORM



(*d*) NO LONGITUDINAL RESISTANCE REQUIRED OF TIES IN THE FIXED PART OF THE RAIL

FIG. 2.—MODEL SHOWING THAT RESISTANCE TO LONGITUDINAL MOVEMENT DEVELOPED BY THE RAIL FASTENINGS NEAR THE ENDS CAN BE SUFFICIENT TO PREVENT CONTRACTION OF THE REMAINDER OF THE RAIL.

The free ends of the continuous rail will always expand a certain extent. To determine this actual expansion for any length of rail, and for any degree of tie resistance, it is simply necessary to determine how much expansion is prevented, and to subtract that from the free expansion. The concept of the increasing resistance to longitudinal movement, from zero at the free ends to a maximum when N ties completely fix the rail, is illustrated in Fig. 1.

It may be demonstrated very effectively with the model shown in Fig. 2, which consists of two elastic bands fastened to cardboard strips which represent equally spaced sections of the rail base. By first stretching the elastics, then placing weights on the strips to obtain various degrees of longitudinal resistance (rail-fastenings), and, finally, releasing the ends, the action will closely approximate that of a rail following a drop in temperature. Using compressed springs instead of the elastic bands would illustrate the case in expansion.

DERIVATION OF FORMULAS FOR EXPANSION (OR CONTRACTION)

Without Joint Restraint.—For simplicity consider the tie resistance to be distributed uniformly along the rail so that $\frac{T}{c}$ is in pounds per linear inch. (A solution by summation of the terms of the arithmetical progression representing the expansion that is prevented in consecutive tie spans shows the error of this assumption to be small, and quite negligible for long rails.) The compressive stress at any point, x in. from the end of the rail, resulting when the tie resistance prevents expansion with a rise in temperature (or tensile stress with a drop in temperature), is, from Fig. 1:

$$s_x = \frac{T\,x}{c\,A} \quad \text{(3)}$$

and may be considered constant in the differential element, dx.

Hooke's law states that this stress is proportional to the strain; and this is true no matter how that strain is produced, whether by a load in the ordinary case with no change in temperature, or, as in this case, by suppressing part of the normal change in length required to leave the rail unstressed at the different temperature. Therefore, the strain in the differential element produced by preventing its lengthening or shortening an amount $d(\Delta l)$ can be substituted in the fundamental formula and equated to the stress in the rail at that point, as follows:

$$s_x = E\,\delta_x = E\frac{d\,(\Delta\,l)}{dx} = \frac{T\,x}{c\,A} \quad \text{(4)}$$

Solving for the change in length that is prevented,

$$d\,(\Delta\,l) = \frac{T\,x\,dx}{c\,A\,E} \quad \text{(5)}$$

The total expansion (or contraction), Δl_r, that is resisted in the half-length, l, is now found by integrating between the limits, $x = 0$ and $x = l$, or,

$$\Delta\,l_r = \int_0^l \frac{Tx\,dx}{c\,A\,E} = \frac{T\,l^2}{2\,c\,A\,E} \quad \text{(6)}$$

(This result of integrating Equation (6) can be obtained without the use of calculus by noting that the sum of all the suppressed unit changes in length is equal to the area of the triangle which represents them as changing uniformly from zero at the free end to a maximum when the rail is fixed. However, it is best to use calculus to illustrate the general method to be used in case the resistance to longitudinal movement of the rail is found to be some empirical curve rather than the simple straight-line variation assumed in this analysis.) The net expansion is the free expansion minus that which is prevented, or,

$$\Delta l = \Delta l_f - \Delta l_r = n \Delta t \, l - \frac{T \, l^2}{2 \, c \, A \, E} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

The maximum end expansion occurs when l becomes equal to the critical length,

$$l' = \frac{E \, n \, \Delta t \, A \, c}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

required to develop the longitudinal force, $F = E \; n \; \Delta t \; A$, which fixes the rail thereafter, so that there is no further increment in the total expansion. If this is not sufficiently obvious from Fig. 1, a convincing proof may be obtained by setting the first derivative of Equation (7) equal to zero for a maximum, and solving for l. Consequently, the tie resistance, T (which is simply a frictional reaction opposing any movement), does not come into action at all in any additional length of rail placed between the critical lengths, l', at each end, as the model action shown in Fig. 2(d) clearly demonstrates.

Substituting the value of this critical length in Equation (7) results in the following simple expression for the maximum end movement of a rail without joint restraint:

$$\Delta l'_{\max} = \frac{E \, n^2 \, (\Delta t)^2 \, A \, c}{2 \, T} = \frac{\Delta l'_f}{2} \quad \ldots\ldots\ldots\ldots\ldots (9)$$

Stated as a theorem:

Theorem 1.—For free ends, the maximum expansion at each end of a long continuous welded rail is one-half of what the free expansion would be in the length that is required to fix the rail.

That this is what should be expected is evident when it is considered that the expansion varies from its normal unit free expansion at the end of the rail, uniformly, down to zero at the point along the rail where the force (independent of the length) required to fix it becomes fully developed.

Note that Equation (9) shows that the maximum expansion: (1) Is independent of the length of the rail; (2) varies as the square of the temperature difference for a given track; and, (3) for different tracks, varies inversely as the tie resistance. This is quite a departure from the time-honored method of computing allowable gaps for 33-ft rails. That the actual expansion follows Equation (9) in principle is strongly supported by the careful observations made on four continuous rails, each about 2 600 ft long, at Mechanicville, and the general observations made in Germany and other countries where long rails have been installed.

Example 1.—For the case in which $n = 0.0000073$, $\Delta t = 100°$ F; $c = 22$ in.; $T = 1\,000$ lb per tie; $A = 9.85$ sq in.; and $E = 30\,000\,000$ lb per sq in.: $F = 216\,000$ lb; $N = 216$ ties; and $\Delta l'_{max} = 1.74$ in. This maximum expansion would occur at each end of a continuous rail, $2l' = \frac{216 \times 22 \times 2}{12} = 792$ ft long. For any longer rail, the additional length would be fixed completely against expansion, so that the total expansion at the ends would remain the same.

Expansion of Rails Shorter Than the Critical Length.—For lengths less than 792 ft, the net expansion can be derived from Equation (7), which reduces to,

$$\Delta l = 0.00073\, l - (0.077 \times 10^{-6})\, l^2 \ldots\ldots\ldots\ldots\ldots (10)$$

for this case.

The theoretical expansions from this equation for various lengths are given in Table 2 to show what can be expected under the assumed conditions.

TABLE 2.—THEORETICAL EXPANSION AS COMPUTED BY EQUATION (10)

Number of 33-ft. rails	Length, L, in feet	Number of ties, N (end to middle of rail)	Δl_f, in inches	Δl_r, in inches	Net expansion, Δl, in inches	Number of 33-ft rails	Length, L, in feet	Number of ties, N (end to middle of rail)	Δl_f, in inches	Δl_r, in inches	Net expansion, Δl, in inches
1.....	33	9	0.145	0.003	0.14	10...	330	90	1.45	0.30	1.15
2.....	66	18	0.29	0.012	0.28	20...	660	180	2.90	1.21	1.69
4.....	132	36	0.58	0.05	0.53	24...	792	216	3.48	1.74	1.74*
6.....	198	54	0.87	0.11	0.76	30...	990	270	3.48	1.74	1.74

* $\Delta l'_{max}$.

The maximum expansion varies inversely as T. Therefore, a value of $T = 5\,000$ lb (claimed possible in certain types of track construction), instead of the $T = 1\,000$ assumed, would give a result only one-fifth as great (that is, 0.35 in.) and $N = 43$ would be the number of ties required on each side of the middle of the rail to prevent further expansion for longer rail lengths. For this case the critical length, l', would be 79 ft.

There probably would be a slight movement of the tie proportional to the resistance for so high a value of T. To show a possible cause of discrepancy in the observed and theoretical rail expansions, as described in this paper, and to show also that it is probably negligible if H, the modulus of horizontal resistance, is about the same as u, the vertical modulus of foundation[5], the following calculation is given. (The modulus of foundation, u, is the load per unit length of the rail required to produce a vertical deflection of the foundation equal to unity. By analogy, the modulus of horizontal resistance, H, is the force per unit length of the rail required to produce a horizontal deflection of the foundation equal to unity.) If, on the average, each tie moved about the same distance to develop its value of tie resistance, T, the additional movement of the end of the rail would be:

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[5] Introduced in the First Progress Report of the Special Committee on Stresses in Railroad Track, *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1191.

$$\Delta l_H = \frac{T}{c\,H} \qquad (11)$$

Substituting $T = 5\,000$, $c = 22$ in., and $H = u = 1\,500$ lb per in. per in. (an average value), in Equation (11), the additional movement is found to be only 0.15 in. The total expansion at the end is, then, 0.35 + 0.15 = 0.50 in., for this case.

A length of continuous rail, 200, 400, or even 1 000 ft long, would give the same theoretical expansion (0.50 in.), that is calculated for this 158-ft length. It follows that, for preventing expansion of the remaining length, only 79 ft at each end of the long continuous welded rail need have the higher resisting and more expensive rigid type of track. Maintenance-of-way engineers contemplating the change to welded joints will be interested in verifying this conclusion. In this connection the writer wishes to make it clear that, although he recognizes the desirability of rigid track for the entire length of the rail, his present purpose is to emphasize and call attention to the basic fact that the longitudinal resistance is developed only as a reaction to movement, and this occurs near the ends of the rail, not at the middle, fixed part.

For rail laid in cold weather, it may even be necessary to provide lateral anchors, in addition to stiff track, to prevent the rail from buckling laterally and from being thrown out of alignment. On the other hand, for tangent rail laid in hot weather, with only the tensile stresses resulting from a drop in temperature, no special type of track may be necessary.

FORMULAS INCLUDING JOINT RESTRAINT

Fig. 3. shows the longitudinal resistance assumed to be increasing uniformly from the initial value, P, representing the joint restraint, to the maximum value, $F = E\ n\ \Delta t\ A$, that is required to fix the rail. As in the case without joint restraint, the change in length that is prevented depends directly on the stress developed in the rail, which is now:

$$s_x = \frac{P}{A} + \frac{T\,x}{c\,A} \qquad (12)$$

but as in Equation (4):

$$s_x = E\,\delta_x = E\,\frac{d\,(\Delta l_r)}{dx} \qquad (13)$$

Therefore,

$$\Delta l_r = \int_0^l \frac{1}{E}\left(\frac{P}{A} + \frac{T\,x}{c\,A}\right) dx \qquad (14)$$

and,

$$\Delta l_r = \frac{P\,l}{A\,E} + \frac{T\,l^2}{2\,c\,A\,E} \qquad (15)$$

The net expansion for rails shorter than twice the critical length, l'' $\left(\text{that is, } l < \frac{c\,(F - P)}{T}\right)$ is:

$$\Delta l = n\ \Delta t\ l - \frac{P\,l}{A\,E} - \frac{T\,l^2}{2\,c\,A\,E} \quad \text{................} (16)$$

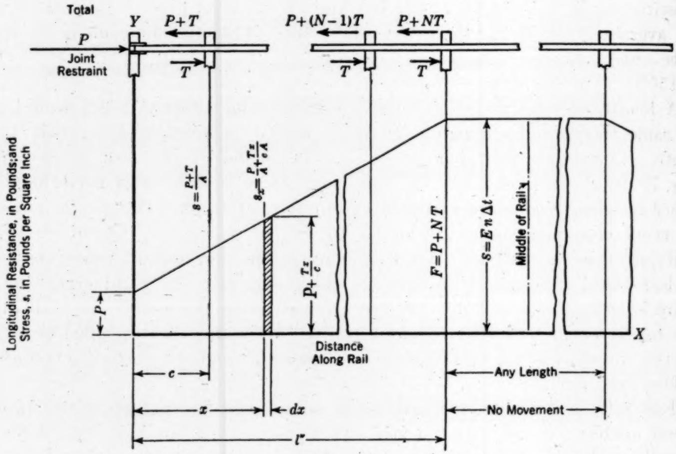
Total
Y
$P+T$
$P+(N-1)T$
$P+NT$
P
Joint
Restraint
T
T
T
Longitudinal Resistance, in Pounds; and Stress, s, in Pounds per Square Inch
$s = \frac{P+T}{A}$
$s_x = \frac{P}{A} + \frac{Tx}{cA}$
$P + \frac{Tx}{c}$
P
$F = P + NT$
$S = E\,n\,\Delta t$
Middle of Rail
X
Distance
Along Rail
c
x
dx
l''
Any Length
No Movement

FIG. 3.—FORCES IN A RAIL WITH RESTRAINED JOINTS.

Substituting the critical length,

$$l'' = \frac{c\,(E\,n\,\Delta t\,A - P)}{T} \quad \text{....................} (17)$$

required to finish fixing the rail, into Equation (16) will result in the simplified expression for maximum end movement with joint restraint, independent of the length, for any length of rail greater than the critical length (see Equation (9)):

$$\Delta l''_{\text{max.}} = \frac{c\,(E\,n\,\Delta t\,A - P)^2}{2\,T\,A\,E} = \frac{c\,(F - P)^2}{2\,T\,A\,E} \quad \text{...........} (18)$$

To Determine Values of P and T by Observation of End Expansions.— The procedure for determining the values of P and T by observing end expansions, is as follows:

(1) Measure Δl_1, the end movement with joint restraint from a fixed reference point, for a known rail temperature change from the laying temperature.

(2) Remove joint bolts, strike joint bars with a hammer to loosen their grip completely so that the ideal condition of free ends will be realized—to warrant applying Equation (9), which was derived with this assumption. Now, measure Δl_2, the end movement with free ends.

(3) Substitute Δl_2 and the temperature difference in Equation (9) (or Equation (18) which reduces to Equation (9) when $P = 0$) and solve for the correct tie resistance, T_2.

(4) Placing this correct value, T_2, and the first measured end movement, Δl_1, with the same temperature difference, in Equation (18), solve for the joint restraint, P; thus:

$$P = F - \sqrt{\frac{2\,(\Delta l_1)\,T_2\,A\,E}{c}} \quad \text{.................(19)}$$

Equation (19) can be simplified so that the ratio of the two measured end movements can be substituted directly in the formula to give the ratio of joint restraint to the total force required to fix the rail. Multiplying the term under the radical by $\frac{\Delta l_2}{\Delta l_2}$, or its equivalent, $\frac{\frac{E\,n^2\,(\Delta t)^2\,A\,c}{2\,T_2}}{\Delta l_2}$, from Equation (9),

$$P = F\left(1 - \sqrt{\frac{\Delta l_1}{\Delta l_2}}\right) \quad \text{...................(20)}$$

COMPARISON WITH OBSERVED EXPANSIONS

General.—For a comparison with general observed expansions in Germany, attention is called to the following quotations[6];

> "In Germany a 1 000′ length of rail moved ¼″ and ⅜″ at the joints. * * * The German engineers say the amount of movement at the ends regardless of the length is apparently only the normal movement for 40 ft. rails." [The movements for 100, 200, and 400-ft lengths were compared.]

In Australia, engineers of the Victorian Railways found that the actual expansion and contraction was about one-half the "theoretical" expansion. This was for rails as long as 250 ft, with standard track construction.

GRAPHICAL SOLUTION

Equation (9) can be transposed and written:

$$T = \frac{K\,(\Delta t)^2}{\Delta l'_{\text{max.}}} \quad \text{......................(21)}$$

in which,

$$K = \frac{E\,n^2\,A\,c}{2} \quad \text{......................(22)}$$

Example 2.—Equation (21) can be solved by reference to Fig. 4. For example, let the observed end movement, $\Delta l' = \frac{5}{16}$ in.; $\Delta t = 58°$ F; $E = 30 \times 10^6$; $n = 7 \times 10^{-6}$; $c = 22$ in.; $A = 12.86$ sq in. By Equation (22) the value of the track constant, K, is found to be 0.20. Then, entering Fig. 4 at $\Delta t = 58°$ F, intersect the curve, $K = 0.20$; proceed horizontally from the intersection to the radial line, $\Delta l' = \frac{5}{16}$ in., to determine the tie resistance, $T = 2\,100$ lb per tie.

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[6] "Thermit Rail Welding," in a brochure pub. by The Metal and Thermit Corporation, New York, N. Y.

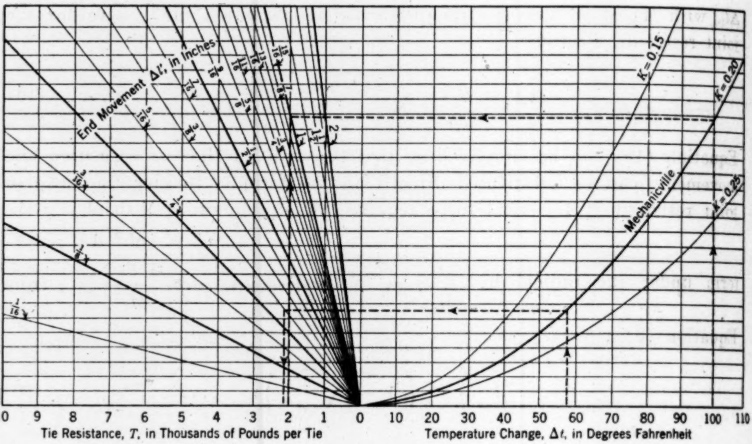

FIG. 4.—GRAPHICAL SOLUTION OF EQUATION (9).

Example 3.—Varying the conditions in Example 2, let $T = 2\,000$ lb per tie; $\Delta t = 100°$ F; and, $K = 0.20$. Proceed vertically from $\Delta t = 100$ to $K = 0.20$, as before; continue horizontally from this intersection to an intersection with the vertical line, $T = 2\,000$; and read the answer, $\Delta l' = 1$ in. on the diagonal lines.

APPLICATION OF THE FORMULAS TO THE MECHANICVILLE, N. Y., OBSERVATIONS

Example 4.—Four continuous half-mile lengths of welded A.R.E.A., 131-lb rail were laid in the main tracks of the Delaware and Hudson Railroad, at Mechanicville, on an extremely hot day, and the closing rail temperature was noted. Subsequently, the maximum observed contraction at one end was 0.75 in. for a temperature drop of 82° F. If the joint restraint were slight, or were removed entirely, Equation (9) (for free ends) applies.

The values of the track constants are: Cross-sectional area of rail, $A = 12.86$ sq in.; $E = 30 \times 10^6$; $n = 7 \times 10^{-6}$; and $c = 22$ in.

Substituting the observed values of $\Delta\, l_{\max.}$ and $\Delta\, t$ in Equation (9) (see heading "Graphical Solution") and neglecting the small tie movements gives a value for the average tie resistance to longitudinal movement of $T = 1\,860$ lb per tie. The average calculated value of T from forty observations (that is, both ends of four rails measured at five different times after installation) was 2 100 for an average movement of $\frac{5}{16}$ in., and an average temperature drop of 58° F.

Using a value of, say 2 000 for T, the maximum contraction for a temperature drop of 100° can now be predicted from Equation (9) (as shown in

Example 3), and results in the value, $\Delta l = 1$ in. This will be the same for any rail (with similar track constants) longer than twice the critical length, and from Equation (9) this length is: $L = 2\,l' = \dfrac{4\,\Delta\,l'_{\text{max.}}}{n\,\Delta t} = 475$ ft. Thus, the actual contraction is only one-half of what the free contraction would be at each end in a 475-ft length of rail, and one-eleventh of what it would be at each end in the 2 600-ft length.

Example 5.—With Joint Restraint.—Since it is quite likely that joint restraint did exist when the observations were made at Mechanicville, it is important to show how the additional data obtained in the suggested procedure for determining the degree of the joint restraint is to be applied in Equations (18) and (20).

Suppose after $\Delta\,l_1$ had been measured equal to 0.75 in., the joint bolts had been removed to allow the end to move freely an additional 0.50 in., making $\Delta\,l_2 = 1.25$ in. The correct value of the tie resistance can now be solved from Equation (9), or more directly by making use of the fact that the maximum end movement for free ends varies inversely as the tie resistance:

$$\frac{\Delta l_1}{\Delta l_2} = \frac{T_2}{T_1} \qquad (23)$$

or, $T_2 = \dfrac{\Delta l_1}{\Delta l_2}\,T_1 = \dfrac{0.75}{1.25} \times 1\,860 = 1\,120$ lb per tie. The number of ties required to develop the force, $F = E\ n\ \Delta t\ A = 222\,000$ lb, would be, for free ends: $N_2 = \dfrac{222\,000}{1\,120} = 198$ ties, and the critical length, $l' = 198 \times \dfrac{22}{12} = 364$ ft.

The number of ties and the length of rail that were originally in use, however, depended upon the degree of joint restraint. Applying Equation (20): $P = 222\,000\left(1 - \sqrt{\dfrac{0.75}{1.25}}\right) = 50\,000$ lb. The number of ties in use with this degree of restraint was: $N_1 = \dfrac{222\,000 - 50\,000}{1\,120} = 154$ ties; and, the critical length: $l'' = N_1\,c = 154 \times \dfrac{22}{12} = 282$ ft. This means that the maximum contraction (0.75 in.) would have remained the same at each end, for any length of welded rail from 564 ft to the actual length of the installation, 2 600 ft.

If the rail had been shorter than twice the critical length required to fix it completely, this fact would have been evident, according to Theorem 1, by its observed free end movement being greater than one-half the calculated free contraction. Equations (7) and (16) would then apply.

CONCLUSION

A complete analysis of the stresses in welded rail would have to include the complicated wheel load stresses in addition to the temperature stresses described in this paper. The various ways in which a rail can be stressed are summarized in Appendix II to show the extent of this subject and the necessity for limiting the present investigation to one particular aspect.

The principal objection to the introduction of continuous welded rail has been the fear of excessive expansion or contraction. The writer's purpose has been solely to answer this objection by presenting a theoretical explanation of the small observed expansions in existing long rails which are not accounted for under the existing "theory," thus establishing a sounder basis for future installations.

It is hoped that, with more observations of expansions, the correctness of the formulas presented in this paper will become established. It is only with this knowledge of what fundamentally takes place at the ends of the rail that the proper approach can be made to the economies resulting from the elimination of rail joints.

ACKNOWLEDGMENTS

The writer expresses his appreciation to his superior, F. W. Gardiner, M. Am. Soc. C. E., for making available the data suggesting the problem, and for the discussions which served as the basis for this paper. Credit is due G. M. Magee, Assistant Engineer, Kansas City Southern Railway Company, for his valuable help in checking the mathematical derivation of the formulas, and for his suggestions.

APPENDIX I

NOTATION

The following letter symbols have been adopted for use in the paper:

A = cross-sectional area of a rail, in square inches;
c = distance between ties; tie-spacing, in inches;
E = modulus of elasticity;
F = force; the total force required to restrain a rail, in pounds;
f = a subscript denoting free expansion;
H = modulus of horizontal resistance;
K = a track constant (see Equation (22));
L = total length of a rail or track;
l = length of a rail from one end to the middle, when the longitudinal resistance is insufficient to develop a force, F, to restrain a rail; Δl_f = change in length due to temperature change (in inches), with free expansion in the length, l; Δl_r = change in length that is resisted or prevented from taking place due to the cumulative effect of the tie resistance; l' = critical length, or length required to develop a restraining force of F lb, or a force sufficient to restrain the rail entirely; l'' = a critical length which includes the effect of joint restraint; Δl_1 = end movement when a joint is restrained, measured from a fixed reference point and corresponding to a known value of Δt; Δl_2 = end movement when a rail is permitted to expand freely;

N = number of ties required to develop the force, F;
n = coefficient of linear expansion, in inches per degree Fahrenheit per inch;
P = initial joint resistance to expansion, or contraction;
r = a subscript denoting "resisted," or "prevented";
s = unit stress;
T = resistance at the ends of each tie, in pounds; T_2 = correct tie resistance;
t = temperature of the rail, in degrees Fahrenheit; Δt = change in temperature from that existing when the rail was laid;
u = vertical modulus of foundation, in pounds per linear inch of rail per inch of vertical deflection;
δ = unit strain resulting from a stress, s.

APPENDIX II

Wheel Load Stresses

Wheel loads may contribute to the stresses in rails in the following ways:

1.—Static Stresses.—The rail is considered a loaded structure on a continuous elastic foundation. Under this assumption corroborated by field tests with extensometers, the stress, s_x, in the extreme fiber at a distance $\pm x$ from the position of the wheel load on the rail, is:

$$s_x = \frac{P}{4\,\beta\,Z}\,e^{-\beta x}\,(\cos\beta x - \sin\beta x) \dots\dots\dots\dots\dots (24)^{7}$$

in which P is the wheel load, in pounds; β is a symbol for conveniently representing the quantity, $\sqrt[4]{\frac{u}{4EI}}$; I and Z are, respectively, the moment of inertia and the section modulus of the rail about the horizontal neutral axis; u is the modulus of foundation; E is the modulus of elasticity for steel; and e is the base of the natural logarithms, or 2.718.

2.—Dynamic Stresses[8].—These are impact stresses caused by low spots in the rail, flat or out-of-round areas on the wheel, rail-joint gaps, differences in the elevation of adjoining rails, and vertical components of the connecting-rod and counterbalance forces in the case of steam locomotives.

3.—Shear Stresses.—Formulas developed by Thomas and Hoersch[9] give values for the high localized shear stresses that form slightly below the contact surface of the rail and wheel. Once the initial crack starts from this

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[7] For derivation of formula see "Applied Elasticity," by S. P. Timoshenko and J. M. Lessells, 1928, p. 141.

[8] "Stresses in Railroad Track" by S. P. Timoshenko and B. F. Langer, *Transactions*, A. S. M. E., November 30, 1931; also, "Rail Stresses and Locomotive Tracking Characteristics Found in Tests on the Great Northern Railway," by J. P. Shamberger and B. F. Langer, *Bulletin 339*, A. R. E. A., September, 1931.

[9] "Stresses Due to the Pressure of One Elastic Solid Upon Another," by H. R. Thomas and V. A. Hoersch, *Bulletin 212*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill. 1930.

cause, it spreads under the constant reversal of bending stresses of much smaller magnitude. Transverse fissures are examples of this type of overstress in the rail.

4.—Web, Fillet, and Bolt-Hole Stresses.—The sudden change of cross-section causes high concentration of stress which may result in minute initial cracks. These stresses can be calculated by photo-elastic methods.

5.—Torsion Stresses.—These stresses may be caused by eccentric vertical loads, and the lateral forces due to swaying, unbalanced centrifugal and superelevation forces, and flange pressures.

6.—Vibration Stresses.—Vertical and lateral oscillations of the truck and rail may produce momentary high stresses.

In addition to the specific references given in the footnotes, data on rail stresses may be found in the Progress Reports of the Special Committee on Stresses in Railroad Track published in the *Transactions* of the Society; the *Proceedings* of the American Railway Engineering Association; and the quarterly reports of the Rails Investigation being undertaken at the Engineering Experiment Station, University of Illinois, at Urbana, Ill.

DISCUSSION

CHESTER F. GAILOR,[10] ESQ. (by letter).—The calculations and the formula presented in this paper appear to be correct and should offer an excellent basis for continuing the study of this subject, as it is very evident that the information available heretofore is inadequate for modern track construction. The following factors entering into this subject should be taken into consideration in future studies and should be given adequate thought by engineers preparing to install rail in long continuous lengths:

(1) Most, if not all, of the rail laid in long lengths in the United States has been installed during the summer months when the expansion of the rail was greatest; therefore, the objections or complications from a practical point of view which might arise from this type of construction are vastly different from those that would arise if the same length of track was laid when the rails were shortest.

(2) Mr. Africano assumes that these long lengths of rail are laid in a perfectly straight line and that this alignment is so maintained. This is not exactly the case as can be ascertained easily and quickly by observing, with an instrument during a very hot day, the lateral variation in the alignment, to say nothing of the possible vertical movement that may occur in a similar manner.

(3) The free ends of long lengths of continuous rail will not expand any appreciable amount provided the rail is adequately fastened at each tie. This fact seems to have been clearly proved on the Delaware and Hudson installations.

(4) It would be well to emphasize the necessity of holding the rail definitely at each tie by means of a fastener that will always maintain a pre-determined gripping strength such as the spring fastenings exert on the rails laid on the Delaware and Hudson Railroad. The gripping strength required on this type of construction, is maintained at 1 500 to 2 500-lb pressure on the base of the rail by each spring fastener. It would appear that disastrous results might be expected if long lengths of rails were installed without similarly fastening the rails at each tie.

(5) The welding of long lengths of rails on electric railway systems has been in practice for many years and the question of expansion or contraction when rails are laid in paved streets is a subject which, to-day, requires only routine attention. However, the fact that the head of rails embedded in street pavement does expand and move laterally can be seen by observing, on a hot day, the effects of this action along the outside of the head of the rail, especially in asphalt pavements. This action is noticeable, even if the base of the rail is embedded firmly in concrete and, necessarily, not subject to such variations in temperature as are found in the head of the rail.[11]

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[10] Cons. Engr., New York, N. Y.

[11] See *Proceedings*, Am. Transit Eng. Assoc. (formerly Elec. Ry. Eng. Assoc.), for information on this subject.

(6) The expansion or contraction of rails might well be better analyzed by assuming that fastenings at each tie are sufficient to hold all pressures or stresses that may develop between such fastenings. In practice, there is little or no evidence that these pressures or stresses are accumulative.

(7) There is need of more definite information on this subject, which should be obtained from a series of practical and laboratory tests, as the practice of laying long lengths of rail on railroad systems is rapidly becoming standard.

Mr. Africano should be commended for the thoroughness of his study and for the value of the information contained in his paper.

E. F. KENNEY,[12] ESQ. (by letter).—The vital feature in this subject of rail expansion is not so much the control of the linear expansion as the control of the lateral movement which might result unless very substantial fastenings are applied to the rail. It has been shown that some types of fastening are ample to take care of this feature, but it is not so certain that the fastenings used on the average railroad will do so safely. Thinking a little further, one wonders what would be the outcome when economic conditions affect the upkeep, even if fastenings are used which, when well maintained, are capable of holding the alignment.

C. W. BALDRIDGE,[13] ESQ. (by letter).—Under the title, "A New Theory of Rail Expansion", Mr. Africano gives a mathematical analysis of the forces resulting from expansion and contraction of steel rails in track, with a discussion of the probable performances of long rails or continuously welded rails. Tests of long rails have been attracting the attention of engineers since 1890 at least. The earliest tests, of which the writer has found record, consisted of an installation of 3 miles of rail on the Lynchburg and Durham Railroad, near Gladys Station, Va. A description of the test[14] shows the installation to have been made between March 19 and April 25, 1892. Three miles of 78-lb rail of the girder type, with the ends fitted tightly together, were jointed by having the bars riveted as tightly as possible. The track was then "buried in" with earth to the level of the top rail on the outside and to the level of the under side of the head on the inside of the track.

The report provoked considerable discussion, and established some values regarding the stresses resulting from such jointing of rails, which approach those calculated in later studies. Unfortunately, the promoter of the test claimed too much for the plan. He announced that continuous rail, buried in as it was in the test, would eliminate all need for track maintenance, and his attempt thus to handle the test resulted in the track getting into very bad condition. When the line was acquired by the Norfolk and Western Railway Company about three or four years after the installation of the test, the rails were removed and bolted-joint track was substituted for them.

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[12] Metallurgical Engr., Bethlehem Steel Co. (Inc), Bethlehem, Pa.

[13] Asst. Engr., A. T. & S. F. Ry., Chicago, Ill.

[14] *Engineering News*, 1892, p. 388.

The history of the test seems to have ended with the publication of a reply by the Editor,[15] to a letter of inquiry regarding the test. In the reply, a list of the articles which had been published concerning the test was given.

Almost contemporary with the test on the Lynchburg and Durham Railroad was a test of long rails made on the Michigan Central Railroad, under the direction of A. Torrey, Chief Engineer of that line.[16] One section was composed of: One rail, 800 ft long; one rail, 500 ft long; two rails, 250 ft long, each; and two rails, 100 ft long, each. Originally, it was intended to construct these long rails by electrically welding 30-ft. lengths, but for a preliminary study Mr. Torrey constructed the rails by holding the individual 30-ft lengths together by means of the splices while the drilling for the turned bolts was done. The rail ends were not planes, but warped surfaces; consequently, the joints could not be held in intimate contact.

The rail was laid in the autumn of 1893 in a main-line freight track where the traffic was in one direction only, and consisted of all the east-bound freight business of the main line of the Michigan Central Railroad. The speed over this track at that time did not exceed 20 miles per hr, but brakes were often set and engines sometimes reversed, as it was in the vicinity of an interlocker.

The points to be determined by the tests were (quoting Mr. Torrey):

> "(1) Whether a long rail would move freely enough on ties to be safe under sudden and extreme changes of temperature;
>
> "(2) What provisions would have to be made to hold a long rail from creeping under traffic in one direction only;
>
> "(3) Having been forced to use rails held in abuttal instead of welded, whether the joint would be stiffer with the rail ends in abuttal than the ordinary joint is;
>
> "(4) Whether the life of the rail would be appreciably greater if the opening between the heads of the rails was eliminated and the splice relieved from the wear occasioned by the movements of the rail due to temperature changes."

Referring to Item (2), it was reported that under the speed of such traffic as this track accommodated, the rails did not drag the ties, since they were securely connected to other ties for a distance of 28 ft at the point of fastening. The answer to Item (3) was found in the result of tests made at the University of Michigan, Ann Arbor, Mich., which began with the assumption that the stiffness of the full rail was 1. The stiffness of the joint with abutted rail ends was $\frac{1}{2}$, and the stiffness of the joint with $\frac{1}{8}$-in. expansion and without the turned bolts was $\frac{1}{3}$, comparatively. The tests were made with the rails upright and inverted, and the proportionate stiffness was the same for each position.

From December 26, 1893, to January 20, 1894, measurements of the movement at the expansion joints were made in the morning. From January 20 to mid-summer, measurements were made at 7:00 A. M., on

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[15] *Engineering News*, March 22, 1906.

[16] *Railroad Gazette*, July 27, 1894, p. 518.

Sundays, and at 7:00 A. M., and at noon, on week days. From December 26, 1893, until April 14, 1894, the theoretically proper range of movement was based on temperatures obtained from the United States Signal Office. From April 15 to mid-summer, the theoretically proper range of movement was based on temperatures obtained from a thermometer embedded in a piece of rail alongside the long rail. In the colder months the rail movement was considerably less than would be forecast from the changes of temperature during those months, if such forecast should be made in the light of the sensitiveness of the rail to changes of temperature, as shown between May 17 and May 20. Mr. Torrey concluded that the rail may or may not assume the air temperatures when the ground is frozen and stated finally that: "* * * defects of construction entered into it [the rail], which would naturally be discarded in its successor, if it has one, and the results as to smooth and noiseless passage of the wheels would probably be more favorable under better conditions."

Another study of longer rails was made public in a committee report on the subject, "Miter Cut Ends and Longer Rails," which was presented at the Convention of the Roadmasters and Maintenance of Way Association of America in the autumn of 1895, in which some comments of interest are to be found.

In recent years interest in the subject of the use of continuous, or very long sections of rail welded together, has resulted in actual tests being made on the Delaware and Hudson Railroad, in four different locations and under somewhat different conditions. Another installation of rails welded together for a full mile has been made on the Bessemer and Lake Erie Railway. These tests are making history and are attracting the attention which such studies deserve.

Although actual tests of continuous rails are necessary to prove conclusively whether or not such installations are dependable and practicable, some information regarding actions which result from contraction and expansion of long rails may be gleaned from the results of the shearing off of track-bolts in rail-joints during periods of severe cold weather.

In 1924, the writer had occasion to investigate a case in which, on a severely cold night following a mild day, nineteen joints in a distance of about 25 miles had sheared off both bolts in one end of the joint, thus allowing the rails to pull apart. The track was laid with 90-lb rail, all-steel joint bars and oil-quenched bolts. Inquiry as to maintenance revealed the fact that the joints had been bolted up as tightly as possible, using wrenches which, by means of pieces of pipe, had been lengthened from 46 to 48 in. The bolt-tightening had been done during an unusually warm spell late in the autumn. A few weeks later a sudden drop in temperature to − 18° F resulted in the shearing of the bolts at joints where for some reason the bolts were less tightly set, thus permitting the rails to slip through the joint bars. Notwithstanding the fact that the track was equipped with single-shouldered tie-plates and only four cut spikes per tie (two for each rail), none of the openings between rail ends widened to more than 5.5 in.

During the present winter (1936–37) a number of joints in track laid with 112-lb, all-steel joints, and high carbon, oil-quenched bolts, have sheared two bolts, allowing the rails to pull apart as far as the contraction would take them. In these cases, the track is also equipped with single-shouldered tie-plates and with four cut spikes per tie. The bolts had been installed by bolting machines when the rail was laid, followed by normal maintenance. Most of the adjacent joints showed that no movement had occurred through the bars, and in most cases the sheared bolts occurred in joints a mile of more apart; yet in these cases the opening between the rail ends was usually about 2 in. and none was reported to have opened more than 2.5 in., even when temperature readings below zero were experienced. These rails were laid using expansion shims based on rails that were tight at 100 degrees.

Taking the coefficient of expansion at 0.0000065 (which is the value generally used for railway rails), the width of opening found in these cases represents the contraction of only 320 ft of rail, and assuming that it was equally effective in both directions, it shows that the contraction of rail steel, due to zero temperature, is overcome by the resistance to movement of only 160 ft of 112-lb rail held by cut spikes in a normal manner.

In order to determine the results, of the forces created by the expansion and contraction of steel due to temperature changes, it is pertinent to begin with a single rail and then to build up a series of rails. It is easy to recognize that a single rail, 39 ft long, resting freely with no connections except the support under it, will have no movement, as a whole, due to contraction and expansion. Connect two rails of the same size, weight, and length, and each rail will exert the same pull upon the other. Add a third rail and the pull of Rail No. 1 upon Rail No. 2 will be equal and opposite to the pull of Rail No. 3, and since the resultant of equal and opposite forces is zero, there will be no movement. It becomes evident, therefore, that as long as there is no break in the connecting joints in a track the forces of expansion or of contraction nullify each other and there is no resultant force to produce movement. It is also evident that if the rails in a track are rigidly connected, the forces acting nullify each other, except for the short distance at each end where the resistance to movement produced by the weight of the rail and the grip of the spikes is less than the unbalanced force of contraction or expansion.

The widths of opening between rail ends, resulting from the shearing of bolts prove that such distances are not great and also that they are shorter for heavy rails than for lighter ones.

These actual results in track go far to confirm the accuracy of the mathematical determination of forces resulting from, and the resistances to movement due to, contraction or expansion of long rails, as given in Mr. Africano's paper. The results found in track as described herein, with the studies made by Mr. Africano, by the Kansas City Southern Railroad Company, by the Rail Committee,[17] and the Track Committee,[18] of the American Railway Engineering Association, and by others, give favorable indication of the practicability of the use of continuously welded rails.

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[17] *Bulletin No. 391*, A. R. E. A., November, 1936, p. 233.

[18] *Bulletin No. 393*, A. R. E. A., January, 1937, p. 493.

George W. Hunt,[19] Esq. (by letter).—First considering the condition of a rail perfectly free to expand, Mr. Africano proceeds to the condition in which the ends are restrained by means of ballast resistance to tie movement, and, finally, to the condition in which the ends are restrained partly by the ballast resistance and partly by the joints connecting the welded section with the adjacent track sections. The equations developed by Mr. Africano are applicable to all lengths of welded track, from a few rail lengths to a number of miles. Their purpose is to show the relation between: (1) Tie resistance; (2) joint resistance; (3) rail movement under various changes in temperature; and (4) to indicate the proper balance between these factors in order to avoid damaging strains and stresses after the rail is welded.

As Equation (18) contains all of the aforementioned factors ((1) to (4)), and represents the third condition, it may be accepted as the general equation. Equation (9) is simply a special case of Equation (18) when the joint restraint, P, is reduced to zero. For analytical purposes, P may be expressed in fractional parts of F, the total force required to restrain the rail, and incorporated in Fig. 4, thus making this graph to include all factors in the problem. For example, Equations (21) and (22) which determine the values of T for the case of no joint restraint, can be restated as follows to take joint restraint into account:

$$T = \frac{K' (\Delta t)^2}{\Delta l''_{\max}} \qquad \text{(25)}$$

in which K' (see Equation (22)) equals $0.5\,K$ when $P = 0.3\,F$; $0.25\,K$ when $P = 0.5\,F$; and $0.0625\,K$ when $P = 0.25\,F$; and $F = 2\,816\,(\Delta t)$. In Fig. 5, the value of T can be read for various degrees of tie restraint. For example, suppose that $P = 0.3\,F$; then, for 130-lb rail, with a temperature change of 100°, $P = 0.3 \times 281\,600$ lb $= 84\,400$ lb. For a 60° temperature change, $P = 0.3 \times 2\,816 \times 60 = 50\,688$ lb. For these two temperature changes Fig. 5 shows that $(\Delta l'')_{max}$ (the rail end movement) has the values shown in Table 3.

TABLE 3.—Values of $(\Delta l'')_{\max}$

Values of tie restraint, T, in pounds	Critical Length, $(\Delta l'')_{\max}$, in Inches, for Temperature Changes of:	
	100° F	60° F
750	$1\frac{3}{8}$	$\frac{1}{2}$
1 000	1	$\frac{3}{8}$
1 250	$\frac{11}{16}$	$\frac{1}{16}$

It is the opinion of the writer that the influence of joint restraint has been under-estimated in the test at Mechanicville, and that if the facts were known the joints at the free ends (so-called) were furnishing part of the resistance, and that the ties were furnishing considerably less than 2 000 lb. The opinion is based on his understanding that the joint gaps at the "free" ends were as much as $\frac{7}{8}$ in., at the temperature drops mentioned in this test.

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[19] Maintenance Inspec., M. of W. Dept., Baltimore & Ohio R. R., Baltimore, Md.

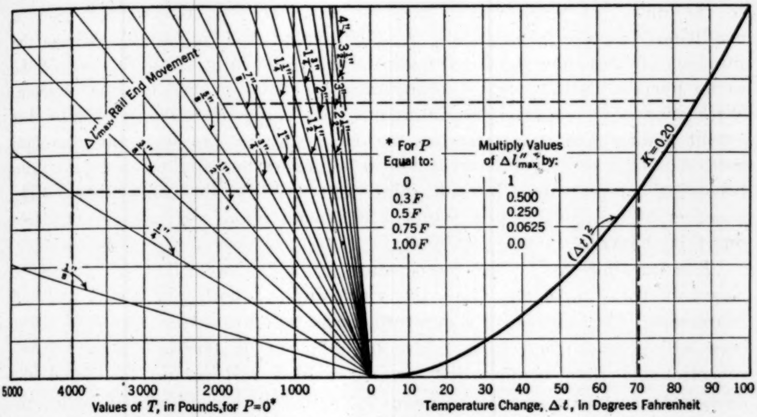

FIG. 5.—RELATION BETWEEN RESISTANCE, TEMPERATURE CHANGE, AND RAIL MOVEMENT UNDER VARIOUS DEGREES OF JOINT RESTRAINT.

Joint restraint in tension is composed of the frictional resistance between bars and rail, plus the bending resistance of the bolts. The frictional resistance in a 6-bolt joint may be taken as $(3 \times 20\,000 \times 2) \div 2 = 60\,000$ lb. The bending resistance of one bolt may be taken as 8 000 lb, or 24 000 lb for three bolts. On this basis the total joint resistance would amount to 84 000 lb, or about 30% of the force, F, required to restrain a 131-lb rail under a temperature drop of 100 degrees. It seems reasonable, therefore, to consider the ultimate value of P in tension as equal to $0.3\,F$. In this connection, it should be remembered that the rail-drilling, bar-punching, and bolt diameter for 131-lb and 112-lb rail sections are proportioned so as to allow only a $\frac{3}{8}$-in. rail gap before bringing the bolts into bending resistance. It follows, therefore, that when the joint gaps exceed $\frac{3}{8}$ in., the frictional resistance of the bars against the rail has been overcome and the bolts are exerting bending resistance. In compression, the ends of the rails come into contact before any bending strain is thrown on the bolts, so that the joint restraint in compression may be taken as 60 000 lb. The fact that a joint gap has greater latitude to enlarge under tension than to contract under compression is one reason why welded track should be laid or closed at temperatures that favor tensile strains, rather than compressive strains. The major reason is that the track is less likely to be thrown out of line by contraction in cold weather than by expansion in hot weather.

Mr. Africano conceives joint resistance as that force at which the joints in the adjacent unwelded section will begin to yield and continue to yield, sufficiently to accommodate $(\Delta l'')_{max}$, the rail-end movement. The actual value of the joint resistance is of little practical importance, serving only to explain the variation in the value of $(\Delta l'')_{max}$ as between theory and practice. In no case could it exceed the value of F; to do so the unwelded track would have to be as stiff as the welded track, a condition scarcely attainable. A high value for it would reduce the number of ties necessary to restrain the rail fully, but this would be to no purpose as long as a special track construction was used throughout. With increasing values it would reduce the value of $(\Delta l'')_{max}$ and to that extent reduce the movement at the rail gaps in the adjacent unwelded section. This objective is desirable as a precaution against tensile strains becoming concentrated in one joint and pulling the rails apart. The question of using expansion joints will depend upon what weight is placed on this hazard.

The question as to the temperature at which welded track should be closed is very important from the viewpoints of both rail stress and rail movement. A range of temperature from 20° below zero to 130° above zero will give a mean temperature of 55°; but as tensile strains are less objectionable than compressive strains, welded rail should be closed and anchored at a temperature between 70° F to 80° F. If this is done, the temperature change above the closing temperature need not exceed 60° and the rail movement in compression will be limited to that temperature change. Similarly, the temperature change below the closing temperature will be limited to 100° and the rail movement in tension to that temperature change.

The value for tie resistance, T, as applied to fairly well ballasted track, has probably been over-estimated in much of the discussion on this subject. There is a temptation to place too high a value on this factor, for rail laid at high temperature, on the theory that with a severe drop in temperature below the closing temperature, the ballast will freeze and thus increase the tie resistance, restrict the contraction movement, and build up the value for F in a shorter distance. This reasoning appears fallacious when it is considered that the rail responds very quickly to temperature change; freezing is a slower process and could scarcely occur in time to offer any resistance to rail movement. In fact, in the absence of sufficient moisture, the ballast may not freeze at all; it may have no higher resistance in cold weather than in warm weather. If the ballast did finally freeze, subsequent to the rail movement, and a thaw should set in, the temperature would have to rise to the closing temperature before any reversal of tie restraint could occur, and considerably above this temperature before the reversal of tie restraint reached any great magnitude. By this time the thaw, in all probability, would have begun to loosen up the ballast. It is felt, therefore, that the process of freezing and thawing can be disregarded as far as influencing the ballast resistance, and that the factor, T, can be accepted as a fairly constant value for any given ballast section, throughout the year.

An approximation of this value can be made for loose ballast by taking one-half the weight of the rail, ties, and ballast, above the bottom of the ties for a distance of 22 in., and multiplying it by the coefficient of sliding friction, taken as unity. For loose ballast, $T = 0.5\ (192 + 175 + 755) = 561$ lb; and, for packed ballast, with the usual degree of cementation, it would seem that 1 000 lb per tie per rail, is all that could be depended on. Table 4 is illustrative of the manner in which $(\Delta l'')_{\max}$, the rail end movement, varies with variation in temperature change, when $T = 1000$ lb and $P = 0.3\ F$. Items Nos. (3) and (4), Table 4, represent the conditions that are said to have existed at Mechanicville, so far as temperature change and rail end movement are concerned. The division of restraint as between ties and joints are the results of the writer's construction and about this there may be a difference of opinion.

TABLE 4.—VARIATIONS IN $(\Delta l'')_{\max}$

Item No.	Tie restraint, T, in pounds (1)	Changes in temperature, Δt, in degrees Fahrenheit (2)	Restraining force, F, in pounds (3)	Joint restraint ($P = 0.3\ F$), in pounds (4)	Number of ties, N, required to develop force, F (5)	Critical length, l'', in feet (6)	Critical end movement, $(\Delta l'')_{\max}$ in inches (7)
(1)......	1 000	100	281 600	Free end	432	792	2
(2)......	1 000	100	281 600	84 480	197	361	1
(3)......	1 000	82	230 910	69 270	162	297	$\frac{11}{16}$
(4)......	1 000	58	163 330	49 000	114	209	$\frac{5}{16}$

Where the welded track is isolated (that is connected at both ends by unwelded rail with the usual joints), these rail end movements, even in the case of Item No. (2) are not alarming. They would not result in any greatly enlarged rail gap, as in all three cases, the value of P is sufficient to distribute this movement through from two to four joints. Insulated joints, however, would present a problem, particularly in Item No. (2). Required to withstand a tensile force of 281 600 lb they would be able to withstand only 84 480 lb, and a rail gap of 2 in. would result if the joint failed. The only recourse, as a matter of precaution, would be to install several ordinary rails on either side of the insulated joint so that this movement could be distributed through several joints, or to adopt some extraordinary means of increasing the joint resistance. A similar problem will be encountered at all switch-point joints, on the frog side as well as at the heel joints of all frogs. Here Condition No. (1), Table 4, would obtain, due to the break in the continuity of the main-track rail by the switch-point, and the 2-in. rail end movement would have to be provided for either by a single expansion joint, or by the more simple expedient of installing about four or five rails with ordinary joints, in advance of the switch-point and heel, to distribute this movement. For obvious reasons, both economical and practical, welding through yards involving main-track turnouts should be avoided, and the welding should be terminated at the entering and leaving turnouts to yards.

Item No. (1), Table 4, shows that $(\Delta l'')_{\max}$ amounts to 2 in. when the joint resistant is zero. It follows, therefore, that if a rail broke at a temperature drop of 100°, there would be a rail gap of 4 in. If no special track construction was used to utilize the tie resistance, the rail gap would be 8 in. This is one reason why it is advisable to bind the rail to the ties throughout, rather than just for a distance, l'', sufficient to develop the force, F.

It is apparent that if a rail broke, or if for any reason a section of rail were removed, it should be replaced by one of identical length, otherwise abnormal rail stress or rail end movement may result when the temperature changes radically in the other direction. The repair of a broken rail at a radical temperature change without restoring the rail gap, has the effect of creating a free-end condition at the break at that temperature as regards rail strain, without providing any means for rail movement when that temperature alters radically. What would happen under a radical change in temperature in the opposite direction is best illustrated by studying some specific case, such as that of a welded track laid at 80° F. A rail breaks at zero temperature, leaving a gap at the break of $2\frac{3}{4}$ in., and is repaired by cutting out a distance sufficient to accommodate a new 39-ft rail and then re-welding. The temperature subsequently rises to 130° F. How will the rail stress and rail end movements at 130° compare with these same factors if the rail had gone through temperature changes and not broken? What happens can be best illustrated by reference to Fig. 6, which shows that, had the rail not broken, the rise in temperature to 130° would have resulted in a unit compressive stress of only (140 800) ÷ 12.86 = 11 000 lb and the rail end movement would have been $\frac{3}{16}$ in.

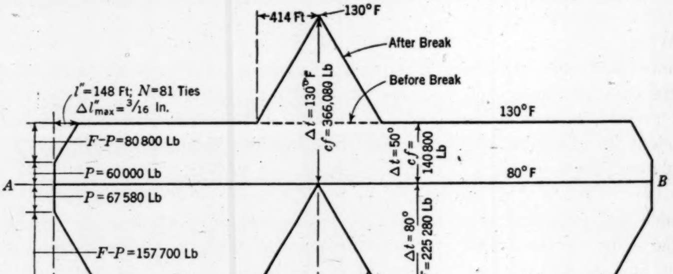
414 Ft
130° F
After Break
Before Break
l''=148 Ft; N=81 Ties
$\Delta l''_{\max} = 3/16$ In.
Δt = 130° F
cf = 366,080 Lb
130° F
$F-P$=80 800 Lb
Δt = 50°
cf = 140 800 Lb
P = 60 000 Lb
80° F
A
B
P = 67 580 Lb
$F-P$ = 157 700 Lb
Δt = 80°
tf = 225 280 Lb
414 Ft
0° F
l''= 290 Ft; N=158 Ties
$\Delta l''_{\max} = 11/16$ In.
After Break
Before Break

FIG. 6.

The breaking of the rail at 0° F and its repair by adding 4 in. to the length would result in a concentrated compressive stress at the break of 366 080 ÷ 12.86 = 28 500 lb when the temperature rises to 130° F. This stress would hold as long as the break was far enough away from

End *A* for the ties and joint restraint to develop a force of 366 080 lb. In Fig. 6 this distance is 148 + 414 = 562 ft. Fig. 7 shows the conditions that would exist had the rail broken 300 ft from the end of the welded section. The theoretical gap at the break would be $1\frac{7}{8}$ in. at zero temperature which it may be assumed, was not restored when repaired. When the temperature rises, a movement of the rail end will occur equal to $1\frac{7}{8} + \Delta l''$.

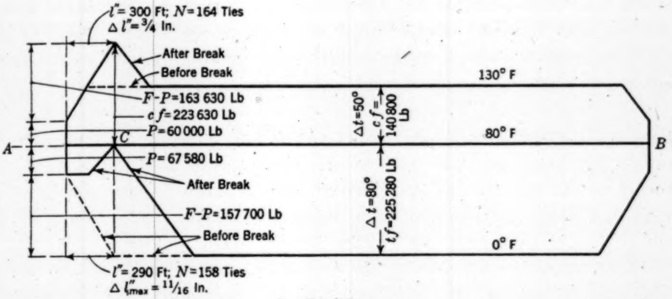

FIG. 7.

When a temperature of 130° is reached, a peak of maximum value for *F* will arise at Point *C*, Fig. 7, determined by the value of the joint resistance plus the tie resistance developed in 300 in. This value will be $60\,000 + (300 \times 12 \times 1\,000) \div 22 = 223\,630$ lb, or equivalent to a maximum unit stress of 18 400 lb per sq in. The rail movement at 130° will be equal to $1\frac{7}{8} + \Delta l''$. From Equation (18) the value for $\Delta l''$ is $\frac{3}{4}$ in., and the total rail end movement will be $2\frac{5}{8}$ in.

Should the break occur at or very close to End *A*, where the only force is the joint restraint, the rails would separate very little if they broke. Even if they separated considerably, the neglect of correction would have no effect on the stress in the welded section. The sole effect of this neglect would be to limit the room for expansion at higher temperatures, in the joints of the adjacent unwelded track.

This discussion shows that the repair of broken rails without restoring the original strained condition, may result in one of two conditions: Either the stress may be radically increased locally, adding to the hazard of buckling track, or the end movement may be increased radically, creating a necessity for expansion joints. Repairs in cold weather would create excessive compressive strains in hot weather, and repairs in hot weather would be likely to create excessive tensile strains in cold weather. It follows, therefore, that welded rail should be repaired wherever possible at or close to the temperature it was laid, and that any work of this kind will require the most rigid rules and careful supervision.

Although rail welded into continuous stretches should reduce greatly the necessity for lining and surfacing track, all such work would have to be done with the utmost care. "Out of face work," such as raising track, cleaning cribs, and re-ballasting, should not be undertaken except at temperatures bordering on that at which the rail was laid, say, between 55° F and 95° F. ("Out of face" is a term used by track forces; it denotes a fairly long piece of track as distinguished from the one or two rail lengths. "Out-of-face surfacing," means the raising and tamping of all joints continuously; "spot surfacing" means just raising a joint here and there.)

With due consideration to the features mentioned, the writer can see no reason why welding of rail into continuous stretches cannot be practiced with perfect safety, with resulting economy, and greatly improved riding conditions.

FRANK B. WALKER,[20] M. AM. SOC. C. E. (by letter).—The mathematical formulas developed by the author are of interest to maintenance-of-way engineers, and the conclusions derived are apparently confirmed, at least in part, by the experience of the Eastern Massachusetts Street Railway Company in maintaining 210 miles of unpaved, welded track. At one time, there were 150 000 welded joints on that street railway.

The major part of the welding on this track was done from 1919 to 1923, but the change from trolley cars to buses during recent years has caused the abandonment and removal of most of it. The rail sections welded were 48-lb, 60-lb, and 75-lb tee-rail (Am. Soc. C. E. standard). The standard track consisted of 6-in. by 8-in. by 8-ft wooden ties, spaced 2 ft on centers; $5\frac{1}{2}$-in. by $\frac{9}{16}$-in. track spikes; and 6 in. to 12 in. of gravel ballast. There were no tie-plates. The rail welded in 1919, 1920, and 1921, had few expansion joints although some of the sections were more than a mile in length, as indicated in the following list:

Location	Feet of track welded
Brockton and Easton	15 600
Boston-Randolph	10 000
Brockton, Crescent Street	6 300
Salem-Danvers Line	8 300
Danvers-Danvers Center Line	6 400
Peabody-Danvers Line	11 600

No serious difficulties due to expansion or contraction were encountered in maintaining such long sections of welded track. Occasionally, during a very cold winter night, especially when the temperature was falling rapidly to near zero, a rail would break. Rail defects, together with high tension due to temperature changes, were believed to have been the cause, as they occurred only in old rail. New rail did not break under these conditions. It may be assumed that the difference in temperature from the time of welding to the time of failure was from 70° F to 100° F. Only one or two of the so-called "sun kinks" occurred during hot weather (see Fig. 8).

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[20] Cons. Engr., Eastern Massachusetts Street Ry., Boston, Mass.

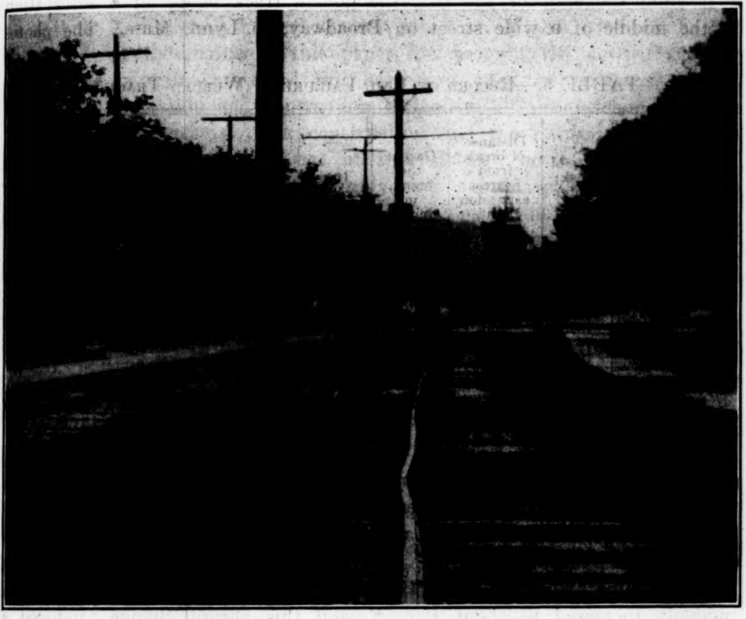

FIG. 8.—VIEW OF A "SUN-KINK" ON A WELDED TRACK.

When the rail broke, or when a sun kink appeared, expansion joints were installed (not necessarily at the break), about 20 rail lengths or 600 ft apart on tangents and at the ends of long curves. After the installation of expansion joints no sun kinks occurred, but rail breaks on old rail did occur occasionally. Sometimes these breaks were at long distances from the expansion joints, and at other times only a few feet, or a rail length, away. Where expansion joints were 660 ft apart, it was thought at first that the expansion would be 6 in.; but it was soon learned that the actual expansion did not exceed one-half the anticipated expansion. This phenomenon is mentioned by the author in connection with the experience on the Victorian Railways in Australia.

Hundreds of expansion joints were examined during hot and cold weather over a period extending from 1922 to 1934, and their movement, due to temperature changes only, did not exceed the 2 to 3 in. When a rail broke, it gave a very loud report and the ends separated by a distance never in excess of 2 in. Table 5 contains information concerning thirteen broken rails, the distance of the break from the nearest expansion joint, and the gap at the break.

The rail in Fig. 8 was a 60-lb tee-rail (Am. Soc. C. E. standard), laid on chestnut ties, 2 ft apart, without tie-plates, with $5\frac{1}{2}$-in. by $\frac{9}{16}$-in. track spikes, and about 12 in. of gravel ballast. There was a stone curb about 2 ft from the ends of the ties, the double track being in a reservation in the middle of a wide street on Broadway, in Lynn, Mass. The photograph (Fig. 8) was taken a few minutes after the sun kink appeared. The rail was welded in the summer of 1920. It was in service through the remainder of that summer, an unusually cold winter, and a late, cold spring

TABLE 5.—Record of Rail Failure in Welded Track*

Item No.	Date welded; summer of: (1)	Date of break; winter of: (2)	Distance of break from nearest expansion joint, in feet (3)	Gap at the break, in inches (4)	Item No.	Date welded; summer of: (1)	Date of break; winter of: (2)	Distance of break from nearest expansion joint, in feet (3)	Gap at the break, in inches (4)
1.....	1921	1925	0†	1.5	8.....	1921	1925	300	1.75
2.....	1921	1925	300	1.0	9.....	1920	1926	500	2.0
3.....	1921	1925	100	2.0	10.....	1920	1926	700	1.25
4.....	1921	1925	400	1.0	11.....	1920	1926	600	1.0
5.....	1921	1925	250	0.5	12.....	1920	1926	400	1.5
6.....	1921	1925	500	1.5	13.....	1920	1926	200	1.5
7.....	1921	1925	0†	1.5	....	......	......	....	....

* In each case the atmospheric temperature was reported as "falling"; and, each expansion joint was oiled.

† Break adjacent to joint.

Some time in early June, 1921, after a cold night, the temperature suddenly increased to about 100° F, and this abrupt change produced a sun kink, in one track only. The kinking of the rails also moved the ties laterally. There was no noticeable bending or displacement of the track spikes. It is believed by the Company Engineers that this sun kink would have occurred even if a more substantial type of rail-fastening, such as larger or more track spikes, screw spikes, or even shoulder tie-plates, had been used. As far as had been observed previously the rail had been in perfect line and surface. There were no expansion joints at the time the sun kink occurred, but expansion joints were installed immediately and no further trouble occurred. The track was discontinued in 1935.

Another kind of difficulty occurred on a hot summer day when a man's leg was almost cut off when he pulled out too many spikes in renewing ties. The rail popped out of the remaining spikes, catching his leg against the adjacent rail.

It is obvious from the author's mathematical discussion and from the testimony of those experienced with maintaining welded track, that ballast must be of sufficient width, depth, and character to withstand the thrusts put upon it during hot weather; and no doubt the construction with tie-plates, and with better means of fastening rail than by ordinary track spikes, is desirable. The rail will withstand, safely, the additional tension induced by a decrease in temperature. It is suggested that no welding be done when the temperature is extremely hot or cold.

Arthur N. Talbot,[21] **Past-President and Hon. M. Am. Soc. C. E.** (by letter).—To many engineers, this paper has brought the news that long stretches of continuous welded rail have been used on railroads without troublesome expansion and contraction difficulties. To those familiar with modern developments in track construction and the nature of the forces applied to the track it was not unexpected that, on a long stretch of continuous welded rail connected firmly to a suitable substructure of ties and ballast, the cumulative anchorage of the ties in the ballast at the two ends will reach a total force sufficient to hold an intermediate section stationary and fixed through the usual range of atmospheric temperature without changes in length, except for local effects. In the substructure used in present installations the rail is secured firmly to each tie in a manner such as to prevent slippage between rail and tie either longitudinally or transversely of track, and the ties are well bedded in ballast. This anchorage must be such as to provide adequately for the great force required to hold the intermediate section in a fixed position during temperature changes equal to those obtained in both summer and winter exposure and also to provide adequate transverse support against lateral buckling during the highest temperatures. These requirements imply a construction differing from the ordinary run of common track, in which the rail can be lifted from the tie a short distance and is held by anchors along its length only roughly and not fixedly with respect to the ties; that is, the rail must be fastened to the tie so that there will be neither longitudinal nor transverse movement between them, and the relation between tie and ballast bed must be one of firmness and security.

It is obvious that an analysis of the action of welded rail under such conditions must be based on known or assumed premises and, as in all reasoning, the validity of the analysis will be contingent on the validity of the premises. A start must be made to learn what underlying bases to use.

In beginning an analysis in this matter, one naturally first makes inquiry as to the possible ways in which the end portions of the track may act. Many questions arise. In developing the anchorage that will fix the unmoving intermediate part of the length of the welded stretch, does the anchorage act elastically? Does the anchoring force push the ties and ballast longitudinally along the track with the changes in temperature, even in small amounts? Is the nature and magnitude of the resistance the same in backward and forward directions—that is, with rising and falling temperatures? Do the ties near the end of the welded stretch, where the movement of the rail under temperature changes is greatest, give a greater resistance (greater rate of anchorage) than the ties at points farther from the end but within the moving length; or does the anchorage resistance have the same value (rate) all along this end portion; or does some other law govern?

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[21] Prof. Emeritus, Coll. of Eng., Univ. of Illinois; and Chairman, Committee on Stresses in Railroad Track, Urbana, Ill.

Whether or not the tie-and-ballast rate of anchorage is uniform along the length of the anchorage stretch, does the magnitude of the length of the anchoring portion vary with the magnitude of the change in temperature of the rail, or is it constant throughout any range of temperature? In other words if the rise in temperature from the laying and fastening temperature is at first 40° and then 40° more, does the 80° increase in temperature result in twice as long an anchorage length as the 40° increase?

Then, there are questions as to the source of the anchorage. Granting that the rail does not slip over the tie (considered as essential in this form of construction) is the anchorage due to friction of the ties on the ballast by reason of pressure of the tie on, and in, the ballast so that the anchorage rate will have a constant value throughout any given anchoring stretch; or does the anchorage utilize some compressive resistance in the ballast, ties, and fittings? If the latter, does the movement of the tie within the ballast bed vary with the longitudinal anchorage force developed? How can the rail at the end of the stretch move without a movement of the ballast? For sensible movements of the rail end, will the tie move sufficiently to leave a crack behind it which would be closed on reversing the temperature change? Furthermore, if the ultimate resistance is reached, what will be the nature of the failure (all materials have their ultimate resistance)? Will it shear along a plane under the ties, or what?

These queries and others, although important, must mostly await further experience and investigation. Without definite and explicit information concerning some of them it is natural that in starting an analysis the easiest and simplest assumptions would be chosen first. The simplest and most probable assumptions to use would seem to be that for a given track: (1) The anchorage rate is uniform in magnitude over the length of the end portion that acts as anchorage restraint; and (2) the value of the rate of anchorage force is constant regardless of the magnitude of the change in temperature from the laying temperature of the welded rail.

These two assumptions are made by Mr. Africano, at least implicitly. They were chosen tentatively by the writer when, in 1935, he began considering the phenomena of long stretches of welded track preliminary to planning tests and observations on the track of the Delaware and Hudson Corporation for the Track Stresses Investigation of the Joint Committee on Stresses in Railroad Track, which is a co-operative investigation of the Society and the American Railway Engineering Association. The assumptions formulated at the time with other statements of the accompanying conditions in part were as follows:

> "To study the general action of the track in a preliminary manner, use may be made of assumed anchorage conditions. The simplest one and the one most likely to come to mind is the assumption that each tie toward the ends contributes the same increment of force and these many increments

together accumulate a force of sufficient magnitude to restrain and anchor the intermediate portion of the rail fully so that it will not contract or expand under the given change in temperature. A second simple assumption may be made, which at first thought may be considered a corollary of the first one although it is not, that the magnitude of the restraining force developed at a tie along the two anchoring stretches of the long rail has a constant value regardless of the magnitude of the temperature change. The validity of these two assumptions can be tested from observations on the action of the track. At any rate they will be useful in discussing the possible magnitude and distribution of the anchorage forces and the movement of the end portions under temperature change."

In the study, the calculus method was first used similar to the method given in the paper, but the premises assumed were so simple that it seemed sufficient and preferable to use the analogy of constant increments of force and of constant acceleration, following the laws of mechanics, and at once write the equations for length of anchorage and for rail movement without the more formal phraseology of the calculus. The force transmitted through the joint at the end of the stretch was included readily in the general consideration. It should be stated that this discussion refers to stretches with a length more than sufficient to give full anchorage. Some of the further statements formulated at that time (omitting the derivations and using the notation of the paper under discussion) were:

"(1) The anchorage force required to restrain the intermediate portions of the length of rail so that it will not change its length with a change in temperature is:

$$F = \Delta t\, n\, E\, A \qquad (26)$$

this force being independent of the length of the fully restrained part and the unit stress there being $s = \Delta t\, n\, E$. (See Equation (2).)

"(2) Calling l'' the length of the end part of the rail (that giving the anchorage to the intermediate and fully restrained portion), under the assumption of a uniformly increasing stress in the rail from zero at the end of the rail to the full stress, s, at the other end of the anchorage and, likewise, to the full anchorage force, $\Delta t\, n\, E\, A$, the movement of the end of the rail will be one-half the full temperature expansion or contraction for the length, l''; that is, $0.5\, \Delta t\, n\, l''$. Likewise, the distribution of the anchorage force along l'' will be per unit of length,

$$\frac{F}{l''} = \frac{\Delta t\, n\, E\, A}{l''} \qquad (27)$$

Likewise, the length of the anchorage under the assumption made, is:

$$l'' = \frac{\Delta t\, n\, E\, A}{q} \qquad (28)$$

Even with the assumptions used further information will be needed.

"(3) From Assumptions (1) and (2) and Equations (26), (27), and (28), certain deductions may be made: (*a*) Assumption (2) implies that for any given change in temperature the full available anchorage force per unit of length (and likewise the anchorage force per tie) will be developed and applied on the anchorage portion, whether the temperature change is small or large; and by Assumption (1) this same maximum available anchorage force per tie is applied uniformly along the anchorage portion; (*b*) the length of the anchorage portion with a given available anchorage unit per length or per tie will vary directly as the temperature change—twice as great a temperature change will require or use twice as long an anchorage length; and (*c*) the movement of the rail end for the same conditions will vary as the square of the temperature change—twice as great a temperature change will result in four times as great a movement at the rail end."

Naturally, the observational tests on welded track of the Stresses in Track Investigation were planned to cover the validity of the two assumptions. As the results of the tests are giving needed cumulative information as to the action of welded track, it seems germane to the discussion of this paper to present some of the results which bear on the assumptions made in the analyses, with a very brief statement of the methods used in the observations.

Reference points and readings to determine longitudinal and lateral movements of the rail along the track had been included in the program undertaken by the engineers of the railroad companies; it was thought not necessary to duplicate these observations. The method of test chosen was based on strain-gage measurements longitudinally along the web of rail (8-in. or 10-in. gage lengths) on gage lines distributed throughout the length of the stretches, with simultaneous measurement of the rail temperatures along the stretch and frequent comparison of the strain-gage with reference bars or comparators. The strain-gage measurements were made at several rail temperatures in summer and winter. The rail temperature was found, accurately and quickly, by an open-ended thermo-couple and portable potentiometer. An invar reference bar of known expansion coefficient, a short piece of rail similar to that in the track, and, later, a compensating comparator giving almost no variation in length with changes in temperature were used in the tests; temperature and temperature correction for both rail and instrumental equipment, of course, were of extreme importance in securing accuracy of data.

As an example of the distribution of the gage lines, their spacing on the mile stretch of the Bessemer and Lake Erie Railroad may be cited. Beginning at the ends of the welded stretch, gage lines were placed on both rails, at distances from ends, of 3, 10, 30, and 58½ ft; then at intervals of one rail length to the tenth rail; then at distances of two lengths to the twenty-fourth rail length; and finally at distances of four rail lengths to about the mid-point of the welded stretch. Additional gage lines were placed at points of special interest by reason of curves and other local conditions.

It was not convenient to make observations at the time of the rail laying; besides, later adjustments by track men and other changes, would have rendered data taken then of doubtful value for the purposes of the investigation. Observations were made in September, 1935, on one stretch of track of the Delaware and Hudson Railroad Corporation at Albany, N. Y., and on two stretches at Schenectady, N. Y.; the length of these welded stretches varied from 0.5 mile to 1.3 miles. Further tests on these stretches were made in January, 1936, August, 1936, and January, 1937.

Observational tests have likewise been made on the mile stretch of welded G.E.O. track of the Bessemer and Lake Erie Railroad Company near Pittsburgh, Pa., in July, 1936, and February, 1937. The rail on the Albany stretch was 130-lb and on the others, 131-lb. The alignment of these stretches contained various curves from 1° 30′ to 7 degrees. In the summer tests, rail temperatures were made as high as 120° F and, in the winter tests, observations were made at rail temperatures as low as 13° F. Tests were also made at medium temperatures. Observations in the early morning, or on cloudy days, were useful to form a basis of comparison for the changes in length in the rail. Generally, check sets of readings were made along the stretches.

A few of the results of the tests on the Bessemer and Lake Erie Railroad will give an idea of the nature of the information obtained in all the tests. In Fig. 9 are plotted the changes in length observed along the mile stretch for two representative series in summer and two in winter. The position along the stretch is shown in terms of rail lengths of 39 ft each. Naturally, the temperatures of the rail at any time varied somewhat as the conditions of shade and exposure varied, and these variations affected the stresses and the movements. The rail temperatures along the track for the series shown were 108° to 118°, 59° to 60°, 26° to 33°, and 13° to 19°, F. As the changes in length from one temperature to another are quite minute, for ease in picturing values the unit strains (inches per inch) have been translated into equivalent stress per square inch in steel for representation on Fig. 9—that is, that stress which would be developed

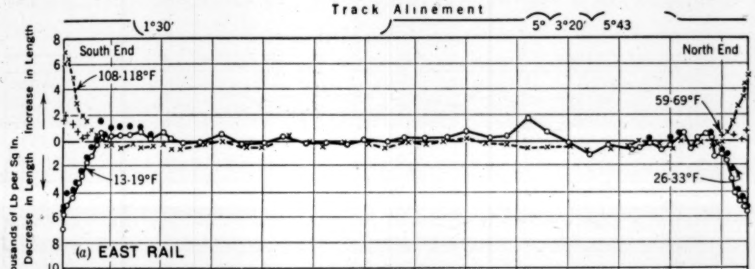

FIG. 9.—CHANGE IN LENGTH IN TERMS OF STRESS (BASE, 53° F, AND WELDED LENGTH, 1.0 MILE).

in the rail if an external force had produced the given change in length without any change in temperature. The observations before sunrise on a day in July with a rail temperature of 53° F, were taken as the basis or zero line for the diagram.

It will be noted that throughout the intermediate part of the welded stretch (about 0.9 mile) very little change occurred—only small local changes—the rail having been held closely to one length through all these variations in temperature. As the differences are generally so small, not all the points for the four series are plotted on the diagram. For the end portions of both rails, and for both the summer and the winter tests, the rails changed length through an average distance of about seven rail lengths. This change in length increases rather regularly from the points at the end of the intermediate portion (which may be thought of as the point of fixation) to the end of the welded stretch, showing a lengthening in summer and a shortening in winter, with some rounding off in the line of the end portion as it approaches the intermediate portion. The data indicate that the magnitude of the winter and summer temperatures has not particularly affected the length of track over which expansion and con-

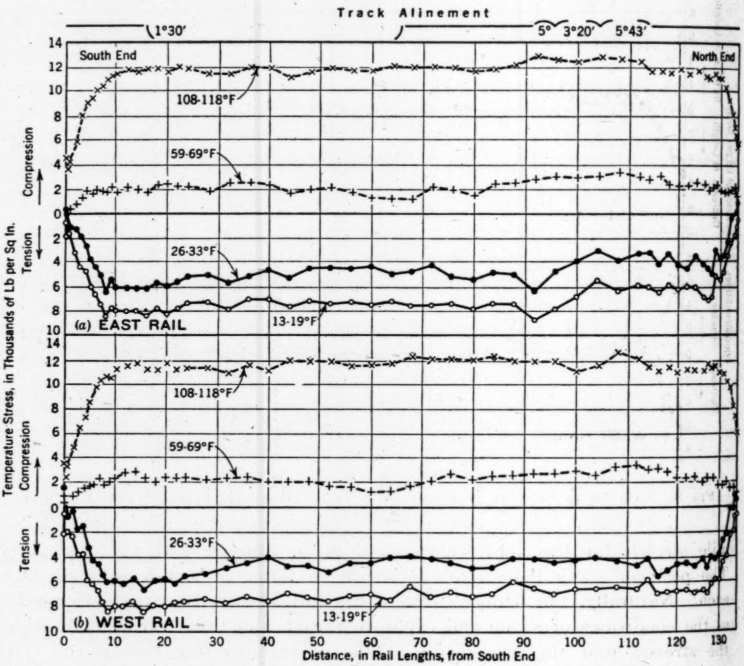

FIG. 10.—TEMPERATURE STRESSES IN RAIL (BASE, 53° F, AND WELDED LENGTH, 1.0 MILE).

traction occurs in each end portion, and that the long intermediate portion remains practically fixed in position with the various changes in temperature at the time the observations were made. It is evident that under these conditions there is no pull or drag on the ties of this intermediate portion, except as may be caused by differences in exposure and other local conditions.

In Fig 10 are recorded the stresses developed in the rail throughout the length of the stretch based on the rail temperatures and strain-gage readings at the time of observation as compared with readings at a base temperature of 53°F, which were taken before sunrise in the summer of 1936 with temperature conditions very uniform over the entire stretch. For the summer observations, at an average temperature of 113° F, the relative compressive stresses over the intermediate 0.9 mile are about 12 000 lb per sq in., and for the winter observation at a temperature of about 16° F, the relative tensile stresses average somewhat less than 8 000 lb per sq in., both values being based upon the aforementioned readings at 53° F; this means that the zero reference line represents the stress at 53° F. If the reference line were placed at 63° F, the temperature at which the rails were originally fastened in the track (the difference of 10° in temperature representing approximately 2 000 lb per sq in.), the corresponding compressive stress on the summer test would be approximately 10 000 lb per sq in. and, for the winter test, a tensile stress of approximately 10 000 lb per sq in. For summer temperatures higher than those observed and for lower winter temperatures, in accordance with other data obtained, the stress may be expected to increase in proportion to the increase in change in temperature from the 63° base. For the winter tests, the end portions show a tapering off of stress over the distance of, say, seven rail lengths to nothing at the end. For the summer tests, there was a similar tapering off in seven rail lengths to a stress of 2 000 to 6 000 lb per sq in. at the end. This end compressive stress was doubtless transmitted through the joint connecting the welded stretch to the regular track beyond. Incidentally, it may be remarked that joints have been found to transmit tensile and compressive forces to the adjacent rail equivalent to as much as 4 000 and 6 000 lb per sq in. of rail section, respectively.

The data given in Figs 9 and 10, corroborated by data of the track tests at Albany and Schenectady, indicate that within the range of temperature observed the magnitudes of the change in temperature for different series having different temperature changes do not particularly affect the length of track over which expansion and contraction takes place at each of the end portions. It is apparent, too, that the central part of the stretch has remained practically fixed in position through the summer-to-winter interval. The change in length and in stress over the end portions of about seven rail lengths, is fairly regular with some rounding off near the point of fixity, although, of course, there are changes in position from time to time at the extreme ends of the welded rail and the movement here is not always regular. The length of the end portions which give anchorage to the intermediate portion, and the stress set up in the intermediate portion correspond to an average anchorage or restraint of about 600 lb per tie per rail in each direction for the temperatures of 113° F and 16° F as counted above and below a laying temperature of 63° F. If the anchorage length remains fairly constant for temperatures above or below the aforementionad range as is indicated to be the case by the data of the various tests, the corresponding anchorage force would be proportionally greater than 600 lb.

It may be added that no indication was observable of any visible movement between the ballast and ties one way or the other at the times of the tests or of any particular change in length of the anchorage developed. It was found, however, that breakage in imperfect welds and readjustment of track in cool weather have, in places, changed the conditions of stress and anchorage considerably. It is not known, of course, what the ultimate shearing strength of the ballast bed may be. The track seems to have kept close to an elastic condition of strength.

As the data of the tests on this stretch of welded rail, and also on the other stretches tested, show that the rate of anchorage is fairly uniform over the end portions, Assumption (1) in the analyses is borne out by the tests. The tests, however, indicate that the length of the average anchorage portion remains about the same for different values of the change in temperature. This conclusion is not in accord with Assumption (2) of this discussion referred to previously as having been made by the writer in his earlier study of the problem, nor with an implied assumption of the paper that in a given track the anchorage force is a constant regardless of the value of the difference in temperature. In other words, the tests indicate that T is not the same for different values of Δt, but varies nearly as Δt.

As a consequence, it follows that Equations (8) and (9) and others in the latter part of the paper are true only when the value of T used corresponds to the given Δt. It also follows that Item (2) in the second paragraph following Theorem 1 (that the expansion varies as the square of the temperature difference for a given track) is not warranted. The tests on the tracks of the Delaware and Hudson Railroad and on those of the Bessemer and Lake Erie Railroad indicate that the expansion at the end of the welded stretch varies as the first power of the change in temperature rather than as the square, and also that the length required to develop the force, F, is largely independent of the magnitude of the change in temperature, referring, of course, to more than slight changes in temperature. Care should be taken in the use of the formulas given in the remainder of the paper where the reader may be led to think that T is implicitly used as constant for a given track. Caution in this respect is suggested in the use of Equations (18), (20), and (21). It may be implied from the wording in Example 4 that the value of T is the same for a temperature change of 100 ° F as for one of 58° F. This assumption is not in accord with the results obtained in the Stresses in Track Investigation. These several inaccuracies in the analysis and its applications affect the usefulness of the "theory" in a way that seems to warrant calling the attention of the reader to them.

These comments are made in the light of what has been learned so far in the observational tests of four stretches of long welded track, all of which are in a sense experimental. It is planned to continue the seasonal tests to learn the changes that occur at later dates when time and traffic may have had their effects. As stated by Mr. Africano further fundamental knowledge of the physical action of welded rail in track is needed. It will take time to secure this information. Much more must be learned of welds before welded track, with its higher investment in welds and substructure, will be generally accepted as a standard for high-grade track. That interest has been aroused in this type of construction is shown by the starting of an investigation on continuous welded rail at the Engineering Experiment Station, University of Illinois, through a co-operative agreement with the American Railway Engineering Association and the Association of American Railroads. The investigators will study the strength, ductility, and durability of continuous welded rails made by different processes. They will doubtless find the quality and uniformity of the metal at the welds and at each side of them as well as the uniformity of surface and line and the wear of the joint under repeated rolling loads for both sample joints and joints cut from track.

It may not be out of place to suggest that Mr. Africano's recognition of the desirability of giving the track anchorage throughout the entire length of the welded stretch might well be made much stronger. Without continuous rigid support, breakage of rail in the intermediate portion of the welded stretch and other accidental events might lead to serious trouble. To make a further comment: The coefficient of linear expansion of steel varies over quite a range; the International Critical Tables give values in the range of rail steel from 0.0000055 to 0.0000072. For 131-lb rail tested at the University of Illinois, Urbana, Ill., the coefficient was found to be 0.0000063. This differs appreciably from the value, 0.0000073, given in the paper.

The author is to be commended for putting forth an analysis in a new field, even without having adequate experimental evidence of the action of continuous rail in railroad track.

Randon Ferguson,[22] Assoc. M. Am. Soc. C. E. (by letter).—As a member of the test party of the Special Committee on Stresses in Railroad Track of the Society and the American Railway Engineering Association, the writer has been engaged on tests on several sections of long welded track. A short discussion on some of these tests has been published elsewhere.[23] Further discussion and description of various installations are also available in the reports of the Committee on Rail of the American Railway Engineering Association.[24]

Although the tests are not complete at the present time certain comments of a tentative nature may be made as to the results. It is doubtful whether the nature of the tie resistance is frictional, as assumed by the author. A frictional resistance would imply that the value of the tie resistance, *T*, is a constant for any given track for all temperature changes if end movement occurs, as it apparently does, and that the length of track giving anchorage would depend on the temperature change. Instead, the tests on the track of the Delaware and Hudson Railroad Company and the Bessemer and Lake Erie Railroad Company indicate that the degree of tie anchorage on a given track varies with the magnitude of the temperature change, and that the length of track giving anchorage remains constant for all changes of temperature both winter and summer. The tie resistance has also been found to vary somewhat near the ends for a given temperature; or, in other words, the force or stress diagram is not necessarily a straight line, as the author indicates in Equation (3). The tracks tested have been found to be quite completely anchored except for the end portions where the periodic expansion or contraction due to temperature occurs over a distance of about five to twenty 39-ft rail lengths.

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[23] *Proceedings*, A. R. E. A., Vol. 37, 1936, p. 954.

[24] *Loc. cit.*, p. 465; and, also, *Bulletin 393*, A. R. E. A., p. 493.

It should be stated for the benefit of readers who are not intimately familiar with the existing installations of continuously welded rail, that as far as known all installations of welded track for standard railroad service have special fittings designed to anchor the rail firmly to each tie. A clip and bolt fastens the rail to the tie-plate on both sides of the rail and the tie-plate is fastened to the tie with screw spikes. This greatly increases the available longitudinal restraint as compared with the usual practice of applying three to six rail anchors per rail. Ordinarily, the rail anchors are effective in one direction only. Of course, the fastenings must be provided over the entire stretch whether or not they are doing any work. Any rail breakage, in a weld or elsewhere, at once requires the track adjacent to the break to be capable of withstanding forces of the same magnitude as those occurring at the normal end portions.

The tests have indicated the development of tie resistances of about 600 lb per tie per rail in each direction, with a rail temperature range of 50° above or below the laying temperature. The limiting value of the tie restraint is not known from the tests. Any appreciable slippage observed has been through the clips, and this occurred in a few cases when a rail broke or the end joints were loosened for adjustment. The ballast in all cases was snugly bedded against the ties with no evidence of the tie working back and forth along the track.

The range of temperature of the rail may be as much as 150° over a period of a year. In the absence of sunlight or protective covering, such as snow, the rail follows the air temperature quite closely. On a bright day, either winter or summer, the rail may be as much as 40° warmer than the air temperature if there is no breeze. With snow around the rail, the rail temperature has been found to be 10° to 15° warmer than the air temperature, thus minimizing the effect of low air temperatures.

A. N. Reece,[25] M. Am. Soc. C. E. (by letter).—In directing attention to factors influencing the amount of rail expansion, that have quite generally been neglected in the past by engineers in the United States, this paper serves a useful purpose. Not only does the new theory that Mr. Africano describes apply to long welded rail installations on the rigid type of track, but it shows equally well that expansion requirements for rails of ordinary length, secured only with cut-spike rail fastenings, are appreciably less than the theoretical requirements based upon the assumption that the rail is free to expand without restraint.

For example, in a 131-lb rail having a cross-sectional area of 12.86 sq in., a force of $12.86 \times 0.0000065 \times 30\,000\,000 = 2\,500$ lb, is required to restrain the rail fully through a temperature change of 1° F. Assuming that six-hole joint bars, with bolts tightened, will develop sufficient frictional resistance so that a force of 75 000 lb is required to slip the rails within the bars, then, for a temperature increase or decrease of $\frac{75\,000}{2\,500}$, or 30° F, there would be no rail end movement. This joint restraint would then be sufficient to prevent rail expansion or contraction through 60° of the seasonal variation in rail temperature, or almost throughout one-half the range. Thus, only one-half the expansion allowance, calculated for the rail without restraint, need be allowed. To secure full advantage of this condition, however, it would be necessary to lay the rail at temperatures ranging from less than 30° of the maximum to more than 30° of the minimum.

The use of longer and continuously welded rail offers a prospect of material improvement in the track structures, and studies, such as that of the author, are invaluable in acquainting engineers with a new field into which they are destined to "travel."

G. M. Magee, Esq.[26] (by letter).—In recent years, the expansive action of rails subject to joint and tie restraint has been given intensive study in Germany by several investigators. In addition to theoretical analyses of the temperature change and restraining forces, considerable research work was conducted to determine the actual values of joint and tie restraints. The adoption of a new track construction as standard on the German National Railways in 1925, provided a tie-plate fastened to each tie with four screw spikes, a compressed wooden shim between rail and plate, and a firm fastening of rail to each plate by two clamp bolts, clamps, and double-coil spring washers. This construction has also been introduced in the United States.

It was found that with this type of fastening, the rail did not slip through the plate, but that where longitudinal movement of the rail did occur, the rail, ties, and ballast within the cribs moved as a unit, presumably slipping along a horizontal plane at the bottom of the ties. In 1929, Ammann described[27] an interesting experiment in which a panel of track was especially constructed on crushed rock ballast, a concrete buttress placed at one end, and known longitudinal forces applied to the rail ends by hydraulic jacks, in order to determine the tie resistance to longitudinal movement of the rail. Later investigations were made on track in traffic use and under different weather conditions. A tie resistance of 881 lb per running meter of rail was observed for 24 wooden ties per 15-m rail length.

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As a result of the analysis of expansion movement with proper consideration of tie and joint restraint the rail lengths in use on this type of track construction were generally increased from 15 to 30 m (approximately, 49 to 98 ft, respectively) by welding two rails when the old rail was serviceable, or by purchasing new rail of 30-m length. Trial installations were also placed of 60 and 120-m lengths.

Long welded installations of rail have been placed in open track in the United States on two railroads. The first installations were placed on the Delaware and Hudson Railroad, using a type of construction similar in principle to the German fastening, but differing principally in that two spring clips and bolts secured the rail to the tie-plate and no shims were used. An installation of long welded rail has also been placed on the Bessemer and Lake Erie Railroad, using the German type of construction.

The new theory of rail expansion described by Mr. Africano explains the apparent phenomena of placing rail with no provision for expansion and shows that, in this welded construction, expansion movements may be analyzed by known engineering principles.

Although, as Mr. Africano points out, the rigid rail fastenings are required only at the ends of long welded rail to restrain the expansion, it should not be thought that these fastenings are not needed at the intervening ties. Their function is a highly valuable one at the intervening ties to prevent excessive opening of the rail in the event of rail breakage in cold weather and to resist track buckling in hot weather. Should the track begin to buckle at a point of weakness, the rail is held from running into this point of weakness and a "sun-kink" is thus prevented in its beginning stages.

The rail stresses due to restraint of the expansion must not be disregarded. Should rail be laid, welded, and secured in the hottest weather, a maximum tension stress due to temperature effects in the coldest weather of approximately 30 000 lb per sq in. should be anticipated. This would be additive to the rail bending stresses and would place too large an added burden on the rail. The laying of this type of track at mean temperatures would equalize the disadvantages of stress due to rail tension in winter and track-buckling forces in summer. The maximum rail-tension stress of 15 000 lb per sq in., due to temperature effect, which would be developed in this method of laying, is probably not too severe for the heavier rail sections that weigh 131 lb per yd, or more.

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[27] "Latest Investigations in Regard to the Effect Which the Longitudinal Forces Exert on the Rails", by Prof. Dr.-Ing. Ammann, Karlsruhe, Germany, *Technical Weekly Traffic Review*, Issue No. 47, Berlin, November 20, 1929.

The effect that wave action of traffic may have on tie resistances for long welded track are not yet well determined. Preliminary measurements indicate that with traffic in one direction, the rail is likely to show somewhat more movement at one end of the long welded rail than at the other. It does not seem, however, that this will appreciably alter the analytical determinations.

There is also some question as to whether the tie resistance to horizontal movement may be considered as an elastic resistance similar to the modulus of track resistance, *u*, to vertical depression. It is to be expected that the tie resistance is more in the nature of a frictional, rather than of an elastic, resistance.

The author is to be commended for his invaluable analysis of this new and important subject.

H. D. HUSSEY,[28] M. AM. SOC. C. E. (by letter).—The assumption is made in this paper that long welded rails are laid in tangent, level track, and the author has developed a method of computing the stress in the rails and the end expansion, for a change in temperature and a known tie resistance. When the rails are laid on a curved alignment, the radial component of the stress in the rails produces a force which tends to displace the ties laterally. This radial force on each tie is:

$$F_r = \frac{2\,s\,A\,c}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

in which, in addition to the notation of the paper, F_r = radial force on one tie, in pounds; and r = radius of curvature. Mr. Africano has demonstrated that $s = 219$ lb per sq in. for 1° change in temperature. It is seen that F_r decreases as the radius of curvature increases, and that, in a straight track, it becomes equal to zero. For example, assume that 131-lb rails are laid on a 1° curve and that the tie-spacing, *c*, is 22 in. Then, by Equation (29), $F_r = 1.80$ lb for 1° change in temperature. For 100° change in temperature, $F_r = 180$ lb. This force must be added to (or subtracted from) the radial force on the tie that is due to a train passing around the curve. When the temperature is higher than that at which the rails were laid, these forces will be additive, but when it is lower, they will counteract each other.

Equation (29) can be used in the computation of vertical forces exerted by rails that are curved in a vertical direction, as in the case of vertical curves at the top of a hill or the bottom of a valley. A horizontal and a vertical curve occurring at the same point is an unfavorable combination.

Research on long welded rails should include the determination of the lateral resistance of ties in track on curved alignment and the action of curved rails.

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[28] Designing Engr., Am. Bridge Co., New York, N. Y.

ALFRED AFRICANO,[29] ASSOC. M. AM. SOC. C. E. (by letter).—Those who have been kind enough to discuss this paper have added immeasurably to its value by clarifying some points perhaps too briefly treated, by contributing their experience with continuous welded rail in support of the theoretical analysis, and, finally (and of equal importance), by emphasizing certain objections to the fundamental assumptions made.

Although most of the discussion indicates an acceptance of the simple mathematical derivation of the formulas as valid for practical purposes (leaving to the judgment of the engineer who uses them the determination of the best average resistance, T, per tie—as the writer intended), the objections offered by Professor Talbot seem sufficiently important to be given first attention.

Of the questions which Professor Talbot raises the principal one seems to be the uncertainty as to whether or not the longitudinal resistance set up at each tie is a constant. It is not a constant. It varies under the influence of so many factors that the writer purposely avoided attempting to assign any quantitative value to those factors at all. If a simplifying average or effective value of the tie resistance under all conditions can be used in such a manner that the theoretical analysis will show a practical agreement with observed end movements, its use is obviously justified.

The experimental data given for the two half-mile rails of the Delaware and Hudson Railroad Company at Mechanicville, N. Y., were the average of forty reported observations of end movements made over a period of 1 yr, for various temperature changes. It is extremely interesting and instructive to plot these individual results against Δt, the temperature change in degrees Fahrenheit from the rail-laying temperature. This has been done in Fig. 11(a). For comparison, the curves of theoretical end movements obtained by the use of Equations (9) and (18) are also shown.

The scattering of the observed points indicates the difficulty of attempting to formulate an exact equation accounting for every variation and, consequently, the desirability of using the simplest possible analysis that will give practical results. Equation (9), based on a constant value for T, apparently gives such results. The important consideration here is that the theoretical or predicted expansion or contraction is approximately 1 in., as observed for the ordinary range of temperature change, and not the large value heretofore calculated by assuming free expansion. That the fear of dangerous free expansion actually existed at the time these rails were laid, is proved by the fact that an elaborate expansion joint was provided by the engineers of the Delaware and Hudson Railroad Company to take up the maximum of 22-in. end movement computed in this way if one end remained fixed and the other received the total expansion.

Equation (9) is plotted with $T = 2\,100$, whereas Equation (18) is plotted with $T = 600$ and a joint restraint of 35 000 lb, or about 3 000 lb per sq in. transmitted by the joint (averaged from Fig. 10 given in Professor Talbot's discussion).

Although joint restraint no doubt exists, it is too erratic to be depended upon and, in the writer's opinion, should not be used at all in designing fasten-

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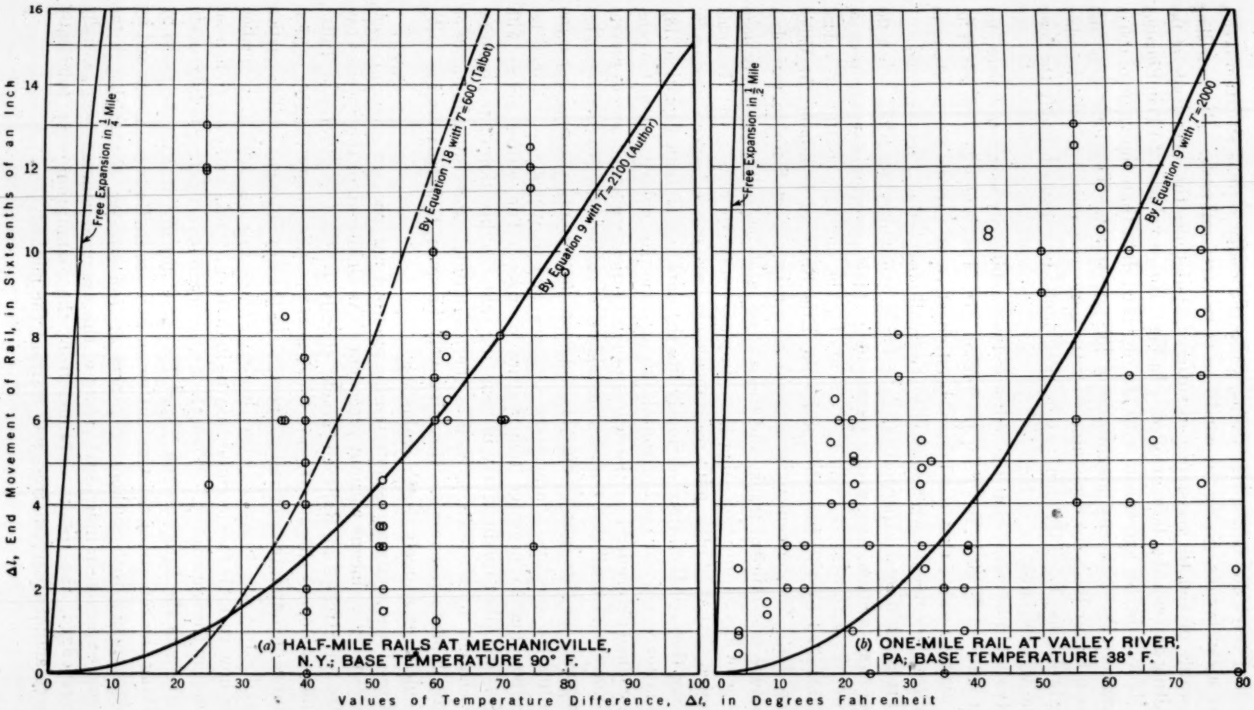


(*a*) HALF-MILE RAILS AT MECHANICVILLE, N.Y.; BASE TEMPERATURE 90° F.

(*b*) ONE-MILE RAIL AT VALLEY RIVER, PA; BASE TEMPERATURE 38° F.

FIG. 11.—COMPARISON OF THEORETICAL WITH OBSERVED END MOVEMENTS OF LONG RAILS

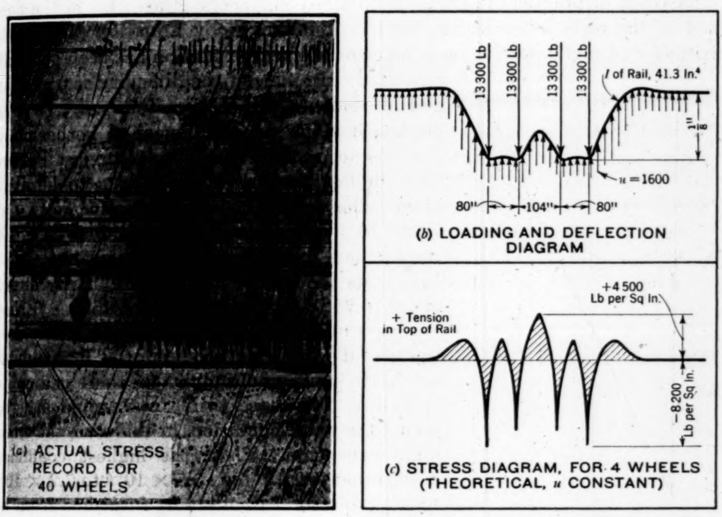
ings and track for new installations, since the really serious situation occurs when a rail breaks, giving the condition of two free-rail ends. By controlling the value of the tie resistance in future installations the magnitude of the resulting gap can also be controlled. In Fig. 11(*a*), a value of 2 100 in Equation (9) shows a better agreement with the observed data than the value of 600 in Equation (18) with any magnitude of joint restraint. The high value also gives a better basis for explaining why the gaps were only 1 in. or 2 in. when rails broke instead of the 3 in. to 6 in. that should be expected if only the lower value was available.

It should be noted that the writer did not arbitrarily assign or "place" the high value of 2 100 lb for T for the reason Mr. Hunt suggests, but solved for the average value that must have been effective at various temperatures to produce the evident correlation with the small observed end movements.

From additional experimental data made available since 1935, when the paper was written, the writer plotted Fig. 11(*b*) showing a comparison of theoretical with observed end movements for the one-mile continuous welded rail installed by the Bessemer and Lake Erie Railroad Company, at Valley River, Pa. The agreement is as good as can be expected for so variable a structure as a railroad track, and the scattering of the points again precludes the use of any assumption other than simple constant tie resistance. It should be noted that very little joint resistance must have existed since Equation (9) passing through the origin obviously fits the data better than Equation (18). The latter would cut the horizontal axis at some particular value of temperature change determined by the degree of joint resistance which acts to prevent any movement until that temperature change is exceeded. No distinction has been made in this graph (Fig. 11) between movements (expansions) corresponding to positive temperature changes and movements (contractions) corresponding to negative temperature changes. The writer agrees with Professor Talbot qualitatively in believing that the direction of the temperature change, up or down, as well as the magnitude, exerts some influence on the available tie resistance. Quantitatively, however, to judge from Fig. 11(*b*), it would seem scarcely worth evaluating.

Use of a simplifying assumption for practical purposes under such circumstances is by no means without precedent. As Chairman of the Special Committee on Stresses in Railroad Track, already referred to, Professor Talbot used an analogous simplification in the case of the exceedingly variable vertical modulus of foundation, or rail support, u, in the development of the equation for rail stress due to static wheel loads.[5] Those familiar with this equation know how variable this modulus may be in different tracks and even from point to point in the same track. When the writer began investigating rail stresses in the tracks of the Interborough Rapid Transit Company, in 1931, he found by studying the reports of the Committee on Stresses in Railroad Track and by correspondence with authors of papers on rail stresses, that although the extreme variation in individual values of this assumed constant was generally acknowledged, the results obtained from the assumption were sufficiently close to actual observed stresses to warrant the method.

To illustrate this point, the similarity is shown in Fig. 12 between actual stresses measured by the writer in 1934 with a scratch extensometer and the theoretical static stresses from Equation (24) by using a constant value of $u = 1\,600$ and superimposing stresses due to the four wheels. The scratch record was made while the 10-car train passed slowly over the instrument, recording actual strains in the steel, and later was magnified 100 times and photographed. Since the recorded stress represents the effect of different values of the vertical elasticity of the track, the agreement based on a constant value is surprisingly good.



(a) ACTUAL STRESS RECORD FOR 40 WHEELS

(b) LOADING AND DEFLECTION DIAGRAM

(c) STRESS DIAGRAM, FOR 4 WHEELS (THEORETICAL, u CONSTANT)

FIG. 12.—ILLUSTRATION OF ANALOGOUS CASE IN WHICH MODULUS OF FOUNDATION, u, AS A CONSTANT GIVES PRACTICAL RESULTS FOR RAIL STRESSES DUE TO WHEEL LOADS

Referring again to Fig. 10, an independent conclusion may be reached as to whether or not the tie resistance varied appreciably for the same, and for different, temperature changes. If it is constant, the accumulating longitudinal force should increase uniformly along a straight line, and this the graph shows.

By noting that the slopes of the lines showing the accumulating stress in Fig. 10 are approximately 30° from the vertical, for three of the four temperature changes shown, the conclusion may be drawn that, as far as practical results are concerned, this assumption is valid for all temperature changes. The rounding off near the top of the sloped line is to be expected, since, as the writer was careful to point out, there is no doubt a slight effect, more pronounced at this point, of the longitudinal resistance being proportional to the longitudinal movement. As the accumulating resisting force approaches that required to fix the rail completely, the movement of the rail diminishes to so small an amount that the tie resistance does not have a chance to come into action, especially if there is any looseness in the ties. In other words, there may be a limiting degree of movement above which the tie resistance is fairly constant and below which the longitudinal restraint is more nearly proportional to the rail movement. In the writer's opinion, the latter effect is a negligible factor in the computation of the end movement.

A point of practical importance to many engineers—the stress in the fixed part of the rail—seems satisfactorily taken care of by Equation (2), but the writer's choice of value for the coefficient of linear expansion has been generally criticized. In view of the difficulty of predicting end movements within more than, say, 75% accuracy, it seems poor judgment to bother about extreme accuracy in this one factor just because more or less precise values happen to be available. The writer chose the value for hard steel, 7.3×10^{-6}, given in the steel handbooks, not to recommend its use particularly, but because it better fits the case of rail steel ordinarily having about 0.75% of carbon with a higher Brinell hardness number than the structural steel which has about 0.33% carbon and the lower value, 6.5×10^{-6}. In addition, where there is a choice of values, the conservative one would be that giving the worst condition, in this case, the maximum expansion. Thus, the highest coefficient in the range for rail steel, 5.5×10^{-6} to 7.2×10^{-6}, which Professor Talbot kindly furnished from the International Critical Tables, should unquestionably be used.

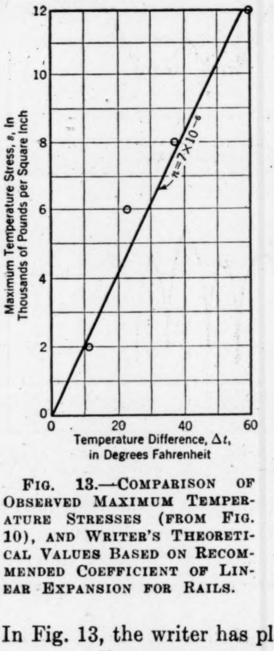


FIG. 13.—COMPARISON OF OBSERVED MAXIMUM TEMPERATURE STRESSES (FROM FIG. 10), AND WRITER'S THEORETICAL VALUES BASED ON RECOMMENDED COEFFICIENT OF LINEAR EXPANSION FOR RAILS.

The handbook value for hard steel checks this maximum coefficient very closely. From the temperature stresses for the fixed part of the rail shown in Fig. 10, it is possible to compute an average value for actual track conditions. In Fig. 13, the writer has plotted the observed temperature stress against the corresponding average temperature differences from the base of 53° F for each case, choosing the straight line that passed nearest the data as the best stress-temperature relationship. The slope of this line is the same as that for the straight-line Equation (2), or $E\,n$. Substituting a constant value of 30 000 000 for E, and solving for n results in the value, $n = 7 \times 10^{-6}$, for the best actual track value of the coefficient of linear expansion in these cases, as against the laboratory test value of 6.3×10^{-6} which Professor Talbot favors. This calculation corroborates the higher value used by the writer, and, in his opinion, shows that the inaccuracy involved is not only negligible, but that the round value of 7×10^{-6} is entirely adequate for the degree of precision warranted and should be used in all computations of end movements for long rails.

The remaining points raised by Professor Talbot can be considered more briefly. It is to be expected that the special problems involved in the maintenance of long welded rails will have to be solved satisfactorily before they come into general use. Nevertheless, since it is known that they do not expand or contract as free rails, and have been in actual use for several years, as pointed out in Mr. Baldridge's valuable contribution of historical background, they are no longer experimental except for these problems. If this paper serves only to show to the Engineering Profession that the existing statistical evidence in favor of the practicability of long welded rails is supported by known engineering principles even if the theoretical analysis may not be completely satisfactory, the writer's intention has been fulfilled.

Regarding the use of calculus for deriving what proves to be a simple second degree equation, this is the usual method for determining the integral or total effect of a variable factor.[30] The writer took special care to show every step of the precise reasoning upon which the resulting formulas are developed. It may seem unnecessary to those who have devoted years of study to a specialized subject.

Figs. 9 and 10 are important in showing conclusively the existence of the fixed intermediate part of the rail as predicted by the photographs of the writer's elastic model shown in Fig. 2. The use of "equivalent stress per square inch" for the measured unit change in length may be confusing to some readers. Since a temperature change requires the length to change to avoid becoming stressed, it might have been more informative to plot the actual data to some convenient scale, especially since its use is desired for developing a general principle. Such data, if obtained near the ends of an intentionally broken rail, along with the corresponding end movements would yield valuable information on the actual maximum holding power of the rail fastenings.

In cautioning readers in the use of Equations (18), (20), or (21) (or for that matter any equation or its application), the writer heartily concurs with Professor Talbot. Both the derivation of the equations and their applications, the writer agrees, are too new to be plucked blindly from a handbook. It is of equal importance to point out that inaccuracies may be conceded to exist, and yet be so negligible as not to affect the usefulness of the theory for all practical purposes.

Mr. Gailor appears to be satisfied with the calculations and formulas as a basis for future studies but states that the possible out-of-alignment of a long rail on a hot day is a different condition from the straight line assumed in the paper. It is true that for the purpose of analysis the writer used the case of straight tangent track. However, even if the rail is laid in the winter, which is not recommended, the buckling that may be caused vertically and laterally by excessive expansion the following summer is not likely to affect the end movement appreciably, and remains as a problem to be solved by providing proper lateral resistance.

Statement (6) by Mr. Gailor seems to show a misinterpretation of one of the fundamental points of the analysis, namely, that the temperature stress set up in a rail when fixed is independent of the length and would be the same whether for one tie-spacing or for one mile. Since it may be of the order of

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[30] "Mechanics of Materials," by L. A. Martin, p. 16. Here a similar use of calculus is made to determine the extension of a cylindrical rod due to its own weight. The interesting and analogous result is that the extension is one-half that which would occur if the total weight were concentrated at the lower end.

200 000 lb it is impossible for any single tie-fastening to resist. The cumulative resistance is the only remaining explanation for the fact that such large forces must have been developed to restrain the long rails in present use. This cumulative feature of the tie restraint has been well established by the experience presented in several of the discussions.

The writer regrets having given the impression that he would have considered using rigid fastenings at the end portions only; yet according to the data submitted by Mr. Walker in Table 5, even ordinary spike fastenings prevented the ends of broken rails from separating "in excess of 2 in." so that the fear of "disastrous results" expressed by Mr. Gailor and Professor Talbot in such a case seems to be unfounded. However, the writer would recommend using some relatively economical type of rigid fastening for the mid-section and the best available rigid fastenings at the ends. In this way the definite end movements occurring with temperature changes are reduced to a minimum, thus avoiding trouble with the insulated joints which, to a great extent, will determine the practical maximum lengths of welded rail. The number of rail breaks for standard rail is of the order of 1 per track-mile in 5 yr of service. The temperature stress may increase this, but it will depend on the experience and judgment of the engineer as to what degree of rigidity and extra expense he will consider absolutely necessary to offset the risk of this occasional failure.

Mr. Kenney believes the vital problem is not so much the control of linear expansion as the control of lateral expansion without, possibly, increasing the maintenance cost. Since the scope of the paper was limited to the discussion of temperature stresses and rail expansion the writer will not attempt to prove any economy in the use of welded rail. Such conclusions depend to a great extent on the additional life of the rail due to elimination of end batter, and this factor requires several years to determine statistically.

The case mentioned by Mr. Baldridge of the shearing of joint bolts following a sudden drop in temperature is significant as it indicates the need for investigating even existing standard track for the possibility that it may act as a continuous rail. The writer has observed frequent cases in which rails have "ganged up" or closed their gaps in warm weather and then have acted as a unit in ensuing cold weather, in one case permitting a gap to open almost an inch.

Applying the analysis to the data given by Mr. Baldridge of a 2.5-in. maximum gap opening for a temperature difference of 100° F it is possible to estimate the length of anchorage and the average tie-resistance that had to come into action in this case. A 2.5-in. gap means that 1.25 in. of contraction occurred in the critical length on each side of the joint. This is the free expansion for 160 ft of rail, as Mr. Baldridge calculates, by substituting $\Delta l = 1.25$ in., $n = 6.5 \times 10^{-6}$, and $\Delta t = 100°$ F, in Equation (1), and solving for l. However, this length, l, represents expansion or contraction without restraint, and since the actual degree of expansion varies from that for a free rail at the gap end, to zero at the opposite fixed end, the average rate of expansion is only one-half the free expansion as the writer demonstrates by Equation (9) and Theorem 1 in the paper. Consequently, the length of the anchorage must have been twice 160 ft, or 320 ft, on each side of the joint.

Assuming 2-ft tie-spacing, the average longitudinal resistance of 160 ties will be available to equal the total temperature stress set up when the rail is finally fixed. The unit stress from Equation (2) is 19 500 lb per sq in., if n is taken as 6.5×10^{-6} and E, as 30×10^{6}. Multiplying by the area of the 112-lb rail (11.05 sq in.), the total temperature stress to be resisted by the ties is about 216 000 lb, which, divided by the 160 ties, gives as the probable effective resistance of each tie 1 300 lb. Equation (9) would have given this value immediately since all other quantities are known.

Mr. Hunt emphasizes the fact noted by the writer that Equation (9) is simply a special case of Equation (18) when the joint resistance, P, is equal to zero. He contributes a useful graphical solution in Fig. 5 for this general case. The writer developed the "special" case first for two good reasons: It is logical to start with the simplest case; and since this tells the maintenance-of-way engineer what maximum gap opening to expect, it is the most important case.

Table 3 is useful to show the effect of joint restraint on the calculated value of the tie resistance. Both Messrs. Hunt and Magee believe the influence of joint restraint was under-estimated, thus leading the writer to what may be considered too high a value for T in the illustrative examples. These examples show that two different values may be obtained by using a definite joint restraint and by considering the gap at a free end. Hence, it should follow that the correct application of Equation (18) at a joint to determine the tie restraint depends upon an accurate knowledge of the degree of joint restraint. It is much simpler to obtain accurate values for the maximum available tie resistances, by reproducing the condition of a free end. Thus, by loosening the joint bolts and noting the free end movement, the elementary Equation (9) can be applied. Purposely breaking the long rail at some point on the fixed length would be an accurate method of obtaining the data to guard against such an accidental contingency. The effect of the joint restraint on the magnitude of the temperature stress seems of little importance since this stress becomes a maximum in the fixed part of the rail regardless of whether or not joint restraint exists. Its only effect on the end movement is to decrease it.

In Table 4 Mr. Hunt demonstrates that an 8-in. gap might result if ordinary track were made continuous. The actual data submitted in one of the discussions for such track showed that the gaps were much smaller so that a higher restraining force must be considered to have been in effect. His calculations and diagrams showing the local stresses set up when a short length of a long rail under temperature stress is replaced, are interesting, but such stresses may not be serious since they would have a good chance of being "ironed" out by vibration and wave action in the rails due to the wheel loads. Creeping of rails produces a similar effect and is no doubt responsible for some of the scattering in the observed points in Fig. 11.

Mr. Walker's photograph of the "sun-kink" (Fig. 8) on a welded rail is extremely interesting, and his experience in eliminating this trouble in the 210 miles of unpaved welded track of the Eastern Massachusetts Street Railway, is a valuable addition to the paper. He reports that the actual expansion in the 660-ft lengths was about one-half the expected 6 in., thus checking the phenomena observed in the Victorian Railways in Australia and corroborating

Theorem 1. The incident of the rail "popping out" of the remaining spikes after some had been removed is worth noting, both for the warning it gives of this possible source of danger, and as an illustration of how the total resisting force is an accumulation of many small forces.

Mr. Ferguson's objections to the use of a uniformly increasing anchorage due to constant tie resistance are similar to those of Professor Talbot which have already been considered. He is correct in stating "the force or stress diagram is not necessarily a straight line"; but the straight line, nevertheless, seems appropriate for the analysis.

Mr. Reece's numerical example showing how joint restraint may cause even short rails to expand and contract less than they would if free is of special interest to those who have noticed discrepancies with the expected gaps based on the standard tables for rail laying.

Mr. Magee agrees that the tie resistance is more in the nature of a frictional than an elastic resistance. To explain end movement where no surface evidence of movement is noticeable, he suggests that slippage may occur along the horizontal plane at the bottom of the ties—the rail, ties, and ballast moving as a unit. However, cases are on record in which sliding of the rail occurred through the fastenings, particularly when the temperature dropped suddenly and the rail broke.

A question frequently raised regarding the magnitude of the buckling forces in curved rail due to temperature stresses is answered by Mr. Hussey's Equation (29) which gives the required radial resisting force in each tie to keep the rail in place. It should be stated that this equation involves the assumption that the half-chord of the curve enclosing two tie spans is equal to the tie span, *c*. This is a close approximation. Radial forces of about the same order may exist when the speed of a train around a curve exceeds that corresponding to the superelevation. The same lateral rail anchors that are already in use for the one radial force may be sufficient to hold the additional radial force due to temperature stress, at least for curves of large radius. For sharper curves, this effect might become serious, and the added factor of a shorter life may make welded rail inadvisable in certain cases.

In the writer's opinion the entire track structure of the future will be altered radically from that called "standard" to-day in the effort to obtain the maximum economy and the smoothest riding from continuous welded rail. A continuous elastic support under the rail in addition to adequate cross-members and a concrete sub-grade should do much as an approach to the more rational structural unit required to parallel the advances made in modern stream-lined trains.

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ECONOMICS OF HIGHWAY-BRIDGE FLOORINGS OF VARIOUS UNIT WEIGHTS

By J. A. L. WADDELL,[1] HON. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JONATHAN JONES, HENRY C. TAMMEN, MILLER McCLINTOCK, JOSEPH G. SHRYOCK, C. CALOR MOTA, W. G. FOWLER, PHILIP A. FRANKLIN, AND J. A. L. WADDELL.

SYNOPSIS

The comparative economics of any two or more types of modern highway-bridge floors is presented in this paper. The method provides a means of answering questions of relative economy in a very few minutes, beyond the peradventure of doubt in regard to accuracy, either to-day or at any future time. The following information concerning each competing type must be known: The thickness of the flooring and its weight per square foot; the weight of metal per square foot of floor, between the stringers and the flooring with a stringer spacing of 5 ft, if there is any such special metal (or, otherwise, the extra weight of metal per square foot, in the floor system, due to a closer spacing of stringers); and the cost per square foot of flooring in place.

Figs. 1 to 8, with Table 1, afford all the information required for making such economic comparisons; and the text describes the conditions that governed the computations for all diagrams and tables. Invariably, both superstructure and substructure costs are considered, under the average or the usual conditions that were assumed for the latter.

Certain unit prices of materials in place were adopted for making the calculations of total costs per linear foot for the more than two hundred and fifty spans that were specially computed in making the investigation. Due cognizance was taken of the fact that the unit prices of both superstructure and substructure material, in place, increase as the total quantities thereof diminish; and certain empirical formulas are furnished for the proper and adequate correction (for this reason) of all such unit prices.

NOTE.—Published in February, 1937, *Proceedings*.

[1] Cons. Engr., New York, N. Y. Mr. Waddell died March 3, 1938.

The bridges covered in the records are for simple-truss and cantilever spans, for both wide and narrow roadways, and for both carbon-steel and silicon-steel structures. Attention was paid, wherever necessary, to the augmenting of total metal weights due to wind loads, and to the effect on costs of both superstructure and substructure from the spreading of trusses beyond the requirement for width of roadway. The curves were determined upon the basis of there being solid rock or an equivalent reasonably hard foundation for piers; and the effect on the comparative economies of using pile foundations has been treated.

Full explanations are provided regarding the utilization of the diagrams and tables; and three practical cases for bridge-floor competitions have been solved in the text. The economic effects of using light floorings on vertical-lift and bascule spans and on suspension bridges have been treated at some length. The entire investigation has been made upon an absolutely unbiased basis; and the method of presenting the cost diagrams represents the acme of impartiality.

INTRODUCTION

In addition to the usual type of reinforced concrete flooring for modern highway bridges, termed herein the "Standard", there are a number of patented floorings, the main object of which is to reduce the dead load on the structure and thus save metal and, therefore, expense. Because these special types are more costly than the ordinary type of reinforced concrete flooring, and, in addition, involve royalties, they are likely to increase rather than to decrease the total cost of structure in short-span bridges.

Other considerations being equal (that is, ignoring all claims for superiority of surface, resistance to skidding, absence of pitch or crown, better drainage, non-collection of snow and ice, etc.), the bridge engineer, in the interests of his clients, will very properly desire to design his structures so as to keep their total cost down to a minimum by selecting the most economic type of flooring. Each patent owner naturally wants people to believe that his special type is not only superior to all other types, but is also the least expensive, when the total cost of the completed structure is in question. Comparing any two types of equal suitability, it is evident that the lighter the flooring the smaller will be the percentage of steel required for both the floor system and the trusses; but the total saving of metal may not be great enough to offset the difference in the costs of flooring for the two types under comparison, after the royalties have been included.

It is evident that the longer the span the greater is the proportionate saving in metal weight of trusses resulting from the reduced dead load. As far as the different types of floor construction are concerned, it is almost axiomatic that the lighter the flooring the greater, generally, is (or should be) its actual cost per unit of area, because the thinner it is the larger must be its volume of expensive steel and the smaller its volume of cheap concrete. This statement does not always apply to varying thicknesses of flooring of the same type.

Since, at present (1937), few, if any, bridge specialists are at all certain about the comparative ultimate economics of the various bridge floorings, the establishment of an absolutely correct and impartial method for settling the question would be a real boon, not only to the Engineering Profession, but also to the promoters of bridge projects and to the financiers whose money is to be spent for the materialization of the desired structures. It is for that reason that this paper, with Figs. 1 to 8, has been prepared.

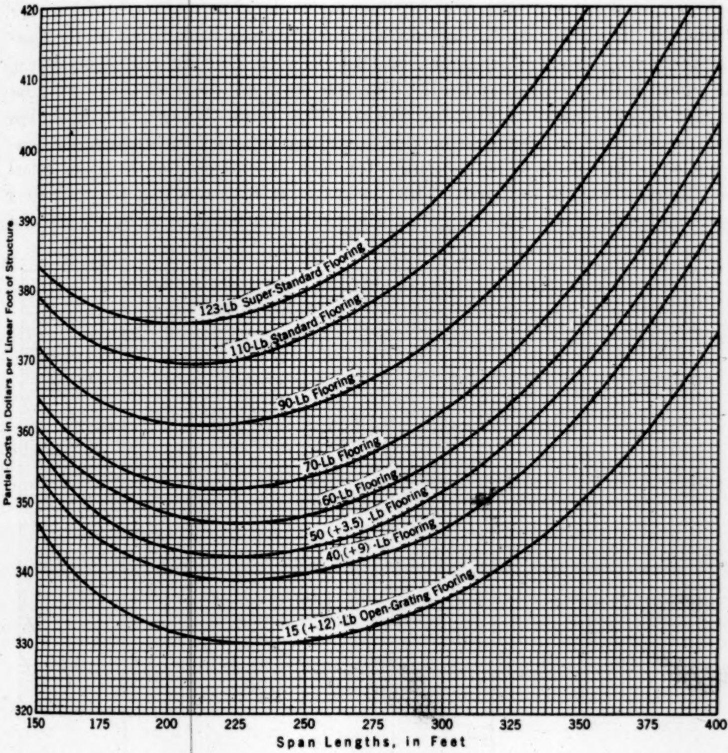


FIG. 1.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF CARBON STEEL; 45-FOOT CLEAR ROADWAY

BASIC ASSUMPTIONS

A series of solid floorings having weights per square foot of floor equal to 123, 110, 90, 70, 60, 50, and 40 lb, as well as an open-grate flooring computed for 15 lb per sq ft, plus an allowance of 12 lb per sq ft for small I-beam cross-girders, was assumed; and, for each one of the series, an arbitrary (but logical) cost per square foot was adopted, including royalty, these costs generally increasing as the weight of floor-slab diminishes.

Next, cost curves were plotted to cover the substructure, the superstructure metal work contained in the floor system proper (on the basis of 5-ft stringer spacing), the lateral system, and the trusses (excluding the metal on piers and in the anchorages) for both simple-truss spans and cantilever bridges, in both carbon steel and silicon steel, and for both wide roadways and narrow roadways. These curves do not contain the costs per linear foot of the flooring, the metal in extra stringers, or that between flooring and floor system, the guard-rails, or the hand-rails; but the costs of these enumerated items (except only the hand-rails) are given in tables, using the assumed unit prices that are adopted in the text for this investigation. In the future, the foregoing costs will need to be computed for each special case as it arises before the diagrams can be utilized. This program, at first reading, may appear a bit complicated; but the examples, given hereinafter, for utilizing the cost curves will make the procedure perfectly clear.

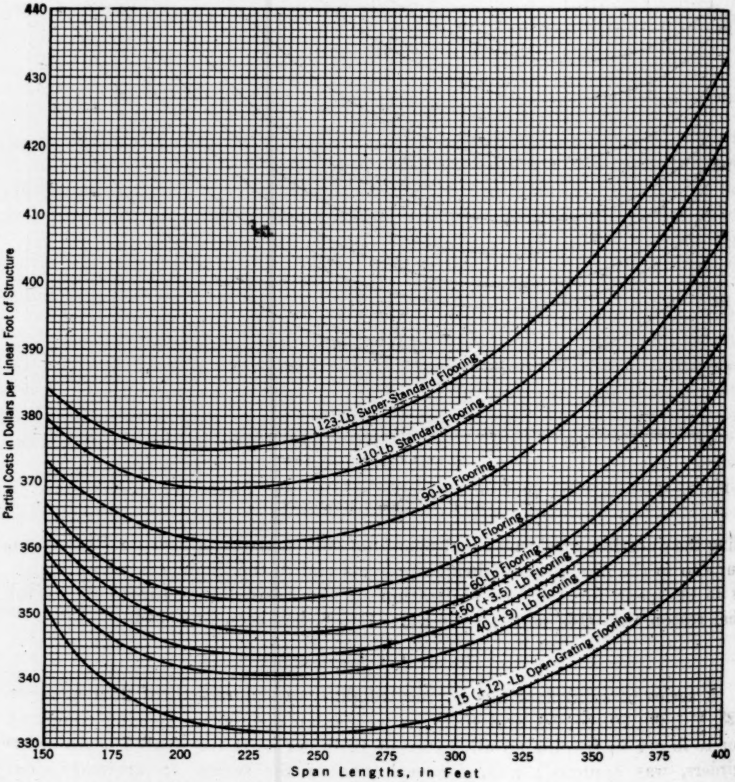


FIG. 2.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF SILICON STEEL; 45-FOOT CLEAR ROADWAY; SPANS, 150 FEET TO 400 FEET.

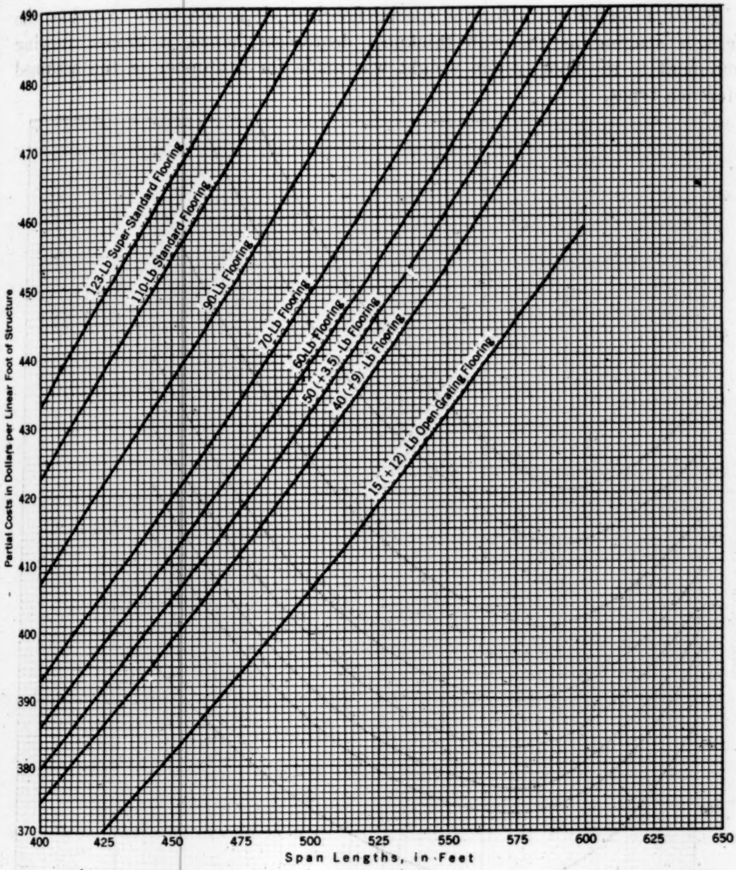


FIG. 3.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF SILICON STEEL; 45-FOOT CLEAR ROADWAY; SPANS, 400 FEET TO 650 FEET.

Because of its common use, a 110-lb, 9-in. slab floor of ordinary reinforced concrete, consisting of an 8-in. structural slab with its thickness increased 1 in. for wear, has been selected as the "Standard" type of non-patented flooring. The 123-lb flooring, designated as "Super-Standard", consists of an 8-in., reinforced concrete slab, covered with a 2-in. thickness of asphalt paving. For both these types, the stringer spacing was assumed to be 5 ft.

The 90-lb flooring represents a 7-in., concrete slab reinforced with trusses of welded-bar or other rigid construction, also carried by stringers spaced 5 ft on centers. This thickness includes 1 in. of wearing surface.

Below a certain limit for the special floorings, it is necessary to reduce the stringer spacing as the slab thickness decreases; hence, a proper varying increment to the floor-system weight was made for a number of the assumed reference types.

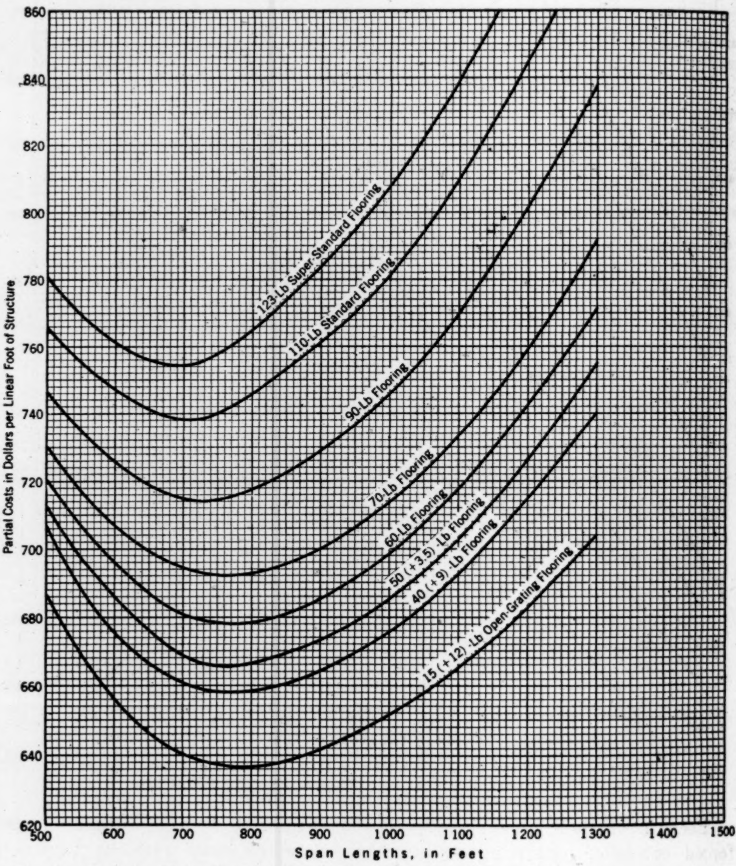


FIG. 4.—PARTIAL UNIT COST OF TYPE *A*, CANTILEVER HIGHWAY BRIDGES OF SILICON STEEL; 45-FOOT CLEAR ROADWAY.

The open-grate flooring of this comparative series was based on using $\frac{3}{16}$-in. metal throughout, but it is better practice to make the straight (or carrying) bars $\frac{1}{4}$ in. thick. This would add 2 lb per sq ft to the weight of the floor, but would permit a saving of 1 lb per sq ft (through slightly wider spacing) in the weight of the small transverse I-beams, making a net increase of 1 lb of metal per sq ft. Consequently, if the heavier carrying bars are used, it will be necessary to allow a rebate of 4 cts per sq ft of floor, but,

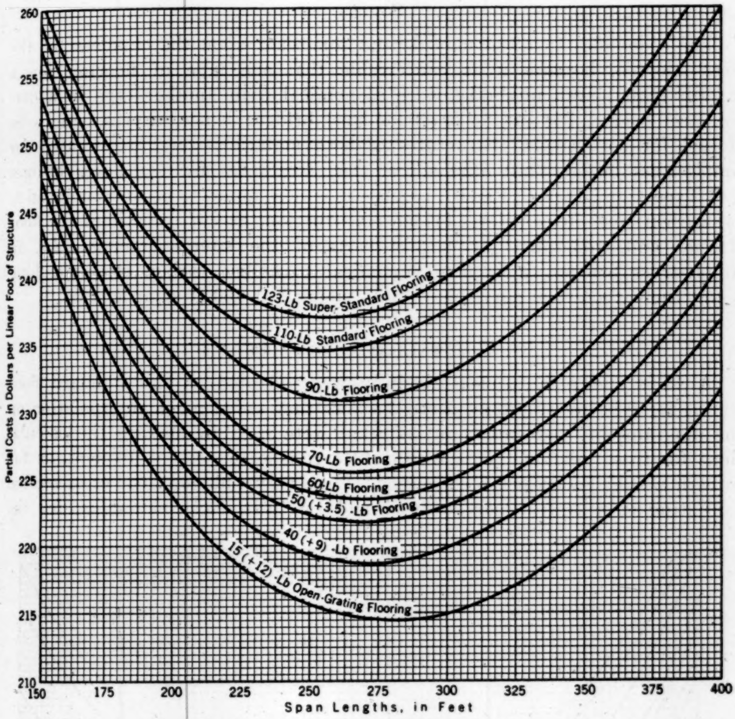


FIG. 5.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF CARBON STEEL; 20-FOOT CLEAR ROADWAY.

on the other hand, to provide for the increased cost of the structure due to the extra load of 1 lb per sq ft. In Table 1 are recorded full data concerning these eight comparable floorings.

TABLE 1.—DATA CONCERNING COMPARABLE FLOOR TYPES

Item No.	Type	Total weight of floor, in pounds per square foot	Cost in place per square foot	Allowance included in cost for extra floor metal as compared with the standard, Item No. 2
1	Super-standard flooring.....	123	$1.00	None
2	Standard flooring..........	110	0.80	None
3	90-lb flooring...	90	1.00	None
4	70-lb flooring...	70	1.50	None
5	60-lb flooring...	60	1.40	None
6	50-lb flooring...	50	$1.30 + 0.18 = $1.48	3.5 lb. @ 5 cts = $0.18
7	40-lb flooring...	40	1.40 + 0.36 = 1.76	9 lb. @ 4 cts = 0.36
8	Open-grate flooring......	15	1.40 + 0.48 = 1.88	12 lb. @ 4 cts = 0.48

STEELS

The economics involved may depend somewhat upon the character of the steel adopted, because it is conceivable that one type of flooring for a certain span length might be more economical than another type in a carbon-steel bridge, but not in a silicon-steel bridge; hence, curves had to be plotted for both steels. Structures of both materials were investigated for simple-truss bridges, but only silicon-steel structures for cantilever bridges (see Type *A* and Type *C*, Fig. 9), because it is now found to be uneconomical to use carbon steel in cantilever-bridge construction, except sometimes for floor systems and, generally, for certain parts of the lateral bracing. As, in simple-truss bridges, it is not economical to utilize carbon steel for long spans, the carbon-steel curves have been stopped at 400-ft span lengths.

ROADWAYS AND SIDEWALKS

A 45-ft and a 20-ft clear roadway were adopted as the standard floor widths for this investigation, partly because they saved much interpolation on the diagrams of "percentage ratios" that were used in making the calculations for truss weights. This width of 45 ft is practically equivalent, as far as both weight and cost are concerned, to the very common floor cross-section consisting of a 40-ft, clear roadway and two 5-ft, clear sidewalks, for the

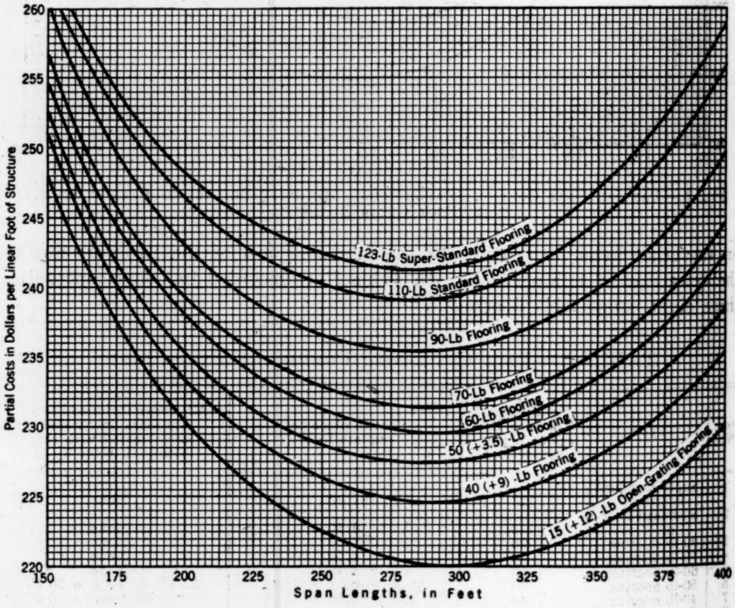


FIG. 6.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF SILICON STEEL; 20-FOOT CLEAR ROADWAY; SPANS, 150 FEET TO 400 FEET.

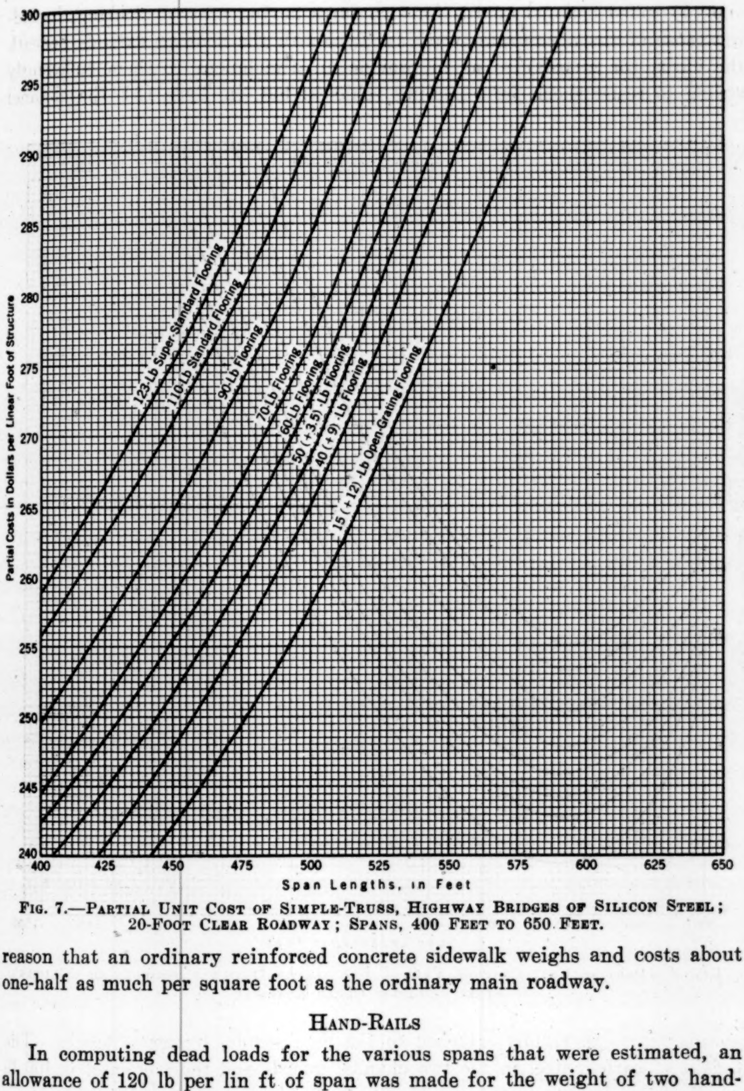


FIG. 7.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF SILICON STEEL; 20-FOOT CLEAR ROADWAY; SPANS, 400 FEET TO 650 FEET.

reason that an ordinary reinforced concrete sidewalk weighs and costs about one-half as much per square foot as the ordinary main roadway.

HAND-RAILS

In computing dead loads for the various spans that were estimated, an allowance of 120 lb per lin ft of span was made for the weight of two hand-rails, but their cost has been (and in the future use of this paper is to be) ignored, as it is common to all the competing structures investigated.

GUARD-RAILS

Two types of guard-rails have been adopted for this investigation: (*a*) 12 by 12-in. curbs of reinforced concrete for structures with solid floors;

and (*b*) rolled channel guards for structures having all-steel flooring, including those of the open-grate type. In Type (*a*), the flooring extends beneath the curbs (or guards) so as to provide an attachment to them sufficiently strong to resist shock from passing vehicles; but, in Type (*b*), the channel guards are to be rigidly attached to the floor system by metal braces. The weight of carbon steel in the two channel guards and their bracing is 100 lb per lin ft, and the cost, in place, is $4 per lin ft.

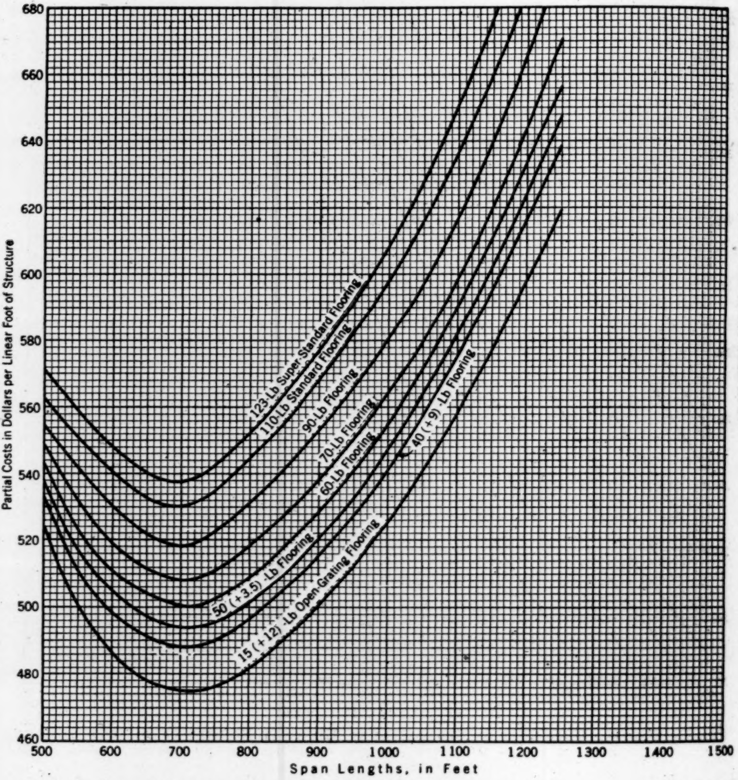
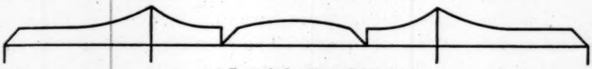


FIG. 8.—PARTIAL UNIT COST OF TYPE *A* CANTILEVER HIGHWAY BRIDGES OF SILICON STEEL; 20-FOOT CLEAR ROADWAY.

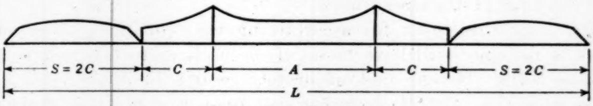
The cost per linear foot of structure for the two reinforced concrete guards and their supporting base is $2 for the 2 cu ft of concrete in the curbs, plus the value of 2 sq ft of the flooring. It is self-evident that there is no need for extending the flooring outside the inner faces of the channel guard-rails.

SPECIFICATIONS

The standard specifications that were used for making the computations are those now current, involving basic tensile unit stresses of 18 000 lb per sq in. for carbon steel and 24 000 lb per sq in. for silicon steel.



(a) Type *A*, Cantilever Bridge



(b) Type *C*, Cantilever Bridge

FIG. 9.

Although all the data in this paper are based on the usual standard (H-20) live loading, the general economic conclusions will apply quite closely to structures designed for lighter live loadings. For the usual H-15 loading, for instance, the 5-ft stringer spacing could be increased for all types of floor, or it could be kept unchanged and thinner floor-slabs utilized. In general, the percentage saving resulting from the use of the lighter floorings will be found a little less for the lighter live loads than for the H-20 loading.

DEAD LOAD REDUCTION EFFECT ON PIERS

Any reduction in dead load saves money, not only on the superstructure, but usually also on the substructure because it is the sizes of shoe-plates that dictate both the width and the length of the pier tops; hence, it was necessary to include the cost of the substructure in making the comparative curves of costs per linear foot of bridge.

LOCATION EFFECT

As bridge costs differ somewhat in various parts of the United States, it was considered advisable to select a particular locality for the comparison; and Kansas City, Mo., was chosen as being centrally located.

ADDITIONAL METAL FOR SUPPORTING THIN FLOORINGS

Thin floorings may require more closely spaced supports than the "standard" flooring (Item No. 2, Table 1). There are two ways of accomplishing this requirement—either by inserting additional stringers, or by adhering to the 5-ft spacing that is used with the "standard" flooring and crossing the stringers with small **I**-beams, spaced as far apart as the strength and the rigidity of the flooring under consideration will permit. The second method is the better one from the point of view of stiffness; and it is sometimes also the more economical.

As none of the usual floorings less than 3 in. in thickness will permit of a support-spacing in excess of 3 ft, and as the cross-**I**-beam construction is usually so much more satisfactory, the writer concluded to utilize it for such thin floorings. For floorings weighing 60 lb per sq ft, or more, or for those $3\frac{1}{2}$ in., or more, thick, he assumed them to be able to carry properly over a 5-ft spacing. For thin floorings ranging from 3 to $3\frac{1}{2}$ in., he adopted a stringer spacing of 4 ft; and for all still thinner floorings the small transverse I-beams were used, their spacing being dependent upon the strength and rigidity of the floorings. The greatest, safe, usable, span length of any flooring is so difficult to compute that tests to destruction may be needed to determine its strength.

Before using this paper for ascertaining the economics of any particular flooring, the designer will have to satisfy himself as to this safe span length, l (less than 5 ft). He can find, accurately enough for his purpose, the corresponding excess weight of floor metal, e, per square foot of floor by the formula,

$$e = 3.5\ (5 - l) \quad \text{..........................} \quad (1)$$

For instance, if $l = 4$ ft, e will be 3.5 lb.

TRUSSES

The truss weights were readily obtained by a method presented[3] by the writer in 1935. In Type *A* cantilever bridges[3] they were the properly adjusted average weights for the entire structure, namely, one suspended span, two cantilever arms, and two anchor arms, the metal in anchorages and on piers being omitted

CORRECTION FOR EFFECT OF WIND LOAD

It was anticipated originally that the computed truss weights would require correction to cover the effect of wind loading upon the sectional areas of certain bottom-chord members for all the various floorings, except the "standard" type, in which this effect had already been included. Such was found not to be the case for the simple-truss spans with the 45-ft, clear roadway; but it held true for the simple-truss spans with the 20-ft, clear roadway, and for all cantilever spans.

These corrections were determined by reference to several actual bridges designed with "standard' floorings, in which the wind loads had increased some of the bottom-chord sections. The weight of extra metal that would be required to care for the wind loading, if the dead load were reduced by using the lightest of all the floorings, was also computed. Cases of both wide and narrow roadways, involving both carbon steel and silicon steel, were selected; and enough of them were introduced to render this method of correction for wind loading perfectly satisfactory. The corrections for intermediate floorings were interpolated by proportionate weights per square foot of flooring.

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[3] "Weights of Metal in Steel Trusses", *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1.

NARROW ROADWAY

Were it not for the aforementioned feature of correction for the effects of wind loading, the comparative economics of the various types of flooring might have been assumed as constant for all widths of roadway; but, unfortunately, this feature is much more potent for narrow roadways than for wide ones; hence, the writer deemed it advisable to prepare the curves in Figs. 5 to 8, inclusive, in order to cover highway bridges of 20-ft clear roadway. The methods of calculation used were the same as those for the structures of the 45-ft clear roadway, except only that carbon-steel floor systems were adopted in otherwise silicon-steel bridges for Figs. 6 and 7 and carbon-steel stringers with silicon-steel cross-girders for Fig. 8, and that the pound prices for metal erected were duly adjusted for the mixed steels. Proper cognizance was taken of the increased weights of metal in floor systems due to the necessary spreading of trusses in long-span, narrow-roadway bridges, in order to comply with standard engineering practice.

UNIT PRICES OF STEEL ERECTED

The following unit prices of steel, erected, were adopted as representing the average current values at Kansas City, including a proper profit to the contractor, for bridges with the "standard" flooring:

Simple-truss structures of carbon steel........	5 cts per lb
Simple-truss structures of silicon steel........	6 cts per lb
Cantilever structures of silicon steel..........	7 cts per lb

For the mixed steels used in computing the curves of Figs. 3 and 4, the average unit prices for bridges with the "standard" flooring ranged uniformly from 5.4 cts at 150-ft spans, to 5.7 cts at 600-ft spans; and for those used in computing the curves of Fig. 5, they ranged from 6.65 cts at 500-ft main openings to 6.85 cts at 1 200-ft main openings.

For structures with decks lighter than the "standard," a correction in the unit prices of the metal work was made, because, as the weight of metal in a structure is reduced, its pound price, erected, will increase. This correction was effected by means of the formula,

$$C' = \frac{C}{4}\left(3 + \frac{w}{w'}\right) \text{ ....(2)}$$

in which w = the weight of metal, in pounds per linear foot, in the bridge with the heavier deck; w' = the weight of metal, in pounds per linear foot, in the bridge with the lighter deck; C = the unit cost of metal, erected, in the bridge with the heavier deck; and C' = the unit price of metal, erected, in the bridge with the lighter deck.

SUBSTRUCTURES

In determining the costs for substructures, it seemed desirable to make a distinction between the assumed profiles of crossings for simple-truss structures and those for cantilever structures. Generally, the grade line is lower in the former than in the latter, and the depths below the water-line of the pier foundations are often less. For the simple-truss bridges, the height of the shafts was taken at 42 ft, and the height of the pneumatic bases at 50 ft, whereas, for the cantilever bridges, both the shafts and the bases were assumed to be 84 ft high.

The all-around batter adopted for all piers was $\frac{1}{2}$ in. to the foot. Dumbbell shafts, as illustrated in plan by Fig. 10, were adopted for all cases, because of their economy in materials and cost. For the simple-truss bridges, with 45-ft roadways, two separate bases were assumed; but for the bridges with 20-ft roadways, and for all cantilever structures, single bases were adopted. The reason for the single bases in narrow structures is that there would probably be no great pecuniary advantage in dividing them; and their use in cantilevers is generally called for because of location over navigable streams where the bed-rock is deep, and where, consequently, the sinking of small bases would be expensive per unit of volume.

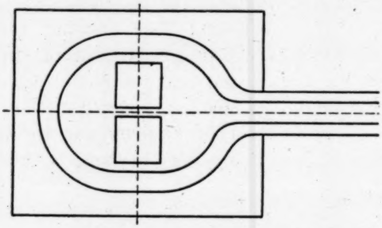

FIG. 10.—PLAN OF PIER TOP.

In computing the costs per linear foot for the substructures of simple-truss bridges, the total value of one pier was estimated, and the amount was divided by the distance between pier centers; whereas in the case of cantilever bridges, the total value of a main pier was divided by one-half the length of the main opening plus one-half that of one anchor arm. This method assumes that one-half of each anchor arm pertains entirely to its anchor pier. As the design for any anchorage varies greatly with the foundation conditions, it is not advisable to assume the latter, but, instead, to designate the cost per linear foot of the substructure the amount computed from the value of one main pier, as previously described. If this is not strictly exact, no harm can be done, because all the competitive costs per linear foot are affected alike.

UNIT PRICES FOR SUBSTRUCTURE

For the masses of materials in the shafts and walls and in the pneumatic bases of piers, the following unit prices were adopted for structures with the "standard" flooring:

Simple-Truss Structures:	
Shafts and walls	$15 per cu yd
Bases	$22 per cu yd
Cantilever Structures:	
Shafts and walls	$17 per cu yd
Bases	$25 per cu yd

For structures with decks lighter than those of the "standard" flooring, a correction in substructure unit prices was applied, as in the case of superstructure metal, because, as the size of the pier decreases, the unit prices of shafts, wall, and bases will increase. This correction was made by means of the formula:

$$C' = \frac{C}{4}\left(3 + \frac{V}{V'}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which V = total volume of shafts, wall, and base or bases, in pier with heavier deck; V' = total volume of shafts, wall, and base or bases, in pier with lighter deck; C = one of the substructure unit prices for pier with heavier deck; and, C' = the corresponding substructure unit price for pier with lighter deck.

In respect to the application of the unit-price generic factor,

$$C' = \frac{C}{4}\left(3 + r\right), \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

each of the sets of curves in this paper, was treated as an independent entity; in other words, these factors do not function from diagram to diagram or from span length to span length, but only, in each diagram and for each span length, between the "standard" flooring and the other floorings.

PIER DESIGNING

In pier designing, all steel pedestals were made square; and, in simple-truss structures, 6 in. clear were left between them. In these designs from the edge of any pedestal base-plate to the edge of the masonry, a minimum distance of 9 in. was allowed. The heavier of the two similar piers was first designed and estimated, and then the dimensions were reduced (always exactly and in like manner) to agree with the lighter superimposed load on the pedestal or pedestals. The interpolation of pier costs for the other types of flooring was done with due reference to the superimposed loads and to the varying unit-prices of materials in place.

For convenience, all piers were assumed to rest on solid rock. Had pile foundations been adopted, a still more economic showing would have been indicated for the light floorings. In computing volumes of shafts, the prismoidal-formula method was used invariably; but for the walls, the simpler method of averaging end areas was deemed to be sufficiently accurate. The permissible load on concrete under pedestals was taken at 600 lb per sq in., according to the latest standard specifications of the American Association of State Highway Officials.

EFFECT OF PILE FOUNDATIONS

It has been stated that, had pile foundations been adopted instead of those of solid rock, a better showing would have been made for the lighter floorings. For example, consider a foundation with 100-ft, large-diameter, timber, friction piles, embedded 10 ft in the concrete. A bearing capacity of 45 tons per pile was assumed, the cost per linear foot of pile in place below the base being $1.00, and the cost per cubic yard of the mass of base (placed by a steel sheet-pile coffer-dam) being $20.00. The costs of the two piers thus computed per linear foot of span, for the "standard" and the open-grate floorings, are $98.44 and $82.20, respectively, indicating an advantage of 16.5% in the substructure for the lightest flooring when compared with the "standard." The longer the span the greater would be this percentage of advantage. The corresponding percentage for piers resting on bed-rock is 14.5%, showing that the lightest flooring is only a little, if any, more economical with pile foundations than with rock foundations.

Effect of Excess Weights in Floor Systems

Before proceeding to show how to use the cost curves for actual cases of competition, it is necessary to indicate how to deal with any excess weight of metal per linear foot of span, as compared with the allowance therefor recorded in Table 1. If a competitor has a flooring so thin that it is not strong enough properly to carry over a 5-ft stringer-spacing, it will be necessary, as previously indicated, to put in either additional stringers, or small transverse **I**-beams, thus adding a number of pounds (*n*) per linear foot to the total load to be carried. It is then necessary merely to add to the cost of the floor the product of *n* by the pound price of the metal erected, and to include the extra weight of the metal per square foot of floor when entering the various diagrams

Diagrams of Partial Costs

In the curves accompanying this paper are recorded the computed partial costs (including substructure, trusses, lateral systems, and floor system) per linear foot of structure, for the various types of bridges and the floorings listed in Table 1. As can be seen from these diagrams, the simple-truss spans vary from 150 ft to 600 ft in length, and the Type *A* cantilever spans from 500 ft to 1 200 ft of main opening, all measured from center to center of piers.

On the lowest three curves of each diagram there is noted the weight of extra floor metal resulting from close stringer spacing or small cross-beams, as assumed in Table 1, as well as the weights of the flooring itself, because the trusses and substructure were designed to carry the weight of this extra metal These three curves, therefore, should be considered as representing floors weighing 53.5 lb, 49 lb, and 27 lb per sq ft.

Type C Cantilevers.—The curves for the Type *A* cantilever bridges can be used for Type *C* cantilever bridges by multiplying the main opening of the latter by 1.43.

Method of Computing the Total Costs of a Structure

References have been made in the text to the fact that the several diagrams record partial costs of bridges per linear foot of structure, and it was indicated that the curves covered only the cost of the substructure and that of the trusses, lateral system, and floor system upon the basis of a 5-ft spacing of floor stringers. In order to find the total cost per linear foot for any case, there must be added to the data given in the diagram the costs per linear foot for: (*a*) The flooring between curbs; (*b*) any extra stringers, at the unit price, erected, of the metal for the entire bridge; or, (*c*) the small cross-girders that rest on the stringers, worth in place only 4 cts per lb; and, (*d*) the steel guard-rails or the reinforced concrete curbs, as the case may be.

TABLE 2.—ADDITIONAL COSTS OF STRUCTURES, SUPPLEMENTING DATA FROM THE CURVES, FOR FINDING TOTAL UNIT COSTS

Types of flooring	Costs of flooring per linear foot of bridge	Costs per linear foot of extra metal in floor system	Costs per linear foot of curbs or guard-rails	Summation, giving total extra costs per linear foot to add to diagram, readings
(*a*) DATA TO SUPPLEMENT FIGS. 1 TO 4, INCLUSIVE, FOR WIDE HIGHWAY BRIDGES				
Super-standard....	45 ft. @ $1.00 = $45.00	None	$2.00 + $2.00 = $4.00	$49.00
Standard.........	45 ft. @ 80 cts. = $36.00	None	1.60 + 2.00 = 3.60	39.60
90-lb............	45 ft. @ $1.00 = $45.00	None	2.00 + 2.00 = 4.00	49.00
70-lb............	45 ft. @ $1.50 = $67.50	None	3.00 + 2.00 = 5.00	72.50
60-lb............	45 ft. @ $1.40 = $63.00	None	2.80 + 2.00 = 4.80	67.80
50-lb in carbon-steel bridges....	45 ft. @ $1.30 = $58.50	45 ft. @ 18 cts. = $8.10	2.60 + 2.00 = 4.60	71.20
50-lb in simple-truss bridges of silicon steel	45 ft. @ $1.30 = $58.50	45 ft. @ 21 cts. = $9.45	2.60 + 2.00 = 4.60	72.55
50-lb in cantilever bridges of silicon steel	45 ft. @ $1.30 = $58.50	45 ft. @ 24.5 cts. = $11.03	2.60 + 2.00 = 4.60	74.13
40-lb............	45 ft. @ $1.40 = $63.00	45 ft. @ 36 cts. = $16.20	4.00	83.20
Open-grating.....	45 ft. @ $1.40 = $63.00	45 ft. @ 48 cts. = $21.60	4.00	88.60
(*b*) DATA TO SUPPLEMENT FIGS. 5 TO 8, INCLUSIVE, FOR NARROW HIGHWAY BRIDGES				
Super-standard....	20 ft. @ $1.00 = $20.00	None	$2.00 + $2.00 = $4.00	$24.00
Standard.........	20 ft. @ 80 cts. = $16.00	None	1.60 + 2.00 = 3.60	19.60
90-lb............	20 ft. @ $1.00 = $20.00	None	2.00 + 2.00 = 4.00	24.00
70-lb............	20 ft. @ $1.50 = $30.00	None	3.00 + 2.00 = 5.00	35.00
60-lb............	20 ft. @ $1.40 = $28.00	None	2.80 + 2.00 = 4.80	32.80
50-lb in carbon-steel bridges....	20 ft. @ $1.30 = $26.00	20 ft. @ 18 cts. = $3.60	2.60 + 2.00 = 4.60	34.20
50-lb in simple-truss bridges of silicon steel	20 ft. @ $1.30 = $26.00	20 ft. @ 21 cts. = $4.20	2.60 + 2.00 = 4.60	34.80
50-lb in cantilever bridges of silicon steel	20 ft. @ $1.30 = $26.00	20 ft. @ 24.5 cts. = 4.90	2.60 + 2.00 = 4.60	35.50
40-lb............	20 ft. @ $1.40 = $28.00	20 ft. @ 36 cts. = $7.20	4.00	39.20
Open-grating.....	20 ft. @ $1.40 = $28.00	20 ft. @ 48 cts. = $9.60	4.00	41.60

These additional costs in wide highway bridges for the flooring types and the unit prices given in Table 1 are presented in Table 2(*a*). These data will supplement Figs. 1 to 4, inclusive, for finding the total unit costs of wide highway bridges. The corresponding data for narrow highway bridges, to supplement Figs. 5 to 8, inclusive, are given in Table 2(*b*).

How to Use the Cost Curves

Assume, for example, that an engineer is contemplating the building of a four-lane, Type *A*, cantilever bridge, having a main opening of 750 ft, and two 5-ft sidewalks, and that he intends using a solid flooring. Two types of floor have been shown him, one by Mr. Smith and another by Mr. Jones. Either type would be satisfactory; hence, his choice is merely a matter of economics.

The flooring offered by Mr. Smith is 3 in. thick, weighs 58 lb per sq ft, and costs $1.23 per sq ft in place; but it has a superior limit of only 4 ft between supporting girders, necessitating 3.5 lb of extra metal per square foot of floor, worth 7 cts per lb in place.

The flooring shown by Mr. Jones, is $2\frac{1}{2}$ in. thick, weighs 45 lb per sq ft, and costs $1.18 per sq ft in place; but it requires, for a stringer spacing of 5 ft, small transverse I-beams, which he justly claims will weigh only 8 lb, and will cost only 32 cts per sq ft of floor.

Mr. Smith's Case.—Turning to Fig. 4, and interpolating for 58 + 3.5 = 61.5-lb flooring, the "partial cost" indicated is $680 per lin ft of bridge. The remaining cost per linear foot is found thus:

Flooring, 45 ft @ $1.23	$55.35
Extra metal in floor system, 45 × 3.5 = 158 lb worth 7 cts per lb, or	11.06
Curb supports, 2 × $1.23 = $2.46. Adding $2.00 for curbs makes	4.46
Summation	$70.87
Plus partial cost from Fig. 4	680.00
Total cost of bridge with Mr. Smith's flooring	$750.87

Mr. Jones' Case.—Turning to Fig. 4, and interpolating for 45 + 8 = 53-lb flooring, the "partial cost" indicated is $666 per lin. ft. The remaining cost per linear foot is found thus:

Flooring, 45 ft @ $1.18	$53.10
Extra metal in floor system, 45 × 8 = 360 lb worth 4 cts per lb, or	14.40
Steel guard-rails, 100 lb @ 4 cts	4.40
Summation	$71.50
Plus partial cost from Fig. 4	666.00
Total cost of bridge with Mr. Jones' flooring	$737.50
Difference in favor of Mr. Jones = $750.87 − $737.50 =	$13.37

To assume another case—a succession of wide, simple-truss, carbon-steel spans of 250 ft each: Mr. Brown thinks he can compete with the standard flooring by quoting a price of $1.30 per sq ft of floor for his type of flooring, which is 3 in. thick and weighs 50 lb per sq ft. It requires a 4-ft spacing of stringers, necessitating 3.5 lb per sq ft of extra metal for stringers, worth 5 cts per lb in place—the same as the 50-lb flooring in Table 1.

Turning to Fig. 1, the "partial cost" for 50 + 3.5 = 53.5-lb flooring is found to be $343.50. The remaining cost per linear foot is found thus:

Flooring, 45 ft @ $1.30	$58.50
Extra metal, 45 × 3.5 = 158 lb @ 5 cts	7.90
Curb supports, 2 × 1.30 = $2.60. Adding $2.00 for curbs makes	4.60
Summation	$71.00
Plus partial cost from Fig. 1	343.50
Total cost with Mr. Brown's flooring	$414.50
For the "standard" flooring, Fig. 1 indicates a partial cost of	$373.20
Table 2(*a*) gives a remaining cost as	39.60
Plus the total cost with "standard" flooring is	$412.80

Mr. Brown, consequently, does not secure the contract.

The following is an illustration of how to apply the diagrams to a comparison of the economics of an open-grate flooring and the "standard" flooring: An engineer has a 4-lane, simple-truss, silicon-steel, highway bridge, with a succession of 380-ft spans and double, 5-ft sidewalks, to design and build, and desires to know what percentage of saving in cost can be obtained by means of an open-grate flooring which has the longitudinal pieces, $\frac{1}{4}$ in. thick. The best quotation he can secure for the flooring is $1.50 per sq ft in place. The open-grate flooring, of course, will be the criterion to use; but, as previously indicated, the thickness of the longitudinal pieces adopted for this type in Table 1 was $\frac{3}{16}$ in. The increased thickness adds 2 lb per sq ft of floor to the dead load, but reduces the weight of the special cross-girders from 12 lb to 11 lb per sq. ft, thus making the net increase in total load, 1 lb per sq ft, or 45 lb per lin. ft of span. Turning to Fig. 2 the "partial cost" in pounds per linear foot, is found to be as follows:

Standard flooring	$411.00
Open-grate flooring (for 17 + 11 = 28 lb)	353.60
Table 2(*a*) gives for the remaining cost of the standard flooring	39.60
Making the total cost for the bridge with standard flooring, $411.00 + $39.60	$450.60

For the open-grate structure the additional cost is found thus:

Flooring, 45 × 1.50	$67.50
Extra metal in floor system, 11 × 45 = 495 lb @ 4 cts	19.80
Guard-rails	4.00
Summation	$91.30
Plus partial cost from Fig. 2	353.60
Total	$444.90

The ratio of costs, therefore, will be $\frac{\$444.9}{\$450.6} = 0.987$. Hence, the saving will be 1.3 per cent.

MOVABLE SPANS

This investigation has thus far ignored an important economic application of light floorings, namely, their use in vertical-lift and bascule bridges. In these cases, regardless of span length, there is a decided saving of money in adopting any of the light floorings, especially an open-grate flooring, because the economy involved is not confined (as in fixed spans) to the floor system and the trusses, but extends to the cables with their connections, the counterweights, the sheaves with their bearings, the towers, the entire operating machinery in vertical-lift bridges and bascules, and to numerous other parts of bascule bridges—all in addition to the saving in cost of substructure.

In the past, almost all bascule spans have been designed with light flooring, first of timber (which is most objectionable because of fire risk) and, later, of some patented type of thin flooring; but many vertical-lift spans have had ordinary flooring of reinforced concrete; and the weight of this flooring constitutes a considerable portion of the total load to be raised and lowered.

The open-grate flooring has an additional advantage for bascule spans, shared by no other type of flooring: When the span is being raised or lowered, the force of the wind blowing through the grating largely reduces the pressure against the floor, thus lessening materially, under certain extreme conditions of operation, the quantity of power required to raise or to lower the moving leaf.

The open-grate flooring has a characteristic that is of considerable economic importance in vertical-lift bridges—it does not have a tendency to collect snow or ice. Therefore, the customary excess-load allowance for snow or ice of 5 lb per sq ft of floor in such structures may safely be reduced, thus effecting quite a saving in total cost of structure and of the operating machinery, in addition, of course, to the numerous economies from the reduced weight of the flooring itself.

The combination of all these savings by the open-grate flooring on both vertical-lift and bascule spans is so large, in comparison with the ordinary reinforced concrete deck, as always greatly to overcome the handicap of the excess unit-price of the lighter flooring. At present (1937) it is not worth while to attempt to investigate the comparative economics of the various light floorings on movable spans; but at some time in the future it might prove otherwise.

TRESTLE BRIDGES

In view of the present, very intensive introduction of elevated highway structures to relieve the congestion of traffic in metropolitan centers, the writer has been urged to treat the economy of floors on such structures.

In general, such structures are composed essentially of short spans, and are similar in layout to elevated railways in cities and to ordinary trestles. In such cases there is absolutely no economic advantage in lightening the floor by increasing, materially, its cost per square foot. The usual thick, reinforced concrete flooring, therefore, is the best type to adopt for both economy and rigidity, providing the matter of lateral skidding is ignored; but should such structures contain spans exceeding 250 ft in length, a lighter flooring might be economical on these longer spans.

SUSPENSION BRIDGES

The economics of the various types of flooring for suspension bridges may be stated in a very few words—the lighter the flooring the less will be the first cost of the structure, and, therefore, the open-grate type would seem to be called for in all cases; but sometimes the designer may desire to rely upon continuity in the flooring for lateral rigidity, because the usual narrow suspension bridge is unquestionably lacking in that desideratum. In such a case, it might be well to use a thin, solid flooring; but the writer's preference would be to adopt the open grate and provide a rigid, substantial lateral system near the plane of the deck.

A study of Figs. 4 and 8, in conjunction with Table 2, should convince any one that heavy floorings are always uneconomical in cantilever bridges, and that the open-grate flooring is specially economical for this type. It is conceded that a thick, solid flooring on the anchor arms would reduce the uplifts at the anchorages, and thus effect a saving in the truss weights of cantilever bridges, as well as in the cost of the flooring itself; but such a saving would not be as great as the gain from the reduced dead loads on the entire main span caused by the open-grate, which reduction affects favorably all the trusses in the suspended span, the cantilever arms, and the anchor arms. It is true that the open grating could be used on the main span, and the heavy flooring on the anchor arms; but the hybrid deck thus produced would be objectionable from the æsthetic viewpoint, and, possibly, also for other reasons.

WARNING

Some one may claim that a fundamental change in the governing unit prices for materials and labor in connection with bridge building will render null and void the service of the cost-curve diagrams; but such is by no means the case, because the relativity of the results will still hold good, and the comparisons of economics will remain reliable, in spite of any such changes. It is true that the unit prices for superstructure and those for substructure do not always vary in the same proportion; but eventually they settle down to an almost constant relationship. Even a permanent dislocation of their relativity, however, would not cause any material incorrectness in the results of the application of the curves to their legitimate function.

Conclusion

Attention is called to the fact that no where in this paper has any reference been made concerning the superiority, or the contrary, of any particular patented flooring; the problem investigated is solely one of economics. A consistent effort has been made to avoid advocating "special interests," with the understanding that discussers, likewise, will confine their comments to the intended scope of the paper.

DISCUSSION

Jonathan Jones,[3] M. Am. Soc. C. E. (by letter).—It should only emphasize the high regard in which the writer holds this paper, to state that its value will ultimately lie more in the method of approach and analysis which it sets forth, than in its statistical data. To the writer, it seems impossible to generalize the problem so that the unit costs involved will lie in the same pattern to-morrow as to-day; even the basic assumption that unit costs of flooring will be somewhat in inverse ratio to their unit weights, may not always hold.

Furthermore, all the factors that have been carefully analyzed in the paper for the general, may not enter into the particular, case. The most interesting case in which, within the writer's knowledge, the designing engineer erred in selecting a light-weight and expensive floor, involved several narrow 210-ft truss spans on adequate old piers. The cost of the piers, therefore, was not a factor in the problem, and to that extent at least the application of Fig. 5 would have been in error. Had the designing engineer, however, applied the logic of the author's method to those factors which did exist in his problem, and to the unit costs which he could foresee at his location, he would inevitably have found that he would have saved several thousand dollars by adopting a "standard" or even a "super-standard" floor.

The relationship between 4 cts per lb for secondary joists and 7 cts or more for an added stringer, would similarly be debatable in a given case. From such considerations comes the writer's feeling that, although the method herein given is invaluable, the most exact data should be applied to each particular case, and the author's data should be considered as illustrative.

The important effect of a variation in the data may be demonstrated by reference to Table 3. Using the author's unit costs, the minimum simple span at which the "standard" and the "50-lb" floor will be of equal cost is shown in Table 3(*a*); and in Table 3(*b*) these spans are re-calculated on the hypothesis that the cost of the "50-lb" floor has been reduced from $1.30 to only $1.15 per sq ft.

There are two generalizations in the paper toward which the writer would suggest considerable caution; these are found under the heading, "Suspension Bridges." With respect to suspension bridges it is stated "the lighter the flooring the less will be the first cost of the structure." The first requirement in approaching the design of any suspension bridge is to place a maximum limit on its flexibility, or deformation under partial live load. The weight of the flooring, and the consequent weight of the cables, have an important influence on stiffness. Granted that the design has been established as just adequately stiff, using a "standard" floor, then, if it is decided to use a lighter flooring and less wire in the cables, it will follow that the structure exceeds the pre-determined limit of flexibility, and the stiffening trusses will have to be increased in section or depth, or both, to restore the rigidity that has been taken out. Some, at least, of the money saved on cables and other main structure, therefore, must be spent on the stiffening system; and this is rather a pity because a dollar spent in the main structure is better invested than a dollar spent for local stiffening. This type of structure, therefore, calls for a complete comparative analysis.

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[3] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

TABLE 3.—EFFECT OF A VARIATION IN DATA, IN THE CASE OF SIMPLE-TRUSS BRIDGES

Item No.	Description	20-FOOT ROADWAY		40-FOOT ROADWAY	
		Carbon steel	Silicon steel	Carbon steel	Silicon steel
	(*a*) SPANS OF EQUAL COST, USING THE AUTHOR'S DATA				
1	Standard 9-in., 110-lb slab	$19.60	$19.60	$39.60	$39.60
2	Special 50-lb floor	34.20	34.80	71.20	72.55
3	Difference	$14.60	$15.20	$31.60	$32.95
4	From Fig	5	6	1	2
5	Read the span length of equal cost, in feet	300	380	270	335
	(*b*) SPANS OF EQUAL COST, IF COST OF THE 50-POUND FLOOR IS REDUCED FROM $1.30 TO $1.15 PER SQUARE FOOT				
6	Difference is less than Item No. 3 by:	$3.00	$3.00	$6.75	$6.75
7	New difference (Item No. 3 less than Item No. 6)	$11.60	$12.20	$24.85	$26.20
8	Span length of equal cost, in feet	230	320	180	240

With respect to cantilevers, the writer does not see the objection, from the æsthetic viewpoint, in using the heavier flooring on part or on all of the anchor arms. The change from the slab floor of the approaches to the special floor of the bridge spans must be made somewhere, and there seems to be no reason not to make it where it will be most beneficial. The prevention or, at all events, the minimizing, of live load reversal on the main truss-to-anchorage pins is proving to be extremely desirable on old cantilevers, even if it calls for longer anchor arms than former practice favored; to augment the downward force at this point by the use of heavy flooring on the anchor arms seems much more important than the question of where the tourist leaves the concrete and enters upon the metallic deck.

This paper is a most important contribution to one of the livest topics, from a money standpoint, now before the structural engineer, and thanks are due to Mr. Waddell for the effort expended upon it.

HENRY C. TAMMEN,[4] M. AM. SOC. C. E. (by letter).—In view of the many new developments in light-weight highway-bridge floorings during recent years and the interest of engineers in such floorings as evidenced by their increasing use, this paper is very timely. It should be of material help in the analysis of the economics of such floorings.

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[4] Cons. Engr. (Ash-Howard-Needles & Tammen), New York, N. Y.

The author develops the relative economy of two floorings by the comparison of sums composed of two items: (1) The construction cost of the flooring and of certain special floor parts affected by the flooring design; and (2) the construction cost of the remainder of the superstructure and of the substructure.

For any flooring the amount of the first item (the cost of the flooring and certain special parts, shown in Table 2) is to be determined by each engineer for himself. It is to be expected that, for the same flooring and for the same structure, different engineers will reach somewhat different results. This might follow from variations in the design procedure (which is not yet standardized for many of the floorings); from variations in fastenings of the flooring to its supports (regarding which there are wide differences of opinion); from variations in minimum metal thicknesses; or from other causes. Some engineers will find it difficult to accept thicknesses of $\frac{1}{4}$ to $\frac{3}{16}$ in., or even less, as proper for a primary element carrying heavy loads such as a flooring, after having thought and worked for years with such members in terms of $\frac{5}{16}$ in. and $\frac{3}{8}$ in., or even greater thicknesses. The question will arise as to whether such thicknesses should be considered satisfactory at all, and also, whether a noncorrosive coating should be applied to the metal or whether the safe life of the flooring making use of such thicknesses should not be considered very much less than that of, say, the standard flooring.

To give a true economic comparison between several floorings, the writer would consider it proper to add to the flooring costs suitable amounts to cover an allowance for amortization of the flooring cost in cases where the safe life cannot be taken the same for the several floorings being compared. Similar allowances should properly be made also for differences in anticipated maintenance costs, including painting, snow removal (where applicable), and other maintenance items. For open-grate floorings the average annual maintenance painting cost may be in excess of 1 ct per sq ft, requiring the annual interest on an investment of 20 or 25 ct per sq ft, or more. For light-weight floorings having only a flat-bottom steel surface to be painted the cost will be only a fraction of the foregoing values, whereas for concrete floorings this cost disappears. Clearly the capitalized maintenance costs may vary substantially with the flooring type and may require consideration in any determination of comparative economy of several floorings.

With reference to the costs making up the second item (shown in Figs. 1 to 8) there should be no question regarding the portion of this cost which represents the superstructure. The superstructure cost is quite definite for each span length and, without doubt, the author's determination of it is as accurate as can reasonably be made. However, there may be material variations for different structures in the portion of this cost which represents the substructure.

Inspection of the cost curves for simple spans shows that minimum costs are found for spans between 200 ft and 300 ft for the various types of floor, for various roadway widths. Clearly, these are the economic span lengths for the grade line and foundation conditions which have been assumed by the author. Granting that, in actual structures, the span lengths in general conform closely to the economic, it would follow that for such economic structures the substructure costs and the saving effected by the use of lighter floorings which would result from the use of the cost curves, are too high for spans less than the foregoing and too small for greater spans. This applies also to the cost curves for cantilever spans where the several curves show minimum costs for spans between 700 ft and 800 ft.

In arriving at his substructure quantities, the author has used increasingly smaller piers with the lighter flooring. Many times the conditions are such that the pier shafts and, more frequently, the base sizes which would be adopted for standard concrete flooring are minimum sizes to give stability for the dead, live, wind, water, ice, and other load conditions for which provision must be made, and would not be decreased even if a light-weight flooring were used.

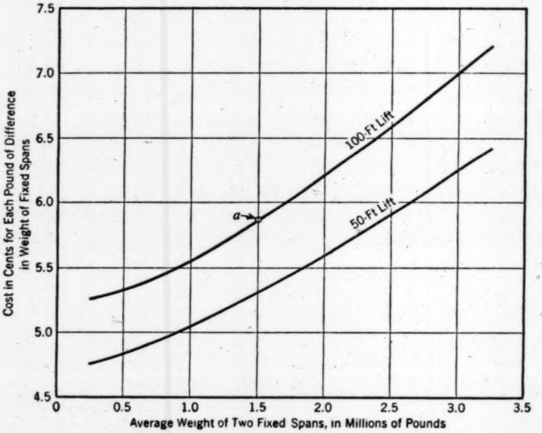

FIG. 11.—DIFFERENCES IN COST OF CONVERTING TWO FIXED SPANS OF DIFFERENT WEIGHTS INTO VERTICAL LIFT SPANS.

To make the cost data more generally useful in the analysis for structures with foundation conditions varying from those assumed in the paper, it would be helpful if the superstructure and substructure costs could be separated, and it is hoped that the author will find it possible to add this information in his closing discussion.

The writer has been particularly interested in the statements made with reference to economy of light-weight floors on movable spans. He has had occasion to make use of light-weight floors many times on such structures. The earlier applications on vertical lift spans involved merely the use of a light-weight aggregate in the concrete floor-slabs, these slabs being supported on closely spaced cross-beams to reduce slab thicknesses to a minimum. Even with the limited weight reduction practicable by this means, it was found possible with economy to ship such light-weight aggregate from its point of production in Kansas City, Mo., as far as San Francisco, Calif., and Canada, and in the latter case also to pay a duty on the aggregate.

Some years ago, the writer prepared estimates of quantities and costs for a group of vertical lift spans; from these he determined the cost of converting an ordinary fixed span into a vertical lift span and, finally, the increase or decrease in this conversion cost resulting from an increase or decrease of 1 lb in the weight of the fixed span. The results, revised to conform to present design practice, are shown in Fig. 11 in convenient form for use. They are based on towers with inclined rear legs supported on truss approach spans and on the conventional method of operation, with the machinery at the middle of the lift span, but should apply very generally also to spans with "tower drives". The make-up of the total cost, in cents, for Point *a*, Fig. 11, is as follows:

Tower and span metal	0.327 lb @ 5.5 ct × 0.75 =	1.35
Counter-weights	1.153 lb @ 1.0 ct × 0.75 =	0.87
Tower sheaves, shafts, and bearings	0.111 lb @ 15.0 ct × 0.75 =	1.25
Ropes	0.044 lb @ 20.0 ct × 0.75 =	0.66
Balance chains	0.052 lb @ 8.0 ct × 0.75 =	0.31
Operating machinery	0.028 lb @ 20.0 ct × 0.75 =	0.42
Motors, engine, house, etc.		1.00
Total		5.86

The same unit prices were used for all other points. The factor, 0.75, used with the assumed unit prices conforms to Equations (2), (3), and (4), given by the author which means that any reduction or increase in cost resulting from a change in quantity of an item is calculated by assuming that the quantity change takes a unit price equal to three-fourths that for the original quantity. In making a comparison of lift-span costs for two floorings, enter the curves with the average weight of the fixed spans for the two floorings. The difference in weight of the two fixed spans multiplied by the unit cost taken from the curves, will then represent the total cost saving in the movable span superstructure by use of the lighter of the two floorings under consideration.

Referring to Fig. 11, the average weight of two fixed spans may be expressed as $\frac{A + B}{2}$, in which A is the weight of the heavier fixed span; and B is the weight of the lighter of the two. Similarly, the total difference in cost may be expressed as $(A - B)\ C_x$, in which C_x is the cost per pound, taken from the curves of Fig. 11. The costs given by these curves include only the superstructure, and the savings shown by them for light-weight floorings are in addition to any savings for fixed spans that may be shown in the analysis presented by the author.

Fig. 11 indicates clearly the value of light weight in the flooring of a vertical lift span. The economy in a bascule span by use of light-weight flooring, in general, will be somewhat less than on vertical lift spans, due to the absence of ropes and to the fact that the dead weight of the span and counterweights has only a minor effect on the operating machinery and power requirements. The latter are determined largely by the wind loads. The saving in counterweight cost on bascules by the use of light flooring, on the contrary, may be several times that for lift spans, due to an increased ratio of counterweight weight to span weight and, frequently, also to a reduction in the unit weight of counterweights. The conditions for bascule spans are so variable that it is not practicable to show, by curves, the effects of weight changes in the span.

Miller McClintock,[5] Esq. (by letter).—Shortly after the first installment of an open steel-mesh flooring was made in the University Bridge, in Seattle, Wash., the writer, attracted by the possibilities of such light-weight construction for the decking of elevated, limited-way, structures, visited Seattle and made observations of the characteristics of the installation. On the basis of these observations and other tests and considerations, he recommended the use of this type of open steel-mesh flooring for a system of elevated limited highways then being designed for the City of Chicago, Ill. The incorporation of this type of light-weight flooring made it possible to reduce the size of various structural units and resulted in substantially lowering the estimated unit costs. The cost element, however, although important, was not the main factor which justified the recommendation. The writer was seeking a flooring completely flat in character and without pitch or crown for the purpose of improving the riding characteristics of the structure. To obtain such a flooring, it was necessary to design a decking which would be self-draining. This function of the open steel-mesh decking was very satisfactory and, in turn, eliminated certain structural costs in the design of drainage and run-off systems. The added economy of self-snow-cleansing was not without importance in the consideration.

A further very important economy was found in the capacity of this open steel-mesh flooring to pass a substantial percentage of normal light and air. In view of the fact that many of the elevated limited way structures, as designed, were contemplated for construction over existing streets, this was an element of very considerable significance. It eliminated in all or in part the necessity for day-time illumination for street surfaces under elevated structures and materially reduced contemplated damages to abutting property owners.

In the writer's opinion, this paper indicates a trend in thinking which should have substantial influence upon future designs of bridges and similar types of structures. There appears to be a particular need to obtain a simpler and less expensive type of construction for the thousands of grade-crossing separations which will be required in the near future.

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[5] Director, Bureau for Street Traffic Research, Harvard Univ., Cambridge, Mass.

JOSEPH G. SHRYOCK,[6] M. AM. SOC. C. E. (by letter).—This timely and useful paper brings out many points of interest to the bridge and structural engineer, and emphasizes the fact that the lighter the floor, the greater will be the difference of cost per square foot over that of the conventional standard designs, and that there is doubtless a point on long-span bridges where the saving in weight of the floor-slab is offset by a sacrifice of lateral stiffness and rigidity of the structure.

All forms of solid floor construction, whether of concrete or steel, or a combination of both, form a horizontal girder capable of taking and absorbing a large percentage of the lateral stresses; this attribute is lacking in the open type floor.

All standard bridge specifications call for a specified $\frac{3}{8}$-in. minimum thickness of material, even for lacing-bars and redundant members, and only permit less thickness for lining and filling vacant spaces.

In 1900, the author presented[7] specifications for steel highway bridges which read as follows: "No metal less than $\frac{5}{16}$ of an inch in thickness shall be used, except for filling plates; and in important bridges this limit shall be increased to $\frac{3}{8}$ of an inch." Even with modern bridge steel, corrosion is still an important problem, and leaves only a relatively small factor for permanent strength on any $\frac{3}{16}$-in. member.

It would be advisable, therefore, to design an open-grate floor to meet standard specification requirements, as to minimum thickness of material and minimum size of structural rivets. To add a supplementary jack system to support the conventional open-grate floor properly, doubles the number of pieces in the floor system and their maintenance as well. Owing to the importance of future maintenance on all major structures, the modern engineering tendency is to use as few members as possible of rolled sections without lacing, thereby not only reducing secondary stresses to a minimum, but the cleaning and painting of the structure as well.

Whether it is of masonry or steel, solid or open, the floor-slab is a structural member of major importance; it receives not only the maximum concentrated wheel loads, but a greater percentage of impact than any other member in the bridge, and, in addition, is subject to abrasion or wear from moving loads.

On long-span bridges, each pound per square foot saved in the dead load of the floor becomes an important factor. Savings, therefore, of as much as 50% of the weight of the standard type of floor would effect economies without undue sacrifice of stiffness and rigidity. To reduce the weight of the floor to only 25% of the standard, however, could scarcely be secured without a sacrifice of these important requirements.

C. CALOR MOTA,[8] ESQ. (by letter).—In a course of bridge design for civil engineering students this paper is indeed a very useful one, and it is especially adapted to the practical use of the bridge designer. The most striking characteristic of the paper is its accurateness and completeness. All possible factors that may cause a change of metal weights in the trusses and floors per linear foot of bridge, are considered carefully and also any variations from the basic assumptions adopted. There is no question but that, in order to obtain the partial unit-cost curves of the paper, an enormous mass of data was necessary which only a designer of long experience could make possible. Table 2 is indeed a valuable tool for computing the total cost of structure per linear foot.

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[6] Vice-Pres., Director, and Chf. Engr., Belmont Iron Works, Philadelphia, Pa.

[7] "De Pontibus", 1900, p. 226.

[8] Head, Dept. of Civ. Eng., Univ. of Puerto Rico, Mayaguez, Puerto Rico.

This paper, similar to that entitled, "Weights of Metal in Steel Trusses"[9], by the same author, constitutes a creditable contribution to technical literature in the economics of bridge engineering.

W. G. Fowler,[10] Assoc. M. Am. Soc. C. E. (by letter).—The problem of designing highway bridge floors has been attracting increasing attention from engineers during recent years, and the importance of this phase of bridge construction is attested by the number and variety of floors introduced during the last decade. To the best of the writer's information, this paper represents the first public attempt to correlate the knowledge relating to the various types of floors and to present it in a rational manner.

The curves and data presented by the author should prove invaluable as a means of making rapid comparisons between various types of floors in a preliminary study. The writer has made some studies of typical bridges with "standard flooring" and the pier heights assumed in the paper, and he finds a close check with the values presented by the author. However, if slightly different assumptions are made for the pier heights, results are obtained which depart rather widely from the costs indicated by the curves of partial unit costs. Because of this fact, the writer feels that these curves could be made much more valuable and easier to understand if the items of superstructure costs were separated from substructure costs. Substructures are usually a special problem for every bridge location, and the writer has never yet been able to set up curves or tables sufficiently comprehensive to be very useful in such studies.

The question of superstructure costs lends itself much more readily to standardization. Many engineers prepare their preliminary studies of a particular bridge structure by making parallel estimates of the superstructure from curves or tables, and of the substructure from special studies adapted to the location; and then they assemble the results in various combinations to determine the most economical arrangement. For this reason, the aforementioned division of data would be very useful.

Philip A. Franklin,[11] M. Am. Soc. C. E. (by letter).—It is with economics of first cost that the author has concerned himself and to that phase of the problem he has confined his paper. Very definitely, he states in his "Introduction" that he is "ignoring all claims for superiority * * *," and, in his closing paragraph, warns the discussers of the paper against advocating "special interests." All this is right and proper and the author has done a thorough and complete job of giving the profession a valuable working tool.

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[9] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1.

[10] Office Engr., Black & Veatch, Kansas City, Mo.

[11] With Bridge Decking Div., Blaw Knox Co., Blawnox, Pa.

The problem of choice of floor still remains and the duty of the engineer is to solve it by balancing the first cost of the various types against their desirability. This involves a knowledge of relative costs as determined by the curves and tables of the paper and also an appreciation of the values of the intangibles not susceptible of reduction to such curves and tables.

Mr. Waddell has simplified the problem by separating the factors into tangibles and intangibles and reducing the tangibles to as efficient a tool for solving first cost as the members of the profession have had presented to them for many years. If the economics of first cost were the ultimate answer to the problem of designing bridges, the consideration of such a paper would be a matter for a convention of purchasing agents and would not be worthy of the efforts of a body of engineers.

The curves and tables are only part of the necessary tools, however, and they must be used in combination with other tools in order to shape the materials entering into the design problem into a product commensurate with the reputation of the engineer. The consideration of this larger problem of the "Ultimate Economics of Highway Bridge Floors" is quite worthy of the efforts of the profession. It should not be hampered by the limitations of mathematics in the discussion, but the merits and shortcomings of all the various types of floors in common use should be reviewed; the factors which lead to desirable features as well as those which produce undesirable results should be listed; and conclusions should be drawn concerning the attributes of the perfect floor.

That there are defects in all the types of floors under discussion is well known to all bridge engineers. The heavier concrete floors are liable to surface abrasion, becoming rough and requiring re-surfacing. It is difficult to keep them from developing cracks due to negative bending, thereby allowing percolation of water to the supporting steel, and causing corrosion in places difficult to inspect and repair. The lighter concrete floors, containing aggregate of lesser weight, relieve the load on the supporting structure, but are still less abrasion-resistant than the heavier types. The filled-grid types are less subject to abrasion, and are still lighter; but they have a continuous metal surface beneath them that presents a maintenance and inspection problem in an inaccessible location. If they are not properly designed they are also likely to crack because of negative bending, with higher repair costs than the plain concrete floors. The inverted trough type of filled floor, if filled with alternate strips of thin and thick filler covered with a bituminous wearing surface, will become wavy under traffic and will require frequent re-surfacing. If this type is filled with concrete, the concrete and the surfacing will crack along the lines where the depth of the filler changes. It also has the same faults as the other all-metal under-surface floors in that it is more or less inaccessible for inspection and maintenance. The all-metal solid floor, with checkered-plate wearing surface, is also subject to corrosion on the under side and is anything but a safe driving surface in wet or icy weather, or even in summer when the grease-drip oils the surface. The markings soon wear off and only a plain steel plate is left.

All the solid surface types can only have good driving surfaces in fair weather. In snowy, sleety, or even in wet weather, they present a surface to traffic different than in dry weather and much more hazardous.

The open-grid floors require a secondary system of supports and complicate the original detailing and shop work as measured by present-day routine methods. Their greatest drawback is their novelty. For instance, the fact that some engineers "view with alarm" the supposed lack of lateral rigidity in the open floor when no consideration has ever been allowed on that score for any of the solid floors, is quite puzzling. Lateral bracing always has been, and probably always will be, designed for the full effect of some extremely high wind, and the effect of any stiffening due to the floor has been taken as an "added safety factor." The steel in the open-grid floors forms a rather effective horizontal girder of very large moment of inertia, with ample riveting to carry horizontal shear between the various longitudinal members. These, in turn, are welded to their stiffeners in the form of the transverse carrying members and these again to the longitudinal stringers forming a three-member laminated system much more resistant to distortion or to disruption than any concrete floor. To the writer, at least, the idea of putting metal in the lateral system, and in the stiffening truss of a suspension bridge, answers the requirement of "a dollar spent in the main structure is better invested than a dollar spent in local stiffening." It is better than to spend it for more costly floor construction which, formerly, was never credited with lateral stiffness.

Another principal fallacy is that water, in itself, is responsible for corrosion. Corrosion is due to water in combination with other compounds and conditions, and the longer these factors are allowed to remain in combination the faster the corrosion proceeds. Conversely, the speedy breaking up of the combination is the best preventive of corrosion. The sunlight goes through an open grid to the supporting stringers and floorbeams. Each passing vehicle fans the grid and the supports with an intense blast of air. The drip from the engines coats the surface of the grid members. The dirt and dust falling through the grid are dried by the ventilation and forced draft and are blown away so that corrosion does not attack either the supports or the much-feared $\frac{3}{16}$-in. metal of the grid.

Naturally, the consultant is interested primarily in his reputation for conservative design; the fabricator's engineer is interested in securing a contract by offering a cheaper structure than his competitor while remaining within the letter of the specification. The inventor, whether or not he is in either of the foregoing classes, is interested in having his "brain-child" accepted in "the best circles" and in securing a royalty for it.

This leaves the prospective owner or the unbiased engineer in the position of having to satisfy himself as to the desirability of the various floors offered, and then to choose from them by balancing the costs against the merits. Since only approximately 10% of the membership of the Society is directly interested in bridges and probably not 10% of these are in either of the aforementioned biased classes, it should be apparent that the only hope for any of the other 90% of the members to "sift the wheat from the chaff" is to make their own investigation into the intangibles not covered in the paper.

J. A. L. WADDELL,[12] HON. M. AM. SOC. C. E. (by letter).—The discussion of this paper, although not as extensive as the writer had hoped, has served to emphasize several important features. The points raised by Messrs. Jones, Tammen, and Franklin are of especial interest.

Messrs. Tammen and Fowler have asked for a separation of the substructure and superstructure costs used in preparing the curves of Figs. 1 to 8. On the average, the substructure for the open-grate flooring was found to cost about $14 per lin ft of bridge less than that for the "standard" flooring for a roadway width of 45 ft, and about $7 less for a 20-ft roadway. Cost differences for intermediate types can be interpolated with sufficient accuracy. For the case mentioned in the second paragraph of Mr. Jones' discussion, the substructure differential should be subtracted from the difference found from the curves of Fig. 1 or Fig. 5.

Mr. Jones emphasizes the value of "the method of approach and analysis" which the paper sets forth, but questions whether the findings are sufficiently general for the curves to be used in actual comparisons of cost. He states: "From such considerations comes the writer's feeling that, although the method herein given is invaluable, the most exact data should be applied to each particular case, and the author's data should be considered as illustrative."

In making his criticism, Mr. Jones has apparently overlooked the fact that Figs. 1 to 8 give merely the combined costs of substructure and superstructure to carry floorings of various weights, and that the costs of the floorings themselves are to be determined and added separately. Tables 1 and 2 give data regarding the costs of various floorings that are fairly representative of present-day light-weight floors; but it is assumed that an engineer, in dealing with a specific problem, will determine the weights and costs of the actual floorings he is comparing, and will use his results rather than those of Tables 1 and 2. Ample provision is made, therefore, for variation in flooring costs and for the future development of new types.

Table 3 of Mr. Jones' discussion, showing the effect of a variation in floor costs, is correct; but this is no argument against the accuracy of the results found from Figs. 1 to 8, for the reasons just stated.

Messrs. Jones, Tammen, and Fowler have referred to factors affecting the accuracy of the diagrams themselves, such as variations in prices and in the substructure. They have, however, over-estimated the relative importance of such variations. Suppose, for instance, a silicon-steel highway bridge of 400-ft simple spans, carrying a 45-ft roadway, is being considered, and that comparison is to be made for the "standard" flooring, a 60-lb flooring, and an open-grate floor. From Fig. 2 and Table 2, the values in Table 4 are obtained, the separate items for substructure and superstructure being taken from the computations on which the diagrams were based.

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[12] Cons. Engr., New York, N. Y. Mr. Waddell died March 3, 1938.

TABLE 4.—COMPARISON OF COSTS

Item No.	Description	Type of Flooring		
		Standard	Sixty pound	Open grate
	(1)	(2)	(3)	(4)
1	Superstructure	$311.40	$282.30	$263.00
2	Substructure	111.00	103.50	97.30
3	Cost, without deck	$422.40	$385.80	$360.30
4	Cost of deck	39.60	67.80	88.60
5	Total	$462.00	$453.60	$448.90
6	Excess cost over open-grate type	+$13.10	+$4.70	0

If a 10% change in superstructure unit prices is assumed, the differential between the "standard" and open-grate floors would be changed by $4.84, and that between the 60-lb and open-grate floors by $1.93. A considerable change in the substructure conditions would mean merely a change in the differential between the "standard" floor and the open-grate flooring ($13.10), or in the differential between the 60-lb floor and the open-grate floor ($6.20). In most instances, such changes in costs would not be great enough to upset the relative economics of the floors being considered; and where the relative standings were affected, it would merely be because the costs for the types being compared were so nearly alike that it would make little difference, as far as cost is concerned, which type was used. In the case under consideration, a difference of $5 per ft would mean only a 1% variation, and even $10 per ft would amount to only 2 per cent. Such differences are well within the ordinary errors of estimating. Furthermore, changes in the general price level can be allowed for by multiplying the results taken from the diagrams by a correction factor, and a major difference in substructure conditions can be allowed for by increasing or reducing the substructure differentials given in the second paragraph of this closure.

Since this paper was published the writer has had occasion to prepare several estimates affording an opportunity for a check on the curves in Figs. 1 to 8, and has found them absolutely reliable.[13] A further study, made in connection with the preparation of this closure, shows that a considerable variation in pier heights makes no appreciable difference in the relative costs of different types of floors.

Mr. Jones is correct in calling attention to the fact that lightening the floor of a suspension bridge will require an increase in the weight of the stiffening trusses. In most cases, however, this means simply a reduction in the saving to be realized by the use of a lighter floor. For a comparatively short span, where the saving in other parts of the bridge is not large, the increase in the stiffening trusses will be small; and, for a long span, on the other hand, where the increase in stiffening truss weight will be considerable, the saving in other parts—floor-system, suspenders, cables, towers, and anchorages—will also be large.

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[13] "Quick Estimating of Economic-Bridge Costs," by J. A. L. Waddell, Hon. M. Am. Soc. C. E., and Shortridge Hardesty, M. Am. Soc. C. E., N. Y., John Wiley & Sons.

Referring to cantilever spans, there would be no objection to using a heavy floor on the anchor spans of a three-span Type *A* cantilever, and a light floor on the central span. For a Type *C* cantilever or a long bridge consisting of an alternation of anchor and cantilever spans, it would be unsatisfactory, from the standpoint of both appearance and traffic, to use one type on the anchor spans and another on the cantilever spans.

Mr. Tammen mentions the fact that different engineers will reach different results for the same flooring, due to variations in design procedure, details, minimum thickness of metal, and allowances for maintenance and amortization. Such considerations do not affect the usefulness of the paper in any way, because it is assumed that each engineer will determine for himself the total costs and allowable span lengths of the floorings he is comparing. Allowances for maintenance and amortization are difficult to determine, since relative costs and life for light-weight floors and older types of heavy floors are not yet known.

Mr. Tammen criticizes the use of thinner metal in the flooring rather than in other parts of the superstructure—and possibly his comment is truly sound; but it must not be forgotten that, if the light-weight flooring should either rust out or wear out, it could easily be repaired or replaced, whereas thin metal in the superstructure proper could not. It is either the manufacturers or the patentees of such floors who have determined the thickness of their products; and if these sections are computed properly for strength, experience will ultimately tell whether the use of the thin sections is truly economical.

Mr. Tammen's data relative to the savings resulting from the use of light-weight floors on movable spans are interesting and valuable, and the writer endorses them heartily.

It is difficult to agree with Mr. Shryock's conclusion that floor weights can be reduced to 50% of the weight of the standard type "without undue sacrifice of stiffness and rigidity," but that a reduction to 25% "could scarcely be secured without a sacrifice of these important requirements," because both the percentages quoted are purely arbitrary. The 540-ft spans of the Marine Parkway Bridge over Rockaway Inlet in New York City, which are paved with open-grating, are perfectly stiff and rigid under traffic. It should also be noted, as stated by Mr. Franklin, that an open-grate floor, when properly detailed and thoroughly connected to the supporting steelwork, forms an effective horizontal girder.

To Mr. McClintock and Professor C. Calor Mota, the writer desires to express his deep appreciation of their hearty endorsements of his technical efforts. Mr. Fowler should not forget that the sole object of the paper is to compare the economics of different types of flooring and not to provide the data for making cost estimates of bridges.

Mr. Franklin's discussion is most important. The writer endorses unequivocally all the points he raises, and specially commends the division of the conditions into "tangibles and intangibles"; moreover he concedes that the latter are, and should be universally considered by far the more important. In selecting the type of flooring for a highway bridge, the prime requisite certainly is safety; and economy in first cost should always be a subordinate consideration. Safety from accidents through skidding should always be a primary consideration, because human life and human welfare are endangered by such accidents.

Mr. Franklin has prepared his discussion very skilfully, in that he has handled the subject of "intangibles" without any reference to patented floorings or special interests. Nevertheless, he states:

> "* * * The consideration of this larger problem of the 'Ultimate Economics of Highway-Bridge Floors' is quite worthy of the efforts of the profession. It should not be hampered by the limitations of mathematics in the discussion, but the merits and shortcomings of all the various types of floors in common use should be reviewed; the factors which lead to desirable features, as well as those which produce undesirable results, should be listed; and conclusions should be drawn concerning the attributes of the perfect floor."

Such a general open discussion would be of truly great value.

The objection to open-grid floors that they "require a secondary system of supports and complicate the original detailing and shopwork" is an economic one, and, therefore, pertains to the "tangible" group of factors; consequently, it is taken care of by the cost-curves and tables of the paper. From the standpoint of safety, the principal advantage of the open floor is its elimination of lateral skidding, even during times of snow and sleet.

The Marine Parkway Bridge, previously mentioned, will provide an excellent measure of the safety, the durability, and the cost of maintenance for open flooring, as compared with the "standard" solid flooring; because, on the three long main spans, open-grate flooring is used, whereas all the approach spans have concrete flooring. Experience during one winter may settle the question of the comparative safety of the two types, but years will be required in order to determine the comparative maintenance costs.

In the writer's opinion, the open flooring has come to stay, and is destined to be one of the standard highway-bridge floors of the future.

In closing this paper, the writer desires again to thank all the engineers who have so kindly discussed it, especially Mr. Jones and Mr. Tammen, whose skepticism concerning accuracy of results has enabled him to show that such skepticism is unfounded, and Mr. Franklin who indicated the existence of "intangible" as well as economic factors in the comparison of bridge floorings.

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NATIONAL ASPECTS OF FLOOD CONTROL
A SYMPOSIUM

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FEDERAL RESPONSIBILITY FOR FLOOD CONTROL

By the Hon. James J. Davis[1]

Since Colonial times, floods have taken their toll annually in various sections of the country. Experience in Pennsylvania is typical, and a few comments on conditions in that State will suffice to indicate the importance of the problem, not only there, but throughout the United States.

Flood Problems in Pennsylvania

Prior to 1937, about thirty major floods had been recorded on the Delaware River, thirty-five on the Susquehanna, and about fifty on the Ohio. (These three streams together drain approximately 95% of the State.) Numerous floods localized on their smaller tributaries have also occurred frequently and have caused serious damage.

The flood of June 1, 1889, is the greatest on record in the Susquehanna Basin. Although the storm was concentrated over the water-sheds of the Juniata and the West Branch of the Susquehanna River, most of the central and eastern part of the State suffered unparalleled losses; at Johnstown, in Cambria County, 2 142 lives were lost, and a large part of the city was destroyed.

The Pittsburgh Flood Commission estimated the flood toll to the City of Pittsburgh alone, from 1898 to 1908, at $12 000 000, of which $6 500 000 was caused by three floods in 1907 and 1908. The total financial loss along the Allegheny River resulting from the 1913 flood amounted to $720 000. Damage in the Shenango and Beaver Valleys from the same flood was $2 100 000. In August, 1915, in the City of Erie, thirty-four people were drowned and damage amounting to $2 000 000 resulted from a flood in Mill Creek. In July, 1931, Norristown suffered a loss of $1 000 000 as the result of floods in two small creeks traversing the Borough. York lost $260 000 in 1884 and $4 360 000 in 1933 from floods in Codorous Creek.

Even these figures, of course, become insignificant in comparison with those for the general floods of 1936, in which Pennsylvania was one of the heaviest sufferers. The flood damages throughout the East during that year are variously estimated at from $250 000 000 to $500 000 000.

Floods can be controlled by storage or retarding basins, diversion channels, stream channel improvements, and levees or embankments. Protection generally may best be secured, not by any one of these methods, but by a combination of several of them. For example, storage basins, together with certain channel improvements, have been recommended for flood protection for the City of Pittsburgh. A combination of channel improvement and levee or dike construction has been suggested along the North Branch of the Susquehanna River to protect the Wyoming Valley. A combination of retarding basins, diversions, and channel improvements has been in successful operation for years at Harrisburg, Pa.

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[1] U. S. Senator from Pennsylvania, Pittsburgh, Pa.

In the Shenango and Beaver Valleys, in Western Pennsylvania, flood control by storage basin retardation alone has been effected by the completion of the State Pymatuning Reservoir Project, although the primary purpose of this reservoir is to provide sufficient water during the dry season for domestic and industrial use.

A carefully determined public policy for control and supervision of the streams of the Commonwealth was adopted and put into effect in 1933, with the enactment of the Dam and Encroachment Act, which made it unlawful for any one to construct any "water obstruction" without a State permit. Since that time, stream channels of a section adequate to discharge the maximum expected floods have been conserved and protected. Channels that had been encroached upon and restricted so that they were unable to pass floods without overflowing, have not been allowed to become worse, and, in various places, serious obstructions to the flow have been modified or removed and the channels have been improved. In communities subject to flood damage the State has made a number of surveys and hydraulic studies. Channel lines, marking the permissible limits of new construction, have been established in Johnstown, York, Manayunk, Sharon, New Castle, Butler, Reading, Norristown, Scranton, and Erie, and existing encroachments on these lines are gradually being removed.

The major flood-control problem in Pennsylvania—protection for Pittsburgh and the Upper Ohio River—was studied under the direction of the Pittsburgh Flood Commission from 1908 to 1912. In the latter year, the Commission recommended the construction of seventeen retarding or flood-storage reservoirs distributed over the drainage areas of the Allegheny and Monongahela Rivers, supplemented by a river wall at Pittsburgh. The cost was estimated to be about one-half the direct loss that would otherwise be caused by flood damage to the city within a 20-yr period. Between 1924 and 1929, United States Army Engineers studied the problem and developed many additional valuable data.

State legislation approved in 1931 authorized the Water and Power Resources Board to take up the study and develop a plan of flood control, and empowered it to proceed on its own authority, or at the request of some outside person or agency, to improve stream channels, build levees and diversion channels, and to construct retarding storage basins for flood control. The Board and its agencies were further empowered to "enter upon, take, appropriate or injure any land or lands", and damages sustained thereby are to be paid by the Department of Forests and Waters. In 1933, the Commission submitted a new plan, based upon all the essential data developed to date, providing for the construction of ten storage reservoirs, seven in the Allegheny water-shed and three in the Monongahela water-shed, at an estimated cost of $57 500 000.

It is natural to ask why a project so meritorious as this one has remained so many years in the planning stage. The answer is plain. The Allegheny River has portions of its water-shed in two States—New York and Pennsylvania. The Monongahela River has its water-shed in three States—Maryland, West Virginia, and Pennsylvania. The proposed reservoirs would be in several States and would affect the flow of the stream in each. It is not possible, therefore, for any city or county, or even for the State of Pennsylvania, to undertake to complete the project in its entirety, or in the comprehensive manner necessary, without assuming more responsibility than it rightly should. National control and action are necessary. Prior to 1936, however, the interest of the Federal Government in large streams was confined, except in a few cases, to the problems of navigation. The main question of concern to the National Government was how much the destruction of a forest, or the building of a bridge or a reservoir, would affect navigation.

More recently, however, a new consciousness has been developing. It is now recognized that flood control is a problem of major national significance, and that it deserves consideration in its own name and on its own merit. It is recognized that what is done to control floods in one part of the country vitally affects the lives, the property, and the well-being of people in other parts.

Federal Participation in Flood Control

The first aid from the Federal Government relating to flood control pertained to the Mississippi River and was provided in the Swamp Land Acts of 1849 and 1850. By these Acts Congress granted to the several States the swamp and overflowed lands within their borders and provided that the proceeds from the sale of the lands must be spent for drainage and flood protection. Congress passed another Act in 1850 that must be given an important place in any consideration of the problems of the Lower Mississippi Valley. This act "directed a topographical and hydrographical survey of the delta of the Mississippi River, with such investigation as might lead to determine the most practical plan for securing it from inundation." The report of Charles Ellet, a Civil Engineer working for the U. S. Army Engineers, was the result of this first official study of flood control by the Federal Government. The Ellet report came in 1852 and concluded that the control of the floods on the Mississippi was the nation's duty and that it was a question that "must be decided by the justice and humanity of the nation." That statement showed a definite trend in official thinking.

The Civil War halted the movement for flood control. In 1878, however, the Rivers and Harbors Bill provided $1 000 000 for aiding navigation on the Mississippi. The money was spent by the Board on the Improvement of the Mississippi River under the supervision of the U. S. Army Engineers. This Board was composed of Army engineers who favored levees for improving navigation. Thus, the Army engineers came to support those who wanted levees to control floods. The combining of the groups desiring flood control and the groups interested in navigation was evident in the debate on the bill creating the Mississippi River Commission in 1879, which put the United States definitely into flood-control work, and which probably stands as the most important piece of flood-control legislation in the entire history of the country.

One of the strongest arguments generally advanced for flood control by the Federal Government is that the United States owns the rivers and has paramount jurisdiction over them. The conclusion that the United States should not permit its property to damage the citizens of any section is the natural outgrowth of ownership. The interstate nature of the flood-control problem has furnished another reason for Federal responsibility. The contrast in size between the gigantic drainage area of the Mississippi River as a whole and the small territory drained by the Lower Valley—the region that suffers most from floods—is striking. The Mississippi Basin contains 1 250 000 sq miles of territory, 41% of the area of the United States. It includes all or parts of thirty-one States, the combined population of which is one-half that of the entire nation. The river system includes 15 000 miles of navigable streams and many thousands of miles of non-navigable ones. The contention that the interstate nature of the problem makes it a national one grows much stronger when it is understood that the area that suffers most from floods contributes little or none of the water that causes them.

The suggestion has frequently been made that the cost of flood control should be borne by the various States in proportion to their responsibility for the floods, but the difficulties of apportioning costs on such a basis appear to be insurmountable The State of North Dakota and the State of Pennsylvania, for example, both contribute water to the Mississippi, but it would hardly be possible to say how much the flow from either State adds to the floods on that stream. The problem is sufficiently complex when only the natural causes of floods are considered; it becomes much more so when the acts of States and individuals in regard to drainage and flood control are taken into account. Works built for flood protection by citizens of one State may pile up the water and spread it over the lands of the citizens of another State. On the other hand, the vigilant have often been inundated because of the negligence of their neighbors. Hence, the demand for Federal control.

Most of the people believe that the Federal Government exists for the practical purposes of protecting the general welfare. In considering the general welfare as related to flood control the humanitarian and the economic phases can not be entirely separated. The great sacrifices, suffering, expenses, and property losses of those at the mercy of the floods are all intertwined.

A consideration of Federal responsibility for flood control calls for thought as to the causes of floods. Apparently, all agree that the causes have many far-flung ramifications. They extend even to the grazing of cattle and sheep far in the interior, when over-grazing of the lands causes rapid run-off and erosion. Even the direction of a furrow in plowing may affect the quantity of water that flows from the surface of a field and the amount of sediment it carries. Paved streets and sewerage and drainage systems have added at least to local flood problems by causing both a heavy and a rapid run-off. Many cities with splendid drainage systems do not realize that they have been hastening floods down upon their neighbors. Even the construction of hard-surfaced and well-drained highways throughout the country has added to the areas from which run-off is heavy and rapid. Many of these highways have been built under Federal specifications and with the aid of Federal funds. Highways, railroads, cities, and industrial and commercial enterprises have aided in piling up the flood-waters by encroaching upon the natural channels of streams with bridges, embankments, piers, terminals, and other structures. In the case of navigable streams such encroachments have been made with the express permission of the Federal Government, which thereby has become an agent in creating these encroachments, and, therefore, in causing destructive floods.

However, the influence on floods of such factors as those just mentioned must be relatively small in comparison with that of the general deforestation, drainage, and development of the vast valley territory. Large areas of swamps and lowlands have been drained, forests have been cleared away, and the land has been used in such a way as to cause much erosion and rapid run-off. The hills pour torrents of turbid waters through man-made gullies into the rivers, filling their beds with mud, and causing them to overflow their banks.

Arguments for Federal Responsibility

Many believe that flood control is the duty of the Federal Government for the reason that no other power exists that is able to cope with the problem. In 1879, the Hon. James A. Garfield stated in the House of Representatives that it was "too vast for any State to handle; too much for any authority less than that of the nation itself to manage." When it is realized that the Federal Government, the States, levee boards, cities, counties, railways, and individuals have been building levees, and that ten agencies in the Departments of War, Interior, Commerce, and Agriculture, to say nothing of the PWA and WPA, have had authority in the development and control of streams, it appears rather remarkable that the task has been performed as well as it has; but a satisfactory public policy can never be developed under these conditions. Each organization naturally outlines its own problems and proceeds to work them out with little reference to those of other groups. This situation has caused a widespread demand for a unified control.

Added to these reasons for Federal responsibility in flood control are important considerations of interstate commerce, including the postal service, public health, and national defense. They are so obvious as to require no elucidation.

Federal control has been delayed because the construction of protective works costs large sums of money, because opponents have claimed that it constituted a reclamation project for the benefit of private property, because sectionalism has invested proposed flood-control legislation with the objectionable "pork-barrel" feature, because there has been disagreement as to the best methods of handling the problem, and because some have held that it was unconstitutional. None of these arguments has weighed so mightily as the general inertia of the public in forcing the issue. Organized representation of the flood-control cause in Washington, D. C., has not been sufficiently persistent or powerful.

Several strong points for the constitutionality of flood control by the Federal Government have been advanced, but the bulwark of most of the arguments has been the jurisdiction of the Federal Government over interstate commerce. The early participation of the United States in flood control was based entirely on the proposition of improving navigation to facilitate commerce on the Mississippi. Congress and the public paid particular attention to the provisions in the early appropriation acts that no part of the money should be spent for levees to prevent injury to lands by overflow. This point of view has long prevailed. However, in 1917, proponents of flood control sought to establish the right to control the Mississippi under the commerce clause on a wider basis than that of merely improving navigation. They frankly stated that it was a flood-control measure. They took the position that legislation for the improvement of navigation had been "predicated upon the power of Congress to regulate commerce", that the word, "navigation", did not appear in the Constitution but had been written in as a part of the interpretation of the commerce clause. They turned their attention to statements of legal authorities and to Court decisions, apparently with satisfactory results.

The general welfare clause has naturally furnished a strong point for those who sought to prove the constitutionality of flood control by the United States. It has been suggested that the Constitution has been expanded by usage and by interpretation to include many things under this head. To many the Government had as much right to make land suitable for habitation by protecting it against floods as it did to give away the public domain. Congress has been very liberal in voting money to relieve suffering among the victims of floods; little question as to the constitutionality of such aid has ever been raised. It has been urged with apparent logic that appropriations for flood protection mean as much for the general welfare as appropriations for the relief of flood sufferers and that they are, therefore, just as constitutional. Chief Justice Story maintained that if the benefit was general, whether it was located in "one State or several", Congress could appropriate money for it because it was for the general welfare.

The problem of flood control must be regarded fundamentally as one of engineering, and the Engineering Profession deserves recognition for the support it has given to the Federal program. Engineers have appeared times without number as witnesses before Congressional committees. They have passed many resolutions in State and National conventions demanding Federal control. No other group of financially disinterested persons has more generally urged complete control of the floods of the Mississippi and its tributaries.

The people of the United States have finally realized that floods will continue to increase in importance. Floods have not become more frequent, but as the population grows, they are becoming increasingly more destructive. Property values and the density of population have already reached such a status that millions of persons now live in areas that are without adequate protection. Flood control is now a national problem and must be considered fully as a Federal responsibility.

PROBLEMS IN DEVELOPING A NATIONAL FLOOD-PROTECTION POLICY

By Abel Wolman[2], M. Am. Soc. C. E.

Synopsis

After every major catastrophe public demand for preventive action reaches its peak and the solutions proposed are as varied as the interests involved. In this respect the March, 1936, floods were no exception. Demands for immediate action, for Congressional appropriations, for financial programs, filled the lay and technical journals. History teaches that in such periods of stress, judgment and logic are unduly influenced by emotion. It is frequently the most unfavorable time in which to formulate policies for control. When public interest is at its height, on the other hand, is the most fruitful period in which to make real progress in the solution of a difficult national problem.

Most thoughtful students of flood-protection procedure would probably agree on the following facts:

(*a*) Engineering studies of past destructive floods are by and large incomplete.

(*b*) Detailed and carefully prepared programs for immediate flood protection are likewise lacking in many areas, although reconnaissance studies are available.

(*c*) In the development of principles and policies much more than engineering information and conclusions is needed.

(*d*) A kind of statesmanship and creative thinking, not yet fully developed, is required, which will balance local and national needs and costs against assessments of benefits.

(*e*) A new sense of social responsibility on the part of local areas is needed.

(*f*) Protection for a local area should be designed not only to meet the flood-control requirements of the area, but for the social and economic advantage of the nation in respect to all water uses.

A large number of reports by Federal or local agencies have been prepared from time to time. The most important undertakings of this type are those known as the "308 Reports" of the Corps of Engineers, U. S. Army. Because of the limitations under which these documents were prepared, they cannot all be considered as of completely definitive character. In such instances adjustment is needed, and, in general, a continuing review is desirable.

The interval between the preparation of reports, where they have been made, and the actual initiation of construction work, is likely to be long.

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Almost a quarter of a century elapsed, for example, between the preparation of a document on the Muskingum River floods and the building of the control structures.

An analysis of national shortcomings in flood-protection measures soon discloses that the problem is highly complex and that it cannot be reviewed upon so simple a basis as flood protection *per se.* Although many engineers and most laymen would urge strongly the correction or prevention of flood damages, even a superficial review of the situation makes it clear that few flood-protection programs should proceed without consideration of the relationship of these measures to other possible uses of water. When such reviews are undertaken, simplicity of approach gives way to complexity of use and to conflicts in purpose and in policy.

Because of these difficulties in orientation the writer considers it wise to present a brief analysis of the various aspects of determining upon a national flood-protection policy and to outline what measures are under way, as of 1937, to clarify some of the problems involved. In this attempt, it will be apparent that considerations other than engineering loom large in the final solution. Nevertheless, any generalized program for water use must rest obviously upon detailed and careful engineering analysis. In periods of stress and haste this axiom has not always been observed. It is a prerequisite to all that is written in this paper.

ENGINEERING ASPECTS

What are the links in the chain of engineering evidence that should be available before a national policy can emerge?

To engineers, of course, the first element of deficiency is in incomplete hydrologic data, not only for floods but for every form of river run-off. The Water Resources Committee of the National Resources Committee, in several of its sub-committee publications, has called attention to these deficiencies in basic hydrologic data. Whether these deficiencies are corrected in the future depends upon such organizations as the Society and upon lay support.

Without such book-keeping of hydrologic data, no policy can be adopted with complete confidence. The program should be nation-wide, continuous, and technically accurate, and the results should be regularly interpreted and reported to the profession. Discontinuous observation, although perhaps better than nothing, does not fully meet the requirements.

Even in 1937, detailed analyses of major floods leave something to be desired, although the "308" reports of the War Department and the publications by the Miami Conservancy District, the United States Geological Survey, and the Bureau of Agricultural Engineering, have added material of incalculable value in recent years. The studies should be extended. Findings are spotty geographically, and methods of analysis and of forecast still need improvement.

In determining the effects of different procedures on the control of floods, in some instances the profession is still divided, and the heat of discussion has so far prevented a calm analysis.

What part detention reservoirs, channel improvements, zoning control of encroachment, forestation, soil conservation, small dams, and so-called "upstream engineering" should play in a general water-resources program, still remains to be demonstrated conclusively. The hiatus between claim and substantiation in fact is wide, and competitive claims are scarcely sufficient basis for a permanent policy. Only detailed field and office analysis of water resources control can disclose the relative merits of the various devices and procedures. In this regard, "newspaper experts" have not been the only delinquents. Members of the Engineering Profession have permitted themselves to be diverted into defensive or offensive controversial fields where their capacities for objective analysis have suffered or disappeared. At times, little or no interest has been exhibited by those individuals who should be most interested, in the multiple-use problem in water-resources development. Not all the present difficulties of approach may be ascribed to misguided enthusiasts among the laymen. After all, engineering guidance for public opinion is more necessary to-day than at any other time in the past. Aloofness, professional provincialism, political partisanship, are all strange bedfellows for engineering judgment. Engineering texts, of course, are filled with principles of caution and logic in the matter of water-resources development; but how frequently are they ignored! For example, the zoning principle suggested by the late Allen Hazen, M. Am. Soc. C. E., many years ago, for application in flood areas, when discussed at all to-day is met with scoffing in some groups and with complete disregard in others. Yet as a flood-protection measure, it may some day have the place in the sun that it undoubtedly deserves.

FINANCIAL ASPECTS

The engineering aspects of water-resources control lend themselves to quantitative determination and decision. This is not the case, however, with the financial aspect of the problem. The simple statement that water-resources development, and flood protection as a unit thereof, should be financed largely by local interests, is sound and clear on paper. No one can take issue with this statement of policy; but its practical application requires discussion.

It is not invariably true that a water-resources program is restricted to the confines of a single State. Often two or more States are involved. Often reservoir sites in one State may be used for the control of flow in three or four down-stream States. In this setting, the concepts of damages, benefits, and allocation of the costs of correction are decidedly confusing.

Any one reviewing the damages presumably caused by destructive floods is at once impressed with the absence of any standard criteria for evaluation. Any two agencies, Federal or local, are likely to arrive at separate estimates of the flood damages in a given area that will differ widely. These deficiencies in approach led the Water Resources Committee to appoint a sub-committee in December, 1936, to study the criteria for the evaluation of flood damages. No one believes that an immediate standardized methodology in this field is desirable, but it should be possible to present certain major criteria of approach that would put such estimates on a more scientific basis. Likewise, no one believes that a methodology must be devised which is a substitute for long experience. An analysis of practical criteria may clarify, however, the concepts even for the experienced observer.

Most people assume that the economic justification for flood protection is simple to evaluate. It is surprising to find how little information is available, not only on specific flood damage, but on the general, special, and intangible benefits that might arise from the initiation of flood-protection measures. Various procedures have been used from place to place, but it is rare that the details of such principles of benefit evaluation are presented so as to be readily subject to check.

Since assessment of local benefit, presumably the base of local participation, is still in a nebulous state, the difficulty of adhering to what appears to be a sound policy becomes more complicated as each flood-protection project comes under review.

The pressing question of how much central responsibility for flood protection the Federal Government should assume, is more difficult to answer when viewed in the light of specific undertakings than when stated as a general principle. One reason for this difficulty is that the mere definition of "Federal" interest is continually undergoing adjustment. This is not solely due, as many believe, to the so-called rise in central bureaucracy. It may be the result and the penalty of increasing national integration. It may be the result and the penalty of growth of population and of the difficulty of separating local from regional hazards and corrective measures.

It is a Jeffersonian principle that government in the United States should result "in the partition of cares, descending in gradation from general to particular, that the mass of human affairs may be best managed for the good and prosperity of all." This precept is valid only until its application under the pressure of such difficulties as those encountered in water-resources development make deviations therefrom necessary. The question is how far such deviations are wise; how much may be accomplished within the framework of State autonomy; how well drainage-basin enterprises may be developed within that framework; and what re-adjustments in theoretical principles, if any, the pressing problems of water resources may make necessary.

In such a complicated problem it is obvious, at least to the writer, that slow motion is a sound principle of action. Heroic changes in procedures of Federal-local responsibility must be approached with caution. At the same time, it may be questioned whether such approaches should be so congealed by precedent no longer applicable, that no comprehensive treatment of water resources is possible.

In the allocation of financial responsibility, it should also be remembered that a simple formula may be a first necessity. The Flood Control Act of 1936 illustrates, however, that in the desire to adopt a simple formula for local participation, many inequities necessarily arise. The percentage to be paid by local areas in terms of land and rights of way may vary from 10 to the maximum legal limit of 50. The intent of such a Congressional Act may be wise, but its application is likely to be inequitable. A logical substitute for the simple formula type of Congressional approach lies in moving administrative control. This gives rise in turn to the charge of increased bureaucracy, although the nature of the water-resources problem would appear to pre-suppose the necessity of avoiding a rigid formula for evaluating local and central responsibility.

When the problem is extended to the evaluation of benefits and costs in multiple uses of a stream, the difficulty of using a formula on a national basis becomes even more apparent. In this field, there may be no escape from continuing administrative adjustment and evaluation. This will no doubt create a variety of geographic apportionments of costs, and necessarily much argument. Here, again, is a fruitful field for the best technical thought of the country.

The formulation of a national fiscal policy with reference to any important public undertaking has always been beset with the necessity of reconciling logical processes with public sentiment and clamor. Changes, supported by local greed, may soon convert a sane national policy into a general "grab-bag" for distributing Federal funds to local projects, sometimes well conceived and involving important national interests, but too frequently badly conceived and purely local in character.

The cold statement of historical facts in regard to national enterprise may be interpreted as disheartening examples of "increasing feeding of local areas out of the Federal trough", or as examples of "the slow evolutionary assumption of Federal financial responsibility for undertakings of national interest." It is the writer's opinion that the correct interpretation lies somewhere between these two extremes.

Experience certainly teaches that before establishing any permanent national policy for financing public works, one should make certain, on the one hand, that the interests of the country as a whole are being adequately protected and advanced, and, on the other, that opportunities are not created for the complete degradation of local responsibility. The tendency, unfortunately, is generally in the latter direction, and safeguards must be provided in the water-resources development policy to avoid the error of expanding Federal power and financial responsibility without local responsibility.

It should be clear from this brief discussion of some of the financial problems involved that a trial-and-error technique is a pre-requisite to ultimate solution. Experimentation in government to-day arouses violent opposition in some quarters, but, carefully organized and carefully controlled, it appears to be a necessary adjunct to the normal processes of desirable water-resources development. A substitute for experimentation is no more desirable in this, than in any other scientific, field where rationalization has long since given way to trial-and-error technique.

More than fifty years ago, Ambassador Bryce made the discerning observation with reference to American political institutions, strongly applicable to the subject in hand, that: "The longer one studies a vast subject, the more cautious in inference does he become."

ADMINISTRATIVE ASPECTS

If future water development should result only from comprehensive study of possible multiple uses or methods, other problems beyond those of engineering and finance present themselves. Decisions must be reached from time to time as to what governmental or private agency should be permitted to undertake the construction of vast water-resources programs, whether for flood protection alone, or for multiple purposes. Controversy again rages in this field as between central and local public bodies and as between public and private agencies.

There are occasions, of course, when this particular administrative difficulty is absent. The development of flood-protection measures within Los Angeles County, California, alone, for example, may present no particular problems of national policy. The type of organization may be controlled locally. The methods of raising funds and assessments may be controversial, but, at the same time, not of national import. When the problem is extended, however, beyond the confines of a single county, or of a single State, and when the best uses of a stream include its development for power, for navigation, for potable water supply, and for flood protection, can some form of national control be avoided? The question is posed without prejudice at the moment. It should be brought clearly into the daylight, and its discussion should not rest wholly upon abstract and emotional concepts of political philosophy.

The administrative difficulties are not all resolved when the responsibility for the initial construction has been agreed upon. Similar difficulties arise in the operation and management of undertakings, where further reconciliation of diverging interests may be continually necessary.

Recent experience with one of the reservoirs in the Far West, is illustrative of a type of problem likely to be multiplied in the future. With a full reservoir, technical experts recommended, on the basis of snow surveys, that the reservoir be emptied so that its capacity would be available for catching spring run-off. If this had not been done the heavy spring run-off, equivalent to the reservoir capacity, would have created damaging floods. Yet compliance with this competent technical advice required extensive negotiations with political parties, private interests, and public agencies, leading ultimately to the Governor of the State for final decision. The engineer cannot ignore the brute facts of public pressure no matter how cloistered he may be.

History has shown that each drop of water may be the source of deadly conflict. The problems cannot be settled in the laboratory, but must be adjusted in terms of public demand. Undoubtedly, the Engineering Profession has the task of educating the public in the field of its operations. It is a task, incidentally, in which so far it has made no great progress; otherwise, the clamor for faulty engineering panaceas would not have reached the heights it has attained in recent years.

SUGGESTED APPROACHES TO THE DEVELOPMENT OF A NATIONAL WATER RESOURCES POLICY

No easy road to the promulgation of a national policy for flood protection or for a national policy of broader character is available. Acceptance of this conclusion has led agencies of all levels of Government, and the Water Resources Committee and its predecessors (the Mississippi Valley Committee and the Water Planning Committee of the National Resources Board), to approach the task experimentally.

The operations of the Miami Conservancy District, the Muskingum Conservancy District, the Corps of Engineers on the Mississippi Valley flood-control work, and of the Tennessee Valley Authority are all indicative of a distinct, although unconscious, desire to direct public action into logical engineering, financial, and administrative channels. The varied experiences of each of these organizations disclose a gradual evolution toward emphasis on multiple uses of streams, on drainage-basin concepts, on Federal-local participations, and on generally broader handling of stream problems than those current in the early years of the century.

Whether for better or worse, people are beginning to view the control of a stream for flood reduction as an integral phase of the use of that stream for power, navigation, irrigation, public water supply, and sanitation. To the writer it is an evolutionary step of merit. It creates complications in all fields of attack, but the objectives appear wise.

To the aforementioned approaches by Federal and local agencies, the Water Resources Committee has added several undertakings, which are briefly presented herein as further examples of multiple-use of streams. In its studies, the Water Resources Committee has kept two basic principles in mind: First, it has included in the deliberations on the uses of a particular stream, the States covered by the drainage basin in question (this was done on the assumption that no real progress in analysis or solution of the problem would be successful without the highest degree of local participation); and, second, it has assumed that no standard method of approach has yet been evolved, which is invariably applicable to the United States as a whole.

The Committee assumed further that different problems would require different structural arrangements for experimental demonstration, and that the cost of carrying out such experimental studies in every instance should be borne by the Federal and local agencies primarily concerned.

The Red River of the North.—The investigation of the Red River of the North resulted largely from a meeting called on June 3, 1935, in Fargo, N. Dak., by the Governors of Minnesota and North Dakota to consider the problems of water conservation, stream pollution, and sewage disposal in the Red River Valley. The call was directed particularly to the members of the State Planning Boards of the three States—Minnesota, North Dakota, and South Dakota.

This conference was followed by a second one in St. Paul, Minn., on November 26, 1935, in which representatives of the National Resources Committee participated. At this meeting, a committee was appointed to develop a co-ordinated plan for the development and protection of water throughout the basin of the Red River of the North. As representatives of the Water Resources Committee, W. W. Horner, M. Am. Soc. C. E., of St. Louis, Mo., and Harlan H. Barrows, of Chicago, Ill., were delegated to act. At the request of the conference, Mr. Horner was designated by the National Resources Committee as Technical Consultant in the preparation of the ultimate water plan.

On July 8, 1936, a report of the Third Inter-State Conference on the Red River of the North Drainage Basin was issued, proposing an adequately developed water plan, on a long-time basis, the primary objective of which is the development of dependable stream flows during the dry periods of the year in quantity ample for urban water supply and for the dilution of wastes. To attain this objective, the plan called for the development of storage in head-water areas, the regulation of stream flow in accordance with a predetermined program, and for improvement of stream channels to minimize water losses.

The report also reviewed the physical characteristics of the basin, its economic conditions and trends, and its water problems, and presented a recommended program of projects to put the co-ordinated plan of water development into action.

This is perhaps the first comprehensive program of river drainage basin improvement carefully developed in the United States, with three State groups participating and with competent engineering control and direction. It offers for the first time a skilfully drawn program for water conservation, susceptible to step-by-step construction in accordance with the financial capacity of the territory to carry the load. It provides for flood control in heavy run-off periods and for low-water supplementation in dry years.

The findings of the report are now (1937) being put into effect by local groups, and where money is being spent by Federal agencies in the territory, it is being spent in accordance with the definitely formulated plan.

The development of such a program implies time and the availability of money. It cannot be accomplished otherwise. The case in point illustrates, however, that it is feasible, with a certain amount of central direction and stimulation, to interest individual States in the preparation of a joint comprehensive program of action. It is hoped that this precedent may be followed with necessary local modification by other groups of States having similar pressing problems of water use. It is not implied, of course, that local pressures may not create deviations from the master plan, but the existence of an agreed master plan generally reduces these hazards to a minimum.

The Rio Grande.—Competitive uses of the water of the Rio Grande had reached such an acute stage in 1935 that it became necessary for the President of the United States to issue an order preventing the release of further Federal funds for any projects on this basin north of El Paso, Tex., until the National Resources Committee had approved the undertaking. An interstate compact between the States of Colorado, New Mexico, and Texas had been in operation for some time and was to expire in June, 1937.

In view of the conflicts of interest these three States, together with the Federal agencies concerned, undertook a detailed study of the water resources of the basin. Both Federal and State groups have contributed money and personnel to the undertaking. The studies should disclose noncontroversial bases for the development of new agreements. They may be substitutes for Court action and may lead to the evolution of interstate agreements for the use of the Rio Grande water without basic controversy. The method of approach is again one for special purposes which might be adopted in other areas. Again, time and money are pre-requisites; it should be emphasized that without these two elements programs and policies cannot be "pulled out of thin air."

Kansas River Flood Studies.—The first contribution to the study of the Kansas River flood problem by the Water Resources Committee, or its predecessors, was made on November 1, 1934, when $25 000 was made available for expenditure under the direction of the Chief of Engineers, U. S. Army, for survey and investigation of the Milford Reservoir site on the Lower Republican River and the Tuttle Creek site on the lower part of the Big Blue River. Before investigations were started, it was decided to hold up the work pending agreement with the Flood Protection Planning Committee for Greater Kansas City, in regard to the conduct of the study, and the appointment of a Consultant by the Committee. The agreement was soon reached, and the surveys went forward during the summer of 1935, with F. H. Fowler, M. Am. Soc. C. E., as the Consultant.

It shortly became aparent that a complete plan for reservoir control of the Kansas floods might require the construction of additional storage on tributaries entering the main stream below the Republican and the Big Blue, and that considerable information regarding flood conditions in Kansas City might be secured from a river model. On the recommendation of the Water Resources Committee additional funds were made available to the U. S. Division Engineer on November 27, 1935. These funds consisted of $22 500 for surveys and borings at supplementary storage sites, and $12 000 for a river model, which was later constructed at the U. S. Waterways Experiment Station, at Vicksburg, Miss.

During the spring of 1936, the Bureau of the Budget rescinded $8 000 of the $22 500, thus preventing temporarily the completion of the schedule of boring at the dam sites. However, shortly afterward, Congress authorized a preliminary examination of the Republican and Smoky Hill Rivers with a view to the control of floods, and the Flood Control Committee of the House of Representatives authorized the Corps of Engineers to review the Kansas River report already published[8]. As a result, $150 000 was made available to the U. S. Division Engineer for additional studies. Of this amount, $8 000 was used to replace the amount rescinded by the Bureau of the Budget. Two series of tests on the river model were completed at Vicksburg during 1936. The results have gone far toward remolding public opinion.

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[8] H. R. Doc. 195, 73d Cong., 2d Sess.

Study of an alternative arrangement of levees, combined with bridges, was initiated by the U. S. District Engineer Office, in Kansas City, after a joint inspection by the U. S. Engineer and the Flood Control Committee in May, 1936.

The development of the final plan should represent the reconciled viewpoint of all interests. The method of approach illustrates the merit of joint review by the Corps of Engineers and the local Flood Control Committee and its engineer advisers.

Potomac River Conservancy District.—The Water Resources Committee in 1934 recommended to the President the establishment of a Potomac River Conservancy District, primarily for the purpose of providing a field laboratory for evolving engineering, administrative and financial principles, and methods of control. In this instance, as in others, the district was to be under the supervision of the interested States, with Federal participation in a minority membership on the commission. Financing of the enterprise was to be largely by local agencies, public and private, with the stimulation of the Federal Government. Its purpose is solely to develop methods of arriving at solutions on major drainage basins. No action had been taken on this proposal, as of March, 1937, but it is presented herein as another device for supplying the nation with additional information on the problem of drainage-basin control and development. It was proposed as a more practical alternative to nation-wide drainage basin authorities, for which the ground is not yet prepared and which only future careful study can demonstrate to be valid or invalid structures for drainage-basin operation.

The National Drainage Basin Study.—Most civil engineers are familiar with the drainage-basin study of the Water Resources Committee. Its purpose is to indicate the outstanding water problems in the various drainage areas of the country, to fit them tentatively into integrated patterns of water development and control, and to present, where existing data make it possible, specific construction and investigation projects as elements of an enlarged plan.

This study supplements and extends the work of the Mississippi Valley Committee of the Public Works Administration and the Water Planning Committee of the National Resources Board, which agencies have been succeeded by the present Water Resources Committee.

The study under the title of "Public Works Planning" was presented to the President by the National Resources Committee in December, 1936. The President forwarded it to the Congress of the United States on February 3, 1937. It is to be followed by the issuance of a volume containing the more detailed findings of the Water Resources Committee. The date of the publication of this latter volume is the early part of March, 1937.

For many basins, the plan contains relatively few construction projects, because of the lack of essential data. In others, fairly complete construction programs are already available. Obviously, none of the plans is either fixed or final. No one supposes that any long-term plan for any drainage area can be formulated, immediately, in detail, even if all requisite hydrologic data were available. The drainage-basin study, however, should result in a significant contribution "to the framework of an enduring, but adjustable national water plan." It should serve to arouse professional and lay interest in water resources integration, in the extension of these preliminary studies under local auspices, in emphasis upon multiple, rather than upon single-use, development of streams, and in relating water development more adequately to the entire economy of a basin.

SUMMARY

In this paper, the writer has attempted to show that the development of a National Flood Protection Policy should not be separated from that of a National Water Resources Policy in general; that the elements of such policy are highly complex; that they run the gamut of engineering, finance, and administration; that they are bound up with the problem of public prejudice and pressure; that the contributions of engineering thought are the key to the ultimate evolution of a national policy; and that such engineering contributions must be made on the field of battle and not in *vacuo*. In order to clarify some of the issues and to evaluate some of the procedures, local and Federal public and private agencies are collaborating in certain studies. As time goes on the results of these studies should disclose certain principles of action and should supply the basis for an ultimate national policy, changing in character, logical in administration, and as free from prejudice and greed as is possible in a world still fortunately consisting of human beings. A perfect syllogism of action is probably not attainable.

THE ECONOMIC ASPECTS OF FLOOD CONTROL

BY NATHAN B. JACOBS,[4] M. AM. SOC. C. E.

SYNOPSIS

Flood control is a problem of applied economics in which the social security and protection of life and property must be evaluated in balancing the annual savings from flood control against the cost of construction and maintenance. Floods damage the homes of those least able to bear the cost, who only live in the path of the inundation because financially unable to move elsewhere. No formula has yet been developed to apportion costs of such works satisfactorily between Government (Federal, State, and Local), and property owners, but each plan represents a problem for solution not only in its engineering aspects but also in distributing the costs.

INTRODUCTION

Engineers are guided largely by the principles of economics. Rigid rules are followed to test the economic worthiness of buildings, bridges, dams, railways, etc., before capital is called upon to make the projects possible. Heretofore, proposals for flood protection have been subjected to the same rigid analysis. Measurable direct damage is estimated and tabulated. To this is added some evaluation of the indirect damages that cannot be definitely determined. From this, the average annual damages incurred within and without the flooded areas, are determined. If the return on capital to be invested in protective works, plus depreciation, maintenance, and operating expenses, is less than the average annual damages sustained, the proposed project is considered to be on a sound economic basis.

It is true that flood control involves questions of economics, because floods destroy wealth and property, disturb the production and exchange of goods, and endanger lives and health. The questions, however, are not so much those of pure economics as applied economics—the science that seeks the improvement of existing conditions and the translation of such knowledge into action. Should a strictly monetary evaluation be the only basis of providing relief from inundation by waters gone wild?

SOCIAL SECURITY A FACTOR IN EVALUATING FLOOD CONTROL

Widespread human suffering and appalling loss of life have a stronger universal appeal than the monetary and property damage suffered by industry, trade, and transportation in major catastrophes. As far back as 1882, President Chester A. Arthur, in a message to Congress, stressed the sufferings of the people of the Mississippi Delta. A former Governor of Louisiana, the Hon. John M. Parker, felt the principle of humanitarianism to be so important that he maintained it to be inhuman to permit three-quarters of a million people to be driven from their homes and made dependent upon the charity of fellow Americans. In 1927, former President Herbert Hoover, Hon. M. Am. Soc. C. E., then Secretary of Commerce, in discussing the 1927 Mississippi flood, said the losses sustained touched the entire nation, and subtracted something from the wages or income of every citizen.

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[4] Pres., Morris Knowles, Inc., Pittsburgh, Pa.

The concept of humanitarianism is forcibly brought to public attention by the wording of the Omnibus Flood Bill, enacted into law in 1936 by the 74th Congress of the United States. This law reads, in part:

"That the Federal Government should improve or participate in the improvement of navigable waters or their tributaries, including watersheds thereof, for flood control purposes if the benefits to whomsoever they may accrue are in excess of the estimated costs, and if the lives and social security of people are otherwise adversely affected."

It will be noted from this passage that as the law now stands there are other considerations to be reckoned with than money or property damage. Under this Act, it is proper for the Federal Government to participate in flood-control projects if the life or social security of the people is in danger. This criterion will be as controlling and persuasive as the financial formula by which it formerly was necessary that projects be economically justified on the basis of past experience. There is inculcated in the minds of this generation, more so than in those of the past, a higher regard for general welfare and human protection.

The construction of flood-protection projects on a monetary economic basis alone is a thing of the past. Where the lives of human beings are endangered and where great suffering may occur as a result of floods, protective works should be constructed. It is too much to expect that people shall move from the path of danger.

In 1906, there occurred one of the worst catastrophes in the history of the United States—the San Francisco earthquake—which rendered 25 000 homeless and caused monetary losses estimated at $600 000 000. The people of that city had the courage to rebuild on the same site, with the knowledge that no man-made protection could be afforded. Human nature is the same the world over, and cities ravaged by floods have been rebuilt just as San Francisco was. It seems to be the spirit of some to reside in danger zones. The fact that floods occur again and again has not acted in the past, and will not act in the future, as a deterrent to the rebuilding of homes and the re-establishment of commercial enterprises and industries in perilous localities. Since it is possible to provide measures of protection against floods, the people who live in flood-plains must be afforded that protection.

Furthermore, the returning to homes, or to the location of former homes, after a flood disaster, may not be so much a matter of choice as of compulsion. City planners claim that much of the territory that is inundated during a flood is not economically and practically adaptable for homes, and that, therefore, the proper procedure is to relocate these families on ground reasonably above the flood-plain. In large cities, however, many of the commercial and industrial enterprises are located as near the river as possible, often for the purpose of obtaining water for condensing or cooling purposes; and many workers live in homes near the industries because of the convenience and economy of being able to walk to work in a short time. Although transportation is now much more readily available and more economical than formerly, the workers continue to live in these houses, possibly because there are no other houses available in the district.

For example, of the area in Pittsburgh, Pa., that was inundated in the flood of March, 1936, about 80% is zoned as "heavy industrial." Under the provisions of the zoning ordinance, no residence may be built in a heavy industrial district. Of course, the law is not *ex post facto*, and residences existing in such a district at the time of its passage (1923) were permitted to remain. However, it may be assumed that no new dwelling has been constructed in the heavy industrial zone since that time, and, further, that no house has been subject to major repair, because permits could not be obtained for either purpose. Many of the houses in the district have already been condemned by the Pittsburgh Housing Association as not fit for human habitation. During the flood, the Association made provisions to prevent tenants from returning to these condemned houses after the water receded; but as homes within the rent range of these tenants were not available in other and higher parts of the city, it was necessary, in order to re-house them, to allow them to return to their original homes. Such a condition may be alleviated by additional housing and proper city planning and zoning, but it cannot be cured.

ENGINEERING ASPECTS

The economic problem of flood control does not have a single answer, such as the solution of an algebraic equation. There are many ways to reduce flood damages. Retention basins, storage reservoirs, levees, channels, and dikes are the common structures provided. The problem is not so much one of design and construction as of determining the most economical type of protective works. This requires a competent and comprehensive engineering study. A comparison of the initial cost of construction of the various plans on the same price basis is necessary. Annual charges must be compared against the average annual benefits to be derived, in order to determine the most economic project. Consideration must also be given to the degree of protection afforded by each plan, as the human and social welfare angle must not be ignored. It is also essential that those who must pay for the protective system have full knowledge of the cost, of what burden they must bear, and of what security the final developed plan provides.

The physical characteristics of the up-stream basins will have considerable bearing upon the plan to be selected. In some localities, sites for retarding dams may not be available. In others, the construction of the dams may be economically impossible; good borrow-pits may not be available for the construction of earth dams; and concrete masonry structures might prove to be far too costly. If the river traverses wide, flat lowlands, levees and channel improvements may be the best solution. In the adoption of any flood-control plan, it should be emphasized that floods cannot be prevented. Protection against all floods will not be furnished by a control system; no one can predict the intensity of the maximum flood to come, but the damage caused by floods of certain heights can be reduced or entirely prevented, and that fact should be stressed by the promoters and designers of projects.

In the matter of design and operation of works to impound flood waters, there are two schools of thought. One holds that such reservoirs should act solely as retention basins, and be automatic in control. Others advocate reservoirs that may be used for many purposes, such as storage for power and water supply and for regulation of the stream during periods of low flow. The degree of flood protection desired establishes the basis for the selection of retention, or multi-service, reservoirs. Multi-service reservoirs may not provide maximum flood control, because rapid and heavy run-off may occur when they are full or nearly so. One or more of the services to be rendered by this type of reservoir must suffer because of the others. On the other hand, if automatically controlled retention basins are constructed, future conservation of water for other uses will be seriously hampered.

Examples of both methods of control are in service. The automatically controlled retention basin is best typified by the reservoirs in the Miami Conservancy District of Ohio. The more recently established Muskingum Watershed Conservancy District, in the same State, provides for protection against floods, by three automatic retention basins and eleven multi-service reservoirs. The Sacandaga Reservoir, constructed in 1930, in New York State, is an example of the multi-service type. It is operated as a storage reservoir to regulate the flow for power interests down stream. At the same time, it acts as a retarding basin to some extent, thereby lessening flood heights of the Hudson River, at Albany and Troy, N. Y.

Another typical multi-use reservoir is the one on the Tygart River, in West Virginia, which was designed primarily to aid navigation on the Monongahela River, but which can and will be used for flood-control purposes as well. It is planned to operate this reservoir so that it is empty near the end of December; from then until April 1, a volume of 100 000 acre-ft of water will be allowed to accumulate. This volume will be drawn on as required during the remaining nine months of the year to assist navigation. Flood water will be impounded during January, February, and March, thus reducing flood heights on the Monongahela River.

The Pymantuning Reservoir, on the Shenango River, in Pennsylvania, was designed primarily for low-water control and for recreational purposes. Since its completion in 1933, however, it has been responsible for lowering flood heights at Sharon, Pa., and other communities. (The reservoir was completed in June, 1933; the project was completed and the gates closed on January 23, 1934.)

The first and foremost thought in providing flood-protection projects should be to obtain maximum security. It is conceded that many multi-service projects are worthy; but if flood protection is the major problem of the community, it should not be sacrificed to the benefit of other uses. On the other hand, if the amount of protection desired permits the use of multiservice reservoirs, these projects should be erected, in order to further the conservation of water as a natural resource. Such reservoirs could be constructed initially for flood control, with provisions for the future installation of power units where they appeared feasible.

The Distribution of Costs

The furnishing of needed funds for flood-protection works is a controversial question. Should flood protection be considered a problem of local interests alone, or should Municipal, State, and Federal Governments share in the cost? Those who live within the flood path do not consider the cost of protective works to be a local problem. It is their contention that the entire cost, or the greater part of it, should be borne by governmental agencies outside the affected area. On the other hand, those who are not directly affected object strenuously to contributing to the cost. Flood disasters, however, have far-reaching effects and, indirectly, the entire nation suffers to a greater or less degree.

Following severe and damaging floods, there generally occurs in the inundated area and surrounding territory a depreciation in property values because of the slowing up of business activities. This is due to the hesitation of industrial organizations and individuals to locate in the community. Losses occur outside the flooded area through the disruption of electric service, water supplies, communication, transportation, and other utility services. The commodity market is disturbed, and prices rise. Counties, towns, and cities are losers to the extent that municipal property is damaged, and also to the extent that depreciation of private property reduces the value upon which taxes can be assessed Floods also cause a loss to the Federal Government, especially where navigation is affected; lock-gate machinery is damaged, erosion takes place around wing-walls, and costly re-dredging of channels is necessary, where silt and débris have been deposited by excessive flows.

The distribution of the cost of a flood-control project among the beneficiaries is a difficult problem. Should the Federal Government bear the entire cost, one-half the cost, or a lesser amount? What should be the share of State and Local Governments, and how should private interests and those immediately affected contribute toward the cost?

Flood-control works have been financed in various ways. The Pymatuning Reservoir, previously referred to, was paid for entirely by funds appropriated by the Commonwealth of Pennsylvania, except for certain land and flowage rights in Ohio, which were acquired by the industries and other interested parties in the Shenango and Beaver Valleys, and donated to the Commonwealth. No doubt the legal difficulties created when one State acquires property in another, prompted this action. In the case of the Sacandaga Reservoir, in New York State, almost 95% of the cost was assessed directly against the water-power properties benefited by the river regulation. Although the project has been of value to navigation and has provided flood protection, only 5% of its total cost has been borne by the public in general.

The Miami Conservancy District System was constructed at a cost of approximately \$30 000 000, without Federal or State assistance. This entire investment will ultimately be repaid by the interests benefited, about one-half by direct assessment on property in the district, and the remainder through tax levies. Of the total direct assessments, 5% is levied on farm lands, and 95% on city and industrial property.

The Muskingum Water-Shed project is being financed in a different manner. The total estimated cost of the work is given[5] as \$34 590 000. Of this amount, \$22 590 000 is to be furnished by the Federal Government from funds of the Public Works Administration for construction purposes. This contribution includes two elements: First, the PWA grant; and, second, the payment for Federal benefits. At the time the project was begun, the estimated construction cost was \$22 000 000. On this basis the PWA grant (30% of construction cost) was \$6 600 000. However, a study was made of the navigation benefits and the savings in future construction work for navigation projects on both the Muskingum and the Ohio Rivers. This benefit was due to the stabilization of the stream flow. Credit was also given for the lesser interruption to river traffic at times of flood level. Federal participation, due to the reduction of flood crests and the benefit to navigation, was computed at \$15 990 000, making the total Federal contribution to the project \$22 590 000. The State of Ohio participated on the basis of the protection afforded State property, including highways. In regard to the latter, it estimated that the benefits would be greater than the cost of the relocations made necessary by the construction of the reservoirs, and, therefore, permitted the Highway Department to co-operate with the Conservancy District in the relocations. The State Legislature further made a direct grant to the District of \$2 000 000. The remainder of the fund, comprising the total cost of approximately \$34 500 000, is being made up by the District itself through assessments on benefited properties, and taxation in the District.

The Miami Conservancy District is a more heavily industrialized area than the Muskingum District, and, therefore, it no doubt could bear better the burden of providing the entire cost without aid from State or Federal Government. Conditions exist throughout the country, however, where the affected area will not be able to bear the entire cost of protective works. This is especially true in agricultural areas.

In making an equitable distribution of the cost of flood-control projects, both direct and indirect benefits must be estimated. The total cost should then be apportioned among the various interests in accordance with the benefits. The percentage of the cost to be borne by each group cannot be set by any given formula; it should be determined in each individual case on the basis of the problems involved.

The formula for Federal participation in the cost of flood control, as set up in the Flood Control Act of 1936, is sufficiently discussed by Lt.-Col. Covell.

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[5] "The Muskingum Flood Control Work": A Symposium, *Civil Engineering*, January, 1936, p. 1.

Whatever may be the basis of cost distribution, it is essential that the full cost of the project be established. It should also be broken down to reveal the cost of land, rights of way, structures, equipment, and all other major elements, not only so that these costs may be distributed among the beneficiaries, but also to permit the etablishment of correct depreciation rates on the various projects.

Benefits of Flood Control

Alleviation of major floods will undoubtedly improve the productive efficiency of the affected areas. When all industry and trade are stopped by flood, it requires weeks, or months, to restore them to normal operating capacity. While repairs are being made, disrupted organizations cannot concentrate upon their normal activities. Their efficiency is lowered, with the result that not only they, but their clients and customers also, suffer to some extent.

With flood-control systems installed, rail and river transportation systems will be free of their worst enemy, and the keeping of transportation systems in a state of efficient operation not only benefits local interests in the flooded area, but also the nation as a whole. Particularly would this be so in case of war. Had the 1936 Pittsburgh flood or the 1927 Mississippi Valley flood occurred in time of war, carefully timed shipments of supplies and materials to the seaports would have been seriously delayed. If the country had been involved in repelling an invasion, the hampering effect of the floods on the efficiency of the military organization might have been even more serious.

The highway system is just as important as the railroad and navigation systems, and flood-protection work will enable the construction of more dependable highways throughout the flooded areas. Airways, too, will be benefited in many cases by the possibility of establishing terminals nearer to the centers of large cities.

If transportation systems do not suffer recurring flood damages, maintenance charges will be reduced, and the money thus saved will be diverted to other and more productive channels.

Floods have done a vast amount of damage to agricultural lands. Whenever crops are destroyed, the loss is felt not only by those directly damaged, but by all who buy the products of agricultural industry. With less soil erosion in these agricultural districts, the farmer will be able to produce his crops without fear of their being washed away; and the improvement in his economic status will be a gain not only for him but for the entire community.

One of the most serious consequences of a disastrous flood is that, in so far as residential properties are concerned, it affects adversely those people who are least able to bear the burden. As previously stated, 80% of the property in Pittsburgh inundated in the 1936 flood was "heavy industrial", but thousands of people lived in this territory. They had not the wherewithal generally to move into better districts. Much of their income must go for replacing and restoring damaged properties. Incidentally, this reduces the demand for consumer goods, particularly food and raiment, and thereby creates further economic maladjustments. Most of this could be avoided by proper provisions for flood control.

There are many other benefits of flood control that might be mentioned here. The danger of interruption of the water supply of river towns will be materially reduced. The value of industrial sites along the river plains will be increased, and better housing facilities can be constructed in their vicinity. Recreational facilities, such as parkways and parks along rivers, can be developed. Rural communities and business enterprises which depend upon tourist trade will profit through the prevention of stream erosion. The artificial lakes created by multi-service reservoirs will themselves provide a source of outdoor enjoyment that is rapidly becoming an important part of American life.

FLOOD CONDITIONS IN NEW ENGLAND

By W. F. Uhl,[*] M. Am. Soc. C. E.

Synopsis

In New England there have been two disastrous floods within the past nine years. The losses due to the flood of November, 1927, are estimated at about $40 000 000, with nearly two-thirds of this in Vermont.

With the estimates virtually complete (February, 1937), the damages due to the March, 1936, flood amount to about $70 000 000, of which $55 000 000 occurred in the Merrimac and Connecticut River Valleys alone.

The problem of preventing or lessening of damages to all kinds of property during floods has received consideration throughout the ages and many solutions have been proposed.

There is no complete practical solution but numerous partial solutions can be offered. Many of these are suitable only where peculiar conditions make them practical. Even where there is a practical solution, it may be not be economically feasible.

In New England, one of the methods which appears economically feasible is to build reservoirs designed for storage, which afford a large measure of flood protection as a by-product.

The statistical data collected on the 1927 and 1936 floods have materially increased the information available on the magnitude of flood flows.

Introduction

Most of the older cities situated on the rivers of New England have developed their present facilities and industries during the last 100 yr. Many of the important water-power centers have experienced their complete development during this period. Previously, floods, although undoubtedly causing much inconvenience, did little damage, as indicated by a lack of historical data regarding them.

Under existing conditions, with large and important cities and their various utilities situated in the river valleys, the flood hazard is becoming more serious and the economics of flood control are being investigated.

It is unfortunate that the records of the behavior of New England rivers are so meager, and it is of increasing importance that all available information be collected, recorded, and properly interpreted for future use in connection with possible mitigation of flood hazards.

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[*] Hydr. Engr., Chas. T. Main, Inc., Boston, Mass.

TOPOGRAPHY OF NEW ENGLAND

The topography of New England is dominated by the two northern ranges of the Appalachian Mountains, the Green Mountains, which practically divide the State of Vermont in two, and then extend across Western Massachusetts into Northern Connecticut as the Berkshire Hills; and the White Mountains, which occupy most of Northern New Hampshire and extend half-way across Northwestern Maine. The general level of these mountains is between 1 000 and 2 000 ft, with some areas above 2 000 ft and the mountain peaks considerably higher.

Five of the largest New England rivers rise within a 50-mile radius in the White Mountains: The Kennebec (drainage area, 5 970 sq miles), the Androscoggin (3 470 sq miles), the Saco (1 720 sq miles), the Merrimac (5 015 sq miles), and the Connecticut (11 320 sq miles). Speaking very generally, each of these water-sheds may be divided into three parts, as follows:

(1) An upland area, where the rivers move sluggishly through plateaus containing a large number of lakes. Valuable storage has been created in many of these lakes by the construction of dams at their outlets. The Rangeley System, on the Androscoggin River, and the Moosehead Lake System, on the Kennebec River, are the most notable examples of this.

(2) A central part, characterized by steep slopes and narrow valleys on the main rivers. Many of the tributaries entering this stretch come off the mountain sides, and are very flashy.

(3) A lowland area in which the slope of the rivers is less pronounced, but more concentrated by ledge barriers. Above these barriers, the valleys widen out, and the banks are low, so that in time of flood large amounts of channel storage space become available. It is at the ledge barriers that the valuable early water powers, such as Holyoke, Mass., Lowell, Mass., and Lewiston, Me., were developed. The tributaries in this stretch are generally sluggish, with practically all the available fall and some storage developed.

The Penobscot River (drainage area, 8 570 sq miles), the largest in Maine, rises in the mountains east of the head-waters of the Kennebec. Compared to the other Maine rivers the level upland part is larger and the water-shed wider in proportion to the length. Natural lakes and reservoirs are scattered over the entire drainage area.

West of the Connecticut lies the Housatonic River (drainage area, 1 930 sq miles), which rises in the Berkshire Hills and flows south across Massachusetts and Connecticut. This river is relatively flat, with a wide valley affording much channel storage in times of flood. The amount of developed storage is small.

The west slope of the Green Mountains is drained by the four Vermont rivers which are tributary to the St. Lawrence River by way of Lake Champlain: Otter Creek (the farthest south), the Winooski, the Lamoille, and the Missisquoi, the drainage areas being 925, 1 076, 710, and 885 sq miles, respectively. Otter Creek is the flattest and most sluggish, with some natural storage in the numerous marshes and flats. The other three streams to the north are more precipitous, with very little storage, either natural or developed.

THE 1927 FLOOD

The floods of November, 1927, and March, 1936, are outstanding events in the recent history of New England. The two floods, both of which exceeded all previous records in their respective sections, were caused by altogether different climatological combinations.

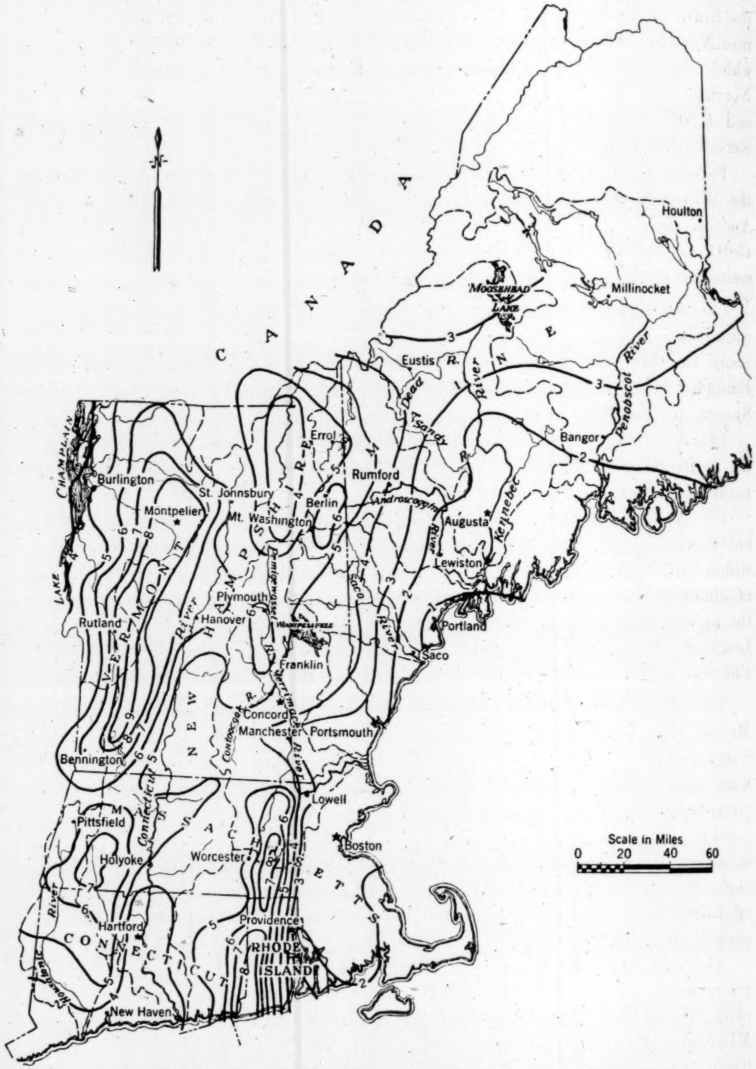
N
CANADA
MAINE
NEW HAMPSHIRE
VERMONT
MASSACHUSETTS
CONNECTICUT
RHODE ISLAND
LAKE CHAMPLAIN
MOOSEHEAD LAKE
WINNIPESAUKEE
Houlton
Millinocket
Eustis
Dead R.
Sandy R.
Kennebec River
Penobscot River
Bangor
Errol
Rumford
Androscoggin River
Augusta
Lewiston
Portland
Saco
Saco River
Burlington
St. Johnsbury
Montpelier
Mt. Washington
Berlin
Pemigewasset R.
Plymouth
Hanover
Franklin
Rutland
Connecticut River
Contoocook R.
Merrimack River
Concord
Manchester
Portsmouth
Bennington
Lowell
Boston
Pittsfield
Holyoke
Worcester
Providence
Hartford
Housatonic River
New Haven
Scale in Miles
0 20 40 60
1
2
3
4
5
6
7
8
9

FIG. 1.—RAINFALL IN NEW ENGLAND, NOVEMBER 3 AND 4, 1927.

The 1927 flood was much more localized than the 1936 flood. The worst of the damage was confined to the State of Vermont, with spectacularly high flows occurring on drainage areas of 1 000 sq miles or less.

On the other hand, the effect of the 1936 flood was felt over most of New England, the areas least affected being the State of Vermont and Northern Maine. It was on the large rivers that the record-breaking flows occurred. Although the total damage in the 1936 flood was about twice as great as in 1927, the devastation wrought by the 1927 flood was much more intense in the localities affected, and the loss of life was much greater.

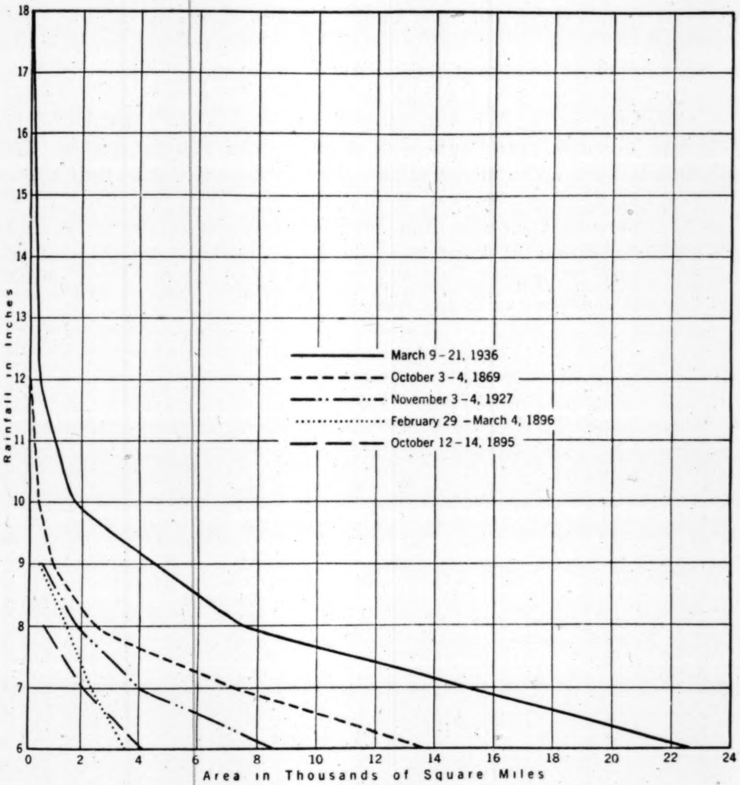

Fig. 2.—Rainfall-Area Relation for Great Storms in New England

The November, 1927, flood resulted from the torrential rains produced when a tropical storm, coming up from the South, was caught between and forced over two cold areas of high pressure, one moving in from the northeast and another from the northwest. When this storm began, the ground was still saturated from the heavy rains of October 18 to 21; the surface storage had been largely utilized; and the rivers had already risen to medium stages.

The rainfall which produced the 1927 flood occurred almost entirely on November 3 and 4 (Fig. 1). The precipitation was greatest in Vermont, with the maximum recorded at Somerset (Elevation 2 080), in the southwestern part of the State (9.65 in. in two days). It is probable that 11 to 12 in. fell in the higher portions of the Green Mountains. Another area of high rainfall was centered near the Rhode Island–Connecticut State line. The 2-day rainfall exceeded 9 in. over an area of about 500 sq miles, 460 of which was in Vermont, in the Green Mountains, and 40 in Rhode Island and Connecticut. The rainfall exceeded 5 in. over an area of about 22 000 sq miles, or one-third the area of New England. Data on the rainfall-area relations of this storm and other great storms in New England are plotted on Fig. 2.

As a result of this great precipitation, the rivers quickly rose to flood stages on November 3 and 4 and filled many of the valleys. Excessively high velocities on the rivers with steep slopes, such as the Winooski and the White in Vermont, caused sudden flood peaks which arrived at night. The inhabitants were taken unawares and many were drowned in their houses. In Vermont, 87 lives were lost, 55 of them in the Winooski River Valley. The total property damage in New England was about $40 000 000, of which $26 000 000 occurred in Vermont. This was equivalent to 3.27% of the taxable wealth of the State in 1920, or $74.30 per capita. About one-half the loss was concentrated in the Winooski Valley.

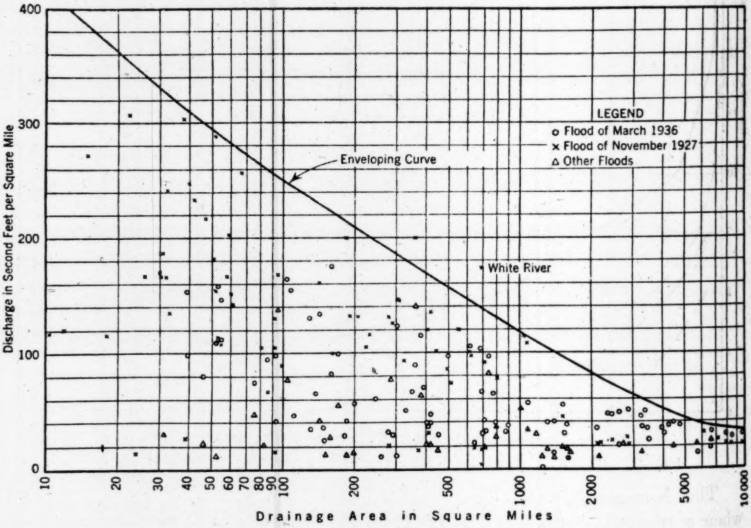

FIG. 3.—PEAK FLOWS OF NEW ENGLAND STREAMS

On the Winooski River, a flow of 116 000 cu ft per sec was observed, equivalent to 114 cu ft per sec per sq mile. On the White River, which drains the eastern slope of the Green Mountains and is tributary to the

Connecticut River, a flow of 140 000 cu ft per sec was observed, equivalent to 202 cu ft per sec per sq mile. These two flows are still (February, 1937) the maximum of record in New England for drainage areas of comparable size. On smaller drainage areas, much greater discharges per square mile were observed. Many of the 1927 flows are given in Table 3, and are plotted on Fig. 3.

The flood of November, 1927, has been described so thoroughly in the report of the Committee on Floods of the Boston Society of Civil Engineers[7] and by H. B. Kinnison, Assoc. M. Am. Soc. C. E.,[8] that further comment regarding it will be restricted to a comparison with the 1936 flood.

The 1936 Floods

The March, 1936, floods were due to an aggravated combination of the conditions that normally cause the annual spring freshets in New England. The fundamental cause was, of course, the abnormally high precipitation in a period of unseasonably warm weather which quickly melted the heavy snow cover. The first storm, March 12 and 13, melted most of the snow and produced flows considerably greater than the usual spring freshet. After a lull, with the rivers bank-full but receding, there came three more days of unusually heavy rain which fell on ground largely bare and frozen, with all the low spots full of water. Practically all of this second storm went rapidly into the streams and built up record-breaking flows. The excessive rainfall was the primary reason for the flood, but various other factors contributed largely to the eventual result.

Climatological Conditions Prior to the 1936 Flood.—In general, the winter of 1935–36 in New England was one of about average precipitation, but lower than average temperature. For New England as a whole, the average temperature for December, 1935, and January and February, 1936, was 20.6°F, which is 3.2° below the mean temperature for these months. This deficiency applied about equally to all six States.

The total precipitation for these three months was 10.32 in., which was 0.41 in. above the normal. The total snowfall, to March 1, as recorded by a number of stations, was 35 in. in the Southern States and 64 in. in the Northern States, or about 5 in. above normal.

The winter was a severe one, characterized by long-continued periods of sub-freezing weather, with few thaws. Data collected by the power companies and winter sports organizations indicate that by March 1 the snow cover in Northern New England was about 2.5 ft thick, with perhaps a foot more in the mountains; in Southern New England, it was from 1 ft to 2 ft thick.

During most winters the snow blanket in Central and Southern New England lies in layers of crust and powdery snow, due to the alternate periods of freezing and thawing. In the winter of 1935–1936, however, the snow came early, and because of the lack of thawing weather the blanket was light and powdery throughout its depth, which made it much more susceptible to quick run-off than usual.

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[7] *Journal*, Boston Soc. of Civ. Engrs., September, 1930.

[8] "The New England Flood of November, 1927", *Water Supply Paper 636-C*, U. S. Geological Survey.

From March 1 to March 10, conditions were generally normal, with one hard freeze on March 2. On March 3, there was a moderate snowfall, adding

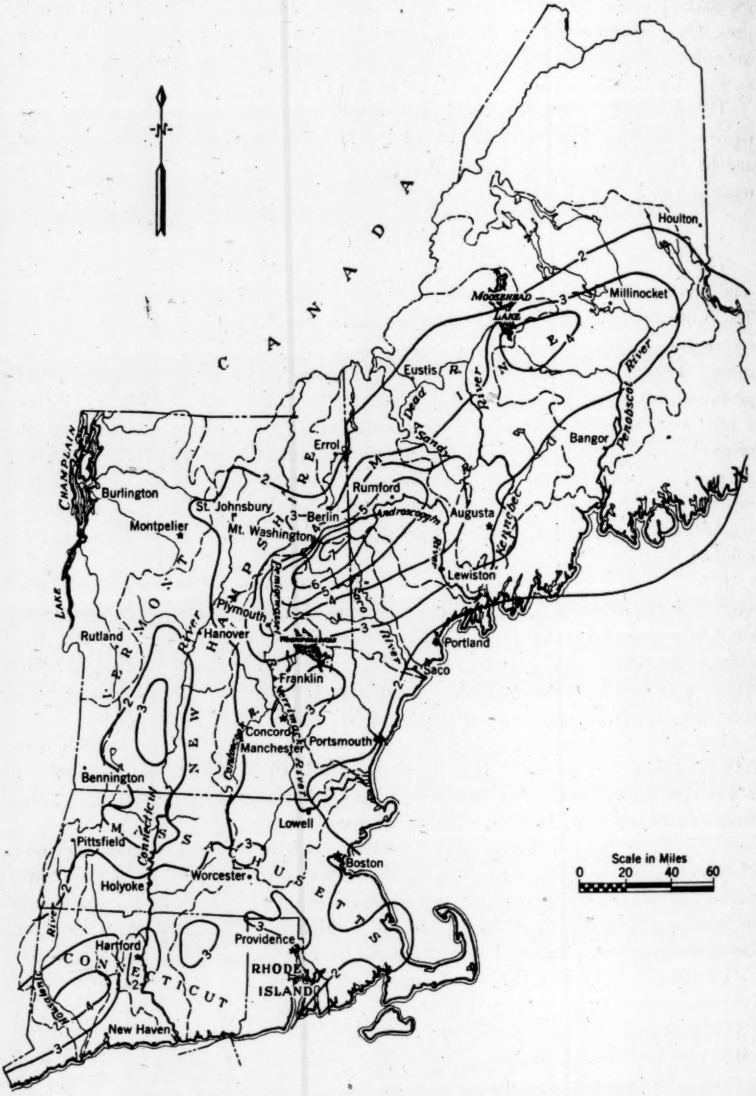

N
CANADA
MAINE
Houlton
Millinocket
Moosehead Lake
Eustis
Dead R.
Sandy River
Penobscot River
Bangor
Errol
NEW HAMPSHIRE
Rumford
Berlin
Androscoggin
Augusta
Kennebec River
Lewiston
St. Johnsbury
Mt. Washington
Burlington
Montpelier
Lake Champlain
VERMONT
Plymouth
Pemigewasset
Hanover
Rutland
Saco River
Portland
Saco
Franklin
Merrimack River
Concord
Manchester
Portsmouth
Contoocook
Connecticut River
Bennington
MASSACHUSETTS
Pittsfield
Lowell
Boston
Worcester
Holyoke
Housatonic River
Hartford
CONNECTICUT
Providence
RHODE ISLAND
New Haven
Scale in Miles
0
20
40
60
2
3
4
5
6
7

FIG. 4.—RAINFALL IN NEW ENGLAND, MARCH 11–13, 1936

a few inches to the existing snow cover. On March 9 a rise in temperature to above normal began, coupled with a light snowfall in Northern New England and moderate rain in Southern New England, on March 9 and 10.

Heavy rain began on March 11 and continued on March 12 and 13. The area of greatest precipitation centered in the White Mountains (see Fig. 4). The maximum rainfall in the 3-day period amounted to 7.78 in. and was reported at the Pinkham Notch Station, which is on the northeast slope of Mt. Washington, virtually on the divide between the Saco and Androscoggin water-sheds. In the same period, the water equivalent of the snow cover that melted in the vicinity of this station amounted to about 3 in. During this storm the rainfall was 7 in. over an area of about 70 sq miles, 6 in. over 450 sq miles, and 5 in. over 1 400 sq miles.

This precipitation was accompanied by an inrush of warm air from the Gulf, causing abnormally high temperatures which melted most of the snow cover in Southern New England and a part of that in Northern New England.

The daily record of maximum and minimum temperature, rainfall, and the quantity of snow cover at eight typical stations in New England are given in Table 1. The quantity and extent of the precipitation is also shown in Table 2, which gives the daily rainfall for twenty-six stations. Only those stations were included which had a record of snow depletion for the period. For each station the water equivalent of the snow depletion has been added to give the total equivalent precipitation during the period.

The combination of rainfall and melting snow resulted in a flood peak of major proportions on March 13, which carried a heavy ice burden. This flood is comparable to those which may be expected once in about every twenty-five years. The ice, being hard and strong, caused much damage to river structures, and in many cases to buildings located on the banks. Serious ice jams also formed on some of the rivers, causing severe valley flooding. In some instances the ice jams acted as diversion dams, causing new river channels to be cut.

The first flood had largely passed down the rivers by March 16, but some rain continued to fall afterward. Then a storm reached New England on March 18 and 19, giving rise to the record-breaking flood of March 19. This storm resulted from the junction of a moderately strong low-pressure area from the Gulf States with another low area coming from Indiana. The former caused an inflow of tropical and marine air which moved slowly to the north and east and reached New England on March 18.

In the second storm, March 17–19, inclusive (see Fig. 5 and Tables 1 and 2), the rainfall was much greater than in the first storm. The area of greatest precipitation was again the White Mountains, and the Pinkham Notch Station again reported the maximum precipitation, 10.76 in. The water equivalent of the snow cover which melted at this station was about 4 in., making a total equivalent precipitation of about 15 in. The rainfall was in excess of 8 in. over an area of about 300 sq miles, and in excess of 5 in. over about 1 000 sq miles. There was also about 5 in. of rainfall over 600 sq miles in Central Massachusetts not included in the areas mentioned in the previous sentence.

Another intense low-pressure area from the Ohio Valley followed on March 21, closely after the southern storm, causing heavy rains in certain sections

TABLE 1.—CLIMATOLOGICAL DATA AT TYPICAL

Drainage Basin	Penobscot					Kennebec					Androscoggin					Saco				
Station	Millinocket, Me.					Winslow, Me.					Berlin, N. H.					Pinkham Notch, N. H.				
Elevation	388 feet					90 feet					1 110 feet					2 000 feet				
March	Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit		Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit		Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit		Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit	
				Max.	Min.				Max.	Min.				Max.	Min.				Max.	Min.
1......	....	..	34	28	19	....	..	..	33	0	....	..	30	27	2	0.02	T†	45.5	24	4
2......	....	..	..	30	−12	....	..	..	30	4	....	..	30	28	—20	....	..	45.5	25	−12
3......	0.81	..	..	26	−7	0.53	..	18	30	6	T†	T†	30	33	—7	0.25	4.0	49	30	12
4......	....	..	..	39	23	....	..	..	45	21	0.04	0.4	28	48	23	0.01	T†	47§	45	9
5......	0.05	0.5	..	36	29	T†	T†	18	41	21	....	..	22	34	25	T†	T†	47	44	24
6......	....	..	..	36	17	....	..	..	41	22	....	..	20	26	14	0.03	0.3	47	25	10
7......	....	..	42	28	−7	....	..	..	33	1	0.02	0.3	20	30	—12	....	..	46.8	24	—9
8......	....	..	..	34	−14	0.15	..	17	35	10	....	..	20	29	—8	....	..	46.5	30	—5
9......	0.06	0.8	..	33	1	0.24	..	..	36	12	0.19	1.5	22	35	17	0.86	7	48.0	32	17
10......	....	..	..	48	25	....	..	..	42	28	0.21	1.5	22	45	25	0.07	..	46.5§	44	24
11......	0.69	..	..	49	30	0.82	..	..	47	31	0.02	..	20	50	29	0.69	..	43.5§	44	38
12......	2.94	..	..	50	36	1.81	..	0	52	37	1.68‡	..	12	51	33	6.46	..	36	44	38
13......	0.05	..	..	51	36	....	..	..	50	35	1.01	..	P*	40	34	0.63	T†	34	43	36
14......	....	..	34	40	29	....	..	..	41	29	0.02	..	P*	37	25	0.08	T†	34.5	33	23
15......	0.19	..	..	41	24	....	..	..	50	25	T†	..	P*	46	21	T†	T†	33	42	18
16......	0.73	..	..	36	34	0.18	..	..	49	33	0.09	..	P*	46	33	0.86	..	32.5	37	32
17......	0.11	..	21	59	34	0.04	..	..	64	34	0.89	..	P*	53	35	0.44	..	31.0	45	33
18......	0.32	..	..	62	42	0.97	..	..	61	45	0.52	..	P*	54	44	6.27	..	24.0	48	42
19......	0.83	..	..	55	46	0.09	..	..	58	47	2.20	..	T†	51	46	4.05	..	20.5	46	36
20......	0.03	..	..	49	37	0.05	..	..	51	30	0.42	..	T†	46	35	0.05	..	18.0	40	33
21......	0.49	..	..	40	29	0.68	..	..	48	35	0.16	..	T†	45	31	1.33	..	16.2	46	32
22......	0.12	..	..	46	37	....	..	..	50	35	2.12	..	T†	38	32	0.64	1.4	16.5	38	30
23......	....	..	..	51	34	....	..	..	54	33	0.04	..	T†	45	29	0.04	0.4	15.5	48	27
24......	....	..	..	51	27	....	..	..	57	25	....	..	..	55	28	....	..	14	50	34
25......	....	..	..	43	29	....	..	..	55	35	0.12	..	T†	43	34	0.28	T†	13	40	32
26......	....	..	..	55	35	....	..	..	60	35	0.02	..	T†	55	32	0.01	..	11	52	30
27......	0.63	0.5	..	49	24	0.62	..	..	59	26	....	..	..	45	25	0.38	..	11	35	27
28......	....	..	..	50	32	....	..	..	54	35	0.57	..	..	49	34	0.34	..	10	44	32
29......	....	..	..	52	33	....	..	..	56	33	0.06	0.6	0.4	59	24	0.01	0.1	7	54	29
30......	0.01	..	..	65	31	....	..	..	69	33	....	..	..	66	34	....	..	3	60	35
31......	0.05	..	P*	62	38	....	..	..	65	39	0.06	..	..	58	45	0.06	..	0	54	28
Total.....	8.11	1.8	..	...	...	6.18	..	..	...	...	10.46	4.3	..	...	...	23.86	..	...	...	...
Average...	....	..	..	45.0	24.9	....	..	..	48.9	26.9	....	..	..	44.0	23.9	....	..	...	40.8	23.5
Mean.....	....	..	..	35.0		....	..	..	37.9		....	..	..	34.0		....	..	...	32.2	
Departure from normal.....	+4.52	..	..	+7.8		+2.90	..	..	+7.4		+7.39	..	..	+6.5		....	..	...	....	
Years of record...	37	..	..	34		41	..	..	32		37	..	..	28		7	..	...	7	

* P = Patches. † T = Trace. ‡ Hail during rain. § Snow settled.

STATIONS IN NEW ENGLAND, MARCH, 1936

Drainage Basin	Merrimack					Connecticut					Housatonic					Champlain				
Station	Plymouth, N. H.					Cavendish, Vt.					Falls Village, Conn.					Northfield, Vt.				
Elevation	500 feet					800 feet					585 feet					876 feet				
March	Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit		Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit		Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit		Precipitation, in inches	Snow fall, in inches	Snow on ground, in in.	Temperature, in degrees Fahrenheit	
				Max.	Min.				Max.	Min.				Max.	Min.				Max.	Min.
1......	....	..	23	26	9	T†	T†	31	32	19	....	..	12	38	18	....	..	23	21	—3
2......	....	..	21	25	—14	0.00	..	31	30	-14	....	..	..	34	—1	0.03	..	..	29	—14
3......	0.25	20	22	34	17	0.27	3.0	33	33	21	0.12	2.0	..	43	25	0.10	1.3	..	33	20
4......	....	..	20	43	10	0.00	..	30	47	5	....	..	..	49	24	....	..	..	49	12
5......	....	..	20	35	20	0.00	..	29.5	39	29	0.15	1.0	..	45	26	....	..	..	43	22
6......	T†	T†	19	30	10	T†	T†	29.5	30	9	....	..	..	31	8	0.02	0.3	..	25	—2
7......	....	..	18	27	—9	....	..	29	25	—7	....	..	..	32	3	....	..	..	25	—7
8......	....	..	17	35	—11	....	..	28	38	—9	....	..	..	41	—1	T†	..	..	35	—6
9......	0.75	0.5	20	32	19	0.26	1.7	29	36	21	0.06	..	..	39	29	0.16	0.4	17	37	27
10......	T†	..	18	41	29	0.05	..	27	44	31	T†	..	..	59	32	....	..	..	45	31
11......	0.46	..	16	42	30	0.26	..	24	45	33	0.12	..	..	53	35	0.09	0	..	50	34
12......	3.46	..	13	46	36	2.66	..	16	46	35	1.80	..	..	53	37	1.12	0	..	49	35
13......	0.33	..	11	42	30	0.30	T†	12	42	32	0.44	..	..	48	26	0.09	0.1	..	47	28
14......	....	..	10	37	21	0.08	1.4	11.5	39	26	0.02	..	..	43	29	0.01	0.5	..	38	20
15......	T†	..	9	44	20	0.00	..	9	50	24	....	..	0	64	27	....	..	..	48	20
16......	0.49	..	8	40	30	0.19	..	8	40	32	....	..	..	60	34	1.34	0	5	41	35
17......	0.31	..	7	45	30	0.51	..	6	43	33	0.11	..	..	62	39	0.29	0	..	53	33
18......	2.12	..	4	50	40	1.52	..	4	51	37	2.34	..	..	60	46	0.59	0	..	55	48
19......	1.82	..	P*	48	40	1.03	..	2	49	36	....	..	..	58	43	0.18	0	..	54	40
20......	....	..	P*	46	33	0.00	..	1	50	35	0.02	..	..	60	36	....	..	..	44	31
21......	1.13	..	..	45	28	0.86	..	T†	47	31	0.05	..	..	58	39	0.90	0	..	44	30
22......	0.31	T†	..	39	32	0.17	..	T†	40	34	0.08	..	..	48	30	0.10	0.3	..	40	33
23......	....	..	..	50	29	T†	..	T†	51	28	....	..	..	55	35	....	T†	..	47	28
24......	....	..	..	55	25	0.00	..	T†	61	28	....	..	..	67	34	0.19	0	..	59	27
25......	0.27	..	..	48	33	0.39	..	T†	51	36	0.35	..	..	65	42	0.34	0	..	43	38
26......	....	..	..	58	26	0.00	..	T†	60	29	....	..	..	64	30	....	0	..	57	29
27......	0.45	..	..	41	24	0.78	..	T†	52	26	0.05	..	..	52	30	0.33	0	..	43	27
28......	0.27	..	..	52	33	0.05	..	T†	53	31	0.88	..	..	56	40	0	0	..	50	32
29......	....	..	..	63	30	0.00	..	T†	65	28	....	..	..	68	24	T†	0	..	63	28
30......	....	..	..	66	28	0.00	..	T†	68	38	....	..	..	72	34	0.01	0	..	64	33
31......	....	..	0	52	34	0.09	..	0	63	38	....	..	0	69	38	0.16	0	0	56	30
Total.....	12.42	2.5	...	...	..	9.47	6.1	..	...	...	6.59	3.0	..	...	...	6.05	2.9	..	...	....
Average...	....	..	..	43.1	23.0	....	..	..	45.8	24.8	....	..	..	53	28.9	....	..	..	44.7	23.8
Mean	....	..	..	33.0		....	..	..	35.3		....	..	..	40.9		....	..	..	34.2	
Departure from normal.....	+9.21	..	..	+4.3		+6.31	..	..	+5.8		+3.42	..	..	+6.7		+3.49	..	..	+7.8	
Years of record...	49	..	..	49		34	..	..	34		46	..	..	20		....	..	..	50	

TABLE 2.—PRECIPITATION AND SNOW DEPLETION AT SELECTED STATIONS IN NEW ENGLAND, MARCH 9–21, 1936

Station	Altitude, in feet	Drainage area	PRECIPITATION, IN INCHES, BY CALENDAR DAYS IN MARCH													Total, March 9–21, 1936	SNOW DEPLETION		Total precipitation + water equivalent of snow depletion, in inches
			9	10	11	12	13	14	15	16	17	18	19	20	21		Inches	Water equivalent, in inches	
Houlton, Me.	364	St. John River	0.04	....	....	0.91	0.65	0.07	....	0.37	0.38	....	0.40	0.09	0.15	3.06	29	7.25	10.31
Millinocket, Me.	388	Penobscot River	0.06	....	0.69	2.94	0.05	....	0.19	0.73	0.11	0.32	0.83	0.03	0.49	6.44	21	5.04	11.48
Eustis, Me.	450	Kennebec River	0.11	....	T*	1.61	0.23	0.11	....	0.61	0.80	0.48	0.65	0.10	1.20	5.90	31	7.76	13.66
Madison, Me.	257	Kennebec River	0.49	....	0.88	2.59	0.19	....	....	0.85	0.26	0.86	0.49	0.02	0.87	7.50	18	4.50	12.00
Lewiston, Me.	182	Androscoggin River	0.57	0.17	0.13	3.07	0.37	....	....	0.02	0.10	0.65	1.60	....	1.56	8.24	18	4.4	12.64
Mt. Washington, N. H.	6 270	Androscoggin River	0.96	0.70	....	1.66	2.27	0.21	....	0.14	1.15	1.32	2.53	0.94	1.01	12.89	19	4.75	17.64
Rumford, Me.	505	Androscoggin River	0.28	....	0.38	5.07	0.39	T*	....	0.81	0.26	1.60	1.29	0.45	1.36	11.89	18	4.50	16.39
Portland, Me.	0	Coastal	0.58	0.12	0.25	1.23	0.24	....	T*	0.03	0.03	0.22	0.73	....	1.17	4.60	15	4.00	8.60
Lincoln, N. H.	1 200	Merrimac River	0.50	0.49	....	2.21	2.49	0.01	....	0.20	0.91	0.19	1.23	0.40	0.05	8.68	29	7.75	16.43
Franklin, N. H.	390	Merrimac River	0.41	0.17	0.19	2.70	0.29	....	....	0.17	0.07	0.97	1.41	....	0.84	7.22	18	4.50	11.72
Wolfboro Falls, N. H.	514	Merrimac River	0.64	0.10	0.15	1.98	0.30	....	....	0.04	0.05	1.13	1.23	0.01	1.25	6.88	25	5.51	12.39
Haverhill, Mass.	50	Merrimac River	0.15	0.14	0.05	1.55	0.07	....	0.03	T*	....	0.44	1.12	....	0.99	4.54	4	1.00	5.54
First Connecticut Lake, N. H.	1 660	Connecticut River	....	....	0.25	0.79	0.03	T*	0.01	1.35	0.04	0.54	0.20	0.01	1.37	4.59	16	5.5	9.09
Bloomfield, Vt.	930	Connecticut River	0.16	....	....	1.14	0.27	0.02	....	0.82	0.61	0.10	0.04	0.23	0.53	3.92	31	7.75	11.67
Bethlehem, N. H.	1 440	Connecticut River	0.22	0.17	....	1.74	0.19	....	....	0.71	0.97	0.41	0.23	....	0.87	5.51	24	6.00	11.51
Hanover, N. H.	603	Connecticut River	0.40	0.02	0.06	1.31	0.10	0.02	0.01	0.39	0.41	0.77	0.25	T*	0.69	4.43	16	4.00	8.43
Newfane, Vt.	450	Connecticut River	0.20	0.05	0.50	2.47	0.20	0..0	....	0.05	0.70	3.45	0.20	0.25	0.75	8.82	15	3.39	12.21
Hoosac Tunnel, Mass.	800	Connecticut River	0.25	0.24	0.05	2.16	0.47	0.08	T*	0.03	0.25	1.19	2.31	0.16	0.55	7.74	18	3.54	11.28
Hubbardston, Mass.	1 030	Connecticut River	0.36	T*	1.43	0.20	0.01	T*	T*	T*	0.61	3.68	0.05	0.21	0.46	7.01	14	3.0	10.01
Collinsville, Conn.	300	Connecticut River	....	....	....	2.91	0.22	....	....	....	....	0.84	1.59	0	0.54	6.10	7	1.75	7.85
Stockbridge, Mass.	850	Housatonic River	0.05	T*	0.12	1.48	0.30	T*	....	T*	0.16	2.08	0.13	0	0.38	4.70	6	1.28	5.98
Cream Hill, Conn.	1 300	Housatonic River	0.05	....	0.52	1.53	0.36	0.06	....	....	0.16	2.18	0.26	0.03	0.41	5.56	12	3.00	8.56
Salisbury, Conn.	1 015	Housatonic River	0.09	....	0.23	1.54	0.40	....	....	....	0.18	2.25	0.21	0	0.47	5.37	10	2.50	7.87
Enosburg Falls, Vt.	422	Lake Champlain	0.15	....	....	0.78	0.45	....	....	0.37	....	0.45	....	....	0.54	2.74	9	2.25	4.99
Cornwall, Vt.	504	Lake Champlain	0.03	T*	....	1.55	....	....	....	0.68	0.75	....	....	0.32	0.45	3.78	16	6.41	10.19
Burlington, Vt.	403	Lake Champlain	0.03	T*	0.03	0.99	0.46	0.04	....	0.78	0.14	0.31	0.10	....	0.32	3.20	6	1.50	4.70

* T=Trace

of New England on March 21, and a third flood peak of minor proportions on March 22 on the Merrimac River and some of its tributaries.

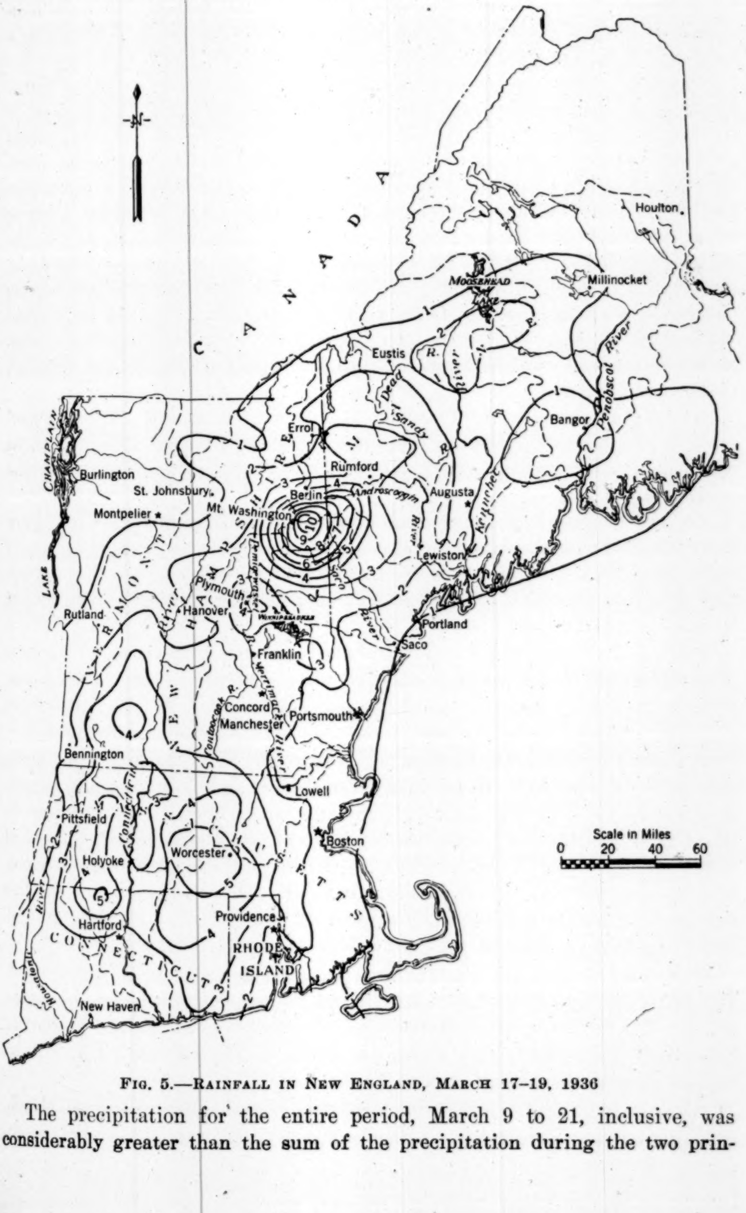

N
CANADA
Houlton
Moosehead Lake
Millinocket
Eustis
Dead R.
Kennebec River
Penobscot River
Bangor
Sandy R.
Errol
Rumford
Berlin
Androscoggin River
Augusta
Mt. Washington
Lewiston
Burlington
St. Johnsbury
Montpelier
Lake Champlain
VERMONT
NEW HAMPSHIRE
MAINE
Plymouth
Pemigewasset
Hanover
Rutland
Winnipesaukee
Portland
Saco
Saco River
Franklin
Concord
Manchester
Contoocook R.
Merrimac River
Portsmouth
Bennington
Lowell
Connecticut River
Pittsfield
Worcester
MASSACHUSETTS
Boston
Holyoke
Providence
Hartford
CONNECTICUT
RHODE ISLAND
Housatonic River
New Haven
Scale in Miles
0
20
40
60
1
2
3
4
5
6
7
8
9
10

FIG. 5.—RAINFALL IN NEW ENGLAND, MARCH 17–19, 1936

The precipitation for the entire period, March 9 to 21, inclusive, was considerably greater than the sum of the precipitation during the two principal storms (March 11 to 13, and March 17 to 19), as at many stations there was some rainfall almost every day. The total rainfall at Pinkham Notch was 21.79 in. The entire snow cover at this station, 30 in. (with an estimated rainfall equivalent of 7.5 in.), melted during the period, making a total equivalent precipitation about 29 in.

The general pattern of the isohyetal lines for the period is similar to that for the two individual storms. The area of greatest precipitation centered in the White Mountains near Mt. Washington and ran across the head-waters of the Merrimac, Saco, and west central portion of the Androscoggin water-shed in a roughly elliptical shape with the axis from northeast to southwest. The 10-in. isohyetal line encloses two large areas: One, of about 1100 sq miles, in the White Mountains; the other, of about 600 sq miles, in the Androscoggin water-shed, with its center at Rumford, Me. The 10-in. isohyetals also enclosed a smaller area in Central New Hampshire, centering on the southeastern end of Lake Winnepesaukee, and another in Central Massachusetts, near Worcester. The extent of the rainfall for the period, March 9 to 21, inclusive, as compared with other great storms in New England, is shown on Fig. 2.

As one of the causes of the flood, the melting snow was next in importance to the rainfall. Data on the quantity of snow that melted, and on its water equivalent, are not nearly so complete as those on precipitation and temperature.

On the Connecticut River water-shed, an extensive survey of the snow cover and its water equivalent was made about March 1. Airplane views of the Connecticut Valley taken between March 21 and 24 indicate that by that time the snow cover was gone from the principal river valleys to within about 50 miles of the Canadian border.

On the Androscoggin River water-shed an excellent series of periodic observations of the snow cover and its water equivalent was taken in connection with the regulation of the storage. A similar set of observations, although not so extensive, was taken on the Upper Kennebec water-shed.

Scattered observations made in other parts of New England agree fairly well with the results in these three river valleys and indicate that, in general, except in Northern New England, the snow cover was equivalent to about 25% moisture; in other words, 4 in. of snow was equal to 1 in. of water. In the Upper Androscoggin and Kennebec River water-sheds the snow was heavier, with a higher moisture content. Observations made in this region about March 1, showed moisture in the layer of snow near the ground, indicating that the ground was frozen little, if at all, in this locality.

Fig. 6 is based largely on data from the stations listed in Tables 1 and 2. The lines show the sum of the rainfall and the water equivalent of the snow cover which melted between March 9 and March 21. The general pattern of these lines is similar to that of the isohyetals of Figs. 4 and 5. The 10-in. line includes about one-half of New England.

An examination of the climatological data for the entire month of March, 1936, shows the abnormality of the conditions which produced the floods.

Based on 98 stations, which cover the entire area of New England, the average temperature for the month was 38.9°, which is 6.5° above the mean for March for the past 49 yr, and higher than the average temperature for

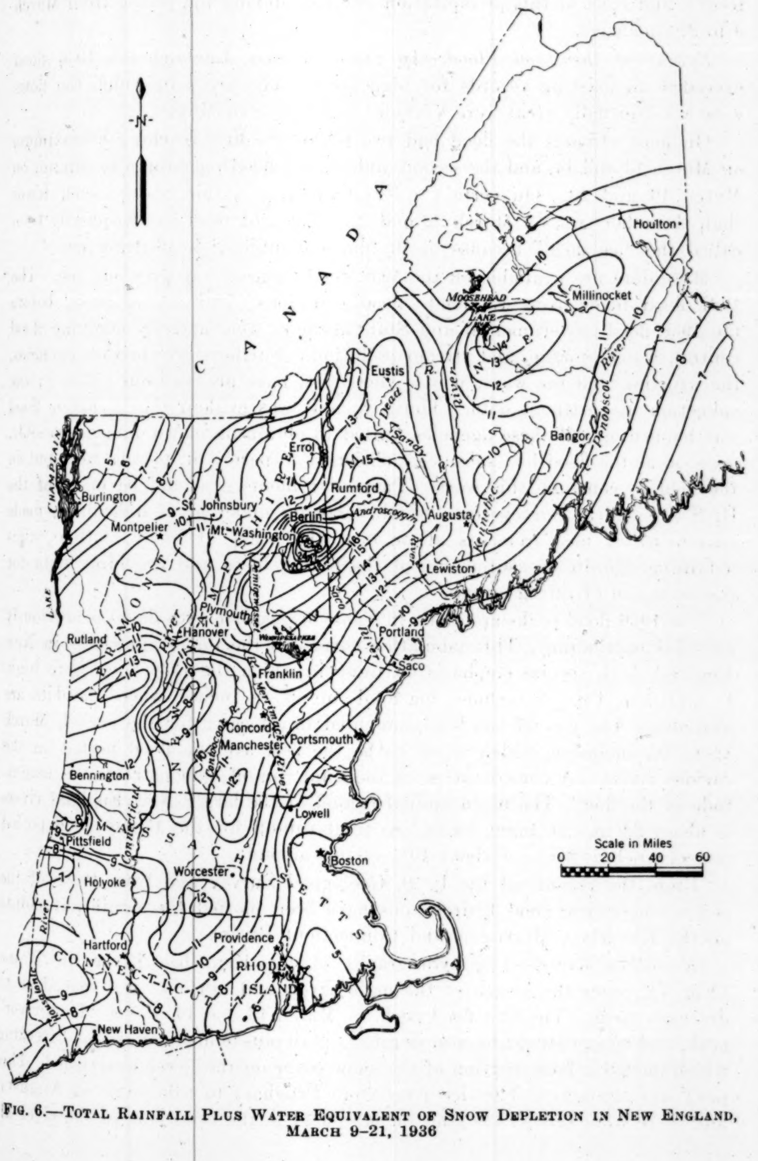

N
CANADA
Houlton
Millinocket
Moosehead Lake
Eustis
Dead R.
Sandy R.
Kennebec River
Penobscot River
Bangor
Errol
Rumford
Androscoggin
Berlin
Augusta
Lewiston
Portland
Saco
Saco River
Mt. Washington
St. Johnsbury
Burlington
Montpelier
Lake Champlain
VERMONT
NEW HAMPSHIRE
MAINE
Plymouth
Hanover
Pemigewasset
Winnipesaukee
Franklin
Rutland
Contoocook R.
Merrimack River
Concord
Manchester
Portsmouth
Bennington
Connecticut River
Lowell
MASSACHUSETTS
Pittsfield
Boston
Worcester
Holyoke
Housatonic River
Hartford
Providence
CONNECTICUT
RHODE ISLAND
New Haven
Scale in Miles
0
20
40
60

FIG. 6.—TOTAL RAINFALL PLUS WATER EQUIVALENT OF SNOW DEPLETION IN NEW ENGLAND, MARCH 9–21, 1936

March in every year of that period except 1903 and 1921. Based on 178 records, the average precipitation for March, 1936, was 8.04 in., which is 4.66 in. greater than the mean for the past 49 yr, and greater than any March on record, and most of this precipitation occurred during the period from March 9 to 21, inclusive.

Extent of the 1936 Flood.—In most of New England the 1936 flood exceeded all existing records for high flows. The areas in which the flows were not unusually great were Vermont and Northern Maine.

On most streams the flood had two peaks, the first reaching a maximum on March 13 and 14, and the second and greater flood reaching a maximum on March 19 and 20. On certain rivers there was a third peak much lower than the other two, on March 22 and 23. The first peak has frequently been called the "ice flood" because of the damage inflicted by floating ice.

More data are available on the 1936 flood than on any previous one. The 1927 flood made New England "flood conscious" and, in addition, before the 1936 flood, governmental and State agencies were actively studying flood control, water storage, and stream pollution. Furthermore, private agencies, the utilities, and the water power companies, have always been active in the collection of pertinent data. Thus, when it was evident that a major flood was happening, all these agencies united to obtain as many data as possible. As soon as the flood had subsided sufficiently to permit it, parties were put in the field to continue this work. Particular mention should be made of the U. S. Geological Survey force which, laboring under great difficulties, made current-meter measurements at or near the peaks at many stations, thus obtaining definite measurements of the higher flows and providing data for the extension of rating curves.

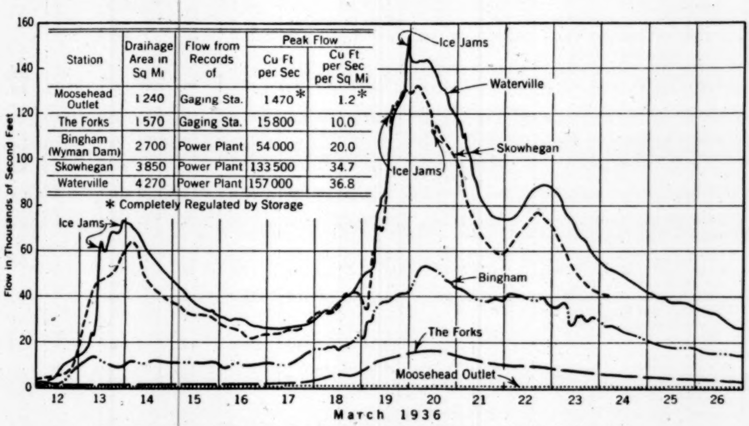
The 1936 flood peaks are given in Table 3, together with data on previously recorded maximums. This tabulation covers all the important rivers in New England, with special emphasis on those in which the 1936 flows were high. In addition, Table 3 includes the total run-off, in inches, where the data are available. The run-off has been computed in all cases for the period, March 12 to 26, inclusive, which seems to be the best average flood period on the various rivers. A consideration of the total run-off demonstrates the magnitude of the flow. The mean annual run-off of the larger New England rivers is about 22 in. In many cases, the total run-off for the 15-day flood period was as much as 9 in., or about 40% of this amount.

From the records of the U. S. Geological Survey and from those of the power companies, flood hydrographs have been plotted for significant points on the Kennebec, Merrimac, and Connecticut Rivers.

Kennebec River.—The hydrographs of the Kennebec River, in Maine (Fig. 7), cover the length of the main river, and are also well spaced as to drainage areas. The first flood peak, of March 13 and 14, was a "down-river" peak, and was produced by a moderate rainfall plus temperatures above freezing which melted a large portion of the snow cover on the lower water-shed. This peak was accompanied by ice jams from Bingham to tide-water at Augusta, and there were further ice jams on March 14 in the tidal part of the river all the way to the mouth. The ice jams were the occasion of much of the damage. The effect of an extensive ice jam in the river above Waterville is indicated on the Waterville hydrograph of March 13.

The peak of this first flood at Waterville, 71 500 cu ft per sec, was not great in terms of the entire water-shed, but from the 1 670 sq miles of intervening drainage area below Bingham, the peak was about 42 000 cu ft per sec, or 37.1 cu ft per sec per sq mile.

In the second flood there were further ice jams, which were caused by up-river ice. These jams can be observed in the Waterville and Skowhegan hydrographs. One of them broke and passed Waterville at 1:45 P. M., March 19, and reached the Augusta Dam at 4:40 P. M., traveling a distance of 20 miles in 3 hr. The peak of the second flood at Waterville, of 157 000 cu ft per sec, amounted to 52.2 cu ft per sec per sq mile, on the 3 000 sq miles of drainage area between Moosehead and Waterville.



Station	Drainage Area in Sq Mi	Flow from Records of	Peak Flow	
			Cu Ft per Sec	Cu Ft per Sec per Sq Mi
Moosehead Outlet	1 240	Gaging Sta.	1 470*	1.2*
The Forks	1 570	Gaging Sta.	15 800	10.0
Bingham (Wyman Dam)	2 700	Power Plant	54 000	20.0
Skowhegan	3 850	Power Plant	133 500	34.7
Waterville	4 270	Power Plant	157 000	36.8

* Completely Regulated by Storage

FIG. 7.—KENNEBEC RIVER HYDROGRAPHS, FLOOD OF MARCH, 1936

These hydrographs of the flood on the Kennebec River show graphically the great value of the storage of the Moosehead System. In the period, March 12 to 26, inclusive, 16 300 000 000 cu ft (375 000 acre-ft) was stored in Moosehead, a raised natural lake, and Brassua, an artificial reservoir, and practically no flow was released. The quantity stored is equivalent to 5.7 in. over the Moosehead water-shed of 1 240 sq miles, or 1.7 in (about 23% of the run-off) over the entire drainage area above Waterville. Another measure of the value of this storage is shown by a comparison of the run-off of the Dead River and Moosehead Lake. The Dead River, an important tributary immediately below Moosehead Lake, with no storage and with a drainage area much like that of Moosehead, had a peak run-off of 32.4 cu ft per sec per sq mile, and a total run-off of 8.5 in. from a drainage area of 878 sq miles. On the other hand, there was practically no flow from the Moosehead drainage area, all the run-off going into storage.

TABLE 3.—NEW ENGLAND FLOOD FLOWS; 1936 PEAK DISCHARGES AND PREVIOUS MAXIMUMS

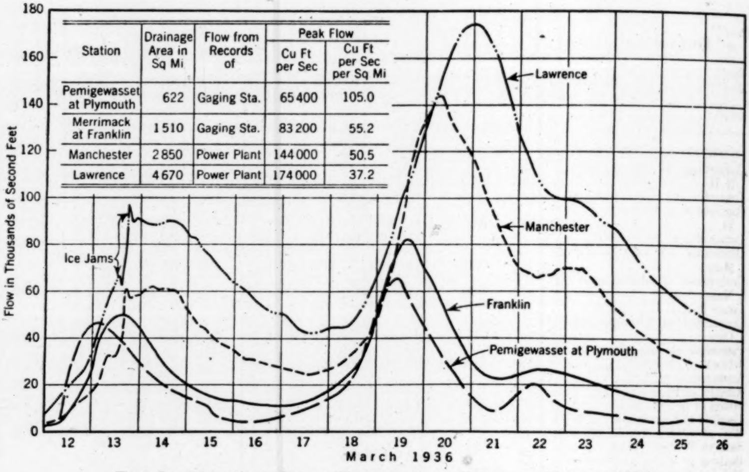
River and station	Drainage area, in square miles	Period of record	Maximum Discharge Previously Recorded			1936 Peak Discharge			Run-off in inches, March 12-26, 1936, inclusive
			Date	Discharge		Date March, 1936	Cubic feet per second	Cubic feet per second per square mile	
				Cubic feet per second	Cubic feet per second per square mile				
St. John at Fort Kent, Me.....	5 690	1926–	5/ 3/33	121,000	21.3	23	82 000	14.4	3.3
St. Croix at Baileyville, Me...	1 320	1919–	5/ 1/33	23 300	17.7	23	17 100	13.0	4.9
PENOBSCOT RIVER BASIN:									
West Branch, near Medway, Me.	2 120	1916–	5/27/28	24 100	11.4	20	5 500	2.6	......
Penobscot at W. Enfield, Me...	6 600	1901–	5/ 1/23	153 000	23.2	21	125 000	19.0	6.1
East Branch at Grindstone, Me	1 070	1902–	4/30/23	35 100	32.8	20	23 900	22.3	......
Mattawamkeag at Mattawamkeag, Me..................	1 500	1902–	5/ 1/23	43 900	29.3	23	30 400	21.7	......
Piscataquis near Foxcroft, Me...	286	1902–	9/29/09	21 700	76.4	20	19 200	67.2	......
Piscataquis at Medway, Me.....	1 170	1924–	9/18/32	35 000	29.9	20	58 400	49.9	......
KENNEBEC RIVER BASIN:									
Kennebec at Moosehead, Me...	1 240	1919–	4/ 9/29	13 600	11.0	3	2 280	1.84	0.2
Kennebec at The Forks, Me....	1 570	1901–	6/18/17	23 700	15.1	20	17 100	10.9	2.0
Kennebec at Wyman Dam, Me.	2 612	....	......	......	......	20	53 900	20.7	......
Kennebec at Bingham, Me.....	2 710	1907–1910 1930–	5/11/09	57 500	21.2	20	55 900	20.6	......
Kennebec at Skowhegan, Me...	3 950	....	......	......	......	20	133 000	33.7	......
Kennebec at Waterville, Me....	4 270	1902–	12/16/01	157 000	36.8	19	157 000	36.5	7.5
Dead at The Forks, Me........	878	1901–1907 1925–	4/30/33	23 800	27.1	20	28 400	32.4	8.5
Carrabasset nr. Northanson, Me.	351	1901–1907 1925–	9/17/32	20 100	57.2	19	23 700	67.5	12.4
Sebasticook nr. Pittsfield, Me...	598	1928–	4/16/34	9 400	......	22	15 000	25.1	......
ANDROSCOGGIN RIVER BASIN:									
Androscoggin nr. Gorham, N. H.	1 390	1929–	5/ 4/29	13 900	10.0	22	19 500	14.0	4.4
Androscoggin at Rumford, Me..	2 090	1892–	11/ 5/27	46 700	22.3	20	74 000	35.4	......
Androscoggin at Gulf Island, Me.	2 600	....	......	......	......	20	128 000	49.3	......
Androscoggin nr. Auburn, Me...	3 260	1928–	4/19/33	50 100	15.4	20	148 000	45.4	9.3
Androscoggin at Brunswick, Me.	3 430	....	......	......	......	21	121 000	35.3	......
Little Androscoggin nr. So. Paris, Me........................	76.2	1913–1923 1931–	4/14/20	3 540	46.4	19	7 160	93.9	......
Swift nr. Roxbury, Me.........	95.0	1929–	4/17/32	13 000	136.8	19	10 500	110.6	......
Presumpscot at Sebago Lake...	435	1887–	......	......	......	22	2 240	5.1	......
SACO RIVER BASIN:									
Saco nr. Conway, N. H.........	386	1903–1910 1927–	10/ 3/07	24 000	62.2	19	45 000	116.5	16.0
Saco at Cornish, Me...........	1 298	1916–	5/ 2/23	23 000	17.3	22	49 400	38.1	11.7
Saco at W. Buxton, Me........	1 572	1906–1916 1919–	5/ 2/23	27 800	17.7	22	59 260	37.7	......
Ossipee at Cornish, Me.........	453	1916–	4/19/23	7 440	16.4	21	14 000	30.9	10.1
Ellis at Goodrich Falls, N. H....	55.4	....	......	......	......	......	8 150	147	......
Salmon Falls nr. Lebanon, Me..	147	1928–	4/19/23	3 550	24.1	19	5 500	37.4	......
MERRIMAC RIVER BASIN:									
E. Br. Pemigewasset nr. Lincoln, N. H.........................	104	1928–	5/ 3/29	8 000	76.9	19	17 000	163.5	20.2
Pemigewasset at Plymouth, N.H.	622	1886–	11/ 4/27	60 000	96.4	19	65 400	105.0	15.4
Pemigewasset at Ayers Is., N.H.	686	....	11/ 4/27	62 300	90.7	19	71 400	104.1	......
Merrimac at Franklin, N. H....	1 507	1903–	11/ 5/27	67 000	44.4	19	83 000	55.2	9.8
Merrimac at Manchester, N. H..	2 850	1924–	11/ 5/27	60 300	21.2	20	144 000	50.4	10.4
Merrimac below Lowell, Mass..	4 424	1846–	4/19/52	104 000	23.5	20	173 000	39.1	9.9
Merrimac at Lawrence, Mass..	4 672	1880–	3/ 3/96	86 900	18.6	20	174 000	37.2	9.6
Bakers nr. Rumney, N. H.	143	1928–	4/12/32	5 280	40.7	19	13 100	133.5	18.6
Smith nr. Bristol, N. H........	85.8	1918–	11/ 4/27	5 800	67.5	19	8 150	95.0	16.6
Contoocook at Penacook N. H..	766	1928–	4/20/33	17 600	23.0	20	45 900	59.8	12.1
Suncook at N. Chichester, N. H.	157	1918–	4/ 7/23	4 300	27.4	19	12 800	81.5	......
Souhegan at Merrimac, N. H...	171	1909–	4/ 8/24	9 300	54.4	19	16 900	98.8	13.9
N. Br. Nashua nr. Leominster, Mass.......................	107	....	......	......	......	18	16 400	152.3	12.1
Concord at Lowell, Mass.......	376	....	......	......	......	......	7 500	20.0	......
Ipswich nr. Ipswich, Mass......	122	1930–	......	......	......	15	2 490	20.4	7.5
Blackstone at Woonsocket, R. I.	416	1929–	11/ 5/27	12 900	31.1	19	15 000	36.1	9.1
Shetucket at S. Windham, Conn.	406	1913–	12/15/20	8 900	21.9	......	16 000	39.4	......
Quinnebaug at Putnam, Conn...	332	1927–	3/ 6/34	5 200	15.7	......	16 500	49.7	10.3
Quinnebaug at Jewett City, Conn.	712	1918–	11/ 5/27	13 100	18.4	......	29 000	40.7	......

TABLE 3.—(*Continued*)

River and Station	Drainage area, in square miles	Period of record	Maximum Discharge Previously Recorded			1936 Peak Discharge			Run-off, in inches, March 12–26, 1936, inclusive
				DISCHARGE					
			Date	Cubic feet per second	Cubic feet per second per square mile	Date March, 1936	Cubic feet per second	Cubic feet per second per square mile	
Connecticut River Basin:									
Connecticut at N. Stratford, N. H.	796	1930–	4/13/33	21 500	27.0	13	28 400	35.7	7.9
Connecticut at S. Newbury, Vt.	2 825	1918–	11/ 5/27	78 000	27.6	19	77 800	27.6	8.5
Connecticut at White R. Junc., Vt.	4 068	1911–	11/ 4/27	131 000	32.2	19	120 000	30.1	8.6
Connecticut at Vernon, Vt.	6 110	....	11/ 4/27	155 000	25.4	19	180 000	29.8	......
Connecticut at Turners Falls, Mass.	7 138	1915–	11/ 4/27	171 000	24.2	19	195 000	27.6	......
Connecticut at Montague Ct., Mass.	7 840	1904–	11/ 5/27	171 000	21.8	19	227 000	28.9	8.8
Connecticut at Holyoke, Mass.	8 390	1880–	11/ 5/27	169 000	20.8	20	222 000	27.4	......
Connecticut at Thompsonville, Conn.	9 637	1928–	11/ 6/27	199 000	20.6	20	282 000	29.3	8.8
Passumpsic at Passumpsic, Vt.	423	1928–	5/19/33	9 540	22.5	18	16 000	37.8	......
White at W. Hartford, Vt.	690	1915–	11/ 4/27	120 000	174.0	18	45 400	65.8	11.8
Ammonoosuc nr. Bath, N. H.	393	....	......	......	......	18	27 400	69.7	......
Ashuelot at Hinsdale, N. H.	420	1907–	3/ 9/20	8 940	21.3	19	16 400	39.0	......
Millers at Erving, Mass.	370	1914–	11/ 4/27	6 350	17.2	19	19 700	53.2	......
Deerfield at Charlemont, Mass.	362	1913–	7/ 8/15	50 600	140.0	18	32 300	89.0	......
Ware at Gibbs Crossing, Mass.	199	1912–	9/17/33	2 960	14.9	19	11 130	55.8	......
Swift at W. Ware, Mass.	186	1910–	4//7/23	2 390	12.8	19	7 580	40.7	......
Quaboag at W. Brimfield, Mass.	151	1909–	3/17/20	1 980	13.1	19	3 760	24.6	8.3
Mid. Br. Westfield at Goss Hts.	52.6	1910–	9/17/33	6 310	120.0	18	8 400	159.8	......
Farmington nr. New Boston, Mass.	92.7	1913–	11/ 3/27	7 900	85.1	18	9 080	97.8	......
Farmington at Tariffville, Conn.	569	1928–	11/21/32	7 610	13.4	......	22 200	39.0	......
Housatonic at Gr. Barrington, Mass.	280	1913–	11/ 5/27	5 690	20.3	19	8 960	32.0	......
Housatonic at Falls Vill., Conn.	644	1912–	11/ 5/27	11 700	18.2	21	14 500	22.5	7.8
Naugatuck nr. Naugatuck, Ct.	247	1918–1924 1928–	......	......	......	......	15 000	60.7	......
Housatonic at Stevenson, Conn.	1 550	1928–	3/ 5/34	23 700	......	......	67 700	43.7	......
Hoosic at Adams, Mass.	46.2	1931–	4/ 1/32	1 030	22.3	18	3 685	79.8	14.5
N. Branch Hoosic at N. Adams, Mass.	39.0	1931–	11/ 4/27	10 218	262.0	18	3 860	98.9	17.6
Lake Champlain Basin:									
Otter Cr. at Middlebury, Vt.	628	1903–1907 1910–1920 1928–	11/ 4/27	13 600	21.7	19	11 000	17.5	......
Winooski at Montpelier, Vt.	420	1909–1923 1928–	11/ 3/27	57 000	135.8	18	20 000	47.7	......
Winooski nr. Essex Junct., Vt.	1 070	1928–	11/ 4/27	116 000	108.2	19	61 000	57.0	......
Dog at Northfield, Vt.	52	1909–1920 1928–	11/ 3/27	8 000	153.8	18	5 700	109.6	11.7
Mad at Mooretown, Vt.	139	1928–	......	......	......	18	9 070	65.3	10.0
Lamoille at Johnson, Vt.	289	1910–1913 1928–1936	11/ 3/27	36 600	126.7	18	10 000	29.9	......
Lamoille nr. Milton, Vt.	692	1929–	4/13/34	16 600	23.0	19	22 000	30.4	......

Merrimac River.—The flow of the Merrimac River is shown in Fig. 8, together with the flow of the Pemigewasset River, at Plymouth, N. H. At Lawrence, Mass., the shape of the first peak on March 13 and 14 is considerably distorted. An ice jam was formed at a bend in the Merrimac River about a mile above Lawrence in the early evening of March 13, and caused first a drop and then a very rapid rise of the water level.

For the period, March 12 to 26, inclusive, the run-off at Lawrence was equivalent to 9.6 in. over the entire water-shed, which is 49% of the mean annual run-off based on 55 yr of record. At Franklin, N. H., the flood run-off was 9.8 in., indicating that the run-off was uniformly distributed over the entire water-shed. This fact is also indicated by the high run-off of tributaries all over the water-shed (see Table 3).



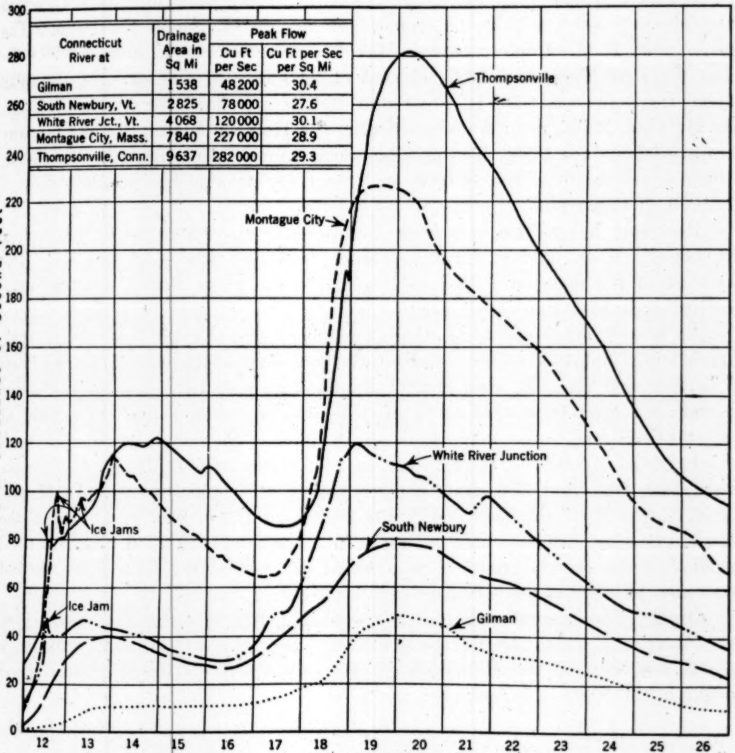
Station	Drainage Area in Sq Mi	Flow from Records of	Peak Flow	
			Cu Ft per Sec	Cu Ft per Sec per Sq Mi
Pemigewasset at Plymouth	622	Gaging Sta.	65 400	105.0
Merrimack at Franklin	1 510	Gaging Sta.	83 200	55.2
Manchester	2 850	Power Plant	144 000	50.5
Lawrence	4 670	Power Plant	174 000	37.2

Fig. 8.—Merrimac River Hydrographs, Flood of March, 1936.

The magnitude of the flood peaks on the main river was considerably reduced by valley storage. Between the Lawrence Dam and the junction of the Winnepesaukee and Pemigewasset Rivers, at Franklin, the valley storage amounted to about 17 500 000 000 cu ft (400 000 acre-ft) equivalent to 1.6 in. over the water-shed above Lawrence. The total amount of reservoir storage now developed above Lawrence is 12 000 000 000 cu ft (276 000 acre-ft).

Connecticut River.—The flow of the Connecticut River at five U. S. Geological Survey Stations is shown in Fig 9. Two floods occurred on this river, one having its peak on March 13 and 14, and the other on March 19 and 20. The first peak was confined largely to the middle and lower parts of the water-shed, as indicated by the fact that it amounted to only 11 000 cu ft per sec, or 7.2 cu ft per sec per sq mile, at Gilman, Vt., as compared with 14.7 cu ft per sec per sq mile at Montague City, Mass. This first peak was not as high relatively as the corresponding peak on the Merrimac River. At Montague City the peak of March 13, which reflects some interference by ice jams, was about 115 000 cu ft per sec, and the average daily flow was 110 000 cu ft per sec, or 14 cu ft per sec per sq mile. In the 56 yr of record on the Lower Connecticut River, flows of this magnitude have occurred only in 1895, 1896, and 1927.

On the Connecticut River, the valley storage was relatively greater than on the Merrimac River. Between the Thompsonville Dam (Windsor Locks, Conn.), and Waterford, Vt., the total valley storage was approximately 42 400 000 000 cu ft (974 000 acre-ft), equivalent to 1.9 in. on the water-shed above Thompsonville. The total quantity of water stored in existing power storage reservoirs in the period, March 12 to 26, inclusive, was about 8 000 000 000 cu ft (184 000 acre-ft) equivalent to about 0.4 in. on the water-shed above Thompsonville.



Connecticut River at	Drainage Area in Sq Mi	Peak Flow	
		Cu Ft per Sec	Cu Ft per Sec per Sq Mi
Gilman	1 538	48 200	30.4
South Newbury, Vt.	2 825	78 000	27.6
White River Jct., Vt.	4 068	120 000	30.1
Montague City, Mass.	7 840	227 000	28.9
Thompsonville, Conn.	9 637	282 000	29.3

FIG. 9.—CONNECTICUT RIVER HYDROGRAPHS, FLOODS OF 1936.

The second peak prevailed over the entire water-shed, the unit run-off being between 27 and 30 cu ft per sec per sq mile for all the stations. For the entire period the run-off at all the stations, except Gilman, was about 8.5 in., and that at Gilman was 7.7 in. This is about 40% of the mean annual run-off (30 yr of record) of 22 in.

LONG-TERM RECORDS

More long-term records are available in New England than in other regions of the United States. The magnitude of the 1936 flood can be best judged by a review of these records on various streams. Reference is made in this connection to Table 3. The following detailed remarks are also of interest.

Connecticut River.—At Hartford, Conn., a record of river stage has been kept since about 1836. Scattered observations of great flows go back to 1683. On March 20, 1801, there was a great flood, with a river stage of 27.5 ft. From contemporary newspaper accounts, it appears that this exceeded any previous stage within the memory of any one living at that time. The next great flood, which occurred May 1, 1854, reached a stage of 29.8 ft. The flood of November, 1927, crested at 29.0 ft. At the peak of the 1936 flood, the stage was 37.3 ft, greatly exceeding any flood for at least 150 yr. At Holyoke, Mass., records maintained by the Holyoke Water Power Company are available since 1880. The previous maximum flood occurred in November, 1927, when the flow was 169 000 cu ft per sec as against the 1936 flow of 222 000 cu ft per sec.

Merrimac River.—Records have been kept by the water power companies, at Lowell, Mass., and at Lawrence since 1846 and 1848, respectively. The average of the maximum flood each year for the 91-yr record, 1846 to 1936, inclusive, is 46 600 cu ft per sec.

At Lowell, in that period, there have been five great floods: 1852, 1895, 1896, 1927, and 1936. The notes of the late James Bicheno Francis, Past-President and Hon. M. Am. Soc. C. E., the Engineer in charge of the water power at Lowell from 1845 to 1885, state that, in 1785, there was a flood of substantially the same magnitude as in 1852. The latter reached a peak discharge of 108 000 cu ft per sec. In 1895, 1896, and 1927, the flood peaks were 18.9, 20.8, and 17.3 cu ft per sec per sq mile, respectively. In March, 1936, the peak of the "ice flood" was 90 000 cu ft per sec, and that of the week following, 173 000 cu ft per sec. In other words, in 1936, the first flood peak was the fourth greatest in a period of about 200 yr, and the second exceeded by 65% any flood on record.

On the Pemigewasset River (drainage area, 622 sq miles), a tributary of the Merrimac which drains the southern slope of the White Mountains, a record has been kept at Plymouth, N. H., since 1886. The five greatest flood peaks in the 50 yr of record are:

Date	Cubic Feet per Second	Cubic Feet per Second per Square Mile
July 15, 1897	32 800*	52.7
April 29, 1923	33 800	54.3
November 4, 1927	60 000	96.4
March 13, 1936	46 200	74.3
March 19, 1936	65 200	105.0

Kennebec River.—At Waterville the approximate height of the great floods since about 1836 has been obtained on a landmark known as the "freshet oak". Elevations of the scars left by the great freshets are as follows:

Date	Elevation, in Feet	Date	Elevation, in Feet
May 22, 1832	104.8	April 30, 1887	102.2
October, 1854	102.8	December 16, 1901	105.14
October, 1869	102.7	March 20, 1936	106.3

---

* Average daily flow. No data available on the flood peak, which presumably was higher.

The flood of May 22, 1832, had an estimated peak of 140 000 cu ft per sec.[10] The flood of December 16, 1901, had a peak flow of 157 000 cu ft per sec, as measured at the Hollingsworth and Whitney Dam about a mile up stream. The flood of March, 1936, also had a peak flow of 157 000 cu ft per sec, as measured at the Hollingsworth and Whitney Dam. Because of the heavy run of ice, the scar left by this freshet was very wide. The elevation given is of the middle of the scar and may not be an accurate record of the maximum flood stage.

Comparison of the 1936 and the 1927 Floods

In spite of the fact that the 1936 flood was greater than any on record on most of the large rivers in New England, the flows per square mile on some of the medium and small water-sheds were greater in the 1927 flood (see Table 3 and Fig. 3).

For drainage areas of about 1 000 sq miles, the maximum flow on record is that of the Winooski River at Essex Junction, Vt., 111 cu ft per sec per sq mile. In 1936, no flow of more than 60 cu ft per sec per sq mile was measured on a drainage area of this size.

For drainage areas of about 500 sq miles, the 1927 unit flows were also much greater. In 1927, the White River (Vermont) had a peak of 174 cu ft per sec per sq mile on a drainage area of 690 sq miles. In the 1936 flood the nearest approach to this unit flow occurred on the Pemigewasset River, at Plymouth (104.8 cu ft per sec per sq mile for a drainage area of 622 sq miles).

As for areas of less than 100 sq miles, in 1927, unit flows of 303 and 257 cu ft per sec per sq mile occurred on drainage areas of 38 and 67 sq miles, respectively—Jail Branch and North Branch, tributaries of the Winooski River. (Detention basins on these two streams were completed in 1935 by the Army Engineers.) No flows of this magnitude were recorded in 1936, the greatest comparable unit flows being 201 and 169 cu ft per sec per sq mile, on two areas of about 30 sq miles each, tributary to the Souhegan River in the central part of the Merrimac water-shed.

Flood Damages

The amount of damage caused by the 1936 flood has been investigated by a number of public agencies, including the Massachusetts State Planning Board, the New Hampshire Flood Reconstruction Council, and the Providence, R. I., and Boston, Mass., District Offices of the Corps of Engineers, U. S. Army. The work of gathering and tabulating these data is largely completed (1937) although some of the figures given are subject to some further correction. These figures do not include estimates of indirect or consequent damages, such as loss of business.

The total flood damage in New England amounted to about $70 000 000, of which $67 893 000 is included in the nine river basins listed in Table 4, and comprises the areas where the damage was heavy. The areas not included

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[10] H. R. Doc. No. 658, 71st Cong., 3d Sess.

(the St. John and St. Croix River basins in Northern Maine, and the western slope of Vermont, tributary to Lake Champlain) were not badly damaged.

TABLE 4.—ESTIMATE OF DAMAGE CAUSED BY THE NEW ENGLAND FLOODS OF MARCH, 1936

River basin	Drainage area, in square miles	Damages	
		Total	Per square mile
Penobscot	8 570	$369 000	$430
Kennebec	5 970	1 494 000	250
Androscoggin	3 470	3 574 000	1 030
Saco	1 720	1 401 000	810
Merrimac	5 015	20 148 000	4 010
Blackstone	540	1 879 000	3 470
Thames	1 470	3 633 000	2 470
Connecticut	11 320	34 500 000	3 050
Housatonic	1 930	895 000	460
Total	40 005	67 893 000	.....
Average	......	....	$1 700

In its report of April 25, 1936, the New Hampshire Flood Reconstruction Council estimated the total flood damage in that State at $10 417 924. It was divided as follows:

Highways and bridges		$4 257 214
Other public property (including municipal utilities)	$229 855	
Quasi-public property	39 345	
		269 200
Utilities and railroads	$1 983 813	
Less municipal utilities	49 855	
		1 935 958
Industries: Manufacturing, wholesale, and retail establishments		2 868 857
Recreational facilities		10 000
Farms and rural property		426 695
Other private property		650 000
Total		$10 417 924

The population of New Hampshire in 1930 was 465 293, and the taxable wealth in 1922 was $1 283 000 000. The estimated damage amounted to $21.90 per capita, or 0.81% of the taxable wealth.

The Massachusetts State Planning Board has made an estimate of the total damage in Massachusetts. At the present time (March, 1937) this estimate is about complete, and accounts for damages of $35 773 926, made up as follows:

Highways	$2 943 760
Highway bridges	5 085 343
Water supply and sewerage systems	571 030

Other municipal losses	$754 871
Railroads	1 305 631
Other utilities	1 638 555
Industrial and commercial	16 872 255
Dams	714 650
Residential	5 070 078
Crop lands	817 753
Total	$35 773 926

The population of Massachusetts, in 1930, was 4 249 614, and the taxable wealth, $22 552 000 000. The estimated damage amounted to $6.25 per capita, or 0.115% of the total taxable wealth.

For some of the tributaries estimates of damage are virtually complete. Two are given herein, one of the damage on the Millers River water-shed, the other on the Pemigewasset River water-shed.

The Millers River lies on the eastern slope of the Connecticut River, which it enters a short distance above Greenfield, Mass. The drainage area is 393 sq miles, and the total fall of the main river is about 800 ft in 37 miles. There are eight large towns along the river. Some of the rural land is farmed, but a large proportion is woodland. The towns and industries are centered around the many low-head water-power developments.

There were two flood peaks on the Millers River, the first being accompanied by ice jams which caused much damage. The second peak was considerably higher than the first one, and attained a unit discharge of 53.2 cu ft per sec per sq mile on a drainage area of 370 sq miles at Erving, Mass. This is about three times as great as any other peak observed in the 22-yr record. The damages were estimated, as follows:

Highways	$189 531
Highway bridges	471 200
Water supply and sewerage systems	34 980
Other municipal losses	0
Railroads	226 800
Other utilities	95 000
Industrial and commercial	925 450
Dams	158 500
Residential	241 825
Crop lands	9 060
Total	$2 352 346

The total population of the Millers River water-shed is about 50 000, making the per capita damage about $47.

The Pemigewasset River has a total drainage area of 1 085 sq miles on the southern slope of the White Mountains. The water-shed is largely wooded and is sparsely settled. Most of the permanent population is concentrated in a few towns in the lower valley. There is a large summer population, particularly on Squam and Newfound Lakes and in the vicinity of Franconia Notch. The 1936 flood flows are shown on Fig. 9. The damages were estimated as follows:

Commercial	$57 000
Industrial	225 000
Residential	156 000
Municipal	12 000
Utilities	25 000
Railroads	180 000
Highways	112 000
Highway bridges	30 000
Farms	232 000
Total	$1 029 000

Effect of Ice Jams.—A large part of the total flood damage in 1936, particularly in Maine, was caused by the ice that went out with the peak of March 13. Ordinarily, the spring freshets come after a period of warm weather that to some extent melts and rots the ice sheet in the mill ponds. In 1936, however, the freshet came very early, and the ice went out as hard, "blue" ice. As a result, ice jams accompanied the progress of the flood peak down river.

On the Androscoggin and Kennebec Rivers, the ice jams were particularly spectacular and destructive. On March 20, an ice jam dammed the river at the head of the mill pond of the Gulf Island Hydro-Electric Station. This jam was about ½ mile wide and 1¼ miles long, and extended 20 ft above the water level. When it broke, it created a rise of 1.2 ft in the pond level, equivalent to about 500 000 000 cu ft (11 500 acre-ft), in less than 30 min. An 80-ft highway truss rode the ice jam, and finally passed through one of the Tainter gates on the dam, buckling on the spillway crest as it went through.

On the Kennebec River the greatest damage was due to an ice jam in the tidal part of the river. This jam took out the piers of a modern highway bridge, valued at $260 000 and carried the spans some distance down stream, on top of the ice, in a part of the river where the depth was about 50 ft.

On the Connecticut River, an ice jam at the head of the mill pond above Holyoke, Mass., blocked the river so that it finally cut a new channel to the east of the old one.

ECONOMICS OF FLOOD CONTROL

In a period of 200 yr, no other floods in New England have caused damage approaching that of the floods of 1927 and 1936. The question of how much this region is justified in spending on preventive measures presents itself.

In a study of the economics of flood control there are two distinct problems. The first concerns the design and construction of individual river structures that affect the natural flow and the occupancy of the natural floodplains, and the second concerns the construction of works which will prevent or reduce flood flows on the entire river or in certain localities.

The first problem must be solved by the individual owners of properties of various kinds—dams and power plants, bridges, industrial buildings, river walls, railroad and other embankments, and, of course, homes. Damage to river structures during floods indicates that many of them were constructed without adequate engineering design and supervision.

It is difficult to determine how far it is profitable to go in the design of a structure to prevent possible flood damage where property damage alone is involved. Where life is endangered there can be only one answer, but the temporary loss of the use of property or even its partial destruction once in 25, 50, or 100 yr, may not warrant design and construction to prevent all damage.

Attempts have been made to rationalize the second problem, namely, that of flood prevention or reduction, on a purely economic basis. If definite information were available, the solution would be relatively simple; the maximum expenditure economically justified to prevent recurrence of certain flood damages would be the capitalized present worth of the damages over the period of flood frequency causing them.

Because of the lack of definite scientific data on floods which occurred before about 1850, however, it is difficult to establish an accurate basis for computing the frequency of floods of certain magnitudes. This does not mean that no attempt should be made to make economic studies; it does indicate that all possible information should be collected and studied, impartially analyzed, and tested by all known methods.

Estimating the damages due to the 1927 flood on the Winooski River, in Vermont, to have been $13 500 000, the flood frequency, 100 yr, and giving consideration to other floods, the U. S. Army Engineers estimated[11] that the expenditure warranted to provide adequate control of this stream would be $433 000, based on a 3½% interest rate.

It is further stated that the Winooski Valley can be adequately protected from flood damage by a series of detention reservoirs designed solely for flood control, but that the cost "is far beyond the amount which can be economically justified."

This conclusion is the one generally arrived at from a study of flood protection on the larger New England streams. For a specific case on a small stream where the damage may run very high, or where the opportunities for flood control are unusually favorable, it may be that flood-control measures can be economically justified.

PREVENTION OF FLOOD DAMAGES

Most of the practical means of reducing flood damages have to do with such matters as flood warnings, storage reservoirs, detention basins, river channel improvements, or river structures, such as dams, bridges, and levees. Of these items, flood warnings and storage reservoirs are of greatest general interest in New England, since they are the most practical measures. Storage reservoirs may provide flood protection for large areas, although not primarily developed for that purpose.

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[11] H. R. Doc. 785, 71st Cong., 3d Sess.

Since the 1927 flood a few detention basins have been built in New England to protect a comparatively small area. Technically, they are perfectly satisfactory, and will serve their purpose if they are maintained properly, but their cost under New England conditions will be found to be prohibitive except in rare instances. The other items mentioned are principally of local interest, but of great importance, and are receiving consideration in many places.

Flood Warnings.—The U. S. Weather Bureau is the formal agency in New England which maintains flood stations that report to it and from which flood warnings are issued if the occasion arises. It is proper to question the adequacy of this service and to make suggestions which may improve it.

The problem may be somewhat different in New England than elsewhere. The distances are relatively short except on the major streams, and on all these streams there are agencies which have a vital interest in the progress of floods, and, therefore, make it their business to keep informed. In some instances, this agency is an utility operating hydro-electric plants over a considerable part of the river. In other instances, as at Lewiston, Me., and Lowell, Mass., the company owning a water power also has an interest in up-stream storage and, for many years, has maintained a private system of communication to keep informed about threatened rises. Under existing conditions such information is of little value to the general public, since it is not given out except as an inquiring reporter may obtain it for his newspaper. Information concerning two important factors affecting floods is not generally available, namely, the extent and water equivalent of the snow cover and the condition of the ground beneath the snow. Numerous observations of precipitation in the form of snow are made by the Weather Bureau and other agencies. Public utilities owning water-power plants, and the winter sports organizations, take scattered observations on the amount of snow cover, but few measurements are made of its water equivalent.

Estimates by trained observers of the amount of snow cover and its water equivalent, at representative points in important areas, are needed. The District Engineers of the U. S. Geological Survey, whose assistants periodically visit the gaging stations in their charge to change charts, would seem to be in an excellent position to take this work on with little additional expense.

This agency also obtains considerable advance information about other conditions conducive to floods, from the reports of its field crews, who make periodic measurements and inspections on all the important streams and many of the tributaries. Again, this information is not made public except on request.

In some years snow falls before the ground freezes and never completely leaves the ground before the spring flood. Such a condition (more likely to be found in Northern New England), is one of the many factors that sometimes combine to cause the extreme run-off, and, consequently, should be a part of the data available to the forecasting agency.

One of two corrective steps is needed. Either a central forecasting agency should be set up to supplement the work of the Weather Bureau, or that body itself should take on additional duties and correlate all the information collected by private and public agencies, so that it may be made available to the public.

Newspapers and radio commentators invariably exaggerate threatened flood conditions, and the public, unable to judge between facts and exaggeration, usually discounts all such information because of its general unreliability. Regulations providing that none but official information be published or broadcast during a flood crisis might be helpful.

Sometimes, of course, meteorological conditions change so rapidly over small areas, that the event happens before the forecast can possibly reach the affected area. This was the case before the 1927 flood in Vermont.

Storage Reservoirs.—Economically, the most promising method of alleviating flood conditions on New England streams is the construction of storage reservoirs in which the reduction of flood peaks is incidental to their operation for increasing the low-water flow.

Storage reservoirs in New England are normally well down by October, and usually nearly empty by the end of winter, the periods when floods are most likely to happen. The larger reservoirs are not full much more than two months in an average year, and with competent management considerable water can be stored temporarily even on a full reservoir. Any temporary storage reduces the peak flow down stream.

In its report, the Committee on Floods of the Boston Society of Civil Engineers states[12]: "The most desirable policy toward the flood problem here in New England appears to be to encourage in every way the construction of reservoirs for power purposes."

The Advisory Committee of Engineers on Flood Control in the State of Vermont, in a report after the 1927 flood. made substantially the same recommendation, as did the Army Engineers in their reports on flood control on many of the New England rivers.

Effectiveness of Storage Reservoirs for Flood Control.—As a test of the effectiveness of storage reservoirs for flood control, an examination has been made of several existing storage reservoirs to see how much capacity was available at the beginning of various floods of record.

Lake Winnepesaukee (Merrimac Water-Shed).—In Lake Winnepesaukee, 7 000 000 000 cu ft (161 000 acre-ft) of storage is made available by works built about 1845 which permit the lake level to be drawn down 40 in. Restrictions occasioned by the use of the lake in the summer for recreational purposes require that if the gage registers 21 in., or less, the outflow is limited to a daily average of 250 cu ft per sec. Floods begin to cause damage in the Merrimac Valley when the average daily flow at Lawrence reaches about 60 000 cu ft per sec. In the period from 1870 to 1935 there were eight floods of this magnitude, or greater. Table 5 shows the percentage of full reservoir capacity available at the beginning of each. The behavior of Lake Winnepesaukee in the 1936 flood is discussed later in detail.

Moosehead Lake (Kennebec Water-Shed).—Damage due to floods in the Lower Kennebec Valley starts with an average daily flow of about 87 000

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[12] *Journal*, Boston Soc. of Civ. Engrs., September, 1930, p. 464.

TABLE 5.—STORAGE CAPACITY AVAILABLE IN LAKE WINNEPESAUKEE AT BEGINNING OF LARGE FLOODS

Date	Average daily flow at Lawrence, Mass., in cubic feet per second	Percentage of full capacity available at beginning of flood*
April 21, 1870	89 780	18
March 29, 1877	66 600	77
December 12, 1878	76 300	50
April 16, 1895	65 300	77
March 3, 1896	82 200	11
April 9, 1901	62 500	45
March 4, 1902	61 200	16
November 6, 1927	66 600	71

* Capacity, 7 000 000 000 cu. ft. (161 000 acre-ft.)

cu ft per sec, at Waterville, Me. In the period, 1895 to 1926, five freshets of this magnitude or greater have occurred. In four of these floods, the quantity of water stored in Moosehead Lake was an important factor in reducing flood peaks in the Lower Kennebec River (see Table 6).

TABLE 6.—STORAGE CAPACITY AVAILABLE IN MOOSEHEAD LAKE AT BEGINNING OF LARGE FLOODS

Date	Average daily flow at Waterville, Me., in cubic feet per second	LEVEL OF MOOSEHEAD LAKE		Percentage of total capacity available*
		Date	Elevation, in feet	
April 15, 1895	86 200	April 12	1.8	76
March 2, 1896	111 000	February 28	4.75	37
December 16, 1901	151 000	December 12	1.25	83
June 18, 1917	88 500	June 13	7.55	0
May 1, 1923	134 000	April 30	3.6	52

* Capacity, 20 600 000 000 cu. ft. (473 000 acre-ft.).

After 1926 additional storage was created on the Moosehead Lake drainage area by the construction of Brassua Reservoir, with a capacity of 8 600 000 000 cu ft (197 000 acre-ft). The contributing area is 675 sq miles. On March 12, 1936, only 945 000 000 cu ft (21 700 acre-ft) (5% of its total capacity) was stored in Moosehead Lake, and Brassua Lake was empty. Therefore, at the start of the 1936 flood, practically the entire capacity of the two reservoirs was available for the storage of flood flows.

Effectiveness of Power Storage Reservoirs in Reducing 1936 Flood Peaks.—An examination has been made of the amount by which storage reservoirs reduced flood peaks in the 1936 flood, on several streams where sufficient data were available.

Lake Winnepesaukee (Merrimac Water-Shed).—On March 10, 1936, Lake Winnepesaukee was 22% full. In the period, March 10 to 26, inclusive, the total yield of the Lake Winnepesaukee drainage area was 8 860 000 000 cu ft (203 000 acre-ft) (the equivalent of 10.6 in. of run-off). Of this quantity, 7 500 000 000 cu ft (172 000 acre-ft) was stored in the lake, only a few hundred second-feet being released until March 21, two days after the peak had passed Franklin, N. H.

On Fig. 10 is shown the effect of this storage on the average daily discharge of the Merrimac at Franklin, the point where that stream is formed by the junction of the Winnepesaukee River (drainage area, 480 sq miles) and the Pemigewasset River (drainage area, 1 010 sq miles).

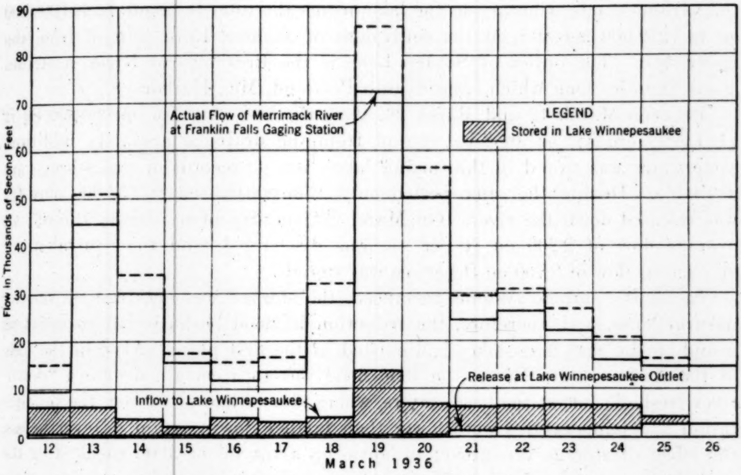

Fig. 10.—Effect of Lake Winnepesaukee Storage, Flood of March, 1936

It seems reasonable to assume that the momentary peak at Franklin, which occurred about 2:00 P. M., on March 19, and amounted to 83 000 cu ft per sec, was reduced by about 9 000 cu ft per sec. The corresponding reduction in gage height is about 3 ft.

At the dams below, the reduction was somewhat less, because of the greater time of transit and the effect of channel storage. It is estimated that at the Sewall Dam and the Garvins Falls Dam, 16 and 28 miles, respectively, below Franklin, the reduction in flow due to Winnepesaukee storage was about 8 000 cu ft per sec, equivalent to a reduction of about 0.75 ft of the flood peak.

Rangeley System (Androscoggin Water-Shed).—During the 1936 flood, 13 000 000 000 cu ft (299 000 acre-ft) was stored in the Rangeley Reservoir System in the period between March 12 and March 26. Releases during the same period totaled only about 420 000 000 cu ft (9 600 acre-ft), and did not begin until March 22, when the flood peaks had passed Rumford, Me. The total quantity stored was equivalent to 5.1 in. of run-off over the water-shed, or to an average flow of 10 000 cu ft per sec for the period.

Because of the number of units comprising the system, it is difficult to estimate the effect of this storage on the flood peaks. However, on the rigorous assumption that the reduction in peak flow was no greater than the reduction in the average flow for the period, it amounted to about 13.5% at Rumford, 84 miles below the lowest reservoir. The corresponding reduction in height over this dam was about 1.5 ft.

Sebago Lake (Presumpscot River).—As an example of complete storage, the behavior of Sebago Lake is interesting. Sebago Lake, a natural lake with an area of 45.6 sq miles, was raised about 8 ft by means of a dam at the outlet, built in 1878. The storage available is about 9 700 000 000 cu ft (223 000 acre-ft) on a drainage area of 360 sq miles. Additional storage in the various ponds tributary to the lake brings the total to about 12 600 000 000 cu ft (289 000 acre-ft), or the equivalent of about a 15-in. run-off from the water-shed. The outlet of Sebago Lake is the Presumpscot River, a stream about 22 miles long which empties into Portland, Me., Harbor.

Between March 12 and March 26, 1936, the lake stored 6 700 000 000 cu ft (154 000 acre-ft), or an 8-in. run-off from the drainage area. In addition, some water was stored in the smaller lakes, but no records of this storage are available. During the same period only 2 300 000 000 cu ft (53 000 acre-ft) was released down the river. On March 22, the day of maximum run-off, an average flow of 2 200 cu ft per sec was discharged, and the equivalent of an average flow of 8 700 cu ft per sec was stored.

In all the storage systems instanced, the storage was obtained by raising natural lakes, and, therefore, the reduction in flood peaks is not so great as would be the case if no lakes had existed in the first place. Also, in the case of the Androscoggin River, the 1936 flood was to a certain degree a "down-river freshet", for, at the head-waters, where the storage is located, the precipitation and temperatures were lower than farther down stream. Nevertheless, the effect of storage was great; to bottle up a run-off of 5 in. until after the peak is past, is of great benefit.

Deerfield River Storage.—The effects of storage were analyzed on one stream, the Deerfield River, where there are no natural lakes. The Deerfield is an important tributary of the Connecticut River, rising in the Green Mountains in Southern Vermont, and flowing south and east to join the Connecticut just below Turners Falls, Mass. The water-shed is mountainous, with steep rocky slopes, and the stream is characterized by flashy floods and high run-off. Two reservoirs, Somerset and Harriman (formerly known as Davis Bridge), with drainage areas of 30 and 184 sq miles, respectively, have been built by the New England Power Company, the former in 1913, the latter in 1924. Somerset Reservoir completely controls the 30 sq miles above it, the storage capacity amounting to 38.9 in. on the water-shed. The Harriman Reservoir largely controls the 154 sq miles below Somerset, the storage amounting to 14.1 in. on the water-shed.

In the 1936 flood, no water was wasted from Somerset Reservoir. At Harriman Reservoir, except for a few hundred second-feet used for power purposes, no water was released until March 27, a week after the crest of the Connecticut River flood. It is estimated that these two reservoirs reduced the flood peak at Shelburne Falls, Mass., where the drainage area is about 500 sq miles, from 72 000 to 48 000 cu ft per sec, with a corresponding reduction of gage height of 3.3 ft.

Table 7 illustrates further the value of storage reservoirs in time of flood, giving the quantity of water stored and the storage capacity remaining after the 1927 and 1936 floods.

Studies of Potential Developments.—At present (1937) many different plans of flood control by means of storage reservoirs are being developed by various State and Federal agencies. The scope of this paper does not permit a detailed analysis of the effect of any of these projects on flood peaks. Such analyses have been made, however, in several cases, and a few of the conclusions are quoted.

TABLE 7.—BEHAVIOR OF NEW ENGLAND STORAGE RESERVOIRS—FLOODS OF NOVEMBER, 1927, AND MARCH, 1936

River and reservoir	Drainage area, in square miles	FULL RESERVOIR CAPACITY			BEHAVIOR NOVEMBER 3-4, 1927			BEHAVIOR MARCH 12-26, 1936		
		In million cubic feet	Inches, depth over drainage area	Corresponding draw-down, in feet	AMOUNT STORED, IN INCHES, ON DRAINAGE AREA		Percentage capacity remaining, Nov. 5	AMOUNT STORED, IN INCHES, ON DRAINAGE AREA		Percentage capacity remaining, March 26
					Maximum, per day	Two days		Maximum, per day	Period	
Kennebec River:										
Brassua Lake	675	8 560	30	5.5	‡	‡	‡	‡	3.0	45
Moosehead Lake	945*	20 630	6.5	9.4	‡	‡	‡	‡	5.3	44
Androscoggin River:										
Upper Dam Reservoir	405	8 890	12.2	9.4	0.6	1.0	17	1.1	7.0	26
Middle Dam Reservoir	104*	5 874	17.3	24.3	2.5	2.6	37	2.3	7.2	10
Aziscohos Reservoir	233	9 590	45.0	17.7	1.0	1.1	48	1.8	11.1	38
Umbagoag Reservoir	353*	3 240	9.5	4.0	0.9	1.1	10	0.7	1.8	7
Presumpscot River:										
Sebago Lake	436	9 700	9.7	9.6	‡	‡	‡	0.8	6.6	
Merrimac River Basin:										
Merrymeeting Lake	12.6	770	16	26.5	‡	0.8	8	‡	9.2	‡
Lake Winnepesaukee	360	7 000	3.7	8.4	‡	1.4	55	1.7	7.0	0
Squam Lake	57.8	2 000	6	14.9	‡	‡	‡	‡	‡	‡
Newfound Lake	96.4	1 300	6	5.7	1.6	3.1	0	‡	‡	‡
Connecticut River Basin:										
First and Second Lakes	83†	3 840†	......	20.0†	1.79	2.36	26.5	1.0	7.0	35
Goose Pond	15	480	20.0	13.8	0.66	1.28	81	‡	8.95	14
Lake Mascoma	126	500	8.5	1.7	0.70	1.12	0	‡	0.77	0
Lake Sunapee	45.1	860	11.5	8.2	‡	‡	‡	2.5	6.1	0
Somerset and Harriman Reservoirs	184†	7 750	....	18.2	3.8	5.2	25	2.4	12.1	19

* Effective. † Combined. ‡ Data not available.

The New Hampshire Water Resources Board has proposed four storage reservoirs on the water-shed of the Merrimac River, to be built primarily to increase the low-water flow of the river and, secondarily, to reduce floods. These reservoirs would have a total capacity of about 18 000 000 000 cu ft (414 000 acre-ft) and a contributing drainage area of 600 sq miles. In a report on the March, 1936, flood[13], Richard S. Holmgren, M. Am. Soc. C. E., Chief Engineer of the New Hampshire Water Resources Board, estimates that the proposed system of reservoirs would have reduced the 1936 peak flow at Manchester, N. H. (drainage area, 2 854 sq miles), from 114 000 to 85 000 cu ft per sec, with a reduction in the height of water at the Amoskeag Dam, at Manchester, N. H., of 3.1 ft. At the Lowell (Mass.) Dam, he estimates that the flow would have been reduced from 173 000 to 145 000 cu ft per sec, with a corresponding reduction in gage height of 3.0 ft.

In 1935–1936, the Army Engineers made a comprehensive report[14] on the Connecticut River on "the efficient development of its water power, the control of floods, and the needs of navigation." In this report they recommend an "initial flood control plan", which would include ten proposed reservoirs operated primarily for the control of floods and, secondarily, for the generation of power. The reservoirs would have a combined contributing drainage area of 951 sq miles, a capacity below the spillway level of 11 700 000 000 cu ft (269 000 acre-ft), and a capacity above the spillway level of 3 670 000 000 cu ft (84 000 acre-ft). The report states (page 85),

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[13] "Extent and Magnitude of the March, 1936, Flood in New Hampshire."

[14] H. R. Doc. 412, 74th Cong., 2d Sess.

> "If the reservoirs of the initial program had been in operation during the 1927 flood, they would have been depleted at the time of the flood under normal operating conditions by 50% of their capacity. * * * Six of the reservoirs would have given almost complete protection at the dam site. * * * On the main stream, the peak at White River Junction would have been reduced by approximately 46% from 148 000 second-feet to about 80 000 second-feet."

Detention Reservoirs.—In Vermont, as a result of comprehensive investigations following the 1927 flood, two detention reservoirs were built on tributaries of the Winooski River and put into operation in 1935. The dams were designed by members of the Corps of Engineers, U. S. Army, and built under their supervision.

As originally planned[15], these reservoirs were large enough for power storage as well as flood control As finally built, however, the dams were lower and their operation was restricted to flood control, with uncontrolled outlets of such size that with the reservoirs full the discharge would still remain within the banks of the stream below. Both dams have wide crests, so that their height may easily be increased if it should prove desirable in the future to provide storage for power purposes.

The Wrightsville Dam is on the North Branch of the Winooski River and controls a drainage area of 71 sq miles. The reservoir has a capacity of 895 000 000 cu ft (20 500 acre-ft) to the spillway crest and 1 000 000 000 cu ft (23 000 acre-ft) to the maximum flow line, equivalent to 5.4 and 6.1 in., respectively, on the water-shed.

The East Barre Dam is on the Jail Branch of the Winooski River and controls a drainage area of 39 sq miles. The reservoir has a capacity of 523 000 000 cu ft (12 000 acre-ft) to the spillway crest and 706 000 000 cu ft (16 200 acre-ft) to the maximum flow line, equivalent to 5.3 and 7.8 in., respectively, on the water-shed.

It should be noted that these reservoirs are in the area in which some of the highest unit flows were recorded in the 1927 flood, that at Wrightsville being 257 cu ft per sec per sq mile and that at East Barre, 303 cu ft per sec per sq mile.

In the 1936 flood, the unit flows were not so high on the Vermont streams; nevertheless, the run-off was high enough to constitute a real test of the effectiveness of these detention reservoirs. At Wrightsville, the maximum rate of inflow, computed from the rise in the reservoir, was 3 960 cu ft per sec (55.8 cu ft per sec per sq mile), and the rate of discharge through the outlet at this time was 870 cu ft per sec (12.3 cu ft per sec per sq mile), a reduction in the flood peak of 78 per cent. The maximum rate of outflow was 960 cu ft per sec. The maximum quantity stored was equivalent to 4.2 in. on the drainage area, or 78% of the total capacity of the reservoir. At East Barre, the maximum rate of inflow was 2 035 cu ft per sec (52.2 cu ft per sec per sq mile), and the concurrent rate of discharge was 427 cu ft per sec (10.9 cu ft per sec per sq mile), a reduction in flood peak of 79 per cent. The greatest rate of outflow was 450 cu ft per sec. The maximum quantity stored was 5.6 in. on the water-shed, or about 95% of the capacity to the spillway level.

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[15] H. R. Doc. 785, 71st Cong., 3d Sess.

It is estimated by the Army Engineers that these two detention reservoirs, regulating 110 sq miles, reduced the peak flow of the Winooski River, at Montpelier, Vt., where the drainage area is 418 sq miles, from 23 700 to 19 200 cu ft per sec.

A computation has been made of the potential effect of the Wrightsville Dam on flows similar to those in 1927. A hydrograph was used based in general on the "standard 100-yr flood",[16] with a total run-off of 5.2 in. in 27 hr and a peak flow of about 250 cu ft per sec per sq mile. With the reservoir in operation the peak flow would have been reduced from 17 000 to about 800 cu ft per sec. The maximum storage would have been 89% of the reservoir capacity.

CONCLUSIONS

Comparatively little definite information is available about floods, the conditions under which they have occurred, and the inconvenience, loss of life, and economic losses they have caused.

Predictions relative to 1 000-yr and even 100-yr floods are based on meager information. Except for a few records covering less than 100 yr, the basic data are largely legendary. The importance of collecting additional data should be stressed.

The determination of future flood flows at a specific point on any river should include a study of the flood records of all streams having similar physical and climatological aspects.

Existing storage reservoirs have appreciably reduced the flood flows on several streams. Additional reservoir sites, many of which can undoubtedly be economically developed, are available on the head-waters of most of the New England rivers and their tributaries.

In some cases, it will be necessary to provide for joint action by all the water users on a river in order to develop the storage economically. Such action can be furthered by legislation similar to that creating the New Hampshire Water Resources Board. This Board can initiate and carry out comprehensive developments under certain conditions. With regard to some of the larger streams, which pass through several States, it may be necessary to set up interstate compacts before some of the available storage sites can be developed economically.

Except for local improvements, which will undoubtedly be carried out regardless of economic considerations, the possibilities of strictly economic flood protection on New England rivers other than by storage reservoirs are believed to be scarce. Such possibilities are increased as the concept of the economics of flood protection is broadened by considerations of public policy.

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[16] H. R. Doc. 708, 71st Cong., 3d Sess., Figs. 9 and 10.

Flood-protection works on the upper reaches of the large rivers, and on their tributaries, will have only a very minor effect on the lower reaches, where much of the damage during extreme floods occurs.

Considering the many combinations of conditions which may cause flood flows on the lower reaches of the larger rivers, calculations of stage reductions due to hypothetical reservoirs, or other practical works, on their upper reaches are somewhat academic since they must be based on meager flood-flow information. It seems sufficient to state that any reduction whatever is desirable, but that the extent of such reduction will vary with conditions.

ACKNOWLEDGMENTS

This paper could not have been written so soon after the 1936 flood without the co-operation of the public and private agencies that have been collecting data and have made available data which were still being analyzed. Particular mention should be made of the following agencies and individuals: The Boston, Mass., Office of the U. S. Weather Bureau; the District Engineers of the Corps of Engineers, U. S. Army, in Boston, and in Providence, R. I.; the District Engineers of the U. S. Geological Survey in Boston, Augusta, Me., and Hartford, Conn.; the New Hampshire Water Resources Board; the Massachusetts State Planning Board; and, the Engineers of the New England Power Company, New England Public Service Company, Union Water Power Company, and Kennebec Water Power Company. The writer is also indebted to his associate, Mr. Truman H. Safford, whose assistance in preparing this paper was invaluable.

THE NEW YORK FLOODS OF 1935 AND 1936

BY ARTHUR W. HARRINGTON,[17] M. AM. SOC. C. E., AND
HOLLISTER JOHNSON,[18] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Although records, relatively long for this region, indicated a favored location in respect to the occurrence of great storms, within nine months, New York State was visited by two major floods, one in July, 1935, and the other in March, 1936. The flood of July, 1935, was caused by a storm of the thunder-shower type of unusually large areal extent in the south-central part of the State. Many small streams had peak discharges of more than 2 000 cu ft per sec per sq mile. The damage, especially severe along the small streams, might well be called dynamic damage. The flood of March, 1936, was caused by moderate rains and melting snow which produced record-breaking discharges, both peak and total, on the larger streams, chiefly in the Hudson and Susquehanna River basins. Damage was caused principally by inundation along the larger streams. More and better hydrologic records are needed as part of the basic data for the design and operation of flood preventive and control projects and the solution of the problems of engineering economics involved.

GENERAL

It is difficult to speak in general terms of New York State, which, in many ways, is a region of contrasts. In topography, geology, climate, vegetative cover, precipitation, and run-off, this State offers wide variations. It is doubtful whether any other State east of the Rocky Mountains can show more divergent conditions.

In general, Western New York is flat, a rich agricultural region of low run-off in which the Genesee River is the principal stream. The central part of the State is deeply furrowed by glacial action, the most striking evidence of which is seen in the Finger Lakes. Many small streams of this part have cut deep gorges and are characterized by swift, turbulent flow. The principal rivers (Fig. 11) are the Susquehanna, which flows south through Pennsylvania and Maryland to Chesapeake Bay, and the Oswego, which drains the Finger Lakes and empties into Lake Ontario. Eastern New York has two more or less distinct mountain ranges, the Adirondacks and the Catskills. The highest peak in the State is Mount Marcy, in the Adirondacks, 5 433 ft above sea level. This part of the State is drained principally by the Hudson and Delaware Rivers and by numerous smaller streams flowing into the St. Lawrence River and Lake Champlain.

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[18] Hydr. Engr., U. S. Geological Survey, Albany, N. Y.

With the possible exception of the Hudson, New York State has no large interior rivers, principally because it is so divided by major water-shed lines that the run-off goes in all directions. The Hudson, Delaware, and Susquehanna Rivers, all of which rise in New York, flow directly into the Atlantic Ocean. The Genesee, Oswego, and Black Rivers are tributary to Lake Ontario. The northern and northeastern parts of the State are in the St. Lawrence drainage basin, and a small area in the extreme western part is tributary to the Mississippi through the Allegheny and Ohio Rivers.

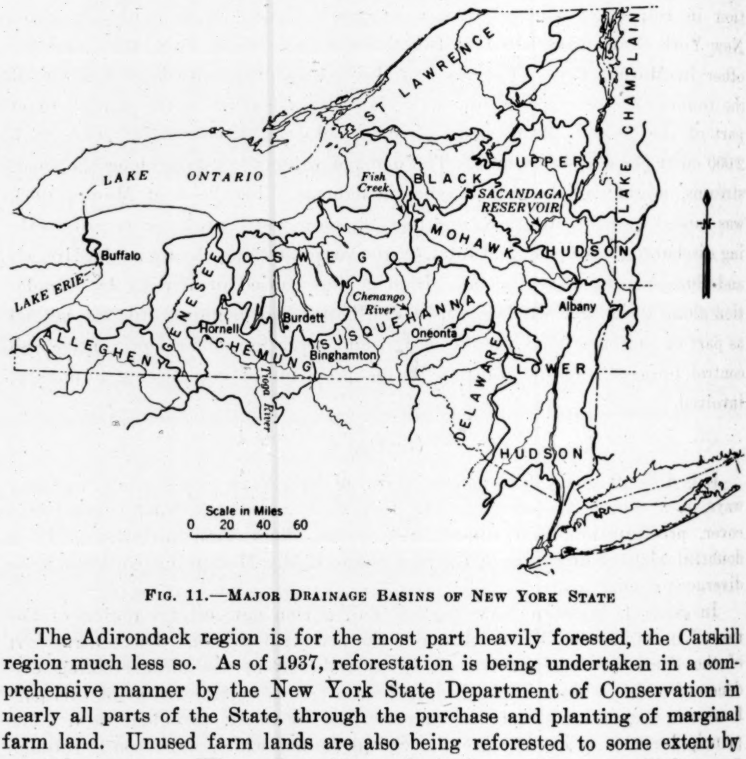
ST LAWRENCE
LAKE CHAMPLAIN
LAKE ONTARIO
Fish Creek
BLACK
UPPER
SACANDAGA RESERVOIR
MOHAWK
HUDSON
Buffalo
LAKE ERIE
OSWEGO
GENESEE
Chenango River
Albany
Burdett
Hornell
SUSQUEHANNA
Oneonta
ALLEGHENY
CHEMUNG
Binghamton
DELAWARE
LOWER
Tioga River
HUDSON
Scale in Miles
0 20 40 60

FIG. 11.—MAJOR DRAINAGE BASINS OF NEW YORK STATE

The Adirondack region is for the most part heavily forested, the Catskill region much less so. As of 1937, reforestation is being undertaken in a comprehensive manner by the New York State Department of Conservation in nearly all parts of the State, through the purchase and planting of marginal farm land. Unused farm lands are also being reforested to some extent by natural processes. What effect this gradual change in cover will have on stream flow is unknown at present. An investigation by the U. S. Geological Survey, in co-operation with the Division of Lands and Forests of the State Department of Conservation, is under way and eventually should yield valuable information on this subject.

It is well known that the areal distribution of precipitation in New York State is variable, but unfortunately only meager information is available for the areas of maximum precipitation, which are wild and inaccessible. The prevailing winds are westerly or southwesterly, and because the Adirondacks act as a barrier, the heaviest precipitation in the State occurs in certain areas on the western slopes of these mountains. Although substantiating records are not available, it seems probable that an annual precipitation of more than 75 in. occurs in some parts of this region.

Variations in run-off are noteworthy and can be illustrated by two examples of extremes. During the eight water-years from October, 1924, to September, 1932, the mean annual run-off from the 75 sq miles of drainage area above the gaging station on the East Branch of Fish Creek, near Constableville, was 48.5 in., whereas, during the same period, the run-off from 2 530 sq miles above the gaging station on the Chemung River, at Chemung, was only 13.4 in. All possible gradations of run-off occur between these extreme conditions.

HYDROLOGIC HISTORY PRIOR TO 1935

At present, the U. S. Weather Bureau maintains about 175 precipitation stations in New York State, co-operative and otherwise. Precipitation is recorded by other agencies at a few additional points. Relatively few of the continuous records extend back farther than about 1889, at which time the New York State Weather Service was created by Act of Legislature. Three of these continuous records, however, date from prior to 1830. The U. S. Geological Survey maintains at present 102 gaging stations in the State, but the average length of record per station, in 1936, was only about 16 yr. Very few records of stream flow were made prior to 1900. Basic data covering periods relatively so short are inadequate for drawing conclusions as to the effect of the works of Man on stream flow, or for determining whether or not the run-off is characterized by significant trends of change through the years.

A study of available precipitation records discloses the fact that, during the period, 1869 to 1934, few 2-day storms, with 6 in. or more of precipitation, occurred in New York State on areas of any considerable extent. In this connection see Fig. 12, which is compiled chiefly from Part V of the Technical Reports of the Miami Conservancy District (1936 revision). The storm of October 8 and 9, 1903, showed a 2-day precipitation in excess of 6 in. at some stations, and, no doubt, other such storms occurred over small areas, but the interesting fact, whatever its significance, is that during this period of 65 yr, for the last 43 yr of which it appears certain that the records were adequate, the State was singularly free from storms carrying heavy precipitation. Except perhaps the Delaware flood of 1903, the history of New York State shows no floods prior to 1935 comparable with the Potomac and Susquehanna floods of 1889, the Ohio Valley flood of 1913, or the Vermont flood of 1927. In spite of storms since 1934, therefore, there seems to be some evidence that New York State, except possibly in its southwestern part, occupies a favored geographic position with respect to the incidence of destructive floods.

THE FLOOD OF JULY, 1935

During the period, July 6 to 9, 1935, heavy thunderstorms accompanied by torrential precipitation occurred in an area in South-Central New York extending roughly eastward from near Hornell, in the northern part of Steuben County, to near Oneonta, in Otsego County. Similar disturbances, generally of a less severe nature, occurred in several other isolated parts of Central and Eastern New York, but the storm's greatest intensity was felt in the area indicated. The fact that the storm centered where it did seems to have had no particular significance, most authorities being agreed that a similar storm might have occurred under the influence of slightly different atmospheric conditions in almost any other part of the State. In most of the area over which the storm centered, all previous 2-day records of rainfall were broken. The maximum official or semi-official 2-day record for this storm was 10.50 in. at Burdett, in Schuyler County, on July 7 and 8, but unofficial records from other points showed precipitation of 12 to 16 in. occurring in as many hours. Unfortunately, the number of reliable precipitation records in the flood area was altogether too small to give an adequate idea of the condi-

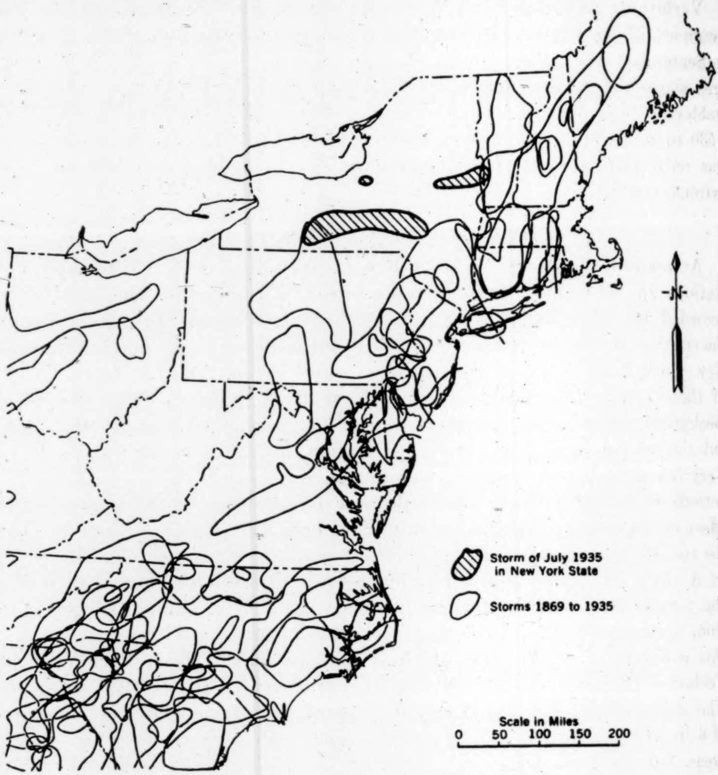

FIG. 12.—LOCATION AND EXTENT OF GREAT 2-DAY STORMS, WITH PRECIPITATION OF 6 INCHES OR MORE

tions, and as the intensity of the storm varied greatly, there is good reason to conclude that greater precipitation occurred at many points than was recorded even unofficially.

One fortunate feature of this storm was that its incidence was such as to cut transversely the main drainage basins of the region in a comparatively narrow zone, with the result that none of the larger streams reached as high a stage as it might have, had the area of the storm been more nearly coincident with its basin. Here, again, there seems to be no reason why the axis of greatest precipitation might not have extended in some other direction than from west to east. Great as the flood damage was, the conditions might conceivably have been such as to produce even more serious results.

The central drainage systems of this region are fed by numerous small streams with generally steep gradients, draining hilly country. Under these conditions and with the extraordinary rainfall, the rate of run-off very quickly approached the intensity of rainfall, and the rise of these streams was very rapid. It was along these small streams that the greatest dynamic damage was done. On the larger streams, losses were caused principally by inundation.

The peak discharges of many small streams with drainage areas of less than 5 sq miles were almost incredibly high (see Fig. 13). There are no gaging stations in the flood area on streams of this size, but subsequent determinations by engineers of the U. S. Geological Survey, using the slope-area method, indicate peak discharges of more than 2 000 cu ft per sec per sq mile. It seems probable that some streams draining 1 sq mile, or less, may have had peak discharges of more than 3 000 cu ft per sec per sq mile. The magnitude of the peak flows was much less in the larger drainage areas, although the Chenango River, near Chenango Forks, draining 1 492 sq miles, broke all existing records with a peak discharge of 82 800 cu ft per sec, or 56 cu ft per sec per sq mile.

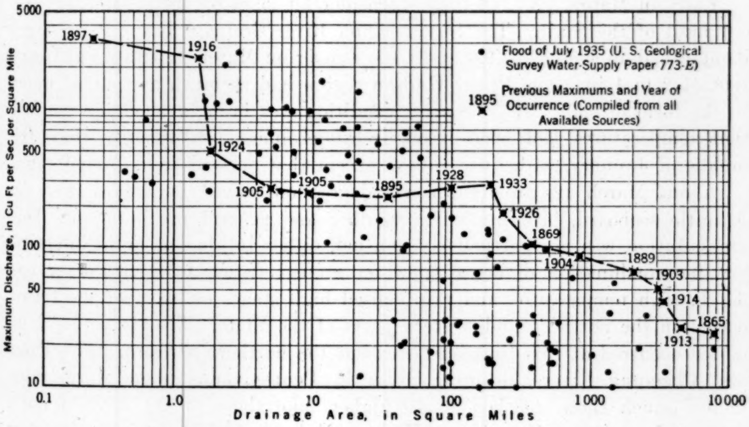

FIG. 13.—MAXIMUM FLOOD DISCHARGES IN NEW YORK STATE.

The outstanding features of this flood, that are of particular interest to engineers, may be summarized as follows:

(1) Large area covered by storm of thunder-storm type.

(2) Unit peak discharges of a magnitude higher than ever before recorded in this part of the United States.

(3) Abnormally heavy and intense precipitation in a hilly country, which caused heavy erosion not only in small stream beds, but even on unbroken hillside surfaces.

(4) Excessive damage in villages situated along small streams, where the encroachment of buildings and other structures restricted the channels and raised the water levels.

(5) Inadequacy of bridge and culvert openings, further complicated by the presence of drift, which resulted in washouts of fills and loss of structures.

(6) Damage by inundation in cities, caused by construction at too low levels, or in unfavorable localities.

A detailed report on the flood of July, 1935, has been published by the U. S. Geological Survey[19].

The Floods of March, 1936

Early in March, 1936, there was considerable snow on the ground throughout most of the State. The Susquehanna River drainage basin above Binghamton had an average of 9.6 in. of snow cover on March 2, with a water content of 2 to 4 in. The Upper Hudson region had as much as 6 in. of water in the form of snow. This general condition was brought about by the fact that temperatures had been below freezing almost continuously since the middle of January, with little run-off except that derived from ground-water.

About March 12, an atmospheric depression moved northward over the Atlantic seaboard, bringing warm weather and general precipitation. This precipitation was not particularly unusual, either in amount or in intensity, but the combination of dense snow on ground frozen to a greater or less depth, high temperature, and rain caused high stages on most of the rivers, except in the northern and western parts of the State. The spring break-up of ice occurred at this time on all except the northern streams. The Tioga and Chemung Rivers, draining areas in Pennsylvania where warm rains melted much snow, reached stages higher than those of the flood of July, 1935. The streams of the Upper Hudson, Mohawk, and Delaware basins were very high. This storm left saturated soils and bank-full streams, and in many sections there was still much saturated snow remaining on the ground.

During the period, March 16 to 19, another atmospheric depression moved slowly northward, bringing from 2 to 3 in. of precipitation. In Central New York this precipitation came as sleet, doing much damage to trees and public utility lines; in the western part of the State it fell as snow. In the southern and eastern areas this rain caused a second flood. Several of the streams in the Mohawk and Upper Hudson drainage basins reached higher stages than during the preceding week and registered new high records at the gaging stations. The high discharges of this second flood were induced largely by the saturated soils, overlying the frost still remaining in the ground, and the relatively high stages still lingering from the first flood.

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[19] *Water Supply Paper 773-E,* U. S. Geological Survey.

The second storm left much snow in the western part of the State; at Buffalo, for example, there was 18 in. on the ground. Warm weather on March 25 and 26, although unaccompanied by rain, caused rapid melting of this snow, with high run-off in the Genesee and other streams. Such stages, although high, were not record-breaking.

It will thus be seen that in the latter part of March, 1936, three separate and distinct floods occurred in New York State. The northern part of the State cannot be considered in the flood area as, in general, no exceptional records were established there. This is also true, although to a less extent, of the western part of the State. The flood stages were most notable in the eastern part of the State and in the Susquehanna and Delaware drainage basins. Moreover, although many new records were established, especially at gaging stations having short-term records, the March floods in New York State were much less outstanding than those occurring in New England, Pennsylvania, Maryland, and other States. The general statement may be made that no New York stream having a drainage area of more than 400 sq miles showed a peak discharge of more than 50 cu ft per sec per sq mile, and few peak discharges of as much as 100 cu ft per sec per sq mile occurred in drainage areas of less than 400 sq miles.

One important and unusual feature of these floods was the occurrence of two storms in close succession, with a resulting long period of high water. High stages existed for more than two weeks, and the total run-off during that time was very large, amounting in some drainage areas to as much as 10 in. over the entire areas. This fact is pertinent in connection with problems of flood control. As the rains extended over a 10-day period, the high stages on the large streams were more notable than on those of smaller drainage areas.

A comprehensive report on the March floods in the northeastern part of the United States is now (March, 1937) in preparation by the U. S. Geological Survey.

OTHER FLOODS

Prior to the floods of March, 1936, the flood of March, 1913, was generally regarded as the outstanding spring flood in New York State. It was caused by heavy rains on saturated, snow-covered ground. The entire rainfall occurred in two or three days. Record high water was reached in the Hudson, Mohawk, Susquehanna, and some Northern New York drainage basins. If the rain of March 11 to 21, 1936, had been concentrated in two or three days, or even if the two storms of March 11 and 12 and March 17 and 18 had coincided, the conditions would have been more like those of the flood of 1913, but because of the greater rainfall the results would have been vastly more disastrous.

In November, 1927, rains of high intensity fell on saturated ground in New England and to a less extent in Eastern New York. Serious floods occurred at that time on a few New York streams, notably those rising in New England.

Flood Control

One notable example of the value of effective flood-control measures was brought out during the floods of March, 1936.

In 1913 the Hudson River, at Albany, reached a peak of about 225 000 cu ft per sec, and caused great damage in that city. Later, the Sacandaga Reservoir, with a total capacity of 866 000 acre-ft, was created through the construction by the Hudson River Regulating District of the Conklingville Dam, near the mouth of the Sacandaga River, the principal tributary of the Upper Hudson. This reservoir was designed to control with the reservoir full a Sacandaga River flood with a peak flow of 50 000 cu ft per sec, as a peak of about 35 800 cu ft per sec had been reached in March, 1913. The contributing area is 1 044 sq miles.

At the time of the first flood in March, 1936, the Sacandaga Reservoir was nearly empty. On March 18, the computed peak inflow reached 64 600 cu ft per sec, but the entire flood flow was controlled without difficulty. As a result the peak discharge of the Hudson River, at Albany, was held to about 165 000 cu ft per sec, and a repetition of the great losses suffered in 1913 was prevented.

The inhabitants of the area affected by the flood of July, 1935, had the unique and altogether unpleasant experience of being visited by two disastrous floods within less than nine months. Under these conditions it is but natural that there should arise an insistent popular demand for flood control, particularly in this area and, to some extent, in other parts of the State. The subject is a live one in New York State as well as in neighboring States, but there is little evidence that the general public appreciates what is needed to bring about effective flood control or even effective flood protection.

It is recognized that some relief can be afforded by cleaning and straightening channels, by removing structures encroaching upon the streams, and by increasing bridge and culvert openings within reasonable limits. These measures, however, are merely of an auxiliary protective nature. The backbone of successful flood-control operations must be storage. Whether this storage is provided in one large reservoir, or in many small reservoirs on head-water streams, the total capacity must be relatively large if floods like those of March, 1936, are to be rendered harmless. The general public seemingly does not understand that large storage reservoirs require large natural sites, which are too often non-existent. Should not the public be better informed as to the limitations imposed by Nature on flood-control projects?

Many streams have flood-plains of greater or less extent, which carry off flood waters without difficulty. Most flood damage is caused by the fact that Man has not only occupied these flood-plains with his structures, but has encroached on the stream beds themselves. Would it not in some areas, particularly in small villages, be cheaper and more desirable to purchase and remove some of these man-made obstructions to flood flows? The opinion is ventured that flood protection involves a problem in engineering economics that may not always have been adequately considered.

The flood of July, 1935, yielded rates of peak discharge to the square mile which before that time had not been considered possible in New York State, and, to this extent, it has provided a new conception of what may be again encountered in any part of the State at any time. It is a puzzling economic problem to determine on what basis designs for flood-control projects should be made. What are the justifiable economic limits?

These floods have emphasized the importance and value of basic data. More and better precipitation stations are needed—stations equipped with automatic recorders, so that rainfall intensities can be studied. Additional gaging stations should also be established, because the best guide for the future is the experience of the past. Long-continued records of precipitation and stream flow at as many strategic points as possible will furnish a basis for better preparation to meet future great floods as they occur. Such floods may be expected, because there is no valid reason why small areas in any part of the State may not experience storm conditions similar to those of July, 1935, or why major parts of the State may not experience conditions such as existed in New England and Pennsylvania during March, 1936.

FLOODS IN THE UPPER OHIO RIVER AND TRIBUTARIES

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MEMBERS, AM. SOC. C. E.

SYNOPSIS

The object of this paper is to describe the outstanding characteristics of the unprecedented flood of March, 1936, in the Upper Ohio River and its tributaries, to point out the causes which combined to bring about the occurrence of this flood, and to outline the history of the present flood-control program for these rivers. Recommendations are made for improving the facilities and technique used in forecasting floods in this region.

FLOOD FACTORS

Topography.—The Allegheny and Monongahela Rivers converge at "the Point", in the City of Pittsburgh, Pa., to form the Ohio River. The headwaters of the Allegheny are in the Allegheny Mountains, in Pennsylvania and New York State, and those of the Monongahela River are in the Allegheny and Appalachian Mountains, in Northwestern Maryland and Northeastern West Virginia. The Allegheny River flows in a southerly direction, and the Monongahela, in a northerly direction, both skirting the western foothills of these ranges.

According to the Pittsburgh Flood Commission Report of 1912,

> "The drainage area of the Allegheny River is 11,580 square miles, and that of the Monongahela River is 7,340 square miles, giving a total of 18,920 square miles. * * *
>
> "The area distribution of the combined basins in percent is as follows:

"New York	10%
Pennsylvania	66%
West Virginia	22%
Maryland	2%
	100%

> "The area lying in Pennsylvania is equal to 27.5 per cent of the total area of the State.
>
> "The basins have a greatest length of 290 miles, an average width of about 65 miles and a least width of 46 miles, across the Allegheny at Kittanning.
>
> "The combined area is equal to 9 per cent of the total area of the Ohio Basin, which is 210 000 square miles. * * *
>
> "The principal Allegheny tributaries on the east, 13 in number, have an average length of 47 miles and an average fall per mile of 22.8 feet. The principal tributaries on the west, 10 in number, have an average length of

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[20] Cons. Engr., Pittsburgh, Pa.

[21] Prof., Dept. of Civ. Eng., Carnegie Inst. of Technology, Pittsburgh, Pa.

37 miles and a fall per mile of 16.7 feet. The Monongahela, on the east has 9 tributaries, with an average length of 45.6 miles and a fall per mile of 31.9 feet; on the west, 7 tributaries, with an average length of 44 miles and a fall per mile of 19.7 feet."

The eastern tributaries of both rivers rise among rough foot-hills at elevations of from 2 300 to 4 800 ft. Those on the west have their source in a high, hilly tableland at elevations of from 1 300 to 2 000 ft.

Although the highest mountains are at the head-waters of the Monongahela, the latter is a mild river even during floods, while in bridge-erection parlance, the Allegheny is a "mad" river. It is, like the Missouri, a treacherous stream; the risk of wash-outs is great during the winter months, and there is no assurance of safety against floods at any time.

It is believed that during the Glacial Period, approximately 75 000 yr ago, the ice flow struck the Allegheny Mountains in Southwestern New York, followed along the Allegheny and Appalachian Mountains, and then followed the Ohio River as far south as Cairo, Ill., depositing enough material in the Eastern Moraine to divert a stream that formerly fed Lake Erie south into what is now the Allegheny. The bluffs of the Upper Allegheny are 700 to 1 000 ft high for many miles, and gradually decrease to a height of 400 to 500 ft at Pittsburgh. The mountains lie within 6 to 10 miles of the stream for many miles at a stretch; their forest cover is gone, the humus is burnt off or washed into the streams, and rocky bluffs are exposed. All these conditions tend to produce rapid run-off.

Rain.—Most of the storms that cause floods in the Upper Ohio basin originate in the Gulf of Mexico. They travel northward up the Mississippi to the vicinity of Cairo, Ill., and thence northeasterly up the Ohio Valley to about Cincinnati, Ohio. From that point they tend to follow one or another of three principal troughs (Fig. 14) which converge at Corry, Pa. Thence, the storms travel in almost an air line to Boston, Mass., and pass into the Atlantic Ocean.

Most of the storms of maximum rainfall travel northeast from Cincinnati. The greatest rain storm of recent years covering a large and continuous area was that of March 23 to 27, 1913, with a rainfall at Dayton, Ohio, of 8.21 in.; Columbus, Ohio, 7.42 in.; Canton, Ohio, 7.39 in.; and Sharon, Pa., 6.56 in. The center of this storm passed 60 miles west of Pittsburgh. On December 14, 1927, a similar storm passed up the Ohio River from Cincinnati to Marietta, Ohio, on the Muskingum River, 297 miles above Cincinnati. Thence, it followed a northerly course through Sharon to Corry, Pa., passing only 40 miles west of Pittsburgh.

The rain storms that bring disastrous floods to Pittsburgh and vicinity are those that continue in an easterly direction from Marietta, swing northeast into the head-waters of the Monongahela River in West Virginia and Northwestern Maryland, and proceed northward along the western foot-hills of the Appalachian and Allegheny Mountains. If the rainfall of March 23 to 27, 1913, or of December 14, 1927, had followed some such path, centering over the Monongahela-Allegheny basin, it would have produced a flood greater than the maximum of record, that of March 18, 1936.

Snow.—In the Allegheny Mountains there is no certainty or regularity in the fall of snow. During the winter of 1907, for example, there was 4 ft of packed snow on the Allegheny Range; in 1909–10 there was 3 ft, which came generally in very light falls; in none of the years from 1930 through 1935 was there more than 12 in. The winter of 1935–36 was unusually cold and severe. The first snow fell early in December, and was followed by frequent heavy falls throughout the season. As late as February 20, 1936, there was 5 ft of light fluffy snow on the range.

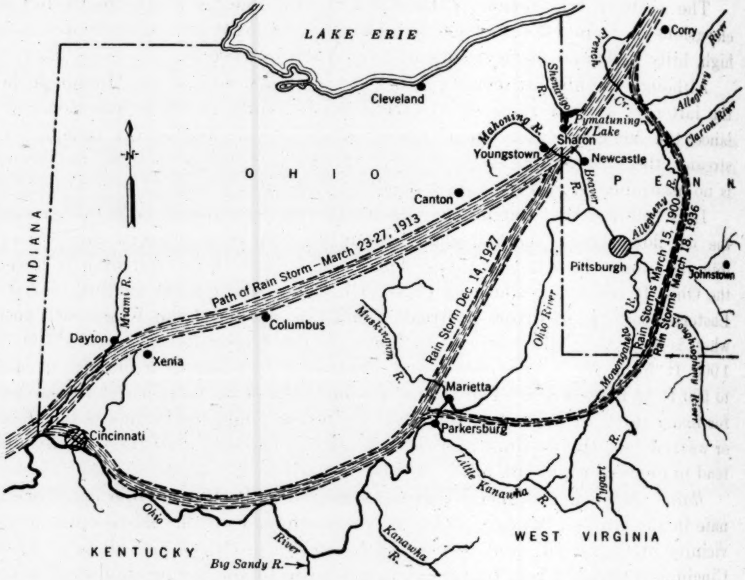
LAKE ERIE
Cleveland
N
OHIO
INDIANA
Miami R.
Dayton
Xenia
Columbus
Canton
Youngstown
Mahoning R.
Shenango R.
French Cr.
Pymatuning Lake
Sharon
Newcastle
Beaver R.
Corry
Allegheny River
Clarion River
PENN
Allegheny R.
Pittsburgh
Johnstown
Path of Rain Storm – March 23-27, 1913
Rain Storm Dec. 14, 1927
Rain Storms March 15, 1900
Rain Storms March 18, 1936
Ohio River
Muskingum R.
Marietta
Parkersburg
Monongahela R.
Youghiogheny River
Tygart R.
Little Kanawha R.
Cincinnati
Ohio River
Big Sandy R.
Kanawha R.
KENTUCKY
WEST VIRGINIA

FIG. 14.—PATHS OF HEAVY RAINFALL IN UPPER OHIO BASIN.

In the last week of February, a thaw occurred, with temperatures of 50° to 59° F in the Upper Allegheny basin and 60° to 66° F in the Monongahela basin. However, the deep snow had drawn the frost out of the ground, and although about 3 ft of snow melted in the low lands, and most of the heavy ice passed out of both rivers, the flood crest of February 28, at Pittsburgh, was only 29.2 ft (flood stage at that point is 25.0 ft).

Again, during the early part of the winter of 1909–10 the bare ground was frozen to an average depth of about 12 in. before the deep snows of 1910 covered the foot-hills and most of the water-shed of the Allegheny River. On the banks of the Clarion River, one of the two largest tributaries of the Allegheny, there was 3 ft of frozen snow. In the early part of March, 1910, the temperature rose, and remained above 40° F continuously for nine days.

Meanwhile, mist and rain fell constantly. A big flood on the Allegheny was predicted. Yet the maximum stage during this thaw and break-up was barely above the flood stage of 25 ft. The blanket of snow had drawn out the frost and had acted as a sponge, holding back the mist, rain, and thawing frozen snow.

Nevertheless, the melting of snow contributes to flood stages, and is always a menace. When there is a blanket of snow, a thaw and a heavy rain storm coming together can produce a flood-stage of more than 30 ft at Pittsburgh. This combination of snow, ice, hard rains, and mild weather was, in fact, the cause of the flood of March 18, 1936, with a stage of 46 ft—the greatest of record at that point.

Ice.—Navigation on the Ohio River as far as Pittsburgh usually continues during the winter months. One of the writers recalls seeing loaded horse-drawn sleds crossing the Ohio on ice from East Liverpool, Ohio, to Newell, W. Va., on December 17, 1904, but such a condition is rare.

One of the severest winters on record was that of 1936. The Allegheny and Monongahela Rivers froze over on December 26, 1935, and by February 21, 1936, there was clear hard ice of a thickness of 20½ in. on the Allegheny and 18½ in. at Lock No. 7, on the Monongahela, 52.6 and 82.3 miles, respectively, from "the Point", in Pittsburgh. It is believed that clear ice froze to a thickness of at least 24 in. in the Upper Allegheny, but no authentic records have been obtained. A similar condition has never been recorded in this section of the United States.

This great body of clear ice with a blanket of deep snow presented a flood menace of unprecedented nature. It was providential that the ice went out of both rivers (except from the head-waters of the Allegheny) on the minor crest of February 28, instead of during the record flood that occurred just nineteen days later.

Forests.—The 1912 report of the Pittsburgh Flood Commission contains descriptions and maps of a large acreage of virgin forest in the Allegheny and Appalachian Mountains. This forest was especially dense in the headwaters of the Allegheny, where there was a fine stand of white pine in eight counties in Northern Pennsylvania and good stands in New York State. In 1937, the only virgin forests left are Federal-and State-owned tracts that include a few parks. The Commonwealth of Pennsylvania has been purchasing forest lands for years, and on October 1, 1934, the State-owned forest land amounted to 1 649 430 acres. In addition, the Game Commission of Pennsylvania has purchased 547 406 acres of forest land for sanctuaries and has turned it over to the possession and protection of the Commonwealth. The Weeks Bill, passed by the Federal Government in 1911, contemplated the purchase of 726 340 acres in Northwestern Pennsylvania to be known as the Allegheny National Forest. By April 26, 1936, forest and surface rights to about 70% of this land had been acquired by the Government.

Notwithstanding the fact that the Federal and State Governments maintain fire towers throughout these areas, and are thoroughly equipped to fight forest fires, the latter remain the greatest cause of forest destruction and the greatest hindrance to reforestation of unproductive farm lands.

Run-Off.—When the flood of March 15, 1907, came it was freely predicted that it would not be equalled for 100 yr. The engineers of the Pittsburgh Flood Commission, however, did not foresee that the day would soon come when disastrous floods would be caused by the carelessness of Man. They did not realize that the virgin forests would disappear so soon, nor that the time would ever come when hundreds of men would be sent into the upper water-sheds to clear and burn great tracts of fallen, dead trees, to locate and construct dirt highways through the mountains, and all but make parks out of the wilds.

To-day, the virgin forests are all but gone. There are thousands of miles of improved highways and scores of mountain parks; and fishermen, hunters, and careless campers are here, there, and everywhere. The lighted cigarette tossed away by the criminally careless smoker has caused a great increase of timber and land fires that have left in their wake destroyed forests and burnt humus—humus that it will take centuries to replace. Hence, the Man-made floods.

THE UNITED STATES WEATHER BUREAU

Every day of the year other than Sundays and holidays the Pittsburgh Office of the U. S. Weather Bureau publishes a weather map issued by the U. S. Department of Agriculture, giving a forecast for Pittsburgh and vicinity, together with observations taken at 8:00 A. M. It is supplemented by a card giving data on river conditions at various points. About 448 maps and from 86 to 100 river bulletins are mailed daily for local guidance.

The force of the local office has been proficient, accommodating, and of untold value to the public, especially to the river interests and navigators. It is with keen regret, therefore, that it must be recorded that on March 16 and 17, 1936, the local office failed utterly to predict the disastrous flood that was sweeping down both the Allegheny and the Monongahela Rivers, until it was too late to cope even partially with the onrushing tide.

A study of the previous maximum flood of record (1907) and that of 1936 reveals that in both cases the U. S. Weather Bureau either was ignorant of the volume of snow in the mountains at the head-waters of the Allegheny and Monongahela Rivers, or did not appreciate that snow is always a flood menace during March.

The following data are abstracted from the Weather Bureau observations and predictions for three days in March, 1907:[22]

March 13, 1907, Observations at 8:00 A. M.:

Maximum temperature in preceding 24 hr, in degrees Fahrenheit..	57
Minimum temperature in preceding 24 hr, in degrees Fahrenheit..	49
Rainfall in preceding 24 hr, in inches..........................	1.46
Stage of rivers at Point, in feet..............................	26

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[22] In 1907, zero stage was at Elevation 696.8 above mean tide.

March 14, 1907, Observations at 8:00 A. M.:

Maximum temperature in preceding 24 hr, in degrees Fahrenheit	58
Minimum temperature in preceding 24 hr, in degrees Fahrenheit	33
Rainfall in preceding 24 hr, in inches	0.68
Prediction made at 3:30 A. M. (of crest stage at Point), in feet	31
Warning issued at 9:00 P. M. (for crest stage), in feet	34

March 15, 1907, Observations at 8:00 A. M.:

Maximum temperature in preceding 24 hr, in degrees Fahrenheit	46
Minimum temperature in preceding 24 hr, in degrees Fahrenheit	31
Rainfall in preceding 24 hr, in inches	0.00
Flood crest, 2:15 A. M. (writers' record), in feet	35.5

Similar data from the observations and predictions for March, 1936,[28] are as follows:

March 16, 1936, Observations at 8:00 A. M.:

Maximum temperature in preceding 24 hr, in degrees Fahrenheit	45
Minimum temperature in preceding 24 hr, in degrees Fahrenheit	37
Rainfall in preceding 24 hr, in inches	0.91
Stage of rivers at Point, in feet	19.8

March 17, 1936, Observations at 8:00 A. M.

Maximum temperature in preceding 24 hr, in degrees Fahrenheit	40
Minimum temperature in preceding 24 hr, in degrees Fahrenheit	32
Rainfall in preceding 24 hr, in inches	1.68
Stage of rivers at Point, in feet	24.7
Prediction, 8:00 A. M., for forenoon of March 18, in feet	32 to 33

March 18, 1936, Observations at 8:00 A. M.:

Maximum temperature in preceding 24 hr, in degrees Fahrenheit	34
Minimum temperature in preceding 24 hr, in degrees Fahrenheit	30
Rainfall in preceding 24 hr, in inches	0.72
Stage of rivers at Point, in feet	42
Prediction (made at 8:00 A. M.) of crest stage, in feet	43.5
Crest reached at 9:00 P. M., in feet	46

Something important was lacking in the Weather Bureau in 1907. It was still lacking in 1936, when the prediction of March 17 was 10 ft in error, and that of the following morning still under-estimated the crest by 2.5 ft. Why these errors?

During the week preceding the flood of March 15, 1907, one of the writers crossed the Tussey Mountains, 16 miles east of Tyrone, Pa., in two places, at elevations of 2 100 and 2 200 ft, respectively. There was about 12 in. of packed snow at the foot of the mountains and 48 in. of light fluffy snow on the summit, equivalent, say, to 4.8 in. of rain. The run-off at that time and place was 70 to 80 per cent.

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[28] In 1936, zero stage was at Elevation 694.0 above mean tide.

Immediately before the thaw of February 24, 1936, which resulted in the flood of February 28, of 29.2 ft, the personal records of one of the writers show that there was 60 in. of light fluffy snow on the crest of the Tussey Mountains. As mentioned previously, it happened that no floods came from the 3 ft of melted snow. However, there was 3 ft of packed snow at Cresson, Pa., at the head-waters of the Conemaugh River in the Allegheny River basin, and about 5 ft on the top of the Allegheny Mountains, which did contribute to the flood of March 18.

The thirty-five local Weather Stations in the Allegheny-Monongahela basin reported the snowfall in their daily reports, but gave only meager data regarding the amount of snow on the ground. This deficiency in the record is in part due to the fact that the majority of the stations are in cities and towns instead of in rural districts. However, the primary reason for the deficiency is that the pay of recorders is rediculously small. The pay per month for reporting the weather, daily, is $10. For rainfall reports the observers are paid at the rate of 50 cts per in. of rain.

The Pittsburgh Office of the Weather Bureau claims that it failed to receive many reports from stations along the Allegheny and Monongahela Rivers on account of broken communications by telephone and wire. It is also stated that those stations are manned by agents having other jobs and that the $10 is so much "extra money." There are twelve such weather stations and six rainfall observation stations in the Allegheny basin. The U. S. Army Engineers report the weather and rain conditions to the Weather Bureau from the locks and dams on the Allegheny and Monongahela Rivers and down the Ohio as far as Wheeling, W. Va., without cost.

The local U. S. Weather Bureau Office erred in its predictions for March 15, 1907, and March 18, 1936, for the one and simple reason that it failed to receive adequate weather and rain reports.

The time has long since come when the Pittsburgh Metropolitan Area should be served by adequate weather stations along the Allegheny and Monongahela Rivers and their main tributaries, each fully equipped with modern appliances and manned by a competent employee on a yearly salary. Properly equipped stations should also be established in the mountains, at 50-mile intervals from the Southern Appalachians to the Northern Alleghenies. The observer at each mountain station should be a young, healthy man, preferably a graduate forester.

All stations, both along the rivers and in the mountains, should have private broadcasting outfits, and the mountain stations should broadcast data at stated periods so the public could also listen in. If the local U. S. Weather Bureau Office had known the actual conditions in the mountains on March 16, 1936, and had had the benefit of frequent, accurate, radio reports, forecasts and warnings could have been issued to the public in ample time to have prevented at least $100 000 000 of the loss that occurred in Pittsburgh and vicinity on March 18. A 24-hr notice of the flood would have saved one single department store, for example, from a direct loss of between $750 000 and $1 000 000.

Three great navigable rivers flow through Pittsburgh, the tonnage on one of them, the Monongahela, being greater than that on any other inland river in the United States.

The railroad tonnage over bills of lading in Pittsburgh is the greatest of any city in the country.

Three Federal trunk-line highways, and many highways of less importance, pass through the community.

The direct recorded loss from the flood of March 18, 1936, in the City of Pittsburgh was equal to the total flood loss in 1936 in any one State except Pennsylvania.

Notwithstanding all these facts, the facilities furnished the Pittsburgh Office of the U. S. Weather Bureau for obtaining intelligent reports from the Allegheny and Monongahela Rivers are pitifully meager. This situation is a disgrace. Pittsburgh's most disastrous flood is yet to come.

If the local Weather Bureau continues to be "starved" financially, and is not equipped by intelligent salaried men for reporting weather and rain conditions throughout the 19 000 sq miles of the contributing water-shed, then the entire Weather Bureau should be taken out of the Department of Agriculture and become a department under the jurisdiction of the Chief of Engineers, U. S. Army.

Pittsburgh Floods

There have been eighty-six floods in Pittsburgh since 1856—somewhat more than one per year. According to estimates compiled by the writers the local flood losses during the three decades prior to 1936 have averaged about $2 000 000 annually. The two floods most disastrous to the Pittsburgh District have already been indicated as those of 1907 and 1936.

Flood of 1907.—At its crest of 35.5 ft,[24] the flood of March 15, 1907, inundated 52% of the "Golden Triangle," the 218-acre central business section. The direct loss to Pittsburgh and its immediate vicinity amounted to $6 000 000, and the property and indirect loss was placed at $50 000 000. The supply of water, heat, and light, and most of the local transportation facilities were maintained, although the steam railroads were paralyzed for three days.

Prior to this flood there was considerable snow on the head-waters of the Allegheny and Monongahela Rivers, but on account of the low temperature its thawing was not the main contributing factor. Between March 12 and March 15, rainfall was general over the entire tributary basin. Precipitation varied from 0.20 in. to 3.25 in. on the Allegheny water-shed and from 1.50 in. to 4.25 in. on the Monongahela water-shed. The crests of the two streams reached the Point 2 hr and 15 min. apart. Had they coincided, it is estimated that they would have produced a flood stage of 40 ft[24] at Pittsburgh.

Flood of 1936.—The flood of 1936 reached its crest during the night of March 17–18, at which time 62.4% of the Golden Triangle was under water. The basements of all the office buildings below Smithfield Street and Upper Grant Street were flooded. All power, elevator, and electric service was shut down. No one was allowed to pass the Federal guard lines into the "Golden Triangle" without a permit from the Director of Public Safety. The business of the city was paralyzed for ten days. Fortunately, so thorough were the regulations of the State Board of Health and the City Department of Health that no epidemic followed the flood in the city and vicinity, and no appreciable increase occurred in the normal death rate.

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[24] Based on zero stage at Elevation 696.8 above mean tide.

No precise estimate is available covering both the direct and indirect damage to the Pittsburgh District during this flood. A preliminary estimate by the Pittsburgh Chamber of Commerce placed the damage at about $200 000 000. No estimates have been made of the losses that occurred between the sources of the Allegheny and Monongahela Rivers and the Point, although hardly a town escaped great damage. The disaster also continued down the Ohio River 968 miles to Cairo Ill.

It is impossible to describe the distress suffered on that memorable day of March 18, 1936, especially among the poorer classes, who lost their meager all along a river-bank frontal of more than 1 000 miles. One illustration, out of thousands that could be cited, must suffice. Sharpsburg, Pa., is a little suburb of Pittsburgh, on the right bank of the Allegheny, 6.7 miles above the Point. When the flood waters had receded sufficiently to start shoveling out the mud from hundreds of homes of the middle and poorer classes so many pianos and pieces of furniture and other household goods were dumped on the sidewalks and streets to be hauled away and destroyed that transportation on the highways was blocked.

As to the causes of the flood: The severe winter and heavy snowfall of 1935–36 have already been mentioned. After the minor flood of February 28, the rivers dropped. From March 1 to March 15 mild weather prevailed; the average temperature was 39° and the total rainfall during the period was 1.28 in. The average river stage was 19 ft. All signs pointed to a mild break-up of winter, with the thermometer reaching 61° on March 15 and the river gage at the Point recording 20.8 ft.

General rains began the next day, however. Precipitation on March 17 and 18 varied from 1.33 in. to 3.76 in. on the Allegheny water-shed and from 2.17 in. to 3.79 in. on the Monongahela. The crest of the Monongahela flood reached the Point at 6:00 P. M. of March 17, and that of the Allegheny arrived three hours later, the Monongahela meanwhile remaining stationary.

The Allegheny River flows faster than the Monongahela, and the fact that the crest of the latter generally reaches the Point first can be accounted for by the shorter length of the stream and by the fact that storm centers usually pass over the head-waters of the Monongahela several hours before reaching the drainage basins of important feeders of the Allegheny.

It is a striking coincidence that the Kiskiminetas River, emptying into the Allegheny River 40 miles north of Pittsburgh, and the Youghiogheny River, joining the Monongahela River 15 miles south of that city, should have supplied the major part of the flow at the crest of both the 1907 and 1936 floods. The rapid travel of the flood crest is especially pronounced in the Youghiogheny, which is a rapidly flowing stream from source to mouth.

The Possibility of Greater Floods

The disastrous flood of 1936, has opened deaf ears, and has quickened the conception of the importance of adequate flood control. The public has come to realize that actual damages were only a fraction of the loss that might have occurred. It may seem absurd to suggest that Pittsburgh is a lucky city, but it is a fact.

Many of the conditions that tend to produce a disastrous flood are present in the drainage basin of the Allegheny and Monongahela Rivers. Among these conditions may be mentioned: (1) Steep summit approaches of the Allegheny and Appalachian Mountains; (2) high percentage of run-off; (3) rapid fall of stream beds; (4) ground frozen 1 ft to 3 ft before deep snow falls; (5) severe winters; (6) deep snows every four to six winters; (7) thick ice every five to eight winters; (8) warm spells either the latter part of February or early in March; and (9) hard rains coming about March 15 to 20.

The combined incidence of these conditions was illustrated in the flood of 1936, but not to the fullest possible extent. For example, according to estimates by the writers:

1.—If the rain of March 17–18, 1936, had not turned into snow above Oil City, Pa., the flood crest stage at the Point on March 18 would have been at least 48 ft.

2.—If the great rainfall of March 23 to 27, 1913, had centered over the drainage basin of the Monongahela and Allegheny Rivers instead of about 60 miles west of the basin, the flood height at the Point would have reached 48 ft.

3.—On December 14, 1927, there was a flood of 30.2 ft at the Point. If the rain storm on that date had passed over the drainage area of the Monongahela and Allegheny Rivers, instead of 40 miles west of the Point, the crest would have reached 45 ft.

4.—In 1936, if the thick ice that came out in the break-up of February 16 and 17 had combined with the floods of February 28 and March 18, the crest would have reached 54 ft.

It is of interest to examine one such possibility in detail. If the floods of February 28 and March 18 alone had combined, there would have been a flood of 54 ft at the Point, with enough ice left in the head-waters to form heavy gorges. The ice in the Allegheny River might first have swept the Highland Park Bridge off its foundations, and then the Sharpsburg Bridge. Next, it might have gorged on Herrs Island, and after doing great damage there, might have broken and moved down through the 16th Street and Pennsylvania Railroad Bridges, only to lift the 9th Street Suspension Bridge off its piers, causing it to turn over and hold the ice gorge like an umbrella in a wind storm.

Before the anchorages of the bridge were stressed to the breaking point, the ice might have been packed solid to the bed of the river and piled up from 15 to 20 ft above the flood height at that time. It would soon have extended to the concrete wall of the elevated section of the Pennsylvania Railroad on the North Side, and by that time the flood in the Golden Triangle would have reached the appalling height of 55 to 60 ft, and the Allegheny River would have been discharging its flood water through the center of the city.

What damage the thick clear ice, rushing through the main streets, would have done to the old buildings and even to the modern sky-scrapers is difficult to state. Its impact against the walls at a velocity of from 8 to 12 ft per sec would certainly have wrought untold damage and destruction. It is certain that most of the old buildings would have quickly collapsed, and this collapse, in turn, would have menaced the stability of the modern buildings. The loss of life and destruction of property would have been appalling. This is no fantastic imagination, but a conclusion which is the product of many years of technical experience in the field. Pittsburgh, the "Lucky City," may some day have such a combination of floods. The following tabulation shows the estimated maximum discharge at Pittsburgh that would have resulted from a combination of the floods of February 28 and March 18, 1936. It is based on gage readings taken March 18, on the tributaries of the Allegheny and Monongahela Rivers:

Estimated Maximum Discharge at Pittsburgh, from the:	Cubic Feet per Second
Allegheny River	473 000
Monongahela River	322 000
Total at Pittsburgh	795 000

This discharge represents a gage height of 54 ft at the Point, according to the extended revised rating curve for Pittsburgh of June, 1936. It is equivalent to a run-off rate of 41.5 cu ft per sec per sq mile, which is 25% in excess of the tabular value published by the Commonwealth of Pennsylvania for a drainage area equal to that of Pittsburgh. The actual run-off rate on March 18, at Pittsburgh, was 97% of the tabular value. The run-off rate on the Susquehanna River, at Harrisburg, Pa., during the same flood, was 21% in excess of the tabular value.

PROPOSED PLANS FOR CONTROLLING FLOODS

Although it is not the intention of the writers to enter upon an elaborate description and analysis of the various groups of retarding reservoirs that have been proposed for flood protection on the Allegheny-Monongahela River System, a brief discussion of the nature and action of these reservoirs is pertinent.

The history of floods in the Pittsburgh District begins in 1754, when George Washington, then a young man, nearly lost his life while crossing the flooded Allegheny River on an improvised raft. The site of this occurrence is now marked by the "Washington Crossing Bridge" at 40th Street, in Pittsburgh. Since then, the problem of controlling floods on the Allegheny and Monongahela Rivers has been discussed on many occasions, but it was not until early in the Twentieth Century that the population and wealth in these river valleys increased sufficiently to justify serious consideration of a comprehensive program of flood control by reservoirs. The flood of March, 1907, rising to a stage of 38.5 ft (based on present pool level) at Pittsburgh, furnished the impetus for the creation of the Flood Commission of Pittsburgh.

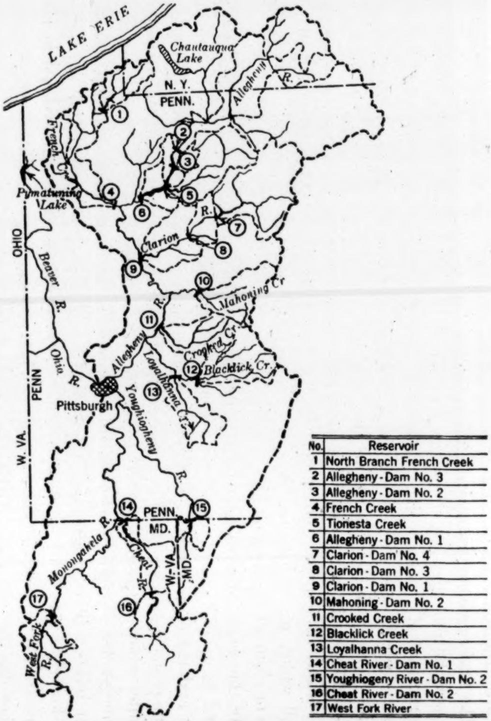
This was essentially a citizen's committee, organized under the auspices of the Chamber of Commerce. The Engineering Committee of the Flood Commission, at the time of the publication of its voluminous report of 1912, was as follows: E. K. Morse, M. Am. Soc. C. E., *Chairman;* the late Emil Swensson, M. Am. Soc. C. E.; the late William Glyde Wilkins, M. Am. Soc. C. E.; George S. Davison, Past-President, Am. Soc. C. E.; the late Paul Didier, M. Am. Soc. C. E.; the late Julian Kennedy; the late Morris Knowles, M. Am. Soc. C. E.; and George M. Lehman, M. Am. Soc. C. E., Engineer in Charge. The Principal Assistant Engineer was Kenneth C. Grant, M. Am. Soc. C. E.

Funds for the work of the Flood Commission were obtained by asking the owners of property affected by floods to contribute at the rate of 1 mill per dollar of assessed valuation. Substantial contributions were made by the City of Pittsburgh and the County of Allegheny. A total of approximately $124 000 was expended from 1908 to 1911 in carrying on extensive surveys and investigations, and a publicity campaign.

The first engineering investigations of the Commission were predicated on the assumption that flood protection would be secured primarily by the construction of river walls. Thorough surveys were made of the river channels within the city limits of Pittsburgh, and hydraulic and economic studies were made to determine the feasibility and cost of such walls, and to establish their most desirable location. It developed from these studies that serious difficulties and uncertainties would be encountered in designing a wall which would provide flood protection without the assistance of reservoirs or channel improvement.

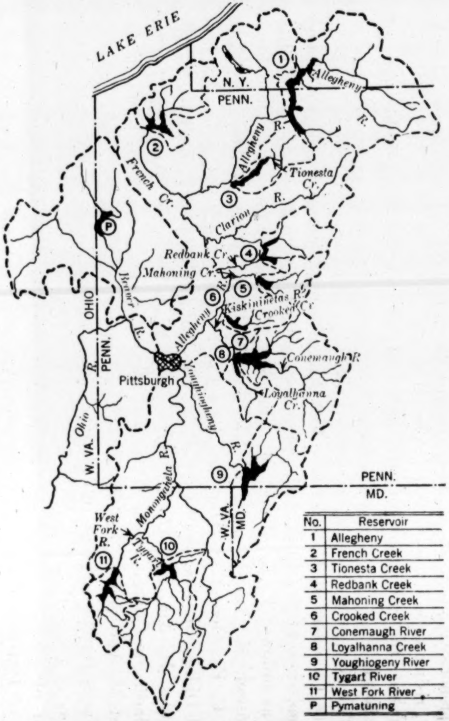
As the work progressed, the possibility and advantage of securing flood control by reservoirs on the head-waters and tributaries of the rivers became more and more apparent, and later investigations included extensive surveys of numerous reservoir sites scattered over the drainage area of about 19 000 sq miles above Pittsburgh. The selection of these sites was based on a reconnaissance including every tributary with a drainage area of more than 50 sq miles. The studies included detailed analyses of costs, and of the flood-lowering effect of various groups of reservoirs. They also developed the possibility of using part of the reservoir capacity to regulate the dry-weather flow of the rivers in the interest of navigation, sanitation, hardness control, and acidity control.

The ingenuity and thoroughness of the hydraulic studies can be admired even after the lapse of a quarter century that has seen great advances in the technique of flood control by reservoirs.


LAKE ERIE
Chautauqua Lake
N. Y.
PENN.
Allegheny R.
French Cr.
Pymatuning Lake
OHIO
Clarion R.
Beaver R.
Mahoning Cr
Allegheny R.
Crooked Cr.
Blacklick Cr.
Ohio R.
Loyalhanna Cr.
PENN
Pittsburgh
Youghiogheny R.
W. VA.
PENN.
MD.
Monongahela R.
Cheat R.
W-VA.
MD.
West Fork R.
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

No.	Reservoir
1	North Branch French Creek
2	Allegheny - Dam No. 3
3	Allegheny - Dam No. 2
4	French Creek
5	Tionesta Creek
6	Allegheny - Dam No. 1
7	Clarion - Dam No. 4
8	Clarion - Dam No. 3
9	Clarion - Dam No. 1
10	Mahoning - Dam No. 2
11	Crooked Creek
12	Blacklick Creek
13	Loyalhanna Creek
14	Cheat River - Dam No. 1
15	Youghiogeny River - Dam No. 2
16	Cheat River - Dam No. 2
17	West Fork River

FIG. 15.—SEVENTEEN-RESERVOIR FLOOD-CONTROL PROJECT, PROPOSED BY FLOOD COMMISSION OF PITTSBURGH, PA., IN 1912.


LAKE ERIE
N. Y.
PENN.
Allegheny R.
Allegheny R.
Tionesta Cr.
French Cr.
Clarion R.
Redbank Cr.
Mahoning Cr.
Kiskiminetas R.
Crooked Cr
Allegheny R.
Beaver R.
OHIO
PENN.
Conemaugh R
Pittsburgh
Loyalhanna Cr.
Youghiogheny R.
Ohio R.
W. VA.
Monongahela R.
PENN.
MD.
W. VA.
MD.
West Fork R.
Tygart R.
1
2
3
4
5
6
7
8
9
10
11
P

No.	Reservoir
1	Allegheny
2	French Creek
3	Tionesta Creek
4	Redbank Creek
5	Mahoning Creek
6	Crooked Creek
7	Conemaugh River
8	Loyalhanna Creek
9	Youghiogeny River
10	Tygart River
11	West Fork River
P	Pymatuning

FIG. 16.—ELEVEN-RESERVOIR SYSTEM PROPOSED BY CORPS OF ENGINEERS, U. S. ARMY, IN 1928

The Flood Commission secured the establishment of numerous stream-gaging stations and rainfall gages. It collected a great mass of information on matters directly and indirectly related to flood control, such as stream flow, rainfall, topography and geology, forestation and agricultural development, location of coal, gas, and oil fields, railroads and other transportation lines, flood damages, and industrial developments. This information, together with the engineering studies and conclusions of the Commission, was set forth in the report of 1912. The final recommendations of the Commission proposed that seventeen reservoirs be constructed on the head-waters or tributaries of the Allegheny and Monongahela Rivers, for the primary flood protection of all communities and industries on the banks of these rivers. The location of these reservoirs is shown in Fig. 15. Flood walls were also recommended as a secondary line of defense.

In order to convey some idea of the merits of the seventeen-reservoir system of the Flood Commission, in comparison with reservoir systems subsequently proposed, it is desirable to review the general topographical and industrial conditions governing the selection of reservoir sites in Western Pennsylvania.

Although the sources of the Allegheny and Monongahela Rivers and many of their tributaries are in the high mountains that border the eastern edge of the combined drainage basin, the major reaches of the main rivers and their larger tributaries are located in the Allegheny Plateau. The even and horizontal skyline of this plateau shows it to be a plain which has been elevated in comparatively recent geologic times. Since this elevation, the rivers and streams of the region have carved a fully developed drainage system of deep and narrow valleys. In the Pittsburgh region, where the characteristic topography is clearly developed the main valleys have a depth of 400 to 500 ft below the hill tops.

The rocks composing the plateau consist, in general, of the soft sandstones, shales, and coal seams of the Pennsylvanian period, and are bedded nearly horizontally, with a gentle dip to the southwest. This dip causes the main river channels to lie to the west of the median line of their drainage basin, so that the tributaries entering from the west are shorter and less numerous than those entering from the east. In general, the rock structure of the Allegheny Plateau is remarkable for the absence of hard and erosion-resistant strata. The lack of such strata causes the gradients of the larger streams to be uniform over long distances, to the joy of the railroad engineer and the despair of the water-power engineer. The absence of waterfalls, canyons, or gorges on the main rivers of such a rugged region is unusual and striking. Except in its extreme northwestern part, the region is not glaciated, and, in general, rock strata are exposed in the river beds or have only a shallow cover. (The latter statement does not apply to the glaciated region or to of glacial gravel.) Because of these topographic characteristics, one might the main channel of the Allegheny River, which contains an outwash train say that on a typical river or stream of this region every cross-section is a potential dam site. In so far as natural conditions are concerned, no site has outstanding advantages over numerous other sites on the same stream. In this district the engineer cannot hope to find a canyon dam site lying below a broad flat valley. If topography were the only factor to be considered, it would be correct to state that storage in this region is very easy to obtain, but expensive per acre-foot.

The great obstacle to the construction of retarding reservoirs on the rivers of Western Pennsylvania is furnished not by Nature but by the works of Man; namely, railroads and industrial developments. Several great trunk railroad lines and many branch railroads follow the easy and uniform grades of the river valleys. In the lower and more densely settled parts of the main valleys, it is almost impossible to displace or relocate the railroads or to build a dam of any height without interfering with important industries and communities. Farther up stream there are sites where a branch railroad can be shifted to an adjacent valley, or where a main railroad can be elevated above a proposed reservoir flow line, although at enormous expense. These considerations are of paramount importance in the selection of reservoir sites.

TABLE 8.—Data on Reservoirs in Seventeen-Reservoir Project Proposed by the Flood Commission of Pittsburgh in 1912

Reservoirs*	Controlled drainage area, in square miles	Storage Capacity			Percentage of main basin controlled
		In acre-feet		In inches of run-off	
Allegheny Basin:					
Loyalhanna	277	94 500		6.4	2.4
Black Lick	414	33 400		1.51	3.6
Crooked	287	75 000		4.9	2.4
Mahoning No. 2	335	54 500		3.05	2.9
			257 400	....	....
Clarion No. 1	1 212	116 000		4.06	10.5
Clarion No. 3		112 000			
Clarion No. 4		35 300			
			263 300	....	....
French (exclusive of North Branch)	1 009	76 500		1.42	8.6
North Branch of French	217	49 000		4.24	1.9
Tionesta	477	83 500		3.29	4.1
			209 000	....	....
Allegheny No. 1	3 795	66 000		1.18	32.8
Allegheny No. 2		112 000			
Allegheny No. 3		61 000			
			239 000	....	....
Total, Allegheny Basin	8 023		969 000	2.26	69.2
Monongahela Basin:					
Youghiogeny No. 2	394	35 600		1.69	5.4
			35 600	....	....
Cheat No. 1	1 399	129 000		3.97	19.1
Cheat No. 2		168 000			
			297 000	....	....
West Fork	366	63 500		3.25	5.0
			63 500	....	....
Total, Monongahela Basin	2 159		396 100	3.42	29.5
Total, both basins	10 182		1 365 000	2.53	53.8

* For location, see Fig. 15.

Of the seventeen reservoirs recommended by the Flood Commission of Pittsburgh, thirteen were located in the Allegheny basin and four in the Monongahela basin. A summary of data regarding these reservoirs is given in Table 8.

Among the excellent features of this system of reservoirs may be mentioned the large percentage of controlled area, and the good distribution of sites throughout the basin. The degree of protection afforded by this system agrees well with the economic limit of protection determined by more recent studies. The design and location of the dams made them well adapted to the storage of April, May, and June run-off to increase the minimum stream flow during the dry months of the summer and fall. The wisdom of not attempting to develop hydro-electric power at the flood-control reservoirs has been confirmed by subsequent studies.

The occasion for subsequent revision of the original seventeen-reservoir plan arose primarily for two reasons:

(1) Among the largest and most effective reservoirs of the system were those on the Clarion and Cheat Rivers. The former river is the second largest tributary of the Allegheny and the latter is the second largest tributary of the Monongahela. After 1912 the Associated Gas and Electric Company constructed the Piney Hydro-Electric Plant on the Clarion River, and the West Penn Power Company constructed the Lake Lynn Hydro-Electric Plant on the Cheat River. These Companies, it is said, also own property rights in many or in all undeveloped reservoir sites on the respective rivers and have plans for the future power development of these rivers by chains of reservoirs extending from source to mouth. Among the flood-retarding reservoirs recommended by the United States Engineers since 1928 there are none on the Clarion and Cheat Rivers. This indicates a conclusion that existing and proposed hydro-electric developments have eliminated the possibility of constructing flood-retarding reservoirs on these streams. The writers do not concur in this conclusion. They consider that flood protection for the great industrial district of Western Pennsylvania is of such paramount importance as to justify, if necessary, the use of condemnation proceedings to acquire flood-control-reservoir sites on the Clarion and Cheat Rivers. Such proceedings although conducted in the public interest, would give full consideration to the legal rights of the Power Companies. The reservoirs would be among the most valuable and effective ones of the entire system. For some years, the Pennsylvania Water and Power Resources Board has refused to grant permits for further power developments on the Clarion River, because of the fact that public welfare might require the development of one or more sites on this river for flood-control purposes. For these reasons, the writers believe that no final program for flood protection in the Allegheny-Monongahela Valley should be adopted until additional consideration has been given to the development of flood storage-reservoir sites on the Cheat and Clarion Rivers.

(2) The Flood Commission's original plan contained several reservoirs having a storage capacity equivalent to less than 2 in. of run-off from the

controlled water-shed. This capacity is only about one-third or one-fourth of that required to absorb the entire run-off of the water-shed during a great flood. Advances in the art of flood-control engineering have made it evident that the storage of a retarding reservoir can be designated as of "superior" nature only if the reservoir is capable of absorbing completely, or almost completely, any flood that may originate above it. Storage of less amount than this must be regarded as of 'inferior" nature, because of the possibility that the reservoir may be operated so as to fill during the early period of a great flood and before the arrival of the crest. For such a reservoir, it is not possible to set up an operating schedule that will avoid this difficulty in a satisfactory manner. If the reservoir is operated for safe control of the largest floods, it will be very ineffective in smaller floods, and *vice versa.* In the light of more recently acquired knowledge it appears best, therefore, to modify the Flood Commission's original plan so as to substitute storage of a superior type for that of an inferior type.

Although the work of the Flood Commission was supported by influential and public-spirited citizens, many years were spent in an unsuccessful attempt to arouse the public to take the necessary legislative and financial steps to bring about the actual construction of the reservoirs. If another disastrous flood had occurred within a few years after the publication of the 1912 report, it is probable that the reservoirs would have been built. However, the great storm of March, 1913, which caused such terrible damage on the lower tributaries of the Ohio River, did not produce excessively high stages on the Allegheny and Monongahela Rivers, and for the following quarter century, no flood of extraordinary nature occurred on these streams.

In 1927, that portion of the work of the Flood Commission dealing with protection by river walls was reviewed and amplified by the City of Pittsburgh. The Department of Public Works prepared detailed plans for a river wall extending along the front of the Golden Triangle and along a considerable length of the banks of the Ohio River within the city limits. These plans proposed the deepening of the rivers by dredging and also provided for terminal wharves, boulevards, and other civic improvements associated with the river walls. Accurate hydraulic studies were made to determine how flood heights would be affected by the proposed improvements. Before the completion of the report on these studies, the work was brought to a close by conditions arising within the City Administration.

Within the past decade the engineering supervision of flood-protection studies in the Pittsburgh region, as in other parts of the United States, has passed from private engineers to the engineers of the United States Army. Moreover, the old idea that each community or group of communities should pay in full for its own flood protection is being cast aside by the public in favor of Federal financing. These considerations have terminated the technical labors of the Pittsburgh Flood Commission.

Through the efforts of the Flood Commission, an appropriation of $50 000, one-half from the State of Pennsylvania and one-half from the Federal

Government, was obtained in 1924 to re-study the flood-protection problem and to bring up to date all previous studies. This work was delegated to the United States Engineers, who, after making extensive surveys, designs, and investigations, submitted their findings in a report dated 1928. In March, 1937, this report was still unpublished, but it is said to have furnished the physical basis for the several subsequent reports by the United States Engineers on the same subject. It is the understanding of the writers that none of these subsequent reports is based on surveys and studies that can compare in elaborateness and completeness with those of the report of 1928.

The report of 1928 recommended the construction of eleven reservoirs: Eight in the Allegheny basin and three in the Monongahela basin. The location of these reservoirs is shown in Fig. 16, and pertinent data are given in Table 9.

It is to be noted that the total storage of this eleven-reservoir system is almost twice that of the original seventeen-reservoir system of the Flood Commission. The individual dams are much higher, and their storage is definitely of the "superior" type. In the 1928 report it is stated that this eleven-reservoir system would reduce future flood damage to a negligible amount on the Allegheny and Monongahela Rivers and on a limited reach of the Upper Ohio River. The estimated total cost was $96 378 000. The proposed dams would be equipped with manually controlled gates to permit the conservation of the spring run-off. No hydro-electric power development is contemplated at the reservoirs.

TABLE 9.—DATA ON RESERVOIRS IN ELEVEN-RESERVOIR SYSTEM PROPOSED BY THE CORPS OF ENGINEERS, UNITED STATES ARMY, IN 1928

Reservoirs*	Controlled drainage area, in square miles	STORAGE CAPACITY		Percentage of main basin controlled
		In acre-feet	In inches of run-off	
Allegheny Basin:				
French Creek	541	140 000	4.8	4.66
Allegheny	2 190	930 000	7.96	18.9
Tionesta	483	133 000	5.17	4.16
Red Bank	460	143 000	5.83	3.96
Mahoning	327	79 000	4.53	2.82
Crooked	278	95 000	6.4	2.4
Conemaugh	1 427	283 000	3.72	12.3
Loyalhanna	267	79 000	5.55	2.3
Total, Allegheny Basin	5 973	1 882 000	5.93	51.4
Monongahela Basin:				
Youghiogheny	1 020	340 000	6.2	13.9
Tygart	1 220	327 500	5.04	16.6
West Fork	366	62 500	3.2	5.0
Total, Monongahela Basin	2 606	730 000	5.27	35.5
Total, both basins	8 579	2 612 000	5.72	45.3

In studying this system, the following comparison between certain features of the Allegheny and Monongahela Rivers, will be of interest:

Comparison of Head-Waters.—

Allegheny River.—There is a favorable site for a large reservoir on the head-waters of the main river (Allegheny Reservoir, 10 miles above Warren, Pa.). It contains no important towns or industries, and a single-track railroad running through it can be diverted to another valley.

Monongahela River.—There is a favorable site for a large reservoir on the head-waters of the main river (Tygart Reservoir, 3 miles above Grafton, W. Va.). It contains no important towns or industries, and a single-track railroad running through it can be diverted to another valley.

Comparison of Largest Tributaries.—

Allegheny River.—The Kiskiminetas River, its largest tributary, enters a short distance above Pittsburgh, Pa., and has an extremely important effect on floods. The valley is completely occupied by a main railroad line with heavy traffic (Pennsylvania Railroad Freight Line). A favorable site for a large reservoir is 1 mile above Saltsburg, Pa. A railroad relocation above the proposed reservoir level is physically feasible, but would be enormously expensive.

Monongahela River.—The Youghiogheny River, its largest tributary, enters a short distance above Pittsburgh and has an extremely important effect on floods. The valley is completely occupied by main railroad lines with heavy traffic (Baltimore and Ohio and Western Maryland). A favorable site for a large reservoir is 2 miles below Confluence, Pa. A railroad relocation above the proposed reservoir level is physically feasible, but would be enormously expensive.

Next Largest Tributary.—

Allegheny River.—The Clarion River, the second largest tributary, enters at a moderate distance above Pittsburgh. It has been considered unavailable for flood control because of an existing large hydro-electric plant (Piney), with other reservoir sites controlled by a power company. The writers do not concur in this conclusion. A flood-storage reservoir on this river would be of immense value in a flood-control system.

Monongahela River.—The Cheat River, the second largest tributary, enters at a moderate distance above Pittsburgh. It has been considered unavailable for flood control because of an existing hydro-electric plant (Lake Lynn), with other reservoir sites controlled by a power company. The writers do not concur in this conclusion. A flood-storage reservoir on this river would be of immense value in a flood-control system.

Smaller Tributaries.—

Allegheny River.—Favorable reservoir sites on smaller tributaries (French Creek, Tionesta Creek, Red Bank Creek, Mahoning Creek, Crooked Creek, and Loyalhanna Creek), but the total storage of these reservoirs is much less than is desirable for the general plan.

Monongahela River.—A favorable reservoir site exists on one smaller tributary (West Fork), but its total storage is much less than is desirable for the general plan.

In 1929 and 1930, the Consulting Engineers of the Flood Commission of Pittsburgh, Ross M. Riegel and H. A. Thomas, Members, Am. Soc. C. E., reviewed the 1928 report of the United States Engineers, and commended the excellence of the engineering surveys, designs, and cost estimates described therein. They called attention, however, to the fact that a reservoir system similar to that described in the report, but of reduced capacity, could be built at a cost of about $58 000 000 (that is, at about 60% of the cost of the eleven-reservoir system), and would be capable of eliminating 87% of the flood damage. They concluded that the expenditure of an additional $38 000 000 to eliminate the last 13% of flood damage would be an uneconomic investment. The concurrence of the United States Engineers in this general conclusion is indicated by the selections made in 1936 of reservoirs for construction under Federal appropriations.

Messrs. Riegel and Thomas showed that the uneconomically high storage capacity and cost of the eleven-reservoir system can best be reduced by eliminating certain of the reservoirs, or by constructing smaller reservoirs at points farther up stream. In general, it is not desirable to decrease the heights of the dams, since this would change the nature of the storage from superior to inferior. It is generally agreed that the first step toward cheapening the system should be the elimination of the proposed large reservoir on the Youghiogheny. Although this reservoir would be of great value for the protection of the Pittsburgh District, its cost is saddled with an enormously expensive railroad relocation.

The reservoir on the Tygart River is now (1937) under construction, and it appears certain that some of the other reservoirs will be built in the near future. The decision as to which are to be started first is yet to be made. The Flood Control Act of 1936 provides that the Federal Government shall carry the construction costs of the authorized projects, while local interests must provide without cost to the United States all lands, easements, and rights of way (within certain limits) necessary for the construction of the project. Legislation providing for the payment of that portion of the cost assessed to the local interests has been enacted by the Legislature of Pennsylvania.

PYMATUNING RESERVOIR

Experience with the recently completed Pymatuning Dam demonstrates the effectiveness of the reservoir method of flood control under conditions encountered on the rivers of Western Pennsylvania. This dam was constructed by the Commonwealth of Pennsylvania to provide an adequate supply of water for manufacturing and domestic use during periods of low discharge on the Shenango River. It is located 86 miles north of Pittsburgh and controls a drainage area of 160 sq miles. The capacity of the reservoir is 8 400 000 000 cu ft and its total cost was $3 750 000.

The first year's operation of the Pymatuning Dam has proved its effectiveness in improving dry-weather flow and in controlling floods. The industrial city of Sharon, Pa., is located 25 miles below the dam, at a point where the drainage area is 640 sq miles. On July 1, 1935, the discharge of the Shenango River at Sharon dropped to 9 cu ft per sec. Although the Pymatuning

Reservoir was only partly filled at this time, the gates were opened and sufficient flow was released to supply the water needed for industrial and domestic purposes in Sharon, Newcastle, and other manufacturing communities. This action prevented the complete shut-down of many important industries from July 1 to December 1, 1935.

Since its completion the Pymatuning Dam has prevented two floods. Shortly prior to January 26, 1937, the level of Pymatuning Lake was 2 ft below the spillway. About that time the occurrence of warm weather and rainfall produced floods on all the rivers of the region, the stage at Pittsburgh, for example, being 34.2 ft on January 26. On this day the level of Pymatuning Lake rose to 0.86 ft over the spillway. Because of this storage of peak run-off, the maximum flood stage at Sharon on January 26 was only 0.36 ft.

When the great storm preceeding March 18, 1936, swept over the Middle States and New England, the Pymatuning Reservoir was not full. The gates were closed and impounded the flow so that no flood damage occurred in the Shenango Valley, and no industries were shut down on account of high water. This was the only river valley in Pennsylvania which did not suffer damage during this flood period. Even at Newcastle, 21 miles below Sharon, where the drainage area is 804 sq miles, there were no serious flood losses.

The owners of important industries in the Shenango Valley claim that the Pymatuning Dam project paid for itself during the first year of its life. In general design and construction this dam is similar to those proposed for flood-control on the Allegheny and Monongahela Rivers.

THEORETICAL CONSIDERATIONS

No reference has thus far been made to the exact effect the various proposed reservoir systems would have on a particular flood. The 1912 report of the Pittsburgh Flood Commission and the 1928 report of the United States Engineers both contain the results of extensive computations purporting to answer such questions. However, a close study of the different methods used in these two computations indicates that in neither case is it possible to have confidence that the final results are accurate. The fact is that, in the present state of the science of hydraulics, no accurate, reliable, and practical analytical method exists for determining the manner in which flood waves from several tributaries will combine to form a single flood wave on a main river.

From considerations based on the volumes under the hydrographs of the tributary floods, together with approximate estimates of the velocity of crest movements, it is possible to form a rough idea of how the tributary waves combine into a single main wave. This was essentially the method used in the 1912 study of the Flood Commission and was the best or only method known to the Engineering Profession at that time. It is, however, subject to errors of large or uncertain magnitude.

In more recent years, a method of flood-wave routing, usually known as the Watkins method, was developed in studies on the Tennessee River under

Colonel L. H. Watkins[25]. This method was used by the United States Engineers in their 1928 studies for flood protection of the Pittsburgh District. It assumes the river to be divided into several reaches extending between successive gaging stations, and the flood period to be divided into several intervals, each ordinarily of 24-hr duration. Each reach during each interval is made to satisfy certain requirements of inflow, outflow, and storage. It has been shown by one of the writers[26] that the errors and uncertainties of this method, as it is ordinarily applied, are probably as great as or greater than those of the simpler approximate method used in the 1912 report of the Flood Commission.

Since little has been published regarding the fundamental hydraulic principles of flood-wave routing, and since a knowledge of these principles is of vital importance in a study of flood protection by reservoirs on a river system the main streams of which are several hundred miles long, it is appropriate to include a brief discussion of the theory at this point. At any cross-section of a river, let z = the elevation of the water surface above datum; v = the mean velocity of the water; t = the time; g = the acceleration of gravity; x = the horizontal distance from the source of the river or from some arbitrary origin; d = the mean depth of the water (cross-sectional area divided by surface width); and s = the hydraulic slope, or loss of head per foot of travel.

The ordinary law that the hydraulic slope of an open channel varies as the square of the velocity is then expressed by the formula, $s = k\,v^2$, in which k is the "conveyance factor" of the channel, involving the effect of both the hydraulic radius and the surface roughness. At any given cross-section, k is a function of z. In general, v and z are functions of both x and t.

In the previously mentioned *Bulletin*[26] it is shown that at any cross-section of a river, the flow must satisfy two conditions: (1) The law of conservation of energy; and (2) the law of continuity or conservation of matter. The differential equations expressing these two laws, respectively, are as follows:

$$\frac{\partial z}{\partial x} + \frac{v}{g}\,\frac{\partial v}{\partial x} + \frac{1}{g}\,\frac{\partial v}{\partial t} + k\,v^2 = 0 \text{ ....} (1)$$

and,

$$d\,\frac{\partial v}{\partial x} + v\,\frac{\partial z}{\partial x} + \frac{\partial z}{\partial t} = 0 \text{ ....} (2)$$

In Equation (1) the names of the terms, in order, are: The "surface slope term", "velocity head term", "acceleration head term", and "hydraulic friction term." In Equation (2) the names of the terms, in order, are, the "prism storage term", the "wedge storage term", and the "rate of rise term."

The derivation of the Equations (1) and (2), being based on fundamental mechanics, is rigidly exact, but the actual integration to obtain numerical solutions of practical problems is by far too intricate and laborious to be workable. To obtain a solution that is workable from the standpoint of human patience and endurance, it is necessary to replace the infinitesimals, ∂x, by finite reaches of the river expressed in miles, and the infinitesimals, ∂t, by finite time intervals expressed in hours. Further, the necessity of labor saving requires that these finite space and time intervals must be made very coarse.

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[25] H. R. Doc. 185, 70th Cong., 1st Session, Appendix B, Pt. II, pp. 40–52.

[26] "The Hydraulics of Flood Movements in Rivers", by H. A. Thomas, M. Am. Soc. C. E., *Engineering Bulletin*, Carnegie Inst. of Technology, Pittsburgh, 1934.

Unfortunately, this coarseness utterly destroys the accuracy and reliability of the computations. Moreover, to keep the intricacy of the computations within the resolving power of an ordinary human mind, it has been the practice to neglect some of the terms of the fundamental differential equations. For example, the "Watkins method" neglects the second and third terms of Equation (1) and the second term of Equation (2). The fact that the neglected terms may be of great actual importance adds to the fundamental unreliability of these ordinary analytical methods of flood-wave routing. In the writers' opinion, the results of long and tedious computations by these methods are apt to be less accurate and reliable than the results of approximate estimates based on simple assumptions of the type used in the 1912 report of the Flood Commission.

The successful use of mechanical integration in other fields of scientific or engineering practice suggests that it might be possible to construct some integrating machine which would solve the fundamental differential equations of flood motion by a mechanical process. At the Carnegie Institute of Technology, much study has been devoted to the design and operation of devices suitable for this purpose. The most successful machine has been a model channel, of length suitable for indoor construction and with a greatly exaggerated vertical scale, so that depths of flow can be read with the necessary precision. The term, "integrating model", is used to designate a device of this type. Curves of the river are not represented in such a model, the effects of curve friction being taken care of in the roughness adjustment. The construction of each cross-section of the channel of an integrating model to duplicate the area-elevation curve at the corresponding section of the prototype offers only mechanical difficulties. The principal new technical feature involved in the construction of integrating models is the reproduction of the hydraulic friction effect as represented by k.

In the models thus far constructed, the hydraulic friction effect has been produced by sheet-metal baffles extending into the channel from the bed and sides. A technique has been developed for designing these baffles so as to make k vary with z in the model channel in any desired manner[27]. By this process the rating curves of the river can be reproduced in the integrating model, thereby satisfying all the requirements to make the model reproduce the flood-wave hydrographs of the prototype with perfect fidelity.

In such a model of a river system, each important tributary has its own cam-controlled inlet, by which the hydrographs of the tributary floods are reproduced. Several preliminary models representing parts of river systems and involving these principles have been built and tested at Carnegie Institute of Technology, and in 1936, the Pittsburgh Office of the Corps of Engineers recommended construction of a large integrating model of the entire Allegheny-Monongahela River System. At present, work on the construction of this model is under way.

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[27] "Model Studies of Flood Wave Movement in the Rivers of the Upper Ohio Watershed", by John W. Hackney, Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science, Carnegie Inst. of Technology, 1935.

This model will provide an accurate and reliable method of determining how much various flood crests will be lowered by the retarding reservoir system. It will also be used for flood-forecasting. Provided a sufficient number of automatic recording gages are established in the field, and properly manned, this will make possible early and accurate forecasts of high river stages.

AN IDEAL ORGANIZATION FOR THE RIVER AND FLOOD SERVICE OF THE UNITED STATES WEATHER BUREAU

BY MONTROSE W. HAYES,[28] ESQ.

SYNOPSIS

Flood forecasts may be based on river-gage relations, or on rainfall data, or on both. The demand for flood forecasts has grown rapidly, and the standard of refinement insisted upon, is constantly increasing. This makes necessary the establishment of additional observation stations and the development of more precise methods of prediction. A plan for improving the river and flood service of the United States Weather Bureau is presented.

Flood forecasting on a systematic basis seems to have had its beginning in France in 1854. The extent of the network of stations used is not known, nor is it clear, from the literature available to the writer, what degree of success was attained in making the predictions. However, the obstacles that undoubtedly were in the way were overcome, for the flood-forecasting service in France has not been interrupted since that beginning. Similar work was undertaken in Italy and Bohemia about 1866, and in the United States in 1871, the latter date being one year after the establishment of the National Weather Service.

At present, two distinctly different methods are used by the Weather Bureau in forecasting flood stages. The older and more refined method is based on gage relations and, more recently, on discharge data. The latter method utilizes largely, or altogether, reports of rain that has fallen or is expected to fall.

In forecasting from gage relations and discharge data, intervening rains must be given weight, but the underlying factors are up-river stages. It may seem unnecessary to state that the farther a reach is from its source water, the greater the time range that is possible in making a flood forecast, but the statement is made to bring out more forcefully the difference in the two general methods of forecasting. On the larger streams, or on the reach of any stream several days removed from the regions of the flood-producing rains, there is sufficient time to make the final forecast in refined terms, both as to stage and as to time. This is done now in a satisfactory manner and has been done for as long as 64 yr. The term of the forecasts ranges from 2 or 3 days in the upper valleys to as many as 3 or 4 weeks in the Lower Mississippi basin.

As a rule, it is impracticable to use gage relation or discharge data in making flood forecasts for regions into which flow numerous small streams, that may be considered source-water streams. The gaging of these numerous streams, and of the channels that are water carriers only during heavy rains, is not economically practicable. In general, moreover, the results would be of little value in forecasting. Therefore, it becomes necessary to rely upon reports from rain-gage stations, distributed over the catchment basins in such a way as to provide adequate information on the amount and intensity of the rain, and its geographic distribution. In much of the country east of the Appalachian Mountains, flood forecasts to be timely enough to be of real benefit must be made from rainfall reports.

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[28] Chf., River and Flood Div., U. S. Weather Bureau, Washington, D. C. Mr. Hayes died November 16, 1936.

The time when the work of the Weather Bureau was principally of academic interest has long since passed, and for many years the field offices, as well as the Central Office, in Washington, D. C., have had difficulty in meeting the calls made on them for general weather information. At offices situated on rivers the demands for river stage and flood forecasts have grown concurrently with the requirements for other service, and as the use of rivers increases these demands will continue to grow. The development of streams for power, navigation, and irrigation, the larger consumptive use of water by growing cities, and the greater use of the flood-plains by industry, all make flood forecasts not only desirable but absolutely necessary. Too, contrary to popular opinion, the successful operation of flood-control works, as well as their construction, requires forecasts of ice-forming weather, of impending rains, and of floods.

The standard of refinement of flood forecasts is set by those who use them. This statement may cause some surprise, but its truth can easily be demonstrated. If the interests along a river can be protected by 2-day forecasts verified in stage with an accuracy of about 2 ft, it would be useless to spend the money necessary to provide 3-day forecasts with an order of accuracy of less than 0.5 ft. On the other hand if the latter and more accurate forecasts were needed, an effort would be made to obtain the money that would make them possible. Because of the unprecedented heights of the floods of March, 1936, when, in inundated cities, each 6 in. of rise meant an additional loss of enormous extent, the Weather Bureau now finds itself facing unprecedentedly widespread demands for river stage and flood forecasts of longer range and greater refinement. In providing them many obstacles will have to be overcome. Changes in the present plan of operation will be necessary, and some outstanding deficiencies must be met. The simpler requirements need not be reviewed here. Others, more complex or difficult of attainment, are presented to show why the Bureau can not immediately strengthen its river service to the extent known to be desirable. These requirements are as follows:

(*a*) The establishment of more and better placed rainfall stations, especially in head-water regions.

(*b*) The installation of an adequate network of recording rain-gages to enable the forecaster to know the intensity of the rainfall. At present, most of the rain-gages used in flood work are of the eye-reading, 8-in. type.

(*c*) Surveys of the amount and condition of snow in the eastern mountains. Few data of this kind are available at present. Reliable and prompt rainfall reports are not sufficient when the mountain regions hold a great quantity of water in the form of snow, which is likely to be released by the rain.

(*d*) Arrangements for a more reliable transmission of rainfall and river-stage reports from the sub-stations to the district centers. In the eastern floods of March, 1936, the failure of wire communication was almost entirely responsible for any lack of timeliness and accuracy in the warnings issued. Except in very unusual cases the telephone and telegraph wires answer all purposes with a great degree of satisfaction, but it is in the unusual cases—the emergencies—that the greatest need for the reports exists. A river forecaster without information from the drainage area above him is helpless. The problem is difficult to solve. It has been suggested that radio stations should be established in flood-producing regions that are without reliable wire communication, and that they should be manned by Weather Bureau employees to transmit reports promptly under all conditions, to the forecasting center. This, of course, would be a solution, but it is believed that the cost would not be justifiable; certainly, it would not be, until it could be shown definitely that the Weather Bureau had exhausted every other and less costly means for having reports transmitted satisfactorily. One plan now (1937) being investigated would utilize amateur radio operators to transmit reports in times of emergency. Another proposal is to find hydrologically satisfactory locations for rain-gages, and for some river gages, from which reports can be sent by automobile to telegraph or telephone offices. This latter procedure would be expensive, but considerably less expensive than the maintenance of Weather Bureau radio stations.

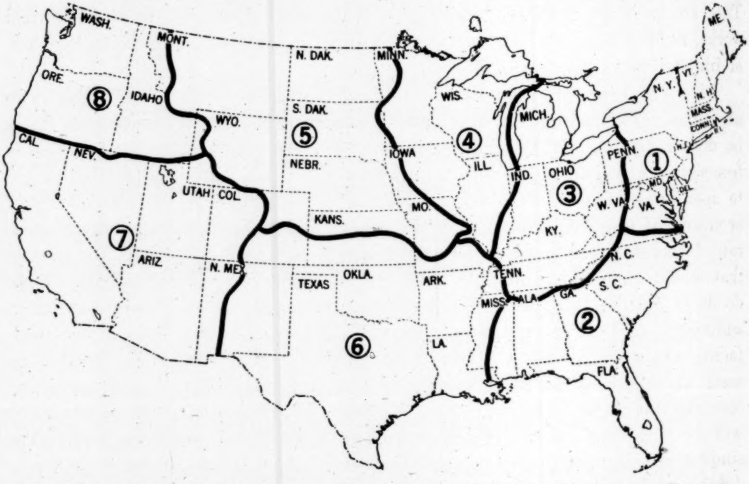

FIG. 17.—PROPOSED DIVISION OF THE UNITED STATES FOR FLOOD-FORECASTING SERVICE.

(*e*) Division of the country into eight sections for river administrative and forecasting purposes (Fig. 17), and the establishment of a staff of men in each section, under a section head, to handle the river work. These men should be charged with: (1) Placing and supervising the operation of the sub-stations; (2) developing and putting into effect a plan for transmitting reports to the forecasting centers; (3) co-ordinating all phases of the work, and co-operating with other organizations engaged in river work; (4) investigating the requirements for forecasts and arranging to meet those requirements; and (5) developing formulas for forecasting. Through a close co-operation with the U. S. Geological Survey, the Weather Bureau is now obtaining discharge data on most of the rivers of the country. This information can be used to advantage, in combination with the data supplied by the Weather Bureau itself, in developing more accurate formulas for river-stage and flood forecasts. However, the officials in charge of the Weather Bureau stations, who have multitudinous duties to perform, have not the time to devote to all the intricacies of the river work. It is obviously necessary to have a number of well-qualified men who can give all their time to studying these data and developing forecasting formulas.

The foregoing list of requirements for a river-forecasting service of a high order of accuracy is not merely something to be desired and never attained. On the contrary, it represents a plan set up by the Chief of the Weather Bureau in 1935. A modest beginning toward its accomplishment was made on July 1, 1936. Small staffs have been placed both in the Missouri Valley and in the Upper Mississippi Valley, and other parts of the country will be taken care of, as it becomes possible to do so.

At present, no definite decision has been reached as to who shall actually make the forecasts. There are sixty-four forecasting centers, and it is thought the usual station personnel can handle the forecasting with the facilities that will be provided by the river and flood staffs. The staffs, however, will be mobile, and in emergencies they will be available to do anything that will aid in issuing timely and accurate warnings of impending floods.

FEDERAL PLANS FOR FLOOD CONTROL

By W. E. R. Covell,[29] M. Am. Soc. C. E.

Synopsis

It was natural that the watercourses and their valleys should have had a great influence in the early settlement, trade, and transportation of the United States since they were the lines of least resistance to the vast unexplored inland country. Later, devastating floods occurred in the valleys and protective measures were sought. The first flood protection sponsored by the Federal Government was in the Mississippi Valley followed by protection in the Sacramento Valley, California. In 1925, Federal interest in river planning of broad scope was manifested, which later resulted in the "308" reports of the Corps of Engineers, U. S. Army, covering the possibilities of navigation, flood control, water power, and irrigation. By June 30, 1935, nine Federal flood-control projects were in force which were followed by twenty-seven additional projects throughout the country to June 22, 1936, when the general Flood Control Act was approved. A Federal policy was thereby established, 219 main flood-control items were authorized in 46 major basins and localities, and 232 localities were designated for further investigation. Fourteen reservoirs were authorized for the protection of Pittsburgh, Pa., and the Ohio Valley, based upon the "308" reports. The general features of the reservoir system, which contemplates control of flood waters at or near their source, are presented.

Introduction

It was natural that the watercourses should have had a great influence in the early settlement, trade, and transportation of the United States. They were the lines of least resistance to the vast unexplored country that lay inland; they provided transportation for the settlers and power for the first mills; and their fertile valleys produced rich crops and afforded topographically favorable town sites.

They were not an unmixed blessing, however. When the valleys were first settled, they were accepted in the condition in which they were found, because the country was new. Later, devastating floods were experienced, which went their way to the sea in the channels which Nature had provided. When the primary banks could not confine the flows they spread to the flood-plains of the inhabited valleys.

Man's ingenuity was aroused, and he sought to protect himself and his property. The progressive development of the valleys was accepted as a necessity and protective measures were early devised to keep the flood-waters within bounds. To-day such work is still in progress. The extreme floods of recent times and the heavy losses sustained thereby have aroused the country at large as never before to a realization of their effect upon welfare and progress. In the last analysis, economic justification will probably govern the extent to which flood protection will eventually be provided.

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Flood Control on the Mississippi River

The largest drainage basin of the country is that of the Mississippi River, with an area of 1 240 000 sq miles. The total area of the Continental United States is 3 026 789 sq miles. This basin includes all or parts of thirty-one States and about 20 000 sq miles of Canada. The first levee along the Mississippi River was built by the French in about 1718. Levees were gradually extended with the growth of settlements, under provisions of the land grants, which required the grantee to construct and maintain a levee line along the river front of his property.

The floods of 1849 and 1850 caused widespread damage in spite of the levees. National interest was aroused by the pleas for Federal aid and resulted in the Swamp Land Act of 1850, which granted to the several States all unsold swamp and overflowed lands within their limits to provide funds to reclaim the districts subject to overflow. In 1850, also, Congress authorized a topographical and hydrographic survey of the delta of the Mississippi River, with such investigation as might lead to the determination of the most practical plan for securing it from inundation. This work was placed in charge of Captain A. A. Humphreys, later assisted by Lieutenant H. L. Abbot, both of the Corps of Topographical Engineers, U. S. Army. Another survey and report of a similar nature, authorized by Congress in the same year, was made by Charles Ellet, a Civil Engineer employed by the Corps of Engineers.

The formation of the Mississippi River Commission was authorized by Congress in 1879. It consisted of seven commissioners of whom three were required to be from the Corps of Engineers, U. S. Army, one from the United States Coast and Geodetic Survey, and three from civil life. Two of the latter must be civil engineers. Between 1890 and 1917, the periodic appropriations made by law for expenditure by the Commission were divided among levees, revetments, and dredging, but the levees were considered as an adjunct to river improvement. It was not until the Flood Control Act of March 1, 1917, that by law, flood control as such became a definite part of the work of the Commission. The Act of 1917 provided for co-operation by local interests which were required to provide the necessary rights of way, pay not less than one-third of the cost of levee construction, and to assume the entire cost of maintenance after completion. The cost of river improvement (revetment and dredging) was to be paid for entirely from Federal funds.

In 1927, the Lower Mississippi River experienced the highest flood of record to that time. More than 200 breaches in the levees occurred, although it is of interest to note that only a few were in levees built by the Commission, and only one in a levee built to the Commission's standard dimensions. Public interest was aroused by the catastrophe, which submerged about 20 000 sq miles of the valley and cost the lives of about 250 persons. The Flood Control Act of 1928 resulted, based on the report of the late Major General Edgar Jadwin, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., then Chief of Engineers. The project for the valley of the Mississippi River, as outlined in this Act, provides for the control of flood waters from Rock Island, Ill., to the Head of Passes near the Gulf of Mexico, by means of levees. Tributaries and outlets, in so far as they are affected by the Mississippi River, are also included within the scope of the project. (The alluvial valley of the Mississippi River consists of the St. Francis, the Yazoo, the Tensas, and the Atchafalaya basins, as well as the alluvial lands around Lake Pontchartrain.) The plan called for raising existing levees by about 3 ft and strengthening them, and permitting flood waters in excess of the safe capacity of the leveed channel to find their way to the Gulf through the natural floodways in the lowlands adjacent to the river, which have always carried the waters of extraordinary floods. Levees were planned on the edges of these natural floodways, to restrain the waters within them as far as practicable and to protect the maximum area of land that it was economically justifiable to reclaim. The flood waters would not escape into the floodways until they reached a dangerous stage in the main leveed channel.

An appropriation of $325 000 000 was authorized by the Act of 1928 for carrying on this work. The Act further provided for local co-operation in the project to the extent that States or levee districts were required to maintain the flood-control works, except the controlling and regulating structures, after their completion; provide all the necessary rights of way without cost to the United States; and contribute 33⅓% of the cost of the project work between Rock Island, Ill., and Cape Girardeau, Mo. A later Act, approved April 23, 1934, provided for the reimbursement of States or local levee districts for the cost of the rights of way or easements required for the levees which the Federal Commission was obligated to provide under the Act of May 15, 1928, provided the cost was reasonable.

The general project for flood control on the Lower Mississippi River adopted by the Act of May 15, 1928, has been amended by the Flood Control Act of June 15, 1936[30], and modified in accordance with the report of February 12, 1935, of the Chief of Engineers, U. S. Army[31]. The adopted modifications of the project consist of the following: (1) The abandonment of the Boeuf Floodway, and in lieu thereof the construction of the Eudora Floodway (west of the Mississippi River, and extending from the latitude of Eudora, Ark., into the Red River back-water area), with a control structure at its head; (2) the construction of a rear protection levee extending from the head of the Eudora Floodway north to the Arkansas River, located so as to afford adequate space for the escape of flood waters without endangering the levees on the east side of the river; (3) the maintenance of the present river levees between the head of the

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[30] Public Doc No. 678, 74th Cong.

[31] H. R. Committee on Flood Control, Doc. No. 1, 74th Cong.

Eudora Floodway and the northern junction with the protection levee, at the 1914 grade and section, except in front of densely populated areas, as a part of a ring levee; (4) the construction of a floodway extending from the Mississippi River north of Morganza, La., to the Atchafalaya River back-water, with a control structure at its head; (5) the raising of the levees from the head of this floodway to the head of the Atchafalaya River to full standard grade and section; (6) the immediate completion of the guide levees in the Atchafalaya basin to afford full protection to all lands outside of these levees; (7) the construction of an additional outlet to the Gulf of Mexico, west of Berwick, La.; (8) the increase of the discharge capacity of the leveed channel of the Atchafalaya River and of its outlets; (9) a six-year program for the improvement and regularization of the Mississippi River, including the continued maintenance of the navigation channel provided in the previous project; and (10) the flood control of the St. Francis and Yazoo Rivers.

It was contemplated that the unfinished parts of the levees and structures authorized by the Act of 1928 and not modified by the Act of May 15, 1936, would be completed as planned. The total estimated cost of all works added to the previous project, including the acquisition or reimbursement for land rights, was $313 000 000, or $245 000 000 in excess of the unappropriated balance of the appropriation previously authorized.

Federal Interest in Other Streams

The first expansion of Federal interest in flood control to another locality of the United States was authorized by the Flood Control Act of March 1, 1917. It resulted in a general project for flood control, including channel enlargement, cut-offs, by-pass weirs, outfall gates, and levees, for the Sacramento River, in California. The project, later modified by the Flood Control Act of May 15, 1928, represents the outgrowth of the early activities of the California Débris Commission created by an Act of Congress in 1893, and consisting of three officers of the Corps of Engineers, U. S. Army. The Act of 1917 required that the State of California share equally with the United States in the cost of the works, and in addition, that all rights of way, easements, and lands be provided free of cost to the Federal Government. The Flood Control Act of May 15, 1928, required the State or other local interests to contribute one-half the cost of constructing the levees to Commission standards and to complete the project.

In 1925, Federal interest was manifested in river planning of a broad scope. The River and Harbor Act approved on March 3 of that year directed the Secretary of War and the Federal Power Commission jointly to prepare and submit to Congress an estimate of making such examinations, surveys, or other investigations, as in their opinion were required for those navigable streams of the United States, and their tributaries, whereon power development appeared feasible and practicable, with a view to formulating general plans for the purposes of navigation and for the prosecution of such improvements in combination with the most efficient development of the potential water power, the control of floods, and the needs of irrigation. The streams and their tributaries to be investigated were designated in a Federal report[32], and the River and Harbor Act of January 21, 1927, directed the Chief of Engineers to prosecute the investigations. The resulting reports have become quite generally known as the "308" reports.

The work covered practically all the principal drainage systems of the United States. The surveys first authorized, involved an estimated cost of $7 322 400. In addition, the Flood Control Act of May 15, 1928, authorized the expenditure of $5 000 000 of flood-control funds for surveys of tributaries of the Mississippi River. In particular, the Chief of Engineers was thereby directed to make a comprehensive and detailed investigation of the possible alleviation of Mississippi River flood conditions by means of reservoirs, operated either primarily for that purpose or for some other purpose or combination of purposes, on tributaries of the Mississippi that were included in the "308" list. Further study or review of the "308" reports is authorized by Section 6 of the River and Harbor Act of August 30, 1935. By June 30, 1935, a total of $10 254 123.13 had been allotted. The work was conducted by the Corps of Engineers, through thirty-six of the forty-three District Offices. The resulting reports, together with other reports by the Corps of Engineers, furnished the basis for the flood-control projects adopted by the Flood Control Act of June 22, 1936. In addition to flood control proper, the possible influence of potential water power and irrigation developments on flood control has been covered in the "308" studies.

It is interesting to note the status of Federal flood control in the United States and Territories as of June 30, 1935. Under Federal Public Works and Emergency Relief programs, nine flood control projects were in force, seven of them in various parts of the United States and two in Alaska, in addition to the initial Mississippi and Sacramento River projects. These projects were: Winooski River System, Vermont (reservoir system); Wallkill River System, New York (channel improvement); South Canadian River, New Mexico (reservoir); New River, West Virginia (reservoir); Tygart River, West Virginia (reservoir); Muskingum River System, Ohio (reservoir system and channel improvement); Los Angeles and San Gabriel Rivers, California (flood-control system); Salmon River, Alaska (dikes and clearing); and Lowell Creek, Alaska (dam). The total estimated cost of these projects was more than $100 000 000, including the cost of construction and land and damages. Most of them required local co-operation, principally with respect to the payment for land and damages.

Between June 30, 1935, and the approval of the general Flood Control Act of 1936, a total of twenty-seven additional flood-control projects was approved for construction under provisions of the Emergency Relief Appropriation Acts of 1935 and 1936. These projects were scattered throughout the country and consisted of levee, channel, cut-off, flood-wall, and reservoir improvements at a total estimated cost of about $22 351 000. Payment for lands and damages was the responsibility of local interests. Notable projects in this list were the Sardis Reservoir, in the Yazoo River basin, Mississippi (a part of the modified plan for Mississippi River flood control); and the Possum Kingdom Reservoir, on the Brazos River, Texas (one of the thirteen reservoirs planned by the Brazos River Conservancy and Reclamation District).

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[32] H. R. Doc. No. 308, 69th Cong., 1st Session.

THE FLOOD CONTROL ACT OF 1936

The Flood Control Act of June 22, 1936, opens a new era for flood control in the United States. For the first time in the history of the nation, a Federal policy with respect to flood control for the country at large is adopted by law. The first four sections and Paragraph 1 of Section 5 of the Act are contained in the Appendix.

Analysis of the provisions of the Flood Control Act discloses the following general features: (*a*) Flood-control projects are authorized in 46 major basins and localities throughout the United States; (*b*) the Act contains 219 main flood-control items, some of which involve several sub-items; (*c*) the types of projects include channel improvements, dikes, reservoirs, levees, cut-offs, diversions, floodways, and flood walls; and (*d*) the estimated total cost of the projects authorized is from $396 532 880 to $397 532 880, of which $300 571 325 is for construction and from $95 961 555 to $96 961 555 is for land and damages. In addition to the authorization of definite flood-control projects, the Act provides for preliminary examinations and surveys for flood control at 222 localities throughout the United States, and for surveys, studies, and reports at 10 Southern and Western localities where there are indications that opportunities may exist for useful flood control in combination with hydro-electric power development whenever sufficient markets become available to absorb the power.

The Act of 1936 does not repeal the Flood Control Act of 1928, or any provision of any law amendatory thereof. The sum of $310 000 000 is authorized to be appropriated for the flood-control projects adopted by the Act. Not more than $50 000 000 may be expended during the fiscal year ending June 30, 1937, and provision is made for relief of unemployment, in constructing the projects.

The Flood Control Act of 1936 represents national stream planning on a large scale. The most meritorious projects of the country at large are included in the initial program of development. The further possibilities are recognized. The ultimate development will likely be determined by economics of cost and benefit.

The Federal policy with respect to local co-operation in projects has been traced in this paper from the original flood-control project on the Mississippi River to the present time. Such co-operation has largely attended the development of flood control and is a feature of the Act of 1936 which provides that when the cost of the land, rights of way, and damages exceeds the construction cost of a project, the resulting participation of the Federal Government and the local interests is on an equal basis. Local interests are required to maintain and operate all the flood-control works after completion.

The project for the Ohio River basin authorized by the Flood Control Act of June 22, 1936, provided for two features, namely:

"Reservoir System for the Protection of Pittsburgh: Construction of reservoirs for the Allegheny-Monongahela basin as in comprehensive plan for the protection of Pittsburgh and for the reduction of flood heights in the Ohio Valley generally, as set forth in House Document Number 306, 74th Congress, 1st Session, and in the report on the Allegheny-Monongahela Rivers and tributaries on record in the Office of the Chief of Engineers; estimated construction cost, $20 646 000; estimated cost of lands and damages, $34 569 000.

"Reservoir System for the Reduction of Ohio River Floods Below Pittsburgh: Construction of reservoirs including the completion of the Bluestone Reservoir now under way, which together with the reservoirs for Pittsburgh flood control, constitutes a comprehensive plan for flood control on the main stream of the Ohio River and on the tributary stream below the reservoirs, as set forth in House Document Number 306, 74th Congress, 1st Session; estimated construction cost, $19 616 800; estimated cost of lands and damages, $10 519 600."

The report mentioned as being on record in the Office of the Chief of Engineers is an unpublished report, and House Document No. 306, 74th Congress, 1st Session, is a published report, both prepared in accordance with House Document No. 308, 69th Congress, 1st Session.

Projects in the Ohio Basin

The Ohio River is formed by the confluence of the Allegheny and Monongahela Rivers at Pittsburgh, Pa., and flows in a general southwesterly direction 981 miles to enter the Mississippi River at Cairo, Ill. The river discharge varies widely, the maximum and minimum at the mouth being 1 600 000 (prior to January, 1937) and 22 000 cu ft per sec, respectively. The basin, which embraces parts of fourteen States, is 800 miles long and has an area of approximately 204 000 sq miles. The topography is greatly diversified. In the northern and western areas the surface is generally flat to rolling, whereas in the eastern and southern areas it is rugged and mountainous. The normal annual precipitation for the basin is 45 in. varying from 83 in. in the southeastern mountainous part to 31 in. in the northwestern part.

Because of its large and extensively developed coal fields, the basin is one of the principal industrial regions of the United States. The outstanding development is that of the iron and steel industry, centered at Pittsburgh, Pa., and extending up the Monongahela River and down the Ohio River. Cincinnati, Ohio, Louisville, Ky., and Wheeling, W. Va., are other large centers of industry on the river. Outside the industrial regions, agriculture and coal mining are the principal activities.

High stages occur frequently on the Ohio River and its tributaries, the range in stage from low water to high water attaining 70 ft (prior to January, 1937) in some reaches of the main river. The bottom-lands, however, are generally narrow, and most of the cities, roads, etc., are above the ordinary flood stages. Nevertheless, between 1876 and 1936 twenty-three floods have risen 5 ft, or more, above ordinary flood stage, with damage to cities, towns, industries, railroads, and highways. A large part of this damage is due to the flooding of basements and the lower floors of buildings, with consequent injury to goods and furniture therein. Crop damages are of less importance. Prior to the flood of March, 1936, the average annual flood loss on the main stem of the river, exclusive of Pittsburgh, was estimated at $1 860 000. (About two-thirds of this loss is at or above Cincinnati, because of the large industrial development along the upper river, and the relatively greater flood heights there experienced.) It was also estimated that an effective system of flood control would afford an annual benefit of about $1 000 000 to the lands bordering the Ohio River, because of increased value for urban or farm use.

Reports have shown that the construction of levees or walls for the protection of towns and cities on the Ohio River against high floods appears to be economically justified in only certain cases. The cost of levees for the protection of agricultural lands would greatly exceed any possible benefits.

Because of the intensive development of the valleys, the construction on the main rivers, of dams of sufficient height to afford flood storage and effective power development would inflict widespread and costly flowage damage. For the control of floods in the Mississippi Valley, the most effective location for such a dam would be near the mouth of the Ohio River, but investigation of the most practicable site, has shown it to be inadvisable. The same is true of other possible sites in the more highly developed upper parts of the Ohio Valley. Flood control on the Ohio River, therefore, must primarily be obtained by works on its tributaries.

A compilation of the most favorable plans for the comprehensive development of the water resources of the Ohio River basin, as developed in the "308" surveys, shows that about 250 water-power developments, supplemented by 32 flood-control reservoirs, and levees and channel improvements on tributaries, may eventually be warranted. The total installed hydro-electric capacity of such a comprehensive system would reach 8 600 000 kw. So large an amount of power could not be absorbed for many years, and, obviously, such a development could only be made progressively, as the growth of the power market justified the individual installations. The total cost of such a comprehensive development is estimated in the "308" reports at $1 700 000 000, and the annual charges at $160 000 000. The possible future annual returns from power are estimated at $169 000 000; the annual benefits from flood control (largely on the tributaries), at $8 600 000; and the annual benefits to water supply and navigation, at $500 000.

A possible comprehensive reservoir plan for the alleviation of floods on the Mississippi River, through works designed for the control of floods on its tributaries was investigated in connection with the "308" reports. It included 81 reservoirs on the tributaries of the Ohio, with a capacity of 25 300 000 acre-ft. The estimated cost of these 81 reservoirs was $517 000 000.

Three large flood-control reservoir projects included in the comprehensive plan are now (1937) under construction in the basin. One of these, on the Tygart River, a tributary of the Monongahela River, is not only for the purpose of flood control but also to assure an adequate supply of water for the operation of the locks on the Monongahela during periods of extreme drought, such as occurred in 1930. The dam is being built by the War Department with funds allotted by the Public Works Administration. The second project is a system of reservoirs for the control of floods on the Muskingum River and its tributaries, which has been undertaken by the War Department in co-operation with the Muskingum Conservancy District. This project fulfills the purpose of two of the reservoirs contained in the comprehensive plan. The third project is the Norris Dam, on Cove Creek, a tributary of the Tennessee River, nearing completion by the Tennessee Valley Authority.

The Chief of Engineers in 1935 approved the selection of fourteen additional flood-control reservoirs in the Ohio Basin, the immediate construction of which appeared to be warranted. They will be built by the Federal Government after appropriation of funds, provided local interests furnish the necessary lands and flowage, hold and save the United States against damage, and take over the works after completion. These works include nine reservoirs above Pittsburgh, forming the comprehensive plan for the protection of that city, three on the tributaries of the Kanawha, and two on the Licking River, which enters the Ohio opposite Cincinnati. The total estimated cost of the nine dams above Pittsburgh was $55 215 000, of which $20 646 000 was for construction and $34 569 000 was for lands and damages. At one of the project reservoirs on the tributaries of the Kanawha (the Bluestone Reservoir on the New River), power production can apparently be combined with flood control. The construction of this reservoir and appurtenant power installation at an estimated cost of $13 000 000 was recommended in a comprehensive report on the Kanawha River[33], provided a definite agreement could be entered into for the disposal of the power at prices that would protect the public interest. The total cost of the other two flood control reservoirs on the Kanawha was estimated at $9 849 000 of which $2 886 000 was for construction and $6 963 000 was for lands and damages. The total cost of the two proposed reservoirs on Licking River was estimated at $7 345 000, of which $5 046 000 was for construction and $2 299 000 was for lands and damages. The total estimated cost of the construction of the reservoirs recommended, exclusive of the Bluestone Reservoir, was $28 578 000, and the total estimated cost of lands and damages was $43 831 000. The estimates were made at the time the "308" reports were prepared.

The system of fourteen reservoirs, together with the reservoirs now under construction on the Tygart and Muskingum Rivers, may be expected to reduce the peak of a great Ohio River flood, similar to that of 1913, by 7.5 ft at Pittsburgh, 3 ft at Cincinnati, 1.6 ft at Louisville, and about 6 in. at Cairo. Lesser floods, in general, would be reduced to a greater degree. The direct annual benefits from this system of reservoirs were estimated (prior to the great floods of March, 1936, and January, 1937) at $4 100 000, and the annual carrying charges at $3 416 000. The flood of March, 1936, is the highest of record in the Upper Ohio River and resulted in new data as to the probable frequencies of high stages. These data indicate that the flood-control benefits will be greater than previously estimated. The drainage area to be controlled by the system of fourteen reservoirs is 15 936 sq miles of the 204 000 sq miles total for the Ohio River basin. The gross capacity of the reservoirs amounts to 3 169 510 acre-ft.

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[33] H. R. Doc. No. 91, 74th Cong. 1st Session.

The immediate local effect and benefit of a flood-control reservoir is directly tangible and certain, but its effect on flood heights at the more remote localities on the main stem of the Ohio River is less certain. The just apportionment of the cost of structures of this character between the Federal Government and the local interests presents a considerable problem. The apportionment under which the construction cost would be borne by the United States and the land damages by local interests is considered advantageous, in that it assigns to local interests that part of the cost which they can most effectively undertake.

Prior to 1936, great floods had been experienced at Pittsburgh, notably in 1762, 1763 (the succeeding year), 1809, 1810 (another succeeding year), 1816, 1832, 1884, 1902, and 1907. The flood menace has ever been present, but the risk has been accepted and the flood losses taken as they occurred. In March, 1936, the Pittsburgh area experienced its greatest flood of record; the river reached a stage of 46 ft which was 21 ft above flood stage, 7½ ft higher than the great flood of March 7, 1907, and 12.6 ft higher than that of March, 1913, which was an extremely high flood for the Ohio River as a whole. It had been predicted as the result of engineering studies by the Flood Commission of Pittsburgh published in 1912, that Pittsburgh would some day experience a 43-ft flood. The flood studies of the War Department indicated that a 47-ft flood stage was possible under certain conditions. From 1854, when continuous records began, to the close of 1936, 93 flood stages, or an average of 1.1 per yr, have occurred at Pittsburgh. According to the all-time record dating back to January 9, 1762, the eleven highest floods crested at stages ranging from 35 to 46 ft. The month of March is the most prolific for occurrence of floods at Pittsburgh; January ranks second, February third, and December fourth.

With the benefit of the continuous record of floods over a long period of time, floods of various heights may be given a frequency rating according to the law of probability The flood of March, 1936 (46 ft), may be expected to occur once about every 270 yr, and the flood of March, 1907 (38.5 ft), once about every 26 yr. At first thought, these frequencies appear to lend some comfort to unfortunates situated in the stricken areas. Although it is entirely possible that this generation may never witness a recurrence of the March, 1936, flood, the menace is ever present and high waters may occur close together. This possibility was borne out in 1762 and 1763, when stages of 39 and 40.9 ft, respectively, occurred at Pittsburgh.

The Pittsburgh area lies within the influence of both the Allegheny and Monongahela River Systems, the flood waters of which converge to the head of the Ohio River. The time of arrival of the flood crests from these river systems, at Pittsburgh, has an important bearing upon the height of the flood stages, which is also influenced by the peculiarities of the distribution of precipitation over the separate drainage areas. The differences in latitude and altitude and related temperatures over the basin are ordinarily redeeming factors, although under some circumstances these factors may become completely nullified. The respective natural peculiarities of the adjoining Allegheny and Monongahela River basins have been instrumental in minimizing the number of floods experienced at Pittsburgh. However, there is always the possibility that conditions may be such that the converging flood waters from the two basins may arrive at Pittsburgh at nearly or at exactly the same time, with disastrous results.

The possibility of flood control by means of reservoirs in the Allegheny-Monongahela basin has been the subject of numerous investigations beginning with that of the Flood Commission of Pittsburgh. The principal object of these flood-control investigations has invariably been to determine the feasibility of the control of floods at or near their source, since it is readily recognized that such control would be productive of the most widespread benefits. In effect, a part of the drainage area would be practically cut off from the remainder by means of flood detention reservoirs. Such reservoirs should have sufficient capacity to retain practically the entire inflow thereto until it can be released so as to reach Pittsburgh after the high water from the intermediate, uncontrolled drainage area above that city has gone on down the Ohio River. As the Monongahela and Lower Allegheny River Valleys cannot advantageously be utilized for reservoirs, resort must be had to locations on the head-waters and the principal tributaries. The individual reservoir sites that have been considered in the investigations have been selected after a study of the physical and economic features of the basin as a whole although, in this respect, the basin offers little choice of sites, and with a view to distributing the individual units so that as many of them as possible would become effective no matter what the particular storm and consequent run-off conditions might be.

As a result of previously mentioned investigations and reports by the U. S. Corps of Engineers, the Tygart River Reservoir was authorized and is now (1937) under construction at an estimated cost of $15 700 000. This reservoir will have a capacity of 314 000 acre-ft for the control of spring floods, equivalent to a run-off of 5 in. of rainfall from the tributary area of 1 183 sq miles. The reservoir is also designed to provide an adequate low-water discharge for navigation purposes on the Monongahela River. As a flood-control reservoir, it will serve as the initial unit in any flood-control reservoir system which may be provided in the Allegheny-Monongahela River basin.

The project system of flood-control reservoirs for the Allegheny-Monongahela River basin above Pittsburgh adopted by the Flood Control Act approved June 22, 1936, consists of nine units, in addition to the Tygart Reservoir. This system may be considered tentative. Revision of the "308" studies by bringing them up to date may indicate that additional reservoirs or other facilities may be economically justified. The total capacity of the project system, including the Tygart Reservoir, would be 2 272 500 acre-ft and the total amount of the drainage area controlled would be 7 487 sq miles. The basin above Pittsburgh comprises 19 117 sq miles, which would leave 11 630 sq miles, or 61% of the basin, uncontrolled by reservoirs.

Investigation of the probable effectiveness of the reservoir system indicates that the flood of March, 1907 (stage, 38.5 ft at Pittsburgh), would have been reduced to a stage of 31.5 ft and the flood of January, 1913 (stage, 34.3 ft) to 27.9 ft. For the great flood of March, 1936 (stage, 46 ft), studies indicate that the reduction in stage would have been about 7.7 ft. Slightly reduced benefits resulting from the reservoir system would also extend down the Ohio River to and below Wheeling, W. Va.

The total cost of the nine reservoirs of the system was estimated at the time the "308" reports were made, to be $55 215 000, of which $20 646 000 would be for construction and $34 569 000 for land and damages. It is evident that the Allegheny-Monongahela River basin does not represent virgin territory in which reservoirs may be built cheaply at favorable physical locations. Railroad lines early pre-empted the main valleys in a region of extensive natural resources. In every reservoir site, except one (Crooked Creek), existing rail development would be affected, and in some cases extensive relocation would be required. Numerous small villages would also have to be abandoned.

The benefits that would result from flood control by means of reservoirs should outweigh the cost of providing the facilities. Although a flood of the magnitude of March, 1936, causes widespread damage of very great amount, it is of infrequent occurrence, so that the average annual damage from such floods is relatively small. The floods of small heights, although they have a high frequency, also result in relatively small annual damages. It is the floods of intermediate height; that is, neither extremely high nor just barely flood stage, that result in the greatest amount of average annual damages, since the damage per flood is relatively high and they occur fairly often.

The unprecedented flood of March, 1936, in the Upper Ohio River area presented some new evidence of cause and effect of extreme floods. The project reservoir system, however, was planned to accommodate certain hypothetical floods. These represent the probable results of the translation of large storms of record, such as that of March, 1913, which involved only a part of the Upper Ohio River basin, to more effective positions on the basins. Some adjustment of the reservoir system may be found desirable in view of the March, 1936, flood. This matter is now (1937) under investigation as provided for under the various Acts of Congress. The City of Johnstown, Pa., (in the Allegheny River basin) experienced a devastating flood in March, 1936. Flood-control measures for that area are also under consideration. A reservoir, or reservoir system, above Johnstown would supplement the reservoir system for the protection of Pittsburgh and the Ohio Valley. The existing reservoir system project, however, would have to be modified by Congressional action to include such facilities.

The flood-control work of the Corps of Engineers, U. S. Army, has been and will continue to be prosecuted in obeisance to the mandates of Congress. Flood control is essentially a matter of engineering and economics, and it is recognized that it presents many problems which must be solved effectively, perhaps boldly, and economically. The endeavor has been made to formulate projects which are comprehensive, co-ordinated, practical and economic. Revisions and additions may be required to a certain extent. A high standard of engineering is imperative in dealing with flood problems lest failure of works designed to withhold floods result in greater catastrophes than the floods themselves. In the investigation and design of flood-control structures the Corps of Engineers has upon occasion retained many eminent engineers from civil life, as well as eminent geologists, in a consulting capacity. After all the years since the early settlement of the United States, it appears that at last flood control has attained a proper place among the numerous activities of the Federal Government, for the welfare of the country. However, States, political subdivisions thereof, and other responsible local agencies must also do their part.

APPENDIX

ABSTRACT OF THE FLOOD CONTROL ACT[34] APPROVED JUNE 22, 1936

AN ACT

Authorizing the Construction of Certain Public Works on Rivers and Harbors for Flood Control, and for Other Purposes.

Be It Enacted by the Senate and House of Representatives of the United States of America in Congress Assembled,

DECLARATION OF POLICY

Section 1. It is hereby recognized that destructive floods upon the rivers of the United States, upsetting orderly processes and causing loss of life and property, including the erosion of lands, and impairing and obstructing navigation, highways, railroads, and other channels of commerce between the States, constitute a menace to national welfare; that it is the sense of Congress that flood control on navigable waters or their tributaries is a proper activity of the Federal Government in cooperation with States, their political subdivisions, and localities thereof; that investigations and improvements of rivers and other waterways, including watersheds thereof, for flood-control purposes are in the interest of the general welfare; that the Federal Government should improve or participate in the improvement of navigable waters or their tributaries, including watersheds thereof, for flood-control purposes if the benefits to whomsoever they may accrue are in excess of the estimated costs, and if the lives and social security of people are otherwise adversely affected.

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[34] Public Doc. No. 738, 74th Cong., H. R. 8455.

Section 2. That, hereafter, Federal investigations and improvements of rivers and other waterways for flood control and allied purposes shall be under the jurisdiction of and shall be prosecuted by the War Department under the direction of the Secretary of War and supervision of the Chief of Engineers, and Federal investigations of watersheds and measures for run-off and waterflow retardation and soil erosion prevention on watersheds shall be under the jurisdiction of and shall be prosecuted by the Department of Agriculture under the direction of the Secretary of Agriculture, except as otherwise provided by Act of Congress; and that in their reports upon examinations and surveys, the Secretary of War and the Secretary of Agriculture shall be guided as to flood-control measures by the principles set forth in Section 1 in the determination of the Federal interests involved; Provided, That the foregoing grants of authority shall not interfere with investigations and river improvements incident to reclamation projects that may now be in progress or may be hereafter undertaken by the Bureau of Reclamation of the Interior Department pursuant to any general or specific authorization of law.

Section 3. That hereafter no money appropriated under authority of this Act shall be expended on the construction of any project until States, political subdivisions thereof, or other responsible local agencies have given assurances satisfactory to the Secretary of War that they will (*a*) provide without cost to the United States all lands, easements, and rights-of-way necessary for the construction of the project, except as otherwise provided herein; (*b*) hold and save the United States free from damages due to the construction works; (*c*) maintain and operate all the works after completion in accordance with regulations prescribed by the Secretary of War: Provided, That the construction of any dam authorized herein may be undertaken without delay when the dam site has been acquired and the assurances prescribed herein have been furnished, without awaiting the acquisition of the easements and rights-of-way required for the reservoir area: And provided further, That whenever expenditures for lands, easements, and rights-of-way by States, political subdivisions thereof, or responsible local agencies for any individual project or useful part thereof shall have exceeded the present estimated construction cost therefor, the local agency concerned may be reimbursed one-half of its excess expenditures over said estimated construction cost: And provided further, That when benefits of any project or useful part thereof accrue to lands and property outside of the State in which said project or part thereof is located, the Secretary of War with the consent of the State wherein the same are located may acquire the necessary lands, easements, and rights-of-way for said project or part thereof after he has received from the States, political subdivisions thereof, or responsible local agencies benefited the present estimated cost of said lands, easements, and rights-of-way, less one-half the amount by which the estimated cost of those lands, easements, and rights-of-way exceeds the estimated construction cost corresponding thereto: And provided further, That the Secretary of War shall determine the proportion of the present estimated cost of said lands, easements, and rights-of-way that each State, political subdivision thereof, or responsible local agency should contribute in consideration for the benefits to be received by such agencies: And provided further, That whenever not less than 75 per centum of the benefits as estimated by the Secretary of War of any project or useful part thereof accrue to lands and property outside of the State in which said project or part thereof is located, provision (*c*) of this section shall not apply thereto; nothing herein shall impair or abridge the powers now existing in the Department of War with respect to navigable streams: And provided further, That nothing herein shall be construed to interfere with the completion of any reservoir or flood control work authorized by the Congress and now under way.

Section 4. That consent of Congress is hereby given to any two or more States to enter into compacts or agreements in connection with any project or operation authorized by this Act for flood control or the prevention of damage to life or property by reason of floods upon any stream or streams and their tributaries which lie in two or more such States, for the purpose of providing, in such manner and such proportion as may be agreed upon by such States and approved by the Secretary of War, funds for construction and maintenance, for the payment of damages, and for the purchase of rights-of-way, lands, and easements in connection with such project or operation. No such compact or agreement shall become effective without the further consent or ratification of Congress, except a compact or agreement which provides that all money to be expended pursuant thereto and all work to be performed thereunder shall be expended and performed by the Department of War, with the exception of such reasonable sums as may be reserved by the States entering into the compact or agreement for the purpose of collecting taxes and maintaining the necessary State organizations for carrying out the compact or agreement.

Flood Control Act of 1936

Section 5. That pursuant to the policy outlined in Sections 1 and 3, the following works of improvement, for the benefit of navigation and the control of destructive flood waters and other purposes, are hereby adopted and authorized to be prosecuted, in order of their emergency as may be designated by the President, under the direction of the Secretary of War and supervision of the Chief of Engineers in accordance with the plans in the respective reports and records hereinafter designated: Provided, That penstocks or other similar facilities, adapted to possible future use in the development of adequate electric power may be installed in any dam herein authorized when approved by the Secretary of War upon the recommendation of the Chief of Engineers.

DISCUSSION

Fred C. Scobey,[35] M. Am. Soc. C. E. (by letter).—In the Symposium under discussion reference is made to the use of snow surveys, in forecasting floods. When sufficient snow-survey data have been accumulated for any major water-shed then, of course, excessive snow depth and water content will be indicative of a heavy run-off. This information can be directly applied to streams in which floods are always, or nearly always, dependent on snow melting, such as the Colorado River of the Far West. For streams in which floods are developed by torrential rains alone, or by hard rains on top of snow, such as occur in the Gila River water-shed, of course a snow survey does not apply or merely indicates a contributing factor.

In 1936, snow surveys were extended in the West, by many agencies acting through the U. S. Bureau of Agricultural Engineering as a clearing house, until now they cover nearly all the Western States. The purpose of this information is to forecast run-off by the Bureau of Agricultural Engineering through its Irrigation Division. Primarily, the interest of this agency lies in irrigation and supplemental power, although, of course, the information developed is equally valuable to hydro-electric interests, to those responsible for city water supplies, and to the operators of the flood-control reservoirs that lie in the Southwest and other parts of the country.

The writer believes that the snow-survey data, thus collected, will induce its extension until all the major streams, at least in the West, have sufficient courses established and under observation to enable their performance for the following season to be known. The purpose is not in terms of a snow survey to determine merely how much snow lies on the ground, but wholly as a point on a definite curve from which one can forecast the probable run-off from that stream during the following year. A long-time accumulation of such points gives a curve of reasonable accuracy.

Howard T. Critchlow,[36] M. Am. Soc. C. E. (by letter).—It is scarcely necessary to emphasize the importance of the collection of fundamental data to which Mr. Wolman refers as the bookkeeping of the water-resources problem. There is an economic aspect to this problem, namely, that its solution requires money and that such money must be obtained, in most cases, from Government agencies. The writer has had a little experience with trying to get money from State legislatures for this purpose in order that the State may co-operate with the Federal Government in the collection of this necessary information. Engineers who appreciate the importance of these fundamental data, should make that fact known to the people who have control of the "purse strings" in the Federal Government and in the State Government. It does not require large sums, but it does require sufficient money to increase the facilities for collecting this information. This subject is discussed at some length,[37] in a report of the National Resources Committee on "Deficiencies in Hydrologic Data."

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[36] Engr. in Chg., State Water Policy Comm., Trenton, N. J.

T. T. KNAPPEN,[38] M. AM. SOC. C. E. (by letter).—One gains the impression from reading the interesting group of papers in this Symposium that flood-control engineering is as yet a poorly developed branch of the profession. The economic aspect of the problem seems to be the least developed in the opinion of most of the authors, although the engineering side provokes a rather varied discussion. There is manifested a feeling that the problem must be dealt with on broader lines than in the past, that technically it must be considered as a major problem in stream planning and development, and that, economically, new values must be reflected in the determination of justification if the popular demand for adequate protection is to be met.

These demands point the need for a critical overhauling of the methods of handling these problems in the past and, to-day, there is every evidence of this development in that section of the profession connected with flood control. Essentially, the problem may be divided between: (1) Hydrological studies, concerned with snowfall, rainfall, run-off, and flood characteristics; (2) planning, which includes the investigation and design of flood-control structures, such as reservoirs, walls, dikes, floodways, and channel improvements; (3) flood routing, which is used to determine the effect of such works on flood stages; and (4) economic studies, which determine the justification for the construction of flood-protection works and relative merits of the various plans considered.

Hydrological Studies.—The recent experience with disastrous floods on many of the important rivers of the United States has modified the American engineer's ideas as to the probabilities of their occurrences. Many engineers now are dubious of the value of frequency studies developed on the short recorded experience available, particularly in view of the constantly changing condition of water-shed development. A natural development is an investigation in the meteorological field that will indicate the possibilities of maximum precipitation conditions and a possible upward revision of intense rainfall frequencies. Another major consideration is the change in water-shed development that has been occurring over many years, the determination of its effect on run-off, and a consideration of curative or alleviating measures. The necessity for more basic data along these lines is apparent to every engineer dealing with these problems, and it is to be hoped that the curtailments that have resulted in the partial cessation of these activities in many parts of the country will come to an early end.

Planning.—Rarely to-day does one hear of proponents of levees, reservoirs, or channel improvements declaring that their method is the only one to use. On the other hand most engineers realize that any major plan may combine all of these, as well as other methods, such as floodways, outlets, and water-shed control. Furthermore, it is becoming more apparent, as emphasized by several of the authors, that flood control cannot be separated from other developments in water conservation and must be considered in civic planning and regional development plans.

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[37] Rept. of the Special Advisory Committee on Standards and Specifications for Hydraulic Data; Submitted by the Water Resources Committee on April 18, 1936, to the National Resources Committee.

[38] Associate, Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

Mr. Uhl has indicated that, in the New England States, flood control can be secured as an incident to power development with a reasonable degree of safety. Others may question the dependability of such control, but regardless of individual opinion, at least in many instances economic planning requires that the two be considered together. Another important factor is the need for increased stream flow for sanitary reasons which may justify a greater measure of conservation than power alone. The reasonable combination of these needs with those for flood control, together with a careful economic valuation of available storage and its most beneficial use, is an important regional problem. The solution of such problems has an important bearing on the planning of flood-control structures.

Flood Routing.—Only through accurate flood routing can the effect of flood-control works be determined and their economic justification established. Messrs. Morse and Thomas take a rather pessimistic view of the developments along this line and suggest that model studies under way at the Carnegie Institute of Technology, in Pittsburgh, Pa., offer the best solution to this problem. They seem quite hopeful that these models will give accurate results in the determination of the effectiveness of proposed works. Although the writer feels that the use of these models is unquestionably a wise move and that they will give valuable results, he is also of the opinion that recent developments in the analytical handling of these problems have eliminated their objections partly and that excellent results are now being obtained with the methods developed on the Muskingum flood-control project, in Ohio.

It should be emphasized that the analytical flood-routing methods can be checked by applying them to known past floods. When properly made, these checks have resulted in accurate reconstruction of past floods. This procedure seems parallel to that used in the model studies, in which the model is adjusted for known past floods.

The analytical method is based upon the principle that constant ratio exists for any reach of the river, between the valley storage in the reach and a weighted flow determined from inflow and outflow. The equation may be written,

$$K = \frac{t\,[0.5\,(i_2 + i_1) - 0.5\,(o_2 + o_1)]}{X\,(i_2 - i_1) + (1.0 - X)\,(o_2 - o_1)} \quad \ldots\ldots\ldots\ldots (3)$$

in which K = ratio of storage increment in the reach, in day-second-feet, to corresponding weighted flow increment, in cubic feet per second; t = time unit of computation, in days, or in fractions of a day; X = fraction of weighted flow increment that is derived from the inflow increment; i_1, i_2, etc. = total instantaneous inflow, in cubic feet per second, to a reach at the beginning of successive time units (t); and, o_1, o_2, etc. = corresponding instantaneous outflow to a reach at the beginning of successive time units. The numerator in Equation (3) is the storage increment, in day-second-feet, and the denominator is the corresponding weighted flow increment, in cubic feet per second. If the denominator is rewritten in the form: $(o_2 - o_1) + X\,[(i_2 - i_1) - (o_2 - o_1)]$, it is apparent that the first term represents the "prismatic" storage below the normal surface slope, and the second term represents the "wedge" storage increment. It will be noted that this eliminates at least part of the objection expressed by the authors to the analytical method. This method was brought to its present development by Thomas S. Burns, M. Am. Soc. C. E., and Frank B. Harkness and G. T. McCarthy, Assoc. Members, Am. Soc. C. E., in the flood-routing studies on the Muskingum project.

The analytical flood-routing method as now applied is believed to give fair results on the main stems of important rivers but in most cases its application is not practicable on the tributaries particularly where many streams of about the same relative importance are coming together. For such cases, some method of developing modified hydrographs for reservoir systems is needed, which will account for the location and the collection characteristics of the controlled areas as well as their extent. Possibly, the technique of model analyses can be developed for the accurate solution of such cases. At the same time there is a need for an analytical method that can be used at least for preliminary studies.

Economic Studies.—Mr. Jacobs states that: "The construction of flood-protection projects on a monetary economic basis alone is a thing of the past. Where the lives of human beings are endangered and where great suffering may occur as the result of floods, protective works should be constructed." This statement is qualified later in the paper by the following: "It is also essential that those who must pay for the protective system have full knowledge of the cost, of what burden they must bear, and of what security the final developed plan provides."

On the other hand Mr. Uhl states,

> "Attempts have been made to rationalize the second problem, namely, that of flood prevention or reduction, on a purely economic basis. If definite information were available, the solution would be relatively simple; the maximum expenditure economically justified to prevent recurrence of certain flood damages would be the capitalized present worth of the damages over the period of flood frequency causing them."

Somewhere between these points of view the answer to the problem must lie. Taking the case of an individual community planning to protect itself, it can determine its policy knowing the costs and the degree of protection. The people of that community may see more to the problem than the capitalized value of average annual damages. They may do as the people of Dayton did, after the 1913 flood—envision a blighting of their community, an almost complete abandonment of the flooded areas, with the loss in private investments therein far in excess of the capitalized annual direct and indirect flood losses based on frequency calculations.

Cities such as Pittsburgh, Pa., Hartford, Conn., Binghamton, N. Y., Springfield, Mass., Wheeling, W. Va., Cincinnati, Ohio, Louisville, Ky., and Paducah, Ky., can take little satisfaction from statistical computations that show the average annual losses to be small based upon frequency studies. They are faced with the plain facts of tremendous property depreciation and restricted development, creating a situation in which the assurance of protection against future floods has a value in excess of the capitalized average annual value of flood losses.

The writer would like to suggest that the answer to the problem lies neither in the abandonment of all economic rules nor in the strict application of the criterion of capitalized value of average annual flood losses in the accepted sense, but rather that it may be found in the determination of benefits which may be concerned with the restoration or maintenance as well as the appreciation of property values depreciated by flooding, with an added evaluation of the benefits to Government property and of the benefits due to elimination of indirect damages outside the flooded areas. This attitude is based on the assumption that the intangible values not ordinarily evaluated, such as fear of loss of life, mental distress, tendency to abandon flooded areas, etc., as well as direct flood losses, are reflected in such depreciation.

The evaluation of these benefits although difficult, may be worked out and the need for flood protection established in such places as the Ohio River, Connecticut River, and Susquehanna River Valleys where recent disastrous floods have demonstrated the vulnerability of the valley towns. On the other hand, may there not be many other places as vulnerable that have not felt the disaster of a major flood to "drive home" the necessity for, and value of, flood protection? Despite the previous floods, the citizens of Pittsburgh were not sufficiently flood-minded before 1936, to demand protection and to be willing to pay at least part of the costs. Prior to the 1937 flood, it would have been most difficult to convince the people of Louisville or Paducah that their cities were in danger of serious flooding. Certainly, there is a necessity for the development of sound engineering and economic methods for determining need for protection, not only for places which floods have attacked, but also for those places which, although equally vulnerable, have not felt the scourge of a major flood in recent times.

Assuming that the need for flood protection can be evaluated on a broad enough basis to justify protection for the more populated sections, it should be possible to apply the developed data to the internal economics of various projects. There are many problems to be solved therein, such as the relative balance between reservoirs, channel work, and levees, the economic size for any reservoir, the height to which levees should be built, etc. The solution of these problems requires the careful development of the basic data by zones and elevations as well as extensive flood-routing studies to assign values to the units under consideration. Furthermore, frequency studies are necessary as a basis for the determination of average annual values.

The economic problem is not a simple one in any case and on large river systems becomes highly involved; but only by the application of thorough field investigations and office studies can adequate and sound economic, as well as engineering, studies be developed.

M. C. TYLER,[39] M. AM. SOC. C. E. (by letter).—An excellent account of the situation with respect to flood-control work by the Federal Government has been given by Colonel Covell. He has traced the legal and legislative procedure through which the United States has passed to reach the stage at which the Congress has authorized many worthy and beneficial projects for the protection of lives and property from the ever-recurring ravages of excessive flood flows caused by periodic extraordinary precipitation.

It was inevitable that, sooner or later, the people would demand that the National Government assume some obligation for the protection of citizens from the injuries and devastations caused by floods which frequently arouse so much human interest, first in one section of the country and then in another. Flood-control works, of necessity, affect the navigation of rivers in some manner, and it is logical that the Federal Government should accept its proper function with respect to flood control.

The Constitution of the United States confers upon Congress the power "to regulate commerce with foreign nations, and among the several States, and with the Indian tribes," and by this provision there was transferred from the States to the Federal Government the control of all the navigable waters of the country for the purpose of navigation. At first, it was seriously doubted whether the power to regulate, comprehended the right to improve; that is, whether the improvement of rivers and harbors was a subject of national concern and of constitutional appropriation. In May, 1824, Congress passed the first Act to improve navigation entitled, "An Act to Improve the Navigation of the Ohio and Mississippi Rivers." About that time the luminous decision of Chief Justice Marshall, in the case of Gibbons *vs.* Ogden, established clearly and indubitably the power of Congress with respect to the waterways of the country which could carry interstate commerce, and removed completely the doubts of earlier years. That decision of the Supreme Court of the United States marked the birth of a permanent Federal policy of river and harbor improvement—a policy which has grown from year to year with the increase of population and needs of commercial transportation. Now, flood control also has been made a national activity by Federal law.

The general flood-control bill approved June 22, 1936, and the Overton Flood Control bill approved June 15, 1936, were studied for a period of almost two years by committees of Congress. They were based on the reports of the Chief of Engineers containing the results of studies extending over a long period of years. The Mississippi River Commission is continuously engaged in studying the flood-control problem on the lower river, and advance planning for additional works necessary to provide full protection has continued with the prosecution of the works authorized for construction by Congress. The voluminous and comprehensive data presented in the reports of the Army Engineers[40] provide the principal basis for national planning with respect to the development of water resources. They permit the preparation of a program for such development on a scale limited only by advisable expenditures. This program may go hand in hand with the development of land resources, reforestation, and measures for the prevention of erosion and retardation of run-off. It is very seldom that major flood-control structures conflict or interfere with other water uses.

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[39] Brig.-Gen., Corps of Engrs., U. S. Army; Asst. to Chf. of Engrs., War Dept., Washington, D. C.

The Flood-Control Act of June 22, 1936, embarked the Federal Government on a flood-control program which is destined to develop in an economic and orderly manner so as to afford great and lasting improvements throughout the land for the preservation of lives and property and the betterment of living conditions generally.

Since the enactment of the general Flood Control Act, the Army Engineer organization has been working vigorously in making detailed plans for beginning construction by contract of the flood-control works authorized. The War Department will be ready, when funds for actual construction are appropriated, to begin work without delay. In the design of dams and appurtenant works, the Chief of Engineers is fortunate in having the assistance of many of the ablest and most experienced consulting engineers in the United States, and hopes to secure the services of others as the need develops. Much help is also being afforded through co-operative arrangements with other Departments and with the hydraulic laboratories of universities.

Although the planning of flood-control works by the Federal Government is in a most favorable and constructive situation, this does not mean that everything desired is already accomplished. There is much hard work to be done, and innumerable difficult problems to be solved. However, the ground-work has been well laid. Questions of local co-operation as to financing appear difficult, but they can be solved with patience, judgment, and co-operative thinking.

Gordon R. Williams,[41] Assoc. M. Am. Soc. C. E. (by letter).—In discussing the economics of flood control, Mr. Uhl states that, on a purely economic basis, the justified expenditure for the prevention of flood damage caused by a flood of a certain magnitude and frequency "would be the capitalized present worth of the damages over the period of flood frequency causing them." It seems desirable to clarify the procedure implied for the determination of future damages for which the present worth is to be obtained. There may be situations in which too much reliance on the figure for past damages would erroneously justify too large an expenditure for flood protection and then, again, it might lead to an entirely inadequate expenditure. As an example of the former situation consider the Winooski River Valley which is mentioned by Mr. Uhl.

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[40] House Doc. No. 308, 69th Cong., 1st Session.

[41] Associate Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C. Submitted with the approval of the Director, U. S. Geological Survey.

The reconstruction that occurred in the Winooski River Valley after the flood of 1927 was undertaken with the intent of avoiding, as far as possible, a repetition of former damages. Consequently, highways were relocated on higher ground, concrete and steel truss bridges with large openings replaced light bridges with inadequate openings, spillways of dams were enlarged, and many people rebuilt their homes on higher ground. The result is that if the flows of 1927 occur again, the damage in the Winooski River Valley will not be as great as it was in 1927. In other words, some of the justified expenditure for the control of flood waters was spent at the time of reconstruction. Of course, if flood-control measures are adopted immediately after a flood there may be a different policy in regard to expenditures for reconstruction, but if reconstruction has already taken place weight should be given to the situation just considered. A similar situation may exist in other communities where there has been a permanent decrease in the value of property which lies in the path of flood waters.

In contrast to the foregoing example are the more numerous arguments for using, as a basis for computing justified expenditures, a value that exceeds past damages from the greatest flood. Again, taking the Winooski River as an example, one finds, that the U. S. Army Engineers computed that an expenditure of $433 000 was justified for protection against the 100-yr flood. However, this sum should be increased because a structure which protects against the 100-yr flood also protects against damage caused by the smaller but more frequent floods. Actually this was done and the justified expenditure was increased to $1 055 000.

The problem of increasing the justified expenditure because of a possible increase in the value of property which will be benefited by flood control is analogous to the one of determining, theoretically, the justified expenditure for a trunk-line highway. Roughly, the procedure is to take a traffic count of vehicles that use existing facilities, and then compute the resulting revenue in taxes that will come from the sale of gasoline used by the vehicles. Obviously, this does not give a true picture of the situation which will result from the construction of the road because there is little doubt that the volume of traffic will exceed the original traffic count and, in addition, there will be an almost indeterminate saving to private individuals and to industries that benefit by the construction of the road. The original estimate of justified expenditure must be increased, but how much?

A comparable situation exists in flood-control planning. If reasonable protection is given to river valleys, the cities and towns affected may grow, industries may be attracted, property values will doubtless increase, and the wealth of the individual citizens will be augmented. In such cases the figure for the past flood damages becomes entirely inadequate as a basis for determining the price to pay for future protection.

This discussion has considered the purely economic justification without any allowance for the humanitarian aspect stressed by Mr. Jacobs. There is no question but that this consideration should be stressed, but not to the point where a community assumes a burden which may even threaten its financial ability and thereby interfere with the fulfillment of its normal commitments. Experience may prove that in some cases the soundest method to pay for future damages will be to establish an insurance fund maintained at least in part by small premiums paid by those who cannot, or will not, move out of danger.

ARTHUR T. SAFFORD,[42] M. AM. SOC. C. E. (by letter).—Engineers working on designs of river structures, officials and operating men in hydro-electric power stations, and owners and managers of industrial plants located on New England rivers will find the paper by Mr. Uhl extremely valuable, not only because of the important data, but because of its arrangement and the accurate conclusions contained therein. One must have had the experience of living and designing structures in and on New England rivers to interpret properly flood records. One needs only to read over the newspaper accounts and early reports of the 1927 and 1936 floods, to realize how little was known at the time about their causes, and the lessons to be drawn from them. Unless records of river heights and conditions given out during a great flood are correctly reported by newspapers, or correctly broadcast, the result is to confuse and make it more difficult for the authorities to know when and how to act in protecting the public.

This is one of the many papers contributed by engineers far enough removed from the hysteria produced by the floods, to make them valuable as a permanent record. Even a year after the Great Flood (which name, for many years, will be given to that of March 20, 1936), opinions are expressed and plans made for eliminating future floods, which in many cases have no more basis than the hope that such a result is possible.

The writer's official responsibilities during the 1927 and 1936 floods were confined to the Merrimac River, at Lowell, Mass.; but he was in the employ of the Water Power Company there, during the floods of 1895 and 1896 mentioned by Mr. Uhl; and was familiar with the flood marks of 1785 and 1852. At Lowell, there are on record six or more floods during 150 yr which registered 10 to 13.5 ft on the Pawtucket Dam, and about 15 ft of back-water, but which did little damage beyond the quiet flooding of streets, cellars, and mill basements. Machinery and household goods, and goods in process left on the machines were inevitably wet; but the protective margin of a few feet above the peak of these floods was sufficient to leave intact all hydraulic structures of the Company, due to the foresight of Mr. Francis, by installing the Francis gate,[43] before the famous 1852 flood, as a result of his careful studies of former floods.

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[42] Engr., Proprietors of Locks and Canals on Merrimack River; Cons. Hydr. Engr., Lowell, Mass.

[43] *Civil Engineering*, February, 1937, p. 143.

The flood of March 20, 1936, on the contrary, which registered a final height of 102.02 ft, or 20.02 ft above the crest of the Pawtucket Dam, and 6.5 ft above any previous peak, produced an additional 8 ft of backwater. For the first time the Merrimac River was not held within its ordinary flood channel, but broke through at several points, went entirely out of bounds, and isolated large sections of the city. It was only prevented from wrecking parts of the business center of the city, by a skeleton dam, 3.5 ft above the Francis gate, and a timber and sand-bag dam across the railroad right of way into the city from the north (see Fig. 18).

After the water subsided, it was found that the city had had a very narrow escape from lasting injury. The great mass of fill accumulated over a period of 100 yr and more over low areas on the banks of the river or dumps was gone; many neighboring buildings were wrecked; certain streets in the path of the flood were left with half the buildings either moved or wrecked; the great Northern Canal wall of the Proprietors of Locks and Canals (which had been overtopped for the first time in its history) suffered serious injury in several places; and most of the conduits carrying electric power to the river mills were so thoroughly wet that for nearly a year thereafter, weak places kept appearing and had to be replaced.

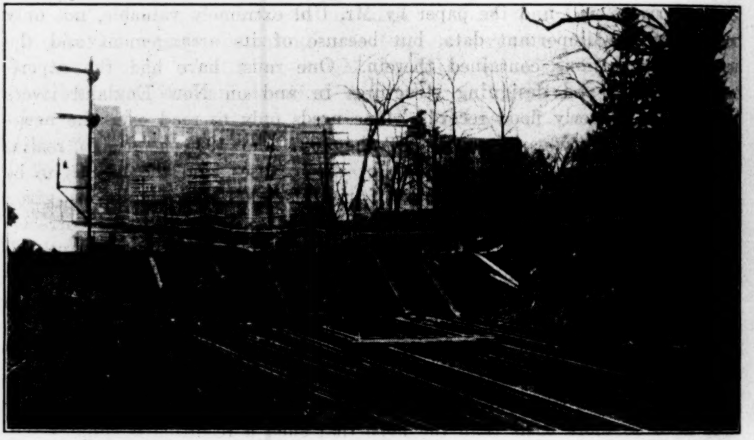

FIG. 18.—SAND BAG AND TIMBER DAM ACROSS THE BOSTON AND MAINE RAILROAD WEST OF LOWELL RAILROAD STATION.

All these structures belonging to the Water Power Company and the industrial plants on the river, were repaired before the end of the year, although much of it had to be done by continuous week-end operations. Fig. 19 shows the repair work in progress, the unfinished part indicating the nature of the break.

From personal examination and published reports by utilities, insurance authorities, and others, apparently these same conditions, both during and after the Great Flood, obtained at Concord and Manchester, N. H., and at Lawrence and Haverhill, Mass.

The fact that, in the 1936 flood, the stage was 6.5 ft higher, and the quantity of water 65% greater than any previous flood, with existing margins of safety wiped out, would appear to make some improvement necessary if these communities are to survive without the anxiety of a repetition of this situation.

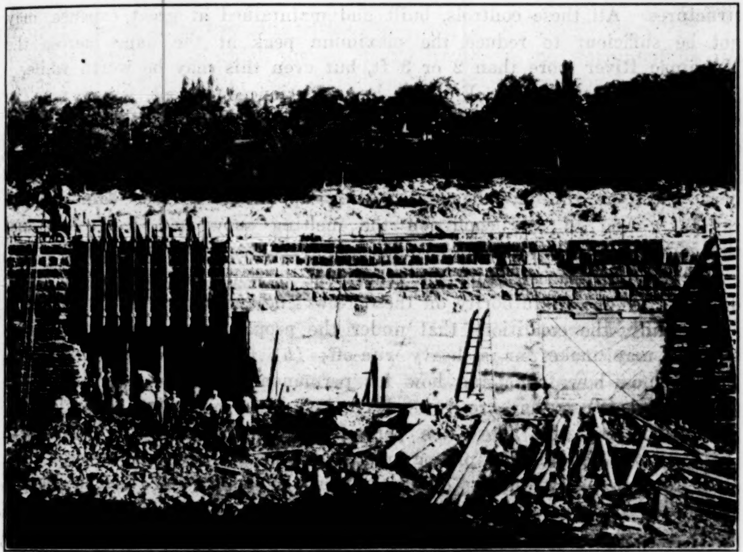

FIG. 19.—ABOUT THREE-QUARTERS OF CANAL LINING REPLACED; REMAINING QUARTER OPEN SHOWING CHARACTER OF BREAK.

Any study of flood control for New England rivers would seem to require a knowledge of what quantity and height of water had occurred prior to this 1936 flood; what additional burdens this flood had placed on the industrial cities; and what reasonable improvements could be made in the nature of reservoirs, channel improvements, and dikes, to bring the local control down to the situation as it prevailed during 150 yr or so prior to 1936. For Lowell and the Merrimac River, this means a peak run-off of not more than 100 000 cu ft per sec instead of 173 000 cu ft per sec; a depth on the Pawtucket Dam of not more than 13.5 ft instead of 20 ft; and a corresponding reduction of 8 ft in the height of the backwater below the city.

Assuming flood reservoirs to be actually empty and 100 000 cu ft per sec to be the maximum at Lowell, and the 17 500 000 000 cu ft of natural flood pondage above Lowell, Manchester, Concord, and Bristol, N. H., to be not reduced, the equivalent of two additional Lake Winnepesaukees would be required.

Mr. Uhl wisely refers to several "partial" solutions, and they should be more generally considered as such. The promise of more than this simply deceives the public who must pay the bill, either by assessment or taxation. These structures may be storage reservoirs, improved channels and dikes, and free channel bridges instead of reinforced concrete arched structures. All these controls, built and maintained at great expense, may not be sufficient to reduce the maximum peak at the dams across the Merrimac River more than 2 or 3 ft, but even this may be worth while.

In recent years, New England has experienced storms which may be called torrential, and such storms appear to be inevitable both locally and over areas of several thousand square miles. They make the White Mountains and the Green Mountains look more and more like the roof of a barn with the run-off, ultimately, being approximately the same as the rainfall. Add to this condition the melting snow and ice cover, and the conditions existing prior to March 20, 1936, are re-created.

As a lesson learned from the 1936 flood, it would appear that it is the duty of any one in authority on these New England rivers: (*a*) To understand, fully, the conditions that under the proper combination of circumstances, may make for a heavy run-off; (*b*) to have complete records; (*c*) to know how much and how far permanent structures will stand the strain; and (*d*) to have ready at least a skeleton on which to build temporary barriers well above the permanent structures. Office records in the form of diagrams and photographs should be available and studied from time to time, as freshets (even mild ones), occur; and lessons learned from each freshet should have their place in the records.

In addition, an important element in preparing for New England floods is time—time to transfer human beings, livestock, and necessary and valuable possessions to higher ground. In Vermont, in 1927, besides property damage approaching $40 000 000, there was a large loss of human life (87 were drowned, 55 in the Winooski River Valley), for the most part due to the suddenness of the rise of the water and the difficulty in maintaining communications.

Very few lives have been lost in floods of the Merrimac River although the time of rescue often has been short. The peak of the 1936 flood at Plymouth, N. H., was on March 19, from 8:00 A.M. until noon, and at Lowell on March 20, from 5:00 P.M. to 11:00 P.M.; but the Lowell peak had been forecast early on the morning of March 19, the Francis gate had been dropped, and work had begun on the protective dams above that point by 7:00 A.M.

The reports and predictions of flood heights from the up-stream stations, maintained by the public service companies and other authorities, were so frequent and reliable that not only could the highest points to be reached be estimated, but the length of the peak. There was ample time to warn officials of the city, the U. S. Works Progress Administration, Red Cross, State Police, and the National Guard; and as a result of the splendid co-operation which American people always have given when they know what and when there is an emergency, the City of Lowell did not have an appalling loss of life. It speaks for the value of the operating forces of the public utilities, beyond that of temporary activities. Most of the information broadcast by radio, in so far as it was insufficient or mistaken, probably increased the anxiety and may have done more harm than good. The radio can be made a splendid addition to the telephone and telegraph, if in the hands of proper authorities.

Some conclusions, which appear to the writer to be warranted by the experiences of the March, 1936, flood, and its record-breaking heights, flow, and currents, are:

(1) Engineers and others, concerned with flood work, should think in terms of maximum run-off, possible from heavy rains, warm weather, and melting snow, and not of what heights had been reached through a limited period of years. They should be "conditioned" to the idea of the 100-yr flood. The precipitation and water equivalent of the Pinkham Notch record for March, 1936 (29 in.) may occur the next time in almost any of the mountain valleys of New England.

(2) More records should be taken every half hour, or so, for the upper rivers and principal tributaries, as well as the main river stations, in order to be able to predict whether peaks will be distributed or be concentrated.

(3) Abutments should be higher; if the last few feet interfere with approaches to power or gate-houses, or if there are railroad cuts, such as those at Lowell, Mass., and Manchester, N. H., concrete ribs, with holes drilled for plank flash-boards, may be substituted. At the Old Guard Locks, at Lowell, an additional 3.5 ft above the Francis gate was taken care of without excitement by that form of construction, built after a study of the 1927 flood appeared to warrant it.

(4) Every growing industrial city or town, situated on a New England river, should have a map showing flooded areas for the maximum flood. This should be used as a picture in the acceptance of streets, in building bridges, in laying out sewers and in granting building permits. The occurrence of ice jams, accumulating pressure, and wrecking bridges and other structures, is always a possibility before the winter is over. No structures should be built which will add to the flood menace of any district.

(5) The valley storage which obtained during the 1936 flood from Lawrence, Mass., to Franklin, N. H. (estimated at 17 500 000 000 cu ft) was about 50% greater than the combined storage in Lake Winnepesaukee and other reservoirs that were already full and discharging before the danger of a higher peak was over. There is no advantage in depending upon flood reservoirs unless they are nearly empty before the extreme test which they are expected to meet.

(6) The duty of setting permanent markers on New England rivers for the 1927 and 1936 floods, should not be allowed to be forgotten. The fact

that the 1936 flood carried 65% more water than ever before, may make these records good for a century or more.

(7) Federal and State operating activities (such as the Weather Bureau, Geological Survey, U. S. Engineer Corps, State police, etc.) should have on file the records of the 1936 flood; and, at times of future floods, they should refer to the unusually good records of the utilities operating hydro-electric power stations as well as to their own. Snow records should be much increased in number.

This discussion does not do justice to Mr. Uhl's paper and is too personal. Having lived through the strain of the week and more of the floods that occurred from March 13 to 20; and afterward waiting for the full extent of the damage to be known and another two months before repair work could be begun, it is not easy to study any problem of flood control for the Merrimac River impersonally; but it leaves one with a profound respect for the forces of Nature, and the difficulty of competing safely with them.

The declivity of New England streams is in their favor. The Pemigewasset River, at Plymouth, could not have passed 65 000 cu ft per sec on March 19, 1936, or the Merrimac River, at Lowell, 173 000 cu ft per sec the next day, except for the slope maintained to produce the high velocities. Velocities as great as 20 ft per sec were not uncommon.

Local flood control by dikes, and river channel improvements on limited areas, should include the sewerage problem, because of many individual outlets into the river. When such outlets "back up" they can be even more dangerous to the public than quick flooding by clean water.

From the insurance standpoint: If the damage to the Merrimac Valley from the 1936 flood was roughly $20 000 000, and a flood of such intensity occurred only once in 100 yr, the sum of $400 000 at 4% compound interest would pay the equivalent at the end of 100 yr; if once in 200 yr, the sum of $8 000. There would appear to be a field for flood damage insurance, as it might be written by companies operating on a national scale, to protect at least a part of the investment in industrial plants, stores, warehouses, and their contents against loss.

W. G. HOYT,[44] M. AM. SOC. C. E. (by letter).—No city of the United States of equal size is situated so unfortunately with reference to rapid concentration of flood waters as is Pittsburgh, Pa.; yet, with respect to flood damage, Messrs. Morse and Thomas describe Pittsburgh as a "lucky city," a description that will be concurred in by all engineers who have studied cause and effect of major floods in the basins of the Allegheny and Monongahela Rivers However, there seems to be no valid reason why Pittsburgh's luck will always hold. Eventually, Nature's roulette wheel will record a double zero. When that occurs it is sincerely to be hoped that a catastrophe will be averted by means of a "superior type" of reservoir control combined with an efficient system of flood-forecasting.

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[44] Hydr. Engr. (Prin.), Conservation Branch, U. S. Geological Survey, Washington, D. C.

Whether or not a super-storm can occur in the Monongahela and Allegheny River Basins simultaneously is a question which, perhaps, meteorologists will some day be able to answer. It is true that no such outstanding storm has thus far been recorded—the Miami (Ohio) storm of March 24 and 25, 1913, and the storm of January, 1937, having had their centers to the west, and the storm of March 17 and 18, 1936, to the southeast.

If any of these storms had been centered over the Upper Ohio River Basin greater floods would have resulted than the record-breaking flood of March, 1936. However, even if it is assumed that precipitation over the basin which has been recorded in the past is an index of future possibilities, Pittsburgh has thus far apparently been fortunate, and especially so in March, 1936. During February, 1936, there were three thaws in the Monongahela River Basin and one in the Allegheny River Basin which materially reduced the water content of the snow then on the ground. Conservative estimates indicate that the reduction in water content may have amounted to between 1 in. and 2 in. in depth over the Allegheny River Basin and to between 2 in. and 3 in. over the Monongahela River Basin. The minor flood peaks resulting from these thaws broke up and removed the unusually thick channel ice. To some extent at least flood run-off must have been reduced by increased infiltration as a result of the reduction of frost in the ground. All these conditions were favorable to Pittsburgh.

Messrs. Morse and Thomas refer to the turning of rain to snow above Oil City, Pa., during the storm of March 17 and 18, 1936. An analysis of the snowfall, rainfall, and run-off during the flood period discloses how important this freak of temperature was in lowering the flood crest. Apparently, much of the rain turned to snow not only in the Allegheny River Basin above Oil City, but also in the Monongahela River Basin above Morgantown, W. Va. The effect of this occurrence was materially to reduce the resultant run-off from that storm. That this condition existed is evidenced by the fact that in the upper basins of both the Monongahela and Allegheny Rivers the run-off during the period, March 20 to 31, after the flood crest had passed Pittsburgh, was in excess of the precipitation, indicating a postponement of run-off because of the snow of March 17 and 18, equivalent to possibly 2 in. in depth of water over the basins. The peak run-off during the period, March 20 to 31, in these areas, was greater than the peak run-off during March 17 and 18, when Pittsburgh was experiencing its maximum stage—a stage very considerably lower than that which would have occurred if the rain had not turned to snow over nearly half the drainage basins.

Messrs. Morse and Thomas seem to have departed from an excellent presentation of facts into the realm of conjecture when they referred to "the man-made floods." It is true, that at Pittsburgh, Man is occupying the valley and encroaching on the channel that Nature has designed to carry the floods and, to this extent, the flood damage is man-made. Getting into the way of a flood and being damaged by it however, are far different from making a flood. No evidence is presented to indicate that the flood of March, 1936, was increased as a result of changes made in forests or other vegetative cover of the basin, or as a result of any of Man's activities.

Do engineers play fair with the public who look to them for facts and advice if they lead the public to believe that Man is accountable for the excessive run-off of a major flood, such as that of March, 1936, in the Upper Ohio River or that the run-off following major storms, involving the unusual timing of the meteorologic events which create floods, can be controlled materially, except by engineering structures?

LT. COL. J. D. ARTHUR, JR.,[45] CORPS OF ENGINEERS, U. S. ARMY (by letter).—The present status of Federal flood-control projects has been fully outlined by Colonel Covell. It is to be noted that the system of reservoirs for flood control on the Ohio River, either authorized or under construction, is distributed over a wide area, on many tributaries. Little has been published as to the methods of operating this system to provide the maximum benefits.

The operation of the fourteen reservoirs now (1937) under construction in the Muskingum Valley will be placed in the hands of the Muskingum Watershed Conservancy District, a political sub-division of the State of Ohio. It is only human to assume that it will be operated to provide the maximum protection to the residents of the Muskingum Valley. However, will such operation provide the maximum protection to the residents along the Ohio River below Marietta? Engineers know that the Miami Conservancy District System provides full protection to the residents of that Valley, but they likewise know that, with its uncontrolled outlets, benefits to localities along the Ohio River will not necessarily accrue. In fact, it is easy to imagine conditions under which it might prove to be decidedly harmful.

When the system is extended to other tributaries, each with a local flood problem in its valley, does it not follow that, unless there is a single office with full authority to control operations on all tributaries, the system will not be operated for the greatest good of the greatest number? The construction of the system must be the primary objective, but studies should be initiated at once of the best methods of operation.

The problem is a difficult one, and it is not simplified by the requirements of local participation in costs. If the residents of the Muskingum Valley, or of Pittsburgh, for example, are taxed for a portion of the costs, might they not well ask why they should be interested in what happens at Cincinnati? Perhaps this phase of the subject did not receive proper attention when the policies enunciated in the Flood Control Act of 1936 were adopted.

JOHN H. MEURSINGE,[46] ASSOC. M. AM. SOC. C. E. (by letter).—Except China, there is probably no country in the world which has suffered so much from floods as the low lands along the North Sea, which are better known as Holland, or The Netherlands.

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[45] Dist. Engr., U. S. Engr. Office, Zanesville, Ohio.

[46] Structural Designer, General Petroleum Co., Los Angeles, Calif.

The first levees in Holland were built by the Romans to protect their military highways. As the construction of levees has been continued ever since, a protective system has been developed, which probably has no equal in the world. It is small wonder that some Dutch engineers do not consider the levee system of the Mississippi as very effective; but in an interesting comparison between flood problems in Holland and in the United States,[47] C. E. W. van Panhuys, District Engineer for the Netherlands Government, states that engineers in the United States have done everything technically and financially possible for such a young country.

In connection with this Symposium, the question may be raised as to whether American engineers could benefit by Holland's long experience with floods. The answer is "yes," always bearing in mind the fact that the Dutch engineers are mainly concerned with delta problems, so that dams and reservoirs are of minor importance to them. The purpose of the collapsible dams constructed on the Meuse River about 1917, was to improve navigation in summer. They are made to disappear during the flood season.

Furthermore, the area considered is rather small—about one-five-hundredths of the Mississippi River water-shed. However, it is not the size that counts, but the effectiveness of the protection system. Since about 1880 human lives have scarcely ever been lost in Holland on account of floods.

For each river, the Dutch engineer defines two channel sections: The so-called winter section and the summer section. Both are protected by levees. The small dikes along the summer section simply protect the valuable grazing grounds in the winter section against the summer flash floods. The winter section is from 1 500 to 3 000 ft wide, and the summer section from 450 to 675 ft wide. Within these sections no encroachments are allowed except brick kilns, which may excavate enough clay to equal the volume of the buildings they occupy.

Occasionally, a severe flood will overtax the winter section and spillways are provided to meet this emergency. To understand the function of these spillways the geography of the Meuse-Rhine delta must be considered. Most of the lands have elevations which vary between the low-water and high-water stages of the rivers. If a dike collapses much land would be threatened with submersion except for the fact that it has been divided into many different diking districts or "polders," each of which are protected by minor levees. Such a "polder" is an administrative unit, administered by the property owners themselves. They are under the supervision of the provincial authorities, however, who create and consolidate the districts if desirable. The district maintains all its levees, makes minor repairs, and operates the pump-houses which take care of the drainage of the seepage water.

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[47] "Overstroomingen langs Ohio en Mississippi", *Ingenieur,* Royal Inst. of Netherlands Engrs., March 5, 1937.

In case of a dangerous flood, each "polder" sends its guards to the dikes. Their services come under the control of the engineers of the Netherlands Government who take charge of the entire river as long as the emergency lasts. If a dike threatens to collapse, or actually breaks, the engineers have the same authority as is generally granted to military commanders in time of war.

The advantage of many diking districts is obvious. When a spillway discharges its water over the dike, it submerges one "polder" only. This district is occasionally sacrificed for the benefit of the others.

In spite of these spillways, breaches in dikes still occur. Hence, each district has been provided with a so-called "terp." This is nothing but a properly located levee, which is high and large enough to take care of the population and live stock of the "polder" in case the district happens to be flooded. Frequently, the church and some public buildings are located on the "terp"; and it hurts no one's feelings when the church is converted into a cow barn during times of emergency.

The methods of making emergency repairs have changed very little in the past hundred years. The first sign of danger is an increased seepage flow in the dike. This water is likely to wash away so much of the material that all the fingers of all the Dutch boys would be unable to stop it. Mats made of willow branches, staked down on the inside talus of the dike, are more effective. They keep the material in its proper place, at the same time allowing a free flow of seepage water. This method does not cause any increased water pressures inside the dike, which might produce a breach.

Although breaches seldom can be stopped until the floods have passed, occasionally the "hay-wagon" may solve the problem. One or several wagons tightly packed with hay are dumped, wagon and all, in the breach. Whereas a more solid material would be swept away immediately by the in-rushing waters, the hay permits water to flow through, and the wagons may form a foundation on which sand bags can be piled for temporary repair.

Most of the breaches in Holland have occurred in the dikes along the Meuse, the most troublesome river in The Netherlands. In its low stage the run-off may be as low as 7 000 cu ft per sec, and in its flood stage as high as 100 000 cu ft per sec.

In 1923, the construction of a series of collapsible dams was begun to improve navigation conditions in summer. While this construction was at its peak a run-off occurred which was 15% greater than the previous recorded maximum. It flooded all the works and broke levees everywhere.

The cause of this unusual occurrence has been explained in a report by the late Dr. C. W. Lelij,[48] submitted when he was District Engineer for The Netherlands Government. Dr. Lelij's investigation showed that deforestation and newly cultivated lands had had no important influence on this run-off, because the average run-off of the floods had been decreasing in the previous fifty years.

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[48] "Rapport betreffende de Verbetering van de Maas voor groote Afvoeren", door Dr. C. W. Lelij, Hoofd Ingenieur van den Rijkswaterstaat, s' Gravenhage, *Algemeene Landsdrukkerij*, 1926.

This remarkable flood was caused by the following conditions: From November 24 to December 20, 1925, much snow had fallen in the mountains of the Meuse water-shed. Immediately thereafter a heavy thaw had set in, and simultaneously a heavy rain soaked the entire shed. Remarkably enough, the peak of the storm moved with the peak of the flood from the south to the north.

In other words the determining factor in this flood was the weather; and as no one has yet learned how to control the weather, the Dutch engineers (who have accumulated an enormous mass of run-off data) are as much "in the dark" concerning the possible maximum run-off as their American colleagues. The authors of the Symposium suggest the gathering of more data, but the writer believes that the value of data should not be overemphasized, because some of the floods of the future may be much worse than data will ever reveal.

This is not the only important consideration stressed in Dr. Lelij's report. It calls attention to the fact that the bottom of the winter section of the Meuse River was rising at a rate of 2 mm (0.08 in.) per yr. In the past 750 yr, this bottom has risen 5 ft.

At some other places conditions were worse. At the end of the Nineteenth Century many railroad bridges had been built. To compensate for the piers the area of the winter section had been increased by excavating the channel. In 1926, when Dr. Lelij's report was published those places had been entirely filled in again. This condition, undoubtedly, had contributed its share to the higher floods.

American engineers probably never will know the possible maximum run-off of the Ohio or Mississippi Rivers. In the centuries to come the lands adjacent to the river, which are flooded regularly, will become higher also; and the floods, subsequently, will grow more severe. Undoubtedly, there will be flood disasters as long as people keep making homes along rivers. As a perfect protection system seems an impossibility, it may be advisable to consider first a more adequate protection of human lives. It may be worth while to spend some money in creating an efficient emergency organization that could be mobilized on short notice. Places of refuge, where the flood victims could find temporary shelter, may fit in to this plan, and although from a technical and economic standpoint the results may not seem great, the most important part of the job would have been done at a relatively small cost.

H. K. Barrows,[49] M. Am. Soc. C. E. (by letter).—A clear picture of flood conditions in New England, a very full discussion of the interesting features of the flood of March, 1936, and much information about earlier great floods are contained in the comprehensive and excellent paper by Mr. Uhl.

The flood of November, 1927, centering principally in Vermont, Northern New Hampshire, and Massachusetts, was due to excessive rainfall, reaching a maximum of probably 12 in. in the Green Mountains and resulting in especially high run-off from the smaller streams. Some noteworthy data are given in Table 10(a). The flood hydrograph[60] in these cases was approximately triangular in form, beginning at about 2% of the peak flow, rising uniformly to 100% at the twelfth hour, and then decreasing uniformly to 20% of the peak flow at the thirty-sixth hour. This would correspond to total flood run-offs, as shown in Column (4) Table 10(a), ranging from about 17 in. for the smaller drainage area to 6.5 in. for White River (695 sq. miles).

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[49] Prof., Hydr. Eng., Mass. Inst. Tech.; and Cons. Engr., Boston, Mass.

TABLE 10.—NEW ENGLAND FLOOD DATA

Stream (1)	Drainage area, in square miles (2)	Maximum discharge, in cubic feet per square mile (3)	Approximate total flood discharge, in inches (4)
(a) FLOOD OF NOVEMBER, 1927			
Ellis River, near Jackson, N. H	28	528	17.0
Jail Branch, East Barre, Vt	38	303	9.7
Baker River, near Wentworth, N. H	52	288	9.2
White River, near West Hartford, Vt	695	202	6.5
Winooski River, near Essex Junction, Vt	1 000	111	3.6
(b) FLOOD OF MARCH, 1936			
North Branch of the Hoosac River, near North Adams, Mass	39	99	17.6
Middle Branch of the Westfield River, near Goss Heights	53	160	....
East Branch of the Pemigewasset River, near Lincoln, N. H	104	164	20.2
Deerfield River, near Charlemont, Mass	180*	180	....
Connecticut River, near Montague, Mass	7 840	28.9	8.8
Merrimac River, near Lowell, Mass	4 424	39.1	9.9

* Omitting the controlled area of 182 sq miles up stream from Harriman Reservoir.

As will be noted from Fig. 3, the unit flow of White River is materially in excess of the enveloping curve presented by Mr. Uhl, indicating the unusual magnitude of this peak flow for such an area. This is probably due, in part at least, to the shape of the drainage area of this stream, which has three approximately parallel tributaries tending to concentrate flood flows in the lower river.

The flood of March, 1936, was due in part to heavy precipitation and in part to melting snow, and it covered a much larger area than that of 1927, resulting in what may be called a "main river" flood of unusual magnitude, but with tributary streams at lesser peak flows than in 1927. Table 10(b) shows a few of the peak flows and total flood run-offs.

The Lower Connecticut River flows exceeded those of 1927 by about 33% at Montague City, Mass., and 42% at Thompsonville, Conn. On the Merrimac River, at Lowell, Mass., the peak flow of March, 1936, exceeded by 67% that of the previous highest (in April, 1852).

The writer has made a study of flood frequency on the Lower Connecticut River, a graph of which is shown in Fig. 20. Records of yearly flood flows are available for the period, 1880 to 1936, at three Massachusetts stations, Holyoke, Sunderland, or Montague City. A record of flood stages at Hartford, Conn., extends back to about 1801 and by a plot of flood flow, in cubic feet per second per square mile, at one of the foregoing three stations to flood stage at Hartford, it was possible to relate the latter to the flood flow at these stations (about 8 000 sq miles of drainage area).

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[60] Rept. of the Cons. Engr. to the Advisory Committee of Engrs. on Flood Control, Vermont, 1928.

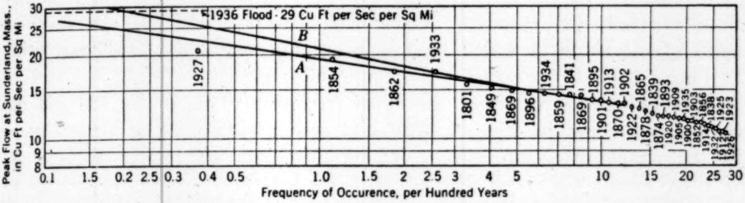

FIG. 20.—FLOOD FREQUENCY CURVE, CONNECTICUT RIVER, AT SUNDERLAND, MASS., 1801–1935.

In this manner all peak flood flows of importance were determined approximately for the period 1801-1936, or 136 yr, upon which the plot of Fig. 20 was based. Two interpretative extension curves are shown on Fig. 20. Curve *A* gives weight to the five or six upper points whereas Curve *B* is a more conservative interpretation, giving little weight to the 1927 point and to the other relatively high points.

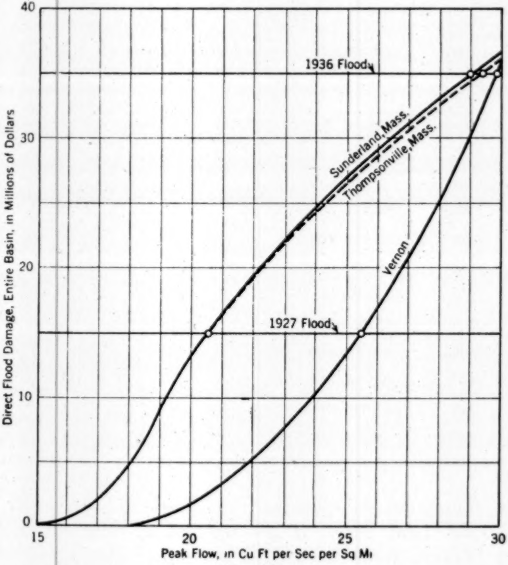

FIG. 21.

This shows the flood of 1936 (not plotted as a point) to have been the actual greatest based upon observations for about 150 yr of record, and by inference from the plot it would have a frequency of about once in 500 yr. The 1927 flood shows a frequency of about once in 90 yr.

On Fig. 21 are plotted peak flood flows in the Lower Connecticut River, at Sunderland and Thompsonville, against direct flood damages; a curve is also shown using Vernon, Conn., as an index station. In each case these curves have three points for their definition: (1) Damages in the flood of 1936; (2) damages in the flood of 1927; and (3) the stage or point of discharge at which flood damages are negligible. This last point has been fixed by personal experience and judgment.

Combining the results of the upper extension curve in Fig. 20 and the Sunderland curve in Fig. 21, Table 11 was prepared. The various steps in the computations are obvious and require no further explanation. As will be noted, the lesser half of the flood stages contributes mostly to the average yearly flood damages because, although flood damages are not relatively high, their greater frequency of occurrence effects a relatively larger yearly increment of damage. A flood of the order of that of March, 1936, with about 29 cu ft per sec per sq mile, has a high damage ($35 000 000), but its frequency is so rare that the yearly increment of damage from such a flood is relatively small.

TABLE 11.—DETERMINATION OF YEARLY DIRECT FLOOD DAMAGES, CONNECTICUT RIVER BASIN

Peak flood discharge, in cubic feet per second per square mile (8 000 sq. miles)	Flood frequency years, (from Fig. 20)	Direct damage, in dollars	Increment of damage, in dollars	Average frequency, in years	Yearly increment of damage, in dollars	Average yearly flood damage, in dollars
15	19	0				0
			700 000	23	30 000	
16	27	700 000				30 000
			1 500 000	31.5	48 000	
17	36	2 200 000				78 000
			2 500 000	41.5	60 000	
18	47	4 700 000				138 000
			4 300 000	55	78 000	
19	63	9 000 000				216 000
			4 000 000	72	56 000	
20	81	13 000 000				272 000
			6 600 000	105	63 000	
22	130	19 600 000				335 000
			5 400 000	165	33 000	
24	200	25 000 000				368 000
			4 200 000	250	17 000	
26	300	29 200 000				385 000
			3 800 000	365	10 000	
28	430	33 000 000				395 000
			4 000 000	510	8 000	
30	590	37 000 000				403 000
			3 000 000	695	4 000	
32	800	40 000 000				407 000

Roundly, the average yearly flood damages on the Connecticut River, as determined by this study, are $400 000. This is direct damage only and allowance for indirect losses of all kinds would double this figure, or possibly raise it to $1 000 000 yearly.

Flood protection on the Connecticut River including a system of reservoirs as well as dike or levee systems in certain of the lower cities, such as Hartford, Springfield, etc., will probably cost about $50 000 000. This would mean a yearly carrying charge of $2 500 000 to $3 000 000 for such works. It is obvious, therefore, as Mr. Uhl states, that a complete plan of flood relief on the Connecticut River is not economically justifiable based upon flood benefits alone. Other benefits from the use of reservoirs besides those of flood relief, principally from the use of water for power purposes, but also from the point of view of recreational and sanitation benefits, must bear a substantial portion of the cost of a reservoir system to justify its construction.

Mr. Wolman's statement of the factual situation in regard to flood protection is in general accord with the views of the writer.

Engineering studies of past destructive floods are generally incomplete, due to the lack in the past of effective agencies to procure and co-ordinate at or about the time of the flood the necessary information to serve as a basis for its study. Fortunately, there are some exceptions, however, as illustrated by the Ohio flood of 1913, where local agencies obtained much valuable data—particularly by the work of the Miami Conservancy District and its excellent set of engineering reports. Even in this case the data of flood damages—particularly those of an indirect nature—are uncertain. During the past decade the U. S. Geological Survey has developed a policy of procuring data of flood run-offs, following up all major floods, and is covering this field admirably.

In general, it is not likely, therefore, that much more specific data of value relating to the earlier destructive floods can be obtained, due to a dearth of definite information from the past.

The lack of detailed and carefully prepared complete programs for flood relief is quite obvious, in most sections of the United States, and little attention has been given as yet to the consideration of other water uses, in co-ordination with flood control.

As Mr. Wolman states, every major flood brings a popular clamor for action which if hastily acted upon, without careful and deliberate planning, is bound to mean wasteful and perhaps ineffective measures.

The inhibitions of the Copeland Act of 1936, which was directed solely at flood-control measures, have had an unfortunate effect in causing almost a complete lack of comprehensive planning of river developments thus far in accord with all the other aspects of river conservation and control. Moreover, in many sections of the country flood-relief benefits are not sufficient to warrant the expenditures required for this purpose alone.

Although the destructive aspects of a river in flood must be reckoned with, the development of its constructive and useful qualities prevailing most of the time should certainly not be overlooked and flood-relief measures are only a part of the general plans that should be made and consistently followed in the full development of the streams of the country.

E. D. Hendricks,[51] M. Am. Soc. C. E. (by letter).—New York State has a varied topography. Its surface drains into the Mississippi, through Chesapeake Bay, Delaware Bay, and New York Bay, into the Atlantic Ocean, Lake Erie, the Niagara River, Lake Ontario, and the St. Lawrence River. It contains the lowest divide into the great central basin between Canada and the Gulf of Mexico. The writer has been familiar with the Oswego and Mohawk drainage areas for many years, both from personal observation and from the records that have been kept since 1898, the year when the U. S. Deep Waterways survey was started across New York State from the Hudson River to Lake Ontario. This survey was followed by the New York State Barge Canal surveys and the record has been continued with a greater or less continuity until the present time.

For the ten years (1927–1937) records in the Oswego River drainage area have been kept by Foster B. Crocker, M. Am. Soc. C. E. The control of the principal tributaries of the Oswego River, namely, the Seneca, Clyde, and Oneida Rivers, is vested in the State Department of Public Works as these streams are canalized and are part of the New York State Barge Canal System.

As a result of the intense rainfall of July, 1935, and of the much greater volume of run-off that occurred in March, 1936, this drainage area was subjected to the two major floods generally described by Messrs. Harrington and Johnson.

Flood control that has been accomplished in New York State prior to the great floods of 1935 and 1936, except in isolated cases, has been incidental to other purposes. The isolated instances with which the writer is familiar are the Hornell and Canisteo flood abatement projects on the Canisteo River, which were completed before 1927 and which, in the case of the 1935 flood, proved inadequate. The Sacandaga Reservoir was as much a water-storage project for power interests as a flood-control project and a large part of its cost is being paid for by power corporations. In the writer's opinion the deepening of the Hudson River down stream from Albany, making that city a seaport, had as much effect in reducing the flood level at Albany as the Sacandaga Reservoir.

The 1936 flood was the greatest of which there is a record on the Mohawk River in respect to the rate of flow. However, it did a relatively small amount of damage compared to other floods, such as those of October, 1903, March, 1904, March, 1910, March, 1913, and March, 1914.

The floods of 1904, 1910, and 1914 were accompanied by ice gorges. The flood of 1913 on the Mohawk had climatic conditions similar to that of 1936—a period of warm weather early in March followed by high water, followed in turn by a rain in the latter part of the month on saturated or frozen soil. The physical conditions, however, were greatly changed in 1936; that is, the canalization of the Mohawk River had been completed, and a number of additional storage reservoirs were in operation on the tributaries, the level of which reservoirs had been drawn down early in March. The result was that although a greater quantity of water was retained in reservoirs on the head-waters, a smaller quantity was retained on the flood-plains of the river, resulting in an increased discharge at the mouth with much lower elevations up stream; for example, the estimated discharges, in cubic feet per second, were: 1913, 110 000, and 1936, 130 000.

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[51] Senior Claims Engr., State Dept. of Public Works, Albany, N. Y.

Neither the reservoirs on the Mohawk nor the canalization were for the purpose of flood control. However, nearly all flood damage has been eliminated between Schenectady and Utica, a distance of 80 miles, where prior to the canalization the ground floors of property located within the flood-plain were flooded every few years.

Although the primary purpose of the control of the Seneca and Oneida Rivers was to render them navigable with a minimum depth of 12 ft, an indirect benefit has been to reduce flood elevations and to reclaim large tracts of land, particularly in the Montezuma marshes through which the Seneca River flows. The property owners along these streams, however, have had a different opinion; and as a result have filed claims against the State alleging that as a result of the canalization of the aforementioned streams and their control by the State, flood conditions have been aggravated with increased damage to them. Test cases have been tried in the New York State Court of Claims for the years 1935 and 1936, and in the preparation of the State's defense a great mass of data concerning previous floods has been collected, together with an intensive study of climatic conditions, rate of inflow, progress of the flood, yield of the tributary streams, etc.

The drainage area of the Oswego River is shown on Fig. 11. It consists of 5 120 sq miles at Oswego. A daily discharge record is kept by the U. S. Geological Survey at that place. The State maintains a discharge station at Fulton, 11 miles up stream (area, 5 017 sq miles). At Three Rivers, 23 miles up stream from Oswego, the Oneida (area, 1 493 sq miles) and Seneca (area, 3 445 sq miles) Rivers unite to form the Oswego.

The State maintains a gaging station at the Caudenoy Dam, 12 miles up the canalized Oneida River, where the drainage area is 1 377 sq miles. This dam maintains the water surface in Oneida Lake, 4 miles farther up stream, and is provided with a flood-gate which, with a guard-gate above Lock 23, permits the lowering of the water surface in the lake during the winter months and the control of the outflow from the lake.

Seneca River has its first dam at Baldwinsville, 12 miles up stream from Three Rivers. This dam is supplied with a 30-ft flood-gate. Nine miles up stream from Baldwinsville, the State has a gaging station consisting of two recording gages situated at each end of a 4 300-ft rock-cut channel, known as State Ditch. The discharge from the drainage area of 3 073 sq miles is computed by the slope-area method. Thirty miles up stream is the junction with the Cayuga-Seneca Canal. The main canal proceeds westerly up the Clyde River, and 6 miles farther up stream the State maintains a gaging station at Clyde, which has a drainage area of 880 sq miles. Thirty miles farther up the Clyde, at Lock 30 at Macedon and the westerly limit of the water-shed, the water brought into the drainage area from Lake Erie and Genesee River is measured through calibrated sluice-gates. This quantity averages about 135 cu ft per sec during the navigation season, April to November, inclusive. It varies between 100 cu ft per sec and 170 cu ft per sec, depending on the number of lockages at Pittsford, 17 miles west of Macedon.

Returning to the main channel of the Seneca River: The Cayuga-Seneca Canal proceeds up the Seneca River southerly a distance of 4 miles to the outlet of Cayuga Lake, the surface of which is controlled by six Tainter gates. The outflow through these gates has been calibrated and a daily record is kept. The drainage area above this station is 1 572 sq miles. The canal turns westerly from Cayuga Lake, following the course of the Seneca River to Waterloo, a distance of 8 miles, where the control works regulating the surface of Seneca Lake are maintained and where a daily record of the outflow from the lake is kept. The drainage area above Waterloo is 749 sq. miles.

In addition to these gaging stations maintained by the State the U. S. Geological Survey, under the supervision of Mr. Harrington, as District Engineer, maintains three stations on the Oswego River water-shed, all of which are near the head-waters, in addition to the one at Oswego at the outlet of the drainage area (see Table 12).

TABLE 12.—GAGING STATIONS MAINTAINED BY THE UNITED STATES GEOLOGICAL SURVEY

Station		Drainage Area	
At (1)	On (2)	Name (3)	Area, in square miles (4)
Taberg, N. Y.	The East Branch of Fish Creek	Oneida River	189
Auburn, N. Y.	The outlet of Owasco Lake	Seneca River	208
Ithaca, N. Y.	Fall Creek	Cayuga Lake	124

Storm of July, 1935.—The writer is convinced, from personal observations and Court testimony, that the storm of July 7 and 8, 1935, was more extensive than is shown by Fig. 12[52], which is based on reports from rainfall stations of the U. S. Weather Bureau. In the writer's opinion, the precipitation was concentrated at or near the higher elevations where there are no Weather Bureau stations, and passed over the low divides or valleys with a heavy precipitation to concentrate with double the precipitation on the hills beyond; that is, extending from the New York-Vermont line east of Mt. McGregor, the area of high precipitation extended westward along the southern escarpment of the Adirondacks to Stewart's Landing, 20 miles northwest of Gloversville. South of the Mohawk the storm concentrated on the peaks of the Cherry Valley Hills, on the Mohawk-Susquehanna divide, and Oak Hill, which is the divide between the Mohawk and the Schoharie Rivers. Thus, streams entering the Mohawk from the south and west of the Schoharie, show peak flows of more than 400 cu ft per sec per sq mile which resulted from a rainfall of not more than 6 hr duration.

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[52] *Water Supply Paper 773-E*, U. S. Geological Survey.

However, the water-shed of the Oswego River is well supplied with rainfall stations in addition to the U. S. Weather Bureau stations, most of which are maintained by the New York State Department of Public Works and by the Oswego River Watershed Corporation. In addition to precipitation records, snow survey records are kept.

The run-off that resulted from the rainfall of July 7 and 8, 1935, does not appear to have been equalled for summer flows on the Seneca River in the period in which records have been kept—that is, for 37 yr. Fortunately, no unusual run-off occurred on the water-sheds of the Oneida and the Clyde Rivers. The precipitation on the Oneida water-shed was less than one-half that on the Upper Seneca and the precipitation on the Clyde water-shed was no more than that from an ordinary summer storm. In fact, at least one-third of the area draining into Cayuga and Seneca Lakes did not experience a really destructive rain.

The remainder of the Cayuga and Seneca Lake areas had a rainfall that resulted in an inflow at the rate of 80 000 cu ft per sec for 8 hr on Seneca Lake, from a drainage area of 749 sq miles. Cayuga Lake had a rate of inflow of 100 000 cu ft per sec for 8 hr from a drainage area of 823 sq miles. Due to the storage utilized in both lakes, the peak outflow from Seneca Lake was 4 400 cu ft per sec and that from Cayuga Lake was 16 000 cu ft per sec. The elevation reached in Cayuga Lake was 386.0 and in Seneca Lake, 449.34. The peak discharge at Baldwinsville was 12 000 cu ft per sec. The greatest damage on the navigable waters resulted to truck gardeners cultivating thousands of acres of muck land in the former Montezuma marshes which had been rendered tillable by the canalization of the Seneca River between Cayuga Lake and Jack's Reef.

Greater damage occurred on the streams flowing into the lakes and has been described elsewhere by Mr. Johnson.[52] Over these streams the New York State Department of Public Works exercises no control.

The discharge at Jack's Reef or at Baldwinsville had been exceeded a number of times in the late fall or spring months, both before and since 1918, the date of completion of the Barge Canal There is no record of a greater summer run-off. The summer elevation in Cayuga Lake had been exceeded several times prior to canalization, but with an outflow of less than 4 000 cu ft per sec. In August, 1922, the lake reached an elevation of 386.1; and although that was a summer of heavy precipitation, the high water was as much the result of the method of operation as of the climatic conditions. Any elevation in excess of 385.0 results in damage to property surrounding the lake and particularly to property in Ithaca at the head of the lake.

The term, "method of operation," is used advisedly. In 1922, the control of the navigable waters of the State were under a bureau separate from the State Engineer Department. Since 1927 both the Division of Canals and Division of Engineering have been included in the Department of Public Works under Frederick Stuart Greene, M. Am. Soc. C. E., as Superintendent. Not only has there been an increase of accurate records kept by Mr. Crocker; but the long-continued sequence of the records has resulted in a sound engineering basis on which a proper method of operation has been predicated.

Flood of March, 1936; Oswego River Water-Shed.—The climatic conditions causing the flood of March, 1936, seem to have been similar in many respects to those that occurred in March, 1865, as shown by the report[63] of J. P. Goodsell, Division Engineer of the Middle Division, New York State Canals, including the Erie, Oswego, Cayuga, and Seneca Canals and the Oneida River improvement. Mr. Goodsell reported that the measured flow at Jack's Reef was 19 435 cu ft per sec on March 22, 1865, and that this was the greatest flow ever experienced in the Seneca River. The greatest flow preceding this was in March, 1857, when the discharge at Jack's Reef was 15 269 cu ft per sec.

There was another great flood on April 10 and 11, 1873, in which Cayuga Lake reached the same elevation as was reached in 1865, but no discharge measurements were reported. The peak discharge at Oswego for the 1865 flood has been estimated as 42 000 cu ft per sec. The peak discharge at Phoenix has been stated (Empire Street Railway Company *vs* the State of New York) as 30 000 to 35 000 cu ft per sec. The Town of Phoenix is below the junction of the Oneida and Seneca Rivers and has a drainage area of 4 940 sq miles, about 500 sq miles greater than at the gaging stations on the Seneca and Oneida Rivers.

The peak discharge at Phoenix is usually about equal to that at Oswego. However, in the 1865 flood, 170 ft of a dam situated on the river 4 miles up stream from Oswego, was washed out which could well account for the difference of 7 000 cu ft per sec. The highest flood elevation recorded in Oneida Lake prior to the construction of the Barge Canal was 376.0, with an estimated peak outflow of 12 500 cu ft per sec. This peak probably occurred in the flood of 1865 and checks very well with the peak at Phoenix and the discharge from the Seneca River, at Jack's Reef. The effect of the control, incidental though it was, is disclosed in Table 13. The low navigable stage of Cayuga Lake is 381.5 and that of Oneida Lake is 369.9. During the winter of 1936 the level of Cayuga Lake was drawn down to 380.3 and that of Oneida Lake to 368.5, in preparation for the

TABLE 13.—COMPARISON OF ELEVATIONS AND DISCHARGES, FLOODS OF 1865 AND 1936

Date	Cayuga Lake elevation, in feet above sea level	Seneca River discharge, in cubic feet per second	Oneida Lake elevation, in feet above sea level	Oneida River discharge, in cubic feet per second	Oswego River discharge, in cubic feet per second
1865..........	389.3	19 435	376.0	12 500	35 000
1936..........	386.7	22 500	375.0	11 700	37 000

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[63] Annual Rept., State Engr. and Surv., January 23, 1866.

expected high water that did occur. Damage did result from high water in the Oswego River drainage area, and claims have been filed against the State, but it is anticipated that they will be dismissed.

Messrs. Harrington and Johnson call attention to the importance of basic data depending on long-continued records. This discussion is primarily for the purpose of illustrating the value of numerous long-continued records and their intelligent use by a skillful engineer. The reduction of a flood elevation by 1 ft or 2.5 ft may appear negligible in sections where flood rises greatly exceed those in New York State; but this lowering reduced by hundreds of thousands of dollars the damage suffered. The streams discussed traverse regions of long settled and thickly populated communities and the last foot of rise usually causes the maximum damage.

To spend great sums on construction without the help of basic data is to insure not only waste, but greater injury than the benefits realized.

The reference by Messrs. Harrington and Johnson to the importance of considering the engineering economics of each problem is often lost sight of by some enthusiastic advocates of only one phase of the problem of flood control. Thus, there are the reforestry proponents claiming deforestation to be the primary cause of floods. This cannot be true of the discharge areas discussed herein. There is a greater forest cover now than there was in 1910 (from the writer's personal observation). The 1935 Graphic Compendium of the New York State Planning Board indicates that lumbering reached its peak in New York State in 1869, and that there has been a great reduction in improved farm areas since 1875. Man-made changes, apparently, have relatively little or no effect in these two instances compared to climatic disturbances over which Man has no control and no definite knowledge of the cause.

EDWARD W. BUSH,[54] M. AM. SOC. C. E. (by letter).—Combined flood control and hydro-electric developments are engineering subjects that will keenly interest the profession for some time to come. The papers of this Symposium are very valuable contributions on the subject. Many schemes are under consideration for building groups of reservoirs for the dual purpose of flood control and electric development, but practically no information has been presented by their proponents as to the procedures that necessarily must be followed by those operating such dual systems so that the flood-control protection will receive proper safeguarding and, at the same time, those relying on the generated electricity will be satisfied with the services obtained.

In the few cases where procedures have been briefly mentioned the proponents have stated, what to them seems obvious, that the reservoirs will be drawn down just before the freshet seasons arrive and after they are filled by the freshets the impounded waters will later be used to generate electricity. This is quite a simple solution if the rain storms and the weather could be foretold accurately, or could be regulated, but seemingly quite fantastic when the actual facts are recognized. Those wanting protection against flood damages need the reservoirs empty except when in use for holding back flood waters, and when so filled they should be drawn down as soon as the outflow would not cause damage so they are set ready for the next freshet when it comes. Those using the electricity and those selling it to obtain funds for retiring the cost in whole or in part of the reservoirs will desire plenty of water in the reservoirs at all times. It would seem that these conflicting interests cannot be reconciled.

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[54] Engr., Aetna Casualty and Surety Co., Hartford, Conn.

The Connecticut River well illustrates the uncertainty of the time of high freshets. During the past 300 yr, the highest freshet came in the middle of March, 1936, the next highest freshet came in the first week of May, 1854, and the next highest came in the first week of November, 1927, at a time when very low water has often occurred in past years. Fairly high freshets have occurred at one time or another in about every month of the year.

It is highly improbable that any official or board of officials in charge of the operation of a system of reservoirs constructed for a dual purpose could at any critical time determine in advance whether or not to empty the system, or to keep it filled or partly filled if it happened to be in that condition. At such a time the power users and sellers could readily estimate the dollar value to them of any stored water and would strongly urge their claims for such, but those relying on the reservoirs for the protection against flood damages would probably not be organized and, therefore, not able to present forcefully their advantage from having empty reservoirs when what might be termed, "border line" situations arose; and any freshet, when it reached its crest, might only be one of moderate size.

After several of such false alarms it is expecting too much of human nature to believe that those in control would operate a system so that the flood prevention side would always receive the full protection needed. This seems to be a fundamental consideration that should not be lost sight of. In most of the cases when a critical situation is forming there will not be time to convene a board of consulting experts to determine whether or not the reservoir gates should be opened or closed, and even if convened the experts would probably not know the correct answers.

IVAN E. HOUK,[55] M. AM. SOC. C. E. (by letter).—Comprehensive treatments of flood-control problems presented in this Symposium show that the disastrous flood occurrences of recent years are gradually causing the development of a long needed, national, flood-prevention movement. Some of the Federal plans outlined by Colonel Covell may be delayed for many years, possibly until future flood calamities provide new impetus. Nevertheless, if only a few are carried to completion at this time, they will result in saving of life and property which can scarcely be evaluated on a purely monetary basis.

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[55] Senior Engr., U. S. Bureau of Reclamation, Denver, Colo.

Probably one of the most urgent needs in connection with present flood-control work is an adequate forecasting system. Officials of the U. S. Weather Bureau have rendered excellent service on some of the larger rivers where appreciable time intervenes between the occurrence of storm rainfall and the arrival of flood peaks. However, much remains to be accomplished in developing adequate forecasting organizations for small tributary streams in the upper parts of the drainage areas where maximum flood stages occur only a few hours, or only one day or two days, after the most intense precipitation.

The requirements for an ideal river and flood service mentioned by the late Mr. Hayes,[56] if provided in sufficient number in each river section, should furnish the necessary facilities for forecasting maximum stages during most flood-producing storms. Naturally, they would not be adequate for forecasting cloudburst floods in isolated mountainous drainage courses, where crest stages arrive almost simultaneously with the first run-off (as at Heppner, Ore., on June 14, 1903, when two hundred people were drowned, one-third the town washed away, and approximately $250 000 worth of property destroyed).

Cloudburst floods of the Heppner type occur so suddenly and erratically that their prediction probably never will be practicable. In such cases residents along the lower sections of the streams usually must depend on telephone warnings from up-stream locations. During the Cherry Creek flood of August 3, 1933, caused by cloudbursts south of Denver, Colo., and the resulting failure of the Castlewood Dam, telephone and radio warnings were so effective that only two lives were lost.[57] Probably the only way to insure warnings of advancing cloudburst floods, if such warnings should ever be considered necessary, would be to install automatic float-gages at up-stream locations, and to connect them electrically with the forecasting office, so that alarms would be sounded when dangerous stages were reached.

Automatic recording river and rainfall gages are desirable for use in forecasting floods in small up-stream drainage areas. However, they are not essential. About 1917 the writer was forecasting floods in the Miami Valley of Southwestern Ohio, where the most remote parts of the watershed were not more than a hundred miles from the Central Office. Cooperative arrangements were made with the Weather Bureau observers whereby each inch of rainfall and each foot of river rise were reported by telephone or telegraph during storm periods. The observers were paid additional compensation on the basis of the number of special reports submitted. Consequently, they watched their gages and made their reports promptly. The system worked so satisfactorily that accurate forecasts of river heights could be issued well in advance of the arrival of the crest stages, not only for the principal cities of the Valley, but also for the various construction activities which were being carried on by the Miami Conservancy District.

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[56] Mr. Hayes died November 16, 1936.

[57] "Failure of Castlewood Rock-Fill Dam", by Ivan E. Houk, *Western Construction News*, September, 1933, pp. 373 to 375.

The real value of installing recording rainfall and river gages in forecasting systems for small drainage areas lies in the satisfactory data they supply for subsequent detailed studies of hydrologic conditions. The forecasters will want to make such studies after the floods have passed, when they have time to analyze all factors carefully. During the occurrence of the storm rainfall and the arrival of the flood stages they will be too busy to make detailed quantitative analyses of run-off phenomena. At such times, the most important matter is to get the necessary rainfall and run-off reports, promptly, so that forecasts can be quickly formulated and issued in time to be of value. Consequently, telephone, telegraph, or radio transmission facilities must be available.

Mr. Hayes mentioned the desirability of conducting snow surveys in the Eastern mountains. The methods to be used in such surveys probably should be somewhat different from those used in Western United States, since the information will be used in forecasting flood-stages during the winter and spring months instead of total run-off during the spring and summer seasons. Probably the accurate measurement of the water content of the snow at selected stations throughout the eastern drainage areas will be more satisfactory for flood-forecasting work than the establishment of snow courses such as are used in the West. Measurements of the water content should also be made much more frequently than snow surveys are usually conducted.

JOHN E. FIELD,[58] M. AM. SOC. C. E. (by letter).—In proposing five changes in the present plan of operating the U. S. Weather Bureau, the late Mr. Hayes listed snow surveys as Requirement (*c*). The method of making snow surveys proposed herein applies only to those areas where the snow banks last well into the summer and where the spring floods come from melting snows and are probably confined to the Rocky Mountains, and the Wasatch and Coastal Ranges. Even limited to those areas snow water affects the territory from the 104th Meridian, about the eastern boundary of Montana, Wyoming, Colorado, and New Mexico, to the Pacific Ocean, and especially to those areas where irrigation is practical.

In the building of the hydraulic-fill dam of the Terrace Reservoir, in the Alamosa River, in Southern Colorado, covering the years 1908 to 1912, it was necessary to know the probable run-off of the river a week or more in advance, to be able to estimate the probable peak flow from melting snow, and to know when the maximum had passed, because any quantity greater than 1 200 cu ft (the capacity of the outlet turned) must be stored or carried over the dam by flumes. By observing the snow banks, about 10 miles distant, in the canyons and gulches, or in the high mountains of the water-shed (10 000 to 12 000 ft above sea level), the writer found several that were indicative of the run-off to be expected in the days immediately following the observations. The weather conditions, of course, were taken into account in forecasting the run-off for each day as cloudy weather lessened the melting and clear days augmented it.

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[58] Cons. Engr., Denver, Colo.

At the end of four years and with the records of the flow of the river, the writer felt he could predict the season's run-off quite satisfactorily, from the appearance of the snow banks in May.

Being called upon to estimate the probable supply for the City of Denver, Colo., in March, 1933, and the predictions later proving to be much in error, it occurred to the writer that a study each spring of the snow banks on the city water-shed would furnish a more reliable basis for prediction than precipitation, snow surveys, and soil moisture records, which had been used in the 1933 estimates.

The Denver Board of Water Commissioners was induced to test the theory; it furnished telescopic lenses for the cameras and designated Harry L. Potts, M. Am. Soc. C. E., to gather the data. A program was devised (since much improved by Mr. Potts) fixing the points from which photographs were to be taken, at least once each month, and then co-ordinated with stream flow and weather records.

These records are depended upon by the Board of Water Commissioners of the City of Denver, in the operation of the city system and particularly in determining whether the City can safely sell stored water to proprietors of the irrigation canals below, who are in great need of water in July and August.

It is the hope of the writer that the current research will show the method to be both simple and reliable and, supplemented by some snow measurements, will be valuable to all who are concerned with water from the high mountain areas. It is his wish that some Federal Department will adopt the method and that it will lead in time to the publication monthly of the probable run-off on each stream, in a manner similar to the crop reports now being furnished by the Government to the public.

C. S. Jarvis,[59] M. Am. Soc. C. E. (by letter).—It is indeed gratifying to note the progress achieved recently toward recognizing and sponsoring flood control, stream regulation, and water utilization as both national and local problems. It has long represented "a consummation devoutly to be wished", and earnestly sought by leaders in the Engineering Profession, by far-sighted and progressive economists, legislators, and community officials. Now that such generous provision has been made for the flood control problems in several regions of the United States, it is desirable to continue such activities until equitable distribution has been accomplished among the several States.

It has long been advocated, although occasionally challenged by those reluctant to "fall into line," that all storage development under intelligent operation should result in positive, measurable benefits to flood control and stream regulation. Notable confirmation is accorded in a recent paper by Eric Chester Hillman, with discussions, entitled "The Effect of Flood

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[59] Cons. and Hydr. Engr., SCS, Washington, D. C.

Relief Works on Flood Levels Below Such Works".[60] Among the conclusions[61] is the following: "The effect of eliminating storage capacity will be to accelerate the flood wave and increase the flood peak below the area from which the floods have been excluded."

In the case of the Nottingham flood protection plan, the reduction of about 10 000 acre-ft of flood-plain storage, due to new protecting levees, resulted in speeding up the flood crest by 7 hr, and a rise of 0.4 ft during a freshet, with maximum flow of 20 000 cu ft per sec. Obviously, increase of storage, therefore, would retard and reduce flood crests proportionately.

Conversely, the Bon Accord Dam,[62] impounding 400 000 000 cu ft (9 183 acre-ft) at full supply level, with a surface area of 614 acres and maximum storage depth of 45 ft at the spillway level, reduced a maximum inflow of 16 000 cu ft per sec to a regulated outflow not exceedingly 10 500 cu ft per sec for the maximum observed flood from 121 sq miles, including Pretoria, South Africa.

Even if, in an extreme case, a reservoir may be full or nearly full to spillway level at the arrival of a flood crest, there is generally considerable regulation available due to storage capacity above the spillway crest, together with back-water effects along contributing stream channels. Ordinarily, intelligent operation of storage reservoirs will provide nearly maximum regulation during the critical seasons, and satisfactory flood routing in the lower courses.

JOSEPH JACOBS,[63] M. AM. SOC. C. E. (by letter).—Every member of the Society who feels a concern in Federal flood-control activities and policies must have read this highly informative Symposium, with great interest and appreciation. Because of the persistent, and not infrequently unnecessary, encroachment of the works of Man upon the flood planes of the rivers of the United States, and because of the increased property values attaching to urban, suburban, and rural developments along its streams, floods are a steadily growing menace and, year by year, are becoming a more serious economic problem both nationally and locally. The writer has always felt a deep interest in the matter and, abstracting largely from material which, from time to time in the past, he has prepared on the subject, he submits the following as an expression of his views concerning a proper approach to the problem of a permanent Federal policy in respect of flood control.

In the discussion of flood control there is frequent reference to the so-called 308 Reports, prepared under direction of the Corps of Engineers, U. S. Army, and it is not an uncommon view, even among engineers, that these reports provide all the preliminary information needed concerning American rivers and that they constitute an adequate basis for the formulation of a national flood-control policy. Commendable as was the undertaking, and excellent as was its execution, it is believed that the 308 Reports were by no means exhaustive finalities as to the matters with which they dealt; nor were they adequately inclusive territorially. This is particularly true as to flood control which apparently received far less consideration than did the navigation, power, and irrigation phases of the reports. No one familiar with the matter, and least of all, the Army engineers, will contend that the reports were in any sense final. Neither sufficient funds nor sufficient time was available to cover the country fully. Necessarily, there were many streams that must have been only cursorily investigated and many others, no doubt, that were not then, and have not yet been, investigated at all. It is believed, too, that the 308 investigations were not directed toward the specific problem of a permanent Federal policy as to flood control but dealt rather with the engineering aspects of individual projects. There is yet much important investigational work that ought to be done before any final Federal policy should be formulated in detail.

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[60] *Journal*, Inst. of Civ. Engrs., Vol. 2, 1935–36, p. 393.

[61] *Loc. cit.*, p. 409.

[62] *Loc. cit.*, p. 432.

[63] Cons. Engr., Seattle, Wash.

During the next thirty or forty years probably as much as $2 000 000 000 will be expended on flood-control works and, unquestionably, a large part of this expenditure will devolve upon the Federal Government. Certainly, an activity of that potential magnitude is worthy of the most careful consideration by the responsible agencies, in order that sound technical and legislative policies respecting it may be established and that maximum economy and facility in planning, in execution, and in final operation of the completed works, may be achieved. The evolvement of these governing policies should be based upon definite knowledge of what the flood-control problem is in all its major requirements and ramifications, and when the necessary surveys have been made and the factual data assembled they should be interpreted, in their broad economic aspect, by men technically competent for such a task.

Congress, under the Commerce Clause of the Constitution, assumes jurisdiction over the navigable waters of the United States and generously provides for navigation requirements. To the extent that floods have affected navigation the Government has regarded them as being "affected with a Federal interest" and has made appropriations therefor on that basis. As to floods on non-navigable streams, these, in general, have been regarded as lacking in "Federal interest" and, therefore, have not been considered a matter of Federal concern or responsibility. The Federal Government seems now to recognize that it must retreat from the negative policy that heretofore has characterized its attitude toward flood control and assume a larger responsibility concerning it. This salutary change of attitude is manifest in the Flood Control Act of 1936 and, although the general flood-control measure of 1937 failed of final enactment, 'the new Federal purpose of a larger responsibility in flood-control matters was also manifest in that measure. The entire subject of flood control is now in a state of flux, and it needs sane guidance toward a policy not temporary or ephemeral, not based on hasty considerations, but one that is sound and enduring because it is based upon a thorough-going investigation of all phases of the problem. No such thorough investigation has yet been made.

The chief significance of the Flood Control Act of 1936 is not so much in its detailed provisions, some of which are deemed open to question, nor in its project authorizations, but rather in the incidence of a new Federal attitude toward flood control. It is the writer's feeling that some of the provisions of the Act are not well considered. It appears to him that the measure must have been rather hastily drawn, probably with greater consideration for providing early unemployment relief than with a view to formulating a definite, Federal flood-control policy. Those provisions of the 1936 Act, the desirability of which the writer questions, are as follows:

(*a*) It authorizes the appropriation of $310 000 000 for the construction of a very large number of specifically designated projects. It is seriously doubted whether there could have been sufficient investigation to warrant a Congressional approval of the large number of projects involved in this authorization. Presumably, the projects are subject to further investigation and presumably, too, for those that are finally found to lack merit, the actual appropriation will not be made; but is such wholesale approval in advance of final investigation a desirable Congressional procedure? The writer thinks not.

(*b*) It provides that the local agency concerned shall, up to 50% of the total project cost, furnish all required lands and rights of way. The usual defense of this provision is that it will result in lesser right-of-way costs, but that need not be the case if the local agency is required to pay its full and fair share of all project costs, whatever they may be. Rights of way constitute as legitimate a part of a project as do its structural elements, and its cost should not be treated as a thing apart but should be included with other costs without distinction. The inconsistency of the present provision is obvious when one realizes that right-of-way costs may easily range from practically nothing to more than one-half of the total project cost and they cannot, therefore, except by accident, be a fair measure of that portion of the total cost that should be borne by the local agencies. There are many projects for which the local agencies should bear the major part of the total cost and others for which they should bear practically none of the cost, and these facts should be recognized in making cost allocations. An instance is recalled of a $2 500 000 projected Federal institution whose location was largely predicated on a free furnishing of the land required for its accommodation, land that could cost scarcely more than 3% of the total project cost, and the communities of three States were asked to submit offers. How impractical and archaic it is to determine the location of an important Federal enterprise on such a basis! It is to be hoped that the Federal Government will avoid making such a procedure part of its flood-control policy.

(*c*) It provides that the local agencies shall hold the United States free from damages due to the construction works and that they shall maintain and operate all the works after completion. Why should these onerous responsibilities necessarily be imposed upon the local agencies? It seems to the writer that the allocation of these responsibilities and their costs, as well as the allocation of initial construction costs, should be predicated on the theory of conferred benefits. If the Federal interest in a project is dominant, as it might easily be on a navigable stream, or if the project is of an interstate or international character, as some of them will be, then it would seem most practical for the Federal Government to operate; but this does not necessarily mean that the Federal Government should bear all the costs of operation. Other conditions may clearly indicate that operation should be by the local agency and a final flood-control policy should contemplate and provide for these variable contingencies.

As stated previously, the writer thinks that there is much investigational work yet to be done before a final flood-control policy should be fixed. There are deficiencies in factual information and there are differences in judgment as to the best Federal policy respecting flood control. These deficiencies cannot be supplied, these differences cannot be composed, and the best final policy cannot be determined, without additional facts developed from actual further surveys and investigations. It is suggested, therefore, that the Federal Government ought now to undertake a comprehensive, nation-wide, flood-control survey and investigation, and that the formulation of a final flood-control policy should await the results of that investigation. Some of the factual material that such an investigation should provide, and some of the questions it will have to deal with, are embraced in the following items:

(1) Sufficient basic data to afford a broad-gage perspective of the entire problem including, for all important drainage basins and for the country as a whole, the extent of the flood menace, the average annual losses due to floods, an outline of the character of corrective measures that will need to be applied, and something of an estimate as to the probable gross initial and gross annual cost of these corrective measures.

(2) A careful assembly of all dependable data as to actual flood losses. The validity of flood-control economics is so closely related to a correct evaluation of this factor that a special effort should be made to secure authentic information concerning it. Many wholly unreliable estimates of flood-control losses are extant, both above and below their true value.

(3) An appraisal of the causes of the present unpreparedness for major floods. To what extent is a deficiency of hydrologic data a contributing cause and to what extent is there a failure adequately to heed the warnings afforded by the data already available? To what extent is there unwarranted encroachment, beyond real economic requirements, on the natural flood planes of American rivers? To what extent is there a lack of adequate legislation, both Federal and State, and a lack of an adequate plan of finance, to make possible the construction and operation of needed flood-control works?

(4) What principles should govern in framing a Federal flood-control law? Should such a law provide for, and require, the co-operation of local agencies and provide for an allocation of cost on the basis of benefits conferred and, if so, should this allocation apply only to construction cost, or should it apply also to the initial investigation cost and to the cost of operation and maintenance? Should not the law be devised to serve either a single or a multiple political jurisdiction; that is, should it not be sufficiently flexible to permit the construction of works for a restricted area or for an entire drainage basin that may extend beyond the boundaries of a single county or a single State, or even beyond a national boundary as might be the case for international streams?

(5) What is the best basis for a rational and equitable appraisal of benefits to Federal and local agencies resulting from the construction of flood-control works? This is not a simple matter but, no doubt, some general principles can be evolved that will facilitate solution for each particular case.

(6) What agency should maintain and operate the flood-control works and what principles should control in determining this question?

(7) Should there be State legislation permitting a requirement, by the State, that certain storage capacities be reserved for flood-control purposes in any new storage reservoirs that may be built, the extra expense entailed thereby to be borne by the beneficiaries concerned?

Doubtless there are other pertinent items that could be added but the foregoing list is sufficiently extensive to indicate that there is a considerable body of factual data and material to be acquired, and that the proposed comprehensive investigation is truly an important and necessary preliminary for the best formulation of a final flood-control policy. It is the writer's judgment that the cost of such an investigation will approximate $25 000 000 and that a time interval of five years may be required for its completion. It should be emphasized, however, that the prosecution of this work need not in any way embarrass or delay the construction of individual projects that may require earlier attention, but the legislation authorizing them should not be regarded as representing any final Federal flood-control policy.

What provision should be made for prosecuting the proposed investigation? An interpretation of the data provided by the investigation, and the translation of such data into a sound, Federal flood-control policy, will require not only a high order of technical skill and understanding but, equally so, a high order of statesmanship. It is suggested that there be created a Federal Flood Control Commission whose duty it shall be to direct the investigation, and to render a report of its findings, which shall include a specific recommendation as to a definite Federal policy respecting flood control. The personnel of such a Commission should include representatives of those few Federal agencies that now have a major interest in flood control, and it should also include men from the outside who, by training and experience, have special qualifications for such an inquiry. It is the writer's belief that such a comprehensive investigational program as here proposed would, in material degree, receive the financial, co-operative aid of the States.

W. M. Dawley,[64] M. Am. Soc. C. E. (by letter).—Under the heading, "Projects in the Ohio Basin," Colonel Covell refers to flood-stages in the Ohio River that attained a gage height of 70 ft (prior to January, 1937), with consequent damage due to the flooding of basements and the lower floors of buildings. There is an Indian tradition of a much higher flood at Cincinnati which, if repeated, would inundate the entire business section of the city, with consequent damages far in excess of those due to the 1937 flood.

In the fall of 1789, Gen. Josiah Harmar sent Maj. John Doughty with a body of troops, and discretionary power, to locate a fort in the Miami country. As a result Fort Washington was built between Third and Fourth Streets produced east of Eastern Row (later Broadway) in Cincinnati, Ohio. Some friendly native Shawnee Indians advised against the location chosen for the site for the fort, stating that it had been flooded by the Ohio River, and designated a point on the hillside to which the waters had risen, which point was at an elevation of 105 ft above the low-water mark.

Since the Indians then were a migratory people, it is not probable that a knowledge of the location of this specific high-water mark would be handed down for more than one or two generations, so this flood must have occurred in the latter part of the Eighteenth Century, possibly coinciding with the high water referred to in the press comments on the 1936 flood in the Allegheny River as being the highest of record, except that of 1763. It occurred at a time when the entire drainage area was wooded and uncultivated.

This information was gleaned before 1900 from the papers and personal correspondence of Col. Richard C. Anderson, Principal Surveyor of the Virginia Military District, in Ohio, which were then in the possession of the Rev. R. G. Lewis, of Chillicothe, Ohio. The documentary evidence is not at the moment available, but if it can be verified, it would seem that regulatory measures now proposed would have to be revised.

Howard M. Turner,[65] M. Am. Soc. C. E. (by letter).—A comprehensive description and general discussion of the 1936 flood is presented in Mr. Uhl's admirable paper. The immediate cause of this great flood in most of the New England States was the storm of March 17 to 19, the run-off from which was superimposed on a very large underlying flow due to melting snow and the run-off from the previous storm of March 12 and 13. A consideration of this particular storm and run-off is thus of interest in analyzing the flood.

The writer questions whether the high rainfall in the second storm (March 17 to 19) covered as large an area as is shown in Fig. 5. The rainfall on the summit of Mt. Washington for these three days was 5 in.

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[64] Engr., Land and Tax Dept., Erie R. R., Cleveland, Ohio.

[65] Cons. Engr., Boston, Mass.

and not 8 to 9 in. Any such rainfall as that shown in Fig. 5 would have produced very high flows in the streams draining Mt. Washington to the west. The flow of the Ammonoosuc River, however, was only 56% of that in the 1927 flood. The pattern of isohyetal lines was probably more like that shown for the previous storm and that of 1927. It is even quite possible that the 10-in. rainfall may have covered only a very small area, and the area covered by 8 in., given as 300 sq miles, may have been only a fraction of this, as indicated by the lower rainfall at Randolph, N. H. It would be of great value if a rain-gage at an altitude similar to that at Pinkham Notch could be established on the west side of the Mt. Washington range.

In Fig. 2, showing the rainfall-area relation for several great flood-producing storms in New England, the total rainfall for 13 days in the 1936 storm is compared to other storms of much shorter duration. Considering the storm of March 17 to 19 by itself, it is interesting to note that a plotting of this 3-day storm on the rainfall-area relation chart shows it to be much less than that of the other great storms.

Figures of the total run-off for the entire two-week period of high water from March 12 through March 25 show the enormous quantity of water that flowed down the rivers. The run-off of the concentrated flow during the second peak is also of interest. The writer has made an analysis of this concentrated flood on certain rivers according to the method advanced in the report of the Committee on Floods of the Boston Society of Civil Engineers.[66] This report presented the general theory that the length of the base of the flood hydrograph at a given station for the run-off of a storm within the "concentration period" for that station was constant regardless of the size of the flood, the peak flow thus being proportional to the total flood run-off. Thus, each point on a river has its own flood hydrograph curve which is a measure of its flood characteristics. By this method of analysis the total run-off during a flood is divided into two parts, the "base flow" which is not dependent on the storm rainfall producing the flood, and the "flood run-off" due to the storm rainfall within the concentration period of the basin. From this flood run-off "flood characteristic curves" were developed. These curves were determined from the hydrograph of the flood run-off above the base flow set on a unit basis by dividing the time and quantity of the flow by the square root of the drainage area, and the quantity of flow, in addition, by the number of inches of flood run-off, thus giving a flood hydrograph reduced to a 1-in. flood run-off from 1 sq mile of drainage area. This "flood characteristic curve" is somewhat similar to the "distribution graphs" of the unit hydrograph method of analysis developed by L. K. Sherman,[67] M. Am. Soc. C. E., which express the shape of the flood hydrograph above the base flow in terms of percentage of peak flood run-off, instead of in terms of flow for a 1-in. run-off, and of the actual time instead of time reduced to a unit basis. The base flow is also computed differently, but in cases that the writer has analyzed by both methods the difference is small, and the two curves are very similar.

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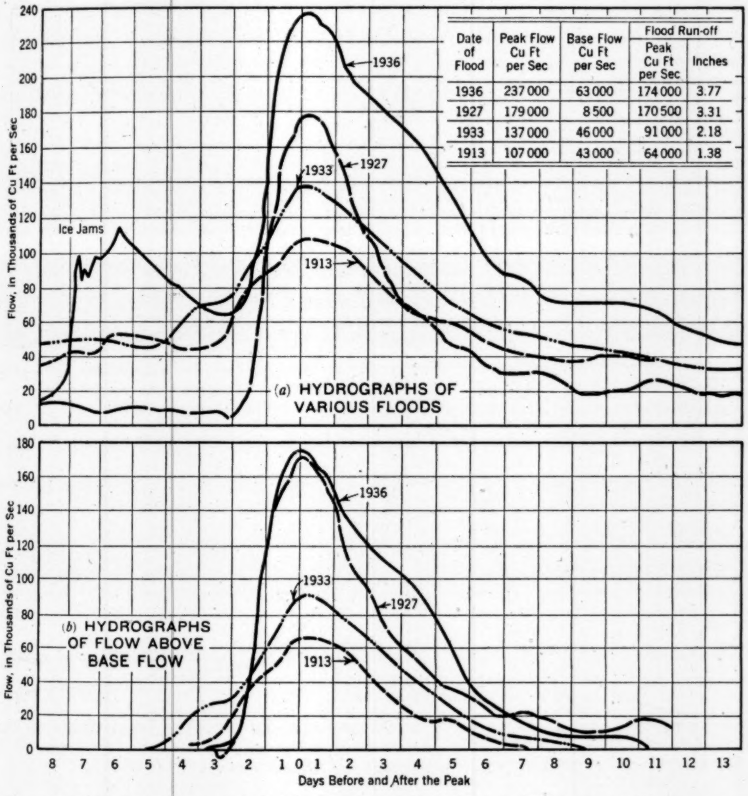
[66] *Journal*, Boston Soc. of Civ. Engrs., Vol. XVII, No. 7, September, 1930.

[67] "Stream Flow by the Unit Graph Method," *Engineering News-Record*, Vol. 108, 1932, p. 501; and "Studies of Relations of Rainfall and Run-Off in the United States," *Water Supply Paper 772*, U. S. Geological Survey.

The writer has analyzed the 1936 flood on the Connecticut River, at Sunderland, Mass., and on the Merrimac, at Lowell, Mass., comparing it with other floods. Fig. 22(*a*), shows the flood hydrographs of 1913, 1927, and 1933, on the Connecticut River, at Sunderland (8 000 sq miles), and that of 1936 at Montague City, Mass. (7 940 sq miles). Fig. 22(*b*) shows the same flood hydrographs after deducting the base flow in each case. The similarity of the length of the base of the hydrograph for these different floods is clearly shown.



Date of Flood	Peak Flow Cu Ft per Sec	Base Flow Cu Ft per Sec	Flood Run-off	
			Peak Cu Ft per Sec	Inches
1936	237 000	63 000	174 000	3.77
1927	179 000	8 500	170 500	3.31
1933	137 000	46 000	91 000	2.18
1913	107 000	43 000	64 000	1.38

FIG. 22.—FLOOD HYDROGRAPHS, CONNECTICUT RIVER AT SUNDERLAND, MASS. (1936, MONTAGUE CITY).

There is some variation possible in the value taken for the base flow. It has been assumed as constant throughout the flood period and, in most cases, equal to the flow existing just before the flood rise began. Usually, this is slightly less than the flow remaining at the end of the flood period, as the flood flow tends to diminish after the peak at a slower rate than it increases to the peak. In some cases, where the run-off before the flood was larger than afterward, it has been taken at somewhat less so as to include the entire flood run-off. A small difference in the quantity of base run-off does not greatly affect the characteristic curve. It may be argued that for the 1936 flood this was not a base flow, strictly speaking, but was a part of the first flood that had not yet run off. This is undoubtedly true, although an examination of the hydrographs and of the rates of diminution of snow on the ground during the period between the two storms seems to indicate that even if the storm of March 17 to 19 had not occurred, the flow prior to that storm would probably have continued high. In this case it would seem to be immaterial just what made up this base flow, as it is about the same before and after the total flood period.

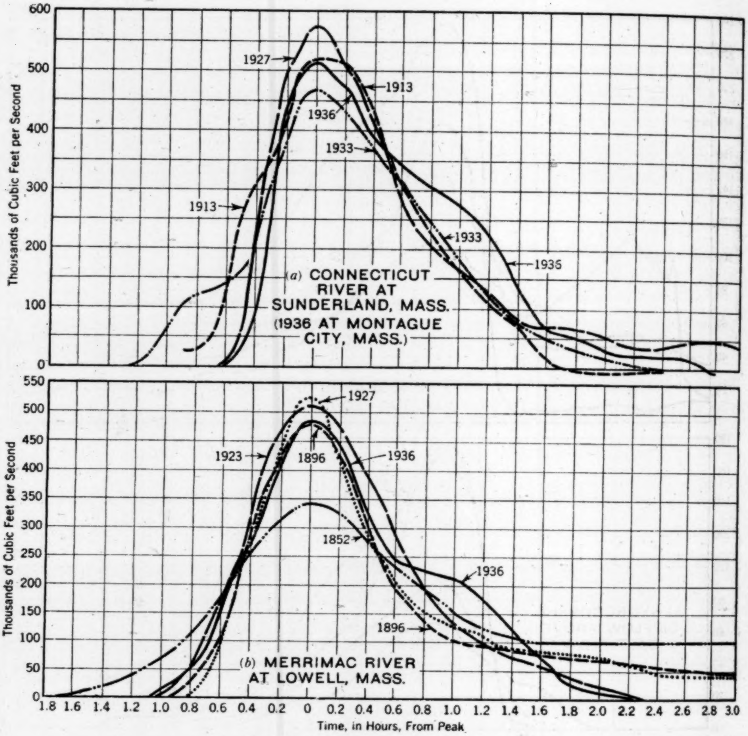
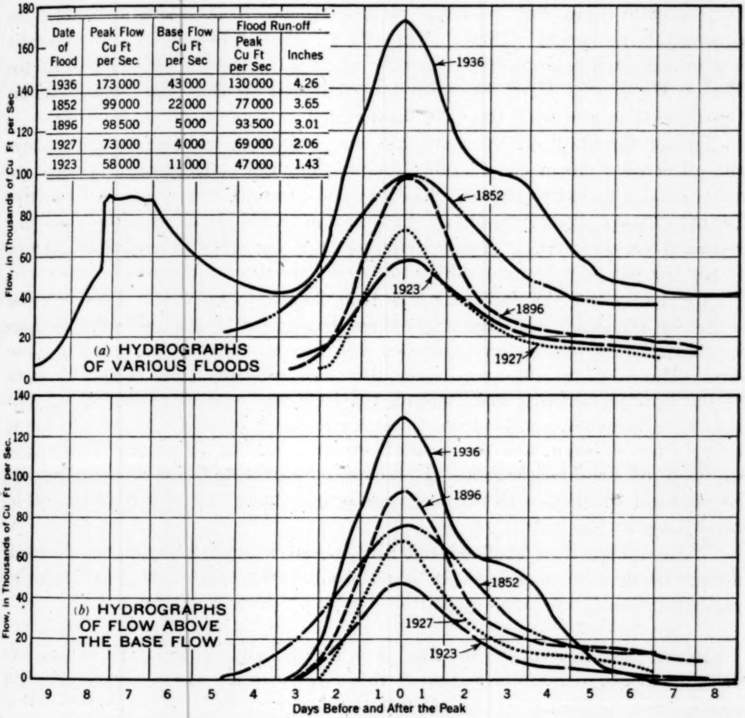


FIG. 23.—CHARACTERISTIC FLOOD CURVES.

From these "flood run-off" curves the flood characteristic curves shown in Fig. 23(*a*) have been computed. The similarity of these curves for the four floods, with flood run-offs varying from 1.38 in. to 3.77 in., is apparent. That for 1936 is somewhat affected by the additional run-off due to the rainfall of March 21, which shows clearly on the flood hydrograph as a bulge on the curve after the peak. If allowance was made for this condition, the curve would show about the same peak as the curve for 1927.

The 1936 flood on the Connecticut River appears, therefore, as a strictly characteristic flood run-off due to the storm of March 17 to 19, including the consequent melting snow caused by it, superimposed on a very large base flow. The latter was probably largely melting snow although in part it was the remainder of the run-off from the previous storm. The flood run-off, 3.77 in., above the base flow was greater than the previous record run-off of 1927 (3.31 in.), and this was added to a base run-off of 8.0 cu ft per sec per sq mile (equivalent to 0.3 in. of run-off per day). The question of what is the total run-off including the base flow will depend on the period taken. It seems preferable to express the flood run-off in inches for the total, but



Date of Flood	Peak Flow Cu Ft per Sec	Base Flow Cu Ft per Sec	Flood Run-off	
			Peak Cu Ft per Sec	Inches
1936	173 000	43 000	130 000	4.26
1852	99 000	22 000	77 000	3.65
1896	98 500	5 000	93 500	3.01
1927	73 000	4 000	69 000	2.06
1923	58 000	11 000	47 000	1.43

FIG. 24.—FLOOD HYDROGRAPHS OF MERRIMAC RIVER, AT LOWELL, MASS.

the base flow as a rate, continuing as it does before and after the flood period.

Fig. 24(*a*) shows the same hydrographs of various floods at Lowell. These hydrographs are taken from those given by the Committee on Floods of the Boston Society of Civil Engineers,[68] to which has been added the hydrograph of the 1936 flood. Fig. 24(*b*) shows these floods after deducting the base flows. On the Merrimac River the flood run-off above the base flow was greater than any previously recorded flood (1852) by 17%, and 40% greater than any previous recorded flood from a storm within the concentration period (1896). Fig. 23(*b*) shows the flood characteristic curves also taken from the report[69] of the Committee on Floods of the Boston Society of Civil Engineers, re-arranged so that the peaks come at the same time and with the 1936 flood added.

Except for the flood characteristic curve of 1852, which apparently shows the effect of a storm considerably longer than the concentration period, the same similarity exists and the flood of 1936 presents the same characteristic. In this case a flood run-off of 4.26 in., in itself much greater than any recorded previous flood, was superimposed on the high base flow of 10.8 cu ft per sec per sq mile, equivalent to 0.4 in. per day of run-off.

Mr. Uhl notes the fact that the 1936 flood generally was relatively less severe on the small drainage areas than that of 1927. Excluding differences in rainfall this was generally due to two factors: First, the storm of three days (March 17 to 19) was longer than the concentration period of many of the smaller streams; and, second, the base flow was a small part of what may be expected for a flood on a small stream; 8 to 10 cu ft per sec per sq mile is not a large flow for a small stream, but on the larger rivers it is in itself of flood magnitude. The base flow of the Connecticut River, at Sunderland (63 000 cu ft per sec) was larger than 27% of the annual 24-hr flood flows in the last 55 yr.

With a base flow of flood magnitude and a superimposed flood run-off greater than any on record, it is no wonder that the 1936 flood exceeded previous records by large quantities on many New England rivers.

RALPH W. POWELL,[70] M. AM. SOC. C. E. (by letter).—It is scarcely possible to exaggerate the value of this Symposium. Between the time the papers were presented orally (October, 1936) and their appearance in *Proceedings* (March, 1937) another flood occurred on the Lower Ohio River, in most respects more serious than any described in the Symposium. Therefore, some of the statements in the Symposium were already "out of date" when printed, but the importance of the subject was still further emphasized. The fact that it is now a national problem is clear from the wording of the Flood Control Act of 1936.

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[68] *Journal*, Boston Soc. of Civ. Engrs., Vol. XVII, No. 7, September, 1930, Fig. 16, p. 384.

[69] *Loc. cit.*, Fig. 18, p. 394.

[70] Assoc. Prof. of Mechanics, Ohio State Univ., Columbus, Ohio; formerly, Hydr. Engr., Muskingum Watershed Conservancy Dist., New Philadelphia, Ohio.

With the exception of those few non-navigable streams that discharge directly into the ocean or into the Great Lakes, the control of floods on any stream in the United States is now "a proper activity of the Federal Government in co-operation with States, their political subdivisions, and localities thereof."

One minor correction might be offered to Mr. Jacobs' valuable paper. Under the heading, "Engineering Aspects," he states that three of the Muskingum reservoirs are automatic retention basins. Probably the three reservoirs meant are the Mohawk, Mohicanville, and Bolivar. These are "dry reservoirs," but the outlets are gate-controlled. Of course, with the gates open, the reservoirs would still give a certain volume of automatic storage just as they did in the January, 1937, flood when the gates were kept open due to legal reasons; but, normally, the gates will be closed as soon as it appears that the safe capacity of the channel down stream would otherwise be exceeded.

The Pleasant Hill Reservoir has an automatic outlet (orifices in an intake tower), but it is not a simple retention basin, as it has 13 500 acre-ft of permanent storage that can be released through two 3.5 by 7-ft sluice-gates. Since these gates are normally under a head of about 45 ft, they could, if it were desired, afford an appreciable controlled discharge in addition to the automatic discharge through the orifices. The original plans called for gates in the orifices also, but this was ruled out by the consulting board as one more complication in an already complicated design.

Of the plotting of discharge per square mile against drainage area (as in Fig. 3, presented by Mr. Uhl, and in Fig. 13, presented by Messrs. Harrington and Johnston) there is no end, and the graphs are always interesting. The latest to come to the writer's attention is by Victor H. Cochrane,[71] M. Am. Soc. C. E. It would seem to the writer, however, that they would be much more valuable if the data were limited to the maximum annual floods over the entire period of record of a large number of streams. Then a line or a smooth curve which had 1% of the points outside it might be thought of as representing a 100-yr probability; one with 5% of the points outside it, a 20-yr probability; etc. It must always be remembered, however, that drainage area is only one of the factors determining probable peak discharge. The arrangement of tributaries, whether fan-shaped and converging on the point considered, as at Pittsburgh, Pa., and Dayton, Ohio, or with various concentration periods, as in the Mississippi River at New Orleans, La., plays an extremely important part. As C. R. Pettis, M. Am. Soc. C. E., has shown,[72] the average width of the drainage area is probably a better criterion than the total area, and a plotting of peak flow against average width might be the best way to represent the data.

In spite of its obviousness, furthermore, attention should be called to the fact that it is only in the design of channel improvements, levees, bridge openings, etc., that the unmodified peak flow is important. If the flow is to be controlled by reservoirs, the essential item is the volume of the entire flood hydrograph. Fortunately this can be estimated much more accurately than the peak flow, since for safety it must be taken as equal to, or nearly equal to, the entire rainfall over the drainage area; and the data on rainfall are much more extensive than those on run-off.

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[71] *Engineering News-Record,* November 25, 1937, p. 867.

[72] "Floods in the United States," U. S. Geological Survey, *Water Supply Paper No. 771,* p. 36.

In this connection the writer feels that Messrs. Morse and Thomas have erred in their treatment of the effects of forests. The run-off from light and medium rains is greatly affected by the cover, and a good forest should definitely reduce the average annual flood. For the great floods upon which the design of adequate flood control must be based, however, it is doubtful whether the retention would be a large enough part of the entire precipitation to affect greatly the design. For example, the January, 1937, flood at Louisville, Ky., would probably have been almost as great if the same rainfall had fallen 200 yr earlier. Certainly accurate quantitative data on the subject are greatly to be desired.

With the remarks of Messrs. Morse and Thomas on "flood routing" and on the importance of the experiments at the Carnegie Institute of Technology, at Pittsburgh, the writer is in hearty accord. Some have felt that the Burns-Harkness-McCarthy method is a satisfactory solution of the problem, but the writer's experience is that the ratio of storage increment to the corresponding weighted flow increment is not the same for great floods as it is for medium-sized floods, and that it is not the same for winter floods as for summer floods. To be able to compute the effect at all points down stream, of storing or discharging at given rates from each reservoir, is so necessary for flood forecasting as well as for the design of control systems, that the problem is probably the most important one confronting flood control engineers. Fortunately, sufficiently complicated systems are now in existence on such rivers as the Tennessee and Muskingum, to try out, by full-scale experiment with small floods, any theory or method of computation proposed.

Franklin F. Snyder,[73] Jun. Am. Soc. C. E. (by letter).—In the part of their paper discussing the U. S. Weather Bureau relation to the 1936 flood at Pittsburgh, Messrs. Morse and Thomas compare a stage prediction made at 8:00 A.M. on March 17, 1936, for the forenoon of March 18, with the observed stage at 8:00 A.M., March 18, and arrive at an error of 10 ft. From the data now available it appears that the forecast made on 8:00 A.M., March 17, was not more than 3 ft in error. The aforementioned type of comparison and conclusion is often made and illustrates the care that must be taken to make clear the basis for any particular forecast. Studies for the improvement of existing flood forecasting methods, made co-operatively by the Pennsylvania Department of Forests and Waters, the U. S. Weather Bureau, and the U. S. Geological Survey, were begun in 1937, and, in this connection, some interesting aspects of the 1936 flood at Pittsburgh, Pa., have been developed.

Procedures of river forecasting on head-water areas are based on data accumulated only a short time before the forecast is released. Progress is being made in weather forecasting, and particularly in air-mass analysis, which indicates possibilities of forecasting rainfall quantitatively, but, until such developments reach a state of practical application, forecasts, as issued, must be understood to apply to flow from rain already fallen.

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[73] Hydr. Engr., Dept. of Forests and Waters, Commonwealth of Pennsylvania, Harrisburg, Pa.

The interval between the time of critical rainfall and that in which the resulting flood can be identified in the principal channel system is appreciable in most cases. The unit graph procedure provides an opportunity to demonstrate this point.

In analyzing the March, 1936, flood at Pittsburgh, the Allegheny River was studied at Dam No. 3 (drainage area, 11 530 sq miles) which is 14.5 miles up stream from the junction. On the Monongahela River, Dam No. 2 (drainage area, 7 340 sq miles), 11.2 miles up stream from the junction, was used. The surface run-off hydrographs were computed at these two stations and added to the Pittsburgh base flow to obtain the computed hydrographs of Fig. 25.

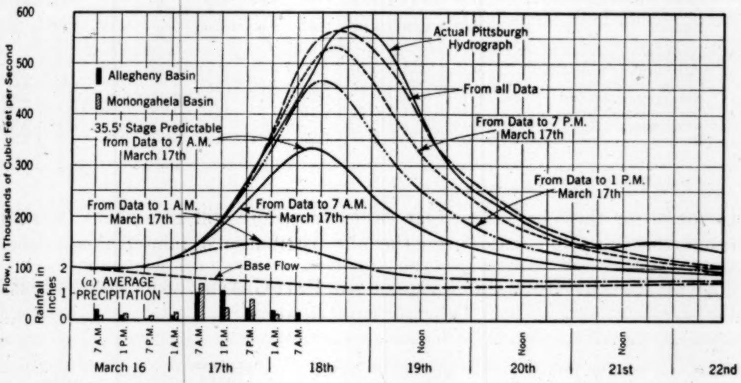

FIG. 25.—FLOOD OF MARCH, 1936, AT PITTSBURGH, PA.

Nineteen selected precipitation stations were used on the Allegheny Basin. They gave an average depth of 3.94 in. of precipitation for the period, March 16–19, 1936, of which the last 0.63 in. was classified as snow and was eliminated from the study for the period after 7:00 A.M. on March 18. The unit graphs used were designed for 6-hr rainfall data, the basin precipitation being divided into 6-hr intervals according to the precipitation recorded automatically at the Piney Plant of the Pennsylvania Electric Company, on the Clarion River (see Table 14).

Snowfall equivalent to 0.50 in. of water was added and assumed to melt in proportion to the precipitation for lack of a better method. This gave 3.81 in. of water depth to be accounted for and, with an initial loss of 0.25 in. deducted from the rainfall of the first period, left 3.56 in. The estimated volume of surface run-off was 1.95 in., giving a surface run-off factor of 54.8 per cent. This factor is not a real one since only enough run-off occurred from the precipitation above Franklin, Pa. (mostly melted snow) to maintain a recession flow of 42 000 cu ft per sec from the previous rise to give a peak of 50 000 at midnight, March 17–18.

Another way to express the run-off factor, more in accord with the actual conditions (that is, the area above Franklin eliminated), is based upon a depth of precipitation plus water equivalent of snow of 5.00 in., with a surface run-off volume of 3.75 in., giving a run-off coefficient of 75% for the lower area. It will be found for this illustration that either procedure will give about the same result, the controlling factor being the 6-hr distribution of rainfall as established

TABLE 14.—PRECIPITATION AND RUN-OFF DATA, IN INCHES, MARCH 16–19,1936

	SIX-HOUR PERIODS ENDING AT:									Total
	March 16			March 17				March 18		
	7:00 A.M.	1:00 P.M.	7:00 P.M.	1:00 A.M.	7:00 A.M.	1:00 P.M.	7:00 P.M.	1:00 A.M.	7:00 A.M.	
Allegheny Basin:										
Piney precipitation	0.50	0.18	0.05	0.17	1.32	1.43	0.54	0.41	0.33*	4.93
Basin precipitation	0.39	0.14	0.04	0.13	1.02	1.10	0.42	0.32	0.25*	3.81
Initial loss	0.25	...	...	...	...	...	...	...	...	0.25
Run-off	0.08	0.08	0.02	0.07	0.56	0.60	0.23	0.18	0.13	1.95
Monongehela Basin:										
Pittsburgh precipitation	0.28	0.35	0.16	0.20	0.85	0.39	0.28*	0.09*	*	2.60
Elkins precipitation	0	0	0.08	0.23	1.50	0.41	1.06	0.26	*	3.54
Average precipitation	0.14	0.18	0.12	0.21	1.18	0.40	0.67	0.17	...	3.07
Basin precipitation	0.16	0.21	0.14	0.25	1.37	0.46	0.78	0.20	...	3.57
Initial loss	0.16	...	...	...	...	...	...	...	...	0.16
Run-off	0	0.17	0.11	0.20	1.11	0.37	0.63	0.16	...	2.75

* Snow.

TABLE 15.—SIX-HOUR ORDINATES OF THE UNIT GRAPHS, AND CORRESPONDING DISTRIBUTION GRAPHS

Period	ALLEGHENY RIVER		MONONGAHELA RIVER		Period	ALLEGHENY RIVER		MONONGAHELA RIVER	
	Rainfall Concentrated in Lower Valley		Rainfall of Average Distribution Over Valley			Rainfall Concentrated in Lower Valley		Rainfall of Average Distribution Over Valley	
	Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow	Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow		Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow	Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow
1	8.0	0.63	2.9	0.36	15	20.8	1.68	22.3	2.81
2	31.2	2.50	10.8	1.36	16	16.8	1.35	19.2	2.42
3	66.5	5.35	23.0	2.90	17	13.4	1.08	16.7	2.11
4	118.0	9.51	40.3	5.09	18	10.7	0.86	14.5	1.83
5	176.3	14.22	64.7	8.16	19	8.3	0.67	12.5	1.58
6	177.7	14.35	90.7	11.45	20	6.3	0.51	10.7	1.35
7	155.5	12.54	92.8	11.70	21	4.6	0.37	9.1	1.15
8	118.0	9.52	79.3	10.00	22	3.0	0.24	7.7	0.97
9	88.5	7.14	65.2	8.22	23	1.9	0.15	6.3	0.80
10	66.6	5.38	53.3	6.72	24	0.6	0.05	5.0	0.63
11	50.4	4.06	44.0	5.55	25	....	....	3.8	0.48
12	39.4	3.18	36.5	4.60	26	....	....	2.7	0.34
13	32.0	2.58	30.6	3.86	27	....	....	1.6	0.20
14	25.8	2.08	26.0	3.28	28	....	....	0.6	0.08
Totals	....	....	....	....	..	1 240.3	100.00	792.8	100.00

by the Piney record. However, since the depths used are those for the entire area, the unit graph applied is that designed for conditions prevailing when the greater run-off comes from the lower basin.

Twenty-five precipitation stations were used on the Monongahela Basin and, for the period, March 16–19, 1936, gave an average of 3.54 in., of which the last 0.47 in. was classified as snow assumed to have fallen after midnight of March 17–18. The water content of melting snow was assumed at 0.50 in. With an initial loss of 0.16 in., the remaining 3.41 in., together with a surface run-off factor of 80.5%, provided the desired volume of 2.75 in. of surface run-off. The precipitation period was divided into 6-hr intervals according to the average of the recorded rainfall records of the U. S. Weather Bureau stations at Pittsburgh, and at Elkins, W. Va., as shown in Table 14. The unit graph applied was that designed for use with a fairly uniform distribution of run-off.

Six-hour ordinates of the unit graphs and their corresponding distribution graphs are given in Table 15. The first value is to be plotted at the middle of the first 6-hr run-off period and the successive values at 6-hr intervals.

No time correction was applied to the computed surface run-off data for the time lag between Dam No. 2 and Dam No. 3 and Pittsburgh. The actual Pittsburgh hydrograph is also shown, and is based on a Pittsburgh rating obtained by gage relations from the U. S. Geological Survey Station at Sewickley, Pa.

The foregoing example is presented to illustrate the situation that must be taken care of when forecasts are to be issued at regular intervals. Certainly this must be done on head-water areas where, if the forecast were delayed until the cessation of rainfall, the opportunity for preparation would be largely lost. The illustration is not intended as a demonstration of the unit graph procedure nor as a justification or criticism of any actual forecasts. A considerable part of the study was prepared by R. J. MacConnell, of the Pennsylvania Department of Forests and Waters.

W. F. UHL,[74] M. AM. SOC. C. E. (by letter).—To the many engineers who have contributed discussion to his paper, the writer is indebted. Mr. Williams' discussion of the basis for computing flood damages, which he illustrates by reference to the Winooski River, is helpful. The writer agrees with Mr. Williams that the humanitarian aspect of flood control should not be stressed to the point of causing a community to assume a burden which threatens its financial ability.

By his account of the flood at Lowell, Mass., Mr. Safford has described the effect of the greatest flood on record on one of the old water-power cities of New England. The damage that occurred, and the problem of flood control as it affects Lowell is probably typical of the situation in New England cities situated on the larger rivers. In his study of flood frequency on the Lower Connecticut River, Professor Barrows has made use of one of the longest records of flood stages in the United States—that of the Connecticut River, at Hartford, Conn. The fact that he has developed two extension curves from these data illustrates the difficulty of establishing an accurate basis for computing the frequency of floods of certain magnitudes.

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[74] Hydr. Engr., Chas. T. Main, Inc., Boston, Mass.

Mr. Bush questions whether it is possible to operate reservoirs, built for the dual purpose of power storage and flood control, in such a way that the flood-control aspect will be fully safeguarded. In such a dual-purpose reservoir, unquestionably some of the effectiveness of a given reservoir for power-storage purposes will have to be sacrificed to the requirements of flood control. In New England, at the present time, there are several instances of power-storage reservoirs subject to regulations dictated by the recreational uses of the reservoir, which impair their effectiveness as power-storage reservoirs; for instance, the operation of Lake Winnepesaukee is subject to State regulations which limit the draft during the summer months. It is a fact that the recreational uses of these reservoirs have been safeguarded by these regulations. Therefore, it seems reasonable to suppose that, in the case of reservoirs built for the dual use of power storage and flood control, it would be possible to control the operation by regulations which would safeguard the requirements of flood control.

The writer agrees with Mr. Turner that the isohyetal lines of very high rainfall in the second storm, as shown in Fig. 5, probably did not extend as far west as is shown. An exact determination of the area covered by the 10-in. and the 8-in. rainfall is an interesting subject of speculation, but there are not sufficient data available to arrive at a definite answer to the problem. Mr. Turner's study of the major peak of the 1936 flood on the Connecticut and Merrimac Rivers [66] is a valuable contribution to the subject of New England floods.

The writer wishes to point out that his paper was written while the work of collecting and analyzing the data on the March, 1936, floods was still in process. As a result, many of the figures given in the paper, particularly those of 1936 peak discharges, are subject to change. In general, the changes are so small that it seems unnecessary to revise the original data. The revised values are available in some of the more recent papers on this subject.[75]

ARTHUR W. HARRINGTON,[76] M. AM. SOC. C. E., AND HOLLISTER JOHNSON,[77] Assoc. M. AM. SOC. C. E. (by letter).—Since the presentation of the writers' paper, the U. S. Geological Survey has completed a number of papers and reports pertinent to the subject of this paper. One series, of three parts, concerns "The Floods of March, 1936." [78] In these volumes are presented records of stage and discharge at river-measurement stations, contents of storage reservoirs, peak discharges and stages with comparative data for other floods, crest stages, precipitation, the depth and water content of snow on the ground, the results of studies of rainfall and run-off, and many other kinds of flood information.

The writers are grateful to Mr. Hendricks for his discussion of the floods of 1935 and 1936, with special reference to their occurrence and relation to previous floods in the basins of the Mohawk and Oswego Rivers.

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[75] See, especially, "The Floods of March, 1936, Part I, New England Rivers," *Water Supply Paper 798*, U. S. Geological Survey.

[76] Dist. Engr., U. S. Geological Survey, Albany, N. Y.

[77] Hydr. Engr., U. S. Geological Survey, Albany, N. Y.

[78] "Part 1, New England Rivers," *Water Supply Paper No. 798;* "Part 2, Hudson River to Susquehanna River Region," *Water Supply Paper No. 799;* and "Part 3, Potomac, James, and Upper Ohio Rivers," *Water Supply Paper No. 800.*

Professor Powell has a very interesting idea of plotting drainage area against the discharge per square mile for the maximum annual floods over the entire period of record of a large number of streams and representing the 100-yr probability by a line or smooth curve which would have 1% of the points outside it; the 20-yr probability by the line with 5% of the points outside it, etc. However, some difficulties might be encountered in establishing a criterion by which to judge the distribution of the points along the line and thus determine the slope of the line. If some reasonable criterion was not used it would be easily possible to produce an absurd interpretation that would show increases in unit discharges with increases in drainage areas which, of course, is generally not true. It is the writers' belief that, if, before plotting, the discharges per square mile were corrected for channel storage (thus representing the unit rate of run-off into the channel rather than the unit rate of run-off in the channel past the measuring point) the resulting chart would represent more nearly unit rates of run-off which might be expected to occur in streams in the same locality but with perhaps much less channel storage.

It appears that there is unanimity of opinion among engineers and others as to the need for the development of a national flood-protection policy and that one of the essentials for the evolution of such a policy is the availability of the hydrologic data which are so basic and fundamental in any study and investigation of the control of water. Including the aforementioned series,[78] the U. S. Geological Survey has prepared reports on ten notable and outstanding floods which have occurred since 1934. These reports[79] were planned as an effort to present such basic hydrologic data (especially that of precipitation and stream flow) as were available at the time of publication.

MERRILL BERNARD,[80] M. AM. SOC. C. E. (by letter).—It is gratifying to report that field headquarters for five of the hydrologic regions planned by Mr. Hayes were established early in 1938. Other gains in expanding and strengthening the service have been made, one of the most important being a co-operative program between the Commonwealth of Pennsylvania, through its Water and Power Resources Board, the U. S. Weather Bureau, and the U. S. Geological Survey. This co-operative project is providing the means to establish a modern forecasting service on the Allegheny, Monongahela, Susquehanna, and Delaware Rivers.

Modern methods of estimating stream flow from rainfall extend the analysis into the relatively small water-sheds comprising the river basin. The procedure involves short concentration periods, and rainfall rates of short duration and high intensity. Obviously, rainfall rates recorded as averages for 24 hr cannot be used in studies on water-sheds of 50 to 500 sq miles having concentration periods of 2 to 6 hr. Therefore, the first step in the aforementioned co-operative program has been to establish, over the river basins comprising the project, a net of approximately 130 recording rain-gages having an average density of 400 sq miles per gage.

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[79] "Flood in the La Canada Valley, California, January 1, 1934," *Water Supply Paper No. 796–C;* "Flood on Republican and Kansas Rivers, May and June, 1935," *Water Supply Paper No. 796–B;* "The New York State Flood of July, 1935," *Water Supply Paper No. 773–E;* "Major Texas Floods of 1935," *Water Supply Paper No. 796–G* (includes the floods of May, June, and December); "Major Texas Floods of 1936," *Water Supply Paper No. 816* (includes the floods of June, July, and September); "The Ohio-Mississippi Floods of January–February, 1937"; and "Major New Mexico Floods of 1937."

[80] Mr. Hayes died on November 16, 1936. This closing discussion on his paper was prepared by Merrill Bernard, M. Am. Soc. C. E., who succeeded Mr. Hayes as Chf. of the River and Flood Div., U. S. Weather Bureau, Washington, D. C., on March 5, 1937.

These recording rainfall stations have been classified as: (1) Those that are located primarily to provide data for the hydrologic studies from which forecasting methods are to be developed; and (2) those that are to serve ultimately as reporting rainfall stations under the established forecasting procedure. Stations in the first group have been located so as to give proper weight to geographic position in the net, area representation, elevation, and aspect, without depending upon local attendance. Those in the second group have been located at strategic points where qualified 24-hr attendance is available and dependable communication assured.

Practically all the rainfall stations are accessible over paved highways and have been located on three circuits over which experienced hydrologic engineers travel on schedule. These engineers are charged with the duties of servicing the gages, assembling the charts, and tabulating the data for immediate use. During the winter months they will conduct snow surveys and keep themselves informed of day-by-day changes in run-off conditions.

In general, the procedure at the forecasting centers will be as follows:

(*a*) A day-by-day estimate of run-off potentialities based upon the discharge from index water-sheds, the estimate to include the possible contribution to run-off from an existing snow mantle;

(*b*) A day-by-day check on storm approach, utilizing the general weather service of the Weather Bureau, and particularly the weather maps prepared at frequent intervals at the aerological stations;

(*c*) With the beginning of a critical storm period, the reception of hourly (or other-period) rainfall by telephone and telegraph, or by amateur and automatic radio broadcasting;

(*d*) The preparation of an isohyetal map for each period showing the progressive accumulation of rainfall depth over the basin;

(*e*) Under a well organized and largely mechanical technique, the conversion of rainfall into stream-flow hydrographs;

(*f*) The conversion of rates of stream flow to river stage at points down stream for which a forecasting service is rendered; and,

(*g*) As quickly as rainfall has ceased, the prompt announcement, through established channels, of flood-peak stage and the hour of the arrival of the flood's crest.

Mr. Snyder has given an interesting picture of the March, 1936, flood at Pittsburgh, Pa., based upon a critical review of the meager data available and an application of the unit graph method of estimating stream flow.

The unit hydrograph is found to be an excellent tool for forecasting, as his discussion indicates. As data are made available through the newly established network of recording rainfall gages, refinements in the method and time-saving devices in procedure can be expected to develop. His study constitutes an important step in the establishment of a modern forecasting plan for Pittsburgh.

In closing the incident of the forecasts of the March, 1936, flood at Pittsburgh, it seems appropriate to comment upon the courage which it must have taken to issue an unprecedented forecast based upon the most meager data, and dependent upon a method in which judgment gained from personal experience was the predominating factor.

Mr. Scobey has called attention to the important place snowfall has in the genesis of floods, particularly on Western streams where flow is comprised largely of run-off from snow melting.

In the mountainous States of the West forecasts of the seasonal yield of the snow mantle, expressed as a percentage of the normal for the basin, are determined from surveys made toward the end of the snowfall season. The obvious need for a quantitative forecast—that is, a forecast in terms of the hydrograph of flow—has stimulated an interest in the Weather Bureau's investigations of snowfall gages, the most promising type being one in which the snow is melted as it falls into the gage with a salt brine, the catch stabilized against wind action with a flexible shield, and the contents of the gage protected against evaporation by an oil film.

The snow course lends a real significance to the record by virtue of its length. Likewise, the snowfall stations of the Bureau will utilize sampling areas, carefully selected as typical, upon which are placed batteries of storage snow gages, the average record of the group being considered applicable to the part of the basin represented by the snowfall station. The average water depth for the group (usually five gages) is determined by the simple expedient of weighing, under which procedure the increment increase in weight between weighings becomes the snowfall for the period, expressed in inches. Batteries of such gages have been established at twenty stations having elevations greater than 7 000 ft. Gratifying experience to date (1938) indicates that they have the following advantages over other methods of determining the water equivalent of snowfall under Western conditions: (1) They measure snow as it falls and before it is subjected to prolonged wind action; (2) they secure the record against loss through poor attendance or failure to reach the station before a critical period of melting; (3) they function at all elevations, including the transition zone from snowfall to rainfall; (4) they are inexpensive to install and require a minimum of attendance; (5) they are not dependent upon tree shelter for stability of record; and (6) precipitation in the form of snow can be recorded as daily precipitation through the simple procedure of daily weighing.

One hesitates to take up the gauntlet in Mr. Meursinge's good-humored jibe at American engineers for their temerity in attempting to determine possible maximum run-off of the Ohio and Mississippi Rivers. Such optimism has evidenced itself in the newly established hydro-meteorological studies of the Weather Bureau set up co-operatively with the U. S. Corps of Engineers. The study undertakes to provide the meteorological and hydrological background of the great storms of record and the justification for their transposition to basins other than those embraced actually by the critical storm area.

Mr. Houk has sensed the weakness in any head-water forecasting service to meet the situation developing around the highly intense cloudburst type of storm which can develop locally and may not be associated with a period of general rainfall.

Two circumstances may mitigate toward a practical solution of such forecasting problems:

(*a*) *Plans for an Expanded Network of Recording and Reporting Rainfall Stations.*—The density of the co-operative network in Pennsylvania is approximately 1 gage to 400 sq miles. Recent estimates for networks on problem areas in the Mississippi Basin provide a recording rain-gage to each 100 sq miles on the average.

(*b*) *The Rapid Expansion of the Network of Aerological Stations of the U. S. Weather Bureau.*—These stations, located at the active airports of the United States, serve aviation and issue a weather map at 6-hr intervals from transmitted signals which include the results of upper-air soundings.

It is planned to make full use of these aerological stations in the river and flood forecasting service of the Weather Bureau.

Head-water flood-warning services are being inaugurated (1938) by the Bureau in co-operation with the Cities of Cumberland, Md.; Johnstown, Pa.; and Elkins, W. Va. In these instances no immediate attempt is being made to forecast flood stages, but a carefully organized warning service is intended which will utilize municipal departments, railroad and utility company communication facilities, and the services of municipal officials and employees whose interests insure an alert and efficient attendance.

The snow work of the Weather Bureau has been commented upon briefly in the discussion by Mr. Scobey. Mr. Field has emphasized the need for quantitative forecasts of run-off from melting snow. Under conditions in the Central and Eastern States, where periodic melting may remove the snow mantle several times during the winter months, it is necessary to know, day by day, the potential run-off in an existing snow cover, and an adequate forecasting system must provide for the measurement of the factors that may combine to release such run-off. Under Western conditions, where there is no appreciable loss through the winter, the problem of quantitative forecasting will be largely that of measuring factors, such as temperature, concurrent rainfall, and ground surface conditions, as well as snowfall measurements at a sufficient number of points in a basin to give dimensional significance to the average depth of snow and water equivalent. The gathering of such data involves remote observational points at high elevations where attendance is difficult or impossible and emphasizes the obvious need for automatic recording seasonal snow gages.

The greatest obstacles to be overcome in the re-organization and expansion of the Bureau's river and flood forecasting service is a general lack of understanding of its functions, inadequate financing, and the difficulties of adjusting cost to benefit. The responsibility as well as the expense of a flood-forecasting service should shift in varying degrees from the Federal Government only, to a co-operation with other interests as head-waters are approached and as problems become more and more localized. This transition in economic responsibility is accompanied by marked changes in method. Thus, in flood control, levee systems only give way to combinations with reservoirs and organized land-use practices over large areas. Likewise, methods of river forecasting which have met successfully the conditions of lower river flow must be elaborated to take into account the many factors influencing smaller stream flow, the principal one of which is rainfall.

On the larger river systems the objective for the service is an integrated forecast from head-waters to outlet. This is best understood by conceiving the principal channel of the river as divided into reaches of known channel storage. It can be stated, then, that the discharge from the lower end of any reach throughout a fixed interval of time is equal to the discharge into the upper end from the reach above, plus the inflow of tributary streams entering throughout the length of the reach, plus or minus any change in storage that has occurred within the time interval. The travel of the flood wave through the reach must be taken into account. This takes the form of a progressive distortion in the shape of the storage prism throughout length and through time, and, under fixed conditions of inflow and outflow, may prove to be fairly constant. The most effective method of forecasting on a large river such as the Mississippi must begin at the head-waters, and on all important tributaries simultaneously, and must provide the means of transmitting hydrologic data and estimates of flow progressively down stream to the important forecasting centers along the main river stem.

The average annual flood loss to the nation as reported to the Weather Bureau is about $100 000 000. It is believed that this sum can be reduced materially by increasing the period of the forecast so that greater opportunity is provided for the removal of perishable goods and property and for the evacuation and protection of areas doomed to inundation by the oncoming flood. The cost of providing the additional preparation opportunity will prove negligible when compared with the resulting reduction in loss of life, human misery, and property damage.

The answer to the problem of minimizing national flood hazard does not lie in the permanent evacuation of the flood plains of the great rivers. Man has profited greatly by utilizing these fertile lands during such time as the rivers have not needed them to pass their flood waters to the sea. He should, and does, however, pay a rental for his occupancy, either in the continuing cost of protection works or in the periodic repair of flood damage. This cost could be reduced and, in many cases, nearly eliminated through a co-ordinated program of flood protection, flood warning, and flood insurance—all within the realm of practicability, as demonstrated in the several European countries pioneering in this field.

As the modern automatic fire alarm system is to be credited largely with the immunity now enjoyed from the catastrophic conflagrations that occurred all too frequently through the past half century, so should a modernized flood-forecasting service pave the way to a more secure and intelligent use of the river valleys of the United States.

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THE PASSAGE OF TURBID WATER THROUGH LAKE MEAD

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SYNOPSIS

Turbid water carrying a considerable load of fine silt was discharged from Lake Mead, above Boulder Dam, in Arizona and Nevada, at three different periods during 1935 when the reservoir was 70 to 90 miles long and contained from 4 000 000 to 5 000 000 acre-ft of water. Apparently, it flowed through the reservoir essentially unmixed. Chemical analyses of the water entering into, and discharged from, the reservoir corroborate the conclusions drawn from the observations of silt as to the occasional discharge of essentially unmixed water. The phenomenon is ascribed to the greater specific gravity of the incoming water relative to the generally clear water at the surface of the lake, due probably, in part, to its silt load. A practical significance is suggested with respect to the possibility of increasing the passage of fine silt through a reservoir, thereby prolonging its effective life. References are made to other known occurrences of similar phenomena.

INTRODUCTION

The apparent resistance of water to mixing, under certain conditions, is a matter of common knowledge based on many observations. A river having water of one color receives a tributary having water of another color,

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NOTE.—By decision of February 1, 1936, by the Board on Geographical Names, the reservoir formed by Boulder Dam on Colorado River in Arizona and Nevada, was named Lake Mead, in honor of the late Elwood Mead, M. Am. Soc. C. E. This paper is published by permission of the Director, U. S. Geological Survey. The paper was published in April, 1937, *Proceedings*.

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and the two waters flow for miles side by side in the same channel without very much mixing. The Gulf Stream flows for hundreds of miles through the Atlantic Ocean and retains such definiteness of boundaries (identified by color, temperature, and organic matter), that a ship's bow may be situated within, and its stern outside, the stream. The late John R. Freeman,[3] Past-President and Hon. M. Am. Soc. C. E., calls attention to rivers that discharge turbid water into the clear water of lakes. The phenomenon may be seen as the current cascades to considerable depths practically unmixed with the clear lake waters through which it is falling. The flowing of one water through, or by the side of, another water without rapid mixing might be illustrated by many examples.

There is one type of the failure of water to mix which has not been so commonly observed and has not been adequately recorded in engineering literature, but which may be of great interest and perhaps practical value to engineers. This type relates to the flow of turbid water through a reservoir of great length and its discharge at the dam with its conspicuous characteristics unchanged. Obviously, as the discharge of turbid water from a reservoir does not occur at all times when such water is flowing into it, there must be certain conditions of the inflowing water related to its specific gravity, viscosity, temperature, size, and degree of dispersion of the silt particles (or other physical qualities and combinations of them), that are prerequisite to such discharge. Although such discharges have been mentioned occasionally in engineering literature, the conditions under which they occur have not, as far as the writers are informed, been thoroughly described or studied. Mr. H. F. Robinson[4] quotes a report made in 1914 by Rollin Ritter, Assoc. M. Am. Soc. C. E., that "the silt-laden waters, being heavier than the clear, oftimes appeared at the tunnel outlet when the reservoir surface was clear and a small or moderate flood was entering the reservoir." Such a flow of turbid water through a long reservoir has been observed at Lake Murray, about 30 miles long, on the Saluda River, in South Carolina. L. M. Lawson,[5] M. Am. Soc. C. E., has mentioned the discharge of silt through the gates of Elephant Butte Dam on the Rio Grande, in New Mexico. Mr. L. R. Fiock[6] states that flows of turbid water from the Elephant Butte Reservoir, the head of which has been 30 to 40 miles above the dam, have occurred in 7 yr out of 20 yr of its operation. The movement of turbid water through the lake formed by Boulder Dam has also been mentioned.[7]

Measurements by Mr. H. M. Eakin,[8] of the sediments deposited in many reservoirs, appear to indicate similar flows of unmixed waters through reservoirs of various lengths and characteristics. He reports that his measurements disclose two types of deposition of sediment in a reservoir—in delta areas adjacent to points of stream inflow and in the deeper parts of the reservoir. The deposition of sediment in the deep parts near the dam are explained by him on the hypothesis that turbid water passed through the reservoir from the point of entrance.

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[3] "Hydraulic Laboratory Practice", John R. Freeman, Editor, p. 322, A. S. M. E., 1929.

[4] "The Silt Problem of the Zuni Reservoir", Discussion by H. F. Robinson, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 873.

[5] "Movement of Silt, Elephant Butte Reservoir", by L. M. Lawson, *Reclamation Record*, Vol. 10 (1919), p. 411.

[6] *Transactions*, Am. Geophysical Union, 1934, Pt. 2, p. 472.

[7] *Reclamation Era*, October, 1935.

[8] "Silting of Reservoirs", by Henry M. Eakin, *Technical Bulletin 524*, U. S. Dept. of Agriculture, Soil Conservation Service, p. 7.

Colorado River and Lake Mead

The Colorado River normally carries large quantities of silt. Under natural conditions this silt moves down the river channel, in part continuously and in part by stages of alternate deposition and erosion. Except as modified by gradual degradation or aggradation of the channel, as much silt would normally pass one point on the river over a considerable period of time as would pass another point, due allowance being made for accretions from tributaries; but within short periods there would not be such equality of silt movement.

Lake Mead was created by the construction of Boulder Dam, about 600 ft high, above the stream bed (731 ft from the lowest point in the foundation to the highest point of the dam), in the Black Canyon of the Colorado River. The lake thus formed has an initial capacity, at spillway level, of 30 500 000 acre-ft, which is about the total run-off of the river in two average years. When the reservoir is filled, the lake will have a length of about 115 miles and a surface area of about 145 000 acres. It is irregular not only in outline but in cross-section. Within it are several box, or near-box, canyons through which the Colorado River flows, ranging from 2 to 10 miles in length, separated by more open valleys into which the lake widens. The lake, therefore, may be considered as composed of several great pools, notably at or near Las Vegas Wash, the Virgin River, Hualpai Wash, and Grand Wash, separated by the relatively narrow necks formed by the canyons. The making of this lake has changed the natural processes of the river within the 115 miles of the lake and, of course, throughout the course of the river below the lake. The annual floods in the river below the lake, with their eroding and transporting action, have been eliminated. In this part of the river the range in stage and discharge is now relatively small. The conditions in the basin above the lake are unchanged, of course, and most of the silt that enters the lake is now (1937) deposited and remains there.

After the filling of the lake had progressed for a short time, the effluent became essentially clear, and, in general, it has continued so since. However, turbid water was discharged from Lake Mead three times during the eleven months from February 1, 1935, when storage began, to December 31, 1935, although the quantity flowing through the openings in the dam throughout the period has been controlled at less than 10 000 cu ft per sec. During these months the water was flowing through the tunnels at the base of the dam, and, when the turbid discharges occurred, the lake was from 70 to 90 miles long and contained from 4 000 000 to 5 000 000 acre-ft of water; there was no significant change in the quantity of outflow. The turbid discharges represent the conveyance through, and the discharge from, the lake of as much as 6 000 000 tons of silt. At these times there has been not only the ocular evidence of unmixed or partly mixed water flowing through the lake, as disclosed by its turbidity, but this evidence has been substantiated by measurements of the size of the particles of suspended matter and by the determination of the dissolved matter, as indicated by the sulfates, contained in the water entering into, and discharged from, the lake.

Measurements and Observations

The loads of suspended matter carried by the Colorado River are computed for the Grand Canyon Gaging Station, about 265 miles above Boulder Dam, and for the Willow Beach and Topock Gaging Stations, about 10 and 115 miles, respectively, below the dam. The positions of these stations with respect to each other and to the lake are shown by Fig. 1. The percentages by weight of suspended matter are given in Table 1 for the Grand Canyon, Willow Beach, and Topock Gaging Stations during the period, March 1 to October 31, 1935. The loads, in tons per day, are shown in Fig. 2 (*c*) for the Grand Canyon and Willow Beach stations.

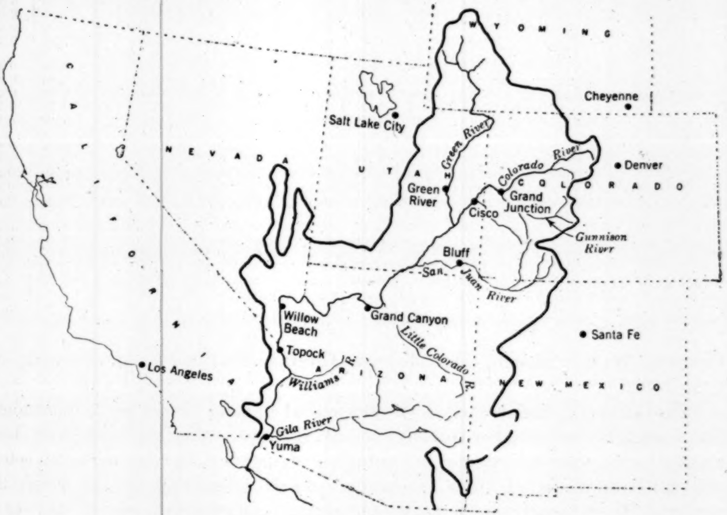
WYOMING
Cheyenne
Salt Lake City
Green River
NEVADA
UTAH
Colorado River
Denver
Green River
COLORADO
Grand Junction
Cisco
Gunnison River
Bluff
San Juan River
Willow Beach
Grand Canyon
Santa Fe
Topock
Little Colorado R.
Los Angeles
ARIZONA
Williams R.
NEW MEXICO
Gila River
Yuma

FIG. 1.—MAP OF COLORADO RIVER DRAINAGE BASIN.

Comparisons of the loads carried past the Grand Canyon and Willow Beach Stations, give an approximate measure of the quantity of material deposited in the lake. Comparisons of the loads carried past the Willow Beach and Topock Stations give a measure of the scouring of the river channel that occurs between those two stations. The results for the Topock Station are not shown in Fig. 2, because with the scales used the curves for Topock could not be distinguished from those for Willow Beach. One of the writers[9] has presented data on the quantities of material carried past the Grand

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[9] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 284.

TABLE 1.—Suspended Matter (Percentage by Weight), in Colorado River, Gaging Stations During the Period

Day of month	March			April			May			June		
	G.C. (1)	W.B. (2)	Top. (3)	G.C. (4)	W.B. (5)	Top. (6)	G.C. (7)	W.B. (8)	Top. (9)	G.C. (10)	W.B. (11)	Top. (12)
1	0.46	0.09	0.37	0.93	0.07	0.44	0.85	0.04	0.26	1.70	0.06	0.32
2	0.44	0.10	0.39	1.06	0.06	0.35	0.90	0.04	0.25	1.90	0.06	0.30
3	0.48	0.10	0.40	1.26	0.06	0.31	1.04	0.03	0.28	1.58	0.04	0.28
4	0.35	0.07	0.39	1.24	0.06	0.32	1.22	0.03	0.26	1.45	0.07	0.28
5	0.30	0.07	0.35	1.31	0.07	0.27	1.28	0.03	0.22	1.31	0.06	0.29
6	0.28	0.05	0.40	1.38	0.07	0.30	1.04	0.03	0.24	1.12	0.05	0.30
7	0.32	0.05	0.32	1.40	0.14	0.25	0.99	0.05	0.22	1.08	0.05	0.26
8	0.44	0.06	0.34	1.48	0.32	0.25	1.06	0.04	0.21	1.10	0.05	0.28
9	0.46	0.08	0.36	1.42	0.38	0.40	0.88	0.04	0.21	1.28	0.04	0.29
10	0.54	0.06	0.55	1.60	0.53	0.46	0.73	0.04	0.20	1.28	0.06	0.34
11	0.44	0.06	0.44	3.16	0.95	0.46	0.68	0.05	0.20	1.25	0.05	0.24
12	0.41	0.06	0.40	2.16	0.70	0.66	0.68	0.05	0.19	1.26	0.06	0.32
13	0.35	0.06	0.42	1.58	0.72	0.82	0.77	0.07	0.24	1.14	0.05	0.28
14	0.38	0.06	0.30	1.31	0.95	0.77	0.80	0.10	0.27	1.22	0.04	0.26
15	0.36	0.06	0.38	1.12	0.81	0.99	0.98	0.06	0.22	1.06	0.04	0.32
16	0.43	0.05	0.44	0.89	1.15	1.00	1.13	0.06	0.19	1.09	0.04	0.31
17	0.44	0.05	0.38	0.81	0.96	1.00	1.45	0.06	0.22	1.03	0.04	0.28
18	1.72	0.05	0.32	0.84	0.62	1.24	1.52	0.07	0.27	1.01	0.06	0.27
19	1.55	0.06	0.43	1.11	0.45	0.91	1.36	0.05	0.25	0.82	0.04	0.28
20	1.16	0.03	0.30	1.46	0.24	0.70	1.32	0.05	0.24	0.79	0.04	0.26
21	0.94	0.04	0.31	1.38	0.31	0.60	1.26	0.04	0.27	0.72	0.05	0.28
22	0.89	0.05	0.33	1.38	0.04	0.54	1.22	0.06	0.20	0.55	0.04	0.32
23	0.79	0.04	0.30	1.23	0.04	0.42	1.38	0.08	0.23	0.54	0.04	0.32
24	0.70	0.06	0.32	1.20	0.05	0.48	1.48	0.07	0.40	0.58	0.04	0.30
25	0.61	0.16	0.27	1.10	0.04	0.30	1.48	0.08	0.47	0.56	0.04	0.24
26	0.60	0.52	0.28	1.06	0.40	0.26	1.08	0.08	0.45	0.44	0.05	0.32
27	0.64	0.41	0.57	1.26	0.08	0.26	1.01	0.09	0.35	0.48	0.05	0.23
28	0.61	0.34	0.70	1.40	0.05	0.57	1.01	0.09	0.38	0.44	0.05	0.26
29	0.56	0.18	0.57	1.08	0.05	0.26	1.26	0.06	0.30	0.37	0.04	0.26
30	0.59	0 30	0.58	0.92	0.06	0.29	1.38	0.07	0.38	0.36	0.07	0.27
31	0.72	0.31	0.54	....	....	....	1.54	0.06	0.30	....	....	....

Canyon, Willow Beach, and Topock Gaging Stations from February to June, 1935.

The sizes of the particles of suspended matter have been determined from samples collected regularly at all stations. The samples have been treated with sodium oxalate for complete dispersion of the material before the determinations of sizes are made. As it is carried by the river, the material is evidently in a state of partial coagulation, which may be a function of: The kind of material carried; the chemical composition of the water in the river at the time the sample is collected; and, perhaps, other factors associated with the source of the material. Because of the impossibility of duplicating the field conditions, it has seemed desirable to disperse all samples, completely, before the determinations are made, in order to obtain reasonably comparable results. However, the dispersion, desirable though it may be for uniformity, may obscure or destroy certain natural conditions that have significance with respect to the passage of unmixed or imperfectly mixed water through Lake Mead.

Laboratory tests using Colorado River water for the settling medium show that a large percentage of the undispersed material has a much faster settling rate than the same material in a state of complete dispersion. It now appears, therefore, that it would be desirable to know the sizes of the

AT THE GRAND CANYON (G.C.), WILLOW BEACH (W.B.), AND TOPOCK (TOP.) MARCH 1 TO OCTOBER 31, 1935.

Day of month	JULY			AUGUST			SEPTEMBER			OCTOBER		
	G.C. (13)	W.B. (14)	Top. (15)	G.C. (16)	W.B. (17)	Top. (18)	G.C. (19)	W.B. (20)	Top. (21)	G.C. (22)	W.B. (23)	Top. (24)
1	0.36	0.04	0.26	0.16	0.04	0.18	1.43	0.03	0.14	5.09	0.005	0.26
2	0.35	0.05	0.29	0.25	0.03	0.20	1.83	0.02	0.14	4.65	0.007	0.21
3	0.33	0.06	0.26	0.17	0.05	0.21	2.80	0.86	0.18	3.44	0.02	0.27
4	0.28	0.07	0.36	0.23	0.03	0.22	5.02	0.99	0.24	1.98	0.007	0.24
5	0.28	0.04	0.26	0.36	0.04	0.16	2.54	0.68	0.92	1.28	0.01	0.20
6	0.30	0.05	0.26	1.82	0.04	0.16	1.94	0.17	0.91	0.88	1.57	0.28
7	0.22	0.08	0.26	1.54	0.05	0.28	2.00	0.42	0.78	0.69	1.60	0.58
8	0.23	0.04	0.24	0.72	0.04	0.39	1.64	0.94	0.40	0.54	1.64	1.62
9	0.23	0.05	0.26	0.44	0.05	0.20	1.12	0.70	0.81	0.45	1.99	1.71
10	0.20	0.04	0.26	0.46	0.04	0.16	0.97	0.57	1.00	0.42	2.16	1.79
11	0.17	0.04	0.26	1.72	0.07	0.21	0.72	1.08	0.92	0.31	1.86	2.12
12	0.19	0.03	0.18	1.13	0.05	0.22	0.64	1.37	0.77	0.26	1.19	2.34
13	0.16	0.03	0.18	0.97	0.04	0.17	0.56	0.71	1.03	0.22	0.34	2.00
14	0.16	0.02	0.17	1.11	0.04	0.22	0.65	0.27	1.33	0.19	0.004	1.24
15	0.16	0.03	0.21	1.16	0.01	0.25	0.54	0.01	0.79	0.16	0.002	0.51
16	0.15	0.04	0.23	0.98	0.02	0.17	0.56	0.03	0.28	0.14	0.005	0.30
17	0.13	0.01	0.16	0.76	0.02	0.19	0.40	0.01	0.32	0.13	0.009	0.25
18	0.13	0.01	0.19	1.35	0.03	0.23	0.33	0.02	0.22	0.12	0.003	0.28
19	0.14	0.03	0.16	0.53	0.02	0.24	0.41	0.01	0.20	0.11	0.02	0.26
20	0.30	0.02	0.20	0.40	0.03	0.18	0.31	0.01	0.21	0.11	0.02	0.27
21	0.39	0.04	0.20	0.44	0.02	0.20	0.21	0.01	0.22	0.12	0.02	0.31
22	0.62	0.04	0.21	0.54	0.03	0.25	0.16	0.03	0.24	0.12	0.02	0.30
23	0.60	0.04	0.22	0.48	0.04	0.15	0.15	0.01	0.20	0.11	0.01	0.34
24	0.35	0.04	0.23	0.50	0.02	0.14	0.14	0.01	0.24	0.12	0.01	0.40
25	0.24	0.03	0.26	0.52	0.02	0.16	0.12	0.01	0.23	0.14	0.01	0.26
26	0.27	0.03	0.26	4.81	0.02	0.15	0.15	0.01	0.20	0.15	0.01	0.27
27	0.54	0.03	0.40	2.56	0.02	0.15	0.24	0.01	0.22	0.16	0.01	0.28
28	0.46	0.02	0.23	2.74	0.04	0.15	0.90	0.01	0.26	0.20	0.02	0.26
29	0.23	0.02	0.31	1.86	0.02	0.19	2.14	0.02	0.26	0.49	0.02	0.27
30	0.22	0.03	0.24	1.69	0.02	0.13	4.14	0.01	0.21	0.44	0.02	0.24
31	0.19	0.03	0.22	2.14	0.02	0.14	....	....	....	0.26	0.01	0.33

particles as they are actually carried in suspension, inasmuch as it is evidently such sizes that may be most significant with respect to the passage of turbid water through a reservoir. It is probable that determinations should be made of the sizes of the material both as received from the river and after complete dispersion.

The larger sizes of particles have been measured and separated by sieves. However, if a considerable proportion of such matter passes a 200-mesh sieve (diameter of particles 74 microns), further determinations of size are made by observing times of settling in tall cylinders and computing sizes by Stokes' law. The settling method, of course, is not exact, because the separations represent quantities of particles that settle at given rates rather than quantities of particles that have given diameters, and each size group obtained contains particles that are smaller than the size for which the separation was made and that have been carried down with the larger sizes. According to Stokes' law the settling rates are, approximately, 26 ft, 4 ft, and 0.3 ft per hr for 50-micron, 20-micron and 5-micron sizes, respectively.

Fig. 2(*c*) shows the daily load of suspended matter at the Grand Canyon and Willow Beach Gaging Stations during the year ending December 31, 1935. During January, 1935, before the flow of the river was regulated by the dam, the load of suspended matter at the Willow Beach Station was

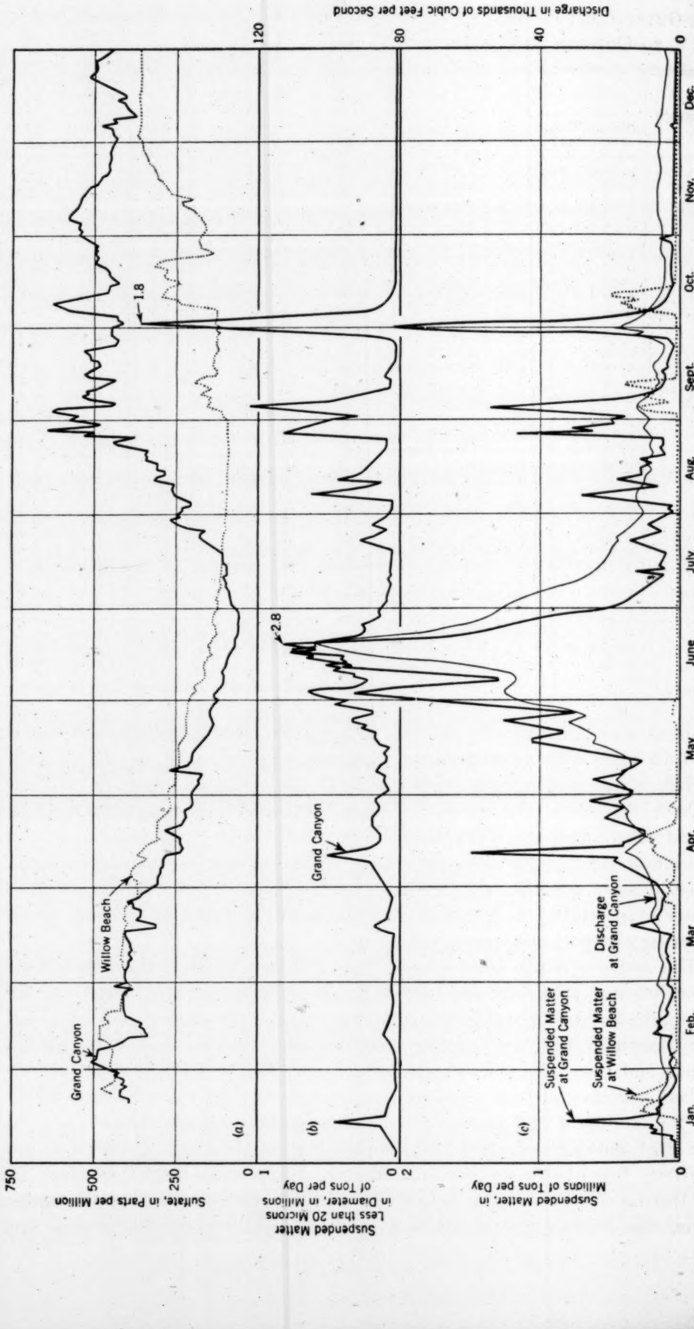

Fig. 2.—Sulfate Suspended Matter, and Discharge at Gaging Stations on Colorado River Near Grand Canyon and Willow Beach, Arizona.

practically the same as that at the Grand Canyon Station and averaged about 100 000 tons per day. After regulation was begun the load of suspended matter at the Willow Beach Station was fairly constant during the first part of February and nearly all of March, averaging about 10 000 tons per day. The nearly horizontal line in Fig. 2(*c*) shows the load of silt at the Willow Beach Station and is a measure of the load picked up between Boulder Dam and Willow Beach by the clear water discharged from Lake Mead. The source of this load has been recently discussed by C. P. Vetter,[10] M. Am. Soc. C. E. The peaks rising above this horizontal line are measures of the load of silt that is being discharged from the reservoir.

There was a slight increase in the suspended load at Willow Beach for a few days, during the latter part of February. However, as shown in Fig. 2, the first outstanding discharge of turbid water from Lake Mead occurred in March and April. In the latter part of March the water discharged at the dam became slightly turbid and continued so for six or seven days. Then, after a week of relatively clear water, the river again became turbid to a greater degree than previously and continued to be turbid for ten days or more. The river became clear in the latter part of April and remained clear until September. The large increase in the load of suspended matter at Willow Beach, in March and April, followed an increase in suspended matter at the Grand Canyon Station. Each subsequent increase at Willow Beach can also be related definitely to a rise at the Grand Canyon Station. Each such increase at Willow Beach appeared about eight days after the rise at Grand Canyon. The records show similarly that each increase at the Willow Beach Station was followed about one day later by an increase at the Topock Station.

During May, June, July, and August, 1935, the river above the lake was carrying considerable suspended matter, which apparently settled completely in the lake, as the river below it was relatively clear during most of that period. The slight turbidity at the Willow Beach Station in that period appeared to be caused by local scouring of the channel and to be accompanied by small eddies or "boils"; and when observed from a cable car 50 to 60 ft above the surface, the river seemed to have spots of relatively clear water interspersed with spots where "boils" were rising from the bed; but the turbidity was too great for actual observation of the bottom. Some of the "boils" did not appear to reach the surface; others came to the surface and created turbulence over rather large areas. It seems likely that this action is normal for the river and that the load of suspended matter normally carried in the lower river at low stages consists generally of material scoured from the river channel.

On September 3, 1935, the river suddenly became turbid at the Willow Beach Station, with practically no change in stage, and continued in this condition until the morning of September 14, when it was again practically clear. During this 12-day period more than 2 000 000 tons of suspended matter was carried past Willow Beach. On September 5, 1935, the river at the Topock Station showed an increase in suspended matter, and for the next twelve days the load at the Topock Station was comparable to the load at Willow Beach, but there was a marked decrease on September 16. The appearance of turbid water at Willow Beach and Topock apparently was related to the large increase in suspended matter in the river water at the Grand Canyon Station, which began August 26 and continued for fourteen days.

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[10] "Why Desilting Works for the All-American Canal"? by C. P. Vetter, *Engineering News-Record*, Vol. 118, p. 321, (March 4, 1937).

The river at the Willow Beach Station again became turbid on October 6, 1935, and remained in that condition for eight days. On October 14, it had returned to a condition of slight turbidity. During this 8-day period more than 2 500 000 tons of suspended matter was carried past Willow Beach. On October 8, a large increase in suspended matter was observed at the Topock Station, followed by a decrease on October 14 and a return on October 15 to about the conditions of the first seven days of the month. These changes in the quantity of suspended matter below the reservoir apparently resulted from an increase in discharge and suspended matter observed at the Grand Canyon Station for a few days beginning September 27.

SIZES OF PARTICLES

Particular attention is called to the curve in Fig. 2 showing variations in sizes of particles, because it seems likely that the greater specific gravity due to the turbidity of the water, especially when the particles are very fine, may be an important factor in the movement of turbid water through the reservoir.

Fig. 2(*b*) shows the quantity of material with effective diameters of less than 20 microns that is carried daily past the Grand Canyon Station. Whenever there was an increase in the load of suspended matter at the Willow Beach Station there had been a prior increase in the quantity of material less than 20 microns in diameter at Grand Canyon. In each of the periods of turbid discharge, the suspended matter at Willow Beach consisted chiefly of small particles with slow rates of settling, more than 90% of the material being less than 20 microns in diameter.

There will naturally be speculation as to the reason why the high water of May and June, with its heavy silt load at Grand Canyon, did not produce a turbid discharge from Lake Mead. A possible explanation may lie in the fact that, at that time, a high percentage of the silt after dispersion (and probably the percentage was still higher before dispersion) was larger than 20 microns and that this large portion of the silt, with its fast settling rate, carried down with it practically all the finer silt. Determinations of size show that the material carried at Grand Canyon during this period was in a state of partial coagulation and that the sizes of the particles carried were larger than the analyses of the dispersed samples indicate.

During those parts of the period from February to August, when the loads at Willow Beach were small, the suspended material carried past this station consisted chiefly of particles, having diameters greater than 50 microns, picked up in the river channel between the dam and the station. During the same period a slightly larger load was carried past the Topock Station, but the sizes of the particles at the Topock Station were similar to those at the Willow Beach Station. Similar scouring of the river channel at certain periods has been indicated by the results obtained for several years before Boulder Dam was completed. During certain periods of the year (usually in the late fall and winter) before storage in Lake Mead was started, the load at the Topock Station was considerably greater than that at the Grand Canyon Station, and the differences could not be attributed to inflow from tributaries between the stations. It has seemed likely that at these times the river (which probably was not carrying a capacity load at the Grand Canyon Station) had eroded, from the bed and banks, material that had been previously deposited. It is significant that during a period of ten years the samples collected at Grand Canyon have often had less than 0.05% of suspended matter, whereas the samples collected at Topock have rarely had less than 0.10% of suspended matter. At such times of low percentage of suspended matter at the Topock Station, the load has consisted chiefly of particles more than 50 microns in diameter which would drop out of suspension rather easily.

Increases in discharge and suspended matter at the Grand Canyon Station, particularly in the late summer, are frequently caused by rains of high intensity in regions of little vegetation, resulting in rapid run-off of water, high in both dissolved and suspended matter. Herman Stabler,[11] M. Am. Soc. C. E., has presented data indicating that 60% of the suspended matter carried past the Grand Canyon Station came from 36% of the drainage basin lying above that point. The increase in discharge at Grand Canyon on August 26, 1935, was caused largely by an increase in the flow of the Little Colorado River, which drains a large arid region and enters the Colorado 26 miles above the Grand Canyon Station. The material carried by the Little Colorado during certain other summer floods has consisted chiefly of fine particles, and it appears likely that the Little Colorado brought into the Colorado a large part of the material carried past the Grand Canyon Gaging Station on August 26 and a few days subsequent. There is evidence that the Little Colorado and perhaps the San Juan may have brought to the Colorado much of the silt that appeared at Willow Beach during the three periods of turbid flow in 1935, but the evidence is not complete.

DISSOLVED MATTER

Changes in concentration of dissolved matter have been estimated approximately from the changes in sulfate, the principal acid radicle present in the Colorado River water. The determinations of sulfate have shown that a rapid increase in concentration of dissolved matter in the main river usually accompanies an increase in the quantity of suspended matter. The variations in sulfate at the Willow Beach Station, shown in Fig. 2(*a*), are of significance, therefore, in confirming the evidence afforded by the silt that, at times of silt discharge, water entering the reservoir is going through the reservoir without complete mixing. This evidence of absence of mixing is in agreement with the observations on the Rio Grande, where, it is reported by Fiock and Scofield[12] that the soluble salt content of the outflow increases materially during the periods of turbid flow. The simultaneous increases in sulfates and suspended matter at the Willow Beach Station would seem to indicate that they are definitely related to each other. The interval of about eight days between the increase at Grand Canyon and the increase at Willow Beach appears to be a reasonable time for water to flow the 275 miles from the Grand Canyon Station to the Willow Beach Station, including 70 to 90 miles through Lake Mead, with due allowance for the velocity of flow through the lake, resulting from an average slope of 3 or 4 ft to the mile of the bed of the reservoir, on which the turbid water probably flows beneath the quiescent clear water.

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[11] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 277.

SPECIFIC GRAVITY

Simple experiments can be made in the laboratory to show that water of greater specific gravity, whether such specific gravity is obtained by decrease in temperature, the solution of soluble material, or the suspension of solid material of such fineness that it will stay in suspension for several hours, will find its way to the bottom of a dish containing water of less specific gravity. If there is any flow of water through the dish, the water of greater specific gravity will move along the bottom to the outlet.

Causes for the differences in specific gravity should be considered on the assumption that the relatively rapid flow of unmixed water through a long reservoir is due to: Differences in specific gravity; the slopes of the surfaces of water of greater specific gravity; or the slope of the sediment on which the flow takes place. In the studies reported in "Hydraulic Laboratory Practice,"[3] the phenomena were discussed wholly on the basis of differences in temperature, although the differences in specific gravity of the water used in the laboratory experiments appear to have been obtained generally by adding a solution of salt and not by changing the temperature. As a result of those studies it was concluded that water of greater specific gravity due to lower temperature might flow essentially unmixed through a basin or reservoir containing water of less specific gravity, and that the depth at which the flow would take place would be determined by the specific gravity of the colder water. This same effect as related to temperature was discussed[13] by Armin Schoklitsch in 1922.

A new aspect, evidently not previously studied extensively and, perhaps, not pertinent in the foregoing studies,[3] is presented in connection with the Colorado River and doubtless many other silt-laden streams, by a water made heavy because of a load of fine silt that is not quickly dropped even when the velocity is reduced to zero. This fine silt is associated so closely with the water by which it is carried that it apparently increases the specific gravity of the water. The range in specific gravity due to the presence of fine silt may be much greater than that due to differences in temperature. Under some conditions, therefore, water of an inflowing stream, even if it is higher in temperature, may not remain on the surface of a reservoir but may sink to the level determined by its greater specific gravity due to its load of fine silt. Mr. Fiock[8] has stated that the temperature of the outflow from the Elephant Butte Reservoir during periods when turbid water is passing through the reservoir, not on its surface, is about 6° higher than the temperature of the reservoir water. When the outflowing water becomes clear, its temperature drops to that of the reservoir water. Unfortunately, there are no records of the temperature of Colorado River water as it enters into, or flows from, Lake Mead; nor of the water in Lake Mead at various depths. No attempt has yet been made to find the depth of the flow of unmixed turbid water through Lake Mead, but it is tentatively assumed that the flow of turbid water is just on top of previously deposited silt and that it follows the original channel of the river.

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[12] *Transactions*, Geophysical Union, 1934, Pt. 2, p. 472; also "The Movement of Silt through the Elephant Butte Reservoir", by C. S. Scofield. (Unpublished memorandum, February 17, 1936).

[13] "Einfluss der Durchleitung von Flusswasser durch Seen auf deren temperatur", Sonderheft to *Wasserkraft*, 17, 1922.

The differences between the specific gravity of the incoming water and of the lake water may be greater than the difference in specific gravity caused by differences in temperature. The specific gravity of pure water is 0.993 at 100° F, and has a maximum of 1.000 at 39.2° F. The specific gravity of the water that flowed through Lake Mead in October, 1935, ranged from 0.995 to 1.008, showing a greater variation than could be caused by the most extreme natural differences in temperature. Approximate values for the specific gravity of the turbid water that passed Lake Mead and the percentages of suspended matter in the water as determined on samples collected at the Willow Beach Gaging Station are given in Table 2.

TABLE 2.—SPECIFIC GRAVITY AND PERCENTAGE OF SUSPENDED MATTER IN WATER OF COLORADO RIVER AT WILLOW BEACH GAGING STATION, OCTOBER, 1935

Date (1)	Specific gravity* (2)	Suspended matter (percentage by weight) (3)	Date (1)	Specific gravity* (2)	Suspended matter (percentage by weight) (3)
October 5	0.995	0.004	October 10	1.008	1.76
October 6	1.005	1.38	October 11	1.007	1.76
October 7	1.002	1.32	October 12	1.004	1.14
October 8	1.004	1.40	October 13	0.999	0.38
October 9	1.003	1.00	October 14	0.996	0.01

* Determinations made at 84.2° F, referred to pure water at 39.2° F.

On the basis of present information it appears, therefore, that the phenomenon of flow of essentially unmixed turbid water through a long reservoir at one season or another is not uncommon, and that it may be affected by temperature, dissolved salts, size of particles of silt, and the degree of coagulation of dispersion of the silt particles.

The observations herein recorded relating to the flow of turbid water from Lake Mead during 1935 may have peculiar value, because they have been made systematically throughout the period when stored water was discharged through tunnels at, or near, the base of the dam, and, therefore, was drawn from the bottom of the lake. Early in 1936, while water was still being discharged through these tunnels, some of the water passing the dam was drawn through the intake towers from a level about 250 ft above the base of the dam. Under these conditions on April 22, 1936, the river at Willow Beach became turbid shortly after noon, the silt appearing in the main current at first and in 10 min the entire section was filled with turbid water. At this time, the discharge below the dam was about 10 000 cu ft per sec, of which about 5 000 cu ft per sec came through the diversion tunnel and about 5 000 cu ft per sec came through the outlet works. In this period of turbid flow about 2 400 000 tons of silt was carried past Willow Beach. The gate in the tunnel was closed on May 1, and the river at Willow Beach became clear on May 2.

CONCLUSIONS

The passage of silt in considerable quantities through a reservoir may have much more than scientific interest and importance. One of the great unsolved problems pertaining to the detention of water in reservoirs relates to the gradual loss of capacity caused by the deposition of silt. If any considerable quantity of silt can be carried through the reservoir its effective life will be correspondingly increased. In the three periods in 1935 of turbid flow from Lake Mead described herein 6 000 000 tons, or about 2.5% of the estimated average annual load of silt brought to the lake, has passed through it. The quantity of silt carried through other reservoirs on rivers having different silt and run-off characteristics has not been recorded and, perhaps, may reasonably be assumed to be smaller than that carried through Lake Mead. However, the quantity of silt thus carried may well be of sufficient importance to warrant further consideration.

The question may naturally be raised as to the possibility of increasing the discharge of silt from reservoirs created by future dams by modifying the positions of and approaches to the discharge openings or the manner of their operation. It is suggested that a study of the conditions pertaining to the flow of silt through a reservoir may be worth while. Engineers who make future designs will surely want to know the conditions that make such passage of turbid water possible, in order that full consideration may be given to any advantages that may be practicable of attainment by modifications in the details of design of the dams or in the plans for operation.

The record of silt flow described herein may be important not only to engineers in their evaluation of silt problems related to many other reservoirs built, or to be built, on silt-laden streams, but also in connection with certain geologic and engineering studies related to the deposition of silt and possibly to the origin of subaqueous channels. It is hoped that there may be brought into the discussion other definite information related to the little understood phenomena associated with the flow of water through reservoirs, and to the possible applications of the phenomena that may have practical significance with respect to engineering problems.

DISCUSSION

O. A. Faris,[14] M. Am. Soc. C. E. (by letter).—Turbid water showing in the discharge from large reservoirs, in the opinion of the writer, is the result of disturbance of silt which was virtually in place; that is, it was no longer in suspension, but had settled to the reservoir bottom soon after entering the slack-water and formed a "flocculent" or "honeycomb" structure of a consistency, ranging from thick cream to heavy molasses. It flows readily on the slope of the reservoir bottom, by virtue of a greater specific gravity than water, and, finally, reaches the dam.

Water is discharged from Lake Kemp, a reservoir on Wichita River, near Wichita Falls, Tex., through a battery of concrete pipes with inverts at or below the elevation of the river bed. The control gates are about midway the length of the conduits, allowing the up-stream half of the pipes to be exposed to pressure at all times. There is evidence that the liquid mud flows from the place of deposition in the old river channel, and down the channel within the reservoir until blocked by the closed outlet gates. On opening the gates, the first rush of water carries a heavy load of silt, but after discharging all mud within easy reach of the conduits, the discharge becomes clear. Water from this reservoir flows down the river channel several miles where it is diverted into the main canal of the irrigation system.[15]

The general arrangement which provides for the outlet at or below the elevation of the bottom of the stream channel and the diversion of water to canals at some distance down stream from the storage dam is favorable for the discharge of deposited silt of the aforementioned character. In most of the Texas reservoirs the highest elevations of the bottom, in a cross-section, are on the immediate banks of the old stream channel; therefore, none of the silt that is deposited on the wide flat areas between the elevated ridge near the channel and the toe of the slope from the upland, can flow into the old channel and ultimately reach the outlet structure. It is believed to be advantageous to clear reservoir sites of brush, trees, and stumps, and to construct channels with a view to conveying the liquid mud, resulting from the deposition of suspended silt, into the old stream channel and thence through the outlet gates.

As further evidence that deposited silt, in the form of liquid mud, flows down the slopes on reservoir bottoms, attention is called to conditions in Medina Reservoir, near San Antonio, Tex. In September, 1930, this reservoir was empty. A careful inspection revealed that the hummocks on the large flat areas in the reservoir bottom were void of silt deposit although they had been submerged by water to depths ranging from 40 to 100 ft. for long periods of time. On the other hand, all depressed areas on the flats contained silt with maximum depths in the lowest places and tapering to zero depth up the slopes.

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[14] Engr.-Appraiser, Federal Land Bank, Houston, Tex.

[15] "The Silt Load of Texas Streams", by Orville A. Faris, M. Am. Soc. C. E., *Technical Bulletin 382*, U. S. Dept. of Agriculture, 1933, pp. 37–39.

The dry material per cubic foot of deposit of "flocculent" and "honeycomb" structure, taken from the bottom of Lake Worth, near Fort Worth, Tex., and composed largely of colloidal sizes, contained 18.7 lb. of oven-dry material per cu ft of deposit. On the basis of 2.65 for the specific gravity of the dry material, water occupied 88.59% of the space and the absolute voids amounted to 88.66 per cent. A sample from the bottom of Lake Kemp, consisting of sizes crresponding to clay particles, contained 37.05 lb of oven-dry material per cu ft of deposit. On the basis of 2.658 for the specific gravity of the dry material, the space occupied by water was 76.76% and the absolute voids amounted to 77.63 per cent.

The water capacity, by volume of undisturbed soils, below the water-table, ranges from 31.3% for very fine sand to 49% for marly loam. Comparing the water capacity of marly loam with that of the sample silt deposit which contained about 89% water, by volume, it is seen that the latter has a greater capacity by 40%, than the former. This greater capacity is due to the difference in structure of the mass of soft silt deposited, from suspension, under water and remaining submerged.

The excess water held in the silt deposit by virtue of structure is liberated so slowly upon exposure of the deposit to the sun and atmosphere that it is practically all dissipated by evaporation and is not available as storage water.

The unconsolidated character of material deposited from suspension, under water and remaining below the water-table, will continue indefinitely. This is evident from the soft character of low-lying material in peat marshes. This soft material was deposited ages ago and still retains the "honeycomb" structure on account of its position below the water-table.

The release of the excess water can be accomplished only at the surface of the deposit, therefore, in large reservoirs, consolidation of such deposits can not occur until the stored water is drawn down exposing the soft material to the action of the sun and atmosphere.

PAUL A. JONES,[16] M. AM. SOC. C. E. (by letter).—The studies regarding transportation of silt through large reservoirs will, no doubt, be of great interest to engineers, and it is hoped that the work begun by the authors may be continued. However, it is the writer's opinion that studies made concerning the turbidity of water passing Boulder Dam in 1935, or even until the lake has been completely filled, should not be given much weight in connection with transportation of silt in the future.

There were certain topographic conditions that, no doubt, greatly affected the turbidity of the flow at Boulder Dam, especially during 1935. The canyon above the dam, in general, is in the shape of an arc, with the inside of the arc on the side of diversion through Tunnel No. 1, on the Nevada side. It is reasonable to assume that currents of water act similarly to currents of air, and will move toward the inside of the curve instead of following the original channel. Cold or heavy air will follow the inside of a curve, or the shortest distance, as may be observed in a tunnel constructed on a curve. It would seem fair to expect the greatest number of contraction cracks in the concrete lining of a tunnel on the outside of the curve, but the reverse is true, as the cold air in a draft follows the inside of the curve.

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[16] Constr. Engr., U. S. Bureau of Reclamation, Glendive, Mont.

At Elevation 720 at Boulder Dam, which is about 30 ft above the top or back of Tunnel No. 1, the contractor constructed a railroad grade down the canyon on the Nevada side, principally from tunnel muck which was of a reddish brown color. Below this grade, on the same side of the canyon, at about Elevation 650, was a comparatively large sand-bar. Just above the entrance to the canyon, and also on the inside of the arc, blow-sand drifts extended in general to approximately Elevation 800. About three miles above the entrance to the canyon, there are at present (1937), or there will be, two rock islands which were connected by a blow-sand ridge, and the original current flowed east of the island farthest east.

If a check is made of the dates of turbidity of flow through the dam and lake-level elevations, it will be found that the first turbidity occurred soon after the sand-bar was well covered; the second turbidity occurred soon after the railroad grade was well covered, and was much redder in color than the usual Colorado River water; and that, a turbidity occurred soon after the ridge between the two rock islands was well covered, and at about the time that a change in current from the east side of the small island to the west side, would be expected. At the level of diversion through the towers, the canyon has little blow-sand or silt, and it is not likely that turbidity below the dam will again occur to the extent observed in 1935.

This discussion is not written to contradict any of the statements made in the paper, but as a possible explanation for the amount of turbidity in 1935 when the reservoir was being filled for the first time, and the current was being changed gradually to new and unstable ground, due to the fact that the canyon immediately above the dam takes the form of an arc, with the inside of the arc on the side of diversion, or the Nevada side.

It is well understood that river water carries large quantities of collodial silt in suspension, and studies relative to this material, no doubt, would be of great benefit to the Engineering Profession. For example, there is a shoal in San Francisco Bay, just west of Mare Island, which is composed entirely of colloidal silt deposited where the fresh waters of the rivers come in contact with a certain percentage of saline water from the ocean. Since the effect of tides is noted as far up the Sacramento River as the City of Sacramento, and the collodial silt is not deposited until it reaches a point nearly 100 hundred miles distant, and is there precipitated by salt, it is reasonable to expect that a large quantity of fine silt will be transported through Lake Mead.

Carl S. Scofield,[17] Esq. (by letter).—Hydraulic engineers are fortunate in having made available in precise form the information concerning a phenomenon that has been reported hitherto only in general terms.

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[17] Agriculturist in Chg., Div. of Western Irrig. Agriculture, Bureau of Plant Industry, U. S. Dept. of Agriculture, Washington, D. C.

Observations that different waters may move together in the same stream channel with little blending, and that water may flow into, or through, a reservoir with little or no mixing, have been reported before but only circumstantially. The findings reported by Messrs. Grover and Howard, therefore, challenge attention and invite further inquiry. The reasons underlying the phenomenon described remain obscure.

The assumption that two different solutions brought together in the same container would automatically blend into one homogeneous mixture runs counter to daily experience. One naturally uses a spoon to stir a cup of coffee after the cream and sugar are added. In the case of reservoirs such mixing as occurs may be due in part to the effects of currents set in motion by inflowing water, by wind action on the surface, or in part to "inversions" caused by temperature changes that affect the specific gravity of the water. For example, if the surface water of a reservoir becomes colder than the deeper water, its specific gravity becomes higher and inversion occurs.

When turbid stream water enters a reservoir in the summer, if its temperature is higher and its salinity is lower than that of reservoir water, the normal behavior is for the silt to settle to the bottom and for the clear water to spread over the reservoir surface. On the other hand, if the inflowing turbid water is much more saline than the reservoir water the higher specific gravity due to salinity tends to offset the lower specific gravity due to temperature so that the water tends to stay with the silt and to move into the reservoir along the bottom.

Furthermore, the behavior of silt in relation to its transporting water appears to be influenced by the relative conditions of salinity. If the concentration of dissolved salts in the inflowing water is less than that of the reservoir water, flocculation and deposition of the silt appear to be hastened. If, conversely, the concentration of dissolved salts in the inflowing turbid water is higher than that of the reservoir water, the tendency is to retard the flocculation and deposition of the silt. The combined effect of silt and of dissolved salts on the specific gravity of water, of course, is much greater than the effect of temperatures, at least within the ranges that ordinarily occur in streams and reservoirs.

The foregoing are largely theoretical considerations. Engineers do not, as yet, have enough facts to explain convincingly why the incoming silt is sometimes deposited at the head of a reservoir and, at other times, is carried far along the bottom or even all the way through the reservoir. One phenomenon would probably be regarded as quite as "normal" as the other if one knew more of the facts and factors involved. Both are known to have occurred at Boulder Dam and at Elephant Butte Dam.

IVAN E. HOUK,[18] M. AM. SOC. C. E. (by letter).—The authors have contributed some valuable information regarding the character and quantity of suspended material carried through Mead Lake. The paper is especially noteworthy since outflow conditions at the reservoir during the period covered by the silt observations will never again be experienced. During 1935 lake water was released through one of the diversion tunnels at about the normal low-water surface of the Colorado River. About May 1, 1936, the tunnel flow was shut off; not only by closing the regulation gates, inside the tunnel, but also by permanently sealing the up-stream end of the tunnel with a 50 by 50-ft Stoney gate. Since that time water has been released through cylinder gates in the intake towers. These gates are located at heights of approximately 250 and 400 ft above the tunnel gates. They will be used in all future regulation of reservoir outflow for irrigation and power purposes.

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[18] Senior Engr., U. S. Bureau of Reclamation, Denver, Colo.

Unfortunately, no measurements of river water temperature were made at the up-stream end of Mead Lake during 1935. However, temperatures of river water at the dam site have been observed since April, 1930. These measurements were being made, daily, during the period covered by the authors' silt observations. After storage began on February 1, 1935, the temperature measurements were made in the reservoir outflow. Fig. 3 shows the results of the observations; also the daily air temperatures, mean monthly air temperatures, and mean monthly water temperatures.

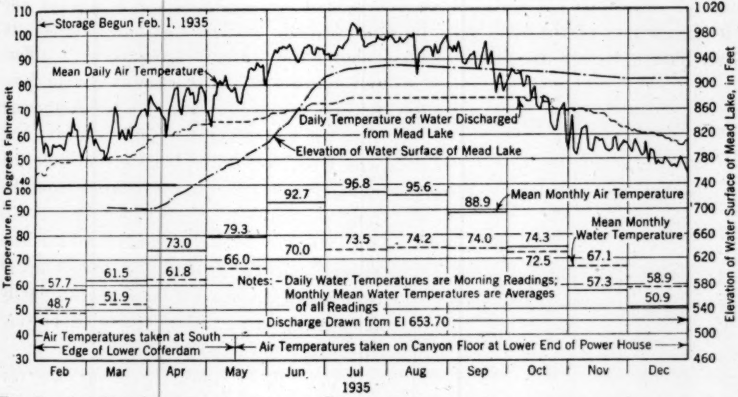

FIG. 3.—AIR TEMPERATURES AT BOULDER DAM COMPARED WITH TEMPERATURES OF WATER DISCHARGED FROM LAKE MEAD DURING PERIOD FROM FEBRUARY 1 TO DECEMBER 31, 1935.

A comparison of the daily water temperatures during 1935 with those measured during previous years shows that the reservoir inflow temperatures must have been approximately the same as, or slightly lower than, the lake temperatures at the elevation of the tunnel gates, during the periods of silt movement through the reservoir. Consequently, the inflow densities must have been approximately the same as, or slightly greater than, the lake densities at the tunnel level; and the silty inflow undoubtedly followed the original river channel along the lake bottom to the open gates as assumed by the authors. Since the gates were closed silty inflow probably has moved along the lake bottom to the level of its density, then spread out, and gradually come to rest at that level. The fine silt particles, which might have been discharged with the outflow had the tunnel gates remained open, are now settling to the lake bed and are gradually becoming consolidated under the lake pressures.

Measurements of water temperature at different depths in Mead Lake were begun in June, 1936, and are being continued at monthly intervals. The securing of samples of the lake water at different depths, for chemical and silt analyses, is now (1937) being considered. When such data become available, it should be possible to discuss the movement of turbid water through the reservoir much more adequately. The writer believes that, some time in the future, silty flood water, having the same density as the lake water at the elevation of the cylinder gates, may enter the reservoir, flow through the lake at the level of its density, and be noticeable in the reservoir discharge.

The conditions under which turbid water may flow through a reservoir instead of along the bottom, or instead of being rapidly diffused at the up-stream end of the lake, may be discussed on the basis of the temperature distribution within the lake. Fig. 4 shows the water temperature at different elevations in Elephant Butte Reservoir on June 18, 1934. Assuming that the water had the same salinity and was free from silt at all elevations, the density curve can be calculated from the temperature curve. Such a curve will show approximately constant densities from the bottom of the reservoir up to about Elevation 4 270, then gradually decreasing densities as the temperature increases toward the surface.

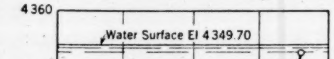

FIG. 4.—TEMPERATURE DISTRIBUTION IN ELEPHANT BUTTE RESERVOIR ON JUNE 18, 1934.

Silty flood flows, entering the reservoir with a density greater than the water at the lake bed, would be expected to flow along the original river channel beneath the body of the lake. Similar flows, entering the reservoir with a density the same as the lake water below Elevation 4 270, would be expected to spread out laterally and vertically through the entire reservoir section below that elevation. However, silty floods entering the reservoir with a density equal to the lake water at some elevation above 4 270, as, for instance, at Elevation 4 300, would be expected to spread out laterally at that level and to flow through the reservoir at approximately that elevation without diffusing through appreciable vertical distances, just as the water did in the laboratory experiments described by Mr. Freeman, and cited by the authors.[3] In any case, the appearance of silt in the reservoir outflow is probably dependent upon the continuance of a relatively large silty inflow at the upper end of the reservoir until the silty flow has had time to reach the outlet gates.

Temperature measurements in existing reservoirs show that during some periods of the year, usually during the winter months, lake densities are approximately constant through the entire vertical section. At such times flood flows, entering the reservoir with a density the same as the reservoir water, would be expected to diffuse laterally and vertically through the entire cross-section of the reservoir. Consequently, under such conditions, only exceptionally large and long-continued inflows would cause movement of the turbid water to the outlet gates.

The writer doubts whether it will ever be desirable to design irrigation reservoir outlets so as to secure a maximum outflow of turbid water. As mentioned by Mr. Fiock, in the reference cited by the authors,[a] the silt discharged at such times would be deposited in the canals, or carried through the canal systems, and deposited on the farm lands, either of which would be objectionable. However, investigation of the problem is well worth while from the scientific standpoint, even though the studies may produce no results of great economic value. The authors have rendered a service to the Engineering Profession in bringing the matter up for discussion.

William P. Creager,[19] M. Am. Soc. C. E. (by letter).—The contents of this paper are of exceedingly great interest. It is hoped that more data of like character will be forthcoming for other cases.

When water containing silt in suspension enters a relatively clear reservoir, the chances of its passage through such reservoir essentially unmixed, depend upon: (1) The volume of the incoming water; (2) the slope of the bed of the reservoir; (3) the roughness of the bed of the reservoir; (4) the length of the reservoir; (5) the specific gravity of the incoming water; and (6) the size of the particles in suspension.

For the same specific gravity, the greater the volume of inflow the deeper will be the stream of unmixed water; and this factor, together with the slope and roughness, affects its velocity of flow just as in an ordinary stream. It is obvious that the greater distance the unmixed stream has to travel, the greater will be the opportunity for the suspended matter to settle.

All these conditions must be taken into consideration for a particular problem and additional information similar to that furnished by the authors will provide the basis of the constants of flow.

The specific gravity of the incoming water depends upon: (*a*) Its temperature; (*b*) its dissolved matter; and (*c*) its suspended load. The temperature of the water entering the lake may be greater than that of the lake with a resulting tendency to keep on the surface. However, the authors have shown that, for this case, the variation in specific gravity due to temperature is negligible in comparison with the variation due to suspended load.

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[19] Cons. Engr., Buffalo, N. Y.

Although the quantity of dissolved material undoubtedly affects the specific gravity of the incoming water, there is no evidence in Fig. 2 to show that it had much influence on the discharge of silt from Lake Mead, as indicated by the following notes: (1) There was no increase in dissolved matter prior to the April discharge of silt; and (2) dissolved matter has no chance to settle, as does suspended matter. Therefore, one would expect that, if the greater specific gravity of the water due to dissolved matter were predominant, there would be some similarity between the inflow and outflow of dissolved matter. However, in August, the outflow was reduced while the inflow was increasing; whereas, in November, the reverse is true.

The writer wishes to discuss the suspended load, first, on the basis of size of particles; second, on the basis of total volume in suspension; and, finally, on the basis of the percentage of material in suspension.

Small particles settle in water more slowly than large ones. Therefore, other conditions being the same, it would be expected that turbulence caused by fine particles would find its way through the lake more readily than that caused by large particles. However, the quantity of fine particles does not seem to be the predominating feature at Lake Mead, for the following reason: The variation in size of particles entering the lake is shown in Fig. 2(*b*) of the paper. The authors state that "whenever there was an increase in load of suspended matter at the Willow Beach Station there had been a prior increase in the quantity of material less than 20 microns in diameter at Grand Canyon." However, the reverse of this statement is not the case. For instance, a considerable discharge of suspended matter from the lake occurred after the April increase in quantities of material less than 20 microns, but in June there was a still greater increase in fine particles with no resulting discharge of suspended matter from the lake. This same argument may be applied to show also that neither the total quantity of suspended matter entering the lake nor the total flow is the predominating feature affecting discharge of suspended matter from the lake.

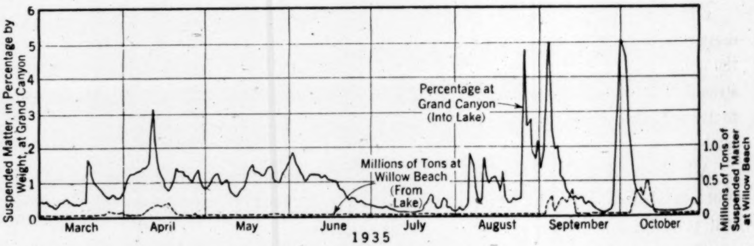

FIG. 5.—SUSPENDED MATTER, PERCENTAGE BY WEIGHT, INTO THE LAKE AND TONS OUT OF THE LAKE.

The authors state that "there will naturally be speculation as to the reason why the high water of May and June, with its heavy silt load at

Grand Canyon, did not produce a turbid discharge from Lake Mead." The writer believes that the answer is to be found in his Fig. 5, in which the percentage of suspended matter entering the lake (taken from Table 1) and the tons of suspended matter discharged from the lake (taken from Fig. 2(*c*)) are plotted. It will be noticed that there is a distinct agreement between these plottings. There was always a discharge of silt from the lake when the percentage of suspended matter entering it was greater than about 2 and no discharge of silt from the lake when the percentage was less than 2. The authors' speculation will be answered when it is noticed that the higher water of May and June with its "heavy silt load" by volume actually had a relatively small percentage of silt, and therefore, a relatively low specific gravity.

The fact that, in March, silt was carried through with only 1.7% of suspended matter whereas, in June, it did not carry through with 1.9%, may be accounted for by the fact that, in March, the percentage of dissolved matter was greater and the temperature probably less, resulting in greater specific gravity for the same percentage of suspended material.

It is not possible, with the data available, to form final conclusions; but the facts presented in the paper indicate that:

1.—It requires a certain specific gravity of the incoming water to carry the silt through the lake.

2.—At Lake Mead, the predominating feature affecting specific gravity was the percentage of total suspended load, although the percentage of fine particles and dissolved matter and the temperature may have had a minor influence.

3.—For the physical conditions at Lake Mead, it requires a specific gravity corresponding at least to that for about 2% of total suspended matter.

Harold K. Palmer,[20] M. Am. Soc. C. E. (by letter).—An unexpected phenomenon of scientific interest is described in this paper. Although it might be that the useful life of this large project would be increased slightly by proper operation, as the authors suggest, any possible increase in life would be small because the percentage of silt fine enough to be carried through such a long reservoir would be small. The coincidence of the increase in specific gravity of the water at Willow Beach observed from October 5 to 14, 1935, with the period of turbidity, suggests that this increased specific gravity is probably the explanation of the passage of turbid water through the lake. The authors show that the increased specific gravity is almost entirely due to the concentration of fine material, and Figs. 2 (*b*) and 2 (*c*) show that periods of turbidity follow sharp peaks of fine material observed at Grand Canyon. Therefore, a solution should be sought in the behavior of two bodies of water of different specific gravity when brought into contact with each other.

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[20] Chf. Draftsman, Los Angeles County Sanitation Dists., Los Angeles, Calif.

In collaboration with A. M. Rawn, M. Am. Soc. C. E., the writer made an extensive series of experiments on the mixture of fresh water and sea water.[21] This investigation dealt with the behavior of a lighter water introduced at the bottom of the body of heavy water which is the opposite of the conditions encountered in Lake Mead, but some of the results can be easily applied to both cases.

In the experiments with sea water it was found that the mixture of the two waters occurred in two stages, the first being a violent mixture caused by the turbulence along the plane of contact as the fresh water rose rapidly to the surface of the salt water, followed by the second stage when the somewhat diluted fresh water spread over the surface of the salt water. The term, "initial dilution", was used to express the degree of mixing that had been affected at the end of the first stage, when the water reached the surface. All the experiments revealed the fact that if this initial dilution was small (about nine parts of salt water to one part of fresh), the final mixing was very slow and the field would spread to large dimensions before disappearing at a theoretical dilution ratio of 225 to 1. It was also found that if the natural spreading of the field was interfered with in any way, such as being confined by a sea wall, further dilution was much slower and the fresh water tended to travel without much further mixing.

The authors show that about eight days after the specific gravity of the water at Grand Canyon showed an increase from 0.995 to from 1.005 to 1.008, the turbid water appeared at Willow Beach. The specific gravity of sea water is 1.026. Reducing this figure to 1.016 would be equivalent to the change in specific gravity noted at Grand Canyon and would be accomplished by a dilution of approximately two parts of salt water with one of fresh. Such a mixture would be quite stable and could be expected to continue its travel for a long distance without great dilution.

When the heavy water encounters the quiet water of Lake Mead, in a confined canyon where the cross-section increases gradually, its velocity will be reduced without turbulence, and the cross-section of the moving stream must increase in the same proportion. In the narrow box canyons which occur in Lake Mead, the bottom stream is confined and the plane of contact with light water will be narrow, thus making any mixing very slow. The velocity of the heavy water under the quiet light water is a function of the difference of specific gravity and the slope of the bottom, which latter is a constant for any given location in the lake. The velocity of the stream of heavy water will be a direct function of the difference in gravity whereas its depth will be inversely proportional to its velocity. Friction between the two classes of water complicates the problem, and the inertia of the heavy water prevents any motion until the difference in gravity attains some minimum value; but once motion is started the same inertia will maintain it until this difference has decreased to a value less than was required to initiate the movement. This fact would explain the movement in surges. Furthermore, since the increase in specific gravity is due to a mechanical mixture, rather than to a solution, the settling of the fine particles in the quiet water of the bottom during periods of negligible movement would tend to decrease the force required to start the flow.

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[21] "Predetermining Extent of Sewage Field in Sea Water", *Transactions* Am. Soc. C. E., Vol. 94 (1930), p. 1036.

The actual behavior could best be found by means of experiments with a model, and it would also be well to make regular specific gravity observations at Grand Canyon, and at Willow Beach beginning a few days after heavy water was noted at Grand Canyon. Sub-surface samples from different parts of the lake, especially at both the upper and lower ends of the wide sections, should yield interesting data regarding the difference in behavior in the box canyons and the open sections of the lake.

When the difference in specific gravity is very small, the depth of the stream of heavy material should be great and, in the valleys, it would spread well out of the old stream bed, thus increasing the plane of contact so much that mixing would be increased enough, practically, to stop the flow beyond the valley. Any model for this study should contain alternate box canyons and wide valleys to study this effect. With this information it might be possible to determine the shapes of reservoirs which would be expected to pass turbid water through them, and compare these with Lake Mead, or with any other reservoir which passes turbid water through it.

MORROUGH P. O'BRIEN,[22] ASSOC. M. AM. SOC. C. E. (by letter).—Recent theories dealing with the suspension of solid material in turbulent flow, treat the problem as one of statistical equilibrium in which the transverse components of velocity carry more material upward than downward, because of the increase of concentration in the downward direction, and thus counteract the constant settling tendency. Provided that the upward velocity components exceed the settling velocity of the material, a steady state of suspension may be attained. From this viewpoint a possible explanation of the flow of turbid water through Lake Mead may be that the greater density of the water carrying suspended material caused it to follow the bottom at a sufficiently great velocity to result in a turbulent flow capable of re-suspending the solid particles as they settled out. Thus, the inflowing stream might be held to the bottom in its course through the reservoir. This phenomenon is essentially different from the flow of cold or saline waters because the mixture is not permanent.

An approximate computation of the order of magnitude of the quantities involved may be made by assuming that the usual friction equations apply without allowance for the energy dissipated in the circulation set up in the overlying water. The resulting equation would be,

$$V = \sqrt{\frac{g_m}{k} \times \frac{p}{100} \times \frac{s-1}{s} R\,S} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

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[22] Prof. of Mech. Eng., Coll. of Eng., Univ. of California, Berkeley, Calif.

in which V is the mean velocity; g_m is the gravitational force per unit mass; k is a friction coefficient; R is the hydraulic radius; S is the slope of the bottom; p is the percentage concentration; and s is the specific gravity of the material in suspension. If the layer of water is thin in proportion to its width, R may be replaced by T, the thickness. The discharge is bVT, in which b is the width. When $S = 0.0007$, $s = 2.65$, $k = 0.003$, and $b = 400$ ft, the depths and velocities for three days of high concentration are as given in Table 3. At best, these values are only rough approximations not only because of the simplified theory used, but also because of insufficient data. No allowance was made for differences in temperature between the reservoir and the inflowing stream, the width was assumed as 400 ft, and the slope is only an average value, and yet the results are within reason. The Reynolds number is high and these flows should be turbulent, but the vertical density gradient would probably reduce the turbulence below that which would occur in a clear stream of the same depth and mean velocity.

TABLE 3.—COMPUTED VELOCITY AND THICKNESS OF SILT CURRENT.

Description	April 11	September 4	September 30
Percentage by weight	3.16	5.02	4.14
Discharge, in cubic feet per second	15 600	8 630	18 100
Computed thickness, T, in feet	21.5	12.5	22
Computed velocity, in feet per second	1.8	1.7	2.1

At a velocity of 1.5 ft per sec (which is below the computed velocities) the flow would traverse the 90-mile length of reservoir in a little less than four days. During this interval, material as small as 5 microns would settle out of still water, and the stream would lose its identity before reaching the tunnels unless the material is kept in suspension by the transverse velocities of the turbulence. The approximate average magnitude of these transverse velocities would be $0.15 \times 1.5 = 0.22$ ft per sec, or 80 ft per hr and some particles larger than 50 microns might be transported.

Using the same methods one may compute a lower limit of discharge and concentration which would maintain particles of a given size in suspension, but the result would be greatly affected by the temperature. If the incoming stream is colder than the water in the reservoir, capacity for maintaining particles in suspension is increased both because of the increased density of the water and the decreased settling velocity of the particles. If the inflow is warmer, the capacity for suspension is reduced and this effect may account for the absence of turbid outflow during the high-water period of May and June.

It is to be hoped that this paper will arouse sufficient interest among those concerned with reservoir operation that information will be obtained to form the basis of a general theory. It appears possible to develop a combination of theory and experiment which would permit computation of the flow of sediment through reservoirs with known inflow conditions, but to do so will require extensive field data.

JOHN C. PAGE,[23] M. AM. SOC. C. E. (by letter).—Since the authors completed their paper, additional information has been obtained from field records and the observations of employees of the U. S. Bureau of Reclamation which may throw further light on this interesting phenomenon.

Storage in, and controlled release of water from, Lake Mead was started on February 1, 1935. The water discharged from the reservoir through the tunnel at river grade was clear until February 15, when the water became muddy and did not clear until the following day. This occurred again on February 19 and from February 23 to February 25. In all instances, there were storms on the lower water-shed preceding the discharge of the turbid water. The silt content in the first two runs was light, being recorded as approximately 10 000 ppm at the gaging station 1¼ miles below the outlet.

No measurements of silt content were taken for the third run, but samples were taken in the reservoir, just up stream from the inlet of the diversion tunnel, to determine the elevation of the silt-laden stratum. Sampling across the channel showed that this layer occupied the bottom of the original river channel, being about 2 ft in depth in the center. The surface of the stratum was level and was marked so definitely that a change of 6 in. in the elevation of the sampler changed the sample from clear to cocoa-colored. The dried silt from a sample taken about the middle point of this layer amounted to about one-fourth in volume of the total sample. At the time of this third occurrence, a streak of muddy water, visible on the surface, extended through the center of the reservoir for approximately 3 miles below Boulder Canyon, clear water appearing on both sides. Undoubtedly, it extended a greater distance, but was not visible. Boulder Canyon is 20 miles up stream from the dam.

Turbid water again flowed from the reservoir on March 24 and April 21. The temperatures of the water at the outlet of the tunnel increased from 52° F to 63° F during this period. The silt content varied from about 3 100 to 11 400 ppm, measured at the rating station below the dam and checked closely with the silt measurements taken at Willow Beach. Nine samples were taken at different depths for each reading. The silt content was found to be uniform for all samples, indicating that a thorough mixing occurred at the outlet gates. Previous to the passage of the water through these gates, and under similar conditions of silt load in the river, the silt content of the surface sample was about 78%; and the middle sample 85% of the bottom sample.

No measurements were made during the turbid releases in September and October, 1935, or from April 22 to May 1, 1936. Water was being released both from the bottom of the reservoir and through the gate-opening in the base of one of the intake towers 260 ft above the old stream bed during the April–May period. No turbid water came from the Arizona canyon wall outlet works which connected with the intake tower gate. The muddy water emerging from the low-level tunnel on the Nevada side and the clear water from the outlet works on the Arizona side did not mix completely until after they had passed the gaging station. During the period of this flow, the temperature of the water was 50° F from the bottom of the reservoir, 58° F from the outlet works, and 55 to 57° F at the gaging station.

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[23] Commr., U. S. Bureau of Reclamation, Washington, D. C.

Recently, samples have been taken between the dam and the up-stream coffer-dam, and another set approximately 4 000 ft up stream from the dam. The silt stratum near the dam has its surface at about Elevation 650, whereas the silt surface farther up stream is about Elevation 695. The question has been raised concerning the possibility of silt flowing over the coffer-dam. The top of the up-stream coffer-dam is at Elevation 720, the reservoir bottom at approximately Elevation 615 near the dam, and the bed of the old river channel about Elevation 640. It is possible that the surface of the silt stratum was at one time higher than the top of the coffer-dam and since that time has lowered, due to the settling of the silt when undisturbed.

Temperature measurements recently taken near the dam at the time of the silt sampling show an abrupt rise of from 10° F to 20° F near Elevation 650. From this elevation downward the higher temperature is maintained until the bottom is reached. Measurements of specific gravity and electrical conductivity also show pronounced increases near Elevation 650. A brief summary of these data is given in Table 4.

TABLE 4.—CHARACTERISTICS OF WATERS OF LAKE MEAD MIDWAY BETWEEN UP-STREAM INTAKE TOWERS, MAY 21, 1937.

Elevation, in feet, above mean sea level	Clarity	Temperature, in degrees Fahrenheit	Conductivity (ohm reciprocal)	Specific gravity
1 066*	Clear	65.7	0.00128	1.0010
690	Clear	48.5	0.00141	1.0013
665	Clear	48.4	0.00141	0.0014
655	Clear	50.1	0.00142	1.0014
640	Muddy	60.1	0.00166	1.1732
620	Muddy	63.5	0.00161	1.1787

* Surface

Although there is not sufficient information available to arrive at definite conclusions concerning the phenomenon, it is evident that, under certain conditions, silt-laden water will pass through a reservoir without intermingling, although the length of the reservoir is as much as 100 miles. The silt that passes through Lake Mead is colloidal in character and differs greatly from the type normally carried by the stream. Under normal conditions the silt is abrasive, dark in color, and so heavy that 98% of it will settle out in a 2-hr period. The silt that has passed through the reservoir has a clay-like consistency, is lighter in color, and is difficult to remove from the water even by filtering. It is the writer's opinion that this lighter material comes from localities where the soil is predominantly clay and contains a high percentage of salts. The material is carried to the river, generally, by cloudbursts. Its ability to pass through the reservoir seems to be derived from its colloidal and chemical characteristics.

The abrupt change in temperature at Elevation 650 in the waters near the dam may be due to the silt stratum, but the increased temperature is possibly due to the presence of warm springs. These springs, having a temperature as high as 130° F, were encountered in the canyon during construction. The heavy layer of colloidal clay in suspension may have an insulating effect which tends to delay rapid temperature diffusion.

JOHN H. BLISS,[24] ASSOC. M. AM. SOC. C. E. (by letter).—The passage of silty water through a reservoir of clear water in a more or less definite stratum, with little diffusion of the two, is a phenomenon which has not received much attention but which merits considerable study. Messrs. Grover and Howard have presented a very interesting paper on its occurrence through the waters of Lake Mead, and have suggested the possibility of utilizing such phenomena to reduce silt accumulations within the storage area of reservoirs.

Since its completion in 1915, there have been frequent passages of silt through the Elephant Butte Reservoir of the Rio Grande Project, in New Mexico, most of them occurring in the summer and fall when the flow of the Rio Grande was comparatively low and the torrential discharge of tributaries above the reservoir was relatively great.

The silt-laden waters that seem to be chiefly responsible for this phenomenon are contributed by two large intermittent tributaries, namely, the Rio Puerco, which drains an area of about 5 000 sq miles on the west side of the Rio Grande and empties into it about 45 miles above the head of the reservoir; and the Rio Salado, which drains an area of about 1 500 sq miles also west of the river and enters a few miles below the Rio Puerco. As their names imply ("Puerco" meaning "soiled" or "dirty," and "Salado" meaning "salty"), both streams carry extremely silty water of high saline content during periods of run-off. Analyses of the waters of the Rio Grande at the San Marcial Gaging Station at the head of the reservoir indicate that these tributaries may carry 150 tons or more of silt and a concentration of total dissolved salts of more than 3 tons per acre-ft of water discharged. It is interesting to note that these two streams, particularly the Puerco, have their sources immediately to the east of those arid regions drained by the Little Colorado River and the southern branches of the San Juan River, which latter streams the authors believed to be the chief contributors of the finely divided silt causing the turbid flows through Lake Mead. In this connection the authors' reference to silt passage through the Zuni Reservoir on the head-waters of the Little Colorado may also be cited.

The phenomenon of the difficulty with which two waters of different density may unite is of rather common occurrence. Any one who has seen a muddy torrential flow enter a stream of clear water has probably observed that the two flows may often be readily distinguished for considerable distances down stream. The San Acacia diversion dam of the Middle Rio Grande Conservancy District is constructed across the Rio Grande about two miles below the mouth of the Salado and ten miles below the Rio Puerco. The gate-tender at the dam has noted several instances when the flows of these tributaries have appeared at the dam practically unmixed with the main river discharge. On one occasion, the line of demarcation between the two flows was described as quite noticeable, being almost as distinct as if they had been separated mechanically. It was further stated that such tributary waters might appear on the right, or entrance, side of the channel, or they might pass under the main stream and appear on the opposite side.

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[24] Engr., State Engr.'s Office, Santa Fé, N. Mex.

Since 1931, officials of the Rio Grande Project and the Division of Western Irrigation Agriculture, of the Bureau of Plant Industry, U. S. Department of Agriculture, in co-operation, have obtained and made complete analyses of the waters of the Rio Grande at several stations, including San Marcial above the reservoir, Elephant Butte immediately below the dam, and Leasburg diversion dam about fifty miles below. Although the saline content of the water is the primary consideration in these analyses, incidental silt determinations have been made at the first two of these stations. Periodic sampling at these points, together with an endeavor of the U. S. Bureau of Reclamation to keep a daily record of all appreciable silt passages through the lake, provide the means of studying some of the details of these occurrences. L. R. Fiock, Project Superintendent, has records of turbid flows through the reservoir for somewhat less than half the years since its construction. There is reason to believe, however, that there may have been other occasions when tributary waters have passed through the reservoir unobserved. At such times the quantity of silt carried may not have been sufficient to be noticeable, the clue to such passage being an increase in salt content of the water released from the reservoir.

During the six years that complete analyses have been made of the waters passing Elephant Butte only five periods out of more than twenty major tributary discharges were recorded as silt flows through the reservoir (see Table 5). However, as the sampling period was monthly (since 1924, weekly), the periods between the dates of sampling are long enough so that no records have been obtained during periods when silt flows of short duration might have passed through the gates. As an example, in 1931, monthly samples only were taken at Elephant Butte, except that in September a two-day flow of silt through the reservoir was recorded. During the latter part of September and October of that year there were at least four tributary discharges which might have caused turbid discharge. At Leasburg Station, where eight samplings were taken during that period (silt determinations not being made), all but one sample showed an increase in total dissolved salts carried. On October 12, the salt concentration of the water had increased 100%, the increase on other days of sampling being less marked. The data are not conclusive, because the discharge from the

TABLE 5.—DATA REGARDING TRIBUTARY DISCHARGES TO ELEPHANT BUTTE RESERVOIR, 1931 TO 1936

Dates	(a) SAN MARCIAL RECORD				(b) RECORDS BELOW RESERVOIR	
	Silt, in tons per acre-foot	Salt, in tons per acre-foot	TRIBUTARY DISCHARGE		Elephant Butte dates	Remarks
			In cubic feet per second	Percentage of river discharge		
September 17–20, 1931	..	2.0	800	30	September 22–23	Average silt, 55 tons per acre-ft.; salt, 1.42 tons per acre-ft. (85 tons per acre-ft., normal).
September 25–October 23, 1931	..	..	*	..	....	No record at Elephant Butte except sample of October 5, which showed trace of silt and slight increase in salt content; increased salt content at Leasburg through most of period (no silt record).
July 13, 1932	..	..	1 800	45	....	No samples taken below dam.
July 25–29, 1932	..	..	800	50	August 1	Trace of silt and slight salt increase recorded.
August 18–22, 1932	..	..	1 500	75	....	No samples below dam during period.
August 29–30, 1932	..	..	3 500	90	....	No samples below dam; water clear at Elephant Butte, September 5.
June 18–25, 1933	48–124	0.98–1.83	3 800	65	June 23–30	Silt, 0–67 tons per acre-ft.; salt, 0.85–1.38 tons per acre-ft. (80 tons per acre-ft. normal). Increased salt content in water at Leasburg during period.
July–August, 1933	..	..	Three rises up to 2 000	..	....	No samples during period.
September 14, 1933	148	1.96	Peak of 14 000	..	....	No samples below dam.
August 28–29, 1934	..	..	6 500	100	....	Elephant Butte sampled September 1, and September 8; no record of silt passage.
September 25–26, 1934	..	1.5	2 900	100	....	Elephant Butte sampled September 29; no record of silt passage.
August 4–6, 1935	84	2.20	3 400	55	August 9–10	Average silt, 55 tons per acre-ft.; average salt, 1.64 tons per acre-ft.; (0.75 ton per acre-ft., normal).
August 21–22, 1935	170	2.14	5 700	85	August 25–26	Average silt, 21 tons per acre-ft.; average salt, 1.10 tons per acre-ft. (0.70 ton per acre-ft., normal).
August 30–31, 1935	130	1.85	4 400	100	....	Not sampled below dam during period.
September 27–30, 1935	71	1.54	2 300	60	October 5	Silt, 106 tons per acre-ft.; salt, 1.51 tons per acre-ft. (0.75 ton per acre-ft., normal).
July–September, 1936	..	..	†	..	....	No published record of any silt or salt flows through reservoir during period.

* Three periods of flow—about 75% of total discharge of river.

† Four rises of from 1 000 to 5 000 cu ft per sec from Rio Puerco; those from Rio Salado are unrecorded.

reservoir was sharply reduced about September 19 and stopped entirely on October 5, thereby changing considerably the regimen of flow at the Leasburg Station; but the indications are that enough saline tributary flow found its way through the reservoir to increase the salt concentration of the water below the dam appreciably. Whether this water was silty or even murky is not recorded, but at least, there was not enough coloration so that turbid discharge was noted and silt samples were taken at the outlet gates.

Since the discharges of the Salado and Puerco always involve increased salt as well as silt concentration of the river water, it is impossible to state to what extent each of these two factors contributes toward this phenomenon. As the authors have suggested, it is probable that, because of the greater quantity carried, silt has much the more pronounced effect of the two in increasing the specific gravity of the waters. There appear to be instances, however, in which flows have passed through Elephant Butte carrying increased quantities of salts and but very little or no silt. Although data are insufficient for proof, it seems probable in such cases that the silt gradually settles from the influent in its passage through the lake, leaving only the dissolved salts to maintain the increased density of the solution and pass through the gates.

As Mr. Fiock[25] has inferred, temperature apparently has little effect upon the specific gravity of the turbid influent. The average temperature of the waters near the bottom of Elephant Butte is about 57° F, whereas the average summer temperatures of the discharges into the reservoir are several degrees higher—in the 60's or 70's. The effect of temperature, therefore, must be slight in comparison with other factors, as its tendency is in a direction opposite to that of the observed phenomena.

W. F. Resch, Project Hydrographer, has informed the writer that soundings were made during one period of silt passage which indicated that the flow beneath the clear waters of the reservoir followed a definite stratum with little mingling of the two layers. As the outlet gates are at the bottom of the dam, it is probable that the silt flows do occur on, or close to, the flow line through the lake.

The data, although not definite, indicate that it takes the turbid water four to five days to find its way through the reservoir, a length of 30 to 35 miles, suggesting an average velocity under water of 0.5 ft per sec, or less.

The conclusions to be drawn from a study of the passage of silt through the Elephant Butte Reservoir would seem to be: (1) That any discharge into the reservoir whose waters are of sufficient density, due to very fine suspended silt (and also to salt), will potentially pass through it in an essentially unaltered condition; (2) those flows which are below this minimum will tend to pass through the lake, but will gradually lose their identity and may or may not appear at the outlet gates; and (3) under certain conditions, discharges may find their way through the reservoir, progressively leaving their silt load behind, and may appear below the dam with only an increased salt content to mark their passage. These conclusions are tentative and are subject to confirmation by further investigation and study. They are substantially in accord with the authors' findings, however. The bearing of Conditions (2) and (3) upon the passage of appreciable quantities of silt through reservoirs is slight.

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[25] *Transactions* Am. Geophysical Union, 1934, Pt. 2, p. 472.

The suggestion that, by proper attention to gate location and the approach conditions thereto, turbid waters might be encouraged to pass through reservoirs and thereby reduce accumulations of silt within their high-water lines, is one which should receive considerable study. It is related to, and, must be investigated in conjunction with, the problem of silt conservation and the aggradation of stream channels.

If a reservoir could be operated so that peak releases could be made whenever turbid waters were available at the outlet gates, considerable quantities of silt might be prevented from settling in the reservoir area. Under ordinary operating conditions, however, it is unlikely that enough change in releases could be made economically to remove much additional silt above that ordinarily carried through the gates. Furthermore, as no greater concentration of silt can be passed through a reservoir than appears in the influent above, the problem resolves itself into one of whether the inflowing water or the reservoir volume usurped by the deposited silt is the more valuable. Under certain conditions, the disposal of even part of the silt burden of a turbid stream behind retention or storage dams would seem to be advantageous, particularly in those arid Western States where consumption of large quantities of water for irrigation has resulted in the serious problem of rapidly aggrading stream channels which threaten the very existence of agriculture along their banks.

A compilation of data pertaining to the quantity of water above and below Elephant Butte Reservoir since 1930, showing conditions at the time of appreciable tributary inflows to the reservoir, is given in Table 5. The data are not sufficient to show the sources of the rises, but those listed are believed to have come chiefly from the Rio Puerco and Rio Salado. The dates of such tributary flows, their average discharges in cubic feet per second, and their percentages to total river flows as listed, are approximate values for comparative purposes only, the discharges being so variable and the time element so uncertain that exact figures are difficult to obtain. The data from which comparisons may be made of changes in silt and salt quantities in their passage through the reservoir are not good, either, because of the extreme variations of these quantities during the various stages of tributary flow. The need for additional and more definite information is evident.

B. H. MONISH,[28] ESQ. (by letter).—The failure of fluids of different densities to mix when flowing in contact with each other at appreciable relative velocities has been recognized as a problem for many years, but it is only comparatively recently that active interest has been shown in the phenomenon. This is particularly true in the field of applied hydraulics. The authors of this paper are to be commended on a clear presentation of carefully collected data. Workers in fluid dynamics have made mathematical analyses of the stability of streams of superposed fluids of different densities, and it was along the lines suggested by one of these analyses that the writer attempted a study of the data presented in this paper.

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[28] Assoc. Engr., National Bureau of Standards, U. S. Dept. of Commerce, Washington, D. C. Publication of discussion approved by the Director, National Bureau of Standards, U. S. Dept. of Commerce.

As a criterion of mixing between any two superposed fluids of different densities, when the density and velocity vary continuously from one fluid to the other, L. Prandtl[27] has given the expression:

$$\theta = \frac{-g\dfrac{d\rho}{dy}}{2\rho\left(\dfrac{du}{dy}\right)^2} \quad\text{(2)}$$

in which θ = a dimensionless coefficient which exceeds a certain value when no mixing occurs; $\frac{d\rho}{dy}$ = the density gradient between the fluids; $\frac{du}{dy}$ = the velocity gradient between the fluids; ρ = the density of the fluid at that element at which $d\rho$ and du are taken; and, g = the acceleration due to gravity.

This criterion cannot be applied directly to the problem at hand, since the density and velocity gradients are not obtainable from the data. Hence, it is necessary to modify the criterion so that it will be expressed in terms of observed quantities. Linear velocity and density gradients were assumed over some characteristic (but unspecified) length, L, obtaining instead of Equation (2), the expression,

$$\theta = -\frac{gL}{2\,U^2} \times \frac{\Delta\rho}{\rho} \quad\text{(3)}$$

or, modifying the criterion by the factor, $-\frac{1}{2}$,

$$\theta = \frac{gL}{U^2} \times \frac{\Delta\rho}{\rho} \quad\text{(4)}$$

in which U is the mean relative velocity between the two fluids and $\Delta\rho$ is the difference in density between the current and the still water. Equation (4) will be recognized as the product of a Froude number and a density ratio, two familiar dimensionless quantities.

Since L is not determinable from the data, it was postulated that it is some dimension similar to Prandtl's "mixing path", or the distance through which a particle of fluid moves into the surrounding fluid when effecting a transfer of energy, and that this distance is dependent on the mean velocity, U, of the current, and the kinematic viscosity, ν, of the liquid composing the current, as follows: $L \sim \frac{\nu}{U}$, or,

$$L = k\frac{\nu}{U} \quad\text{(5)}$$

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[27] "Einfluss stabilisierende Kräfte auf die Turbulenz", von L. Prandtl, Vorträge aus dem Gebiete der Aerodynamik und verwandter Gebiete, p. 1, Aachen, 1929.

in which k is a numerical constant. This expression merely amounts to assuming that the Reynolds number for this phenomenon is constant. Substituting Equation (5) for L in Equation (4) and again absorbing the constant in θ:

$$\theta = \frac{g\,\nu}{U^3}\left(\frac{\Delta\rho}{\rho}\right) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which $\Delta\rho$ = the density difference between the stream and the still water in the reservoir; ρ = the density of the lighter liquid (reservoir water); and, ν = the kinematic viscosity of the lighter liquid.

Again taking advantage of the fact that all observations were made in the same reservoir and on the same liquids, Equation (6) can be further simplified by absorbing the constant quantities, g and ν, in the constant. Recognizing that any particle which is entirely borne, surrounded, and supported by a liquid over the range of action studied should be considered a part of that liquid and as affecting the properties of that liquid, $\frac{\Delta\rho}{\rho}$ could be computed from the data given, but it is sufficient to note that this ratio is expressed with sufficient accuracy over the range of values of n considered by the expression, $\frac{\Delta\rho}{\rho} \sim n$, or,

$$\frac{\Delta\rho}{\rho} = k_2 n \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which n = the silt load, in percentage by weight, of the total sample; and k_2 = a numerical constant. This leaves only U to be converted into the form of some quantity given in the data studied.

In any given channel, U will be dependent on the flow, the exact proportionality being unknown, but unimportant for immediate purposes, since the order in which the values of the final criterion and of the critical number arrange themselves will be unchanged by any relation that may logically be assumed. As a matter of convenience, the relation was assumed that would hold if the flow were in accordance with the Froude law, $Q \sim U^3$; or,

$$Q = k_3 U^3 \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which Q is the flow of the river, in cubic feet per second; and k_3 is a numerical constant. Substituting Equations (7) and (8) in Equation (6) (absorbing the constants, k_2 and k_3 in C):

$$C = \frac{n}{Q} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which C is a numerical criterion of mixing.

Ten peak flows were selected from Fig. 2 of the paper and C was computed by Equation (9). The results are given in Table 6. It will be noted that whenever there was a silt flow through the reservoir, C was greater than 200×10^{-6}, and, in each case, when there was a peak flow at Grand Canyon which was not followed by a silt flow through the reservoir, C was less than 140×10^{-6}. Thus, the critical value of C for the reservoir condition in 1935 was between these values. The critical value of C for the final conditions of a full reservoir, or for some other reservoir, will not necessarily lie between these same limits, but the same form of criterion will hold.

TABLE 6.

Date (1935) (1)	Q, flow at Grand Canyon, in cubic feet per second (2)	n, percentage of silt carried at Grand Canyon (3)	$C \times 10^6$ (4)	Unmixed current through Lake Mead (5)	Date (1935) (1)	Q, flow at Grand Canyon, in cubic feet per second (2)	n, percentage of silt carried at Grand Canyon (3)	$C \times 10^6$ (4)	Unmixed current through Lake Mead (5)
March 18	7 440	1.72	230	Yes	June 16	102 000	1.09	10.7	No
April 11	15 800	3.16	200	Yes	August 6	13 000	1.82	140	No
May 18	25 100	1.52	60.6	No	August 26	13 000	4.81	370	Yes
May 24	31 600	1.48	46.9	No	September 4	13 000	5.02	386	Yes
June 2	50 200	1.90	37.8	No	October 1	17 700	5.09	288	Yes

An analysis of recorded silt flows through Elephant Butte Reservoir, New Mexico, for 1919, 1923, 1927, 1929, 1931, 1933, and 1935 shows that the same form of criterion applied to that reservoir for each of these years except 1929. The critical value of C found for this reservoir is approximately twice that found for Lake Mead. The cause of this discrepancy is being made the subject of further study, but undoubtedly this difference is due in part to the different sizes and settling rates of the silt particles entering the two reservoirs. A river that carries a large percentage of fines in its silt load would not drop as much of its load on entering a reservoir as one carrying a smaller percentage of fines and, hence, would maintain a density great enough to cause a silt flow through the reservoir with a comparatively small total silt load.

The criterion as used is dimensional and is not unique, and, therefore, is not a true criterion in the strictest sense; nevertheless, it is felt that the significance shown by the ratio, $\frac{n}{Q}$, in the cases tested is not entirely fortuitous.

As the bottom gates of Boulder Dam are now closed and, in all probability, the quantities of water discharged will never be great enough to raise the silty water to the outlet towers in easily detectable quantities, it would be necessary to detect a silt flow through the reservoir in some other manner, and there is no reason to believe that such flows will not occur in the future, the muddy water forming a submerged pool near the dam. This might be done by lowering a photo-electric turbidimeter unit into the reservoir or (as most silt flows through a reservoir occur in the summer, when the river water is warmer than the bottom reservoir water) by lowering a resistance thermometer. If it was desired to study the application of Prandtl's theory of flow as quoted, or some of the similar theories, it would be necessary to make velocity and density determinations over cross-sections of the current and the immediately surrounding reservoir. Since, to the best of the writer's knowledge, no device has been developed for indicating on the surface the density of water at any considerable depth, it is necessary to resort to sampling for density determinations, which would allow analyses to be made to determine what caused the density difference and from what tributary or part of the system the contributing causes arose.

The whole forms an interesting problem. Any possibility of increasing the life of a reservoir by training silt flows through it must certainly wait on a fuller understanding of the phenomenon, an understanding to be obtained only by careful observation and study of such phenomena whenever and wherever they occur. Theories and laboratory studies must be substantiated by field data before full reliance can be placed on them.

The authors mention the flow of unmixed or of partly mixed river water through Lake Mead. This is entirely in accordance with theory. As may be seen from Equation (2) the vertical distance, y, over which density and velocity gradate, may have any value from nearly zero to infinity. Conditions might exist in which a stable current would be accelerated, causing mixing until the relations of $\frac{d\rho}{dy}$ and $\frac{du}{dy}$ were such that stability was again reached. After such acceleration and mixing, stable conditions might not be reached a second time, and one would have the case of a silt flow through part of the reservoir followed by mixing and eventual deposition at an intermediate point between the river and the dam.

The author's estimate of the silt carried through Lake Mead by the recorded flows is conservative. Experiments made at the National Bureau of Standards[28] show that clear water will scour a sand bed faster than water carrying fine silt, due to the cementing action on the bed material of the fine particles carried in the turbid water. The method used by the authors of subtracting the load scoured from the river bed below the dam from the total load carried by the silty water gives results which are low by an appreciable amount.

D. M. Forester,[29] M. Am. Soc. C. E. (by letter).—The passage of turbid waters through reservoirs is a phenomenon which has seldom been considered and, in fact, generally not known. Quite often it is assumed that the discharge of turbid water or silt through the discharge gates has been the result of current changes and slides within the reservoir. The flow of turbid water with its apparent resistance to mixing is a common occurrence in settling or clarification basins of water treatment plants. During the early stages of the filling of Lake Mead, when light conditions were satisfactory, the writer could distinctly see the flow of turbid water along the old river channel when observed from an altitude of 300 or 400 ft above the stream bed.

The records of the water treatment plant at Boulder City, Nev., with its intake approximately 1 000 ft down stream from the discharge portals, very closely confirm the observation at the Willow Beach gaging station. These records do not, however, indicate a noticeable variation in the total hardness content of the water during the periods of high turbidity. Analysis of Colorado River water since February, 1935, have recently become available for public inspection in the offices of the U. S. Geological Survey.[30] These analyses show wide variation between the inflow to Lake Mead and the outflow from Boulder Dam. During the first year of storage in Lake Mead the range of the total dissolved solids at the Willow Beach gaging station was 57% of that at Grand Canyon. During the second year the range was 39 per cent. The first-year range at the Grand Canyon gaging station was from 242 ppm to 1 332 ppm and, at Willow Beach, from 276 ppm to 897 ppm. During the second year, the range at Grand Canyon was from 258 ppm to 1 215 ppm and at Willow Beach from 393 ppm to 762 ppm. The gate in the tunnel was closed on May 1, 1936, three months after the beginning of the second year.

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[28] "Experimental Study of the Scour of a Sandy River Bed by Clear and Muddy Water", by Chilton A. Wright, M. Am. Soc. C. E., *Research Paper No. 907, Journal of Research*, National Bureau of Standards, Vol. 17, No. 2, August, 1936.

[29] With U. S. Bureau of Reclamation, Yuma, Ariz.

The dissolved solids and the suspended solids entering and flowing from the lake appear to vary in somewhat similar manner and both indicate that the mixing action between the inflowing water and the water within the reservoir is not pronounced over a short time period.

A statement by O. A. Faris, M. Am. Soc. C. E.,[31] that the flow of silt along the reservoir bottom is due to its having a greater specific gravity than the main body of water in the reservoir, would appear to be conclusive. However, the discharge of turbid water does not occur at all times. At Lake Mead, from March 1 to October 31, 1935, the inflow was never less than 0.11% by weight (1 100 ppm) of suspended solids, the maximum being 5.09% (50 900 ppm) and averaging 0.91% (9 100 ppm). The outflow suspended solids content varied from 2.16% to 0.002%, averaging 0.17%, indicating a deposition of 0.74 per cent. At periods of maximum inflow of suspended solids and corresponding maximum outflow, the indicated deposition was much higher.

From present available information it would appear that the flow of turbid water through a long reservoir is dependent upon the quantity of suspended solids and their particle size. The degree of concentration would directly affect the specific gravity of turbid water. The flow would also be affected, to a lesser degree, by temperature conditions and perhaps by dissolved salts.

Comparison of deposition of the various particle sizes was not given by the authors; however, it is reasonable to assume that deposition of suspended solids would be in proportion to their size.

Any steps which may ultimately lead to the maintenance and conservation of original reservoir capacity cannot be otherwise than valuable. It is the writer's opinion that other economic values, especially in irrigation and municipal supplies, usually would outweigh the advantages secured by increasing the silt discharge through the normal, or low-level outlets.

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[30] *Western Construction News*, Vol. XII, No. 6, June, 1937, p. 220.

[31] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 251.

A. D. LEWIS,[32] ESQ. (by letter).—Several features treated in this paper have been described by the writer in connection with similar occurrences in South Africa.[33] It should be noted that for several years past the standard practice at Lake Arthur Reservoir has been to remove a portion of the incoming silt through the valves during the flood by a process which is termed "bottom-flow tapping." There cannot be the least doubt that in certain circumstances there is a flow of flocculated silt along the bottom of the reservoir under the comparatively clean water. Observations as to the conditions of the actual flow along the bottom at different points along its course are exceedingly difficult, especially in the case of reservoirs in South Africa which are fed by very flashy floods in intermittent rivers. As far as local observations have gone, it appears that in small floods the depth of the flow of concentrated silt along the bottom is seldom greater than 3 ft within 0.75 mile from the dam walls, and its velocity is probably less than 0.5 ft per sec. The analysis of the silt that reached and passed through the valves given in *Communication No. 5*[33] showed that most of it was flocculated clay. After deflocculation, 78% consisted of particles of smaller diameter than 5 microns, and 83% of these particles were of smaller diameter than 2 microns.

The first conclusion one is inclined to draw is that bottom flow is essentially associated with flocculated clay. The experiments and observations in *Communication No. 5,* suggest some reasons why bottom flow should be possible with flocculated clay in certain states of concentration. When the flocculated clay occurs in concentrations greater than a lower limit which lies between 1.5% and 5% and less than an upper limit which is about 25%, it appears to form with the water enclosed a separate mobile liquid with a defined boundary. It is possible that in concentrations greater than the lower limit the flocculated particles are so close together that they are capable of some cohesion, which increases as the concentration increases and makes the escape of the enclosed water more difficult and the settlement of the particles in still water slower. While the flocculated mass is between these approximate limits of concentration it has a mobility which permits its flow as a separate liquid with a sharp boundary surface. The conditions of flow would naturally be governed by such factors as the concentration and temperature of the liquid and the slope and shape of the bottom of the reservoir.

Flow becomes very sluggish and suddenly almost ceases near the upper limit of concentration, which the writer has termed the "critical point," and the concentration at this critical point is generally about 19 lb per cu ft. At this point the process of settlement might be said to give way to a process of compaction. In concentrations less than about 1.5%, the top particles settle at a rate of about 2 ft per hr without impeding each other, and they pile up on the bottom into a mass which is probably not very mobile and, therefore, not capable of producing bottom flow. In concentrations between 1.5% and 5%, a sharp surface forms at a position which is farther from the bottom as the concentration increases. In concentrations greater than 5%, the sharp surface forms at the top and no independent settlement of particles is possible even at the top; the silt settles as a whole with a sharp upper surface. If the liquid entering the reservoir, and having a concentration of flocculated silt greater than the lower limit, can glide smoothly under the clear water already in the reservoir without dispersion or mixture with the clean water (which would reduce the concentration to less than 1.5%), then it is possible to picture the mobile liquid flowing along the bottom of the reservoir with just sufficient turbulence to prevent settlement, and a concentration beyond the "critical point" of about 25%, until it is checked at the dam, if no great discharge is taking place through the outlets of the dam. There, in the stiller water, settlement will occur, and the time of settlement to the "critical point" is represented by the formula:

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[32] Director of Irrig., Union of South Africa, Pretoria, Union of South Africa.

[33] *Communication No. 5* to the Second Congress on Large Dams, Washington, D. C., 1936, entitled "Silting of Four Large Reservoirs in South Africa."

$$t = \frac{c\,L}{15} + 1.2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which t is the time of settlement, in hours, to the "critical point"; c is the initial concentration, expressed as pounds of dry silt in 1 cu ft of silty water; and L is the original length of the silty column, in inches.

If the silty concentration is not drawn off within this time, the mass will lose its mobility and will not flow through the outlets. A certain degree of concentration by settlement at the dam wall is necessary; otherwise, an unnecessary quantity of water will be wasted in getting rid of the the silt. In the actual practice of bottom-flow tapping at Lake Arthur Reservoir (although, actually, a maximum of 25% concentration has been obtained in the liquid extracted), in endeavoring to secure the maximum extraction of silt with the minimum wastage of water by regulating the opening of the bottom valves, it has seldom been possible to achieve a greater concentration than 15% over a long period. If, therefore, the water entered the reservoir with a concentration of 5% of flocculated clay, it would be necessary to waste about one-third of it in order to get rid of the silt that reaches the wall by means of bottom-flow tapping; but it should be remembered that, even if this silt had remained in the dam, it would have entrapped a considerable volume of water which could never be extracted as clear water through the valves.

With the foregoing picture of the process of bottom-flow tapping in mind, it is interesting to examine Table 1 and Fig. 2 of the paper. In no case was there any bottom-flow discharge when the percentage of suspended matter entering the reservoir was less than 1.7 per cent. There was no such discharge during the four months of high flow from May to August; all the discharges were during small floods in the remainder of the year. This might be due to the fact that the saline content of the water entering the reservoir was sufficient to cause flocculation during the latter periods, but not sufficient during the former period, as would appear to be indicated by Fig. 2(a).

If, however, there was flocculation during the former period, the concentration was usually less than 1.5% and, therefore, too low to permit bottom flow. According to Table 1, if the last six days of August, which had their effect in September, are excluded, on only eight days in the four summer months did the concentration exceed 1.5 per cent. The highest concentration during this period was on June 2, namely, 1.90%, and as the flood at that time was about 50 000 cu ft per sec, it is possible that even if the clay were flocculated, the turbulence on entering the reservoir would have been sufficient to reduce the concentration to less than 1.5 per cent.

It is assumed that throughout 1935 all the discharges occurred through outlets very low in the dam, which would permit extracting the silt below the level at which concentration would occur in still water above the dam. It is understood that, at a later date, all discharges were at a much higher level, and if the foregoing "picture" is correct, it is to be expected that the bottom-flow discharge will not again take place until the reservoir has been filled with settled silt nearly to the level of the outlets that are in use.

In view of the nuisance that such silt may be when the water is drawn off for domestic purposes or for driving turbines, a practical consideration is that it is desirable to have low-level valves that can be opened sufficiently to discharge at a concentration of about 15% when bottom flow is taking place, and, secondly, that when clear water is required from the reservoir for domestic or other purposes reliance should not be placed solely on valves at low levels which might tap the bottom flow, but a series of valves should be provided at such levels that clear water can be tapped at higher levels during periods of bottom flow.

G. C. Dobson,[34] M. Am. Soc. C. E. (by letter).—After reading this interesting paper, it occurred to the writer that an examination of the many reservoir silting surveys made by the Section of Sedimentation Studies, Division of Research, United States Soil Conservation Service, should throw considerable light on the subject of the passage of silt-laden water through reservoirs. It would seem that, if this silt-laden water passed through the reservoir as an underflow or density current along the bottom, the distribution of silt within the reservoir would be entirely different than if it was diffused generally with the water already stored in the reservoir. If this phenomenon of underflow appeared only rarely, under certain special conditions, but occurred where these special conditions involved very heavy silt loads, the arrangement of silt in the bottom of the reservoir should indicate this condition also.

Fifty-five detailed reservoir surveys, distributed fairly well throughout the United States, have been made in the three years, 1935, 1936, and 1937. The objectives of these surveys did not include the study of underflows or density currents so that any evidence found on this subject is incidental. A thorough analysis of this mass of data and a gathering of exhibits to prove certain theories was not the object of this examination, the intention being rather to review these data and, in a general way, to reveal what they indicate as to the existence of this phenomenon not only in the Southwest, but in other sections of the United States. Several of these reservoirs are too small to be of any benefit in this study. Some others are of such pronounced "channel" type that they are of little value in studying this distribution of silt. However, there are a sufficient number left to give a fair picture of the conditions throughout the country.

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[34] Asst. Head, Sedimentation Studies, Div. of Research, Soil Conservation Service, U. S. Dept. of Agriculture, Washington, D. C.

Sufficient time has not elapsed since the impounding of Lake Mead to warrant a silting survey, although a complete original-capacity, contour survey has been made for this purpose by the Soil Conservation Service. A detailed survey of Elephant Butte Reservoir was made in 1935 in cooperation with the U. S. Bureau of Reclamation. This reservoir has been noted by a number of writers as the outstanding example of underflow currents in the United States.

In studying the profile of Elephant Butte Reservoir, showing the maximum depth of silt above the valley floor, it is noted that because of the great fluctuations in the elevation of water surface and the consequent flooding and drying of the area above the narrows, little can be learned from this section of the profile. Thus about 28 miles of the upper part of the profile is eliminated, leaving about 15 miles from the lower end of the narrows, which are about five miles long, to the dam where the distribution of silt should show the effects of density currents if they exist. At the lower end of the narrows the maximum depth of silt on the valley floor is 30 ft. This depth decreases to 12 ft at a point 7 miles down stream where it begins to increase somewhat irregularly until it reaches a depth of 20 ft just above the dam.

The most noticeable point, however, in the arrangement of silt in this reservoir is shown on the cross-sections taken between the dam and the narrows. Regardless of the depth of the silt, or the width or shape of the cross-section, the upper surface of the silt is practically level.

Pausing for a moment to consider what one would expect to find in the distribution of silt, it would seem that, if the silt-laden water flowing into a reservoir entered into general diffusion with the water already stored, a section across the reservoir should show a distribution of velocity which, at least, would be somewhat similar to that within a section taken across the river above its entrance to the reservoir. The area of the reservoir section would be so large compared to the area of the river section that the resulting average velocity would be very low, and it would be difficult even to surmise whether the difference between maximum and average velocities would be more or less pronounced than within the river section. Furthermore, it would seem logical to suppose that the part of the reservoir section formed by the submerged river channel would affect the distribution of these velocities, but that this effect would become less and less as the reservoir sections became larger. In the case of a meandering stream channel in a submerged alluvial bottom, where the channel formed only a small percentage of the total reservoir cross-section, the influence of the channel would be lost, and the slow currents moving down the reservoir would tend to take the shortest route, disregarding the meanderings of the channel. If this is true, the depth of silt within a cross-section would be affected considerably by the depth of water although not necessarily directly proportionally to it; probably it would be more nearly proportional to some power of the depth, slightly greater than one. There would be a tendency for the low places, particularly the channel, to fill faster so that the upper surface of the silt would tend to approach a level line in cross-section.

If the silt-laden water were to pass through the reservoir as an underflow, it would seem very probable, because of the greater density, that at any section within the reservoirs this underflow would fill the low places, and its upper edge, if it were clearly defined, would be level from bank to bank. If, as supposed, these underflows occur only at flood time, they would be of sufficient volume to overflow the submerged channel, but the cross-sectional area of the underflow would form only a small part of the area of the total cross-section. The influence of the submerged channel on this flow would be much more marked than on a diffused flow so that, it seems probable, the maximum velocity of this underflow would follow the submerged stream channel even in its meanderings in a submerged alluvial valley. Deposition of sediment from an underflow would undoubtedly occur more or less continuously in its flow down the reservoir and, on a cross-section, the depth of this deposition should bear about the same relation to the depth of the underflow that it would to the total depth of the reservoir in diffused flow. Here, again, there would be a tendency to fill the low places faster, causing the upper surface of the silt to approach a level line. The point to be noted, however, is that, in this latter case, the tendency would be much stronger than in the former and it would seem probable that in a comparatively short time underflow conditions would produce a level floor of silt in the reservoir, while in the case of a diffused flow, the tendency would be so slow that it might not reach this condition until the reservoir was practically full of silt.

Because of the great difference in the effective velocities in these two types of flow one would expect to find a marked difference in the longitudinal distribution of the silt. The low velocities of diffused flow should produce heavy deposits in the upper part of the reservoir gradually decreasing in depth, possibly extending all the way to the dam. The pattern would depend, of course, on many factors, such as the length of travel and general shape of the reservoir. The higher velocities of density flow would be expected to carry the silt much farther into the reservoir on an average and probably all the way to the dam, it might even form the deepest deposits immediately above the dam.

Since the level floor of silt was the most striking feature found in the survey of Elephant Butte Reservoir, this condition was looked for in other reservoirs. In only two other cases was it found to a pronounced degree. These examples were the San Carlos Reservoir, on Gila River, in Arizona, and the Gibraltar Reservoir, near Santa Barbara, Calif. Another striking feature was noticed in the profile of the Gibraltar Reservoir. This reservoir is only about four miles long. The upper half of the profile is very irregular for reasons somewhat similar to those described for Elephant Butte, but in the lower 7 000 ft, the depth of silt increases down stream from 27 ft to 85 ft at the dam. The gradient of the channel in this reservoir is very steep which probably accounts for this rapid increase in silt toward the dam. The lower 7 000 ft of this reservoir seems to be rapidly approaching a level floor of silt longitudinally as well as transversely. The down-stream slope of the silt surface is only about 0.00076 whereas the effective slope of the original channel is about 0.0062.

Aside from these three reservoirs, the surveys show no marked cases of level silt surfaces. At the other extreme is Black Canyon Reservoir, on Payette River, near Emmett, Idaho, where the cross-sections show the deepest deposits of silt at the points of greatest depth, and where these depths of silt are only slightly greater than proportional to the total depth of water. This condition is shown generally throughout the reservoir. In profile, heavy deposits are found in the upper parts of the reservoir as would naturally be expected of a stream carrying sand bed load. These delta deposits extend a considerable distance down into the reservoir. From the lower end of the delta to the dam, there is a gradual lessening in the average depth of the silt.

A condition prevailing in many reservoirs, particularly in those of the Piedmont Region in the Southeast, is a distribution of silt indicating in general a diffusion of the silt-laden water with the water already stored in the reservoir, but showing evidence also that at times there have been underflows. It must be admitted that this evidence is not very pronounced and might be due to other causes. In many of these reservoirs, without regard to length, or the varying shapes of the cross-sections, the profiles, when taken along the thalweg or lowest points of the submerged channel, show a surprisingly constant depth of silt. This is generally true regardless of the meanders of the channel in a submerged alluvial plain. On the cross-sections the depth of silt within the submerged channels seems to be considerably greater than one would expect as a result of settlement from diffused flow. Outside the channel, the distribution of silt seems to be nearly proportional to the depth of water in the reservoir, but there are some places where deposition on the banks of the submerged channel is greater than at other points on the valley floor, indicating that there is some tendency of the silt to deposit in formations similar to natural levees characteristic of the banks of alluvial streams. In a few cases, this heavier deposition was found on what appeared to be a natural levee in the original bottom of the reservoir. One explanation of this might be that for some unknown reason the velocity of the underflow at these points was greatly decreased, resulting in an enlargement of its cross-section and the overflowing of the submerged channel with a still greater reduction in the lateral velocities causing rapid deposition on the near banks. However, as these apparent natural levees of silt are not generally prevalent and are not very pronounced, their value as evidence is questionable.

Another very noticeable feature in these reservoirs is that a negative gradient in the profile of the original channel, as when emerging from a deep pool, is parallel to the silt surface, which shows as great a depth at the high point as at the low. If the distribution of silt in a reservoir was greatly affected by mass flow of the silt itself, it would be expected that the pools in the original channel would be filled and the high points would show less silt. Further evidence bearing on this question of mass flow of the silt is to be found on the cross-sections of the great majority of these reservoirs where almost uniform layers of silt of considerable depth exist on slopes many times steeper than the gradient of the river or the floor of the valley.

The phenomenon discussed in this paper is too little understood by the Engineering Profession. The authors have rendered a worthy service in presenting the subject in such a thought-provoking manner.

William W. Rubey,[35] Esq. (by letter).—The flow of muddy or saline or cold water through lakes is of interest to all who are concerned with the natural processes of streams. The formation, in 1937, of a committee of the National Research Council for the study of density currents is evidence of the wide interest in this subject among engineers, physicists, and geologists. To geologists, bottom currents through bodies of standing water are important because, if such currents are common to-day, they must have influenced materially the character and distribution of sediment deposited on (or swept from) the beds of lakes, estuaries, and seas in the past. The best means of appraising the importance of these currents in the past is a knowledge of the conditions that accompany their formation to-day. This paper with its data on muddy flows through Lake Mead is thus a very welcome contribution.

From what is known about the conditions that determine whether fluids of different density will mix turbulently or flow separately, it appears that, for simple homogeneous fluids, the controlling factors are the differences in density and velocity of the two fluids,[36] and that, for complex gradational fluids, the controls are the density gradients and velocity gradients in transitional layers.[37]

The available data for Lake Mead reveal nothing whatever about the thickness of a possible transitional layer or layers between lake water and muddy current. Studies in progress by the U. S. Bureau of Reclamation at Lake Mead, and by others elsewhere, should eventually furnish the required information on this point; but until this information has been assembled, it seems permissible to consider the lake water and the muddy current as two essentially homogeneous fluids, only one of which has an appreciable velocity. Thus, when the density difference between them is great enough and the current velocity low enough, the two fluids are separated by a surface on which waves move down stream. Increasing the current velocity or decreasing the difference in density breaks up these waves and causes mixing.

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[35] Geologist, U. S. Geological Survey, Washington, D. C.

[36] "Handbuch der Ozeanographie," by O. Krümmel, Vol. 2, p. 62, 1911.

[37] "On the Stability of Superposed Streams of Fluids of Different Densities," by S. Goldstein, *Proceedings*, Royal Soc., London, Vol. 132, Ser. A, pp. 524–525, 1931.

By this concept, the boundary between lake water and muddy current is determined by the same principles that govern wind waves at a water surface[38] and current ripples on a sandy stream bed.[39] At high winds the tops of the waves break into fine spray; at high current velocities the sand ripples are swept away and the sand is picked up and carried along, mixed with the stream water. Just as the differences of density between muddy currents and lake water are much less than the differences of density between water and air or between sand and water, so the critical velocities that cause instability and mixing of the muddy currents should be much less than the critical velocities that make wind waves or sand ripples unstable.

The available data permit no close estimate of velocities of the muddy currents that moved through Lake Mead in 1935, but published and unpublished records of the U. S. Geological Survey and the Bureau of Reclamation afford a basis for computing approximate densities of the inflowing and outflowing waters. Inasmuch as water densities were almost certainly one of the fundamental factors that controlled the existence of muddy currents that year, these data appear worth recording in connection with this paper.

The principal variations in density or specific gravity of the river water were those caused by variations in the content of suspended matter. Values of density at different concentrations of suspended matter, calculated on the basis of 0.996 gram per cu cm for pure water (at 84.2° F) and 2.6 grams per cu cm for suspended solids,[40] agree fairly well with the authors' observed values (see Table 2), giving a maximum disagreement of 0.0022 gram per cu cm, and an average of about 0.0010 gram per cu cm. (These disagreements are probably due, in part at least, to differences in salinity.) Assuming these densities for water and solids, 1% of suspended matter should make a density increase of 0.0062 gram per cu cm, 3%, an increase of 0.0188 gram per cu cm, and 5%, an increase of 0.0217 gram per cu cm.

Two other relatively minor causes of variation in density of the river water need to be considered—salinity and temperature. The effects of salinity were estimated (see heading in paper, "Dissolved Matter") on the assumptions that daily sulfate determinations are approximately proportional to total dissolved matter and that total salinity affects density about the same as if it were all sodium sulfate (0.0011 gram per cu cm for

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[38] "The Physics of Solids and Fluids," Section by L. Prandtl, pp. 225, 266, London, 1930; also, "Physics of the Earth, V Oceanography," by N. H. Heck, M. Am. Soc. C. E., and others, *Bulletin,* National Research Council, pp. 207, 212, 1932.

[39] "On Ripples and Related Sedimentary Surface Forms," by W. H. Bucher, *American Journal of Science*, Vol. 57, 4th Ser., pp. 165, 178–179, 199–207, 1919; also "Treatise on Sedimentation," by W. H. Twenhofel and others, pp. 458–460, 1926.

[40] "Denudation," by R. B. Dole and Herman Stabler, M. Am. Soc. C. E., *Water Supply Paper 234,* U. S. Geological Survey, p. 80, 1909.

1 000 ppm). The resulting estimates of density are believed to be in error by not more than 0.0006 and commonly about 0.0003 gram per cu cm.

Observations on temperature of the river water at Grand Canyon Gaging Station began April 20, 1936, and unpublished records are available from that date through July 31, 1937. For the period, April 20 to July 31, for which there is a 2-yr record, the 1936 and 1937 records show closely comparable water temperatures day by day, and it seems fair, therefore, to assume that water temperatures changed in about the same manner in 1935. The detailed records show fair agreement with uniform changes from 43° to 75° between March 1 and July 1, from 75° to 81° between July 1 and August 1, from 81° to 75° between August 1 and September 1, and from 75° to 52° between September 1 and October 31. Densities estimated from uniform rates of temperature change between these respective dates show a maximum of 0.0006 and a mean departure of about 0.0002 gram per cu cm, from densities estimated from the detailed temperature records. For the Willow Beach Gaging Station below Lake Mead, the water temperatures used were those of the water discharged from the lake during the same period.[41]

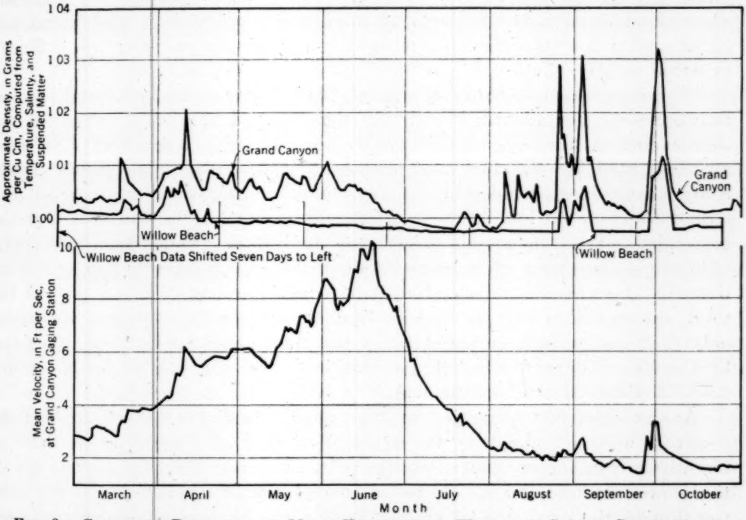

FIG. 6.—COMPUTED DENSITY AND MEAN VELOCITY OF WATER AT GAGING STATIONS ON COLORADO RIVER ABOVE AND BELOW LAKE MEAD, MARCH 1, TO OCTOBER 31, 1935

By combining the effects of the three variables (suspended matter, salinity, and temperature), the daily densities of the water at the two gaging stations above and below Lake Mead were estimated for the period, March 1 to October 31, 1935. These calculated values of density are believed to be in error by not more than about 0.002 and commonly about 0.001 gram per cu cm.

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[41] See Fig. 3, p. 737.

Fig. 6 shows clearly that the density of the river water at the upstream (Grand Canyon) station was not the sole factor that caused muddy currents through Lake Mead. Likewise, if the water discharged from the lake between periods of muddy water is taken as representative of the lake water, the difference in density between river water and lake water was not the sole factor either. It is true that densities greater than 1.011 grams per cu cm at Grand Canyon were always accompanied by muddy discharges through the reservoir. However, from the duration of individual muddy flows, it is clear that, at times, densities of 1.005 to 1.008 grams per cu cm (and density differences of 0.004 to 0.010) were sufficient, whereas on June 2, for example, a calculated density of 1.0104 grams per cu cm and a density difference of 0.012 grams per cu cm were not sufficient to cause muddy flows.

Of course, these densities of the rapidly flowing water at Grand Canyon Station were not identical with the densities of the same water in the slower current through the reservoir down stream. Important differences are suggested by the fact that the muddy discharges at Willow Beach Station contained considerably less suspended matter than was carried past Grand Canyon Station. This suggestion is confirmed by the fact that the suspended matter in muddy discharges at Willow Beach consisted chiefly of particles less than 20 microns in diameter (see heading, " Sizes of Particles "), whereas this finer sediment at times was only a small part of the total suspended matter at Grand Canyon Station up stream. Evidently, only the finer particles brought in by the river could be kept in suspension by the slower current through the reservoir, and the coarser sediment was dropped somewhere along the way.

Accordingly, the densities of the muddy water after it reached the reservoir were estimated on the assumption that all suspended particles at Grand Canyon larger than 20 microns in diameter (Fig. 2) settled out in the slower current below. These estimated densities show no closer relationship to the existence of muddy flows than do densities calculated from total suspended matter at Grand Canyon. Density differences of less than 0.002 gram per cu cm (estimated from the finer material only) resulted in muddy flows in March and April, but differences of 0.003 to 0.004 gram per cu cm did not cause muddy flows in June and August.

The authors carefully show that the sizes of suspended matter at Grand Canyon were determined in the laboratory from carefully dispersed samples, whereas in the natural river water the finer sediment was partly flocculated into larger masses which settle more rapidly. Inasmuch as the salinity of the river water changed during this period, the degree of flocculation presumably changed also. Thus, the finer material brought into the reservoir in June and August may have been flocculated into larger masses that were dropped by the slower current, with the coarser sediment. If so, the density of the muddy water, after it reached the reservoir, cannot be estimated from the quantity and size of sediment at Grand Canyon without data on variations in the degree of flocculation also.

Whatever may have been the effect of variations in the degree of flocculation, variations in the velocity of the muddy currents themselves were presumably a contributing factor in the formation of currents through the reservoir, for the velocity of these currents would be expected to vary somewhat depending on the density and stage of the incoming water. However, reliable estimates of rate of flow through the reservoir are difficult to obtain from existing data. The earlier muddy flows of 1935 required about 5 to 7 days, and the later flows about 7 to 9 days, to travel from Grand Canyon to Willow Beach. During this time the length of the lake increased many miles, and variations in velocity through it cannot be estimated with the degree of accuracy required to test their effect.

Lacking reliable data on the velocities of currents through the lake, it seemed that the velocities of incoming water up stream might possibly be a factor in the formation of the muddy currents. Accordingly, the mean velocities at Grand Canyon Station were read from daily gage-height records. These mean velocities (see Fig. 6) are probably fairly accurate because, during the period, February 26 to November 3, 1935, the cross-section and mean velocity of the stream at Grand Canyon Station were measured on 117 days (from unpublished records of the U. S. Geological Survey).

On comparing water densities at the two stations with mean velocities at Grand Canyon Station (Fig. 6), it is evident that low or intermediate velocities above the lake accompanied flows of muddy water through it and that, in general, high velocities accompanied the absence of such flows during May and June. It is equally evident, however, that the mean velocities and densities at Grand Canyon Station afford no satisfactory criterion for the existence of the muddy currents. The period of low velocities and high densities, August 6 to 18, was not followed by muddy discharge through the reservoir, although velocities were then lower than, and densities were as great as, those that caused muddy flows in March and April.

The evidence presented herein is largely negative. The available data show no well-defined relationship between the existence of muddy currents through Lake Mead, on the one hand, and the density, density difference, or density and velocity at Grand Canyon, on the other. It seems clear that, in future investigations of the conditions that accompany muddy currents through a lake, direct observations on the degree of flocculation of sediment in the incoming water, and on the velocity or the slope of the muddy currents themselves, will also be desirable.

J. C. STEVENS,[42] M. AM. SOC. C. E. (by letter).—In calling to the attention of the profession one more interesting phenomenon with which the silt problems of the West are replete, the authors of this paper have "started something."

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[42] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

The silt problems in the West are manifold. How and where does silt originate? How is it transported? What space does it occupy in reservoirs? What part do the cultural activities of Mankind play? What steps can be taken to hold it intact in the basins? Can it be prevented from entering reservoirs, or can it be removed therefrom once it has entered?

The phenomenon so interestingly described by the authors is only one of a great number of silt problems that should be "run to earth" and made controllable in some degree by human agencies.

The records of 1935–36 indicate that during moderate and low-water periods of inflow, a slug of fine silt brought into the reservoir by a tributary may remain intact and pass entirely through the reservoir. At Lake Mead these slugs appeared to originate in the basin of the Little Colorado River. There is no reason to doubt that the same part could be played by the San Juan River or by the Virgin River if a summer cloudburst could produce sufficient run-off with an attendant quantity of fine silt.

The spring floods on both the Colorado River and Rio Grande originate from melting snows in the upper reaches of the water-shed. The sediments at such times are predominately coarser than those on the lower tributaries. In June, 1935, the average daily load passing Grand Canyon was about 1 800 000 tons per day of which only about 400 000 tons, or 22%, was less than 20 microns in diameter. The flood of September 29 to October 5, however, brought in a maximum of about 2 000 000 tons per day, of which 1 800 000, or 90%, was less than 20 microns in diameter. The latter produced noticeable silt discharge whereas the former did not.

It is hoped that more data will be published regarding the silt flow through Elephant Butte Reservoir. The papers by Messrs. Lawson[5] and Fiock,[6] referred to by the authors, do not give details. The writer is of the opinion that if the silt flows of the past were compared with up-stream summer floods it would be found that most of these silt flows follow floods on Rio Puerco and Rio Salado, and, as in the case of the Colorado River, they will not coincide with the spring floods which carry the coarser material.

There appears to be no doubt that the phenomenon of silt discharge results from an intimate mixture of water and fines of such coherency that the specific gravity of the entire mobile prism is increased until it flows along the lowest channel of the reservoir, quite independent of the temperature differences between the reservoir and flowing water. The slowness with which fluids of different densities mix has been well demonstrated by some experiments on both liquids and gases made at the University of California. Diffusion among mixtures of varying densities was found to be surprisingly slow.

It appears that the stream of water-silt only issued from the reservoir through the tunnels with entrances at stream level. Since the tunnels were closed, no silt has been discharged, indicating that, at the dam, this mixture has been unable to rise the 250 ft necessary to flow through the penstocks or the outlet works.

Although the total quantity discharged was only 2.5% of that coming into Lake Mead in 1935, the fact that any at all was discharged is too important not to be investigated further. These fines probably would not occupy nearly so much space in the reservoir as an equal weight of coarser material; nevertheless, the question arises as to whether it might not have been worth while to have left gates in the tunnels that could be opened at such times, instead of having them plugged permanently. On other reservoirs, the proportion of silt that could thus be discharged might have greater significance and would raise the question as to the wisdom of maintaining stream-level outlets.

Another phase of this problem concerns the quality of the water passing through a reservoir in this manner. Such a concentration of fines may render the water unfit for irrigation on certain soils as it makes them "slick" and effectively seals them against air and water drainage. Prolonged use of such concentrates or intermittent use over long periods of time is quite certain to be detrimental to almost any type of soil; hence, from this standpoint, it might be better not to permit such silt slugs to pass through a reservoir.

Each basin must be studied on its own merits in these regards, in which the geological, chemical, mechanical, and physical factors must be analyzed carefully. All these problems of silt are of tremendous importance to the West and it is "high time" that an orderly, well-planned approach to their solution was begun without further delay.

In its report on "Drainage Basin Problems and Program, 1936," the National Resources Committee included as one of its principal study projects a thorough investigation of the silt problem in the Colorado River Basin, and recommended an early allotment of $1 000 000 with the understanding that several million dollars would ultimately be required for an extended period of research.

In order to secure co-ordinated results, it would seem essential that such an investigation be headed by an agency or board composed of the chiefs of the several Federal bureaus now engaged, independently, in investigating some phase of the silt problem, as well as a number of non-Federal engineers and scientists, to the end that the elements of engineering, hydrology, chemistry, plant biology, geology, and soil mechanics will be adequately represented and that co-ordinated conclusions will be secured.

The answers to some angles of this problem are being sought independently by certain Federal agencies, among them the Soil Conservation Service, the Forest Service, the U. S. Geological Survey, the Bureau of Reclamation, the Bureau of Agricultural Engineering, the Bureau of Plant Industry, and the Grazing Division. There is little or no co-ordination of effort among these agencies, however. A co-ordinating board would concern itself solely with the research phase of the problem and would enlist the services of all agencies concerned, including the several hydraulic laboratories. To the latter could be assigned special problems, while the field research could be conducted by Federal agencies. Such an investigation of the silt problems in the Colorado River Basin would be equally valuable for the Rio Grande Basin, and all other silt-producing basins in the Arid West.

C. S. JARVIS,[43] M. AM. SOC. C. E. (by letter).—Experience with operation and maintenance of numerous small reservoirs had led a number of engineers, including the writer, as early as 1914, to expect the passage of heavily silt-laden water as a separate stream through the clear water of large reservoirs. Regarding the small reservoirs as hydraulic models, they concluded that large reservoirs should respond to the treatment so commonly used for maintaining the capacity of small storage projects in arid regions, where both the erosion rate and the water values are relatively high. As early in his career as 1897, the late Arthur P. Davis, Past-President, Am. Soc. C. E., had proposed the use of such expedients as hydraulic giants and sluicing for the removal of such silt.[44]

Serious consideration was being given in 1914 to the plans for installing a flushing conduit to extend several miles up stream from the dam to the narrowest section of the storage basin, with a view to trapping the sediment and conveying it through the deepest part of the reservoir to the outlet. The prevailing opinion among the consultants was that the heavier, silt-laden inflow would follow the valley trough and would convey a considerable portion of the suspended load to the river channel and irrigation canals, thereby prolonging the life of the storage project and adding fertility to the irrigated land.

A corollary to the axioms recognizing the impoverishment of soils through erosion would acknowledge the values in soil fertility to be restored through desilting reservoirs by sluicing and spreading the suspended load upon the land surface.

In the writer's discussion[45] of "The Silt Problem of the Zuni Reservoir," a plan was outlined for maintaining a turbid outflow to keep pace with, and correct, the silting process. At that time (1920), the passage of turbid water through Elephant Butte Reservoir, if observed at all, had not been widely heralded. It is recognized that disadvantages would attend the silt-laden outflow, due particularly to increased wear of turbines and to silting of canals supplied from the reservoir.

Computations in connection with Fig. 2 disclose the fact that the occasions when turbid water was discharged from Lake Mead were a few days after heavily silt-laden inflow had occurred at Grand Canyon, with suspended matter representing 2% to 5% by weight. By inference, one may assume that it requires a pronounced difference in specific gravity of inflowing and impounded water to build up the inertia or momentum sufficiently to maintain the unmixed flow through the reservoir. As described by Mr. Fiock,[6]

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[43] Cons. and Hydr. Engr., SCS, Washington, D. C.

[44] "Irrigation near Phœnix, Arizona," by Arthur P. Davis, *Water Supply Paper No. 2*, 1897, pp. 81–83.

[45] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 883.

"Observations have revealed that the silt-laden water flows along the bottom of the lake in a very thin sheet (less than 5-ft depth), or through the lowest portions of the floor of the Reservoir much as water itself flows under air, and apparently does this without diffusing into the water already in the lake, as it retains certain distinct characteristics which are evident and have been observed as it is discharged from the outlets."

If the 6 000 000 tons of silt discharged from Lake Mead during 1935 represent 2.5% of the average annual load brought into the lake, then the total annual silt inflow is about 240 000 000 tons, or nearly 660 000 tons per day, or 7.6 tons per sec. On the basis of 16 000 000 acre-ft of annual inflow, or nearly 22 600 cu ft per sec, the weight of average inflow, in tons per day, is 61 517 000. Therefore, it appears that the average load of suspended matter represents $\frac{660\,000}{61\,517\,000} = 1.07\%$ of the weight of water.

If it is necessary to maintain 2% or more of silt content in order to insure turbid flow through the lake, it would require some kind of turbulence, such as may be produced by dredging pumps or hydraulic giants, to displace the required 7 tons or more of sediment per sec, during the intermittent periods of operation. Obviously, for such conditions, the dredging would continue not more than one-half the time to keep pace with sedimentation, or proportionately less for partial benefits.

Let it be assumed that suitable types of floating dredging plants have been provided to agitate and to bring into suspension one-fourth of the annual silt inflow, or 60 000 000 tons, representing nearly 2 tons per sec throughout the year, but capable of greater output during actual working hours. Also, assume 1 000 000 acres as susceptible of irrigation from Lake Mead (about 500 000 acres in Imperial Valley, 100 000 acres in the Yuma Project, and 200 000 acres in Mexico, a total of 800 000 acres in 1936), the major portion of 60 tons per acre per yr would be en route to enrich the irrigated land.

If the amortized cost of installation, together with maintenance and operating expense for the necessary floating plant and mechanical equipment, should amount to \$2 000 per day, or \$730 000 per yr, the charge per acre served would be only \$0.73. The actual value of the fertile elements delivered with the turbid water should be several-fold more than the cost, while other benefits realized would be the prolonged usefulness of reservoir capacity, and the checking of accelerated channel erosion below Boulder Dam resulting from the recent clarification of effluent.

R. E. REDDEN,[46] ESQ. (by letter).—The Engineering Profession should indeed be grateful to Messrs. Grover and Howard for the presentation of facts and conclusions relative to the transportation of silt through reservoirs and the worth-while suggestion that a study be made of the little understood phenomena. It is in answer to the authors' invitation for discussion and related information that the writer submits data and conclusions on the passage of a flood through Echo Reservoir in Utah.

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[46] Professional Engr. and Geologist, Coalville, Utah.

On August 6, 1934, a mud flow produced by a heavy local storm which hit at Rockport, Utah, about thirteen miles up the Weber River from Echo Reservoir, entered that body of water at 10:30 P.M., and 10 hr later, after apparently following the river channel, passed through the needle-valve gates below the dam. The flood water passing through the gates was reddish-brown in color, very thick, with suspended matter, and gave off a putrid odor.

From the automatic water-stage recorders on the river, and from data gathered by witnesses (including the writer) average velocities for the flood in the river above the reservoir and through the reservoir have been computed. The average velocity of the flood water above the reservoir was 4.4 ft per sec., whereas the average velocity through the clear water in the reservoir was 0.22 ft per sec.

The greater specific gravity of the flood water, due to the character of the suspended matter, is ascribed for its not mixing with the clear water of the reservoir. The intensely putrid odor of the flood water as it poured into the stilling-basin from the gates, is attributed to the gases locked up in the mass of suspended matter and is additional evidence that the two waters remained unmixed.

Floods are not uncommon in the mountainous valleys of Utah. They are of rather short duration, carry variable loads of débris and, in general, do no great damage; however, a few mud flows have taken human lives and caused considerable property damage. In the main, these mud flows have not been caused by the much over-worked word, "over-grazing." Burned over and over-grazed areas add to the intensity of the mud flow if centered within the local area of maximum precipitation. Field observations of mud flows suggest that they may occur on any steep area during a general or local storm where a localized belt within the general storm area receives a maximum precipitation in the form of hail, which is characteristic of flood-producing storms. Hail, as such, is not readily absorbed by the soil mantle; hence it piles up as temporary immobile precipitation, which is partly melted and lubricated by the "tapering-off" rain, a few minutes later, and rushes down the slopes, dragging with it a portion of the soil mantle to join together below in one mad rush down the local canyons.

Floods entering reservoirs built on streams in Utah have generally deposited their loads on the bottom of the reservoir; however, many instances are known in which the silt-laden waters have appeared at the outlet gates. These records show that this phenomenon is more noticeable on certain drainage areas, which gives rise to the question of the difference in the soil mantle, and which, in turn, is dependent upon the geology of the water-shed area.

It is expected that future studies will show that flood loads from certain formations of a particular geological age have a large influence upon the phenomenon of floods passing through reservoirs of clear water without intermixing.

Weber River and Echo Reservoir.—Weber River heads in the Uinta Mountains in Utah and drains an area of 732 sq miles above Echo Reservoir. It has a maximum discharge of about 4 000 cu ft per sec. The reservoir was built across the channel of Weber River and completed for a storage capacity of

74 000 acre-ft in 1930. On the day of the flood the reservoir held 4 100 acre-ft. The depth of the water at the dam and outlet was 30 ft.

During the spring run-off, the river, at times, carries considerable material in suspension. However, during these periods of high-water run-off, the water discharging from the needle-valve gates during the filling of the reservoir has remained clear, with few exceptions. Only part of the high water entering the reservoir is discharged for irrigation rights below the dam. The remainder is stored in the reservoir. The old river channel meanders over the floor of the reservoir and the length of the submerged channel, when the reservoir is at full stage, is about 4.5 miles. The temperature of the water discharging from the reservoir on July 26, 1934, ten days before the flood, was 60° F, whereas the temperature of the water in the reservoir, 0.5 ft below the surface, was 65° F. The maximum temperature of the atmosphere was 94° F, and the minimum was 44° F. No temperature tests were made on the day of the flood. During the late summer, when the reservoir is being emptied, the discharge water is slightly turbid.

Record of the Flood.—Between 3:30 and 3:50 P.M., on August 6, 1934, a typical thunder storm centered 1.5 miles west of the valley at Rockport. This heavy storm was part of a general storm which covered an area of about 100 sq miles. Floods came down eight short canyons west of the State highway and spread out over the narrow river valley. These mud flows lost their bed load before entering the Weber River, but retained their suspended load down the river and, for the most part, through the reservoir.

The distance from the center of the flood area on the Weber River to the U. S. Geological Survey water-stage recorder at Coalville, Utah, was 12 miles. The recorder showed the first abrupt rise at 8:05 P.M. The record of stream flow for 12 hr, beginning at 8:00 P.M., is as shown in Table 7.

TABLE 7.—FLOOD RUN-OFF, WEBER RIVER AND CHALK CREEK, UTAH

August 6, 1934	DISCHARGE, IN CUBIC FEET PER SECOND		August 7, 1934	DISCHARGE, IN CUBIC FEET PER SECOND	
	Weber River, at Coalville, Utah	Chalk Creek		Weber River, at Coalville, Utah	Chalk Creek
8:00 P.M.	26	....	2:00 A.M.	73	9.0
8:05 P.M.	227	....	4:00 A.M.	63	....
9:00 P.M.	246	1.5	6:00 A.M.	56	....
9:30 P.M.	....	56.0	8:00 A.M.	49	....
10:00 P.M.	201	40.0	9:05 A.M.	33	....
12:00 P.M.	105	14.0	9:30 A.M.	....	3.0

The average velocity of the flood water in the river over the distance of 12 miles was 3 miles per hr, or at the rate of about 4.4 ft per sec. The distance covered by the river from the gaging station to the upper end of the reservoir was nearly 4 miles, making the time of the entry of the flood into the clear water of Echo Reservoir at 10:30 P.M.

A storm of similar intensity struck on Chalk Creek, 8 miles east of Coalville, a short time after the heavy storm at Rockport (see Table 7). Chalk Creek enters the Weber River 0.5 mile below the gage, making very little difference in the time at which both floods entered Echo Reservoir.

The thick muddy water entered the clear water of the reservoir at 10:30 P.M., and passed through the needle-valve gates below the dam at 8:30 A.M. The length of the submerged river channel was about 7 900 ft, which would establish an average velocity of 730 ft per hr, or nearly 0.22 ft per sec for the flood through the reservoir.

The quantity of flood water, as determined from the water-stage recorders, which flowed into the upper end of the reservoir before any of it appeared below the dam, was 102 acre-ft. A body of water 7 900 ft long by 110 ft wide by 5 ft deep would contain approximately 102 acre-ft. The channel of the Weber River before the reservoir was built would average very close to 110 ft wide and 5 ft deep. It seems not far wrong to state that the flood water followed the old river channel through the reservoir.

The color of the flood water above and below the reservoir was reddish-brown. When it was whorled out of the needle-valve gates and into the stilling-basin the violent agitation seemed to liberate a very putrid odor which was stifling near the stilling-basin. It was noted that clay particles, which apparently had remained in suspension through the reservoir, settled out near the lower end of a protected section of the stilling-basin. Evidence was against these particles being in the stilling-basin before the flood water came through. The unlocking of the gas and the breaking up of whatever affinity these clay particles had for each other may shed some light on some of the factors affecting these phenomena.

Conclusions.—When the thunder-storm hit at Rockport, the downpour of hail and rain fell on soil weathered from the thin-bedded clay shales of the Upper Jurassic Age which are reddish-brown in color and contain a high percentage of colloidal clay. This fine material absorbs a large quantity of water and remains in suspension for many hours. The rolling, slipping action which the clay, sand, and coarse material received on its way down from the mountain sides and steep hollows, produced a mud flow having a maximum density of fine particles suspended in an emulsion of colloidal clay and water, which, in turn, supported the bed load of coarser material. Only the suspended load was carried down the river. The flood lost little of its load down the river, as was noted the following day. This dense liquid mass, more or less tied together, seemed to confine itself to the submerged river channel through the reservoir and failed to mix to any extent with the clear, less dense water, notwithstanding the marked decrease in the velocity of the flood water through the reservoir as compared to the velocity in the river above it.

Had the mud flow been composed of different materials, a product of a different geologic age, it would have lost its identity in the reservoir as some other floods have done.

The writer agrees with the authors that further study on the flow of water through reservoirs will be of practical significance with respect to engineering problems. It seems reasonable that some changes in design and position of the outlet works may be needed where culinary water is to be taken from reservoirs to be built on streams which are subject to seasonable floods.

RAYMOND A. HILL,[47] M. AM. SOC. C. E. (by letter).—Judging from similar conditions at Elephant Butte Reservoir on the Rio Grande, little is to be gained by the passage of silt through Lake Mead. From the former reservoir, water carrying large quantities of suspended matter has been discharged for several days in almost every year, but the total quantity of silt so removed is only a small part of the accumulation in the reservoir. Even if the life of Elephant Butte Reservoir thus becomes greater than would have been the case if no silt flows had occurred, it will make little difference whether the lost capacity is replaced, for example, in 1960, or a year or two later. Furthermore, only the finest particles are held in suspension and these tend to seal the soil when such water is used for irrigation. Likewise, to increase the quantity of silt that may naturally pass through Lake Mead would probably not be welcomed by irrigation water users, and definitely would be disadvantageous to those communities that will take their domestic supplies from the Colorado River.

TABLE 8.—QUALITY OF WATER, COLORADO RIVER

Period	Discharge, in acre-feet	TOTAL DISSOLVED SOLIDS		PERCENTAGES		MILLIGRAM EQUIVALENTS PER LITER						U. S. Geological Survey Laboratory No.
		In parts per million	In tons per acre-foot	Na + K	Cl + NO_3	Ca	Mg	Na + K	CO_3 + HCO_3	SO_4	Cl + NO_3	
(a) AT GRAND CANYON STATION												
June 1-10	985 000	335	0.46	30.6	14.0	2.60	1.23	1.69	2.52	2.14	0.75	14 616
11-20	1 680 000	289	0.39	26.2	11.8	2.60	0.90	1.24	2.62	1.62	0.55	14 661
21-30	1 320 000	243	0.33	26.3	12.4	2.20	0.77	1.06	2.18	1.35	0.48	14 662
July 1-10	726 000	281	0.38	30.5	16.9	2.25	0.99	1.42	2.07	1.77	0.76	14 663
11-20	451 000	378	0.51	34.9	18.8	2.75	1.15	2.09	2.34	2.44	1.09	14 695
21-31	338 000	571	0.78	37.2	19.3	3.79	1.73	3.27	2.72	4.43	1.70	14 696
August 1-10	214 000	692	0.94	41.8	23.6	4.14	1.97	4.40	2.90	5.27	2.48	14 697
11-20	179 000	834	1.13	44.6	22.9	5.14	2.22	5.91	3.52	6.56	2.97	14 805
21-31	188 000	1 099	1.49	37.7	20.0	6.89	3.45	6.25	3.57	9.66	3.27	14 806
September 1-10	178 000	1 173	1.59	39.7	19.4	7.49	3.21	7.04	3.66	10.85	3.46	14 871
11-20	124 000	1 124	1.53	42.4	22.8	6.49	3.37	7.26	3.29	9.95	3.87	14 872
21-30	152 000	1 107	1.50	42.6	23.7	6.14	3.37	7.06	3.28	9.54	3.94	14 873
October 1-10	172 000	1 184	1.61	45.8	20.3	6.69	3.13	8.30	3.75	10.56	3.62	14 976
11-20	103 000	1 114	1.51	43.6	25.0	6.19	3.54	7.50	3.39	9.43	4.21	14 977
21-31	110 000	1 232	1.67	45.0	26.3	6.44	4.03	8.55	3.54	10.33	4.89	14 978
(b) AT WILLOW BEACH STATION												
August 1-10	202 000	299	0.41	32.4	17.8	2.20	0.99	1.53	1.87	2.08	0.83	14 700
11-20	202 000	292	0.40	29.7	16.9	2.40	0.99	1.43	1.84	2.06	0.75	14 801
21-31	220 000	284	0.39	29.6	16.2	2.20	0.99	1.34	1.75	2.04	0.70	14 802
September 1-10	198 000	380	0.52	22.5	14.6	3.24	1.32	1.33	2.28	2.77	0.84	14 803
11-20	198 000	361	0.49	25.9	16.9	2.94	1.23	1.45	2.00	2.73	0.83	14 804
21-30	197 000	297	0.40	28.8	15.0	2.40	0.99	1.37	1.74	2.27	0.68	14 874
October 1-10	196 000	466	0.63	33.3	15.5	3.49	1.40	2.44	2.61	3.58	1.12	14 973
11-20	195 000	621	0.84	38.9	18.9	4.14	1.73	3.73	2.54	5.29	1.80	14 974
21-31	214 000	457	0.62	37.3	18.6	3.14	1.40	2.69	2.05	3.79	1.31	14 975
November 1-10	193 000	479	0.65	36.5	18.7	3.29	1.48	2.74	2.10	4.04	1.39	14 988
11-20	192 000	654	0.88	39.2	21.6	4.19	1.97	3.97	2.39	5.62	2.17	14 989
21-30	187 000	797	1.08	40.5	22.6	4.84	2.47	4.98	2.69	6.85	2.76	14 990

Although the passage of silt through Lake Mead, therefore, is of itself unimportant, the phenomena which produced such silt flows are important.

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[47] (Quinton, Code & Hill-Leeds & Barnard), Los Angeles, Calif.

The impression might be gained from the authors' presentation that water flowed through Lake Mead without mixing only at times when the discharge at Boulder Dam was turbid. Actually, it is evident from the chemical analyses of the water entering the reservoir and discharged from it that the flow of saline water through Lake Mead is of usual, rather than unusual, occurrence. The writer, therefore, would re-phrase the first conclusion of the authors to read: "The passage of water in considerable quantities through a reservoir without mixing has much more than scientific interest and importance."

Variations in Salinity.—For several years the U. S. Geological Survey has taken daily samples of water from the Colorado River at Grand Canyon Station above Lake Mead and at Willow Beach Station below the reservoir. The percentage of sulfates has been determined for each sample and complete water analyses have been made of composites of each consecutive 10-day group of samples. Table 8(*a*) contains the analyses of the composites made up from daily samples taken at Grand Canyon Station during the months of June to October, 1935, inclusive; Table 8(*b*) contains similar data for the waters at Willow Beach Station from August 1 to November 30, 1935, inclusive. The constituent ions are recorded in milligram equivalents per liter, rather than in parts per million, for convenience of geochemical classification. The total discharge of the Colorado River at each station for the successive 10-day periods is also recorded in acre-feet.

The data from Table 8 are shown graphically in Fig. 7, which is an enlargement of part of a geochemical chart. This form of plotting is not in general use and some explanation of it is probably warranted. The small inset is an outline of a geochemical chart which esentially consists of two trilinear diagrams and a diamond-shaped diagram adjoining them. The left-hand triangle is used for plotting the percentage reacting values of the cations; the right-hand triangle is similarly used for plotting the percentage reacting values of the anions. Since the percentage reacting values of the cations and of the anions always total unity, the cations in any water can be represented by a single point on one trilinear diagram, and the anions by a single point on the other equilateral triangle. When these are projected to an intersection on the diamond-shaped figure, the resulting position represents the geochemical grouping characteristic of the water. The diamond, itself, is made up of two trilinear diagrams having a common base, and the sum of the perpendiculars from any point likewise equals unity. For comparison, the range of the points plotted at the enlarged scale is indicated on the inset.

It may be demonstrated that: (*a*) On each trilinear diagram, and on the diamond-shaped diagram, the mixture of two waters must plot on the straight line connecting the plotted positions of each of the waters; and (*b*) the plotted position of the mixture must be at the center of gravity when all quantities are expressed in ton-equivalents. Ton-equivalents bear the same relation to total tons as milligram equivalents bear to parts per million, and the magnitude is fixed by the following relation:

$$T_e = \frac{Q \times m_e}{736} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

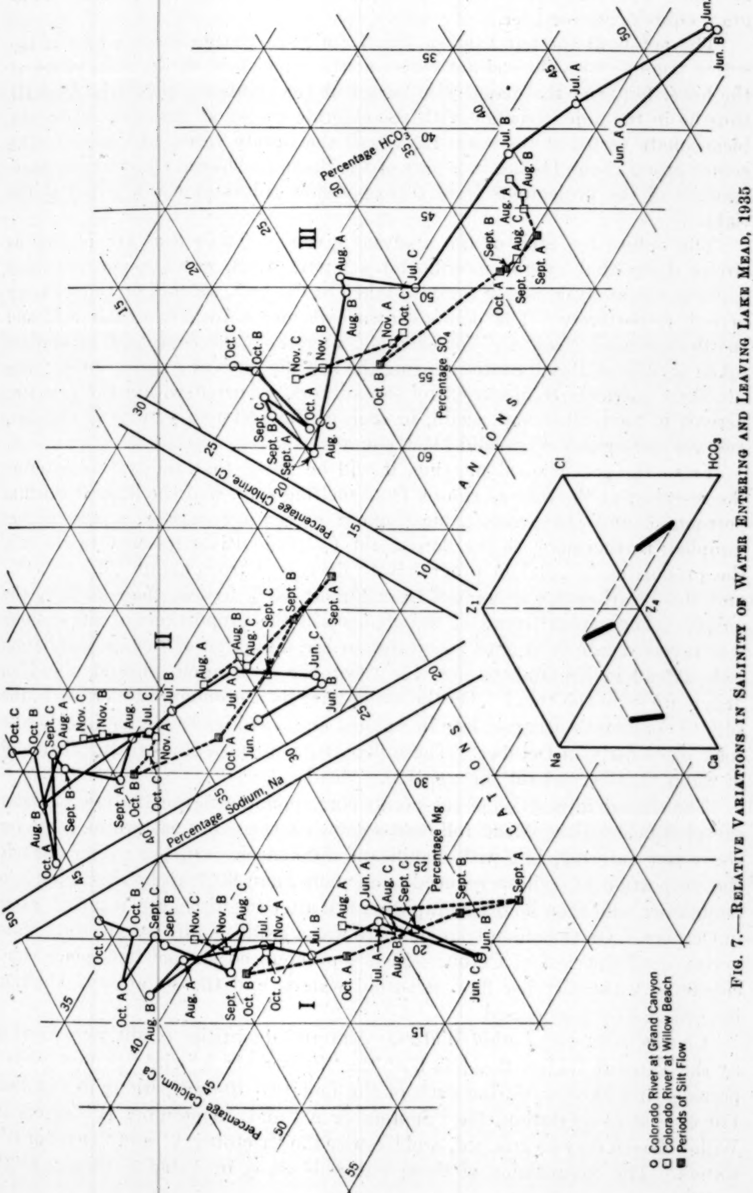

FIG. 7.—RELATIVE VARIATIONS IN SALINITY OF WATER ENTERING AND LEAVING LAKE MEAD, 1935

in which T_e = ton-equivalents; Q = quantity, in acre-feet; and m_e = milligram equivalents per liter.

The points at the left hand of Fig. 7 (in Zone I) thus lie in a part of the cation triangle with the sodium vertex at the upper left, the calcium vertex at the lower left, and the magnesium vertex at the right; the points in Zone III thus lie in the anion triangle with the chloride vertex at the upper right, the bicarbonate vertex at the lower right, and the sulfate vertex at the left. The center group (Zone II) lies in a part of the diamond-shaped chart at the intersections of the projections from corresponding points at the left and at the right.

The values for each of the analyses given in Table 8(*a*) are plotted as circles designated by the month and a letter, A, B, or C, depending upon whether the analysis was of samples taken in the first, second, or third 10-day period, respectively. The points shown by squares are from Table 8(*b*) and the designation, "Aug. C," for example, represents the composite of samples taken at Willow Beach Station during the last ten days of August, 1935. The silt flows particularly discussed by the authors, occurred within the first two periods in September and, again, in each of the first two periods of October, and are distinguished by solid black squares.

From the geochemical plotting, it will be noted that the waters entering the reservoir at the time of run-off from melting snow had the highest calcium percentage and the greatest bicarbonate percentage of any of the waters sampled; furthermore, as the flow of the Colorado River lessened in the fall, the proportion of calcium progressively decreased from about 55% to 34%, and the bicarbonates decreased from almost 55% to less than 20% of the anions. The general trend of the changes in the proportions of the cations was reversed late in August just prior to the discharge of turbid water from Lake Mead in September; also, the "October A" sample showed a sudden departure from the trend. On the anion triangle a pronounced increase in the sulfate percentage occurred late in August, and a sudden reduction in chlorides took place early in October. These departures furnish clues to the character of water that carried silt through Lake Mead.

The change in position of the points corresponding to the analyses of water sampled below Lake Mead followed somewhat the same pattern as those for the water entering, but with significant differences. On the cation triangle the proportion of sodium reduced from more than 30% to less than 25% in September, and then increased rapidly until after the discharge of turbid water in October. On the anion triangle there was a gradual shift toward the sulfate vertex until the first of October; then a pronounced change took place coincident with the October flow of turbid waters, and the sulfate and chloride percentages both increased.

Characteristics of Turbid Waters.—Material quantities of silt were carried by the water at Willow Beach in both the "September A" and "September B" periods, and likewise during each of the first two 10-day periods in October. For ease of computation, the "September A" and "September B" waters at Willow Beach may be grouped, and likewise the "October A" and "October B" waters. The composition of these waters is given in Table 9, Columns (2) to (5).

The authors have concluded from other data that about eight days elapsed between the time that very silty water passed Grand Canyon Station and the time when this water reached Willow Beach. Accordingly, water that passed Grand Canyon late in August and early in September should have been characteristic of the water leaving the reservoir during the time of the turbid flow in September; but from Fig. 7 it is obvious that either the "August C" water at Grand Canyon Station did not produce the turbid outflow from Lake Mead in September, or considerable mixing or dilution took place. Furthermore, with reference to the cation plotting, it is apparent that part of the turbid water at Willow Beach in September had a very low percentage of sodium, because the average sodium percentage of the September turbid water was less than the sodium percentage of even the spring flood waters.

TABLE 9.—ANALYSIS OF COLORADO RIVER WATER

Description	AVERAGE QUALITY OF WATER AT WILLOW BEACH, IN 1935, FROM:				SPRING FLOOD WATER IN RESERVOIR RELEASE		WATER THAT CARRIED SILT THROUGH LAKE MEAD, IN 1935, FROM:			
	September 1 to September 20, inclusive		October 1 to October 20, inclusive				September 3 to September 14, inclusive		October 6 to October 14, inclusive	
	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Cations:										
Calcium	3.09	53.8	3.82	45.0	2.22	51.2	10.64	59.1	6.73	37.4
Magnesium	1.27	22.1	1.56	18.4	0.87	20.2	4.70	26.1	2.84	15.8
Sodium	1.38	24.1	3.10	36.6	1.24	28.6	2.66	14.8	8.43	46.8
Total	5.74	100.0	8.48	100.0	4.33	100.0	18.00	100.0	18.00	100.0
Anions:										
Carbonates	2.13	37.1	2.57	30.3	2.12	49.0	2.22	12.3	3.80	21.1
Sulfates	2.75	47.9	4.44	52.3	1.56	36.1	13.04	72.5	10.67	59.3
Chlorides	0.86	15.0	1.47	17.4	0.65	14.9	2.74	15.2	3.53	19.6
Total	5.74	100.0	8.48	100.0	4.33	100.0	18.00	100.0	18.00	100.0
Quantity of water, in acre-feet	396 000		391 000		....	....	....	....	....	....
Quantity of salts, in ton-equivalents	3 090		4 500		....	....	....	....	....	....

These apparent anomalies make possible the calculation of the quantity and character of the water that carried silt through the reservoir in September. After repeated trial and error, it was found that a mixture of 355 000 acre-ft of spring flood water, carrying 2 080 ton-equivalents of salt, with 41 000 acre-ft of water, loaded with 1 010 ton-equivalents and having a high calcium sulfate content, would produce the quantity and quality of water measured at Willow Beach from September 1 to September 20, 1935. The spring flood water was evidently itself a mixture, in equal quantities, of the water that entered Lake Mead in the last ten days of June and the first ten days of July, 1935. The composition of these waters is shown in Table 9, Columns (6) to (9).

In October, 1935, the passage of silt through Lake Mead was produced by a water of markedly different characteristics from those of the water which produced the silt flow in the preceding month. From Fig. 7 it will be noted that there was a marked increase in sodium percentage at the time of turbidity in October, as distinguished from a decrease when silt was discharged in September. The direction of shift of the "October A" and the "October B" points for Willow Beach on divisions of Fig. 7 is toward the "October A" point for Grand Canyon Station. These directions likewise depart from those for the ten days immediately preceding and immediately following, which also is indicative of some change in conditions coincident with the passage of silt through the reservoir. There thus seems to be justification for the assumption that the water which carried the silt through Lake Mead in October was, in part at least, closely similar to the "October A" water at Grand Canyon. Actually, the water which produced the silt flow was found to be that given in Table 9, Columns (10) and (11).

A simple mixture of this water and of the spring flood water would not satisfy the criterion that the mixture of "October A" and "October B" water at Willow Beach Station should lie at the center of gravity of the points corresponding to the spring flood water and the October silty water entering the reservoir. It was observed that the clear water released from Lake Mead between the times of the two silt flows had substantially the same proportion of anions as the turbid water released in September. This indicated that some of the calcium sulfate water which carried silt through the reservoir in the first part of September was still passing through the reservoir without its silt load in the latter part of that month.

This assumption is borne out by the fact that the various criteria can be satisfied by mixing with the water entering the reservoir in October, some of the water characteristic of the September run. Actually, the 4 500 ton-equivalents of salts carried past Willow Beach from October 1 to October 20 were evidently made up of 1 630 ton-equivalents of salts of the character carried by the water which entered the reservoir about July 1, 510 ton-equivalents of salts characteristic of those in the water which came into Lake Mead late in August, and 2 360 ton-equivalents of salts carried by water which passed Grand Canyon about the end of September, 1935. The corresponding quantities of water are as follows: "June C" water, 137 000 acre-ft; "July A" water, 137 000 acre-ft; September silt-run water, 21 000 acre-ft; and 96 000 acre-ft of the water which directly caused the flow of turbid water in October, 1935.

Minimum Silt Percentage.—In September, roughly 40 000 acre-ft of water carried 2 000 000 tons of silt through Lake Mead, equivalent to an average of 50 tons per acre-ft. It would appear from this that the water which brought the silt to Willow Beach Station was that which passed Grand Canyon Station on August 26 and on September 4, and that little of the suspended load at Grand Canyon on the intervening days found its way through the reservoir. Again, in October, roughly 2 500 000 tons of silt were carried through by less than 100 000 acre-ft of water, or more than 25 tons per acre-ft. The days of high silt percentage at Grand Canyon were September 30 to October 3, inclusive, during which time the discharge of the Colorado River was almost as great as the quantity of water that apparently passed through the reservoir with its silt load. In general, the data indicate that silt was not carried through Lake Mead when the water entering the reservoir contained less than 2.0% of suspended matter, and then only if the proportion of fine particles was relatively large.

Passage of Clear Saline Water.—Reference was made in the foregoing discussion to the passage of clear saline water through Lake Mead without material mixing except at the outlets. In the writer's opinion, this is of far greater practical importance than the removal of silt that might otherwise deposit in the reservoir.

With reference again to Fig. 7, and particularly to the points on the segment of the anion triangle, it is apparent that the clear waters released from the reservoir in August, in September, in October, and in November, 1935, were not characteristic of the great body of water that entered during the flood run-off in the spring. The points for the flow at Willow Beach in the month of August lie roughly on a line between the "June C" and the "July C" water at Grand Canyon Station, indicating that the water in the river several miles below Boulder Dam was a mixture of spring flood water and of the flow entering the reservoir late in July. The clear water at Willow Beach in the last ten days of September, 1935, definitely must be a mixture of the spring flood water and some other water having a much higher sulfate percentage. Actually, the data indicate that some of the calcium sulfate water which produced the turbid flow early in September was continuing to flow under the lake without its silt load in the latter part of that month. The water at Willow Beach near the end of November appears to be somewhat similar to the water entering the reservoir in September, but the concentration of salts in this water is less than two-thirds of the concentration in the water entering. There must have been, therefore, dilution or mixing of the water characteristic of that entering Lake Mead late in September, with other water which came into the reservoir during spring floods.

It is not necessary that this mixing should take place in the reservoir, and probably this was not the case. The outlets at Boulder Dam presumably drew both from the saline water which was flowing along the bottom toward the dam and from the large mass of water lying above the saline flow. This could be true, of course, regardless of whether one outlet or a series of outlets were in service, although it would be more likely to occur with outlets open at different elevations.

This phenomenon of the passage of saline water along the floor of the reservoir is also suggested by the upper graph in Fig. 2 of the paper, in which it is shown that the quantity of sulfates at Willow Beach steadily increased from September to the end of November, 1935, approaching the concentration of sulfates in the water at Grand Canyon Station, regardless of the fact that most of the water in Lake Mead entered there during months of run-off from melting snow.

Various estimates have been made of the quality of water that will be available for domestic and other uses from the Colorado River after regulation in Lake Mead. Such estimates have naturally been based upon the assumption that there would be complete mixing of the relatively pure flood waters with the saline low-water flow of the Colorado River. Analysis of the data now available (1938) indicates, on the other hand, that the saline waters tend to flow through the reservoir rather than to mix, and that the spring flood waters tend to "ride on top." Since May 1, 1936, the tunnel gates at Boulder Dam have been closed and the lowest outlet is far above the original stream bed. The question naturally arises: Will the normal release from Lake Mead be materially more saline than the weighted average inflow and will the best water be wasted at the time of spill or release for flood control?

NATHAN C. GROVER,[48] M. AM. SOC. C. E., AND CHARLES S. HOWARD,[49] ESQ. (by letter).—Since the publication of this paper interest in density currents has been manifested through correspondence, published discussions, and the creation of a specific committee (Interdivisional Committee of the National Research Council on Density Currents) for their further study. The discussions have served to emphasize the inadequacy of information with respect to the phenomena of density currents and the probable theoretical and practical value of further study of them. Several interesting and important aspects have been presented. In general, the discussions have disclosed: That there have been instances of flows of turbid water through reservoirs under a variety of conditions that cannot be evaluated because of lack of observational information; that there may be many instances of stratification of water in reservoirs due to differences in density, in many of which turbidity is not involved; and, that there is an important and unexplored field of scientific research in relation to density currents and stratification in reservoirs which may have both scientific and practical significance.

Mr. Faris has described a flow of turbid water through Lake Kemp and mentioned the presence of silt in depressions in Medina Reservoir. Messrs. Page and Forester have given valuable information concerning the visibility of the flow of the turbid water through the center of Lake Mead during the period before the depths of water were too great. Messrs. Faris, Dobson, and Lewis have described similar flows of turbid water through other reservoirs and discussed conditions of deposition in reservoirs which are indicative of other such flows which may not have carried through to the places of discharge. Messrs. Jones, Scofield, Creager, Bliss, and Stevens have discussed the conditions under which the flows have occurred in Lake Mead and other reservoirs and Mr. Palmer has suggested the similarity of the conditions causing turbid flows and those of lack of mixing or of stratification of fresh and salt water. Mr. Houk and Professor O'Brien have discussed the phenomenon on the basis of temperature differences. Mr. Monish has suggested a modification of Prandtl's expression for the criterion of mixing, and Mr. Rubey has made several computations to show the combined effects of the quantities and nature of the suspended matter, and the salinity and temperature of the water. He has also considered the effects of variations in velocity as it was measured at the Grand Canyon gaging station about 160 miles above the head of the lake.

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[49] Chemist, U. S. Geological Survey, Washington, D. C.

Mr. Jarvis has referred to earlier considerations of the subject and has suggested the practicability of creating turbulence as a means of putting the silt into suspension for passage out of the reservoir. The information presented by Mr. Lewis in his discussion and in the paper to which he refers,[33] is very valuable in outlining the efforts that have been made to eliminate silt from certain reservoirs in South Africa.

Mr. Page has presented additional information concerning conditions in Lake Mead during certain periods when the flows of turbid water took place. On the information available, the writers do not now agree with Mr. Page that the higher temperature of the relatively warm water near the bottom of the lake is explained by the warm springs known to be present near the dam site. The source of the heat producing this higher temperature in the lake water is not positively known, but water of the observed temperature appears to the writers to have extended over too large an area to be considerably influenced by the relatively small discharge of water from warm springs. It may be significant that the temperature of this stratum of warm silty water is 68° to 70°F which is approximately the mean annual temperature at and near Boulder Dam.

The writers believe that the submerging or covering of certain areas mentioned by Mr. Jones was of little or no importance in causing the silt flows since the material carried past Willow Beach during the flows was much finer than would be expected to result from sliding or scouring in those areas.

Mr. Monish has discussed the extent of mixing that might be expected under different conditions of flow and suspended matter. He has computed limits for the critical values of mixing which seemed to hold for the flows of turbid water observed at Lake Mead in 1935, and has shown that these values might not be the same for another year, nor for another reservoir. He has computed his values from records obtained at the Grand Canyon station which were the best available to him. Such computations, based on records of water and silt that actually came through the reservoir, would be of interest since it appears to be reasonably certain that the sizes and conditions of flocculation of the particles are of importance in their effect on rates of settling. Mr. Rubey has made a careful study of the available data and was not able to explain the flows of turbid water by consideration of densities or density differences, or by the densities and velocities of the water measured at the Grand Canyon gaging station. He has emphasized the need for observations on the degrees of flocculation and the velocities or slopes of the turbid water within the reservoir. Mr. Stevens has suggested that an agency or board be created to co-ordinate investigations of the Colorado River Basin, and that the results of comprehensive studies in the Colorado River Basin would have value in connection with studies of the Rio Grande and other silt-bearing streams of the arid West. Professor O'Brien has suggested that some of the solid particles were first carried to the bottom and, later, were resuspended as a result of turbulent flow and then were carried through the lake in the turbid flows. Mr. Lewis has suggested that the matter in solution causes flocculation during periods of turbid flow, which, in turn, causes bottom flow.

Mr. Hill has presented tables and graphs to show that the water flowing out of Lake Mead during part of 1935 was a mixture of the water that entered during the spring flood with other water in the lake. He has raised the question: "Will the normal release from Lake Mead be materially more saline than the weighted average inflow and will the best water be wasted at the time of spill or release for flood control?" Mr. Hill has also suggested that the passage of the more concentrated water through the lake may be of greater practical importance than the passage of turbid water.

Mr. Redden described a flood on the Weber River in Utah, in August, 1934, after which turbid water passed through Echo Reservoir, and stated that although the river often carries considerable suspended matter the water discharged at such times was usually clear, and that of many known instances when silt-laden waters appeared at the outlet gates of reservoirs in Utah, the phenomenon is more noticeable on some rivers than on others.

Samuel B. Morris, M. Am. Soc. C. E., has furnished, by personal letter to the writers, a general description of some observations made on Morris Reservoir, in California:

> "The Morris Dam completed in 1934 creates a reservoir in San Gabriel Canyon 20 miles east of the City of Pasadena. The reservoir is four miles long and of maximum depth of 250 feet. Storage capacity is 38,360 acre-feet and surface area 417 acres. Following winter storms the water in the reservoir clarifies and becomes stratified. Temperatures at the bottom of the reservoir will generally be 50 to 55 degrees while surface temperatures will be 5 to 10 degrees warmer by April. In the spring of 1935 a flood discharge of warm water entered the reservoir and flowed along the bottom to the flood release outlets at the base of the dam discharging a muddy stream while the surface water in the reservoir remained clear. After two or three days the reservoir became cloudy throughout and of constant temperature.
>
> "Apparently the warm silt-laden flood water was heavier than the colder water in the reservoir. After two or three days, however, sufficient of the silt was precipitated to cause the warm water to be lighter in specific gravity than the cool water above it. This caused the warm water to rise and the reservoir 'turned over' thoroughly mixing its contents and giving a constant temperature throughout. This is similar to the condition which usually occurs in the late fall of each year when the surface water becomes cooler than the deeper water."

Since the publication of the paper, two reports concerning the delta of the Colorado River[50] have appeared. These reports present considerable information on the nature of the silt carried by the Colorado River, which will be of value in future studies of silt movement.

In the spring of 1937 the National Research Council appointed an Interdivisional Committee on Density Currents which may serve as the co-ordinating agency suggested by Mr. Stevens. Meetings of this Committee were held in June, 1937, to formulate a program of field and laboratory studies. Intensive programs were planned for the Elephant Butte Reservoir and Lake Mead. During the summer of 1937, observations were made at the Elephant Butte Reservoir, but there have been no apparent movements of turbid water through the reservoir during that year. The results obtained during the year show variations in concentration of the water in different parts of the reservoir, but the complete data are not available to show the extent or the nature of the stratification.

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[50] "The Colorado Delta," by Godfrey Sykes, *Publication 460*, Carnegie Inst. of Washington; pub. jointly by the Carnegie Inst. of Washington and the Am. Geographical Soc. of New York, 1937; also, "Delta, Estuary, and Lower Portion of the Channel of the Colorado River, 1933 to 1935," by Godfrey Sykes, *Publication 480*, Carnegie Inst. of Washington.

Samples of water and records of its temperature were collected in August and September, 1937, in co-operation with the U. S. Indian Service, on the Gila and San Carlos Rivers above the San Carlos Reservoir and at the outlet of Coolidge Dam. Although some of the samples of the inflowing water had more than 1% of suspended matter (of which a large part consisted of particles with diameters less than 20 microns), no suspended matter passed through the dam during this period. The mean daily discharge of the San Carlos River during the period was less than 100 cu ft per sec for all but four days. The flow of the Gila was somewhat larger, and in the two months of the study more than 250 000 tons of suspended matter was carried past the Calva gaging station at the head of the San Carlos Reservoir. The failure of turbid water to pass through the reservoir may have been due to the small quantity of suspended matter and the small flow.

No turbid water has passed through Boulder Dam since the gates at the bottom of the reservoir were closed on May 1, 1936. Periodic observations have been made by the U. S. Bureau of Reclamation to ascertain the temperature and position of turbid water in different parts of the lake. Most of these observations have been made in the canyon section immediately above the dam, but, since the summer of 1937, additional observations have been made in Boulder Canyon, 20 miles above the dam, and in Virgin Canyon, 50 miles above the dam. The observations show that there has been a body of turbid water at the bottom of the lake, as mentioned by Mr. Page. Temperature observations made as a part of the sampling program show a marked increase in the temperature of the water at the points of transition from clear to turbid water. Samples of the turbid water show that the quantities of solid material increase with the depth. The bottom samples have been so thick that they could be poured into a bottle only with difficulty. Mechanical analyses of the suspended material taken from the bottom of the lake at several places show that the material consists chiefly of very fine particles, more than 90% being less than 20 microns in diameter. In July, 1937, there was about 20 ft of the silty water in Virgin Canyon and about 35 ft in Boulder Canyon. Observations made in Virgin and Boulder Canyons, in September and November, 1937, and in January, February, and March, 1938, showed varying quantities of turbid water in both canyons.

On the basis of several observations made at different parts of the lake Table 10 has been prepared showing approximate elevations of the lake bottom and of the top of the silt layer during the period July 1, 1937, to March 12, 1938.

These results indicate fluctuation at the upper stations in the level of the top of the silt which probably represents flowing of silt toward the dam. The observations made at Pierces Ferry in February, and at the other stations in February and March, seem to indicate such a flow of turbid water. The notes submitted with the description of the sampling program for February included the following:

"As the head of navigation was approached there was a distinct line between the clear and turbid water. There were stretches in this vicinity constantly bubbling as if the air entrapped in the rocks on the bottom of the lake was still being released. Where these bubbles came up they carried with them a cloud of muddy water to the surface.

"When the heavy rains starting around March 4 began to bring up the flow of the Virgin and Colorado Rivers, an attempt was made to pick up its effect in the lake by sampling on March 5, 9, 11, and 12. In the sampling of March 9, at Virgin Canyon there was noticed an upstream surface current. At 259′ depth below the surface there was no noticeable deflection of the sampler cable but at 275′ depth below the surface the sampler cable showed a very decided downstream inclination as if encountering a strong downstream current."

Observations made at Boulder Dam and at Cape Horn, about 3 200 ft above the dam, show considerable fluctuation in the level of the top of the silty water. The data for Boulder Dam indicate that it may be difficult to observe, accurately, the elevation of the bottom of the lake, because the material deposited on the bottom appears at times to be consolidated, and, at other times, to be so stirred up or loosened that the sounding weight penetrates it to a greater depth. During the periods of observation the top of the silty water near the dam has been below the top of the old diversion dam, which is still (1938) in place.

TABLE 10.—ELEVATION OF LAKE BOTTOM AND TOP OF SILT LAYER AT SEVERAL POINTS IN LAKE MEAD DURING THE PERIOD, JULY, 1937, TO MARCH, 1938

(Surface of Lake Approximately 1 100 Ft Above Sea Level During the Period)

Period	Pierces Ferry		Virgin Canyon		Virgin River, Lower Narrows		Boulder Canyon		Black Canyon			
									Cape Horn		Boulder Dam	
	Silt layer	Lake bottom	Silt layer	Lake bottom	Silt layer	Lake bottom	Silt layer	Lake bottom	Silt layer	Lake bottom	Silt layer	Lake bottom
1937:												
July 1	....	....	....	....	....	....	....	....	....	....	645	625
July 7	....	....	....	....	....	....	....	....	....	....	649	625
July 20–21	....	....	827	807	....	....	742	705	....	....	....	....
August 5	....	....	....	....	....	....	....	....	....	....	704	....
September 8–9	....	....	819	801	....	901	734	709	....	....	....	....
October 5	....	....	....	....	....	....	....	....	707	658	667	621
October 28–29	....	....	....	....	....	....	....	....	717	663	709	625
November 16–17	....	....	809†	801	....	....	726	708	....	....	....	....
December 2–3	....	....	....	....	....	....	....	....	713	663	687	615
1938:												
January 4	....	....	....	....	....	....	....	....	....	....	684	616
January 7	....	....	....	....	....	....	....	....	712	666	....	....
January 11–12	....	....	811	808	....	892	729	708	....	....	....	....
February 2	....	....	....	....	....	....	....	....	....	....	681	613
February 10–11	955*	948	....	....	....	....	....	....	710	666	....	....
February 24–25	....	....	812	810	....	892	725	715	....	....	....	....
March 1–2	....	....	....	....	....	....	....	....	709	668	679	615
March 4	....	....	810	810	....	....	....	....	....	....	....	....
March 9–12	....	....	848	813	902	896	754	709	....	....	731	614

* At head of navigation (about 110 miles above Boulder Dam) on this date there was considerable silt at bottom (Elevation 1 068 ft).

† November 16: No silt layer in Las Vegas Wash area of lake; bottom elevation, 936 ft; no silt layer in Callville Wash area of lake; bottom elevation, 1 026 ft.

The samples collected in Virgin Canyon show that the water in contact with the silt was more concentrated in September than in July. The quality of the water in the September samples from the bottom of the lake in Virgin Canyon was similar to that in a turbid flow observed at Grand Canyon a few days prior to the sampling in Lake Mead. These comparative data would seem to indicate that there had been a density flow through the lake, from the head of the lake, as far as Virgin Canyon, at least, and that the time required for the flow over that distance was about six days. Samples collected in Boulder Canyon in September indicate a possible movement of turbid water to that point, but the evidence for Boulder Canyon is not quite so satisfactory as that for Virgin Canyon. In addition to the samples taken in the deeper parts of the lake, samples of water have been taken at the surface in several parts of the lake and, for some sampling periods, at several depths in order to obtain information on the dissolved constituents at different parts of the lake and at different depths.

The stratification, or its opposite, the thorough mixing of the water in a reservoir, and the season of the year when such mixing occurs, may affect greatly the chemical quality of the water, whether wasted or drawn from the reservoir for use, and may determine the level from which it might be desirable to draw the water. In the summer of 1937, the surface water in Virgin Canyon, Boulder Canyon, in the lake between these canyons, and in the wide part of the lake at the mouth of Las Vegas Wash had practically the same concentration and was similar in composition to the water which flowed past the Grand Canyon station during the spring flood. At considerable depths below the surface during the same period the water had a concentration of 700 to 800 ppm of dissolved matter, which is practically the same as that of the water released from the reservoir during the fourteen months ending February 28, 1937. The presence of such a large body of water of uniform chemical composition seems to indicate that at some time, and by some means, the water in the lake became well mixed. During the summer of 1937 this large body of mixed water had a temperature of about 53°F and the uniformity of this temperature in different parts of the lake may be another indication of the thoroughness of the mixing. Temperature measurements made by the Bureau of Reclamation show that in January, 1937, the water at all depths at the dam had a temperature of about 55° F. During the summer the water at the surface of the reservoir was warmer, of course, and, in July, midday temperatures of the water at the surface were as high as 90° F.

The depth-temperature curves in the vicinity of the dam show temperatures decreasing from the surface to a depth of about 200 ft where about 52° to 54° F were observed during the summer months. Below about 200 ft the same temperature was observed as far down as the top of the layer of turbid water where there was a sharp increase in temperature. It is not known how great an effect the mixing caused by a temperature "turn-over" may have on density currents and stratification; but the evidence seems to indicate that in the early winter of 1936–37 the entire mass of water in the reservoir came to a temperature that was essentially uniform, and it seems likely that the water throughout practically the entire reservoir was of essentially uniform chemical composition.

It is not known whether the turbid water (if there was any at that time) lying at the bottom of the lake was involved in the "turn-over" of the winter of 1936–37; but it seems probable that its density may have been so great that little or none of it was so involved. The water brought into the lake by the spring floods carried considerable quantities of suspended matter, but usually those floods carry larger particles, chiefly, which settle quickly. The spring run-off was low in dissolved matter and its temperature was somewhat higher than that of the water previously stored in the reservoir. It seems likely, therefore, that the incoming flood water with its lower density, because of lower content of dissolved matter and higher temperature, spread out over the surface of the lake and mixed only slightly with the main body of stored water.

Observations made by the Bureau of Reclamation during January, 1938, at different points in the lake showed that temperatures ranging from 56° to 59° F, at the surface and in the top 100 ft of the lake, were quite uniform, and that at greater depths they varied between 53° and 56° F. In March, the temperature of the water at most depths was between 52° and 55° F. It seems likely that the top layers of water (possibly the upper 100 or 150 ft, at least) were mixed during a "turn-over" in the winter of 1937–38, but the available data are not sufficient to show that the "turn-over" affected greater depths. Samples of water collected at different depths show variations in the quantity of dissolved matter which would indicate that the mixing of the water was not thorough. The values of specific conductance given in Table 11 indicate the degree of concentration of the water at different depths.

The stratification throughout the lake is apparent from these data. The average values given are representative of values having a small range. At all sampling points the more highly concentrated water is found near the bottom of the lake, but the total quantity of such concentrated water is a small fraction of the entire body of water in the lake.

In a region where there is sufficient range in temperature to cause a "turn-over," the yearly cycle of a storage reservoir of large capacity that carries over

TABLE 11.—AVERAGE VALUES OF SPECIFIC CONDUCTANCE OF WATER AT DIFFERENT DEPTHS AT FOUR POINTS IN LAKE MEAD

(Specific Conductance,* $K \times 10^5$)

Approximate elevation (feet above sea level)	Black Canyon				Boulder Canyon				Virgin River Narrows				Virgin Canyon			
	1937		1938		1937		1938		1937	1938			1937		1938	
	Oct.	Dec.	Jan.	Mar.	Sept.	Nov.	Jan.	Mar.	Sept.	Jan.	Feb.	Mar.	Sept.	Nov.	Jan.	Mar.
1 000 to 1 100	84	88	92	104	71	81	96	103	76	98	103	103	85	110	109	110
850 to 950	115	114	116	121	117	116	121	121	119†	99†	128†	184†	121	133	135	127
800	121	120	119	134	119	127	124	133	....	....	....	....	162†	138†	141†	155†
700	145	124	129	143	114†	123†	167†	166†	....	....	....	....	....	....	....	....
625	156†	132†	147†	153†	....	....	....	....	....	....	....	....	....	....	....	....

* Specific conductance × 8 = approximate parts per million of dissolved solids.

† Samples from the bottom of the lake.

considerable depths of water from year to year appears to the writers to consist of at least two phases: (1) The annual "turn-over," which so thoroughly mixes the water near the surface that it becomes essentially uniform in chemical qualities and in temperature, and which does not involve, perhaps, a stratum at the bottom of the lake (where the water with its load of silt may be of such density that it does not mix with the remainder of the water at the time of the "turn-over"); and (2) the inflowing flood water that contains less dissolved matter and is less dense than the water already in the lake and, therefore, spreads over and rests upon it. During the summer, also, temperature differences develop which lead to more or less definite stratification which persists until the next "turn-over," when the cycle starts again. Into this cycle that is relatively uniform in the times and types of changes, there come the non-uniform and erratic turbid discharges carrying such fine silt and of such relatively great density that they flow as fairly definite streams along the bottom of the reservoir under the lighter clear water.

The apparent mixing of the lake water at times, and its failure to mix at other times, is of interest in relation to density currents which may not involve flows of turbid water but, instead, may involve flows of water of such varying concentrations of dissolved salts that they may be of concern to the users of the water discharged from the lake, as Mr. Hill has shown.

Any serious study of the problem of stratified flow will require the closest co-operation between laboratory and field. Data obtained from observations made under field conditions furnish the only sure knowledge of such complex phenomena. Because of the complexity, field measurements must be directed by laboratory experiments and theories in order to obtain necessary pertinent facts without an undue waste of time and effort.

Theories and criteria of stratified flow have been developed in laboratories, and values of the criteria, seemingly independent of the size of the surrounding apparatus, have been obtained from laboratory experiments. One such theory was described by Mr. Monish who made an attempt to apply it to the data presented in the paper. The application involved several assumptions as to density and velocity distribution, which are not immediately susceptible to proof or disproof and which could be verified only by their determination under field conditions, including measurements of the magnitude and direction of currents and of the densities and viscosities of the waters in the reservoir so as to chart and define, completely, the strata under consideration. In order to expedite the task of obtaining such data and the necessary correlation as to places of measurements, it would be necessary to devise a combined current meter, direction indicator, and distant-indicating hydrometer—a difficult but not impossible task. A sonic depth-finder might be an aid in determining deposition of silt in a large and deep reservoir.

Assuming that these measurements have been made and have confirmed the criterion of stability found by the laboratory, the next step might be an attempt to control the currents.

Further study should include a large amount of field work to ascertain the position and extent of the stratum of turbid water and of the strata of water of different densities throughout lakes of various sizes. In a reservoir as large as Lake Mead, observations must be made in wide and narrow parts of the lake as well as systematically at the lower and upper ends. The continued study of the characteristics of the incoming water will be of fundamental importance in the interpretation of the results obtained from the studies within the lake. It would seem desirable to have observations made in reservoirs which would be typical of classes so as to have information on the effect of climate, topography, reservoir characteristics, and inflow characteristics on the occurrence of the phenomenon of density currents.

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PRACTICAL USE OF HORIZONTAL GEODETIC CONTROL

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Synopsis

An argument for the increased use of the horizontal geodetic control system, is offered in this paper. It contains: (1) An "Introduction," including suggestions to the engineer when he is beginning to use this control; (2) a description of the origin and use of such a system, followed successfully on the Panama Canal for twenty-four years; (3) an analysis of the errors developed by using the system of control in the Canal Zone; (4) an analysis of the errors developed by using the same system near the Fortieth Parallel of Latitude; and (5) the writer's conclusions.

Introduction

Recent Progress in Geodetic Control Work.—Since 1930 there has been a large increase in the activities of those organizations working on geodetic control in the United States. The main triangulation net of the United States Coast and Geodetic Survey is nearing completion, giving chains of triangulation stations spaced at not more than 25-mile intervals. In addition to this main net many intermediate triangulation chains and traverses have been established by various other organizations, and, more recently, by the State Relief Agencies. Some States have begun to tie their extensive highway surveys to the National Net.

All work already done, and much more that is planned for the near future, either has been, or is soon to be, reported on the basis of geodetic co-ordinates. It is time that engineers interested in surveying engage in some discussion to determine the best methods of using this control system. At present (1937), the only engineers who use the geodetic control, as it should be used, are those employed in the Mapping Divisions of the U. S. Coast and Geodetic Survey, the United States Geological

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Survey, the Hydrographic Office of the United States Navy Department, and other Federal agencies that produce maps covering large areas.

Engineers Do not Use Geodetic Control.—The American engineer and surveyor refuses to work with geodetic positions. The published information on the subject has many statements explaining the need for Statewide plane co-ordinate systems, such as the following (11)[3]: "Of course, if all engineers would use geodetic computations, there would not be need for such aids, but they do not favor this method of handling the situation."

The reason for this reluctance to use geodetic control is apparent to any engineer who has attempted to adapt it to every-day survey work. If he finds a geodetic control point in the vicinity of his work he generally wants to use it. Then he goes through the following cycle: He looks up the position of the point in the geodetic reports and finds it given in degrees, minutes, seconds, and in decimal parts of seconds; in some cases the seconds are given in meters. In any case, he must go to the textbooks for information as to the number of meters in a second or in a minute of latitude and longitude, before he can base his survey on the point. There he finds: (*a*) Pages of material on the methods used in reconnaissance for the triangulation; (*b*) more pages on the methods of observation; (*c*) lengthy descriptions on the measurement and accuracy of base lines; (*d*) some sketchy information as to the computations; (*e*) detailed description of the relatively simple operation of precise leveling; and (*f*) a reference to the Government publication from which the information given, was taken. Finally, he abandons the system, reasoning that if the computation of seconds to meters is too complicated for the author of a textbook on surveying, it is too complicated to use.

The publications of the Government organizations covering the subject (see Appendix), if taken as a whole and carefully studied, give a complete picture of the necessary process; but, unless one has all of them at his disposal and studies them thoroughly, he is likely to become confused and decide that geodetic control is to be used for surveys covering large areas only. The beginner will not find a clear step-by-step explanation as to how to tie a local survey to geodetic co-ordinates, except by the use of plane co-ordinates.

Plane Co-Ordinates Used as Substitute.—Realizing the necessity for tying their work to the national net, some of the more progressive cities have made geodetic control surveys of their area; but as a basis for actual work they have substituted plane co-ordinates, which are about to be adopted as a standard practice. This is unfortunate because the objections to the use of plane co-ordinates are numerous, and increase with the distance from the origin. In addition to the fact that it is fundamentally wrong to use plane co-ordinates in surveying areas on the spherical surface of the earth as if it were a plane, they are also confusing to the surveyor who has always used the meridian as his line of reference. True azimuths must be corrected when using plane co-ordinates, the correction depending on the position of the point with reference to the origin of co-ordinates. Errors are certain to occur where two co-ordinate systems join.

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[3] For reference to figures in parentheses, see "Bibliography", in Appendix.

SIMPLICITY OF HORIZONTAL GEODETIC CONTROL MUST BE EMPHASIZED

In order that the engineer may be encouraged to use this geodetic control in his every-day work (and it would be an aid to every engineer having any kind of survey work to do), its simplicity must be emphasized. A comparable example is in the use of geodetic levels. Their advantages are instantly admitted and they are used whenever possible, even where profile levels must be run for miles to base some project on the common sea-level datum. If the horizontal geodetic system were made as simple to understand as the vertical system, it would be used by all engineers.

Horizontal Geodetic Control Is Easy to Understand.—Surveyors and engineers have generally understood and used the fundamental principles of geodesy; that is, that the earth is an approximate sphere, that north is always to the pole, that the meridians of longitude converge, and that parallels of latitude are curved lines when run on the earth's surface. Children are taught in the elementary grade schools that the earth is a sphere and that points are located on it by measuring from the equator and the meridian through Greenwich, in terms of degrees and minutes of latitude and longitude. Therefore, it is easy for any layman to understand that the corners of his lot or farm are at a certain latitude and longitude, based on these well known references.

When engineering students are taught in college that they are to ignore all that they have previously learned about the shape of the earth; and that they are to consider it a plane surface for surveying purposes, because the solutions of geodetic problems are too difficult, they are led a step backward; a step that is entirely unnecessary. They should be told instead that: (1) All points located by the U. S. Coast and Geodetic Survey are tied to the equator and to the Greenwich meridians by an average of many observations; (2) it is their duty as surveyors to tie their surveys to such points, wherever possible; and (3) the computations to do this are no more difficult than those involved in ordinary surveying.

Information to Beginners.—For those who have never made any study of geodetic control methods the following simple facts will be an aid.

The computation of geodetic positions is a distinct and separate operation from adjustment of any kind; that is, no least squares computations are involved in the simple computation of geodetic positions. Having the geodetic position of one point and the geodetic azimuth and the distance from that point to another, the geodetic position of the second point and the geodetic back azimuth are computed in the standard printed forms.

The computations for difference in latitude are made by multiplying the length of the line between the points by the cosine of the azimuth times a factor taken from tables. The purpose of this factor is to convert the meters of length into seconds of arc, at the latitude of the initial station. This latitude difference is then corrected by three other values all of which vary, in some degree, with the sine of the azimuth (that is, they depend on the difference in longitude). The computation for difference in longitude is made by multiplying the length of the line by the sine of the azimuth times the secant of the latitude, and by a factor, taken from tables at the latitude of the new station. This factor converts the meters into seconds. No corrections are made to the difference in longitude.

Although the standard forms for geodetic computations of both the Coast and Geodetic Surveys and the Geological Survey are well arranged for the work as done by those Bureaus, they are not a good guide to the beginner. The computations are done in steps as follows: (*a*) Having the geodetic azimuth from one known point to another known point by a previous computation, the geodetic azimuth to the unknown point is computed by a simple addition or subtraction of the angles at the known point (spherical excess corrections are necessary in the case of very large triangles); (*b*) the difference in latitude is then computed by the steps and corrections shown on the standard form, and is followed by the computation for the difference in longitude; and (*c*) the difference in azimuth is computed last. On short lines it is not necessary to apply all the corrections to the difference in latitude as the values found will be negligible. The entire operation requires only a knowledge of logarithms and simple trigonometry.

Astronomic azimuths and positions are those that are determined by observations on the sun, moon, or stars. Geodetic azimuths and positions are those that are computed and carried forward by geodetic computations. Astronomic azimuths and positions should check with the geodetic azimuths and positions, except for small corrections.

CONTROL SYSTEM OF PANAMA CANAL ZONE

Origin.—Since 1911, the Section of Surveys of the Panama Canal Zone has used a system of geodetic control as the base for all surveys. It has been found to be of great value in avoiding confusion, as all surveys are reported with positions of latitude and longitude. No special study is necessary by those using the system. In the vicinity of the Canal surveys are made by five different Canal or other Government organizations as well as by some private engineers. It is possible for each of these agencies to use the work done by any or all the others because all surveys are tied to the one common Panama-Colon Datum.

At the beginning of the construction of the American Canal, and for the several years of the preliminary work, a situation existed on the Isthmus, with respect to survey control, that was much worse than that in the United States prior to the establishment of the Standard North American Datum. There were many different systems of surveys that joined and overlapped each other without a common base. On the Atlantic side of the Isthmus there was a base line and a triangulation system which were based on an astronomical determination of a point on the Colon Light-House. On the Pacific side, there was another system of triangulation based on an astronomical determination of the south tower of the Panama Cathedral. The two astronomical determinations were originally made by outside departments for coast surveys. There was no definite cross-tie between the two systems, although there were many survey lines covering the construction area, established by both the French and the American engineers. This situation led to much confusion especially where the work of two separate divisions met.

In 1910 and 1911 a base line, 3 785.454 m long, was surveyed on the Tavernilla Dump (now covered by Gatun Lake) and measured to a probable error of 1:700 000. A triangulation system, called the primary system for local purposes, consisting of twenty-three stations (including the ends of the base line and the stations of the old Atlantic and Pacific Systems) was carefully located and measured. The triangles of the primary system had an average closure of 1.98″ and a maximum closure of 5.12″. The computations and adjustments were made in Washington, D. C., by the U. S. Coast and Geodetic Survey, by which it was classed as secondary work. Serious discrepancies were found between the positions of the two systems on the opposite sides of the Isthmus. After having made all possible corrections to these positions, dual positions were computed through the primary system (using the initial astronomic value of each of the old systems) for the Balboa Station in the center of the Canal Zone. These two positions were then averaged and the mean position of the central point was adopted as the common datum, called the Panama-Colon Datum.

Approximately seventy supplementary triangulation points were established, and the angles observed, computed, and tied to the primary system; these angles were observed with lesser care than those of the primary system and the computations were made locally; they are considered the secondary system. These triangulation stations were the base for all surveying done on the construction after 1911 and since, in the maintenance and operation of the Panama Canal. They are also used for the control of that part of the military maps covering the vicinity of the canal, and as a base for extending triangulation surveys covering both coasts. Since the establishment of the main control system, it has been necessary to extend it for particular points, and to produce control to desired places. There are now (1937) about 250 triangulation points (most of which are second-order points) in the area around the Canal Zone; the controlled area averages about 4 sq miles per station. This complete system is comparable to the geodetic control net of the United States and all the supplementary work that has been done to extend it.

When this triangulation system was established it was intended to be used as a control on which a rectangular system of surveys would be based. The proposed plan was similar to that of the General Land Office surveys (although not in method), modified somewhat to resemble the systems of rectangular co-ordinates that were then being adapted to some of the large cities in the United States. Monuments for the 2-km intersections of the co-ordinate lines were to be located in the field. This plan failed when it was less than one-half completed, mostly because of the impossibility of locating or using the intersection points on the rough topography of the Canal Zone. Shortly afterward, it became necessary to do considerable surveying to settle land claims and to locate some of the topographical features in the outlying parts of the Zone. To complete this work, the surveyors, some of whom had previously worked on the triangulation system and were thus familiar with geodesy, adopted the present system as a practical substitute for the planned scheme.

Control of Maps and Municipal Surveys.—In working from the triangulation net in the Canal Zone the procedure varies with the class of surveying to be done. For most of the surveys in the occupied parts of the Canal Zone, at the terminals, and along the Panama Railroad, the Section of Surveys has placed control points on the curbs of the streets and in other accessible places. These points are located by ordinary transit and chain traverse with an accuracy of 1:5 000. Since there is no private ownership of property on the Canal Zone, there is no use in maintaining a higher accuracy than is necessary for the control of ordinary municipal work in a suburban district of an American city.

For work involving the boundaries between the Republic of Panama and the Canal Zone, or for the boundaries of the military and naval reservations, transit traverses are tied directly to the triangulation system. Such surveys are run with a varying accuracy, depending on the location and importance of the lines. For the computations of positions on all the transit traverse lines the procedure is as follows:

(1) All triangulation stations are recorded in a card index. The card for each station gives the position of the station and the azimuths and distances to the observed stations (see Table 1).

TABLE 1.—SAMPLE CARD FROM FILE OF TRIANGULATION STATIONS
(Ancon; Primary; Elevation, 654.0 Feet)

Latitude	8° 57′ 25.526″	784.2 m	2572.75 ft
Longitude	79° 33′ 08.813″	269.2 m	883.26 ft
	Azimuth:	Back Azimuth:	Station:
	125° 51′ 23.51″	305° 50′ 18.62″	Gordo
	170° 05′ 04.07″	350° 04′ 51.19″	Iron
	Distance, in Meters	Log Distance:	Station;
	15 627.73	4.1938958	Gordo
	14 630.51	4.1652595	Iron

The positions are given with the seconds reduced to feet; this reduction is made very easily from tables which give the length, in feet, to three decimal places for the seconds of both latitude and longitude for every minute of latitude. As the variation between minutes is small, interpolation between minutes is satisfactory for most purposes. If a more accurate determination is required, the formula for any latitude is taken from *Special Publication No. 5* of the U. S. Coast and Geodetic Survey (6).

The surveyor consults this card index before beginning a survey from a station and uses the correct azimuth. On closing his survey on another station his azimuth will be in error (in addition to the errors made in running the line) an angle equal to the difference in geodetic azimuth because of the difference in longitude of the two stations. No lines are run locally that are closed on the same station; that is, closed circuits. In computing the traverse this error, with all the field azimuth errors, is balanced evenly over the angles to close on the two triangulation stations. Thus, the azimuths in the computations are approximately correct, depending on the evenness of the course lengths; if some lines are much shorter than others, or if a series of short lines is followed by a series of long lines, errors will be introduced. For most of the work the errors in running the line are far greater than those that develop from this difference in azimuth between stations. It is scarcely realized by the men doing the work that there is any difference.

After the angles are balanced the line is then computed in the ordinary manner by converting the azimuths into bearings and using the sine and cosine to obtain the latitudes and departures. The line is then adjusted to close on the triangulation stations by distributing the error of closure by any method that is justified by the accuracy of the field work.

TABLE 2.—FEET PER SECOND AND PER MINUTE OF LATITUDE AND LONGITUDE

Latitude	Latitude		Longitude		Latitude	Latitude		Longitude	
	Feet in 1′	Feet in 1″	Feet in 1′	Feet in 1″		Feet in 1′	Feet in 1″	Feet in 1′	Feet in 1″
(a) In the Panama Canal Zone; Latitude, 9° N					(b) In the United States; Latitude, 40° N				
8° 56′	6 047.36	100.789	6 013.73	100.229	40° 00′	6 071.33	101.189	4 669.51	77.825
8° 57′	6 047.36	100.789	6 013.46	100.224	40° 01′	6 071.34	101.189	4 668.37	77.806
8° 58′	6 047.37	100.789	6 013.18	100.220	40° 02′	6 071.36	101.189	4 667.24	77.787
8° 59′	6 047.37	100.789	6 012.91	100.215	40° 03′	6 071.38	101.190	4 666.10	77.768
9° 00′	6 047.38	100.790	6 012.63	100.211	40° 04′	6 071.40	101.190	4 664.96	77.749
9° 01′	6 047.38	100.790	6 012.36	100.206	40° 05′	6 071.41	101.190	4 663.83	77.731
9° 02′	6 047.39	100.790	6 012.08	100.201	40° 06′	6 071.43	101.191	4 662.69	77.712
9° 03′	6 047.39	100.790	6 011.81	100.197	40° 07′	6 071.45	101.191	4 661.55	77.693
9° 04′	6 047.40	100.790	6 011.52	100.192	40° 08′	6 071.47	101.191	4 660.43	77.694
9° 05′	6 047.41	100.790	6 011.25	100.188	40° 09′	6 071.49	101.192	4 659.28	77.655

For the computation of lines using this system it is necessary to have a table giving the value, in feet, to the nearest hundredths of feet, of the minutes of latitude and longitude, for each minute of latitude. Such a table, a section of which is shown in Table 2, covering the two degrees of latitude of local interest, consists of one sheet slightly larger than letter size; the table, mounted on cardboard, is easily consulted by computers. When, in summing up the feet of latitude or longitude, the values go above those for 1′, the proper value taken from Table 2(*a*) is subtracted or added to the sums and the next minute placed in the minute column.

Table 2(*a*) can be prepared in several ways. The method that gives the more accurate value in feet is, to make an inverse geodetic calculation to obtain the distance, in meters, between two points on the meridian or on the parallel 60″ apart, at the latitude desired. In the case of a line with an azimuth of 0°, or 180°, the computation for obtaining the distance in latitude reduces to the simple division of the distance between the stations, in seconds, by the factor taken from the tables, as all the corrections contain a multiple of the sine of the azimuth, which is zero in this case, and the other term is divided by the cosine which is unity. The value found by the inverse calculation is in meters and is converted to feet to be included in Table 2(*a*). The computation for the distance in longitude is made in the same manner; the operation of making the inverse calculation requires only one multiplication and one division in this case, in addition to the reduction of meters to feet. These two tables (Tables 2(*a*) and 2 (*b*)) can also be prepared by the use of tables that can be found in *Special Publication No. 5* (6), which give the lengths of the seconds of both latitude and longitude for each minute of latitude in meters. This latter method does not give accurate values in the last decimal place because of a lack of sufficient significant figures in the values given in *Special Publication No. 5* of the U. S. Coast and Geodetic Survey (6).

Methods Used for More Accurate Surveys.—Triangulation is used for locating points with a higher degree of accuracy than by the transit traverse method, or for determining the positions in a truly geodetic manner. Quadrilaterals, using the stations of the existing system, are observed and adjusted. The observations are made with a repeating theodolite, reading to 10″, six complete sets of angles, both direct and reversed, and horizon closures, are turned. The standards of secondary triangulation are obtained without difficulty. The geodetic positions are computed by the use of a standard form (No. 9–902) prepared by the U. S. Geological Survey (4). Values for the corrections can be found in *Bulletin 650* (4), or in *Special Publication No. 8* (5).

ANALYSIS OF ERRORS

Errors in the Computations of Traverses.—The foregoing system of computation for traverse positions does not give exact geodetic positions, but it does give positions that are nearer to the truly geodetic than can be determined with the survey methods used. The line that would yield the nearest to the true geodetic positions of the points would be one with many equal courses. The balanced azimuths would then make the line follow the curve of the earth and the computed positions, to be nearly geodetic. The line creating the worst condition would be that containing one long straight course. The direction of the line also affects the difference between the positions as computed by this system and the true geodetic positions. Lines running north or south are affected very little by the geodetic corrections, being only concerned with the difference between the spheroid of reference and a true sphere, whereas lines running east or west must be corrected for convergence of meridians. For these reasons the computations for errors were made on the basis of a line running due west consisting of one single course.

Table 3(*a*) was prepared by taking an even degree of latitude and longitude as a base and computing geodetic positions of points at varying distances on a straight line beginning at the base, with an azimuth of 90 degrees. These positions are placed in Columns (2) and (4). They would represent triangulation stations at the various distances apart. The difference in azimuth is placed in Column (6), Table 3(*a*), and shows the maximum difference that would appear in the azimuths of lines run between stations at the same distances apart as those in Column (1). The positions of the same points on the same straight line are then computed by the use of the sine, cosine, and table method (see Column (7)). The difference between these positions and the true geodetic positions is shown in Column (8), Table 3(*a*). It will be noted that the errors in longitude are very small. The proportional error is shown in Column (9).

TABLE 3.—MAXIMUM ERROR POSSIBLE

Distance, in thousands of feet	GEODETIC POSITION				Azimuth difference	Longitude by sine and table* (seconds are expressed in feet)	Latitude error, in feet	Proportional error, one in:
	Latitude 8° 59″ plus seconds:	Seconds of latitude, in feet	Longitude	Seconds of longitude, in feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	(*a*) IN THE PANAMA CANAL ZONE; LATITUDE 9° N.							
0......	00.00000″†	6 047.38	79° 00′ 00.0000″	0.00	000	79° 00′ 0000.00	− 0.00	0
1......	59.99996″	6 047.376	79° 00′ 9.9790″	1 000.00	0′ 1.56″	79° 00′ 1000.00	− 0.004	250 000
5......	59.9991 ″	6 047.29	79° 00′ 49.8950″	5 000.00	0′ 7.80″	79° 00′ 5000.00	− 0.09	56 000
10......	59.9962 ″	6 047.00	79° 01′ 39.7898″	3 987.37	0′ 15.61″	79° 01′ 3987.37	− 0.38	26 000
20......	59.9850 ″	6 045.87	79° 03′ 19.5798″	1 962.11	0′ 31.22″	79° 03′ 1962.11	− 1.51	13 000
30......	59.9629 ″	6 043.64	79° 04′ 59.3697″	5 949.50	0′ 49.04″	79° 04′ 5949.50	− 3.74	8 000
50......	59.9061 ″	6 037.92	79° 08′ 18.9495″	1 898.95	1′ 18.05″	79° 08′ 1898.96	− 9.46	5 200
100......	59.6246 ″	6 009.54	79° 16′ 37.899 ″	3 797.90	2′ 36.11″	79° 16′ 3797.92	−37.84	2 600
	(*b*) IN THE UNITED STATES: LATITUDE, 40° N.							
0......	00.00000″‡	6 071.33	79° 00′ 00.0000″	....	....	79° 00′ 0000.00¶	0.00	0
1......	59.9998″§	6 071.31	79° 00′ 12.8493″	1 000.00	0′ 8.26″	79° 00′ 1000.00¶	0.02	50 000
5......	59.9951″§	6 070.83	79° 01′ 04.2466″	330.49	0′ 41.30″	79° 01′ 330.49¶	0.50	10 000
10......	59.9802″§	6 069.33	79° 02′ 08.4934″	661.00	1′ 22.59″	79° 02′ 660.98¶	2.00	5 000
20......	59.9209″§	6 063.32	79° 04′ 16.9865″	1 321.97	2′ 45.19″	79° 04′ 1321.96¶	8.01	2 000
30......	59.8048″§	6 051.57	79° 06′ 25.4794″	1 982.93	4′ 7.78″	79° 06′ 1982.94¶	19.76	1 500
50......	59.5053″§	6 021.28	79° 10′ 42.4663″	3 304.94	6′ 52.97″	79° 10′ 3304.90¶	50.05	1 000
100......	58.0224″§	5 871.22	79″ 21′ 24.933 ″	1 940.41	13′ 45.94″	79° 21′ 1940.29¶	200.00	500

* Latitude, by cosine and table, equals 9° 00′ 00″. † 9° 0′. ‡ 40° 0′. § 39° 59′. ¶ Latitude by cosine and table, equals 40° 00′ 00″.

Table 3(*a*) shows the maximum errors that will be introduced in any line run between triangulation stations which are the distances apart shown in Column (1). It is clear that this maximum error will not be reached in practice as it would require running one straight course between triangulation stations. If this were done the balanced azimuths would be the average of the forth and back azimuths of the line and, if this were used in the sine, cosine, and table method, the line would check exactly. To show a more practical determination of the errors it was assumed that the ends of the aforementioned line were to be triangulation stations 100 000 ft apart, and that a traverse was run between them with points as shown in Column (1), Table 4(*a*). The angles were then balanced over these courses and the traverse was computed in the usual way, using the sine, cosine, and Table 2. The unbalanced traverse is compared with the true geodetic positions and these errors are shown in Columns (7) and (8), Table 4(*a*). The line was then corrected to close on the triangulation point by a simple distribution of the error of closure according to the lengths of the courses, thereby lessening the errors to the values, as shown in Columns (10) and (11). Table 4(*a*) shows the errors in this particular traverse; it would be possible to arrange a traverse that would give any closures and errors desired, but this was considered to be an example of the worst condition ever likely to be encountered in the field. Most of the traverses run will have a better distribution of length than this, and will have errors that will be much less than those given in Table 4(*a*).

TABLE 4.—ERRORS ON EXTREME EXAMPLE

Station (1)	Distance, in thousands of feet (2)	Balanced azimuth*; 89°: (3)	Increment of latitude, in feet (4)	Position latitude (seconds in feet); 8° 59′: (5)	Geodetic position (seconds in feet); 8° 59′: (6)	Error, in feet (7)	Proportional error, one in: (8)	Balance to close, in feet (9)	Final error, in feet (10)	Proportional error, one in: (11)
(*a*) IN THE PANAMA CANAL ZONE; LATITUDE 9° N										
0.....	0	....	....	....	6 047.38	00.00	......	....	00.00	.....
1.....	1	59′ 40.5″	− 0.09	+6 047.29	6 047.38	− 0.09	11 000	+ 0.17	+ 0.08	12 000
5.....	4	59′ 21.0″	− 0.76	+6 046.53	6 047.29	− 0.76	6 400	+ 0.87	+ 0.11	45 000
10.....	5	59′ 01.5″	− 1.44	+6 045.09	6 047.00	− 1.91	5 200	+ 1.74	− 0.17	53 000
20.....	10	58′ 42.0″	− 3.78	+6 041.31	6 045.87	− 4.56	4 400	+ 3.48	− 1.08	18 000
30.....	10	58′ 22.5″	− 4.73	+6 036.58	6 043.64	− 7.06	4 200	+ 5.22	− 1.84	16 000
50.....	20	58′ 03.0″	− 11.37	+6 025.21	6 037.92	−12.71	3 900	+ 8.70	− 4.01	12 500
100.....	50	57′ 43.5″	− 33.08	+5 992.13	6 009.54	−17.41	5 700	....	....	.....
(*b*) IN THE UNITED STATES; LATITUDE 40° N										
0.....	0	....	....	+0 000.00†	0 000.00†	....	......	....	....	.....
1.....	1	58′ 17″	− 0.50	+6 070.83‡	6 071.31‡	− 0.48	2 100	+ 0.92	+ 0.44	2 300
5.....	4	56′ 34″	− 3.99	+6 066.84‡	6 070.83‡	− 3.99	1 300	+ 4.61	+ 0.62	8 000
10.....	5	54′ 50″	− 7.51	+6 059.33‡	6 069.33‡	−10.00	1 000	+ 9.22	− 0.78	13 000
20.....	10	53′ 07″	− 20.02	+6 039.31‡	6 063.32‡	−24.01	830	+18.44	− 5.57	3 600
30.....	10	51′ 24″	− 25.01	+6 014.30‡	6 051.57‡	−37.27	800	+27.66	− 9.61	3 100
50.....	20	49′ 41″	− 60.02	+5 954.28‡	6 021.28‡	−67.00	750	+46.10	−20.90	2 400
100.....	50	47′ 57″	−175.26	+5 779.02‡	5 871.22‡	−92.20	1 100	....	....	.....

* Original azimuth, 90° 00′ 00″. † 40° 00′ plus. ‡ 39° 59″ plus.

Use of This Same System in the United States.—The personnel of the Section of Surveys is hired locally or is requisitioned from the United States from young men with some college training and with general field experience. No attempt is made to find men trained for geodetic work. These men work satisfactorily with this system after a short period of instruction. They soon become familiar with the common geodetic operations and understand the principles on which their surveys are based. The superiority of the system is such that these new men soon wonder why a similar system is not used generally in the United States.

One of the reasons for the adoption and the success of this system of surveys on the Canal Zone is the fact that, being near the equator (the Canal Zone is between Latitudes 8° 55′ and 9° 25′), the effects of geodetic corrections are much less than they are 30° or 40° farther north. Tables 2(*b*), 3(*b*), and 4(*b*), similar to Tables 2(*a*), 3(*a*), and 4(*a*) for the Ninth Parallel of Latitude, were prepared to show the errors that would be developed in using the same system at the Fortieth Parallel of Latitude.

CONCLUSIONS

The conclusions that can be made from the arguments given in this paper are:

(1) Plane surveying is out of date, and any mention of it should be avoided. Any system of surveying that does not fit in with the positions of the National Geodetic Net is doomed to eventual failure.

(2) If it were clearly understood by the engineers of the United States that the computation of geodetic positions had nothing to do with least squares adjustments of any kind, and that these computations were very easy to make, geodetic control would come into common use.

(3) A system such as that described herein is a valuable aid in performing all kinds of field engineering work.

(4) For the greater part of engineering work the lines can be run between the triangulation stations, ignoring the geodetic corrections, because the errors of the measurements in the field are more than those introduced by ignoring the geodetic corrections.

(5) For the few occasions when it is necessary to determine positions that require the use of trangulation and the computation of geodetic positions these positions can be determined by the average engineering organization without special training or instruction.

ACKNOWLEDGMENTS

The Canal Zone system of triangulation was begun under the direction of C. M. Saville, M. Am. Soc. C. E., then Assistant Engineer in charge of the Third Division of the Office of the Chief Engineer of the Isthmian Canal Commission. The method of using the system was begun under his direction and has been continued by the Section of Surveys of The Panama Canal, under the direction of Mr. O. E. Malsbury, from 1913 to 1926, and Mr. R. Z. Kirkpatrick, from 1926 to 1936, as Chiefs of Surveys. Various extensions of triangulation and improvements of tables were made by the Fortifications Division of The Panama Canal, under the direction of Assistant Engineer R. C. Jones, and by the Dredging Division of the Panama Canal, under the direction of the Superintendent, J. G. Claybourn, M. Am. Soc. C. E.

APPENDIX

BIBLIOGRAPHY

The books and publications that were reviewed in the preparation of this paper, and from which information was taken, are:

(1) **The Theory and Practice of Surveying,** by the late J. B. Johnson, M. Am. Soc. C. E., and Leonard S. Smith, M. Am. Soc. C. E.

(2) **Surveying Theory and Practice,** by Raymond E. Davis, M. Am. Soc. C. E., and Francis S. Foote, and W. H. Rayner, Assoc. Members, Am. Soc. C. E.

(3) **Topographic Instructions of the United States Geological Survey,** by C. H. Birdseye, M. Am. Soc. C. E., *Bulletin 788,* U. S. Geological Survey.

(4) **Geographic Tables and Formulas,** compiled by Samuel S. Gannett, *Bulletin 650,* U. S. Geological Survey.

(5) **Formulae and Tables for the Computation of Geodetic Positions,** *Special Publication No. 8,* U. S. Coast and Geodetic Survey.

(6) **Tables for a Polyconic Projection of Maps and Lengths of Terrestrial Arcs of Meridain and Parallel,** *Special Publication No. 5,* U. S. Coast and Geodetic Survey.

(7) **General Instructions for the Field Work of the United States Coast and Geodetic Survey,** *Special Publication No. 26,* U. S. Coast and Geodetic Survey.

(8) **Use of Geodetic Control for City Surveys,** by Hugh C. Mitchell, *Special Publication No. 91,* U. S. Coast and Geodetic Survey.

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(10) **Relation Between Plane Rectangular Coordinates and Geographic Positions,** by Walter F. Reynolds, *Special Publication No. 71,* U. S. Coast and Geodetic Survey.

(11) **Manual of Plane Coordinate Computations,** by Oscar S. Adams and Charles N. Claire, *Special Publication No. 193,* U. S. Coast and Geodetic Survey.

(12) **Grid System for Progressive Maps in the United States,** by William Bowie, M. Am. Soc. C. E., *Special Publication No. 59,* U. S. Coast and Geodetic Survey.

DISCUSSION

PHILIP KISSAM,[3] ASSOC. M. AM. SOC. C. E. (by letter).—A most important and basic conception has been presented by Mr. Sheldon. He has indicated the prime necessity of establishing the position of all horizontal survey points in relation to a single fundamental datum. This principle cannot be emphasized too strongly, and its general adoption at some time in the future is inevitable. All surveys will become in fact parts of one great survey and the inter-relationship between all survey points will be capable of quick determination. Although the advantages of such a condition are too many to enumerate herein, a moment's thought will convince the reader that every effort should be made to attain this aim. As Mr. Sheldon states: "A comparable example is in the use of geodetic levels. Their advantages are instantly admitted and they are used whenever possible, even where profile levels must be run for miles to base some project on the common sea-level datum." A common vertical datum co-ordinates only one dimension, a common horizontal datum co-ordinates the other two. It appears to be obvious that far greater utility of a similar nature will be found in the use of a horizontal datum.

Moreover, two indirect benefits will obtain when a survey is placed on a horizontal datum. It will be usually necessary to connect it with two or more known control points. With such a connection a check on accuracy will be obtained and the precision of results can be raised by adjustment. Furthermore, the permanent marking of surveys will be encouraged, as it will be at once apparent to the engineer in charge that points of such a survey will have a lasting value and it will be well worth while to place monuments at the stations and to record the descriptions and data relating to these monuments. The cumulative effect of monumenting surveys on a standard datum will further advance the acceptance of the system.

Mr. Sheldon's description of the surveying methods in use in the Canal Zone illustrate pointedly the rapid adoption and the utility of such a system when it is known and available. He states that it has been used as a base for all surveying on canal construction since 1911, for control of part of the military maps, as a base for extending triangulation, for municipal construction, and for the boundaries of the Canal Zone and military and naval reservations.

Without question, the standard datum that should be adopted is the North American Datum of 1927, as advocated by Mr. Sheldon; but the method of expressing positions appears to be open to question. Thus far, only two methods have been developed which offer the requisite characteristics; both have sufficient advantages in their favor to merit careful consideration. Mr. Sheldon advocates the use of geographic positions. The U. S. Coast and Geodetic Survey advocates State plane co-ordinate systems based on definite projections especially adapted for the purpose. Such projections have already been designed by that Bureau for every State and in many States they are already in use.

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Geographic positions must be computed in either case for the fundamental first and second-order triangulation; but with the adoption of standard plane co-ordinate systems, the plane co-ordinates of any geographic position can be computed according to the plane system existing in the area. Likewise, the plane or "grid" azimuth can be computed from the geodetic azimuth at any known position.

Although Mr. Sheldon's arguments advocating the use of geographic positions are sound, the arguments he has chosen against plane co-ordinate systems seem to the writer to be valid only when local co-ordinate systems are considered. State-wide systems have not been carefully weighed; in fact, they have been only mentioned by reference to *Special Publication No. 193,* of the U. S. Coast and Geodetic Survey (11).[2] In most areas, these State systems will produce a higher degree of accuracy than the method proposed. Does not the proposed system require reduction from geographic positions to plane co-ordinates, computation by plane co-ordinates, and, finally, reduction back to geographic positions with accompanying loss of accuracy?

It may be stated justly that in many commercial surveys a high degree of accuracy is frequently not of major importance; but lack of precision is not the basic objection to the system proposed. There is a certain fact which has not been given sufficient consideration in Mr. Sheldon's paper. It is a fact not susceptible of mathematical deduction nor scientific demonstration. It was the reason which caused the U. S. Coast and Geodetic Survey, finally, to adopt plane co-ordinates after a long and careful study of the question. It was the basic argument that influenced the Federal Board of Surveys and Maps to recommend the use of plane co-ordinates to its twenty-four member Federal mapping agencies. The fact is that engineers making surveys over a limited area or of low precision refuse to use geographic positions, and are entirely justified in so doing.

To the writer it appears that the cause for this stand is not the lack of a simple geodetic manual, nor the lack of training received by engineers as students, nor any other cause that can be remedied. If a remedy were available, the writer would recommend plane co-ordinates for the present and geographic co-ordinates for the future, because it would take so many years to change the thought in the engineering schools and in the profession that the general adoption of a common datum might be endangered. Mr. Sheldon describes in especially clear and forceful paragraphs, and carefully prepared tables, the simplicity of a certain system of geodetic reduction for small surveys or for surveys of the usual commercial precision. The recommended system can be criticized only for its lack of accuracy. His suggestions could not be better chosen as guides to develop the proper attitude toward geodetic reduction. If the writer could convince himself that geographic positions should be used for the type of surveys under discussion (very large precise surveys must usually be reduced by geodetic methods), he would still recommend the use of plane co-ordinates if only as a stepping stone to the later adoption of geographic positions.

Unfortunately, however, the increase in cost necessitated by the use of geographic positions precludes their adoption for commercial purposes. If the engineer can be shown that he will receive concrete advantages by using a standard datum, he may be persuaded to extend his survey to control monuments; but the extra advantages to him of expressing the stations of his survey in terms of geographic positions instead of plane co-ordinates are so limited, if existing at all, that it is impossible to justify any of the necessary increase in computing costs.

The State plane co-ordinate systems advocated are considerably better than the local systems mentioned by Mr. Sheldon. The State systems are based on carefully devised projections; in general they have a minimum dimension of about 180 miles, a maximum dimension frequently equal to the largest dimension of the State, and a maximum scale correction of one part in ten thousand. It appears that only a small percentage of engineers making ordinary surveys would be required to work in an area where such systems adjoined, and those few who might have to do so could utilize one of the systems throughout their survey. One of the great uses for a standard datum will be for the description of property. In fact it is the writer's hope that the demand for control for property surveys will be so great that it will ensure sufficient control for all surveys. It will be only the greatest exception when a property survey will cross from one State system to another. The boundaries of the systems have been especially arranged to reduce further the possibilities of this occurrence.

In conclusion, it is well to emphasize the fact that Mr. Sheldon has presented an important and timely paper, worthy of the careful study of all civil engineers. He shows the importance of a universal horizontal datum and directs attention to the simplicity of geodetic reduction for ordinary surveys. However, in spite of offense to the mathematical sense that plane co-ordinates invariably produce and also in spite of the difficulties encountered when two such systems join, the accuracy and practical advantages they offer make them invaluable as a means of establishing the use of a universal horizontal datum and are probably the best method of utilizing such a datum in any case.

The question at issue could be stated as follows: Is the increased cost to the private surveyor, of computing geodetic positions for "run-of-the-mill" surveys, justified by the elimination of the errors and difficulties which may occur at the infrequent boundaries of adjoining plane co-ordinate systems? The writer believes that the cost is not justified.

Ralph Z. Kirkpatrick,[4] Esq. (by letter).—After having had an initiatory part in its development, the writer inherited, administratively, the horizontal geodetic control system in the Canal Zone, which is described accurately in Mr. Sheldon's paper.

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[4] Chf. of Surveys, Panama Canal, Balboa Heights, Canal Zone.

In any work of sufficient importance the method is likely to warrant the expense of the requisite secondary and tertiary triangulation and traverses. An interesting application was in the 1911–12 rectangular Canal Zone Land Survey, a sectionalization of lands into 2-km squares. Unfortunately, when 40% of the intersections had been determined in the field, the project was abandoned due to the adoption of the then new military policy of nationalizing all lands and expropriating by purchase all settlers' lands, crops, and improvements.

The flexibility of the system has caused its use in the Canal Zone in virtually all city, cadastral, and topographical surveying (such as the Madden Road and Dam project), whether in the jungle or elsewhere, by the Army, Navy, and the Canal Administration. An interesting and early adaptation was in determining the errors in boundaries of certain Colombian rectangular land grants owned by the Panama Railroad Company. The lines had been run by magnetic bearings and measured inaccurately. Studies of the old documents were made and certain long-time accepted boundary points were tied into the geodetic control, and new controlled traverses were run through the accepted points. Eventually, a position was found for "Point A" (the initial point of beginning in the descriptions of the 1855 grants), for which all conditions were met. It was then learned that a mile-post (mentioned in the description), had been moved many years before. Comparison of the true azimuths through certain of these points revealed a compass variation of 7° 14′ east, virtually checking the variation (7° 15′) on the 1855 map; with that checked, the steps in correcting the boundaries in the field became mere routine.

E. B. Roberts,[5] M. Am. Soc. C. E. (by letter).—A problem is treated in this paper which is vital to surveying and mapping programs—a problem, therefore, the economic importance of which can scarcely be over-emphasized. The author is heartily endorsed in his plea for general use of geodetic control in the co-ordination of surveys. It is thought that this principle has the support of most informed engineers, and that the problem, therefore, is one of means to its end. The means advocated in this paper are open to strong criticism.

Because of its scope, the national geodetic datum is computed and defined in terms of geodetic co-ordinates. The reference surface being a spheroid, the azimuthal and distance relations of points on the surface cannot be determined by plane geometry. Geodetic computation of positions is necessary, as well as other special treatment. Triangulation of any magnitude does not achieve full value unless adjusted by least squares. The checking of geodetic and astronomic determinations in horizontal control surveys involves highly technical problems, and the differences are not always small, as stated by Mr. Sheldon.

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[5] Hydrographic and Geodetic Engr., U. S. Coast and Geodetic Survey, Washington, D. C.

These considerations are not pertinent to the problem, however, because the establishment of the national datum, and the performance of control surveys on a scope demanding geodetic treatment, are provided for by competent Federal and State organizations. The problem is how practicably to handle local surveys, mainly traverses, so they may be defined in terms of, or relative to, the national datum, a sufficient number of control points on that datum being presupposed to be available. Three possible methods are:

(1) Use of geodetic co-ordinates and pure geodetic computations;
(2) Use of geodetic co-ordinates and plane survey computations; and
(3) Use of rectangular co-ordinates on an arbitrary reference plane related to the national geodetic datum.

Method (1).—This method appeals to the geodesist because of its technical perfection. It is true that geodetic position computation in itself is simple enough to be understood by competent surveyors. It is the only form of geodetic computation that would be indicated in general for local surveys. Its use, however, for the computation of points through the mass of traverses comprising ordinary surveying would be laborious and generally unwarranted in view of the development of Method (3). For this reason Method (1) is difficult to popularize.

Method (2).—After stressing the technical unsoundness of plane co-ordinate surveys and the simplicity of geodetic position computation, the author tacitly acknowledges the laboriousness of geodetic position computations for traverse work, and proceeds to advocate Method (2). Its advantages are the use of the geodetic datum, and simplicity, with no laborious computations, and no artifice, such as a plane co-ordinate grid. Its shortcomings in the matter of accuracy are serious.

It involves linear adjustment of azimuth discrepancies and position closure errors in traverses between geodetic control points. Geodetic corrections are grouped with accidental errors for adjustment. This is illogical, because the geodetic corrections have no linear characteristics; that is, they have no accidental distribution.

Table 4 is an inadequate criterion of proportional errors, because it deals only with the special case of straight traverses. It is not an example of the worst condition ever likely to be encountered. Although it is true that most traverses would have better length distribution than this extreme example, it is also true that they would have poorer distribution of relative directions. In the straight traverse used as an example the linear distribution of closure errors tends to approximate the distribution of geodetic corrections. In a meandering or circuitous traverse this is not so. In the case of a traverse closing on its own starting point the algebraic sum of the geodetic corrections is zero, resulting in their complete disregard by the linear adjustment. A moment's consideration makes it clear that Table 4 has no meaning for other than straight traverses.

It is unnecessary to develop this reasoning further. Even if the proportional errors shown by Table 4 were critical, they would still be too large for acceptance for common use in the United States; in fact, they are greatly in excess of those found in modern plane co-ordinate projections.

Method (3).—In plane co-ordinates, linear traverse adjustments concern only accidental errors and, in this case, are logical. Position computational methods are technically rigorous. The source of error lies in the impossibility of correctly representing a spheroidal surface on a plane surface. The error, however, is definitely known, and can be kept small.

The Lambert and Transverse Mercator grids established for the various States by the U. S. Coast and Geodetic Survey make geodetic control available for use on plane rectangular co-ordinate projections the maximum scale errors of which are kept within satisfactory limits. As an example, the entire State of North Carolina was covered by a single projection, only at the very extremities of which did the scale error become slightly greater than 1:10 000. Obviously, throughout the greater part of the State the errors are much smaller. Even so, the scale error, everywhere, is known and readily corrected if necessary.

Admittedly an artifice, and an indirect approach to the problem, such a projection nevertheless is finding ready acceptance. It leaves to competent organizations the performance and computation of necessary geodetic surveys, and the definition of the control points in terms of the projection. The local surveyor has nothing new, either to learn or to do, but to accept a ready-made grid instead of a local, unrelated grid of his own. All the control points will eventually be on it, and his work will be that of plane survey computation in its simplest form. Nevertheless, it will be on the national datum.

From the foregoing it is argued that plane surveying is definitely not "out of date." It is, in fact, finding wider use in connection with the projections described, and bringing the use of geodetic control more and more "into the picture."

H. W. HEMPLE,[6] M. AM. SOC. C. E. (by letter).—Geodetic control expressed on a geographic base has no superior when accurate methods of measurement, and exact methods of computation, are used. Mr. Sheldon is to be commended for endorsing it for local surveying purposes. However, he uses an approximate method of computing whereby plane surveying methods are applied to a geographical base, and the resulting data are expressed in terms of geographical positions. His field of activity is in the vicinity of the equator, and, therefore, he is able to do this without undue violence, because of the small rate of convergence of meridians, and because the area covered by his surveys is of limited extent. Since he can do this, the author immediately assumes the premise that such a method should be generally adopted throughout the United States, and concludes that there is no need for the adoption of the plane co-ordinate systems such as have been devised recently by the U. S. Coast and Geodetic Survey for each of the States.

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[6] Hydrographic and Geodetic Engr., U. S. Coast and Geodetic Survey, Washington, D. C.

The term, "errors," is used rather loosely in the paper. To develop the spheroid representing the earth's surface on a plane it is necessary to apply certain mathematical corrections which are definitely known. These corrections can be applied to measurements made on the earth's surface, in order to relate them to the plane, and they are not errors in the sense of mistakes, blunders, or personal peculiarities of the observer, which apply to surveying observations. Mr. Sheldon considers these corrections, which are mathematically determined, in the same sense as errors of observation.

Table 3, entitled "Maximum Error Possible," purports to show the greatest possible errors which would occur for traverses computed by his method. Table 3 is misleading, however, because traverses do not usually extend in a straight line, but consist of a number of broken courses. He applies a correction for azimuth closures equally to all such courses. In this correction is included the effect of convergence of the meridians. This effect is a variable quantity, depending for any given latitude upon the length of line and its direction. In any given latitude, and for lines of equal length, it should be applied as a maximum to lines in an east-and-west direction, and as a minimum of no correction to lines extending north and south. On a line which bears eastward the convergence should be positive, and on a line which bears westward, it should be negative. Mr. Sheldon, on the other hand, distributes this effect, which is included in the azimuth closure, equally on all lines, even to those running in a north-and-south direction and to those lying in different quadrants. These effects of the convergence of meridians are not always small, and for traverses covering some distance the discrepancy due to their non-consideration, is likely to be considerable. A little study shows that to adopt such a system with its attendant discrepancies generally throughout the United States will lead to endless confusion.

To state that no consideration should be given to the convergence of meridians, because these effects are far overshadowed by the type of surveying accomplished and the accuracy obtained, is to take a rather limited viewpoint. Surveys of an accuracy of 1 part in 10 000, or better, are being executed by engineers and surveyors constantly, and work of this order of accuracy is not exceptional. Instrument manufacturers report that their greatest demand is now for 20″ and 30″ instruments, whereas several years ago the minute transit was most generally sold. This means that the Engineering Profession is becoming alive to the utility and ease of execution of accurate surveys. Certainly as the land becomes more thickly populated, more accurate surveys will be a necessity, and this being the case, the surveying system accepted should be adapted to such needs.

In advocating the method of computing advanced in this paper, Mr. Sheldon states that loop closures should be avoided, and that the courses should be of equal length. Why adopt a system of computation where restrictions such as these are placed upon the field work? Any computing system recommended should be adapted to the various conditions to be encountered in ordinary field operations. The terrain is not always such that courses of equal length can be obtained. It is not always possible to have triangulation stations conveniently located at the ends of a traverse, and it is frequently necessary to make a loop and close on the same station. Certainly, if the conditions of the field work are such as these, the work should not be hampered unnecessarily by restrictions imposed by the method of computation.

The writer has computed three actual traverses: (1) By the method advocated by Mr. Sheldon; (2) by the method of the State plane co-ordinate system; and (3), by geodetic methods (see Table 5).

TABLE 5.—COMPARATIVE ACCURACY OF SURVEYING COMPUTING METHODS.

Traverse No.	State	Length of traverse, in feet	Criteria	(1) Method advocated by Mr. Sheldon	(2) State plane co-ordinate system	(3) Geodetic method
1	Nebraska	80 700	Azimuth difference	9′–48″.3	0′–59″.0	0′–64″.4
			Error of closure	1 to 4 000	1 to 11 700	1 to 11 700
2	New Jersey	27 600	Azimuth difference	2′–14″.4	0′–47″.1	0′–49″.4
			Error of closure	1 to 7 220	1 to 12 800	1 to 12 800
3	Nebraska	69 000	Azimuth difference	7′–55″.2	0′–05″.7	0′–10″.3
			Error of closure	1 to 11 300	1 to 30 200	1 to 30 200

These traverses were chosen at random. The field work was done in accordance with instructions for second-order surveys specifying an error of closure of 1 to 10 000, or better. These results show that the State plane-co-ordinate systems and the geodetic methods of computing are rigid, and that the same field observations give the same closures. If third-order specifications for field work had been adopted, the results from the method advocated by Mr. Sheldon would give closures of less than 1 to 5 000 and the work would not conform to third-order accuracy. For this method, also, it is impossible to determine, for any given traverse, what proportion of error is introduced by the method of computation, and what is due to the field observations. If this method were adopted universally, and closures such as those illustrated were obtained for traverses between fixed triangulation stations, then lines run between stations established on different traverses, would be subject to large discrepancies, and the resulting confusion would be intolerable for any rigid control system.

If an extensive system of triangulation is required, the geodetic methods of computation should be used. These computations are not especially difficult or involved, and should be readily understood by any graduate of an engineering school, who has pursued the usual mathematical courses. The resulting data are expressed in terms of latitude and longitude. These terms can be readily converted to the plane co-ordinate system adopted for the State, and on that projection any traverse can be computed with greater ease to any required degree of accuracy than would pertain to computations of traverse by geodetic methods or by the method advocated by Mr. Sheldon.

These State systems of plane co-ordinates were adopted at the urgent request of practical engineers who wished to make use of the advantages accruing to their surveys through connection to the national net of geodetic control, but wished the mathematics of relating their surveys to such points to be as simple as possible, consistent with attaining the accuracy desired.

In devising the plane co-ordinate systems for the various States, two general forms of projection were used. For those States the greatest extent of which is in an east-and-west direction, the Lambert conformal projection was adopted, and for those States lying mostly in a north-and-south direction, the transverse Mercator projection was used. The earth's spheroid at sea level was developed on these projections for areas the extent of which was determined by the limiting scale factor necessary to bring the sea-level surface to that of the plane. In most cases this limiting correction, or scale factor, was kept less than 1 part in 10 000. This limitation necessitated more than one zone in most of the States. These scale factors vary according to the distance from lines of exact scale which, in the Lambert projection, consist of two standard parallels of latitude, and in the transverse Mercator projection of two lines on either side of the central meridian and parallel to it. For an area 158 miles in width, the maximum scale factor is less than 1 part in 10 000. The scale factor varies as the distance from a line, and not from a point of origin, as stated by Mr. Sheldon.

For any surveys within these zones which are connected to the national control net, and are expressed in terms of State plane co-ordinates, the distortion due to placing the surveys on the projection is, in most cases, less than 1 part in 10 000. If the scale factors are applied to the linear measurements, which is not a difficult matter, then the limit of accuracy is dependent almost solely upon the standards of the field measurements.

It is true that at the junctions of the plane co-ordinate systems, there is a jump from one zone to the other. To obviate any difficulties due to traverse lines extending from one zone to another, a sufficient overlap is provided at the junctions, so that any traverse may be placed on either one zone or the other. It is a simple matter to convert the co-ordinates of one zone to those of the other at the point of overlap.

In making use of plane co-ordinate systems for the computation of traverse surveys, a connection is made at each end of the line to triangulation or traverse stations previously located on the national system of control surveys; or the traverse may be a loop and tie to the same initial point at the end of the survey. The positions of the stations to which connections are made are expressed in terms of x and y-co-ordinates with a grid azimuth to a definite point near-by, all on the plane co-ordinate system adopted for the State. The grid azimuth is carried through the traverse from the starting point to a closure at the station at the end of the traverse. The azimuth closure is then distributed equally among the angles of the traverse. The linear measurements are reduced to the horizontal, and if they are above about Elevation 500 they should also include the correction to reduce to sea level. If the maximum of accuracy is desired, the scale correction should also be applied. As previously stated, in most cases, this scale correction is less than 1 part in 10 000 and may be ignored if the accuracy desired does not exceed that criterion. The survey is then adjusted by the method of "latitude and departure" which is so generally known to surveyors and engineers.

The United States is not alone in the adoption of the plane co-ordinate systems for general surveying purposes. Such projections have long been in use in many European countries. Most of these countries are thickly settled and land values are high. Methods of a high degree of accuracy are required for property surveys. These countries have found a plane co-ordinate base adequate for their needs. Should not this country benefit by the experience of others? As the population of the United States becomes greater and land prices increase, surveying standards pertaining to property must be on a higher plane than has generally prevailed to this time. Plane co-ordinate systems, mathematically determined and related to the spheroid, to which field surveys can be referred by utilization of the simple methods of plane surveying, are entirely logical and consistent, and furnish a simple and readily understood method to which engineers and surveyors may turn with confidence that the accuracy desired will be obtained.

These systems are finding ready acceptance among engineers and surveyors. Two States—New Jersey and Pennsylvania—have already adopted laws legalizing the use of the State plane co-ordinate system for property description purposes. Several other States are also considering similar laws.

As to Mr. Sheldon's conclusion that plane surveying is out of date, if by this statement he means relating surveys to a State plane co-ordinate system, the writer takes issue with him. These systems are mathematically logical, and the surveying data expressed thereon are consistent among themselves. Surveyors need have no hesitancy in relating their surveys to such systems with assurance that, if the recommended methods of computing are followed, the accuracy of their data will be limited only by the standards of the field measurements.

J. C. CARPENTER,[7] M. AM. SOC. C. E. (by letter).—Any argument for the universal use of horizontal geodetic control should receive the unanimous endorsement and active support of the entire Engineering Profession. This basic scheme for reference of surveys is the only satisfactory method, and its general adoption seems so logical that it is difficult to understand the indifference displayed by the majority of civil engineers. Before engineers can attain the objective of the universal use of horizontal geodetic control they must enlist, not only engineers, but administrators, executives, and legislators, in a campaign to promote the use of this extremely valuable datum of reference. Engineers are the only members of society who can be brought to realize the value of horizontal control, and the responsibility for this educational work will fall on the shoulders of a small group who are enthusiastically determined to carry the campaign to its logical conclusion. Papers of the type presented by Mr. Sheldon are necessary to bring the subject to the attention of engineers throughout the country.

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[7] Senior Highway Engr., U. S. Bureau of Public Roads, Fort Worth, Tex.

Perhaps the "wish was father to the thought" in Mr. Sheldon's statements that the main triangulation net of the U. S. Coast and Geodetic Survey is nearing completion. About 65 000 miles have been completed in the field and partly computed and published. The final spacing of 25 miles between arcs, will require 117 000 miles. Hence, the actual work is scarcely half finished. True, the projection of the remaining cross-traverses will be easier and less expensive, but there will be some work necessary on the older nets and a large volume of computation and adjustment required so that a conservative estimate places the project now (1937) at about 50% complete. The only way to complete the remainder of the work is an educational campaign among engineers, first of all, and an insistent demand from members of the profession for the completion of the net to a 25-mile spacing.

Similarly, Mr. Sheldon's optimistic statement that some States have begun to tie their extensive highway surveys to the national net is questioned. A large mileage in North Carolina has been tied to the network. Texas made a start on one survey but has never finished that one project and seems indifferent to the entire method. If the chief engineers of the forty-eight State Highway Departments of this country would spend enough time on a study of the possibilities contingent upon the use of geodetic control for highway surveys, and then give the scheme a fair trial in the field, they would leave a permanent legacy of basic control for all planning operations, all mapping procedure, cadastral surveys, and engineering surveys of every character which would be as valuable as the transportation facilities they provide, and, undoubtedly, its permanency would be equal to, or in excess of, the surfaces now being constructed. Geodetic coordinates tied in to the equator and to Greenwich meridian, converted to x and y-co-ordinates for ready use by the most imcompetent land surveyor, spaced along the highways, monumented, and the co-ordinates stenciled in the monuments, will bring the use of this system to a practicable possibility.

Mr. Sheldon has correctly stated the reasons engineers fail to use the network as a control, even if they find a monument within usable distance of their survey, but there is no excuse for the modern engineer to avoid the use of the control, for the Coast and Geodetic Survey has developed a simple and practical method of conversion from spherical to plane coordinates and when the x and y-values are known, it becomes a matter of the use of familiar latitude and departure surveying. O. S. Adams[8] has written a clear and convincing description of the procedure that has been developed by the Coast and Geodetic Survey for the engineers of the United States to use in making their surveys of permanent value. Mr. Sheldon has listed publications that may be consulted by interested surveyors. There are two very important publications of the Coast and Geodetic Survey, which are not mentioned, namely, *Special Publication No. 194*,

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[8] "State-Wide Systems of Plane Co-Ordinates," *Civil Engineering*, January, 1937, p. 33.

"Manual of Traverse Computation on the Lambert Grid," and *Special Publication No. 195*, "Manual of Traverse Computation on the Transverse Mercator Grid." These excellent publications outline in clear and concise form the procedure to be followed in using plane co-ordinates. Every detail is covered and each possible problem is illustrated by examples. Plane co-ordinate tables for the State may be obtained from the Coast and Geodetic Survey and any engineer who may decide to use the Geodetic network can obtain complete information and detailed instructions from that agency.

Mr. Sheldon has described the system in use on the Canal Zone and his description admirably illustrates the difficulties, probabilities of error, and complications in closure that may be expected when an isolated area is to be covered by a system of control. This is the best possible argument for the adoption of the plane co-ordinate control adopted by the Coast and Geodetic Survey for all surveys in the United States. This Canal Zone system of control is not suitable for general use in the United States. Without doubt, the national geodetic net will ultimately be used as a control for all surveys, but the realization of this condition will be brought about through the use of the plane co-ordinate systems developed by the U. S. Coast and Geodetic Survey for all the States of the Union. Conformal conic projections flattened to a plane will provide simple, easily used surfaces and allow engineers to use ordinary surveying methods and the co-ordinates will be right-angle x and y-distances, in feet. When the procedure is so simple and easy to apply there is no excuse for any engineer to become frightened and fail to take advantage of the control survey. With the completion of the triangulation system and publication of the positions, together with the plane co-ordinates based on these new State systems, the adaptation of this system for all surveys will be very much facilitated.

Air-mapping has developed rapidly during the twenty years, 1917–37. No doubt many important improvements in methods and technique are just "in the offing." There are numerous agencies operating throughout the United States and the pictures being obtained are undoubtedly superior to any existing maps; but the horizontal control that is necessary to make these pictures display, accurately, the correct and true position of the topography is not in existence and there is some question as to the value of these pictures in the permanent mapping method for the entire country. It is a case of "the cart before the horse" and, in this instance, the cart may be found to be of limited value.

The geodetic network must be completed by the agencies of the National Government. Its completion is the first important step in the development of a logical plan for mapping, planning, construction, and maintenance of public and private works. Engineers are competent to explain the immense value and vital necessity of this work and, therefore, the "Practical Use of Horizontal Geodetic Control" warrants the attention of all members of the profession, and its application should be actively encouraged for all surveys.

George D. Whitmore,[9] M. Am. Soc. C. E. (by letter).—The proposition set forth by Mr. Sheldon, to compute all horizontal surveys on the spherical co-ordinate base, unquestionably has considerable merit. He convincingly urges engineers to use the ideal spherical co-ordinate basis for preserving survey results, and for rendering such results universally usable. It is believed, however, that the procedure he proposes is too far ahead of the times. Perhaps the end result which he describes could be better accomplished by getting to it more gradually—by easy stages or steps.

Many surveyors and engineers of the present time, for example, although understanding generally the mathematics and theoretical value of rectangular co-ordinate systems as computing and plotting bases for survey results, nevertheless do not clearly realize all the every-day practical advantages to be gained by using even this simple type of co-ordinate system. Hence, a majority of engineering surveys to-day are executed and used without benefit of any kind of a co-ordinate base—spherical or plane. From the standpoint of the engineers' education, therefore, it might be preferable to stress, for a few years at least, the importance and advantages of any system of co-ordinates, rather than to try to plunge at once into full use of geographic co-ordinates. After all, the use of co-ordinates, whether they are spherical or rectangular, is only a means to an end. The primary purpose of any co-ordinate system is to show by a simple mathematical expression the relationship of each point to all other points of the system. The units of measure used, whether feet, meters, or degrees of arc, are only a means of writing this expression.

Most engineers appreciate the necessity for some kind of co-ordinate basis in constructing maps covering large areas, especially if several map sheets are involved. If the maps cover extremely large areas, such as a State, the necessity of spherical co-ordinates is not questioned. If the maps cover smaller areas, such as a city or county, the chances were, until quite recently, that the maps would have been based on some kind of a local plane co-ordinate system. Thus, in strictly mapping operations, the need for a co-ordinate basis is not questioned. Either through inertia or lack of complete understanding, however, engineers have not as a body carried this same reasoning into their engineering surveys. It seems apparent to the writer that what is good practice for mapping is also good practice for surveying.

Perhaps the principal reason why engineers have been slow in using co-ordinate systems in surveys is their realization of the shortcomings of local plane co-ordinate systems, so well explained by Mr. Sheldon. It so happens that there is now available, however, a type of plane co-ordinate system which overcomes many of the disadvantages of such local systems, but which retains many of their advantages. This is the State Plane Co-Ordinate System developed and published for each State by the U. S. Coast and Geodetic Survey in 1934. So much has already been written regarding these State-wide plane co-ordinate systems that there is no need to repeat the descriptions herein. It might be stated in summary, however, that these systems extend over large areas without serious distortion in either distances or positions. Such distortion as does exist in distances or co-ordinates is exactly known from the relation between the spheroid and plane, and, hence, can be determined instantly in case more precise distances or positions are reqired. These systems, on the other hand, have all the advantages of local plane co-ordinate systems in simple, easy computations. They have also the principal advantage of spherical co-ordinates in that they are related directly to all the basic control surveys of the country; it is a matter of only a few moments of computing to convert plane co-ordinates to geographic, and *vice versa.*

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[9] Chf., Surveys Section, Eng. Service Div., TVA, Chattanooga, Tenn.

Thus, it seems apparent that the State-wide plane co-ordinate systems, representing a nice compromise between the universal geographic and the local plane systems, will be the basis, if any, used by engineers in the computation of their surveys during the next several years. Regardless of the real merit of getting all surveys on the universal spherical base, it will take several years and much more propaganda than a few papers in the technical journals to convert engineers to the actual practice.

The experience of the surveying organization of the Tennessee Valley Authority, which adopted the State-wide plane co-ordinate systems soon after their publication, is gratifying. Until that time, all routine reservoir surveys were computed on local plane co-ordinate bases. All valley-wide projects (usually extending over parts of seven States), such as basic planimetric maps, topographic maps, and regional maps, were plotted on the geographic co-ordinate base. Extensive mapping projects similar to these latter are still plotted on the geographic co-ordinate system, with the principal control surveys being computed in terms of spherical co-ordinates. The traverses serving as detailed control for such maps are computed either on the State plane system or the spherical system, whichever happens to be most convenient. Since such map sheets show the projection lines for both systems, either basis may be used in plotting.

All reservoir maps and surveys, however, since they extend over smaller areas (each reservoir being usually entirely within one State), are now computed on the State plane co-ordinate bases. Such surveys include traverse control; large-scale topographic maps of dam sites, camp sites, and town sites; cadastral surveys and maps for land acquisition; transit-tape traverse of the final reservoir property boundary; ranges for measuring future silt deposits; transmission line surveys, and many other types. In the computing unit, where a large volume of computations are handled, using both geographic and plane systems, there is practically unanimous opinion that the State-wide plane systems fulfill a long-felt need, that they must displace local plane co-ordinate systems, and inevitably will be used by practical engineers rather than the more cumbersome and difficult spherical system.

R. M. Wilson,[10] M. Am. Soc. C. E. (by letter).—Many questions that arise in the use of horizontal-control surveys are included in this interesting paper. One of the most important of these is: In what form should the results of control surveys be expressed to meet the requirements of the many kinds and classes of dependent surveys? The basic net of first- and second-order triangulation, reaching into all parts of the United States, provides a rigid framework by which to determine the correct horizontal position on the earth's surface of all other surveys that are connected with it.

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[10] Chf., Section of Computing, U. S. Geological Survey, Washington, D. C.

It is well understood that the geographic co-ordinates of a triangulation station identify, exactly, the position of that station on the surface of the earth. It is true that the rectangular co-ordinates of that station also identify its position exactly, provided the rectangular co-ordinate system is based on a formal plane projection of the geodetic reference system.

The standard rectangular co-ordinate systems for the various States developed by the U. S. Coast and Geodetic Survey are based on mathematically accurate principles. They provide the means to transform geographic co-ordinates into rectangular co-ordinates, and *vice versa,* for any particular point in a survey, without in the least jeopardizing the identity of the position to which the co-ordinates refer. It is apparent, therefore, that co-ordinates in either form represent equally well the position of a point as determined by geodetic surveys. One form is merely a translation of the other. It should be noted that the results of first- and second-order triangulation contained in the more recent *Special Publications* of the Coast and Geodetic Survey give the co-ordinates in terms of latitude and longitude and also in terms of x and y referred to the standard State systems.

It is left to the discretion of the user, therefore, to decide which form of co-ordinates will best suit his particular requirements. It is not at all necessary, however, that an engineer must accept one form for all surveys to the total exclusion of the other. In general, geographic co-ordinates are desirable in projects that are geographic in scope, that is, those that involve large parts of the earth's surface. Rectangular co-ordinates are the natural form in which to express detailed and intensive surveys within small areas, and surveys made in connection with the development of local engineering projects.

Mr. Sheldon mentions the U. S. Geological Survey as one of the organizations using geodetic co-ordinates. The topographic "quadrangle" maps issued by the Survey are integral parts of the topographic map of the United States, and it is logical that the map should be subdivided according to uniformly spaced meridians of longitude and parallels of latitude. The units are thus arranged in an orderly and systematic manner throughout the country from Maine to California and from Florida to Minnesota. There is no discontinuity in arrangement because of State lines. It is obvious, therefore, that geographic co-ordinates are the best form in which to express the positions of control-survey stations used in the standard mapping operations of the Geological Survey, because this mapping is geographic in scope. The system just described was selected by the Geological Survey before 1880. Even if the State rectangular co-ordinate systems had been available for use at that time, any system based on them could not have been considered as comparable in usefulness to the one actually chosen and in use to-day for subdividing the topographic map of the United States.

The Geological Survey recognizes the value of the State rectangular co-ordinate systems, however, and is now publishing many of its standard topographic maps with marginal marks to show where the grid lines of these systems may be drawn. Any engineer can then draw the grid lines, and so plot upon the map, or scale from it, any positions expressed in rectangular co-ordinates in which he may be interested.

The kind of transit traverse described by Mr. Sheldon, with its results computed in terms of geographic co-ordinates, is not new. The Geological Survey began using it for the control of topographic mapping as early as 1893. During the 20 yr since 1916 a total of about 100 000 lin miles has been done, and in 1935 alone the Geological Survey executed 9 204 lin miles of transit traverse. The field and computing methods have become more accurate and efficient with the passing of time, so that these control surveys have remained consistent with the increasing accuracy of the mapping based on them. Frequent azimuth observations are made by the simple astronomic methods familiar to surveyors to avoid any appreciable accumulation of azimuth error in the traverse.

The conversion factors given by Mr. Sheldon in Table 2 are the reciprocals of the M and P factors used by the Geological Survey to express the results of its traverse in the form of geographic co-ordinates. The proved usefulness of these surveys justifies the attention that Mr. Sheldon has given them.

The standard co-ordinate systems developed by the Coast and Geodetic Survey are designed to be State-wide in application. Nevertheless, any individual engineering project or survey that involves an entire State is extensive enough to be considered geographic in scope, so that geographic co-ordinates ordinarily would be appropriate to use for it.

In surveys extending over an entire county, or over a compact group of several counties, rectangular co-ordinates are convenient to use. Then, because the systems are continuous throughout large zones in each State, any county survey usually will be consistent and continuous with surveys made in adjacent counties. Discontinuity exists only along State boundary lines and along the lines separating the large zones; but the systems provide an overlap, so that no ordinary survey need be separated into parts because it lies across a State or zone line.

Many county surveys, and nearly all city surveys, are already referred to some rectangular co-ordinate system. Most surveys for general engineering projects, for mining or other local projects in which accurate and intensive work is required, use rectangular co-ordinates. The State systems provide the means to express the results of these surveys in common terms, so that they may blend harmoniously and so that each may have the benefit of the results of others adjacent to it. To use these systems no real change is needed in the methods already employed in plane surveying beyond accepting the co-ordinates for the origin and the azimuth of an initial line as given by the basic control surveys. Of course, the same object—harmony—could be attained by using geographic co-ordinates, but these may not be convenient to use in local projects. The State systems provide for harmony without losing the advantages of plane-surveying methods.

It would be ridiculous to use figures expressing latitude and longitude in setting column foundations for structures or for calculating volumes in earth-work. It would be equally ridiculous to struggle with plane rectangular co-ordinates, referred to the State systems, in plotting a map of a continent or in computing a far-reaching arc of triangulation. Nevertheless, the standard State co-ordinate systems can hold, in rigid relationship with one another, surveys that differ in extent and scope even as widely as these.

JULIUS L. SPEERT,[11] JUN. AM. SOC. C. E. (by letter).—The author gives voice to a complaint that undoubtedly has been felt by many engineers, but has seldom been mentioned. It is unfortunate that the relative simplicity of many kinds of surveying computations should be masked by complicated explanations or instructions. This condition is probably due to the fact that, as far as the writer knows, the theory of surveying computations is to be found, in general, in only two types of publications; namely, college textbooks, and Government publications. By its very nature, a college textbook must approach its subject gradually, emphasizing the method of approach rather than the ultimate conclusion. The voluminous explanation necessary in this type of treatment makes the book unsuitable as a practical working manual. Similarly, Government publications on surveying, as a rule, are scholarly treatises, written by, and of value to, scholars and mathematicians, but too frequently unintelligible to the average surveyor, whose knowledge of higher mathematics is somewhat limited and who has little desire to become involved in its ramifications.

If there were available a concise manual of instructions in which the method of performing the computations was clearly explained and the significance of each step was briefly stated, there would be less tendency for the young engineer and the practical surveyor to shun precise methods of computing and recording surveys in favor of the simpler methods of plane surveying. The mathematical justification for the various steps should be included in the appendix to this manual, where it would be available to the mathematically minded but would not confuse and bewilder those less gifted.

The author's proposed remedy is to simplify the use of geodetic control by substituting for the more rigorous computations what amounts almost to the methods used in plane co-ordinate computation. For the Canal Zone, where the maximum convergence of meridians amounts to not more than 5′, this method appears to be satisfactory. However, examination of Table

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3(*b*) and Table 4(*b*) shows that for latitudes in the United States excessive errors might readily be introduced into surveys of any appreciable extent. The author offers no simple method for evaluating these errors.

There can be little doubt that the development of the State plane co-ordinate systems, published by the U. S. Coast and Geodetic Survey in *Special Publication No. 193* (11)[2] constitutes the greatest advance in recent years in the simplification of surveying computations. These systems permit computations as simple as those of plane surveying and yet make it possible to tie any located point rigidly to the national geodetic-control net as well as to every other point tied thereto. Although a true, undistorted representation of the earth's surface is impossible on a plane, the State systems permit a precise evaluation of the error involved in the computed position of any point, thereby, in effect, eliminating that error by making possible a compensation for it.

To make the geodetic-control net more readily available for the use of plane co-ordinates, the U. S. Geological Survey has recently published, in pamphlet form, a set of formulas and tables by means of which the conversion of geodetic co-ordinates to plane co-ordinates on the State systems may be greatly simplified.[12] These formulas and tables are designed to be used with a computing machine and natural functions, thereby dispensing with cumbersome logarithms, and greatly reducing the labor and time required for the conversion. Complete detailed instructions are included in the pamphlet and, for those interested in the theory, the derivation of the formulas is explained in a short appendix.

With all the facilities now available for tying any survey to the national geodetic control net, there should be little excuse for an isolated, uncontrolled survey, and there is no longer any need for a part-way compromise between geodetic control and its practical application.

R. C. SHELDON,[13] ASSOC. M. AM. SOC. C. E. (by letter).—Although he was pleased with the several interesting and instructive discussions that were presented, the writer would have been more pleased if the opinions of those engineers interested in small local surveys had been presented.

Several of the discussers have stated that the system of survey computations as presented can be criticized for its lack of accuracy. The system is a combination of geodetic and plane computations and is used in such a manner that the accuracy of the reported position of any point is only limited by the accuracy of the survey methods, or by the accuracy of the basic control system. This is accomplished by the following steps:

(1) Geodetic positions are obtained for scattered points covering the area. This is the base control system, comparable to the control net, both primary and secondary, of the U. S. Coast and Geodetic Survey.

(2) Surveys run between these scattered points are usually computed by plane methods. The computed positions are reported as geodetic positions,

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[12] "Formulas and Tables for the Transformation of Geodetic to Plane Co-Ordinates on the Lambert and Transverse Mercator Projections," by J. L. Speert, U. S. Dept. of the Interior, Geological Survey, 13 pp.

[13] Asst. Engr., Section of Surveys, The Panama Canal, Balboa Heights, Canal Zone.

the residuals beyond any minute line being carried in feet. This plane calculation gives positions which are not the same as those that would be obtained by a geodetic computation of the same traverse. This discrepancy was called an error in the paper. Table 3(*a*) and (*b*) gives the maximum possible discrepancy. Table 4 shows that this maximum is greatly reduced in practice.

(3) In cases where the points of the base system are so far apart that the error, as shown by Table 3 (the rigid use of which is modified by Table 4), approaches that of the specified error of the survey, additional triangulation points are put in and geodetic computations are made to obtain their positions. It is seldom necessary, locally, to place any of these additional points.

Mr. Hemple states that Table 3(*a*) and (*b*) is misleading because traverses do not usually extend in a straight line, but consist of a number of broken courses. The discrepancy between positions determined by geodetic computations and those found by plane computations, as well as the convergence of meridians, is caused by the fact that straight lines run on the surface of the earth by transit or other means are curved lines when referred to the sphere as a whole. With the standard assumption that the meridians of longitude all converge at the pole and the parallels of latitude are all parallel to the equator, the effect of this curvature of straight lines varies with their direction, being nothing for lines running due north or due south and a maximum for lines running due east or due west. The degree of this curvature also varies with the latitude, being nothing at the equator and a maximum at the pole. This latter variation would be small over the extent of latitude covered by the type of surveys under discussion, and can be ignored.

To present a clear picture of the relation between convergence of meridians, azimuth discrepancies, geodetic computations, and plane computations, it can be stated that a geodetic computation is the plane computation of a line that has been corrected for convergence of meridians in sections of equal length, those sections being of infinitesimal length.

From the foregoing and from an examination of the formula for computation of geodetic positions given in *Special Publication No. 8* (5)[2] of the Coast and Geodetic Survey, it is apparent that the discrepancies between plane and geodetic computations as well as the effect of convergence of meridians are almost entirely dependent on the difference in longitude of the ends of the line concerned. It is impossible to find any group of lines, running in any direction, that will have a difference in longitude greater than that of a line of equal length running due west.

Table 3(*a*) and (*b*) gives the maximum difference between positions determined by plane computation and positions found by geodetic computation, at the various distances from the point of origin. Table 3(*b*) would be entirely too rigid to use as a criterion for spacing points of the primary or secondary system on the Fortieth Parallel of latitude. The writer prepared Table 4 to show that this maximum difference was greatly reduced in practice by balancing the angular difference, due to convergence of meridians, and the error of traverse closure, due to neglect of geodetic corrections. Table 4 shows that for a straight-line traverse of 100 000 ft, the maximum error, from Table 3(*b*), would be 1:500, the unbalanced error of closure would be 1:1 100, and the position error of the worst point would be 1:2 300. Thus, the proportional errors shown by Table 4 were roughly one-half those of Table 3 for unbalanced traverses and one-fourth those of Table 3 for balanced positions. Mr. Hemple has submitted Table 5 showing the results of calculations of three separate traverses. The writer has prepared Table 6 to show the errors that would have been expected in these traverses if Tables 3 and 4 had been used as a guide. It would appear that Table 4 was a conservative estimate of the worst condition ever likely to be encountered in practice.

TABLE 6.—COMPARISON OF EXPECTED TO ACTUAL ERRORS

Traverse No.	Length, in feet	Maximum error expected (Table 3(*b*))	Unbalanced error expected (Table 4(*b*))	Unbalanced error obtained (Table 5)
1	80 700	1 : 750	1 : 1 500	1 : 4 000
2	27 600	1 : 1 500	1 : 3 000	1 : 7 220
3	69 000	1 : 900	1 : 1 800	1 : 11 300

It is unfortunate that Mr. Hemple did not extend Table 5 one place farther and show the discrepancy between the final positions as computed by the geodetic methods and as determined by the writer's method. This final position is the ultimate object of any survey. The writer feels certain that the discrepancy of this final position, in all three cases, would be less than the specified accuracy of the survey.

In the three examples given, even if the plane method of calculation suggested by the writer were satisfactory, he would not advocate its use on traverses of the type shown; that is, on traverses of sufficient importance to justify:

(1) The measurement of horizontal angles over these distances with sufficient accuracy to obtain closures consistently of less than 1′;

(2) Chaining with the accuracy and care necessary to secure 1:10 000 closures;

(3) Observing and computing the temperature corrections necessary; and,

(4) The reduction of the calculations to sea level.

The percentage of the total cost of the survey of computing either all or several sections of the traverse by a geodetic computation will be very small.

It is to be clearly understood that the methods presented in the paper are used as an extension of an existing system of primary and secondary control and are not used to obtain that control.

Both Mr. Roberts and Mr. Hemple gave the fact that loop traverses were not permitted in the proposed system as one of its disadvantages. This writer agrees with Professor Kissam that the fact that it does prohibit loop traverses is one of its advantages. Loop traverses have so many of the same qualities of loose ended lines that they are prohibited on any survey that is based on any general control system. On any survey unimportant enough to be based on one tie to the base net, or to the supplementary net, the error taken from Table 3 without any correction for angle balancing would not be large. For example, from Table 3(*b*), a line running for a distance of 10 000 ft from its origin must be at least 20 000 ft long and would be in error at its extremity less than 2 ft—an error of 1 part in 10 000 ft of line and an error of position of 1: 5 000.

Mr. Roberts criticizes the linear adjustment of azimuth discrepancies and position closure errors in traverses between geodetic control points. There would be many ways of correcting these azimuth discrepancies and geodetic corrections. The one chosen by the writer was the least accurate but was used because it was far simpler than any other. It requires no extra effort, the geodetic corrections and azimuth corrections being combined with the accidental errors for adjustment. Since these discrepancies and azimuth corrections are kept well below the specified accuracy of the survey the exact balancing of these small corrections is not important.

Traverses that are tied to any base net must originate at one of the points on this net and terminate at, or tie to, another unless they are the objectionable loops previously mentioned. In order to do this they must have a general direction, which is that of the line between the stations, although there may be courses in the traverse that will run exactly opposite this general direction. In this manner all traverses between points on the base system take on generally the characteristics of a straight line between the points. Any adjustment that corrects the closing error has a tendency to bring the positions of the several points nearer the geodetic positions. The fact that, in cases where the individual course runs opposite to the general direction of the traverse, the correction is applied in the opposite sign from the true correction is not as damaging as it would seem, because the corrections applied to the predominant number of courses in the traverse are applied in the right direction and tend to correct the discrepancies of all the positions.

As Mr. Wilson states, the method of transit traverse and computation given in the paper is not new and has been used by many organizations for years. The U. S. Geological Survey uses astronomical observations for azimuth to keep the effect of convergence of meridians at a minimum whereas, in the Canal Zone system, more frequent ties to the base system are used to accomplish the same result.

Professor Kissam asks the question, "Does not the proposed system require reduction from geographic positions to plane co-ordinates, computation by plane co-ordinates, and, finally, reduction back to geographic positions with accompanying loss of accuracy?" The system can best be explained as a plane co-ordinate system in which the spaces between successive minute lines of latitude and longitude are made the planes. The small plane areas that are enclosed by the minute lines are thus made to join at the lines, and there is no distortion of discrepancy when crossing from one to another. The only reduction that is necessary is the one that is required on any plane co-ordinate system; that is, the positions of the points on the base geodetic system must be converted to the plane co-ordinate system used. In the proposed system this reduction is made very simple, only the conversion of the seconds of latitude and longitude to feet being necessary. All points are used with that residual beyond any minute being given in feet.

Many of the discussers have advocated the State-wide plane co-ordinate systems that have been devised by the Coast and Geodetic Survey. These systems have met with much success in the years since the writer began preparing the paper under discussion. Their use is now advocated by many important organizations. They are, as Mr. Hemple points out, the law in at least two States, all of which is part of the reason why the writer feels some hesitancy in presenting the criticism of these State-wide plane co-ordinate systems that should have been made years ago. A greater part of this hesitancy is due to the fact that where they have been adopted and are in use, they will give surveyors and engineers a method of tying their surveys, that will be entirely satisfactory for most uses.

There are some points to these plane co-ordinate systems, however, that should be given full consideration before they are adopted in any State and required by law for land boundary description.

The use of the State-wide plane co-ordinate systems, or any other system that does not use the parallels of latitude or the meridians of longitude as a key, requires the introduction of the *X* and *Y*-designation, which is an added factor that engineers, surveyors, but more especially laymen, must learn to understand.

The State-wide systems still have the same objectional feature that is inherent in any plane co-ordinate system, in which the planes are continued for any distance; that is, a grid azimuth must be used. This is certain to cause considerable confusion especially in counties that already have grid azimuths for their own local plane system that will be different from the new State-wide grid. For many years to come, it will be necessary in those places, to mark every azimuth with some identification to show whether it is on the old grid, the new grid, or true azimuth. The Engineering Profession, which is just beginning to understand that there is a true and constant north, is to be now taught, that north is not north, but something else at right angles to the grid system. Although it is a relatively easy problem to identify the system on which any point may be recorded, as the figures will usually be much different, the identification of an unmarked azimuth will be difficult.

One of the advantages of any control system is that any point on a large project is identified and compared to any other point in the system by its co-ordinate positions. In intrastate work the plane system of co-ordinates covering that State does this very well. The tendency now is to forget political boundaries, however, and to base projects and surveys on natural features. Mr. Whitmore mentions that the TVA covers parts of seven

States. Seven different State systems of co-ordinates are used and then the geodetic system is used to correlate them all. If a system as advocated by the writer had been adopted, the same amount of field and office work would have placed the entire project on the one system.

The projections of the State-wide plane co-ordinate systems have been arranged so that the curved surface of the earth covering entire States has been squeezed in at the centers and spread out at the edges to form a plane. The degree of distortion caused by this squeezing has been kept below the ratio of 1: 10 000. For all surveying work for which it is desired to have an accuracy finer than this, certain corrections must be made. This fact will tend to place a restriction on the more accurate surveys. Surveys that on an unrestricted basis would be run to finer accuracies than 1: 10 000 will be run on this standard because of the inclination on the part of engineers not to use any more corrections than necessary. In this manner, progress in more accurate surveying will be retarded.

The use of State-wide plane co-ordinate systems has been mentioned by both Professor Kissam and Mr. Whitmore as a possible approach to the eventual universal use of geodetic control. The writer believes that, rather than being a stepping stone, the use of the State-wide plane co-ordinate systems will act as a wall stopping any future approach to that universal use. Plane surveyors will be content to follow the systems that are given them, and will become, if possible, a lower ranking branch of the profession than they are at present; and the knowledge and practice of geodesy will be confined to the employees of the U. S. Coast and Geodetic Survey.

The basic criticism of the writer to the State-wide systems of plane co-ordinates, or any other system of co-ordinates except geodetic, is that they are unnecessary. When the control system of the U. S. Coast and Geodetic Survey is completed, giving points of which the geodetic co-ordinates are known, at intervals of not more than 25 miles, it will be necessary for some organization, either local, State, or national, to place a number of supplementary points between these basic control points, for the use of local surveyors and engineers. In Pennsylvania, this spacing of supplementary control has been placed at a maximum of 5 miles.[14] These supplementary points will have to be established regardless of the system of computations used. In order to be of unquestioned authority they will have to be located with sufficient accuracy to justify their computation on a geodetic basis. If they are to be placed close enough to be useful to the local surveyor without prohibitive expense they will be close enough so that the maximum error introduced in any line run between them (computed by a plane computation as described in the paper and shown in Table 3(*b*)) will be less than the specified error of the surveys desired. For example, in Pennsylvania, with the maximum spacing of secondary control at 5 miles, the maximum error for the worst case would be the unbalanced, or loop, traverse run for 2.5 miles from the point of origin and returned. From Table 3(*b*) this would give a maximum error of 1: 3 500, approximately, which would be the worst case possible and would seldom be encountered. Most lines would be run so that the errors caused by neglect of geodetic corrections would be negligible.

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[14] **"A Standard System of Plane Coordinates for Pennsylvania," by the Pennsylvania State Planning Board, Harrisburg, Pa.**

Professor Kissam has revealed the fact that the cost of computing surveys on a truly geodetic basis would be prohibitive for any commercial survey. However, the cost of computations on the system as presented by the writer would be very little. The first and second-order control will have geodetic computations in any case. The spacing of the secondary control will be governed by practical reasons so that it will be used. The cost of running and computing the third order, or traverses for use, will be the same in any case, for the writer's system, for plane co-ordinates, or for uncontrolled surveys.

Mr. Carpenter has revealed the fact that the main control net of the U. S. Coast and Geodetic Survey is not nearing completion, but is, in fact, little more than half finished (1938). It would be probably more important at this time to find ways and means of completing this important work and the enormous amount of supplementary control that will be necessary before it can be properly used on any system, than it is to find ways to use it after it is completed. Private engineers and small political units cannot be expected to spend very much money on any type of control other than local. A small part of the money wasted on inaccurately controlled and duplicated surveys would have completed this control system many years ago.

Laws have been passed in two States legalizing the use of State-wide systems of plane co-ordinates for land boundary descriptions. These laws were passed after considerable effort on the part of those interested, and will undoubtedly be legal for a long time. These laws would have been of more permanent value if they had been drawn in a more general manner, so that the primary requirement had been a survey based on geodetic control, leaving the method of making the connection between the geodetic control and the survey to the engineers and surveyors.

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PRESSURES BENEATH A SPREAD FOUNDATION

By D. P. KRYNINE[1], M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. O. K. FRÖHLICH, DONALD W. TAYLOR, JACOB FELD, G. P. TSCHEBOTAREFF, A. HRENNIKOFF, ROBERT G. HENNES, T. A. MIDDLEBROOKS, A. A. EREMIN, M. M. BUISSON, FREDERICK J. CONVERSE, AND D. P. KRYNINE.

SYNOPSIS

An attempt has been made to confine the scope of this paper as nearly as possible to the limits defined by the title. Hence, discussion of tangential stresses (shears) and of pile foundations has been omitted entirely. In the "Introduction" an outline of stress distribution formulas is given, with special emphasis on the use of the "concentration factor," a conception applied to soil mechanics only since 1934.

Part I contains a description of a method of determining pressures under both uniformly and non-uniformly loaded foundations, except the case in which unit pressures on a part of the loaded area are infinite. This graphical process, termed the "reduced area method," may be applied in the case of a structure of arbitrary shape, and of an earth mass with an arbitrary concentration factor. Part II deals with the rigidity of structures and earth masses principally from the standpoint of their interaction.

Each of the two parts of the paper can be read independently. Only elementary mathematics, including trigonometry, has been used, the proofs of statements that require the application of calculus being presented in Appendices I and II.

Notation.—The letter symbols introduced in this paper are assembled for convenience of reference in Appendix III.

INTRODUCTION

Every theory of stress distribution within a body applies necessarily to an ideal material. Earth is not such a material, and theoretical formulas for semi-infinite isotropic bodies as used in this paper, may furnish data for an estimation of stresses within an earth mass, only if the latter is more or less homogeneous and isotropic to a considerable depth. Since stresses within an earth mass cannot be computed accurately, but can only be estimated,

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NOTE.—Published in April, 1937, *Proceedings*.

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quick graphical methods are advanced in this paper to replace cumbersome analytical procedures.

Stress Distribution Formulas.—Consider the infinitely large mass in Fig. 1. A concentrated vertical force, P, is applied to its horizontal surface at Point A, thus creating unit stresses, s, within the mass, which decrease in intensity at a rate proportional to the square of the distance from the origin (Point A). At any point of the mass (O, in Fig. 1), the unit stress produced by the force, P, at Point A, will be:

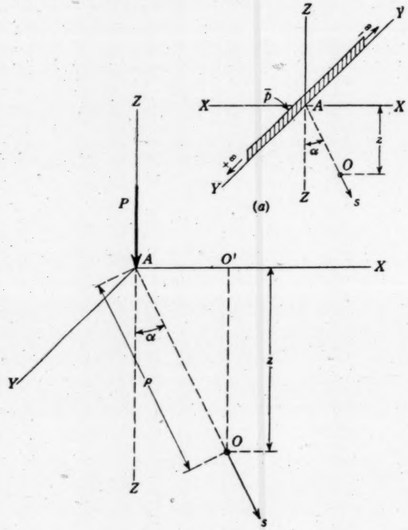

FIG. 1.

$$s = \frac{n\,P}{2\,\pi\,\rho^2} \cos^{n-2} \alpha \;\ldots\ldots (1)$$

Equation (1) has been proposed by John H. Griffith, M. Am. Soc. C. E.[2], and Dr. O. K. Fröhlich,[3] of Holland. Full credit for the development of a comprehensive theory of the concentration factor should be given to Dr. Fröhlich. The "concentration factor," n, is somewhat greater than 3 in the case of clays, and 6 or 7 in the case of coarse sands. It equals exactly 3 in the ideal case in which the mass under consideration follows Hooke's law, and is isotropic and incompressible, so that it may change its shape but not its volume. In such a case:

$$s = \frac{3\,P}{2\,\pi\,\rho^2} \cos \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Equation (2) may be obtained from the formulas developed by Boussinesq.[4] If the mass is loaded at its horizontal surface with an infinitely long unit load, expressed as $\overline{p}$ units of weight per unit of length, and acting along the axis, Y-Y (Fig. 1(a)), the stress distribution in the plane, XAZ, and in every vertical plane parallel to it, will be the same. The stress, s, at Point O, will be:

$$s = \frac{2\overline{p}}{\pi\,\rho} \cos \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equation (3) represents the two-dimensional (plane) analogue of Equation (2). This is the so-called "Michell's radial distribution."[5] It shows that, in the case of a plane stress distribution within an elastic isotropic body, the stress intensities are inversely proportional to the distances from the origin of stresses, A.

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[2] "Pressures under Substructures", *Engineering and Contracting,* March, 1929, pp. 113–119.

[3] "Druckverteilung im Baugrunde", 1934; also, *De Ingenieur,* April 15, 1932.

[4] "Application des Potentials a l'Etude de l'Equilibre et du Movement des Solides Elastiques", Paris, 1885; also, Progress Report of the Special Committee on Earths and Foundations, Am. Soc. C. E., *Proceedings,* Am. Soc. C. E., May, 1933, p. 777.

Since α is the angle formed by the stress, with the vertical, the vertical pressure, p_z, at Point O (Fig. 1), is:

$$p_z = s \cos^2 \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

To find the two-dimensional analogue of the general formula, Equation (1), the vertical pressure at Point O caused by the action of a straight line (Axis Y-Y, Fig. 1), with a load, $\bar{p}$, per unit of length, bearing on the boundary, must be considered. A proposed method is developed in Appendix I. The value of the vertical pressure at Point O (Fig. 1) would be:

$$p_z = \frac{n_1 \bar{p}}{\pi \rho} \cos^n \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

From Equation (5) the value of the plane stress in a general case is:

$$s = \frac{n_1 \bar{p}}{\pi \rho} \cos^{n-2} \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

For the sake of brevity and for the purposes of this paper only, the coefficient, $n_1 = f(n)$, may be termed the "plane concentration factor," with values corresponding to n, as shown in Table 1. When $n = 3$, the value of n_1 equals 2, and Equation (6) becomes identical with Equation (3). Equations (1) and (6) can be applied in the case of any mass regardless of whether or not it follows Hooke's law. Since in this paper stresses are not determined from strains, the resultant of stresses coming to a point from two or more different sources may be determined by superposition.

TABLE 1.—VALUES OF CONCENTRATION FACTORS

Values of the concentration factor, n	$\frac{n}{2\pi}$	Corresponding values of the plane concentration factor, n_1	$\frac{n_1}{\pi}$	Values of the concentration factor, n	$\frac{n}{2\pi}$	Corresponding values of the plane concentration factor, n_1	$\frac{n_1}{\pi}$
3………	0.477	2.00	0.637	6………	0.955	2.95	0.939
4………	0.637	2.36	0.751	7………	1.114	3.20	1.019
5………	0.796	2.67	0.850	8………	1.27	3.44	1.090

PART I.—THE "REDUCED AREA" METHOD OF DETERMINING PRESSURES UNDER STRUCTURES

Pressures under uniformly loaded structures (vertical unit load, p) will be first determined; and for the purposes of Part I it will be assumed that the unit soil reaction, p, is also uniformly distributed throughout the foundation, and that it is vertical. Non-uniform loading, except the case of infinite unit loads, will also be discussed; and the corresponding soil reaction at a point of the earth's surface will be assumed vertical and equal to the unit load at that point.

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[5] *Proceedings*, London Math. Soc., Vol. 32, p. 35 (1900); also, "Theory of Elasticity", by S. Timoshenko, p. 82 (1934).

THREE-DIMENSIONAL PROBLEM

Referring to Fig. 2, the problem is to determine the vertical pressure, p_z, at Point O. The horizontal projection, O', of this point on the boundary plane is selected as a center, and the loaded area, M' N', is subdivided into zones by concentric circles, as shown in the plan view. The radii of these circles, although arbitrary, should be chosen so as to make the division curves pass through marked breaks in the perimeter of the loaded

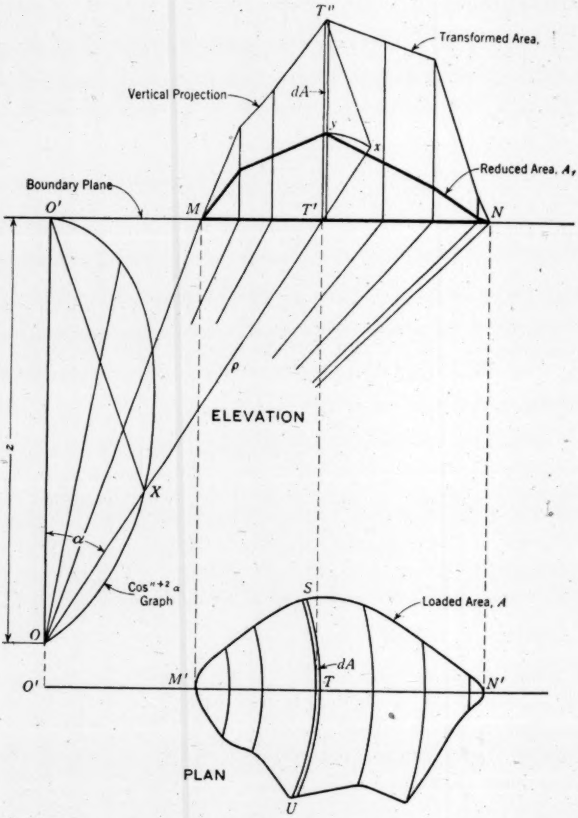
T''
Transformed Area.
Vertical Projection
dA
y
x
Reduced Area, A,
Boundary Plane
O'
M
T'
N
ρ
ELEVATION
z
X
α
Cos^{n+2} α Graph
O
S
Loaded Area, A
dA
O'
M'
T
N'
PLAN
U

FIG. 2.—VERTICAL PRESSURE AT A POINT DETERMINED BY THE REDUCED AREA METHOD (THE THREE-DIMENSIONAL PROBLEM).

areas. Credit should be given to Messrs. Kögler and Scheidig[6] who first proposed subdividing a loaded area into circular rings, and not into rectangles, as is usually the practice. The area of an elementary zone, dA, practically equals the length of the curve, STU, times the width of that zone. The length of the curves is measured simply by "stepping off" small openings of a compass. Plotting the vertical ordinates, $T' T'' = S\,T\,U$, a figure, $M\,T''\,N$, shown in the vertical projection (Fig. 2), would be obtained. The area, $M\,T''\,N$ (Fig. 2) evidently equals the original loaded area. To take advantage of this "transformed" area the following graphical construction is proposed: The stressed condition at Point O would not change if the horizontal loaded area, A, weighing p units per square unit of area, is replaced by the transformed area; also if A of the same unit weight, p, is concentrated as a vertical line load between Points M and N. Actually, a hypothetical load acting at a point of an arc (Fig. 2), can be moved to any other point of the same arc without influencing, in any way, the value of the vertical pressure at Point O. This is because the values of ρ and α to be used in Equations (1) and (4) for computing the vertical pressure, are the same at all points of that arc. Hence, for computing the vertical pressure at Point O, all the loads acting on an arc (Fig. 2) may be concentrated at an arbitrary point along that arc. This is precisely what is done in constructing the "transformed area" by bringing all the loads of different arcs (Fig. 2) to their respective points of intersection with an arbitrary line, $M'\,N'$. An elementary strip, dA, of that vertical figure, produces an elementary vertical pressure, dp_z, at Point O, which can be determined using Equations (1) and (4):

$$dp_z = \frac{n\,[p\,dA]}{2\,\pi\,\rho^2} \cos^n \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

or, taking into consideration the fact that $\rho = \overline{O\,T'} = \dfrac{z}{\cos \alpha}$:

$$dp_z = \frac{n\,p\,dA}{2\,\pi\,z^2} \cos^{n+2} \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The vertical pressure, p_z, at Point O, would be obtained from Equation (8) by summation, thus:

$$p_z = p\,\frac{n}{2\,\pi}\,\frac{1}{z^2}\,\Sigma\,dA \cos^{n+2} \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The area, $\Sigma\,dA\ \cos^{n+2}\alpha$, which will be termed the "reduced" area, A_r, may be determined as follows: An arbitrary vertical distance, O-O' is taken as a unit, and a graph of $\cos^{n+2}\alpha$ plotted. For doing so, radii vectors emanating from Point O and making different angles, α, with the vertical are drawn, and distances equal to $\cos^{n+2}\alpha$ are plotted along each radius vector. The ends of the radii vectors would then form a smooth curve which is the graph in question. For extensive use a table of values of

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[6] *Die Bautechnik*, Vol. 6, No. 17, p. 231 (1928).

$\cos^{n+2}\alpha$ will be found helpful. The following tabulation shows, for example, values of $\cos^{n+2}\alpha$ for the case in which the Boussinesq formula holds ($n = 3$):

in degrees	$\cos^5 \alpha$	α in degrees	$\cos^5 \alpha$
10	0.927	50	0.110
20	0.735	60	0.031
30	0.487	70	0.005
40	0.264		

In Fig. 2 the unit vertical distance referred to, accidentally equals the depth, z. The point of intersection, X, of a radius vector, OT', is joined with the top of the graph, $\cos^{n+2}\alpha$ (Line $O'X$). An ordinate, such as $T'T''$, is then reduced, tracing $T''x$ parallel to $O'X$ to intersect the continuation of the radius vector, OT', at Point x. Plotting $T'y = T'x$ and repeating this operation for all points of the line, MN, the "reduced area," A_r, is obtained and measured by a planimeter. This value is then introduced into Equation (9) to replace $\Sigma\, dA \cos^{n+2}\alpha$:

$$p_z = p\,\frac{n}{2\,\pi}\,\frac{A_r}{z^2} \quad \text{.......................}(10)$$

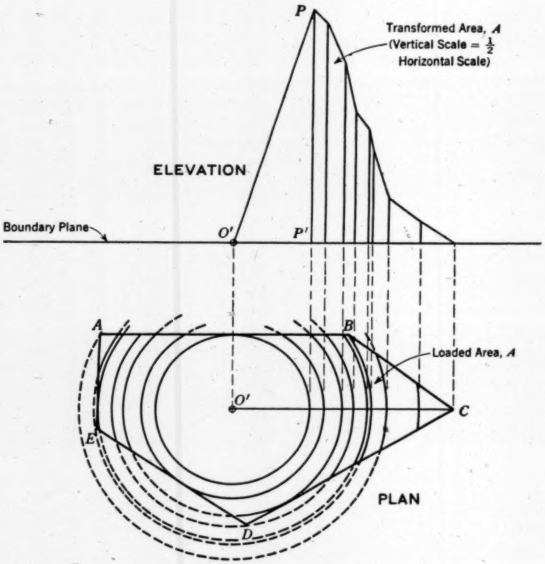
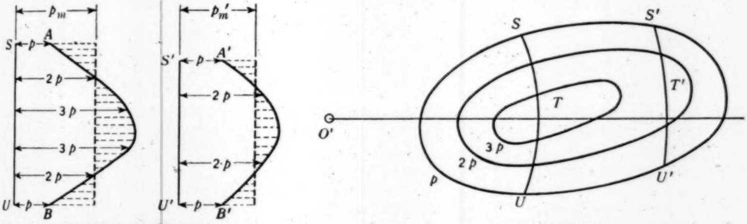

P
Transformed Area, A
(Vertical Scale = $\frac{1}{2}$ Horizontal Scale)
ELEVATION
Boundary Plane
O'
P'
A
B
Loaded Area, A
O'
C
E
D
PLAN

FIG. 3.—CONSTRUCTION OF THE TRANSFORMED AREA.

Since the ratio, $\frac{A_r}{z^2}$, is an abstract number, the scale of the drawing (see Fig. 2) has no bearing on the result. The value, $\frac{n}{2\,\pi}$, may be taken from Table 1. If there is infinite pressure at a part of the loaded area (for instance, at its perimeter), the "reduced area" would also have infinite ordinates. In such a case it is difficult to measure the "reduced area" graphically, and the proposed method does not work.

In a particular case when the horizontal projection, O', of Point O, falls within the loaded area (Area $A\ B\ C\ D\ E$, Fig. 3), the largest possible complete circle is drawn within the area, from Point O' as a center. The ordinate, $P\,P'$, of the transformed area equals the length of the circumference of this circle; and $O'P$ is a straight line. The remainder of the procedure is as explained previously herein. As shown in Fig. 3, the construction may be confined to one side of Point O', plotting the ordinates of the "transformed" area as total lengths of the arcs of equal diameters (shown in solid lines in the plan, Fig. 3). Ordinates of the transformed area may be plotted on a reduced scale. In such a case the resultant vertical pressure, p_z, should be increased accordingly.

If the structure is non-uniformly loaded, for instance, as shown with isolines in Fig. 4(*b*) the problem may be easily reduced to the case of uniform loading with a given unit load, p. For doing so, first all arcs of the loaded area, such as $S\ T\ U$, are rectified, and pressures at each point plotted as ordinates. This procedure, as shown in Fig. 4(*a*), is analogous to the construction of a profile from a set of contour lines as in surveying. Let p_m be the average pressure acting at the arc, $S\ T\ U$; and if the latter is imagined in the form of a circular ring one unit wide, the total load carried by the arc in question would be $S\,T\,U \times p_m \times 1$. Since the vertical pressure p_z, at Point O caused by the arc, $S\ T\ U$, does not depend on the way in which pressures are distributed along that arc, but solely on the value of the total load carried, an arc of unknown length, x, loaded uniformly with an intensity, p, would cause the same pressure, p_z, at Point O, in which case the following condition is fulfilled: $S\,T\,U \times p_m \times 1 = x \times p \times 1$, from which $x = S\,T\,U \dfrac{p_m}{p}$. This means that all ordinates of the transformed area, are to be modified (increased in the case of Fig. 4) in the proportion, $\dfrac{p_m}{p}$, the value of p_m being individual for each ordinate.



(*a*) CROSS SECTIONS (*b*) PLAN

FIG. 4.—PRELIMINARY GRAPHIC OPERATIONS IN THE CASE OF NON-UNIFORM LOADING.

Two-Dimensional Problem

In Fig. 5 a two-dimensional problem is solved for the general case of a non-uniformly distributed load, all the pressures being finite. At each point of the loaded area, *M N*, there is a vertical unit load, p_1, and the values of the ratio, $\frac{p_1}{p}$, are plotted as ordinates, such as *T′ T″*. In this manner an area, $M\,T''\,N = A$, is obtained which is analogous to the "transformed" area in the three-dimensional problem. The total load acting along *M N*, equals $p\,A$; and the elementary vertical, $p\;dA$, applied at Point *T′*, produces, according to Equation (5), an elementary vertical pressure, dp_z, at Point *O*, equal to:

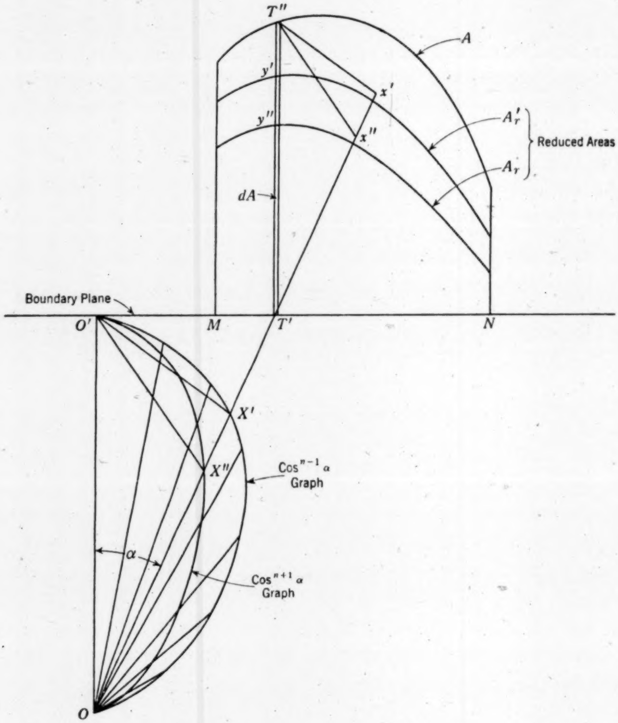


Fig. 5.—Vertical Pressure at a Point Determined by the Reduced Area Method (the Two-Dimensional Problem).

$$dp_z = \frac{n_1\,[p\,dA]}{\pi\,\rho}\cos^n\alpha = p\,\frac{n_1}{\pi}\,\frac{1}{z}\,[dA\,\cos^{n+1}\alpha]\;\ldots\ldots\ldots\ldots(11)$$

Equation (11) is derived directly from Equation (5). Following a procedure analogous to that used when discussing the three-dimensional problem:

$$p_z = p \, \frac{n_1}{\pi} \, \frac{A_r}{z} \quad \text{.......................(12)}$$

in which A_r is the "reduced" area. To construct the reduced area, A_r, it is necessary to use the graph of $\cos^{n+1} \alpha$. For instance, if the Boussinesq formula holds, the equation would be $\cos^4 \alpha$. The values of $\frac{n_1}{\pi}$ may be taken from Table 1. If Point O is located somewhere under the loaded strip, MN, a vertical line passing through O (Fig. 5) would divide the drawing in two parts. A graph, $\cos^{n+1} \alpha$, is constructed separately for each part, the vertical line passing through O being the axis of symmetry of this double graph. The remainder of the procedure is as explained previously.

To determine the horizontal pressure, p_h, at Point O (Fig. 5), the sum of the principal stresses, s, should be found first, and then the vertical pressure p_z, subtracted from it. Substituting $p \, dA$ for p in Equation (6) and remembering that $\rho = \frac{z}{\cos \alpha}$, the elementary stress at Point O would be:

$$ds = p \, \frac{n_1}{\pi} \, \frac{1}{z} \, dA \, \cos^{n-1} \alpha \quad \text{...................(13)}$$

Hence, the sum of the principal stresses, obtained from Equation (13) by summation, is:

$$s = p \, \frac{n_1}{\pi} \, \frac{A'_r}{z} \quad \text{.......................(14)}$$

in which A'_r is the "reduced" area, constructed by using the graph, $\cos^{n+1} \alpha$. If the Boussinesq formula holds, the graph of $\cos^2 \alpha$ is to be used. Finally:

$$p_h = s - p_z = p \, \frac{n_1}{\pi} \, \frac{(A'_r - A_r)}{z} \quad \text{................(15)}$$

Since both ratios, $\frac{A'_r}{z}$ and $\frac{A_r}{z}$, are abstract numbers, the scale of the drawing (Fig. 5) has no bearing on the results. To make this fact clearer, it is necessary to recall that Area A is the sum of the products of dimensionless ratios, $\frac{p_1}{p}$, multiplied by certain widths; hence, A is a length, dimensionally.

The problem of finding the horizontal pressure, p_h, in the case of a three-dimensional stress distribution is more complicated than that solved in Fig. 5 for the case of a plane stress distribution and will not be discussed herein.

Normal stresses under non-rigid structures can also be determined by using the properties of the "angle of visibility" of the foundation. Application of this method to the study of plane stress distribution within an elastic isotropic mass has been discussed by the writer elsewhere.[7]

Example.—Fig. 6 illustrates the determination of the vertical pressure, p_z, at a depth, $z = 100$ ft, under the entering angle, X, of a structure uniformly loaded with 2.5 tons per sq ft. The value of the reduced area, A_r, constructed under the assumption that the Boussinesq formula holds, is 6 560 sq ft, and the vertical pressure: $p_z = p \, \frac{n}{2\,\pi} \, \frac{A_r}{z^2} = 2.5 \times 0.477 \times \frac{6\,560}{100 \times 100}$ $= 0.78$ ton per sq ft. Note that the vertical scale used to construct both the transformed and reduced areas in Fig. 6 is one-half the horizontal.

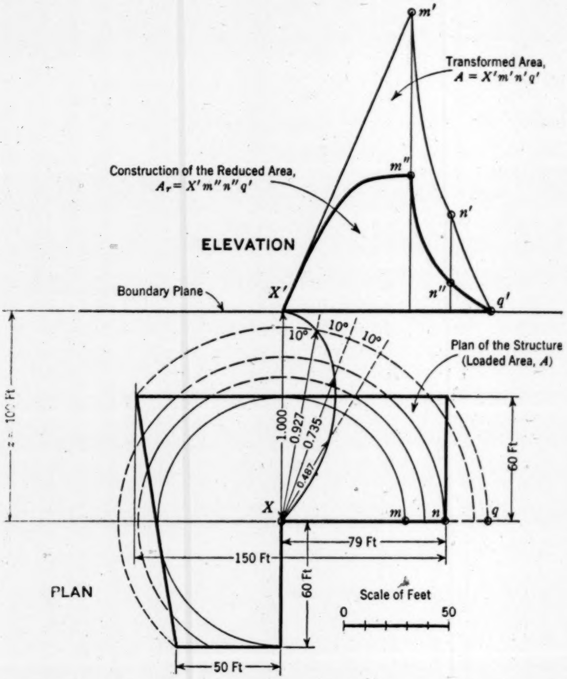

m'
Transformed Area, A = X'm'n'q'
m''
Construction of the Reduced Area, A_r = X'm''n''q'
n'
ELEVATION
Boundary Plane
X'
n''
q'
10°
10°
10°
Plan of the Structure (Loaded Area, A)
z = 100 Ft
1.000
0.927
0.735
0.487
60 Ft
X
m
n
q
79 Ft
150 Ft
PLAN
60 Ft
Scale of Feet
0
50
50 Ft

FIG. 6.—APPLICATION OF THE REDUCED AREA METHOD.

PART II.—PRESSURES UNDER RIGID FOUNDATIONS

In the case of an actual structure, soil reactions are in equilibrium, not with unit loads acting at the loaded surface as assumed in Part I, but with stresses within the structure close to the surface of contact with the earth. It follows that for finding stress intensities within the earth mass beneath a foundation, it is first necessary to establish the manner in which stresses are distributed at the base of the given structure and to consider the surface of contact of the structure with the earth as being loaded with those stresses acting as unit loads. The method which has been discussed in Part I, will be then fully applicable. Unfortunately, the problem of stress distribution at the base of a structure has not yet been solved definitely. Therefore, in presenting Part II of this paper, the writer limits his task to the review of available information in this field, both theoretical and experimental, and advances several ideas of his own. This discussion will be limited to uniformly loaded foundations only.

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[7] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1288.

RIGIDITY OF STRUCTURES AND EARTH MASSES

A structure will be termed "absolutely rigid" if it can be visualized as consisting of thin vertical slices so strongly bound together that they act as a unit. An opposite case would be a "non-rigid structure," which can be visualized as consisting of thin vertical slices with no lateral bonds so that each of them can settle down individually. The degree of rigidity in actual engineering structures is some value between these two limits. To form an accurate idea of the rigidity of the entire structure one should know how to express this rigidity numerically. At the present time this cannot be done in all cases.

Earth masses may also possess varying degrees of rigidity which could be expressed by a number as, for instance, the coefficient of pressure at rest, K, or the concentration factor, n. A mass that develops considerable horizontal stress under vertical load at rest (such as clays) may be considered as non-rigid. Compact sands, on the contrary, may be considered as rigid masses. If one knew how to express the rigidity of both the structure and the earth mass by comparable coefficients, one could attempt to attack the problem of the mutual rigidity of the two masses in contact, replacing the structure with an "equivalent mass." Because of the uncertainty of the manner of expressing the rigidity, however, both structures and earth masses will be identified in this paper, only as "rigid" and "non-rigid," the former approaching the state of absolute rigidity, as already defined.

If the earth mass supporting a structure is more or less homogeneous and isotropic to a considerable depth, the settlement of the structure is due to strains which gradually decrease with the depth. This is a case of "contact settlement" when the seat of settlement resides close to the base of the structure. If there is a soft layer at a certain depth, the structure may undergo an additional settlement owing to consolidation of that soft layer. It may be said that in this case the seat of settlement resides "in deeper strata."

When a deformable body moves through a medium offering resistance, its action is definable in two steps: (*a*) It moves as a non-deformable solid body; and (*b*) resistances of the medium produce deformations (or strains) in it. In a similar manner, a contact settlement is characterized by: (I) A downward movement of the structure; and (II) deformations of the structure due to interaction with the earth mass (Fig. 7). To understand the nature of these deformations the structure will first be assumed non-rigid; thus the phenomena at the surface of contact would occur practically as if the structure were replaced by a non-rigid earth mass. Afterward, rigidity will be restored to the structure, and corresponding changes in the stress distribution over the base of the foundation will be discussed. In a general way, stresses close to the surface of contact of two masses are the same in both masses, according to the well-known physical law of equality of action and reaction. Strains, however, are different because the less resistant of the two masses should yield. Hence, according to these considerations, a rigid mass under a non-rigid structure should bend the latter upward, as shown diagrammatically in Fig. 7(*c*). A case of settlement of this type was reported by Dr. Scheidig[8] when some strucures constructed on loose sand, settled at the edges. The writer understands that the buildings in question were erected rather hastily. Settlement I (Fig. 7(*c*)) occurred at once, owing to the compression of the loose sand; and probably when the latter became compact, the insufficiently rigid masonry was deflected upward. It should be noticed that if a loaded thin elastically isotropic layer is underlaid with rock or similar material, the formation of cupolas is also possible.[9] The writer understands, however,

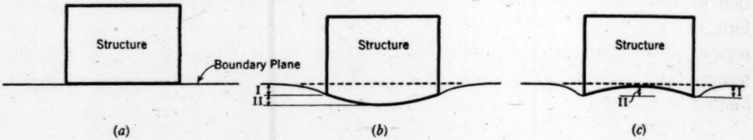

FIG. 7.—EXAMPLES OF CONTACT SETTLEMENT.

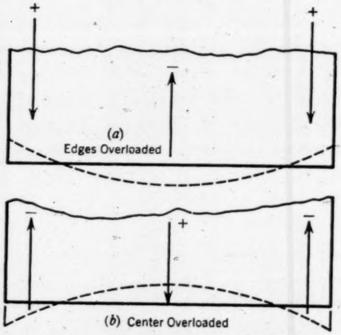

FIG. 8.—ACTION OF ABSOLUTELY RIGID STRUCTURE.

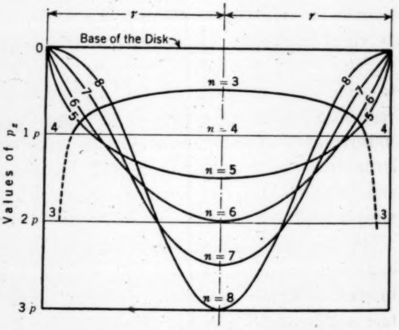

FIG. 9.—GRAPHICAL REPRESENTATION OF EQUATION (17).

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[8] "Die Berechnungsgrundlagen durchgehender Fundamente und die neuere Baugrundforschung", *Die Bautechnik*, Vol. 9, No. 19 (1931).

[9] "Theorie der Setzungen von Tonschichten", von Charles Terzaghi, M. Am. Soc. C. E., and O. K. Frŏhlich, p. 8 (1936). Other references on the subject are also given in the footnote, pp. 8 and 9 of that book.

that there was no such underlying stratum in the case of Dr. Scheidig's observations, and that the edge settlement in this case should be explained by the inter-penetration of the masses in contact.

Although the problem shown in Fig. 7(c) cannot be solved using methods of the theory of elasticity, the latter furnishes all necessary data for the problem of settlement of a non-rigid structure of finite size, supported by an equally non-rigid mass (Fig. 7(b)). With Fig. 7(b) and Fig. 7(c) in mind it is now easy to determine the character of stress distribution at the base of a rigid structure, at least, qualitatively. In order to keep the base of the structure even, additional forces should be applied. Thus, an absolutely rigid structure may be over-loaded at the edges (Fig. 8(a)), or at the center (Fig. 8 (b)), according to the rigidity of the underlying mass (a non-rigid underlying mass in the former case and a rigid mass in the latter). Fig. 8 (b) furnishes an explanation as to why, in all loading experiments on sand masses, the maximum stress in the soil is shown under the center of the loading plate.

DISTRIBUTION OF NORMAL STRESSES UNDER AN ABSOLUTELY RIGID FOUNDATION

The problem of an absolutely rigid structure built on elastic isotropic soil has been solved by methods involving the theory of elasticity. Boussinesq[10] and later Schleicher[11] determined the stress distribution at the base of a rigid round disk placed on an elastic isotropic mass from displacements of its points. Boussinesq's basic assumptions are: (1) That there is no friction at the plane of contact; and (2) the plane of contact is horizontal. Professor Michael A. Sadowsky,[12] of the University of Minnesota, applied the potential theory to solve the three-dimensional problem. According to these investigators, if r is the radius of the disk in question, the pressure at a point distant, $c\,r$, from the center of the disk, would be:

$$p_z = p \frac{1}{2\sqrt{1 - c^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The pressure, p_z, equals $0.5\,p$ at the center of the disk ($c = 0$) and becomes infinite at the edges ($c = 1$). Since there are no absolutely rigid structures, even the slightest settlement at the edge of the loaded disk relieves the strain considerably, so that practically all stresses at the base of the loaded disk are finite. The influence of rigidity of a circular foundation slab in the distribution of pressures on the contact surface, has been considered by H. Borowicka[13] who assumed that both the structure and the earth mass are elastically isotropic with different values of the modulus of elasticity and Poisson's ratio. To express the stress distribution in a case of an absolutely rigid round disk placed at the top of an earth mass, with a concentration factor, n, the writer proposes the following empirical formula (see Fig. 9):

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[10] "Application des Potentials a l'Etude de l'Equilibre et du Movement des Solides Elastiques", Paris, 1885, p. 158; also, "Theory of Elasticity", by S. Timoshenko, p. 338.

[11] "Die Verteilung der Bodenpressungen unter starren Gründungskörpern", *Der Bauingenieur*, Vol. 14, Nos. 17–18, p. 242 *et seq.* (1933); also, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 8 (1928).

[12] "Distribution of Pressure Concentrated on a Small Surface", *Transactions*, Am. Soc. M. E., Vol. 54, Paper APM 54–21, p. 221 (1934).

[13] *Proceedings*, International Conference for Soil Mechanics and Foundation Engineering. Vol. II, Paper E-8.

$$p_z = p\left(\frac{n}{2} - 1\right)\left(1 - c^2\right)^{\frac{n}{2}-2} \quad \ldots\ldots\ldots\ldots\ldots (17)$$

Substituting $n = 3$ in Equation (17), the resulting form is identical to Equation (16). The value, $n = 6$, results in a parabolic distribution very similar to that found in numerous loading experiments on sand masses as reviewed elsewhere:[14]

$$p_z = 2\,p\,(1 - c^2) \quad \ldots\ldots\ldots\ldots\ldots (18)$$

Should the concentration factor, n, of the constrained earth mass increase to 8, the stress distribution at the base of the experimental disk would be defined by the curve:

$$p_z = 3\,p\,(1 - c^2)^2 \quad \ldots\ldots\ldots\ldots\ldots (19)$$

and this creates a pressure at the center of the disk equal to 300% of the average pressure, p, which is quite close to the result obtained by M. L. Enger,[15] M. Am. Soc. C. E.

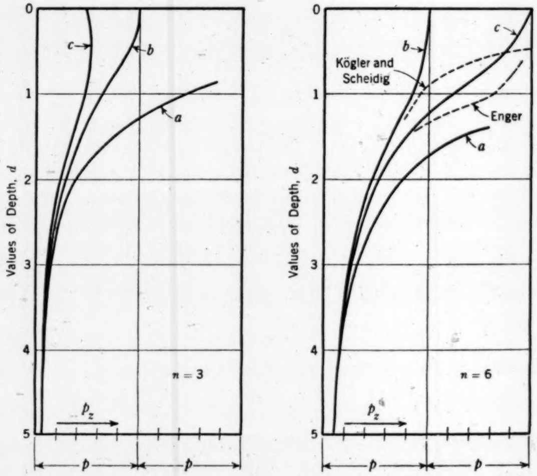

FIG. 10.—VERTICAL PRESSURES AT VARIOUS DEPTHS ALONG THE CENTER LINE OF A LOADED CIRCULAR DISK

Equation (17) has been designed on the assumption that the lower value of the concentration factor, n, equals 3. This is true of the isotropic masses for which the basic formulas given in the "Introduction," have been designed. Other geological conditions (for example, that of a hard layer underlaid by a soft one) are not covered by the formulas of this paper, and the solution of the corresponding problems should be sought elsewhere.

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[14] "Distribution of Stresses Under a Foundation", by A. E. Cummings, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1077.

[15] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 372.

The problem of stress distribution under a rigid structure is complicated by the formation of the so-called "disturbed zone." It follows from the small-scale experiments that the cross-section of this zone under a circular disk is semi-circular or ellipsoidal with a top segment removed. These results cannot yet be generalized for the case of full-sized structures, however, because of the lack of corresponding observations. In order to investigate to what depth the rigidity of a loaded plate in a small-scale experiment affects the earth mass, Fig. 10 has been prepared. For two soils, with concentration factors, $n = 3$ and $n = 6$, respectively, curves representing the vertical pressure, p_z, at various depths along the center line of a round loaded disk, have been traced. Pressures p_z have been plotted horizontally. The load is the same in all cases, but distributed differently: Curves (*a*) correspond to a concentrated load acting directly at the surface of the soil; Curves (*b*) correspond to uniform distribution; and Curves (*c*) correspond to the case of an absolutely rigid plate, respectively. For the latter case, Equations (16) and (18) have been used (see Appendix II for mathematical details). At a depth of about twice the diameter of the loaded part ($z = 4r$ in Fig. 10), Curves (*a*), (*b*), and (*c*) practically coincide, which accounts for the well known statement that, at that depth, a loaded plate acts practically as a concentrated load. The average experimental curves of Kögler and Scheidig and Enger[16] (see Fig. 10), are located partly to the right of Curve (*c*); and this means that in the upper part of the experimental mass, approximately down to a depth, r, the concentration factor, n, exceeds 6 for which value the curve, *c*, in Fig. 10 has been traced. Afterward, the experimental curves swing rapidly to join Curve (*b*); and at a depth of about $2r$ there is no longer any influence of the rigidity of the loaded plate—at that depth the mass behaves as if the base of the experimental disk were uniformly loaded (Curve (*b*), Fig. 10).

Undoubtedly, the stress distribution at the base of an infinitely wide rigid structure should be uniform; it cannot be otherwise. To reconcile this simple logical statement with the results previously obtained, Fig. 11 has been prepared. The stress distribution at the base of a rigid narrow structure is represented in Fig. 11(*a*) and Fig. 11(*c*), for the cases of a non-rigid and a rigid earth mass, respectively. The stress distribution under a wide rigid structure is tentatively shown in Fig. 11(*b*) and Fig. 11(*d*), for the cases of a non-rigid mass and a rigid earth mass, respectively. It should be noticed that the curve in Fig. 11(*b*) is very close to that found theoretically by K. Wieghart[17] for the case of a beam supported by an elastically isotropic mass.

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[16] The average values in question are those computed in the paper entitled "Distribution of Stresses Under a Foundation", by A. E. Cummings, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1078.

[17] Handbuch der Physik, Edited by R. Grammel, Vol. 6, pp. 180–181 (1928); for original work by K. Wieghart, see *Zeitschrift f. angew. Mathematik und Mechanik*, Vol. 2 (1922).

Pressures Under Actual Engineering Structures

To check theoretical considerations, pressure at the base of actual structures should be measured. This can be done by embedding pressure-measuring cells at the base of the structure during its construction. There are only a few published examples of such measurement, although such embedments have been reported from many places. From the three cases known to the writer one refers particularly to sewers[18] and two others will be discussed subsequently.

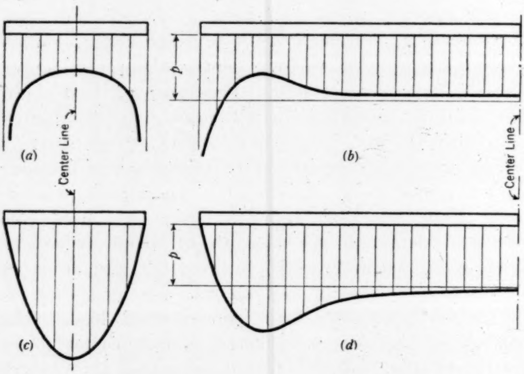

FIG. 11.—STRESS DISTRIBUTION AT BASE OF RIGID FOOTINGS, BOTH NARROW AND WIDE (A TENTATIVE SKETCH).

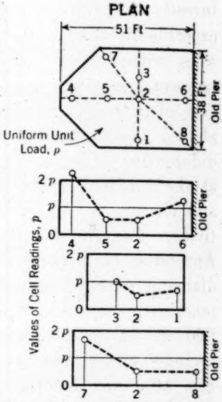

FIG. 12.—PRESSURES BENEATH A BRIDGE OVER THE RHINE RIVER IN GERMANY.

During the construction of the new bridge over the Rhine River, connecting Mannheim and Ludwigshafen, Germany, eight Goldbeck cells were placed beneath a pier,[11] and it was discovered that the foundation was overloaded at the edges (see Fig. 12). The foundation material was gravelly, and since European engineering thought had been accustomed to the idea that in the case of similar foundations the maximum pressure should be at the middle, there was an interesting discussion of the case.[19] The uncertainty was emphasized by the fact that the new pier was constructed close to the old one, which could have a certain influence on the stress distribution. In the case under consideration, the pier was founded on a caisson. Soil reaction could not lift the heavily loaded edges of the caisson, and, apparently, the foundation material was forced somewhat, into the concrete fill of the chamber. Consequently, the stress at the middle of the structure was relieved (see Fig. 7(*c*)). The writer believes that a similar condition may occur in other cases of caisson foundations if the latter may be considered as non-homogeneous structures consisting of a rigid shell containing a less resistant mass.

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[18] "Observed Soil Pressures on the Deep Sewers of the North Toronto System", by C. R. Young, M. Am. Soc. C. E., and W. B. Dunbar, *Bulletin No. 145*, Univ. of Toronto (1935).

[19] "Sohldruckverteilung unter Pfeilern", by F. Kögler, *Der Bauingenieur*, Vol. 14, Nos. 37–38, p. 473 *et seq* (1933); also discussions by Messrs. Scheidig and Schleicher, pp. 482 and 483.

About 1933 a building was constructed by the Agricultural and Mechanical College of Texas.[20] Six Goldbeck cells were embedded under a footing as shown in Fig. 13. The soil beneath the building is clay with small varying quantities of sand and coarse material. The maximum pressure during the construction was at the center of the footing (Cell No. 4), but this shifted gradually during a period of about seven months to Cell No. 3. It might be concluded, at first, that the concentration of stresses near the middle of this structure, as shown by Fig. 13, is in disagreement with the hypothesis advanced, as, for instance, with the stress distribution shown in Fig. 11(*a*). Apparently, Fig. 11(*a*) predicts an overloaded edge under the given soil conditions. The confusion is clarified, however, if one realizes that the footing action is not included in the scope of this paper, which discusses only cases of the interaction between two masses. In the case under consideration, however, there is an interaction between three masses: A column, a base, and an earth mass (see

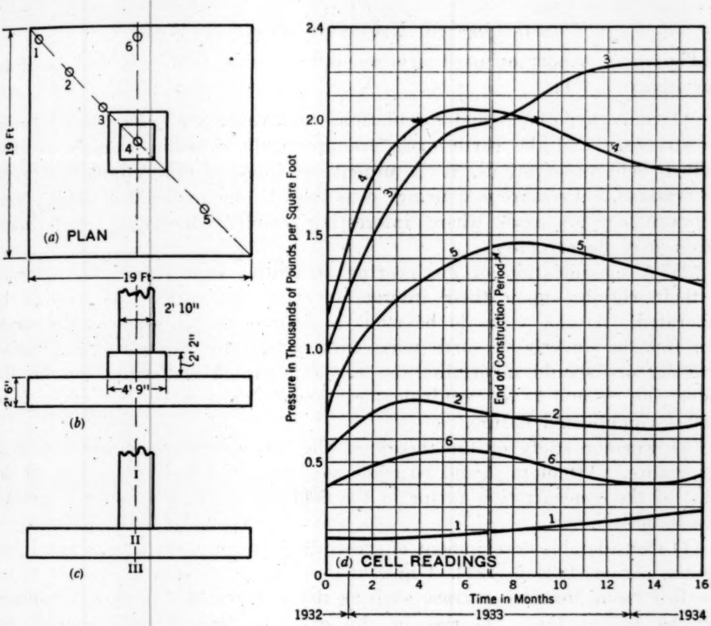

FIG. 13.—PRESSURES BENEATH A FOOTING OF A BUILDING AT TEXAS AGRICULTURAL AND MECHANICAL COLLEGE.

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[20] "The Distribution of Soil Pressure Beneath a Footing," by F. E. Giesecke, M. Am. Soc. C. E., and others, *Bulletin No. 43*, Texas Eng. Experiment Station, 1933.

Fig. 13(*c*)). Should the rigid mass, I, Fig. 13(*c*), act directly at the surface of the mass, III, the edges probably would become overloaded; but the inclusion of the intermediate mass, II, apparently changes the situation. A very important conclusion from the Texas experiment is the possibility of pressure fluctuation at the base of a structure, which is probably due to a progressive change in the physical properties of the earth mass.

Outline of the Necessary Research Work

The most important necessary research in this field may be stated in four items:

(1) Since hypotheses and speculations concerning the interaction between the structure and the earth mass are practically worthless unless checked against field observations, it is urgent to increase the number of such observations. Pressure-measuring cells should be embedded under new structures in all cases where interesting results concerning such interaction may be expected.

(2) A classification of engineering structures concerning the influence of their rigidity upon their interaction with the earth mass, should be elaborated. Attempts should be made to express the rigidity of the structure and the underlying earth mass with comparable numerical coefficients. Investigators should determine the cases in which structures act on the underlying ground as an "equivalent earth mass" and when this action is of a more complicated nature.

(3) Further study of the theory of the "concentration factor," n, is of importance. Attempts should be made to develop methods of measuring the value of the concentration factor in the field before (and not after) a structure is built.

(4) Considerable development of stress distribution theories was noted during the period, 1934 to 1936. Unfortunately, these theories refer only to the so-called "semi-infinite" masses, such as those shown in Fig. 1. It is necessary to extend these theories in the case of "finite" masses, such, for example, as embankments. Some progress in this field has been made since the introduction of photo-elastic analysis. Data analogous to those obtained in the study of the properties of elastic bodies in contact should be obtained for "finite masses" of all kinds.

Most of the research outlined in Items (1) to (4) cannot be done by soil mechanics investigators alone. It requires, also, the most intensive co-operation of structural engineers.

Some Remarks on the Design of Foundations

Bending Moment.—When a structure is designed under the assumption of uniformly distributed soil reaction (as generally done in practice) there is no bending in the loading plate except in cantilever units; but if the "picture," afforded by Fig. 11(*b*) and Fig. 11(*d*), is correct, only the central part of a wide structure can be designed under that approximate assumption. According to Fig. 11(*b*) and Fig. 11(*d*) there should be bending moment, positive or negative, near the perimeter of a bearing surface, the sign of the bending moment depending on the physical properties of the local soil.

Settlement of a Structure Due to Consolidation in Deeper Strata.—To design a structure so that it will not fail by settlement, the designer should be aware of the geological condition of the earth mass to a considerable depth, say, about twice the width of the structure. This is because a possible consolidation of a deep soft layer may cause a serious settlement. Consolidation in deeper strata, however, has no effect on the interaction of the structure and the earth mass. In this case the entire situation is controlled by the same physical law which permits, for instance, the playing of games of ball or cricket on a moving steamship without disturbance. In an analogous sense, when consolidation in deeper strata begins, the entire mass, including the structure, would settle, and there is no reason whatever to believe that the stressed condition at the base of the structure would be disturbed. Settlement in such a case is assumed to be uniform. A similar idea, however, in a somewhat different form, has been advanced by Professor A. Casagrande.[21]

Interrelation of Stresses.—Serious thought should be given to the question of whether the actual method of designing a structure without taking into consideration the influence of the foundation on the stress distribution in the structure itself should be discontinued. Undoubtedly, the structure and the earth mass underlying it form a single statically indeterminate system; and the stress distribution in the mass influences that in the structure, at least at its lower part. This principle has been realized by dam builders; and it is to be hoped that an analogous procedure will be adopted for other branches of structural engineering.

CONCLUSIONS

1.—Stresses within a semi-infinite isotropic mass (see Fig. 1), caused by a concentrated force acting at the boundary, can be determined by using Equation (1). In the case of a two-dimensional (plane) stress distribution, Equation (6) should be used.

2.—Stresses caused by loads distributed at the boundary of an isotropic mass may be determined by superposition.

3.—The "reduced area method" supplies a means of determining pressures in an isotropic mass beneath uniformly loaded foundations. It consists in subdividing the loaded area into circular zones, rectifying them, to obtain the "transformed area" and reducing the latter graphically, according to the position of the load with respect to the point where pressures are to be determined.

4.—The "reduced area method" can also be applied in the case of non-uniformly loaded isotropic masses, if all unit loads are finite. Otherwise, the graphical measurement of the "reduced area" is difficult.

5.—The degree of "contact settlement" that may be expected of a structure built at the top of an earth mass is chiefly controlled by the mutual rigidity of both masses in contact and perhaps by other factors, such as the size and the shape of the structure.

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[21] *Journal*, Boston Soc. of Civ. Engrs., Vol. 23 (1936), p. 15.

6.—The phenomenon of "contact settlement" of a non-rigid structure supported by a rigid earth mass consists in a downward movement, and in an upward deflection, of the structure.

7.—In the case of a narrow, absolutely rigid, structure, there is: (*a*) An overloading at the edges in the case of a non-rigid (for instance, a clay) foundation; or (*b*) an over-loading at the center in the case of a rigid (for instance, a sand) foundation.

8.—An empirical formula, Equation (17), concerning the stress distribution at the base of a narrow rigid structure, has been proposed.

9.—In the case of a wide engineering structure, there should be a certain tendency toward a uniform stress distribution at the center of the foundation. Apparently, the edges of a rigid structure are always overloaded, and there is always bending along its perimeter. Such moment will be positive or negative, depending on the physical properties of the soil beneath the foundation.

10.—There is a "disturbed zone" under a rigid structure apparently possessing a higher concentration factor than the remainder of the mass. The state of constraint along the center line of a round loaded plate in small-scale experiments goes only to a depth equal to one-half the width of the loaded area. Between this depth and that equal to twice the width of the loaded area, the stresses in the mass act as if the load were uniformly distributed through the foundation. At, and beyond, a depth twice the width of the loaded area the loading plate acts as a concentrated load.

11.—The stressed condition at the surface of contact of the structure with the earth mass does not depend on the consolidation in deeper strata.

12.—Field observations available concerning the stress distribution at the base of a structure, are insufficient. Hence, statements advanced in Part II of this paper (except those based on firm physical laws) are rather of a hypothetical nature. Further research is needed before they can be generalized and adopted unconditionally.

APPENDIX I

Determination of the Value of the "Plane Concentration Factor," n_1

The given force, P, and another unknown force, X, acting along the direction of the radius vector, $A\ O$, in Fig. 14, would produce the same stress at Point O, if Equation (20) is satisfied:

$$\frac{n\ P}{2\ \pi\ \rho^2} \cos^{n-2} \alpha = \frac{n\ X}{2\ \pi\ \rho^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)0}$$

from which,

$$X = P \cos^{n-2} \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Equation (21) holds also if a line load, $\overline{p}$, is substituted for the concentrated force, P. Hence, the value of the plane stress, s, at Point O, caused by the action of a line load, $\overline{p}$, at A (Fig. 14(a)) equals the vertical pressure at Point O' (Fig. 14(b)), distant vertically ρ from Point A, and subject to the action of a line load, $\overline{p}\cos^{n-2}\alpha$, at the boundary. In Fig. 14(b) the distance, $A\,M = \rho\tan\phi$, and $M\,M'$ is its differential. Hence, the value of this stress, s (see Equations (1) and (6)) is:

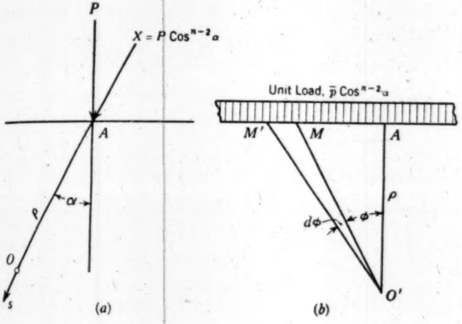

FIG. 14.

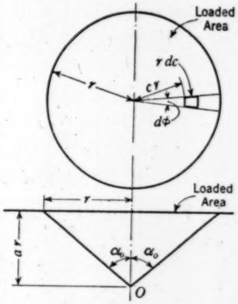

FIG. 15.

$$s = 2\int_0^{\frac{\pi}{2}} \frac{n\left[\overline{p}\cos^{n-2}\alpha\right]\left[\dfrac{\rho}{\cos^2\phi}\,d\phi\right]}{2\,\pi\left(\dfrac{\rho}{\cos\phi}\right)^2}\cos^n\phi = \frac{n_1\,\overline{p}}{\pi\,\rho}\cos^{n-2}\alpha$$

in which:

$$n_1 = n\int_0^{\frac{\pi}{2}}\cos^n\phi\,d\phi \text{ ....(22)}$$

In turn, the integral in Equation (22) equals:

$$\int_0^{\frac{\pi}{2}}\cos^n\phi\,d\phi = \frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(\dfrac{n+1}{2}\right)}{\Gamma\left(\dfrac{n}{2}+1\right)} \text{ ....(23)}$$

in which Γ is the gamma function. If the concentration factor, n, is not an integer, the value, n_1, is determined by using Equation (23). Otherwise, it is better to compute that value as follows: When n is an odd number, Equation (22) becomes:

$$n_1 = \frac{2\times4\times6\ldots(n-1)}{1\times3\times5\ldots(n-2)} \text{ ....(24a)}$$

and, when n is an even number:

$$n_1 = \frac{1 \times 3 \times 5 \ldots . (n-1)}{2 \times 4 \times 6 \ldots . (n-2)} \frac{\pi}{2} \qquad \text{(24b)}$$

Equations 24(a) and 24(b) check with formulas of plane stress distribution developed by Dr. O. K. Fröhlich.[22]

APPENDIX II

FORMULAS FOR TRACING CURVES IN FIG. 10

Equation (1) was used in plotting Curves (a), Fig. 10. Making $\alpha = 0$ and $\rho = a\, r$;

$$p_z = \frac{n\,[\pi\, r^2\, p]}{2\,\pi\,[a\ r]^2} = p\, \frac{n}{2}\, \frac{1}{a^2} \qquad \text{(25)}$$

in which a is a coeffcient $= \frac{\rho}{r}$.

The well-known formula[23] for the case of a uniformly loaded circular area was used in plotting Curves (b), Fig. 10; thus:

$$p_z = p\,(1 - \cos^n \alpha_0) \qquad \text{(26)}$$

in which α_0 is the angle formed with the vertical by the radii vectors joining the given point, O, with the perimeter of the loaded area (see Fig. 15).

Referring to Fig. 15 the area of the loaded element at the boundary is:

$$dA_c = [c\ r\ d\phi]\,[r\ dc] = r^2\, c\ dc\ d\phi \qquad \text{(27)}$$

The vertical pressure in the case of constraint is to be determined: (a) Using Equation (16) in the case of $n = 3$; and (b), using Equation (18), when $n = 6$. Thus, if $n = 3$:

$$p_z = \int_0^{2\pi}\int_0^1 \frac{\dfrac{3\,p}{2\,(1-c^2)^{\frac{1}{2}}}\,[r^2\, c\ dc\ d\phi]}{2\,\pi\,(a^2\, r^2 + c^2\, r^2)} \times \frac{a^3\, r^3}{(a^2\, r^2 + c^2\, r^2)^{\frac{3}{2}}}$$

$$= \frac{3}{2}\, p\ a^3 \int_0^1 \frac{c\ dc}{(1-c^2)^{\frac{1}{2}}\,(a^2 + c^2)^{\frac{5}{2}}} \qquad \text{(28)}$$

Substituting t^2 for $(1 - c^2)$ and remembering that the limits of t are 1 and 0, when $c = 0$ and $c = 1$, respectively:

$$p_z = -\frac{3}{2}\, p\ a^3 \int_0^1 \frac{dt}{(a^2 + 1 - t^2)^{\frac{5}{2}}} = p\, \frac{1 + 3\,a^2}{2\,(a^2 + 1)^2} \qquad \text{(29)}$$

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[22] "Druckverteilung im Baugrunde", p. 28 (1934).

[23] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equation (23), p. 1090.

and if $n = 6$;

$$p_z = \int_0^{2\pi}\int_0^{1} \frac{6\,[2\,p\,(1 - c^2)]\,[r^2\,c\,dc\,d\phi]}{2\,\pi\,[a^2\,r^2 + c^2\,r^2]} \times \frac{a^6\,r^6}{(a^2\,r^2 + c^2\,r^2)^3}$$

$$= p\left\{2 - a^2 + \frac{a^6}{(a^2 + 1)^2}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

To obtain this result, a new variable has been substituted for c^2.

APPENDIX III

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[24] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = loaded area; $d\,A$ = a small element of a loaded area; A_r = reduced area as applied to vertical pressure; A'_r = reduced area as applied to horizontal pressure; A_c = area of a circle;

a = a coefficient $= \dfrac{\rho}{r}$;

c = a coefficient to express the distance from the center of a circular loaded plate, in terms of its radius;

K = coefficient of pressure at rest, or the ratio of principal stresses in a non-loaded semi-infinite earth mass;

n = a concentration factor; n_1 = a plane concentration factor, n_1 being a function of n;

P = a concentrated vertical force acting at the boundary of an earth mass (see, also $\bar{p}$);

p = pressure per unit area; a uniformly distributed load; p_m = average pressure acting along an arc of the loaded area; p_n = non-uniformly distributed load; p_z = vertical unit pressure at any point; p_h = horizontal unit pressure at a point; $\bar{p}$ = a unit line load.

r = radius of a circular loaded plate, or disk; as a subscript, r, denotes "reduced";

s = unit stress, or sum of principal stresses;

t = a substitution factor $= (1 - c^2)$;

z = depth of a point below the ground surface;

α = angle of a force, or of a stress with the vertical; α_0 = one-half the angle at the vertex of a regular cone;

Γ = gamma function;

ρ = radius vector of a point (distance from the origin);

ϕ = auxiliary angles in Figs. 14 and 15.

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[24] A. S. A.—Z10a—1932.

DISCUSSION

DR. ING. O. K. FRÖHLICH[25] (by letter).—The practical significance of Part II of Professor Krynine's valuable paper consists in the emphasis placed upon the influence of the rigidity of structures on the distribution of soil pressures in the contact surface. Equation (16) represents the result of the treatment of the "classical" case of the stress distribution at the base of a circular and absolutely rigid disk, with a symmetrical load, placed on the horizontal surface of a semi-infinite, elastically isotropic mass. Boussinesq[10] has given this result in a very intuitive manner, which, in the writer's opinion, is worth mentioning here for two reasons: First, because it represents, for the practical engineer, a mnemo-technical means of reproducing Equation (16); and second, because it may be used to compute the stress distribution in the two-dimensional case. According to Boussinesq the unit stress at the base of the rigid circular disk (of radius, r) at any point, $c\,r$, of the contact surface is obtained by distributing the total load, $P = p\,r^2\,\pi$, equally over a hemisphere ($2\,r^2\,\pi$), which is assumed to be erected over the base circle of the disk, and by taking that part of this load, which is cut from the hemisphere by a vertical cylinder, having the unit surface at the given point as the base. It is easily seen that, in the center, where $c = 0$, the stress is $0.5\,p$; at the edge where $c = r$, it becomes infinitely great and at any point, $c\,r$, Equation (16) is satisfied.

In the case of a rigid strip (width, $2\,b$) of infinite length (the case of plane stress), a hemi-cylinder, having a diameter, $2\,b$, must be introduced instead of the hemisphere. The total load, $\bar{P}$, per unit of length equals $2\,b\,p$ and the load per unit of the cylinder surface is then $\dfrac{2}{\pi}\,p$. In a manner similar to the three-dimensional case, the formula for the pressure, p_z, at a distance, cb, of the axis of the loaded strip, becomes,

$$p_z = p\,\frac{2}{\pi\sqrt{1 - c^2}} \qquad \text{(31)}$$

which is analogous to Equation (16).

H. Borowicka[26] and A. Habel[27] have dealt with the influence of the rigidity of foundation slabs and strips on the distribution of the soil pressures in the contact surface. One of the characteristic results of these investigations is that, under equally distributed loads of slabs and strips for any degree of rigidity, the pressures under the edges are infinitely great, which is a consequence of the assumption, made in these theoretical computations, that Hooke's law remains valid beyond any limit. In this respect, the practical question arises as to the value that the pressure may attain under the edge of a foundation.

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[25] Cons. Engr., The Hague, Holland.

[26] "Druckverteilung unter einem gleichmässig belasteten, elastischen Plattenstreifen, welcher auf der Oberfläche des elastisch-isotropen Halbraumes liegt," von H. Borowicka, International Assoc. for Bridge and Structural Eng., Second Congress, Final Rept., 1937.

[27] "Die auf dem elastisch-isotropen Halbraum aufruhende, zentral symmetrisch belastete elastische Kreisplatte," von A. Habel, *Der Bauingenieur,* 1937, H. 15–16.

An attempt to answer this question of the magnitude of the critical stress under the edge of a foundation was presented by the writer in 1934.[28] All the aforementioned theoretical computations of the distribution of the soil pressures under loaded plates or strips, having a certain degree of rigidity, relate to very simple cases which, in practice, are very seldom encountered.

To show how the practical problem of determining the influence of the rigidity of a given structure on the distribution of the soil pressures may be attacked, the following example will be considered: In Fig. 16 the four foundations, symmetrical about a center line, consist of infinitely long strips, supporting a rather rigid structure. The loads, P_1, P_2, may be determined as usual under the assumptions applying to unyielding foundations. This is a problem of pure statics, the solution of which is well known. The assumption that the structure is infinitely long simplifies the computation inasmuch as it reduces the three-dimensional, to a plane-stress, problem. The equations, given in the paper and in the monograph mentioned[28] enables the designer to determine the settlements, z_1 and z_2, if the concentration factor and the compressibility of the soil are known.

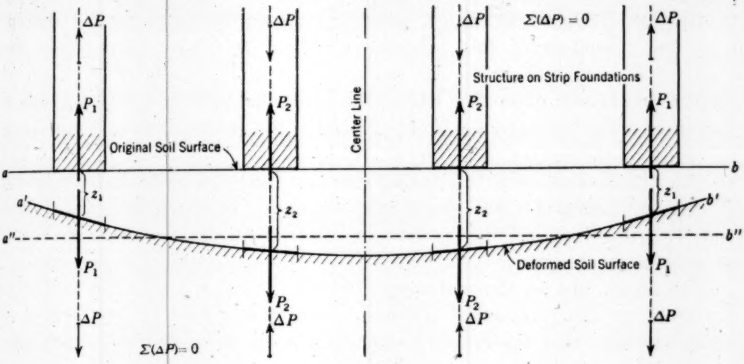

FIG. 16.

As the originally straight surface line, ab, of the soil changes into the curve, $a'\,b'$, it is evident that the distribution of the total load, ΣP, which originally was P_1-P_2-P_2-P_1, is changed by the interaction of structure and underground. How the supplementary system of forces, $\Sigma(\Delta P) = 0$, acts on the structure and on the ground, is shown in Fig. 16. The unknown value of ΔP must satisfy the condition that the deformation of the structure becomes identical with the deformation of the surface line of the soil. Especially, if the structure is absolutely rigid, the supplementary system of forces, $\Sigma(\Delta P) = 0$, must flatten the curve, $a'\,b'$, into a horizontal straight line, $a''\,b''$ (dotted in Fig. 16). As the settlements, due to the action of the supplementary forces, vary directly with ΔP, the numerical work involved is not so tedious as one would suppose at first glance. The last step of this procedure is to compute the so-called secondary stresses in the structure, caused by the four supplementary forces, ΔP, which is again a problem of pure statics.

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[28] "Druckverteilung im Baugrunde," Equation (5), p. 84.

Donald W. Taylor,[29] Assoc. M. Am. Soc. C. E. (by letter).—The author is to be commended for advancing the question of stress distribution under rigid footings and rigid mat foundations, and for emphasizing the fact that the common practice of assuming uniform distribution of pressure is not logical and may be unsafe in some cases. Most engineers will agree with his recommendation that extensive use should be made of pressure-measuring cells under new structures to furnish data on actual distributions.

Professor Krynine has made free use of the concentration factor, a parameter which should be adopted only when the extent to which it is applicable is held clearly in mind. The same point may be said to apply to Dr. Fröhlich's work[3] which, however, contains many valuable conceptions for a student of this subject. Fröhlich has shown that under the edges of a rigid footing on the surface of a sand deposit a condition of plastic flow must exist for which the concentration factor will have a maximum value of perhaps 6 or 8. Under the footing, progressing from the edge to the center, this factor will decrease from point to point and also it will decrease with increasing depth. Thus, the concentration factor at different points for the given case may vary as much as from 8 to 4 and with this in mind it may be seen that diagrams such as Fig. 9 may be very misleading.

The author makes the statement that consolidation of deeper strata has no effect on the interaction between a structure and the earth mass on which it rests; that the entire system may be assumed to settle uniformly. This can no more be possible than that the middle support of a rigid continuous beam can settle without increasing the load which the outer supports must carry. In Fig. 17(*a*) consider the loaded area to be the surface of a sand deposit where there is a complete lack of rigidity, and for simplicity let it be loaded uniformly and let the load be applied instantaneously. Before the underlying layer has had time to compress appreciably the settlement, which will occur entirely within the sand, will be as shown; and the stress thrown to the buried layer, according to the usual conceptions of spreading out of stress, will be as indicated. From this distribution of pressure on the buried layer, it is evident that the

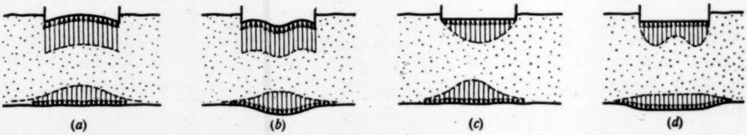

Fig. 17.

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compression in this layer will be greatest at the center. The loaded area must undergo a settlement approximately equal to this compression and after a period of time will have settled about as shown in Fig. 17(*b*). Since the surface stress distribution does not change, the stress at the compressible layer is probably not radically altered. For comparison, in Fig. 17(*c*), let the footing be rigid. When suddenly loaded the stress distribution under it and the stress transmitted to the clay will be as shown. It is evident that in this case there is a tendency toward greater settlements at the center of the loaded area, and yet the rigid footing requires that the settlement must be uniform. The only possible result is that as settlement proceeds there must be a readjustment of the two stress distributions of Fig. 17(*c*), since the stress distribution at the compressible layer is caused by the stress at the loaded area and also must be such as to give uniform settlement. Fig. 17(*d*) is a rough indication of probable distribution for the rigid footing after a period of time.

JACOB FELD,[30] M. AM. SOC. C. E. (by letter).—Because of the clarity of exposition as well as the completeness of the paper, the author's contribution to the study of soil mechanics is one of outstanding merit. The limitations of the results are carefully stated and no universal solution of a very important problem is claimed.

The broad application for Equations (1) and (6) is possibly open to criticism. The derivation of Equation (1) is based on a summation of stresses resulting from strains. As long as the summation of strains resulting from stresses does not exceed that at the elastic limit of the isotropic mass, methods of superposition are permissible. Conclusions 1 to 5 are worth careful study and attention.

It might be proper to point out that the stresses, s, are acting in the direction of the radius vector, joining the point of loading with the point at which the stress is being determined; also, that the graphical method described gives the stress at any one chosen point in the earth mass and must be repeated for each point at which knowledge of the stress is desired.

As a practical problem, the vertical intensity of loading is required. A continuation of the basic formulas of the paper, as is done later for the explanation of the graphical method gives: The vertical unit stress at any point in an earth mass due to a point load, P, at the surface is,

$$p_z = \frac{P}{z^2}\frac{n}{2\pi}\cos^{n+2}\alpha \qquad \text{(32)}$$

The vertical unit stress at any point in an earth mass due to a uniform linear load, $\overline{p}$, at the surface is,

$$p_z = \frac{\overline{p}}{z}\frac{n_1}{\pi}\cos^{n+1}\alpha \qquad \text{(33)}$$

Equations (32) and (33) are derived from Equations (1) and (6) of the paper. For various values of n and α, tables based on the values for n_1 listed in Table 1, can be prepared for ready use, as shown in Table 2. By the use of Table 2, upon an assumed value of n, and with a given loading, stress conditions at various depths along radial lines 15° apart can be computed and stress intensity contours plotted.

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TABLE 2.—VALUES OF $\frac{n}{2\pi}\cos^{n+2}\alpha$ AND $\frac{n_1}{\pi}\cos^{n+1}\alpha$.

	(a) VALUES FOR EQUATION (32)				(b) VALUES FOR EQUATION (33)			
n	$\alpha = 0$	$\alpha = 15°$	$\alpha = 30°$	$\alpha = 45°$	$\alpha = 0$	$\alpha = 15°$	$\alpha = 30°$	$\alpha = 45°$
3	0.477	0.401	0.232	0.085	0.637	0.545	0.359	0.160
4	0.637	0.517	0.269	0.080	0.751	0.632	0.366	0.133
5	0.796	0.625	0.292	0.070	0.850	0.690	0.358	0.106
6	0.955	0.723	0.303	0.060	0.939	0.735	0.343	0.083
7	1.114	0.815	0.306	0.049	1.019	0.771	0.323	0.064
8	1.27	0.897	0.301	0.039	1.090	0.797	0.299	0.048

Except in unusual cases, or at shallow depths, there is seldom any necessity to go into great details such as are outlined in the example illustrated by Fig. 6. Taking the data of that example, a very rapid approximate computation, using a slide-rule only, can be made as follows: Divide the area into three areas by the horizontal and vertical lines through Point X, and locate the center of gravity of each area:

$A_1 = 79 \times 60$; the center of gravity is 39.5 right and 30.0 up; α is

$$\tan^{-1}\sqrt{\frac{39.5^2 + 30.0^2}{100^2}} = 26° \, 20'; \text{ and, } A_1 \cos^5\alpha = 2\,740.$$

$A_2 = 55.25 \times 60$; the center of gravity is 27.63 left and 29.0 down; α is

$$\tan^{-1}\sqrt{\frac{27.63^2 + 29.0^2}{100^2}} = 21° \, 50'; \text{ and, } A_2 \cos^5\alpha = 2\,280.$$

$A_3 = 65.75 \times 60$; the center of gravity is 32.96 left and 31.0 down; α is

$$\tan^{-1}\sqrt{\frac{32.96^2 + 31.0^2}{100^2}} = 24° \, 20'; \text{ and, } A_3 \cos^5\alpha = 2\,480.$$

(The foregoing values are obtained from the correct location of the centers of gravity of the two trapezoids. It is close enough even to assume the centers 30 ft up and 30 ft down from the axis.)

The summation of $A \cos^5\alpha$ is 7 500, and the vertical unit pressure 100 ft below Point X is 0.89 ton per sq ft. This may seem to be in considerable error, the author's value being 0.78, but at a depth of 100 ft, the weight of the soil is approximately 5 tons per sq ft, and the totals are, therefore, practically identical. Much closer approximations, of course, can be determined by further subdivision of the loaded area.

G. P. TSCHEBOTAREFF,[31] M. AM. SOC. C. E. (by letter).—An interesting review of the most important recent developments in the study of pressures beneath foundations, including a method for the graphical determination of pressures within a soil mass, is contained in this paper. Special attention is given to the so-called "concentration factor." One of the welcome features of the paper is a list of questions, related to pressure distribution studies, which define the problems in need of elucidation by further research work. Most of the branches of soil mechanics and foundation studies have reached a stage of complex development calling for closer co-operation between the various research centers and practicing engineers if further progress is to be made rapidly. Such co-operation might be advanced considerably by a clear formulation of the problems requiring solution in the various fields. The publication of detailed lists of such questions should help to co-ordinate the efforts of the individual investigators and to stimulate them.

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The purpose of this discussion is to add a few more points to the outline of the necessary further research work, as given by Professor Krynine, and to offer some comments concerning the possible methods of approach to the solution of these problems.

Problem 1.—When Should the Laws of Pressure Distribution and When —Alternatively—Should the Probable Irregularities in the Compressibility of a Soil Be Adopted as a Basis for Foundation Design?—All estimations of pressure distribution are based on the assumption that the soil is a homogeneous material of uniform compressibility. Systematic investigation is required in different localities to ascertain what types of soil are, as a rule, uniform—at least horizontally—so that the foregoing assumption can be considered sufficiently valid for purposes of practical design. Very little data are available on this important point. In addition to pressure-measuring cells embedded in the foundations, this investigation could best be done by systematic settlement observations on numerous points of all new structures to be erected on compressible deposits. The necessity of such observations for all purposes of foundation research has been stressed repeatedly in technical publications by numerous authors, but little has been done actually. It becomes more and more evident that such full-scale observations require the participation of national or local governments, or of other public institutions, to allow their successful organization on a sufficiently large scale.

The writer's experience with settlement observations of fourteen structures in Egypt and some records from other localities to which he has had access, tend to show that there may be in some cases a relation between the geological history of the deposit and the horizontal uniformity of its compressibility. Of course, many more observations and their comparisons are required in different countries before anything definite may be said on the subject, but the following outline may be attempted at present.

Cohesive soils which had been precompressed in the past by previous alternate exposures to drying and flooding showed a fairly regular horizontal distribution of the observed settlements over the area of the structure. This indicated a similarly regular distribution of the pressures within the soil. They appeared to follow a definite law very closely, approaching the one derived from the Boussinesq formulas. Presumably, the previous higher shrinkage and swelling pressures had had a tendency to make these clays and silts more homogeneous in respect of the degree of their resistance to the later smaller pressures from the structures. This may be tentatively explained, as follows: The shrinkage limit, that is, the density down to which a soil is compressed by drying, is fairly low for all fully inorganic cohesive soils and varies over a comparatively small range. (For the Nile clays and silts of the cases referred to the water content at the shrinkage limit varied from about 14% to 28% of the weight of the solid substance.) The soil which has shrunk most will also tend to expand most when wetted again. In the early stages of the life of a deposit of the type considered, adjoining zones of unequal compressibility may exist in the same layer since its sedimentation. (The age of the deposits referred to varies from about 600 to more than 2 000 years.) In each cycle subsequent repeated drying and flooding would cause, first, the considerable shrinkage of all zones of any compressibility and the appearance of numerous vertical shrinkage cracks. Then, during flooding, these shrinkage cracks would be silted in and, simultaneously, the soil around these cracks would start swelling. After a few cycles of this kind the swelling would be partly prevented by the newly added soil which had filled these cracks. The zones most compacted by the shrinkage would have a tendency to swell most and, being prevented from doing so, would as a consequence exert a horizontal pressure on the adjoining less compact zones which would be greater than the counter-pressure caused by the lesser degree of swelling of the latter. The effect of this one-sided excess of pressure should be to cause a further compaction of the weaker zone and an expansion of the stiffer one until equilibrium would be reached gradually, corresponding to a uniform resistance of both zones to horizontal pressure. This repeated process should have a generally homogenizing influence on fairly large areas of a deposit. The degree of its resistance to further vertical compression should thereby be rendered also more uniform.

The only case of a horizontally irregular degree of compressibility, as evidenced by a measured tilting to one side of a uniformly loaded structure erected on a sedimentary cohesive deposit of the type cited, was recorded during this series of observations on a very recent formation (less than 150 yr of existence). Local hydraulic conditions made it appear very doubtful that it could have been subjected during this short period to the homogenizing process of repeated shrinkage and swelling as tentatively described.

Older and deeper lying but non-prestressed cohesive deposits of different origin in other countries exhibited a varying behavior. Some showed obviously irregular settlements, giving evidence that they were governed by the soil irregularities and not by the pressure distribution. Others exhibited a regularity of settlement distribution in plan which indicated a soil material of almost perfectly isotropic compressibility characteristics. The writer has not come into contact with a sufficient number of settlement records of such cases to allow him to make comparisons for the purpose of establishing the factors governing this difference of behavior. He would only like to stress once more the necessity of gathering numerous complete and detailed records on the subject and of making careful comparative studies of the relevant factors. All the achievements of theoretical investigations and small-scale experiments on isotropic materials will be of little use in the final analysis unless one knows when Nature permits the application of the results obtained to engineering practice. Only full-scale observations can give a reliable answer to this question.

It may be expected that both cohesive and non-cohesive soils pre-stressed by glacier loads would exhibit in many cases a horizontally uniform degree of compressibility. Whenever the great pressure of a glacier was applied to a large area of an underlying older deposit, in compressing such area the glacier would probably plane or squeeze out the stiffer zones first, producing thereby a homogenizing effect on the later compressibility of the whole layer. The writer has not come in contact with records of settlement observations on such deposits and, therefore, makes the suggestion as a tentative one.

Glacier loads would probably be the only homogenizing force that could act effectively on a granular deposit if it was originally sedimented in an irregularly loose condition. Drying would be of no effect on such soils. All the records known to the writer concerning settlements on non-prestressed loose granular deposits (that is non-compacted sandy loams, sands, and gravels of a type where appreciable settlements were measured) appeared to give evidence pointing toward a pronounced horizontal irregularity in the degree of their compressibility.

Further numerous field observations may possibly lead to the deduction of reliable criteria for the guidance of engineers—presumably of a geological nature—indicating the types of soil deposits for which designs may be based on laws of pressure distribution and on what deposits probable horizontal irregularities in the compressibility of a soil should be considered primarily. The practical importance of such criteria is obvious.

Problem 2.—The Depth of the Foundation Beneath the Soil Surface Should Have Considerable Influence on the Value of the "Concentration Factor," the Exact Extent of This Influence Requiring Experimental Investigation.—Several attempts have been made to express mathematically, by means of the so-called "concentration factor," the observed fact that stresses in sand are more concentrated beneath the center of the foundation resting on it than would have been the case if the laws of pressure distribution in an isotropic solid body were valid for sand. The difference in concentration of pressures appears to depend mainly on the nature and the state of the soil material beneath the edges of the foundation; that is, assuming that the foundation rests on the soil surface and that the soil consists of cohesionless sand, a lateral yielding, followed by loss of the supporting power of the soil, is known to occur near the sand surface beneath the edges of the foundation. This phenomenon, in turn, causes a greater concentration of pressures beneath the center of the foundation than is the case on a more cohesive type of soil.

The degree of cohesion of a soil appears to be taken by the investigators of the "concentration factor" as the essential criterion for the choice of its value. For the case of a foundation resting on the soil surface the value of the "concentration factor" is generally assumed to vary from $n = 3$ (clays) to $n = 6$ (sands).

However, the weight of the overlying layers may also be important. The resistance to lateral yielding of the soil at the edges of the foundation depends on its shearing resistance. This resistance, in turn, may be due to the cohesion of the soil, or to its frictional resistance, the latter depending on the weight of the overlying layers. One may assume that at a certain depth of foundation beneath the soil surface this frictional resistance to lateral yielding would become so great that a sand would behave like a solid material with a "concentration factor" of $n = 3$. This may be a probable explanation for the case of the bridge caisson on the Rhine, mentioned by Professor Krynine (see Fig. 12), where pressure cells embedded at its base indicated greater counter-pressures of the sand at the edges than at the center of the caisson, as this should be the case for solid materials where no lateral yielding around the edges is possible. In addition to the weight of the upper layers, pressures transmitted by friction from the external side walls of the caisson may have acted here as an extra surcharge on the area around its base.

Small-scale tests made by Dr. O. Faber, in England, on soil reactions against foundation plates, give interesting indirect evidence on this subject.[32] These tests showed that, for a surcharge of 1.46 tons per sq ft around a circular foundation plate 1 ft in diameter (this surcharge representing the equivalent of about 30 ft of overlying soil), the soil reactions against the edges of the foundation plate were equal to about 60% of the average pressure assumed to be distributed uniformly over the entire test-plate area. For the case of no surcharge, the value of the reactions at the edge reached only 20% of the average pressure.

This increase in the value of the soil reaction against the foundation edge from 20% to 60% of the average pressure, caused by the surcharge around the foundation edge, indicates a substantial increase of resistance against lateral yielding of the soil beneath the edges and, therefore, confirms the views expressed herein concerning the considerable influence on the "concentration factor" of the depth of the foundation below the soil surface. Further experimental investigation of the limit and intermediate values of this influence is desirable. Such investigations should take into account the degree of density of the sand, which is also likely to affect the "concentration factor."

Problem 3.—The "Concentration Factor" Formulas Should Express the Physical Fact That the Influence of the Foundation Boundary Conditions on the Pressure Distribution Decreases with Larger Loaded Areas.—The preceding discussion tends to show that the value of the "concentration factor" depends mainly on the extent of the lateral yielding of the soil around the foundation edges; that is, the "concentration factor" should be governed by the conditions at the foundation boundary. The influence of these boundary conditions on the general pressure distribution obviously decreases with an increase of the size of the loaded area. In respect to soil reactions this circumstance is well illustrated by Fig. 11 of the paper, but it does not seem to have been taken into account in any of the "concentration factor" formulas thus far proposed.

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[32] *The Structural Engineer*, London, March, 1933, pp. 120 and 122, Figs. 3 and 5.

It appears to the writer that the value of the "concentration factor" for a stress component, caused by pressures transmitted to the soil by a foundation element near the center of the foundation area, should be different from the value for a stress component, depending on an element near the edges of the area. In other words, a "concentration factor" changing in value with the distance from the center of the foundation toward its edges, should be used when making the integration of the stress components transmitted from the various foundation elements. The graphical method of integration proposed by Professor Krynine might be adapted to provide the possibility for taking care of this further important variable.

Conclusion (a).—In some cases the study of the geological history of a deposit, combined with full-scale settlement observations of numerous structures, may supply a criterion concerning the probable horizontal uniformity of its compressibility and, therefore, also a criterion as to how far future foundation designs may be reasonably based on pressure distribution laws.

Conclusion (b).—The estimation by means of the "concentration factor" of the pressure distribution beneath a foundation is more complex than would appear from the formulas so far proposed for this purpose by several authors. These formulas do not take into account variables of which the "concentration factor" is a function and which should strongly affect its values. These additional variables; that is, the surcharge of the overlying soil layers around the foundation and the size of the loaded area itself, might possibly cause, in many practical cases, a pressure distribution in sand approaching closely that of fully cohesive soils ($n = 3$), for which numerous auxiliary tables, computation graphs, and other accessories have already been developed, greatly simplifying and accelerating pressure computations.

Conclusion (c).—Much experimental and theoretical research is still required on this important subject, and Professor Krynine is to be complimented on having furthered its investigation.

A. HRENNIKOFF,[33] ASSOC. M. AM. SOC. C. E. (by letter).—Another step in the development of soil mechanics is marked by this paper, which is a valuable contribution to engineering knowledge. The writer has no suggestions to make in connection with the clear presentation of the author's interesting method, and he will limit himself to the discussion of the assumptions of the theory of a stress concentration factor, on which Professor Krynine's method is based.

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It may seem peculiar that, although the new science of soil mechanics deals with the same material as the classical theory of earth pressure on retaining walls, its development has proceeded along quite different lines, and little use has been made of the findings of the old theory. In this discussion the writer will endeavor to correlate the old and the new theories and to examine in detail some of the principles underlying the latter.

The classical earth-pressure theory involves consideration of a material with mechanical qualities that are characterized by the coefficients of friction, $f = \tan \phi$, and of cohesion, c. According to this theory, if at any point in the earth mass a normal unit stress, s_n, exists on any plane, the shearing unit stress on this plane cannot exceed the value,

$$s_s = f\, s_n + c \quad \text{.......................} (34)$$

although it may be less. This formula, known as Coulomb's formula, represents merely a statement of the fact that the maximum value of the shearing unit stress, at all possible on any plane, at any point, in a granular material, may be considered as the sum of two parts, one of which, $f\, s_n$, is proportional to the normal unit stress on the same plane, and the other, c, is independent of it. Coulomb considered the coefficients, f and c, as constant throughout the entire mass of earth, but for this discussion such a limitation is unnecessary, and the coefficients, c and f, will be thought of as quantities constant for different planes at the given point, but varying, if necessary, from point to point. There can scarcely be any quarrel with Equation (34) when so interpreted, since one cannot conceive of any cause that would contribute to the shearing resistance in a manner such that it could not be included either in one or the other of the two parts of the binomial of the formula.

Equation (34) shows the fundamental distinction between a solid body and an accumulation of granular particles constituting the soil. Although in the solid body any value of the shearing stress below the breaking point, imposed by the conditions of continuity, is possible, in the granular material there is a definite limit, and as soon as the limiting value of the shear is reached sliding occurs.

As is shown in standard texts[34] this limitation of the shearing stress results in a limitation of the ratio of the principal stresses. In a cohesionless material (that is, in a material for which c is zero) the least ratio of principal stresses, consistent with Equation (34), is,

$$k = \left(\frac{s_2}{s_1}\right) = \tan^2\left(45^\circ - \frac{\phi}{2}\right) \quad \text{................} (35)$$

in which s_1 is the major principal stress, and s_2 is either of the two remaining principal stresses. In a cohesive material the limiting ratio, k, depends on c, and on the value of the major principal stress; it is less than Equation (35), and may be as low as zero.

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[34] For example, "Earth Pressure, Retaining Walls, and Bins," by the late William Cain, M. Am. Soc. C. E., New York, John Wiley & Sons.

The writer wishes to emphasize the fact that Equation (35) and its counterpart for the cohesive earth are derived by application of the equations of statics to an infinitesimal wedge of earth, and that they are not influenced by the deformations of the material, elastic or otherwise, and by possible variation of the coefficients, f and c, from point to point.

It follows from the preceding discussion that a granular material may act as a solid body only as long as the ratio of principal stresses at any point, found by application of the theory of elasticity, proves to be larger than the least possible value of k; but as soon as the ratio, $\frac{s_2}{s_1}$, tends to fall below k, the particles slip, the stresses re-adjust themselves, retaining the least ratio, k, between them, and the earth mass ceases to act as one solid body. When that happens, the only equations of the theory of elasticity that hold are those based on statics, while the continuity equations become invalid. This phenomenon of slipping must not be overlooked when the equations of elasticity based on continuity, such as the Boussinesq solution, are applied to earth.

The major principal stress in earth under a foundation assumes a value governed by the load and the two other principal stresses—values governed by k. Thus, if a foundation produces a vertical uniform pressure, p, on the foundation bed (an assumption which is not always true), the major principal stress immediately under the foundation is vertical and equal to p; and the other two principal stresses acting horizontally are equal to $k\,p$, unless a larger value is required by the conditions of continuity. Minor principal stresses less than $k\,p$ are impossible because that would require the existence, on some planes, of a coefficient of friction greater than $f = \tan \phi$.

With these principles in mind the question of the admissibility of the Boussinesq solution will be considered. This will be done first in relation to the two-dimensional problem, and, subsequently, some remarks will be added concerning the three-dimensional problem.

The solution has been derived for a solid body obeying Hooke's law. According to this solution the major principal stress acts in a radial direction (see Equation (3)) and is equal to,

$$s_1 = \frac{2\,\bar{p}}{\pi\,\rho} \cos \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

and the remaining principal stresses, s_2 and s_3, are equal to zero. Whether or not this solution is applicable to earth depends on the following two questions: (1) Does the earth obey Hooke's law; and (2) can the minor principal stresses be equal to zero? Only in a case of positive answers to these two questions can the Boussinesq solution be applied properly to earth.

As to the first of these conditions, the applicability of Hooke's law is conceded by most investigators. However, it must be remembered that only a part of the total deformation may be subjected to this law, namely, the part that occurs before any sliding takes place. The part of the deformation that accompanies sliding, and involves a change of shape without any change of volume, naturally bears no proportional relationship with the stress, since the stress may be considered constant during this phase.

The second condition required by the Boussinesq solution (that is, the vanishing of the minor principal stresses) will now be considered in detail. Three broad cases should be distinguished:

(*A*).—*Cohesionless Material.*—In cohesionless material the minor stresses cannot vanish unless the major stress is zero, as is evident from Equation (35); consequently, the Boussinesq solution is not applicable to such material.

(*B*).—*Dry Cohesive Material.*—It is possible for the minor stresses in dry cohesive earth to vanish, provided c is sufficiently high and the major stress is sufficiently low. The greatest value of the major stress for which the minor stresses may vanish, can be found from the relation[34],

$$s_1 = 2\,c\,\tan\left(45^\circ + \frac{\phi}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Equation (37) is derived from statics without any special assumptions. A conclusion, then, may be drawn that once the major stress under the action of a line loading exceeds the limiting value (see Equation (37)) the solidity of the earth is broken, which, in turn, means that the Boussinesq solution does not hold. Taking an example of stiff clay with $\phi = 7^\circ$ and $c = 1\,500$ lb per sq ft,[34] the value of s_1 from Equation (37) equals 3 390 lb per sq ft, a value that is not very high. The writer does not insist on the accuracy of the assumed coefficients for clay in his numerical example, but simply wishes to emphasize the fact that in conditions encountered in practice the dry cohesive earth is not likely to behave as one solid body, at least for some depth below the foundation; but, at a greater depth, where the major principal stress decreases sufficiently, it may act as a solid body. This conclusion is corroborated by the testimony of several investigators, referring to the region immediately beneath the foundation as a disturbed zone. From the writer's point of view this zone is merely a part of the earth mass in which the particles have undergone some sliding, and in which, consequently, the theory of elasticity does not hold.

(*C*).—*Cohesive Earth Saturated with Capillary Water.*—If the foundation bed of a cohesive earth is situated between the levels of capillary and gravitational waters, a state of uniform pressure due to a capillary action is superimposed on the state of stress caused by the load, increasing by the same amount both the major and the minor principal stresses. This condition is favorable to a behavior of earth as a solid continuum, and obviates the necessity of forming a disturbed zone. However, in order that the capillary pressure may properly materialize, the earth must be nearly saturated with capillary water. The writer does not think that mere presence of moisture in a cohesive earth would make it behave in a manner essentially different from dry earth, although the coefficients, c and f, may be affected by moisture content. No capillary pressure is present below the level of gravitational water. Capillary action in sand and gravel is negligible.

Failure of the Boussinesq solution to agree with the experiments led the investigators to adjust it by an arbitrary introduction of a quantity, $(n-2)$, instead of unity in the power of the cosine in the expression (Equation (36)) for the radial stress. The coefficient, 2, also had to be changed into $n_1 = f(n)$ due to considerations of statics; thus, the author's Equation (6) was obtained. In this empirically modified Boussinesq solution all other results of the original theory are retained; the radial stress is still considered as the major principal stress, and the two other principal stresses are held equal to zero; the empirical constant, n, termed the concentration factor, varies in magnitude depending on the nature of soil material. It is this modified theory that has been used by the author in the development of his graphical solution.

The weakness of the modified theory lies in its failure to take proper account of the earth disturbance, a fact leading to several apparent inconsistencies:

1.—As has been demonstrated, the minor principal stresses, caused by the loading, may be equal to zero only in cohesive material when it is lightly loaded, or nearly saturated with capillary water. When such a material is nearly dry, or completely submerged below the gravitational water level and, at the same time, is fairly heavily loaded, the minor stresses do not vanish, and it does not seem justifiable to disregard them. In cohesionless material the minor stresses cannot be equal to zero under any circumstances. One might think that this criticism would not apply if several line loadings were superimposed; however, the objection would still hold, at least for the region near the boundary of the loaded area.

2.—It seems improbable that the same value of n should properly describe the state of stress both near the foundation where the soil is disturbed and outside the zone of disturbance. The writer feels that the stresses at a point depend not only on the geometric location, but also on whether the point in question lies inside or outside the disturbed zone. It also appears that the size and shape of such a zone should have an important bearing on the settlement of the foundation and on the law of distribution of pressure over the base.

3.—The use of the principle of superposition in conjunction with the modified Boussinesq theory seems incorrect. As has been stated, the introduction of the empirical exponent, $(n-2)$, was due to one of two causes: (*a*) The failure of the earth to obey Hooke's law; or (*b*) earth disturbance. Either one precludes the application of the principle of superposition. That the statement is true with respect to the first reason needs no explanation. The earth disturbance, however, has the same effect: First, because the boundary of the disturbed zone is not the same for the three loads—Loading 1, Loading 2, and Loading $1+2$; and second, because in the disturbed zone, the minor principal stresses do not combine vectorially, but keep a constant ratio, k, with the combined major stress.

It may be argued that superposition is an additional empirical assumption which is to be used with the empirical formulas of the modified Boussinesq theory, but such an assumption stands as a contradiction to the physical nature of the phenomenon which it purports to describe. Suppose, for a moment, that Equation (6), describes truly the state of stress in certain soil under a line loading, and that the value of the concentration factor is determined from experiments with such a loading. If then the same soil is subjected to several line loadings, and the state of stress is analyzed by the formulas derived by superposition which involve the same value of n, then a disagreement will be noticed between the observed and the computed values of stress. The disagreement may be removed by a proper modification of the value of n accompanied by an admission that the magnitude of the concentration factor depends not only on the character of the soil, but also on the character of the load and on the depth. Thus, the assumptions of the modified Boussinesq theory lead to a deduction of variability of the concentration factor, with its dependence on several variables besides the character of the soil. This complicates the matter considerably. The difficulty, of course, is the natural consequence of an attempt to describe a physical phenomenon by an uncongenial formula and an unsuitable assumption.

The writer feels that the criticism of the use of superposition in the modified theory strikes at its very root, and unless it is removed, the theory fails, since superposition is the very essence of its application.

4.—The reasons for the particular significance attached by most investigators to p_z, the normal unit stress on the horizontal plane, are not clear. The primary object of stress analysis in the earth under a foundation is the determination of the foundation settlement, caused by the earth deformation, but this deformation is manifestly a function of all three principal stresses, and not of p_z alone. Nor can p_z always be considered nearly equal to the major principal stress. The writer thinks that the pressure exerted by the actual foundation on a bed of cohesionless material may be inclined at an angle of friction with the normal. If that is true, the major principal stress over a large part of the foundation bed in sand and gravel must be far from vertical.

The foregoing criticisms have been made primarily as they apply to the two-dimensional problem. They also apply with minor qualifications to the three-dimensional problem, in regard to which the writer wishes to make the following additional comment:

5.—In the original Boussinesq solution the radial stress is not a principal stress, except for an incompressible elastic solid body, and none of the principal stresses is zero. In the empirically modified solution, Equations (1) and (4), the radial stress is treated as the major principal stress and the other principal stresses are considered as non-existent. Although the author does not make any use of this interpretation, if it is adopted the original Boussinesq solution ceases to be a special case of the modified theory, except for the aforementioned physically impossible material. From these considerations it follows that if a certain earth material exhibits, in its mechanical characteristics, a great similarity to an elastic solid body, such a material is not subject to analysis by the modified Boussinesq theory, a conclusion which is as unexpected as it is logical. The modified theory, therefore, must be supplemented by the list of soils to which it is applicable.

Conclusion.—The Boussinesq theory has been chosen as a starting point in soil mechanics by many investigators. After some adjustments, an apparent agreement with the experiments has been reached, and the investigators have been eager to proceed with further development, whereas the nature of underlying assumptions apparently has received only casual attention. This discussion, by emphasizing some seeming inconsistencies, is intended to bring forward the necessity for critical examination of the basic principles on which the theory rests. The writer appreciates the qualitative value of the original Boussinesq theory in describing the stress behavior of the earth mass, and is inclined to concede a limited field for its applicability in quantitative determination of stress when dealing with cohesive materials, but he feels unable to explain the numerous apparent contradictions inherent in the modified theory. If the writer's diagnosis is correct, and the contradictions indicated by him are irreconcilable, a further elaboration of the theory of the concentration factor is futile.

ROBERT G. HENNES,[35] ASSOC. M. AM. SOC. C. E. (by letter).—This contribution by Professor Krynine to the growing literature on soil mechanics is of more than usual interest, not only because of the excellence of material and presentation, but also because the division of his paper into two parts illustrates rather aptly an important gap in the existing knowledge of this branch of the subject. Part I deals with the determination of numerical stress values in an ideal material under given conditions of loading, which is assumed quite generally to apply with acceptable accuracy to the case of stress distribution in a deeply underlying soil stratum. Part II deals in a more general sense with stress distribution over the surface of contact of the structure with the earth. For the average structure the stress distribution in the intermediate "disturbed zone" is of at least equal importance, but this region constitutes also an "unexplored zone," which needs more investigation before a satisfactory theory is available to the profession. In the meantime, the engineer must resort to the statistical, as opposed to the scientific, approach in dealing with what Professor Krynine terms "contact settlement."

Those who frequently have occasion to determine stress distribution in the subsoil should welcome the graphical methods presented in Part I.

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[35] Asst. Prof., Civ. Eng., Univ. of Washington, Seattle, Wash.

Others, who only seldom undertake this task, can profitably study at least that part of the author's method which presents Kögler and Scheidig's use of annular elements, especially when confronted with non-uniform loads, or with loaded areas of irregular shape.

It is natural, but unfortunate, that most valuable contributions to the science of soil mechanics seem to widen progressively the gap between the specialist and the average civil engineer. It is the impression of the writer that many engineers without formal education in soil mechanics would read this, and similar papers, more profitably if the fundamental assumptions were treated from the physical as well as from the mathematical viewpoint.

Consider, for example, the case of a uniform load placed on the horizontal surface of a large supporting body. If the load could be transmitted through this material without spread, the resulting situation would be as shown in the free body diagram of Fig. 18, the entire load, *W*, being carried by the prism *ABCD*, which would, consequently, be shortened. If the material possessed any shearing strength, however, this compression would set up shearing stresses along the surfaces, *AD* and *BC*, thus transmitting part of the load to the adjacent regions, in violation of the assumptions. If the material had no shearing strength, the underlying material would flow from under the load until the vertical stress across any horizontal plane became equalized, and the body would float, as in Fig. 19. If the material possessed some shearing strength, but less than the maximum stress imposed, there would be flow until the maximum shearing stresses were relieved to a point within the capacity of the material to withstand them. The quantity of flow depends, in part, upon the location of the over-stressed material. If it cannot move without displacing a considerable quantity of adjacent material, the flow may be limited to the relatively small amount required to compress the adjacent under-stressed soil sufficiently to transfer to it the excess load. Because this quantity depends upon the compressibility of the material, the computation of stress distribution is greatly simplified if it is first assumed that the stresses nowhere exceed the shearing strength of the material.

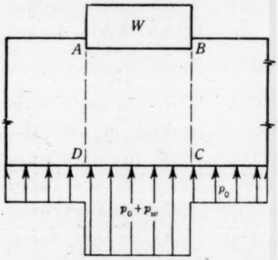

FIG. 18.

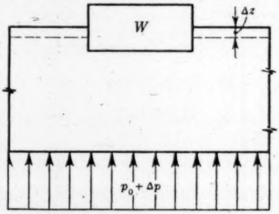

FIG. 19.

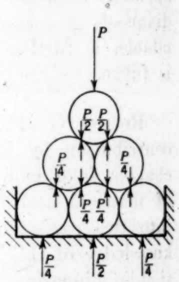

FIG. 20.

Next, consider a pile of incompressible rollers arranged as in Fig. 20. It is obvious that the load, P, would be divided equally between the two rollers in the second layer, and that in the third layer the middle roller would be subjected to a load of $0.5\,P$ while the two end rollers would each carry a load of $0.25\,P$, horizontal stress components being disregarded. If the same stress distribution were assumed to occur in a pile of elastic rollers, the application of the load, P, would result in a compression of each roller, and in the third layer the middle roller would experience twice the vertical compression of the end rollers, and hence lose contact with the overlying rollers which transmit the load. Since this is contradictory, it seems that for elastic materials the normal stress distribution on a horizontal plane will be more uniform than that represented by the example of the inelastic rollers. An intermediate case would be that of a material which, like soil, offered relatively more resistance to compression at higher temperatures. It should be noted further that, in the foregoing example, lateral forces have been neglected. As the end rollers experienced lateral displacements under stress, the middle roller would regain its proper share of the load, as the overlying rollers moved into more stable positions with equal bearing on their supports. Since vertical stress distribution is thus seen to be a function of lateral strain, it follows that divergence between the stress-strain relationships along horizontal and vertical axes would affect that nature of normal stress distribution. The general problem is further simplified, therefore, by assuming that the material has the same physical properties in all directions, and that it is a solid homogeneous mass.

In general, the foregoing assumptions are those of Boussinesq in determining the stress at a point in a semi-infinite, elastically isotropic solid, loaded by a single concentrated normal force applied at a point on the horizontal boundary plane.

With the foregoing concepts, and with a knowledge of Boussinesq's assumptions, one can visualize, in a general way, the application of this type of equation to lightly stressed soil strata, especially when the concentration factor is included. If those concepts are approximately correct, however, it is difficult to understand how the concentration factor can be made to cover the situation that arises when the shearing strength of the soil is exceeded at one or more points, involving such factors as the flow properties and the compressibility of the material. Yet this is the situation that commonly exists in the disturbed zone, at least under the edge of the footing where shearing stress is maximum. Even the lightest surface load, therefore, is likely to cause a trace of flow at the edge of the footing. If the footing is perfectly flexible, the flow continues until the edge of the footing has settled to such a depth that the weight of the adjacent earth has sufficiently served to relieve the intensity of the maximum shearing stress, or to increase the frictional resistance to flow. If the footing is perfectly rigid, the edges cannot settle independently; thus incipient failure of the soil at the edges is accompanied by transfer of some of the load to the central part of the footing, and the condition of uniform loading no longer holds. If this view is acceptable, Fig. 11(*a*) and Fig. 11(*b*) generally do not hold for surface loadings, whereas for footings at considerable depth, the effect of confinement might give even clay sufficient rigidity to correspond to Fig. 11(*c*) and Fig. 11(*d*). It would still be possible, of course, for the intensity of the soil reaction at the center of the footing to be relieved by the actual settlement of the soil, as in the case of consolidation of underlying clay strata.

T. A. MIDDLEBROOKS,[36] ASSOC. M. AM. SOC. C. E. (by letter).—The factors that influence the stresses in earth masses are treated most completely in this paper, which reveals clearly the fact that little is known about the true distribution of stresses in a foundation, especially where there is a wide difference between the rigidity of the structure and the foundation material. A quick realization of this fact by the Engineering Profession in general will be most helpful in speeding the accumulation of badly needed field data. The author's "reduced area" method of determining pressures under structures is a decided improvement over the usual procedure, and should meet with widespread usage.

Recognition of the influence of the rigidity of structures and earth masses on the stress distribution is of major importance. Professor Krynine has given a thorough presentation of the effect of this influence and he rightly states that the degree of rigidity in actual structures will be some value between the two extreme limits. The writer agrees with the statement that a clay can be considered as a non-rigid mass; but, on the other hand, he believes that a compact sand will act as a rigid mass only when it is supported by non-yielding strata. In the case of a footing resting on a compact sand stratum which, in turn, is supported by a non-rigid clay stratum, the sand, in this case, could not be correctly considered as a rigid mass. The same is true of a sand dam resting on a clay foundation. In cases where the clay is underlaid, as well as overlaid, with sand, the top sand should not be treated as a rigid mass, although the sand beneath it should be considered rigid. Working on this basis, the writer has been able to obtain some close checks on the stability of earth fills resting on clay in the foundation.

Interrelation of stresses in the structure and foundation are of utmost importance. The stresses in an earth fill have a marked effect upon the stress in the foundation. This fact has been ignored by numerous investigators in an attempt to devise a simple formula or method for analyzing the stress in a foundation. The writer has found that it is essential, in the design of earth embankments, that the fill and foundation be considered as a whole in determining the stresses in the foundation.

Considerable errors will occur in the computed foundation stresses if the stress in the structure is not taken into consideration, as would be the case if it is assumed to be perfectly rigid. It is evident that the greatest error in these stresses occurs when the actual rigidity of the structures and the foundations is approximately the same. Errors are also introduced if it is assumed that the vertical load on the upper boundary of the foundation material is equal to the weight of the material directly above it at every point, and that the shearing stress along this upper boundary is zero. It is only logical that "dam builders" should be first to recognize the importance of these principles, since the difference in the rigidity of the fill and foundation of a dam is much smaller than it would be for a footing on the same foundation.

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In making use of computed foundation stresses, does the author consider satisfactory the generally accepted method of determining the probable settlement of a structure by computations based on the use of the vertical forces and the change in volume of the compressible strata directly beneath the loaded area only? In order to determine accurately the probable settlement beneath a footing, the volume change from consolidation throughout the entire pressure bulb should be used. Furthermore, the maximum principal stresses, rather than vertical stresses, should be used in computing the consolidation. This method will not be as simple to use as the generally accepted method, but too much simplification of a complex problem can easily lead to undesirable errors in the results. Therefore, since the engineer is attempting to determine the stresses more accurately, he is also justified in using a more nearly correct method of computing the consolidation.

A. A. EREMIN,[37] ASSOC. M. AM. SOC. C. E. (by letter).—A most interesting method of graphical computation of foundation stresses under spread footings is developed in this paper. It is simple and helps to visualize the distribution of foundation pressures. Professor Krynine's graphical construction may also be applied in the computation of foundation stresses with equations developed by Messrs. F. Kögler and A. Scheidig.[38] Using the author's notation, the vertical foundation pressure, p_z, at depth, z, as expressed by Messrs. Kögler and Scheidig is:

$$p_z = \frac{3\,P}{2\,\pi\,z^2}\,\frac{(\cos\alpha - \cot\phi\sin\alpha)\cos^4\alpha}{1 - \cos\phi} \quad\ldots\ldots\ldots\ldots (38)$$

in which ϕ is the angle of repose of the foundation material. Considering the principle of superposition of loads, the foundation pressures due to the load on an elementary area, dA, from Equation (38) are,

$$p_z = p\,\frac{3}{2\,\pi\,z^2}\sum\frac{dA(\cos\alpha - \cot\phi\sin\alpha)\cos^4\alpha}{1 - \cos\phi} \quad\ldots\ldots\ldots (39)$$

The summation of the last term in Equation (39) may be determined by a graphical construction similar to that shown in Fig. 2.

It is interesting to note that instead, of the term, n, used by the author in the equations for foundation stresses, Messrs. Kögler and Scheidig used the angle of repose, ϕ, to express the physical qualities of the foundation material.

M. M. BUISSON,[39] ESQ. (by letter).—The two quite distinct parts into which this paper is divided, are: (I) The description of a graphical method for computing the stresses in soils; and (II) a study of stresses that develop under a rigid foundation.

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[38] *Bautechnik*, 1929, p. 268.

[39] Chef du Service CCI, Bureau Veritas, Paris, France.

Part I.—The Method.—In some cases, it has been possible to derive analytical formulas giving the values for horizontal and vertical normal pressures and for principal stresses at any point in a half space. By dividing the loaded area into rectangular, or square, elements, the vertical pressure can be computed easily, as, for instance, the method developed by Dr. Wilhelm Steinbrenner.[40] In some cases, Professor Krynine's method will be an easier one for the solution of similar problems, taking the value of the concentration factor, n, into account, with sufficient accuracy. This is an application of a graphic integration method which has been well known for many years, and which makes it easy to solve for the principal stress trajectories.

Moreover, since failure trajectories are isoclinic lines that meet the principal stress trajectories at an angle, $\frac{\pi}{4} - \frac{\phi}{2}$, one may draw easily such failure lines, and thus determine the failure conditions in cases where a purely analytical solution of the problem would not be possible.

In two-dimensional cases, if a semi-infinite plane is loaded and if $n = 3$, the failure lines are parabolas with their foci at the half-plane limit. The axis of the parabola is a straight line, inclined to the vertical by an angle equal to the friction angle.[41] In the case of an infinite loaded strip, it is known that the principal stress trajectories are hyperbolas with their foci at the edges of the loading strip. From this knowledge, failure trajectories can be drawn easily.

Failure would occur if the equilibrium of a certain part of the loaded body should become unstable. This condition will decide the position of extreme failure trajectories and also the loading conditions corresponding to an actual failure along these lines. When it is possible to determine the form of the plastic zones analytically—and, above all, to locate the point of intersection of the marginal plastic zone with the loaded surface—(the abscissa of this point is a function of the applied load)—one may locate the extreme failure trajectory also, in an analytical manner, since this line must necessarily meet the aforementioned point of intersection.[42]

In a particular case, a uniformly loaded strip would show, contrary to expectation, that the extreme plastic-zone lines are not in themselves failure lines; this indicates, incidentally, that the bases of Prandtl's computation are not exact.

Furthermore, if the stress distribution can be studied analytically, it is always possible, to choose from a set of possible failure trajectories, one along which actual failure conditions will be attained. This can be done graphically using a method similar to that applied in determining the slopes of earth dams. Without any difficulty, one may compute the load necessary to produce such a critical instability, whether the loading is rigid or semi-rigid.

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[40] *Die Strasse,* October, 1934; also a special publication by that Journal entitled "Bodenmechanik und neuzeitlicher Strassenbau," p. 75 (1936).

[41] "Etude de la plasticité dans le cas d'une bande de largeur indéfinie chargée uniformément," by M. M. Buisson, Travaux, Mars, 1937.

[42] "Conditions de stabilite du sol," by M. M. Buisson, Travaux, Decembre, 1936.

Following the same general trend of thought, the writer would attach great importance to the systematic determination of failure conditions in most of the common loading cases. This can be done easily by means of Professor Krynine's graphical method.

On the other hand, actual tests have shown[43] that if an excessive load is applied to an earth mass, and both the earth mass and the superstructure fail, the failure trajectories in both bodies in contact are always prolongations of each other. They do not possess a common tangent at the boundary, however, but will meet at an angle, unless the angles of internal friction are the same in the soil and in the superstructure. It is worthy of note that, in cases where it was possible to trace lines for these failure trajectories, with sufficient accuracy, they were almost the same as the theoretical lines, corresponding to $n = 3$.

Part II.—The Stresses.—Concerning Part II of the paper, which relates to the study of the stresses that would develop under a rigid foundation, the writer would insist, urgently, on the great necessity of making actual tests and measurements on the foundations themselves.

Numerous important factors which are not yet completely understood are then in operation. First, the basic hypotheses must be checked. Only by actual testing or observation can one obtain reliable results. It is difficult indeed to determine, even approximately, the mutual reactions of both bodies in contact; that is, the superstructure and the supporting soil. Many results seem contradictory only because due attention has not been paid to an analysis of peculiar cases. Professor Fröhlich is credited with having studied the rare tests existing at the time and comparing them, case by case, with the theoretical results obtained by Boussinesq.

In this manner he arrived at the most interesting conception of the "concentration factor," n, thus providing a means of generalizing the formulas for the Boussinesq stress distribution.

Experimental tests and measures, however, are very difficult to carry to a successful conclusion, the principal reason being the presence of important, although secondary, phenomena. In general, pressures are measured by using Goldbeck cells. For small-scale models, it seems better to resort to small manometric capsules, closed in by a rubber diaphragm. It is necessary to conduct the tests in such a manner that the diaphragm cannot become displaced; otherwise, the readings will be affected. For instance, if the movable diaphragm had been pressed into the ground and if, subsequently, one wanted to return it to its original place, the necessary force is not at all in proportion with the actual pressure at the capsule point. Mr. Leo Rendulic[44] made use of a similar method for the measurement of hydrostatic pressures that are developed by various principal stresses in soil cylinder samples.

Actually, this force will be much greater than the pressure caused by any arching effect that may develop in the soil. If sufficient accuracy is to be attained, one must press the diaphragm beyond its definite stationary position and then release the force gradually, noting the pressure at the very moment when the initial position has been reached.

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[43] *Annales des travaux Publics de Belgique.*

[44] "Spannungszustände in der Umgebung eines Hohlraumes," by Leo Rendulic, *Die Wasserwirtschaft*, Vol. 27, June, July, and August, 1934; also, "Ein Grundgesetz der Tonmechanik und sein experimenteller Beweis," *Der Bauingénieur*, Heft 31–32, 1937.

Nevertheless, even when such precautions are taken, errors can be made. The best way would be to mark a capillary zone on a tube, and then to make use of a tank filled with air under compression. The pressure could be exerted by water, for instance, as the measured pressures will be always low. If the air compression is kept constant, the meniscus will stay at the mark, and then the measured pressure will be the actual pressure in the soil.

With such apparatus in operation, one could detect such stress variations as those indicated by Professor Krynine. It is normal that such variations should occur because after loading the soil, its properties will be altered locally. If, previous to tests, the soil behaved as an isotropic and homogeneous body, it will no longer be so after the tests due, among other causes, to unequal settlements. It is obvious that factors for isotropic properties always express extreme conditions that are never absolutely obtained in actual soils.

Tests will show changes of two different types: (*a*) Rapid stress variations; and (*b*), slow changes in stress alterations.

The rapid variations will transform the curves (which appear continuous in Fig. 13 of the paper) into saw-toothed lines with maxima and minima. With the passage of time the difference between these maxima and minima will decrease. The curves in question reflect the process of re-grouping of soil particles. The slow changes are a result of the adjustment of the soil structure to the external load.

In testing some models of wharf retaining walls the writer used apparatus such as the aforementioned to extend the research to other tests more especially intended to study the form of failure curves, and to determine the conditions under which failure occurred. A certain number of capsules were placed in the soil, more or less at random, and only at the end of the tests could a satisfactory experimental procedure be devised. As new tests concerning this question are being conducted, the writer is not yet in a position to furnish definite results.

The original tests were performed in a large basin, 3.93 ft wide, 3.28 ft deep, and 11.5 ft long. Such a basin may be flooded or emptied, at will. A glass panel on one side permitted photographs to be taken which indicate, with the profile of the quay model, the particles in the soil.

By fixing the photographic apparatus in a definite steady position many pictures could be taken at various loading stages. By superposing such pictures taken at different times, one could easily trace (very approximately) the trajectories of movement in the soil particles as well as displacements of the wall itself; thus, the actual failure curves were derived in quite a satisfactory manner. Capsules were placed in the soil at different levels, as the soil was placed in the basin. Datum marks were fixed to measure vertical movements at the soil surface, as well as movements of the loading sheets, and of the quay model. These movements were measured by dials with an accuracy that could easily reveal 0.005 mm (= 0.002 in.).

Some test results were recorded during a loading test on a relatively long strip of homogeneous clay, 10 cm (3.94 in.) wide. The load was applied through rigid plates placed end to end along the entire width of the basin. A layer of fine sand, about $\frac{3}{8}$ in. thick, was laid between the plates and the clay, which provided for drainage through the soil immediately beneath the plates. Many difficulties had to be overcome:

(1) In some places, the pressure differences that were recorded during the application of the load were negative instead of positive as one would have expected. Such phenomena seem to be due to suction that occurs near the capsule, from the flowing clay. The capsules were practically fixed by seating them in the soil mass. It follows, therefore, that such readings cannot yield precise information as to stress distribution.

(2) To date (1937) test measurements have been made only at, or near, the critical loads, when the p_h-value (as recorded by a vertical capsule set 10 cm deep and 10 cm from the plates) is practically the compressive resistance of the same clay as that measured on cylinder samples. This value would be greater only if the material could consolidate.

(3) One may overload the soil beyond the critical loads obtained by rapid application, if the sample is permitted to consolidate after each partial loading. In a particular test, clay possessing a moisture content responding to the Atterberg liquid limit, offered a shearing resistance of 10 gram per sq cm (20.4 lb per sq ft). Failure under a rapid rate of loading took place under the action of a weight of 60 grams per sq cm (122.6 lb per sq ft). In a slow test process a load amounting to more than 120 grams per sq cm (245.3 lb per sq ft) would be required for the same purpose.

(4) Although the following statement bears only a distant relation to stress redistribution under rigid plates, it is useful to emphasize that, as the load approaches its critical value, one detects a considerable increase in the permeability. This is accompanied by a large and almost immediate settlement under the plate and, at the same time, a settlement occurs to the right and left of the plate. Thus, the extent of consolidation settlement beyond the plate, is much greater than the swelling that should have occurred as computed from the instantaneous settlement under the plate and from the constant-volume soil deformation.

Long before failure, however, one can note that instantaneous consolidation is occurring to a large extent, as the uplifted volume is practically nil, although the settlement under the plate is rather large; this is absolutely contrary to results derived from measurements of the permeability coefficient determined in the usual manner.

Observations on the movements of soil surfaces show that the flow influence varies with the distance to the plate; for instance, at a distance of 3 cm (1.18 in.) the maximum settlement was noted about 150 hr after the loading was completed, although at 20 cm and 30 cm (7.87 in. and 11.81 in.) such a maximum was recorded 70 hr after loading; the next swelling to occur was stabilized 200 hr after loading. One may conclude that the length of time necessary for flow stabilization in clay is almost equal to, or longer than, the time necessary for consolidation itself. Indeed, during the tests, the consolidation was not yet complete as the plate continued to settle.

The form of the settlement curve was readily adapted to logarithmic plotting, settlements being in proportion to logarithms of time. Thus, in this testing time, the settlements were due at once both to flowing and to consolidation processes, the effects being equal but opposite beyond the plate. After a certain time had elapsed the importance of the continuous flowing process eclipsed the consolidation noticeably.

The volume of soil raised on both sides of the plate was then equal to the product of the depth of settlement by the area of the plate. In this case, again, the settlement-to-time relationship was adaptable to logarithmic plotting. From the foregoing considerations, it seems that the logarithmic law, as given by Professor Buisman[45] for long-time settlements in eudiometric tests, would apply also to settlements from plastic flow.

Conclusions.—Research in the United States should be directed toward an extension of the ideas herein described, in order to obtain true information as to the behavior of the soil under foundations. This would not minimize, in the least, the merits of Professor Krynine's paper; but it is necessary in this connection to emphasize the urgent need of an experimental verification of theoretical results. At present, experimental data are much less available than theoretical research. It seems timely to minimize the importance of the latter and to strive for a rational balance with the importance of confirmatory tests.

FREDERICK J. CONVERSE,[46] ASSOC. M. AM. SOC. C. E. (by letter).—The method, described by Professor Krynine, for obtaining the pressure of any point within the soil below a loaded area, contains three important viewpoints which should be included in any method of pressure determination. In the first place, recognition of the fact that the soil cannot always be considered as an elastic isotropic medium is given by the use of such formulas as Equations (1) and (9), containing the "concentration factor," n. Secondly, the method can be easily adapted to non-uniformly distributed pressures at the surface. In the third place, the method can be used with equal facility for the case of single spread footings and for groups of foundations.

The work involved in the graphical solution presented by the author may be shortened somewhat by the following procedure: Consider a unit pressure to be applied at Point O', Fig. 2, and construct contours of equal vertical pressure in the soil below Point O', using Equation (8) with $\bar{p}\, dA$ equal to unity. Move the "transformed area," $M\ T'''\ N$, vertically downward until the horizontal through $M\ N$ passes through Point O, as shown in Fig. 21. The percentage of the unit pressure, indicated by the position of Point T' on the iso-pressure chart, multiplied by the height, $T'\ T'''$, gives the vertical component of the pressure at Point O due to a concentrated load, $T'\ T'''$, at the surface. This may be plotted vertically above Point T' and represents a point on the "pressure curve" (see Fig. 22). Its value corresponds to that of $T'\ y$ in Fig. 2, multiplied by $p \frac{n}{2\, \pi\, z^2}$. In a similar manner, other points on the "pressure curve" may

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[45] "Results of Long Duration Settlement Tests," by A. S. Keverling Buisman, *Proceedings,* International Conference on Soil Mechanics and Foundation Engineering, Vol. I (1936), p. 103 (Paper F-7).

[46] Asst. Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

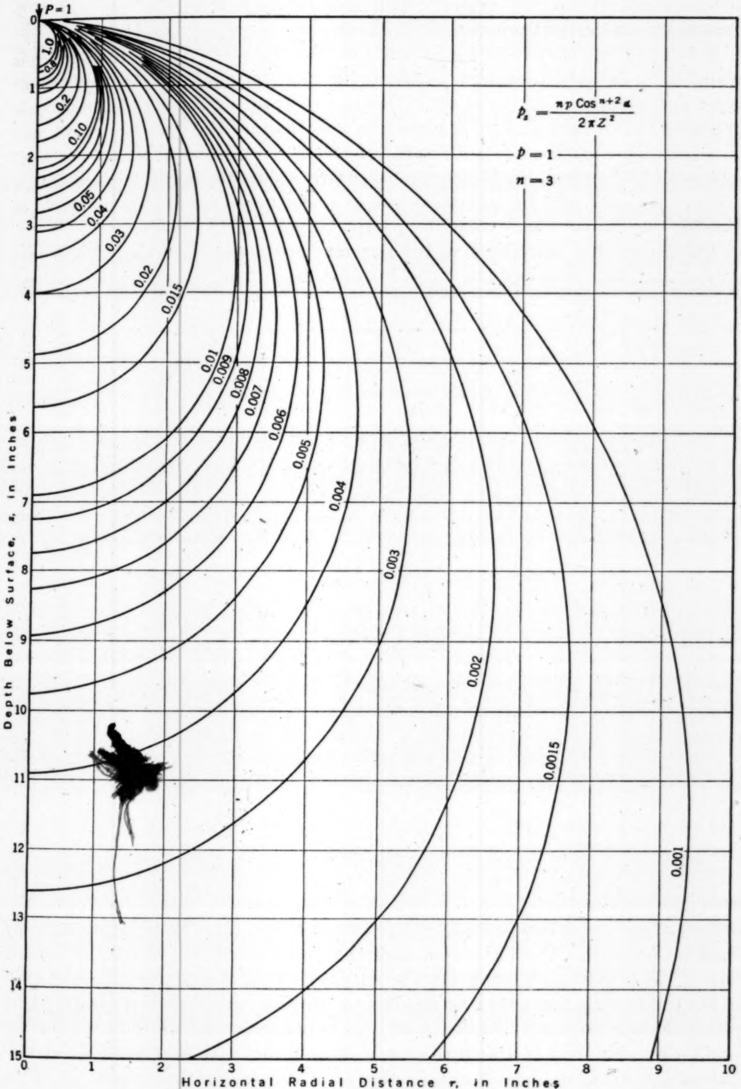

FIG. 21.—ISO-PRESSURE LINES OF VERTICAL COMPONENT OF PRESSURE WITHIN THE SOIL DUE TO A CONCENTRATED UNIT LOAD AT THE SURFACE

be found and the curve drawn. The area between this curve and $M\,T'\,N$ gives the total vertical pressure at Point O.

It is apparent that if an iso-pressure chart for a unit load at the surface is available, it is only necessary to draw the transformed area on transparent paper and place it over the chart at the desired position, using the same scale for depth that was used in the plan. In this manner the pressure curve may be readily constructed at any depth, once the "transformed area" has been obtained. If several iso-pressure charts are available, each representing a different value of n in Equation (8), the engineer may exercise his judgment in choosing the proper chart to fit the particular soil conditions.

Still more time may be saved if the data are presented in tabulated form without actually drawing the "transformed area" and the "pressure curve."

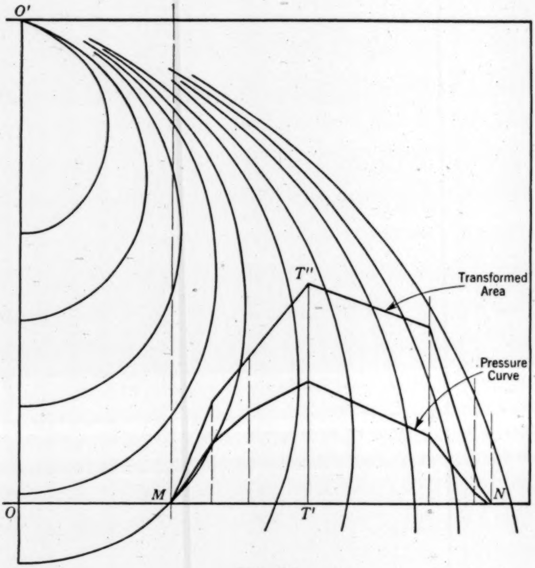

FIG. 22

In fact, even the circles used by the author to establish points on the "transformed area" need not actually be drawn if a transparent polar co-ordinate chart is at hand. With the center of the polar chart placed on the footing plan at the point below which the pressure is desired, the concentric circles will divide the footing areas into zones, and the number of degrees of arc swept by each zone may be read from the chart. It is not even necessary to redraw the foundation plan, if it is drawn to scale. Blue-prints may be used if the actual scale of the drawing is determined—that is, if the scale of the original drawing is corrected for shrinkage.

The author's example supported by Fig. 6 has been used as an illustration of the work involved when the graphical constructions are entirely eliminated.

The plan shown in Fig. 6 was used without redrawing, the scale being considered at 80 ft per in. Table 3 shows all the calculations for pressure at Point X at depths of 100 ft and 320 ft. The iso-pressure chart used was constructed from Equation (8) with $n = 3$. It is shown in Fig. 21.

TABLE 3.—CALCULATIONS FOR AUTHOR'S EXAMPLE, POINT X

(Scale, 1 in. = 80 ft.; Formula, $p_z = \dfrac{p\, n \cos^{n+2} \alpha}{2\, \pi\, z^2}$; $n = 3$)

Zone No.	Zone width, in inches	Radius of center line, in inches	Length, Center Line of Zone		Average unit pressure at surface, in tons per square foot	Product, Columns (2), (5), (6)	100-Foot Depth = 1.25 Inches on Chart		320-Foot Depth = 4 Inches on Chart	
			Degrees	Inches			Unit pressure (Chart)	Unit pressure, in tons per square foot	Unit pressure (Chart)	Unit pressure, in tons per square foot
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	0.20	0.10	270	0.47	2.5	0.235	0.30	0.0705	0.0300	0.0071
2	0.20	0.30	270	1.41	2.5	0.705	0.28	0.1970	0.0295	0.0208
3	0.20	0.50	270	2.36	2.5	1.180	0.20	0.2360	0.0285	0.0336
4	0.15	0.675	270	3.18	2.5	1.191	0.155	0.1845	0.0275	0.0328
5	0.15	0.825	133	1.92	2.5	0.720	0.125	0.0900	0.0265	0.0191
6	0.1	0.95	74	1.23	2.5	0.308	0.095	0.0293	0.0258	0.0080
7	0.1	1.05	31	0.57	2.5	0.142	0.082	0.0116	0.0250	0.0036
8	0.1	1.15	8	0.16	2.5	0.040	0.065	0.0026	0.0245	0.0010
Total	....	....	....	....	....	....	....	0.8215	....	0.1260

There is an important advantage in being able to use an independent iso-pressure chart, aside from the simplification of calculations. At present, Boussinesq's formula, as modified by the "concentration factor," gives the best available information as to pressure distribution in soils. If the soil is stratified within the pressure range, still further modifications must be made. The iso-pressure chart may be modified to suit the particular conditions of the site, and to take advantage of advancing knowledge of stress distribution, without affecting the foregoing method of analysis in any manner.

D. P. KRYNINE,[47] M. AM. SOC. C. E. (by letter).—The discussion of this paper has brought to the attention of the profession several important ideas and has permitted the elucidation of several obscure facts in a more or less adequate manner.

Difference Between the Boussinesq and Fröhlich Stress Distribution.—The writer always has been of the opinion that there is no strain at a certain distance from the loaded area; and this opinion, although contrary to that held by most engineers is in accordance with that of Boussinesq who states:[48]

> "* * * besides it is evident that the sections of the body located at infinity, or rather beyond a certain distance from the loaded portion [contact area] keep their shape and their dimensions."

Thus, what conventionally is meant by "infinity" may be a rather short distance, according to the properties of the soil and the value of the load. Obviously, this statement should be checked experimentally. Furthermore, Boussinesq visualizes the semi-infinite body as a hemisphere of a radius, r, tending toward infinity;[49] but he states categorically that his analysis does not apply to the total loading of the horizontal plane bounding this body from above,[50] since in this case the settlement is infinite. To make this fact clearer, it should be remembered that according to a general belief the settlement of a loaded rigid or non-rigid circular disk, as measured at its center, is proportional to its diameter. Admittedly, the body upon which the disk is placed is assumed to be elastically isotropic. If the diameter of this disk becomes infinite, the settlement likewise will be infinite. Such a situation is not covered by the Boussinesq analysis as stated by Boussinesq himself; and this leads to an opinion that there should be a certain limit of loaded area to which this analysis (particularly the method of superposition) may be applied. Furthermore, according to one of the basic requirements of the theory of elasticity, strains under consideration must be small. In using the principle of superposition in the case of larger areas, this fundamental condition is not satisfied. In other words, application of the principle of superposition to larger areas does not seem to be justified; and, as far as earth masses are concerned, this statement may be proved by experience.

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[48] "Application des Potentiels," p. 51, Lines 8–11.

If a part of the horizontal surface of an earth mass is loaded, earth under a rigid structure is "disturbed," except when the unit load is very small. Experiments and some field observations prove this statement.

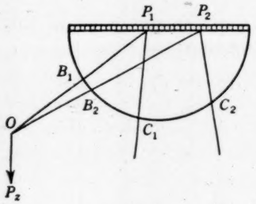

FIG. 23

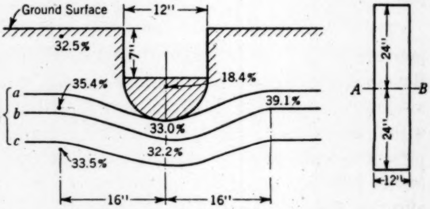

FIG. 24.—LOADING EXPERIMENT AT EAST HARTFORD, CONN. PERCENTAGES SHOW MOISTURE CONTENT AFTER EXPERIMENT

When the Boussinesq formula is applied (see Fig. 23), it is assumed that the "disturbed zone" has no influence on the value of the stresses at Points B_1 and B_2 and that pressures at Point O may be added by superposition. This procedure is not quite accurate. Dr. Fröhlich attempted to increase or to concentrate all the stresses that radiate rather vertically (as at Points C_1 and C_2) and to decrease those that radiate rather horizontally (see Fig. 23). Pressures at Point O are added by superposition.

Beyond question, the Boussinesq formula is the only scientific tool to use for computing stress distribution within a mass. Other formulas proposed are simply practical corrections of the Boussinesq formula which, however, does not fit small-scale loading experiments on sands. The Fröhlich-Griffith formula fits the latter experiments, but there are objections against its application to larger areas. These objections, as revealed in the discussion, are examined subsequently.

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[49] "Application des Potentiels," p. 53, Lines 1–10.

[50] *Loc. cit.*, p. 191, Lines 17–27.

Formation of a "Disturbed Zone."—The "disturbed zone" possesses physical properties that are different from the earth mass. It is a kind of pedestal for the structure and moves down with it during the entire settlement period. Professor Hennes states that the "disturbed zone" needs more investigation. The writer welcomes this statement and believes, in addition, that this study should follow a certain order, beginning, for instance, with the observations of pressure at the base of full-sized structures as formulated in one of the writer's conclusions. It is generally assumed that the basic cause of the formation of the "disturbed zone," in dry cohesionless sands, is the lateral escape of particles. The edges thus become unloaded and the central part stressed. This is the opinion expressed by Professor Tschebotareff in his able discussion. Sometimes it is believed that in certain cases there is no "disturbed zone" at all. For instance, Mr. Hrennikoff thinks that such is the case when the foundation bed is situated between the levels of capillary and gravitational water; and, generally, that the formation of the "disturbed zone" is due to mutual sliding of particles.

To these two causes of the formation of a "disturbed zone" (lateral escape and mutual sliding) the writer wishes to add "packing in" of the particles if there is a more or less considerable settlement and if the structure is wide. It is obvious that if a foundation is 200 ft wide, particles at the center will not travel 100 ft in either direction to escape at the edges. They will stay where they are and will tend to penetrate downward, forming the "disturbed zone." The phenomenon of "packing in" may be observed even in the case of small loaded areas as may be seen from the following field experiments made by the writer.

A wooden platform, 4 ft by 1 ft in size, was loaded uniformly with 12 tons of pig iron that remained on the platform for more than two weeks. The experiment was made in East Hartford, Conn., the soil being fine, cohesive, silty sand.[51] An excavation through the middle of the loaded area, where the stress distribution could be considered plane, revealed a clear-cut, semi-circular "disturbed zone" (Fig. 24). Thin natural sand layers of a color different from the remainder of the mass were bent (but not broken) by the action of the load, and from their shape it could be concluded that the "disturbed zone" was formed by the packing in of soil particles from beneath the loaded platform. There was also some bulging around the load. Another experiment in Stratford, Conn.,[52] was made at the bottom of a rectangular excavation, 13 ft deep, the soil being fine sand with the water-table 1 ft below the bottom of the excavation. Two posts, *A* and *B* (Fig. 25), 1 sq ft in cross-section, were loaded to 5 tons each, and the settlement of the posts and of the adjacent area was observed. Post *A* was simply placed on a sand surface, whereas previous to placing Post *B*, the corresponding 1 sq ft area was enclosed in a coffer-dam of 1½-in. boards driven 3 ft deep. The earth from this coffer-dam was not excavated. There was bulging arount Post *A*, whereas the entire area surrounding Post *B* simply settled. Since in the latter case there was no bulging, and no possibility of the particles escaping laterally, it was concluded that the entire volume of soil corresponding to the settlement sank until it was packed both into the coffer-dam and into the "disturbed zone" in the same manner as in the East Hartford experiment.

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[51] *Engineering News-Record*, Vol. 109 (1932), p. 782.

[52] Annual Rept., Connecticut Soc. of Civ. Engrs., 1934, pp. 70–81.

The writer believes, furthermore, that in the case of a wide structure the "disturbed zone" is not geometrically similar to that under a smaller model. Other opinions as to the formation of a "disturbed zone" below a rigid loading plate, are: (*a*) Fröhlich's "plastic" zone [53] with an elastic nucleus inside (Fig. 26) which, in the belief of the writer, may occur as an exception only when there is no settlement and the loading plate itself is non-rigid; (*b*) Terzaghi' rough mechanical model of the "disturbed zone," [54] composed of sand grains glued with asphalt, the bonds between asphalt and sand being broken as the increasing load reaches a certain limit (Fig. 27(*a*)); (*c*) A. Nádai's disturbed zone [55] in a metallic plate which is formed when a rigid metallic punch is applied to the plate and is revealed upon heating and re-crystallization (Fig. 27(*b*)). A "disturbed zone" under a rigid tool, working in metal, has been reported on several occasions. It follows that the main cause of the formation of a "disturbed zone" is not the lateral escape of particles (which is merely a possibility), but rather "packing in" as mentioned previously.

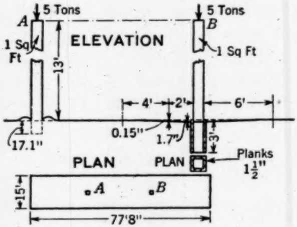

FIG. 25.—LOADING EXPERIMENT AT STRATFORD, CONN.

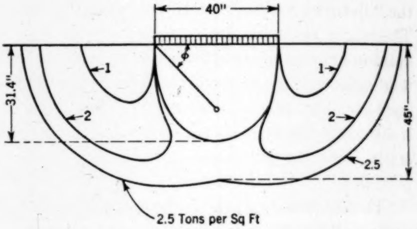

FIG. 26.—GRADUAL FORMATION OF A DISTURBED ZONE AFTER FRÖHLICH

Concentration Factor.—This item appears in almost every discussion. It is mentioned by Dr. Fröhlich, who is the original author of this theory. Mr. Taylor rightly states that the free use of the "concentration factor" is permitted only when the extent to which it is applicable is held clearly in mind. The writer wishes that such sound judgment might also be the guiding principle in applying the Boussinesq formula. "Under the footing," states Mr. Taylor, "progressing from the edge to the center, this factor will decrease from point to point and also it will decrease with increasing depth. Thus, the concentration factor at different points for the given case may vary as much as from 8 to 4 * * *." Mr. Feld indicates that the broad application of Equations (1) and (6) containing the "concentration factor" is possibly open to criticism.

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[53] "Druckverteilung im Baugrunde," p. 76 (1934).

[54] "Theorie der Setzungen von Tonschichten," von Charles Terzaghi, M. Am. Soc. C. E., and O. K. Fröhlich, p. 13, Fig. 8 (1936).

[55] "Plasticity," by A. Nádai, p. 249, Fig. 312 (1931).

Professor Tschebotareff discusses the matter in a very comprehensive manner and comes to the conclusion that a "concentration factor" changes in value with the distance from the center to the foundation toward its edges. At a certain depth of foundation beneath the soil surface the resistance to the lateral yielding would become so great that a sand would behave like a solid material with a "concentration factor," $n = 3$. Mr. Hrennikoff states: "It seems improbable that the same value of n should properly describe the state of stress both near the foundation where the soil is disturbed and outside the zone of disturbance." Professor Hennes expresses an analogous opinion stating that it is difficult to understand how the "concentration factor" can be made to cover the situation which arises in the "disturbed zone."

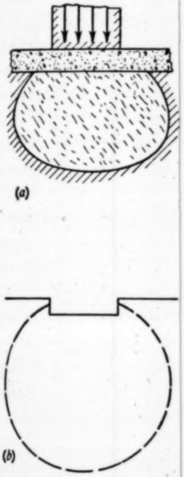

FIG. 27.—DISTURBED ZONE: (a) TERZAGHI; (b) NÁDAI

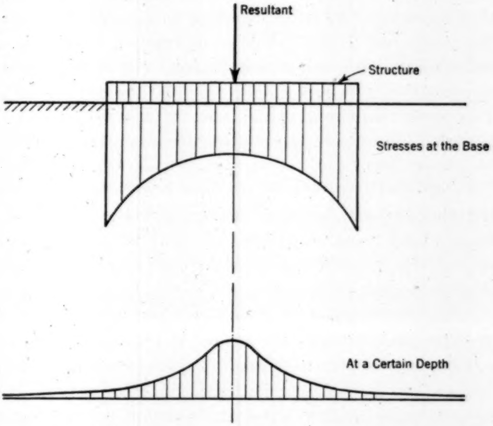

FIG. 28.—APPLICATION OF SAINT VENANT'S PRINCIPLE TO THE COMPUTATION OF STRESSES IN AN EARTH MASS

The writer wishes to make it clear that he never intended to suggest that stresses within the "disturbed zone" could be computed using Fröhlich's formulas. It is impossible to do so since these formulas have been designed for homogeneous and isotropic bodies as carefully specified in the paper, and the "disturbed zone" represents nothing but a foreign body within a relatively homogeneous and isotropic mass. As a rule, stresses never are computed close to sections where forces are applied. In textbooks on elasticity, points of application of forces are generally circled to show that at that section there is a zone that does not obey the formulas developed. In an analogous manner the progress report of the Committee of the Society on Earths and Foundations, leaves a blank space close to the ground surface when tracing a pattern of isobars.[56]

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[56] *Proceedings*, Am. Soc. C. E., May, 1933, pp. 782–783.

As to the value of the "concentration factor" which changes from 8 at the edge of a foundation to 4 at its center, the writer would be very willing to examine this statement if he knew precisely what the center of a foundation is. To clarify this idea, consider Point X in Fig. 6. It practically coincides with the centroid of the plan of the structure; and the vertical pressure under this point, if computed according to the Boussinesq formula, is greater than under any other point of the structure. Hence, Point X is the center of the foundation shown in Fig. 6; on the other hand, it is located at the very edge. Is the "concentration factor" at that point a maximum or a minimum?

Undoubtedly, there is an "edge action" or "edge disturbance" along the perimeter of a round foundation; and this is shown by curves in Fig. 11(*b*) and 11(*d*). Owing to the phenomenon of "packing in," it is also very probable that there is stress concentration at the middle of such a foundation and Fig. 10 was prepared to show this phenomenon. It may be seen therefrom that the value of the "concentration factor" at the middle of a round experimental plate was more than 6 and not 4. Fig. 10 shows also that the "concentration factor" decreases with the depth. The writer was the first to establish this fact.[57] His general opinion concerning the "concentration factor" was published in 1936, as follows:[58] "The physical nature of this new conception is not very clear as yet; and perhaps this is simply a temporary tool which will be put aside when the theory has advanced in this province." In this paper the writer recommends field research as the only way to solve this problem. Apparently, mere speculation or mathematical formulas are of no help in clarifying the bases of such a theory.

In designing the "reduced area method" the writer carefully avoided recommending any numerical value of the "concentration factor," leaving its determination to the eventual field research as stipulated in the "Conclusions." The two values of the "concentration factor" given at the beginning of the paper have been secured in small-scale experiments and cannot be used in practice.

If field research shows that variable values of the "concentration factor" should be used in computations (and this is not certain as yet) the "reduced area method" will still hold. Mr. Feld rightly remarks that the graphical method described must be repeated for each point at which the knowledge of the stress is desired. It is equally easy to apply the method in question to different points either using the same value of the "concentration factor" or changing it from point to point. In any event, a single value of the "concentration factor" will suffice for smaller areas; and it should be noted that curves in Fig. 9, to which Mr. Taylor objects, have been traced for this case.

Summary.—It is premature to argue about the law that governs the distribution of the "concentration factor" around the base of a structure before the corresponding field research is done. Even the incomplete field evidence at the disposal of the writer does not agree in any respect with the speculations advanced by most of the discussers.

Test of Fröhlich's Formulas by Saint Venant's Principle.—Stresses and strains may be distributed arbitrarily within the disturbed zone without influencing the general trend of stress distribution in deeper strata which always shows overloading close to the center, if the foundation is uniformly loaded and round or rectangular in shape. Actually, according to Saint Venant's principle, stresses at a point remote from the loaded part at the surface of a body are practically the same as if that part were replaced by a statically equivalent system. Therefore, in all cases of loading, the pressure distribution curve tends to have its major ordinate close to the direction of the resultant of forces acting at the given loaded area although stresses at the base of the structure or close to it may be thrown toward the edges (see Fig. 28). Upon application of the principle of superposition to a uniformly loaded circular disk both the Boussinesq and Fröhlich formulas give satisfactory results; namely, in both cases stresses are concentrated under the center of the disk as they should be according to Saint Venant's principle. This means that in both cases the principle of superposition holds, at least approximately.

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[57] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1089.

[58] *Proceedings*, International Conference on Soil Mechanics, Vol. 3, p. 74 (Paper E–15).

Principle of Superposition.—It has been already shown that, apparently, the principle of superposition always involves an approximation, and the discussion which follows is a proof of this statement. Generally, the problem of superposition as applied to soils is not new. Its origin may be traced to the work of Bastian,[59] about 1906. Kögler and Scheidig [60] discussed the problem of superposition in soils and came to the conclusion that it applies approximately. In numerous "loading experiments" made in the past fifty years, the principle of superposition was used freely. The test procedure was to: (1) Measure the pressures caused by the unloaded earth (mostly sand) at the bottom of the experimental box (Reading *A*); (2) measure the pressures caused by both the earth and the load (Reading *B*); and (3) subtract Reading *A* from Reading *B* to obtain the pressure caused by the load alone. It is obvious that this subtraction is the application of the principle of superposition in an implicit form. An exception to this procedure is noted in the experiments by Faber which have been introduced in the discussion by Professor Tschebotareff. However, the results of Faber's tests check with others, quantitatively at least. Model experiments show that small disks settle practically proportionally to the square root of their areas; or (which is the same), proportionally to their diameters. This rule cannot be applied, however, in comparing contact settlements obtained in small loading tests with those observed under actual structures. A large structure settles less than it should according to that rule; and, in the writer's opinion, application of the principle of superposition to any earth mass furnishes stress excess at some points of the foundation. The writer greatly appreciates the able discussion of the principle of superposition by Mr. Hrennikoff. Especially fascinating is his statement that the principle of superposition does not hold in the case of earth disturbance, because the boundary of the disturbed zone is not the same for the three loads: Loading 1, Loading 2, and Loading 1 + Loading 2. This very potent argument, which appears to contradict Fröhlich's theory of "concentration factor," in reality proves that the principle of superposition never holds, except perhaps for very light loadings. This is because the disturbed zone is being formed in all cases of heavy loadings, including the case of metals (see Fig. 24 and Fig. 27(*b*)).

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[59] "Organ für die Fortschritte des Eisenbahnwesens," 1906.

[60] "Die Bautechnik," 1927–1929.

The mistake made by Fröhlich and all others (including the writer) who have written about the "concentration factor" was the lack of a clear-cut statement that the use of the principle of superposition in this case (as in others, incidentally), is an approximation.

Principal Stresses: Coulomb Formula.—At a greater depth, soil particles are compressed by the action of principal stresses; no shear failure is possible there except small movements and changes in the shape of individual grains. Hence, Mr. Middlebrooks' question, as to whether the major principal stress or the vertical pressure is to be taken into consideration in computing consolidation, is to be answered in favor of the action of the major principal stress; and these stresses are to be computed at all points where their action may produce a more or less considerable effect. A doubt as to why vertical pressures are used instead of principal stresses in computing settlements, is expressed also by Mr. Hrennikoff. In this connection the writer wishes to state that principal stresses are to be computed from the combined action of the weight of the earth mass and the given load; and this furnishes lines of major principal stresses which at greater depths possess almost vertical tangents so that the actual error in replacing the major principal stress with the vertical pressure may be insignificant in many cases. The action of the minor principal stress is thus disregarded.

Some explanations as to Equation (35) should be included under this caption. This formula states the "condition of plasticity" of a granular mass expressing the least ratio of principal stress as a function of the angle of friction. Without arguing about the angle of friction (which, after all, is indefinable), suppose that ϕ is the correct value of the angle of friction at a given point located at a great depth and that Equation (35) is not satisfied. Then, according to Mr. Hrennikoff, a very small sliding should occur; but this sliding would not modify the principal stresses, s_1 and s_2, at the given point since they depend only on the value of the given load and its position. Hence, the situation still remains the same, and the sliding should continue indefinitely. What happens in reality is not so much the mutual slip of particles, as the change in shape of individual grains due to the shearing stress developed in each of them considered as a separate body. As to the Coulomb formula (Equation (34)), it is valid beyond question, at a given point, but only under given circumstances, since the actual value of c is also variable. Both coefficients, f and c, have been introduced to represent, in a more or less tangible form, some elaborate phenomena; and it is inadvisable to interpret them too literally and to base categoric statements on formulas containing these values.

Discussion of Fig. 17.—Mr. Taylor is to be commended for preparing Fig. 17 which conveys a number of useful ideas. At the top, Fig. 17(*a*) shows that there is no continuity of strains at the upper layers of a sand mass loaded with a non-rigid load. The break at the edges of the loaded area indicates that this is not yet a body in the sense used in mechanics, but merely an agglomeration of particles. The upward movement shown in the sketch is simply a dynamic process, or movement of particles which have not been deprived of all degrees of freedom as in a body. Hence, such terms as "modulus of elasticity" or "modulus of deformation" cannot be applied to the upper strata of a sand mass such as that shown in Fig. 17(*a*). This mass becomes a body only at a certain depth. The stress distribution at the bottom of Fig. 17(*a*) is that which the writer obtained by using Saint Venant's principle. The most ardent defender of the validity of the principle of superposition in sands would not be able to prepare a more convincing sketch than Fig. 17(*a*). The remainder of Fig. 17 contains still more useful information as to settlement in deeper strata. Stresses in a rigid structure are always influenced by this settlement, whether or not it is even. Investigators in soil mechanics must correct their views on the given subject to accord with this valuable statement by Mr. Taylor.

Small Strains; Stress-Strain Compatibility.—Admittedly, the theory of the semi-infinite elastically isotropic body is based on the assumption that strains are very small, as already stated at the beginning of this discussion. In turn, stresses may be very great and even infinite, as at the edges of an absolutely rigid plate (Equation (16)), provided, however, that the corresponding strains are kept below a reasonable limit and are continuous. Apparently, the latter conditions are not satisfied in some papers on soil mechanics [26, 27] mentioned by Dr. Fröhlich.

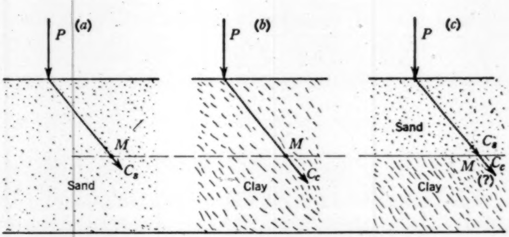

FIG. 29.—STRESS-STRAIN COMPATIBILITY

An example of the lack of stress-strain compatibility is given in Fig. 29 which, for the sake of simplicity, is treated as a plane problem. The plane stress distribution does not depend on the elastic constants of the material; hence, the stress at Point M is the same whether the earth mass is of sand or of clay (Fig. 29(*a*) and Fig. 29(*b*)). The situation is quite different if Point M is in a plane that separates sand and clay (Fig. 29(*c*)). Since the elastic constants of these two materials are different, there should be two different displacements, C_s and C_c, at Point M. Since in reality there is only one displacement, this means that there should be some stress re-arrangement at the plane in question. This circumstance may be of special interest to Messrs. Middlebrooks and Buisson who are working on the stress determination at a surface separating two different materials.

Equivalent Load at the Center of Gravity of a Loaded Portion.—Mr. Feld proposes to sub-divide the loaded area and to apply equivalent loads at the center of gravity of each sub-division. Apparently, this method may indicate some excess in pressure if the point where Pressure p_z is determined is not far from the vertical line passing through the center of gravity of the uniformly loaded foundation. Such is the case of the example in Fig. 6, in which the difference in values computed by Mr. Feld and by the writer is about 14% taking the writer's value as 100. If the point where p_z is to be determined, is outside the foundation, Mr. Feld's results may be smaller than those of the writer, although the difference may be practically negligible. For the sake of simplicity, the latter statement will be proved for the case of a uniformly loaded strip of a length, a, situated at the boundary of an elastically isotropic body ($n = 3$, $n_1 = 2$). Assume that the radius vector to the equivalent line load, $\bar{p} = p\,a$ (Fig. 30), makes an angle, α_0, with the vertical. Using Equation (33) by Mr. Feld, substituting $\alpha = \alpha_0$; making p_z equal to the pressure of the strip load; and remembering that $dA = da$:

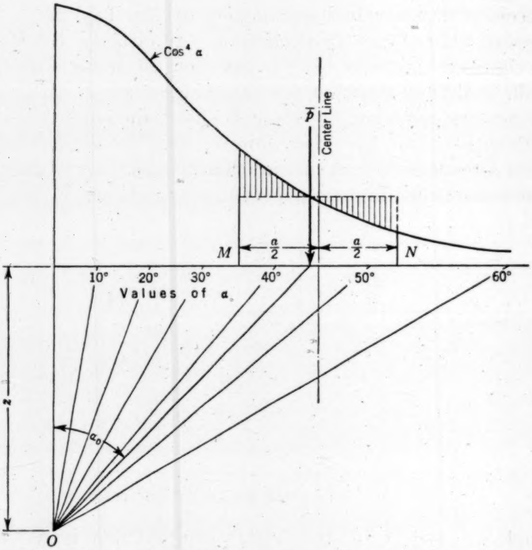

FIG. 30.—COMPARATIVE ACTION OF CONCENTRATED AND DISTRIBUTED LOAD

$$\frac{2\,\bar{p}}{\pi\,z}\cos^4\alpha_0 = \frac{2\,p}{\pi\,z}\,\Sigma\,da\cos^4\alpha \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Multiplying and dividing the right side of Equation (40) by a:

$$\cos^4\alpha = \frac{1}{a}\,\Sigma\,da\cos^4\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

The right side of Equation (41) is the average ordinate of the sub-divided area, $\Sigma\,da\cos^4\alpha$, corresponding to the loaded portion, $M\,N$. Equation (41) shows that a line load, $\bar{p} = p\,a$, and the distributed strip load, $p\,a$, produce equal vertical pressures at Point O, if the line load, $\bar{p}$, passes through the centroid of the part of the area, $\Sigma\,da\cos^4\alpha$ (corresponding to the loaded part, $M\,N$) (see Fig. 30). The vertical passing through this centroid may not pass through the middle of the loaded part, $M\,N$, however.

Graphic Integration.—Graphic integration of expressions of the type of $\Sigma\, d\omega\, f(\alpha)$, (in which $d\omega$ is an element of area and $f(\alpha)$, a trigonometric function), is a simple operation if the given area, ω, is sub-divided into circular rings, and if these rings are represented as "transformed areas," in the manner described in this paper. An example is given by Mr. Eremin. Since the angle designated by Kögler and Scheidig as ϕ is assumed to be constant, the right side of Equation (39) must be broken into two integrals:

$$p_z = c\,[\Sigma\, dA \cos^5\alpha - \cot\phi\, \Sigma\, dA \sin\alpha \cos^4\alpha] \dots\dots\dots (42)$$

in which c is a constant to be determined by Equation (39). The integral, $\Sigma\, dA \sin\alpha\cos^4\alpha$, may be obtained graphically from the integral, $\Sigma\, dA \cos^5\alpha$, by multiplying each ordinate of the curve which bounds the area expressing the latter integral, by the variable, $\tan\alpha$; thus:

$$[dA \cos^5\alpha]\tan\alpha = dA \sin\alpha\cos^4\alpha \dots\dots\dots\dots\dots (43)$$

Professor Converse proposes an interesting simplification of the writer's "reduced area method." Instead of drawing the "reduced area," Professor Converse draws a "pressure curve" (which, after all, is the reduced area, all ordinates of which are multiplied with the coefficient, $\frac{p\,n}{2\,\pi\,z^2}$). The possibility of a complete elimination of graphical construction also merits attention. Simplifications of the "reduced area method" are always welcome because this method may be applied not only for determining vertical pressures in an earth mass, but also for solving several other problems of mechanics. Mr. Buisson's remark that the method of graphical integration has been known for years is true, of course; but the principal feature of the writer's method consists not in the application of the graphical integration as such, but in its combination with the transformation of the loaded area, and this considerably simplifies the procedure.

Failure Lines.—The writer has been much interested in Mr. Buisson's studies of "failure lines" which according to his statement are close to the trajectories of maximum shearing stress. In a personal conversation with the writer, Hardy Cross, M. Am. Soc. C. E., expressed the following idea: Since the shearing stress and the shearing value of the earth material are not constant along such a trajectory, equilibrium will be broken first at some part of the theoretical "failure line." Then, this part of the "failure line" will be "out of service," and the body under consideration will therefore be transformed to a different condition of stress. Subsequent failure will be along a line or surface which may well be quite different from the original line. In about 1922 the writer visited a locality in Russia where previously many landslides had occurred, and was shown a mass of land about to slip. There was a crack at the top, and the entire wedge was literally hanging at the remainder of what may be called "shearing surface." It is obvious that the stress condition at this surface was not the same as determined by theory, exactly as Professor Cross states.

Two questions arise in the light of these statements: (*a*) Why are "failure lines," as observed in model experiments, more or less close to the trajectories of maximum shearing stress? and (*b*) do small scale models truly represent "failure lines" in an actual case? These questions will be answered simultaneously.

It should be noticed that "failure lines," "slip lines," or "Lüders' lines," as observed in the case of metals also "have a shape similar to the shearing stress trajectories in the most highly stressed portion under a concentrated load.* * * How nearly exact this is, however, has not yet been investigated."[61] The situation being thus rather obscure, only hypotheses can be advanced. In the writer's opinion a quick application of an excessive load to a small model causes instantaneous failure in the "highly stressed portion," both in metals and in soils, which may be compared to an explosion; and shear occurs along the trajectories of maximum shearing stress, which offer least resistance. Apparently, this model action has no analogy in building practice since cases of quick loading (without impact) are exceedingly rare. Another type of model action as described by Mr. Buisson is slow changes in loading. The earth mass consolidates and flows plastically; and, finally, under a "critical load," fails. In this case particles move plastically along certain trajectories close to those of maximum shearing stress.

What happens in reality is that a building of a constant (and not variable) weight begins to tip and after a certain interval of time (in some cases 24 hr), stops. There is no doubt that this is a shear action consisting of both a separation of a part of the mass from its remainder, and plastic flow. Small models as described in the discussion do not explain such failures; and, obviously, "failure lines" in these cases are not trajectories of plastic flow but rather a succession of shearing surfaces as described by Professor Cross.

Experiments conducted by Mr. Buisson are of great value for students of physical properties of soils. For instance, they show clearly the influence of consolidation on the shearing value, or the fact that a soil at the liquid limit still may possess a shearing value. The method of recording the path of particles by taking successive pictures and superposing them is very interesting. W. S. Housel, Assoc. M. Am. Soc. C. E., applied a similar method recording the movement of particles by a time-exposure photograph.[62]

Other Remarks.—Some of the discussions of this paper are veritable contributions to the field of soil mechanics, and only a few items will be mentioned in addition to those the writer has already discussed. Professor Tschebotareff gives a comprehensive discussion of the influence of some geological factors on the design of structures, and makes valuable additions to the list of research projects cited by the writer. Professor Hennes discusses the necessity of "statistical" approach to questions of settlement. The writer believes that similar methods may be adequate in spreading popular knowledge of soil mechanics after corresponding items have been studied formally.

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[61] "Plasticity," A. Nádai, p. 251 (1931).

[62] *Engineering News-Record,* November 24, 1932, p. 626.

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EARTHQUAKE RESISTANCE OF ELEVATED WATER-TANKS

By ARTHUR C. RUGE,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. N. H. Heck, Mason A. Stone, H. C. Boardman, Franklin P. Ulrich and Dean S. Carder, and Arthur C. Ruge

Synopsis

The problem of building elevated tank structures that are truly earthquake-resistant is considered in detail; and a new type of construction, embodying increased flexibility combined with snubbing action, is suggested as a practical solution. Extensive model studies, directed toward obtaining engineering data for design purposes, are described and the results summarized. The writer concludes that the commonly used statical method of design against earthquakes cannot be applied safely to elevated tank structures, and that the suggested new type of construction has a safety factor in stress of at least 2 over the standard type of tower. Experiments in which reproductions of actual earthquake motions were applied to a model of a 60 000-gal, 100-ft structure show that dangerous stresses are not avoided by designing against a statical horizontal force of one-tenth the weight of the tank and contents. Rules are given for designing structures of the new type suggested, and reasonable limits of applicability are set.

Introduction

Purpose of Paper.—Consideration of the large number of failures and near failures of elevated water-tanks during the earthquake of March 10, 1933, at Long Beach, Calif., has led to serious study of the earthquake resistance of these structures by many engineers. The importance of the problem is already so well known as to require but passing comment. The actual money value of the structures is in itself not so very important; it is in connection with the fire, life, and health hazards generally accompanying earthquakes that the effective value of a water supply system increases out of all proportion to its actual cost.

Note.—Published in May, 1937, *Proceedings*.

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A very important class of elevated tank structures is that comprising the gravity sprinkler-supply systems for industrial plants and for many other buildings of moderate height. Elevated sprinkler tanks generally vary in capacity from 50 000 gal to 150 000 gal and in height from 75 ft to 150 ft. This paper is largely concerned with structures that vary within these ranges of size, but more specifically with independent tank structures of all-steel construction. Some of the results, however, obviously apply also to structures of reinforced concrete, wood, or mixed construction.

The purpose of this paper is to present the principal results of an extended series of model studies which were directed toward: (1) The determination of the stresses produced in elevated water-tank structures by earthquake motions; (2) the determination of the effect of strengthening the structures by present methods of statical design; and (3) the investigation of a new type of tower design which was developed in the course of the research. That the experiments described herein were restricted to a small part of a very large and important field will be obvious to the reader. Without a comparatively large fund at hand for the work, practical results could be obtained only by narrowing the range of the research.

Nature of the Problem.—At first glance, the design of an earthquake-resistant elevated water-tank seems to be a simple engineering problem. It is not uncommon to find practical engineers reasoning that, since competent engineers can design buildings of six or eight stories to resist moderate earthquakes with considerable confidence, the design of a water-tank tower should present no difficulty, since it is a much simpler structure and of a mass far inferior to that of a building. This kind of reasoning, unfortunately, is the result of a misconception to the effect that a relatively limber, top-heavy structure such as an elevated tank will behave dynamically like a low, rigid building. Indeed, the practical treatment of earthquake-resistant design from the standpoint of dynamics has been, for the most part, of a very superficial nature in the past. The tendency has been to simplify the design methods on the basis of intuition without first attempting a rigorous analysis.

First, it is important to realize that earthquakes manifest themselves as inexorable motions, rather than as forces. The expression, "earthquake force", has become so widely used that many engineers conceive of the problem of quake-proof design as one of simple stress analysis in which the "forces" are determined definitely by the violence of the earthquake alone. The "forces" produced by the earthquake motion depend primarily upon the characteristics of the structure suffering the motion. Various frequencies, or periods, are associated with the inexorable motions of an earthquake, and, although the actual motions may be highly irregular, certain periods may sometimes appear more prominently than others.

If a structure is acted upon by a ground motion which has a period five or six times that of the structure, then it is possible, with reasonable accuracy, to compute the stresses in the structure on the basis of the accelerations of the ground motion. In this case, the forces acting upon the structure are taken as the product of the various masses and the ground acceleration. This simple rule, however, cannot be extended to structures such as elevated water-tanks, which commonly have natural periods within the range of the periods associated with major earthquakes. Here, it is necessary to proceed with extreme caution; nor can one rely upon intuitive reasoning unless it is backed by a thorough grasp of the dynamics of vibrating systems.

The type of tower now in common use was not developed for earthquake resistance, but merely to fulfill the purpose of elevating the water supply on a structure which has to resist a certain intensity of wind force in addition to carrying the vertical load. For this use it is practically ideal; but there is no reason to accept it as the ideal structure for withstanding earthquake motions, which actual experience and repeated laboratory tests have proved it certainly is not.

The dynamic behavior of elevated tanks is not amenable to simple mathematical analysis because of the complications introduced by the action of the water in the tank. The system is analogous to one made up of a gravity pendulum suspended at the top of an elastic vertical frame work (see Fig. 1), except that the water action in the tank is more complex than that of the pendulum. It will be seen subsequently from the experimental results that the water motion exerts a cushioning effect upon the deflections in a tank tower of moderate size.

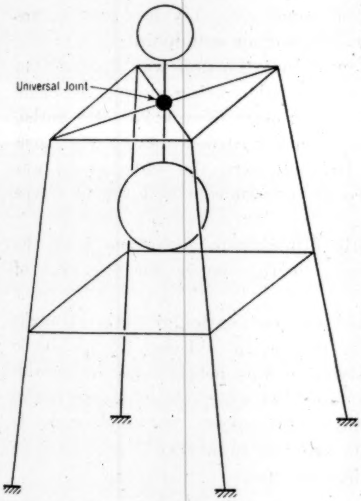
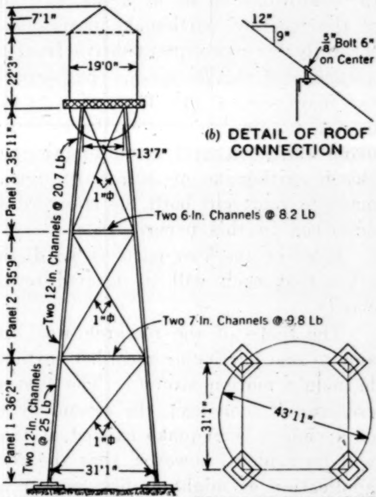


FIG. 1.—ANALOGUE OF ELEVATED WATER TANK



(a) ELEVATION (b) DETAIL OF ROOF CONNECTION (c) FOUNDATION LAYOUT

FIG. 2.—STRUCTURAL DESIGN OF PROTOTYPE TESTS IN PART I

It is to be noticed in Fig. 1 that a compound pendulum (not a simple pendulum) represents the water and the tank. This feature corresponds to the restraining action of the tank walls upon the water mass which forces part of the water to move with the tank as it vibrates. A simple pendulum would correspond approximately to a shallow, saucer-shaped tank of water, in which there is little restraint on the lateral motion of the contents.

The problem of determining the earthquake resistance of elevated water-tanks can be solved mathematically in a qualitative way only, not because the mathematical process is unknown, but because of the tremendous work involved in obtaining results of practical value to engineers. Those who have attempted to calculate the effects of irregular transient disturbances on vibrating systems will recognize at once the advantages of the experimental approach to the earthquake problem. The model method, to be sure, is not the only practical way to avoid mathematical difficulties, but, at present, it appears to be the most convenient one. The greatest advantage of the model method lies in its directness, with the attendant simplicity in interpretation of results.

It will be seen that a system is involved which is dynamically complex in itself; it is subjected to ground motions of a transient and complex nature. The problem is to obtain information which has engineering significance in the sense of being truly quantitative rather than merely numerically precise. That there can be a tremendous difference between precision and true quantitativeness is a fact easily overlooked in research. Thus, the results obtained in the simple harmonic motion studies described herein can be regarded as reasonably precise, but from the engineering standpoint their quantitative aspect is rather poor, because they do not directly represent the conditions set up by actual earthquakes. On the other hand, the results of the recorded earthquake-motion tests, although probably less precise, are certainly far more quantitative from the engineering standpoint.

Scope of the Research.—The experimental investigation was divided into two main parts: (1) Tests of a 1:46.5 scale model, using simple harmonic motion to simulate earthquake motion; and (2) tests of a 1:25 scale model, using the horizontal ground-motion components recorded during the Long Beach earthquake of March 10, 1933. In each part, the same model was made to represent both the standard type of construction and the new type described in this paper.

Each of the two parts is so distinctly different in technique from the other that each will be treated separately in this paper for the sake of clarity.

The scope of the research was limited to one particular class of tanks and to one particular recorded earthquake motion in addition to the simple harmonic motion studies. The first restriction was necessitated by considerations of economy; the second by the fact that only one complete record of a violent earthquake existed at the time the research was conducted. It will be evident, however, that the results are not so narrowly restricted in application as might at first appear to be the case.

PART I.—EXPERIMENTS IN SIMPLE HARMONIC MOTION

STANDARD DESIGN; MODEL SCALE, 1:46.5

Considerations Relative to Building and Testing the Model.—A standard 60 000-gal hemi-spherical bottom tank on a 100-ft tower was chosen as the first object of study. It was situated about 33 miles from the epicenter of the Long Beach earthquake (March 10, 1933), and about $6\frac{1}{2}$ miles from the point where the earthquake motion used in this research was recorded. Several bracing rods in the top panel were permanently stretched and one rod was broken during the earthquake. The general design of this structure is shown in Fig. 2. The results obtained therefrom, although probably typical of a fairly wide variety of elevated tanks, cannot be applied indiscriminately to other designs. However, they do point to certain general important conclusions as to the inadequacy of present design methods.

The behavior of a water-tank supported on a steel tower is controlled by two kinds of physical forces: (1) Gravity (or weight) forces, which affect the motion of the water within the tank; and (2) elastic forces produced by the tower, which affect the motion of the tank as a whole. These two kinds of forces interact in a complex manner when the tower is subjected to vibratory motions. Therefore, in constructing a model of such a system, it is necessary to maintain the same relation between these two physical forces in the model as exists in the prototype if true similarity is to be attained; that is, gravity forces and elastic forces must be scaled down in the same proportion. This condition precludes the possibility of using a simple geometrically scaled model. It becomes necessary to adopt a common artifice of model technique—scale distortion—to preserve similarity. This may be accomplished by using different scales for the tower and the tank; but there is a simpler and more practical solution.

As far as the interaction between the tower and the tank is concerned, it is easily seen that the tower could be replaced by any other elastic system capable of producing the same horizontal deflection per unit horizontal force applied at the tank, without affecting the behavior of the tank during an earthquake. In other words, the tower merely provides an elastic restoring force proportional to the deflections of the tank, tending to restore it to its position of statical equilibrium. In building a model, it is advantageous to make use of this fact in order to avoid a costly tower model which might prove to be of doubtful accuracy. Accordingly, it was decided to support the model tank on two flat steel bars acting as simple vertical cantilevers, the length of the bars being made adjustable in the laboratory to permit an accurate setting of the stiffness. A symmetrical four-post tower, being equally stiff in all directions, can be simulated in the model by making the supporting bars of the proper stiffness in one direction only and then testing the model in that direction. In this manner the model "tower" can be made strong enough to hold the tank weight safely, and at the same time flexible enough to permit very large deflections without danger of failure.

It is not intended, of course, to determine the ultimate—or failing—strength of the prototype, because the engineer is interested primarily in building structures so that they will not be stressed above the elastic limit. The results of a model study of this kind, therefore, must be interpreted as valid for the prototype to the point where its elastic limit is reached. It is neither practical nor very useful to attempt to duplicate the ultimate strength conditions in a model, because the range of experimentation would thus be greatly curtailed indeed unless one were willing to build a great many models, each of which would have to be discarded as soon as its elastic limit was exceeded.

The point next in importance to be considered is the choice of scale for the model. There is no simple rule for the determination of what constitutes the ideal scale. The upper limit of model size is fixed largely by considerations of laboratory convenience, expense, and capacity of the available testing equipment. In the lower limit the choice of scale is restricted by the difficulties in recording the minute motions accurately and also in controlling the testing conditions. Other difficulties with very small scale models are brought about by the effect of surface tension and viscosity in the liquid which tends to destroy the similarity, and by the attendant speeding up of the model motions which renders accurate recording and timing more difficult. Traffic vibrations also are likely to become troublesome when very small scales are used.

After considerable study it appeared that a scale of about 1 to 50 would be satisfactory. This proved to be the case. The model of the tank itself was made of No. 26 gage, sheet aluminum, the bottom and roof being spun and the joints rolled together. The resulting tank model gave the proper ratio between weight of shell and weight of water, which is a necessary condition for similarity.

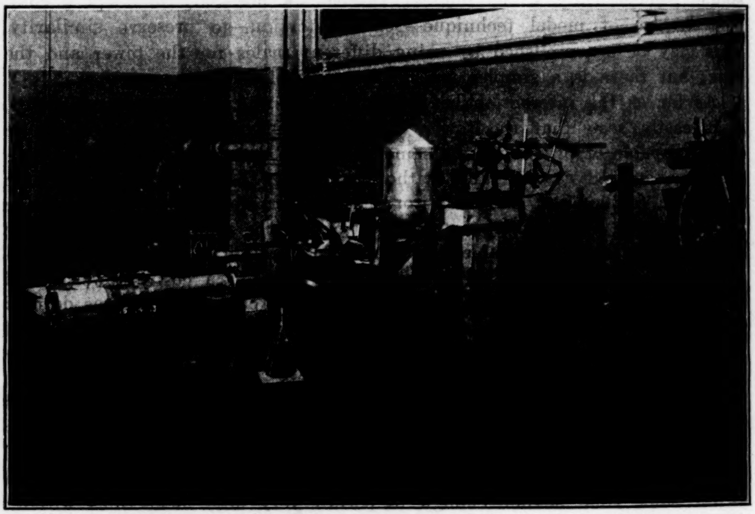


FIG. 3.—MODEL ON FIXED TABLE, SHOWING ARRANGEMENTS FOR ADJUSTING STIFFNESS AND MAKING FREE VIBRATION TESTS

The complete model mounted on a stationary table for stiffness adjustment and free vibration tests is shown in Fig. 3. It will be seen that the legs are clamped at the base so that the stiffness of the model tower is easily adjusted to the desired value. The method of determining the stiffness is also shown in Fig. 3. A known force is applied horizontally at the center of gravity of the tank by means of a weight-and-pulley arrangement, the resulting deflection of the tank being measured by a micrometer microscope reading on a fine scratch on an index attached to the tank.

The tank was attached to the tower legs by means of spring hinges soldered to the top of the legs and to the bracket on the tank. Each hinge consisted of a strip of spring steel, 0.003 in. thick, having an unsupported length of about 0.01 in. This type of hinge is practically frictionless and, at the same time, remarkably rugged. It exhibited no deterioration during the entire set of tests. The tank bracket was riveted to the shell.

It should be remarked that in determining the stiffness of the model tower it is necessary to have the proper quantity of water, or equivalent weight, in the model tank, because the vertical load on the legs has an effect upon their stiffness which needs to be considered. The following relations between model and prototype are easily deduced from model theory[2]: If the length scale $= \lambda$, the time scale $= \sqrt{\lambda}$, and the stiffness scale for the tower $= \lambda^2$. For the actual model the length scale, λ, was 46.5. This gives a time scale of 6.82; that is, a duration of 1 sec in the model corresponds to 6.82 sec in the prototype. The following list of weights, in pounds, shows the relations between the laboratory model and the full-sized tank:

	Weight, in pounds
Weight of water in prototype tank (3 in. from top)	503 000
Weight of empty tank, balcony, etc.	31 000
Equivalent tower weight concentrated at center of gravity of tank	11 000
Total concentrated weight at center of gravity of tank	545 000
Weight of water in model tank	5.00
Weight of empty tank	0.24
Equivalent tower weight constructed at center of gravity of tank	0.11
Attachments	0.04
Total concentrated weight at center of gravity of tank. (The model weight corresponds with the prototype weight; thus: $5.39 \times (46.5)^3 = 543\ 000$ lb)	5.39
Stiffness of prototype tower (calculated), in pounds per inch of deflection at center of gravity of tank.	16 000
Stiffness of model tower (as adjusted in laboratory), in pounds per inch of deflection at center of gravity of tank; and $7.37 \times (46.5)^2 = 15\ 940$ lb per in.	7.37

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[2] *Civil Engineering*, August, 1935, pp. 457–458.

The stiffness of the prototype tower was computed by applying a horizontal force of 1 000 lb at the center of gravity of the tank to one bent of the tower and calculating the resulting deflection of the tank (0.125 in.). The corresponding angular tilt of the tank was also computed and was found to be only 8 sec under this load; consequently, it is obvious that the rotation need not be considered in building the model, which further simplifies the laboratory work by permitting the use of vertical instead of battered legs.

The foregoing scale factors establish the magnitude and period of the artificial earthquake motion to be applied to the model in order to simulate a given earthquake. For instance, an earthquake in Nature, having a period of 1 sec and an amplitude of 2 in., would be simulated in the model by a motion having a period of $\frac{1}{6.82}$ sec. and an amplitude of $\frac{2}{46.5}$ in.

The shaking-table was loaded with a dead weight of approximately 300 lb, so that the model weight was less than one-fifteenth of the dead weight of the table. This is an important consideration because, if this ratio is not made large, the reactions of the model may seriously affect the table motion. Of course, in the region of resonance this effect is unavoidable, but during the early stages of the motion the effect of the model on the table motion will be quite small, and, after all, it is the early part of the motion that is of greatest interest.

The interpretation of the model results in terms of the full-sized structure is quite simple. The model is so constructed that its deflections are related to those of the prototype according to the length and time scales. Therefore, the deflections that would exist at any instant in the full-sized structure may be determined easily. Having once computed the stresses in the tower, corresponding to a known deflection, the determination of the magnitude of the stresses at any given instant is a matter of simple arithmetic. It is obvious, then, that a record of the motion of the model tank relative to its base is sufficient for the determination of both the stresses and deflections that would be produced in the prototype, as well as the time at which they would occur.

Artificial Earthquake Motion.—In this part of the research an earthquake motion of cosine form was adopted, that being the most convenient to work with in the writer's laboratory. When the table is vibrating by itself, the shaking-table motion is so slightly modified by damping that, over a range of 15 or 20 cycles, the attenuation is quite negligible. When a model is placed upon the table, the record of the table motion appears as a damped cosine wave. This is due to the absorption of part of the energy of the table by the model.

The range of periods chosen for these tests covers the generally discussed band of earthquake frequencies, although there is considerable yet to be learned about the upper and lower limits of earthquake periods. Apparently, the commonly accepted "destructive periods" of approximately 0.75 sec to 1.5 sec, are based largely on the destruction to buildings, the natural periods of which often fall close to that range. It is not to be assumed, however, that severe ground motions of much longer periods do not exist during severe earthquakes; and it is certain that there are much shorter periods present.

Notation.—The symbols used throughout this paper are defined where they first appear. An effort has been made to conform as nearly as practicable with "Symbols for Mechanics, Structural Engineering, and Testing Materials",[3] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

Functions of the "Dummy Tank".—It was considered to be of interest to compare the behavior of the actual water-tank with that of an equivalent solid mass in order to determine the effect of the water on the motions. Consequently, a small inverted pendulum was constructed and its period adjusted to the proper value. This pendulum is referred to as the "dummy tank" because it behaves in exactly the same manner as the actual tank if the water was replaced by a solid dummy mass of equal weight. In other words, the motions of the dummy tank represent the effects of earthquakes upon a water-tank which is frozen solid.

The dummy tank was made to serve another important purpose at the same time, namely, that of controlling the accuracy of the laboratory work. The calculation of the theoretical motion of the dummy pendulum being a relatively simple matter, it is possible to check the correctness of the work at any time. Since the dummy pendulum and the actual tank were always tested simultaneously, it is safe to assume that if the theoretical dummy motion checks the recorded motion satisfactorily, the motions of the actual tank must also be correct.

The proper natural period, P, for the dummy pendulum is found as follows:

$$P = 2\pi \sqrt{\frac{W}{g\ k}} \qquad \text{.......................} \quad (1)$$

in which W = weight of tank plus contents (assumed to be solid); g = acceleration due to gravity; and k = spring factor of tower (restoring force per unit deflection). From the foregoing lists of weights and scale relations: $P = 2\pi \sqrt{\frac{545\,000}{32.2 \times 12 \times 16\,000}} = 1.863$ sec for the prototype. Then, since the time scale is $\sqrt{\lambda} = 6.82$, $P_m = 1.863 \div 6.82 = 0.273$ sec for the model. The correctness of the model-scale relations is easily verified by calculating P_m, using the actual model constants; thus: $P_m = 2\pi \sqrt{\frac{5.39}{32.2 \times 12 \times 7.37}}$ $= 0.273$ sec. (The dummy pendulum was adjusted by trial to a period of 0.271 sec, within 1% of the theoretical.)

It is easily proved that the dynamic response of a pendulum of this type is dependent only upon its natural period, and independent of its physical dimensions. Therefore, it was not necessary to make the mass and spring factor of the dummy identical with those of the tank model. The dummy pendulum was made by attaching a small mass (about 0.5 lb) at the top of a vertical flat steel bar. This type of small pendulum was adopted in order to reduce the total effect of the models upon the table motion to a minimum.

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[3] A S A—*Z10a*—1932.

Laboratory Procedure.—The laboratory work was divided into two main parts—free vibration tests and shaking-table tests. In the free vibration tests, the model was mounted on a rigid stationary table and the tests were made by suddenly releasing the model from a pulled-back position. The resulting motions of the tank were recorded photographically. Fig. 3 shows the general arrangement used for recording the free vibrations. The pull-back and release were accomplished by means of standard link fuses. One end of the fuse was clamped to a small bracket (see at the top of the left tower leg); the other end was clamped to a bracket attached to the table, the latter bracket being arranged with screw adjustment to permit varying the amount of pull-back at will. The shield over the fuse kept the light of the flash from striking the recording paper. A 110-volt circuit, shorted across the fuse, gave a very fast release.

At the right of the tank is a swiveled mirror and lens arranged to trace the model motions on the recording drum, the source of light being just above the front of the drum (extreme right of Fig. 3). Immediately to the right of the recording mirror is another lens for recording the time intervals which are marked by the passage of a periodic electric spark. The spark-gap is shielded, but the terminals where the leads connect may be seen in Fig. 3.

The shaking-table motions were produced in a manner similar to that of the free vibrations, except that it was necessary to use a short length of resistance wire, instead of a link fuse, in order to deflect the stiff springs of the table. A special form of wire link was developed for this purpose, which proved to disintegrate very rapidly under a 110-volt current with no detectable effect upon the table motion. This was accomplished by reducing the length of the wire to be burned to about $\frac{1}{8}$ in.

The set-up for the shaking-table test has been amply described elsewhere[4]. The table itself was suspended entirely by piano wires and actuated by springs. Lateral guy wires leading to the floor prevented any tendency of the table to twist due to unbalanced forces.

Three mirror-and-lens systems, which received light from three illuminated slits beneath the recording drum, were arranged so that one recorded the motions of the shaking-table relative to the floor, while the second recorded the motions of the tank relative to the shaking-table (the deflections); and the third recorded the deflections of the dummy pendulum relative to the table. A fourth lens system recorded the time marks. The records were made on bromide paper.

Fig. 4 represents a part of a typical record. The axes of the three motions were separated purposely for the sake of ease in reading the records.

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[4] *Civil Engineering*, August, 1935, p. 458, Fig. 7.

Just before releasing the table, the drum was revolved once with the lights on, and the timer off, to provide base lines for measuring the deflections. The entire record (39 in. long) being made in about 5 sec, it was necessary to time the recording procedure carefully to prevent awkward overlapping. The switchboard which controlled the table release, lights, timer, and recording drum, is seen in Fig. 3.

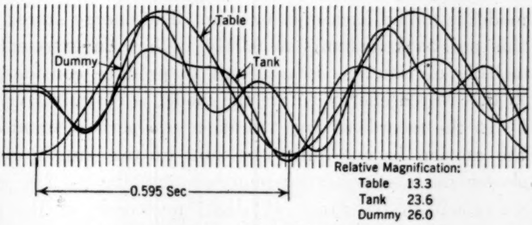


FIG. 4.—PART OF A TYPICAL RECORD

Free Vibration Characteristics.—As the pull-back amplitude is varied, the resulting free vibrations prove to be very uniform in character. Analysis of the records shows the system to be remarkably linear, in view of the complex action of the water inside the tank. Fig. 5 is a plot showing various successive maximum amplitudes of free vibration, expressed in terms of the percentage of the original pull-back before releasing. (In this and all succeeding diagrams all data are given in terms of the full-sized tank in order to avoid confusion as to scale relations.) In a perfectly linear system such as one made up of springs and weights, these curves would all be horizontal straight lines. In the water-tank the curves are quite straight, but all show more or less deviation from the horizontal. However, the percentage deviation within any reasonable working deflection of the tank is quite small. On the basis of this observation one might expect that the system would exhibit nearly linear characteristics in the shaking-table tests, and this was found to be the case.

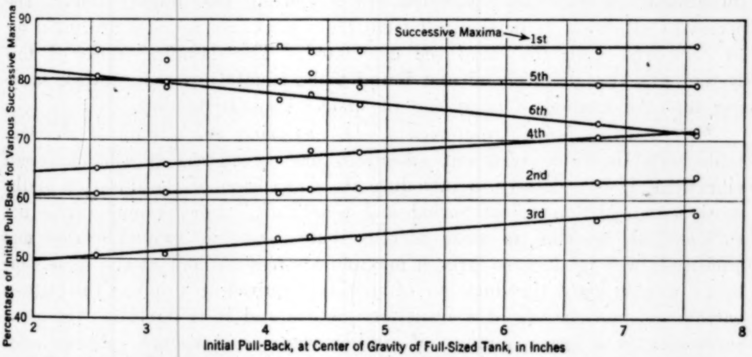


FIG. 5.—FREE VIBRATION BEHAVIOR OF TANK WHEN SUDDENLY RELEASED FROM PULLED-BACK POSITION

The curves in Fig. 5 are not extended below deflections of 2 in., because it was found that as the deflections became quite small the characteristics of the pull-back fuse began to affect the resulting free vibrations sometimes by as much as 5 or 6 per cent. It is easily seen why this can be true when it is realized that, corresponding to a pull-back in Nature of 1 in., the model is pulled back only 0.02 in. The energy stored in the model tower for this pull-back amounts to 0.0016 in-lb. An extremely short lag during the melting of the fuse is sufficient to produce considerable percentage change in this stored energy before the model is entirely free.

In the records of free vibration two periods stand out distinctly, although the motion is not quite the sum of two simple harmonic motions. The slower mode of vibration consists principally of a swinging motion of the water within the tank. A slight movement of the tower accompanies this mode. The second and faster mode consists of a swaying back and forth of the tank as a whole, while a large part of the water moves with the tank. This mode resembles the action of a simple inverted pendulum. These two periods were estimated from the records to be 0.39 sec and 0.24 sec, respectively, for the model. (The corresponding periods for the full-sized structure are 2.7 and 1.6 sec.)

It is interesting to compare these periods with that of the dummy tank model which has a period of 0.27 sec. The change from 0.27 to 0.24 sec and the addition of the secondary period of 0.39 sec are created entirely by the freedom of the water to move relative to the tank.

In order to verify the accuracy of the model and laboratory work, free vibration tests were also made with the tank filled with a mixture of sand and sawdust, of volume and weight equal to that of the water (5.00 lb). This mixture was solid enough to act as an inert mass in the tank so that it duplicated the conditions assumed for computing the dummy tank. The period of vibration as determined from these tests was found to be 0.272 sec as compared with the 0.273 sec calculated. The damping ratio of this system was also investigated and found to be small (less than 1.015), showing that the method of mounting the model was satisfactory.

Forced Vibration Characteristics.—In analyzing the results of shaking-table tests in which transient vibrations are considered as well as forced vibrations, it is necessary to introduce the conception of the duration of the motion in addition to its period and amplitude. For example, it is not sufficient to be able to state merely that a certain ground period and amplitude will produce so large a maximum stress without at the same time being able to state the duration of motion required to produce this stress; earthquake motions are of a transitory nature and it is doubtful whether a succession of a larger number of similar waves ever occurs. Accordingly, the data have been studied in the light of the maximum stresses (or deflections) that occur up to various periods of time, the measure of time adopted being the number of ground cycles elapsed since the start of the motion. This arrangement of data gives a clearer view of the matter than one in which time intervals are expressed in seconds, because it is the number of ground vibrations rather than the absolute duration of time that is of greatest interest.

Fig. 6 presents the results of the shaking-table tests in a condensed form. The maximum amplitudes of the tank and dummy are shown as functions of the ground period, holding the maximum acceleration of the ground constant at 3% of that due to gravity. (These data are given in terms of the maximum ground acceleration because many engineers are accustomed to compute earthquake stresses on the basis of acceleration alone. The value, 3% of gravity, was chosen as a base for plotting the results because the structure would have a safety factor of 2 when subjected to a constant horizontal acceleration, 0.03 g; that is, according to the statical method of earthquake design the structure would have a safety factor of 2 for any earthquake having a maximum acceleration of 0.03 g.) Because the system was found to be so nearly linear it is possible to derive other values of acceleration by direct proportion. Of course, this is slightly approximate at certain points on the curves and probably would not hold true for very large accelerations (say, greater than 0.15 g) even if the elastic limit point were indefinitely high.

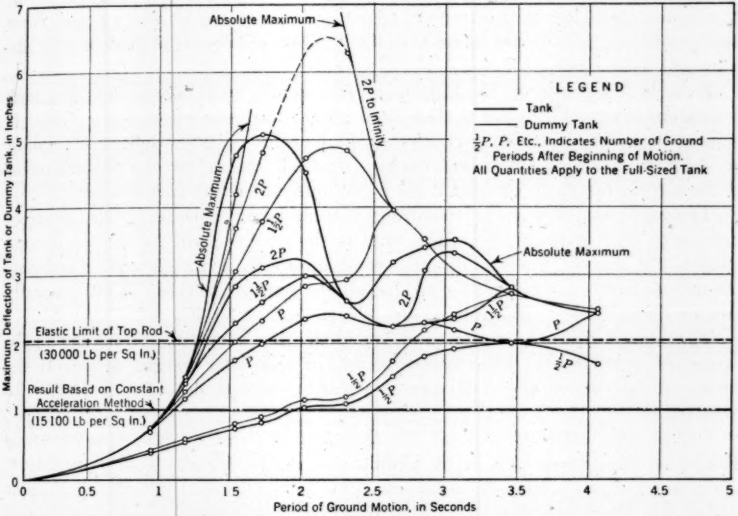


FIG. 6.—RESULTS OF SHAKING-TABLE (MAXIMUM GROUND ACCELERATION = 3% OF GRAVITY; FOR OTHER VALUES USE DIRECT PROPORTION)

One of the most outstanding characteristics of these curves is the extreme difference between the form of the early maximum curves and that found in the absolute maximum curves. First considering the actual tank, one notes in the absolute maximum response the appearance of two distinct resonance points. These points correspond to the natural periods observed in the free vibration tests. (The maxima in Fig. 6 do not coincide with the natural periods—2.7 and 1.6 sec—because of the method of plotting the results. If Fig. 6 is replotted, keeping the ground amplitude constant instead of the maximum acceleration, it will be found that the maxima then appear at the natural periods of the system.) The resonance point corresponding to the natural period of the water is suppressed, a natural result of the non-linearity in this region. In the early stages of the motion, these two resonance points are almost entirely obscured by the presence of the transient vibrations.

Equally outstanding in Fig. 6 is the rapid building up of the tank deflections, even at ground periods well away from the resonance points. It is seen that, for the most part, two ground cycles produce deflections almost as great as a series of several cycles. Indeed, the relatively flat character of the curves for *P* (1.5 *P* and 2 *P*) is obviously of great significance.

The response curves for the dummy tank begin with a pattern roughly similar to that of the water-tank; but, as the duration of the motion is prolonged, the curves for the dummy tank approach a form having a single resonance point corresponding to its single natural period. The generally greater deflections of the dummy are clearly shown in Fig. 6.

The elastic-limit deflection and the deflection resulting from the application of a horizontal force equal to 3% of the dead weight acting at the center of gravity of the tank, are also shown in Fig. 6 to form a basis for comparison.

Comparison with Results of the Statical Method of Earthquake Stress Analysis.—The results given by simple statical theory are shown on Fig 6, but a more direct comparison between the actual conditions and the simple theory is given in Fig. 7. The maximum acceleration of the ground (expressed as a percentage of gravity) necessary to produce elastic-limit stresses during one and two ground vibrations is plotted as a function of the ground period. It is obvious that the simple theory based on a constant acceleration fails to fit the true conditions, even as a rough approximation.

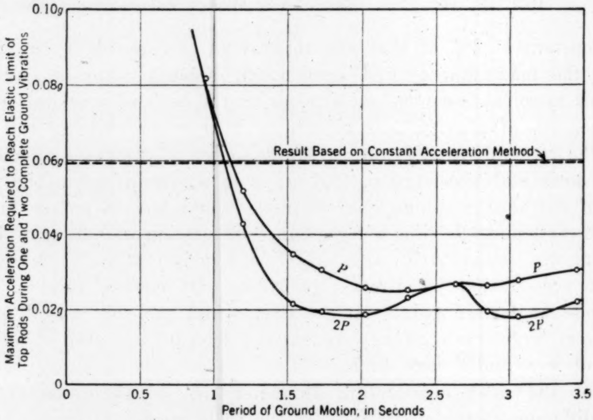


FIG. 7.—COMPARISON WITH SIMPLE THEORY OF DESIGN

The curves cross in the vicinity of the 1-sec ground period, but this is a coincidence rather than a result of the assumptions upon which the simple theory is based. The inadequacy of this type of tower is shown strikingly in Figs. 6 and 7.

Conclusions Pertaining to the Standard Design.—The most important conclusion to be drawn from the results of this part of the research concerns the inadequacy of the generally accepted method of designing elevated tank towers against earthquakes. It is clearly shown that the assumption of a static horizontal force proportional to the maximum ground acceleration leads to large errors over practically the entire band of possible earthquake frequencies. If earthquakes did not exhibit periods shorter than 5 or 6 sec, the fixed-force method of analysis could be considered fairly satisfactory for tank design provided a reasonable factor of safety was adopted. Unfortunately, the more violent motions of the ground during an earthquake are known to contain periods considerably shorter It would be possible, of course, to make the simple theory fit elevated tank design fairly well by building the towers of such stiffness that the natural period of the structure would be shortened, say, to 0.25 sec; but with a mass of 500 000 lb elevated approximately 100 ft above the ground, such a design would be extremely expensive. In other words, unless the natural period of the structure can be reduced to at least one-third or one-fourth that of the shortest important earthquake period to be expected, the dynamical effect of the ground motion can not safely be ignored.

It will have been observed from Figs. 6 and 7 that the effects of earthquakes upon the structure are expressed in terms of motions or deflections, the unit stresses produced by these motions being incidental to them and not controlling them. It may be seen that, in any given tank tower, the best effect that can be had from a moderate amount of strengthening is obtained by strengthening those members which are inherently weak until all the members of the structure reach ultimate stresses simultaneously. Then the structure achieves the maximum possible safe deflection; this means that, for the tower studied in this research, an increase in the size of the rods in the upper panels would strengthen the tower dynamically to some extent (since they are the highest stressed rods); but that, paradoxically enough, if all the rods were increased in cross-section by the same percentage, the tower would have the same strength dynamically as it had originally, because the safe deflection would be unchanged.

The conclusion that large tower deflections can be produced in short periods of time over a wide range of earthquake frequencies is obvious from the data of Fig. 6. This conclusion leads further to the consideration that an actual earthquake motion may introduce dangerously large amplitudes even if the condition of resonance or near resonance is entirely lacking.

It is clear that although they indicate in a general way the effect to be expected, the simple harmonic motion tests do not offer a practical means of arriving at quantitative engineering data on the effects of actual earthquake motion. On the basis of the familiar Fourier harmonic analysis, the simple harmonic test can be regarded as a reliable "measuring stick" for comparing the dynamic behavior of one system with that of another system; but beyond demonstrating the possibility of the process mathematically for certain structures one cannot practically go very far toward converting the results of simple harmonic motion tests in terms of the effect of a given irregular earth motion.

Spring Element Design; Model Scale, 1:46.5

Considerations Relative to Development of Earthquake-Proof Tank Structures.—A study was made of a number of possible types of tank and tower construction, with a view toward finding a type or types suitable for active seismic regions. Some of these types, of course, appear to be fantastic or impractical, whereas others are uneconomical. Among the former may be listed such arrangements as roller supports for the foundations or tanks, various ways of hanging or suspending the tank so as to decrease its vibrations, etc. Among the uneconomical designs are those which seek to give the structure adequate quake-resistance by providing an extremely stiff or rigid tower. The data in Table 1 will serve as an illustration. Thus, it is clear that it is quite uneconomical to attempt to give very short natural vibration periods to heavy tanks on towers of this height. For capacities of 100 000 gal and more, and for towers of greater height, the situation is very much worse.

TABLE 1.—Comparative Designs, 60 000-Gallon Tank on a 100-Foot Tower (Distance from Column Base to Balcony = 108 Feet)

Item No.	Design (1)	Number of panels (2)	Weight, in tons, more than Item No. 1 (3)	Natural vibration period, P, in seconds (4)
1........	Standard design*....................	3	0	1.50
2........	Design for a constant acceleration of $0.1g$	3	8.25	1.13
3........	Extra heavy design..................	3	10.25	0.99
4........	Extra heavy design..................	4	11.50	0.93

* Diagonal rods, 1 sq. in. in area.

If the natural period cannot be reduced to about 0.5 sec, it is doubtful whether the tank is very much more likely to survive a violent earthquake than a standard design built for wind resistance only. The odds will depend upon the nature of the particular ground motion to which it may be subjected and upon the foundation conditions.

The natural conclusion from a prolonged study was that, barring fantastic and impractical designs, the best way to obtain adequate quake-resistance is to provide the tower with a greater range of elasticity; that is, if the tower can be built so that the safe deflections are large enough, there will be no danger of failure. This does not necessarily imply that the tower must be extremely limber or weak, although a certain degree of limberness is almost always created by increasing the allowable deflection of a structure. This concept, of course, is not a new one, but it has been most often discussed in relation to buildings; and, in this connection, the writer wishes to emphasize that the present discussion is in no way intended to apply to buildings.

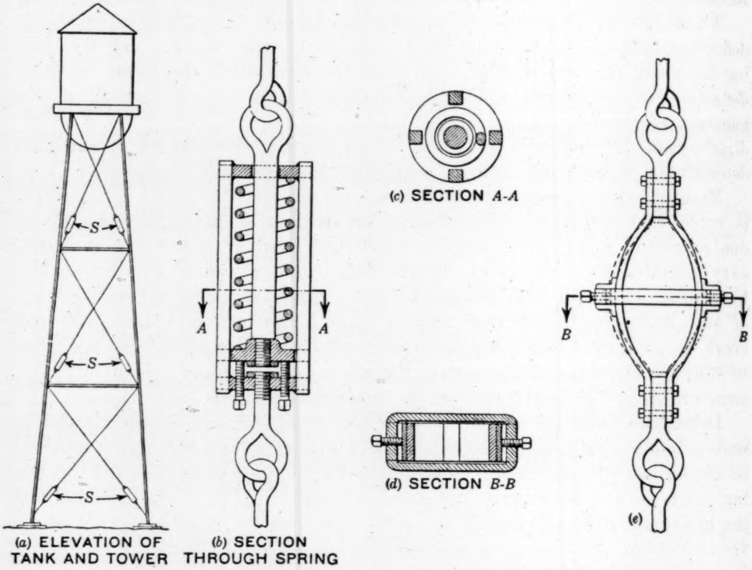
There are two general methods of increasing the allowable or elastic deflection range of a structure such as a water-tank tower: (1) By making the tower of some type of open framework in which the forces resisting deflection are produced by bending stresses set up in some or all of the members of the framework; and (2) by artificially increasing the allowable direct-stress deformations of some or all of the members of a statically determinate tower. Of course, combined systems will also suffice.

Method (1).—The first method has one serious drawback—if the tower is made with sufficient deflectibility for quake resistance, the deflections due to strong winds are likely to be undesirably large and the tank will sway considerably with every light wind. This difficulty can be avoided by adding a system of diagonal bracing to the tower, the bracing being of such strength that it will safely stand ordinary wind force but will break or collapse when a serious earthquake occurs, leaving the tank free to sway as far as need be during the quake, and yet not endangering the structure as a whole (if it is made inherently stable).

Difficulties arise in the design of a fool-proof collapsible bracing system, since a bracing failure on one side only or on one panel only would probably be disastrous. Furthermore, questions arise as to the advisability of taking chances on hurricane or near-hurricane winds which would break the bracing and leave nothing to resist them but the bending action of the frame (which, for towers about 100 ft high, is rather small if the design is economical). The replacement of bracing (or of breakable links in the bracing) after an earthquake does not present any serious objection. All in all, it may be stated that this method is at least worthy of serious consideration and perhaps it would be well worth while to study the behavior of models representing various towers of this general type.

Method (2).—The second method can best be explained by reference to Fig. 8 which illustrates schematically a device developed during this research. If spring elements such as those in Figs. 8(*b*) and 8(*e*) are introduced at Points *S* into the bracing of the tower (Fig. 8(*a*)), it will be seen that the allowable elastic deflection of the tower can be increased at will within very wide limits. Furthermore, it will be noticed that the spring elements are provided with a means of applying arbitrary initial stresses to the springs, so that up to the limit of the initial stresses they are perfectly rigid, thus preventing undesirable sway of the tank from ordinary wind forces. When an earthquake of serious magnitude occurs, however, the initial stress is quickly overcome and the elements act as simple springs to the point at which the coils close up tight, when they again become perfectly rigid.

It is evident that this method of construction has the advantage over the first method in that here extremely high winds or hurricanes will never lack adequate tower resistance as long as the main members of the tower are large enough to take the stresses. Furthermore, a slight dissymmetry will not be disastrous, as compared with the effect of breaking the bracing on one side of the tower.



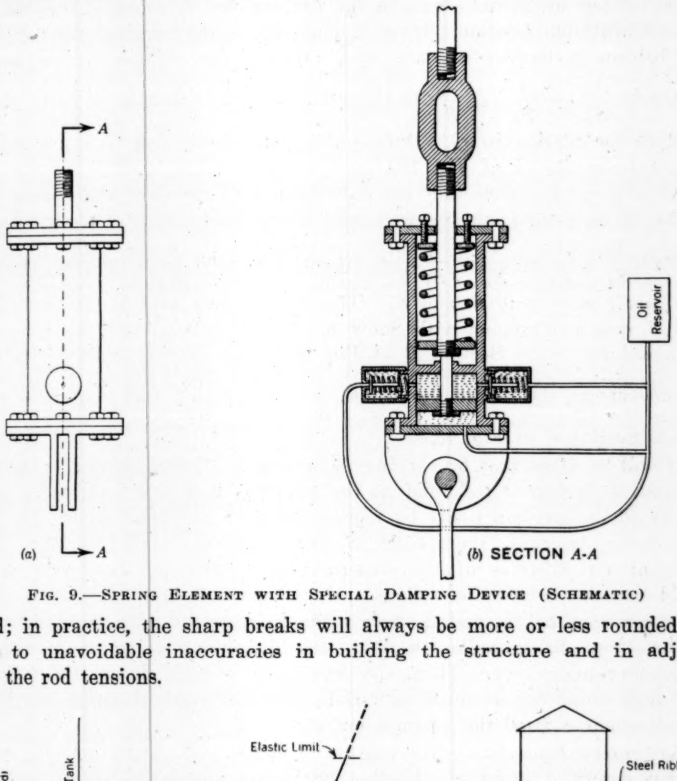
(a) ELEVATION OF TANK AND TOWER (b) SECTION THROUGH SPRING

FIG. 8.—SPRING ELEMENT CONSTRUCTION (SCHEMATIC)

There are so many possible types and combinations of spring elements that no attempt will be made to catalog them all. One special type that is of interest, however, is shown in Fig. 9, in which a damping device has been added to the spring element as a means of decreasing the amplitudes of vibration. It is surprising to find how little damping force is needed to produce a very beneficial effect in the behavior of this type of construction.

Description of Model Technique.—In dealing with an entirely new type of construction, where there are no definite data at hand on which to base design, it is desirable to investigate a number of possible designs under similar conditions. To build a series of models, each representing some variation in design, would obviously be a very expensive and difficult procedure. Therefore, it was decided to conduct this preliminary study by means of a single model which could be adapted to represent a number of designs and, at the same time, would entail a minimum of time and expense.

In order to understand how this was accomplished, it is first necessary to examine the statical, or load-deflection, characteristics of a structure equipped with spring elements. Fig. 10 shows the typical theoretical load-deflection curve for the new style of tower. This curve is obviously idealized; in practice, the sharp breaks will always be more or less rounded off due to unavoidable inaccuracies in building the structure and in adjusting the rod tensions.



(a) (b) SECTION A-A

FIG. 9.—SPRING ELEMENT WITH SPECIAL DAMPING DEVICE (SCHEMATIC)

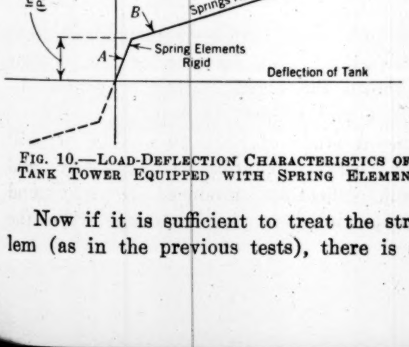


FIG. 10.—LOAD-DEFLECTION CHARACTERISTICS OF A TANK TOWER EQUIPPED WITH SPRING ELEMENTS

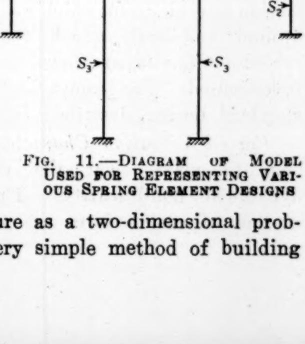


FIG. 11.—DIAGRAM OF MODEL USED FOR REPRESENTING VARIOUS SPRING ELEMENT DESIGNS

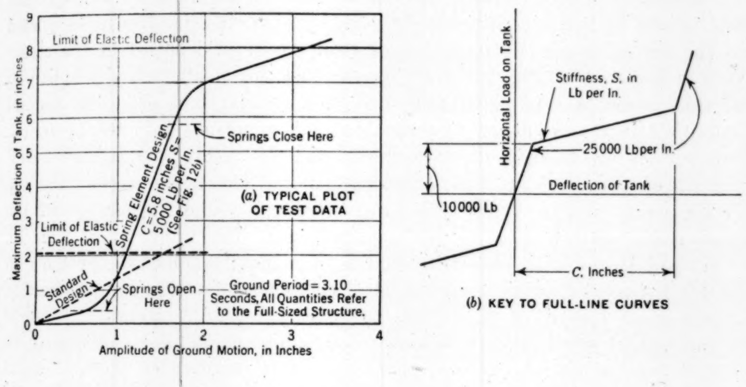
Now if it is sufficient to treat the structure as a two-dimensional problem (as in the previous tests), there is a very simple method of building its dynamical equivalent. Fig. 11 shows how the model was constructed by simple changes in the original model of the standard design. Screws *a* are set to put an initial stress in the flat springs, S_1, when the system is in its equilibrium position. If, then, one deflects the tank over to the left, say, Ribbon r_L buckles at once, and, at first, the stiffness of the tower system is $\frac{1}{S_3} + \frac{1}{S_3} + \frac{1}{S_2}$ to the point where the initial stress in S_1 is equal to the tension in r_R. During this part of the deflection the model represents Section *A* of Fig. 10.

As soon as the deflection proceeds farther to the left, the spring, S_1, begins to separate from Screw *a* and the stiffness of the tower system becomes $\frac{1}{S_3} + \frac{1}{S_3} + \frac{1}{S_2} + \frac{1}{S_1}$, in which the term, $\frac{1}{S_1}$ dominates because S_1 is small relative to S_2 and S_3. The stiffness now remains at this value until S_1 comes in contact with Screw *b*. During this part of the deflection the model represents Section *B* of Fig. 10. Any further deflection to the left encounters the original stiffness, $\frac{1}{S_3} + \frac{1}{S_3} + \frac{1}{S_2}$, and, therefore, represents Section *C* of Fig. 10.

It will be noted that by varying the settings of Screws *a* and *b* and the stiffness of Springs S_1, S_2, and S_3 (by altering their free lengths), a wide variety of designs can easily be represented with the same model. In the tests herein described, the springs, S_2 and S_3, were not changed since they represent the stiffness of the structural framework of the tower which would not change for the various spring-element designs considered. The adjustments of S_1, S_2, and S_3 were made independently by using the conventional weight-and-pulley loading, the deflections being read with a micrometer microscope. Then, the set-up was assembled and the settings of Screws *a* and *b* were made by "trial and error" until the measured load-deflection curve fitted the assumed curve.

Laboratory Procedure.—The routine of shaking-table tests was the same as that described under the heading "Standard Design." Several possible designs were chosen for study, and each design was put through a series of simple harmonic motion tests in which both period and amplitude were varied. For each design, curves were drawn showing the maximum deflection of the tower as a function of ground amplitude for a given ground-motion period, from which could be determined the largest simple harmonic ground amplitude which the structure could safely withstand at that period. Fig. 12(*a*) shows a characteristic plot for one of the designs investigated. For comparison is shown the corresponding curve for the standard design described in Table 1 and supporting text.

Forced Vibration Characteristics.—A study of Figs. 10 and 12 (*a*) will help to make clear the relation between statical non-linearity and dynamical non-linearity. The tank deflections produced by a ground motion of cosine form are not proportional to the amplitude when the period is held constant, as was the case with the simple standard structure. This phenomenon is due to the fact that the natural vibration period of the structure changes as the amplitude of its vibration varies. Thus, from



(b) KEY TO FULL-LINE CURVES

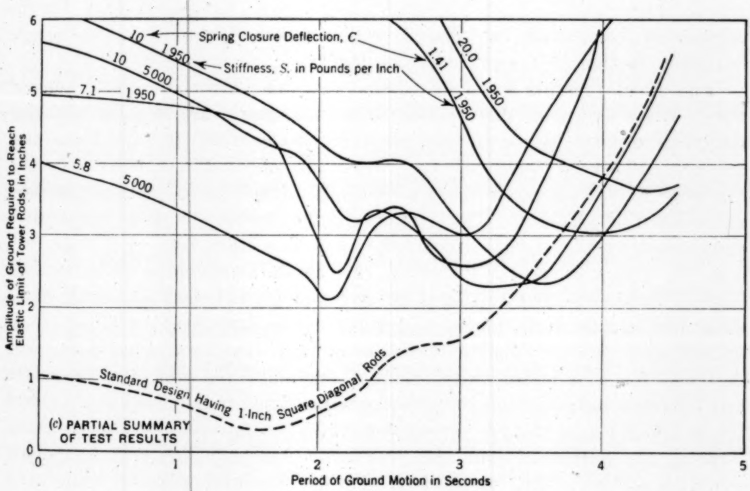


FIG. 12.—EARTHQUAKE RESISTANCE OF SPRING ELEMENT DESIGNS COMPARED WITH THAT OF STANDARD DESIGN

Fig. 10, it is evident that over the range, *A*, the natural period is constant and is equal to that of the same structure without spring elements. As soon as the amplitude of vibration passes over to the range, *B*, however, the natural period becomes progressively longer with increasing amplitude because the effective stiffness (that is, the ratio of the horizontal load to the corresponding deflection) becomes progressively less. This state of affairs continues until the amplitude of vibration begins to enter the range, *C*, when further increase in amplitude will result in progressive shortening of the period that the structure had at the end of the range, *B*. However far the deflection may go in the *C* range, the natural period can never be as short as the period in the *A* range.

In such a system, then, it is obvious that the amplitude of the ground motion can be just as critical as its period. Thus, in Fig. 12(*a*), the period of the ground motion is much longer than the natural period of the structure in Range *A* of Fig. 10. As a consequence, for small ground amplitudes of this period, a very small tank deflection occurs; but when the ground amplitude increases enough to throw the tank amplitude into Range *B*, the two periods are brought closer together, and the result is a sudden building up of vibration due to a quasi-resonance. The break-over at the top of the curve in Fig. 12(*a*), of course, represents the disappearance of the quasi-resonance condition as the amplitude goes into Range *C* of Fig. 10.

It will be noted at once that studies of markedly non-linear systems in simple harmonic motion, such as the spring-element tower designs, cannot give a clear picture of the effects to be expected from irregular motions of an actual earthquake. Indeed, these experiments were restricted to simple motions only because at that time actual earthquake motions could not be reproduced in this laboratory; but it was recognized that, for the purpose of comparing one design with another dynamically, the simple harmonic motion is a serviceable yardstick.

Fig. 12(*c*) presents a partial summary of the results of several hundred tests. Also, shown on this diagram, for comparison, are the corresponding results of tests on the standard design previously investigated. These data are represented in terms of the amplitude of simple harmonic ground motion required to reach the elastic limit of the tower rods for various ground-motion periods. The points on the curves in Fig. 12(*c*) were determined by the absolute maximum tank deflection reached during each test. In almost every test, however, this absolute maximum deflection was reached within the first two or three cycles of ground motion. Thus, if the data were restricted to motions of three cycles duration, the figure would change very little. The rod-bracing of the tower in these tests was designed for a safe horizontal load of one-tenth the weight of the tank and contents acting at the center of gravity of the tank. Had the standard tower bracing (see dotted curve, Fig. 12(*c*)) been designed for the same loading, the ordinates would be practically unchanged in magnitude, but would be shifted a little to the left. Thus, the minimum point would move from a period of 1.6 sec to about 1.3 sec, with no change in magnitude.

The superiority of the spring element design over what is generally thought of as the principal part of the earthquake spectrum is very marked, but for earthquake periods beyond about 3 sec the standard design compares favorably with it.

Some preliminary experiments were made with a frictioning device in order to study the effect of damping. The results indicated that surprisingly small damping forces would suffice to raise the low points on the curves well above the curve for the standard design. These tests were not sufficiently quantitative to warrant extended discussion.

Conclusions Pertaining to Spring Element Design.—These preliminary experiments suggested that the new form of construction has very definite possibilities as a means of overcoming the earthquake hazard for elevated water-tanks. The conclusions to be drawn are quite limited by the very nature of the tests:

(*a*) For earthquakes characterized by short-period vibrations (not more than about 2.5 or 3-sec periods) the simple spring-element design can be considered a satisfactory solution. For such earthquakes the data in Fig. 12(*c*) show that the greater the closure deflection, *C*, the more violent shocks the structure can withstand.

(*b*) For earthquakes in which moderately large long-period vibrations are present (say, between 3 and 6 sec) dangerous stresses may be reached; and, without damping, the spring-equipped structure may be no better off than one of standard design. For such earthquakes the data in Fig 12(*c*) seem to indicate that closure deflections of more than 7 or 8 in. may be undesirable.

(*c*) The presence of a slight degree of damping action would greatly help the structure in the long-period range without appreciably reducing its effectiveness in the short-period range.

(*d*) In order to obtain design data for engineering purposes, tests should be made with actual earthquake motions. The model preferably should be three-dimensional and large enough to permit accurate determination of the damping forces required for practical design. (In the model of 1:46.5 scale the measurement of damping forces is made difficult by the fact that the force-scale is $(46.5)^3$, or 100 500; that is, a force of 1 000 lb in Nature corresponds to less than $\frac{1}{6}$ oz in the model. Although there is no difficulty in measuring very small elastic forces in a frictionless structure such as that shown in Fig. 11, it is difficult to produce and measure frictional forces below the level of 0.5 oz with reasonable accuracy. No such difficulty appears in measurements of deflections, however, since the scale of deflection is equal to the scale of the model).

PART II.—EXPERIMENTS WITH ACTUAL EARTHQUAKE MOTION

Model Characteristics and Testing Procedure

Introductory—Four important questions were left unanswered by the research discussed in Part I:

(*a*) How will the new construction behave in three dimensions?

(*b*) What will be the effect of actual irregular earthquake motions as compared with the simple harmonic motion previously used?

(*c*) How much damping is needed to avoid excessive stresses?

(*d*) How does the proposed structure compare with the standard type of structure in dynamic behavior when subjected to actual earthquake motions.

Before further model studies could be directed profitably toward answering these questions, it was necessary to construct a shaking-machine capable of reproducing actual earthquake motions. The design of this machine[5] was begun in the fall of 1934. Construction was completed in the summer of 1935; and by September, 1935, the operation of the machine was considered quite satisfactory for conducting the research.

The research described in Part II should be regarded as only a part of the program of tank-tower investigation that was originally planned. Limited funds made necessary the curtailment of the planned program, with the result that the data are not nearly so extensive as could be desired. Sufficient engineering data have been obtained, however, to make possible for the first time rational design of a fairly wide range of sizes of elevated water-tank structures for seismic regions.

It is not intended to suggest that the spring element design is the only practical solution to the problem. It is the only solution which the writer has (to date) considered sufficiently practical for actual construction.

The Model.—A three-dimensional model was constructed on a scale of 1:25 to represent a 60 000-gal tank on a 100-ft tower. Spring elements were built into the model to simulate the new type of construction to be investigated.

The basic theory applying to models of water-tank structures was discussed[6] by the writer in 1935, who then showed that perfect geometrical scaling will result in dissimilarity unless the model tower is built of a material having a modulus of elasticity of $\frac{\rho_m}{\rho\lambda}$ times that of the steel in the prototype (ρ and ρ_m being the densities of the liquids in the prototype and model tanks, respectively; and λ is the scale of the model). This means that if mercury was used as the liquid in the model, and brass ($E = 13\,800\,000$ lb per sq in.) for the structural parts and tank shell, then a geometrically scaled model with $\lambda = 29$ would be entirely satisfactory (with a minor correction for the density of the brass).

A consideration of the hazards and difficulties involved in the use of a mercury-filled model led to the decision that this procedure should be avoided. This was done by means of a scale distortion analogous to that applied in earlier studies. The expense of building an accurately made geometrical-scale model of a water-tank and tower would have been very great, and the problem of simulating riveted connections exactly to small scale has not been solved. Fortunately, in tank towers of conventional design, the structure is so nearly statically determinate that it makes little difference whether the joints are rigid or pinned, as far as the dynamical behavior of the structure is concerned. Usually, they are somewhere in between, probably being nearer the rigid condition, except where the struts are actually pinned to the columns, as is sometimes the case. The eccentricity of the members coming together at a joint is often of more consequence in affecting the tower stiffness than the degree of rigidity in the joint itself

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[5] *Civil Engineering*, March, 1936; and *Bulletin*, Seismological Soc. of America, July, 1936.

[6] *Civil Engineering*, August, 1935, p. 455.

It was decided to make the joints of the model rigid (the prototype joints were riveted) and to preserve the eccentricities as they are in the actual design chosen for study. In this manner the net tower stiffness can be simulated closely by a simple model.

In order to arrive at the model-member design required for a given over-all length scale, λ, the model theory referred to in Part I is merely extended to the individual parts of the structure.[7] The following relations are easily obtained:

For structural members,

$$\frac{A}{A_m} = \frac{\rho}{\rho_m} \frac{E_m}{E} \lambda^3 \qquad (2)$$

$$\frac{I}{I_m} = \frac{\rho}{\rho_m} \frac{E_m}{E} \lambda^5 \qquad (3)$$

and, for thickness of tank and roof plates,

$$\frac{t}{t_m} = \sqrt[3]{\frac{\rho}{\rho_m} \frac{E_m}{E} \lambda^4} \qquad (4)$$

in which A = area of member in prototype; A_m = area of member in model; t = plate thickness in prototype; t_m = plate thickness in model; I = moment of inertia of prototype member; and I_m = moment of inertia of model member. The thickness scale for the tank and roof plates is correct only if the principal shell distortions are due to the bending of the plates and not to direct stresses. This is obviously the case for tanks of the type studied herein.

It is important to realize that these relations, although resulting in true dynamical similarity, do not result in similarity of unit stresses, from which it is to be concluded that a model built according to these relations properly represents the prototype, in so far as statical and dynamical deflections are concerned, to the point where elastic limit stresses are reached either in the model or in the prototype. Beyond this point true similarity ceases to exist, and the interpretation of the model results becomes a matter of judgment. Once the model deflections are known, it is a simple matter to calculate the corresponding stresses in the prototype as explained in Part I.

To satisfy, simultaneously, both the $\frac{A}{A_m}$ and the $\frac{I}{I_m}$-relations for $\lambda = 25$ (the scale chosen) and $\frac{\rho}{\rho_m} = 1$ (water in the model tank) is a practical impossibility for the sections comprising a standard tank tower; but there is a simple solution of the problem. If the scale of $\frac{I}{I_m}$ is made correctly, the area scale can be corrected by artificially giving to the members the compliance (force per unit elongation) they would have if the area scale could be maintained. If C and C_m are the compliance of prototype and model, respectively, it can be shown that for the model in question the following relation must be satisfied[7]:

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[7] *Bulletin*, Seismological Soc. of America, July, 1934, pp. 170 to 230.

$$\frac{C}{C_m} = \frac{\rho}{\rho_m} \lambda^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

In the present problem it was found that, when the proper $\frac{I}{I_m}$-ratio was maintained, the areas of the model sections were invariably much too large. Consequently, the introduction of additional spring action into the members was necessary. Instead of altering the compliance of all the members of the frame, it was found much more convenient to attach appropriate springs to the diagonal rods only in order to correct for the stiffness of the tower as a whole. The reader will observe that the necessary calculations are straightforward, the model springs being chosen so that with a horizontal load acting at the tank, the total deflection in any one panel was made the same as it would have been had all the members in the panel been corrected.

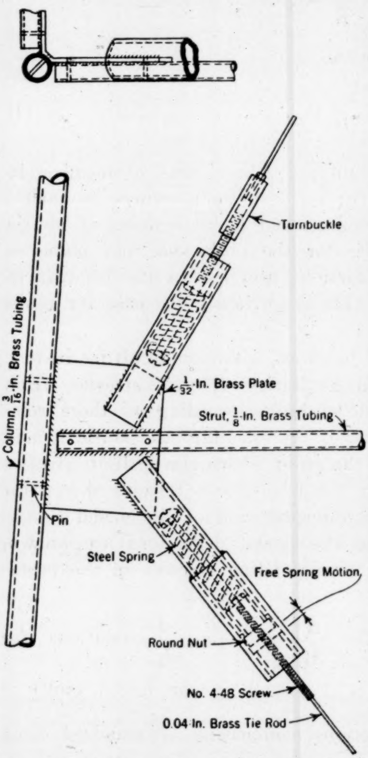
Turnbuckle
$\frac{1}{32}$-In. Brass Plate
Strut, $\frac{1}{8}$-In. Brass Tubing
Column, $\frac{3}{16}$-In. Brass Tubing
Pin
Steel Spring
Free Spring Motion
Round Nut
No. 4-48 Screw
0.04 In. Brass Tie Rod

FIG. 13.—TYPICAL MODEL JOINT DETAIL

Fig. 13 illustrates the construction of a typical joint; the correction spring is above the joint and the spring element below. The individual spring elements were all completely calibrated, assembled, and set for initial load and closure point before they were put into the tower. All connections were joined thoroughly with medium hard solder. The pins shown were inserted for construction purposes only. The nuts and turnbuckles were secured with solder after final adjustment to prevent their loosening during vibration of the model.

The adjustment of the tie-rods gave the only real difficulty in the entire work of building the model. This had to be done largely by gaging

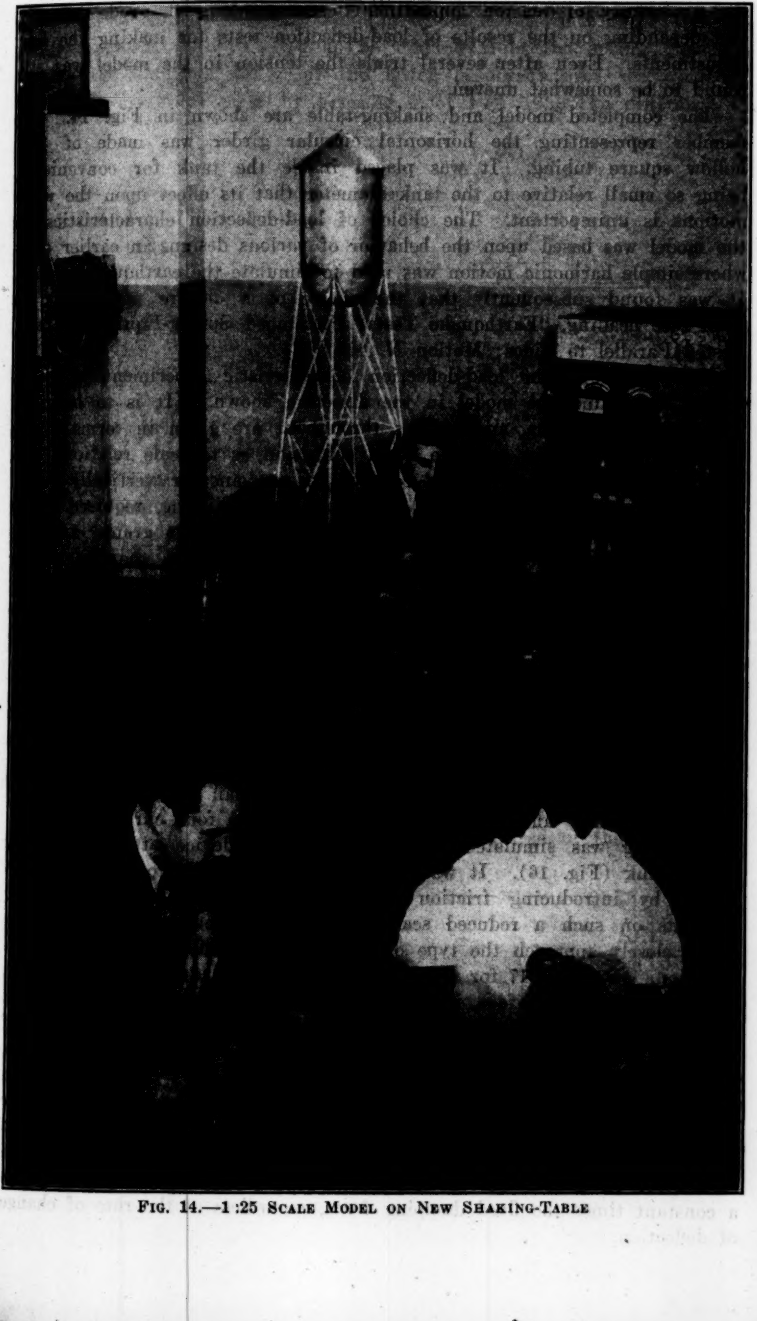

FIG. 14.—1:25 SCALE MODEL ON NEW SHAKING-TABLE

the uniformity of tension approximately by touching the rods laterally, and depending on the results of load-deflection tests for making the final adjustments. Even after several trials the tension in the model was still found to be somewhat uneven.

The completed model and shaking-table are shown in Fig. 14. The member representing the horizontal circular girder was made of ¼-in. hollow square tubing. It was placed inside the tank for convenience, being so small relative to the tank diameter that its effect upon the water motions is unimportant. The choice of load-deflection characteristics for the model was based upon the behavior of various designs in earlier tests, where simple harmonic motion was used to simulate the earthquake motion. It was found subsequently that the choice of a closure point was not ideal (see heading, "Earthquake Tests; Undamped Spring-Equipped Tower, Tested Parallel to Sides; Motion N. 40° E.").

Fig. 15 shows the load-deflection characteristic experimentally determined by pulling the model in the direction shown. (It is to be noted that loads, deflections, and periods throughout are given in terms of the full-sized structure, in order to avoid confusion as to scale relations.) A slight shifting of these curves relative to one another vertically would make them coincide very well, the maximum shifting required being about 1 200 lb. In the model, this amounts to about 35 grams, and represents the variations in adjusting the initial tension in the twenty-four tie-rods. Considering the small scale used, it is doubtful whether much better uniformity of rod tension can be obtained practically, and there is no reason to believe that an actual full-sized structure would be more uniformly adjusted as to tension by a field gang.

Shaking-table tests, first in the *A*-direction and then in the *C*-direction, showed no appreciable difference in dynamic characteristics. Indeed, this is what one would expect, since the natural period of the structure varies approximately with the square root of the area beneath the load-deflection curve, which means that for amplitudes greater than 2 or 3 in. the period is not appreciably affected by the variations in the four curves.

Damping was simulated by a solid friction device attached directly to the tank (Fig. 16). It was not considered possible to obtain reliable results by introducing friction into the twenty-four individual spring elements on such a reduced scale. The solid friction used in testing would closely approach the type of damping contemplated for actual construction. (See Fig. 17 for sectional assembly of a large unit equipped with damping device.) In fact, it is possible that a satisfactory element might be built more economically with pure solid (sliding) friction instead of the piston and cylinder arrangement shown in Fig. 17.

The choice of solid friction (or the equivalent) as a damping medium instead of the more conventional damping proportional to velocity (or some power of velocity) is justified by a consideration of the stresses that might be set up by the latter in the case of fast ground movements. Constant-force damping limits the stresses to those produced by deflection, plus a constant times the fixed damping force, regardless of the rate of change of deflection.

It will be noticed that the damping device shown in Fig. 16 does not exactly represent the damping to be derived from the spring elements, because the damping action of the latter is absent over the range of tower deflections indicated by Section *A* of Fig. 10. The difference in effect is small, however, and simply means that the damping force observed in the laboratory tests should be increased by 8%, or by 10% at the most, to secure the equivalent damping force required of the spring ele-

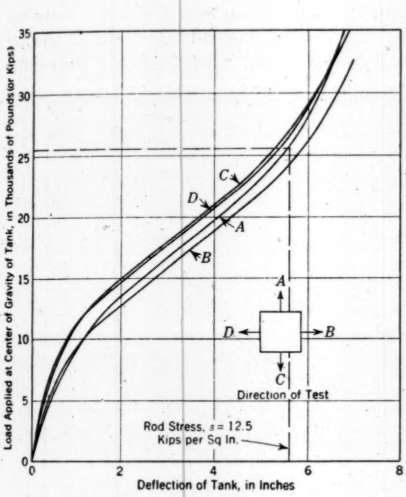

FIG. 15.—LOAD-DEFLECTION CURVES FOR TOWER EQUIPPED WITH SPRINGS

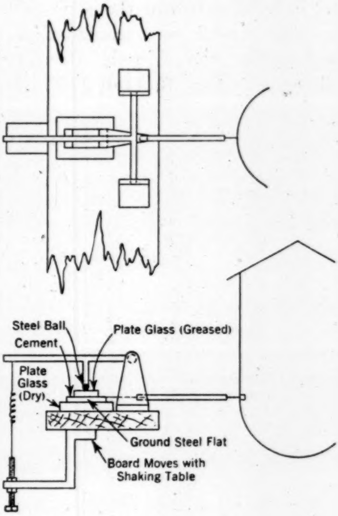
Steel Ball
Plate Glass (Greased)
Cement
Plate
Glass
(Dry)
Ground Steel Flat
Board Moves with
Shaking Table

FIG. 16.—DIAGRAM OF FRICTION DEVICE

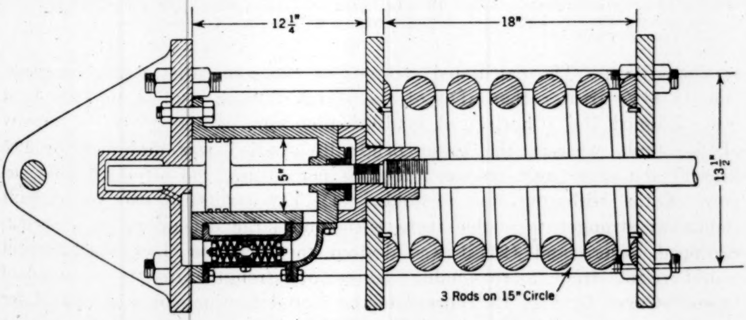
12 1/4"
18"
5"
13 1/2"
3 Rods on 15" Circle

FIG. 17.—SHOCK-ABSORBING ELEMENT FOR TANK TOWER

ments for the same degree of earthquake resistance. (This has been generously allowed for in Conclusion (8), "General Conclusions Drawn from Research.")

The structural design assumed for this investigation is shown in Fig. 18. This design gives a calculated stiffness to the tower of 23 kips per in. of deflection before the springs begin to act.

The model was transformed into a standard tower for later tests by connecting the tie-rods directly to the joints instead of through spring elements. Fig. 19 (full line) shows the measured load-deflection curve for this condition. The calculated stiffness of the structure in this investigation is 44% greater than that of the standard tank studied in Part I; it was designed for a horizontal load of 43.5 kips at the center of gravity of the tank, whereas the tower originally studied was designed for 19.5 kips at the same unit stresses (18 kips per sq in., rod stress). The purpose of the stiffening was twofold: (1) The structure had to be built somewhat stronger to enable it to withstand large deflections as a spring-equipped tower; and (2) it was desired to learn just how much benefit would result from a reasonable degree of strengthening in a standard tower. Curve C, Fig. 19, represents the load-deflection curve computed for the design shown in Fig. 18. (The calculations were made assuming no initial tension in the tie-rods.)

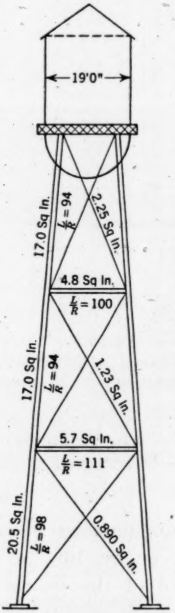

FIG. 18.—STRUCTURAL DESIGN OF PROTOTYPE; TESTS, PART II

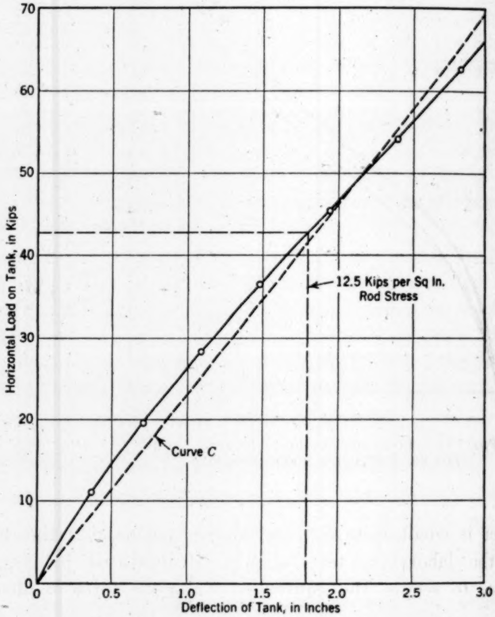

FIG. 19.—LOAD-DEFLECTION CURVE, STANDARD TANK; TESTED AT 45° TO THE TOWER SIDES

Testing and Recording Procedure.—The model (see Fig. 14) was first tested statically to obtain the data in Fig. 15. Then it was mounted on the table of the shaking-machine so that the motion of the base was applied parallel to the tower sides. Later, the tower was turned through 45° so that the motion was applied diagonally for reasons given subsequently. All tests (including those for load deflection) except one, were made with the tank full of water.

The motions used for testing the model dynamically were taken from the data on the Long Beach earthquake of March 10, 1933. These valuable data consist of tabulations of ground displacements at intervals of 0.120 sec for the three components of that earthquake, as calculated from the accelerograms recorded at the Subway Terminal Building, in Los Angeles, Calif. The two horizontal components were plotted on cross-section paper and, after eliminating some very long period waves which were obviously unimportant, a polar plot was made for each component. The polar plots were made into optical cams[8] for driving the machine by cutting them along the plotted lines representing the ground motion. The cam for the component, N. 40° E, is seen in Fig. 14. No tests were made with the vertical component. (There is no direct evidence of dangerous earthquake stresses in the columns of steel tank towers due to vertical motions. Such column failures as have occurred appear to have resulted from poor details and not from true column failure in the proper sense of the word. However, it would be very desirable to extend the investigation to include earthquake effects in the vertical direction. From this standpoint a spring-equipped tower would behave substantially like a standard structure.) The two horizontal components are referred to throughout as the Los Angeles motions, N 40° E and S 50° E. The reader will understand that they are motions of the Long Beach shock as recorded in Los Angeles (37 miles from the epicenter).

A number of free vibration tests were made with the model as a spring-equipped structure (with and without damping), and as a standard structure. As in earlier tests, the procedure was a sudden release from a pulled-back position.

The procedure used in recording the motions during tests was essentially the same as that followed in earlier studies. For each test, the table motion, the deflection of the tank relative to its base, and time marks, were recorded simultaneously on one photographic record. Free vibration tests were recorded in the same manner.

The arrangement for testing and recording is shown in Fig. 20. A rigid wooden framework moving with the shaking-table carries the recording mirrors and lenses so that the tank deflections are recorded directly. The same frame also carried the friction device shown in Fig. 16.

Free Vibration Tests.—All free vibration tests were made diagonally to the tower, in order to be consistent with most of the earthquake tests. The maximum amplitudes of successive swings in the free vibration of the standard structure are given in Fig. 21(*a*) in terms of the percentage of the original pull-back before releasing, plotted against the pull-back amplitude. One observes here about the same degree of linearity as in the case of the smaller model investigated previously. The exact form of the free vibration is not the same as before because the present model represents a stiffer structure. The absence of damping action is shown by the persistence of large amplitudes in the 5, 6, and 7 maxima. (By "damping" is meant the dissipation of energy (usually in the form of heat) by the action of forces resisting motion.)

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[8] *Bulletin*, Seismological Soc. of America, July, 1936.

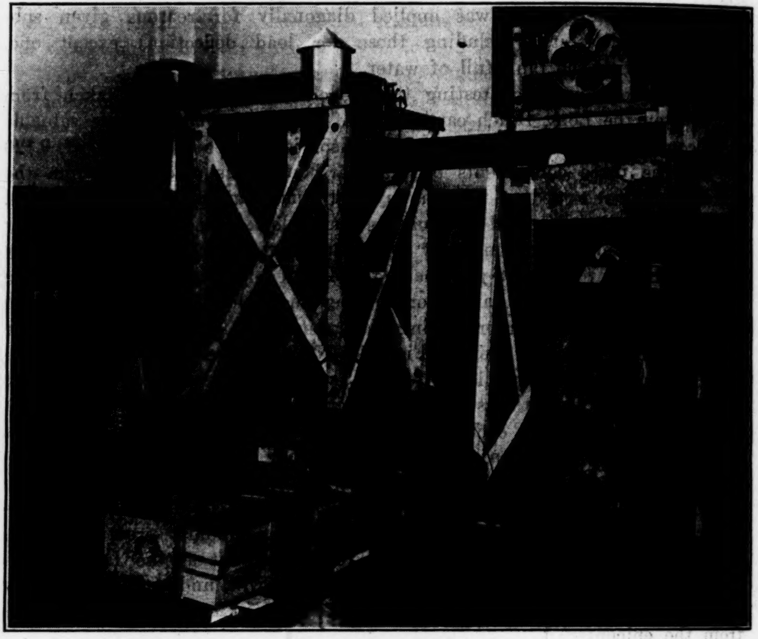

FIG. 20.—SET-UP FOR TESTING AND RECORDING ACTUAL EARTHQUAKE MOTION EXPERIMENTS

It must not be inferred that because the amplitude has dropped to 60% of the initial pull-back by the seventh swing that the stored energy has dropped to 60% of the original. The original energy has merely been changed from all potential to part potential and part kinetic (that is, the water is surging when the tank stands still at the end of a swing), the total energy being scarcely diminished in the first five to ten swings. Similar pull-back tests made by the United States Coast and Geodetic Survey[9] on full-sized structures show very little decrease in energy even after twenty to thirty swings of the tank. The absence of appreciable damping in standard tank structures and their foundations is the greatest hindrance to making them earthquake resistant by any ordinary means. From the free vibration tests, the principal period of vibration was found to be 1.3 sec (2-in. amplitude). The secondary period (mainly due to the water swinging in the tank) is about 2.7 sec.

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[9] *Special Publication No. 201,* U. S. Coast and Geodetic Survey, pp. 75-95.

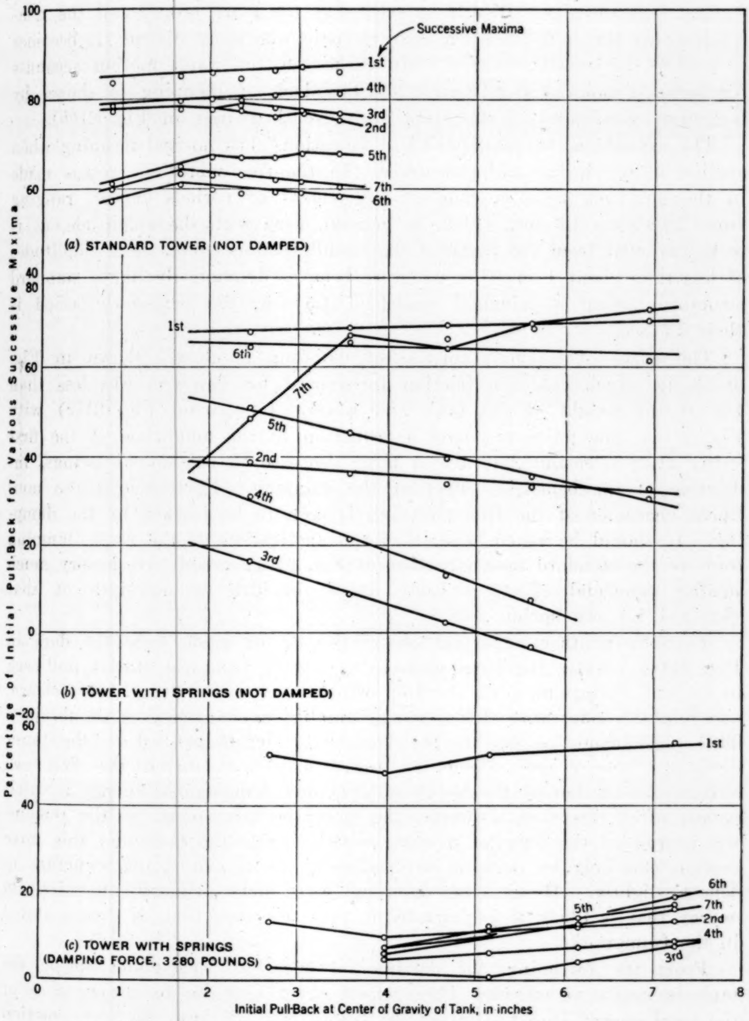

FIG. 21.—FREE VIBRATION OF TANK WHEN SUDDENLY RELEASED FROM A PULLED-BACK POSITION

Fig. 21(*b*) shows graphically the free vibration characteristics of the spring-equipped structure with no damping acting. Here, a pronounced non-linearity is observed, as evidenced by the great deviation of some of the plotted lines from the horizontal. Comparing Fig. 21(*b*) with Fig. 15, one can obtain a further conception of the relation between dynamical and statical non-linearity. It will be seen that the more pronounced the non-linearity of the load-deflection characteristic, the more difficult it becomes to predict dynamical behavior. (In each case, the water motion accounts for some dynamical non-linearity.) The lack of damping is shown by the large amplitudes of the sixth and seventh swings in Fig. 21(*b*).

The expression, "natural period of vibration," has no real meaning when applied to the spring-equipped tower. In the free vibration records made in the pull-back tests, a complete assortment of periods appear, ranging from 3.5 sec to 1.5 sec, which, in general, vary with the amplitude, as is to be expected from the shape of the load-deflection curve. For amplitudes of less than about 1 in., this structure behaves dynamically like a standard structure having a principal period of 1.5 sec; the secondary period is then 2.7 sec.

The effect of a small amount of damping is clearly shown in Fig. 21(*c*), in which case the friction force used for damping was less than 1% of the weight of the tank and water. Comparing Fig. 21(*c*) with Fig. 21(*b*), one notes not only a reduction in the amplitude of the first swing after releasing, but also a large decrease in subsequent swings, indicative of the beneficial effect of the damping. Furthermore, the non-linear character of the free vibration is seen to be masked by the damping. It should be noted, also, that the application of the same damping force to the standard tank structure of Fig. 21(*a*) would have a very much smaller beneficial effect, probably about one-fifth or one-sixth of that obtained for the spring tower.

There is another important observation to be made from the data in Fig. 21(*c*). After the large drop in amplitude from the initial pull-back to the first swing and from the first swing to the remainder of those shown, one might wonder why the decay in amplitude changes pace so abruptly. This phenomenon is due to the fact that friction applied to the tower dissipates energy only when the tower deflects. During the first two swings, the motion of the tower is large and considerable energy is dissipated; after this time, however, the water contains most of the remaining energy in the form of a wave motion inside the tank and this wave motion can only be damped out indirectly as it causes the structure to deflect slightly. Hence, the damping force acts primarily to dissipate energy that is likely to be harmful to the structure; that is, energy stored in the framework.

From the foregoing, the detrimental effect of baffle-plates inside the tank is easily visualized. Their effect could only be to throw more of the total energy into the structural frame and less into the wave motion of the water. In the case of very large and flimsy storage tanks, the capacity of the tank shells to withstand the surging motion of the water may well be questioned. The shells of the standard elevated tanks of capacities not greater than 100 000 or 150 000 gal are most likely quite strong enough to stand the water forces, provided the strength of the roof is properly utilized to support the upper part of the shell.

SPRING ELEMENT DESIGN; SCALE OF MODEL, 1:25

Earthquake Tests; Undamped Spring-Equipped Tower, Tested Parallel to Sides; Motion N 40° E.—The behavior of the undamped spring tower was first investigated by applying the Los Angeles, N 40° E, earthquake motion to its base, the direction of motion being parallel to the sides of the tower. To insure the safety of the model, the amplitude of this motion was reduced by the factor, 0.72, of the actual earthquake amplitude. This factor is referred to throughout this paper as the "Amplitude Factor" and signifies the ratio of the testing amplitude to the amplitude of the actual earthquake as recorded by the accelerograph in the Subway Terminal Building, in Los Angeles.

Tests made by applying the described motion at various speeds are plotted in Fig. 22(*a*). The maximum deflection of the tank (relative to its base) occurring during the application of the earthquake motion is plotted against the time factor. The "Time Factor" represents the ratio of the duration of the earthquake in the test to the duration of the actual earthquake as recorded by the accelerograph. Thus, a time factor of 1.10 denotes that T sec of actual earthquake occupied 1.10 T sec in the test; that is, a time factor greater than 1.00 indicates that the motion was more slowly applied than the actual earthquake. Since as previously mentioned, all data in this paper have been expressed in terms of the full-sized structure, no attention need be paid to scale relations, and all the curves may be interpreted directly as if the tests had been made on the full-sized structure.

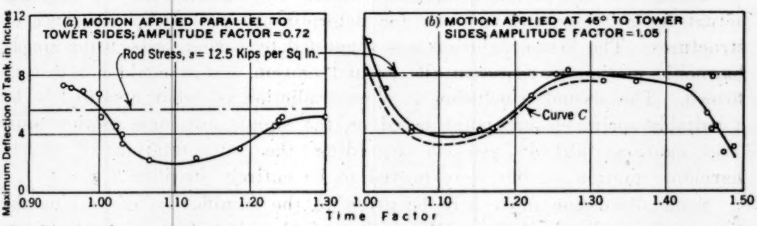

FIG. 22.—MAXIMUM DEFLECTION-TIME FACTOR CURVES, TOWER WITH SPRINGS; NOT DAMPED; LOS ANGELES MOTION, N 40° E.

The horizontal broken line in Fig. 22, and in subsequent diagrams, is a reference line for comparing the various sets of tests. It represents the deflection at which a rod stress of 12 500 lb per sq in. is developed, and by noting the position of the curve of tank deflection relative to this line one can interpret the results in terms of stress. Fig. 15 will also be found helpful in this respect. The reference stress line was chosen at the break-over point of the load-deflection curve (see Fig. 15). To be consistent, the same stress was used as a reference line for the standard tower data (see Figs. 19 and 25). It is for comparison only and does not represent "safe" or "working" stress.

Fig. 22(*a*) indicates that without damping the spring elements cannot be considered very satisfactory because, although dangerous deflections are not reached anywhere on the curve, they certainly might be if the amplitude factors were raised from 0.72 to 1.00, or more. Two important conclusions were drawn from these tests. It was evident: (*a*) That damping should be present if the Los Angeles earthquake records can be accepted as representing a possible kind of motion to be found in Nature; and (*b*) that it would have been much better to have chosen a structure capable of about double the maximum safe deflection originally chosen. In other wards, instead of having the springs close at a deflection of about 6 in., they should close as far out as the structure can safely be deflected (15 in. is a practical figure from the standpoint of column bending and spring design, although 20 in. does not seem unreasonably large.)

These two conclusions emphasize clearly the desirability of applying actual irregular ground motions for determining earthquake resistance of structures. The first conclusion was suggested by earlier tests, using simple harmonic motion to represent the ground motion, but it could not be confirmed. The second conclusion is a contradiction of what appeared to be a suitable spring-closure point based on the simple harmonic motion tests. The cautions already stressed regarding the interpretation of simple harmonic motion studies were proved to be entirely justified.

Some discussion must here be given to the significance of varying the time and amplitude factors. Variation of the time factor can be interpreted in two ways: (1) With a given structure, varying the time factor makes it possible to find what effect the earthquake would have if it happened faster or slower, but with the character of the motion preserved as recorded; and (2) with a given earthquake, varying the time factor has the same effect as changing the natural vibration period of the structure. In other words, by varying the time factor the experimental results can be generalized considerably, whereas if only a time factor of unity is used the results can be applied only to one particular earthquake acting on one particular structure.

The significance of varying the amplitude factor is obvious. It makes it possible to generalize the results in another direction, so that the effect of the same motion changed in amplitude can be seen.

Fig. 23(*a*) represents a series of tests following those previously described, the purpose being to find how the behavior of the structure would vary with changing amplitudes of the same earthquake motion. The time factor was held constant. These results show that the maximum tank deflection is very sensitive to small changes of earthquake amplitudes when no damping is present. They emphasize the correctness of Conclusions (*a*) and (*b*), cited in discussing Fig. 22(*a*). No attempt was made to carry the amplitude factor near to unity for fear of over-stressing the model.

Earthquake Tests; Undamped Spring-Equipped Tower, Tested Diagonally; Motion N 40° E.—Since there were not sufficient funds available to permit building and testing a model to represent the design suggested by the tests just described, it was decided to turn the model so that the motion could be applied diagonally. In this position, the safe deflection is increased by $\sqrt{2}$, and it was thought that the results would be interpretable.

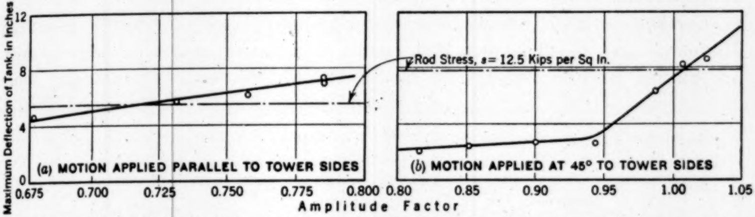

FIG. 23.—MAXIMUM DEFLECTION-AMPLITUDE FACTOR CURVES, TOWER WITH SPRINGS; NOT DAMPED; LOS ANGELES MOTION, N 40° E; TIME FACTOR, 1.02

Fig. 22(*b*) (full line) is a plot of the results of a series of tests with earthquake motion at N 40° E applied diagonally to the tower and with an amplitude factor of 1.05. The reference stress line will aid in comparing with Fig. 22(*a*). In order to determine how well the diagonal tests could be used for interpreting the behavior parallel to the tower sides, Curve 1 was computed from the data of Fig. (22*a*), as if the structure were linear; that is, it represents the behavior of the tower under the same conditions, as computed by compounding the tests parallel to the sides. The agreement indicates that it is probably safe to accept the results of the diagonal tests as also applying to tests parallel to the sides of the same tower, by resolving into components as for a linear system.

Of course, a direct interpretation can be made by noting that the diagonal tests merely represent a tower with a load-deflection characteristic somewhat different from Fig. 15. It makes no difference in what direction this characteristic is taken to be; the behavior is dependent only on the moving mass and the character of the load-deflection curve.

From this study, it was concluded that applying the test motion diagonally would serve the purpose quite well. Conclusions (*a*) and (*b*), cited in connection with Fig. 22(*a*), were further substantiated.

A set of tests was then made to find the effect of changing the amplitude factor when shaking the structure diagonally. The results are plotted in Fig. 23(*b*). The form of the curve here closely resembles that obtained by varying the amplitude of the simple harmonic motion applied in earlier experiments (see Fig. 12(*a*)). Fig. 23(*a*) would show the same characteristics had the curve been extended far enough to the left. As long as the deflections of the structure do not pass the upper break in Fig. 15

(or the corresponding point for diagonal tests), the shape of the part of the load-deflection curve beyond this break has no effect upon the results, and one can just as well imagine that the break has been moved farther out to the right, as it should be. (The upper break in Fig. 15 is caused by the closing up of the spring elements. Its position can be varied between wide limits by a proper choice of springs.)

Earthquake Tests; Damped Spring-Equipped Tower, Tested Diagonally; Motion N 40° E.—The principal results of a large number of experiments are plotted in Fig. 24(*a*). As in Fig. 22(*b*), the testing conditions were

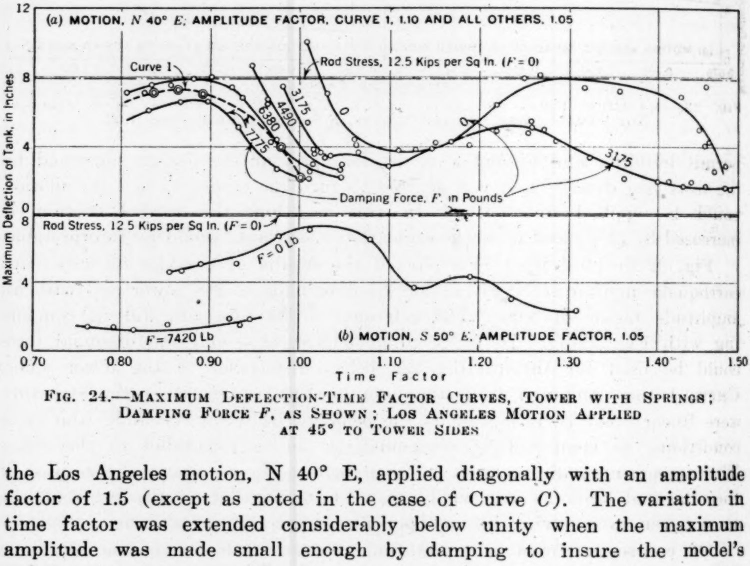

FIG. 24.—MAXIMUM DEFLECTION-TIME FACTOR CURVES, TOWER WITH SPRINGS; DAMPING FORCE *F*, AS SHOWN; LOS ANGELES MOTION APPLIED AT 45° TO TOWER SIDES

the Los Angeles motion, N 40° E, applied diagonally with an amplitude factor of 1.5 (except as noted in the case of Curve *C*). The variation in time factor was extended considerably below unity when the maximum amplitude was made small enough by damping to insure the model's safety.

Within the limits of the damping force applied in these tests the maximum tank deflection progressively decreased as the damping was increased, the beneficial effect tapering off as the damping became greater. Since the maximum stresses set up in the tower members result from the maximum tower deflection and the frictional force acting together, it is obvious that there must be some optimum frictional force beyond which the reduction in maximum amplitude due to a further increase in friction will be more than offset by the additional stresses produced by the increase in the friction itself. That this point was not reached in the tests plotted in Fig. 24, will be seen by noting that increasing the damping force, *F*, from 6 380 lb to 7 775 lb (or 1 395 lb) reduced the maximum deflection by about 1.2 in., which by Fig. 15 amounts to a reduction of more than 3 500 lb in the pull on the tower due to deflection; that is the net gain in reducing tower stress corresponds to a horizontal load of more than 2 000 lb at the tank.

The reasoning just given leads to the important consideration that most likely there are load-deflection characteristics much more desirable than that chosen for this particular study. Thus, it seems fair to conclude from the data obtained that the same structural design, equipped with somewhat more limber springs and adequate damping, would exhibit considerably lower stresses under the same earthquake conditions. The economic considerations involved in the choice of design could not be investigated with the funds available. Here is perhaps the most inviting field for future research, and it should be explored thoroughly if the new type of construction is to be applied to any extent in practice.

One set of tests was made with the amplitude factor increased to 1.10 to observe the sensitivity of the damped structure to amplitude changes. Curve 1 in Fig. 24(*a*) is a plot of the results of these tests (frictional force = 7 775 lb). The maximum of this curve is only about 10% greater than that of the corresponding curve for amplitude factor (= 1.05), indicating that the amplitude sensitivity has been reduced to a reasonable value by the damping action (compare Figs. 23(*a*) and 23(*b*)).

Limited as the data of Fig. 24(*a*) of necessity had to be, they demonstrate definitely the possibilities of the damped spring-element type of construction as a practical engineering approach to the problem of building earthquake-proof tank towers, and perhaps some other types of engineering structures as well.

Earthquake Tests; Spring-Equipped Tower, Tested Diagonally; Motion S 50° E.—The effect of the S 50° E component of the Los Angeles motion on the spring-equipped structure was found to be much less than that of the N. 40° E component. Fig. 24(*b*) is a plot of the results of tests made under the same conditions as those in Fig. 24(*a*) except for the different earthquake component. The tank motion proved to be so much more gentle than that for the other component that, after exploring the response of the undamped structure over a wide range of time factors, it was decided to limit the damping tests to one set (7 420 lb of frictional force). The significance of these curves is obvious. (Compare the relative effects of the two components on the standard structure, Figs. 25(*a*) and 25(*b*)).

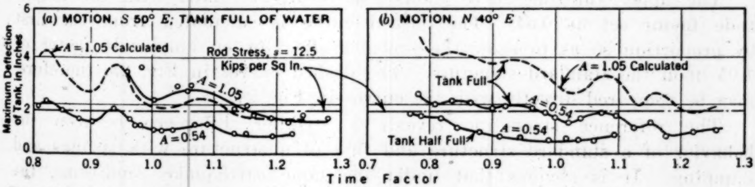

FIG. 25.—MAXIMUM DEFLECTION-TIME FACTOR CURVES, STANDARD TOWER; NOT DAMPED; LOS ANGELES MOTION APPLIED AT 45° TO TOWER SIDES; AMPLITUDE FACTOR, *A*, AS SHOWN

Standard Tower; Scale Model, 1: 25

Earthquake Tests; Standard Tower, Tested Diagonally, Motion S 50° E.—It was felt that this research would be somewhat inconclusive without some experiments giving a comparison of the dynamical behavior of a standard rod-braced tower with that of a spring-equipped tower under identical earthquake conditions. Accordingly, the model was made to represent the standard structure described in connection with Figs. 18 and 19. The S 50° E component was first applied because it was expected to be the gentler of the two. The motion was applied diagonally to the tower to allow the maximum possible safe deflection. (In the case of the standard tower, the load deflection characteristic is practically identical in all directions, so that the tests described herein apply equally well to other directions.) The amplitude factor was fixed at 1.05 as before and a set of tests was made by varying the time factor from 1.28 to 0.96, at which point it was decided that it might be dangerous to the model to proceed farther without reducing the amplitude factor (see Fig. 25(*a*), upper full-line curve).

In order to explore the response for smaller time factors, the amplitude factor was reduced to 0.54 and tests were made over a wide range of time factors. The results are plotted in the lower full-line curve in Fig. 25(*a*). Knowing the approximate linearity of the structure from statical and free vibration tests, it was assumed that the results obtained with the reduced amplitude factor could be extended fairly well to larger amplitudes by computation. The dashed curve in Fig. 25(*a*) shows how this assumption agrees with the facts. This curve was obtained by simple proportion from the lower full-line curve, each ordinate of the former being $\frac{1.50}{0.54}$ times the corresponding ordinate of the latter.

Earthquake Tests; Standard Tower, Tested Diagonally; Motion N 40° E.—It seemed wise to apply the N 40° E component with the reduced amplitude factor of 0.54 also, because of the danger of overstressing the model. Fig. 25(*b*) is a plot of the results obtained with this motion.

The upper full-line curve shows the results observed with the amplitude factor set at 0.54. The dashed curve is calculated from the first by proportion so as to show the probable effect of an amplitude factor of 1.05 upon the standard structure. The dashed curves in Fig. 25, therefore, may be compared directly with the curves in Fig. 24.

The reference stress-line reveals a striking difference between the behavior of a standard structure and that of a structure with springs and damping. It is obvious that under the same earthquake conditions, the stresses in the standard tower reach values two or three times as great as those in the spring-equipped structure having about 7 000 lb of friction damping. This situation is brought about as follows: The rigidity of a standard tower, by giving it a relatively short natural period of vibration, results in considerably smaller earthquake deflections than are produced in a moderately damped spring-equipped tower with its longer period.

On the other hand, the rigidity of a standard structure is so much greater than that of a spring-equipped tower, that the beneficial effects of the decreased deflections are more than offset by the large forces required to produce them.

In Fig. 25, one is impressed by the small decrease in maximum tank amplitude as the time factor changes from 1.00 to 1.15. Recalling the second interpretation of the time factor variation stated in discussing Fig. 22, it is clear that, as the time factor goes from 1.00 toward the right, the results indicate the probable behavior of a structure stiffened even further than the one chosen for the present research. Thus, at Time Factor 1.15 the results may be interpreted as closely representing the effect of the earthquake upon a tank structure having a natural period $\frac{1}{1.15}$ times that of the structure described in Figs. 18 and 19, and since the natural period varies inversely with the square root of the tower stiffness, the new stiffness would be $(1.15)^2 \times 23\,100 = 30\,500$ lb per in. This is 1.9 times as stiff as the standard structure studied in earlier investigations. To achieve this stiffness requires a surprising area of steel.

Thus, it must be concluded that the degree of direct strengthening that is generally applied cannot be expected to insure the ability of the structure to withstand the earthquake motions in question. (In this connection it is interesting to note that if the areas of the rods in the lower two panels, Fig. 18, are doubled, leaving the top rod areas as shown, the resulting earthquake unit stresses will be increased by more than 15 per cent. The rod areas in Fig. 18 have been chosen to give a safe deflection much greater than that achieved in the usual "quake-proof" structure, in which no particular attention is given to balanced design. Furthermore, the unit stresses used in this design were very conservative—20 kips per sq in. rod stress at 10% gravity loading. If higher unit stresses are used, the natural period will be lengthened.) Although only the motions of one particular earthquake were available for these studies, the fact that the same conclusion was reached from the results of the application of simple harmonic motion in earlier experiments indicates that it cannot be ignored in any serious discussion of quake-resistant design for elevated water-tanks.

That strengthening of towers, as now practiced by some designers, does generally serve to produce a better design with sounder structural details is not to be gainsaid; indeed, improvement in that direction is commendable and much needed. It must not be imagined, however, that the process has a rational dynamical basis.

Earthquake Tests; Standard Tower with Tank Half Full, Tested Diagonally; Motion N 40° E.—The lower full-line curve in Fig. 25(*b*) represents a series of tests made with the tank half filled with water. All other conditions were kept the same as for the tests plotted in the upper full-line curve. These results help to answer the much-asked question about the behavior of a partly filled tank.

The reduction in deflection in the half-full tests must not be attributed directly to the change in mass. It is brought about solely by the shortened period resulting from the decrease in the ratio of mass to tower stiffness. Had the stiffness of the structure at the same time been reduced by one-half, the behavior would be closely represented by the upper full-line curve in Fig. 25(*b*). The dynamical behavior is purely a matter of the natural periods of the structure and is not directly dependent upon its physical proportions.

That this is so may be verified by replotting the curve for the half-full tank, giving each point the same ordinate but multiplying its time factor by $\sqrt{2}$. This procedure is equivalent to treating the results as applying to a full tank with a natural period $\frac{1}{\sqrt{2}}$ times that of the actual full tank studied. The result will be a curve to the right of Time Factor 1.08 coinciding very well with the experimental curve for the full tank.

Extending this reasoning in the direction of added mass instead of decreased mass, it is evident that unless the tower stiffness increases in proportion to the capacity of the tank, the natural period will be greater than that of the structure taken as the prototype in the present research, which is the same as working to the left of Time Factor 1.00 in Fig. 25. Keeping in mind the fact that the natural period of the structure tested in these studies was about 1.3 sec, with a stiffness 1.44 times that of the original tank, one cannot avoid the conclusion that, for larger tanks and for tanks on higher towers, the natural periods cannot economically be made short enough to help matters very materially. (Mr. D. S. Carder has presented valuable data[10] on observed tank periods. In this connection, it is important to bear in mind that a small amount of initial tension in the rod-bracing can produce a natural period for small amplitudes which is considerably shorter than the natural period for amplitudes such as exist during an earthquake.) Besides, one must consider the fact that non-rigid foundations will invariably lengthen the period of a structure, which might annul, partly or totally, the beneficial effect of the increased tower stiffness.

It should not be inferred that the lengthened natural period brought about by the introduction of spring elements is a desideratum; in itself it is to be regarded rather as an unavoidable, and not necessarily desirable, effect resulting from the necessity of providing a greater range of deflection to the structure. The increased limberness makes it possible to introduce damping action into the system and requires a far smaller damping force than would be needed for a stiff structure.

COMPARISON OF EARTHQUAKE STRESSES IN STANDARD AND SPRING-EQUIPPED TOWERS

In comparing the earthquake stresses in the standard and spring-equipped structures one must take account of the fact that, in Nature, the two components of ground motion occur simultaneously, whereas they were treated separately in the laboratory work. This means, of course, that the orientation of the structure in question relative to the recorded components of motion is of importance. Since it is beyond the scope of this paper to compare the two structures under all possible orientations, the writer has chosen to compare them only when oriented in such a manner that the standard structure has every advantage over the spring-equipped structure.

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[10] *Bulletin*, Seismological Soc. of America, January 1936, Tables I and II; also, *Special Publication No. 201*, U. S. Coast and Geodetic Survey, pp. 75–95.

A study of Figs. 24 and 25 and of the laboratory records shows that, if the recorded ground-motion components are simultaneously applied parallel to the tower sides for both structures, the standard tower will be stressed about 25% less than at its worst possible orientation, whereas the spring-equipped tower is oriented so as to receive within 5% of its worst possible stresses. The comparison will be made on the basis of an amplitude factor of 1.05.

The stress calculations involve only straightforward statical analysis, once the deflections are known from the experimental data (the gravity effect must not be ignored in computing the stresses). The deflections of the standard tower will be substantially the same regardless of the direction of the applied earthquake motion, because the tower stiffness is substantially the same in all directions. The maximum deflections of the damped spring-equipped tower, however, will be somewhat greater parallel to the sides than in the diagonal direction for the same earthquake motion. In the following data, the deflections parallel to the sides are assumed to be 35% greater than the diagonal deflections for the spring tower, which the writer considers a generous allowance. Data are also given for the case in which no such assumption need be made. In the absence of complete data to determine the value of 35%, the writer has set what seems to be an upper limit, based on his experience with the system. He prefers the more direct interpretation of stresses given in the last paragraph of this section, since the element of estimation is there eliminated entirely.

For the standard tower under the conditions specified, the rod stress at a time factor of 1 would be more than 28 kips per sq in.; and for the worst case ($P = 0.82$) the rod stress would be about 40 kips per sq in., which is beyond the normal yield point. It is not necessary to postulate any great increase in the violence of the ground motions closer to the epicenter in order to explain the failures which occurred, knowing that few of the existing structures could safely withstand as much deflection as the structures studied herein, and that most of the larger ones corresponded to time factors of considerably less than unity.

As a matter of fact, some rods were stretched permanently in a tank tower situated 3.7 miles from the Subway Terminal Building, in Los Angeles, where the motions used in this research were recorded. This structure is a 100 000-gal tank on a 100-ft tower, and, at the time of the Long Beach earthquake, was braced for wind velocity of 100 miles per hr. At that time, it corresponded to a time factor of about 0.9. The calculated maximum rod stress for this structure, based on the experimental data in Fig. 25, proves to be 56 kips per sq in. for an amplitude factor of

1.05 and a time factor of 1.00, or 53 kips per sq in. for the earthquake as it was recorded. (These high values, of course, are fictitious, since the elastic limit of the material would be passed at lower stresses.) Failure might easily occur, therefore. Several other tank structures within a few miles of the Subway Terminal Building also showed evidence of very high stresses. Since, in these cases, there were indications of flaws in material, workmanship, or details, they are not cited herein as numerical examples.

The stresses set up in the damped spring-equipped structure, under identical earthquake conditions (allowing 35% extra deflection), prove to be quite low in comparison. With a damping force of about $\frac{7\,500 \text{ lb}}{\sqrt{2}}$ (parallel to sides) the rod stress at a time factor of 1 would be less than 10 kips per sq in. (compared with more than 28 kips per sq in. in the standard tower). In the worst case ($P = 0.88$) the rod stress would be less than 20 kips per sq in. (compared with a stress beyond the yield point in the standard tower). Thus, even with a generous margin in favor of the standard structure, the damped spring-equipped tower shows a minimum safety factor of at least 2 over the standard tower. A 25% increase in damping would raise this safety factor to above 2.25; or, if a more direct interpretation is desired (in which no assumption need be made as to deflection increase parallel to sides), the following will suffice: Given a tower whose load-deflection characteristic parallel to the sides is the same as that of the tested structure in a diagonal direction, and given a damping force of 7 500 lb, the rod stress in the worst case ($P = 0.88$) would be less than 19 kips per sq in. (minimum safety factor, greater than 2). It is to be remembered that the assumed orientation is such as to give the advantage to the standard structure by perhaps 20%, which was ignored in calculating the safety factors.

General Conclusions Drawn from Research

It is not intended to list herein a set of conclusions that may be read casually and without a careful study of the supporting text. Such a procedure would be worse than useless; it might even be dangerous. Certain general conclusions, however, can be drawn from the results of this research, provided the limitations and cautions already set forth have been thoroughly digested. These conclusions are:

(1) The standard type of elevated tank tower now in common use is poorly adapted for withstanding earthquakes of destructive intensity, both on acount of the absence of damping action and of its inability to withstand large deflections.

(2) A moderate degree of strengthening does not, and can not, improve the earthquake resistance of a standard tower very materially; that is, if the recorded motions of the Long Beach earthquake of 1933 can be taken as indicative of what may be expected in future earthquakes. The data from these motions indicate that a properly balanced strengthening is somewhat

better than a straight wind design, but in the absence of a more general knowledge, of possible earthquake motions, this conclusion cannot, of course, be accepted without reservations. (By "balanced strengthening" is meant strengthening of the tower bracing members in proportion to the loads they carry, as distinguished from the common practice of using the same rod size in two or more panels. The safe deflection of a standard 100-ft tower can sometimes be increased as much as 50% by a balanced design of the bracing.)

(3) It would be impractical to provide sufficient safe deflection to insure the safety of the structure under all conditions by using springs without damping. Given a safe deflection of about 15 in., the undamped spring tower would probably fare better, on the average, than the standard structure but its safety factor would not be dependable.

(4) The presence of sufficient damping in a properly designed spring-equipped tower gives it a safety factor in stress of at least 2 over the standard structure which has been designed for a loading equal to $0.1g$. Damping also provides a safety factor in the duration of violent motion that the structure can withstand—a quantity difficult to express numerically but of great importance. The damping required in the particular design investigated was found to be small enough to be easily achieved in practical design (see Fig. 17).

(5) The results obtained are restricted in scope, and they cannot be extended safely to apply to structures of much greater or much smaller proportions than the tower actually studied. One must not resort to "engineering judgment" in dynamical problems unless one has had considerable experience with them. Practical limits for applying the present results might be 50 000 to 100 000-gal capacities and 150 to 100-ft tower heights, respectively.

(6) The introduction of the proper spring elements does not endanger the stability of the tower structure. Considered as a three-dimensional framework, it is necessary to investigate the effect of possible dissymmetry due to accidental differences in action between the elements on opposite sides of the tower. Although the model did not exhibit serious torsional effects from lack of symmetry, a good engineering design should specify horizontal panels of light rod-bracing at the strut levels, in order to eliminate any undesirable twisting effects which might be produced if some of the damping devices failed to act properly. (The conventional spider can be adapted for this purpose.)

(7) Baffle-plates inside an elevated tank would be detrimental within the limits of the tank size specified in Conclusion (5).

(8) For designing structures within the foregoing prescribed limits of size, the following rules should result in a safe design:

(*a*) Design the structural frame for a static horizontal load of one-tenth the weight of the tank and water applied at the center of gravity of the tank. The columns and struts should be designed so that the rod-bracing would fail first if the structure were tested to destruction by static horizontal loading.

(*b*) Provide a set of springs such that, with the initial compression set for a horizontal force at the tank equal to one-fiftieth of the weight of the tank and water, the tower can deflect at least 15 in. parallel to its sides before the springs close.

(*c*) Design the stiffness of the springs such that at the point of closure the horizontal load is approximately equal to the design load used in Conclusion (8*a*).

(*d*) Design the individual spring elements so that the total tower deflection is about equally distributed between the panels (to avoid high bending stresses).

(*e*) See that the bending stresses in the tower members are not excessive. Use pin-connected struts if necessary, and reduce eccentricity to an absolute minimum. The riser pipe should be equipped with flexible joints if necessary and must be able to follow the tower motions.

(*f*) The damping forces should be such that the energy dissipated by damping during a motion of the tank (parallel to the tower sides) from its equilibrium position out to the maximum safe deflection and back again, is approximately equal to 50% of the energy stored elastically when the structure is at the maximum safe deflection (parallel to the tower sides).

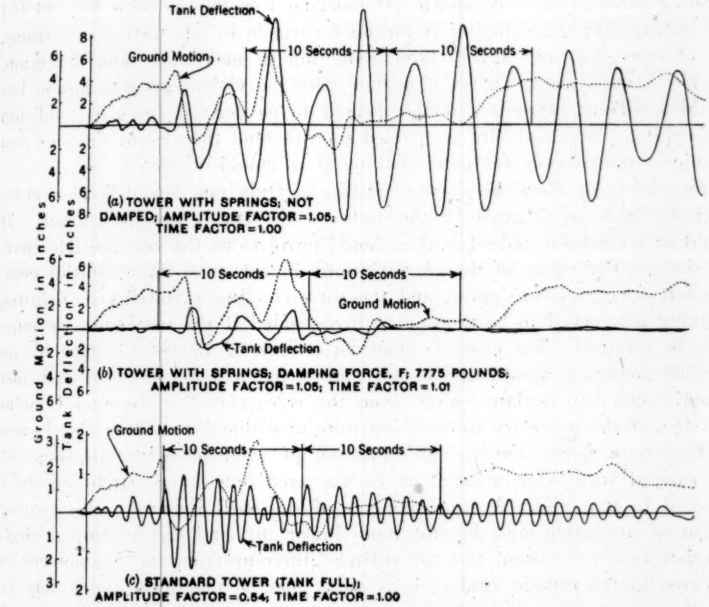
(*g*) The total damping force should be distributed among the spring elements in proportion to the stresses produced in the various rods by a static horizontal load at the center of gravity of the tank. The damping force need not be the same for an outward deflection as for the return to equilibrium, but that for the return stroke should not exceed about 85% of the wind-load initial compression setting; otherwise, all the rods may sag at times.

(*h*) The damping force should be fairly independent of velocity. It would be a good rule to make the damping force corresponding to a tank velocity of 30 in. per sec. (relative to the base) not more than 25% greater than that calculated by Conclusion (8*f*). The damping may be produced by simple sliding friction if desired, in which case the kinetic friction should be made equal to that computed by Conclusion (8*f*).

REMARKS

Relation of Tank Deflection to Ground Motion.—The graphs in Fig. 26 are reduced tracings of records made by applying the Los Angeles motion, N 40° E, to the undamped spring tower, the damped spring tower, and the standard tower, respectively. The time factor is nearly identical for all cases and the scales given are self-explanatory. A careful study of the nature of the tank motion relative to the ground motion in the three diagrams will be found helpful to those who are not well versed in dynamics.

In the first place, one observes that there is no simple relation between tank response and earthquake motion. The notion that statical force equal to the mass of the tank and water times the maximum ground acceleration can be used safely for design against earthquake is seen to be entirely without basis.



(a) TOWER WITH SPRINGS; NOT DAMPED; AMPLITUDE FACTOR = 1.05; TIME FACTOR = 1.00

(b) TOWER WITH SPRINGS; DAMPING FORCE, F; 7775 POUNDS; AMPLITUDE FACTOR = 1.05; TIME FACTOR = 1.01

(c) STANDARD TOWER (TANK FULL); AMPLITUDE FACTOR = 0.54; TIME FACTOR = 1.00

FIG. 26—TYPICAL RECORDS, LOS ANGELES MOTION, N 40° E. APPLIED AT 45° TO TOWER SIDES

Next to be observed, is the serious consequence of the lack of damping shown in Fig. 26 (*a*) and Fig. 26 (*c*). In both cases, the structure continues to vibrate vigorously after the main ground motions have subsided. The cases chosen for discussion are by no means the worst observed in this respect; on the contrary, they are quite typical. What might have happened had there been several more large ground swings immediately following those in the early part of the Long Beach earthquake can only be conjectured, but if such swings happened to be nearly "in step" with the already large tank vibrations shown in Fig. 26 (*a*) and Fig. 26 (*c*), collapse would have been almost certain.

It is to be regretted that the accelerograms taken in Long Beach proper could not be completely analyzed because of the overlapping of the components on the record. Consequently, there is no rational method by which to estimate the relative intensities at Long Beach and Los Angeles except by comparison of destructive effects, and this is admittedly not very satisfactory. However, it is fairly certain that the motions used in the present research cannot be considered as representing the worse conditions to be designed against in an active earthquake zone, such as the West Coast. Hence, the desirability of adopting a generous and real safety factor is apparent.

In Fig. 26 (*b*) the beneficial effects caused by the addition of a moderate damping force are clearly shown. Comparing Fig. 26 (*a*) with Fig. 26 (*b*) and noting that the damping is the only variable in the testing conditions, one observes not only a very large reduction in maximum tank deflection, but also a quick decay in the vibration after the violent ground motion has subsided. Thus, there is a double benefit to be derived from the addition of damping. The structure is enabled to withstand the violent motions and is quickly made ready for more shaking if it should come.

Possibility of More Economical Designs.—Only one load-deflection characteristic was investigated in the tests of the spring-equipped tower. It would be remarkable indeed if this should prove to be the best possible characteristic. The effect of design variations in spring stiffness, initial compression, spring closure point, and the corresponding variations in required damping force need to be thoroughly investigated if the maximum economy is to be attained. For example, consider a tank of 150 000-gal capacity on a 125-ft tower; it would undoubtedly prove far more economical and just as satisfactory to deviate widely from the rules stated in General Conclusion (8), if the necessary information were available for guiding the design.

Extremely Large Tanks.—The question of what procedure to adopt in the case of very large tanks is to be answered only by research especially directed to that end. As the diameter of the shell increases, the surging action of the water must become more important, and the dynamical characteristics of the system may be entirely different from the characteristics observed in the present study. It is quite possible that baffle-plates may be found necessary to prevent dangerous stresses in the shell in some instances.

Size of Riser Pipe.—It has been assumed in the foregoing discussion that the riser pipe is small (say, 6 in. to 12 in. in diameter), so that if made of steel and provided with flexible joints, if needed, it can be forced to conform to the configuration of the swaying tower during an earthquake. Large risers, which in effect tend to act as stand-pipes of considerable stiffness, would be rather difficult to provide for in the spring type of construction. The very considerable vertically distributed mass of water in large risers presents a problem to which the results thus far obtained do not apply.

The Element of Chance.—The extent to which pure chance enters into the destructive effect of earthquakes is perhaps not generally appreciated: The orientation of the structure relative to the direction of the most violent motion; the location of the structure in the shaken region; the relation of the natural vibration period to the frequency spectrum of the earthquake; hidden structural defects; characteristics of tower parts at stresses beyond the elastic limit—all these are factors vitally affecting the behavior of the structure and all are matters of chance over which the engineer has no control whatever.

A careful consideration of the foregoing chance factors leads to an understanding of the difficulties confronting those investigators who have undertaken to formulate design rules based upon field studies of the effects of destructive earthquakes. For instance, there is the fact that a tank tower designed for a static force of $0.1g$ (the only one so designed to the writer's knowledge) survived the Long Beach quake safely. This fact, apparently, has led a number of engineers to accept, tentatively, the $0.1g$ rule as satisfactory for elevated tanks.

Now, had there been, say, forty or fifty tanks of random sizes designed by this rule and a like number designed for wind only, and had they all been fairly well scattered over the destructive region, then there would be a real basis for drawing conclusions as to the superiority of one type to the other from field studies. As matters stand, however, about the most that can be concluded definitely from examination of the effects of the Long Beach quake is that certain tanks did fail totally or partly; and, by examination of the damaged structures, weak points in the details can be brought to light and corrected in the future. To credit the methods of design for saving those structures which did not fail, would lead to some awkward contradictions.

Tanks for Municipal Supply.—It is unfortunate that few municipal storage tanks are small enough to come within the scope of the research described in this paper. As far as importance to the community as a whole is concerned, these structures should receive by far the most serious consideration. It is to be hoped that it will be possible some day to extend present knowledge to this vitally important field.

ACKNOWLEDGMENTS

The writer wishes to acknowledge gratefully the helpful co-operation of all who have contributed to the success of this research project.

The Associated Factory Mutual Fire Insurance Companies and the Freeman Engineering Corporation initiated the research and helped to make possible all the investigations described in Part I, also contributing a very considerable sum toward the construction of the new shaking-table. The engineers connected with these organizations recognized the serious lack of dependable design data and questioned the propriety of applying rule-of-thumb design methods unless they could be shown to produce safe structures. Their helpful and sympathetic attitude has been a source of inspiration as well as a means of keeping the work on a firm practical basis.

Thanks are due to the Chicago Bridge and Iron Works and to the Pittsburgh Des Moines Steel Company, which provided a large part of the funds for the experimental work described in Part II. These funds were given for the purpose of increasing current knowledge of earthquake effects upon tank structures, with no thought of financial gain except to earthquake-ridden communities.

The U. S. Coast and Geodetic Survey has given invaluable aid in providing data on ground motions and on the behavior of actual structures

studied in California, including the data on the Long Beach earthquake of March 10, 1933. Its interest in the work is most gratifying.

The studies, which extended over a period of two years, were made by the writer in the Laboratory of Engineering Seismology of the Massachusetts Institute of Technology. By providing funds for the design and construction of the new shaking-table and for the services of the writer for the duration of the entire research, the Institute has carried by far the greatest part of the financial burden. The Division of Industrial Cooperation has administered the project without making the customary charges for overhead expense to the Institute. In addition, the writer has profited by the valuable advice and assistance of Dr. Vannevar Bush, C. B. Breed, M. Am. Soc. C. E., Professor A. V. deForest, and many other members of the Institute staff.

Especial thanks are due to the four engineers without whose loyal assistance and hard work this research project could not have been carried out with the same degree of economy and success—Messrs. Chester H. Hosmer, Albert L. Grass, George Parmakian, and George W. Hines, all graduates of the Institute.

The data in Table 1 were computed by Mr. H. A. Sweet, Engineer with the Associated Factory Mutual Fire Insurance Companies, of Boston, Mass. Figs. 8 and 9 are reproduced from the drawings used in securing the patents, which are to be administered on a non-profit basis by that organization. These patents cover the general concept of applying spring elements to engineering structures for the purpose of increasing their resistance against earthquakes. The purpose of these patents is to make it possible to prevent the improper use of the new construction; otherwise, no restrictions will be imposed.

DISCUSSION

N. H. Heck,[11] M. Am. Soc. C. E. (by letter).—During recent years attention has been given to earthquake-resistance design of structures. In several places these investigations have been supplemented and based largely upon the results of tests of models of the structure on a shaking platform. At the Massachusetts Institute of Technology, at Cambridge, Mass., a light model can be tested on a platform which may be put into any desired kind of motion, including the reproduction of actual earth motion.

As indicated in Professor Ruge's paper the U. S. Coast and Geodetic Survey has not only made it possible to reproduce the earthquake motion through double integration of the acceleration records of the Long Beach (Calif.) earthquake, obtained at Los Angeles, but it has determined the periods of many tanks of different types and designs, thereby adding greatly to the validity of his conclusions. It will soon be possible to know the degree of accuracy of the deduced earthquake motions through shaking platform tests of the accelerometers made in May, 1937. Although the records have not yet (1937) been studied the results appear promising.

In outlining the special problem Professor Ruge has performed a service which extends to a wider field in showing that the earthquake manifests itself as inexorable motion rather than as direct force, and that the forces exerted depend primarily on the characteristics of the structures undergoing the motion. The complexity of his apparently simple problem becomes obvious when pointed out and indicates the important place taken by tests of a suitably designed model on a shaking platform.

The validity of the term, "artificial earthquake motion" (see heading, "Artificial Earthquake Motion"), may be questioned. Apparently, Professor Ruge has in mind that no earthquake record is yet available that contains the wide range of so-called destructive periods, so he uses the best substitute therefor. When supplemented by the use of earthquake motions actually observed, this procedure is the best available at present.

It should be understood that the Coast and Geodetic Survey has not yet found it possible to integrate all the strong-motion accelerograms that it has, and until this has been done the record cannot be applied to controlling the motion of a shaking platform. Lack of personnel has prevented this work and a special effort is being made to find whether the records can be integrated by the differential analyzer at the Massachusetts Institute of Technology. Furthermore, Frank Neumann, Seismologist, U. S. Coast and Geodetic Survey, is developing a torsion pendulum analyzer which deals with the specific problem. Attention may be called to the fact that although all the strong-motion accelerometers in California are not able to record the greatest possible range of earth motion, twelve instruments are equipped with a double-magnification system so that if the instrument survives a record will be obtained even if the acceleration is 0.5 *g*, or more.

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[11] Chf., Div. of Terrestrial Magnetism and Seismology, U. S. Coast and Geodetic Survey, Washington, D. C.

In determining the vibration periods and in recording earthquakes in actual structures, the Coast and Geodetic Survey is concerned with design, and is anxious that every possible application be made of the information that it has obtained.[12]

Nevertheless, it must be remembered that it is not engaged in structural design. In some respects this may be inconvenient for the engineer, but on the other hand it has the advantage that results are obtained without any desire to support any particular theories, and this may prove of great advantage in the future. Accordingly, the writer will not discuss Professor Ruge's proposed design further than to state that the Long Beach earthquake proved the inadequacy of present design to resist severe earthquakes and that these structures will have to be designed in some other way in order to survive.

No exception is taken to Professor Ruge's remarks as to the element of chance (see heading, "Element of Chance"), except to note that he fails to emphasize one element that may be the key to the great variety of observed effects—the varying conditions of the geological substratum. This can cause changes in the periods and amplitude of the motion and the amount of energy. However, this merely emphasizes the expressed view that valid conclusions can be based only on a large number of cases.

It should be emphasized that the records of the Long Beach earthquake, at Long Beach, and at Vernon, Calif., and the records of the Lower California earthquake of December 30, 1934, at El Centro, contain valuable information which could be revealed by integration, since each record represents motion of a unique geological formation.

MASON A. STONE,[13] ESQ. (by letter).—Being of interest not only to the limited number of designers who specialize in the elevated water tank, but to those who are called upon to design steel structures carrying heavy loads at the top, Mr. Ruge's paper, of course, has a broader application than the title indicates. Among these steel structures, for instance, are viaducts with a heavy deck traffic, towers in chemical plants carrying tanks or machinery, elevated storage bins, hoisting towers with heavy sheaves or hoisting engines at their tops, etc.

The water tower, or rather the tower carrying the elevated tank, as the author shows, is further complicated by the action of the water surges. Therefore, it is necessary to consider as superimposed on the natural vibration period of the tower the period and amplitude of the motion of the ground due to the earthquake and the period and momentum of the water. A rational method of designing the structure would require the evaluation of the stresses set up by these three motions.

Whether or not such a mathematical analysis is possible, it is highly desirable that the conclusion be checked by experiments with actual models, and it is fortunate, therefore, that the apparatus for conducting such tests is available and that experiments with such models are actually being conducted. This paper must be considered as a progress report of such experiments. It is valuable: (1) Because it demonstrates the errors inherent in designing from the simple theory based on a constant acceleration; (2) because it indicates the difficulties and expense of attempting to meet the problem by increasing the stiffness of the tower; (3) because it suggests a mechanical solution whereby the structure, within certain limits of height of tower and capacity of tank, may be made flexible enough to absorb the stresses; and (4) because it reveals the tendency of the tower to continue vibrating in resonance with the earthquake shock, or after the earthquake shock has ceased, and shows how such vibrations may be damped.

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[12] "Earthquake Investigations in California, 1934–35," *Special Publication No. 201*, U. S. Coast and Geodetic Survey.

[13] Structural and Mech. Engr., New York, N. Y.

The data given will enable existing towers to be examined for earthquake-resistant properties and, within the limits given, to be re-designed by the addition of the springs and dampening device so that they may be safe. However, the paper affords no general solution for the problem of designing the elevated tank tower. The stresses in the tower are the result of the amplitude of the vibration at the top, caused by the period and amplitude of the earthquake motion, and modified by the momentum of the water in the tank.

Although the stiffness of the tower may be computed, its period of vibration depends on the initial stresses set up in the members during the erection and the stresses due to the weight of water in the tank. Fortunately, reports are available containing data on the stiffness and period of vibration of a number of tank structures. In this connection, however, it may be remarked that the type of tower selected for the experiment and designated as the standard design is a very poor one from the standpoint of earthquake resistance. Apparently, the batter of the legs was considered solely from the standpoint of resisting the overturning moment due to wind. If one is to design for earthquake stresses a much broader base for the tower would seem to be indicated. On the other hand, for a structure of uniform strength to resist a horizontal force at the top a parabolic curve concave on the outside is indicated. This form is approximated in the design of the towers for the support of high-tension transmission lines. The tower with a wider base also recommends itself in connection with the increasing foundation loads due to earthquakes.

There is a considerable record of the magnitude and velocity of earthquake motion, but it is not readily accessible to the ordinary structural designer as it is contained in the reports of the Imperial Japanese Earthquake Commission and the Seismological Society of America which are not to be found in the average library. It is regrettable that Mr. Ruge did not include some statement of the period and amplitude which might be expected in earthquake motions. His statement that there was only a record of one earthquake available for study (that is, for cutting the necessary optical cam) might have been amplified by the explanation that the ordinary seismograph does not give the necessary information; an accelogram is required.

In the tank under consideration, the action of the water is shown to have a dampening effect upon the vibration of the structure, but this might not be true for all proportions of tanks and towers. It would seem a profitable line of investigation to determine the period and forces set up in water tanks of various proportions and capacity, at least for simple harmonic motions. At present, the proportions of the tank are usually determined by the most economical use of the material.

In this connection it may be suggested that waves set up by using only one component of the earthquake motion at a time may not be representative of the actual action of the water during an earthquake. Undoubtedly, the actual motion of the tank is roughly elliptical. Instead of being reciprocal, therefore, the wave would have a circular motion around the tank; it might have a different period and might develop different forces from those determined in experiments using only a reciprocal motion of the shaking-table.

When more is known about the action of the water, it may be possible to determine optimum proportions of tank and tower for the minimum vibration. The possible effects of the riser pipe also appear to merit consideration. Of course, if the pipe is secured at the bottom of the tank and to a foundation in the ground, and if it is also tied to the tower by horizontal braces at the panel points, a vertical column of water will somewhat reduce the center of gravity of the system as a whole. Furthermore, the stiffness of the pipe will add to the stiffness of the structure, although not very materially as it is situated at its neutral axis. In this case the stresses in the connection at the bottom of the tank and the foot elbow on the foundation should be investigated; but the pipe itself might be expected to resist any deflection that the legs of the tower are called upon to take.

It has occurred to the writer, however, that if the pipe is provided with a universal joint at the top and swiveled at the bottom connection so that it may swing freely as a pendulum, its natural period will probably be different from the structure as a whole and from that of the earthquake, and, therefore, it may have a beneficial dampening effect. This idea occurred to the Japanese when building pagodas. They introduced a heavy timber suspended from the top and appear to have had beneficial results with this type of construction.

The idea of designing a structure 100 ft high to withstand a deflection at the top of 15 in. is somewhat startling to a structural engineer. The secondary stresses at the connections between the horizontal struts and legs would have to be given careful consideration unless the author's suggestion of eliminating them by pin connecting the struts was adopted. Furthermore, the stresses in the riveted connection between the tank and the legs may possibly be relieved by the deflection of the tank sheets, but this action might result in the failure of rivets, or sheets.

The author's conclusions for the design of tanks and structures within the limits indicated appear to be valid. Of course, the damping device would have to be one that would not be affected by frost or that would not deteriorate with age and neglect.

H. C. BOARDMAN,[14] M. AM. SOC. C. E. (by letter).—By his painstaking and well directed efforts, Mr. Ruge has developed useful information in a field where it is much needed. The following excerpts from his paper are especially significant:

(*a*) "The type of tower now in common use was not developed for earthquake resistance, * * * ; there is no reason to accept it as the ideal structure for withstanding earthquake motions, which actual experience and repeated laboratory tests have proved it certainly is not." (See heading, "Introduction: Nature of the Problem.")

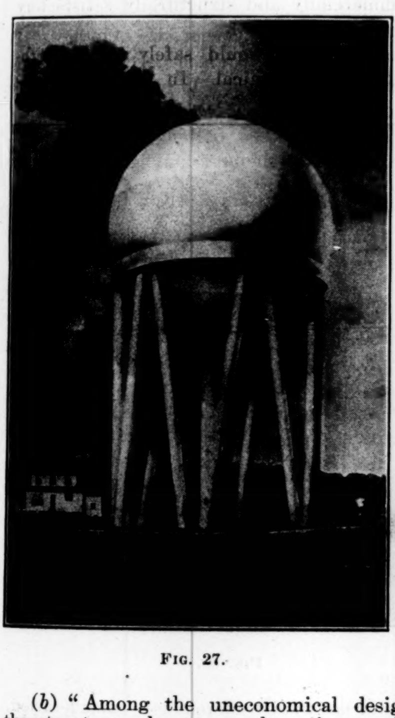

FIG. 27.

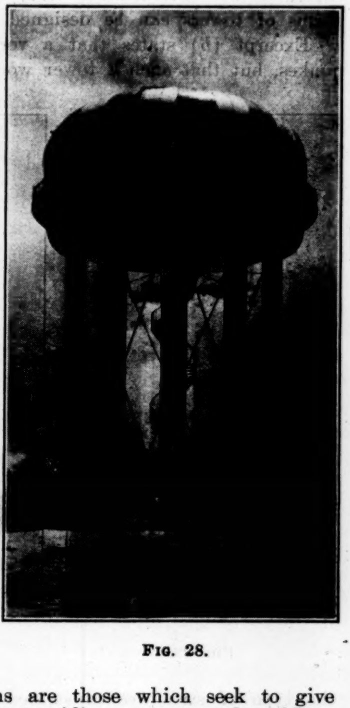

FIG. 28.

(*b*) "Among the uneconomical designs are those which seek to give the structure adequate quake-resistance by providing an extremely stiff or rigid tower. * * * If the natural period cannot be reduced to about 0.5 sec, it is doubtful whether the tank is very much more likely to survive a violent earthquake than a standard design built for wind resistance only." (See following Table 1.)

(*c*) "The natural conclusion from a prolonged study was that, barring fantastic and impractical designs, the best way to obtain adequate quake-resistance is to provide the tower with a greater range of elasticity; that is, if the tower can be built so that the safe deflections are large enough, there will be no danger of failure." (See following Table 1.)

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[14] Research Engr., Chicago Bridge & Iron Company, Chicago, Ill.

(*d*) "There are two general methods of increasing the allowable or elastic deflection range of a structure such as a water-tank tower: (1) By making the tower of some type of open framework in which the forces resisting deflection are produced by bending stresses set up in some or all of the members of the framework; and (2), by artificially increasing the allowable direct-stress deformations of some or all of the members of a statically determinate tower." (See following Table 1.)

Taken at its face value, Excerpt (*a*) suggests that towers of the conventional type be not used in earthquake zones. Apparently, the author did not intend to convey this meaning. Nevertheless, it is well worth careful consideration, since other commercially and structurally satisfactory forms of towers can be designed.

Excerpt (*b*) states that a very stiff tower would safely resist earthquakes, but that such a tower would be uneconomical. In this connection, the type of structure illustrated by Fig. 27 is offered for consideration. It is stiff, but, nevertheless, offers promise of economy.

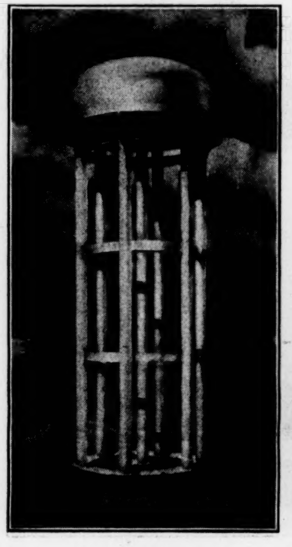

FIG. 29.

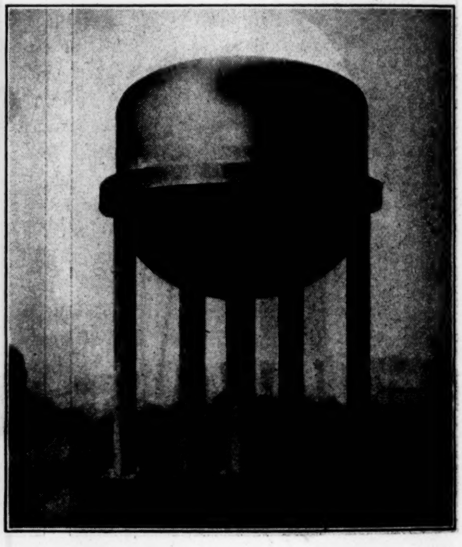

FIG. 30.

Excerpts (*c*) and (*d*) present the author's conclusion that the most practical method of providing earthquake protection is to make the tower flexible, either by use of an open framework with the members subjected to bending stresses, or by increasing the direct stress deformations of some or all of the members by the use of springs or similar devices. It is pertinent to re-emphasize what the author has stated, namely, that many tanks and towers of the conventional design have withstood earthquakes with little damage, except the breaking or stretching of some of the rods, which were easily replaced. Why, then, would it not be feasible to use longer panels and incline the rods more steeply, so that greater horizontal translations of the tower would be required to stretch the rods to the yield point and through the plastic range? Fig. 28 is offered as an example of such a design. Over a period of years, springs would almost surely be subject to serious maintenance troubles, and, therefore, they should not be used if designs not requiring them can be developed. Figs. 29 and 30 illustrate designs of the open type, without diagonals or struts.

FRANKLIN P. ULRICH,[15] M. AM. SOC. C. E., AND DEAN S. CARDER,[16] ESQ. (by letter).—In mentioning structural characteristics and the superficial nature of past earthquake design, Professor Ruge has touched upon a subject which is of considerable interest and for which a large amount of observational data has recently been obtained.

The strong-motion records obtained by the U. S. Coast and Geodetic Survey show the complexity of earthquake motions. The Los Angeles record of the Long Beach shock (used by Professor Ruge) shows periods ranging from 0.12 to 4.8 sec with only a few periods greater than 2.5 sec.[17] Only a few periods were of the same order as the fundamental period of this particular tank which was later measured and found to be about 1.5 sec.

The Coast and Geodetic Survey has made vibration observations on nineteen selected steel water tanks having capacities of 50 000, 75 000, and 100 000 gal, and an elevation of 100 ft above the ground. Vibration observations on eighteen other tanks, including the prototype of Professor Ruge's models, have also been made.[18] For the most part, however, this discussion will be confined to the nineteen tanks selected for comparison at the time the tests were made. All the tanks were filled or nearly filled with water.

Three orders of periods of vibration have been recognized:

(1) The most important type of motion is probably translational vibration which has periods ranging from 0.95 to 1.75 sec depending on the effective water load, the soil conditions, and the elastic properties of the tower. Other things being equal, towers reinforced from 0.08 to 0.15 g for side loads have periods 10% to 30% less than standard towers designed to withstand a 100-mile wind. Towers on piling in yielding soil have periods as high as 20% greater than similar towers on firm ground.

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[15] Chf., Seismological Field Survey, U. S. Coast and Geodetic Survey, San Francisco, Calif.

[16] Asst. Observer, U. S. Coast and Geodetic Survey, San Francisco, Calif.

[17] "Destructive Earthquake Motions Measured for First Time," by N. H. Heck and Frank Neuman, *Engineering News-Record*, June 22, 1933.

[18] "Earthquake Investigations in California, 1934–1935," by U. S. Coast and Geodetic Survey, *Special Publication No. 201*, p. 75; and "Observed Vibrations of Steel Water Towers," by D. S. Carder, *Bulletin*, Seismological Soc. of America, Vol. 26, No. 1, January, 1936, p. 69.

The range of periods of translatory vibration lies within the frequency range of some of the possibly destructive earth waves as observed on accelerograms of the Long Beach earthquake. It is to be understood that observed periods of these tanks apply to low-amplitude vibrations. On the tanks where some of the tie-rods went out of action or failed during the earthquake, the periods undoubtedly became variable and much longer, according to Harry A. Williams,[19] Assoc. M. Am. Soc. C. E., perhaps in sufficient amounts to throw the structure out of resonance with the earth. Broken or stretched tie-rods would further serve temporarily to damp the structure. On the other hand, if the rigidity of a tank tower is to be increased, the purpose should be to strengthen the structure rather than to decrease its period, because destructive earth waves have been known to have periods less than 1 or 1.5 sec. For instance, the highest acceleration registered during the October 31, 1935, earthquake at Helena, Mont., was associated with periods of about 0.15 sec, even though a wave with a 1-sec period was also in evidence. At Long Beach, waves having a 0.3-sec period contained the highest acceleration.[17]

(2) Rotary or torsional vibration has been recorded on all tanks of this group. Observed periods of this type of motion usually range from 0.30 to 0.45 sec on standard structures, and from 0.20 to 0.30 sec on towers with an 0.08 g to 0.15 g side-load design. The magnitude of this period is dependent on the elastic properties and the steel load of the tower and perhaps also on the type of soil. It is independent of the water load because the water within the tank does not partake in this type of vibration.

Torsional vibrations of water tanks should not be overlooked in experimental procedure because it is quite possible that large torsional motions may be set up in resonance with destructive earth motions having the same frequency range.

(3) A period of the order 2.7 sec, assigned by Professor Ruge to swinging motion of water within the tank, has been recognized on several tank-tower vibrograms. Observed periods are 2.63 sec on one 50 000-gal tank, 3.0 sec on two 75 000-gal tanks, and 2.85 to 3.0 sec on five 100 000-gal tanks. One of the 100 000-gal tanks which had bracing to withstand an 0.08 g side load, had a period of this order, the same as similar standard tanks, supporting the belief that this period is due to water motion in the tank. However, the observed periods on tanks having different capacities cannot be explained satisfactorily by simple gravitative oscillations of water within the container. Furthermore, a supposition that water in the tank re-acts upon the tower in such a way as to produce a period approximating the translational period of the tower has considerable experimental support.

Strengthening a tank, primarily, to reduce its period of vibration is not the entire solution to making the tank more resistant to earthquakes. It is quite possible to have the reduced periods of the tank equally as destructive, or more so, than the original periods, especially if the reduced period happens to be close to one of the destructive ground periods.

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[19] "Dynamic Distortions in Structures Subjected to Sudden Earth Shocks," by Harry A. Williams, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 838.

It might be possible to avoid resonance between ground and tank, if the tank had a definite fixed period and if the destructive ground periods for that particular location are known. To date (1937) information on destructive ground periods is too meager to form any definite theory. Observations made by the Coast and Geodetic Survey show that tank periods fluctuate with different types of ground and with changing tank conditions, such as depth of water in the tank or looseness of tie-rods. Tests on one tank showed that the period varied from about 0.36 sec for an empty tank to about 1.0 sec for a filled tank. In another case the period was 1.44 sec with tight tie-rods and 1.66 sec, or more, with loose tie-rods.

The question of merits or objections of spring elements, which Professor Ruge proposes to introduce in the tie-rods, is a structural problem and is beyond the scope of this discussion, but there is little room for doubt that a provision for some kind of damping will reduce, materially, the earthquake hazard of any structure. That standard elevated water tanks are poorly damped has been demonstrated on several of these structures on which a side force of 2 000 to 4 000 lb was suddenly released and the resulting vibrations recorded at the platform of the tank. The ratio of two successive amplitudes (damping ratio) of towers with taut tie-rods is never more than 1.02. One tank tested in this manner had loose tie-rods and the damping ratio was higher—about 1.1. The translatory period of the latter tank at first was 1.74 sec which decreased to 1.65 sec as the amplitudes became smaller. A minute after the release of the side force, vibrations due to elastic properties of the tower became subordinate, and a motion with a period of 2.85 sec was dominant for several minutes until the end of the record. If the idea is correct that this longer period is contributed by swinging action of water in the tank, it appears that this type of motion continued long after structural vibrations had diminished to normal amplitudes. Later, the tie-rods of the tower were tightened and the tests were repeated. The period was then invariable at 1.45 sec and structural vibrations disappeared much more slowly, and the longer 2.85-sec period was not recognized on the record.

The statements in the preceding paragraph should not be interpreted as an argument in favor of loose tie-rods in existing structures, but it does seem that some method of breaking up the high resilience of these structures, without destroying their resistance to dynamic side loads, is desirable. It is possible that Professor Ruge's idea of spring elements, with damping, is a solution.

It is unfortunate that more earthquake data are not available and that Professor Ruge was unable to continue his tests under a wider range of conditions. However, additional tests would probably show results comparable to this series. The errors of small models cannot be avoided and no doubt they are within the limits of error of present-day structural science. It is hoped that several full-sized tanks, embodying the spring design, can be constructed so that comparisons can be made with the present design, and their relative capabilities to withstand earthquakes can be determined.

ARTHUR C. RUGE,[20] ASSOC. M. AM. SOC. C. E. (by letter).—Attention has been called by Captain Heck to the need for further analysis of strong earthquake motions, and he has stated the attitude of the U. S. Coast and Geodetic Survey toward the problem of earthquake-resistant design. Those who are studying the problem recognize the wisdom of the Survey in confining its efforts to collecting data needed for design rather than engaging in the study of design itself, since it thus performs a needful service that could not be done adequately by any existing private institution. Furthermore, the desire to avoid attempting to prove particular theories gives to the work an impersonal character which adds tremendously to its value.

The writer hopes to extend the analysis of earthquake records in cooperation with the Coast and Geodetic Survey along the lines discussed by Captain Heck, and preliminary work in that direction has been in progress for some time. It may be stated herein that the present type of strong-motion accelerometers has been shown to give reliable results and that a convenient means of analysis by integration has been developed.

Mr. Stone notes that the damped spring-element concept can be extended profitably to many other structures. In fact, the patent covering the device does enumerate such uses, and there is every reason to believe that it could be applied satisfactorily in many instances where important structures must be protected against earthquake motions, although the details of the application would be varied considerably to suit individual cases.

The statement of Mr. Stone to the effect that the period of vibration depends upon the initial stresses in the members perhaps needs to be qualified somewhat. It is true that for very small amplitudes the period is considerably affected by the initial stresses in the rods. As soon as the amplitude reaches values of some importance (say, more than 0.25 in.) the initial tension has very little effect upon the period—in fact, any earthquake of destructive character will necessarily induce amplitudes large enough to make the dynamical effect of initial tension negligible.

The comment is made by Mr. Stone that the type of tower selected for experiment is a poor one from the standpoint of earthquake resistance, and a broader tower base is suggested. The tower referred to is of a common type and represents what is considered good practice for sprinkler tanks of moderate size and height. Although a wider base might be desirable for optimum efficiency, one cannot, in general, make the base spread very great unless the efficient use of the plant property is of no importance. Of course, the geometrical shape of the panels does not directly enter into the dynamical conditions stated in Conclusion (8), and the designer will normally choose the most economical layout.

As to the absence of a statement of the period and amplitude to be expected in earthquake motions, the omission was intentional. The writer feels that, unless prefaced by an elaborate discussion, any statement of that kind is almost certain to be misinterpreted by the very readers it was intended to help. A glance at the ground motion shown in Fig. 26 will perhaps serve to make this clear.

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[20] Asst. Prof. of Eng. Seismology, Mass. Inst. Tech., Cambridge, Mass.

Mr. Boardman has suggested several interesting designs for tank towers. Such designs are probably best suited to large capacity tanks and, as such, have a very special interest, since for these types the ordinary spring element design would be uneconomical or impracticable. The arrangement shown in Fig. 28 would be well suited for the introduction of damping devices in the diagonals, and a careful design of the columns might permit large deflections. In such a case, the columns might be stiff enough to provide most, if not all, of the spring action and the dampers could be relatively simple friction devices.

The point regarding the use of steeper slopes for the rods to obtain a maximum of elastic tower deflection is well taken, and if the economy is not thereby destroyed, the dynamic design is simplified. Rods of steeper slope mean heavier columns and longer rods to handle; but there may be a gain in economy in some cases. However, the writer wishes to emphasize that some damping would be required in any event unless the structure can safely withstand extremely large deflections, and that there is no cheaper way of improving the dynamical behavior than by the introduction of damping. The presence of the springs in a design merely reduces the necessary damping and makes it convenient to produce the damping action.

The designs shown in Figs. 29 and 30 would doubtless need damping systems that could take the form of simple friction devices actuated by a light diagonal system, the spring action being provided by the columns. A tower such as that in Fig. 27 would have to be designed as a short-period system and no data exist for obtaining the stresses.

Messrs. Ulrich and Carder have contributed a helpful discussion to the paper. The field tests made by the U. S. Coast and Geodetic Survey were most helpful in the interpretation of the laboratory results. It may be stated that the model periods agreed with the prototype periods as closely as could be expected, and that the differences can logically be attributed largely to the degree of initial rod tension in the field.

The question of torsional vibration is properly raised, but the writer is of the opinion that even in conventional towers it would be only a secondary effect, since it would have to be set up principally by whatever lack of symmetry exists in the structure. In a structure equipped with the proper spring elements any torsional vibration would be quickly damped out since the elements come into action from torsion as well as from linear deflection of the tower.

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BACK-WATER AND DROP-DOWN CURVES FOR UNIFORM CHANNELS

BY NAGAHO MONONOBE [1], M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. J. C. STEVENS, CHESLEY J. POSEY, A. A. KALINSKE, BORIS A. BAKHMETEFF, P. CHARLES STEIN, AND NAGAHO MONONOBE.

SYNOPSIS

Most of the existing formulas for determining back-water and drop-down curves in open channels, are applicable only to channels of simple geometric cross-section. Moreover, these formulas usually yield inaccurate results because in their derivation the effects of variation of velocity head are neglected, or the coefficient, *C*, in Chezy's mean-velocity formula is considered constant. In the present study, rational back-water and drop-down formulas are derived in such a manner that they are applicable to a wide variety of commonly used cross-sections. These formulas are based on accurate mean-velocity expressions of the exponential type, and make full allowance for velocity-head effects. The results of numerous tests in experimental channels are presented, and these results are compared with values computed by existing formulas and by the new formulas derived by the writer. In a thorough treatise entitled "Hydraulics of Open Channels", B. A. Bakhmeteff, M. Am. Soc. C. E., has also introduced the hydraulic exponent to express the characteristics of a channel cross-section. The writer's manuscript was prepared independently and has the merit of simplifying the computation in its application to various cases. This statement does not imply a criticism of Professor Bakhmeteff's treatment, but is intended merely to establish a line of demarcation, in the hope that discussers will avoid confusing the two treatments.

1.—FUNDAMENTAL FORMULAS FOR BACK-WATER AND DROP-DOWN CURVES

In a channel having constant discharge, uniform cross-section, and uniform slope, the flow is said to be non-uniform if the water surface is not parallel to the channel bed. In this case the longitudinal profile of the water surface

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NOTE.—Published in May, 1936, *Proceedings*.

[1] Director, Experiment Station of Public Works, Home Dept., Japanese Govt.; Prof., Civ. Eng. Dept., Tokyo Imperial Univ., Tokyo, Japan.

takes the form of a curve. If the flow is of the "tranquil" type this curve theoretically extends up stream an infinite distance and gradually approaches the straight-line profile for uniform flow at the given discharge If concave upward the curve is called a back-water curve, and if concave downward it is called a drop-down curve. However, the same differential equation is applicable to both cases, the difference being only in the boundary conditions. The determination of the surface profile for cases of non-uniform flow is a problem of much practical importance.

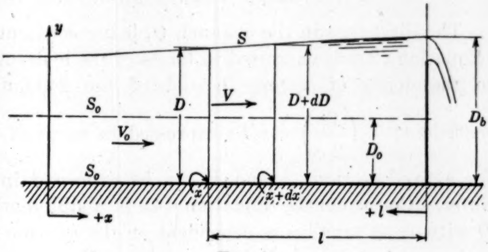

FIG. 1.

Placing the origin of the co-ordinates at any section of the channel, with the X-axis along the lowest line of the channel bed (positive in the downstream direction), and the Y-axis perpendicular to the X-axis, the general equation of motion for non-uniform flow is expressed as follows (see Fig. 1):

$$S = S_0 - \frac{dD}{dx} = \frac{V^2}{C^2 R} + \alpha \frac{d}{dx}\left(\frac{V^2}{2g}\right) \ldots\ldots\ldots\ldots (1)$$

in which S = slope of the water surface when the flow at Section x is non-uniform; S_0 = slope of the channel bed = slope of the water surface for uniform flow; D = depth of water at Section x for non-uniform flow; D_0 = depth of water for uniform flow; D_b = depth of water at weir; D_s = depth of water at point of maximum drop-down; R = hydraulic radius at Section x; C = a coefficient in the velocity formula; α = a correction factor for velocity head $= 1 + \alpha' =$, say, $\frac{10}{9}$.

A general type of mean velocity formula is as follows:

$$V = C\, R^m\, S^p{}_f = C\, R^m\, S^{0.5}{}_f \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which $p = 0.5$ in most cases, and S_f = the slope, S, defined by the frictional head for V and R.

In the Chezy velocity formula, C varies considerably with the depth of water and, if Equation (2) is substituted in Equation (1),

$$S = S_0 - \frac{dD}{dx} = \frac{V^2}{C^2\, R^{2m}} + \alpha\, \frac{d}{dx}\left(\frac{V^2}{2g}\right) \ldots\ldots\ldots\ldots (3)$$

or,

$$S = \frac{Q^2}{C^2\, R^{2m}\, A^2} + \alpha\, \frac{Q^2}{2g}\, \frac{d}{dx}\left(\frac{1}{A^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which Q = discharge; and A = hydraulic area.

A relation between D and x can be found by solving either Equation (3) or Equation (4), if S_0 and C do not vary. Since they do vary for rivers and streams, Equations (3) and (4) are not strictly applicable to natural watercourses. However, if the stream is considered as divided into several reaches, in each of which S_0 and C are nearly uniform, and if the relation between D and x is applied to each reach separately, back-water curves can be obtained more conveniently from either Equation (3) or Equation (4), than from Equation (1), since the latter must be solved by trial.

2.—Various Formulas for Back-Water Curves

The discharge in the channel, Q, being constant, S in the velocity formula (Equation (2)) is expressed in terms of the hydraulic radius, R, corresponding to the depth of water, D, and of the roughness coefficient or velocity coefficient; $\frac{d}{dx}\left(\frac{V^2}{2\,g}\right)$ can be expressed in terms of the hydraulic area, A, and for a fixed section, A and R can be expressed in terms of D, the depth of water. Consequently, Equation (4) is a differential equation that combines D with x, its type being dependent on the relation expressed in Equation (2), and on the shape of the cross-section. Hence, the back-water curve derived by integration is necessarily subject to these two factors.

In determining the back-water and drop-down curves, therefore, it is essential to adopt the velocity formula most appropriate for the channel under consideration and to establish the relation between D and R as accurately as possible. Many of the existing formulas are deficient in one or both of these requirements and, moreover, these formulas do not include the term which expresses a change of velocity head. When the variation of mean velocity is wide, the latter omission introduces a serious error, especially in the case of drop-down curves.

Table 1 contains various types of velocity formulas and channel sections adapted to the determination of back-water and drop-down curves in current use.

TABLE 1.—Types of Existing Back-Water Formulas

Item No. (1)	Investigation (2)	Year of publication (3)	Velocity formula, V, equals: (4)	Type of channel section (5)	Variations in velocity head: (6)	Ratios of $\frac{d}{d_0}$	
						Back-water curve (7)	Drop-down curve (8)
1	Grashof-Bresse.	1860	$C\sqrt{D_0\,S_0}$	Broad rectangle	Considered	$\frac{1}{0.999}$ to $\frac{1}{0.07}$	0.999 to 0.00
2	Dupuit-Masoni	1863	$C\sqrt{R\,S_0}$	Common rectangle	Considered	None	None
3	Rühlmann.....	1880	$C\sqrt{D_0\,S_0}$	Broad rectangle...	Neglected	1.01 to 6.0	0.99 to 0.00
4	Tolkmitt......	1881	$C\sqrt{R\,S_0}$	Broad parabola...	Considered	1.005 to 5.0	0.995 to 0.00
5	Schaffernak....	1913	$C D_0^{0.75}\,S_0^{0.5}$	Broad rectangle...	Neglected	1.013 to 5.0	0.2 to $\frac{1}{1.04}$
6	Ehrenberger...	1914	$\left\{\begin{array}{l}23.78\;D_0^{0.776}\;S^{0.458}\\22.11\;D_0^{0.58}\;S_0^{0.43}\end{array}\right.$	Broad rectangle...	Neglected	1.000 to 5.0	None
7	Baticle........	1921	Q^2 varies as $S\,z^5 = S_0\,z_0^5$.	Approximate trapezoid	Neglected	1.001 to 10.0	None
8	Kozeny.......	1928	$C D_0^{0.7}\,S_0^{0.5}$	Broad rectangle...	Considered	None	0.10 to 0.95
9	Schoklitsch....	1930	$C D_0^{m}\,S_0^{0.5}$	Broad rectangle...	Neglected	1.01 to 5.0	None

Most of the expressions for mean velocity in Column (4), Table 1, are considerably in error because the characteristics of each section of the actual channel vary. Furthermore, in some cases (Items Nos. 3, 5, 6, 7, and 9, Table 1), the important variations in velocity head are neglected. In Item No. 7, Table 1, the symbols, z and z_0, denote $A^2 R$ and z in uniform flow, respectively.

3.—General Solutions

Existing methods of calculating back-water and drop-down curves are scarcely accurate enough for practical purposes; furthermore, the hydraulic theory upon which they are based is very imperfect. The reasons why existing methods are unsatisfactory are, as follows:

Case (*A*).—In order to be able to express the back-water and drop-down curves by functions having a finite number of terms, some of these methods use the Chezy velocity formula with constant C, and consider only broad, shallow, channel sections. Generally, it is impossible to express the back-water and drop-down curves by a function having a finite number of terms unless $\frac{1}{m}$ in Equation (2) is an integer and the broad section is a rectangle.

Case (*B*).—When m has any arbitrary value and when the channel has a broad shallow section, the back-water and drop-down curves may be expressed by functions involving infinite series; but the calculation of the numerical table (Columns (7) and (8), Table 1) is quite complicated in comparison with Case (*A*). The Schaffernak (Item No. 5)[2] and Schoklitsch (Item No. 9)[3] formulas are the simplest in this case. In the Rühlmann formula (Item No. 3)[4] an infinite series is used, laboriously and needlessly, in spite of the fact that the formula should belong to Case (*A*). In the cases of most channels with cross-sections that are not broad in comparison with their depths, the determination of the back-water and drop-down profiles by rigorous mathematical methods involves difficulties which are practically insurmountable.

Unless the Chezy formula is used, the back-water function generally requires the use of infinite series for its expression. Moreover, if the change in the velocity head is considered, two infinite series are derived from one kind of channel. In this case it is extremely laborious to construct tables of the function for all types of sections.

In the following, the writer derives some exponential formulas for the hydraulic area, wetted perimeter, and hydraulic radius, which are sufficiently accurate for practical purposes. The change in velocity head is taken into consideration, and the most appropriate velocity formula is used for each channel. In every case the desired back-water or drop-down function is expressed in terms of two sets of series.

Case 1.—The Most General Case of Non-Uniform Flow.—The hydraulic area, A, the wetted perimeter, P, the hydraulic radius, R, and the average velocity, V, are expressed most generally as functions of D, the depth of water measured from the lowest line of the channel bed; thus, with Equation (2):

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[2] "Hydraulik", by Forchheimer, Third Edition, p. 208.

[3] "Wasserbau", by Schoklitsch, 1930, Vol. 1, p. 103.

[4] "Hydraulik", by Forchheimer, Third Edition, p. 207.

$$A = a_0 + aD^s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

$$P = b_0 + bD^k \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

and,

$$R = \frac{A}{P} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5c)$$

in which the subscript, 0, refers the symbol to the condition of uniform flow.

Since the amount of flow in the channel is unchanged, $Q = VA = V_0 A_0$ and, therefore:

$$S_f = \frac{1}{C^2}\left(\frac{V_0 A_0}{A}\right)^2 R^{-2m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which $\frac{V_0^2}{2g}\left(\frac{A_0}{A}\right)^2 = \frac{V^2}{2g}$. By substituting Equations (2) and (5) into the general differential Equation (1):

$$S_0 - \frac{dD}{dx} = \frac{V_0^2}{C^2}\left(\frac{A_0}{A}\right)^2 R^{-2m} + \alpha\,\frac{V_0^2}{2g}\,\frac{d}{dx}\left(\frac{A_0}{A}\right)^2 \ldots\ldots\ldots (7)$$

Replacing A, R, and P, by their values in Equations (5) and writing $\frac{a}{a_0} = a_1$; and $\frac{b}{b_0} = c$:

$$S_0 - \frac{dD}{dx} = S_0\left(\frac{1 + a_1 D^s_0}{1 + cD^k_0}\right)^{2m+2}\left(\frac{1 + cD^k}{1 + cD^k_0}\right)^{2m}$$

$$- \frac{\alpha C^2 S_0}{g}\left(\frac{1 + a_1 D^s_0}{1 + cD^k_0}\right)^{2m} S_{a1}\left(\frac{1 + a_1 D^s_0}{1 + a_1 D^s}\right)^2 \frac{D^{s-1}}{1 + a_1 D^s}\,\frac{dD}{dx} \ldots\ldots (8)$$

Equation (8) is the most general differential equation that expresses the back-water and drop-down functions; but integration of the equation and the calculation of numerical tables become extremely complicated with the expression in this form, since each term in the right-hand side of the equation expands into a double series with respect to D.

Case (2).—In Which the Hydraulic Area, A, Is Monomial.—In common channels or rivers, if D is measured from the lowest line, the monomial expression, $A = aD^s$, for hydraulic area is sufficiently accurate even when $a_0 = 0$. Therefore, when $A = aD^s$; $P = b_0 + bD^k$; and,

$$R = \frac{A}{P} = \frac{a\,D^s}{b_0\,(1 + cD^k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

then,

$$\left(\frac{A_0}{A}\right)^2 = \left(\frac{D_0}{D}\right)^{2s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

$$S_f = \frac{V^2_0}{C^2}\left(\frac{A_0}{A}\right)^2 R^{-2m} = S_0\left(\frac{D_0}{D}\right)^{2s(m+1)}\left(\frac{1 + c\,D^k}{1 + cD^k_0}\right)^{2m} \ldots\ldots (10b)$$

and,

$$\frac{V^2_0}{2g}\,\frac{d}{dx}\left(\frac{A_0}{A}\right)^2 = -\ \frac{S_0\,C^2\,S}{g}\left(\frac{c_0 + cc_0D^k_0}{D^s_0}\right)^{-2m}\left(\frac{D_0}{D}\right)^{2s}\frac{1}{D}\,\frac{dD}{dx}\ .(10c)$$

in which $c = \frac{b}{b_0}$; and, $c_0 = \frac{b_0}{a}$.

Let,

$$\frac{S_0\,\alpha\,C^2\,S}{g}\left(\frac{c_0 + cc_0\,D^k}{D^s_0}\right)^{-2m} = D_k \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

then, Equation (7) becomes,

$$S_0\ - \frac{dD}{dx} = S_0\left(\frac{D_0}{D}\right)^{2s(m+1)}\left(\frac{1 + cD^k}{1 + cD^k_0}\right)^{2m} - \frac{D_k}{D_0}\left(\frac{D_0}{D}\right)^{2s+1}\frac{dD}{dx}$$

and, therefore,

$$S_0\ dx = \frac{\left[1 - \frac{D_k}{D_0}\left(\frac{D_0}{D}\right)^{2s+1}\right]dD}{1 - \left(\frac{D_0}{D}\right)^{2s(m+1)}\left(\frac{1 + cD^k}{1 + cD^k_0}\right)^{2m}} = F\ (D)\ dD \ldots\ldots (12)$$

The second term in the denominator of Equation (12) may be expressed by a convergent infinite series, and, as a consequence, the function, $F(D)$, can be expressed in terms of a double series of $\frac{D}{D_0}$; but the labor for calculating the numerical table is far greater than in Case (3), which follows.

Case (3).—In Which the Hydraulic Area, A, and Wetted Perimeter, P, Are Monomials.—In this case the differential equations become reasonably simple, since,

$$A = aD^s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

$$P = bD^k \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

and,

$$R = \frac{A}{P} = \frac{a}{b}\,D^{s-k} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13c)$$

It seems somewhat irrational to express P as a monomial function of D. This is especially true in the case of sections such as the rectangle or trapezoid, in which the base width has a finite value. However, in the cases of these sections, as in the cases of most other sections in common use, actual trial shows that it is possible to choose values of b and k in Equation (13b) that will make this equation express the value of P with reasonable accuracy over a considerable range of depths. The fact that the use of this monomial expression for P does not introduce errors of appreciable magnitude into the computation of back-water and drop-down profiles, will be made evident by reference to the many experiments described subsequently herein.

4.—GENERAL FORMULAS FOR BACK-WATER AND DROP-DOWN CURVES

Referring to Case (3), Article 3, and following the form of Equation (13): $\frac{A}{A_o} = \left(\frac{D}{D_o}\right)^s; \frac{P}{P_o} = \left(\frac{D}{D_o}\right)^k; \frac{R}{R_o} = \left(\frac{D}{D_o}\right)^{s-k}$; and $V_o = C\,R^m{}_o\,S^{0.5}{}_o$. Consequently,

$$\frac{V^2{}_o}{C^2}\left(\frac{A_o}{A}\right)^2 R^{-2m} = S_o\left(\frac{D}{D_o}\right)^{-2s}\left(\frac{R_o}{R}\right)^{2m} = S_o\left(\frac{D}{D_o}\right)^{-2s-2m(s-k)} \quad \ldots\ldots(14)$$

and,

$$\alpha\,\frac{V^2{}_o}{2\,g}\,\frac{d}{dx}\left(\frac{A_o}{A}\right)^2 = -\,\alpha\,\frac{V^2{}_o}{g} \times s\left(\frac{D}{D_o}\right)^{-2s-1}\frac{d}{dx}\left(\frac{D}{D_o}\right)$$

$$= -\,K\,D_o\left(\frac{D}{D_o}\right)^{-2s-1}\frac{d}{dx}\left(\frac{D}{D_o}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

in which $K = \alpha\,s\,\frac{V^2{}_o}{g\,D_o}$.

Back-Water.—Substituting Equations (14) and (15) into the general differential Equation (1):

$$S_o - \frac{dD}{dx} = S_o\left(\frac{D}{D_o}\right)^{-2s-2m(s-k)} - KD_o\left(\frac{D}{D_o}\right)^{-2s-1}\frac{d}{dx}\left(\frac{D}{D_o}\right)$$

Hence, by making $\frac{D}{D_o}$ equal to y, the quantity, $d\left(\frac{D}{D_o}\right)$, will equal $dy = \frac{dD}{D_o}$; and, finally,

$$\frac{S_o}{D_o}\,dx = \frac{dy}{1 - y^{-2s-2m(s-k)}} - K\,\frac{y^{-(2s+1)}}{1 - y^{-2s-2m(s-k)}}\,dy \quad \ldots\ldots\ldots(16)$$

Since m is dependent upon the velocity formula used, and s and k are determined by the channel section:

$$\frac{S_o\,dx}{D_o} = \frac{dy}{1 - y^{-r}} - K\,\frac{y^{-(2s+1)}}{1 - y^{-r}}\,dy = \frac{y^r}{y^r - 1}\,dy - K\,\frac{y^{(r-2s)-1}}{y^r - 1}\,dy \ldots(17)$$

in which $r = 2s + 2m\,(s - k)$. Integrating Equation (17) from x to $x + l$ (that is, from D to D_b), the distance, l, from the weir to the point where the depth of water, D, is known, and the rise of the water surface is $D - D_o$ (see Fig. 2):

$$\frac{S_o\,l}{D_o} = \Phi_1\left(\frac{D_b}{D_o}\right) - \Phi_1\left(\frac{D}{D_o}\right) - K\left[\Phi_2\left(\frac{D_b}{D_o}\right) - \Phi_2\left(\frac{D}{D_o}\right)\right] \ldots(18)$$

in which $D_b = D_{x+1}$; $D = D_x$; $K = \alpha s\,\frac{V^2{}_o}{g\,D_o}$; $\alpha = \frac{10}{9}$; $\Phi_1 = \int \frac{y^r}{y^r - 1}\,dy$; $\Phi_2 = \int \frac{y^{(r-2s)-1}}{y^r - 1}\,dy$; and $y = \frac{D}{D_o} > 1$.

Theoretically, the influence of back-water extends backward from the weir an infinite distance. At the point where $l = \infty$, $D = D_0$, or $y = l$. Since both Φ_1 and Φ_2 become infinitely great at this point, their limits of integration are taken to be $y = y_1 = 1.001$; that is, $\frac{D}{D_0} = 1.001$

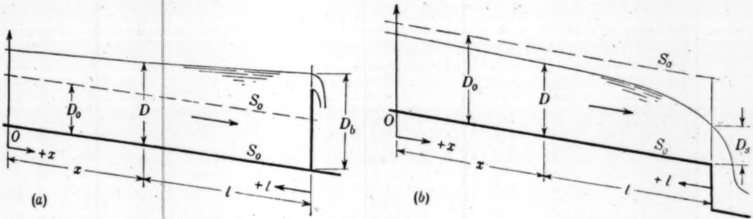


FIG. 2.

Drop-Down.—In this case, $\frac{D}{D_0} < 1$, and for convenience in integration let $z = \frac{1}{y} = \frac{D_0}{D} > 1$. Then, Equation (16) becomes:

$$\frac{S_0\,dx}{D_0} = -\frac{z^{-2}\,dz}{1 - z^r} + K\frac{z^{2s-1}\,dz}{1 - z^r} = \frac{z^{-2}\,dz}{z^r - 1} - K\frac{z^{2s-1}\,dz}{z^r - 1} \quad \ldots\ldots.(19)$$

In Equation (19), $r > 1$; and $z > 1$, and both terms on the right-hand side can be developed into a convergent series of z. Integrating from x to $x + l$ (that is, from D to D_s):

$$\frac{S_0\,l}{D_0} = \Psi_1\left(\frac{D_0}{D_s}\right) - \Psi_1\left(\frac{D_0}{D}\right) - K\left[\Psi_2\left(\frac{D_0}{D_s}\right) - \Psi_2\left(\frac{D_0}{D}\right)\right] \ldots.(20)$$

in which $\Psi_1(z) = \Psi_1\left(\frac{D_0}{D}\right) = \int \frac{z^{-2}}{z^r - 1}\,dz$; and, $\Psi_2(z) = \Psi_2\left(\frac{D_0}{D}\right) \int \frac{z^{2s-1}}{z^r - 1}\,dz$. In this case, also, Ψ_1 and Ψ_2 become infinitely great when $z = 1$, and the lower limit of the integration is taken as $z = \frac{D_0}{D} = 1.001$, or $y = \frac{D}{D_0} = 0.999$.

5.—CALCULATION OF BACK-WATER FUNCTIONS

The Back-Water Curve.—In the expression, $r = 2s + 2m\,(s - k)$, $s =$ the numerical value of the power in the function for hydraulic area; $m =$ the exponent of hydraulic radius in the velocity formula; and $k =$ the exponent of the wetted perimeter in the velocity formula. For open channels and conduits, $1 < r < 6$ and $0.5 < s \leqq 2.0$. The quantity, $\frac{1}{y^r - 1}$, of both terms on the right-hand side of Equation (17) is developed into a convergent series, and their integrals, Φ_1 and Φ_2, may be expressed as follows:

$$\Phi_1 = y - \sum_{n=1}^{\infty} \frac{y^{-(nr-1)}}{nr - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots.(21a)$$

and,

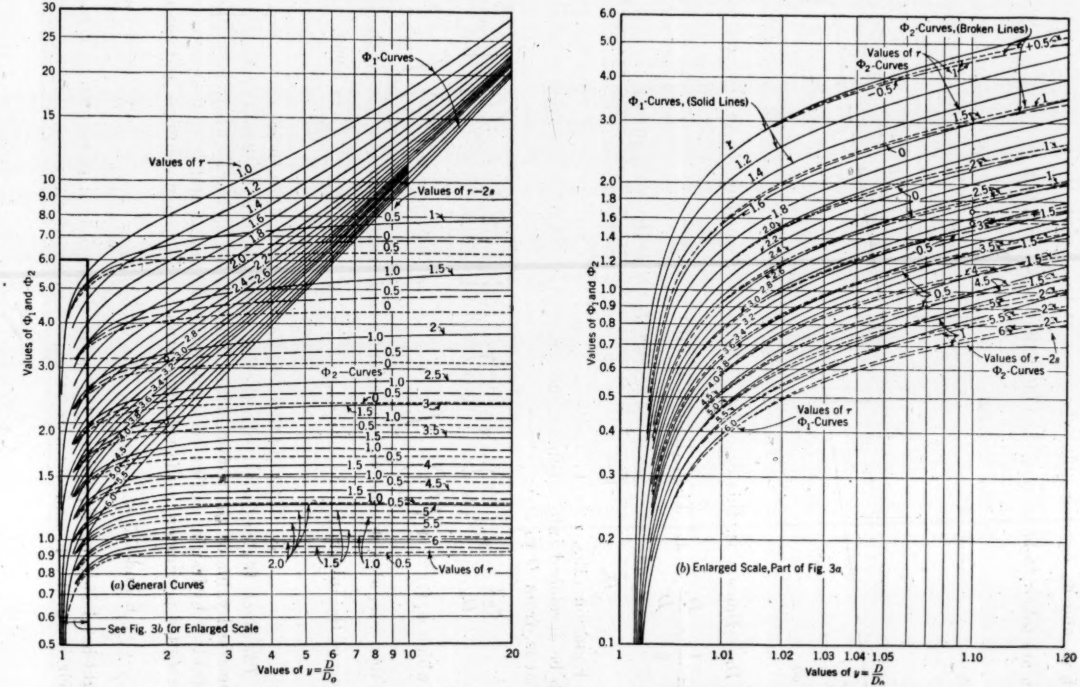


FIG. 3.—BACK-WATER CURVES; VALUES OF Φ_1 AND Φ_2 FOR SUBSTITUTION IN EQUATION (18).

$$\Phi_2 = - \sum_{n=1}^{\infty} \frac{y^{-(nr+2s)}}{nr + 2s} \qquad \text{(21}b\text{)}$$

However, in actual numerical calculation, when y is nearly equal to 1, the rate at which the series converges is very slow. When both r and s are integers or possess special values, Equations (21) can be expressed in a finite number of terms, and in preparing an actual numerical table, Φ_1 and Φ_2 are calculated directly for these special cases and the values of the functions for intermediate values of r and s are obtained by interpolation. The results are shown by groups of curves in Fig. 3, in which, for substitution in Equation (18):

$$\Phi_1 \left(\frac{D}{D_0}\right) = \int_{1.001}^{y} \frac{y^r}{y^r - 1}\, dy \qquad \text{(22}a\text{)}$$

and,

$$\Phi_2 \left(\frac{D}{D_0}\right) = \int_{1.001}^{y} \frac{y^{(r-2s)-1}}{y^r - 1}\, dy \qquad \text{(22}b\text{)}$$

If groups of Φ_1 and Φ_2 curves are constructed by plotting the integrated values thus obtained, the back-water profile applicable to almost any channel can be calculated precisely. Back-water curves published in the past were for a single case of r and s; for instance, in the Grashof-Bresse formula, a broad rectangular section is used so that, with $A = B\,D$, and $P = B$, $s = 1$, and $k = 0$. Since $V_0 = C\,R^{0.5}_0\,S^{0.5}_0$, the exponent, m, $= 0.5$. Therefore, $s = 1$, and $r = 3$.

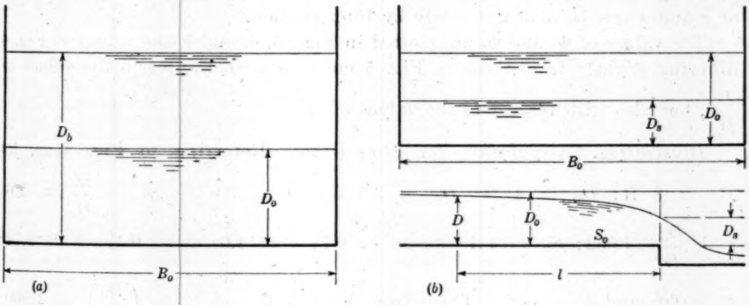


FIG. 4.—EXAMPLES.

In the Tolkmitt formula (Item No. 4, Table 1), a broad parabolic section is used in the Chezy velocity formula so that $m = 0.5$ and $A = \frac{2}{3} BD$; $P = B$; and $R = \frac{2}{3} D$. Consequently, $s = \frac{3}{2}$; $k = 0.5$; and $r = 4$. In the Rühlmann formula (Item No. 3, Table 1) Φ_2 is neglected when $s = 1$ and $r = 3$.

Illustrative Computation for Back-Water.—Referring to Fig. 4(*a*) let $B_0 = 8$ ft; $D_0 = 2$ ft; $D = 2.2$ ft; $D_b = 4$ ft; $S_0 = 1{:}1\,000$; $C = 50$; $m = 0.7$; $s = 1.0$; $k = 0.38$; $r = 2.87$; and $K = 0.212$. Then, from Fig. 3, selecting proper values of Φ_1 and Φ_2, Equations (22) yield: $\Phi_1\left(\frac{D_b}{D_0}\right) = 3.16$; $\Phi_1\left(\frac{D}{D_0}\right) = 1.68$; $\Phi_2\left(\frac{D_b}{D_0}\right) = 2.12$; and $\Phi_2\left(\frac{D}{D_0}\right) = 1.58$. Substituting these values in Equation (18): $\frac{S_0\, l}{D_0} = 3.16 - 1.68 - 0.212\ (2.12 - 1.58) = 1.365$; and, $l = 2\,730$ ft.

Drop-Down Curve.—In this case the drop-down functions are: $\Psi_1\ (z) = \int \frac{z^{-2}\, dz}{z^r - 1}$; and, $\Psi_2\ (z) = \int \frac{z^{2s-1}}{z^r - 1}\, dz$, and each of them can be expressed as infinite series, thus:

$$\Psi_1\ (z) = -\sum_{n=1}^{\infty} \frac{z^{-(nr+1)}}{nr + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23a)$$

and,

$$\Psi_2\ (z) = -\sum_{n=1}^{\infty} \frac{z^{-nr+2s}}{nr - 2\,s} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23b)$$

For integral or special values of r and s, the integration leads to expressions having a finite number of terms, so that if values of the functions are computed by these expressions, the values of the functions for intermediate values of r and s can be obtained easily by interpolation.

The values of Ψ_1 and Ψ_2 are plotted in Figs. 5, 6, and 7, the values in Fig. 6 differing slightly from those in Fig. 5 not only with respect to the values of $\frac{D}{D_0}$, but also with respect to the values of s.

Illustrative Computation for Drop-Down.—Referring to Fig. 4(*b*), let $B_0 = 8$ ft; $D_0 = 4$ ft; $D_s = 2.5$ ft; $D = 3.8$ ft; $Z = \frac{D_0}{D_s} = 1.6$; $z = \frac{D_0}{D} = 1.052$; $S_0 = 1{:}1\,000$; $m = 0.7$; $s = 1.0$; $k = 0.47$; $r = 2.74$; $C = 60$; and $K = 0.27$. Then, from Figs. 5, 6, and 7: $\Psi_1\left(\frac{D_0}{D_s}\right) = 1.900$; $\Psi_2\left(\frac{D_0}{D_s}\right) = 2.34$; and $\Psi_2\left(\frac{D_0}{D}\right) = 1.45$. Substituting these values in Equation (20): $\frac{S_0\, l}{D_0} = 1.900 - 1.395 - 0.27\ [2.34 - 1.45] = 0.265$; and $l = 1\,060$ ft.

General.—In Figs. 3 to 7, the values of Φ_1, Φ_2, and Ψ_1, Ψ_2, which are shown by groups of curves, are obtained by ordinary integration with respect to a number of sets of values of r and s which furnish directly integrable equations.

Intermediate values of r and s are determined by interpolation. For the back-water curves the direct integration is performed with respect to the following special cases: (*a*) For Φ_1: $r = 1, 2, 3, 4, 5$, and 6; (*b*) $r = 3$ is for Φ_1 in the Grashof-Bresse (Item No. 1, Table 1) and Rühlmann formulas (Item No. 3, Table 1); (*c*) $r = 5$ is for Φ_1 of the Baticle formula (Item No. 7, Table 1); (*d*) values of r and corresponding values of s for integrals of Φ_2 (see Fig. 3) are listed in Table 2; (*e*) $r = 3$, $s = 1.0$ are for Φ_2 of the Grashof-Bresse formula; and (*f*) $r = 4$, $s = 1.5$ are for Φ_2 of the Tolkmitt formula (Item No. 4, Table 1).

TABLE 2.—VALUES OF r, WITH CORRESPONDING VALUES OF s, FOR INTEGRALS OF Φ_2 AND Ψ_2

Values of r	VALUES OF s, FOR:		Values of r	Values of s for Φ_2 and Ψ_2	Values of r	Values of s for Φ_2 and Ψ_2	Values of r	VALUES OF s, FOR:	
	Φ_2	Ψ_2						Φ_2	Ψ_2
(1)	(2)	(3)	(1)	(2)	(1)	(2)	(1)	(2)	(3)
1.0	0.25 0.50 0.75	0.50 0.75 1.00	2.0	0.50 0.75 1.00	3.0	0.75 1.00 1.25 1.50	5.0	1.50 2.00	1.50 2.00
1.5	0.25 0.50 0.75	....	2.5	0.75 1.00 1.25	4.0	1.00 1.50 2.00	6.0	2.00	1.50 2.00 ....

In the drop-down the direct integration is performed with respect to the following special cases: (*g*) For Ψ_1: $r = 1, 2, 3, 3.4, 3.5, 4, 5$, and 6; (*h*) $r = 3$ is for Ψ_1 of the Grashof-Bresse and Rühlmann formulas; (*i*) $r = 3.4$ is for the Kozeny formula (Item No. 8, Table 1); (*j*) $r = 3.5$ is for the Schaffernak formula (Item No. 5, Table 1); (*k*) $r = 4$ is for the Tolkmitt formula; and, (*l*) values of r, with corresponding values of s, for integrals of Ψ_2 are listed in Table 2, in which the combination, $r = 3$ and $s = 1.0$ is the value of Ψ_2 for the Grashof-Bresse formula. Direct integration is possible for seven forms of the integral, Φ_1, twenty-two forms of the integral, Φ_2, eight forms of the integral, Ψ_1, and seventeen forms of the integral, Ψ_2. The integrations were performed by Reidi Ito, River Engineer of the Department of Home Affairs, Japan.

6.—HYDRAULIC AREA, WETTED PERIMETER, AND HYDRAULIC RADIUS

Various shapes of sections which are commonly used for channels, however complicated they may be, can be approximated accurately enough by a combination of straight line, parabolic, and circular curves. Then, establishing the mathematical relation of A, P, and R, or s and k, to the depth of water for these three cases, the factors, s and r, may be obtained for a required depth of water in the case of an irregular section by curves or by direct calculation.

(*a*) *Prismatic Section.*—In the cases shown in Fig. 8 the hydraulic area, A, may be expressed in general terms by:

$$A = a_0 D + a D^2 \qquad \text{(24)}$$

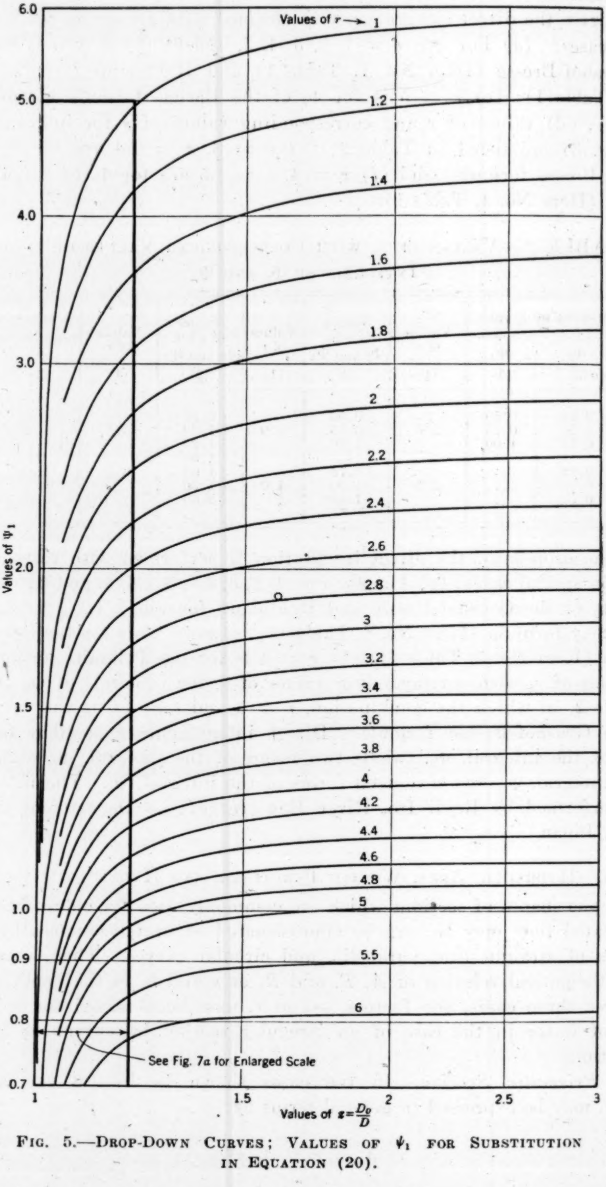


FIG. 5.—DROP-DOWN CURVES; VALUES OF ψ_1 FOR SUBSTITUTION IN EQUATION (20).

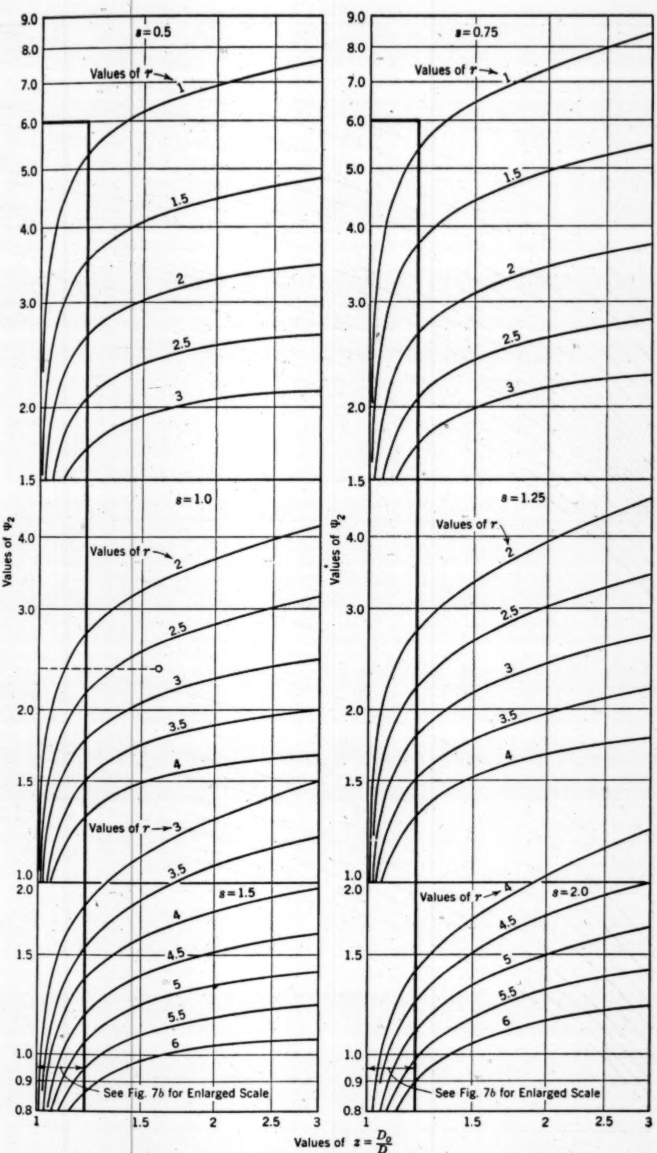


FIG. 6.—DROP-DOWN CURVES; VALUES OF ψ_2 FOR SUBSTITUTION IN EQUATION (20).

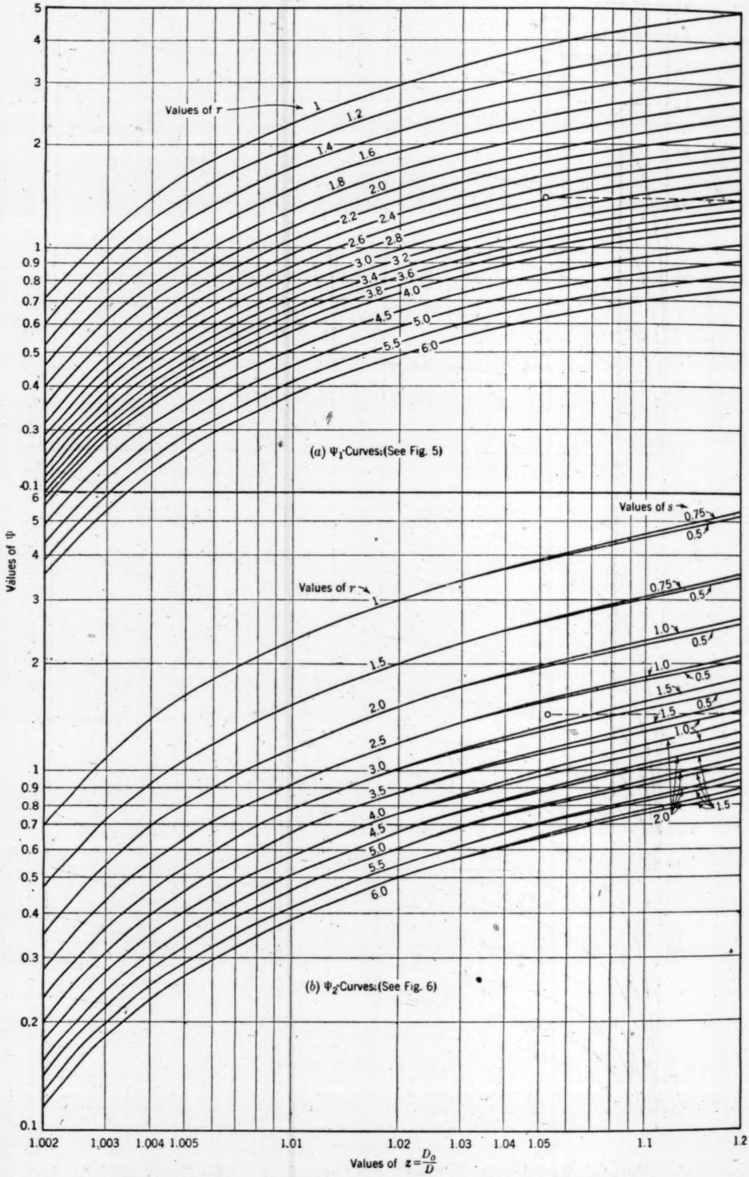
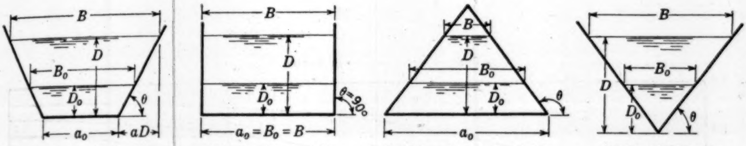


FIG. 7.—DROP-DOWN CURVES, FIGS. 5 AND 6, TO AN ENLARGED SCALE.



(*a*) Trapezoidal; $a = \cot\theta > 0$ (*b*) Rectangular; $s = 1.0$ and $a = 0$ (*c*) Triangular; $a = \cot\theta < 0$ (*d*) Inverted Triangle; $s = 2.0$ and $k = 1.0$

FIG. 8.—REGULAR CHANNEL CROSS-SECTIONS.

in which $a = \cot\theta$; θ = the angle made by the slope of the bank and the extension of the base line of the channel; and the values of a_o are as shown in Table 3.

TABLE 3.—VALUES OF a_o AND θ IN EQUATION (24)

Description	Trapezoidal section (Fig. 8 (*a*))	Rectangular section (Fig. 8 (*b*))	Triangular section (Fig. 8 (*c*))	Inverted triangle (Fig. 8 (*d*))
a_o	a_o	$B = B_o$	a_o	0
cot θ	+	0	−	+

From Equation (24):

$$\frac{A}{A_o} = \frac{a_o\, D + a\, D^2}{a_o\, D_o + a\, D_o^2} = \frac{y + \dfrac{a}{\mu}\, y^2}{1 + \dfrac{a}{\mu}} = y^s \text{ ....(25)}$$

in which $y = \dfrac{D}{D_o}$; $\mu = \dfrac{a_o}{D_o}$; and $\dfrac{a}{\mu} = \dfrac{D_o}{a_o} \cot\theta$.

The expression involving the wetted perimeter, P, is written as:

$$P = a_o + 2\, D \sqrt{1 + a^2} \text{ ....(26)}$$

from which,

$$\frac{P}{P_o} = \frac{a_o + 2\, D \sqrt{1 + a^2}}{a_o + 2\, D_o \sqrt{1 + a^2}} = \frac{1 + 2\, y \sqrt{\dfrac{1 + a^2}{\mu^2}}}{1 + 2 \sqrt{\dfrac{1 + a^2}{\mu^2}}} = y^k \text{....(27)}$$

Then, s and k, the exponents to y which depend on the ratios, $\dfrac{D}{D_o}$, are computed for various values of $\dfrac{a}{\mu}$ and $\dfrac{1 + a^2}{\mu^2}$, with y varying between 0.1 and 20. The curves are plotted as shown in Fig. 9. For example, referring to Fig. 8(*b*), let a_o = 8 ft; D_o = 2 ft; D_b = 4 ft; D = 2.2 ft; values of y at two sections considered are 2.0 and 1.1; $\mu = 4$; $\dfrac{a}{\mu} = 0$; $\dfrac{1 + a^2}{\mu^2} = \dfrac{1 + 0^2}{4^2}$

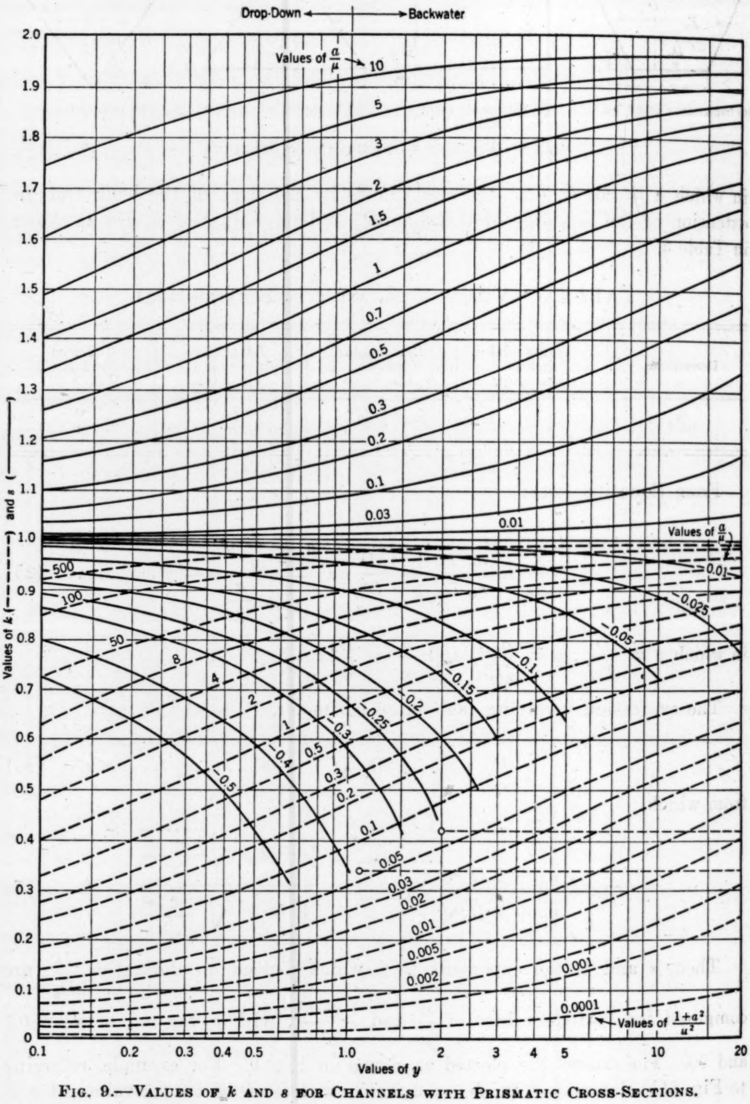


FIG. 9.—VALUES OF k AND s FOR CHANNELS WITH PRISMATIC CROSS-SECTIONS.

$= 0.0625$; and $s = 1.0$, a constant. From Fig. 9, $k = 0.34$ to 0.42, a mean value of 0.38.

(*b*) *Parabolic Section.*—In the case of the parabolic section shown in Fig. 10(*a*), $A = \frac{2}{3} BD$; $A_0 = \frac{2}{3} B_0 D_0$; and $B = B_0 \sqrt{\frac{D}{D_0}}$. Therefore, $\frac{A}{A_0} = y^{\frac{3}{2}} = y^s$; and, $s = 1.5$, a constant. Finally,

$$P = 2\,D_0 \left[\sqrt{y\left(y + \frac{B^2_0}{16\,D^2_0}\right)} + \frac{B^2_0}{16\,D^2_0} \log_e \frac{\sqrt{y} + \sqrt{y + \frac{B^2_0}{16\,D^2_0}}}{\frac{B_0}{4\,D_0}} \right]$$

and,

$$\frac{P}{P_0} = \frac{\sqrt{y\left(y + \frac{1}{16}\frac{B^2_0}{D^2_0}\right)} + \frac{1}{16}\frac{B^2_0}{D^2_0} \log_e \frac{\sqrt{y} + \sqrt{y + \frac{B^2_0}{16\,D^2_0}}}{\frac{B_0}{4\,D_0}}}{\sqrt{1 + \frac{1}{16}\frac{B^2_0}{D^2_0}} + \frac{1}{16}\frac{B^2_0}{D^2_0} \log_e \frac{4\,D_0}{B_0}\left[1 + \sqrt{1 + \frac{B^2_0}{16\,D^2_0}}\right]} = y^k \ldots (28)$$

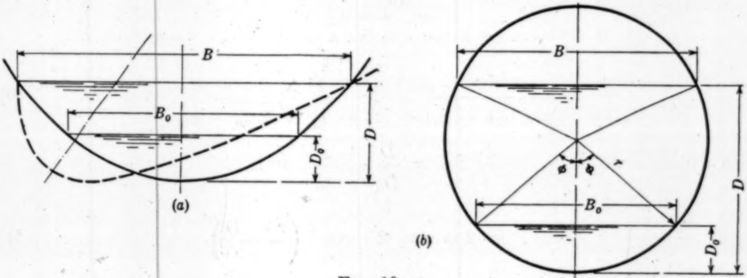


FIG. 10.

For values of y ranging from 0.1 to 20, the exponent, k, corresponding to the ratios, $\frac{B_0}{D_0}$, are calculated and shown by groups of curves in Fig. 11. For example, referring to Fig. 10(*a*), let $B_0 = 80$ ft; $D_0 = 20$ ft; $\frac{B_0}{D_0} = \frac{80}{20} = 4.0$; $D_0 = 40$ ft; $D = 22$ ft; and $y = 2.0$ and 1.1. Then, as shown by the circles and dotted lines in Fig. 11, Exponent k varies from 0.62 to 0.645, with a mean of 0.632.

(*c*) *Circular Section.*—In the case of the circular section shown in Fig. 10(*b*), r is the radius of the circle; ϕ equals one-half the central angle of the wetted perimeter; and

$$A = r^2 \cos^{-1}\left(\frac{r - D}{r}\right) + (D - r)\sqrt{2\,Dr - D^2} \ldots\ldots\ldots (29)$$

so that,

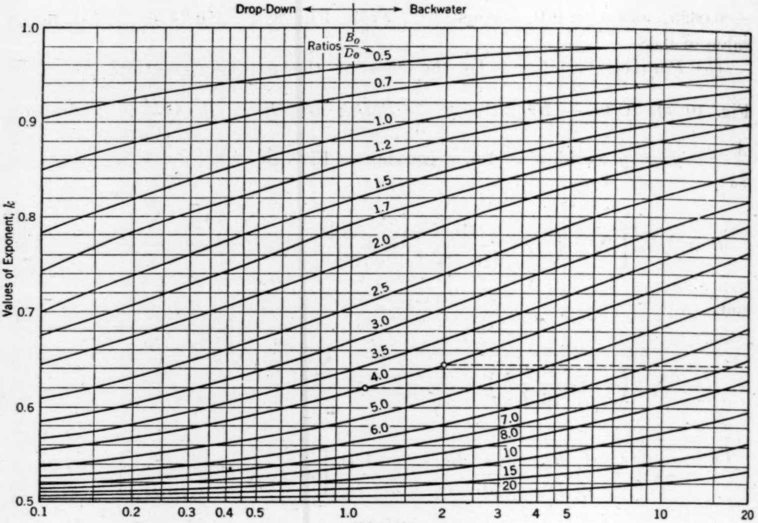


FIG. 11.—VALUES OF k FOR CHANNELS OF PARABOLIC CROSS-SECTION.

$$\frac{A}{A_o} = \frac{\cos^{-1}(1 - x) + (x - 1)\sqrt{2\,x - x^2}}{\cos^{-1}(1 - x_o) + (x_o - 1)\sqrt{2\,x_o - x_o^2}} = y^s \ldots\ldots\ldots (30)$$

in which, $x = \frac{D}{r}$; $x_o = \frac{D_o}{r}$; $y = \frac{D}{D_o} = \frac{x}{x_o}$; and,

$$P = 2\,\phi\,r = 2\,r\cos^{-1}\left(\frac{r - D}{r}\right) \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Consequently,

$$\frac{P}{P_o} = \frac{\cos^{-1}(1 - x)}{\cos^{-1}(1 - x_o)} = y^k \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

For values of y ranging from 0.1 to 20, the exponents, s and k, corresponding to the ratios of $x_o = \frac{D_o}{r}$ are calculated and shown in Fig. 12. For example, referring to Fig. 10(*b*), let $r = 9$ ft; $D_o = 4.5$ ft; $D_b = 13.5$ ft; $D = 5.4$ ft; $x_o = \frac{D_o}{r} = 0.5$; $Y = \frac{D_b}{D_o} = 3.0$; and $y = \frac{D}{D_o} = 1.2$. Then, as shown by the circles in Fig. 12, Exponent s lies between 1.28 and 1.40 (mean, 1.34) and Exponent k lies between 0.55 and 0.63 (mean, 0.59).

(*d*) *Error Involved in Assuming That Section Indices, s and k, Are Constant.*—Generally, the values of s and k in Equations (13*a*) and (13*b*) in any section differ slightly with changes in the depth of water, D, but, in this paper, the average value for the range in which D changes is used in order to facilitate the integration. To study the degree of error produced by this assumption, comparisons are made between two examples in which rectangular sections are used for the back-water experiments and the results obtained from the Dupuit-Masoni formula.

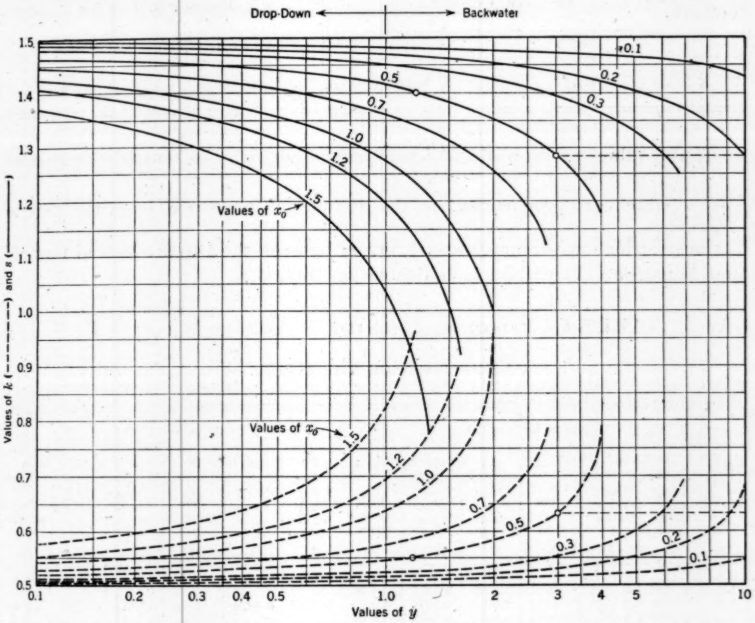


FIG. 12.—VALUES OF s AND k FOR CHANNELS OF CIRCULAR CROSS-SECTION.

Referring to Fig. 4(a), for Example 1, let $\frac{B_o}{D_o} = 2.5; Y = \frac{D_b}{D_o} = 2.0; y = \frac{D}{D_o} = 1.01$; and $k = 0.54$ to 0.46 (mean assumed as 0.5). For Example 2, let $\frac{B_o}{D_o} = 3.0$; $Y = 5.0$; $y = 1.01$; and $k = 0.60$ to 0.40 (mean assumed as 0.5). For convenience in comparing them with the Dupuit-Masoni formula (Item No. 2, Table 1), let $V = C\ R^m\ S^{0.5}$ and $m = 0.5$. Then, $r = 2\,s + 2\,m\,(s - k) = 2.5$;

$$K = \alpha\, s\, \frac{V^2_o}{g\, D_o} = \beta\, \frac{R_o}{D_o} = \beta\, \frac{B_o}{B_o + 2\, D_o} \quad \ldots\ldots\ldots\ldots(33)$$

and,

$$\beta = \alpha\ \frac{C^2\, S_o}{g} = \alpha\ \frac{V^2_o}{g\, R_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(34)$$

If k is considered a variable, the Dupuit-Masoni formula results, and the relation between β and K are as shown in Table 4.

TABLE 4.—RELATION BETWEEN K AND β, EQUATIONS (33) AND (34)

Values of:	EXAMPLE; 1 Y = 2.0		EXAMPLE 2; Y = 5.0	
	$\beta = 0.3$	$\beta = 0.9$	$\beta = 0.3$	$\beta = 0.9$
K	0.17	0.50	0.18	0.54

If the values of the ratio, $\frac{D}{D_0}$, are compared with the numerical values of $\frac{S_0 \, l}{D_0}$ (which are proportional to the distance, l, measured from the weir to various sections up stream from it), errors between the Dupuit-Masoni formula and Equation (17) will be as indicated by Table 5.

TABLE 5.—EFFECT OF ASSUMING AN AVERAGE VALUE OF K; COMPARISON OF THE RATIO, $\frac{S_0 l}{D_0}$

Ratio, $y = \frac{D}{D_0}$	EXAMPLE 1.— $Y=2.0$						EXAMPLE 2.— $Y=5.0$					
	$\beta=0.3$ and $K=0.17$			$\beta=0.9$ and $K=0.5$			$\beta=0.30$ and $K=0.18$			$\beta=0.90$ and $K=0.54$		
	Using exact value of k	Using mean value of k	Percentage error	Using exact value of k	Using mean value of k	Percentage error	Using exact value of k	Using mean value of k	Percentage error	Using exact value of k	Using mean value of k	Percentage error
(1)	(2)	(3)	(4)	(2)	(3)	(4)	(2)	(3)	(4)	(2)	(3)	(4)
2.0	......	......	......	......	......	......	3.1800	3.1724	−0.24	3.1381	3.1306	−0.24
1.7	0.3742	0.3732	−0.27	0.3539	0.3527	−0.34	......	......	......	......	......	......
1.5	0.6526	0.6517	−0.14	0.6086	0.6074	−0.20	3.8261	3.8224	−0.10	3.7365	3.7327	−0.10
1.3	0.9849	0.9853	+0.04	0.8972	0.8968	−0.04	......	......	......	......	......	......
1.2	1.1990	1.2016	+0.22	1.0710	1.0723	+0.12	......	......	......	......	......	......
1.1	1.5049	1.5123	+0.49	1.3005	1.3052	+0.36	4.6624	4.6764	+0.30	4.4009	4.4110	+0.23
1.05	1.7710	1.7828	+0.67	1.4835	1.4911	+0.51	4.9211	4.9435	+0.46	4.5710	4.5867	+0.34
1.01	2.3274	2.3496	−0.95	1.8361	1.8498	+0.75	5.4608	5.5020	+0.75	4.8939	4.9205	+0.54

Evidently, errors produced by integration, using average values of k which are considered as constants, are insignificant, being usually within 1 per cent. Generally, the errors tend to increase in the up-stream direction where the change in the depth of water is slight. If it is desired to find the shape of the back-water curve more accurately for theoretical reasons, groups of curves similar to those in Fig. 3 are drawn by using the average values of k only for the integration, r being determined by using actual values of k for the depths of water in a given channel. It is possible to find the values of ϕ_1 and ϕ_2 from the curves corresponding to r and s thus obtained; but (except when the weir is unusually high) the calculation is simplified considerably if the average values of k are used.

7.—CALCULATED EXAMPLES OF THE BACK-WATER AND DROP-DOWN IN PRISMATIC CHANNELS

In order to demonstrate the method of computing the back-water and drop-down curves presented in this paper, two general examples are offered.

Example (A).—Back-Water: Case of Trapezoidal Channel.—Referring to Fig. 8(*a*), let the side slope = 1 on 1; $D_b = 30$ ft; $D_o = 6$ ft; $\theta = 45°$; $a_o = 30$ ft; $S_o = 1{:}1\,600$; $A_o = 6 \times 30 + 6 \times 6 = 216$ sq ft; and $R_o = \dfrac{216}{30 + 2 \times 6\sqrt{2}} = 4.6$ ft. It is necesary to use a velocity equation in the form of Equation (2) in which C remains constant irrespective of R. Now, if the Manning formula is used (Equation (2), with $m = \frac{2}{3}$ and $p = \frac{1}{2}$), taking Kutter's roughness coefficient as $n = 0.020$, the mean velocity is: $V_o = \dfrac{1.486}{0.02} \times 4.6^{\frac{2}{3}} \times \dfrac{1}{\sqrt{1\,600}} = 5.15$ ft per sec. Therefore, $Q = V_o\,A_o = 5.15 \times 216 = 1\,112.4$ cu ft per sec.

If the depth of water is increased to $D_b = 30$ ft by a weir; then $y = \dfrac{D_b}{D_o} = \dfrac{30}{6} = 5$. By Example (1) of Article 6, if $\theta = 45°$ then, $a = \cot\theta = 1$; $\mu = \dfrac{a_o}{D_o} = \dfrac{30}{6} = 5$; $\dfrac{a}{\mu} = \dfrac{1}{5}$; and, $\dfrac{1 + a^2}{\mu^2} = \dfrac{1 + 1}{25} = 0.08$. It is required to find l, the distance from the weir to the section where the depth of water is 18 ft; thus, $y = \dfrac{D}{D_o} = \dfrac{18}{6} = 3$; and $m = \dfrac{2}{3}$. Hence, from Fig. 9 may be selected the values listed in Table 6(*a*); and with

TABLE 6.—ILLUSTRATIVE EXAMPLE; VALUES READ FROM CURVES

Symbol	EXAMPLE (*A*)		Symbol	EXAMPLE (*B*)	
	For depth, $D = 18$ feet	For depth, $D_b = 30$ feet		For depth, $D = 12$ feet $\left(\frac{D_o}{D} = 1.25\right)$	For depth, $D_s = 10$ feet $\left(\frac{D_o}{D_s} = 1.5\right)$
(1)	(2)	(3)	(4)	(5)	(6)
(*a*) VALUES FROM FIG. 9			(*c*) VALUES FROM FIG. 9		
y	3	5	$\frac{a}{\mu}$	0	0
s	1.31	1.36	$\frac{1+a^2}{\mu_2}$	0.25	0.25
k	0.51	0.56	s	1.0	1.0
$r - 2s$	1.07	1.07	k	0.47	0.45
r	3.69	3.79	$r - 2s$†	0.707	0.733
K	0.194*	....	r	2.707	2.733
(*b*) VALUES FROM FIG. 3			(*d*) VALUES FROM FIGS. 5 AND 6		
Φ_1	3.62	5.68	Ψ_1	1.82	1.92
Φ_2	1.80	1.80	Ψ_2	2.08	2.28

* Equation (15). † Equals $2m\,(s - k)$.

these, the integrals, Φ_1 and Φ_2, listed in Table 6(*b*) may be selected from Fig. 3. Substitution in Equation (18) then yields: $\frac{S_0\, l}{D_0} = 5.68 - 3.62 - 0.194\ (1.80 - 1.80) = 2.06$; and, therefore, $l = 2.06 \times \frac{D_0}{S_0} = 2.06 \times 6 \times 1\,600 = 19\,776$ ft. In this case the last quantity of Equation (18) (which is multiplied by K) is insignificant, but it can not be overlooked when $\frac{D_b}{D_0} < 2$.

Example (B).—Drop-Down: Case of Rectangular Channel.—Referring to Fig. 4(*b*), let $D_0 = 15$ ft; $D_s = 10$ ft; $B_0 = a_0 = 30$ ft; $S_0 = 1{:}2\,500$; $n = 0.02$; $C = \frac{1.486}{n} = 74.3$; $A_0 = 30 \times 15 = 450$ sq ft; $R_0 = \frac{450}{30 + 2 \times 15} = 7.5$ ft; and, by Equation (2) with $m = \frac{2}{3}$ and $p = \frac{1}{2}$ (the Manning formula), $V_0 = 74.3 \times 7.5^{\frac{2}{3}} \times \frac{1}{\sqrt{2\,500}} = 5.69$ ft per sec. Consequently, $Q = 450 \times 5.69 = 2\,560$ cu ft per sec; and $K = \alpha\, s\, \frac{V_0^2}{g D_0} = \frac{10}{9} \times 1 \times \frac{5.69^2}{32.2 \times 15} = 0.074$.

If D_s is equal to 10 ft (that is, if it is nearly equal to the critical depth, at the cross-section over the fall): $\mu = \frac{a_0}{D_0} = 2$; $\alpha = \cot\theta = 0$: $\frac{a}{\mu} = 0$; $s = 1$; and, $\frac{1 + a^2}{\mu^2} = 0.25$. (See Figs. 4(*b*) and 8(*b*).)

Hence, to find l, the distance from the fall in the channel to the section where $D = 12$ ft: $y_0 = \frac{D_s}{D_0} = \frac{10}{15} = \frac{2}{3}$; and $y = \frac{D}{D_0} = \frac{12}{15} = 0.8$. From Fig. 9 may be selected values listed in Table 6(*c*); and, with these, the integrals, Ψ_1, and Ψ_2, listed in Table 6(*d*) may be selected from Figs. 5 and 6.

Substitution in Equation (20) then yields $\frac{S_0\, l}{D_0} = 1.92 - 1.82 - 0.074\ (2.28 - 2.08) = 0.085$; and, therefore, $l = 0.085 \times 2\,500 \times 15 = 3\,190$ (ft).

8.—Experimental Flumes

In general, the deciding factors in establishing the shape of the back-water curve are the shape of the channel cross-section and the degree of roughness. The former is the more important and the inadequacy of previous formulas is due, in large measure, to ignoring its relative effect.

In the experiments described herein, both the cross-sectional shape and the surface roughness of the experimental channels were varied. Moreover, due to the shortness of the effective length of the flume in this experiment, it was impossible to determine the height of the water surface covering the total length of back-water limits by a single experiment. For example, when $Y = \frac{D_b}{D_0} = 2$, the back-water profile is equivalent to a curve constructed by connecting many back-water curves plotted in the ranges from: $\frac{D_b}{D_0} = 2$ to $\frac{D_b}{D_0} = 1.6$; 1.6 to 1.2; 1.2 to 1.05; etc. Hence, in the experiments reported herein, the entire back-water curve was traced by connecting several back-water curves. The curve thus obtained contains from three to nine experimental values which indicates that a considerable number of measurements is required for channels of easy slope, S_0. On the other hand, if S_0 is too steep, the velocity of the water becomes very great and the flow unsteady; in extreme cases, the water shoots ahead so violently that the bed slope up stream from the weir is necessarily fixed at 1:500 and that of the drop-down at from 1:500 to 1:2 000. In order to study the influence of roughness, two kinds of sections were devised for each type of channel section: (1) A smooth flume in which the inner surfaces were planed carefully; and (2), a rough flume in which the inner surfaces were lined with $\frac{3}{8}$-in. mesh made of No. 20 galvanized wire. In addition, the outer surfaces of the flume were submerged in water to avoid deforming the flume by alternate wetting and drying. When the flow was variable, the distance, l (from the weir to the section up stream, where the depth of water is D), was measured, and the back-water and drop-down are represented by the relative values of $\frac{S_0\, l}{D_0}$.

The typical sections of the channels studied in the writer's laboratory are shown in Fig. 13, the dimensions being as indicated. For measuring the discharge a common weir was used; and the height of the water surface was read with an ordinary engineers' level and a needle-gauge readable to the nearest 0.1 mm (0.00394 in.).

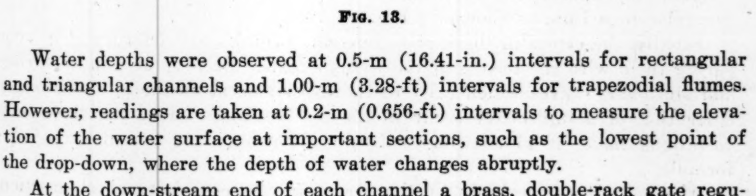


FIG. 13.

Water depths were observed at 0.5-m (16.41-in.) intervals for rectangular and triangular channels and 1.00-m (3.28-ft) intervals for trapezodial flumes. However, readings are taken at 0.2-m (0.656-ft) intervals to measure the elevation of the water surface at important sections, such as the lowest point of the drop-down, where the depth of water changes abruptly.

At the down-stream end of each channel a brass, double-rack gate regulated the water level so that it was nearly the same at the down-stream end in one set of measurements as it was at the up-stream end, in the preceding set. The regulation was effected by means of slits.

9.—The Measurement of Back-Water

Smooth Rectangular Channels.—Referring to Figs. 8(*b*) and 13(*a*), let: $B_0 = 7.874$ in.; $S_0 = 1{:}500$; $D_0 = 1.9685$ in.; $D_b = 3.937$ in.; $V_0 = 1.61$ ft per sec; and $Y = \frac{D_b}{D_0} = 2.0$.

Using Manning's formula for uniform flow, with various slopes, and assuming C to be constant, the writer found by actual measurements that the value of m was also nearly a constant, and was equal to 0.65.

The effective length of the channel was 11 m (36.09 ft). The dam at the lowest down-stream section was adjusted so that $D_b = 2\ D_0 = 3.937$ in., and the elevation of the water surface was measured at 0.5-m intervals (1.64 ft). Then the dam was re-adjusted to make $D_b = 80$ mm (3.15 in.) and the water surface elevations were measured again. This process was repeated until four series of measurements had been made for different values of D_b; then, each set was plotted and the back-water curve from 3.937 in. to 2.087 in. (the depths of water above the weir), was measured. The result thus obtained is shown by dots in Fig. 14(*a*); corresponding data pertaining to rough channels are shown.

Computation of the Ratio, $\frac{S_0\,l}{D_0}$.—In a rectangular channel such as Fig. 4(*a*), in which $s = 1.0$, the mean value of k between $Y = 2.0$ and $Y = 1.01$ may be determined as follows: $\frac{(1 + a^2)}{\mu^2} = \frac{1 + 0}{4^2} = \frac{1}{16}$; and, from Fig. 9, $k = 0.38$. Therefore, $r = 2\,s + 2\,m\,(s - k) = 2.00 + 2 \times 0.65\,(1 - 0.38) = 2.81$; $r - 2\,s = 0.81$; and, $K = a\,s\,\frac{V_0^2}{gD_0} = 0.55$.

After reading the values of Φ_1 and Φ_2 for $r = 2.81$ and $r - 2\,s = 0.81$ which correspond to various values of $y = \frac{D}{D_0}$ from Fig. 3, the values of $\frac{S_0 l}{D_0}$ can be computed by Equation (18) and arranged as in Table 7(*a*). Next, in order to compare the errors in various back-water formulas that have been used heretofore (see Table 1), the values of $\frac{S_0 l}{D_0}$ computed from these formulas are likewise tabulated, and the back-water curves are drawn, for general comparison, as shown in Fig. 14.

Finally, the errors in the various current formulas are determined by comparing them with the results of the writer's tests. Because of minute surface and stationary waves which exist in the experimental channels, variations of a fraction of a millimeter occur in the test readings. In some cases these variations make it almost impossible to determine the actual error of a formula.

Although the magnitude of the errors in the experimental observations was very small, the effects of these errors on the back-water curves for small channels were quite remarkable. In the preliminary tests the measured values were quite erratic, due to deflections in the channel structure between beam

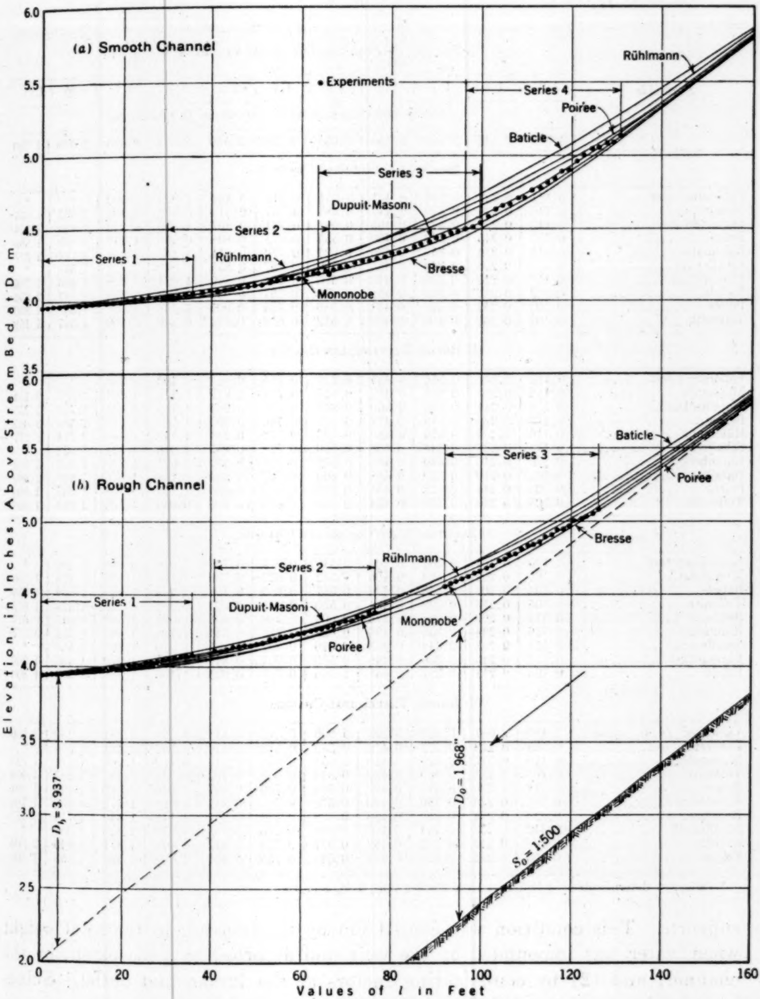


FIG. 14.—BACK-WATER MEASUREMENTS; TEST VALUES COMPARED WITH RESULTS BY VARIOUS INVESTIGATORS; RECTANGULAR CHANNEL (SEE FIG. 4(a)).

TABLE 7.—BACK-WATER MEASUREMENTS; COMPARISON OF THE RATIO, $\frac{S_0\,l}{D_0}$, DETERMINED BY EXPERIMENT AND BY FORMULAS

Formulas	RATIOS, $\frac{S_0\,l}{D_0}$, FOR THE FOLLOWING VALUES OF $y = \frac{D}{D_0}$:										
	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.05	1.01
	AND FOR THE CORRESPONDING VALUES OF D, IN INCHES:										
	3.740	3.543	3.347	3.150	2.953	2.756	2.559	2.362	2.165	2.070	1.990
(*a*) SMOOTH RECTANGULAR CHANNEL											
Laboratory test	0.111	0.222	0.344	0.461	0.581	0.714	0.853	1.020	1.200	*	*
Mononobe	0.110	0.224	0.340	0.459	0.584	0.707	0.844	0.992	1.194	1.333	1.655
Dupuit-Masoni	0.113	0.227	0.345	0.465	0.591	0.724	0.867	1.030	1.242	1.409	1.724
Bresse	0.107	0.215	0.326	0.439	0.555	0.675	0.808	0.956	1.147	1.299	1.569
Rühlmann	0.116	0.234	0.359	0.486	0.623	0.772	0.941	1.148	1.449	1.714	2.277
Schaffernak	0.112	0.228	0.344	0.465	0.590	0.727	0.855	1.070	1.328	1.558	*
Ehrenberger	0.110	0.216	0.330	0.446	0.570	0.701	0.849	1.021	1.270	1.466	1.766
Baticle	0.102	0.210	0.329	0.458	0.600	0.757	0.954	1.198	1.576	1.943	2.690
Poirée	0.103	0.211	0.327	0.451	0.586	0.735	0.905	1.106	1.368	1.553	1.800
Tolkmitt	0.101	0.203	0.306	0.409	0.512	0.618	0.726	0.840	0.966	1.047	1.158
(*b*) ROUGH RECTANGULAR CHANNEL											
Laboratory test	0.113	0.231	0.355	0.485	0.619	0.764	0.922	1.106	1.388	*	*
Mononobe	0.115	0.233	0.355	0.482	0.615	0.759	0.919	1.110	1.384	1.627	2.134
Dupuit-Masoni	0.119	0.241	0.368	0.500	0.640	0.792	0.963	1.166	1.464	1.722	2.256
Bresse	0.113	0.229	0.348	0.472	0.603	0.740	0.903	1.092	1.361	1.595	2.065
Rühlmann	0.116	0.234	0.359	0.486	0.623	0.772	0.941	1.148	1.449	1.714	2.277
Schaffernak	0.112	0.228	0.344	0.465	0.590	0.727	0.855	1.070	1.328	1.558	*
Ehrenberger	0.110	0.216	0.330	0.446	0.570	0.701	0.849	1.021	1.270	1.466	1.766
Baticle	0.102	0.210	0.329	0.458	0.600	0.757	0.954	1.198	1.576	1.943	2.690
Poirée	0.103	0.211	0.327	0.451	0.586	0.735	0.905	1.106	1.368	1.553	1.800
Tolkmitt	0.105	0.213	0.322	0.434	0.550	0.670	0.805	0.959	1.166	1.333	1.669
(*c*) SMOOTH, RIGHT-TRIANGULAR CHANNEL											
Laboratory test	0.100	0.201	0.302	0.404	0.510	0.615	0.725	0.848	0.986	1.102	*
Mononobe	0.100	0.201	0.303	0.405	0.512	0.622	0.736	0.854	1.007	1.118	1.390
Baticle	0.104	0.209	0.315	0.424	0.537	0.655	0.784	0.934	1.139	1.309	1.655
Tolkmitt	0.105	0.213	0.322	0.434	0.550	0.672	0.805	0.960	1.166	1.335	1.673
Bresse	0.113	0.229	0.348	0.473	0.604	0.740	0.903	1.093	1.362	1.547	2.073
Rühlmann	0.116	0.234	0.359	0.486	0.623	0.772	0.941	1.148	1.449	1.714	2.277
Schaffernak	0.112	0.228	0.344	0.465	0.590	0.727	0.885	1.070	1.328	1.558	*
Ehrenberger	0.111	0.216	0.330	0.446	0.570	0.701	0.849	1.021	1.270	1.466	1.766
Poirée	0.103	0.211	0.327	0.451	0.586	0.735	0.905	1.106	1.368	1.553	1.800
(*d*) ROUGH, TRAPEZOIDAL CHANNEL											
Values of D, in inches †	7.480	7.087	6.693	6.299	5.906	5.512	5.118	4.724	4.331	4.130	3.980
Laboratory test	0.105	0.212	0.322	0.436	0.552	0.678	0.814	0.992	1.194	1.370	*
Mononobe	0.110	0.217	0.327	0.436	0.554	0.681	0.818	0.979	1.204	1.373	1.723
Tolkmitt	0.107	0.211	0.320	0.429	0.544	0.663	0.792	0.939	1.132	1.285	1.578
Bresse	0.112	0.226	0.345	0.467	0.595	0.729	0.887	1.070	1.326	1.499	1.991
Ehrenberger	0.111	0.216	0.330	0.446	0.570	0.701	0.849	1.021	1.270	1.466	1.766
Rühlmann	0.116	0.234	0.359	0.486	0.623	0.772	0.941	1.148	1.449	1.714	2.277
Schaffernak	0.112	0.227	0.344	0.465	0.589	0.728	0.885	1.070	1.328	1.558	*
Baticle	0.106	0.217	0.332	0.452	0.577	0.712	0.872	1.078	1.374	1.640	2.203
Poirée	0.103	0.211	0.323	0.451	0.586	0.735	0.905	1.106	1.368	1.553	1.800

* Value not determinable. † Different from Tables 7(*a*), 7(*b*), and 7(*c*).

supports. This condition was caused (1) by the increase in the total weight when water was impounded by the weir and absorbed to some extent by the channel; and (2) by complicating factors at the intake and outlet. Subsequently, however, these defects were mostly corrected; the experiments were conducted under almost ideal conditions; and the results coincided fairly well with the writer's calculated values.

For example, comparing the experimental and calculated values of various formulas of back-water (see Fig. 14(*a*)) for the case of a smooth rectangular flume, the Dupuit-Masoni formula is found to be the most rational, but the values of $\frac{S_0 l}{D_0}$ are a little greater, due to using the Chezy formula in which C is considered as a constant (that is, m being assumed equal to 0.5 in Equation (2)). However, this formula (Item No. 2, Table 1) agrees more nearly with experiments than any of the others.

The Dupuit-Masoni formula was adopted for a study of the relation derived by Bresse, and the section was a broad one. Errors due to these two assumptions are negative in sign and their magnitude is far greater than the positive errors due to assuming a positive value of C. Consequently, the ratios, $\frac{S_0 l}{D_0}$, by the Bresse formula are slightly too small.

The Rühlmann formula (Item No. 3, Table 1) is derived by neglecting the change in velocity head in the Bresse formula. It yields values considerably in excess of those found by the writer, using a smooth channel with a steep slope and a relatively high velocity. This is particularly true toward the up-stream end of the profile where the rise in water surface is slight and, hence, this formula is not applicable when it is desired to find the position of a point which is to be regarded as the upper limit of the back-water.

The Schaffernak formula (Item No. 5, Table 1) is based upon almost the same assumption as the Rühlmann formula. It was found to coincide somewhat better than the Rühlmann formula with experimental results, since m is equal to 0.75, C being regarded as a constant. Values of $\frac{S_0 l}{D_0}$ by this formula are assumed to be considerably greater because the rectangle is assumed to be broad and the velocity head is disregarded.

In applying the Ehrenberger formula (Item No. 6, Table 1), the rectangle is assumed to be broad, which gives rise to negative errors, but those caused by neglecting the velocity head tend to offset them so that, in general, positive errors are obtained. However, the errors were found to be relatively smaller than in the case of the Rühlmann and Schaffernak formulas.

In the Baticle formula (Item No. 7, Table 1) the velocity head is disregarded, and the irrational assumption is made that $A^2 R = z^5$, for a rectangular channel. The errors thus introduced are so great as to make this formula entirely unreliable.

The Poirée formula has no theoretical basis in hydraulics, but for a rectangular channel its errors were found to be rather less than those of some of the more complicated formulas.

The Tolkmitt formula applies to parabolic sections, and the effect of applying it to rectangular sections is to introduce serious negative errors.

Rough Rectangular Channels.—To simulate roughness, the flume was lined with wire netting. Other characteristics were (see Fig. 4(a)): $B = B_0 = 7.874$ in.; $S_0 = 1:500$; $D_0 = 1.97$ in.; $V_0 = 0.886$ ft per sec; $D_b = 3.937$ in.; and, in Equation (2), $m = 0.70$ and $s = 1.0$ in Equation 5(a). Therefore, $k = 0.38$; $r = 2.87$; and $K = 0.16$.

The back-water curves obtained by experiment and from various formulas are shown in Fig. 14(*b*), and values of $\frac{S_0 l}{D_0}$ are shown in Table 7(*b*). The back-water distance in this case is somewhat greater than that for a smooth flume.

In the velocity formula as applied to rough channel sections, $m = 0.70$ actually; but in the Dupuit-Masoni formula, $m = 0.50$ so that the errors are greater than those for smooth surfaces, and values of $\frac{S_0 l}{D_0}$ are always too large.

In the Bresse formula, errors in the ratio, $\frac{S_0 l}{D_0}$, are rather smaller than those in the preceding formula because the errors in assuming and those introduced by assuming the rectangles to be broad cancel each other.

Values computed by the Rühlmann formula agree remarkably well with laboratory tests because the degree of roughness is great and the velocity is small; hence, the effect of neglecting the velocity head is considerably less serious than in the case of smooth channels. Moreover, errors due to the illogical nature of the formula and those introduced by assuming the rectangle to be broad, cancel each other. In this case the errors are smaller in comparison with those for the smooth channel, and the calculated values agree especially well with experimental values in the lower part of the stream where the velocity is small. However, the cancellation of various errors in this manner is rather accidental and can not be expected to occur in a general case.

Smooth, Right Triangular Channel.—The characteristics of the flume for this case were: $S_0 = 1{:}500$; $D_0 = 1.97$ in; $D_b = 3.937$ in.; $V_0 = 0.866$ ft per sec; and, $Y = 2.0$. Furthermore, in case of a triangular section, $s = 2.0$; $k = 1.0$ (and both these indices are fixed); and, $m = 0.70$. Therefore, $r = 2 \times 2 + 1.4\ (2 - 1) = 5.4$; and, $r - 2\,s = 1.4$.

Back-water curves obtained by experiment and those obtained by calculations using the writer's formula, and other formulas, are shown in Fig. 15 and values of $\frac{S_0 l}{D_0}$ are shown in Table 7(*c*).

Originally, the assumption, $A^2 R = z^5$, upon which the Baticle formula is based, was considered suitable for triangular or approximately trapezodial sections, and in spite of the fact that it neglects velocity head, this formula yields results that are fairly in accord with experiment. This illustrates the predominant importance of the shape of the section.

Positive errors due to assuming a broad section are quite serious in applying the Bresse formula to smooth, triangular channels. Since the Chezy formula is also involved, introducing other positive errors, the total is quite considerable.

The Rühlmann, Schaffernak, and Ehrenberger formulas are based on the assumption of a broad section; and the velocity head is neglected, so that the errors are all positive and considerable. Serious errors, likewise, are introduced in working with the Poirée formula.

Rough Trapezoidal Channel.—The characteristics of the flume for this case (see Fig. 13(*c*)) were: a_0 = 11.81 in.; $a = \cot \theta = \cot 45° = 1$; $S_0 = 1{:}500$; D_0 = 3.937 in.; D_b = 7.874 in.; $Y = 2.0$; Q = 0.625 cu ft per sec; and V_0 = 1.453 ft per sec.

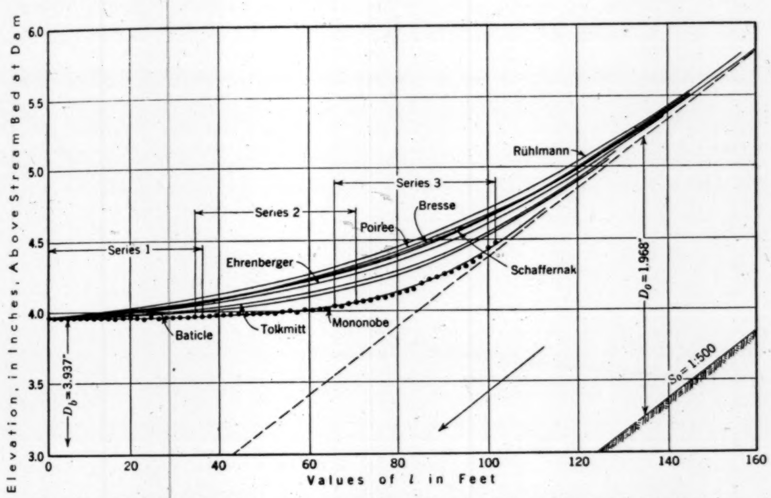


FIG. 15.—BACK-WATER MEASUREMENTS; TEST VALUES COMPARED WITH RESULTS BY VARIOUS INVESTIGATORS; SMOOTH, RIGHT-TRIANGULAR CHANNEL (SEE FIG. 8(*d*)).

The total length of the channel was 12 m (39.37 ft), but the cross-section was large in comparison with the length. The effects of the inflowing water at the up-stream end and of the latticed weir at the lower end extend quite a distance into the channel, reducing the length of a stream in which there is uniform flow, and the effective length is only about 30 ft. Moreover, in order to obtain the same ratio, $\frac{S_0\, l}{D_0}$, it was necessary that the channel be longer than a flume of less depth of water so that nine series of tests were required. Due to the skill of the operators, the results were more uniform than was expected (see Fig. 16).

In the writer's method of computation: $a = 1$; $\mu = \frac{a_0}{D_0} = 3$; $\frac{a}{\mu} = \frac{1}{3}$; and $\frac{1 + a^2}{\mu^2} = \frac{2}{9}$.

Referring to Fig. 9, the average values of s and k between the limits, $y = 1.01$ and $Y = 2.20$, are $s = 1.28$ and $k = 0.535$. In the velocity formula, m equals 0.7 so that $r = 3.60$; $r - 2\,s = 1.04$; and, $K = 0.286$.

Therefore, the integrals, Φ_1 and Φ_2, for any arbitrary values of $\frac{D}{D_0}$ may b read from the curves of Fig. 3. With given values of s and r introduced in Equations (21), the values of $\frac{S_0\, l}{D_0}$ (see Table 7(d)) can be determined easily, and such back-water curves as those shown in Fig. 16 can be plotted. For comparison the back-water curves based on various formulas are computed and these are shown by solid-line curves in Fig. 16.

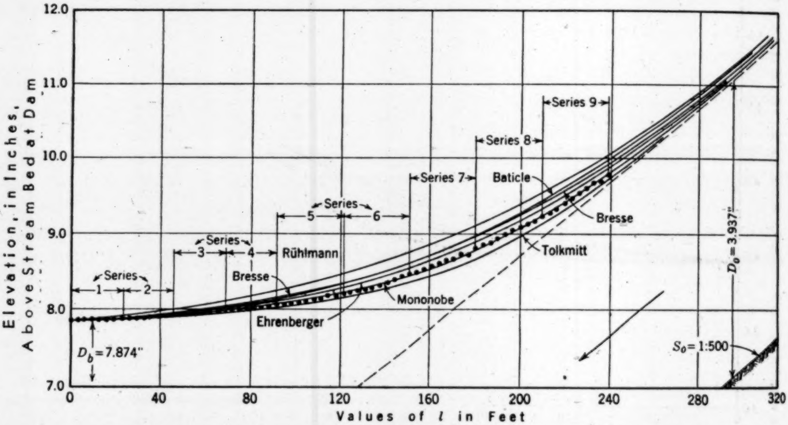


FIG. 16.—BACK-WATER MEASUREMENTS; TEST VALUES COMPARED WITH RESULTS BY VARIOUS INVESTIGATORS; ROUGH TRAPEZOIDAL CHANNEL (SEE FIG. 13(c)).

The Tolkmitt formula was derived for application to the parabolic section. Because of the analogy of this section to the cross-section of the ordinary river channel, this formula gives results that are fairly in accord with actual conditions. Positive errors are produced because $m = 0.5$ was used in the velocity formula, but errors due to the difference between sectional shapes are negative and relatively large so that the total errors are negative and are not large. In applying the Bresse formula to rough trapezoidal channels errors due to the shape of the section and the Chezy velocity formula are both positive and, therefore, fairly large.

Use of the Rühlmann formula introduces positive errors due to the shape of the section, the velocity formula, and the fact that the velocity head is neglected. The resulting errors are positive and the largest of those produced by the several formulas.

The velocity equations involved in both the Schaffernak and Ehrenberger formulas are closer to actual conditions than that in the case of the Rühlmann formula, and errors are also considerably less.

In the case of the Baticle formula, the assumption as to the shape of the cross-section is closer to actual conditions than in the case of the Rühlmann formula; but the errors are greater.

10.—MEASUREMENT OF DROP-DOWN

In the experiments to study the drop-down, the same channels were used as in the case of the back-water except that the depth of water was more shallow and the velocity and the slope of the water surface were increased. Tests were made at depths, D_0, of 3.937 in. and 5.906 in.; and the slopes were $S_0 = 1{:}500$, $1{:}1\,000$, and $1{:}2\,000$.

Smooth Rectangular Channels.—Referring to Figs. 8(*b*) and 13(*a*), $B_0 = 7.874$ in.; $D_0 = 3.937$ in.; $D_s = 2.46$ in.; $S_0 = 1{:}1\,000$; $V_0 = 1.385$ ft per sec; $Q = 0.298$ cu ft per sec; and $Z = \frac{D_0}{D_s} = 1.60$.

The height of the water surface is measured at 50-cm (1.64-ft) intervals, but near the lowest end of the channel where the slope of the water surface is steep, readings were taken at 20-cm (7.874-in.) intervals; four observations, for different values of $\frac{D_0}{D_s}$, were plotted; and the drop-down curve from $D_s = 2.461$ in. to $D = 3.839$ in. was traced.

The critical depth, D_c (the limit between jet flow and ordinary flow) was, $D_c = \sqrt[3]{\frac{Q^2}{g\,B^2}} = 2.232$ in. Therefore, in order to prevent the occurrence of the transition from ordinary flow to jet flow so as to interfere with the drop-down, the depth, D_s, at the lowest end of the channel was selected as 2.46 in.

Computation of the Ratio, $\frac{S_0\,l}{D_0}$.—In a rectangular channel, such as Fig. 4(*a*), in which $m = 0.70$; and $s = 1.0$, the average value of k between $Z = 1.60$ and $z = 1.01$ is determined from Fig. 9, as follows: $a = 0$; $\mu = \frac{B_0}{D_0} = 2$; $\frac{1 + a^2}{\mu^2} = \frac{1}{4}$; and $k = 0.47$. Therefore, $r = 2.74$, and $K = 0.202$.

The integrals, Ψ_1 and Ψ_2, for a series of z-values are read from Figs. 5, 6, and 7, for $r = 2.74$ and $s = 1.0$; the ratio, $\frac{S_0\,l}{D_0}$, is obtained by substituting these values in Equation (20); and l is computed by simple, algebraic transposition. If the curve corresponding with r and s lies between two adjacent curves of the integrals, Ψ_1 and Ψ_2, the problem is solved by interpolation.

As is evident from Figs. 6 and 7(*a*), the values of Ψ_2, expressing the effect of the change of the velocity head, are relatively quite important for drop-down measurements as compared with back-water measurements. It is only by multiplying them by the coefficient, $K<1$, that the values of Ψ_2 are made smaller than Ψ_1 which expresses the friction loss (see Figs. 5 and 7(*b*)). Consequently, it is clear that all the current drop-down formulas in which the velocity head is neglected, are entirely inadequate.

Referring to Figs. 5 and 7(*a*), the increase in Ψ_1 as compared with the increase in z is very small when z exceeds a certain value. For instance, the difference of Ψ_1 between $z = 3.0$ and $z = 2.0$ is almost equal to zero. Accordingly, the actual distance between the two sections is very small, which means that the water surface falls so considerably as to outlaw the formulas now in current use. Moreover, the application of this method of computation is limited to the part of the channel in which the flow is normal; it is not applicable to the parts of the channel shallower than D_C, such as the case in which $z \gtreqless 2$ at the lower end of the channel.

Test Measurements for the Drop-Down Curve in Smooth Rectangular Channels.—Test values for drop-down, with corresponding values derived by current formulas in use, are plotted in Fig. 17(*a*), and corresponding values of $\dfrac{S_0\, l}{D_0}$ are shown in Table 8(*a*).

TABLE 8.—DROP-DOWN MEASUREMENTS; COMPARISON OF THE RATIO, $\dfrac{S_0\, l}{D_0}$, DETERMINED BY EXPERIMENT AND BY FORMULAS

Values of $z = \frac{D_0}{D}$ (1)	Values of D, in inches (2)	Laboratory test (3)	Mononobe (4)	Tolkmitt* (5)	Bresse (6)	Kozeny (7)	Schaffernak (8)	Rühlmann (9)
(*a*) SMOOTH RECTANGULAR CHANNEL								
1.50	2.628	0.0053	0.0046	......	0.0039	0.0030	0.0113	0.0154
1.40	2.815	0.0132	0.0130	......	0.0131	0.0102	0.029	0.0390
1.30	3.030	0.0376	0.0390	......	0.0324	0.0257	0.057	0.0771
1.25	3.150	0.0628	0.0635	......	0.0494	0.0392	0.082	0.1059
1.20	3.280	0.0916	0.095	......	0.074	0.059	0.117	0.146
1.15	3.425	0.140	0.138	......	0.114	0.0924	0.165	0.204
1.10	3.580	0.201	0.196	......	0.182	0.151	0.248	0.299
1.05	3.752	0.307	0.344	......	0.323	0.275	0.423	0.488
1.01	3.898	......	0.765	......	0.714	......	......	0.969
(*b*) ROUGH RECTANGULAR CHANNEL								
1.9	2.073	0.0026	0.0033	......	0.0018	0.0013	0.0032	0.0040
1.8	2.190	0.0064	0.0069	......	0.0045	0.0036	0.0072	0.0096
1.7	2.317	0.0118	0.0122	......	0.0092	0.0073	0.0127	0.0171
1.6	2.462	0.0195	0.0205	......	0.0158	0.0128	0.0209	0.0277
1.5	2.628	0.0350	0.0346	......	0.0268	0.0214	0.0323	0.0431
1.4	2.815	0.0613	0.0578	......	0.0448	0.0354	0.0500	0.0667
1.3	3.030	0.0956	0.0952	......	0.0757	0.0602	0.0780	0.1048
1.2	3.150	0.152	0.164	......	0.134	0.107	0.138	0.174
1.1	3.280	0.292	0.321	......	0.270	0.219	0.269	0.326
1.05	3.425	0.449	0.507	......	0.439	0.370	0.440	0.516
1.01	3.580	......	1.005	......	0.896	......	......	0.997
(*c*) SMOOTH, TRIANGULAR CHANNEL								
1.25	3.156	0.0052	0.0063	0.0100	0.0214	0.0146	0.0250	0.285
1.20	3.28	0.0137	0.0132	0.0248	0.0515	0.0413	0.060	0.0686
1.15	3.424	0.0258	0.0245	0.0528	0.0996	0.0816	0.108	0.128
1.10	3.58	0.053	0.051	0.0988	0.178	0.151	0.191	0.223
1.05	3.75	0.117	0.115	0.202	0.337	0.283	0.367	0.410
1.01	3.9	0.285	0.307	0.494	0.769	......	......	0.892
(*d*) ROUGH TRAPEZOIDAL CHANNEL								
1.40	4.215	0.0030	0.0019	......	0.0054	0.0049	0.0177	0.0236
1.30	4.545	0.0121	0.0098	......	0.0224	0.0177	0.0457	0.0617
1.25	4.72	0.0269	0.0175	......	0.0368	0.0291	0.0707	0.0905
1.20	4.915	0.0458	0.0310	......	0.0584	0.0478	0.1057	0.1306
1.15	5.14	0.0713	0.0532	......	0.0953	0.0772	0.154	0.189
1.10	5.36	0.097	0.096	......	0.158	0.131	0.237	0.283
1.05	5.615	0.220	0.191	......	0.290	0.255	0.414	0.472
1.01	5.85	......	0.490	......	0.660	......	......	0.954

* Blank spaces indicate values not determinable.

In the Kozeny formula, the velocity head is considered and $m = 0.70$ in the velocity formula, but the section is assumed to be a broad rectangle so

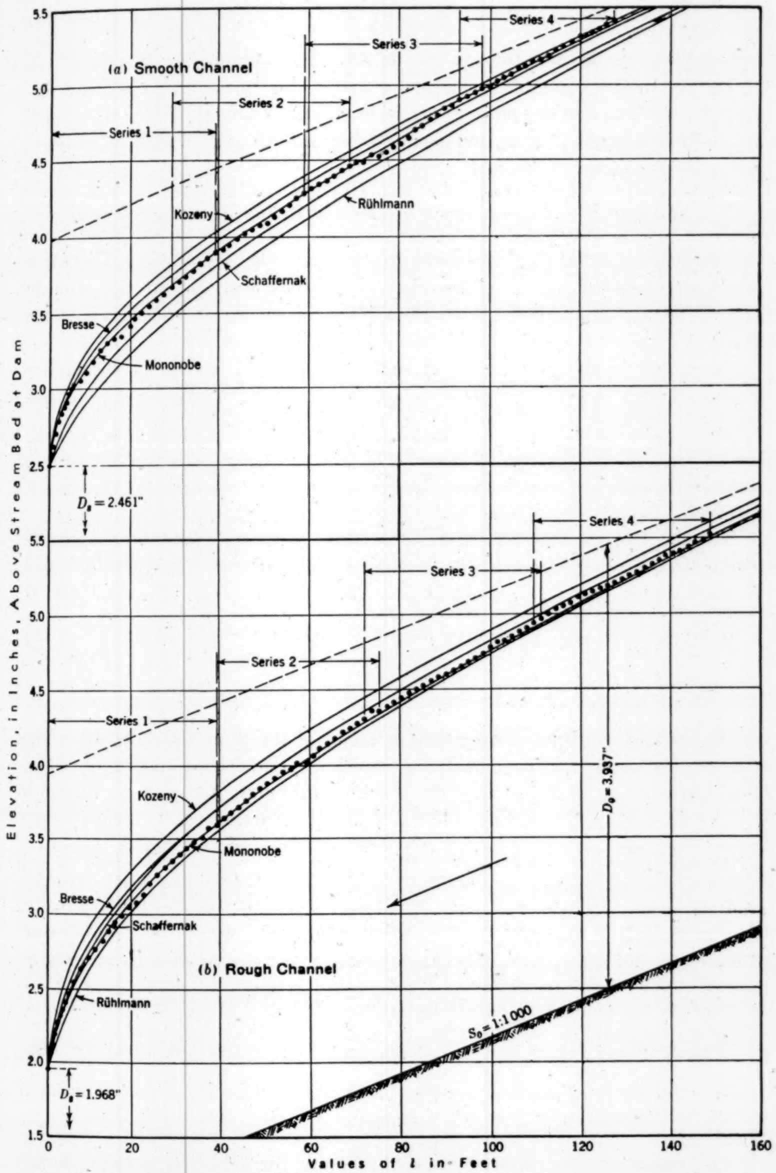


FIG. 17.—DROP-DOWN MEASUREMENTS; TEST VALUES COMPARED WITH RESULTS BY VARIOUS INVESTIGATORS; RECTANGULAR CHANNEL (SEE FIG. 4(a)).

that $\frac{S_0 l}{D_0}$ becomes very small. In other words, negative errors are produced and they become greater toward the low end of the flume.

In the Bresse formula, the negative errors due to assuming a broad rectangular section, and the positive errors due to assuming $m = 0.5$, cancel each other. The resulting errors are negative, but rather smaller than those introduced in the Kozeny formula.

The value of $\frac{S_0 l}{D_0}$ is increased considerably in the Rühlmann formula due to neglecting the velocity head and to assuming that $m = 0.5$. This error is slightly reduced by the assumption of a broad section, but the ultimate errors are very great, being more than 200% in the down-stream part of the flume.

In regard to the Schaffernak formula, $m = 0.75$ is too great a value for a channel such as that used in the experiment because the degree of roughness is relatively slight, and negative errors are produced for $\frac{S_0 l}{D_0}$, so that the resulting errors are considerably less than those involved in the Rühlmann formula.

Rough Rectangular Channel.—For this test, the smooth rectangular channel was made to simulate roughness by lining it with wire netting. Other characteristics were: $B_0 = 7.874$ in.; $D_0 = 3.937$ in.; $S_0 = 1{:}1\,000$; $Q = 0.180$ cu ft per sec; $V_0 = 0.863$ ft per sec; $D_s = 1.968$ in.; $Z = \frac{D_0}{D_s} = 2.0$; and $D_c = \sqrt[3]{\frac{Q^2}{g\,B^2}} = 0.136$ ft.

The actual depth of water at the low end of the flume was 1.67 in. which was almost equal to the calculated values of the critical depth of water. For convenience in computing the back-water curve, the section at which the depth of water, $D_s = 1.968$ in., is used as the starting point. Applying the writer's formula: $s = 1.0$; $m = 0.70$; $a = 0$; $\mu = \frac{B_0}{D_0} = 2$; $\frac{1 + a^2}{\mu^2} = \frac{1}{4}$; and the average value of k, from $Z = 2$ to $z = 1.01$, is $k = 0.46$. Finally, $r = 2.75$; and $K = 0.078$. Values of the integrals, Ψ_1 and Ψ_n, can be read directly from Figs. 5, 6, and 7. Then, similar to the case of back-water computations, it is possible to calculate the values of $\frac{S_0 l}{D_0}$ from Equation (20).

Experimental values and calculated drop-down curves obtained by using various formulas are shown in Fig. 17(b), and values of $\frac{S_0 l}{D_0}$ are given in Table 8(b).

In computing drop-down by the Rühlmann formula, the negative errors due to assuming a broad cross-section, the positive errors introduced by neglecting the velocity head, and the assumption that $m = 0.5$, cancel each other, and the errors are negligible except in the low end of the flume, but the cancellation of the errors is rather accidental.

Accidentally, the algebraic sum of the positive errors introduced by the Schaffernak formula due to neglecting the velocity head, and the negative error involved in assuming that $m = 0.75$, is almost equal to the error caused by assuming that $m = 0.5$. For this reason the result agrees well with that of the Bresse formula.

Smooth Triangular Channel.—The characteristics of the flume for this test were: $S_o = 1:1\,000$; $D_o = 3.94$ in.; $D_s = 3.03$ in.; $Z = \frac{D_o}{D_s} = 1.30$; $V_o = 1.083$ ft per sec; $Q = 0.1165$ cu ft per sec; and $D_c = \sqrt[5]{\frac{2\,Q^2}{g}} = 0.243$ in. Experimentally D_c was found to be 2.98 in., but for convenience the backwater is assumed to occur at the section where $D_s = 3.02$ in.

The computation by the writer's method is as follows: $s = 2.0$ (a constant); $k = 1.0$ (a constant); $m = 0.70$; $r = 5.4$; and $K = 0.247$.

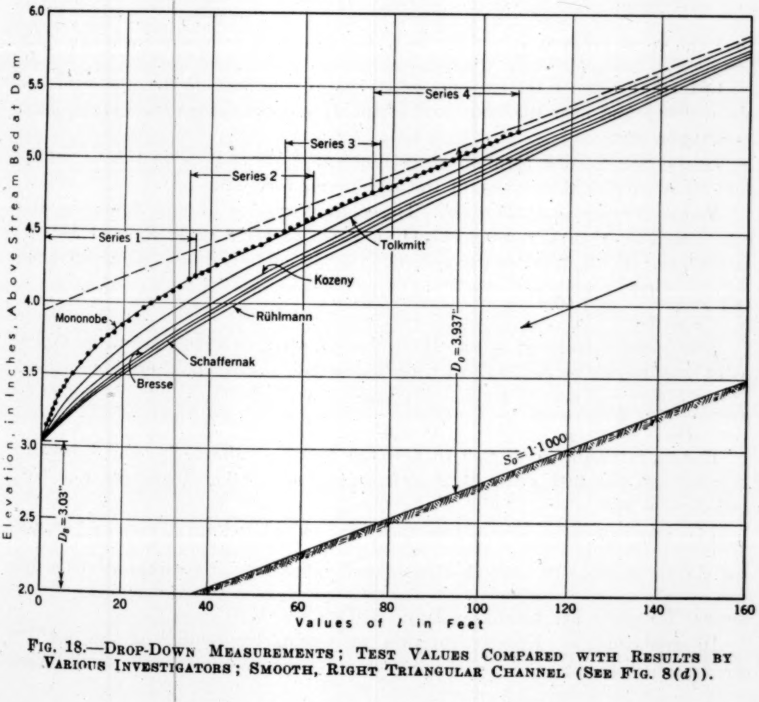


FIG. 18.—DROP-DOWN MEASUREMENTS; TEST VALUES COMPARED WITH RESULTS BY VARIOUS INVESTIGATORS; SMOOTH, RIGHT TRIANGULAR CHANNEL (SEE FIG. 8(*d*)).

As before, the integrals, Ψ_1 and Ψ_2, are read from Figs. 5, 6, and 7, and the ratios, $\frac{S_0\,l}{D_0}$, are computed by Equation (20). The experimental as well as the calculated drop-down curves based on various formulas are shown in Fig. 18 and the calculated values of $\frac{S_0\,l}{D_0}$ based on various formulas are shown in Table 8(*c*).

For the triangular channel section in general, positive errors are introduced by ignoring the velocity head, by assuming rectangular or parabolic sections, and by assuming that $m = 0.5$. Being all positive in sign these errors become quite serious.

The Kozeny formula involves the velocity head and m is assumed equal to 0.70; but the effect of assuming a broad channel section offsets the favorable conditions, and errors greater than 150 or 200% are produced. This indicates once more the importance of the shape of the cross-section in computing the drop-down.

Because of the assumption that $m = 0.5$, errors are introduced in the Bresse formula which are even greater than those in the Kozeny formula.

The Rühlmann formula applied to smooth triangular channels, involves excessive errors due to various causes, which are all positive. The result is that the total error is a serious one; for example, at the point where $\frac{D_0}{D} = 1.01$ (that is, near the extreme upper end of the drop-down), the errors are about 200% and near the lower end of the drop-down these errors are as much as 400 per cent.

In the Schaffernak formula, m is assumed equal to 0.75, so that errors are somewhat smaller than those of the Rühlmann formula.

Rough Trapezoidal Channel.—The characteristics of the flume for this case (see Fig. 13(*c*)) were: $a_0 = 11.81$ in.; $a = \cot\theta = 1$; $S_0 = 1{:}500$; $D_0 = 5.91$ in.; $V_0 = 1.837$ ft per sec; $Q = 1.338$ cu ft per sec; $D_s = 3.937$ in.; and $Z = \frac{D_0}{D_s} = 1.50$.

The critical depth of water, D_C, was computed as 4.016 in. and by experiment was found to be 3.937 in. By the writer's method, therefore, $a = 1$; $\mu = \frac{a_0}{D_0} = 2.0$; $\frac{a}{\mu} = \frac{1}{2}$; and $\frac{1 + a^2}{\mu^2} = \frac{1}{2}$.

Hence, referring to Fig. 9 to find the average values of s and k between $Z = 1.5$ and $z = 1.01$, $s = 1.31$; $k = 0.57$; and $m = 0.65$. Therefore, $r = 3.58$; and $K = 0.30$.

The test values for drop-down and those calculated from various formulas are shown in Fig. 19, and the calculated values of $\frac{S_0\,l}{D_0}$, obtained by using several formulas are found in Table 8(*d*).

In applying the Kozeny formula to rough, trapezodial flumes positive errors are introduced by assuming broad cross-sections, and small negative errors by assuming that $m = 0.70$. However, the total errors are less in this case than in the use of the other formulas, because the velocity head is taken into consideration.

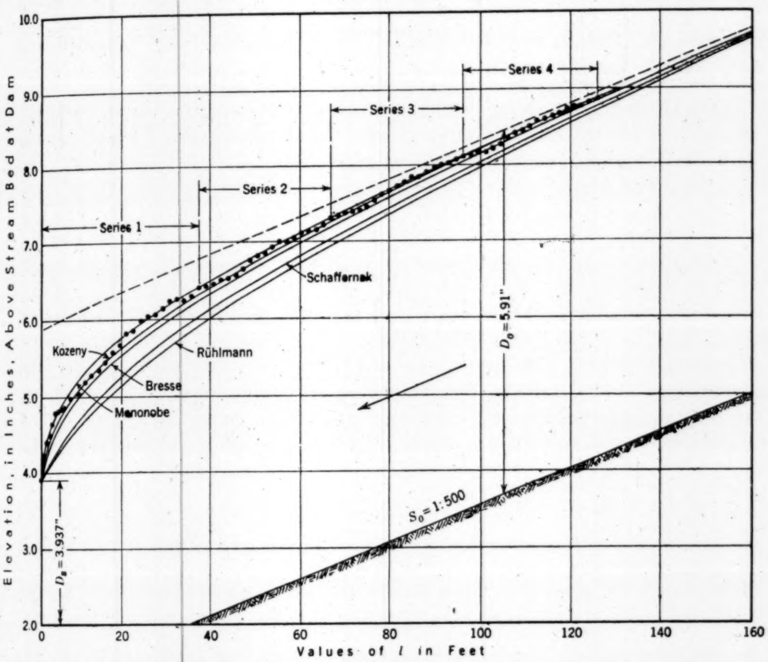


FIG. 19.—DROP-DOWN MEASUREMENTS; TEST VALUES COMPARED WITH RESULTS BY VARIOUS INVESTIGATORS; ROUGH TRAPEZOIDAL CHANNEL (SEE FIG. 13(c)).

The velocity head is considered in the Bresse formula, but since m is assumed equal to 0.5, errors are introduced which are greater than those by the Kozeny formula.

In using the Schaffernak formula, $m = 0.75$ is too great, and insignificant negative errors are introduced, but because the velocity head is ignored, the positive errors are still too great.

The total errors involved are greatest in the case of the Rühlmann formula because it introduces most of the unfavorable conditions cited.

11.—CONCLUDING REMARKS

In formulas for back-water and drop-down which are in current use the errors that enter because of illogical assumptions are:

(1) *Errors Introduced by Assuming C in the Chezy Formula as a Constant.*—

(a) In back-water computations, the actual value of C increases considerably with an increasing depth of water in a down-stream direction. Hence, if C is assumed constant and equal to its value for the original uniform flow, the computed surface slope, S, becomes steeper than the true slope. The depth of water is constant at the weir so that the computed distance, l, between the weir and the section where the depth of water is D, becomes too small. Moreover, S_0 and D_0 are fixed values so that the ratio, $\frac{S_0\, l}{D_0}$, is too small, and the error involved is negative.

(*b*) In drop-down computations the computed velocity is more than the actual if C for the case of uniform flow is used; consequently, the drop computed is too great and positive errors are introduced.

(*2*).—*Errors Created by Neglecting the Change in Velocity Head,* $\frac{d}{dx}\left(\frac{V^2}{2g}\right)$.—

(*a*) In the case of back-water, the velocity, V, decreases in a down-stream direction and the value of $\frac{d}{dx}\left(\frac{V^2}{2\,g}\right)$ is negative. Neglecting this factor, the computed slope, S, is steeper than the true slope corresponding to the fixed quantity of flow, Q. Since the depth of water at the lower end of the flume is constant, the calculated water level at points toward the upper end of the flume is too high; consequently, the elevation of the back-water is too high, so that $\frac{S_0\, l}{D_0}$ becomes too small, and negative errors are produced.

(*b*) In the case of drop-down, $\frac{d}{dx}\left(\frac{V^2}{2\,g}\right)$ is positive; and if this term is disregarded, the computed slope, S, that is necessary to maintain the flow, Q, becomes small and the drop-down in the surface becomes much too great. Positive errors greater than 100% are often introduced in the values of $\frac{S_0\, l}{D_0}$, especially near the lower end of the flume.

(*3*).—*Errors Caused by Assuming the Channel Section to Be Very Wide.*—

(*a*) In the case of back-water, the effect of assuming a wide cross-section is the same as assuming that the wetted perimeter, P, is equal to the width of the surface. That is, the hydraulic mean depth, R, is assumed to be greater than its true value. Thus, the value of S in Equation (1) becomes too small, and positive errors are introduced.

(*b*) The result of using too small a value of S, in the case of drop-down, is to make the computed water-surface slope steeper than the true surface slope and, consequently, the computed value of $\frac{S_0\, l}{D_0}$, is too great.

The foregoing conclusions, and supporting arguments, indicate that formulas for drop-down and back-water in current use, result in ratios, $\frac{S_0\, l}{D_0}$, which are either too great or too small. The effect of adopting illogical assumptions is to superpose positive and negative errors. In certain cases these errors may cancel to some extent, but more often they build up into serious discrepancies of one sign or the other. For this reason many of the old formulas have yielded data involving seriously large errors, especially in the case of drop-down.

ACKNOWLEDGMENTS

For careful reading of the manuscript, for making valuable suggestions, and for important corrections, the writer wishes to acknowledge, sincerely, his indebtedness to Harold A. Thomas and Sherman M. Woodward, Members, Am. Soc. C. E.

DISCUSSION

J. C. STEVENS,[5] M. AM. SOC. C. E. (by letter).—The computation of any form of water-surface profile—on sustaining or adverse slopes, under supercritical or sub-critical flow conditions, in either rough or smooth uniform channels—may be made quickly by the simple step-by-step procedure fully described and illustrated by the writer in his discussion[6] of the paper by Arthur E. Matzke, Jun. Am. Soc. C. E.[7]

The functions set up by the author are only applicable to channels of uniform cross-section (that is, rectangle, trapezoid, triangle, etc.), whereas by far the greater number of surface profile determinations must be made for channels of irregular cross-section. For the latter case computations must be

TABLE 9.—SURFACE PROFILE IN SMOOTH RECTANGULAR CHANNEL
(Width, $B = 0.656$ ft; discharge, $Q = 0.173$ cu ft per sec; Kutter's $n = 0.009$; and $S_o = 0.00200$)

Depth of flow, D, in feet (1)	Hydraulic area, A, in square feet (2)	Average velocity, V, across a section (3)	Velocity head, $\frac{V^2}{2g}$ (4)	Specific energy, ϵ (5)	Increment of specific energy, $\Delta \epsilon$, between adjacent depths (6)	Wetted perimeter, P, in feet (7)	Hydraulic radius, R, in feet (8)
0.328	0.215	0.805	0.0100	0.3380	....	1.312	0.164
0.312	0.204	0.847	0.0112	0.3232	0.0148	1.280	0.160
0.296	0.194	0.892	0.0124	0.3084	0.0145	1.248	0.156
0.279	0.183	0.945	0.0139	0.2929	0.0155	1.214	0.151
0.262	0.172	1.005	0.0158	0.2778	0.0151	1.180	0.146
0.246	0.161	1.074	0.0180	0.2640	0.0136	1.148	0.140
0.230	0.151	1.145	0.0204	0.2504	0.0136	1.116	0.135
0.213	0.140	1.235	0.0237	0.2367	0.0137	1.082	0.129
0.197	0.129	1.340	0.0279	0.2249	0.0118	1.040	0.124
0.181	0.119	1.453	0.0328	0.2138	0.0111	1.016	0.116

TABLE 9.—(*Continued*)

Depth of flow, D, in feet (1)	Chezy's coefficient, C (9)	Friction slope, S'_f, at a given depth point (10)	Average friction slope, S_f between two depth points (11)	Difference, $S_o - S_f$, between bed slope and friction slope (12)	Distance, Δl, between adjacent depth points (13)	Distances, l, Along the Bed of a Channel, Measured from the Weir: Computed (14)	By laboratory test (15)
0.328	115	0.00030	....	....	....	......	......
0.312	115	0.00034	0.00032	0.00168	8.8	8.8	9.2
0.296	115	0.00039	0.00037	0.00163	8.9	17.7	18.2
0.279	115	0.00045	0.00042	0.00158	9.2	26.9	28.1
0.262	115	0.00052	0.00049	0.00151	10.0	36.9	37.6
0.246	114	0.00063	0.00058	0.00142	9.6	46.5	47.6
0.230	114	0.00075	0.00069	0.00131	10.4	56.9	58.5
0.213	113	0.00091	0.00083	0.00113	12.4	69.3	69.6
0.197	113	0.00114	0.00102	0.00098	12.0	81.3	83.3
0.181	113	0.00145	0.00130	0.00060	18.5	99.8	98.0

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[6] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 666.

[7] "Varied Flow in Open Channels of Adverse Slope", by Arthur E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 651.

made by a trial method one of which was suggested by the writer[8] in 1925. For uniform sections a simple direct computation will yield results amply accurate for all practical purposes without going through the maze of mathematical processes so laboriously set forth by the author. In fact, the simple step-by-step formula used by the writer is fundamentally more accurate than that offered by Professor Mononobe, because there are no basic assumptions and approximations entering into its composition. The only limitation is that sections shall be taken sufficiently near each other so that the surface profile may be considered a straight line between them.

Assumptions that wetted perimeters and areas are exponential monomials, and that the Chezy coefficient is constant, are not at all necessary; neither is it necessary to find neutral and critical depths and velocities. One merely fortifies himself with a table of Kutter coefficients (or Manning's, or any other set of coefficients that he prefers) and a slide-rule and performs the simple operations shown in Table 9. This table shows the calculated profile of the back-water curve under discussion for the smooth rectangular channel. The discharge is not given, but presumably may be calculated from $A_0 V_0 = 0.173$ cu ft per sec. From the neutral slope, radius, and velocity given, one may find that Kutter's $n = 0.009$. Using the formula,[6]

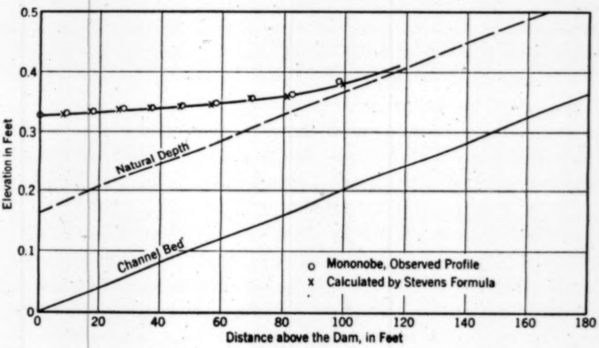

FIG. 20.—COMPARISON OF PROFILES FOR A SMOOTH RECTANGULAR CHANNEL

$$\Delta l = \frac{\Delta \epsilon}{S_f - S_0} \quad \text{.......................} (35)$$

in which Δl = the distance between adjacent depths; S_f = friction slope; S_0 = bed slope; and, $\Delta\epsilon$ = increment of specific energy between adjacent depths, or:

$$\Delta \epsilon = \left(D_1 + \frac{V^2_1}{2\,g}\right) - \left(D_2 + \frac{V^2_2}{2\,g}\right) \quad \text{.............} (36)$$

The depths given, in inches, in Table 7 were converted to feet and used in Table 9. The slope, S'_f, is the friction slope at the depth point, whereas

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[8] *Engineering-News Record*, October 1, 1925, p. 550.

S_f is the average friction slope between depths. For such problems very little difference is found whether one averages slopes, areas, hydraulic radii, or velocities between depth points.

In the application of Equation (35) the specific energy ϵ has increasing values down stream, because the bed slope is greater than the friction slope. Therefore, values of $\Delta\epsilon$ are negative as are also values of $(S_f - S_0)$, making Δl positive up stream. Columns (14) and (15), Table 9, show how closely the calculated distances above the dam agree with the distances observed in the laboratory.

Fig. 20 shows the profile plotted from "Laboratory tests", Table 7, and the profile calculated by Equation (35). The agreement is practically perfect as such hydraulic calculations go.

CHESLEY J. POSEY,[9] JUN. AM. SOC. C. E. (by letter).—A unique method of solving the general equation of motion for steady non-uniform flow in uniform channels is presented in this paper. It is the only method yet published that takes into account the effects of changing velocity head, friction, and channel shape, without any necessity for dividing the reach into steps. The velocity head is assumed to be equal to a constant, α, times the theoretical velocity head based on the average velocity in the cross-section. At present, there is no basis for any other assumption. Consideration of the friction factor is based on an exponential type of formula; no more satisfactory formula is available. The assumption is made that the friction loss for a stream flowing at Velocity V, and Depth D, not the neutral depth, is the same as if there were neutral flow at that velocity and depth. This assumption is open to question, because of the inherent differences between convergent, parallel, and divergent flow. In uniform channels, however, the angle of divergence is not likely to be great enough for serious errors to arise from this source. Consideration of the effect of the shape of the channel is made possible by expressing the area and wetted perimeter as monomial exponential functions of the depth. Simple logarithmic plotting will demonstrate the high degree of approximation that is possible, over a wide variety of channel shapes.

Proof of the accuracy of the author's method is evidenced by the remarkable agreement obtained in the laboratory tests, reported in the paper. These tests covered as wide a range of the variables in question as could be included in a laboratory investigation without unreasonable cost. Data secured from tests in larger and longer channels would be desirable to establish a further check. Additional information might be secured as a by-product of some other investigation, and it is to be hoped that experimental workers and practicing engineers will take advantage of opportunities to secure and report further data on back-water and drop-down curves.

The author's method possesses definite advantages over those methods that require the use of steps. A vital part of the curve may be determined quickly without the tedious computation of steps intervening between it and the control point. Furthermore, theoretical curves such as those presented by the author may be used in the study of the shapes of the various possible surface curves, and in determining under what circumstances they will occur and from what feature their position along the length of the channel is fixed. The author applies his method to the two most common types of back-water curves. A complete discussion of the twelve possible types has been given[10] by Sherman M. Woodward, M. Am. Soc. C. E.

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Integration, taking into account the shape of the channel, is made possible by the author's empirical Equations (13). Before giving these formulas the author sets up Differential Equation (8), based on Equations (5) in which the hydraulic area, the wetted perimeter, the hydraulic radius, and the average velocity "are expressed most generally as functions of D, the depth of water measured from the lowest line of the channel bed." As can be seen from Equations (24), (28), (29), and (31), Equations (5) are not the most general; as approximations it is doubtful whether they are much better than Equations (13). Since neither Equation (8) nor Equation (12) is integrated, it is difficult to see why Cases 1 and 2 are included in the paper. Their presence, for which there is only a slight logical justification, makes the paper more formidable than it need be.

To determine the values of s and k, the area and wetted perimeter may be plotted logarithmically as functions of the depth, and straight lines approximating the relations over the range desired, drawn in by eye. The values of s and k are determined by the slopes of these lines. This graphical method has the advantage of enabling one to judge the closeness of fit obtained. In cases where there is no question but that a high degree of approximation will be secured, Figs. 9, 11, and 12 may be used.

The writer has been unable to check certain values from these diagrams and believes they may contain errors. For the example of the rectangular section for which the author gives $s = 1.0$ and $k = 0.38$, the writer obtains $s = 1.0$ and $k = 0.41$. For the example of the circular section, for which the author gives $s = 1.34$ and $k = 0.59$, the writer obtains $s = 1.29$ and $k = 0.65$. These results were obtained by the graphical method and can be checked by any one familiar with logarithmic plotting; for this reason it is considered unnecessary to reproduce the graphical work herein.

The author's method is simple and straightforward. It may justly be considered the culmination of the work started by Belanger and Bresse. In making the method useful to practicing engineers, it will be necessary to simplify the charts of Fig. 3, re-drawing them to a larger scale, and separating the curves for ϕ_1 and ϕ_2, as has been done with ψ_1 and ψ_2 in Figs. 5, 6, and 7.

A. A. KALINSKE,[11] ESQ. (by letter).—Undoubtedly, the most important contribution by the author is the presentation of data from certain carefully conducted experiments on back-water and drop-down curves and his discussion of the different varied-flow expressions developed by others. Although he does not go into any great detail as to experimental procedure, which is always of interest to those engaged in experimental hydraulics, his general description indicates considerable painstaking experimental and analytical effort. His "Concluding Remarks" should be of particular interest to those concerned with errors introduced by making various assumptions when setting up or using varied-flow formulas.

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[11] Instructor in Hydr. and San. Eng., State Univ. of Iowa, Iowa City, Iowa.

In developing his expression for back-water and drop-down surface curves the author demonstrates effectively that the most complicating item in the development of any varied-flow expression is the difficulty of obtaining a simple relation between the height of water above the channel bottom, the cross-sectional area of water flowing, and the hydraulic radius. To express A and R in terms of depth always leads to extremely complicated notations and mathematical expressions. Nearly all varied-flow expressions developed have led to such integrations that special tables and curves must be used if a problem is to be solved in any reasonable time. Furthermore, the special notations used require considerable time to master, and, frequently, the fundamental hydraulic principles are obscured. This makes it particularly difficult to present the subject of varied flow to students of hydraulics.

The author's back-water and drop-down functions are quite involved and probably will not be of great use to practicing engineers, or of great value in presenting the general subject to students. In trying to apply the various functions and also other existing varied-flow formulas to non-uniform flow in circular sewers, the writer encountered considerable difficulty, and lost much time in obtaining answers to specific problems.

The author's formulas for total length of back-water and drop-down curves (Equations (18) and (20), respectively) can be used for any length of reach as long as the relations between depth, hydraulic radius, and area remain constant and the channel slope does not vary. However, in many types of cross-sections, the constants in the type of monomial used by the author, in expressing the relation between area, hydraulic radius, and depth, vary enough so that good accuracy in outlining the surface curve can only be obtained by solving for distances between successive depths. Furthermore, if a number of points are desired on the surface curve, computation of its total length by solving for the distance between successive depths does not complicate matters, especially if the reaches need not be too short in order to obtain good precision.

A considerable number of mathematical operations and notations can be omitted without the sacrifice of any precision or lengthening of time of computation by dispensing with an expression for hydraulic radius in terms of depth (Equation (13c)). Since the hydraulic radius, relatively, changes the least from depth to depth of any of the variables, it is a convenient term to average in any reach; and in the development of the varied-flow integral from the fundamental Bernouilli equality (Equation (3)), worth while simplifications result if the velocity is the variable instead of the depth. Note that the term, $\frac{d\,(V)^2}{2\,g\,dx}$, in Equation (3), is equal to $\frac{V\,dV}{g\,dx}$. For many common types of cross-sections, whether regular or irregular, for a particular range of depths, the area and depth can be conveniently expressed in a straight-line relationship of the type, $D = a\,A + b$. Substituting $\left(\frac{Q}{V}\right)$ for A, the value of $\frac{dD}{dx}$ in Equation (3) becomes $-\frac{a\,Q\,dV}{V^2\,dx}$. The straight-line relationship is much more simple to use since the slope of the line, a, can be so easily obtained; for any given reach, it is equal to $\frac{D_1 - D_2}{A_1 - A_2}$.

Making the foregoing changes and substitutions in Equation (3), the following integral results:

$$dx = \frac{\alpha\,V\,dV}{\frac{g\,V^2}{C^2\,R^{2m}} - S_0\,g} - \frac{a\,Q\,dV}{V^2\left(\frac{V^2}{C^2\,R^{2m}} - S_0\right)} \qquad \text{(37)}$$

which, on integration, gives an expression for the distance between any two depths (or velocities):

$$L = \frac{\alpha\,C^2\,R^{2m}}{2\,g} \log_e \left(\frac{V_1^2 - V_u^2}{V_2^2 - V_u^2}\right) - \frac{a\,Q}{S_0}\left(\frac{1}{V_1} - \frac{1}{V_2}\right) - \frac{a\,Q}{2\,S_0\,V_u} \log_e \frac{\frac{V_1 - V_u}{V_1 + V_u}}{\frac{V_2 - V_u}{V_2 + V_u}} \qquad \text{(38)}$$

In Equation (38), $V_u = C\,R^m\,S_0^{0.5}$, in which R is the average value of the hydraulic radius between the depths chosen. If the surface curve in the reach approaches a straight line, R is an arithmetical average; if it is considerably concave or convex, the value of R can be slightly decreased or increased to make it approach more truly the average in the section. Using only a slide-rule and arithmetical averages of R, computations were performed on the experimental back-water and drop-down curve data given by the author (see Table 10). Equation (38) applies to either back-water or drop-down curves on sustaining slopes; V_1 is the up-stream velocity and V_2, the down-stream velocity.

TABLE 10.—COMPUTATIONS PERTAINING TO A SMOOTH RECTANGULAR CHANNEL

D-inches	*V*	*R*	*C*	Increment in *L*	VALUES OF *L*		
					Computed	Test	Mononobe
(*a*) BACK-WATER CURVES; $S_0 = 0.002$							
3.937	0.805	0.164	152	0	....	....	....
2.953	1.074	0.140	152	47.4	47.4	47.6	47.8
2.165	1.453	0.116	152	49.5	96.9	98.4	97.0
(*b*) DROP-DOWN CURVES; $S_0 = 0.001$							
2.461	2.200	0.126	155	0	....	....	....
3.030	1.790	0.143	155	13.5	13.5	12.4	12.8
3.752	1.446	0.160	155	99.0	112.5	101.0	110.0

The values for C (Table 10) were computed from data given by the author; the value of m in the back-water problem was 0.65 and in the drop-down problem, 0.70. Note that the increments used are relatively large, and the values obtained are comparable in accuracy to those of the author. Although the expression itself appears quite formidable, the computations are not tedious and can be performed very quickly on a slide-rule. It is also of advantage to be able to use the same expression for either back-water or drop-down curves instead of different functions as is done by the author.

For a rectangular section, the value of a, of course, remains constant; also, for wide trapezoidal sections, it does not change greatly in value for different ranges of depth. For circular sections, a does not change much in its value in a range of depths between 10% to 90% of the diameter. For triangular and narrow trapezoidal sections the more accurate relation between depth and area is of the type, $D = k\, A^{0.5}$.

Use of this parabolic relationship complicates the second term of the integral in Equation (37); however, direct integration is quite possible, and is not beyond the scope of the mathematical equipment of the average engineer. The straight-line and parabolic reationship will take care of all types of channels, and there seems no real necessity to complicate matters by using monomials of the type introduced by the author.

The intricate mathematical expressions presented by Professor Mononobe are perhaps justified in that they provided one means for obtaining theoretical surface curves which coincided almost exactly with the experimental results. However, it seems that the same effect could have been accomplished by much simpler methods. If, in outlining surface curves by computing distances between successive depths, there would be a loss of precision and time, then the author's functions would be of distinct help, however, since such is not the case it seems that nothing in particular is gained for any engineer or student of hydraulics to familiarize himself with the mathematics presented so that he can use the author's methods with confidence.

From the standpoint of presenting the subject of back-water and drop-down curves to those unfamiliar with the hydraulics of varied flow, the author's discussion of the fundamentals could be improved if it were pointed out how the momentum principle applies and how it can be used as a basis for the development of the fundamental equations just as readily as Bernouilli's theorem. For instance, since the acceleration (or retardation) of the water at any point is equal to $\frac{V\,dV}{dx}$, it can be readily shown that the change in velocity head with respect to distance is equal to the ratio of the water acceleration to the gravity acceleration. From Equation (3) it can be seen that this ratio is equal to the difference between friction slope and slope of the water surface. The acceleration term is very useful in indicating what must be done with eddy and turbulence losses for flow at a decreasing velocity. The writer wonders whether there was any need to take account of such extra losses in the author's experimental data on back-water. No definite mention of it is made in the discussion of the test results.

Boris A. Bakhmeteff,[12] M. Am. Soc. C. E. (by letter).—One cannot fail to be impressed by the painstaking efforts and the time which Professor Mononobe and his collaborators must have invested in the preparation of this paper. In the early stage of his studies on non-uniform flow[13] more than twenty-five years ago, studies which led to the methods subsequently developed in "Hydraulics of Open Channels",[14] the writer was confronted with the maze of the different formulas then extant and went through a process of comparison similar to that so ably presented in Professor Mononobe's work. Obviously, to make the general Bélanger formula (Equation (1)) usable for practical purposes, it must be integrated in relation to cross-sectional forms and resistance formulas as they actually occur in engineering practice. That requires, first, stepping outside the limits of the idealized broad rectangular and parabolic channel sections, as treated by the earlier pioneers, as well as doing away with the assumption of a constant Chézy factor value, C, as used by Bresse, Rühlman, and others.

Professor Mononobe correctly indicates that "monomial forms" should be used for this purpose, and recourse is taken to exponential expressions, which is in accordance with earlier suggestions of the writer, suggestions of which Professor Mononobe had no previous knowledge.

It appears to the writer, however, that the thought of giving monomial expressions separately to the area, wetted perimeter, and other elements, as suggested in Equations (13), is not so fortunate. In fact, in such general form, the assumption is somewhat misleading, and, moreover, it tends to eclipse the real significance of exponential expressions. The essence of the method lies in the fact, that any curve, $y = f(x)$, the value of which increases with x (Fig. 21), may be replaced within certain limits ($A - B$, Fig. 21), with sufficient approximation, by an exponential expression of the type, $y = a x^n$.

With reference to Equation (1), the area and the other elements do not enter singly, but in the combination,

$$A \, C \sqrt{R} = K \quad \ldots\ldots\ldots\ldots (39)$$

constituting thus a combined parameter, which connects the bottom slope with the discharge in uniform motion,

$$Q = K_0 \sqrt{S_0} \quad \ldots\ldots\ldots\ldots (40)$$

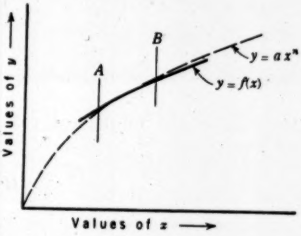

Fig. 21

and which the writer qualifies as the "carrying capacity."[15] This value, properly substituted into Equation (1), makes it,

$$S = S_0 - \frac{dD}{dx} = \frac{Q^2}{K^2} - \alpha \frac{S_0}{\sigma} \frac{K^2_0}{K^2} \frac{dD}{dx} \quad \ldots\ldots\ldots\ldots (41)$$

in which σ is the critical slope.

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[12] New York, N. Y.

[13] "On Varied Flow of Liquids in Open Conduits" (in Russian), St. Petersburg, 1912.

[14] Engineering Society's Monograph, McGraw-Hill Co., 1932.

[15] "Hydraulics of Open Channels", Engineering Society's Mongraph, McGraw-Hill Co., 1932, p. 13.

For an open conduit, Equation (39) is a function increasing with the depth and, within certain limits, can be expressed monomially, in the sense of Fig. 21, leading to a simple relation,

$$\left(\frac{K_2}{K_1}\right)^2 = \left(\frac{D_2}{D_1}\right)^n \qquad (42)$$

with a single value of n = the hydraulic exponent.

In pursuing his course further, Professor Mononobe arrives at Equation (18), with integral functions of the type:

$$\Phi_1 = \int \frac{y^r}{y^r - 1}\, dy \qquad (43a)$$

and,

$$\Phi_2 = \int \frac{y^{(r-2S)-1}}{y^r - 1}\, dy \qquad (43b)$$

Practical application depends on knowing the numerical values of these functions and, therefore, appropriate graphical charts are given in the paper. According to Professor Mononobe, numerical evaluation of the function values in certain cases was direct, which is assumed to mean quadratures, whereas in other cases recourse had to be taken to converging series. Now, in historical retrospect, Equation (18) is generally similar in form to the relations traditionally used by Dupuit, Bresse, and subsequent investigators. With Professor Mononobe's notation the basic relation,[16] Equation (4), which is equivalent to Equation (18), and in the form used by the writer would read:

$$\frac{S_0\ l}{D_0} = \left(\frac{D_b}{D_0} - \frac{D}{D_0}\right) - (1 - \beta)\left[B\left(\frac{D_b}{D_0}\right) - B\left(\frac{D}{D_0}\right)\right] \qquad (44)$$

in which $\beta = \dfrac{S_0}{\sigma}$ is taken to be a coefficient, whereas the varied flow function, B, is the numerical value of the integral,

$$B = -\int_0^v \frac{dy}{y^n - 1} \qquad (45)$$

an expression into which Professor Mononobe's function, Φ_1, is reduced by the simple transformation,

$$\int_0^v \frac{y^r}{y^r - 1}\, dy = \int_0^v \frac{y^r - 1 + 1}{y^r - 1}\, dy = y + \int_0^v \frac{dy}{y^r - 1} \qquad (46)$$

The substantial difference between Professor Mononobe's technique and that of the previous investigators (including the writer), is that, whereas heretofore the customary manner was to present the function values in tabular form, the paper gives them graphically in chart form. Obviously, tables and charts are equivalent, the preference for one or the other being a matter of individual taste. An illuminating analogy would be to replace the customary tables of logarithms by graphical charts.

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[16] "Hydraulics of Open Channels", p. 88.

In the light of the preceding, the writer does not quite understand in what way Professor Mononobe's approach avoids the mathematical complications supposedly inherent in earlier methods, and in what manner it simplifies computations as claimed.

With regard to details, confusion may be avoided by correcting possibly a typographical error, in using an identical symbol, *C*, in Equation (1) for the generalized Chézy factor, and for what is obviously meant to be a numerical constant in Equation (2).

The attempts of Professor Mononobe to complement his analysis with experimental work should be more than welcomed. In fact, although the varied-flow theory has been the subject of repeated treatment, scarcely any experimental data are available, the best probably being still those assembled in the classical opus by Darcy and Bazin.[17] The usual difficulty lies in the necessity of having very long flumes, and Professor Mononobe's manner of using short canals is ingenious. It is to be hoped that Professor Mononobe's experiments will once more draw attention to this much neglected field of research, and that, in particular, observations on a larger scale will become available.

P. CHARLES STEIN,[18] ESQ. (by letter).—The method for determining back-water and drop-down curves in uniform channels, described by the author, lends itself to a relatively simple routine for computing these problems. The errors introduced by the approximation that the area and the wetted perimeter are monomial functions of the depth are of small order and should be well below the range of error incident to the selection of a friction factor.

That the assumption of monomial relations between area and depth and between wetted perimeter and depth does not introduce appreciable error, even for a circular section, may be seen from Fig. 22 for the example given by Professor Mononobe (see text following Equation (32)), in which the depth in a circular conduit, 18 ft in diameter, ranged from 30% to 75% of the diameter. The heavy dashed lines in Fig. 22 show the actual variation of area and wetted perimeter and the full lines represent the monomial approximations.

It is to be noted that the range in depths of a back-water curve cannot extend across the critical depth. In practical problems, even in channels not well adapted to the monomial approximation over their entire range, the range will usually be such that the assumption of monomial variation will agree closely with the actual variation of the hydraulic elements with respect to depth.

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[17] "Recherches Hydrauliques". Paris. 1865.

[18] Graduate Student in Hydraulics, Univ. of Iowa, Iowa City, Iowa.

Removed from its seemingly complex mathematical treatment, the author's method involves very simple reasoning and is adapted to a relatively easy computing procedure. Having expressed the area and the wetted perimeter as approximate exponential functions of depth, Professor Mononobe converts the differential equation of Bernoulli's theorem for a sloping channel (Equation (1)) into a form in which all hydraulic variables are functions of depth and, after grouping terms, integrates the resulting equation to obtain Equation (18) which defines the back-water curve in a uniform channel. A similar formula, Equation (20), defines the drop-down curve.

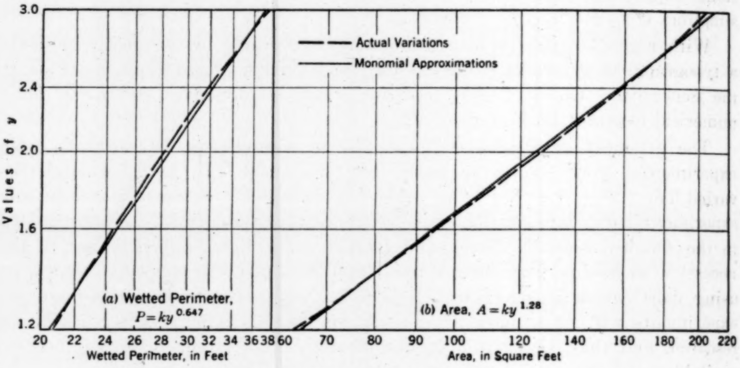

FIG. 22.—RELATION OF WETTED PERIMETER AND AREA TO DEPTH

Equations (18) and (20) are solved readily by means of the curves in Figs. 6, 7, and 8. The parameters necessary for using the charts are $\frac{D}{D_o}$, r, and s. The term, r, is defined as:

$$r = 2s + 2m\,(s - k) \qquad\qquad (47)$$

in which s is the exponent of depth in the approximate area relation; k is the exponent of depth in the wetted perimeter relation; and m is the exponent of R in the friction formula used, Equation (2). One other term, K, is necessary in solving Equations (18) and (20), and it is found from the relation:

$$K = \alpha\, s\, \frac{V_o}{g\, D_o} \qquad\qquad (48)$$

in which V_o and D_o denote the velocity and depth, respectively, which would prevail if, for the discharge under consideration, the surface slope equalled the bottom slope.

An easy method of determining the exponents, s and k, is to compute them by means of the logarithmic differences of the values at the end points of the range. Thus, in determining the area exponent in Equation (13*a*):

$$\log A_2 = \log a + s \log D_2 \text{..................} (49a)$$

and,

$$\log A_1 = \log a + s \log D_1 \text{..................} (49b)$$

Subtracting, and solving for s:

$$s = \frac{\log A_2 - \log A_1}{\log D_2 - \log D_1} \text{........................} (50)$$

For the circular conduit mentioned in the text following Equation (32): $A_2 = 204.7$; $A_1 = 64.2$; $D_2 = 13.5$; and $D_1 = 5.4$. Reading the logarithms on an ordinary slide-rule: $s = \frac{2.311 - 1.807}{1.130 - 0.732} = 1.267$. Similarly, for the wetted perimeter, this type of calculation results in: $k = 0.646$.

It is seen that the exponents thus obtained agree closely with those derived by the graphical method. In the graphical method, the eye is guided in large measure by the location of the end points.

The author's method, wherein he assumes that the average of the slopes at the ends of the range is equivalent to the slope of the straight line best approximating the curve, is open to criticism inasmuch as it may give undue weight to an extreme slope at one end of the range. The logarithmic method is simpler than the method given in the paper and is probably less subject to error.

NAGAHO MONONOBE,[19] M. AM. SOC. C. E. (by letter).—In closing this paper, the writer wishes to add nothing further to the comments so generously offered by the several discussers. The assumption may properly be made that the method yields results that are safe for practical purposes.

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[19] Director, Experiment Station of Public Works, Home Dept., Japanese Govt.; Prof., Civ. Eng. Dept., Tokyo Imperial Univ., Tokyo, Japan.

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Paper No. 1999

READJUSTMENT OF TRIANGULATION DATUM

BY JULIUS L. SPEERT,[1] JUN. AM. SOC. C. E.

SYNOPSIS

The readjustment of the basic triangulation of the United States to the 1927 North American Datum created a problem to all organizations whose surveys were built on the basic system. Complete recomputation and readjustment being out of the question because of their prohibitive cost, the United States Geological Survey investigated methods of readjusting its earlier triangulation to fit the new datum by applying corrections to the old datum values. Since the corrections are usually very small, approximate methods of determination give reasonably precise results.

Several methods of readjustment have been developed. When the subordinate triangulation is tied to the new datum through a single base only, three methods are available: (1) Analytical determination of the new positions by applying formulas for translation, rotation, and linear scale change; (2) a graphical method for determining corrections by plotting lines of equal position changes; and (3) a mechanical method requiring an easily constructed transparent rotating guide. When the subordinate triangulation is tied to the new datum at three or more points, the new positions are determined graphically by plotting lines of equal position changes. The corrections to length and azimuth of the geodetic line are determined analytically by formulas, or graphically on a nomographic chart.

The methods herein described are applicable also to the adjustment of new triangulation.

INTRODUCTION

When the United States Coast and Geodetic Survey began its readjustment of the basic triangulation system of the United States in 1927, it created a problem of major importance to practically all other surveying organizations in the country. All triangulation of lesser importance that

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NOTE.—Published in May, 1937, *Proceedings*.

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had been computed from, or based on, the arcs of the main plan, no longer fitted these arcs as they were shifted or changed by the readjustment. When the task of bringing all its accumulated triangulation data into agreement with the new 1927 North American Datum was undertaken by the United States Geological Survey, it was realized immediately that direct recomputation from the original field notes was out of the question, because neither time nor money would be available for the execution of such a huge project. To avoid the necessity of complete recomputation, various methods have been proposed and tried wherein the changes in the main control system have been studied, in order to ascertain what changes must be introduced into the subordinate triangulation to regain the harmony that had existed on the old datum, or to establish harmony where none had existed before. In dealing with changes of position, direction, and length, instead of with those quantities themselves, the magnitudes of the terms in the computations are kept so small that simplifications can be introduced readily without diminishing the accuracy of the final results.

Much of the old Geological Survey triangulation was originally adjusted as spurs from single bases. Many of these arcs have since been made parts of closed circuits by later ties to other arcs but, in most cases, the closing errors have not been distributed throughout the work. The adjustment of the closures of these circuits simultaneously with the readjustment to the 1927 North American Datum also was undertaken.

WILSON METHOD

The first rational attempt to readjust the old Geological Survey triangulation to the 1927 North American Datum by means of corrections to the previously computed values, was made by R. M. Wilson, M. Am. Soc. C. E., Chief of the Computing Section of the Survey. His method was fully and clearly described in a paper presented before the Federal Board of Surveys and Maps on April 11, 1933. The following is a brief summary of his method.

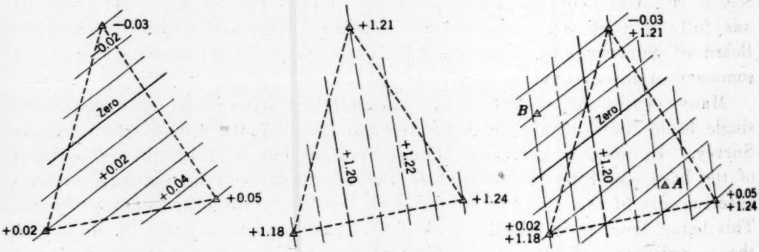
Many chains or networks of triangulation have been extended from single basic lines of higher order triangulation of the Coast and Geodetic Survey. In such a net, any change in the length, azimuth, or position of the base could be transferred to the remainder of the network without affecting any of the adjusted angles of the earlier figure adjustment within it. This being true, it is necessary only to determine the amount of change in the co-ordinates of the ends of the base, and the corresponding change in azimuth and length of the base. The entire system can be translated bodily to bring one end of the base into its new position; the system can then be rotated about this point to bring the base into its correct azimuth. If, finally, a uniform scale factor is applied to all lines of the system, the other end of the base can be brought into its correct position, and the entire system will be in harmony with the new datum. By means of formulas developed by Mr. Wilson, the translation, rotation, and scale factors may be applied to all points in the triangulation net to determine their positions on the new datum, and all lines may be corrected by the uniform azimuth and scale changes.

This method is limited in its application to those systems of triangulation that are developed from a single base and have no other ties to the new datum. In many nets, however, other ties have been made, either at the time of the original survey, or subsequently. In such nets, the values obtained by this method would fit the original base perfectly, but would be likely to show discrepancies at the other tie-points. For these conditions, other methods of treatment were found necessary.

"Isodiffs"

For those triangulation nets in which three or more ties to the new datum are available, a graphical method for the determination of the position corrections has been developed by C. L. Nelson, Assistant Topographic Engineer, U. S. Geological Survey.

On a scale map or tracing diagram of the triangulation net to be adjusted, all available ties to the new datum are indicated, and the proper corrections noted which are needed to bring the old values for latitude and longitude to the new datum. By interpolating along lines between the various ties, corrections may be obtained for intermediate points between the fixed tie-points. When enough of such values have been determined, it is possible to draw lines on the tracing representing lines of equal corrections, or of equal changes from the previous positions. This is very similar to drawing contours on a map when the elevations have been determined at a number of fixed points. It will be necessary to have one set of lines for latitudes and another set for longitudes. These lines have been named "isodiffs" (or lines of equal differences) and are also called "isolats" and "isolongs," respectively. Fig. 1 illustrates the construction of the "isodiffs" for the relatively simple case of three tie-points.



(a) ISOLATS. (b) ISOLONGS. (c) COMBINED ISODIFFS.

FIG. 1.

When the number of ties is greater than three, straight-line interpolation between tie-points sometimes leads to inconsistencies. For instance, in the illustration in Fig. 2, if an attempt were made to interpolate along both diagonals, different values might be obtained for the intersection. Under such conditions, it is advisable to assume a value for the intersection which would be a weighted mean of the values obtained from the respective diagonals, and to proceed with the interpolation from that point as if it were a tie-point.

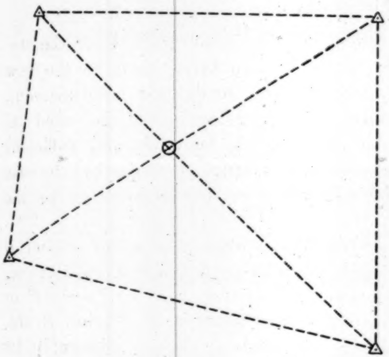


Fig. 2.

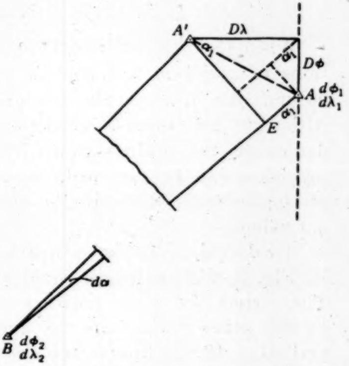


Fig. 3.

With the graph of "isodiffs" constructed on the tracing, it is necessary only to superimpose it on the plot of the triangulation plan, and the corrections to be applied to each station in the system may be read from its position on the graph, just as the elevation of any point on a contour map may be read from the contours, this being the purpose for which the graph is made.

For example, in Fig. 1(*c*), the corrections to be applied to the old co-ordinates of Station *A* in the system to be adjusted, are read as + 0.03 for latitude and + 1.22 for longitude. Similarly, the corrections to the co-ordinates of Station *B* are − 0.02 and + 1.18, respectively..

The construction and application of the "isodiff" chart will be expedited if different colors are used for the "isolats" and "isolongs," as well as to indicate the latitude and longitude corrections, respectively. For instance, red may be used for latitude corrections and "isolats," and blue for longitude corrections and "isolongs."

It will be noted that the judgment of the adjuster is an important factor in the sketching of satisfactory "isodiffs," and different men might conceivably obtain different sets of lines from the same data; but experience has shown that for practically all cases where three or more ties are available, excellent results can be obtained by this method. The "personal factor," furthermore, is not as serious a disadvantage as it might appear, since it is inherent in almost any method of adjustment in which a number of independent conditions must be satisfied. Even the application of the standard method of least squares is not altogether free from it.

The proper application of the "isodiff" graph gives corrections to be used in obtaining new values for the positions of the triangulation stations, but does not yield any new information concerning the lines connecting the stations. For this purpose, the following formulas have been developed by the writer.

NEW FORMULAS

When the required position corrections have been determined for the stations in a system and applied to the old values to bring them to the new datum, the lines connecting the stations are ready for readjustment. Although the inverse geodetic position computation could be used to determine the desired values for the lines on the new datum, sufficient precision can be obtained, at a considerable saving of time, by the use of approximate methods for the determination of the corrections to the old values.

To do this, consider a line connecting Triangulation Stations A and B in Fig. 3, with a length equal to s and azimuth at Station A equal to α_1. The corrections to the position of Station A (as determined by "isodiffs" or by any other means) are $d\phi_1$ and $d\lambda_1$ and to the position of Station B, $d\phi_2$ and $d\lambda_2$. If the line is translated by $-d\phi_2$ and $-d\lambda_2$, no change in its length or direction will take place. Then, the resulting relative changes in latitude and longitude at Station B will both be 0, and the changes at Station A will be $\delta\phi = d\phi_1 - d\phi_2$ and $\delta\lambda = d\lambda_1 - d\lambda_2$. These latter values represent the net change within the line, AB, or the relative change of one end with respect to the other, and are the values that cause whatever change there may be in the length or in the azimuth of the line.

Before they can be used in the computations, $\delta\phi$ and $\delta\lambda$ must be converted from seconds of arc to meters by the formulas:

$$D\phi = Q\ \delta\phi \qquad \text{(1a)}$$

and,

$$D\lambda = R\ \delta\lambda \qquad \text{(1b)}$$

The conversion factors, Q and R, represent the length on the earth's surface of 1" of arc of latitude and longitude, respectively. The factor, Q, varies from 30.77 m at Latitude 25°, to 30.90 m at Latitude 50°, and a mean value of 30.83 m has been assumed as sufficiently precise for all points in the United States. The value of R varies more rapidly (see Table 1).

If $D\phi$ and $D\lambda$ are plotted at Station A to an enlarged scale, Point A' (Fig. 3), represents the new position of Station A with respect to Station B (to the distorted scale), and the total displacement of the station is represented by AA'. If this is projected on AB, then AE represents the linear change in AB (to the scale of the plot) and $A'E$ is a measure of the rotation of the line. The angular rotation at Station B (to bring BA to BA') $= -d\alpha = \dfrac{\overline{A'E}}{\overline{A'B}\sin 1''}$, approximately, $= \dfrac{\overline{A'E}}{s \sin 1''}$, approximately;

TABLE 1.—VALUES OF R

Latitude, ϕ, in degrees (1)	Length, R, on the earth's surface, of 1 second of longitude, in meters (2)	Latitude, ϕ, in degrees (1)	Length, R, on the earth's surface, of 1 second of longitude, in meters (2)	Latitude, ϕ, in degrees (1)	Length, R, on the earth's surface, of 1 second of longitude, in meters (2)	Latitude, ϕ, in degrees (1)	Length, R, on the earth's surface, of 1 second of longitude, in meters (2)	Latitude, ϕ, in degrees (1)	Length, R, on the earth's surface, of 1 second of longitude, in meters (2)
25	28.04	30	26.80	35	25.36	40	23.72	45	21.90
26	27.81	31	26.53	36	25.05	41	23.37	46	21.52
27	27.57	32	26.25	37	24.73	42	23.01	47	21.13
28	27.32	33	25.96	38	24.40	43	22.65	48	20.73
29	27.07	34	25.66	39	24.06	44	22.28	49	20.33
..	.....	..	.....	..	.....	..	.....	50	19.92

TABLE 2.—VALUES OF FACTORS C AND K

Factor C (1)	log s (2)	Factor K (3)	Factor C (1)	log s (2)	Factor K (3)	Factor C (1)	log s (2)	Factor K (3)	Factor C (1)	log s (2)	Factor K (3)
43.4	4.000	20.61	33.4	4.114	15.86	23.4	4.269	11.11	13.6	4.504	6.46
43.2	4.002	20.52	33.2	4.117	15.77	23.2	4.272	11.02	13.4	4.511	6.36
43.0	4.004	20.42	33.0	4.119	15.67	23.0	4.276	10.92	13.2	4.517	6.27
42.8	4.006	20.33	32.8	4.122	15.58	22.8	4.280	10.83	13.0	4.524	6.17
42.6	4.008	20.23	32.6	4.125	15.48	22.6	4.283	10.73	12.8	4.531	6.08
42.4	4.010	20.14	32.4	4.127	15.39	22.4	4.287	10.64	12.6	4.537	5.98
42.2	4.012	20.04	32.2	4.130	15.29	22.2	4.291	10.54	12.4	4.544	5.89
42.0	4.015	19.95	32.0	4.133	15.20	22.0	4.295	10.45	12.2	4.551	5.79
41.8	4.017	19.85	31.8	4.135	15.10	21.8	4.299	10.35	12.0	4.559	5.70
41.6	4.019	19.76	31.6	4.138	15.01	21.6	4.303	10.26	11.8	4.566	5.60
41.4	4.021	19.66	31.4	4.141	14.91	21.4	4.307	10.16	11.6	4.573	5.51
41.2	4.023	19.57	31.2	4.144	14.82	21.2	4.311	10.07	11.4	4.581	5.41
41.0	4.025	19.47	31.0	4.146	14.72	21.0	4.316	9.97	11.2	4.589	5.32
40.8	4.027	19.38	30.8	4.149	14.63	20.8	4.320	9.88	11.0	4.596	5.22
40.6	4.029	19.28	30.6	4.152	14.53	20.6	4.324	9.78	10.8	4.604	5.13
40.4	4.031	19.19	30.4	4.155	14.44	20.4	4.328	9.69	10.6	4.612	5.03
40.2	4.033	19.09	30.2	4.158	14.34	20.2	4.332	9.59	10.4	4.621	4.94
40.0	4.036	19.00	30.0	4.161	14.25	20.0	4.337	9.50	10.2	4.629	4.84
39.8	4.038	18.90	29.8	4.163	14.15	19.8	4.341	9.40	10.0	4.637	4.75
39.6	4.040	18.81	29.6	4.166	14.06	19.6	4.346	9.31	9.8	4.646	4.65
39.4	4.042	18.71	29.4	4.169	13.96	19.4	4.350	9.21	9.6	4.655	4.56
39.2	4.045	18.62	29.2	4.172	13.87	19.2	4.354	9.12	9.4	4.665	4.46
39.0	4.047	18.52	29.0	4.175	13.77	19.0	4.359	9.02	9.2	4.674	4.37
38.8	4.049	18.43	28.8	4.178	13.68	18.8	4.364	8.93	9.0	4.683	4.27
38.6	4.051	18.33	28.6	4.181	13.58	18.6	4.368	8.83	8.8	4.693	4.18
38.4	4.053	18.24	28.4	4.184	13.49	18.4	4.373	8.74	8.6	4.703	4.08
38.2	4.056	18.14	28.2	4.187	13.39	18.2	4.378	8.64	8.4	4.713	3.99
38.0	4.058	18.05	28.0	4.190	13.30	18.0	4.383	8.55	8.2	4.724	3.89
37.8	4.060	17.95	27.8	4.194	13.20	17.8	4.387	8.45	8.0	4.735	3.80
37.6	4.063	17.86	27.6	4.197	13.11	17.6	4.392	8.36	7.8	4.746	3.70
37.4	4.065	17.76	27.4	4.200	13.01	17.4	4.397	8.26	7.6	4.757	3.61
37.2	4.067	17.67	27.2	4.203	12.92	17.2	4.402	8.17	7.4	4.768	3.51
37.0	4.070	17.57	27.0	4.206	12.82	17.0	4.407	8.07	7.2	4.780	3.42
36.8	4.072	17.48	26.8	4.210	12.73	16.8	4.412	7.98	7.0	4.793	3.32
36.6	4.074	17.38	26.6	4.213	12.63	16.6	4.418	7.88	6.8	4.805	3.23
36.4	4.077	17.29	26.4	4.216	12.54	16.4	4.423	7.79	6.6	4.818	3.13
36.2	4.079	17.19	26.2	4.219	12.44	16.2	4.428	7.69	6.4	4.831	3.04
36.0	4.081	17.10	26.0	4.223	12.35	16.0	4.434	7.60	6.2	4.845	2.94
35.8	4.084	17.00	25.8	4.226	12.25	15.8	4.439	7.50	6.0	4.860	2.85
35.6	4.086	16.91	25.6	4.230	12.16	15.6	4.445	7.41	5.8	4.874	2.75
35.4	4.089	16.81	25.4	4.233	12.06	15.4	4.450	7.31	5.6	4.889	2.66
35.2	4.091	16.72	25.2	4.236	11.97	15.2	4.456	7.22	5.4	4.905	2.56
35.0	4.094	16.62	25.0	4.240	11.87	15.0	4.462	7.12	5.2	4.921	2.47
34.8	4.096	16.53	24.8	4.243	11.78	14.8	4.467	7.03	5.0	4.938	2.37
34.6	4.099	16.43	24.6	4.247	11.68	14.6	4.473	6.93	4.8	4.956	2.28
34.4	4.101	16.34	24.4	4.250	11.59	14.4	4.479	6.84	4.6	4.975	2.18
34.2	4.104	16.24	24.2	4.254	11.49	14.2	4.485	6.74	4.4	4.994	2.09
34.0	4.106	16.15	24.0	4.258	11.40	14.0	4.492	6.65	4.2	5.015	1.99
33.8	4.109	16.05	23.8	4.261	11.30	13.8	4.498	6.55	4.0	5.036	1.90
33.6	4.111	15.96	23.6	4.265	11.21	....	.....	.....	....	.....	.....

but $\overline{A'E} = D\phi \sin \alpha_1 + D\lambda \cos \alpha_1$. Therefore,

$$d\alpha = -\frac{1}{s \sin 1''} (D\phi \sin \alpha_1 + D\lambda \cos \alpha_1)$$

or,

$$d\alpha = -K (D\phi \sin \alpha_1 + D\lambda \cos \alpha_1) \ldots\ldots\ldots\ldots\ldots (2)$$

in which $K = \frac{1}{s \sin 1''}$. It is readily apparent that practically the same rotational change occurs at both ends of the line, and $d\alpha$ must be applied to α_1 as well as to α_2.

Line *AE*, Fig. 3, represents, in meters, the change in the length of the line, *AB*. If this is multiplied by a factor, *C*, which is the tabular difference (in units of the last place) in logarithms per meter of *s*, then the change in $\log s = d (\log s) = - C \times \overline{AE}$; but $\overline{AE} = D\lambda \sin \alpha_1 - D\phi \cos \alpha_1$, and therefore:

$$d (\log s) = C (D\phi \cos \alpha_1 - D\lambda \sin \alpha_1) \ldots\ldots\ldots\ldots\ldots (3)$$

For 6-place logarithms, $C = \frac{10^6 \log e}{s}$. Values of Factors *C* and *K* may be read from Table 2.

When the total latitude correction to both ends of the line is appreciable (as is the case between some of the old astronomic datums in the West and the 1927 North American Datum), and the difference in longitude between the ends of the line is large, another term must be added to the length correction to allow for convergence of meridians. This need becomes apparent when one considers that the distance between meridians decreases rapidly with an increase in latitude. For instance, if two points at the same latitude, but on different meridians, were shifted north by the same amount, the differential change between them would be zero in latitude and zero in longitude; but since the movements of the points were not parallel, the distance between them would be decreased appreciably, whereas the azimuth would remain practically unchanged.

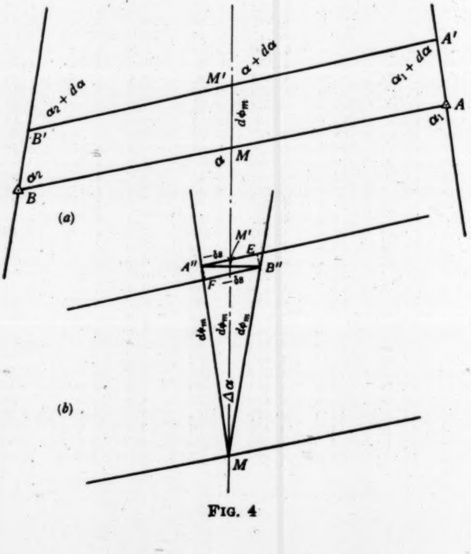


FIG. 4

To determine the change in length due to convergence, consider Fig. 4(*a*). Points A and B are moved northward along different meridians to A' and B', respectively. If M is the mid-point of the line, $\overline{AB}$, $\overline{MM'} = d\phi_m$, or the mean change in latitude of Stations A and B. In meters, $\overline{MM'} = Q \times d\phi_m$.

The next step will be clearer if the line is assumed to be broken at M, and the two halves are moved separately as shown in Fig. 4(*b*), so that MB'' is parallel to BB' and equal to $d\phi_m$, and MA'' is parallel to AA' and equal to $d\phi_m$. For the small angles involved, the arc, $A''\,M'B''$, may be treated as a straight line. Then, $\overline{B'B''} = \overline{BM}$; $\overline{A'A''} = \overline{AM}$; and, $\overline{AB} = \overline{A'A''} + \overline{B'B''}$. The shortening of the original line would then be the distance by which the two halves overlap, or, $-\ \delta s = \overline{A''E} = \overline{B''F} = \overline{A''B''} \sin (\alpha + d\alpha)$; but $\overline{A''B''}$ (in meters) $= Q\ d\phi_m\ \Delta\alpha$ arc 1′. Since $d\alpha$ will usually be very small compared to α, $\sin \alpha$ may be substituted for $\sin (\alpha + d\alpha)$ with negligible error. Therefore,

$$-\ \delta s = Q \text{ arc } 1'\ d\phi_m\ \Delta\alpha \sin \alpha \qquad (4)$$

By substituting Q arc $1' = 0.009$, approximately, and noting that $\Delta\alpha \sin \alpha$ will always be negative, the correction becomes,

$$\delta s = 0.009\ d\phi_m\ \Delta\alpha \sin \alpha \qquad (5)$$

Since δs has been derived in meters, it should be applied to the change in length of s (Equation (3)) to give:

$$d(\log s) = C\ (D\phi \cos \alpha_1 - D\lambda \sin \alpha_1 + 0.009\ d\phi_m\ \Delta\alpha \sin \alpha) \qquad (6)$$

Although the original formulas were derived on the basis of a differential change at Station A, the same might have been done, instead, for the change at Station B, or preferably at the mid-point, M, and the results determined in the same form. It seems reasonable, therefore, to use the mean azimuth of the line instead of that at either end, and the final formulas for the corrections become:

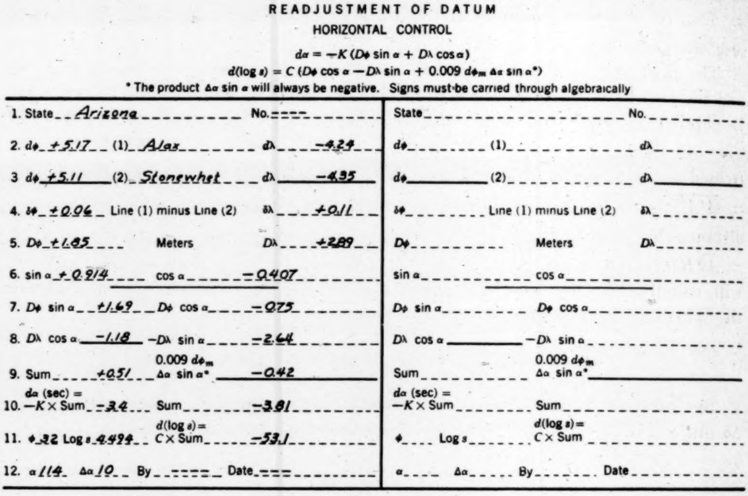
$$d\alpha = -\ K\ (D\phi \sin \alpha + D\lambda \cos \alpha) \qquad (7)$$

and,

$$d(\log s) = C\ (D\phi \cos \alpha - D\lambda \sin \alpha + 0.009\ d\phi_m\ \Delta\alpha \sin \alpha) \qquad (8)$$

in which $D\phi = 30.83\ \delta\phi$; $\delta\phi = d\phi_1 - d\phi_2$; $D\lambda = R\ \delta\lambda$; $\delta\lambda = d\lambda_1 - d\lambda_2$; $d\phi$, $d\lambda$, $d\alpha$, and $d(\log s)$ = corrections to old datum values for latitude, longitude, azimuth, and log meters, respectively; $d\phi_m$ = the mean of $d\phi_1$ and $d\phi_2$; α = the mean azimuth of the line between Points (1) to (2), (or between Points A and B) $= \alpha_1 + \dfrac{\Delta\alpha}{2}$; and, $\Delta\alpha$ = the convergence of meridians, in minutes of arc, $= \alpha_2 - \alpha_1 \pm 180°$.

To facilitate the computations, the form in Fig. 5 has been prepared. It is arranged to hold the computations for four geodetic lines (an example is shown in Fig. 5), which may be performed conveniently in the following order, using a slide-rule for all multiplications:



READJUSTMENT OF DATUM
HORIZONTAL CONTROL

$d\alpha = -K\,(D\phi \sin\alpha + D\lambda \cos\alpha)$

$d(\log s) = C\,(D\phi \cos\alpha - D\lambda \sin\alpha + 0.009\, d\phi_m\, \Delta\alpha \sin\alpha^*)$

* The product $\Delta\alpha \sin\alpha$ will always be negative. Signs must be carried through algebraically

1. State Arizona No. ----
2. $d\phi$ +5.17 (1) Alax $d\lambda$ −4.24
3 $d\phi$ +5.11 (2) Stonewhet $d\lambda$ −4.35
4. $\delta\phi$ +0.06 Line (1) minus Line (2) $\delta\lambda$ +0.11
5. $D\phi$ +1.85 Meters $D\lambda$ +2.89
6. $\sin\alpha$ +0.914 $\cos\alpha$ −0.407
7. $D\phi \sin\alpha$ +1.69 $D\phi \cos\alpha$ −0.75
8. $D\lambda \cos\alpha$ −1.18 $-D\lambda \sin\alpha$ −2.64
9. Sum +0.51 $0.009\, d\phi_m\, \Delta\alpha \sin\alpha^*$ −0.42
10. $d\alpha$ (sec) = $-K \times$ Sum −3.4 Sum −3.81
11. ϕ 32 Log s 4.494 $d(\log s) =$ $C \times$ Sum −53.1
12. α 114 $\Delta\alpha$ 10 By ---- Date ---

State No.
$d\phi$ (1) $d\lambda$
$d\phi$ (2) $d\lambda$
$\delta\phi$ Line (1) minus Line (2) $\delta\lambda$
$D\phi$ Meters $D\lambda$
$\sin\alpha$ $\cos\alpha$
$D\phi \sin\alpha$ $D\phi \cos\alpha$
$D\lambda \cos\alpha$ $-D\lambda \sin\alpha$
Sum $0.009\, d\phi_m\, \Delta\alpha \sin\alpha^*$
$d\alpha$ (sec) = $-K \times$ Sum Sum
ϕ Log s $d(\log s) =$ $C \times$ Sum
α $\Delta\alpha$ By Date

FIG. 5.—SAMPLE FORM FOR COMPUTATIONS. (COMPLETE FORM PRINTS COMPUTATIONS FOR FOUR GEODETIC LINES).

The State in which the line occurs and the reference number of the computation are entered on Line 1 of the form; the names of the stations involved, together with their respective latitude and longitude corrections, on Lines 2 and 3; and the mean latitude and azimuth, log distance, and convergence, on Lines 11 and 12, as indicated. All these values may be taken by inspection from the available records for the two stations involved. For third-order triangulation, with positions given to the nearest 0.01″ of latitude and longitude, azimuth to the nearest second, and log s to six decimal places, it is sufficient to use ϕ and α to the nearest degree, log s to three decimal places, and $\Delta\alpha$ to the nearest minute on the form.

Next, $\delta\phi$ and $\delta\lambda$ are determined by algebraic subtraction and entered on Line 4, and $D\phi$ and $D\lambda$ are computed and entered on Line 5. Note that R is taken from Table 1, using ϕ as the argument; sin α and cos α are taken from tables, using α as the argument, and are entered on Line 6.

With $D\phi$ set on the slide-rule, $D\phi$ sin α and $D\phi$ cos α are determined and entered on Line 7. Similarly, $D\lambda$ cos α and $- D\lambda$ sin α are entered on Line 8; $d\phi_m$ is determined by inspection from Lines 2 and 3; and the term, $0.009\, d\phi_m\, \Delta\alpha \sin\alpha$ is entered on Line 9.

The sums on Lines 9 and 10, Fig. 5, are obtained algebraically as indicated; Factors C and K are taken from Table 2, using log s as the argument, and applied to the sums as indicated to obtain $d\alpha$ and $d(\log s)$ on Lines 10 and 11, respectively. The initials of the computer and the date are entered on Line 12.

Since C and K are direct functions of $\frac{1}{s}$, to find either, for a line for which the value of log s is beyond the limits of Table 2, find the value desired for the log s in the table with the same mantissa, and multiply or divide by the proper power of 10; for instance, C and K for $\log s = 2.200$ are 100 times C and K for $\log s = 4.200$, respectively; and for $\log s = 5.200$, the values are 0.1 times those for $\log s = 4.200$, respectively. In other words:

When log s has a characteristic of:	Multiply Factors C and K, Table 2, by:
2	100
3	10
5	0.1

Table 2 gives C for 6-place logarithms. For each additional place, multiply the tabular value by 10; for each place less than 6, divide by 10. Table 2 is arranged for intervals of 0.2 for C and 0.095 for K.

With all the necessary information before him, the average computer should be able to determine the corrections for a geodetic line in from 3 to 5 min. The work can be speeded considerably by setting up a number of geodetic lines on the forms and computing them together, completing each operation on the entire group before proceeding with the next. If a colored pencil is used for recording the last step in the computations, namely, $d\alpha$ and $d(\log s)$, these values will stand forth conspicuously, and may be found more readily when needed.

NOMOGRAPHIC CHART

In order to check the values of $d\alpha$ and $d(\log s)$ computed by the formulas, the nomographic chart of Fig. 6 has been prepared, on which the same corrections may be determined graphically. It has scales for $\delta\phi$ (the differential change in latitude within the line); $\delta\lambda$ (the differential change in longitude at the various latitudes); sin α; cos α; log s; and the desired corrections, $d\alpha$ and $d(\log s)'$. In addition, there is an auxiliary chart (Fig. 7) for the determination of the supplementary correction to $d(\log s)'$, due to the convergence of meridians.

To use Fig. 6, proceed as follows: To determine $d\alpha$, mark on the proper scales, $\delta\phi$ (Point a), $\delta\lambda$ (Point b) for the proper latitude, sin α (Point c), cos α (Point d), and log s (Point e). On the $\delta\lambda$-scale, project the intersection of the horizontal line for ϕ and the curved line for $\delta\lambda$ to the main horizontal line (Point g), using as a guide the fine vertical lines.

Connect log s with sin α (Line ce) and draw a parallel line (ah) through $\delta\phi$ to intersect the vertical index line containing the sin and cos scales. Call this intersection Point (h). Connect log s with cos α (Line de) and draw a parallel line through $\delta\lambda$ (Line gk) to intersect the vertical index line on the right half of the chart. Call this intersection Point (k). The line joining Points (h) and (k) intersects the $d\alpha$-scale *at* Point l, indicating the value of $d\alpha$.

To determine d(log s), the markings for $\delta\lambda$ (Point g), sin α (Point d), and cos α (Point c) are the same as before: $\delta\phi$ (Point a) must be set off on the opposite side of the $\delta\phi$-scale, and log s (Point e) must be marked on the lower log s-scale.

Connect log s and cos α (Line ec) and draw a parallel line (ah) through $\delta\phi$, intersecting the left index line as before. Call this intersection Point (h). Connect log s and sin α (Line de), and draw a parallel line (gk) through $\delta\lambda$, intersecting the right index line as before. Call this intersection Point (k). The line joining Points (h) and (k) intersects the d(log s)$'$-scale at the preliminary value for d(log s) Point (l).

Fig. 7, an auxiliary chart for the convergence term, has scales for log s, α, $\Delta\alpha$, $d\phi_m$ (the mean latitude correction), and d(log s)$''$. To use this chart, draw a line (ab) connecting log s with $\Delta\alpha$. Through the intersection of this line with the diagonal, draw a line from $d\phi_m$ (Point c), to intersect the right vertical index (Point d). Through this intersection and α (Point e) on the diagonal scale, draw a line (def) to meet the left scale at d(log s)$''$, which will always be opposite in sign to $d\phi_m$. This value for d(log s)$''$ should be added to the preliminary value, d(log s)$'$, obtained from Fig. 6, to derive the final value of d(log s).

For use in the office, the nomographic chart has been drawn on a double-mounted plane-table sheet, 18 by 24 in. in size, and covered with a sheet of transparent cellulose acetate. All marks and lines for each problem are drawn in pencil on this cover, from which they may be easily removed later with a soft eraser, leaving the chart clean for the beginning of another problem. Parallel lines are drawn by the usual methods, using a parallel rule or a pair of triangles.

A complete problem (determination of $d\alpha$ and d(log s)) can be solved in about 2 min by one familiar with the charts.

Two-Point "Isodiffs"

Examination of the Wilson method indicates that two tie-points are sufficient to control the adjustment of a triangulation net if no others are available. Since this is true, it should be possible to draw "isodiffs" to give, graphically, the same adjustment that would be obtained by the application of the Wilson formulas. Interpolation along the line between the two tie-points, however, gives only the spacing of the "isodiffs", but no indication of their directions. It should be possible to compute, by the formulas, the position changes of one additional point on the system, and

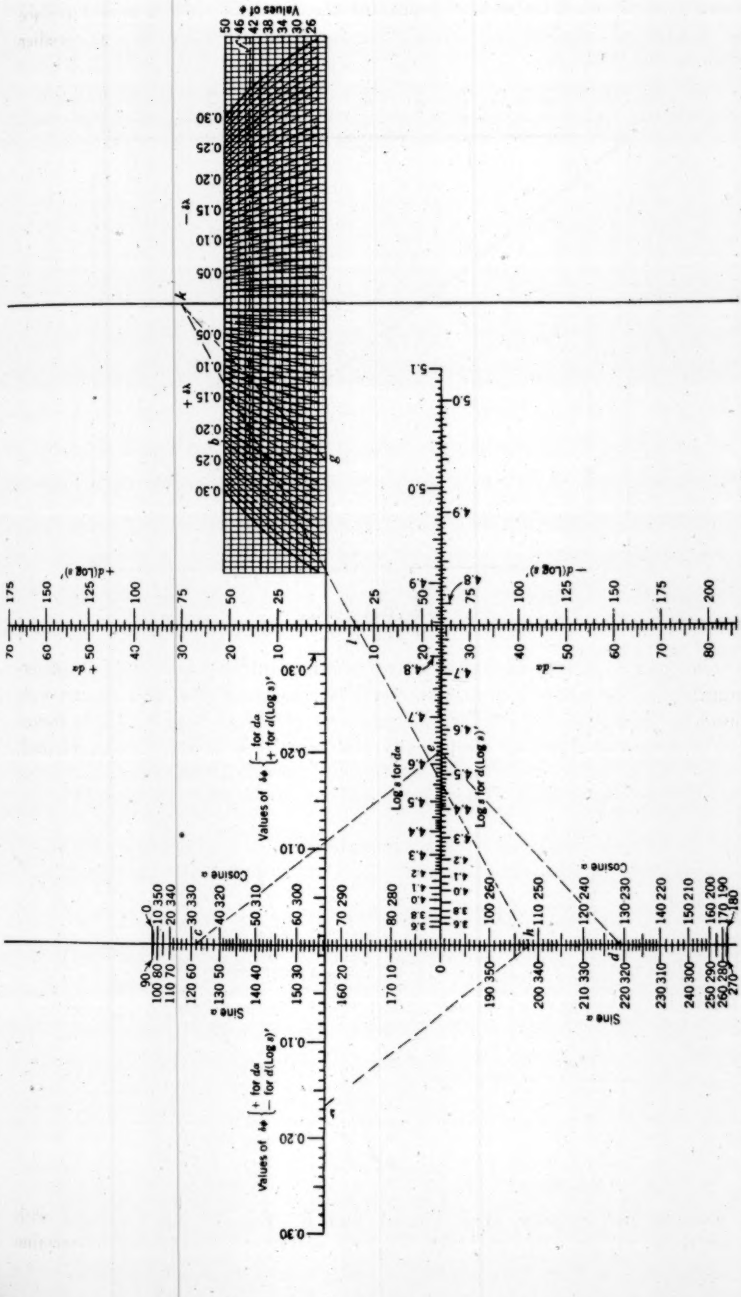


FIG. 6.—NOMOGRAPHIC CHART FOR THE DETERMINATION OF THE SWING AND CHANGE IN LENGTH OF A LINE RESULTING FROM THE SHIFTING OF ITS ENDS.

to use that as the third control point for the "isodiffs". This would replace the analytical method for computing position changes by a speedier graphical method.

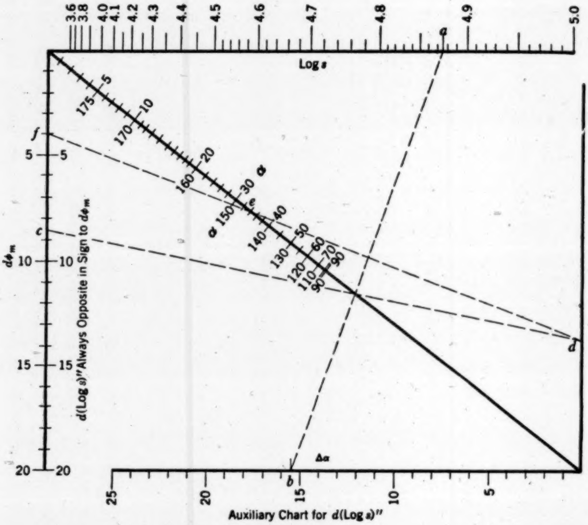


FIG. 7.

The computation of the third control point, requiring, as it does, the determination of the various constants, could be dispensed with if a means were found to draw the "isodiffs" from only two points as a base. This means can be found by applying graphically the principles of the Wilson method.

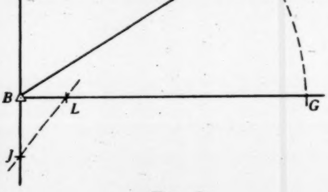


FIG. 8.

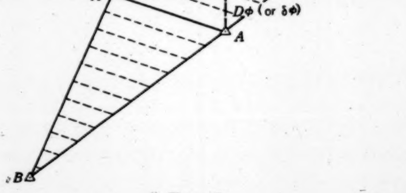


FIG. 9.

Assume, for instance, that Ties A and B (Fig. 8), are available with corrections of $d\phi_1$, $d\lambda_1$ and $d\phi_2$ $d\lambda_2$, respectively. As before, determine

$\delta\phi$, $\delta\lambda$, $D\phi$, and $D\lambda$. If $D\phi$ and $D\lambda$ are plotted at A to an enlarged scale as before, Point A' becomes the new relative position of Station A, and Point E the projection of Point A' on Line AB. Then, AE represents the change in length of AB and $A'E$ is a measure of the rotation of the base. Since the entire system must receive the same treatment as the base, all lines must be swung by the same amount, $K \times \overline{A'E}$, around Station B as a pivot, and reduced (or expanded) by the same ratio, $\frac{\overline{AE}}{s}$.

For convenience, consider Point F on the same meridian and Point G on the same parallel as Station B, so that $\overline{BF} = \overline{BG} = s$. At Point F the relative change in latitude is produced by the scale change only and equals $\frac{-\overline{AE}}{Q}$, and the spacing of "isolats" along Line BF equals $-\frac{sQ}{\overline{AE}} = m_\phi$. At Point G the relative change in latitude is produced by the rotation factor, $A'E$, only, and equals $\frac{\overline{A'E}}{Q}$, and the spacing of "isolats" along Line BG equals $\frac{sQ}{\overline{A'E}} = p_\phi$. Then, $\frac{m_\phi}{p_\phi} = -\frac{sQ}{\overline{AE}} \times \frac{\overline{A'E}}{sQ} = -\frac{\overline{A'E}}{\overline{AE}}$. It is now possible to draw the "isolats" by either of two methods; (1) By interpolation along Lines BF and BG, using the intersections on Line AB as a check; or (2) by the simpler method of interpolating along AB and using the directions, indicated by the slope ratio, $\frac{m_\phi}{p_\phi} = -\frac{\overline{A'E}}{\overline{AE}}$. The "isolats" will be straight, parallel lines, uniformly spaced.

Similarly, the relative longitude change at Point F is produced by $A'E$ only, and equals $\frac{\overline{A'E}}{R}$, and the spacing of the "isolongs" along BF equals $\frac{sR}{\overline{A'E}} = m_\lambda$; also, the relative longitude change at Point G is produced by AE only, and equals $\frac{\overline{AE}}{R}$, and the spacing of the "isolongs" along Line BG equals $\frac{sR}{\overline{AE}} = p_\lambda$. Then, $\frac{m_\lambda}{p_\lambda} = \frac{sR}{\overline{A'E}} \times \frac{\overline{AE}}{sR} = \frac{\overline{AE}}{\overline{A'E}}$. The "isolongs" may be drawn by the same methods as those used for the "isolats", and they, also, will be straight, parallel lines, uniformly spaced.

It will be noted that the slope of the "isolongs" is the negative reciprocal of the slope of the "isolats", which, by the principles of analytical geometry, shows them to be perpendicular to each other. By similar triangles it may be shown that the normal spacing of the "isolats" is $\frac{sQ}{\overline{AA'}}$, and that of the "isolongs", $\frac{sR}{\overline{AA'}}$. Therefore, the normal spacing of "isolats" is to the normal spacings of "isolongs" as $\frac{sQ}{\overline{AA'}}$ is to $\frac{sR}{\overline{AA'}}$ or as Q is to R.

Special attention may now be called to certain fundamental features of the "isodiff" diagram. Whenever a net of triangulation is shifted so that no change occurs in any of the previously adjusted angles of the net, the following conditions must result:

(*a*) The "isolats" will be straight, parallel lines, uniformly spaced;

(*b*) The "isolongs" also will be straight, parallel lines, uniformly spaced, and they will be perpendicular to the "isolats"; and,

(*c*) The normal spacing of the "isolats" will be to the normal spacing of the "isolongs" as Q is to R for the mean latitude of the project.

Conversely, it would appear that, when a graph of "isodiffs" satisfies Conditions (*a*), (*b*), and (*c*), the shape of the triangulation net being adjusted is not changed or warped. In other words, the net may have been shifted, rotated, and expanded (or contracted), but the angles have not been altered. This will be true if one other condition is satisfied; namely, if the direction of increasing values of the "isolats" is taken as north, the values of the "isolongs" should increase to the westward. If this pattern is kept in mind when drawing "isodiffs" for the general problem of several tie-points, an effort can be made to adhere to it as far as possible, in order to prevent or to minimize distortion in the system that is being adjusted.

To summarize the simpler construction for "isodiffs" when only two ties are available for a system: At Station A, plot $D\phi$ and $D\lambda$ to locate Point A'. Measure Lines AE and $A'E$. Along the meridian through Station B (BF), lay off $BJ = -A'E$. Along the parallel through Station B(BG), lay off $BL = AE$. Interpolating along AB for the "isolat" intersections, draw "isolats" parallel to JL. Interpolating along AB for the "isolong" intersections, draw "isolongs" perpendicular to JL. These "isodiffs" may be extended beyond Points A and B, respectively, by continuing the same spacings. As a partial check on the work, it may be noted that the normal distance between "isolats" is to the normal distance between "isolongs" as Q is to R.

From these "isodiffs" the same position corrections should be obtained as by the Wilson formulas; and the uniform azimuth and distance corrections may be applied to all lines in the same manner.

ROTATING GUIDE

Another method has been developed by Mr. Nelson for the graphical application of the principles proposed by Mr. Wilson. On a stiff transparent medium, such as celluloid or cellulose acetate, lay off Line AB (Fig. 9). At Station A, plot $D\phi$ and $D\lambda$ to locate Point A'. Then, AA' represents the actual movement (to the distorted scale) of Point A. With the system pivoted about Station B, all points on the system will receive the same relative movement with respect to B, the magnitude of the movements varying only with the distance of each point from the pivot. If BA and BA' are extended indefinitely, any point the original position of which is on BA would fall on BA' after adjustment, and the path of its movement would be parallel to AA'.

To use the adjustment guide, pass a pin or thumb-tack through Point B on the guide and Point B on the map, so that the guide is free to pivot about that point. Rotate the guide about the pivot so that each station to be adjusted will, in turn, fall on BA, extended. Project the station to BA' by a line parallel to AA'. Measure the differences in latitude and longitude for each station to the scale of the guide, and divide by Q and R, respectively, to obtain $(d\phi)'$ and $(d\lambda)'$. The projection to BA' may be made by eye if a number of lines are ruled on the guide parallel to AA'.

When a number of stations are to be adjusted by this method, time can be saved if scales for $(d\phi)'$ and $(d\lambda)'$ for the mean latitude of the project, are prepared in which one unit of $(d\phi)'$ is to one unit of $(d\lambda)'$ as Q is to R. With the aid of these auxiliary scales, $\delta\phi$ and $\delta\lambda$, for Point A, may be plotted directly on the guide without first converting them to meters, and $(d\phi)'$ and $(d\lambda)'$ may be scaled in seconds instead of in meters. Of course, the corrections at Station B must be added to those obtained by the use of the guide, in order to secure the final corrections for each station being adjusted. The uniform azimuth and distance corrections may be applied to each line in the proper manner.

CONCLUSION

It is readily apparent that all the methods of adjustment herein described are based on approximations of varying degrees. However, since the actual quantities considered are so small, they may be determined only approximately and still be well within the accuracy of the original survey. The mathematical computations will seldom require more than three significant figures, and a 10-in. slide-rule will usually be found ample; for the graphical work, ordinary plotting precision will satisfy most requirements.

The weakest link in the entire chain of readjustment would appear to be the construction of the "isodiffs" when several ties are available. Moreover, these methods do not correct errors or inaccuracies that may exist in the original computations. In the readjustment of good field work that has been rigidly adjusted to a good datum, the change of datum will not be likely to distort the originally adjusted figure to any great extent, so that the "isodiffs" should be sketched with little difficulty and will be fairly regular lines, well spaced. For such work there should be little question about the suitability of readjustment by "isodiffs".

If the field work itself contains appreciable discrepancies, or if it was not properly adjusted, or adjusted to a poor datum, the corrections at the tie-points might not be expected to be consistent. The "isodiffs" for such a readjustment might be badly curved and poorly spaced, but the readjustment based on them would probably be no worse than the original work and would make the survey available on the new datum, whereas its usefulness might otherwise be lost completely.

These methods, furthermore, permit the closing of circuits that formerly consisted of independent links tying to common stations, but with different co-ordinates. In some problems of this type, it is a simple matter to sketch the "isodiffs" between the fixed ties, making due allowance for the break.

Another method that sometimes gives good results is to assume as a position for each common station a weighted mean of its positions on the respective triangulation nets tying to it, the weight depending on the strength of the respective nets and the distance of the common station from each of the fixed tie-points. The common station may then be used as a fixed tie-point in the adjustment.

A third method, somewhat longer than either of the foregoing but, perhaps, more rigid, involves a partial preliminary adjustment. Each link in the circuit is, in turn, adjusted to the datum of any one of the links, regardless of the fixed tie-points. When the entire circuit has been reduced to a single unified datum it may be adjusted to the final datum through the fixed tie-points.

Other appropriate methods will doubtless suggest themselves to the computer, according to the specific problems on hand.

The use of these methods for readjustment of old work suggests an application to new work that might warrant the consideration of the computer. If a chain of quadrilaterals (or other suitable figures) is extended from one fixed base to close on another, it should be possible to begin with the fixed positions at one end of the chain and compute through individually adjusted figures to obtain new values for the fixed positions at the other end. The necessary corrections to bring the closing stations to their proper positions may then be determined, and "isodiffs" drawn for the chain, using the corrections determined for the closing end and zero corrections for the other end. Then, all intermediate positions could be corrected by the indicated amounts, and all lines corrected for azimuth and scale by the formulas. By this means, latitude and longitude conditions, as well as length and azimuth conditions, would be satisfied for both bases. Although the results might not agree perfectly with those of a complete, rigid least-squares adjustment, it is believed the differences would be very slight unless the strength of the chain varies appreciably within its length.

The choice of methods for readjustment of datum may be summarized briefly, as follows:

(*a*) When two ties to the new datum are available, determine position corrections by: (1) The Wilson formulas; (2) the rotating adjustment guide; or (3) the two-point "isodiffs". Apply uniform azimuth and length corrections to all lines.

(*b*) When three or more ties are available, construct an "isodiff" graph for the determination of position changes. Determine azimuth and length corrections by the formulas or by the nomographic chart; or use both for a check.

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STRUCTURAL ANALYSIS BASED UPON PRINCIPLES PERTAINING TO UNLOADED MODELS

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SYNOPSIS

When the stresses in a structure are investigated by the use of unloaded models, known deformations are produced at a section at which the stresses are desired, and the corresponding displacements are measured at the point at which the load is applied in the actual structure. The ratios between the latter and the former deformations give the required stresses for a unit load, and when the original deformation is taken as unity, the model forms the influence line of stress. To obtain a moment, a known or unit rotation is produced at the section at which the stress is required; a normal displacement is produced to obtain a shear or a reaction; and an axial displacement is produced to obtain a direct tension or compression. The geometrical analysis of these operations without the actual use of models leads to fundamental simplifications, and it is the purpose of this paper to establish the few simple equations necessary for the use of this method of analyzing statically indeterminate structures.

DEFINITIONS AND NOTATION

The fundamental beam with a constant moment of inertia used in this analysis is the member, *AB*, shown in the several illustrations. The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[2] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

a = a definite value of x; $a' = L - a$ = a definite value of x'; a'' = distances measured along a cantilever from the support;

f = the intercept at End A of the tangent to the elastic curve, at End $B = \alpha_{BA} L$; similarly, $f' = \alpha_{AB} L$;

NOTE.—Published in January, 1937, *Proceedings*.

[1] Buenos Aires, Argentine Republic.

[2] A.S.A.—*Z10a*—1932.

I = moment of inertia;

K_A = the sum of the relative stiffness values of the ends of all members meeting at a joint; and, therefore, $K_A = S + S_A$; and, $K_B = S' + S_B$;

k = stiffness of a member $= \frac{I}{L}$;

L = span length;

M = bending moment; M_{AB} = moment at End A of Beam AB; M_{BA} = moments at End B of Beam AB; and are positive in the same directions as their corresponding rotations (α);

m = ratio of end rotations; for rotations at Ends A and B, respectively: $m = \frac{\alpha_{BA}}{\alpha_{AB}} = \frac{f}{f'}$; and, $m' = \frac{\alpha_{AB}}{\alpha_{BA}} = \frac{f'}{f}$; for rotations originating at A and B, respectively;

N = a substitution factor (see Equation (18));

P = total concentrated load applied at any point of the actual structure;

R = a ratio $= \frac{\Delta}{L}$;

S = relative stiffness value $= (1 - 0.5\ m)\, k$ = the resistance to rotation of the left end of a member; $S' = (1 - 0.5\ m')\, k$ = the resistance to rotation of the right end of a member; S_A = the sum of the relative stiffness values of all the members meeting the left end of a member at a joint, but not including the end of the member in question. Since the right ends of the members connect with the left end of a member under consideration, $S_A = S'_1 + S'_2 + S'_3 + \ldots +$, etc.; S_B is a quantity similar to S_A, for the right end of a member, and, correspondingly, $S_B = S_1 + S_2 + S_3 + \ldots +$, etc.;

V = total shear;

W = total uniform load on a beam;

x = variable abscissa of the influence line, measured from the left support; $x' = L - x$;

y = variable ordinate of the influence line; y_c = the ordinate at the center of the beam; y_m = the mean value of y for any influence line;

α = the angular rotation of the end of a member relative to the final direction of the axis. It is positive when the rotation is such that tension is produced in the "upper" fibers. As will be seen from the analysis all members have a "left" and a "right" end, both of which are definitely determined; therefore, the terms "upper" and "lower" are also definitely determined;

Δ = the deflection of the left end of a member relative to the right end and is positive when the left end is "raised" relative to the right end;

ϵ = energy; internal work exerted against the work involved in the diminishing reaction (see Equation (26));

θ = the angular rotation of the end of a member relative to the original direction of its axis, and is positive in the same direction as α.

INTRODUCTION

In hypothetical models, as in actual models, the deformations are assumed to be indefinitely small, so that the projection of any deformed member or part of it on its original axis is equal to its original length; that is, in Fig. 1, the distance, *A-B*, remains unchanged after rotation at Point *E*.

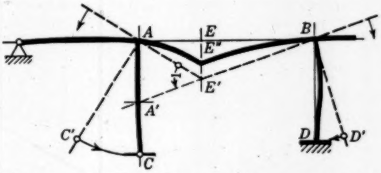


FIG. 1.—A RIGID FRAME; INFLUENCE LINE FOR MOMENT AT POINT *E*.

The present analysis will be limited to skeletons or frames with vertical columns and horizontal beams, each of constant moment of inertia and all of the same homogeneous material, with constant modulus of elasticity. As in other, similar methods of analysis, the structure is first restrained against side-sway. The structure is then released, the effect of side-sway computed, and the stresses are corrected. For example, let *AB* be a beam within the typical skeleton shown in Fig. 1 in which the bending moment, M_E, at Point *E* is required. If the skeleton were merely simply supported at Points *A* and *B*, a unit rotation at Point *E* would produce the deformation, $C' A E' B D'$, in the model as shown in broken lines, in Fig. 1. Then (with the angles between members fixed at all joints, but with the joints free to rotate) Points C' and D' and the ends of all other beams and columns, are returned to their original position. The shape of the frame ($C A E'' B D$), will then represent the final influence line for bending moment.

It will be seen from Fig. 1 that all branches of the influence line for M_E are simple geometrical curves, produced by the rotation of the broken ends at Point *E* and by the restraint at the far ends, except $A E'' B$, which is a combination of the triangle, $A E' B$, corresponding to free supports at *A* and *B* and the eventual restraint at Points *A* and *B*. These rotations, restraints, and displacements produce typical bending moments in the several parts of the model, which admit of an easy analysis of the curves formed. It should be kept in mind, however, that influence lines may be considered as geometrical curves which can be obtained by geometrical methods. Equations (1) to (21) which follow, have been obtained experimentally by applying visible displacements and unit rotations to models composed of elastic splines. The resulting curves have been analyzed in Appendix II. The basic theory of unloaded models is derived in Appendix I.

FUNDAMENTAL EQUATIONS OF SIMPLE INFLUENCE LINES

When an elastic spline (see Fig. 2(*a*)), pin-connected at Points *A* and *B*, is rotated about Point *A* through an angle equal to $\frac{f'}{L}$, the corresponding rotation at Point *B* is equal to $\frac{f}{L} = \frac{f'}{2\,L}$; or $m = \frac{f}{f'} = 0.5$. This experiment may be checked by abstract theory: Rotation at Point *A* produces a bending moment in the spline which is a maximum at Point *A* and which decreases uniformly to zero at Point *B*. At Point *A*, the reaction of the beam, loaded with that triangular moment area, is twice that at Point *B* and, because the inclination of the tangents to the elastic line are proportional to the reactions of this moment area, $f' = 2f$.

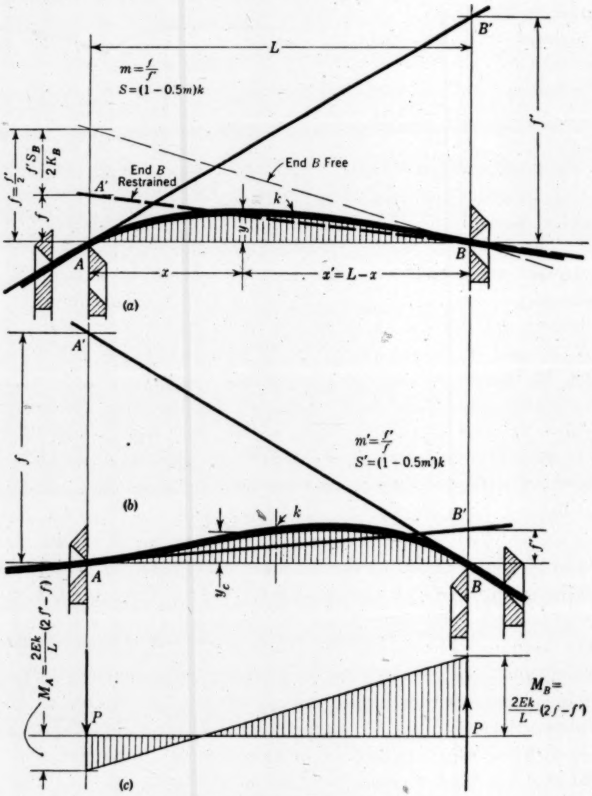

L
B'
$m = \frac{f}{f'}$
$S = (1 - 0.5m)k$
f'
$f = \frac{f'}{2}$
$\frac{f'S_B}{2K_B}$
f
End B Restrained
End B Free
A'
k
y
A
B
x
$x' = L - x$
(a)
A'
f
$m' = \frac{f'}{f}$
$S' = (1 - 0.5m')k$
(b)
k
B'
f'
A
B
y_c
$M_A = \frac{2Ek}{L}(2f' - f)$
$M_B = \frac{2Ek}{L}(2f - f')$
P
P
(c)

FIG. 2.—SIMPLE INFLUENCE LINES

However, when other members join at Point *B* to create a total relative stiffness, S_B, the rotation of the spline is restrained proportionately as shown by Line *BA'* (Fig. 2(*a*)) and, therefore:

$$m = \frac{f}{f'} = \frac{0.5\,k}{S_B + k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly, rotating the spline about Point *B* and restraining it at Point *A* (see Fig. 2(*b*)):

$$m' = \frac{f'}{f} = \frac{0.5\,k}{S_A + k} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Referring back to Fig. 2(*a*), when the spline is released at Point *B* so that the value of *f* increases, the strain on the spline at Point *A* is reduced by one-half the corresponding increase; and since, when Point *B* is fixed and cannot rotate, $S = k$ (for $f = 0$ and $m = 0$):

$$\frac{S}{k} = 1 - \frac{m}{2} \qquad (3a)$$

and, similarly,

$$\frac{S'}{k} = 1 - \frac{m'}{2} \qquad (3b)$$

The curves shown in Figs. 2(*a*) and 2(*b*) are typical simple influence lines; they are obtained by applying to the spline, *AB*, bending moments at Points *A* and *B* and, consequently, may be analyzed as the moment lines of Beam *AB* loaded by areas of moments that change uniformly from:

$M_A = 4K\theta_A - 2K\theta_B$

$$\frac{M_A}{E\,I} = \frac{2\,(2\,f' - f)}{L^2} \qquad (4a)$$

at Point *A*, to:

$$\frac{M_B}{E\,I} = \frac{2\,(2\,f - f')}{L^2} \qquad (4b)$$

at Point *B* (see Fig. 2(*c*)). Thus, one may derive the very simple equation for general influence lines in terms of end rotations:

$$y = \frac{x\ x'}{L^3}\,(x\,f + x'\,f') \qquad (5)$$

At the center of the beam:

$$y_c = \frac{1}{8}\,(f + f') \qquad (6a)$$

and the mean value is:

a parabola approx.

$$y_m = \frac{2\,y_c}{3} = \frac{f + f'}{12} \qquad (6b)$$

When the beam, *AB*, is simply supported or pin-connected at Point *B*: $m = 0.5$; $S = 0.75\,k$;

$$y_c = \frac{3\,f'}{16} \qquad (7a)$$

$$y_m = \frac{f'}{8} \qquad (7b)$$

and,

$$y = \frac{x\,x'\,f'}{L^3}\left(\frac{x}{2} + x'\right) \qquad (7c)$$

and when End B is fixed: $m = 0$; $S = k$;

$$y_c = \frac{f'}{8} \quad \text{.........................} (8a)$$

$$y_m = \frac{f'}{12} \quad \text{.........................} (8b)$$

and,

$$y = \frac{x\,(x')^2\,f'}{L^3} \quad \text{.........................} (8c)$$

Similarly, when End A is simply supported: $m' = 0.5$; $S' = 0.75\,k$;

$$y_c = \frac{3f}{16} \quad \text{.........................} (9a)$$

$$y_m = \frac{f}{8} \quad \text{.........................} (9b)$$

and,

$$y = \frac{x\,x'\,f}{L^3}\left(x + \frac{x'}{2}\right) \quad \text{.........................} (9c)$$

When End A is fixed: $m' = 0$; $S' = k$;

$$y_c = \frac{f}{8} \quad \text{.........................} (10a)$$

$$y_m = \frac{f}{12} \quad \text{.........................} (10b)$$

and,

$$y = \frac{x^2\,x'\,f}{L^3} \quad \text{.........................} (10c)$$

Equations (1) to (10) are the fundamentals required for analyzing frames. They apply to all classes of influence lines and lead to new and useful analytical and graphical interpretations of these curves. Of special interest is the fact that the relative stiffness of end panels can be easily calculated from Equations (8) and (10) and that, therefore, the statical indeterminancy due to restraint at the support need no longer be a serious factor in computations. Side-sway is not taken into account in these equations, but the necessary correction can be made easily.

LINEAR DISPLACEMENT

Fig. 3(a) shows a beam or column in its normal, unstrained position. If End A, with all of the part of the structure to the left, is translated vertically upward a distance, Δ, Beam AB will deform as shown in Fig. 3(b). The members to the left of Point A are considered to be restrained so that they cannot move parallel to the original position of Member *AB*, but are free to move perpendicular to that axis, except that all members move the same distance, Δ. All members are considered free to rotate as much as their relative stiffnesses, *S*, will permit.

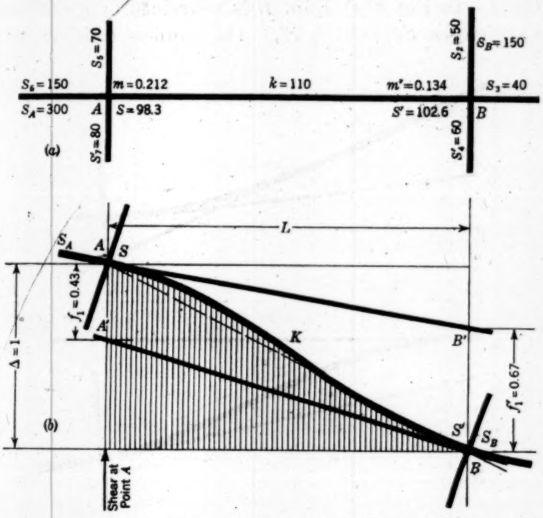

$S_5=70$
$S_B=150$
$S_2=50$
$S_6=150$
$m=0.212$
$k=110$
$m'=0.134$
$S_3=40$
$S_A=300$
A
$S=98.3$
$S'=102.6$
B
$S_7=80$
$S_4=60$
(a)
S_A
A
S
L
$f_1=0.43$
$\Delta=1$
A'
K
B'
$f_1'=0.67$
(b)
S'
S_B
B
Shear at Point A

FIG. 3.—LINEAR DISPLACEMENT IN A RIGID FRAME

Joints *A* and *B*, then, tend to resist rotation and exert a counter rotation as shown in Fig. 3(*b*), so that the algebraic sums of the deformations are:

$$\frac{f}{\Delta} = -\frac{S_B}{K_B} + \frac{m\ S_A}{K_A} \qquad \text{(11)}$$

and,

$$\frac{f'}{\Delta} = \frac{S_A}{K_A} - \frac{m'\ S_B}{K_B} \qquad \text{(12)}$$

For example (referring to Fig. 3), let $k = 110$; $S_A = 70 + 150 + 80 = 300$; and $S_B = 50 + 40 + 60 = 150$. If Joint *A* is displaced vertically a distance, $\Delta = 1$, the rotations are: At Joint *A*, $\frac{f'}{L}$; and, at Joint *B*, $\frac{f}{L}$. The influence line for shear at End *A* of Beam *AB*, then will be determined as follows: $m = \frac{0.5 \times 110}{110 + 150} = 0.212$; $S = 110\left(1 - \frac{0.212}{2}\right) = 98.3$; $m' = \frac{0.5 \times 110}{110 + 300} = 0.134$; $S' = 110\left(1 - \frac{0.134}{2}\right) = 102.6$; $K_A = 98.3 + 300 = 398.3$; and $K_B = 102.6 + 150 = 252.6$. The net rotation, therefore, will be: At Joint B, $-\dfrac{f}{\Delta} = \dfrac{150}{252.6} - \dfrac{0.212 \times 300}{398.3} = 0.434$; and, at Joint A, $\dfrac{f'}{\Delta} = \dfrac{300}{398.3} - \dfrac{0.134 \times 150}{252.6} = 0.674$.

Special Cases.—In Fig. 4(a) Joint B is restrained by a stiffness, S_B, composed of the stiffnesses, $S_2 + S_3 + S'_4$. The stiffness ratio, $K_B = S'_1 + S_B$.

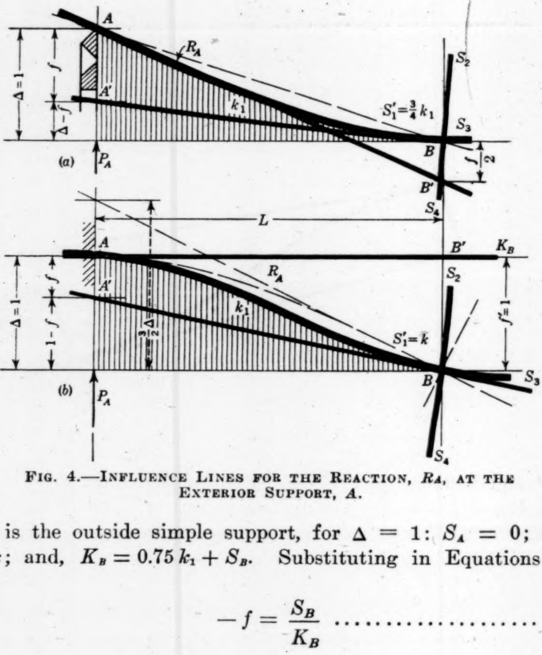

A
R_A
$\Delta = 1$
f
$\Delta - f$
A'
k_1
S_2
$S'_1 = \frac{3}{4} k_1$
S_3
B
$\frac{f}{2}$
B'
S_4
(a)
P_A
L
A
B'
K_B
R_A
S_2
$\Delta = 1$
f
A'
k_1
$1 - f$
$\frac{3}{2}\Delta$
$f' = 1$
$S'_1 = k$
B
S_3
(b)
P_A
S_4

FIG. 4.—INFLUENCE LINES FOR THE REACTION, R_A, AT THE EXTERIOR SUPPORT, A.

If End A is the outside simple support, for $\Delta = 1$: $S_A = 0$; $m' = 0.5$; $S_1' = 0.75\,k$; and, $K_B = 0.75\,k_1 + S_B$. Substituting in Equations (10) and (11):

$$-f = \frac{S_B}{K_B} \qquad \text{(13a)}$$

and,

$$f' = -\frac{S_B}{2\,K_B} \qquad \text{(13b)}$$

On the other hand, if Point A is fixed (see Fig. 4(b)), for $\Delta = 1$: $S_A = \infty$; $\dfrac{S_A}{K_A} = 1$; $m = \dfrac{k_1}{2\,K_B}$; and $m' = 0$, substituting these values in Equations (10) and (11):

$$-f = \frac{S_B}{K_B} - \frac{k_1}{2\,K_B} = 1 - \frac{3\,k_1}{2\,K_B} \qquad \text{(14a)}$$

and,

$$f' = 1 \qquad \text{(14b)}$$

Equations (13) and (14) may also be obtained from Fig. 4 without reference to Equations (11) and (12); for example, in Fig. 4(*b*) if Support *B* were free, an upward displacement, Δ, at End *A* would rotate *AB* at Joint *B* through an angle, $\frac{3\,\Delta}{2\,L}$. If the resisting stiffness of the remaining three members meeting at Joint *B* is then applied, a counter rotation occurs, equal to $\frac{3\,\Delta\,S'_1}{2\,L K_B}$. Finally, for $\Delta = 1$ and $S'_1 = k_1$: $-f = 1 - \frac{3\,k_1}{2\,K_B}$ (see Equation (14)).

TRANSVERSE FORCES

The force necessary to produce a given linear displacement of Support *A* with respect to Joint *B* of the unloaded beam, *AB*, or the shear at the supports, corresponding to known bending moments at the joints, is $P = \frac{M_A - M_B}{L}$ in which, according to Fig 2(*c*), $M_A = \frac{2\,E\,k_1\,(2f' - f)}{L}$; and, $M_B = \frac{2\,E\,k_1\,(2f - f')}{L}$. Making the necessary substitutions and simplifying:

$$P = \frac{6\,E\,k_1\,(f' - f)}{L^2} \qquad (15)$$

INFLUENCE LINES FOR BENDING MOMENTS

The influence line for bending moment at any section, *E*, of a beam, *AB* (see Fig. 5), is obtained by applying a unit rotation at Point *E*. When Joints *A* and *B* are simply supported the unit rotation deforms the member, *AB*, as shown in Fig. 5(*a*), producing the broken beam, *AE′B*, so that $\Delta = a$ and $\Delta' = a'$.

The restraining stiffness of the connecting members of the two joints, however, exerts a counter rotation, producing the curve with end rotations, $a - f$ and $a' - f'$, shown in Fig. 5(*b*), so that, finally, in Fig 5(*c*):

$$f = a\,\frac{S_B}{K_B} - m\,a'\,\frac{S_A}{K_A} \qquad (16a)$$

and,

$$f' = a'\,\frac{S_A}{K_A} - m'\,a\,\frac{S_B}{K_B} \qquad (16b)$$

General Ordinate and Mean Ordinate.—For $x < a$:

$$y = \frac{a'\,x}{L} - N \qquad (17a)$$

and, for $x > a$:

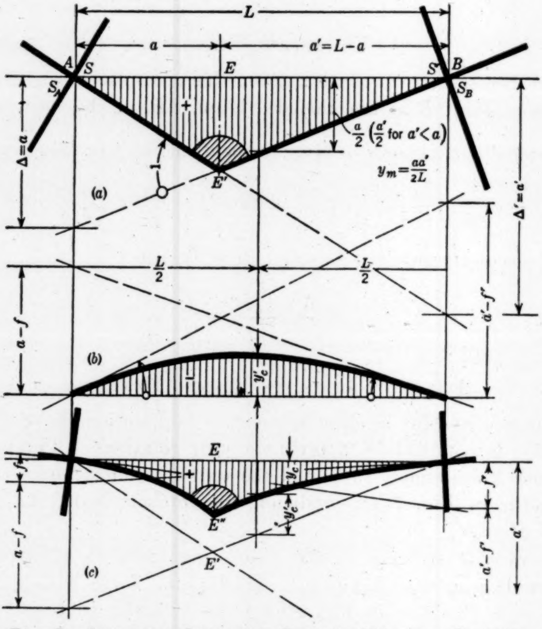
$$y = \frac{a\,x'}{L} - N \qquad (17b)$$

in which,

$$N = \frac{x\,x'}{L^3}\left[\,(a - f)\,x + (a' - f')\,x'\,\right] \quad \ldots\ldots\ldots\ldots\ldots(18)$$

In general, the mean ordinate is expressed by,

$$y_m = \frac{a\,a'}{2\,L} - \frac{L - f - f'}{12} \quad \ldots\ldots\ldots\ldots\ldots(19)$$



L
a
a′=L−a
A
S
E
S′
B
S_A
S_B
+
Δ=a
$\frac{a}{2}\left(\frac{a'}{2}\text{ for } a'<a\right)$
$y_m=\frac{aa'}{2L}$
1
(a)
E′
Δ′=a′
$\frac{L}{2}$
$\frac{L}{2}$
a−f
a′−f′
(b)
−
y_c
E
+
f
y_c
f′
E″
y_c'
a−f
(c)
E′
a′−f′
a′

FIG. 5.—INFLUENCE LINES FOR BENDING MOMENTS, OBTAINED GEOMETRICALLY

When $a = a' = 0.5\,L$, the ordinate at the center, y_c, and the mean ordinate, y_m, are respectively:

$$y_c = \frac{L + f + f'}{8} \quad \ldots\ldots\ldots\ldots\ldots(20a)$$

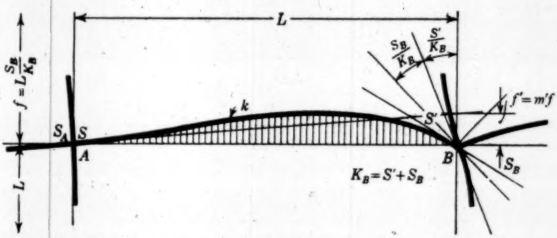
and,

$$y_m = \frac{2\,y_c}{3} - \frac{L}{24} \quad \ldots\ldots\ldots\ldots\ldots(20b)$$

For $x = a$:

$$y = \frac{a\,a'}{L^3}\,(2\,a\,a' + a\,f + a'\,f') \quad \ldots\ldots\ldots\ldots\ldots(21)$$

Here again it is demonstrated that influence lines, considered as purely geometric curves, can be derived by geometric methods alone. For example, the ordinate of Fig. 5(*b*) may be deducted from corresponding ordinates of Fig. 5(*a*) to secure the final curve of Fig. 5(*c*), without becoming involved in the theory of stresses or in stress functions. When $a = L$ and $a' = 0$, the result is an influence line for the end moment, M_B (see Fig. 6), for which the corresponding end rotations are:



L
$f = L\frac{S_B}{K_B}$
L
S_A
S
A
k
$\frac{S_B}{K_B}$
$\frac{S'}{K_B}$
S'
$f' = m'f$
B
S_B
$K_B = S' + S_B$

FIG. 6.—INFLUENCE LINE FOR BENDING MOMENT, M_{BA}.

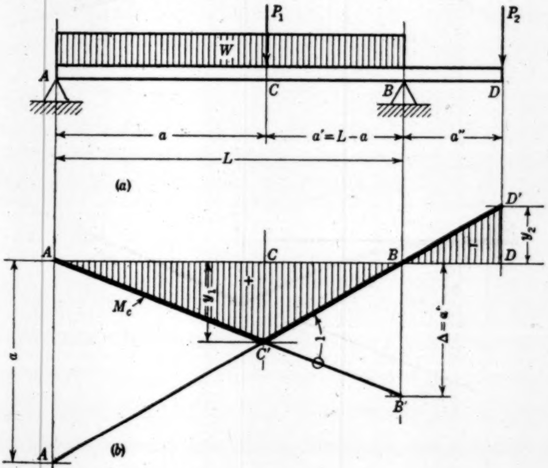
$$f = \frac{L\,S_B}{K_B} \qquad (22a)$$

and,

$$f' = m'\,f \qquad (22b)$$

EXAMPLES

The value of the methods outlined in this paper may be illustrated by means of the following examples.



P_1
P_2
W
A
C
B
D
a
$a' = L - a$
a''
L
(a)
D'
A
C
B
D
y_2
y_1
+
−
M_c
C'
1
$\Delta = a''$
a
B'
A
(b)

FIG. 7.—INFLUENCE LINE FOR BENDING MOMENT AT POINT C OF A SIMPLE BEAM

Example 1.—Fig. 7(*a*) represents a beam simply supported at Points *A* and *B*, with a cantilever extension beyond Point *B*. The bending moment M_c is required. Breaking the hypothetical beam at Point *C* and rotating one of the parts with respect to the other through an angle equal to $\frac{a'}{\Delta} = 1$ (see Fig. 7(*b*)), produces the influence line for M_c, $A C' B$, with major ordinates of: $y_1 = \frac{a\,a'}{L}$; $y_m = \frac{y_1}{2} = \frac{a\,a'}{2\,L}$; and $y_2 = -\frac{a\,a''}{L}$. Consequently,

$$M_C = \frac{W\,a\,a'}{2\,L} + \frac{P_1\,a\,a'}{L} - \frac{P_2\,a\,a''}{L} \quad \ldots\ldots\ldots\ldots\ldots (23)$$

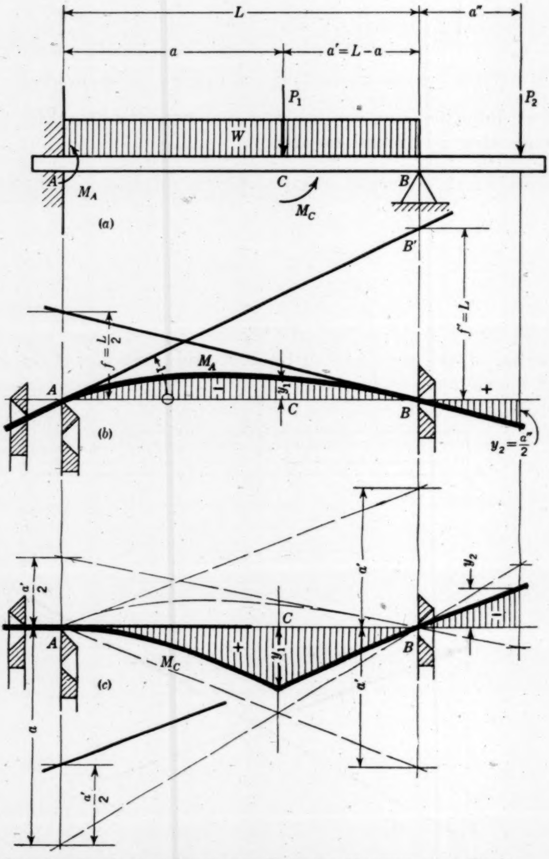


FIG. 8.—INFLUENCE LINES FOR BENDING MOMENT IN A BEAM FIXED AT ONE END

Example 2.—In Fig. 8(*a*), the beam in Example 1 has been fixed at End *A*. The bending moment M_A is required. Rotating the beam at Point *A* through an angle, $\frac{f'}{L} = 1$ (see Fig. 8(*b*)) and then translating Point *B'* back to its original position at Point *B*, $f = \frac{f'}{2} = \frac{L}{2}$ Then, from Equation (7); $y_m = \frac{L}{8}$; $y_1 = \frac{a\,a'\,(0.5a + a')}{L^2}$; $y_2 = \frac{f\,a''}{L} = \frac{a''}{2}$; and,

$$M_A = -\frac{W\,L}{8} - \frac{P_1\,a\,a'\,(0.5\,a + a')}{L^2} + \frac{P_2\,a''}{2} \quad \text{..........} (24)$$

Example 3.—To determine the influence line for the moment, M_C, at any Point *C*, Fig. 8(*a*), the beam is broken at Point *C* and rotated through an angle of $\frac{a'}{\Delta} = 1$, as in Example 1. End *A* is then rotated back until it is once more horizontal, and for this operation considered separately, the hypothetical beam has the form shown by the dotted curved line shown above *AB* in Fig. 8(*c*), ($f' = +\,a'$ and $f = +\,0.5\,a'$), for which Equation (7) gives the ordinates. For the first operation the ordinates are the same as in Example 1. The final ordinates are the differences between corresponding ordinates in each of the two steps, thus: $y_m = \frac{a\,a'}{2\,L} - \frac{a'}{8}$; $y_1 = \frac{a\,a'}{L} - \frac{a\,(a')^2\,(0.5\,a + a')}{L^3} = \frac{a^2\,a'\,(3\,L - a)}{2\,L^3}$; $y_2 = \frac{-\,a\,a''}{L} + \frac{a'\,a''}{2\,L} = \frac{-\,a''\,(3\,a - L)}{2\,L}$; and, finally,

$$M_C = W\,y_m + P_1\,y_1 + P_2\,y_2 \quad \text{.................} (25)$$

It is to be noted that the ordinates in Equation (25) can be obtained directly by means of Equations (16) to (22).

It is to be noted also that the effect of the load on the cantilever section is to produce positive bending moment in Member *AB* when $a < \frac{L}{3}$.

Example 4.—Consider the continuous beam in Fig. 9 for which M_B is required. This beam is elastically determinate, with: $S_1 = k_1 = 80$; $S_2 = 0.75\,k_2 = 45$; $k_B = 80 + 45 = 125$; $\frac{S'_1}{k_B} = \frac{80}{125} = \frac{16}{25}$; and, $\frac{S_2}{k_B} = \frac{45}{125} = \frac{9}{25}$. Referring to Fig. 9(*b*), break the beam at Point *B*; rotate one part through an angle of 1; and holding the angle between the two ends at this joint at 1, return End *C* back to a pin connection in its original position. According to Equation (22*a*), the result is that End *B* of Beam *AB* yields through an angle measured by $f_1 = \frac{20 \times 9}{25} = 7.2$, and (since the beam is fixed at Point *A*) $f' = 0$; $f'_2 = \frac{16 \times 16}{25} = 10.24$; and (since *BC* is simply supported at Point C), $f_2 = \frac{10.24}{2} = 5.12$. From Equation (10): $y_{m1} = \frac{f_1}{12} = 0.6$; and, $y_1 = \frac{14^2 \times 6 \times 7.2}{20^3} = 1.06$. Also, from Equations (7): $y_{m2} = \frac{10.24}{8} = 1.28$. Finally, $y_2 = \frac{-5.1 \times 5.0}{16} = -1.6$; and, therefore, $M_B = 12 \times 0.6 + 15 \times 1.06 + 8 \times 1.28 - 5 \times 1.6 = 25.32$.

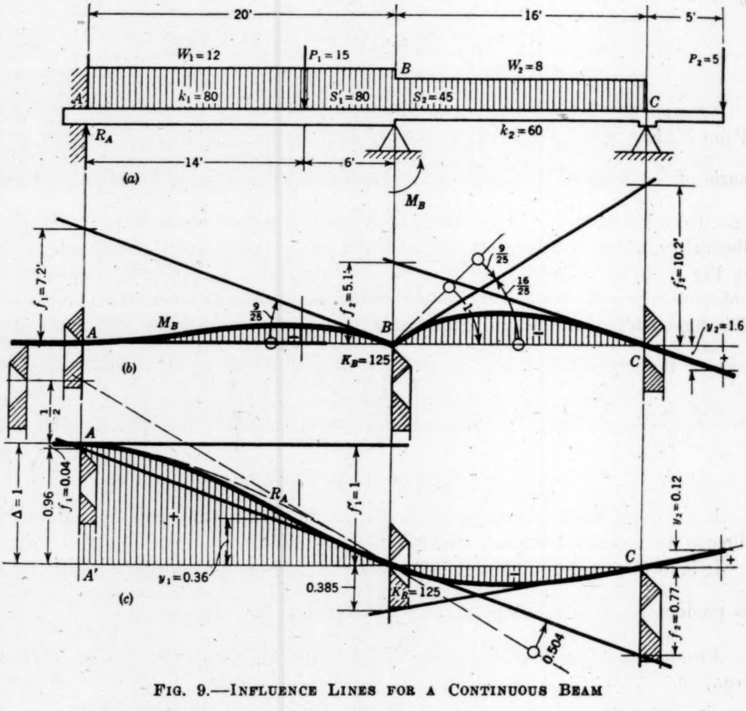

20'
16'
5'
$W_1 = 12$
$P_1 = 15$
B
$W_2 = 8$
$P_2 = 5$
A
$k_1 = 80$
$S'_1 = 80$
$S_2 = 45$
C
R_A
$k_2 = 60$
14'
6'
(a)
M_B
$f_1 = 7.2'$
$f_2 = 5.1'$
$f'_2 = 10.2'$
$\frac{9}{25}$
$\frac{16}{25}$
1
A
M_B
B
$K_B = 125$
C
$y_2 = 1.6$
(b)
+
−
$\frac{1}{2}$
A
$\Delta = 1$
0.96
$f_1 = 0.04$
$f'_1 = 1$
R_A
+
−
$y_1 = 0.36$
A'
(c)
0.385
$K_B = 125$
C
$y_2 = 0.12$
$f_2 = 0.77$
0.504

FIG. 9.—INFLUENCE LINES FOR A CONTINUOUS BEAM

Example 5.—The influence line for the reaction, R_A, in the beam shown in Fig. 9 may be determined by reference to Fig. 9(c). First, deflect End A upward a distance, $\Delta = 1$, without permitting it to rotate. Following the procedure demonstrated in connection with Fig. 4(b) and from Equations (14): $-f_1 = 1 - \frac{3 \times 80}{250} = 0.04$; $f'_1 = 1$; $-f'_2 = \frac{0.96 \times 16}{20} = 0.77$; $f_2 = \frac{f'_2}{2} = 0.385$; $y_{m1} = \frac{1}{2} + \frac{1 - 0.04}{12} = 0.58$; $y_1 = \frac{6}{20} + 14 \times 6 \times \left(\frac{-14 \times 0.04 + 6 \times 1}{20^3}\right) = 0.36$; $y_{m2} = -\frac{0.77}{8} = -0.096$; $y_2 = \frac{0.385 \times 5}{16} = 0.12$; and, $R_A = 12 \times 0.58 + 15 \times 0.36 - 8 \times 0.096 + 5 \times 0.12 = 12.2$.

Example 6.—Consider an unsymmetrical portal frame, loaded by a horizontal force as shown in Fig. 10(*a*). The values, H_A and H_D are required. The stiffness, S, of Columns AB and DC is equal to k, and, therefore, $S'_1 = 20$;

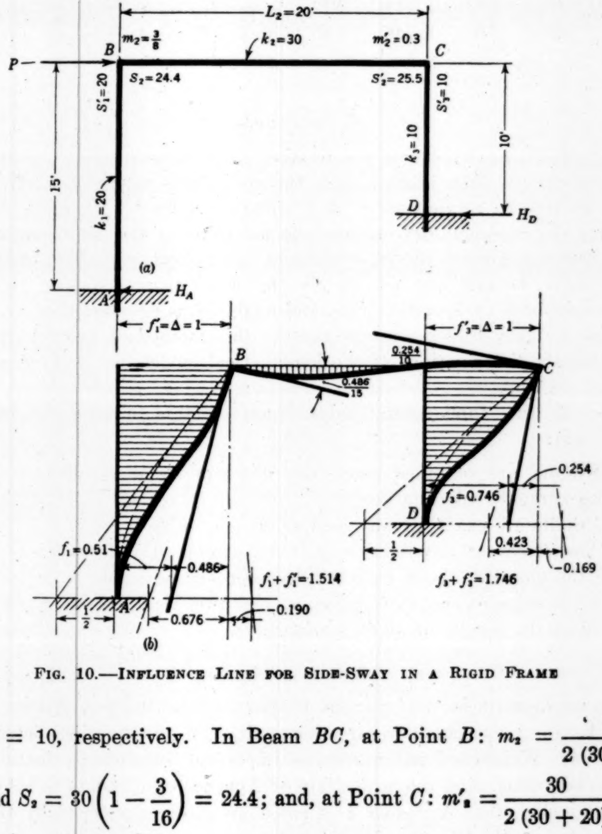

$L_2 = 20'$
$m_2 = \frac{3}{8}$
$k_2 = 30$
$m'_2 = 0.3$
B
C
P
$S_2 = 24.4$
$S'_2 = 25.5$
$S'_1 = 20$
$S'_3 = 10$
10'
$k_3 = 10$
D
H_D
15'
$k_1 = 20$
(a)
A
H_A
$f'_1 = \Delta = 1$
$f'_3 = \Delta = 1$
B
$\frac{0.254}{10}$
C
$\frac{0.486}{15}$
0.254
$f_3 = 0.746$
D
$\frac{1}{2}$
0.423
−0.169
$f_3 + f'_3 = 1.746$
$f_1 = 0.51$
0.486
$f_1 + f'_1 = 1.514$
A
$\frac{1}{2}$
0.676
−0.190
(b)

FIG. 10.—INFLUENCE LINE FOR SIDE-SWAY IN A RIGID FRAME

and $S'_3 = 10$, respectively. In Beam BC, at Point B: $m_2 = \dfrac{30}{2\,(30 + 10)} = \frac{3}{8}$; and $S_2 = 30\left(1 - \dfrac{3}{16}\right) = 24.4$; and, at Point C: $m'_2 = \dfrac{30}{2\,(30 + 20)} = 0.3$; and, $S'_2 = 30\left(1 - \dfrac{0.3}{2}\right) = 25.5$. If the effect of the horizontal force, P, is to deflect the frame sidewise any distance, Δ, with respect to Points A and D (assuming Joints B and C as pin-connected), the next step is similar to that shown in Fig. 3(*b*), with the result that, $f' = \Delta = 1$ and $f = -\dfrac{\Delta}{2} = -\,0.5$. Restoring the rigidity in the joints by applying counter-rotations: $1 - f_1 = \dfrac{3 \times 20}{2(20 + 24.4)} = 0.676$; and $1 - f_3 = \dfrac{3 \times 10}{2\,(10 \times 25.5)} = 0.423$, the adjustment being in the ratio of $m'_2 = 0.3$, and $m_2 = 0.375$. Consequently, $f_1 = 1 - 0.676 + 0.3 \times 0.423 \times \frac{15}{10} = 0.514$; $f_3 = 1 - 0.423 + 0.375 \times 0.676 \times \frac{10}{15} = 0.746$; $f_1 + f'_1 = 0.514 + 1 = 1.514$; and, $f_3 + f'_3 = 0.746 + 1 = 1{,}746$. Therefore, $\frac{H_A}{H_D} = \frac{20}{10} \times \frac{1.514}{1.746} \times \frac{10^2}{15^2} = 0.77$; $\frac{H_A}{P} = \frac{0.77}{1.77} = 0.435$; and, $\frac{H_D}{P} = \frac{1.0}{1.77} = 0.565$.

CONCLUSIONS

When analyzing structures the relative stiffnesses, S and S', of the members are computed first, beginning at the end panels where $S = 0.75\,k$ when the end is free to rotate and $S = k$ when it is fixed. Then, imposing an imaginary displacement or rotation, the rotations at the joints are obtained in the simple manner shown, which at each load give the ordinate by which it must be multiplied to obtain the stress that is sought. Side-sway must be computed by Equation (15) and analyzed as demonstrated by Fig. 10. Until the designer becomes accustomed to this method of analysis it is suggested that the deformations of the hypothetical models be sketched as shown in Figs. 7 to 10.

Some of the fundamental advantages of this method of structural analysis are:

(1) Results are obtained from the diagram by a simple process that eliminates complicated equations;

(2) Results are exact for any load or change of load;

(3) The stress at any section may be computed independently without knowing the moment, shear, etc., at any other section; and,

(4) As a consequence of Conclusion (3) numerical errors at one section do not affect the results of other sections.

ACKNOWLEDGMENTS

Acknowledgment is freely given to Bernard L. Weiner, Assoc. M. Am. Soc. C. E., for a meticulously careful examination of this paper before publication. Mr. Weiner offered numerous important suggestions for clarifying the ideas contained, and the authorship of Appendices I and II is his entirely, except, as noted, that Appendix I is based on theory developed by Henry G. Babcock, Assoc. M. Am. Soc. C. E.

APPENDIX I

THEORY OF THE ANALYSIS OF "UNLOADED MODELS"

Models used for the analysis of structures may be divided roughly into two classes: "Loaded Models" and "Unloaded Models." The first type is simply a small-scale reproduction of the full-sized structure to which loads are applied which are proportional to the loads that will presumably be carried by the actual structure. No loads are applied to the second type of model, but, instead, a known deformation is introduced at one point of the model and the deflection at some other point is measured. The stresses are obtained from the ratio of these two deformations as will be shown subsequently.

The two types of models differ in the manner in which the required information is obtained. In the first case, the action of the real structure is reproduced in the model and the data are obtained by direct measurement of the quantities involved. In the second case, the manipulation to which the model is subjected has no direct counterpart in the actual structure and the results are obtained by a mathematical relationship which exists between the deformations and the loads acting in the prototype A well known example of this latter type is that developed by George E. Beggs, M. Am. Soc. C. E.[3]

The derivation[4] which follows was developed by Henry G. Babcock, Assoc. M. Am. Soc. C. E., as a proof of the validity of the Beggs method, but it applies equally well to any other type of model, or to an imaginary model as is the case in this paper.

Let P (Fig. 11) be a load acting at any point, C, of a structure, in any direction whatever; let R be the component, in any assumed direction, of the reaction of Point A due to P; let R be gradually diminished to zero; let the point of application, A, of R be constrained to move in the line of action of R; let Δ_1 be the deflection of Point A when $R = 0$; and, let Δ_2 equal the component of that part of the deflection of Point C in the direction of P which is due to the removal of the reaction, R (thus, Δ_2 does not include the deflection of Point C due to the application of P while Point A is fixed).

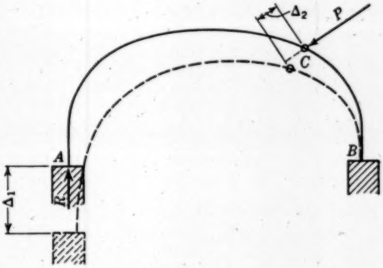


FIG. 11.

As Point C moves through the distance, Δ_2, in the direction of P, the total work done is $P\,\Delta_2$. Part of this work is done against the diminishing reaction, R, and the remainder, ϵ, is the internal work done on the structure. Evidently,

$$P\,\Delta_2 = \frac{R\,\Delta_1}{2} + \epsilon \qquad \text{......................(26)}$$

To evaluate ϵ, remove P and permit Point A to return to its original position. Now, apply a force at Point A, gradually increasing it from zero to $-R$; that is, in a direction opposite to that of $+R$. Since every force produces its own characteristic deflections at every point of a structure, independently of any other forces acting, it follows that the load, $-R$, acting alone will produce the same deflections, Δ_1, at Point A as was produced by the removal of $+R$ when P was acting. It will also produce the same deflections at every other point of the structure and, hence, the same internal work, ϵ; but, without P acting, $\frac{R\,\Delta_1}{2}$ is the total external work and ϵ the total internal. Therefore,

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[3] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

[4] Not previously published.

$$\epsilon = \frac{R\,\Delta_1}{2} \qquad \text{.........................(27)}$$

Substituting in Equation (26), $P\,\Delta_2 = \frac{R\,\Delta_1}{2} + \frac{R\,\Delta_1}{2}$; and,

$$R = P\,\frac{\Delta_2}{\Delta_1} \qquad \text{.........................(28)}$$

Since the ratio, $\frac{\Delta_2}{\Delta_1}$, is independent of the value of P, it may be determined by moving Point A through any arbitrary known distance in the direction of R and measuring the corresponding component of the deflection at Point C in the direction of P.

The preceding derivation can be extended to include moments as well as forces. If a rotation, α_1, is substituted for Δ_1, and a moment, M, for the reaction, R, other symbols remaining the same:

$$M = \frac{P\,\Delta_2}{\alpha_1} \qquad \text{.........................(29)}$$

in which α_1 is in circular measure.

The preceding theory was developed for a reaction, but it can be easily shown that it applies also to the moment, thrust, or shear at any point of a structure. If a model is cut at any section and the two cut ends are moved apart by translation (or rotation) a known amount relative to each other, Δ_1 (or α_1), and Δ_2 (or α_2) measured as before, Equations (28) and (29) still remain true. The moment, thrust, and shear can thus be found directly at any point of a structure. Finding reactions is, therefore, only a special case of finding the stresses at any point in general.

APPLICATION

The theory presented in this paper can be derived without reference to models. The usual theories deal directly with the loads and their reactions and stresses. Here, an indirect method based on the deflections of a model is used, and simplifications result. If instead of discussing more or less abstract deflections, a model is imagined to be manipulated in the same manner as a physical model is deformed by gages, and the relationship between the deflections established mathematically instead of by measurement as in a actual model, the understanding of the theory is simplified.

In the derivations which follow, the customary theories are used only in so far as they are necessary to develop the equations for the deflections. Once these relations are established, stresses are obtained by "model analysis."

APPENDIX II

ANALYTICAL DERIVATION OF EQUATIONS

Fig. 12 shows a simply supported beam subjected to a typical deformation, which forms a simple influence line. It can be easily shown, by the usual methods,

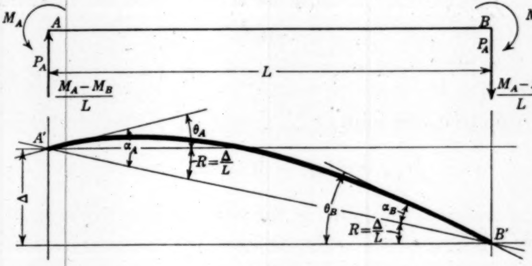
M_A
A
B
M_B
P_A
P_A
L
$\frac{M_A-M_B}{L}$
$\frac{M_A-M_B}{L}$
A'
θ_A
α_A
$R=\frac{\Delta}{L}$
Δ
θ_B
α_B
$R=\frac{\Delta}{L}$
B'

FIG. 12.

that: $\alpha_A = \frac{M_A}{3\ E\ k} + \frac{M_B}{6\ E\ k}$; and, $\alpha_B = \frac{M_A}{6\ E\ k} + \frac{M_B}{3\ E\ k}$; whence, $\theta_A = \frac{M_A}{3\ E\ k} + \frac{M_B}{6\ E\ k} - R$; and, $\theta_B = \frac{M_A}{6\ E\ k} + \frac{M_B}{3\ E\ k} + R$. Solving for M_A and M_B:

$$M_A = 2\ E\ k\ (2\ \theta_A - \theta_B + 3\ R) \quad \text{.................}(30a)$$

and,

$$M_B = 2\ E\ k\ (-\ \theta_A + 2\ \theta_B - 3\ R) \quad \text{...............}(30b)$$

Equations (30) are for the fundamental deformed beam used in the development of the "slope deflection method" and will be found in any standard text on statically indeterminate structures. The system of signs used herein, however, is different from that generally used in these texts.

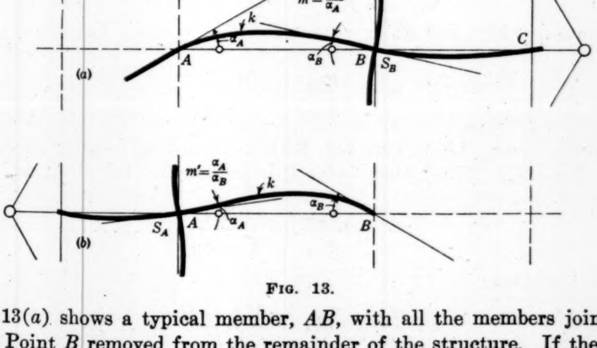
$m=\frac{\alpha_B}{\alpha_A}$
k
α_A
A
α_B
B
S_B
C
(a)
$m'=\frac{\alpha_A}{\alpha_B}$
k
α_B
S_A
A
α_A
B
(b)

FIG. 13.

Fig. 13(*a*) shows a typical member, *AB*, with all the members joining the beam at Point *B* removed from the remainder of the structure. If the End *A* is rotated through an angle, θ_{AB}, this part of the structure will deform as shown. The relation between θ_{AB} and θ_{BA} will now be found (side-sway being prevented so that $\theta = \alpha$) from Equations (30) and Fig. 14 as follows:

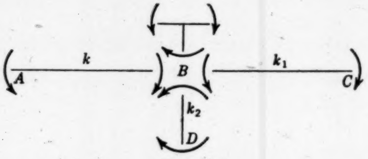

FIG. 14.

$$M_{BA} = E\,(4\,k\,\theta_{BA} - 2\,k\,\theta_{AB}) \,..(31a)$$

$$M_{BC} = E\,(4\,k_1\,\theta_{BC} - 2\,k_1\,\theta_{CB}) \,..(31b)$$

and,

$$M_{BD} = E\,(4\,k_2\,\theta_{BD} - 2\,k_2\,\theta_{DB}) \,..(31c)$$

Since $M = 0$ about any joint:

$$M_{BA} = M_{BC} + M_{BD} \quad \ldots\ldots\ldots\ldots\ldots\ldots (31d)$$

Since the rotation at a joint is the same at the ends of all members meeting at the joint:

$$\theta_{BC} = \theta_{BD} = -\,\theta_{BA} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

and,

$$(k + k_1 + k_2)\,\theta_{BA} + 0.5k_1\,\theta_{CB} + 0.5k_2\,\theta_{DB} = 0.5k\,\theta_{AB} \quad \ldots\ldots (33)$$

By definition, since $\theta = \alpha$ for this case, $\dfrac{\theta_{BA}}{\theta_{AB}} = m$; $\dfrac{\theta_{CB}}{\theta_{BC}} = \dfrac{\theta_{CB}}{\theta_{BA}} = m_1$; and, $\dfrac{\theta_{DB}}{\theta_{BD}} = \dfrac{\theta_{DB}}{\theta_{BC}} = m_2$. . Consequently, it follows that:

$$m = \frac{\theta_{BA}}{\theta_{AB}} = \frac{0.5\,k}{k + \left(1 - \dfrac{m_1}{2}\right)k_1 + \left(1 - \dfrac{m_2}{2}\right)k_2} \quad \ldots\ldots (34a)$$

By definition, $S_1 = \left(1 - \dfrac{m_1}{2}\right)k_1$ (see Equation (3a)); $S_1 = \left(1 - \dfrac{m_2}{2}\right)k_2$ (see Equation (3b)); and $S_B = S_1 + S_2 + \ldots +$, etc. Therefore:

$$m = \frac{\theta_{BA}}{\theta_{AB}} = \frac{0.5\ k}{k + S_B} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34b)$$

Similarly, when AB is removed, with all members joining Beam AB at End A, (Fig. 13(b)), and when Joint B is rotated through an angle, θ_{BA}:

$$m' = \frac{\theta_{AB}}{\theta_{BA}} = \frac{0.5\ k}{k + S_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34c)$$

From Fig. (15):

$$E\,I\,\frac{d^2y}{dx^2} = M_A - \frac{M_A - M_B}{L}\,x \quad \ldots\ldots\ldots\ldots\ldots (35)$$

Integrating twice and remembering that $y = 0$ when $x = 0$ and $x = L$, and substituting the values of M_A and M_B from Equations (30):

$$y = \frac{x\,x'}{L^3}\,(x\,f + x'\,f') \qquad \text{.......................(36)}$$

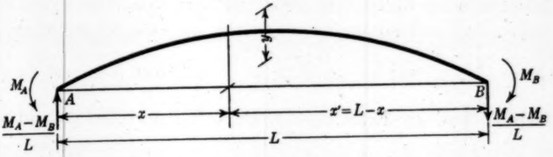

FIG. 15

Equation (36) is the formula of the elastic curve in terms of the end rotations.

Fig. 16 shows two beams, AB and BC, joined at Point B and rotated with respect to each other through an angle of 1, to produce the influence line, M_B. In this case, $\theta_{BA} + \theta_{BC} = \theta = 1$; and, $M_{BA} = M_{BC}$.

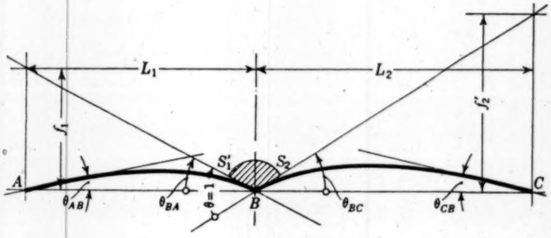

FIG. 16

Writing the values for $M_{BA} + M_{BC}$ from Equations (30), and equating the resulting expressions:

$$k_1\,(2\,\theta_{BA} - \theta_{AB}) = k_2\,(2\,\theta_{BC} - \theta_{CB})$$

or,

$$k_1\left(1 - \frac{\theta_{AB}}{2\,\theta_{BA}}\right)\theta_{BA} = k_2\left(1 - \frac{\theta_{CB}}{\theta_{BC}}\right)\theta_{BC}$$

Since $\frac{\theta_{AB}}{\theta_{BA}} = m'_1$; $\frac{\theta_{CB}}{\theta_{BC}} = m_2$; $\left(1 - \frac{m'_1}{2}\right) k_1 = S'_1$; and $\left(1 - \frac{m_2}{2}\right) k_2 = S_2$:

$$S'_1\,\theta_{BA} = S_2\,\theta_{BC}$$

Since $\theta_{BA} + \theta_{BC} = \theta = 1$:

$$\theta_{BA} = \frac{S_2}{S'_1 + S_2} = \frac{S_2}{K_B} \qquad \text{....................(37a)}$$

and,

$$\theta_{BC} = \frac{S'_1}{K_B} \quad \text{.......................... (37b)}$$

The following equations for side-sway are derived in a manner similar to that used for obtaining Equations (34). Referring to Fig. 3(*b*), using Equations (30) once more and remembering that $R = 0$ for all members except AB, for which $R = \frac{\Delta}{L}$, it follows from $\Sigma M = 0$ about Joint B (repeating the operations which lead to Equations (32), (33), and (34*a*) and (34*b*), that the last equations take the following forms when $R = \frac{\Delta}{L}$ instead of zero:

$$(k + S_B)\,\theta_{BA} - 0.5\,k\,\theta_{AB} = 1.5\,k\,R \quad \text{............. (38a)}$$

and,

$$0.5\,k\,\theta_{BA} - (k + S_A)\,\theta_{AB} = 1.5\,k\,R \quad \text{............. (38b)}$$

Solving these two equations simultaneously for θ_{BA}:

$$\theta_{BA} = 1.5\,k\,R\,\frac{0.5\,k + S_A}{(k + S_A)\,(k + S_B) - 0.25\,k^2} \quad \text{.......... (39)}$$

Subtracting R from both sides of the equation and then dividing by R, simplifying and re-arranging:

$$\frac{(\theta_{BA} - R)\,L}{R\,L} = \frac{\dfrac{0.5\,k}{k + S_B}\,S_A}{k + S_A - 0.5\,k\left(\dfrac{0.5\,k}{k + S_B}\right)} - \frac{S_B}{k + S_B - 0.5\,k\left(\dfrac{0.5\,k}{k + S_A}\right)} \quad \text{. (40)}$$

and,

$$\frac{f}{\Delta} = \frac{m\,S_A}{S_A + k - 0.5\,m\,k} - \frac{S_B}{S_B + k - 0.5\,m'\,k} = \frac{m\,S_A}{S_A + S} - \frac{S_B}{S_B + S'}$$

$$= -\frac{S_B}{K_B} + \frac{m\,S_A}{K_A} \quad \text{.................... (41a)}$$

Similarly,

$$\frac{f'}{\Delta} = \frac{S_A}{K_A} - \frac{m'\,S_B}{K_B} \quad \text{..................... (41b)}$$

The force, P_A, necessary to displace Point A in Figs. 3 and 4 is easily derived. Referring to Fig. 12, $P_A = \frac{(M_A - M_B)}{L}$. Substituting the values of M_A and M_B from Equations (30):

$$P_A = \frac{6\,E\,k}{L}\,(\theta_A - \theta_B + 2\,R) = \frac{6\,E\,k}{L}\,[(\theta_A + R) - (\theta_B - R)]$$

$$= \frac{6\,E\,k}{L}\,(\alpha_A - \alpha_B) \quad \text{...................... (42)}$$

and,

$$\frac{P}{E} = \frac{6\,k}{L^2}\,(-f + f') \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Equation (43) can also be used to obtain the correction for side-sway.

INFLUENCE LINES FOR BENDING MOMENTS

Fig. 17 shows a typical member, AB, broken at Point G and rotated through an angle, θ, by means of the gage. The forces acting at Point G must be the moment, M, and a shear, V, shown in Fig. 18.

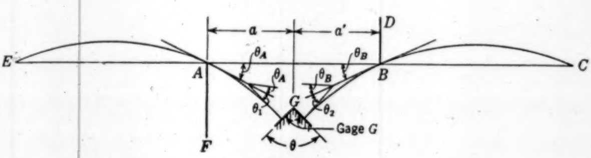

FIG. 17

From Fig. 17, the angle of the gage is:

$$\theta_A + \theta_1 + \theta_B + \theta_2 = \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Furthermore, the deflection normal to AB at Point G, must be the same for both broken members, AG and BG.

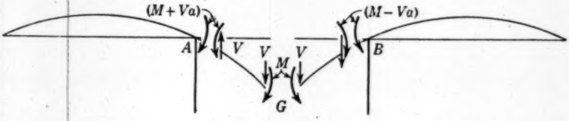

FIG. 18

Using Equations (30), once more and writing $M + V\,a$ for M_A, and $M - V\,a'$ for M_B, the resulting equations can be easily transformed to the following form:

$$M + V\,a = 4\,E\,S_A\,\theta_A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (45a)$$

and,

$$M - V\,a' = 4\,E\,S_B\,\theta_B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (45b)$$

or,

$$\frac{M\,L}{E} = 4\,(a'\,S_A\,\theta_A + a\,S_B\,\theta_B) \quad \ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

and,

$$\frac{V\,L}{E} = 4\,(S_A\,\theta_A - S_B\,\theta_B) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46b)$$

The equations for the angular and linear deflections at Point G, are:

$$\theta_A + \frac{M\,a}{E\,I} + \frac{V\,a^2}{2\,E\,I} + \theta_B + \frac{M\,a'}{E\,I} - \frac{V\,(a')^2}{2\,E\,I} = \theta \;\ldots\ldots\ldots (47)$$

and,

$$a\,\theta_A + \frac{M\,(a')^2}{2\,E\,I} + \frac{V\,a^3}{3\,E\,I} = a'\,\theta_B + \frac{M\,(a')^2}{2\,E\,I} - \frac{V\,(a')^3}{3\,E\,I} \;\ldots\ldots\ldots (48)$$

Substituting Equation (46) in Equations (47) and (48):

$$(k + 2\,S_A)\,\theta_A + (k + 2\,S_B)\,\theta_B = k\,\theta \;\ldots\ldots\ldots\ldots\ldots (49)$$

and,

$$3\,k\,a'\,\theta_A + 2\,(2\,a - a')\,S_A\,\theta_A = 3\,k\,a'\,\theta_B - 2\,(a - 2\,a')\,S_B\,\theta_B \;\ldots (50)$$

Solving Equations (49) and (50) for θ_A and θ_B, and remembering that $a' + a = L$; $L\,\theta_B = f$; $L\,\theta_A = f'$; $S = \dfrac{0.75\,k + S_A}{k + S_A}$; $K_A = S + S_A$; etc., the resulting equations can be put into the form given in Equations (16*a*) and (16*b*).

DISCUSSION

L. J. Mensch,[5] M. Am. Soc. C. E. (by letter).—The many papers on rigid-frame analysis that have been published by the Society have evidently intrigued the author, an engineer of exceptional ability in the art of the design of hyperstatic structures and of extended experience, to contribute his share to the excitement, or "talk-fest", on the subject of rigid-frame analysis.

The honor of having first simplified the analysis of continuous structures belongs to Clapeyron[6]. For the development of his famous formulas, Clapeyron used a method which in the United States has lately been called the "slope-deflection theory." These "slope-deflection formulas", are in common use for all kinds of problems in the well known textbook of Dr. F. Grashof[7], and in many textbooks issued before and afterward.

Professor Grashof treats[8] the problem of a girder, $A B$, partly fixed at the supports by the moments, M_A and M_B, having the slopes, α and β, of the elastic line at A and B, and affected by an uniform load, W, and a concentrated load, P, distant a and b from the ends, A and B, and gives the following formulas:

For moment:

$$M_A = -\frac{P a b^2}{L^2} - \frac{W L}{12} + \frac{2 E I}{L} (2 \alpha - \beta) \quad \ldots\ldots\ldots\ldots (51a)$$

and,

$$M_B = -\frac{P a^2 b}{L^2} - \frac{W L}{12} + \frac{2 E I}{L} (- \alpha + 2 \beta) \quad \ldots\ldots\ldots (51b)$$

and, for shear:

$$V_A = \frac{P (3 a + b)}{L^3} b^2 + \frac{W}{2} + \frac{6 E I}{L} (- \alpha + \beta) \quad \ldots\ldots\ldots (52a)$$

and,

$$V_B = \frac{P (3 b + a)}{L^3} a^2 + \frac{W}{2} + \frac{6 E I}{L} (\alpha - \beta) \quad \ldots\ldots\ldots\ldots (52b)$$

He shows[9] that α and β must be replaced by $\theta_A - R$ and $R - \theta_B$, when Point A is lifted a distance, $R L$, over Point B during the deformation of the girder. He also shows[10] that, when $M_B = 0$, Equation (51b) becomes:

$$E I \beta = \frac{P a^2 b}{4 L} + \frac{W L^2}{48} + \frac{E I \alpha}{2} \quad \ldots\ldots\ldots\ldots\ldots (53a)$$

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[5] Civ. Engr. and Constructor, Chicago, Ill.

[6] *Comptes Rendus*, Vol. 45, Paris, 1857.

[7] "Theorie der Elasticität und Festigkeit," Second Edition, Berlin, 1878.

[8] *Loc. cit.*, p. 78.

[9] *Loc. cit.*, p. 101.

[10] *Loc. cit.* p. 83.

and, from Equation (51*a*):

$$E\,I\,\alpha = M_A + \frac{P\,a\,b\,(b + 0.5\,a)}{3\,L^2} + \frac{W\,L^2}{24} \quad\text{..........(53}b\text{)}$$

When both M_A and M_B are zero, Equations (51) yield the well known formulas:

$$E\,I\,\alpha = \frac{P\,a\,b\,(a + 2\,b)}{6\,L} + \frac{W\,L^2}{24} \quad\text{................(54}a\text{)}$$

and,

$$E\,I\,\beta = \frac{P\,a\,b\,(b + 2\,a)}{6\,L} + \frac{W\,L^2}{24} \quad\text{................(54}b\text{)}$$

Girders of varying section are also treated in this textbook.

In addition to Grashof, there is at least one other old elementary textbook, that of Wilhelm Keck[11], in which may be found all the slope-deflection formulas in use for the solution of continuous structures. The first textbook which the writer can now remember to have treated rigid-frame, multiple-storied structures is that of Professor E. Winkler[12] in which the "slope-deflection" formulas were freely used.

After this revelation of the status of the "slope deflection theory", it is to be hoped that enthusiasts will not further clutter up American literature with the incorrect statement that the "slope deflection theory" is an American development, improvement, or invention, except in so far as the coinage of the phrase, "slope deflection," is concerned.

How did it come about that these important and simple earlier methods have been nearly forgotten by American and European engineers? It seems to the writer that teachers on any subject, in order to acquire at least a quasi-reputation, must produce something new, even if it is not better than the old art, and their students spread the new "light", and, presto, a new philosophy is in fashion. So it was with Mohr's principle of virtual velocities which was soon put out of fashion by Manebrea's and Castigliano's principle of least work (originally invented by Daniel Bernouilli in the Eighteenth Century) which principle found an enthusiastic and able propagandist in Professor Müller-Breslau and, through him, came to be used all over the world. Not so many years passed, however, before it was discovered that this intriguing method was practical only for comparatively simple problems, and could not be used by busy men for highly hyperstatic structures, and the pendulum (but not the habit, so easily) swung backward again and various types of "slope deflection" methods were re-invented.

One of these is the principle of fixed or conjugate points, so well treated by W. Ritter[13] and greatly elaborated for framed continuous structures, consisting of straight and arched members, by A. Strassner[14]. The author's paper is nothing but a variation from Strassner's treatment of this subject. Strassner uses the angles, whereas the author uses the intercepts of the tangents on the verticals at the opposite support, an unnecessary complication. The author has treated only the simplest cases in Strassner's work and has not made the slightest improvement in the formulas or in their derivation. Like Clapeyron, Ritter, and Strassner, he proposes to analyze a structure by beginning at one end and going over its entire length in order to determine the relative angle changes that are necessary for the computation of the unknown bending moments. This is clearly too large and entirely too impractical a task for the large structures that American engineers must design so often, and further simplifications are absolutely necessary so that engineers may be able to arrive at more precise results with comparatively little effort. Strassner gives a hint in one of his examples of an attempt at simplification, but does not go far enough as he has still to go through the entire structure in order to solve that part of it which may require more precise treatment.

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[11] "Vorträge über Elasticitäts-Lehre," Hannover, 1893.

[12] "Vorträge über Brückenbau," Wien, C. Gerold, 1875.

[13] "Graphische Statik," Zurich, 1900.

[14] "Neuere Methoden", Berlin, 1916.

How to solve any girder or column in a large structure quickly and with sufficient accuracy, the writer has shown elsewhere[15]. He obtained a sufficiently close estimate of the values of α and β in Equations (51) by setting,

$$\alpha = \frac{-M_A}{\Sigma m_A E K_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55a)$$

and,

$$\beta = \frac{-M_B}{\Sigma m_B E K_B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55b)$$

in which m_A and m_B are certain factors (given in that discussion) and K_A equals the ratio of $\frac{I}{L}$ for each member adjoining A, except $A\,B$.

The solution of Equation (51) after the values of α and β from Equations (55) are introduced in them, gives the two simple formulas for a girder with an uniform load, W:

$$M_A = -\frac{W\,L}{4}\,\frac{1 + 2\,N_B}{4\,(1 + N_A)\,(1 + N_B) - 1} \quad \ldots\ldots\ldots\ldots (56a)$$

and,

$$M_B = -\frac{W\,L}{4}\,\frac{1 + 2\,N_A}{4\,(1 + N_A)\,(1 + N_B) - 1} \quad \ldots\ldots\ldots\ldots (56b)$$

in which,

$$N_A = \frac{3\,K}{\Sigma\,m_A\,K_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57a)$$

and,

$$N_B = \frac{3\,K}{\Sigma\,m_B\,K_B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57b)$$

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[15] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1050.

The author's treatment of linear displacement is not as simple as that of Strassner and contains the same limitation in the sense that it is strictly applicable only to one-story structures, or to frames which remain vertical in the stories above and below the story which has been displaced, a case that never occurred in the writer's experience. The actual problem is to find the restraint at *A* and *B* when the entire structure is deformed as a girder due to the horizontal load acting at *A*, which is really a difficult problem; possibly, the author may be able to reveal some simplifying suggestions by experimentation. The problem of side-sway in a non-symmetrical single-story bent of two columns can be more easily solved by Strassner's method, which is really only a variation of Clapeyron's method.

The writer has heard engineers state that a method of analyzing highly hyperstatic structures which gives the unknown bending moments within 5% to 10% of their ideal values is necessary. No method, however laborious, known to-day can give such a close result. First, girders and columns have dimensions of from 6% to 20% and more of the spans and story heights, and not enough tests have been made to establish how near the designer guesses when he uses clear spans instead of theoretical spans. Although Strassner shows how to consider the shortening of columns by girders, he uses theoretical spans in his examples.

Another serious disturbance is the variation of the modulus of elasticity, *E*, under high or long-continued loadings. After all, the final intent of these analyses should be the establishment of the factor of safety of designs against failure or undue deformations. All formulas are derived by the assumption of a constant value of *E*.

Under intense loadings the modulus of elasticity of the highly stressed part of the member decreases rapidly, and if it is the end part of a restrained beam, for example, this diminution of *E* will have the effect of decreasing restraint and will modify the moments both in the center of the span and at the ends. The center moment will be increased and the end moments decreased. No clear-cut tests have been made to estimate readily, the decrease of restraint. A very able treatment of this phase of the subject has been offered[16] by E. Mirabelli, M. Am. Soc. C. E.

That much has yet to be learned about the deviation of Hooke's law in such a standard material as structural steel is made evident by tests reported by the National Bureau of Standards[17]. A unit load of 31 000 lb per sq in. was imposed on two heavy stiff columns, 159 sq in. in cross-section and 24 ft long. The shortening was found to be 0.0016 in. per in.; and, therefore, the secant modulus of elasticity was only $\frac{31\,000}{0.0016} = 19\,000\,000$ lb per sq in. The moduli of elasticity decreased when stresses increased beyond 24 000 lb per sq in. It would have been of great importance to know what they would have been had the load remained for a month or a year.

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[16] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1397.

[17] *Journal of Research*, National Bureau of Standards, *Research Paper 873*, March, 1936.

That the deviation of E in reinforced concrete members plays an important rôle was shown by the tests to destruction of Mikishe Abe, M. Am. Soc. C. E.[18], on rigid-framed structures. The ultimate strengths were at least 10% larger than those computed by current analysis when clear spans were used. For this reason highly complicated formulas or processes which require long study will be of limited value in this field.

FREDERICK SHAPIRO,[19] JUN. AM. SOC. C. E. (by letter).—The method of analysis presented in this paper is an excellent variation of the moment-distribution principle. The underlying theory is the same as that of moment distribution, although the problem is viewed from a different angle. When one part of a frame is rotated and the effects of this motion are traced by multiplying the original rotation by factors the values of which depend upon the rotation resistance of the constituents of the frame, the method of moment distribution is being used, even if angular changes instead of bending moments are involved.

The formulas of the paper may readily be converted into more useful ones. Following the notation of the paper,

$$C = \frac{1 - 2\,m}{2 - m} \qquad \text{.........................(58)}$$

and instead of giving the ratio of the angular rotations of the ends of unloaded spans, Equation (58) gives the ratio of the end moments, in which C is the transmission coefficient introduced by R. C. Brumfield, M. Am. Soc. C. E., who developed an analysis of statically indeterminate structures based upon it.[20] It is a very useful value for computing end moments, even for loaded spans, and may be found from the formula:

$$C = \frac{Q}{2\,Q + F_r} \qquad \text{.........................(59)}$$

in which $Q = \frac{1}{k} = \frac{L}{I}$, and F_r is a term called a restraint factor, and is equal to $\frac{1.5}{K_B}$. It is given by,

$$F_r = \frac{1}{\frac{1}{Q_1\,(2 - C_1)} + \frac{1}{Q_2\,(2 - C_2)} + \frac{1}{Q_3\,(2 - C_3)} + \ldots\ldots} \qquad \text{....(60)}$$

A term of the form, $Q\ (2 - C)$, is called a stiffness index. Obviously, each end of a beam has its own transmission coefficient, restraint factor, and stiffness index $\left(= \frac{2}{3\ S_1} \text{ or } \frac{2}{3\,S'_1}\right)$. The stiffness indices found in the denominator of Equation (60) are those for the ends of all the beams meeting at a joint, except that of the beam end for which the restraint factor is to be found.

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[18] *Bulletin 107*, Eng. Experiment Station, Univ. of Illinois, 1918.

[19] Draftsman, Gibbs & Hill, Inc., New York, N. Y.

[20] "The Analysis of Monolithic Structures by Transmission Coefficients", by Frederick Shapiro, *Concrete*, May, June, July, and August, 1936; see, also, "Moving Loads on Beams with Restrained Ends", by R. C. Brumfield, M. Am. Soc. C. E. (Complete manuscript filed for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y.)

The transmission coefficient varies from zero for pin-ended beams to one-half for fixed-end beams of constant moment of inertia. In the interior of interlocking frames it may be approximated by the following method: Compute the ratio of the sum of the $\frac{I}{L}$-values for the other members at the joint to twice the sum of the $\frac{I}{L}$-values of all the members at the joint. If this value is between zero and 0.35, add 0.02; if it is between 0.36 and 0.43, add 0.01; and if it is greater than 0.43, add nothing to obtain the coefficient with less than 1% error. This value may be corrected later.

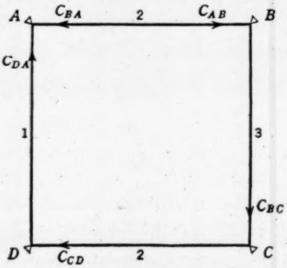

FIG. 19

For example, C_{AB}, in Fig. 19, will be computed. The large numbers on the spans are Q-values or $\frac{L}{I}$-values. Approximately, C_{AB}

$$= \frac{K_{BC}}{2\,(K_{AB} + K_{BC})} + (?) = \frac{\frac{1}{3}}{\left(\frac{1}{2} + \frac{1}{3}\right)}$$

$+ (?) = 0.20 + 0.02 = 0.22$. Using Equation (59) $C_{DA} = \dfrac{Q_{DA}}{2\,Q_{DA} + Q_{AB}\,(2 - C_{AB})} = \dfrac{1}{2 \times 1 + 2\,(2 - 0.22)} = 0.180$; C_{CD}

$$= \frac{2}{2 \times 2 + 1\,(2 - 0.180)} = 0.343; \; C_{BC} = \frac{3}{2 \times 3 + 2\,(2 - 0.343)} = 0.322;$$

and, finally, $C_{AB} = \dfrac{2}{2 \times 2 + 3\,(2 - 0.322)} = 0.222$, which is correct to three decimal places. The change in C_{DA} would be indistinguishable on a slide-rule.

If a bending moment of 1 000 ft-lb were introduced at Point A, it would flow around the frame, becoming 1 000 C_{AB} at Point B, or 222 ft-lb; $222 \times 0.322 = 71$ ft-lb at Point C; $71 \times 0.343 = 24$ ft-lb at Point D; and, $24 \times 0.180 = 4$ ft-lb at Point A, and would continue around the frame again in the same manner.

In computing end moments for loaded spans, the transmission coefficients receive their greatest use. Formulas have been derived for end moments[20] and these involve only constants dependent upon the type of loading, the position of the load on the span, and the transmission coefficients for the span.

The writer and Professor Brumfield have developed charts to give transmission coefficients, restraint factors, and torque splits and moment splits at joints. They have arranged tables to reduce computations to a minimum and to simplify the work. Professor Brumfield has derived formulas for locating loads to produce maximum moment at any given point in a span or under any given load. He has also modified his formulas for use with clear spans and generalized them to include beams of variable moment of inertia, providing the neutral axis may be considered to be a straight line.

The writer has deduced mechanical methods for the determination of transmission coefficients, comparable to those for finding the carry-over factors in the method of distributing fixed-end moments by successive approximations. It might be noted here that the transmission coefficients, summing up by formula the converging series of the Cross method,[21] do in one step what the fixed-end method does by successive approximation. Many short-cuts have been found that may be applied without restricting the generality of Professor Brumfield's formulas. A method of checking computations for continuity, besides the obvious check at each joint for $\Sigma M = 0$, has also been developed[22] by the writer.

The usefulness of direct moment distribution cannot be denied. Although Dr. Gottschalk has presented a method in which the underlying theory is based on a simple conception, its practical application is very limited compared to the system of analysis evolved by Professor Brumfield.

JAMES R. GRIFFITH,[23] M. AM. SOC. C. E. (by letter).—A valuable contribution to the subject of unloaded models is contained in this paper. The principles involved in the determination of influence lines by unit displacements of a model have been somewhat difficult for some engineers to accept. This is particularly true of the influence lines for moment. The mathematical proof in Appendices I and II alone is worthy of consideration. As a mathematical means of computing the numerical value of moment, shear, or thrust at any section of a structure independent of values obtained at any other section, the method is of additional note. When the loads are few and stationary, such a mathematical solution might be more economical in some cases than a solution by model.

In the design of the catenary structures for the terminal electrification of the Illinois Central System, at Chicago, Ill., the writer was one of the first users of the unloaded model. Since that time he has never lost his enthusiasm for the method. As a means of helping students to visualize the action of an indeterminate structure, the method is far superior to any other known to the writer. It has had one serious objection, however, in that the cost of commercial sets for utilizing such models has been prohibitive for the average individual and the smaller offices. The writer has long experimented with various materials and equipment, with cost and availability as the principal objectives.

Anders Bull, M. Am. Soc. C. E., made a real contribution when he described the use of both loaded and unloaded models fabricated of "brass drill rod."[24] The writer has been unable as yet to locate any local supply of this material. Bronze welding rod, of the various materials tried, has thus far best fulfilled the qualifications. It is available in almost every shop equipped to do gas-welding, and the cost is scarcely worth mentioning. Scraps of tin, push pins, and a welding iron are the only other equipment needed.

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[21] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[22] Based on method given on p. 91 of "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., John Wiley & Sons, Inc., N. Y., 1932.

[23] Prof. of Structural Eng., Oregon State Agri. Coll., Corvallis, Ore.

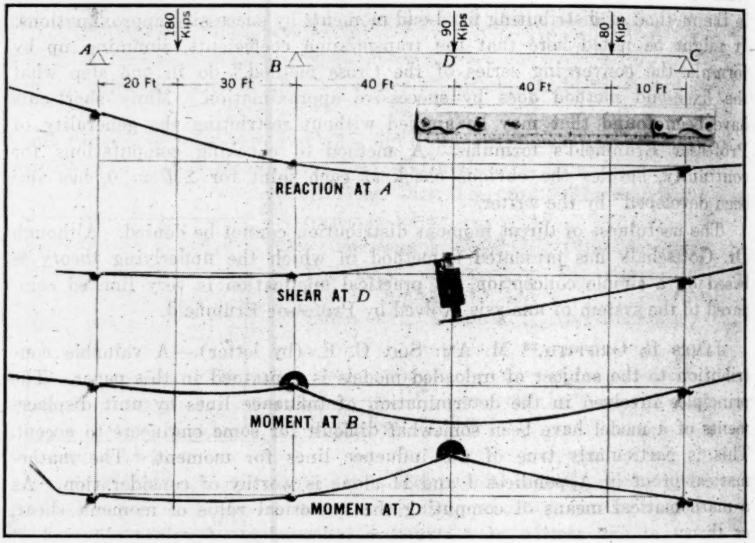

FIG. 20

Fig. 20 shows a series of influence lines constructed by welding wire for the beam of constant section shown. The shear clamp is the development suggested by Ray de Lancey, Jun. Am. Soc. C. E. It is adjustable within a reasonable range of accuracy and is fabricated entirely of tin. The necessary rotation for moment at interior sections is obtained by a tin clamp as shown, which, as in the case of the sheer clamp, can be removed quickly from the wire. Since it was found impossible to fabricate the moment clamps to a predetermined angle within the required accuracy, it was necessary to calibrate them after fabrication.

Theoretically, as the author indicates, the deformation of a model must be very small to avoid error. Practically, even with relatively large deformations, the writer feels that results are obtained well within the limits warranted by the present knowledge of reactions, elastic properties of the finished structure, and rigidity of connections.

CAMILLO WEISS,[25] M. AM. SOC. C. E. (by letter).—Apart from its usefulness, this is an extremely interesting and stimulating paper. By virtue of Appendices I and II, this method may be considered as an ingenious combination of Maxwell's theorem with the slope deflection theory, and it may be permissible to compare it with the latter. Such a comparison is also helpful in appraising the merits of the proposed method and to show under what circumstances it is particularly useful.

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[24] *Engineering News-Record*, December 8, 1927, p. 920.

[25] Structural Engr., Fabricated Steel Constr., Eng. Dept., Bethlehem Steel Co., Bethlehem, Pa.

The author was wise in selecting simple problems to illustrate his method; but because he did so he has not really done his method full justice. Furthermore, the method has certain drawbacks which are not apparent in simple problems.

In Conclusion (3) the author states: "The stress at any section may be computed independently without knowing the moment, shear, etc., at any other section." This is indeed a great advantage. On the other hand, the word, "may", in this Conclusion could be written "must"; and this fact may be a disadvantage at times.

In general, the proposed method will prove most valuable when the loads are movable; when stresses are wanted at only a few points; and when the number of loaded members is comparatively small. For example, in Fig. 21(*a*) all joints are assumed to be free to rotate but are fixed against translation. Suppose that only the moment, M_{AB}, at End *A* of Member *AB*, is desired. To obtain the answer by the slope deflection method, twelve unknown slopes and forty-two unknown moments must

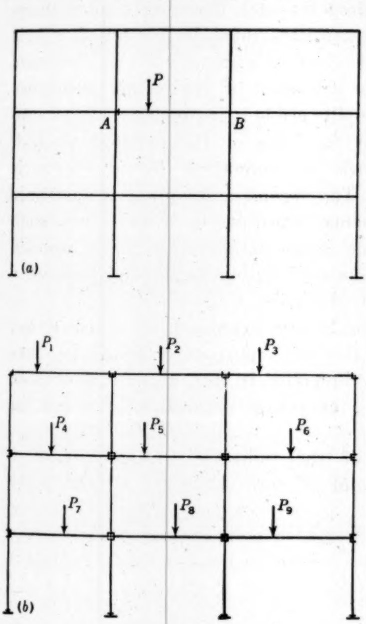


FIG. 21.

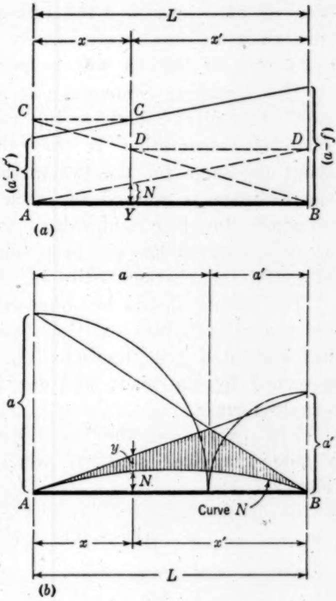


FIG. 22.

be introduced and twelve simultaneous equations must be solved. To obtain the answer by the proposed method, two unknown tangential intercepts and the ordinate of one influence line are the only variables. These three unknown quantities are obtainable from three equations, two of them independent of each other. If Load P is moving along Member $A\,B$, one new ordinate of the influence line is all that is required for each position.

The slope deflection method would necessitate solving a set of twelve simultaneous equations for each position of the load. Therefore, this problem is obviously one for which the proposed method may well be considered superior.

Fig. 21(b) represents the same frame, except that in this case every panel is loaded and the moments at the end of every member (forty-two moments in all) are wanted. To obtain the answer by the slope deflection method again involves fifty-four unknowns and requires the solution of twelve simultaneous equations. To obtain the answer by the proposed method eighteen tangential intercepts and nine ordinates of influence lines (twenty-seven variables), are required for each moment; a total, therefore, of 27 × 42 = 1 134 unknown quantities must be computed. It is true, these unknowns are largely independent of each other. In this case, the choice of method will be primarily a matter of individual preference.

The foregoing comparison is admittedly crude. It implies at least two tacit assumptions, both erring probably in favor of the proposed method. The first assumption is that the work of computing the constants is about the same for the two methods. The second assumption is that only one ordinate is wanted for each influence line; or, in other words, each influence line is considered as only one unknown quantity. The example herein presented has not been selected as typical, but is presented in order to reveal the characteristics of the two methods.

The author should be commended for having expressed, in simple terms, the mutual relations existing between the coefficients of stiffness; for having separated the functions that are inherent in the frame from those governed by the load; and for having expressed these functions, too, in simple form.

The expression for the ordinate of the influence line admits of a simple graphical solution when f and f' are known. Referring to Fig. 22(a):

$$\overline{AC'} = \overline{YC} = (a - f)\,\frac{x}{L} + (a' - f')\,\frac{x'}{L} \quad \text{..........} \quad (61a)$$

$$\overline{BD'} = \overline{YD} = \overline{AC'}\,\frac{x'}{L} \quad \text{..................} \quad (61b)$$

and,

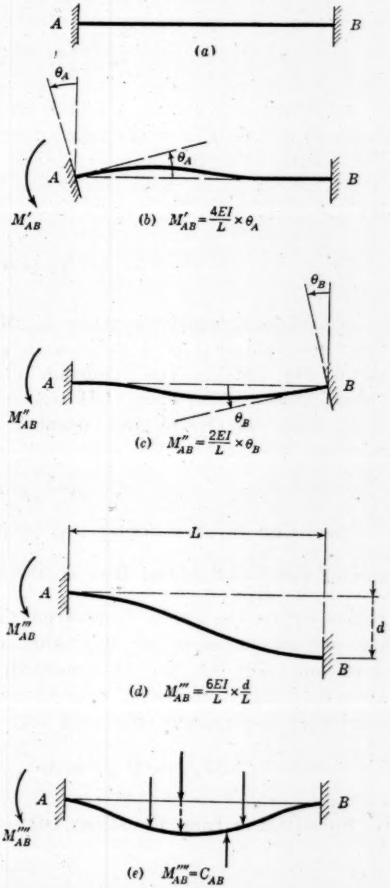
$$N = \overline{BD'}\,\frac{x}{L} \quad \text{........................} \quad (61c)$$

Substituting Equations 61(a) and 61(b) in Equation 61(c), the result is expressed by the author as Equation (18). To construct any point on the curve of this formula, lay off $(a - f)$ above End B, Fig. 22(a), and $(a' - f')$ above End A. The closing line intersects the vertical, Y, at Point C. Projecting Point C horizontally to Point C', Line $C'B$ intersects the vertical through Point Y at Point D. Projecting D to D', Line $D'A$ intercepts the required ordinate, N, on the vertical, Y. Proof of the construction can be obtained from the similarity of triangles. In this manner, Curve N of Fig. 22(b) can be constructed. The remainder of the construction of the influence line in Fig. 22(b) is self-explanatory.

Similar construction will furnish the ordinates of the elastic curve, Equation (5), if the intercept f, is substituted for $(a - f)$ and f' for $(a' - f')$ in Fig. 22(a) and if proper consideration is given to the signs of f and f'.

Appendix II.—Referring to Appendix II, Equations (30) are stated to be the formulas used in the slope deflection method, but "the system of signs is different from that generally used." The signs are not in agreement with the "Notation", nor is it stated, what convention has been adopted. The enlightening derivation of the fundamental formulas, as presented in Appendix II, is an essential part of the paper and the absence of a sign convention may be somewhat confusing. Perhaps the following comments will be of service.

Assume that all slopes, deflections, and moments are absolute numerical quantities. As in arithmetic, a plus sign means "add" and a minus sign means "subtract." Fig. 23(a) represents a restrained but unloaded member, AB. Suppose, it is desired to produce a contra-clockwise restraining moment, M_{AB}, at End A. Apart from temperature stresses, this can be done in any of four (and only four)



(b) $M'_{AB} = \frac{4EI}{L} \times \theta_A$

(c) $M''_{AB} = \frac{2EI}{L} \times \theta_B$

(d) $M'''_{AB} = \frac{6EI}{L} \times \frac{d}{L}$

(e) $M''''_{AB} = C_{AB}$

FIG. 23.

different ways illustrated in Figs. 23(*b*), 23(*c*), 23(*d*), and 23(*e*), and in any combination of these four ways.

A contra-clockwise restraining moment at A can be obtained by:

(*a*) Rotating End A counter-clockwise through an angle, θ_A, as in Fig. 23(*b*). Quantitatively, this moment is:

$$M'_{AB} = \frac{4\,E\,I\,\theta_A}{L} \quad \text{.....................(62a)}$$

(*b*) Rotating End B counter-clockwise through an angle, θ_B, as in Fig. 23(*c*). Quantitatively, this moment is:

$$M''_{AB} = \frac{2\,E\,I\,\theta_B}{L} \quad \text{.....................(62b)}$$

(*c*) Displacing Points A and B with respect to each other so that Point B seems to move in a clockwise direction around Point A. Referring to Fig. 23(*d*), it is immaterial whether Point B was moved down or Point A was moved up, or both points were moved. Quantitatively, this moment is:

$$M'''_{AB} = 6\,E\,\frac{I}{L}\,\frac{d}{L} \quad \text{.....................(62c)}$$

(*d*) Placing transverse loads on Member $A\,B$, so that the moment of their resultant will tend to turn in a clockwise direction about Point A, as in Fig. 23(*e*). The condition illustrated in Fig. 23(*e*) is termed a "beam fixed at both ends." If C_{AB} is the end moment at A for the given load, then the restraining moment is numerically equal to this end moment, or,

$$M''''_{AB} = C_{AB} \quad \text{.....................(62d)}$$

Equations (62*a*) to (62*c*) can be derived from two well-known properties of the $\dfrac{M}{EI}$ -diagram; that is, the principles that the area between two points equals the angle, θ, between the tangents at these points and that the statical moment of this area equals the corresponding tangential deviation. By the law of superposition these four methods of creating a contra-clockwise resisting moment at A are additive, therefore, if all four conditions are present, the total is:

$$M_{AB} = M'_{AB} + M''_{AB} + M'''_{AB} + M''''_{AB} \quad \text{...........(63)}$$

Substituting from Equations (62) and writing K for $\dfrac{I}{L}$ and R for $\dfrac{d}{L}$:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B + 3\,R) + C_{AB} \quad \text{............(64)}$$

Obviously, if any of the four methods of inducing a contra-clockwise restraining moment at A acts in a direction opposite to the direction shown in Figs. 23(b) to 23(e), the corresponding moment must be subtracted instead of being added. For instance, if End B is rotated clockwise, instead of contra-clockwise, and if there are no transverse loads:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A - \theta_B + 3\,R) \qquad (65)$$

Equation (65) corresponds to the condition shown in Fig. 12, as expressed by Equation (30a), and indicates that the foregoing method of treating slopes and deflections as purely numerical quantities, is apparently the convention adopted in Appendix II. This convention is well suited to the author's proposed method, when deformations of known direction are applied.

It might be mentioned that this convention, although admirably adapted to the purposes of the author, is not convenient when the slopes and deflections are unknown in direction. This is why, in the slope deflection method, it is usual to assume that all rotations and deflections are in the same direction (clockwise), and only the known constant, C_{AB}, is treated as an absolute numerical quantity. When, after computation, certain slopes and deflections are found to be negative quantities, it means that their directions are in reality opposite to the direction assumed.

In effect, Equation (64) is the slope deflection equation, without the use of a sign convention. Introducing the convention that clockwise rotation is to be indicated by a "plus" sign, and contra-clockwise rotation by a "minus" sign, and referring again to Figs. 23(b) to 23(e):

$$-M'_{AB} = -4\,E\,K\,\theta_A \qquad (66a)$$

$$-M''_{AB} = -2\,E\,K\,\theta_B \qquad (66b)$$

$$-M'''_{AB} = +6\,E\,K\,R \qquad (66c)$$

and,

$$-M''''_{AB} = +C_{AB} \qquad (66d)$$

Adding Equations (66), $-M_{AB} = -M'_{AB} - M''_{AB} - M'''_{AB} - M''''_{AB}$; $-M_{AB} = -2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) + C_{AB}$; or,

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) - C_{AB} \qquad (67)$$

which is the conventional form of the slope deflection equation.

F. B. FARQUHARSON,[26] ASSOC. M. AM. SOC. C. E. (by letter).—The writer views this paper as a contribution to the field of analysis of structures by models and from this vantage point suggests that it provides a real contribution in a field that receives altogether too little attention from the practicing structural engineer. It has been the writer's experience that the influence line in general takes on no real and lasting significance until experienced and handled in the form of a model. After working with several model methods for a number of years, the re-thinking necessary in reading this paper has served a valuable function in bringing Maxwell's theorem slope-deflection principles, influence lines, and the model and its prototype into a more harmonious relationship.

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[26] Associate Prof. of Civ. Eng., Univ. of Washington, Seattle, Wash.

It is undoubtedly true that the fact that models may only be studied in connection with an existing tangible model, has led many engineers to neglect this approach in favor of one of the many methods requiring only pencil, paper, and a slide-rule as adjuncts to fundamental thinking. Thus, it is probable that the majority of efforts along model lines have been confined to university laboratories and a few of the larger engineering offices.

It is unfortunate that this condition exists since it does not follow that the model method requires the use of an expensive set of equipment. Indeed, it has been the practice of the writer to accompany the discussion of influence lines by practical demonstrations on the blackboard, making use of a long wooden spline and half a dozen small steel pins inserted in holes in the blackboard. In this manner, using a sharp piece of chalk and a meter stick for measurement, influence line results are obtained on fairly complex structures which check theoretical computations to within 15 per cent. Of course, the general usefulness of the model method is much extended by the availability of accurate and more elaborate equipment offered commercially for this purpose.

It has been the writer's experience that much of the indifference to the use of models arises through engineers who have never tried the method. It is suggested that even so complex a problem as that illustrated in Fig. 1 of the paper might be solved with an accuracy within 10% or 15%, through the use of a few carefully selected wooden splines, a dab of glue, and a few brads. The model can be constructed and mounted in an hour. A minute or two will suffice to trace off the influence line from which the moment at Point *E* is observed directly for any single concentrated load and obtained with ease for any load distribution whatever. To be sure, a check solution yielding the exact theoretical influence line may occupy several evenings, but the result is likely to be an enthusiasm for the model as an adjunct to structural analysis.

A. A. Eremin,[27] Assoc. M. Am. Soc. C. E. (by letter).—An interesting method of computing bending moments and shear stresses in rigid frames, based on the Mohr principle, which has been cited by the writer elsewhere,[28] is developed by Mr. Gottschalk. His method has merit, but, it is regrettable that he failed to extend it to the computation of stresses in rigid frames with variable sections. The advantage of rigid frames and continuous beams with variable sections is evident and it is a serious limitation of the method that it can be applied only to structures with constant sections.

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[27] Associate Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[28] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 21.

The computations may be divided into two steps: (1) Computing the stiffness factors, S, at each end of members in the frame; and (2) computing the factors, f. The stiffening factors express the elastic properties of a frame and do not vary with loads on the frame. The factors, f, computed in Step (2), however, vary with the manner of loading and the location of a section in the frame at which analysis of stresses is desired. The final bending moments and shear stresses in the frame are determined by the algebraic expressions involving the Factors f and the loading. The method may be extended to determine stresses in rigid frames with variable sections. Computation of the stiffening factors, S, and loading factors, f, for variable sections may be simplified by preparing tables of constants for the elastic properties of members in frames. However, the final algebraic expression for bending moment and shear stress in members with variable sections, involving the factors, f, will be lengthy and difficult of solution for practical purposes.

In rigid frames more than one story in height the value of S, cannot be obtained by direct computations. It must be assumed, and true values are determined by repeated trial computations. Therefore, the labor required for computing the values of S varies with the experience acquired in deciding on an initial trial value.

A direct computation of shear stresses by Equations (11) and (12) can scarcely be considered a saving of time. In rigid frames, shear stresses at ends of members are generally determined by the formula:

$$V = V_0 + \frac{M_1 - M_2}{l} \qquad \text{.....................} (68)$$

in which V_0 = shear stress at one end of a member assuming both ends freely supported; M_1 and M_2 = the restraining bending moments at the ends of a member; and l = span length of the member. Evidently, the time and effort required for computing Factor f and the shear stresses is not less than that required for computing the restraining bending moments, M_2 and M_1, and shear stress, V_0, in Equation (68).

The author's method is convenient in the construction of influence lines for bending moments and shear stresses in rigid frames. Likewise, it may be used conveniently in the analysis of stresses in rigid frames and continuous beams that have a single statically indeterminate reaction. However, in computing bending moments and shear stresses for each member of a multiple-span rigid frame the method involves lengthy computations and cannot be considered as a time saver.

DAVID M. MACALPINE,[29] ASSOC. M. AM. SOC. C. E. (by letter).—In so far as the author has introduced the little used idea of f, this paper is an interesting one. It is unfortunate, however, that he has chosen an unfamiliar sign convention. For the paper to realize its greatest value to the profession it is imperative that this convention be clearly stated in the closing discussion. It will then be possible to transform the formulas developed to more familiar ones.

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[29] Instructor, Coll. of the City of New York, School of Technology, New York, N. Y.

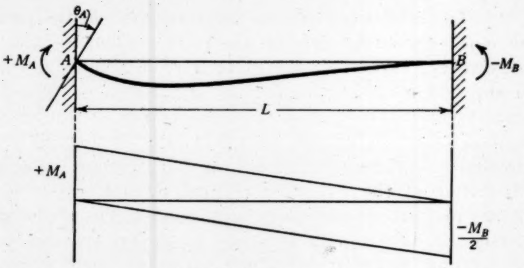

FIG. 24.

There appears to be no need of abandoning any of the several rather widely accepted and publicized sign conventions. One wonders why all these conventions were discarded. If the author had accepted the rather common convention of tension on the bottom fiber being caused by positive moment, Equation (36) would not have been changed. A derivation of this formula, without recourse to the slope-deflection equation, follows: Holding End *B* fixed (see Fig. 24) and rotating End *A* so as to produce tension in the bottom fiber:

$$M = -M_A + \frac{3}{2} M_A \frac{x}{L} \qquad \text{(69a)}$$

and,

$$E\,I \frac{dy_1}{dx} = -M_A\,x + \frac{3}{4} M_A \frac{x^2}{L} + C_1 \qquad \text{(69b)}$$

When $x = L$: $\frac{dy_1}{dx} = 0$; and, $C_1 = \frac{M_A\,L}{4}$:

$$E\,I\,y_1 = -\frac{M_A\,x^2}{2} + \frac{M_A\,x^3}{4\,L} + \frac{M_A\,L\,x}{4} + C_2 \qquad \text{(69c)}$$

When $y_1 = 0$: $x = 0$; and, $C_2 = 0$:

$$y_1 = \frac{M_A\,x\,(x - L)^2}{4\,E\,I\,L} = \frac{M_A\,x\,(x')^2}{4\,E\,I\,L} \qquad \text{(69d)}$$

When $x = 0$: $E\,I \frac{dy_1}{dx} = E\,I\,\theta_A = \frac{M_A\,L}{4}$; $\theta_A = \frac{M_A\,L}{4\,E\,I}$; and $f' = \theta_A\,L$; $\frac{M_A}{4\,E\,I} = \frac{f'}{L^2}$; and,

$$y_1 = \frac{f'\,x\,(x')^2}{L^3} \qquad \text{(70a)}$$

Holding End A fixed and rotating End B so as to obtain tension in the bottom fiber:

$$y_2 = \frac{f\, x^2\, x'}{L^3} \qquad \text{(70b)}$$

Adding Equations (70*a*) and (70*b*), the result is identical with Equation (36) of the paper.

On the other hand, the beam convention is very inconvenient to apply to the slope-deflection equation. A sign convention must fit some definite system if it is to be used from a practical, as well as from a mathematical, standpoint. Such a convention would require that positive angles be measured by counter-clockwise rotation, that distances, forces, and displacements be positive upward and to the right from the origin, and that moments capable of being represented by a positive vector be positive; that is, counter-clockwise external moments in the plane of the paper are positive. In this case positive moments are increasing positive angles. No great harm is done, however, if any other consistent convention is used.

In this convention, the familiar slope-deflection equations are:

$$M_{AB} = M_{F\,AB} + \frac{2\,E\,I}{L} (2\,\theta_A + \theta_B - 3\,R_{AB}) \qquad \text{(71a)}$$

and,

$$M_{BA} = M_{F\,BA} + \frac{2\,E\,I}{L} (2\,\theta_B + \theta_A - 3\,R_{BA}) \qquad \text{(71b)}$$

Ordinarily, $M_{F\,BA}$ is arithmetically negative but should carry a positive sign in the formula. Using Equations (71) and deriving formulas similar to Equations (34*b*) and (34*c*):

$$m = \frac{-\,0.5\,k}{k + S_B} \qquad \text{(72a)}$$

and,

$$m' = \frac{-\,0.5\,k}{k + S_A} \qquad \text{(72b)}$$

These expressions are in accordance with the geometric facts, θ_A being opposite in direction to θ_B. Equations (37*a*) and (37*b*) become, respectively,

$$\theta_{BA} = -\,\frac{S_2}{K_B} \qquad \text{(73a)}$$

and,

$$\theta_{BC} = \frac{S_1'}{K_B} \qquad \text{(73b)}$$

Equations (73) are also in accord with the geometry of the figure. The remainder of the equations follow directly, differing from the author's formulas only in sign.

Referring further to signs, the author has failed to state that he assumes L to be negative when measured to the left of his origin. This is necessary in order to obtain Equation (16) in its present form. This assumption, however, introduces an interesting incongruity in Fig. 2(*a*). For example, both θ_A and θ_B are positive by the author's sign convention. The values of f and f' are then positive and negative, respectively, thereby violating the sign convention as adopted for the paper for L. It is necessary to assume L as positive when measured to the right for θ_A and to the left for θ_B; otherwise, f' is positive when measured upward and f is positive when measured downward.

Referring to Line 2 following Equation (2), the use of the word, "strain," rather than "moment," is open to question. It would seem that "moment" is the more exact term.

The derivation given in Appendix I does not seem to be as satisfactory as the many other derivations of Maxwell's theorem to be found in any text or reference book on the subject. The theorem is so well known and widely publicized that there is some question as to the necessity of a derivation unless a new and better proof is given.

The derivation of the slope-deflection equations, using any of the accepted sign conventions, is also familiar and Appendix II only serves to illustrate the breakdown of the author's sign system (see Fig. 14).

There is a definite need for the establishment of a mathematically consistent, and practical, sign convention, as well as notation, in both the beam theory and stress analysis. Any sign convention that is not self-consistent tends to confusion and delays the establishment and acceptance of a correct one.

Otto Gottschalk,[30] Esq. (by letter).—Interesting suggestions have been presented in the discussion of this paper. The writer's methods have been derived from unloaded model analysis on a primarily geometrical basis, establishing the fundamental relations which make it possible to analyze structures directly, the same as in models, by using the simple stiffness values of the members instead of equations containing abstract stress symbols or a series of numbers.

In contrast with other branches of applied science, structural analysis has lacked that sound, experimental procedure which is characteristic of model analysis (especially models with visible deformations). It has required its own austere development entirely separate from current abstract conception. The fact that one of the discussers considers the geometrical process explained in the paper as only a variation of that by Strassner, and another discusser, a variation of the Cross method, proves the impossibility of judging a new conception merely from appearances, even to those who are most conversant with current theories.

Mr. Mensch contributes an inspiring anthology of former methods and offers suggestions of greatest practical interest. However, in his comments on the paper, he is fundamentally at variance with the writer, probably due to lack of examples in the original presentation. The paper is not a variation of any abstract treatment and does not require one to "analyze a structure by beginning at one end and going over its entire length." On the contrary, a fifth conclusion may properly be added to those cited in the paper: "(5) The analysis may be limited to the structural member under consideration and a few adjacent members." The examples given subsequently herein should satisfy Mr. Mensch that the writer's methods fit the requirements he so clearly establishes.

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[30] Buenos Aires, Argentine Republic.

Mr. Shapiro presents a really interesting example of the rapid convergence of transmission coefficients, which are an essential part of the advantage inherent in Conclusion (5). Thus far, the writer has not encountered the limits in practical application of his geometrical methods which Mr. Shapiro suspects; there is a need, however, for perfecting the application of methods of analytical geometry, especially the evolution of curves, so that the writer's method can be used more easily for curved structures.

Professor Griffith contributes experiences of his own in the handling of models and confirms the writer's experiences which make it possible to deal accurately with visible deformations. In Fig. 20, for example, the angle of the moment clamps at Point *B* is different from that at Point *D*. The necessarily high cost of working commercial sets should make it a habit, where the expense is not warranted, to use the equipment available at the nearest college, just as one would consult its library.

Mr. Weiss deserves sincere acknowledgment for his encouraging comments. His suggestions are of positive value and have been used in the preparation of this closing discussion. In principle, the writer avoids obtaining stresses by equations with unknowns. Special attention should be called to Mr. Weiss' graphical solution for the ordinates of influence lines as demonstrated by Fig. 22.

In a few words, Professor Farquharson shows how the experimentation is indispensable as a guide to intuition in solving problems in statics and his recommendations for teaching influence lines deserve widest acceptance. As a first step students should be taught the habit of having handy wooden or metal splines, in order to check all their results.

Mr. Eremin rightly desires the extension of the geometrical methods shown in the paper to variable sections. These were omitted to simplify the presentation; but they offer no difficulties.[31] Mr. Eremin is incorrect in assuming that the manner of loading has any bearing on the factors, *f*, and that values of *S* cannot be obtained directly in rigid frames more than one story high. Furthermore, contrary to his assumptions, the number of so-called statically indeterminate reactions does not complicate the computations in any respect, and the time saved becomes more marked the more equations there are to be solved by current methods.

Mr. MacAlpine offers an interesting derivation of the general equation of influence lines (Equation (5)) and makes valuable suggestions with regard to the question of signs. The latter cannot be settled entirely satisfactorily within the scope of the paper.

In general derivations, the complications imposed by rotation signs can be avoided by working with *f* and *f'* directly as has been done herein under the heading, "General Influence Lines."

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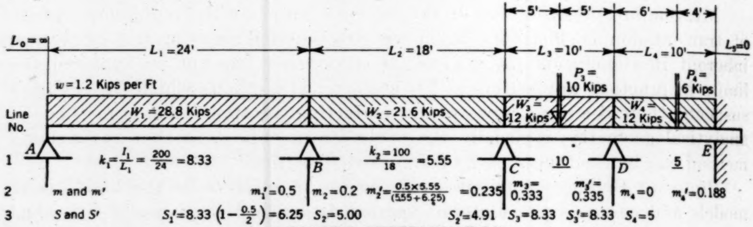
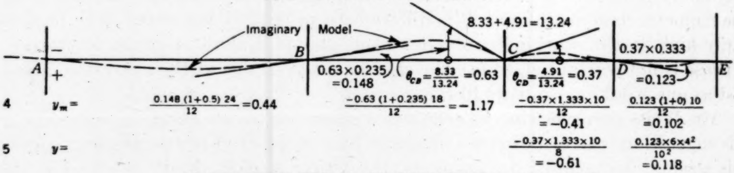
[31] See chapter on "Influence Lines of Haunched Beams" in "Continuous Frames," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., 1932, p. 139; also, the chapter on "Effect of Haunching," in "Concrete Building Frames," Portland Cement Assoc., Chicago, December, 1935, p. 29.

Examples to Clarify Questions Raised in Discussion.—An excellent opportunity to apply the writer's method is afforded by the interesting example of the continuous four-span beam shown in Fig. 25(*a*).[32]


$L_0 = \infty$
$L_1 = 24'$
$L_2 = 18'$
$L_3 = 10'$
$L_4 = 10'$
$L_5 = 0$
5'
5'
6'
4'
w = 1.2 Kips per Ft
P_3 = 10 Kips
P_4 = 6 Kips
W_1 = 28.8 Kips
W_2 = 21.6 Kips
W_3 = 12 Kips
W_4 = 12 Kips
Line No.
A
B
C
D
E
1
$k_1 = \frac{I_1}{L_1} = \frac{200}{24} = 8.33$
$\frac{k_2 = 100}{18} = 5.55$
10
5
2
m and m'
$m_1' = 0.5$
$m_2 = 0.2$
$m_2' = \frac{0.5 \times 5.55}{(5.55 + 6.25)} = 0.235$
$m_3 = 0.333$
$m_3' = 0.335$
$m_4 = 0$
$m_4' = 0.188$
3
S and S'
$S_1' = 8.33 \left(1 - \frac{0.5}{2}\right) = 6.25$
$S_2 = 5.00$
$S_2' = 4.91$
$S_3 = 8.33$
$S_3' = 8.33$
$S_4 = 5$

(*a*) ELASTIC DETERMINATION


Imaginary
Model
8.33 + 4.91 = 13.24
A
B
C
D
E
−
+
0.63 × 0.235 = 0.148
$\theta_{CB} = \frac{8.33}{13.24} = 0.63$
$\theta_{CD} = \frac{4.91}{13.24} = 0.37$
0.37 × 0.333 = 0.123
4
$y_m =$
$\frac{0.148\,(1+0.5)\,24}{12} = 0.44$
$\frac{-0.63\,(1+0.235)\,18}{12} = -1.17$
$\frac{-0.37 \times 1.333 \times 10}{12} = -0.41$
$\frac{0.123\,(1+0)\,10}{12} = 0.102$
5
$y =$
$\frac{-0.37 \times 1.333 \times 10}{8} = -0.61$
$\frac{0.123 \times 6 \times 4^2}{10^2} = 0.118$

(*b*) SOLUTION FOR M_c BY STRUCTURAL ANALYSIS

6 $M_c = W_{ym} = 28.8 \times 0.44 = 12.66$ Kip · Ft $\quad -21.6 \times 1.77 = 25.16 \quad -12.0 \times 0.41 = -4.92 \quad 12.0 \times 0.102 = 1.22$

7 $M_c = Py = \quad -10.0 \times 0.61 = -6.10 \quad 6.0 \times 0.118 = 0.71$

8 Total $M_c = 12.66 - 25.16 - 4.92 - 6.10 + 1.22 + 0.71 = 14.59 - 36.18 = -21.59$ Kip · Ft

(*c*) COMPUTATION OF M_c FOR LOADS W AND P

FIG. 25.—MULTIPLE-SPAN CONTINUOUS BEAM

For the elastic determination (namely, the computation of stiffness values) the analysis may start at one end because of the small number of spans. Line 1, Fig. 25(*a*), shows the stiffness factors, $k = \frac{I}{L}$. In Span *A B*, the coefficient, $m_1' = 0.5$ (see Line 2) and, therefore, $S_1' = (1 - 0.25)\,k_1 = 6.25$ (see Line 3); similarly, in Span *B C*, $m_2' = \frac{0.5\,k_2}{(k_2 + S_1')} = 0.235$; and, $S_2' = k_2\left(1 - \frac{0.235}{2}\right) = 4.91$, etc. Likewise, beginning at the fixed end, *E*: $m_4 = 0$; $S_4 = k_4 = 5$; etc., all of which may be written in the diagram directly from slide-rule readings.

The structure, thus made elastically determinate, may be analyzed now for any stress, M_C (Fig. 25(*b*)), by rotating *C B* and *C D* one unit with respect to each other. Then, $K_C = S_2' + S_3 = 4.91 + 8.33 = 13.24$ and, consequently, for Span *C B*, $\theta_2' = \theta_{CB} = \frac{8.33}{13.24} = 0.63$; and for Span *C D*, $\theta_3 = \theta_{CD} = \frac{4.91}{13.24} = 0.37$. The rotation at Point *B* then is $\theta_2 = \theta_2'\,m_2' = 0.63$

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[32] "Continuous-Beam Analysis by Direct Distribution," by John C. Schulze and William A. Rose, *Engineering News-Record*, December 20, 1934; with discussions in *Civil Engineers' Digest*, June, 1935, by J. D. Gedo, Ben Moreell, Odd Albert, Members, Am. Soc. C. E., F. E. Turneaure, Hon. M. Am. Soc. C. E., and Messrs. Otto Gottschalk, and John C. Schulze.

× 0.235 = 0.148; and at Point D, $\theta_3' = \theta_3\, m_3 = 0.37 \times 0.333 = 0.123$. This is the complete analysis of the structure for M_C.

Line 4, Fig. 25(b), indicates the average ordinates, y_m, with which the uniformly distributed loads must be multiplied. Equation (6b) may be rewritten for spans to the left of Point C:

$$y_m = \frac{\theta' L (1 + m')}{12} \qquad \text{(74a)}$$

and to the right of Point C:

$$y_m = \frac{\theta L (1 + m)}{12} \qquad \text{(74b)}$$

Line 5, Fig. 25(b), shows the y-values for concentrated loads; and Lines 6, 7, and 8, show $M_C = W\, y_m$; $M_C = P\, y$; and the total M_C, respectively.

TABLE 1.—MOMENTS AT THE SUPPORTS OF A CONTINUOUS BEAM, $A\,B\,C\,D\,E$, FIG. 25.

LOADS		Ordinates (letter symbols)	M_B		M_C		M_D		M_E	
Letter symbols	In kips§		$\theta_1' = 0.444$ $\theta_2 = 0.556$		$\theta_2' = 0.629$ $\theta_3 = 0.371$		$\theta_3' = 0.375$ $\theta_4 = 0.625$		$\theta_4' = 1$ $\theta_5 = 0$	
			Values of y, in feet	Values of y, in kip-feet	Values of y, in feet	Values of y, in kip-feet	Values of y, in feet	Values of y, in kip-feet	Values of y, in feet	Values of y, in kip-feet
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
W_1	28.8	y_{m1}	−1.330′	−38.300	+0.443′	+12.758	−0.090′	−2.592	+0.045′	+1.296
W_2	21.6	y_{m2}	−1.000	−21.600	−1.165	−25.164	+0.233	+5.033	−0.117	−2.527
W_3	12.0	y_{m3}	+0.124	+1.488	−0.412	−4.944	−0.417	−5.004	+0.209	+2.508
P_3	10.0	y_{c3}	+0.187	+1.870	−0.618	−6.180	−0.626	−6.260	+0.314	+3.140
W_4	12.0	y_{m4}	−0.031	−0.372	+0.102	+1.236	−0.521	−6.024	−0.990	−11.880
P_4	6.0	y_4	−0.036	−0.216	+0.118	+0.714	−0.600	−3.600	−1.620	−9.720
M_c*	....	....	−60.488		−36.288		−23.180		−24.127	
M_c†	....	....	+3 358		+14.708		+5.033		+6.944	
M_c‡	....	....	−57.130		−21.580		−18.447		−17.183	

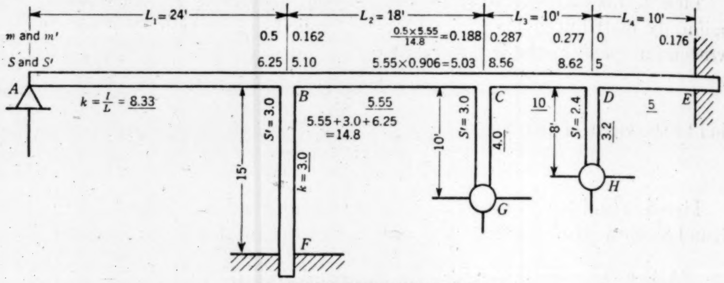
* Minimum. † Maximum. ‡ Total. § 1 kip = 1 000 lb.

Although the explanation, in detail, occupies some space, the analysis shown gives the exact result in only a fraction of the time and work required for any other method now current. If moments at the other supports are needed also, the analysis may be arranged as shown in Table 1.

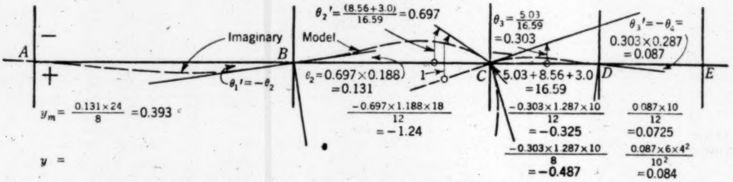
The superiority of the model method becomes even more marked when the continuous beam in Fig. 25 is supported on columns, forming the irregular frame shown in Fig. 26. The stiffness factors, k, of the columns and beams, are underlined. The stiffness, S, of the columns is $S = k$, when the base is fixed and $S = 0.75\, k$ when it is hinged. The computation of m and S to the left, and m' and S' to the right, of the several spans of the beam is shown at the top of Fig. 26(a). The m-values are smaller, and the S-values larger, than those in Fig. 25.

Fig. 26(b) demonstrates the analysis of the structure for M_C, obtaining the influence line of M_{CB} by unit rotation at End C of Span $C\,B$ to which Spans $C\,D$ and $C\,G$ offer resistance. On the left of End C: $\theta_2' = 0.697$; and to the right of End C: $\theta_3 = 0.303$. Therefore, because of rotation at Point B:

$\theta_2 = \theta_2' \, m_2' = 0.131$; and because of rotation at Point D: $\theta_3' = \theta_3 \, m_3 = 0.087$. The values of y_m for uniformly distributed loads, and of y at concentrated loads, are then computed in the same manner as in Fig. 25. With the structure thus analyzed, M_{CB} (see Fig. 26(*c*)) is the sum of the several loads multiplied with y_m and y, respectively; and $M_{CB} = 12.69 - 35.57 = -22.88$ kip-ft.


$L_1 = 24'$
$L_2 = 18'$
$L_3 = 10'$
$L_4 = 10'$
m and m'
S and S'
0.5
0.162
$\frac{0.5\times5.55}{14.8} = 0.188$
0.287
0.277
0
0.176
6.25
5.10
5.55×0.906=5.03
8.56
8.62
5
A
$k = \frac{I}{L} = 8.33$
B
5.55
5.55+3.0+6.25 =14.8
C
10
D
5
E
$S' = 3.0$
$k = 3.0$
$S' = 3.0$
4.0
$S' = 2.4$
3.2
15'
10'
8'
G
H
F

(*a*) ELASTIC DETERMINATION


A
B
C
D
E
−
+
Imaginary
Model
$\theta_2' = \frac{(8.56+3.0)}{16.59} = 0.697$
$\theta_1' = -\theta_2$
$\theta_2 = 0.697\times0.188 = 0.131$
1
$\theta_3 = \frac{5.03}{16.59} = 0.303$
5.03+8.56+3.0 =16.59
$\theta_3' = -\theta_4 = 0.303\times0.287 = 0.087$
$y_m = \frac{0.131\times24}{8} = 0.393$
$\frac{-0.697\times1.188\times18}{12} = -1.24$
$\frac{-0.303\times1.287\times10}{12} = -0.325$
$\frac{0.087\times10}{12} = 0.0725$
$y =$
$\frac{-0.303\times1.287\times10}{8} = -0.487$
$\frac{0.087\times6\times4^2}{10^2} = 0.084$

(*b*) SOLUTION FOR M_c BY STRUCTURAL ANALYSIS

$M_{CB} = W_x \, y_m$ = 28.8×0.393 = 11.32 Kip·Ft −21.6×1.24 = −26.8 Kip·Ft −12×0.325 = −3.90 12×0.0725 = 0.87

$M_{CB} = P_x \, y$ −10×0.487 = −4.87 6×0.084 = 0.50

M_{CB} = 11.32 − 26.8 − 3.90 − 4.87 + 0.87 + 0.50 = 12.69 − 35.57 = −22.88 Kip·Ft

(*c*) COMPUTATION OF M_c FOR LOADS W AND P IN FIG 25

FIG. 26.—UNSYMMETRICAL MULTIPLE-SPAN FRAME

Analysis is begun at the section for which the stress is wanted (see Fig. 26(*b*)), instead of complicating it by beginning at supports; beyond two spans on either side of the section ordinates of influence lines generally become small and may be neglected. Similarly, the elastic determination does not need to start farther away, as is illustrated in Fig. 27. If the support at End A were fixed, the value of S_3' at Point D would be 8.62, exactly the same as if End A

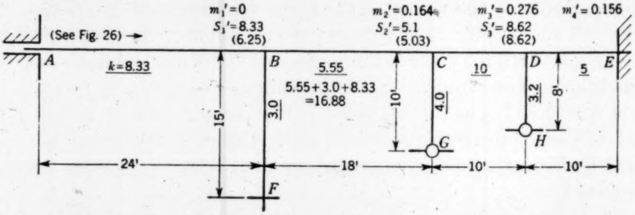
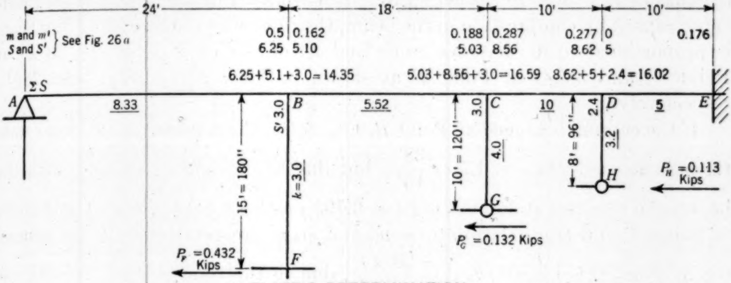

(See Fig. 26)
$m_1' = 0$
$S_1' = 8.33$ (6.25)
$m_2' = 0.164$
$S_2' = 5.1$ (5.03)
$m_3' = 0.276$
$S_3' = 8.62$ (8.62)
$m_4' = 0.156$
A
$k = 8.33$
B
5.55
5.55+3.0+8.33 =16.88
C
10
D
5
E
15'
3.0
10'
4.0
3.2
8'
G
H
F
24'
18'
10'
10'

FIG. 27.—EXTREME CHANGES OF S_1' HAVE NO INFLUENCE ON S_3'

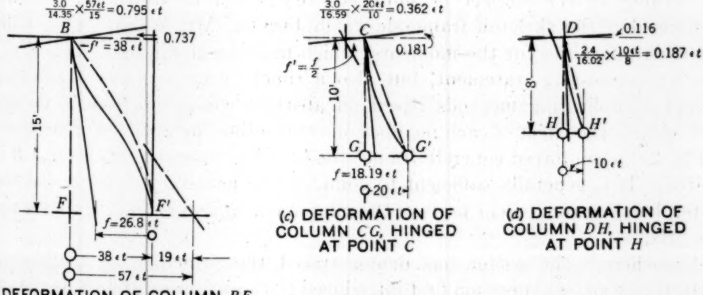
were hinged as in Fig. 26. Assuming, then, at the most two spans away a value of S is wanted: Roughly, by inspection, $\frac{S}{k}$ equals between 0.8 and 0.95. If an interior span is considered, the value will be exact for adjacent spans.

In Fig. 26(a), if Beam $A\ B\ C\ D\ E$ could move freely in a horizontal direction, the influence line for side-sway can be obtained by sliding the beam a unit distance with respect to the column base, as shown in Fig. 10, computing the horizontal thrust at the column base, and deducting the bending moments determined in Fig. 26(c). The analysis is similar to that in Fig. 28(a) except


24'
18'
10'
10'
m and m'
S and S'
See Fig. 26a
0.5 | 0.162
6.25 5.10
0.188 | 0.287
5.03 8.56
0.277 | 0
8.62 5
0.176
6.25+5.1+3.0=14.35
5.03+8.56+3.0=16.59
8.62+5+2.4=16.02
ΣS
A
8.33
S' 3.0
B
5.52
3.0
C
10
2.4
D
5
E
15' = 180"
k=3.0
10' = 120"
4.0
8' = 96"
3.2
H
P_H = 0.113 Kips
G
P_G = 0.132 Kips
P_F = 0.432 Kips
F

(a) ELASTIC DETERMINATION

$$\begin{array}{lccc}
 & \frac{\theta_B}{\epsilon t} & \frac{\theta_C}{\epsilon t} & \frac{\theta_D}{\epsilon t} \\
 & 0.795 \quad \rule{2em}{0.4pt} & -0.795\times 0.162 = -0.129 & \times\,(-0.287) = 0.037 \\
 & -0.068 = (-0.188)\times & \longleftarrow \quad 0.362 & \times\,(-0.287) = -0.104 \\
 & \underline{0.010} = (-0.188)\times & \longleftarrow \quad \underline{-0.052} & = (-0.277)\times \underline{0.187} \\
 & \frac{\Sigma\theta}{\epsilon t} = 0.737 & 0.181 & 0.120 \\
\frac{f}{\epsilon t} & = 38 - 0.737\times 15 = 26.8 & 20 - 0.181\times 10 = 18.19 & 10 - 0.12\times 8 = 9.04 \\
\frac{f'}{\epsilon t} = & -38 & \frac{18.19}{2} = 9.095 & \frac{9.04}{2} = 4.52 \\
\frac{(f-f')}{\epsilon t} = & 64.8 = 777.6\text{ In.} & 9.095 = 109\text{ In.} & 4.52 = 54.3\text{ In.} \\
\frac{6E\epsilon t k}{L^2} = & \frac{6\,000\times 3.0}{32\,400} = 0.556; & \frac{6\,000\times 4.0}{14\,400} = 1.67; & \frac{6\,000\times 3.2}{9\,216} = 2.08 \\
P = \frac{6Ek(f-f')}{L^2} = & \frac{777.6\times 0.556}{1\,000} = 0.432\text{ Kips;} & \frac{109\times 1.67}{1\,000} = 0.182\text{ Kips;} & \frac{54.3\times 2.08}{1\,000} = 0.113\text{ Kips;} \\
e = \frac{2\theta_B - \theta_C}{\theta_B - \theta_C}\times\frac{L}{3} = & \frac{2\times 0.737 + 0.181}{0.737 + 0.181}\times\frac{18}{6} = 10.8\text{ Ft} & 5.38\text{ Ft} & 6.67\text{ Ft}
\end{array}$$


3.0/14.35 × 57εt/15 = 0.795 εt
B
−f' = 38 εt
0.737
15'
F
F'
f=26.8 εt
38 εt
19 εt
57 εt
3.0/16.59 × 20εt/10 = 0.362 εt
C
0.181
f' = f/2
10'
G
G'
f=18.19 εt
20 εt
D
0.116
2.4/16.02 × 10εt/8 = 0.187 εt
8'
H
H'
10 εt

(b) DEFORMATION OF COLUMN BF, FIXED AT POINT F

(c) DEFORMATION OF COLUMN CG, HINGED AT POINT C

(d) DEFORMATION OF COLUMN DH, HINGED AT POINT H

Fig. 28.—Stresses Due to Uniform Changes of Temperature in Beam $A\ B\ C\ D\ E$

that it is simpler because the side-sway is equal at the column tops; the horizontal thrust added at the column bases F, G, and H, respectively, is $0.44 + 0.34 + 0.42 = 1.20$ kips and $M_{CB} = -22.88 + 1.31 = -21.57$ kip-ft.

A further question of practical interest in the case of the unsymmetrical frame in Fig. 26(*a*), is that concerned with the stresses produced by a uniform change of temperature in Beam $A\ B\ C\ D\ E$ of, say, $t = 60°$ F. The coefficient of expansion is assumed equal to $\epsilon = 5.5 \times 10^{-6}$; and the modulus of elasticity is equal to $E = 3 \times 10^6$ lb per sq in. Not considering the microscopic shortening caused by compression in the beam, the side-sway of the ends of any beam is proportional to its distance from End E; or, $10'\,\epsilon\,t$ at Point D; $20'\,\epsilon\,t$ at Point C; and $38'\,\epsilon\,t$ at Point B as shown in Figs. 28(*b*), 28(*c*), and 28(*d*), respectively.

If the column is hinged at Point B, Fig. 28(*b*), the column top, B, will rotate through an angle, $\theta_{BF} = 1.5\,\epsilon\,t\,\frac{38'}{15'}$; but due to its rigid connection with the beams the rotation at Joint B is $\theta_B = 0.795\,\epsilon\,t$, this rotation being transmitted to Points C and D in relation to $-m_2$ and $m_2\,m_3$, respectively. If the column top at Point C is hinged: $\theta_{CG} = \frac{20'\,\epsilon\,t}{10'}$; due to the rigid connection with the beams the rotation at Point C will be $\theta_C = 0.362\,\epsilon\,t$, which is transmitted to Points B and D in proportion to their respective values of $-m_2'$ and $-m_3$. The sum of those partial rotations makes it possible to compute the intercepts of Tangents f and f', the Thrust P at the foot of each column (by Equation (15)) and, finally, the points of inflection of each beam (by Equation (91)), all as demonstrated in Fig. 28.

The horizontal thrusts at Points F, G, and H are, as shown in Fig. 28: 0.432 kips, 0.182 kips, and 0.113 kips, which would seem rather small if the structure were of reinforced concrete. However, the k-values are only proportional to each other and, in practice, all of them would be, say, five times the indicated values, increasing the thrusts to 2.16 kips, 0.91 kips, and 0.56 kips, respectively.

Turning to the subject of multiple-story skeletons mentioned by Mr. Mensch: For the skeleton frame shown in Fig. 21, Mr. Weiss derives 1 134 unknown quantities for the moments, which must be computed. This is not only an interesting statement, but also a timely warning against adopting current complicating methods based on abstract conceptions which do not show clearly the effect of each member on every other member. For example, in Fig. 29 (a reinforced concrete frame) the bending resistance of Beam $A\ B$ is desired. It is generally sufficient to compute the necessary resistance at the center, bending convenient parts of the reinforcement and extending it over the supports.

Elsewhere,[33] the writer has demonstrated the principles of mechanical analysis (a concise expression first introduced to the writer by Hale Sutherland, M. Am. Soc. C. E.) by the physical deformation of models. For present purposes Fig. 30 may be used to illustrate the method of determining M_C at the center Point C of Beam $A\ B$, by this method. The model is composed of steel splines—those for columns reproducing the combined k-values of the upper and lower columns, the lengths of spans of the beams being modified in proportion to their several values of $k = \frac{I}{L}$. The far ends of the column splines are held at the original distance from each other, and at the proper angles, by clamps mounted on a rod which, in turn, is supported on rollers that permit horizontal sliding without friction. In Fig. 30, a unit rotation has been exerted at Point C by means of a special clamp and the experiment has shown that, even where the relative stiffness of structural parts meeting at Points A and B differ extremely, the rolling rod will not sway when a unit rotation is applied at Point C (or even outside of Point C, within the middle-fourth, or even the middle-third of the span). This demonstrates that bending moments may be analyzed at these sections as if the frame did not sway, and that the values of m and S may be computed as shown previously herein for structures fixed against horizontal movement.

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[33] *Journal*, Franklin Inst., Philadelphia, Pa., July, 1926, p. 61, and February, 1929, p. 245; *Der Bauingenieur*, Berlin, 1924, p. 263; 1926, p. 722; 1928, p. 924; and *Beton u. Eisen*, Berlin, 1927, p. 286; 1929, p. 113.

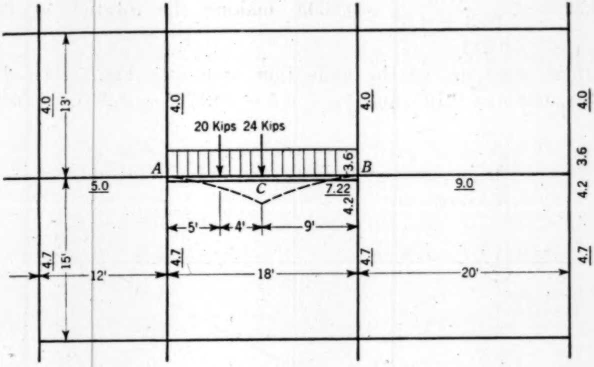

FIG. 29.—TYPICAL REINFORCED CONCRETE FRAME

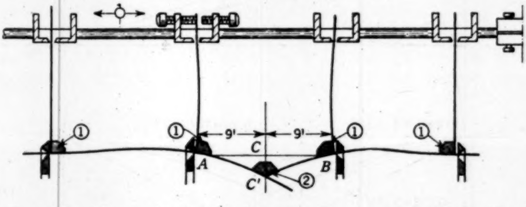

FIG. 30.—MODEL ANALYSIS FOR THE DETERMINATION OF M_C

The structural analysis for M_C is shown in Fig. 31. In a simple beam, $A\ B$, the unit rotation at Point C would rotate Beam $A\ B$ at Points A and B through an angle, 0.5, resulting in the dotted line, $A\ C'\ B$ (Fig. 31(*a*)). To compute the restraint at Point A (Fig. 31(*b*)) assume the beam, $A\ B$, to be free at Point B, making $m = 0.5$ and $S = 7.22\left(1 - \frac{0.5}{2}\right) = 5.41$. Since $S_A = 13.0$, the rotation, $\theta_{AB} = 0.5 \times \frac{13.0}{(13.0 + 5.41)} = 0.363$; and, at the free end, *B*, $\theta_{BA} = 0.5 \times 0.353 = 0.1765$. At End *B*, therefore, the rotation is $0.5 - 0.1765 = 0.3235$ = (Fig. 31(*c*)). Once again, making Beam *A B* fixed at Point *B*, and with m', S', and S_B as computed in Fig. 31(*a*), the rotation, θ_{BA} (see Fig. 31(*c*)) equals $0.3235 \times \frac{16.3}{16.3 + 6.58} = 0.2305$, making the rotation at Point *A*, $\theta_{AB} = \theta_{BA}\, m' = 0.041$.

The total rotations at the ends then, are (see Fig. 31(*d*)) $\theta_A = 0.5 - (0.353 + 0.041) = 0.106$; and, $\theta_B = 0.5 - (0.1765 + 0.2305) = 0.093$, producing the influence line, *A C″ B*. By Equation (20*a*), the ordinate at the center is: $y_c = \frac{L\,(1 + \theta_A + \theta_B)}{8} = 2.70$ ft; the medium ordinate (Equation 20(*b*)) is $y_m = \frac{2\,y_c}{3} - \frac{L}{24} = 1.05$ ft, and, at $x = 5.0$ ft from End *A* (see Equa-

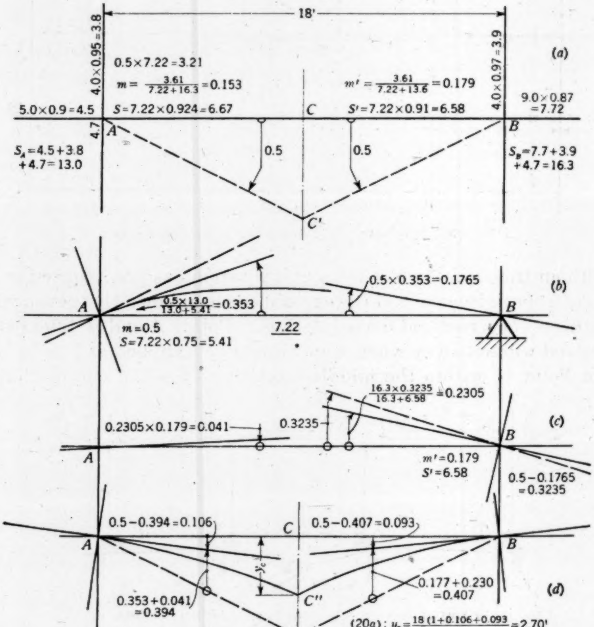
18'
4.0×0.95=3.8
4.0×0.97=3.9
(a)
0.5×7.22=3.21
$m=\frac{3.61}{7.22+16.3}=0.153$
$m'=\frac{3.61}{7.22+13.6}=0.179$
5.0×0.9=4.5
S=7.22×0.924=6.67
C
S'=7.22×0.91=6.58
9.0×0.87 =7.72
4.7
A
B
S_A=4.5+3.8 +4.7=13.0
0.5
0.5
S_B=7.7+3.9 +4.7=16.3
C'
0.5×0.353=0.1765
(b)
A
$\frac{0.5\times13.0}{13.0+5.41}=0.353$
B
m=0.5
S=7.22×0.75=5.41
7.22
$\frac{16.3\times0.3235}{16.3+6.58}=0.2305$
0.2305×0.179=0.041
0.3235
(c)
B
A
m'=0.179
S'=6.58
0.5−0.1765 =0.3235
0.5−0.394=0.106
C
0.5−0.407=0.093
A
B
y_c
0.177+0.230 =0.407
(d)
0.353+0.041 =0.394
C''
C'
(20a): $y_c=\frac{18\,(1+0.106+0.093)}{8}=2.70'$
(20b): $y_m=2\times\frac{2.7}{3}-\frac{18}{24}=1.05$

FIG. 31.—GEOMETRICAL ANALYSIS FOR M_C

tion (17*a*)): $y = 1.05$. Consequently, with the loads indicated in Fig. 29: $M_C = 20 \times 1.05 + 24 \times 2.7 + 18 \times 1.05 = 104.7$ kip-ft.

The solution of Fig. 31, starting either at Point *A* or at Point *B*, is recommended as a means of becoming familiar with the working of geometrical curves; for practical application M_C may be analyzed directly from the *m* and *S* values computed in Fig. 31(*a*). At Point *A*: $\frac{0.5 \times 13.0}{13.0 + 6.67} = 0.330$ and at Point *B*: $\frac{0.5 \times 16.3}{16.3 + 6.58} = 0.356$; and, therefore, the total rotations are, at *A*, $0.330 + 0.179 \times 0.356 = 0.394$; and, at *B*, $0.356 + 0.153 \times 0.330 = 0.407$, leaving $\theta_A = 0.5 - 0.394 = 0.106$ and $\theta_B = 0.5 - 0.407 = 0.093$, the same as was obtained before according to Fig. 31(*d*).

General Influence Lines.—Development of Equations (5) to (10*c*) beyond the form given in the paper, leads to conclusions that should be vitally useful in drafting and in understanding influence lines. The typical influence curve (see Fig. 2) is fully determined by the rotations, $\frac{f}{L} = \theta_B$ and $\frac{f'}{L} = \theta_A$, and may be expressed (see Fig. 32) as,

$$(f, f') = (f, 0) + (0, f') \ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

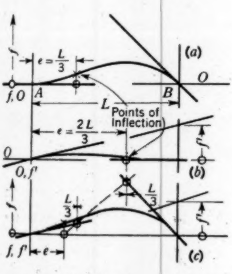

FIG. 32.—GENERAL INFLUENCE LINE

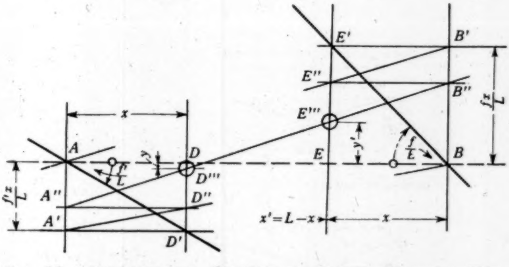

FIG. 33.—SYMMETRICAL ORDINATES, y AND y', FROM END TANGENTS, OR END TANGENTS FROM SYMMETRICAL ORDINATES, y AND y'

which is the sum of one curve rotated through $\frac{f}{L}$ at Point *B* and fixed at Point *A*, and another curve fixed at Point *B* and rotated through $\frac{f'}{L}$ at Point *A*.

The ordinates are found as follows: For the beam fixed at Point *A* $(f, 0)$, use Equation (10*c*); for the beam fixed at Point *B* $(0, f')$, use Equation (8*c*); and the general equation for f, f' (see Equation (5)) is:

$$y = \left(\frac{x^2}{L^2} - \frac{x^3}{L^3}\right) f + \left(\frac{x}{L} - \frac{2\,x^2}{L^2} + \frac{x^3}{L^3}\right) f' \ldots\ldots\ldots\ldots (76)$$

Formulas for influence ordinates for a beam, *A B*, of unit length and various conditions of fixation, are given in Table 2.[34] In connection with Item No. 1,

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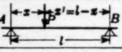
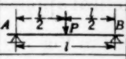
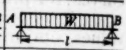
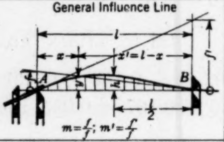
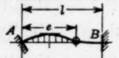
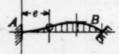
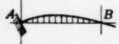
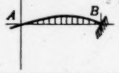
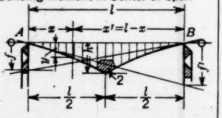
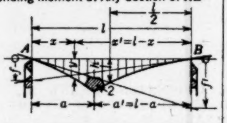
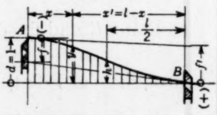
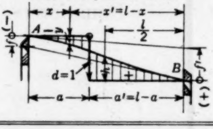
[34] "Einflusslinien," by Otto Gottschalk, in *Die Bautechnik*, Berlin, March 4, 1927, p. 124; see, also, *Der Bauingenieur*, Berlin, 1924, p. 268.

TABLE 2.—ORDINATES OF INFLUENCE LINES FOR STRAIGHT BEAM OF CONSTANT SECTION.

	Item No.	Type of Influence Line		Letter Symbol	Ordinates		
					General y 	In Center h 	Medium y_m 
(a)	1	General Influence Line  l x x′=l−x f′ A B y h l/2 m=f/f; m′=f/f		(f, f')	$\frac{xx'(xf+x'f')}{l^3}$	$\frac{(f+f')}{8}$	$\frac{2}{3}h=\frac{(f+f')}{12}$
(b)	2	Special Cases of No. 1 for Exterior Spans 	Fixed at B $(m=0)$	$(0, f')$	$\frac{x(x')^2f'}{l^3}$	$\frac{f'}{8}$	$\frac{2}{3}h=\frac{f'}{12}$
	3		Fixed at A $(m'=0)$	$(f', 0)$	$\frac{x^2x'f}{l^3}$	$\frac{f}{8}$	$\frac{2}{3}h=\frac{f}{12}$
	4		Free at B $(m=\frac{1}{2})$	$\left(\frac{f'}{2}, f\right)$	$\frac{xx'f'(x+2x')}{2l^3}$	$\frac{3}{16}f$	$\frac{2}{3}h=\frac{f'}{8}$
	5		Free at A $(m'=\frac{1}{2})$	$\left(f, \frac{f}{2}\right)$	$\frac{xx'f(2x+x')}{2l^3}$	$\frac{3}{16}f$	$\frac{2}{3}h=\frac{f}{8}$
(c)	6	Bending Moment in Center of Span  l A x x′=l−x B f y h 2 f′ l/2 l/2		$M_{0.5l}$	$x<\frac{l}{2}; \frac{x}{2}-N$ $x>\frac{l}{2}; \frac{x'}{2}-N$ $N=\frac{xx'}{l^3}\left[\left(\frac{l}{2}-f\right)x+\left(\frac{l}{2}-f'\right)x'\right]$ $x=\frac{l}{2}; \left(\frac{l+f+f'}{8}\right)$	$\frac{(l+f+f')}{8}$	$\frac{2}{3}h-\frac{l}{24}=\frac{(l+2f+2f')}{24}$
(d)	7	Bending Moment at Any Section of AB  l/2 l A x x′=l−x B f y h 2 f′ a a′=l−a		M_x	$x<a; \frac{a'x}{l}-N$ $x>a; \frac{ax'}{l}-N$ $N=\frac{xx'}{l^3}[(a-f)x+(a'-f')x']$ $x=a; \frac{aa'}{l^3}(2aa'+af+a'f')$	$a<a'; \frac{a}{2}-N$ $a>a'; \frac{a'}{2}-N$ $N=\frac{(f+f'-l)}{8}$	$a<a'; \frac{2}{3}h-\frac{a}{6l}(3a-l)$ $a>a'; \frac{2}{3}h-\frac{a'}{6l}(3a'-l)$
(e)	8	Reaction at A  x x′=l−x l/2 A (−) d=1 f y h B f′ (+)		R_A	$\frac{x'}{l}[1+\frac{x}{l^2}(x'f'+xf)]$	$\frac{1}{2}+\frac{(f+f')}{8}$	$\frac{1}{2}+\frac{(f+f')}{12}=\frac{2}{3}h+\frac{1}{6}$
(f)	9	Shear at any Section of AB  x x′=l−x (−) A y l/2 f d=1 h + B f′ (+) a a′=l−a		S_x	$x<a; -\frac{x}{l}+N$ $x>a; \frac{x'}{l}+N$ $N=\frac{xx'}{l^3}(xf+x'f')$	$a<a'; \frac{1}{2}+\frac{(f+f')}{8}$ $a>a'; -\frac{1}{2}+\frac{(f+f')}{8}$	$\frac{a'-a}{2l}+\frac{f+f'}{12}$ $=\frac{2}{3}h+\frac{1}{6}-\frac{a}{l}$

when y or y' are at one of the third points,

$$f = \frac{9}{2}(2y' - y) \qquad \text{(77a)}$$

and,

$$f' = \frac{9}{2}(2y - y') \ldots\ldots (77b)$$

when h and y are at the center or the quarter-points, respectively,

$$f = 12h - \frac{32y}{3} \ldots\ldots (78a)$$

and,

$$f' = \frac{32y}{3} - 4h \ldots\ldots (78b)$$

Items Nos. 6 to 9, Table 2, are the influence lines of simple beams, combined with that in Item No. 1. When $m = \frac{f}{f'} \leqq 0$, the influence line crosses Axis AB at Point x where $\frac{x}{L} = \frac{f}{(f - f')} = \frac{\theta_B}{(\theta_B - \theta_A)} = \frac{-m}{(1 - m)}$. Any straight line, $A'B'$ (Fig. 33), crosses Beam AB at the intersection of the influence line.

Symmetrical Ordinates.—According to Equation (5), the sum of two ordinates at distances, x and $x' = L - x$, from Point A, is:

$$y_x + y_{x'} = 2y_m = \frac{x x' (f + f')}{L^2} = \frac{8 h x x'}{L^2} \ldots\ldots (79)$$

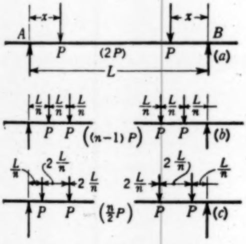


FIG. 34.—SYMMETRICAL CONCENTRATED LOADS

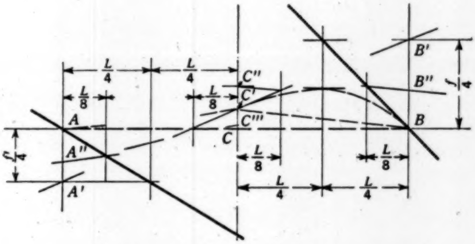


FIG. 35.—INFLUENCE LINE BY PROGRESSIVE CENTER TANGENTS

The stress then produced by two symmetrical loads, P (see Fig. 34a), is $\frac{8 P h x x'}{L^2}$. The relation between two symmetrical ordinates is expressed by:

$$\frac{y_x}{y_{x'}} = \frac{(x f + x' f')}{(x' f + x f')} = \frac{(m x + x')}{(m x' + x)} \ldots\ldots (80)$$

Therefore, when End A is fixed, $f' = 0$ and $\frac{y_x}{y_{x'}} = \frac{x}{x'}$; when End B is fixed, $f = 0$ and $\frac{y_x}{y_{x'}} = \frac{x'}{x}$; when End A is simply supported, $f' = \frac{f}{2}$ and $\frac{y_x}{y_{x'}} = \frac{L + x}{L + x'}$; and, when End B is simply supported, $f = \frac{f'}{2}$ and $\frac{y_x}{y_{x'}} = \frac{L + x'}{L + x}$.

The foregoing coefficients show that when one end of Beam $A\,B$ is fixed, symmetrically located ordinates vary in proportion to their distances from the fixed end; and, when one end is free, they vary in proportion to their distances from the free end, plus L. Consequently, a straight line joining two symmetrical points of the influence line crosses the axis at the fixed support and at a distance, L, beyond the free support, respectively.

When $(n - 1)$ equal loads, P, are acting at equal distances, $\frac{L}{n}$ (see Fig. 34(b)), the sum of the ordinates, according to Equation (79), is:

$$(n-1)\,y_m = 4\,h\left[(n-1) + 2\,(n-2) + 3\,(n-3) + \cdots + \frac{1}{2}\,(n-1)\left(n - \frac{1}{2}\,(n-1)\right)\right]\frac{1}{n^2}$$

For $(n - 1)\,P$ loads, the following apply:

Values of $(n-1)\,P$	Corresponding values of n	Values of $\frac{y_m}{h}$
P	2	1
$2\,P$	3	$\frac{8}{9}$
$3\,P$	4	$\frac{5}{6}$
$4\,P$	5	$\frac{4}{5}$
$5\,P$	6	$\frac{7}{9}$
..	..	..
..	..	..
..	..	..
$n\,P$	..	$\frac{2}{3}$

in which,

$$h = y_c = \frac{(f + f')}{8} \qquad (81)$$

When $\frac{n}{2}$ loads P are acting at a distance $\frac{2\,L}{n}$ from each other, starting at $\frac{L}{n}$ from Points A and B (Fig. 34(c))—so that $x_1 = \frac{L}{n}$; $x_2 = \frac{3\,L}{n}$; $x_3 = \frac{5\,L}{n} \cdots$; and, $x_1' = (n-1)\,\frac{L}{n}$; $x_2' = (n-3)\,L\,n$, Equation (79) yields,

$$y_m = \frac{2\,(n^2 + 2)\,h}{3\,n^2} \qquad (82)$$

For $\frac{P\,n}{2}$ loads, the following apply,

Values of $\frac{P\,n}{2}$	Corresponding values of n	Values of $\frac{y_m}{h}$
P	2	1
$2\,P$	4	$\frac{3}{4}$
$3\,P$	6	$\frac{19}{27}$
$4\,P$	8	$\frac{11}{16}$
$5\,P$	10	$\frac{17}{25}$
..	..	..
..	..	..
..	..	..
$n\,P$	2×number of loads	$\frac{2}{3}$

in which, as before, $h = y_c$, is given by Equation (81).

Slope and Intercepts at A and B of Tangents.—Differentiating Equation (76), or reading directly from Fig. 2(c) considered as the area of the reduced bending moments, the slope of a tangent to the influence line at x from Point A, and at $x' = L - x$ from Point B is given by:

$$\theta_x = \frac{dy}{dx} = \left(\frac{2\,x}{L^2} - \frac{3\,x^2}{L^3}\right) f + \left(\frac{1}{L} - \frac{4\,x}{L^2} + \frac{3\,x^2}{L^3}\right) f'$$

$$= \left[x\,f\,(2\,x' - x) - x'\,f'\,(2\,x - x')\right] \frac{1}{L^3} \quad \ldots\ldots\ldots\ldots (83)$$

Therefore, at the center and at the third-points, respectively:

$$\theta_c = \frac{(f - f')}{4\,L} \quad \ldots\ldots\ldots\ldots (84a)$$

$$\theta_{\frac{L}{3}} = \frac{f}{3\,L} \quad \ldots\ldots\ldots\ldots (84b)$$

and,

$$\theta_{0.67\,L} = -\frac{f'}{3\,L} \quad \ldots\ldots\ldots\ldots (84c)$$

Equations (84) show that the slope of the tangent at the center of the influence line is one-fourth the difference of the slopes of the tangents at Points B and A; at the third-points, it is one-third of the reversed slope at Points B and A, respectively.

The intercepts, f, at Point A and f' at Point B of a tangent at x, equivalent to statical moment of reduced moment area, are:

$$f_x = \left[x^2\,(x - x')\,f + 2\,x^2\,x'\,f'\right] \frac{1}{L^3} \quad \ldots\ldots\ldots\ldots (85a)$$

and,

$$f_x' = \left[2\,x\,(x')^2 f + (x')^2\,(x' - x)\,f'\right] \frac{1}{L^3} \quad \ldots\ldots\ldots\ldots (85b)$$

Therefore, for tangents at the center and at the third-points, respectively,

$$f_c = \frac{f'}{4} \qquad \text{(86a)}$$

$$f_c' = \frac{f}{4} \qquad \text{(86b)}$$

$$f_{0.33L} = \frac{(4f' - f)}{27} \qquad \text{(87a)}$$

$$f_{0.33L}' = \frac{(8f + 4f')}{27} \qquad \text{(87b)}$$

$$f_{0.67L} = \frac{(8f' + 4f)}{27} \qquad \text{(88a)}$$

and,

$$f_{0.67L}' = \frac{(4f - f')}{27} \qquad \text{(88b)}$$

According to Equations (86) the tangent at the center of continuous influence curves crosses ends at one-fourth the distance to the opposite intercept of the end tangent. This leads to a most simple method of designing influence lines the end tangents of which are known (see Fig. 35).

Radius, r, of Influence Line.—Differentiating Equation (83) the curvature at Point x equals,

$$\frac{1}{r} = \frac{d^2 y}{dx^2} = 2\left[(x' - 2x) f + (x - 2x') f'\right] \frac{1}{L^3} \qquad \text{(89)}$$

This makes the radius, r, at several values of x, equal to:

Values of x	Corresponding values of r
0	$-\frac{L^2}{4f' - 2f}$
$\frac{L}{3}$	$-\frac{L^2}{2f'}$
$\frac{L}{2}$	$-\frac{L^2}{f + f'}$
$\frac{2L}{3}$	$-\frac{L^2}{2f}$
L	$-\frac{L^2}{4f - 2f'}$

The radius at the center of the influence curves is equal to L^2 divided by the sum of the end intercepts, $f + f'$, of the end tangents; and the radius at the third-points is equal to $\frac{L^2}{2}$ divided by the far intercept.

Inflection Points.—According to Equation (89) the curvature, $\frac{1}{r}$, is zero at a distance, $x = e$, from Point A. Therefore, the point of inflection is expressed by,

$$e = \frac{L\,(2\,f' - f)}{3\,(f' - f)} = \frac{L\,(2 - m)}{3\,(1 - m)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(90)$$

When Beam $A\,B$ is fixed at Point B, $m = 0$ and $e = \frac{2\,L}{3}$; when it is fixed at Point A: $e = \frac{L}{3}$. Graphically, e is at the intersection with $A\,B$ of the extended straight line connecting the third-points of the end tangents (see Fig. 32). Knowing ordinates y and y' at the third-points,

$$e = \frac{L\,(5\,y - 4\,y')}{9\,(y - y')} \quad\ldots\ldots\ldots\ldots\ldots\ldots(91a)$$

and, knowing $h = y_c$, and $m = \frac{f}{(8\,h - f)}$:

$$e = \frac{L\,(16\,h - 3\,f)}{3\,(8\,h - 2\,f)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(91b)$$

It is of interest to remember that in classical statics, $e = \frac{L\,M_A}{(M_A - M_B)}$; $m = \frac{f}{f'} = \frac{(M_A + 2\,M_B)}{(2\,M_A + M_B)}$; and,

$$M_B = M_A\,\frac{(2\,m - 1)}{(2 - m)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(92)$$

in which M_A and M_B are the bending moments at Points A and B, respectively, due to external loads.

Intercepts f and f'.—Frequently, when using tables containing ordinates of influence lines,[35] f and f' must be calculated from such ordinates. Two symmetrical ordinates (y at x and y' at $x' = L - x$) are expressed in terms of f and f' (see Equation (5)) as follows:

$$f = L^2\,\frac{(y'\,x' - y\,x)}{x\,x'\,(x' - x)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(93a)$$

and,

$$f' = L^2\,\frac{(y\,x' - y'\,x)}{x\,x'\,(x' - x)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(93b)$$

In other words, knowing the ordinates at the third-points, $f = \frac{9\,(2\,y' - y)}{2}$; $f' = \frac{9\,(2\,y - y')}{2}$; $m = \frac{(2\,y' - y)}{(2\,y - y')}$; etc.

If the ordinates, $h = y_c$ and y, at any distance, x, from Point A are known; and, if $x' = L - x$ from Point B:

$$f = \frac{(y\,L^3 - 8\,h\,x\,(x')^2)}{x\,x'\,(x - x')} \quad\ldots\ldots\ldots\ldots\ldots\ldots(94a)$$

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[35] "Kontinuierliche Träger," by Gustav Griot, Zurich, Switzerland, 1924.

and,

$$f' = \frac{(8\,h\,x^2\,x' - y\,L^3)}{x\,x'\,(x - x')} \qquad \text{(94b)}$$

If one intercept and the ordinate, h, at the center are known: $f = 8\,h - f'$; and, $f' = 8\,h - f$. For a simple graphical determination of f and f' from two symmetrical ordinates, see Fig. 33.

Mean Ordinates, y_m.—Stresses from distributed loads, W, are $W\,y_m$. The area determined by the influence line and the neutral axis of Beam $A\,B$ between Points A (with $y = 0$) and x, according to Equation (76) is,

$$y_m\,x = \int_0^x y\,dx = x^2\left[x\,(4\,L - 3\,x)\,f + (6\,L^2 - 8\,x\,L + 3\,x^2)\,f'\right]\frac{1}{12\,L^3} \qquad \text{(95)}$$

For loads, W, uniformly distributed from zero to $\frac{L}{2}$:

$$y_m = \frac{(5\,f + 11\,f')}{192} \qquad \text{(96a)}$$

for loads uniformly distributed all over Span L:

$$y_m = \frac{(f + f')}{12} = \frac{2\,h}{3} \qquad \text{(96b)}$$

for a triangular load over the entire length, with zero at Point A and maximum at Point B:

$$y_m = \frac{(12\,f + 8\,f')}{120} \qquad \text{(96c)}$$

For a triangular load over the half span, $\frac{L}{2}$, from zero at Point A to a maximum at the center:

$$y_m = \frac{(9\,f + 16\,f')}{120} \qquad \text{(96d)}$$

and, for a symmetrical triangular load with its maximum intensity at the center:

$$y_m = \frac{5\,(f + f')}{48} \qquad \text{(96e)}$$

As an example, let Beam $A\,B$ be fixed at both ends, and loaded by a triangular load, W, with its maximum intensity at the center. Then, $f' = L$ for M_A; $f = 0$; and, therefore, $y_m = \frac{5\,L}{48}$ and $M_A = W\,y_m = \frac{5\,L\,W}{48}$.

Flat Arches.—When values of $\frac{f}{L} = \frac{f'}{L}$ are relatively small the influence line becomes similar to a common parabola and a circular arch, the equation of which is,

$$y = \frac{4\,x\,x'\,h}{L^2} = \frac{x\,x'\,f}{L^2} \qquad (97)$$

Likewise, all other relations become simple; that is, intercepts at Points A and B of tangents expressed by Equations (85) become: $f_x = \frac{x^2}{L^2}\,f$; and, $f_x' = \frac{(x')^2}{L^2}\,f$ and, according to Equation (89), $r = \frac{L^2}{8\,h}$, or $h = \frac{L^2}{8\,r}$. A convenient formula for determining the approximate length, l, of the flat arch is:

$$l = L\sqrt{1 + \frac{16\,h^2}{3\,L^2}} \qquad (98)$$

Graphical Design of General Influence Lines.—Of the many graphical methods of designing influence lines suggested by the foregoing equations, the following seem to be most efficient; they may be useful, also, for any other application in engineering or architecture where two tangents at given points A and B, are to be joined by a continuous curve.

Influence Lines by Tangents in Centers (Fig. 35).—Draw vertical lines at the three-quarter points (Fig. 35) and obtain $A\,A'' = \frac{f'}{4}$, and $B\,B' = \frac{f}{4}$; then, the straight line, $A'\,B'$, is tangent to the influence line at the center of $A\,B$ (see Equations (86)). Similarly, drawing the vertical lines at $\frac{L}{8}$, the tangents, $A''\,C'''$ at $\frac{L}{4}$ and $C''\,B''$ at $\frac{3\,L}{4}$, are found for Branches $A\,C'$ and $C'\,B$, respectively, with Line $A'\,B'$ forming the end tangent at Point C'. Where these five tangents are not sufficient to determine the inscribed influence line, further divisions into lengths of $\frac{L}{16}$ might be made, with the tangents at Point A, $\frac{L}{4}$, $\frac{L}{2}$, $\frac{3\,L}{4}$, and Point B serving as end tangents for each of the four parts of lengths, $\frac{L}{4}$. This method is recommended where all the influence line must be drawn.

Influence Lines by Symmetrical Ordinates (Fig. 33).—To determine the exact ordinates at any distance from Point A (for example, at Points D and E, Fig. 33), Equations (5) and (80) are used. Points D and E are projected to the end tangents and Points $A\,A' = D\,D' = \frac{f'\,x}{L}$; and, $B\,B' = E\,E' = \frac{f\,x}{L}$. Point D'' is plotted on Line $A'\,B$ and Point E'' on Line $A\,B'$, making $A\,A'' = D\,D''$ and $B\,B'' = E\,E''$. The straight line, $A''\,B''$, then determines, at Points D and E, the two points of the influence line, D''' and E'''.

On the other hand, knowing any two symmetrical ordinates, y and y', the end tangents may be obtained by reversing the process (see Fig. 33). The straight line, $A'\,B'$ (Fig. 33), crosses Line $A\,B$ at the intersection of the influence line.

Influence Lines of Bending Moments, Reactions, and Shear.—As shown in Figs. 2, 29, and 30, one span of the structure at a time is being deformed from within, in accordance with the stress for which the influence line is wanted; all other spans are deformed through rotation at the ends, and form general influence lines as analyzed herein. The one span having a special influence line is analyzed by considering it free at the ends and then deducting the effect of the continuity.

Unit Displacement in Simple Beam $A\ B$.—A unit angular displacement or rotation at Point E, distant a from Point A and $a' = L - a$ from Point B, breaks the beam at Point E (Fig. 36(a)) so that $\frac{A\ A'}{A\ E} = 1$. Therefore, $A\ A' = A\ E = a$; and, $A\ E'\ B$ is the influence line of the bending moments, M_E.[36]

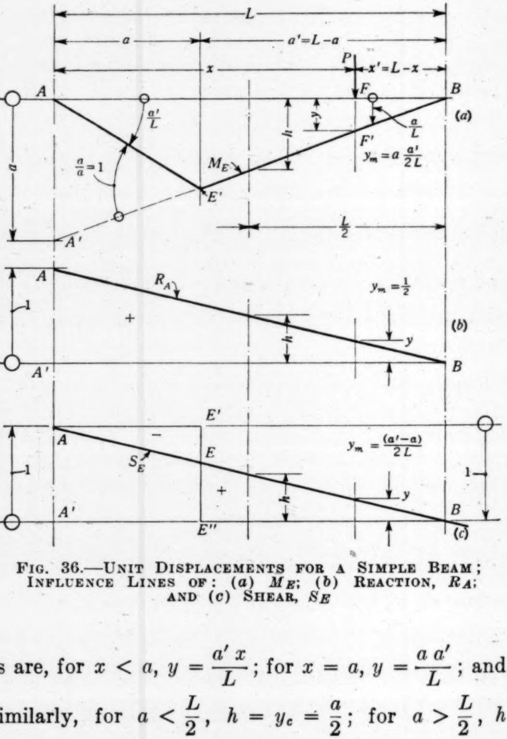

FIG. 36.—UNIT DISPLACEMENTS FOR A SIMPLE BEAM; INFLUENCE LINES OF: (a) M_E; (b) REACTION, R_A; AND (c) SHEAR, S_E

The ordinates are, for $x < a$, $y = \frac{a'\,x}{L}$; for $x = a$, $y = \frac{a\,a'}{L}$; and, for $x > a$, $y = \frac{a\,x'}{L}$. Similarly, for $a < \frac{L}{2}$, $h = y_c = \frac{a}{2}$; for $a > \frac{L}{2}$, $h = \frac{a'}{2}$; and $y_m = \frac{a\,a'}{2\,L}$.

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[36] "Table of Moment Coefficients for One-Span Beams; Simple or Fixed," by S. M. Cotten, *Engineering News-Record,* September 27, 1917, p. 613.

When Point A is translated one unit (Fig. 36(b)) so that $A' A = 1$, Line $A B$ is the influence line of reaction, R_A, at Point A, so that $R_A = P y$, in which $y = \frac{x'}{L}$; $h = \frac{1}{2}$; and, $y_m = \frac{1}{2}$.

A normal unit displacement at Point E will break the beam so that $E' E'' = 1$. The same effect is obtained by a unit settlement, $A' A$ (Fig. 36(c)). Line $A B$ is the influence line for shear, with the reference axis at $A E'$ from Points A to E and at $E'' B$ from Points E to B. The shear is $S_E = P y$, in which the ordinates are, for $x < a$, $y = -\frac{x}{L}$; and, for $x > a$, $y = \frac{x'}{L}$. Likewise, for $a < \frac{L}{2}$, $h = \frac{1}{2}$; and, for $a > \frac{L}{2}$, $h = -\frac{1}{2}$; and $y_m = \frac{(a' - a)}{2 L}$.

It is to be noted that even in the case of the simple beam the influence lines may be applied to advantage, in comparison with purely algebraic methods, especially in solving for bending moments for great variety of loads.

Influence Lines in Restrained Beams.—When Beam $A B$ is part of a continuous structure (Figs. 1 and 29), the rotation at the ends caused by interior displacements (see Fig. 36), will be resisted by adjoining members, making the influence line, $A B$, the sum of one of a simple beam and a general influence line from restraint at the ends. The corresponding equations of ordinates are indicated in Table 2(c), 2(d), 2(e), and 2(f).

Acknowledgments.—The writer wishes to thank the discussers for their interesting comment and (once again) Bernard L. Weiner, Assoc. M. Am. Soc. C. E., whose suggestions have been invaluable, and due to whose unselfish enthusiasm the underlying worth of the paper has been revealed and emphasized.

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HYDRAULIC TESTS ON THE SPILLWAY OF THE MADDEN DAM

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WITH DISCUSSION BY MESSRS. ETTORE SCIMEMI, J. C. STEVENS, F. W. EDWARDS, MORROUGH P. O'BRIEN, AND RICHARD R. RANDOLPH, JR.

SYNOPSIS

Several interesting and unique hydraulic tests have been made in connection with the Madden Dam project located on the Chagres River, in the Isthmus of Panama. A number of features of the design of the spillway section, but principally a plan for dissipating the energy of the overflow at the toe of the dam, were developed from model studies. These studies are described in Part I.[2] In order to check the resulting predictions, apparatus as nearly similar as possible to that used on the models, was installed in the prototype spillway at apportionate positions, as described in Part II.

Field tests other than those on the spillway have also been made.[3] The discharge and loss in head through the needle-valve outlet conduits and the sluice-ways were measured. The efficacy of the air vents for these conduits was determined by measuring the quantity of air flowing through them at various heads and gate-openings.

PART I.—MODEL TESTS

THE PROJECT

Madden Dam is a part of the Panama Canal project, being built primarily to create a reservoir for storing water to use for lock operation and for controlling floods. Power development is small and more or less incidental. The dam is on the same river as that which forms Gatun Lake and is situated about 10 miles up stream from the point where the river flows into the canal.

NOTE.—Published in May, 1937, *Proceedings*.

[1] With U. S. Bureau of Reclamation, Denver, Colo.; formerly Associate Engr., The Panama Canal, Balboa, Canal Zone.

[2] Condensation of a report by the writer to the Governor of The Panama Canal on Madden Dam Spillway Model Tests, dated February 1, 1932.

[3] *The Military Engineer*, Vol. XXVIII, No. 162, November-December, 1936, p. 438.

The principal features of the Madden Dam project consist of the main concrete dam across the Chagres River, the power plant, an earth and gravel-fill dam continuing from the left abutment of the main dam; and several earth and gravel-fill dams across saddles around the rim of the reservoir. The main dam is of the massive concrete straight-gravity type, consisting of an overflow spillway section across the river, with abutment sections on either side (see Figs. 12 and 13). It is about 974 ft long at the top and about 220 ft high at the maximum section from the lowest point of the foundation to the top of the roadway. The spillway is divided into four 100-ft openings by three 12-ft concrete piers. Structural steel drum-gates, 18 ft high, are installed on the concrete crests in these openings.

The flood water passing over the spillway crest has a drop of about 173 ft to the toe at the maximum expected discharge of 260 000 cu ft per sec. The energy of this overflow will be more than 5 000 000 hp, and the velocity at the toe will be about 100 ft per sec. With this energy exerted at the base of the dam it was considered advisable to devise means of dissipation so that the velocity would be checked enough to eliminate erosion of the foundation rock.

THE MODEL

During the summer and fall of 1931, a model of Madden Dam was built and tested at the Hydraulic Laboratory of the Colorado Agricultural College, at Fort Collins, Colo.[4] The model of the spillway was built on a scale of 1:72, or 1 in. (linear dimension) on the model equal to 6 ft on the prototype. The spillway section, with drum-gates installed in the crest, was reproduced accurately to scale. The power house was represented with the needle-valves discharging through the down-stream wall. The contours of the river bed below the dam were reproduced from surveys to represent the original rock and overlying earth and gravel deposits. (Described in terms of equivalent prototype dimensions, this model had four 100-ft gate-openings, a spillway crest at Elevation 232.0, and an apron width of 440 ft between side walls. The radius of the bucket is 60 ft. For a flow of 260 000 cu ft per sec, the tail-water depth is approximately 80 ft.)

MODEL TESTS

Dentated Sill Apron.—In the original proposed design of the spillway a concrete apron, extending about 120 ft down stream from the toe of the dam with a dentated sill developed by Professor Theodor Rehbock[5] at the end, was proposed. This apron was level and at a low elevation in order to follow the average rock level.

Tests on a model of this set-up showed that the sheet of water flowing down the spillway cut through the tail-water, which was of considerable depth, at all stages of flow, with very little surface disturbance or loss in velocity, and struck the dentated sill with terrific force. Fig. 1 shows a cross-section along this apron (through the dentate) with the maximum expected discharge. Velocity measurements were taken at various points with a Pitot tube, and lines of equal velocity, converted to prototype scale, have been drawn representing contours. The high-velocity stream is depressed by the back roll of water above the sill and elevated by the roller below the sill. This latter action, of course, relieves the river bed of erosion velocities. The upward deflected jet continues along the surface, however, at comparatively high velocities and erodes the river banks.

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[4] *Engineering News-Record,* July 14, 1932, p. 42.

[5] *Transactions,* Am. Soc. C. E., Vol. 93 (1929), p. 527.

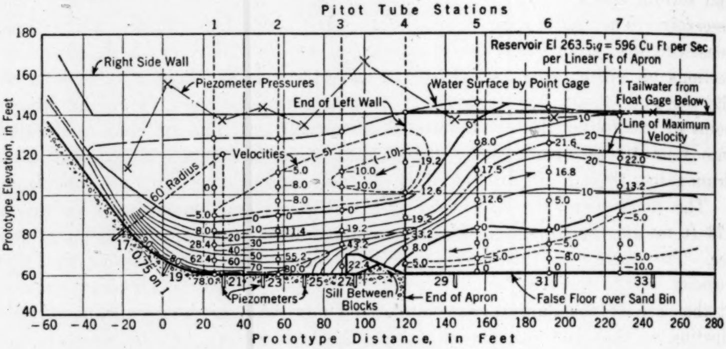

FIG. 1.—VELOCITY CONTOURS ALONG A SECTION THROUGH THE DENTALS; ORIGINAL MODEL DESIGN

The next step in the experiments was to substitute a sill with a solid vertical face for the dentated form. Various heights of this solid sill were tried, and it was found that a height of 6 ft was just as effective in preventing erosion below the apron as the dentated sill 10 ft high. However, the surface flow conditions were unstable, the boil above the solid sill being much more irregular and fluctuating. The flow allowed through the dentates seems to stabilize this boiling, and there is little or no pulsation or surging in the tail-water, making the performance of the dentated sill more satisfactory.

It would seem that a sill under these conditions should be made only high enough to cover the range of the high velocities of the overflow jet along the bottom, and, consequently, its height would vary inversely with that of the dam, because the higher the dam the thinner the overflow sheet will be at the toe. For Madden Dam a satisfactory sill was one-thirtieth of the total height of the dam. The height of the sill varies directly with the depth of water discharging over the crest. In the present case the sill was one-fifth of the expected maximum depth. Another factor, of less importance, is the depth of water that is to be flowing over the sill. The height of the sill is modified directly by the depth of the tail-water because the greater this depth, the thicker the overflowing jet will become in passing through it, and a higher sill is necessitated to cover the range of the thickened jet. A sill, or a series of blocks with no tail-water over them, is practically useless as the jet is merely sprayed into the air almost as high as the original water surface, and falls to the river bed again, gaining nearly all its original velocity. There must be tail-water to float it away, so to speak, after the jet has been deflected from the river bed. Even then, the high velocity is merely transferred to the surface and may erode the banks, and is undesirable.

If any type of block sill is used on this apron, entire dependence is placed on it for stopping the velocity, and a small break starting anywhere would endanger the entire structure. Logs coming over the spillway would be carried down by the high-velocity stream to strike directly against the sill. The proposed dentated sill was 10 ft high and 30 ft wide, involving a large quantity of concrete and form work in its construction. The anchorage would have been difficult and expensive. Furthermore, a royalty would have had to be paid for using this type of sill. With these inherent disadvantages to the use of a block sill, investigation of other devices for dissipating the energy of the overflow was started.

HYDRAULIC-JUMP APRON

Level Apron.—The feasibility of using the phenomenon of the hydraulic jump for energy dissipation was studied first. The dentated sill was removed from the apron. As stated before, this apron was low in the tail-water, being at Elevation 60, and the depths over it at all stages of discharge were apparently too great to allow the hydraulic jump to form, and high velocities persisted along the bottom for a considerable distance down stream. This can be seen in Fig. 2 which shows the distribution of velocities when the tail-water is at its natural depth.

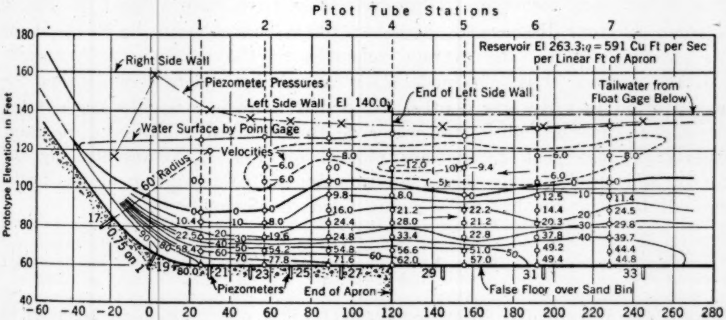

FIG. 2.—VELOCITY CONTOURS FOR FLOW OVER LEVEL APRON, WITH DEEP OR NATURAL TAIL-WATER

It was found that if the tail-water was lowered, the flow over the crest of the dam being kept constant, a certain point was reached at which the surface of the water became very turbulent and rose more or less abruptly from the toe of the dam, thence flowing off in a quiescent state. In other words, an apparently good hydraulic jump formation was taking place.

The tail-water was adjusted to force the beginning of this jump as far as possible back on the toe of the dam without "drowning" it. This adjustment was made for several flows, and the required tail-water noted. A plot of these elevations against the discharge is shown in Fig. 3 as circle points. A check made with a Pitot tube showed that such adjustments of the tail-water were producing the maximum retardation of the velocities along the bottom. A Pitot tube was placed, and remained fixed, at a position within the jump and just off the floor of the apron. The tail-water was then varied and the resultant velocities, as indicated by the Pitot tube, were recorded. These velocities have been plotted in Fig. 4 for three different discharges. Although the velocities from the Pitot tube in this position are not the average velocities of the sections, it is thought that the curves thus obtained, show a relative energy dissipation for the variation in tail-water depths. Note that there is a decided peak in the curves that gives a definite tail-water depth to obtain a minimum resultant velocity. These peak points are plotted on Fig. 3, represented by the crosses, and check closely the first adjustments. Computation of the depths required to form the hydraulic jump, from the formula,

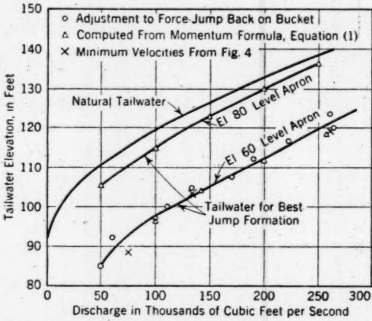

FIG. 3.—TAIL-WATER DEPTHS REQUIRED TO CAUSE THE BEST HYDRAULIC JUMP PERFORMANCE ON LEVEL APRONS AT DIFFERENT ELEVATIONS

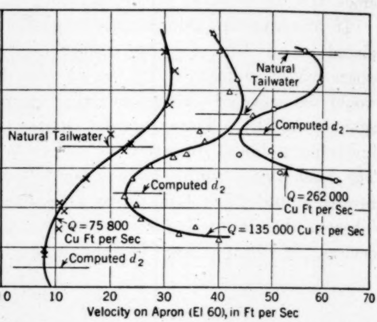

FIG. 4.—VELOCITIES AT VARIOUS DEPTHS OF TAIL-WATER FOR THE SAME FLOW ON A LEVEL APRON

$$d_2 = \sqrt{\frac{d^2_1}{4} + \frac{2V^2_1 d_1}{g}} - \frac{d_1}{2} \quad \dots\dots\dots\dots\dots\dots (1)$$

shows close agreement. These computed points are plotted as triangles in Fig. 3; V_1, or the average velocity before the jump, was obtained from Pitot measurements, and,

$$d_1 = \frac{q}{V_1} \quad \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

was computed, in which $q = C d_c^{\frac{3}{2}}$. (In the notation of this paper an effort has been made to conform with "Symbols for Hydraulics,"[6] compiled by a

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[6] A S A—Z10b—1929.

Committee of the American Standards Association, with Society representation, and approved by the Association in 1929).

It is seen, then, that the natural tail-water at the dam site affords too great a depth over this apron to allow the jump to form. In fact, at natural tail-water depths, the velocities are about as great as can exist along the bottom, as may be seen in Fig. 4. This is contrary to a somewhat general acceptance that a deep tail-water is an assurance for the dissipation of the overflow velocity. With the natural tail-water lowered to allow the hydraulic jump formation, the rate of velocity retardation is much faster and the final velocity, after the jump has been fully completed, is decidedly less than occurs at the same distance down stream under high tail-water. This may be seen by comparing Figs. 2 and 5. Unfortunately, the section does not extend far enough down stream, in Fig. 5, to show this final point of complete recovery of the jump where the velocity has been reduced to a minimum and is practically constant across the section.

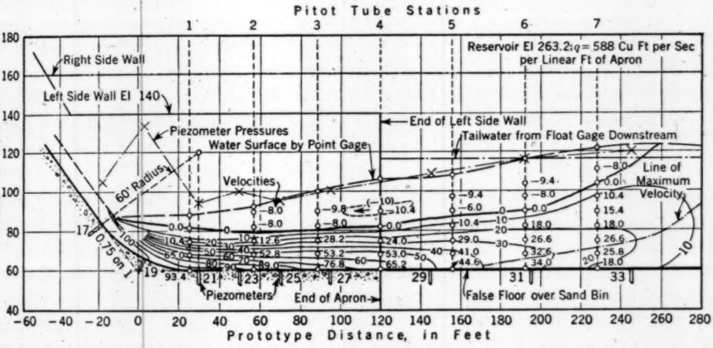


FIG. 5.—VELOCITY CONTOURS FOR FLOW OVER A LEVEL APRON, WITH TAIL-WATER LOWERED TO PERMIT THE FORMATION OF AN HYDRAULIC JUMP

The surface rise in the jump for a flow of this magnitude is very gradual, and the jump action extends for a considerable distance down stream, although the point of beginning is forced back on the toe of the dam. A back flow or roller occurs along the rise. The higher velocities hug closely to the floor until gradually retarded. The flow lines shown in Fig. 5 are typical although for smaller discharges the surface rise is more abrupt and, consequently, the length of the jump is shorter.

The depths required for jump formation, obtained by lowering the tail-water, are considerably less than the actual existent depths over an apron placed at this low level of Elevation 60. Since the natural tail-water at the dam site is fixed, it was necessary to raise the apron to a higher elevation in order to lessen the depths over it so that the jump would not be "drowned out." This alteration was made to the model, the apron being raised 20 ft to Elevation 80 and the foregoing tests were repeated. The jump formed well up on this apron at all flows although indications were that it could be raised 2 or 3 ft higher, which would cause a closer coincidence of the natural tail-water depths to those required to produce the best hydraulic jump. This is illustrated in Fig. 3.

Sloped Apron.—Although a close coincidence may be obtained there would be only one flow, theoretically, where the natural tail-water depth over the level apron would give exactly the depth required for the jump. It was thought that, if the apron was inclined, the jump could move up or down the slope until the depth required was encountered, regardless of the variations in the tail-water. Furthermore, a considerable saving in concrete would result in this case because the sloped apron would more nearly follow the original rock contour.

From the aforementioned experiments of the hydraulic jump on the level apron, it was found that the depths which caused the minimum floor velocities agreed closely with the theoretical depths for the jump formation as computed by the momentum formula. For the sloped apron the computation becomes involved, due to the necessity of considering the gravity component of the water in the jump and nothing satisfactory could be developed from attempts to adapt the formula to this condition. A formula that included this weight component would show that a greater depth is required for the jump, which simply means that it would form farther down the slope where this greater depth is encountered. Even with this knowledge there is no method of predicting a satisfactory tail-water elevation, one that would start the jump well up on the apron at any flow. The difficulty lies in the fact that the datum plane is indeterminate, above which the required vertical depths are to be assumed.

The theoretical position of the jump can be computed but it would not be known whether this position would start the jump well back against the face of the dam and yet not so far back as to "drown it out." Unless the jump starts well up on the apron, full use is not made of the apron length. The calculated position might start the jump some distance down the slope away from the face of the dam and, therefore, for equal protection this length must be added to the pavement down stream.

Apparently, the solution was a case of experimental "cut and try" and a series of tests was made on aprons of various elevations and slopes. Fig. 6 shows tests on aprons of two different slopes. The apron first tested (Apron *A*) had a slope of 1 on 6.6, beginning at Elevation 84. The tail-water below this apron was adjusted so that the jump formed well up on the apron, starting back against the face of the dam. This condition was the first desired prerequisite. It is seen, however, that the required jump depths are less than the actual depths resulting from natural tail-water, indicating that the apron should be raised. This was done until it was found that an apron beginning at about Elevation 97 gave the best average results. The tail-water over this apron was adjusted until the jump was forced back against the face of the dam, as was the first procedure in the tests on each apron. A check was then made of this adjustment by measuring the velocities under various depths of tail-water.

A Pitot tube was placed, and remained fixed, at a point just off the floor and near the end of the apron. The tail-water was raised and lowered until a depth was found which caused the least velocity to be registered.

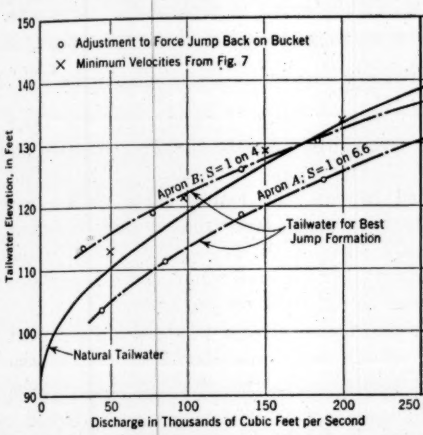


FIG. 6.—TAIL-WATER DEPTHS REQUIRED TO CAUSE THE BEST HYDRAULIC JUMP FORMATION ON SLOPED APRONS AT DIFFERENT ELEVATIONS

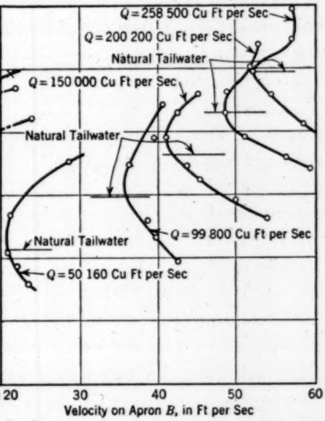


FIG. 7.—VELOCITIES AT VARIOUS DEPTHS OF TAIL-WATER FOR THE SAME FLOW ON A SLOPED APRON

The resulting velocity curves, obtained from the Pitot tube readings, are shown in Fig. 7. The peak points from these curves, indicating the tail-water required for minimum velocities, check closely the tail-water adjustments first made. The latter have been plotted, in Fig. 6, against the discharge to show the range of depths required for the best hydraulic jump formation.

Different slopes were tried, but apparently a slope of 1 on 4 was about as steep as could be used and still retain a good jump formation. On steeper slopes the overflowing jet cut under the tail-water with little surface disturbance or consequent loss of energy. From a study of the curves shown in Fig. 6, indications are that if Apron *A* is raised, a closer average coincidence of the jump depths with the natural tail-water would be obtained than in the case with the raised Apron *B*. This leads to the thought that the shape of the jump curve may be varied by changing the slope of the apron. This curve could then be made to conform closely with any given natural tail-water curve by adopting a proper slope to the apron.

The steepest apron that could be used would result in the greatest economy in this case and since it was found that the depths of the jump over Apron *B*, beginning at Elevation 97.3, averaged closely with the actual tail-water depths, this apron was adopted as satisfactory for the final design of the model. From later studies of the results of these tests, the final construction plans were changed to lengthen the spillway apron by 30 ft and to lower the elevation of beginning of the slope 3 ft.

Both these alterations were seen to be beneficial. The jump curve for Apron *B* shown in Fig. 6 lies above the natural tail-water rating curve at the lower discharges, indicating that the natural depths are not quite sufficient to force the jump well up on the apron at these flows. It may be supposed that, since the tail-water depth is fixed by the rating curve, if the apron is lowered the depth over it is increased and more nearly approaches that required for the jump.

Lowering the apron will improve the jump formation at the lower stages of flow, which occur more frequently. At the higher discharges the required depth of jump will fall slightly below the natural tail-water, but a factor of safety is so obtained in case the rating curve (which was extrapolated for higher flows) is less, or in case the natural tail-water is lowered at the dam site, by the erosion of the gravel beds down stream. Incidentally, the model tests showed that the tail-water could be lowered as much as 20 ft without the jump being swept off the apron.

Extending the apron 30 ft farther down stream would furnish greater protection to the river bed and, as the slope was also flattened, the concrete saved at the upper end was used to make the extension down stream. The apron as built at Madden Dam had a slope of 1 on 4.3, beginning at a point tangent to the bucket (radius, 60 ft) at Elevation 94.3 and extending from this point 158.15 ft down stream with a sill 5.7 ft high, having a sloping face at the end.

Deflector Sill.—At all but the smallest flows the length of the hydraulic jump, or the point down stream where full recovery takes place, is beyond the end of the proposed apron. Theoretically the apron or protective pavement should extend down stream to a point where the bottom velocities have been reduced by the jump to an extent which will not erode the foundation rock. It was found from the tests that a small turn-up sill at the end of the apron would lift the velocities off the river bed below, but would not destroy the functioning of the jump. The thought was that if the sill was too high, impact would result and the stream would be deflected upward, thus "blocking out" the static pressure of the tail-water necessary for the jump. The tail-water might even be swept out between the toe of the dam and the sill. The sill should act only as a deflector, to lift the sheet of water just high enough to clear the bottom, allowing it to continue down stream where the velocities are dissipated in the normal action of the jump.

Expected Jump Performance.—The action of this sill and the functioning of the hydraulic jump on a sloped apron are shown in Fig. 8, which is a composite of some of the results obtained from the model tests on the final design of the sloped apron, under natural tail-water conditions. Water-surface curves, taken by point-gage measurements, are drawn for five different rates of flow, covering the entire range of discharge. Velocities through each cross-section were measured with a Pitot tube. Velocity contours for the lowest flow only (but which are typical) have been drawn in Fig. 8. Energy gradients, representing the maximum velocity heads above the water surface at various stations, are shown for each discharge. The loss in energy through the jump is illustrated graphically, being greater and quicker for the smaller flows. Velocities in the overflowing sheets just before they strike the tail-water were taken with a Pitot tube and were found to agree closely with Equation (2) when d_1 was measured with a point gage. These velocities, especially at the higher flows, amounted to almost the theoretical values since the surfaces of the model were painted and sand-papered to a very smooth finish. On the prototype structure greater loss in friction may be expected. Provision was made at Madden Dam to measure this loss by installing Pitot tubes on the spillway toe. A factor of safety is obtained, however, by keeping the friction losses on the model low.

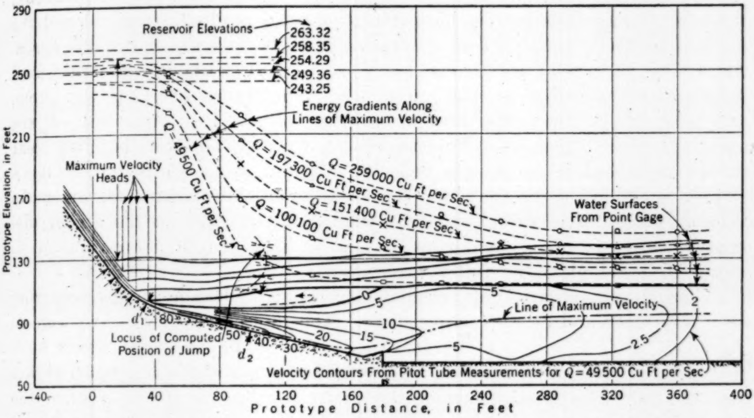


FIG. 8.—HYDRAULIC RELATIONS OF THE FLOW OVER A MODEL OF THE MADDEN SPILLWAY, REPRESENTING NATURAL CONDITIONS FOR THE ENTIRE RANGE OF DISCHARGE

The velocity, V_1, and the depth, d_1, before the jump being measured in each case, the depth required for the hydraulic jump was computed from Equation (1). These depths, or d_2, have been plotted, in Fig. 8, parallel to the slope of the apron and at the points where they are crossed by the tail-water elevations, projected up stream, a line is drawn as the locus of the theoretical position of the jump. It is perhaps significant that these computed positions lie fairly close together near the center of the apron.

As on a level apron, the highest velocities lie along the floor, becoming gradually less toward the top with a reverse flow or back roll on the surface (see Fig. 5). At the beginning of the jump, pressures on the apron, as recorded from piezometers, lie slightly below the water surface, measured with a point gage. This is probably due to the air content of the water. Pressure recorded on the up-stream face of the sill is little greater than the static pressure which indicates only slight impact. Pressure recorded in the curve of the bucket at the toe of the dam is higher than the static depth of water, which may be attributed to the centrifugal force of the stream.

Retardation of the velocity of the overflowing sheet is not instantaneous; the highest velocities persist along the floor until gradually reduced and the jump is not fully recovered until some distance beyond the end of the apron. At a flow of 49 500 cu ft per sec (see Fig 8), although the measured velocity of the water at the toe of the dam before entering the jump, was 89.3 ft per sec, the highest velocity obtained at the end of the apron was 16.8 ft per sec. This remaining velocity, however, is lifted clear of the river bed below by the deflector sill and about 40 ft farther down stream, at which point the jump action is practically completed, the mean velocity of the cross-section was less than 7 ft per sec. At the higher discharges the action is analogous but a greater length down stream is required to complete the full recovery.

There is a similarity of action between the hydraulic jumps occurring on a level floor and on an inclined floor. The rate of velocity retardation on Apron *B*, Fig. 6, seems to be as fast as that along the level apron until the end of the slope is reached, after which the retardation is much slower and the velocities persist farther down stream than is the case when the jump occurs on a level apron. Apparently, the stream is taken down to depths too great to allow the jump to form completely. The steeper the slope of the apron the less perfect is the jump formation. The slope of 1 on 4, adopted here for concrete economy, is about the steepest that can be used and still obtain a satisfactory jump formation.

The jump performance, as discussed in connection with Fig. 8, is typical for the total range of discharge over the spillway. At any flow the beginning of the jump is forced back against the face of the dam and although the distance down stream required for full recovery increases with the discharge and extends beyond the end of the apron, the bottom velocities are lifted free from the foundation rock by the deflector sill so that no erosion results. It was found to be feasible, therefore, to shorten the length of the theoretical spillway apron considerably by placing a sill of the proper dimensions at the end. This apron need only cover the start of the hydraulic jump where the first impact and consequent turmoil occur.

Fig. 9 shows the characteristic relations of the hydraulic jump when it is made to form on a sloped apron at the toe of an over-flow dam. The curves have been obtained from results of these model studies, but it is believed they can be applied, for design approximations, to any installation roughly similar to Madden Dam. The functions represented by the curves are directly proportional to the depth over the crest, and the height of the fall, and the values may be used for any dam where the ratio of these two factors is known.

A datum line is assumed, usually at low flow, or no flow, tail-water elevation. A sufficient length of the apron must be above this elevation in order that the beginning of the jump (which moves up stream at each increase in discharge) may occur on a flat slope. A good jump formation will obtain with the point of beginning on the curved surface of the bucket, but not if it is forced back on the steep slope of the down-stream face of the dam. The jump should begin as far back as possible, however, in order to utilize the full length of the apron. For the maximum expected flow it should begin at the point of curvature of the spillway bucket from the face of the dam.

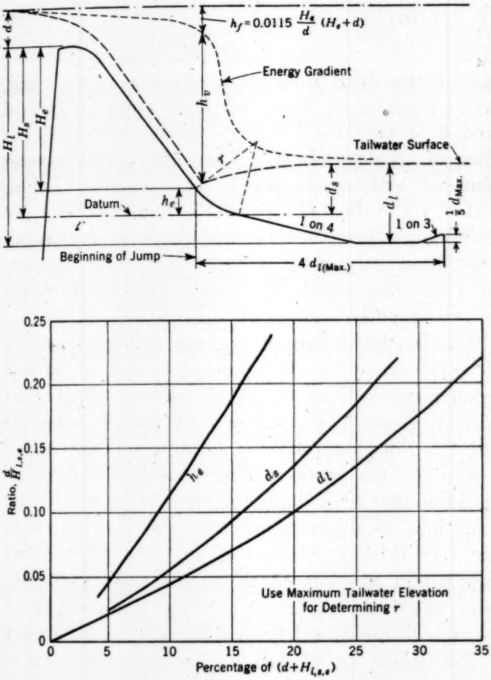
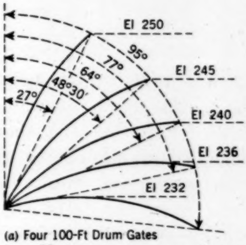


FIG. 9.—HYDRAULIC RELATIONS REQUIRED TO HOLD JUMP FORMATIONS ON THE SLOPED APRON AT TOE OF OVERFLOW DAMS



(a) Four 100-Ft Drum Gates

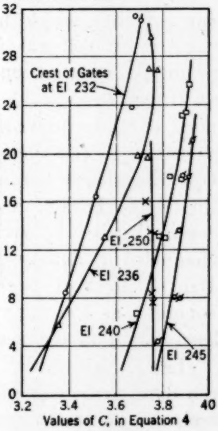


FIG. 10.—COEFFICIENTS OF DISCHARGE OVER CREST, DRUM-GATES AT VARIOUS POSITIONS

The maximum expected tail-water is then used to determine the maximum, d_s, and the maximum, h_e, which locate the point of curvature for the bucket with the radius, r, as shown in Fig. 9. The slope of the apron should not exceed 1 on 4 and should be carried down at least to a depth represented by the function, d_l.

Friction Loss at Toe.—Measurements of the velocity through the overflow jet at the toe of the dam were made with a Pitot tube. They were taken at the same station at the toe, but for various depths of water over the crest. The difference between the indicated mean velocity head from the Pitot tube and the total theoretical velocity head (which is the vertical distance between water surfaces) was taken as the head loss due to friction. The values thus obtained were found to be averaged closely by a curve of the form:

$$h_f = 0.0115 \frac{H}{d_c} (H + d_c) \text{ ....(3)}$$

in which H = the height of the dam, from apex of crest to toe; and, d_c = the depth of water over the crest, from reservoir level to apex of crest. Equation (3) should hold true for friction loss at the toe of any model dam similar to that of Madden Dam. This model was covered with galvanized iron, enameled and sand-papered very smooth. Indications from the tests are that the value, C = 0.0115, increases with the roughness of the surface and would be 0.0135 for models covered with galvanized iron which had not been painted or sand-papered. For concrete dams C = 0.02 was found to average closely the values obtained from the prototype tests, discussed subsequently.

Other Devices.—Variations in the design of the apron were tried but none of them proved successful enough to be incorporated or to warrant a change in the sloped apron layout. Some of the alterations tested were: (*a*) The sloped apron built in steps; (*b*) a channel or pocket below the lip of the bucket cut into the sloped apron; (*c*) a high spillway bucket designed with a vertical drop to the apron paving[7]; and (*d*) transverse channels or dentates cut into the high bucket (later used at Tygart Dam).[8] Most of these alterations were attempted with the idea of saving concrete, as the apron was very thick at the toe of the dam after it had been raised to the necessary height to cause the hydraulic jump formation.

Later, this heavy apron was included in computing the stability of the dam, greatly increasing the factor of safety against earthquake stresses and also against sliding. The simplicity of design, the ruggedness of performance, and the adaptability to conditions at Madden Dam gave preference to the use of the plain sloped apron, with a small deflector sill, to obtain the hydraulic jump; so that investigations of other devices for dissipating the energy of the overflow were discontinued.

OTHER FACTORS OF SPILLWAY DESIGN

Right Bank Excavation.—In addition to the development of the spillway apron, model tests were made to determine other features of the spillway design. The proper treatment of the excavation along the right bank proved difficult of a theoretical analysis. Immediately below the dam site, the river bends abruptly to the left and the original contours of the right bank protruded across the flow from the last spillway gate. The original design contemplated leaving this hillside in place and paving the steep 1 on 1 slope for protection against the spillway flow. The model tests showed that the hillside was too steep for the water to "ride" and, consequently, the flow slipped down into a concentrated jet at the foot of the slope and caused disastrous erosion down stream from the pavement. Alterations were tried in which this slope was made flatter, but little success was attained in attempting to make the water flow in a uniform sheet along a slope of almost any inclination, however slight. A warped surface probably could have been developed that would carry its share of the discharge but still no provision would be made for checking the velocity of the water. A sill placed along the top was not very satisfactory as the tail-water over it became less and less up the slope and the flow was sprayed into the air, only to fall back on the unprotected bank below the apron.

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[7] *Engineering News-Record,* March 14, 1935, p. 376.

[8] *The Military Engineer,* September-October, 1936, p. 331.

It was finally decided that the safest and most positive results would be attained by cutting into the bank and extending the apron across the full width of the spillway. It was found possible to level off gradually and raise the down-stream end of the sloped apron toward this right bank (see Figs. 12 and 13). However, the apron was kept down under the tail-water so that the depth of water over it was sufficient to form the hydraulic jump at any flow.

Tail-Race Walls.—Another problem that arose from the effects of this bend in the river below the dam was the checking of the eddy or reverse flow which naturally formed along the left bank. This eddy is accentuated by the water surface in the hydraulic jump on the spillway apron which is at a lower elevation than the tail-water below the power house. This condition creates a difference in pressure and a cross-flow. The effect is detrimental to the functioning of the jump, causing an unbalanced condition resulting in concentrations of velocity. This reverse flow also tends to deposit sand and gravel in the draft-tube openings and to set up surgings in the tail-race which might hinder the speed regulation of the turbines.

To remedy the condition a division wall was placed down stream from the power house to separate the spillway and turbine tail-race. Theoretically, this wall should be high enough to prevent overflow into the spillway at any discharge and should extend down stream until the water-surface rise in hydraulic jumps levels out. It was found by experimentation that if the height of the wall followed the actual surface rise in the jump, the over-pour on to the top of the jump caused no harm. Accordingly, the wall was built at this point 12 ft below the elevation of the maximum expected tail-water, with a resultant saving in concrete. On the other side of the river, the hydraulic jump is contained by paving along the bank.

Erosion Below Spillway Apron.—Prediction of the nature of the erosion likely to occur below the spillway apron was possible from the model studies. As previously stated, the original gravel deposits in the river bed were represented on the model by building up the contours in sand (see Fig. 1). No attempt was made to graduate this sand to the scale of the model as the objective was to reduce the velocities to a point where no erosion of the foundation rock itself would occur. Thus, erosion tests on the model were qualitative rather than quantitative. It was found that for the smaller flows, up to a quantity representing 50 000 cu ft per sec, the sand contours were scarcely disturbed, but for larger flows the over-burden was washed from the right bank and river bottom below the apron and deposited as a bar across the main river channel. As successive floods free from sediment are passed over the spillway, this bar may be expected to move farther down stream and expose the bed-rock. The maximum expected velocity to which this rock may be subjected is about 25 ft per sec moving along the surface without direct impact. From field tests in which specimens of the foundation rock were subjected to high velocity jets, no erosion is expected at this velocity.

Drum-Gate Operation.—The proper operation of the spillway drum-gates was quickly seen from the model performance. The operation is necessarily fixed by the method adopted for dissipating the energy of the overflow. One of the first prerequisites of a good hydraulic jump formation is to have the water enter the jump in a uniform jet of equal velocity across its entire width. This is readily accomplished at Madden Dam by opening the four spillway drum-gates simultaneously. The design of these gates allows for spilling over the top at any partial opening. If one, two, or three gates are opened, the jump on the apron is broken up by the inpour of the adjacent tail-water. The water below a closed gate will be at a higher elevation than the surface rise of the water in the jump and a cross-flow will result. Although the jump is not completely "drowned out" it was found that concentrations of flow were set up which greatly aggravated the erosion below the apron. Except for the small flows, or for very short periods of time, it would be dangerous to open the gates singly.

Discharge of Drum-Gates.—In the crest of the model, the drum-gates were reproduced to scale. With the gates at several different positions of closure and the pond at various elevations, coefficients of discharge curves were determined by plotting values of C in the formula:

$$Q = C L H^{1.5} \dotfill (4)$$

against the head on the gate crest (see Fig. 10). In the tests the head was measured, in the customary manner, from the highest point on the gate to the water surface up stream where the velocity of approach was negligible.

From these curves the maximum discharge of the spillway was determined, and was found to be in excess of the design assumptions.

The quantity of water that would discharge over the top of the gates in the fully raised position was found. This information was desired as it may prove advantageous to throttle the discharge by raising the gates after the peak of a flood has passed, in order to protect shipping in the canal. Passage is interrupted when the flow exceeds 50 000 cu ft per sec at the point where the river and the canal join down stream. This was one of the factors considered in determining the height of the drum-gates.

Nappes and Pressures on the Crest.—At different gate-openings, with various heads of water flowing over the crest of the gates, the outlines of the surface curves were taken by means of a point gage. The pressures along the crest were obtained by piezometers. Fig. 11 with Table 1 shows a typical set of curves taken for a condition representing the drum-gates fully opened. It will be noted that a vacuum was recorded on the down-stream face of the crest for all the flows tested. Preliminary analysis of the proposed crest by the flow-net method showed this vacuum to exist, and its effect was recognized in the design of the crest. A check of this analysis was made by reproducing the assumed conditions on the model. A very close agreement was found in the pressures, both negative and positive, along the crest. However, the water-surface curve obtained on the model lay considerably below the water surface assumed for the computation, from the apex of the crest down stream.

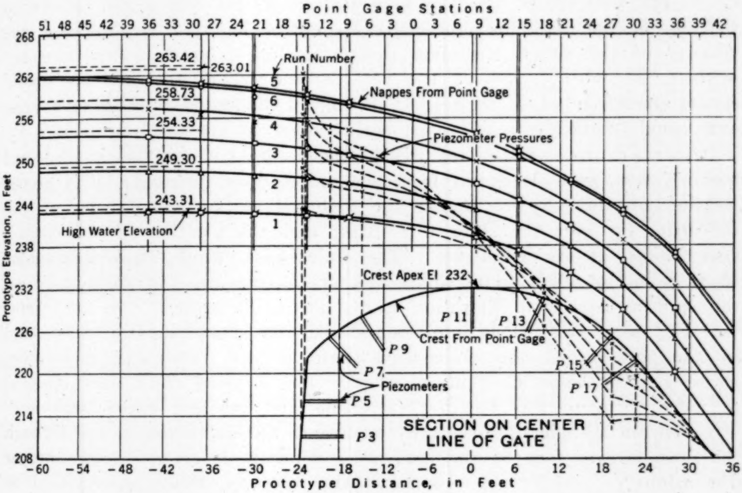


FIG. 11.—NAPPES AND PRESSURES ON SPILLWAY FOR VARIOUS FLOWS

TABLE 1.—TEST RUNS CORRESPONDING TO FIG. 11

Run No. (1)	Ratio of flow, Q, in cubic feet per second (2)	Head on crest, in feet (3)	Run No. (1)	Ratio of flow, Q, in cubic feet per second (2)	Head on crest, in feet (3)
1.........	50 200	11.31	4.........	204 200	26.73
2.........	99 800	17.30	5.........	258 400	31.72
3.........	150 000	22.33	6.........	253 500	31.01

Spillway Retaining Walls.—In order to check the height and stability of end retaining walls of the spillway, nappes and pressures against them were measured at various positions of the gates. The maximum possible case was taken with the forebay at its highest elevation and the gates opening. It was found that as originally designed, the wall needed to be raised slightly and the outline changed in order to follow more closely the shape taken by the overflowing nappe.

At one position of the partly opened gate, this nappe rose nearly to the top of the wall for practically its entire length. Nevertheless, the velocity of the water was such that a negligible pressure was recorded against the wall, except at the very bottom where the jet has finally spread enough to cause impact. This impact amounted roughly to a pressure equal to the depth of water flowing over the crest of the gate.

Other Features.—Other features of the design (not previously discussed herein), that were developed from the model tests included: The proper location of sluice-ways through the spillway section; the shape of the deflecting lip over the sluice-way exits; and the location of the control float intake, at the drum-gate. The sluice-ways, which were rectangular (5 ft 8 in. by 10 ft 0 in. high) discharged into the hydraulic jump on the spillway apron at Elevation 89.67, about the level of low tail-water (see Fig. 13). No serious interference with the jump formation was noted in the tests when the sluice-ways were located between the spillway gates; that is, below the spillway piers. The original plans were changed to move them to this location from the first position which was midway between the piers. Over the exit, where the sluice-way comes through the down-stream face of the spillway section, a deflector lip, the proper dimensions of which were determined by experimentation, was placed to relieve back pressure that may be caused by spillway flow. A check was made to be sure that no vacuum developed in the sluice-ways during the flow over the spillway exits.

Considerable data were obtained on hydraulic jump performance, both on level and sloping aprons, which included velocity contours, water surfaces, and pressures through the jump at various flows. Comparative data on the performance of sills of various heights and shapes were also secured. Performance data were obtained on the action of the other devices previously mentioned for dissipating the energy of the overflow at the toe of ogee dams. Motion and still pictures were taken of nearly every set-up that was tested on the model.

CONCLUSIONS SUPPORTED BY MODEL TESTS

Conclusions pertaining to model tests of Madden Dam may be stated on the basis of the foregoing data, as follows:

(1) The hydraulic jump is very effective in dissipating the energy of the overflow at the toe of ogee dams, the resultant velocities at the end of the jump being little greater than those normally present in the river channel during a given flow, if no dam were present.

(2) The jump can be formed and controlled best by placing a sloped apron down stream, tangent to the bucket at the toe. It will move up or down the slope with variations in tail-water due to different discharge.

(3) The elevation of the apron must be placed relative to the tail-water so that the depths over it will start the jump precisely at the toe of the dam, for all flows, in order that the total length of the apron may be utilized.

(4) The slope of the apron should not be steeper than 1 on 4. A change in the slope alters the shape of the required jump-depth curve. By adopting a proper slope to the apron a close coincidence may be obtained with the natural tail-water depths throughout the entire range of discharges.

(5) The length of the jump, or the distance down stream required for full recovery, varies as its magnitude, increasing with the discharge.

(6) The highest velocities through the jump occur near the floor until they are gradually retarded

(7) The length of the spillway apron then should cover the range of velocity retardation until a point is reached where the velocities are low enough not to cause the river bed to scour down stream.

(8) This apron length may be shortened by placing a sill, with a gradually sloping face, at the end to deflect the floor velocities just free from the river bed below, until they are retarded in the natural action of the jump.

(9) The jump should be protected or enclosed by vertical side walls to a height, at least, which follows the rise in the water surface through the jump, in order to prevent "washing out" by the cross-flow of the adjacent quiet tail-water. It is more desirable to have the side walls high enough to exclude the adjacent tail-water entirely. The walls should extend down stream to the point of full recovery where the water surface reaches the same elevation as that of the adjacent tail-water.

(10) The water entering the jump should be of equal depth and velocity across the entire width; or, in other words, the discharge over the dam crest should be uniform. This is accomplished at Madden Dam, without the use of training walls for each gate, by simultaneously lowering the four drum-gates to the same opening.

(11) The hydraulic jump method is much more advantageous than any other device for dissipating the energy of the overflow below dams. The design is simpler and more rugged when the energy is dissipated by the impact of water on water. The spillway is free from obstructions that may be damaged by logs and other debris coming over the dam. Sills, piers, etc., to break the force of the water are expensive and difficult to reinforce and anchor and are always liable to damage. A break at any point creates a high velocity concentration, which starts erosion that may undermine the entire structure. Difficulties of repair are likely to prove costly as these sills, etc., are usually under a considerable depth of tail-water; if they are not, they are practically useless in dissipating the energy.

Conclusions (1) to (11) have been reached from the tests on the design of Madden Dam. The apron is placed, by experimentation, so that

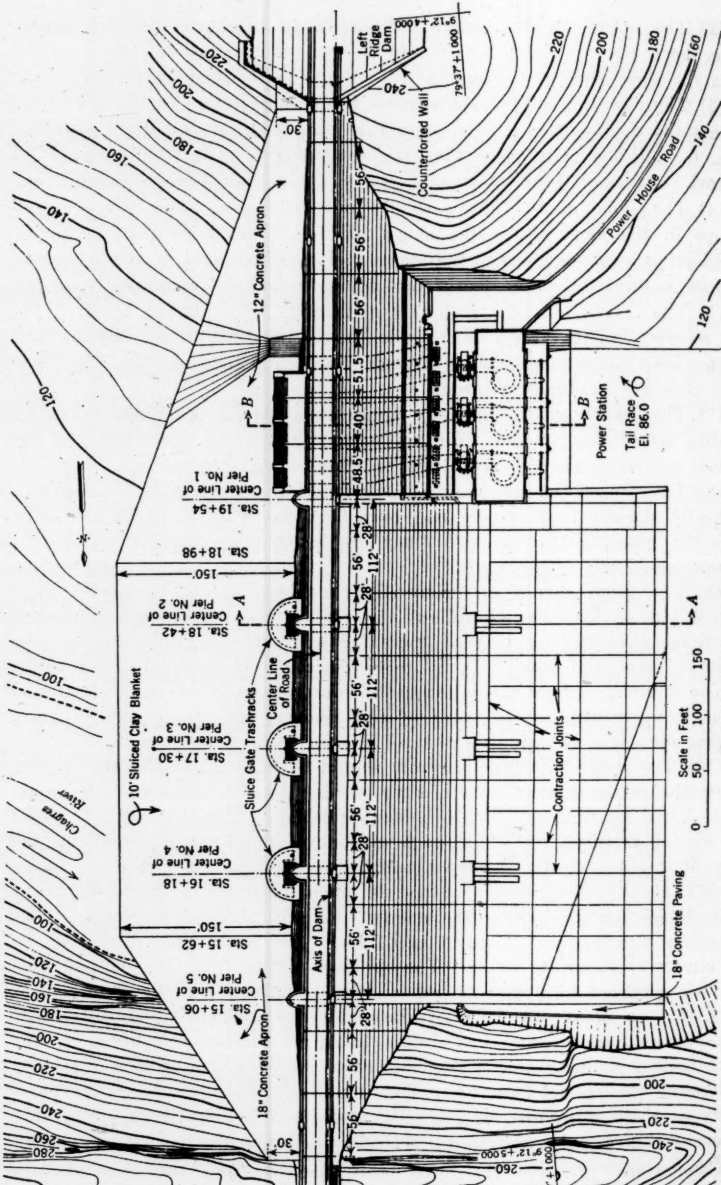
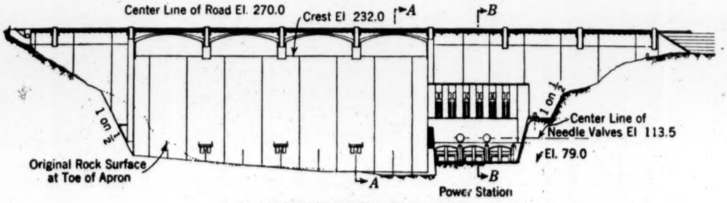
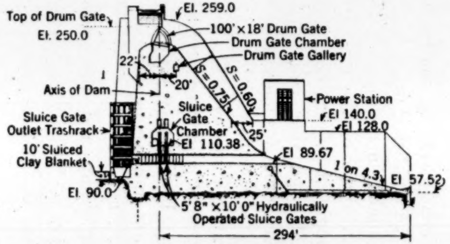

Left Ridge Dam
9°12'+4000
79°37'+1000
Counterforted Wall
Power House Road
12" Concrete Apron
Power Station
Tail Race El. 86.0
Sta. 19+54 Center Line of Pier No. 1
Sta. 18+98
Sta. 18+42 Center Line of Pier No. 2
Sta. 17+30 Center Line of Pier No. 3
Sta. 16+18 Center Line of Pier No. 4
Sta. 15+62
Sta. 15+06 Center Line of Pier No. 5
10' Sluiced Clay Blanket
Sluice Gate Trashracks
Center Line of Road
Axis of Dam
Contraction Joints
Chagres River
18" Concrete Apron
18" Concrete Paving
9°12'+5000
79°37'+1000
Scale in Feet
0
50
100
150
A
B
N
30'
56'
51.5'
40'
48.5'
28'
112'
150'
100
120
140
160
180
200
220
240
260
280

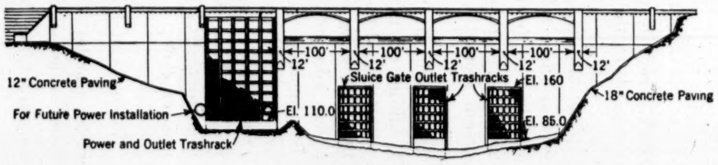
FIG. 12.—PLAN OF MADDEN DAM


Center Line of Road El. 270.0
Crest El 232.0
A
B
1 on 1/2
Center Line of Needle Valves El 113.5
El. 79.0
1 on 1/2
Original Rock Surface at Toe of Apron
A
B
Power Station

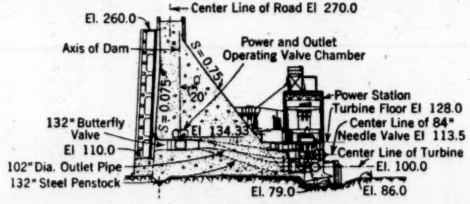
(a) DOWNSTREAM ELEVATION


Top of Drum Gate El. 250.0
El. 259.0
100'×18' Drum Gate
Drum Gate Chamber
Drum Gate Gallery
22'
20'
S=0.75
S=0.60
Axis of Dam
Power Station
El 140.0
El 128.0
Sluice Gate Outlet Trashrack
Sluice Gate Chamber
El 110.38
25'
El 89.67
10' Sluiced Clay Blanket
1 on 4.3
El 57.52
El. 90.0
5'8" × 10'0" Hydraulically Operated Sluice Gates
294'

(c) SECTION *A-A*


100'
12'
100'
12'
100'
12'
100'
12'
12'
12" Concrete Paving
Sluice Gate Outlet Trashracks
El. 160
18" Concrete Paving
For Future Power Installation
El. 110.0
El. 85.0
Power and Outlet Trashrack

(b) UPSTREAM ELEVATION


Center Line of Road El 270.0
El. 260.0
Axis of Dam
S=0.75
Power and Outlet Operating Valve Chamber
20'
S=0.075
Power Station
Turbine Floor El 128.0
Center Line of 84" Needle Valve El 113.5
132" Butterfly Valve
El 134.33
El 110.0
Center Line of Turbine
El. 100.0
102" Dia. Outlet Pipe
132" Steel Penstock
El. 79.0
El. 86.0

(d) SECTION *B-B*

FIG. 13.—SECTIONS AND ELEVATIONS, MADDEN DAM

a balanced effective hydraulic jump occurs well up on the apron at any given flow over the spillway. This is accomplished more easily at Madden Dam than elsewhere, perhaps, as the foundation rock was of sufficient depth below the natural tail-water so that an apron of the proper elevation and slope could be constructed. It is thought, however, that from the results of these tests, principles may be established that will allow the adaptation, at other projects, of this type of apron for obtaining the hydraulic jump. The tendency in later designs of overflow dams has been to use a sloped apron for obtaining the hydraulic jump as a means of dissipating the energy at the toe. Similar aprons were later used at the Norris Dam,[9] and at the Hamilton Dam, in Texas. Preliminary designs for the Marshall Ford Dam, in Texas, and the Friant and Kennett Dams, in California, contemplate the use of this type of apron.

The effectiveness of the hydraulic jump and the possibilities of a sloped apron as a means for controlling it were fully demonstrated by the model studies of Madden Dam.

PART II.—PROTOTYPE TESTS

INTRODUCTION

In order to check the model predictions and also to obtain hydraulic data on a full-sized structure, test apparatus was installed at Madden Dam similar to that used on the model. Piezometers were placed along the spillway crest of one gate in the sides of the spillway piers, and down the training wall. Provision was made to measure the water surface over the crest. Piezometers and Pitot tubes were placed at the toe of the spillway, to measure the head lost in friction, and along the length of the apron to measure the velocity lost through the jump. A gaging station is situated just down stream from the dam for obtaining the quantity of water flowing.

The concrete apron built at the toe of the spillway is constructed on a slope of 1 on 4.3, taking off tangent to the curved bucket at the toe and terminating in a small sloped-face deflector sill. The dimensions are given in Figs. 12 and 13 which show a general layout of the dam. As stated in Part I this apron was adopted after extensive model tests had been made to determine the best method for dissipating the energy of the overflow.

The largest flow that has been passed over the spillway to date (1937) has been about 32 000 cu ft per sec. Complete model data for a direct comparison are not available at this relatively low discharge, but the results obtained are interesting and are indicative of the performance to be expected for greater flows.

NAPPES AND PRESSURES OVER THE CREST

The water-surface profiles of the flow over the spillway crest were obtained by dropping a heavy steel plumb-bob, attached to a steel tape, from a platform which was suspended beneath the archway of the first drum-gate. At approximate 4-ft intervals along the center line of the gate, holes were cut in the floor of the platform and a known bench-mark was established near each hole, from which measurements of the distance to the water surface were taken.

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[9] *Civil Engineering*, April, 1935.

Pressures along the crest were taken from piezometers placed in the concrete and in the steel drum-gate itself, at positions as indicated in Fig. 14. The piezometer openings were connected by pipes to a mercury U-tube in one of the galleries of the dam. When there was no water flowing over the crest, the deflection of the U-tube at a known elevation of the lake, was taken as a datum, and subsequent readings taken during the tests were translated in elevations of the water columns above each piezometer opening.

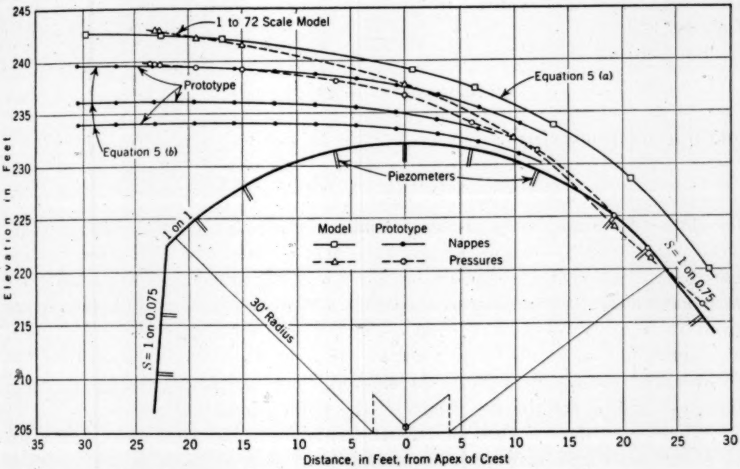

FIG. 14.—NAPPES AND PRESSURES OVER THE SPILLWAY CREST, FOR THE PROTOTYPE AND MODEL

It will be noted that the pressures as recorded by the piezometers on the up-stream face of the dam lie above the measured water surface, indicating impact. The pressures drop off rapidly along the crest due to the centrifugal force, reaching a negative pressure at a point on the downstream side, showing a tendency of the jet to leap the crest. The model tests showed that this tendency increased at higher heads. The vacuum thus produced was taken into account in the stability design of the crest, as its existence was early recognized.

Fig. 14 shows the nappes taken over the prototype crest for three heads. Three other intermediate heads were also taken for this position of the gate fully opened (lowered), and lie consistently between the ones shown. The piezometer pressures are plotted for the higher profile only, in order to avoid confusion in Fig. 14, but the pressures for the lower heads fall in a similar manner below the water-surface nappe in each case. A water-surface curve line of pressures taken on the model, is also shown in Fig. 14. This is the lowest head on which complete data were taken for the model tests. However, there is a remarkable consistency in the curves indicating similarity of performance. Nappes and pressures were also taken with the gate in three different positions of closure.

Many trials were made to obtain an equation for these overflow nappes. For water surfaces over the crest when the drum-gates were fully opened (lowered) the most consistent formulas that could be derived were as follows:

For the model:

$$-y = (0.164\,H - 0.692)\,1.223^{xH^{-0.5}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

and, for the prototype:

$$-y = (0.0733\,H - 0.206)\;1.266^{xH^{-0.5}} \quad \ldots\ldots\ldots\ldots\ldots (5b)$$

in which: $\pm\,x$ and $-\,y$ are co-ordinates of a point on the water surface, with the Y-axis at the up-stream face of the dam and the X-axis as the horizontal line of the reservoir level. The model formula (Equation (5*a*)) was derived from five measured nappes, at heads corresponding to a range from 11 ft to 31 ft over the crest. The prototype formula (Equation (5*b*)), was derived from points on five curves taken up to a maximum head of 7.78 ft over the crest. Each formula showed close agreement with its respective test points, within the range of the points actually taken.

When applied to a head of 7.78 ft (which was the highest head attained over the prototype spillway), Equation (5*a*) showed close agreement with the actual measured water surface on the prototype On the other hand, Equation (5*b*) gave a profile much higher than the actual measured water surface over the model for a head corresponding to 11.3 ft over the crest (which was the lowest head measured on the model). In other words, the model formula could be extrapolated to check the prototype measurements, but the prototype formula, when extrapolated, would not check the model nappes. Equation (5*a*) seems to be more accurate for design approximations, until further tests are taken to check Equation (5*b*).

A row of piezometers was placed along the face of one of the end spillway piers. In general, the pressures recorded on these piezometers fell slightly below the profile of the water surface There was a noticeable disturbance at the end piers of the spillway. The slight drop below static pressure which was recorded along this end pier may be due to the tendency of the flow into the gate to leave the side of the pier as it comes in from the dead end of the spillway. This disturbance could be eliminated, perhaps, by extending the end piers a distance up stream, although in most instances this is not practical. At the intermediate piers, when adjacent gates are opened, there is no disturbance, the flow being symmetrical and uniform on both sides of the pier.

Piezometers were located along the drum-gate just beneath the downstream tip of the top skin-plate. These piezometers, however, did not indicate vacuum for any of the flows attained to date. With the drum-gate fully opened (at which position it conforms with the crest curve), the vacuum (see Fig. 14) noticed at the higher heads occurs at a point down stream from the tip of the gate. At any of the partly raised positions of the gate, there is no indication of a vacuum on the under side, the overflowing sheet of water being quite sufficiently aerated at the ends, through the space behind the square end piers. Piezometers were installed in the left retaining wall of the spillway, but the flows have not yet been high enough to obtain readings from them.

SPILLWAY DISCHARGE

Simultaneously, with the observations being taken at Madden Dam, stream-gage measurements were made at the permanent station about ½ mile down stream. The readings were made with a current meter at 1-ft depth, the mean velocity being obtained by using a correction factor of 0.9. This factor (0.9) was established for the station after a long period of observations in which complete traverses were made. Due to the high velocity through the gaging section of the river it was not possible to make complete traverses during the larger flows, and the 0.9 factor used may be in considerable error for the higher discharges. It is apparent that to measure the discharge in a degree of accuracy comparable with the model tests special apparatus must be used to hold the current meter in the high velocity water so that a complete traverse of the section can be made.

The results as obtained may prove interesting, however. A curve of coefficients of discharge was computed, using the simultaneous discharges taken from a rating curve of the gaging station established from readings taken during these tests. The equation of the coefficient curve for drum-gates fully opened, was obtained from the test points and showed that the coefficient of discharge was a function of the

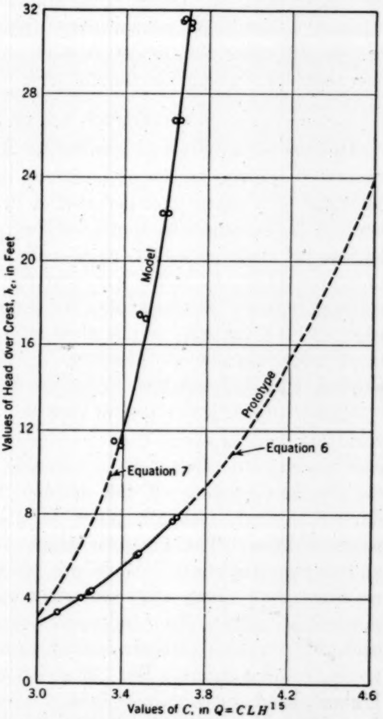

FIG. 15.—COEFFICIENTS OF DISCHARGE

head as well as of the crest shape:

$$C = 2.44 \ h_c^{0.20} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Fig. 15 shows a plotting of this coefficient curve, extrapolated by Equation (6) for heads higher than those tested, for comparison with the model coefficient curve, similarly extrapolated downward by the formula:

$$C = 2.69 \ h_c^{0.094} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

to cover the range of the lower prototype test heads. At the maximum prototype test head of 7.78 ft over the crest, the indicated discharge is about 13% greater than that predicted from the model tests. Some of this flow may be due to the possible error in the stream gaging. There is an apparent definite discrepancy, however, as the two curves have marked different characteristics.

This discrepancy is interesting as it is a generally accepted procedure to use directly, coefficients of discharge obtained from model tests. In some instances a great saving in spillway length can be effected if the true coefficient of the crest is known. The data are yet too meager to attempt a thorough comparative analysis, but it would seem that some factors affecting the model flow, such as capillarity action and viscosity, have not been given full cognizance.

Velocities Along Spillway Apron

During construction a number of fixed Pitot tube nozzles were installed along the spillway apron at positions indicated in Fig. 16. The Pitot tubes consisted of a plate steel pedestal, 4 in. wide by 12 in. long, the plate being bent to a stream-line shape, welded, and filled in with concrete. The bottom of the pedestals were embedded and anchored in the concrete and protruded 14 in. for some to 20 in. for others above the floor of the spillway apron. A piece of 1-in. pipe projected 4 in. from the up-stream edge of the pedestal at heights above the floor varying from 3 in. for those pedestals above tail-water, to 12 in. and 18 in. for those pedestals covered with tail-water.

These projecting pipes or nozzles were connected by 1-in, pipes to a manifold and pressure gage in the basement of the power station. Below each nozzle, but offset 2 ft to one side, were piezometers, opening flush with the concrete surface of the apron. These piezometer openings were conducted through separate pipes to a manifold and pressure gage in the power station. The pressure gages were accurately calibrated previous to and during the tests. With any given flow over the crest, a number of readings were taken simultaneously on both the Pitot nozzle gage and the piezometer gage. The indicated water pressures were then plotted over each Pitot nozzle and piezometer as the kinetic and static energy, respectively, at that point. Fig. 16 shows the plotting of such readings for the highest head yet obtained over the crest. The Pitot nozzles are situated just off the surface of the apron and will register the highest velocity of the water through the hydraulic jump. This was found to be true from tests on the model (see Fig. 8). The indicated velocity, of course, is not the average velocity of the cross-section, but will show the relative change, which was the original objective of the Pitot tubes as installed. The loss in energy is great and occurs mainly just at the beginning of the jump. The flow off the end of the apron is of low velocity, not sufficient to register on the test apparatus. The effectiveness of the hydraulic jump in dissipating the energy of the overflow is vividly illustrated by the sudden drop obtained in the energy gradient curve as shown in Fig. 16.

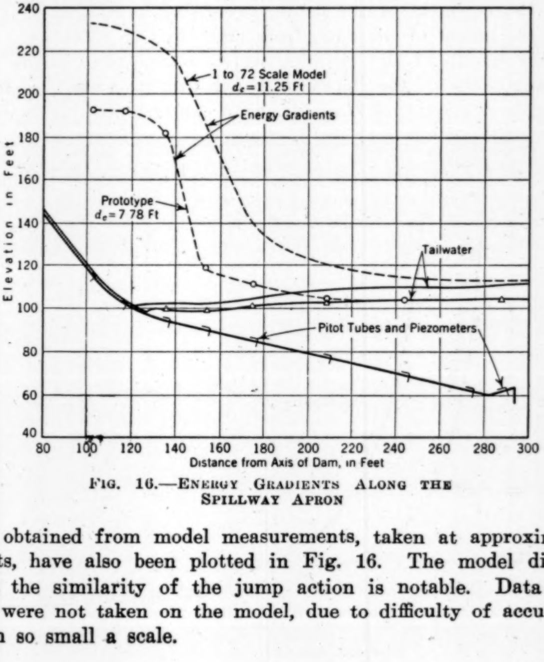

FIG. 16.—ENERGY GRADIENTS ALONG THE SPILLWAY APRON

Curves obtained from model measurements, taken at approximately the same points, have also been plotted in Fig. 16. The model discharge is higher but the similarity of the jump action is notable. Data for lower discharges were not taken on the model, due to difficulty of accurate measurement on so small a scale.

FRICTION LOSS AT THE TOE

From the readings of the Pitot tubes on the apron, which were not covered by the tail-water during the tests, values of the velocity were obtained for five different heads over the crest, with the drum-gates fully opened. These velocities were used to compute the depth of the stream, the rate, Q, having been obtained from stream gaging below the dam. The difference between the velocity head as indicated by the Pitot tube readings and the theoretic velocity head (which is the difference between water surfaces) was taken as the loss in head due to friction on the overflowing sheet of water. These rather crude Pitot tubes will not, of course, measure the true mean velocity of the stream, but a discussion of the friction loss as so indicated by them may prove interesting. It was found that the average of the five readings obtained at different heads over the crest could be expressed by an equation of the following form:

$$h_f = \frac{f_n \, H \, (H + d_c)}{d^q_c} \quad \text{.....................(8)}$$

in which h_f = the friction head loss at a given station on the toe of the dam; f_n = the friction coefficient depending on the roughness of the surface; q = the exponent of the depth, expressing a relation to the friction loss; H = the height of the dam, from crest apex to station on toe (factor involving the length of the channel, which is practically constant for all overflow dams); d_c = the depth of water over the crest, from reservoir elevation to crest apex; and $H + d_c$ = an expression of the velocity.

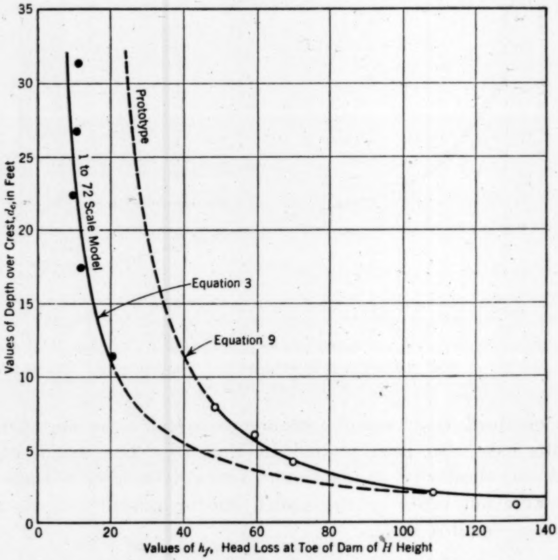

FIG. 17.—CURVES EXPRESSING FRICTION LOSS AT TOE OF OVERFLOW DAMS, AS OBTAINED FROM OBSERVATIONS ON MODEL AND PROTOTYPE

By averaging all test points, $f_n = 0.00934$ and $q = 0.623$ were obtained so that Equation (8) becomes:

$$h_f = \frac{0.00934 \, H \, (H + d_c)}{d^{0.623}} \quad \text{.....................(9)}$$

A curve of Equation (9) with the test points shown, has been plotted in Fig. 17. Extrapolation is made for comparison with a curve similarly obtained from the model tests (Equation (3)).

The prototype curve can be made to fit the model-test points very closely by changing the coefficient, $f_n = 0.00934$ to $f_n = 0.00374$, or 0.4 of the prototype coefficient. The friction loss, then, occurring at the toe of an overflow dam of any size, whether model or prototype, may be expressed by Equation (8) when f_n is a coefficient of friction depending on the roughness of the surface, varying from $f_n = 0.00374$ for the painted galvanized-iron surface of the model, to $f_n = 0.00934$ for the concrete surface of the prototype dam. The formula is expressed in prototype values and the resulting friction loss, h_f, must be divided by the scale of the model (in this case, 72) to obtain the actual value in terms of the model dimensions.

In attempting to check the theory that this difference in energy loss might be due to the roughness of the surface, numerous attempts were made to apply the Chezy formula to the test points. Results varied, depending on assumptions, as to the mean velocity and the mean hydraulic radius. A constant, n, could not be obtained by substitution in Kutter's or Manning's formula. Consistent values were not obtained until the mean velocity, V_m, was assumed to be the average of the velocity at the crest apex (see Fig. 11 and Fig. 14) and at the toe, and taking the mean hydraulic radius, R_m, at the depth where this mean velocity occurs. A formula was obtained in the exponential form which satisfied both the model and prototype test points within 8% ±, as follows:

$$V_m = \frac{1.233}{n} R_m^{0.45} S^{0.57} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which $S = \frac{h_f}{L}$; h_f = the measured friction loss at the toe, from Fig. 17; L = the length of the spillway from the crest apex to the toe (the velocity at the crest apex being $\frac{3\,q}{2\,d_c}$; and, the velocity at the toe being taken from the Pitot tube measurements); $V_m \doteq$ the average of the velocity at the crest and at the toe; R_m = the depth at the point of mean velocity $= \frac{q}{V_m}$; q = the discharge per linear foot of crest; and, n = the friction coefficient corresponding to Kutter's n (taken as 0.009 for the model and 0.016 for the prototype). The values assumed for n seem logical. The model was enameled and sand-papered to a very smooth finish. Form marks left at the horizontal construction joints made the down-stream face of the prototype dam unusually rough. It is indicated, then, that Equation (10) could be used for computing resulting velocities at the toe of dams, by substituting the proper value of n for the surface and solving in the same manner as that used to derive the equation.

When and if further tests are made on the prototype at heads comparable to the model data, the coefficient of the Pitot tubes as used could be obtained from tests on a model, resulting in more accurate data for determination of the friction loss. The comparisons that have been attempted show the possibilities.

Drum-Gate Operation

The correct procedure for operating the drum-gates was derived from the model tests, as previously discussed. It was found that in order to obtain the proper functioning of the hydraulic jump, all spillway drum-gates should be opened to the same amount. To verify this model prediction, an attempt was made to measure the resulting erosion below the spillway apron, with various combinations of drum-gate openings.

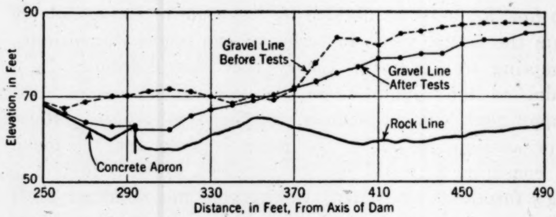

Fig. 18.—Typical Erosion Below Apron

A schedule of tests was run in which, first only one gate was opened fully and allowed to spill for 3 hr, then all four gates were opened partly so that the same quantity of water was spilled as was passed through the

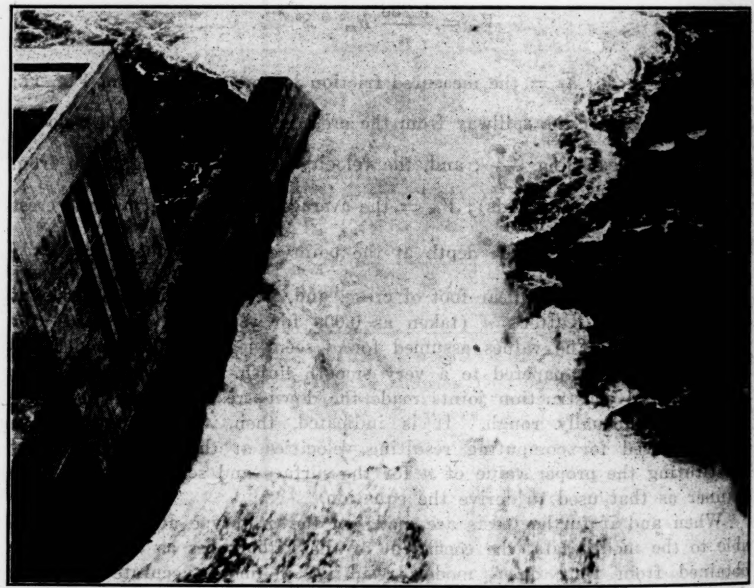

Fig. 19.—View of Spillway Bridge, Showing Interference with Hydraulic Jump on Apron, Caused by the Tail-Water Below Closed Gates; Flow, 7 500 Cubic Feet per Second

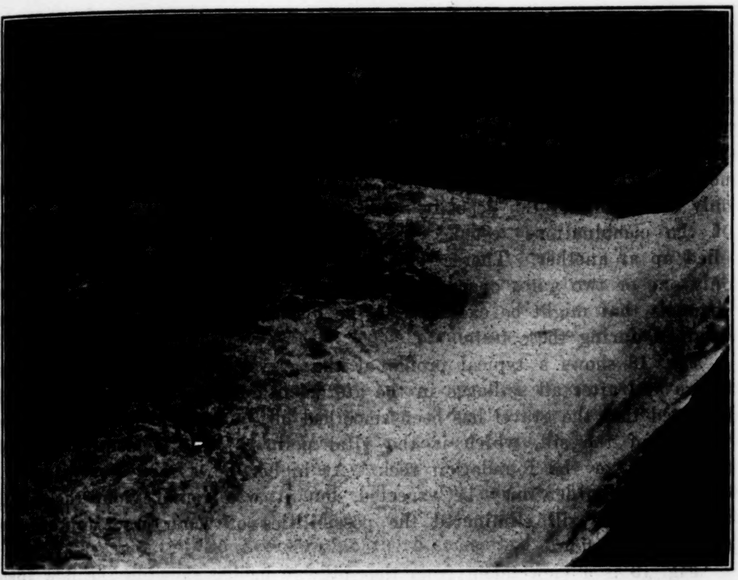

FIG. 20.—VIEW FROM SPILLWAY BRIDGE, SHOWING HYDRAULIC JUMP ON APRON; FLOW, 7 600 CUBIC FEET PER SECOND

FIG. 21.—VIEW OF HYDRAULIC JUMP ON SLOPED SPILLWAY; FLOW, 31 800 CUBIC FEET PER SECOND

one gate. The four gates were then allowed to spill for 3 hr. This same procedure was repeated with two gates fully opened and then with the four gates partly opened to pass the same quantity of flow; and, similarly, with three gates. After each of these settings, soundings of the river bottom were taken in the tail-race below the apron for the purpose of comparison. The change in the profile of the river gravel was so slight, however, that no definitely conclusive comparison could be reached. There seemed to be only a shifting of the deposits, with little movement down stream, at any of the combinations tested. The gravel was moved at one place and piled up at another. There were indications, however, that the flow, with only one or two gates opened, tended to form concentrations and to erode channels that might be excessive if the discharges were greater than those attained during these tests.

Fig. 18 shows a typical profile of the gravel deposits below the apron before and after all spillages in the aforementioned series of tests. It will be noted that the gravel has been smoothed off, but no erosion occurs below the lip of the sill, which is the vital point. Quite a depth of gravel remained over the foundation rock. At higher flows more severe erosion of the over-burden may be expected, but it was found from the model tests that the sill eliminated the possibilities of scour undermining the apron, at any flow.

Fig. 19 is a view taken from the spillway bridge showing the flow from one gate only. Note that the adjacent tail-water crowds the beginning of the jump, causing the flow to concentrate toward the retaining wall. This concentration produced excessive gravel erosion below the apron.

A more positive indication of the effects of one gate operation was obtained from readings taken on the Pitot tubes along the apron, which were situated below Gate No. 1. If this gate were opened alone there was a marked increase in the velocity along the apron as compared with the velocity existing if all four gates are opened partly to pass the same flow. A visual comparison is readily made by referring to Fig. 20 which shows the same flow being passed over all drum-gates, lowered to the same opening.

Obviously, the best method is to operate the gates simultaneously which can be easily done because the drum-gates allow spillages over the top in any position, without detriment. The controls are designed to hold the gate in any position. Except for short periods, or at low heads, the indications are that it would be dangerous to allow spillages from the gates opened individually.

CONCLUSIONS: HYDRAULIC JUMP PERFORMANCES

At the flows over Madden Dam spillway to date, the performance of the hydraulic jump has been consistent with the predictions made from the model tests. The depths furnished by the natural tail-water have caused the jump to begin just at the toe of the dam at any flow. Fig. 21 shows the hydraulic jump at the toe of the dam for the maximum flow attained to date, of 31 800 cu ft per sec. Fig. 20 shows the jump at a lower flow.

The loss in energy in the jump has been almost instantaneous and the resultant erosion of the gravel deposits beyond the concrete apron, almost negligible (see Figs. 16 and 18).

It will be noted from Fig. 17, that the measured velocity at the toe was less on the prototype. This may be attributed to the difference in the roughness of the surface of the model and the concrete dam, as discussed previously.

For other comparative measurements it was found that the nappe curves taken over the spillway crest on the model could be used to predict the prototype nappes within a reasonable degree of accuracy. The comparative pressures on the crest seemed consistent. An apparent increase was obtained in the discharge over the crest of the prototype dam from that predicted from the model. All the model predictions were on the safe side, however.

The foregoing prototype test data were obtained at comparatively low flows over the spillway and there are no corresponding model data available for a direct comparison. Accurate and complete data were not taken on the model for very low heads over the crest for two reasons: (1) The model was built on a rather small scale (1:72) and very low quantities of discharge were difficult to measure; (2) the principal objective of the model experiments was to determine the best method to handle the larger and maximum flows safely, so that few data were obtained on flows less than a prototype equivalent of 50 000 cu ft per sec. For greater flows however, complete and accurate records were obtained at various increments of flow up to the maximum expected discharge of 260 000 cu ft per sec.

The comparisons which have been attempted herein were made from extrapolations, but it will not be until these tests are completed for higher discharges that a comprehensive study can be made. From what has been indicated, many valuable and interesting data could be developed by further testing, which, in comparison with the model data, would establish a more reliable criterion for model similitude.

ACKNOWLEDGMENTS

Acknowledgment is due to James W. Ball, Jun. Am. Soc. C. E., for his assistance in conducting the model tests, and to Mr. W. F. Bingham[10] for his aid in preparing the report on the model tests. Messrs. P. S. O'Shaughnessy[11] and J. C. French assisted in the prototype test program, which was conducted under the administrative direction of Mr. W. L. Hersh. The model tests were conducted under the direction of E. W. Lane, M. Am. Soc. C. E.[10]

The Madden Dam project was completed in 1935 under the administrative direction of Col. Julian L. Schley, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., then Governor of The Panama Canal, assisted by Col. Clarence S. Ridley, Corps of Engineers, U. S. Army, then Engineer of Maintenance; and Lt. Col. William E. R. Covell, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., then Assistant Engineer of Maintenance.

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[10] *Engineering News-Record*, March 14, 1935, p. 373.

[11] *Civil Engineering*, August, 1937, p. 576.

Edward S. Randolph, M. Am. Soc. C. E., was first Designing Engineer and, later, Construction Engineer, on the project. The designs and specifications were prepared by the forces of the U. S. Bureau of Reclamation, in Denver, Colo., in collaboration with the Panama Canal organization. Raymond F. Walter, M. Am. Soc. C. E., Chief Engineer, and John L. Savage, M. Am. Soc. C. E., Chief Designing Engineer, of the U. S. Bureau of Reclamation, served as Consulting Engineers for the design and construction.

DISCUSSION

Ettore Scimemi,[12] Esq. (by letter).—Those who are studying the possibility of applying experimental results obtained from model tests to the prototype, will find this paper of the greatest interest. Experiments of the type demonstrated by the author are now very often made in laboratories, whereas those obtained directly from the original structure are very rare or are lacking entirely. Knowing how very difficult it is to make exact measurements, the writer feels that Mr. Randolph deserves unlimited praise. Especially interesting is the fact that there is good correspondence between the discharges and the distribution of the pressure along the crest of the dam.

For the shape of the spillway of the Madden Dam the author has adopted a circular profile; and yet this profile is neither the most economical nor the most suitable one for a spillway dam. One type that is now quite popular is that recommended by William P. Creager, M. Am. Soc. C. E., about 1917.[13] About 1930 the writer analyzed this profile,[14] but only with regard to the shape of the hydraulic sheet (see Fig. 22); he noted then that the Creager profile is always above the lower nappe of the sheet.

Mr. Creager gives the co-ordinates of this profile in a table which is now included in almost every handbook on the subject.[13] The zero, or the origin, of reference should be referred to the vertex, or the summit, of the dam, as shown in Table 2 and Fig. 23. It is not necessary to describe a complete curve for the up-stream part of this vertex. The lower nappe adapts itself to an arc of a circle equivalent to 0.40 H, in radius, and a rise equal to 0.12 H; the measurements in Fig. 23, of course, are computed for a head equal to unity, as in the Creager method. As to the down-stream part, however, Fig. 23 shows three profiles: The lower one, which

TABLE 2.—Values of $Y = 0.47\ X^{1.80}$

Values of X (1)	Free nappe (2)	Creager (3)	Values of $Y = 0.47\ X^{1.80}$ (4)	Values of X (1)	Free nappe (2)	Creager (3)	Values of $Y = 0.47\ X^{1.80}$ (4)	Values of X (1)	Free nappe (2)	Creager (3)	Values of $Y = 0.47\ X^{1.80}$ (4)
0.10	0.007	0.007	0.007	1.10	0.570	0.567	0.558	2.70	3.05	2.82	2.81
0.30	0.055	0.06	0.054	1.50	1.060	....	0.975	3.00	3.65	....	3.40
0.50	0.140	0.142	0.135	1.70	1.33	1.22	1.22	3.20	4.20	3.82	3.81
0.70	0.260	0.257	0.247	2.00	1.75	....	1.64	3.70	6.00	4.93	4.95
0.90	0.400	0.397	0.389	2.20	2.10	1.96	1.94	4.20	7.25	6.22	6.22
1.00	0.480	....	0.470	2.50	2.65	....	2.45	....	....	....	....

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[12] Professor, Director, Hydr. Inst.; Univ. of Padua, Padua, Italy.

[13] "Masonry Dams", by William P. Creager, 1917, p. 108.

[14] "Sulla Forma delle Vene Tracimanti", *L'Energia Elettrica*, Milan, April, 1930.

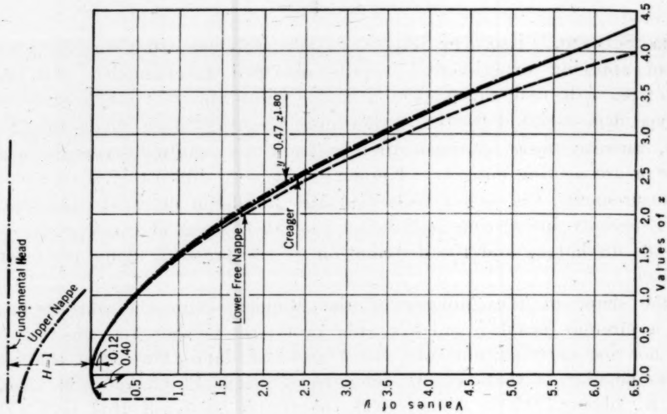

FIG. 23

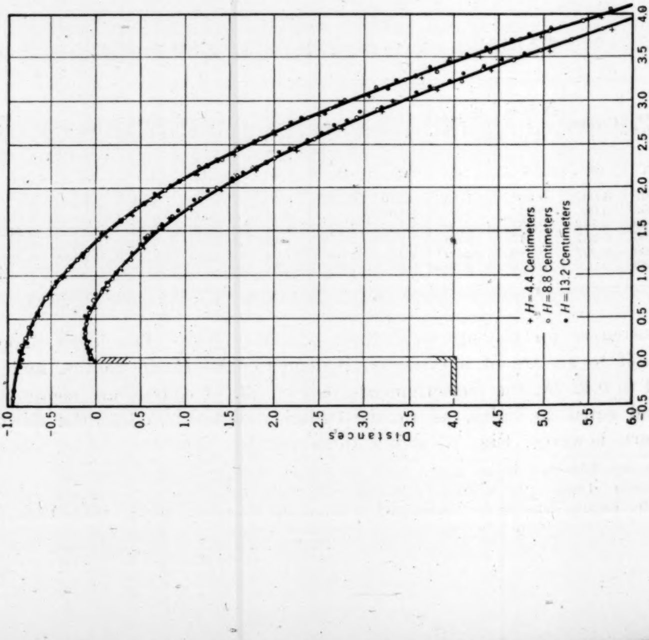

FIG. 22.—EXPERIMENTAL SHEET, UPPER AND LOWER NAPPE

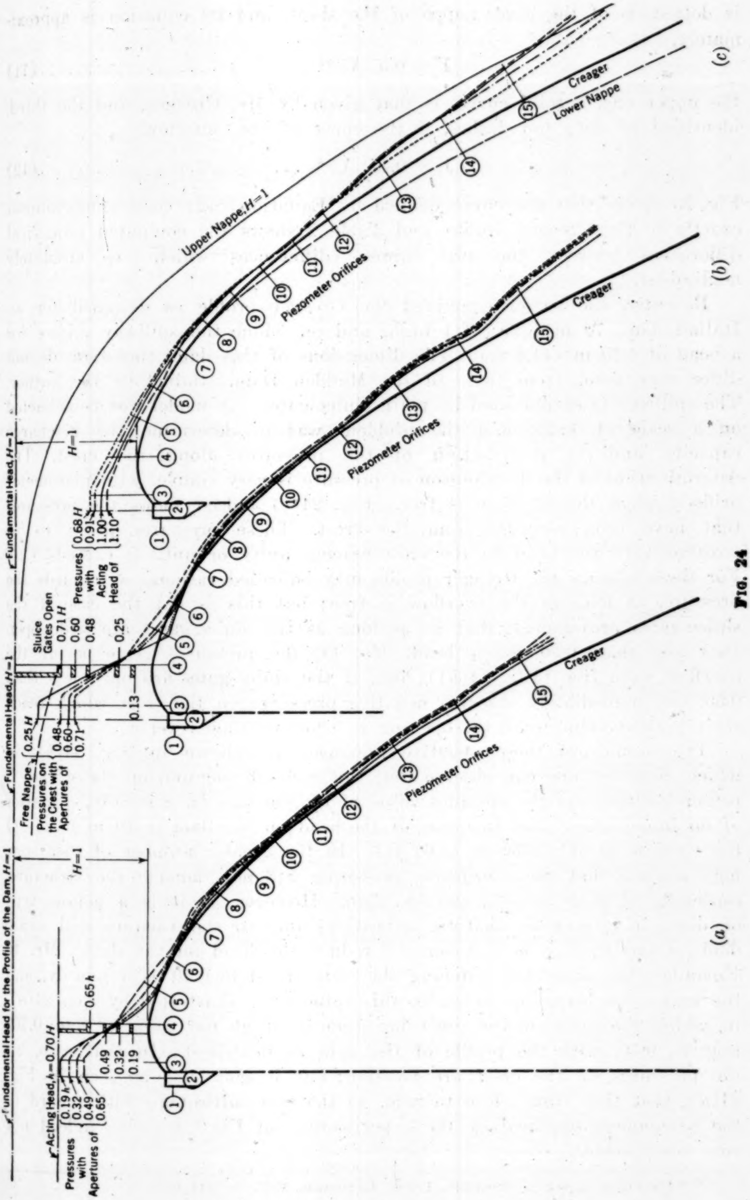
Fundamental Head for the Profile of the Dam, H=1
H=1
Fundamental Head, h=0.70 H
Acting Head, h=0.19 h
0.32
0.49
0.65
Pressures with Apertures of
0.65 h
0.49
0.32
0.19
Piezometer Orifices
Creager
(a)
Free Nappe
Fundamental Head, H=1
0.25 H
Pressures on the Crest with Apertures of
0.48
0.60
0.71
Sluice Gates Open
0.71 H
0.60
0.48
0.25
0.13
Piezometer Orifices
Creager
(b)
Fundamental Head, H=1
Pressures with Acting Head of
0.68 H
0.91
1.00
1.10
H=1
Upper Nappe, H=1
Piezometer Orifices
Creager
Lower Nappe
(c)
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

FIG. 24

is dotted, is of the lower nappe of the sheet, and its equation is approximately,

$$Y = 0.5\ X^{1.85} \qquad \text{(11)}$$

the upper one, a solid curve, is that given by Mr. Creager, and the third, identified by dots and dashes, is the curve of the equation,

$$Y = 0.47\ X^{1.8} \qquad \text{(12)}$$

Fig. 23 shows that the curve defined by Equation (12) corresponds almost exactly to the Creager profile, and Table 2 shows the computed numerical differences between the two curves (differences which are absolutely negligible).

Recently, the writer examined the Creager profile as designed for an Italian dam, 70 m (229.7 ft) high, and on which the spillway allows for a head of 4.40 m (14.4 ft). The dimensions of this dam, therefore, do not differ very much from those of the Madden Dam, studied by the author. The spillway is surmounted by plain sluice-gates. A model was constructed on a scale of 1:25, and the problem was to determine the discharge capacity and the distribution of the pressures along the crest. The determination of the distribution of pressure is very simple, with piezometer orifices, when the overflow is free. Fig. 24(*c*) indeed shows the pressures that have been recorded from the crest. These pressures prove to be positive until the head on the crest reaches unity or until it exceeds 1.10. For these reasons the Creager profile may be called rational as regards the pressure, as long as the overflow is free; but this is not the case if the sluice-gates are opened; that is, as long as the sluice-gates are wide open (not less than 0.50 for a head, $H = 1$), the pressures occur as if the overflow were free (Fig. 24(*b*)); but, if the sluice-gates are open less than 0.50, one immediately observes negative pressures on the crest which reach their highest value when the opening is 0.25, and then reverse.

The profiles of these negative pressures are shown in Fig. 24(*b*), in which, however, one can observe that this zone of vacuum, on the whole, is rather limited, and the absolute value of the vacuum is, generally, a matter of no importance. For instance, if the head on the dam is 10 m (32.8 ft) the vacuum is only 0.50 m (1.64 ft). In the greater number of cases one may assume that these negative pressures will not damage the structure, especially if it is a solid gravity dam. However, if it is a hollow type of dam, it is possible that the vibrations due to the vacuum will cause damage and then it is necessary to reduce the head on the dam. Mr. L. Escande[15] has suggested reducing the water head to 0.70 as a remedy, and the writer has been able to verify this value as is illustrated by Fig. 24(*a*), in which the head on the crest has been kept at not greater than 0.70; that is, 0.70, with the profile of the dam as described. In that case, all the pressures on the crest are positive; but it may also be seen in Fig. 24(*a*), that they come close to zero, so that the advantage with regard to the phenomena obtained by the experiments, on Fig. 24(*a*), is indeed not very remarkable.

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[15] "Barrages", par L. Escande, Paris, Hermann, 1937, p. III–66.

J. C. STEVENS,[16] M. AM. SOC. C. E. (by letter).—One of the projects early adopted by the Special Committee of the Society on Hydraulic Research,[17] is to gather data on the conformity of prototype behavior to model tests. Mr. Randolph has added important data to the Committee's store.

In so many cases the works vary from the design developed by model tests, to such a degree that a direct comparison is impossible. Fortunately, in this case, the prototype was constructed to conform to the design shown by model studies to be the best, and direct comparisons are possible. The construction management is to be complimented for installing the facilities by which such direct comparisons could be made.

It should be recognized that there is no virtue in the sloping deck, as such, to aid in the dissipation of energy. Better results would have been secured had the deck been horizontal at the proper elevation. The sloping deck was used only to save concrete.

Little is known about hydraulic-jump characteristics on sloping beds. A stream flowing down the face of a dam acquires a high kinetic energy at the bottom, which must be destroyed (converted to heat). The hydraulic jump is the most potent factor known in absorbing such energy, and the proportion of total energy that can be destroyed will depend on the ratio of kinetic to potential energy ("kineticity") possessed by the water just prior to the jump and upon the opposing forces that cause the jump. If the jump occurs on a sloping deck the force of gravity on the water prism acts in the direction of flow and, therefore, tends to increase rather than to retard velocity.

In many cases, baffles of various types are placed on the apron to constitute an opposing force and thus aid the formation of the jump and insure its forming on the deck or apron. In this case, the deflector at the end of the apron serves as a retarding force and, to some extent, counteracts the force of gravity on the sloping deck. For high dams and low discharges per unit width, the force of gravity on a sloping apron may be relatively unimportant, but for greater discharges per unit width this force cannot be ignored.

The writer seriously questions the author's Conclusion (2) that the jump can be "controlled best by placing a sloping apron down stream, * * *." This conclusion cannot be supported in theory because little is known of the jump on such slopes. It cannot be supported in practice because the prototype discharge concentrations are too low to serve as a check on the model studies. For the Bonneville Dam with high discharge concentrations model studies showed the sloping deck to be the worst possible form that could be devised.

Fig. 25(*a*) shows the scour resulting from Experiment No. 46, with a deck sloping 1 on 4 from an elevation corresponding to —25. The discharge concentration on the deck, beyond the piers, was 852 cu ft per sec per ft of width. The model river material was scoured to the flume bed—

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[16] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[17] Year Book, Am. Soc. C. E., 1937, p. 8.

a depth corresponding to 50 ft on the prototype. If deeper material had been available it might easily have scoured to twice that depth. Of the 172 experiments made for scour prevention for this dam, Experiment No. 46, with the 1 on 4 sloping deck, showed the maximum scour. Fig. 25(*b*)

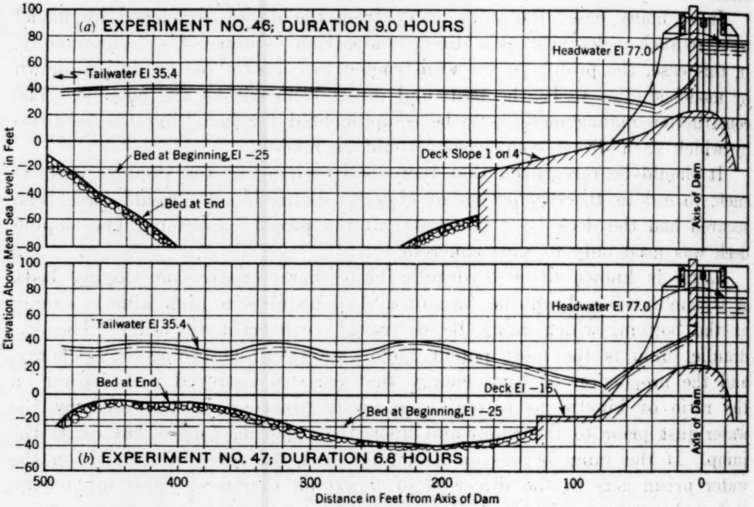

FIG. 25.—COMPARATIVE STUDY OF SCOUR, BONNEVILLE DAM, WITH A DISCHARGE OF 400 000 CUBIC FEET PER SECOND THROUGH EIGHT GATES

shows the results of Experiment No. 47 for the same discharge concentration, but with a horizontal deck corresponding to Elevation —16.0. The maximum scour corresponded to a depth of 15 ft. Although the length of run for Experiment No. 46 corresponded to 9 hr while that for Experiment No. 47 corresponded to 6.8 hr, the flume bed of Experiment No. 46 had been exposed completely and most of the scour had occurred in the first 7 hr. On the Bonneville deck it was necessary to place baffles[18] to insure the jump forming on the deck or apron and to aid in dissipating the energy.

Fig. 3 shows that if a horizontal apron could have been built at Elevation 83 a good correspondence between height of jump and the normal tail-water level could have been secured for practically the entire range of discharge. By proper manipulation of the crest gates as good correspondence might also have been had with a lower apron. This point is well illustrated in Fig. 26 which shows the correspondence between the height of the jump and the tail-water level for the Bonneville Dam. Fig. 26(*a*) shows that with twelve gates open equal distances, the deck should have been placed at Elevation —7.0 for good correspondence, for dis-

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[18] See *Engineering News-Record,* January 14, 1937, p. 61, for description of the baffles used.

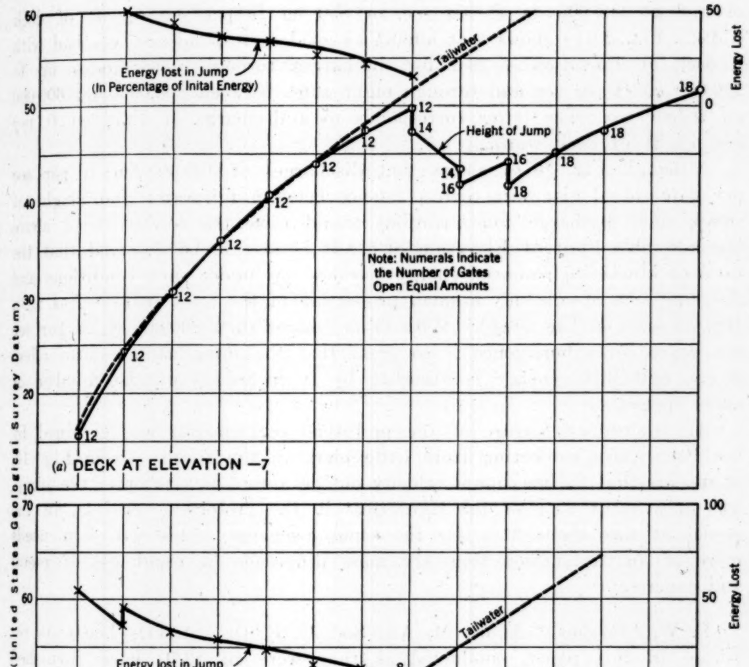

FIG. 26.—CORRESPONDENCE OF TAIL-WATER LEVELS WITH COMPUTED HEIGHT OF HYDRAULIC JUMP

charges to 600 000 cu ft per sec, or 850 cu ft per sec per ft of deck width. Fig. 26(*b*) shows that almost as good correspondence was had with a deck at Elevation — 16.0 for discharges through seven gates up to 200 000 cu ft per sec and through eight gates between 200 000 and 600 000 cu ft per sec. The latter corresponds to a discharge of 1 275 cu ft per sec per ft of deck width.

Although Fig. 26(*b*) shows that discharges of 100 000 cu ft per sec per gate would give fair correspondence between tail-water and height of jump, such discharge concentrations would raise the forebay level above the allowable limit of Elevation 82.5. It is also to be observed that the ratio of kinetic to potential energy becomes low under such conditions and the jump can absorb only a small proportion of the total energy—less than 10% as seen in Fig. 26(*b*). With flows greater than 800 000 cu ft per sec the degree of submergence is so great that the jump cannot form effectively, and little energy is absorbed by it regardless of the number of gates opened.

An interesting feature of the prototype performance was obtained by the Pitot tubes projecting from little piers on the apron. From Fig. 16, it appears that the maximum velocity on the apron, just prior to the jump, was only about 69% of the theoretical in the prototype, whereas, in the model, it was about 91% for the same discharge. This variation needs more of an explanation than the mere difference in roughness of metal and concrete.

F. W. EDWARDS,[19] ASSOC. M. AM. SOC. C. E. (by letter).—The tests reported in this paper, conducted as they were in 1931, have furnished a pattern for numerous later experiments on similar structures.[20] In addition, the paper describes the preparations made for comparative tests on the prototype. Unfortunately, as repeatedly emphasized by the author, the spillway discharges to date have been insufficient in magnitude to make direct comparisons with the model. Extrapolations in studies of this nature are very unsatisfactory. However, the paper should serve to stimulate interest in obtaining data, and thus insure complete measurements on the prototype when the opportunity is presented.

Model Tests.—It is interesting to review the conclusions reached from the model tests of Madden Dam in the light of the results obtained from the model tests of the Conchas Dam in New Mexico,[21] conducted at the U. S. Waterways Experiment Station in 1936.

Conclusions (1), (5), (6), (7), and (10), in Part I of the paper, were entirely substantiated by the Conchas Dam tests. Conclusions (8) and (9), when qualified slightly, were also substantiated by the Conchas Dam tests.

The tests at Conchas Dam indicated that the apron could be shortened by use of a sill (see Conclusion (8)) but, in addition, they indicated the advantages of an end sill when the apron of the stilling-basin extended down stream beyond the point of full recovery for the hydraulic jump. Several types of end sills, including the sloped-face type, were tested, and a stepped sill was selected as the best of those investigated. The essential factor, therefore, is the sill rather than the specific type of sill.

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[19] Chf. of Hydr. Section, U. S. Engr. Office, Second New Orleans Dist., New Orleans, La.

[20] "Hydraulic Model Tests, Madden Dam," *Engineering News-Record*, Vol. 109, 1932, p. 42.

[21] "Model Tests of Conchas Dam Stilling Basin," U. S. Waterways Experiment Station *Technical Memorandum No. 105-1*, July 21, 1936, addressed to The Dist. Engr., U. S. Engr. Office, Tucumcari, N. Mex.

No doubt, the plan suggested in Conclusion (9) produces the very best conditions, but the tests for Conchas Dam indicated that where the stilling-basin is confined within a relatively narrow canyon, the side walls could be sloped off on the down-stream end without seriously interfering with the jump action. As far as stream-bed velocities and erosion down stream were concerned, the essential feature was an adequate apron having an end sill.

Conclusions (2), (3), and (4) were not factors in the design of Conchas Dam, where a horizontal apron was feasible.

The final conclusion, (11), listed by the author in Part I, incorporates more than its first sentence would imply. Although the writer agrees thoroughly with the first part of the paragraph, the remainder appears to delve too much into matter of opinion instead of presenting a definite conclusion based upon the model tests.

There are two schools of thought regarding baffles. One believes that baffles are absolutely essential in combination with the hydraulic jump to help dissipate, sufficiently, the excess energy in the over-fall from a high dam. The other school believes, like the author, that baffles are actually dangerous. It appears that a compromise might be effective in some cases. This was the attitude taken at Conchas Dam where one row of baffles, placed far enough down stream to avoid excessive impact, was used. The stilling-basin selected from the model tests performed satisfactorily when equipped with an end sill alone, but the one row of baffle-blocks was added principally to improve the performance of the basin when the conduits were discharging alone. The baffles definitely improved the action of the jump for high spillway discharges as well, and were considered an additional factor of safety in the design. By placing the baffles relatively far down stream, full advantage was taken of the impact of water on water before giving additional aid with the baffles.

Prototype Tests.—The agreement between corresponding nappes and pressures over the crest of the spillway for the model and its prototype appears to be highly satisfactory. The indicated energy gradients also appear to be in satisfactory agreement. However, the implication regarding coefficients of discharge for the spillway (see Fig. 15) should not be overlooked. Judging from the results of others, whose methods of measuring discharge were superior to those described for Madden Dam, one is inclined to accept the model results at least for the maximum heads. No doubt, viscosity and surface tension do affect coefficients of discharge for very small heads in models, but it seems unlikely that the effect would be great for heads of 0.45 ft (32 ft in Nature), the maximum head on the model of Madden Dam.

It would be extremely interesting and worth while to the Engineering Profession if measurements could be made upon the spillway similar to those made by Albion Davis, M. Am. Soc. C. E., and the late Floyd A. Nagler, M. Am. Soc. C. E., on the Keokuk Dam,[22] and to those made by Louis G. Puls, Assoc. M. Am. Soc. C. E., on the Wilson Dam.[23] These measurements would be especially valuable because of the comparison that could be made with the relatively small model.

In discussing friction loss at the toe of the dam, the author states that, "the difference between the velocity head as indicated by the Pitot tube readings and the theoretic velocity head (which is the difference between water surfaces) was taken as the loss in head due to friction on the overflowing sheet of water." The statement in parenthesis indicates that the correct value for the loss of head was used; but it is incorrect to state that the theoretical velocity head equals the difference in water surfaces, or that the head loss is the difference between the measured and the theoretical velocity heads. To obtain the true loss in head, it is necessary to take the difference between the theoretical energy line (theoretical depth plus theoretical velocity head) and the actual energy line (actual depth plus actual velocity head). The fact that the theoretical depth is different from the actual depth makes the author's statement incorrect.

Although the abscissas in Fig. 17 are entitled, "Values of h_f, Head Loss at Toe of Dam of H Height," it appears that the numerical values shown are for the Madden Dam, of specific height.

MORROUGH P. O'BRIEN,[24] ASSOC. M. AM. SOC. C. E.—The use of the hydraulic jump at the base of a spillway for dissipating energy has been frequently discussed in the technical journals, but several features have not been sufficiently emphasized. The sloping apron of the Madden Dam spillway illustrates one of these points.

In the analysis of the jump, the hydraulic elements are obtained from the summation of pressure force and rate of transportation of momentum. This analysis gives the depth below the jump, but fails to give an adequate explanation of the fact that energy is dissipated in a flow system assumed to be frictionless. The explanation lies in the moments applied which are of such a nature as to produce rotation. At the down-stream section, the same total energy per unit weight is present in mechanical form (potential plus kinetic), but a portion of the kinetic energy is accounted for by the rotation of small masses of fluid and is unavailable. Stated more simply, the adverse slope of the water surface and large bottom velocities combine to bring about a rotation which is the first step in the final conversion of a part of the mechanical energy into heat. The practical significance of this fact is that "drowning" of a jump reduces this moment by eliminating the sloping water surface and results in less effective operation. The sloping apron at Madden Dam prevents drowning and thus increases the energy dissipation over a range of discharges. Only in the event that the back-water curve from the next control down stream produces submergence at each discharge exactly equal to the "jump depth" can a horizontal apron be as effective as a sloping apron which produces a true jump but prevents drowning.

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[22] "Experiments on Discharge Over Spillways and Models, Keokuk Dam," *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 777.

[23] "Spillway Discharge Capacity of Wilson Dam," *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 323.

[24] Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

Information on the jump produced at the bottom of a steeply sloping channel is meager. Fig. 27 shows data obtained in a small laboratory channel by Lieut. B. D. Rindlaub, Corps of Engineers, U. S. Army.[25] The dimensionless variables entering the problem are, $\frac{d_2}{d_1}$; $\frac{l}{d_1}$; and, $\lambda = \frac{Q^2}{g\,d^3}$. The curves show constant values of $\frac{l}{d_1}$ at three slopes. The dashed line, $\frac{d_2}{d_1}$, is a function of λ for horizontal channels.

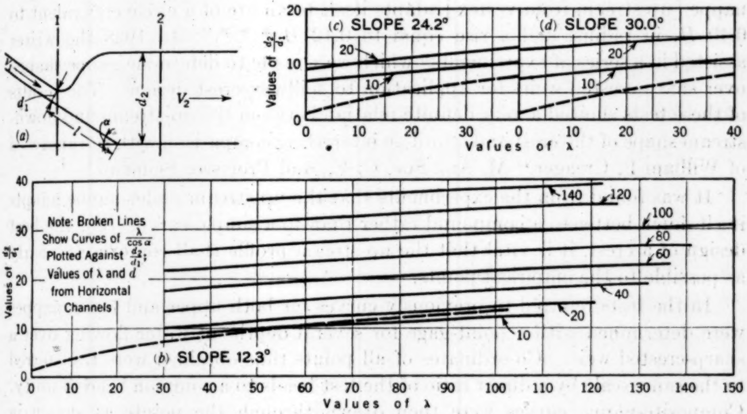

FIG. 27.—RELATIONSHIP OF $\frac{d_2}{d_1}$ TO λ FOR CONSTANT VALUES OF $\frac{l}{d_1}$ AT VARIOUS BOTTOM SLOPES

RICHARD R. RANDOLPH, JR.,[26] ESQ. (by letter).—Professor Scimemi states that the profile of the spillway crest at Madden Dam "is neither the most economical nor the most suitable one for a spillway dam." The adopted crest profile resulted from structural rather than hydraulic considerations. In order to accommodate the drum-gates (which are the largest built to date, 1938) in a well in the crest, it was necessary to broaden the profile beyond one that is theoretically better hydraulically. A drop in the coefficient of discharge resulted, but it was found that the capacity was still ample to handle the maximum expected flow. For most of the time the discharge will be regulated by lowering the drum-gates, and for this condition, with the water passing over the tops of the gates, a consistently high coefficient is obtained (see Fig. 10). A better arrangement in which the theoretical profile was maintained, was later developed by the U. S. Bureau of Reclamation for a similar drum-gate installation at Norris Dam.

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[25] Unpublished Master's Thesis, Univ. of California, May, 1935.

[26] With U. S. Bureau of Reclamation, Denver, Colo.

It would not seem advisable to neglect the shape of the dam crest up stream from the apex, as the down-stream profile is entirely dependent on the preceding curvature. It is presumed that Professor Scimemi has reference only to the illustration as presented in his discussion when he stated that "it is not necessary to describe a complete curve for the up-stream part of this vertex. The lower nappe [up stream from vertex] adapts itself to an arc of a circle equivalent to 0.40 H, in radius, and a rise equal to 0.12 H * * *." In 1928 the writer assisted in a series of experiments[27] which were made to determine nappe shapes over sharp-crested weirs for application to spillway-crest design. The results of these tests showed a very definite relation between the up-stream and down-stream shape of the crest and afford an interesting comparison with the analyses of William P. Creager,[13] M. Am. Soc. C. E., and Professor Scimemi.

It was found from the experiments that the up-stream under-nappe adapts itself much better to a compound rather than to a simple curve. For the best design of a crest, it is vital that the up-stream profile shall conform as nearly as possible to the empirical points.

In the tests referred to previously curves for both upper and lower nappes were determined with a point-gage for several depths of water flowing over a sharp-crested weir. Co-ordinates of all points thus obtained were transferred to the same scale by a direct ratio of the test heads to a common head of unity. Composite-nappe curves were then drawn through the points as shown in Fig. 28. The co-ordinates illustrated were scaled to averaging lines as drawn. The points were obtained by stepping the measurements of the nappes to one common unit head. To design the crest, H was taken as the maximum and the curve was proportioned in terms of this head, as shown. The coefficient of friction, C, in Equation (4) will be 3.97 at the designed head.

The part of the lower nappe curve that is up stream from the apex fits very closely to a compound curve with a radius of 0.5 H tangent at the apex, continuing from 0.18 H out, with a tangent radius of 0.2 H to the weir face, which is 0.287 H out from the vertex (see Fig. 28). Down stream from the apex, the lower nappe curve was found to have the approximate formula,

$$-Y = \frac{0.53}{H^{0.8}} X^{1.8} \qquad (13)$$

or, if $H = 1.0$; $Y = 0.53\,X^{1.8}$.

Equation (13) closely resembles Professor Scimemi's Equation (11) and there is little divergence between the plotted curves. The factor, H, must be introduced in formulas, however, before they can be used for design at any head, as is done in Equation (13). It would not seem necessary to use Equation (12), whose curve would lie above the free nappe curve, for the design of the down-stream part of the dam crest if the crest is designed for the maximum expected head, as in Equation (13).

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[27] "Experiments Made in the Hydraulic Laboratory of the Alabama Power Company," by I. A. Winter, Assoc. M. Am. Soc. C. E., Director, G. J. Hornsby, E. B. Coleman, and Richard R. Randolph, Jr., Assistants. (Not published.)

The coefficient of discharge of the sharp-crested weir used in the experiments was found to be 3.3 for all the heads tested. The distance from the apex of the lower nappe curve to forebay level was scaled from the nappe curves obtained from point-gage readings at four different depths over the weir. If this distance is taken as the head on the crest, a coefficient of discharge of 4.0 resulted in each case. This substantiates the accuracy of the testing and justifies the method of transposing test points to a common head in order to obtain the composite nappe curves.

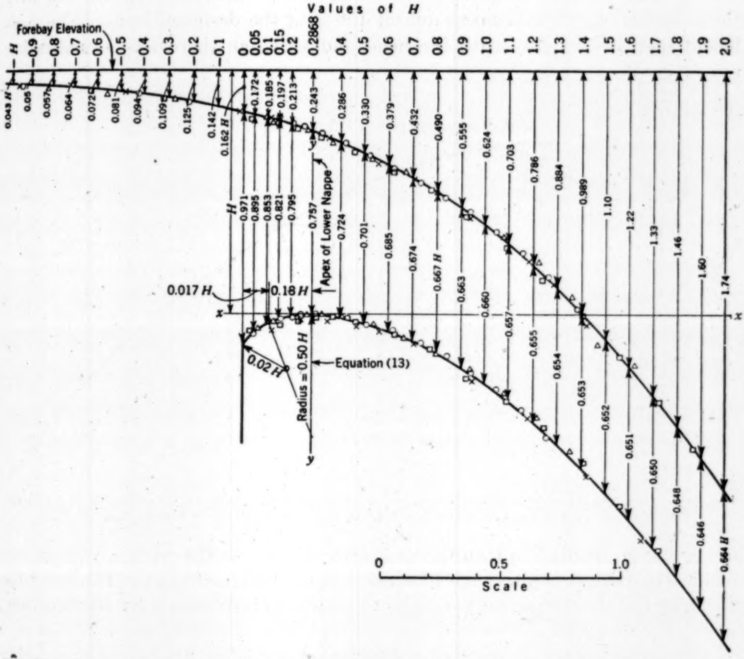
Values of H
Forebay Elevation
−H
−0.9
−0.8
−0.7
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.05
0.1
0.15
0.2
0.2868
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
0.043 H
0.05
0.057
0.064
0.072
0.081
0.094
0.109
0.125
0.142
0.162 H
0.172
0.185
0.197
0.213
0.243
0.286
0.330
0.379
0.432
0.490
0.555
0.624
0.703
0.786
0.884
0.989
1.10
1.22
1.33
1.46
1.60
1.74
H
0.971
0.895
0.853
0.821
0.795
0.757
0.724
0.701
0.685
0.674
0.667 H
0.663
0.660
0.657
0.655
0.654
0.653
0.652
0.651
0.650
0.648
0.646
0.664 H
Apex of Lower Nappe
0.017 H
0.18 H
x
x
y
y
0.02 H
Radius = 0.50 H
Equation (13)
0
0.5
1.0
Scale

FIG. 28.—CALIBRATION OF FREE NAPPE OVER VERTICAL SHARP-CRESTED WEIR. DIMENSIONS OF EMPIRICAL CREST S C AND STABILITY DIAGRAM, MAY 31, 1928

If a spillway crest is made to conform to the profile of the under-nappe of the overflowing sheet, a coefficient of discharge of 4.0 should obtain, and no pressures should exist along the crest. A model of such a crest was made and tested. Actually, a coefficient of 3.97 was obtained indicating only a little drag on the structure. Piezometers along the crest of the model showed no deflection when the exact head of water was overflowing for which the crest profile was designed. When the head was lowered to 66% of the designed head a pressure resulted along the crest and when the head was raised to 133% of the designed head negative deflections were produced in the piezometer columns. Computations were made of the resultant of pressures acting on the crest, assuming two-thirds uplift, and it was found that the stability of the crest was actually threatened when the overflow head was 133% of the designed head.

It has been advocated that crests be designed for heads less than the maximum which will be obtained over them in order to increase the discharge. The danger of this is indicated, and it would be necessary to investigate, thoroughly, the stability conditions as well as possible cavitation effects before such a design is adopted.

The coefficient of discharge was obtained for various depths of flow over the model crest, up to a maximum of 150% of the designed head. The coefficient increases rapidly with the head and although there was no indication of the nappe tending to leap the down-stream face of the crest, a vacuum immediately exists when the overflow exceeds the designed head. The curve for coefficient of discharge was found to be closely approximated by the equation,

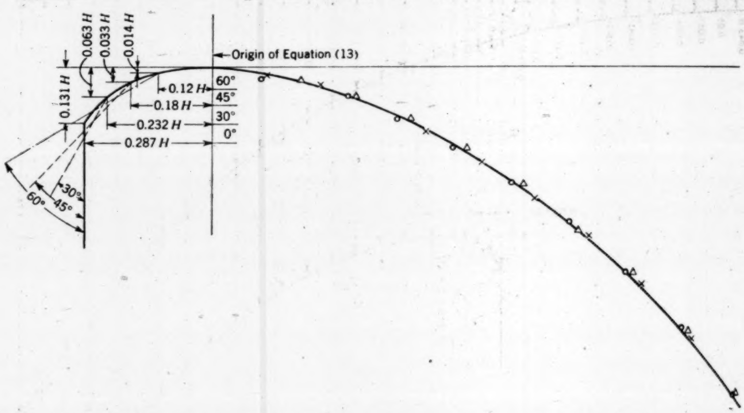

FIG. 29.—CALIBRATION OF NAPPES OVER SHARP-CRESTED WEIR WITH SLOPED FACE. DIMENSIONS OF PIER S C AND EMPIRICAL CREST S C WITH SLOPED FACE, MAY 4, 1928

$$C = 3.97 \left(\frac{h}{H} \right)^{0.17} \qquad \text{(14)}$$

in which, h = any depth over the crest; and H = the designed head of the crest. On the theory that contraction is the same on the sides as on the bottom of an orifice, an empirical pier was laid out using the same curvature for the pier nose as was obtained for the crest. Tests showed that the coefficient of discharge decreased only a little when such a pier was used, the resulting equation being:

$$C = 3.95 \left(\frac{h}{H} \right)^{0.17} \qquad \text{(15)}$$

To ascertain the effects of a sloped-face weir, a series of tests was made with the sharp-crested weir inclined at various degrees with the vertical. The overflow head was kept at the same depth, as nearly as possible, for each of the three inclinations tested.

By transferring the co-ordinates of the measured nappe points to a common head of unity, a composite curve is obtained as given in Fig. 29. It is seen that the sloped faces cut the nappe curve, as obtained for a vertical face, at various points, but the slopes are not tangent to the crest curve. Intersection points of slopes other than those tested may be interpolated.

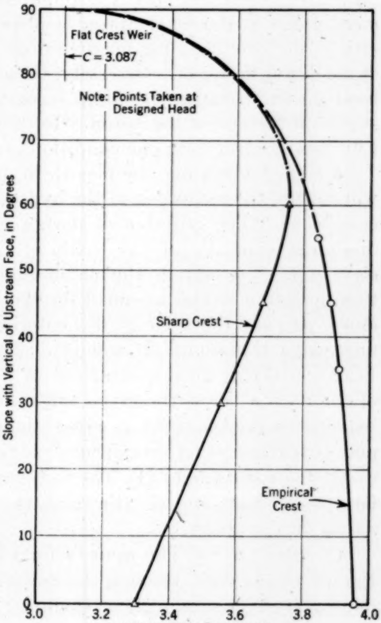

FIG. 30.—EFFECT OF SLOPE TO UPSTREAM FACE OF DAM

The effect of sloping the face was to cause an immediate, although not rapid, decrease in the coefficient of discharge (see Fig. 30). This is probably due to friction of the flow on the long sloping face, as the slopes were carried down to the bottom of the approach channel which was a distance 4 H below the crest. The discharge of the sharp-crested weir increased for inclinations as steep as about 62° with the vertical, probably due to the lessened contraction of the jet. Both curves (Fig. 30) are assumed to drop off to a coefficient of 3.087 for a flat-crested weir although this assumption was not tested.

Discussions.—Mr. Stevens questions Conclusion (2) that the jump can be "controlled best by placing a sloped apron down stream * * *," and states further "that there is no virtue in the sloping deck, as such, to aid in the dissipation of energy." Conclusion (2) stated that the jump could be better "controlled" on the sloping apron; and did not state that the jump was more effective. The writer had previously stated that "the steeper the slope of the apron the less perfect is the jump formation" (see heading, "Hydraulic-Jump Apron: Expected Jump Performance"), but that good dissipation could be obtained on a slope if it were not made too steep, 1 on 4 being approximately the maximum. Control of the hydraulic jump was intended to mean that an effective jump must be made to form well up on the concrete apron for any and all discharges and for various depths of tail-water. To this end it was thought that a sloped apron was better adapted than a level apron. For a level apron the tail-water must furnish a depth over the apron that exactly corresponds to the required jump depth in each and every case and over the entire range of flow. Such a coincidence is very rarely found. The tail-water elevations predicted for the higher flows (and the tail-water for the designed maximum flow is invariably a prediction) are in themselves liable to considerable error. The tail-water curve may also be changed after the dam is built due to old coffer-dams being left in, or due to erosion of the stream bed and lack of sediment deposition by the clear water overflowing the dam, so that in such a case the criterion for design of the level apron is in itself subject to considerable variation.

In the great majority of cases it will be found that the plotted curve of the tail-water depth will cross the required jump-depth curve at an angle. This means that if the level apron is placed at a depth below the tail-water to satisfy the condition for maximum discharge, the jump will be drowned out at the lower flows; or, conversely, there will not be enough tail-water over the level apron to cause the jump formation, and the high velocity sheet may sweep off the end of the apron. In other words, the level apron, theoretically, will usually meet only one condition of flow.

A sloped apron may be placed, in proper reference to the tail-water, which will control the formation of the hydraulic jump for any variation of tail-water conditions. The criterion of design for the apron is usually the maximum discharge capacity of the spillway, resulting in the maximum tail-water elevation. To compute the maximum required jump-depth, D_2, for this condition, a datum is first assumed for the apron level. Referring to Fig. 31, the depth, D_1, and the velocity, V_1, entering the jump are computed at the datum line, using Equations (2) and (3). The jump-depth, D_2, is then found from Equation (1); it must correspond to the depth first assumed for the distance of the datum below the given tail-water elevation. The factors are made to balance by trial and error. Investigation of flows lower than the maximum may reveal one of two conditions existing, if the apron is assumed level at this computed datum line: (1) The tail-water may be too deep and the jump will be drowned out; and (2) the tail-water will be too shallow and the jump will sweep off the apron.

Condition (1).—The remedy for Condition (1) is to raise the apron near the toe of the dam, keeping the down-stream end at the computed maximum depth, D_2. This will allow shallower depths over the sloped apron, which will satisfy the required jump-depths for the lower flows. Tests at Madden Dam indicate that the apron may be raised as much as 0.2 D_2 (maximum) without materially affecting the jump performance at maximum discharge $\left(\text{see Fig. 9; } 1 - \frac{d_s}{d_1}\right)$. The slope of the apron would be 0.0667 if the length is taken as 3 D_2, a conservative value (see Fig. 31). If the apron slope is made steeper than this value, the length of the apron must be increased, as the beginning of the jump at maximum discharge will move down the apron and the distance must be added down stream to afford equal protection.

Condition (2).—The remedy for Condition (2) is to slope the apron. If at some critical lower discharge it is found that the natural tail-water depth is too shallow for jump formation, the apron may be sloped down into the tail-water until the required jump-depth is encountered. The slope may be made as steep as 1 on 4 (maximum). The upper end of the apron must be raised 0.2 D_2 to keep from drowning the jump at maximum discharge (see Fig. 31).

Somewhere along its length, such a sloped apron will furnish the required jump-depth for all stages of flow.

Condition (3).—A third condition may be found to exist in the relation of the jump-depths and the tail-water. The foundation rock may lie so deep under the tail-water that the jump on an apron at this elevation will be drowned out for all stages of discharge. Again, the remedy is to raise and slope the apron. At the toe of the dam the apron is raised 0.2 D_2 (maximum), and sloped down stream to bed-rock. The slope should not be steeper than 1 on 4

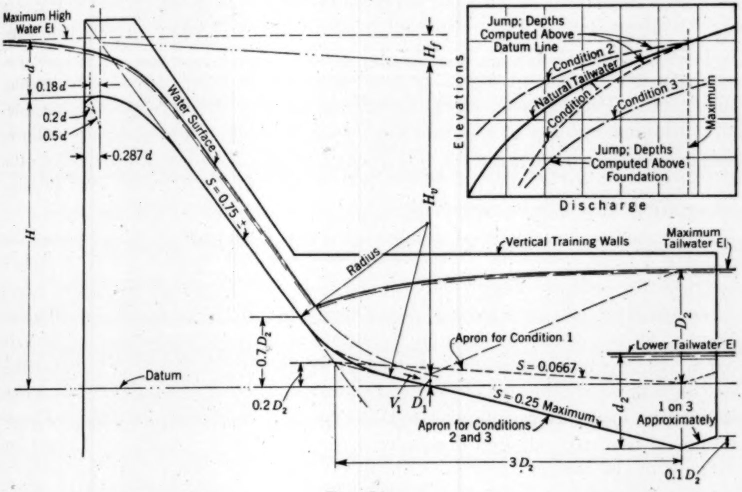
Maximum High Water El
d
0.18d
0.2d
0.5d
0.287d
H
Water Surface
S = 0.75 ±
H_f
H_v
Radius
Datum
0.7 D_2
0.2 D_2
V_1
D_1
Apron for Condition 1
S = 0.0667
S = 0.25 Maximum
Apron for Conditions 2 and 3
Vertical Training Walls
Maximum Tailwater El
D_2
Lower Tailwater El
d_2
1 on 3 Approximately
3 D_2
0.1 D_2
Jump; Depths Computed Above Datum Line
Condition 2
Natural Tailwater
Condition 1
Condition 3
Maximum
Jump; Depths Computed Above Foundation
Elevations
Discharge

FIG. 31

and the length of the apron should be at least $3D_2$ (maximum). The apron is joined to the down-stream face of the dam by a radius tangent at a distance, $0.7\,D_2\left(\text{see Fig. 9; } 0.2\,D_2 + \frac{h_e}{d_i}\right)$ above the datum line (see Fig. 31). The beginning of the jump for maximum discharge will then be at the point of curvature of the bucket radius and the jump will not be drowned out. At less discharges the jump will form lower down the slope, thus affording protection throughout the total range of flow.

This condition was encountered at Madden Dam. The sloped apron was adopted mainly for economy of concrete. However, there were other inherent advantages. On a sloped apron the jump is stabilized, forming in a straight line across the channel. The slope allows for an adjustment of the jump for tail-water variations, if the actual tail-water does not conform to the predicted curve used for the model tests.

As Mr. Stevens stated, a good correspondence was obtained with a level apron located at Elevation 83 (see Fig. 3), assuming, of course, that the tail-water curve is absolutely correct. He further states that "by proper manipulation of the crest gates as good correspondence might also have been had with a lower apron." As applied to the present case, the writer does not agree. With all crest-gates fully opened at the maximum forebay level, an apron at Elevation 83 was required for the jump. On a lower apron the jump would have been drowned out and little dissipation of energy would have been accomplished.

On a plain level apron, the jump is comparatively unstable, moving up and down the apron with slight tail-water variations. It is practically necessary to place baffles or large sills on a level apron, in order to stabilize the jump and assure its formation on the apron. Such baffles and sills do allow the use of a shorter apron but will not function properly unless there is a depth of tail-water over them equal, at least, to the required jump-depth. Their principal capacity is to break up flow concentrations and to diffuse the stream leaving the apron. The dentated sill or the diffuser sill is well adapted to this purpose.

A sloping apron will stabilize the jump, causing it to form in a uniform line across the channel, and if the slope is properly placed in respect to the tail-water, a good "control" of the hydraulic jump is effected and velocity retardation will be quite satisfactory.

Due to the variations of conditions, both structural and operating, encountered for different projects, it becomes practically impossible to formulate set rules for spillway design. The pure hydraulic-jump apron functions best when the flow can be discharged uniformly along the apron, as at Madden Dam. At Bonneville Dam, due to submergence of the spillway crest, gate operation becomes a factor in the design of the stilling-basin. It is for this reason that hydraulic model studies become a practical necessity. Their cost is usually more than compensated for by the economies effected. Proper design is assured and many problems can be solved satisfactorily that are difficult of theoretical analysis.

Mr. Stevens condemns the sloping apron, from comparison of the erosion tests made in the model studies of Bonneville Dam as illustrated in Fig. 25. He offers no proof that the velocity retardation is more effective on the level apron. Even if the velocity at the end of each apron was the same, and if this velocity is directed downward on the river bed, as is the case with the illustrated sloping apron, it is natural that more erosion would occur. The apron would need to be extended down stream to cover a greater part of the jump action so that the velocities off the end are insufficient to cause damage to the river bed. The apron could have been shortened by placing a deflector sill at the end.

It would seem, however, that the sloping apron, as illustrated in Fig. 25, is improperly located to obtain the most effective jump formation. Apparently, the beginning of the jump is forced too far back against the face of the dam, which indicates that lowering the apron would be beneficial. Such a sloped apron at Bonneville may have effected quite an economy in apron concrete. Mr. Stevens does not state the results of tests on the sloped apron using the adopted baffles or a deflector sill similar to one used at Madden Dam.

Mr. Stevens notes from Fig. 16 that the velocity at the toe of the dam is only 69% of the theoretical velocity whereas on the model "it was about 91% for the same discharge"; and questions whether this variation is due to the difference in roughness of metal and concrete. His values of 69% and 91% apparently refer to the velocity-head ratios and not to the velocity ratios, which are about 83% and 95 per cent. This indicates a smaller variance in the velocity ratios than was stated by Mr. Stevens; and it would be even less if the discharges were the same. Values given for the model correspond to a 11.25-ft depth over the crest and those for the prototype are for a 7.78-ft depth. The equivalent model discharge is considerably greater. The percentage of the theoretical velocity head manifested at the toe increases with the discharge so that for a depth of 11.25 ft over the prototype the ratio, 83%, increases and, if the discharges were the same, there would actually be less difference between the comparative prototype ratio and the model ratio of 95 per cent.

Mr. Stevens would probably attribute the difference in this ratio to the air content of the water. For shallow depths of overflow there is no question but that the entrained air will affect the velocity of the sheet; but at greater depths the overflowing sheet may reach the toe of the dam in a solid mass with little or no entrained air present. Fig. 21 shows the condition of the overflow sheet just prior to the jump. The depth over the crest is only 8.0 ft, and yet there is indication of only a slight aeration in the sheet between the piers. For greater depths the only retarding factor for practical consideration would be friction.

Application of the existing friction-loss formulas to flow down steep chutes seems to be impractical; but to state that the velocity of the overflowing sheet has an upper limit of about 80 ft per sec seems equally impractical.

The friction loss will depend on the depth of overflow as well as the length of the chute and the velocity of the water. As a matter of practical usefulness, it would seem desirable to obtain an empirical relation of the governing factors in as simple an expression as possible, as is done in Equation (3). It was with this thought in mind, aside from the idea of checking prototype against model results, that the writer conceived of, and designed, the Pitot tube pedestals placed on the apron at Madden Dam.[28] It is to be hoped that readings on the tubes may be obtained for greater depths over the crest than were available during these tests. The writer has urged the placing of these tubes on the aprons of other dams designed by the U. S. Bureau of Reclamation, and it is also to be hoped that other investigators may take similar steps to obtain needed data on this debatable subject.

Importance of Hydraulic Jump.—It is gratifying to note that Mr. Edwards gives due credit in stating that "the tests * * *, conducted as they were in 1931, have furnished a pattern for numerous later experiments on similar structures." The original report[2] was widely circulated among Government departments and libraries. It may be said that the Madden studies created a "hydraulic-jump consciousness" in relation to its use for stilling-basins at the toe of overflow dams. Certain phases of the laboratory technique have also been widely used, making speedy construction and testing of models possible.

The program for prototype testing pioneered in its comprehensiveness, and it is indeed regrettable that it reached a state of indifferent stagnation before all possible data were obtained.

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[28] *Engineering News-Record,* February 10, 1938, p. 237.

The stilling-basin design for Conchas Dam again demonstrates the value of the use of the hydraulic jump. It becomes a practical necessity, however, to use comparatively heavy sills on a level apron in order to stabilize the formation of the jump.

A sloped apron allows much greater latitude in design assumptions and will stabilize the jump without the use of large blocks or sills, which are susceptible to damage. As part of his design for the stilling-basin at Bonneville Dam, Mr. Stevens includes a caisson to be used for "examination and repair" of the baffles.[18]

Professor O'Brien substantiates the writer's contention that the jump can be "controlled best by placing a sloped apron down stream." One of his closing statements may well be used to summarize this discussion: "Only in the event that the [true] back-water curve from the next control down stream produces submergence at each discharge exactly equal to the 'jump-depth' can a horizontal apron be as effective as a sloping apron which produces a true jump but prevents drowning."

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EFFECT OF DOWEL-BAR MISALIGNMENT ACROSS CONCRETE PAVEMENT JOINTS

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With Discussion by Messrs. L. W. Teller, David J. Peery, L. J. Mensch, W. O. Fremont, George A. Smith, L. E. Grinter, and Arthur R. Smith and Sanford W. Benham.

Synopsis

The subject of joints in concrete pavements, to provide for expansion and contraction, and the means of transferring load across joints, is of particular interest to those engaged in highway construction. Common dowel bars, passing through the joint and embedded in the two adjacent slabs, have long been used for this purpose. The importance of installing the bars correctly, that is, perpendicular to the cross-section of the pavement, will be recognized since dowels, incorrectly installed, will not slip freely in the concrete. This paper describes an investigation to determine the accuracy with which road-building contractors were installing dowel bars, and another investigation which was conducted to determine, by experiment, the degree of precision necessary to assure satisfactory functioning of joints.

Introduction

In accordance with the specifications of the State Highway Commission of Indiana, 1-in. expansion joints are installed in concrete pavement at 80-ft intervals, with a transverse contraction joint in the center of each 80-ft section. As a means of transferring load across joints, dowel bars of intermediate grade steel (0.75 in. in diameter by 24 in. long) at 12-in. centers, are placed in the pavement across all contraction joints and across some types of expansion joints. To prevent bond, the bars are coated with an asphalt paint and are also oiled. Fig. 1 illustrates an assembly of

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Note.—Published in June, 1937, *Proceedings*.

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twenty dowel bars wired to two spacer bars, as installed across expansion and contraction joints in 20-ft pavement.

The importance of placing the bars correctly, that is, perpendicular to the cross-section of the pavement, will be readily appreciated when it is realized that if they are imperfect in this respect, binding between them and the concrete will occur when the joint opens or closes. As a consequence, either the bars will become distorted or the concrete will be damaged in the vicinity of the bars.

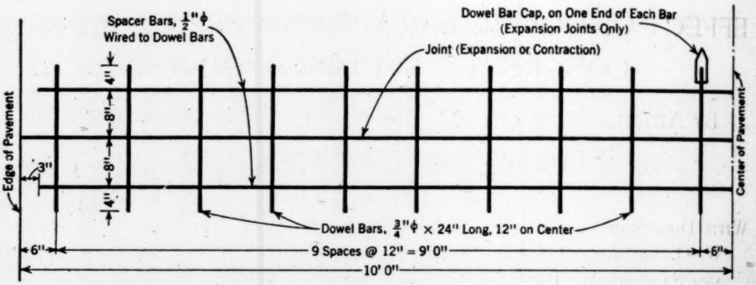


Fig. 1.—Typical Dowel-Bar Assembly in 20-Foot Pavement.

A faulty installation of dowel bars across a contraction joint constitutes a serious hazard. When contraction of the pavement opens the joint cracking or spalling of the concrete may occur. If the concrete is strong enough to resist such damage, however, the joint, instead of being a plane of weakness, may offer more resistance to movement than the tensile strength of the unimpaired cross-section of the pavement. In this case the contraction joint will not fulfill its function and cracks will occur in the pavement at random as if no provision had been made to locate them. In like manner, a faulty installation of dowel bars across an expansion joint will cause damage to the concrete, the bars, or to both, when expansion of the pavement reverses the process and closes the joint. In this case, also, if the concrete and dowel bars offer sufficient resistance to such damage, the joint will not fulfill its function and there will be no provision for expansion. Therefore, to avoid damage to the pavement in the vicinity of expansion and contraction joints the dowel bars must be placed in their proper positions and held in those positions by some method that will prevent their movement during the placing and finishing of the concrete.

During the summer of 1935, the Bureau of Materials and Tests of the State Highway Commission of Indiana, conducted a number of field tests (Special Investigation *A*-7) to determine the accuracy with which dowel bars were being installed by various contractors. When a dowel bar is in perfect position, that is, perpendicular to the cross-section of the pavement, the projected length of its axis on the cross-section is zero. The projected length is equal to the hypotenuse of a right triangle, the altitude of which is the vertical error of the bar, that is, the difference in elevation between the two ends of the bar with respect to the surface of the pavement. The base of the triangle is the horizontal error of the bar, that is, the difference between the distances from the edge of the pavement to the two ends of the bar. Thus, dowel-bar error which, in this paper, is defined as the projected length on the cross-section of the pavement per 22 in. of dowel-bar length, is equal to the square root of the sums of the squares of the horizontal and vertical errors. Although the bars are 24 in. long, the field tests were conducted more easily by choosing points 1 in. inward from each end of the bar to represent the ends of the bar. In conducting the tests, data were obtained, by means of the apparatus described herein under "Methods of Test", from which it was possible to compute the error of each dowel bar in each installation checked. Several different methods of installing the bars were being utilized, namely:

(1) Sheet metal chairs, of the pin type, driven into the sub-grade, with the upper end bent around the dowel bar or spacer bar (see Fig. 2).

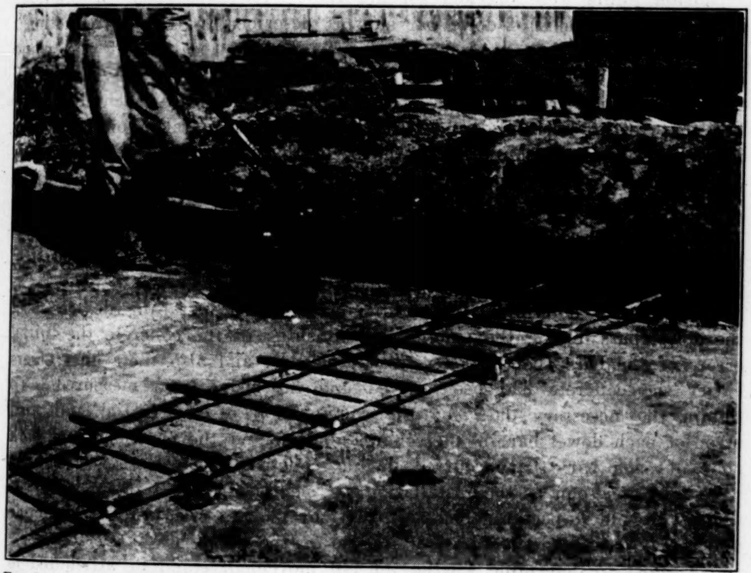


Fig. 2.—Sheet Metal Chairs of the Pin Type Driven into the Sub-Grade (Test No. 6).

(2) Welded assemblies, consisting of heavy, wire, **U**-shaped chairs welded to the spacer bars. The upper ends of the chairs are bent around the dowel bars and the horizontal part of the **U** rests on the sub-grade (see Fig. 3).

(3) Specially designed holders, which span the sub-grade and are supported by the forms. The dowels are thereby held during the placing of the concrete and are left "floating" in the concrete after the removal of the holder. One type of holder is shown in Fig. 4. No satisfactory

photograph is available of one that is supported entirely from the forms (independent of the sub-grade).

(4) Combinations of Methods (1) and (3).

Two claims of superiority for Method (3) have been advanced: First, the holder is supported by the forms, which determine the position of the cross-section of the pavement, thereby eliminating the objections to methods in which the positions of the dowels are governed by the contour of the sub-grade, which is never perfect; and, second, the installation of dowel bars is accomplished more quickly and easily with a holder than with chairs. The first claim, of course, is based on the assumption that the holder is made accurately, is sufficiently strong to resist distortion in handling, and that it is placed on the forms in the proper position.

METHODS OF TEST

Installations of dowel bars across nine contraction joints and across five expansion joints were checked for accuracy. These fourteen installations included the methods of dowel-bar support described in the "Introduction".

The contour of a concrete pavement and the location of the plane representing its cross-section are determined by the tops of the forms, since these lines are the paths of travel of the finishing machine screed. For that reason horizontal and vertical readings were taken on both ends of each bar from a reference line the position of which was definitely fixed with respect to the tops of the forms. This reference line, described briefly, consisted of a steel tape graduated to 0.01 ft, riveted to the lower flange of a 4-in. aluminum I-beam, spanning the sub-grade and resting on the tops of the forms, the points of support being three sharp pointed steel pegs.

In order to determine the rigidity with which installing devices held the dowel bars during the placing and finishing of the concrete the initial errors of the bars (before concrete was placed), and also their final errors (after the concrete had been placed and finished), were measured. To obtain the necessary data for the computation of the initial and final error of each dowel bar in a given installation and the movements of both ends of each bar, that occurred during construction operations, readings were taken as soon as the installation was completed and pronounced satisfactory by the inspector. The operations of placing and finishing concrete and the extent to which workmen stepped on the bars, were observed closely in an effort to determine whether or not movement could be attributed to carelessness. As soon as the finishing machine had passed over the joint for the last time, sufficient concrete was removed from the newly finished surface of the pavement to expose both ends of each dowel bar and permit final readings.

DISCUSSION OF RESULTS

The results of Special Investigation *A*-7 revealed that, in some of the installations checked, the bars were in extremely faulty positions. Twelve of the fourteen installations tested were in 20-ft pavement and two were in 10-ft pavement (widening projects). Consequently, the errors of 260

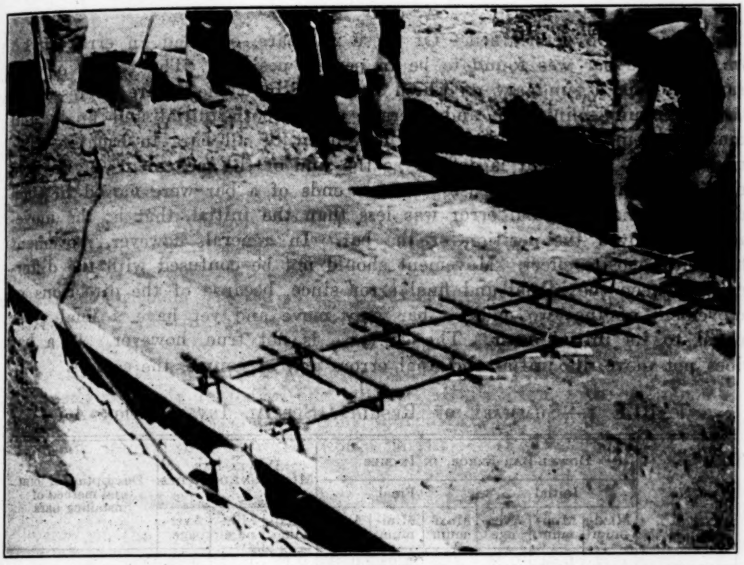


Fig. 3.—U-Shaped Chairs Resting on the Sub-Grade and Welded to the Spacer Bars (Test No. 12).

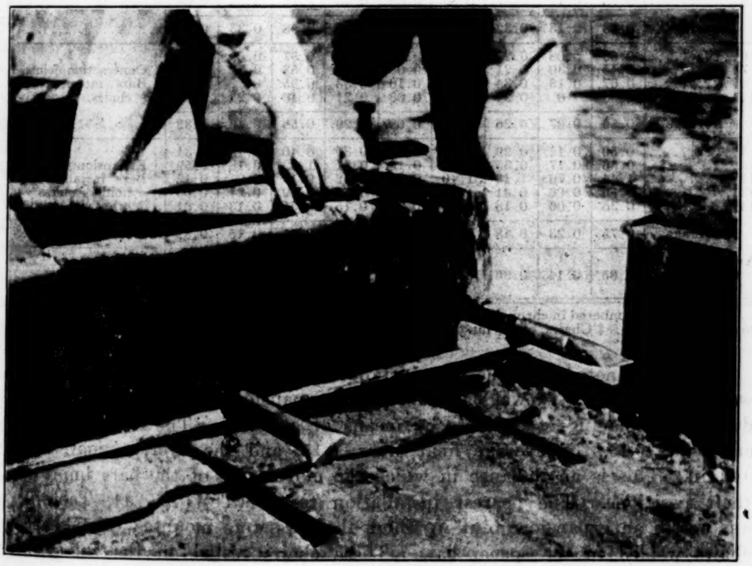


Fig. 4.—View Showing the Use of a Holder (Test No. 14).

dowel bars were determined. Of these 260 bars, one had an error of 1.5 in., and one bar was found to be in perfect position. The results of this investigation, a summary of which is shown in Table 1, are expressed as the maximum, minimum, and average error (both initial and final) and maximum, minimum, and average movement of all bars in each installation tested. Movement is defined as the sum of the movements of the two ends of a bar. In some cases the two ends of a bar were moved in such directions that the final error was less than the initial, that is, the movement improved the position of the bar. In general, however, movement had the opposite effect. Movement should not be confused with the difference between the initial and final error since, because of the directions of movement of the two ends, a bar may move and yet have a final error equal to its initial error. The converse is not true, however. If a bar does not move, its initial and final errors are, of course, the same.

TABLE 1.—Summary of Results, Special Investigation *A*-7

Test No.*	Dowel-Bar Error, in Inches						Dowel-Bar Movement, in Inches			Description of joint and method of installing bars
	Initial			Final						
	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	
1	0.47	0.04	0.18	1.05	0.08	0.41	0.68	0.14	0.43	Contraction joints. Bars installed with holders.
2	0.51	0.06	0.31	0.45	0.06	0.31	1.01	0.10	0.37	
3	0.68	0.10	0.25	0.97	0.21	0.55	1.14	0.60	0.87	
4	0.75	0.35	0.49	0.85	0.28	0.54	0.93	0.23	0.51	
9	0.75	0.11	0.37	0.85	0.03	0.45	1.16	0.60	0.89	
Average	0.63	0.13	0.32	0.83	0.13	0.45	0.98	0.33	0.61	....
6	0.60	0.08	0.27	0.65	0.09	0.29	0.37	0.09	0.23	Contraction joints. Bars mounted on chairs.
7	0.45	0.30	0.21	0.78	0.08	0.32	0.53	0.12	0.29	
8	0.67	0.16	0.34	0.63	0.16	0.33	0.95	0.22	0.48	
12†	0.50	0	0.21	0.50	0.03	0.21	0.40	0.23	0.30	
Average	0.56	0.07	0.26	0.64	0.09	0.29	0.56	0.16	0.32	....
5	0.50	0.12	0.29	0.54	0.07	0.30	0.40	0.15	0.31	Expansion joints. Bars installed with holders unless otherwise noted.
10	0.66	0.17	0.35	1.50	0.10	0.51	1.82	0.18	0.59	
11†	1.44	0.76	1.16	1.40	0.79	1.12	0.17	0.04	0.12	
13‡	0.80	0.06	0.41	1.15	0.27	0.75	1.89	0.24	0.63	
14	0.35	0.06	0.18	0.29	0.02	0.15	0.55	0.13	0.31	
Average	0.75	0.23	0.48	0.98	0.25	0.57	0.97	0.15	0.39	....
Average of all tests	0.65	0.14	0.36	0.82	0.16	0.45	0.84	0.21	0.45	....

* Tests were numbered in chronological order. † U-shaped chairs resting on sub-grade and welded to spacer bars. ‡ Chairs, of the pin type, were used (one to each dowel bar), in addition to the holder.

It will be noted that the best installation tested (Test No. 14, Table 1) was an assembly of dowel bars, across an expansion joint, held in position during the placing of concrete by means of a holder. The average initial error was 0.18, the average final error, 0.15, and the average movement, 0.31 in. This is an instance in which the movements of the bars improved their positions. The poorest installation tested (Test No. 11, Table 1) was across an expansion joint in which the bars were mounted on U-shaped chairs welded to the spacer bars. These chairs rested on the sub-grade. The average error was 1.16 in., initial, and 1.12 in., final. Quite paradoxically this was the best installation so far as movement was concerned

(0.12 in.). The poorest installation so far as movement is concerned was across a contraction joint in which the bars were installed with a poorly designed holder (Test No. 9, Table 1). The average movement was 0.89 in., and the directions of movement were such that the initial error was increased.

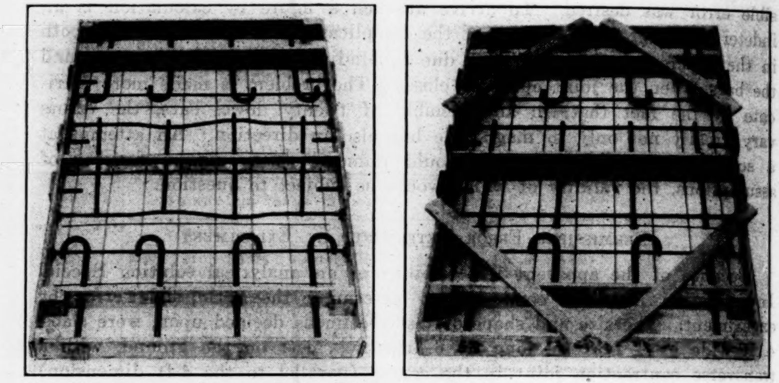
The results of the investigation indicated the need of information concerning the effect on the pavement of given degrees of error in dowel-bar placement. Since it is not feasible to require that every dowel bar in every installation be in perfect position, some figure specifying the permissible error was desired. To arrive at such a figure by calculation is an indeterminate problem because of the complication of stresses induced both in the concrete and in the bars, due to binding between the concrete and the bars, when the joint opens or closes. The problem is made more intricate by the fact that, in an assembly of twenty dowel bars, the errors vary greatly not only in magnitude but also in direction. An attempt at a solution of such a problem would immediately require a number of assumptions, the validity of which would be subject to question.

Permissible Error Determined by Experiment

Because of the apparent impossibility of an analytical solution Special Investigation *A*-8 was conducted to determine the permissible error by experiment. The size and shape of test specimens decided upon, were slabs, 4 ft wide by 4 ft 8 in. long by 6 in. thick, cast on the ground, with a transverse contraction joint in the center (parallel to the 4-ft dimension) through which four dowel bars were placed at 12-in. centers. The slabs were made in groups of five, all five being poured from the same batch of ready-mixed concrete. In one slab the dowel bars were installed in perfect position (with zero error); in the other four slabs, they were purposely set at errors of 0.25 in., 0.5 in., 1 in., and 1.5 in. The second group of slabs, made on a different day, were duplicates of the first group except that 1-in. air cushion expansion joints were provided instead of contraction joints. Groups 3 and 4 were duplicates respectively of Groups 1 and 2 except for thickness, which was reduced to 5 in. In all, the test specimens consisted of twenty experimental slabs, each containing four dowel bars, including two slab thicknesses for each of five errors, for both contraction and expansion joints.

The design of the experimental slabs and the directions of dowel-bar errors will be more readily undertsood by referring to the photograph of an empty form (Fig. 5(*a*)). This form is for the fabrication of a 6-in. slab containing a contraction joint, the dowel bars having an error of 1.5 in. The two outside dowel bars have no error vertically; that is, they are level, but a plan view would show that each one has a projected length on the cross-section of the slab of 1.5 in., and that they converge toward the reader. A plan view would show the two interior bars in perfect position but, due to the difference in elevation between the two ends of each bar, each one has a projected length of 1.5 in. Note the relative positions of these two bars. The end toward the reader, of the bar on the left, is 1.5 in.

higher than the opposite end. The end toward the reader, of the bar on the right, is 1.5 in. lower than the opposite end. As this contraction joint is opened the two interior bars tend to rotate the slab (clockwise from the reader's position), which tendency is resisted by the two outside bars which produce compression (horizontally) in the half of the slab toward the reader and tension in the opposite half. Consequently, opening the joint produces stresses in the concrete and dowel bars which are not only severe but of a complicated distribution. This same arrangement of dowel-bar positions was used in all slabs containing dowel bars in error.



(*a*) CONTRACTION JOINT; DOWEL-BAR ERROR = 1.5 INCHES.

(*b*) EXPANSION JOINT; DOWEL-BAR ERROR = 0.

FIG. 5.—FORM FOR THE FABRICATION OF A 6-INCH SLAB.

The method of forming the contraction joint is also shown in Fig. 5(*a*)). The pre-molded material with which contraction joints are usually filled is simulated by a piece of wood, 0.75 in. thick, having a vertical dimension equal to one-third the slab thickness. A weakened plane was created by installing a piece of No. 24 gage galvanized iron, extending from the wood strip downward to the sub-grade. The dowel bars pass through semicircular openings, 3 in. in diameter, in the sheet metal. Expansion joints were formed by installing a cork filler, 1 in. thick, as shown in Fig. 5(*b*), which also shows the corner bracing used on all forms to prevent distortion during the placing of the concrete. To give the cork sufficient rigidity against bending during the placing of the concrete, it was backed up with a board which was removed after the form was filled. Before the slabs were tested the cork was removed with a chisel. In Fig. 5(*b*) the dowel bars are in perfect position with respect to the cross-section of the slab—that is, zero error. Anchor bars were installed in both ends of the slabs at the time of fabrication, as shown in Fig. 5, for attaching the testing equipment.

In addition to the twenty experimental slabs containing common dowel bars, thirteen slabs were made for the study of another type of load-transfer device. For the purpose of this paper, however, the discussion of

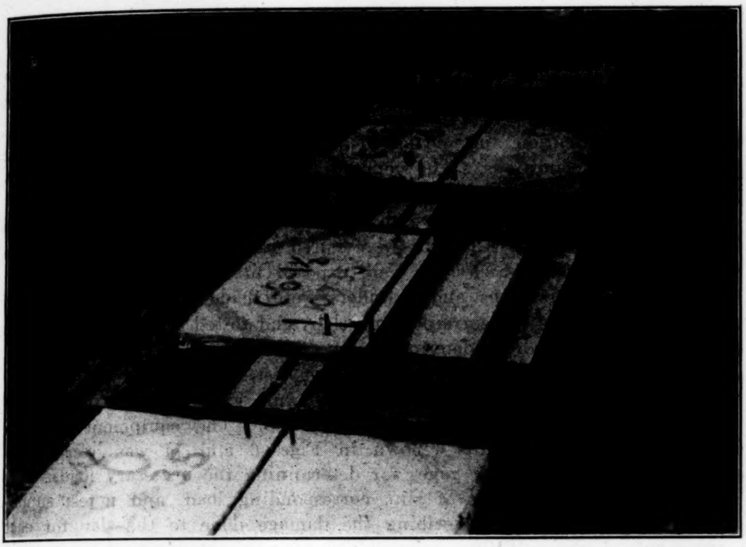


FIG. 6.—EQUIPMENT FOR OPENING A CONTRACTION JOINT.

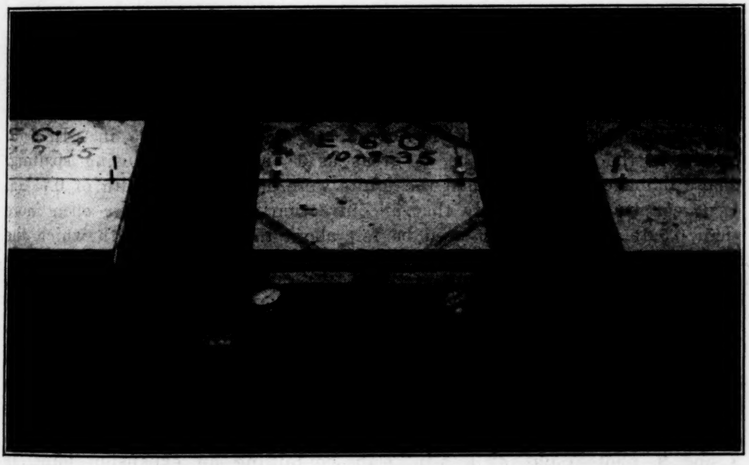


FIG. 7.—EQUIPMENT FOR CLOSING AN EXPANSION JOINT.

results is confined to those obtained with common dowel bars since they are in so much more general use than any proprietary device. The entire investigation required the fabrication and testing of thirty-three experimental slabs.

In the fabrication and curing of the slabs every effort was made to duplicate field conditions. The concrete was of the same cement content and consistency as that used in practice and the slabs were reinforced with standard wire mesh. The sub-grade was tamped and sprinkled before placing the concrete. All slabs were cured under wet burlap during the first ten days, after which they were exposed to the weather. From each batch of concrete, beams and cylinders were made which were cured in the same manner as the corresponding slabs and tested at the same time the slabs were tested, thereby determining the flexural and compressive strength of the concrete in the slabs at the time of test.

At the age of 28 days all contraction joints were opened a distance of 0.75 in. and all expansion joints were closed. The equipment used for opening and closing joints (shown in Figs. 6 and 7) was operated by hydraulic jacks fitted with gages for determining the necessary loads. The test data included records of slip, corresponding load, and notes, supplemented by photographs, describing the damage done to the slab for each increment of slip. Within a few days after all contraction joints were opened and all expansion joints closed, the process was reversed and the contraction joints were closed and the expansion joints opened to their original positions, thus completing one cycle of movement. At the age of one year all slabs were opened and closed nine more times, completing ten cycles of movement, and then pulled entirely apart in order that the cracking and spalling on the faces of the joints could be photographed.

Results and Conclusions

The results of the tests indicate that common dowel bars across contraction joints in 5-in. pavement will cause a very slight spalling of the concrete around the bars if the errors are as great as 0.25 in. In 6-in. pavement the errors may be as great as 1 in. without causing spalling. These damages occurred when the joints were opened to a width of 0.75 in. If it can be assumed safely that a contraction joint will never open more than 0.5 in., little concern need be felt about the accuracy with which the dowel bars are installed. When opened to this width (0.5 in.) the only slab that showed any spalling was the 5-in. slab in which the bars were in error 1.5 in.

The refinement with which dowel bars are placed is more important in expansion joints than in contraction joints. An error of 0.25 in. caused slight spalling and when the joint spaces were closed 0.75 in., to a width of 0.25 in. (which may occur in practice), errors of 1 in. caused the slabs to crack and spall. Typical cracking and spalling are shown in Figs. 8 and 9, photographs of a 5-in. slab, containing an expansion joint, in which the dowel bar error was 1.5 in., after one cycle of movement (Fig. 8) and after ten cycles of movement (Fig. 9).

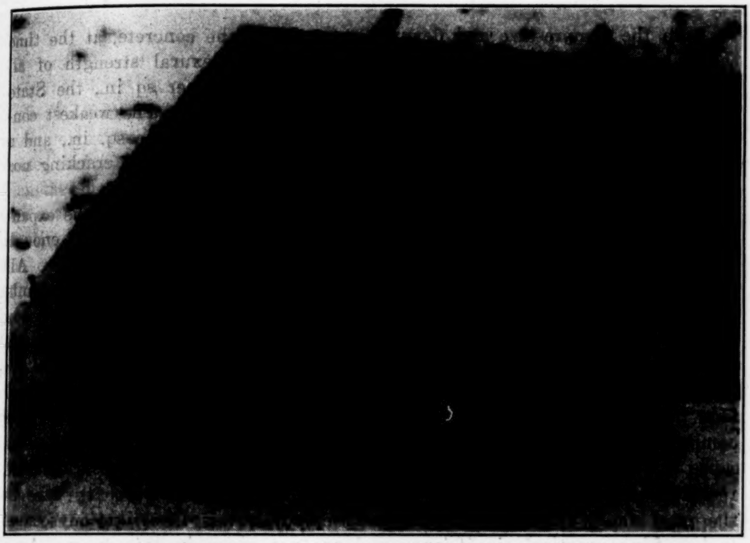


FIG. 8.—DAMAGE DONE TO TEST SLAB BY ONE CYCLE OF MOVEMENT (THICKNESS OF SLAB, 5 INCHES; AND DOWEL-BAR ERROR, 1.5 INCHES.)

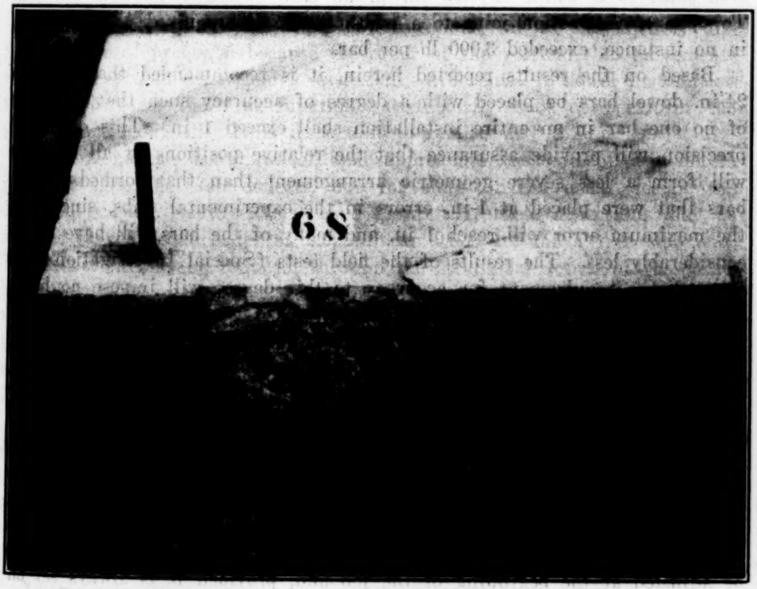


FIG. 9.—SAME SLAB AS SHOWN IN FIG. 8, AFTER TEN CYCLES OF MOVEMENT.

Both the compressive and flexural strengths of the concrete, at the time the slabs were tested, were unusually high. The flexural strength of all test specimens was considerably greater than 550 lb per sq in., the State requirement for opening pavement to traffic in Indiana. The weakest concrete tested had a 28-day modulus of rupture of 670 lb per sq. in., and a compressive strength of 4 010 lb per sq in. Consequently, no cracking nor spalling of the slabs can be attributed to inferior concrete.

The loads required to open or close either the contraction or the expansion joints were less than anticipated and in no case were they great enough to stress the slab dangerously either in direct tension or compression. All damage done to the slabs was plainly due to stresses caused when the joints were being opened or closed, by the wedging and twisting effect of bar misalignment. The loads necessary to open and close the joints were not great enough to produce tensile or compressive stresses approaching the strength of the interior of the slabs—that is, the parts in which no dowel bars nor anchor bars were present. This statement is proved not only by computation, but by observation, since no transverse cracks occurred. Because the slabs were capable of transmitting forces so far in excess of those required to open and close the joints, even when the dowel bars were in the most faulty positions, the loads observed in this investigation seemed to be of no practical significance and are not given in this paper. As a matter of general interest, however, the load necessary to close an expansion joint to a width of 0.25 in. in no case exceeded 4 000 lb per bar. To open a contraction joint to a width of 0.5 in. required a load which, in no instance, exceeded 3 000 lb per bar.

Based on the results reported herein, it is recommended that common 24-in. dowel bars be placed with a degree of accuracy such that the error of no one bar in an entire installation shall exceed 1 in. This degree of precision will provide assurance that the relative positions of all the bars will form a less severe geometric arrangement than that formed by the bars that were placed at 1-in. errors in the experimental slabs, since only the maximum error will reach 1 in. and many of the bars will have errors considerably less. The results of the field tests (Special Investigation *A*-7) show that a requirement for accuracy to this degree will impose no hardship on the contractor. The vertical position of each bar can be checked quickly with a carpenter's level, as shown in Fig. 4, and adjusted if necessary, until the bubble position is approximately the same as when the level is set on the forms. The horizontal position can be made perfect by adjusting each bar so that the distances from each end to one of the forms are equal. Since the total error (projected length on the cross-section of the pavement) of a bar is equal to the square root of the sums of the squares of its vertical and horizontal errors, the vertical and horizontal error could each be as great as 0.707 in. and still comply with this specification. If some type of holder is used to install the bars, it should be adjusted at the beginning of the job and, provided it is always set on the forms in correct position and does not become damaged in handling, all installations should be correct.

DISCUSSION

L. W. TELLER,[3] ASSOC. M. AM. SOC. C. E. (by letter).—A subject that is of importance to every designer and builder of concrete pavements is treated in this commendable paper. The data which the authors furnish from measurements of dowel positions in place, are useful in demonstrating, rather conclusively, what is more or less common belief—that, frequently, good dowel-bar alignment is not obtained in actual construction. The effect of such misalignment upon dowel efficiency is a matter that should receive very careful consideration before an attempt is made to draw broad conclusions or to recommend general tolerances for field construction. Unfortunately, the authors did not include all the data from the experiments that were used to determine the permissible errors that are recommended, and, therefore, no detailed discussion of their test data is possible. Some general discussion of the subject of dowel efficiency is pertinent, however.

A dowel-bar installation really has two functions in a concrete pavement: First, it prevents any gross faulting of the two slab ends and preserves the general continuity of the pavement surface at the transverse joint; and, second, through its shear resistance, it assists the slab end in carrying wheel loads by transferring a part of the load across the joint to the adjacent slab. The installation requirements for the dowels, in order that they may perform these two functions, are of two entirely different orders of precision because the order of the vertical slab movements with which each is concerned is essentially different. A dowel installation that will perform the first function satisfactorily may not perform the second function at all. It is important to realize why this is so.

When a normal heavy wheel load approaches the end of a concrete pavement slab, the slab deflects, but for slabs of the usual design the deflection is quite small. Tests[4] have shown that these deflections may be expected to be of the order of 0.020 in. to 0.030 in. for a free slab end. This deflection may be reduced, through the structural action of the joint, by possibly 0.010 or 0.015 in. Thus, it is apparent that whatever the single dowel is to accomplish must be done during a relative displacement of the two slab ends of only a few thousandths of an inch. The bearing of the dowel on the concrete must be maintained within close limits if the unit is to develop shear resistance and thus transfer load.

In their "Synopsis," the authors state quite correctly that it is important to place the bars accurately. If they are not perpendicular to the cross-section of the pavement, binding between them and the concrete will occur when the joint opens or closes. As a consequence, "either the bars will become distorted or the concrete will be damaged in the vicinity of the bars."

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[3] Senior Engr. of Tests, Bureau of Public Roads, U. S. Dept. of Agriculture, Washington, D. C.

[4] "Structural Design of Concrete Pavements"; Pt. 4, "A Study of the Structural Action of Several Types of Transverse and Longitudinal Joint Designs," *Public Roads*, Vol. 17, No. 7, September, 1936, p. 166.

The bearing stress imposed on the concrete by the round steel dowel bar is very high, and it is the concrete that fails first. The type of failure known as "funneling," observed under certain conditions in the concrete immediately around a dowel, is the direct result of repeated applications of this high bearing stress.

Since the effectiveness of a dowel can be lost entirely if a relatively small amount of "play" is developed between it and its socket, it is apparent that, in the tests reported by the authors, the absence of structural cracking or visible spalling does not necessarily mean that the dowels retained their effectiveness so far as their function of transferring load is concerned.

When a wheel load rests on a slab end at a distance from a corner the critical stress in the concrete is a tensile stress in the bottom of the slab directly under the wheel load. The direction of this critical deformation is parallel to the edge of the slab. The load-transfer feature of a joint installation is intended to relieve this stress and the degree to which it accomplishes this relief is a measure of the efficiency of the joint action.[5] The only apparent way in which one can determine joint efficiency (or change in joint efficiency) is by means of stress determinations that will show, directly, the degree to which the joint is fulfilling its function of relieving the critical edge stress. The authors do not report tests of this kind and without them the effect of dowel alignment on dowel efficiency remains unknown.

The authors do not present the data on the forces required to produce slab displacements for various degrees of misalignment, but state that in no case did they exceed 4 000 lb per bar. From this they reason that the loads "seemed to be of no practical significance." As far as compressive stress is concerned this may be true, but if poor dowel alignment can cause tensile stresses of the order of 50 lb per sq in. in the concrete, then the stress has practical significance. In certain parts of the slab, loads and temperature warping produce tensile stresses at a common point. If joint resistance adds to this, it subtracts directly from the stress resistance available to carry wheel loads. A properly installed plain round dowel should not develop more than about 100 lb of total resistance to longitudinal movement, which is equivalent to about 1 lb per sq in. of concrete stress.

It is not the writer's purpose to criticize the authors nor the tests that they made, but rather the generality of the conclusions which they draw. Until such time as tests are made that measure directly the effect of misalignment of dowels on the structural efficiency of the joints in which they are installed designers should bend every effort toward the development of means for installing them as perfectly as possible and not encourage careless installation in the field by recommendations such as are made in this paper.

DAVID J. PEERY,[6] JUN. AM. SOC. C. E. (by letter).—An investigation of the alignment of pavement dowels supplies information for studying the efficiency of the dowel bars in transferring loads across joints. A homogeneous slab of uniform thickness, supported on an elastic sub-grade, will be much more highly stressed by a load applied at a free edge of the slab than by the same load applied at the interior of the slab. A balanced slab design, in which the maximum stresses at the edges are equal to the maximum stresses in the interior, is approached by thickening the edges or by providing dowel bars to transfer part of the load to the edge of adjacent slabs.

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[5] *Public Roads*, Vol. 17, No. 7, September, 1936, p. 152.

[6] Instructor, Carnegie Inst. of Technology, Pittsburgh, Pa.

If dowels are strong and stiff enough to cause equal deflections of the two slabs at the points of connection, they will transmit about one-half the total load applied at the edge of one slab. This condition, however, does not reduce the critical stress in the loaded slab by one-half, but it does result in a considerable reduction if the dowels are spaced closely. With information available as to the vertical misalignment of dowels in pavements representing common construction practice, it is possible to investigate the effectiveness of dowels in deflecting the points of connection equally.

For effective dowel action the difference in deflections between the edges of the two slabs at the points of connection must be very small in relation to the deflection of the free edge of one slab under a wheel load. The maximum deflection of a free edge of a homogeneous slab under a concentrated load at the edge may be determined by an equation,[7] proposed by H. M. Westergaard, M. Am. Soc. C. E.:

$$z_0 = 0.434 \frac{P}{k\, l^2} \qquad (1)$$

in which, z_0 = maximum slab deflection; P = wheel load; k = sub-grade modulus; and, l = radius of relative stiffness. For a wheel load of 10 000 lb at the edge of a 7-in. slab, with $k = 200$ lb in.$^{-3}$, and $l = 25.73$ in., the maximum deflection is 0.0327 in. This value will be used only to represent the general magnitude of the deflections. The relative deflections between any two dowels would be much smaller.

An investigation of a series of adjacent dowels with similar vertical misalignments will show what relative vertical displacements occur when a joint opens or closes. Assuming a vertical error of 1 in. in installing 22-in. dowels, and an expansion joint movement of 0.75 in., a relative vertical displacement will occur equal to $\frac{1}{22} \times 0.75 = 0.0341$ in. Since this is even larger than the deflection caused by the load, the dowels do not aid in reducing stresses by load transfer. The nature of this distortion is shown in Fig. 10. If a load is placed on either side of the joint, the deflection of the slabs tends to return the left slab to its unstressed position, and most of the load is carried by the slab on the right.

It is evident that dowels which are this much out of line are objectionable because they fail to transfer load. They also cause stresses in the slab by a wedging action, and they resist movement at the joint. If adjacent dowels are inclined in opposite directions, any movement of the joint will cause local over-stresses and elastic failures. Although these may or may not cause complete failure after several cycles of reversal, they will produce enough yielding and play in joints so that effective load transfer is lost.

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[7] "Spacing of Dowels," Highway Research Board, 8th *Proceedings*, p. 156.

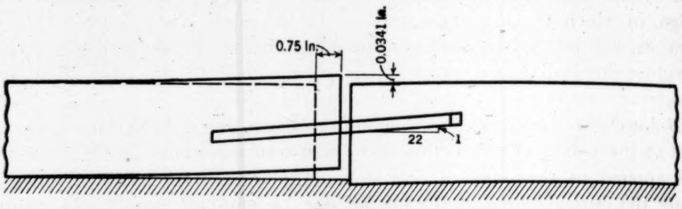

FIG. 10

Any type of pavement joint which transfers load should be designed and installed with great care. As slab deflections under loads are usually less than $\frac{1}{32}$ in., any load-transfer device must be constructed so as to permit very little relative vertical movement between the connected edges of the slab.

L. J. MENSCH,[8] M. AM. SOC. C. E. (by letter).—The art of making joints in concrete construction in general and in pavements in particular is still in the stage of discovery and invention, and is a lucrative field for the patent lawyer and the mountebank. Although many millions of dollars are expended annually on joints, the development of the art has covered a long period of time, and "the trade" has been subject to the losses, faults, and checks to which haphazard knowledge, without adequate tests and proper analysis based on them, is peculiarly liable.

The specifications of State highway departments for joints have lagged seriously behind the best accepted rules of structural engineering and concrete practice and have acted as a deterrent to more scientific development by independent manufacturers of joints. The successful designer of joints must have an unusually thorough training, and long and wide experience, if he is to comprehend and provide successfully for all the variation of stress in dowels and concrete due to many influences, such as warping of the slabs from wet sub-grade or from temperature variation, misalignment of dowels, and movements of the slabs as a whole, vibrations of slabs, cavities at the joints caused by the pumping action of traffic on the water-soaked sub-grade, heavy wheel loads on one side of the joint, etc.

The most satisfactory pavement would be one without joints; this, however, does not seem possible of attainment as the aforementioned influences cause considerable irregular cracking in every direction. The Bates Road tests in Illinois, and the Pittsburg, Calif., tests of fifteen years ago, have taught American engineers that longitudinal joints greatly prevent irregular longitudinal cracks.[9] Nothing so clear was learned by these tests, about transverse joints. In the years following, most pavements were constructed with tongue-and-groove longitudinal joints, thickened edges, and transverse joints rather far apart. After five years these pavements showed many irregular transverse cracks, and after ten years, blow-ups occurred as often as one every three miles, once each year. Early in 1934, the U. S. Bureau of Public Roads required, on all Federal Aid roads, the use of expansion joints not more than 100 ft apart to guard against blow-ups, and the use of contraction joints for crack control. The Bureau also required effective means of load transfer at both expansion and contraction joints, the minimum allowable being the use of $\frac{3}{4}$-in. dowels, 24 in. long, 12 to 15 in. on centers. Even at that time doubt was expressed as to whether $\frac{3}{4}$-in. dowels were sufficiently strong for the purpose.

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[8] Civ. Engr. and Constructor, Chicago, Ill.

[9] "Highway Research in Illinois," by Clifford Older, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1180; and Report of Highway Research at Pittsburg, Calif., 1921–22, Dept. of Public Works, California.

The Highway Department of Indiana like those of many other States which did not specify transverse joints in its pavements, was suddenly compelled to use them, and evidently the authors were astounded when they saw the recklessness and disregard of engineering principles with which dowels were installed in the slabs. They made the tests described in this paper and did a very meritorious job. They not only proved: (1) That dowels were not installed parallel to the longitudinal axis of the road when unusual care was exercised and when special installation devices were used; (2) that the dowels were not parallel when they were welded to two $\frac{5}{8}$-in. cross-bars, showing errors as great as 1 in. on account of the bending of the cross-bars in shipping and handling; but (3) they also proved (what is even to-day scarcely realized by many highway engineers) that the concrete operations and the action of the mechanical strike-off machine (mis-named finishing machine) cause a still greater error in the location of the dowels. Any engineer who watches the modern method of placing concrete in pavements must be surprised that contractors are permitted to dump an entire yard of concrete crudely by the nearly uncontrollable opening of the traveling bucket of the mixer, often producing a wave-like motion of the heavy mass which throws joints and anchoring devices many inches out of line. Furthermore, the top of the dumped concrete is often many inches above the top of the pavement and offers a great resistance to the advancing strike-off machine, which again throws the joint out of line, rarely less than 0.5 in. and often as much as 1.5 in. on top. From his many observations of paving operations the writer is convinced that the errors produced by concreting and by the strike-off machine are much larger than those found by the authors.

Some highly speculative studies on the load transfer of dowels have been published. These studies were not based on any tests and evidently did not agree with the findings of the authors; therefore, they assumed that the subject is impossible of analytical solution. Although the task is certainly difficult and unusual, the writer has shown[10] that a rational description of the action of dowels is possible without the use of too much higher calculus and will now offer an improved and simplified analysis.

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[10] "Joints for Concrete Pavements," Chicago, 1935.

In Fig. 11 let the verticals through Points *A* and *F* represent the original slab faces of an expansion joint with an opening of 1 in., and the verticals through Points *A′* and *B′*, the faces of the slabs after the joint has been compressed; let Line *A B* be the direction of the mis-aligned dowel, and the distance, *B F*, be the error of $\frac{1}{24}$ in. corresponding to an error of 1 in. in 24 in. By closing the joint space, Point *A* moves to *A′* and Point *B* to *B′* and the dowels would be bent into a broken line, *E A′ B′ D*, if the resistance of the concrete proved to be greater than that of the steel of the dowel. Since concrete is the weaker material, the dowel will deform according to a smooth curve, somewhat as shown by *A″ C B″*, having a point of inflection at Point *C*. This bending can be imagined to be caused by a shear, *P*, acting at Point *C* which will produce high compression in the concrete at the under side of the dowel at Point *B″*. The

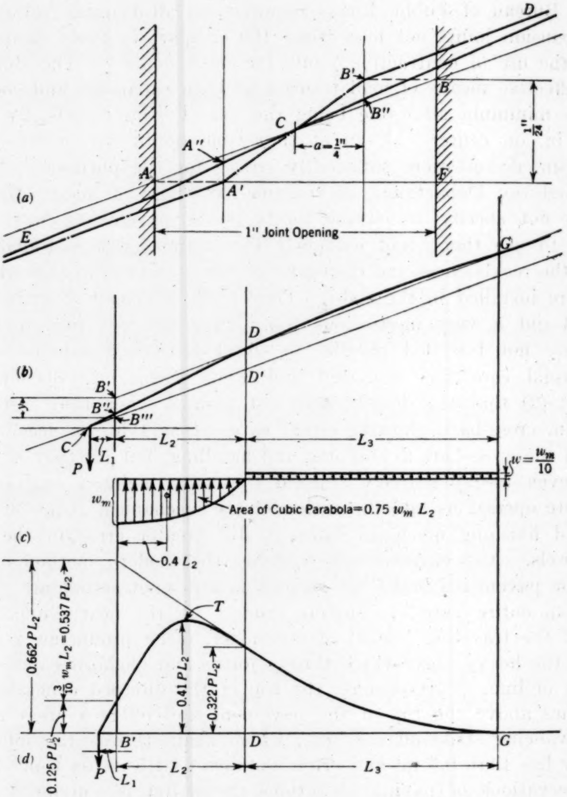
D
B′
B
B″
C
$a=\frac{1''}{4}$
$\frac{1''}{24}$
A″
A
A′
F
(a)
E
1″ Joint Opening
G
D
D′
(b)
B′
B″
B‴
C
P
L_1
L_2
L_3
$w=\frac{w_m}{10}$
w_m
Area of Cubic Parabola=0.75 w_m L_2
(c)
0.4 L_2
T
0.662 PL_2
$\frac{3}{10}$ $w_m L_2$=0.537 PL_2
0.41 PL_2
0.322 PL_2
0.125 PL_2
B′
D
G
(d)
P
L_1
L_2
L_3

FIG. 11.

action of the dowel is very much like that of nails in timber construction. On any reinforced concrete job one can easily observe the shape into which the nails are deformed after they are highly strained, when the lumber is cleaned and assembled for re-use.

The greatest curvature of the nail or dowel is found only a short distance from Point B', and between the latter point and the point of greatest curvature a permanent deformation of the timber is produced. Point D is about twice as far from Point B' as the point of greatest curvature of the nail or dowel; beyond Point D and up to Point G the nail is nearly straight and does not produce a permanent deformation of the timber. Fig. 11(*b*) shows the probable deformation.

The stresses in dowels and in the surrounding concrete in the part, $B'D$, are always high in the tests offered by the authors, being mostly beyond the yield point, and the distribution of the stresses exerted by the dowel on the concrete in the part, $B'D$, can be assumed to be according to a cubic parabola, as shown in Fig. 11(*c*). The writer has previously shown[10] that the part, DG, behaves nearly like a cantilever, projecting beyond Point D at the angle, α, and bent down by an ideal uniform load w, so that the end of the cantilever reaches the level of Point G. By computing many cases, he found that a good assumption is,

$$w = \frac{w_m}{10} \qquad \text{(2)}$$

when w_m is the maximum pressure at Point B'. The dowel can now be considered as a simple beam affected at Point C by the unknown shear, P, loaded in the part, $B'D$, by the upward concrete stresses which are decreased from a maximum value, w_m, at Point B', according to a cubic parabola, to zero at Point D, and loaded by the ideal uniform downward stresses, $\frac{w_m}{10}$ in the part, DG; and the following equations can be written:

The sum of all vertical forces must be zero, or,

$$P + w L_3 = \tfrac{3}{4} w_m L_2 \qquad \text{(3)}$$

The moments about Point B' must be zero:

$$P L_1 + \tfrac{3}{4} w_m L_2 \times 0.4 L_2 = w L_3 (0.5 L_3 + L_2) \qquad \text{(4)}$$

From Equation (2) and Equation (3),

$$w_m = \frac{P}{0.75 L_2 - 0.1 L_3} \qquad \text{(5)}$$

The value of w_m introduced in Equation (4), gives a quadratic equation in L_3 from which:

$$L_3 = -(L_1 + L_2) + \sqrt{(L_1 + L_2) + 10 L_2 (1.5 L_1 + 0.6 L_2)} \qquad \text{(6)}$$

The authors advise assuming $L_1 = 0.25$ in.; by assigning to L_2 various probable values, such as 1.5 in., 2 in., 2.5 in., etc., Equation (6) leads to a very close approximation:

$$L_3 = 1.9 L_2 \qquad \text{(7)}$$

and Equation (5) reduces to:

$$w_m = \frac{P}{0.75\,L_2 - \dfrac{1.9\,L_2}{10}} = \frac{P}{0.56\,L_2} \quad\ldots\ldots\ldots\ldots\ldots(8)$$

It is clear that the deflection of the dowel under the assumed loading must be equal to $B'\,B'''$, which, in the present case, is one-fourth of $\frac{1}{24}$ in., or $\frac{1}{96}$ in. An expression for this deflection, y, can be obtained as follows: The angle, α, for the cantilever, $D\,G$, equals[10]:

$$\alpha = \frac{w\,L_3^3}{8\,E\,I} \quad\ldots\ldots\ldots\ldots\ldots(9)$$

Introducing into Equation (9) the values from Equations (2), (7), and (8),

$$\alpha = \frac{0.153\,P\,L_2^2}{E\,I} \quad\ldots\ldots\ldots\ldots\ldots(10)$$

and the deflection of the dowel at Point C from the rotation, α, is:

$$y_C = (L_1 + L_2)\,\alpha = \frac{0.153\,(L_1 + L_2)\,P\,(L_2)^2}{E\,I} \quad\ldots\ldots\ldots\ldots(11)$$

The deflection of the dowel at Point C from the moments acting in the part, $B'\,D$, is best found by the moment-area method; thus, the moment at Point D is,

$$M_D = \frac{w\,L_3^2}{2} = \frac{P}{5.6\,L_2} \times 1.9^2 \times \frac{L_2^2}{2} = 0.322\,P\,L_2 \ldots\ldots\ldots(12)$$

and the moment at Point B' is

$$M_B = w\,L_3\,(0.5\,L_3 + L_2) = 0.662\,P\,L_2 \ldots\ldots\ldots\ldots(13)$$

This latter moment, M_B, is diminished by the moment of the upward concrete stresses acting in $B'\,D$, or,

$$M_B' = 0.75\,w_m\,L_2 \times 0.4\,L_2 = 0.537\,P\,L_2 \ldots\ldots\ldots\ldots(14)$$

The deflection of the dowel at Point C from the moment area, $B'\,D\,T$, in Fig. 11(*d*) is the moment area times the distance of its center of gravity from C:

$$y_C' = 0.173\,\frac{P\,L_2^3}{E\,I} + 0.317\,\frac{P\,L_1\,L_2^2}{E\,I} \quad\ldots\ldots\ldots\ldots\ldots(15)$$

and the total deflection is found by adding Equation (11) and Equation (15), or,

$$y = 0.326\,\frac{P\,L_2^3}{E\,I} + 0.47\,\frac{P\,L_1\,L_2^2}{E\,I} \quad\ldots\ldots\ldots\ldots\ldots(16)$$

Instead of Equation (16) it will be more convenient to use the approximate formula:

$$y = 0.38\,\frac{P\,L_2^3}{E\,I} \quad\ldots\ldots\ldots\ldots\ldots(17a)$$

and,

$$P = \frac{E\,I\,y}{0.38\,L_2^3} \quad\ldots\ldots\ldots\ldots\ldots(17b)$$

in which E equals 30 000 000 and I is the moment of inertia of the dowel (0.0155 for $\frac{3}{4}$-in. dowel). By assuming various probable values of L_2 (say, from 1 in. to $2\frac{1}{2}$ in.) and of y, the deflection produced by the movement of the joint faces ($\frac{1}{96}$ in. in the present case), the corresponding values of the ideal shear, P, can be computed from Equation (17*b*) and the corresponding maximum pressures, w_m, in pounds per linear inch, which the dowel exerts on the concrete, can be computed by Equation (8). From Fig. 11(*d*) it may be further noted that the greatest moment exerted on the dowel by the concrete pressures is $0.41\,P\,L_2$ which values divided by the section modulus of a $\frac{3}{4}$-in. bar (0.0414) give the unit steel stresses in the dowels. The values thus computed are given in Table 2.

TABLE 2.—STRESSES IN DOWELS AND CONCRETE

Description	VALUES OF LENGTH, L_2, IN INCHES				
	1	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$
Shear, P, in pounds, from Equation (17*a*)	12 600	3 740	2 340	1 560	810
Maximum unit pressure, w_m, in pounds per linear inch, from Equation (8)	22 500	4 350	2 400	1 400	580
Values of $\frac{w_m}{0.75}$, in pounds per square inch, for $\frac{3}{4}$-in. dowels	30 000	5 800	3 200	1 870	773
Moment, $M = 0.41\,P\,L_2$, in inch-pounds	5 160	2 300	1 680	1 280	835
Steel stresses, S_s, in $\frac{3}{4}$-in. dowels, in pounds per square inch	124 000	55 800	41 000	31 000	20 200

Inasmuch as neither P nor L_2 is given, and as it is extremely difficult to measure either the concrete or the steel stresses, it is very difficult to determine which group of values refer to a particular test. The authors state that they observed crushing and splitting in the concrete in the case of a 5-in. slab. They probably also found pulverization of the concrete at Point B' in the 6-in. slabs; hence one may conclude that the stresses in a 6-in. slab were also very high.

In Table 2, the column for $L_2 = 1\frac{3}{4}$ in. shows concrete and steel stresses of 3 200 and 41 000 lb per sq in., respectively. These are by no means unlikely values as it is quite common to observe steps in joints due to the movements of the slabs. Such steps indicate very high stresses in the dowels and concrete. When $L_2 = 2$ in., concrete stresses equal 1 879 lb per sq in., and steel stresses, 31 000 lb per sq in., which are still very high values. The results of this analysis will scarcely be changed when one slab is fixed and the other slab moves the entire 0.5 in.

Assuming that more scientific and more extensive tests may demonstrate that correction factors must be applied to all the foregoing equations, they are unlikely to change values more than 30 per cent. This analysis shows that slab movements of 0.5 in. may cause very high stresses in concrete and anchoring devices, as usually installed, and there is very little strength left for the primary function of the dowels and anchoring devices —the transfer of loads. Searcy B. Slack, M. Am. Soc. C. E., made tests[11] which demonstrated that $\frac{3}{4}$-in. dowels did not transfer any load in actual pavements after they had been in service for a few years.

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[11] *Engineering News-Record*, July 9, 1931.

Imitating modern connectors for timber construction several joint manufacturers are using sleeves over $\frac{3}{4}$-in. dowels in order to diminish the high concrete stresses at Point B'. Table 2 enables one to judge their merits. For $L_2 = 1$ in. and a sleeve diameter of 1.5 in., the concrete stresses would be $\frac{22\,500}{1.5} = 15\,000$ lb per sq in., and the steel stresses, $0.41\,P \times \frac{1}{0.0414} = 71\,000$ lb per sq in.; whereas, for $L_2 = 1.5$ in., the stresses would be 2 900 and 47 800 lb per sq in., on concrete and steel, respectively. The sleeves and dowels being very short one is not entitled to assume larger values of L_2. The writer has heard that the authors made tests to study the sliding of such devices and found resistances to sliding four times as great as for $\frac{3}{4}$-in. dowels without sleeves.

In the writer's own tests on sliding of misaligned $\frac{3}{4}$-in. dowel bars[10], the resistance to sliding was much greater than that found by the authors on account of the greater stiffness of the high-carbon dowels used. The authors proved by their tests that the slip-shod methods used at present to place and hold dowels and anchoring devices before and after concreting, must be abandoned. Positive means are needed to hold dowels, anchoring devices, and the entire joint assembly in correct alignment until the concrete has set.

The closer the expansion joints are spaced, the less is the movement and the less are the stresses caused by misaligned dowels and anchoring devices; and it is very probable that in the near future expansion joints in pavements will be placed about 30 ft apart instead of 80 ft to 100 ft as at present, and some kind of inexpensive contraction joint will be used between the expansion joints.

W. O. FREMONT,[12] M. AM. SOC. C. E. (by letter).—The type of failures described in this paper depends to a great extent on the direct action of the bar on the concrete in the vicinity of the dowel. This action depends on the bending moment, shear force, and longitudinal force in the dowel. Similar functions of the outer forces are produced in the dowels also by vertical loads as well as by the curling and warping of the slabs due to changes of moisture and temperature. Therefore, the effects of the opening or closing of the joint must be studied in conjunction with the effects of all the other factors. If the concrete fails at a certain amount of opening or closing of the joint, it does not mean that it will not fail at a much smaller opening or closing when the vertical load and curling, with warping, are acting in addition.

Conclusions drawn on the basis of joint opening and closing tests alone, therefore, without at least the vertical forces, seem insufficient, if not useless, for practical purposes. It is difficult to believe that the small specimens, cut from the entire pavement construction, truly represent or simulate the actual service conditions. The rotation of the slabs mentioned under the heading, "Permissible Error Determined by Experiment," will be quite different under actual service conditions. The distribution and magnitude of pressure on the sub-grade from the joint-opening forces, or by the closing forces alone, will also be different in actual large pavement slabs. Therefore, the action of the dowels will also be quite different.

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[12] Bridge Designer, State Highway Dept., Lansing, Mich.

The authors state that the dowel bars were coated with asphalt paint and oil. No information is given about the thickness of the coats applied and other details. From theoretical considerations it seems that errors in the alignment of the dowels of the magnitude used in the tests are permissible only because of the clearances between the dowels and the concrete formed by the coating applied. It seems that no exact definition is given as to what is understood in Table 1 under "Dowel-Bar Movements."

In the last paragraph of the paper the authors recommend that "common 24-in. dowel bars be placed with a degree of accuracy such that the error of no one bar in an entire installation shall exceed 1 in." This conflicts with the statement a few paragraphs preceding it, that "errors of 1 in. caused the slabs to crack and spall." According to the authors it required a tension force of 3 000 lb per dowel to open a contraction joint to a width of 0.5 in., which means a tensile stress of 50 lb per sq in. in the concrete slab 5 in. thick. Considering the high tensile stresses in the concrete slab from vertical loads, an addition of 50 lb per sq. in. is appreciable inasmuch as this addition might become larger in the actual pavement slabs farther away from the joint, because of the friction between slabs and sub-grade.

George A. Smith,[13] Esq. (by letter).—Those interested in the design and construction of concrete highways should appreciate the information contained in the paper by Messrs. Smith and Benham and should be grateful to learn that relatively large errors in the alignment of dowels in joints in concrete pavements may not contribute materially to failures at contraction and expansion joints. The design and construction of joints in concrete highways have probably been given as much consideration as any other single factor relating to such work and, although the particular tests reported relate to a single phase of the subject, their findings certainly answer a moot question.

A review of this paper cannot fail to leave the impression that the work involved was ably conducted and that the results, although general, were presented in a clear, concise, and accurate manner. The paper offers little, if any, grounds for critical comment, but does prompt the asking of several questions which, should the studies be continued and the investigation extended, may be worthy of some consideration.

The authors indicate that, when the misalignment of dowel bars is not in excess of 1 in., no particularly serious results need be anticipated. This was based on conditions obtaining after ten cycles of opening and closing, or closing and opening, of the joints. It would be of interest to know whether there was any progressive destructive action noted during the tests, either when the error was small or when large errors existed. There is also the question of what would have happened had the number of cycles been such as would compare with the number that would occur during the life of the pavement.

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[13] With National Park Service, U. S. Dept. of the Interior, Washington, D. C.

From an analysis of conditions that exist when the error in alignment is large, it appears that two types of failure may occur. For a vertical misalignment the failure appears to be in tension, whereas a shear failure, the result of a compressive stress, is indicated when a horizontal error exists. Since the resistance to failure is less in the case of vertical errors, one wonders what the maximum permissible errors in the two directions might be, and if there is actually more likelihood of a failure in the case of a vertical error than for a horizontal error of the same magnitude. From the fact that the tendency to spall was decreased as the thickness of the slab was increased, it would appear that such might be the condition.

It is of interest to note that the effect of errors in alignment is greater at expansion joints than it is at contraction joints. Except for slight differences in the location of the dowels relative to the top and the bottom of a slab and for the fact that only one end of the dowel is free to move in the case of an expansion joint, the conditions are much the same. However, a 3 000-lb pull per bar in the case of a contraction joint, as opposed to a 4 000-lb compression for an expansion joint, would be expected to accentuate the tendency to spall in the case of the contraction joint, as is evidenced by the fact that some spalling is observed occasionally in pull-out tests where bars are axially stressed in tension and the concrete is stressed because of the bond. In the tests reported, an effort was made to reduce the bond. However, the pull required is evidence of the fact that some direct tension was imparted to the concrete when opening a contraction joint, and there is little doubt but that some of the tension was resisted by the concrete immediately adjacent to the joint. One wonders also if confining the movement of the dowel to one part of the slab might not have been in some degree responsible for the greater tendency to spall. Although it is difficult to understand how the condition could produce the effect observed, the increased load required to close an expansion joint, as compared with that for opening a contraction joint, is more readily explainable when reference is made to load-slip data obtained in pull-out tests.

On the basis of their observations, the authors have recommended that 24-in. dowels be placed so that the maximum error will not exceed 1 in. Any deviation from perfect alignment is undesirable, despite the fact that small errors appear to cause no serious results. Consequently, it is the writer's opinion that engineers should require and insist on the making of every reasonable effort to insure all dowels being installed as nearly as possible parallel to the longitudinal elements of the pavement. Particularly should this be the case since, as stated by the authors, there is always some likelihood of there being some movement of the dowels during the placing of the concrete.

L. E. Grinter,[14] M. Am. Soc. C. E. (by letter).—This excellent paper reports the results of tests made to determine a second influence on the action of dowels that may be quite as important as their action in resisting vertical shear caused by wheel loads. Any attempt to design dowels to resist, safely, the shear produced as a wheel load moves across the joint evidently will fail in its major objective if misalignment can produce failure anyway. Strangely enough, the stresses produced by wheel loads and the stresses produced by misalignment are essentially alike, and this statement holds true both for the stresses in the bar and for the stresses that the bar produces in the concrete slab. The result of excessive stresses in either case would be a spalling failure of the concrete as illustrated so clearly in Fig. 9. It must be the purpose of good design to prevent such failure from the action of the various possible load conditions, and it is the purpose of good construction to prevent such possibility from misalignment.

Internal Deformations.—An understanding of the kinds of strains produced by misalignment might aid one in an interpretation of the author's conclusions. It will be evident from Fig. 12 that the influence of a wheel load or of a heaving sub-grade, as indicated in Fig. 12(*a*), is identical with the influence of misalignment, as shown in Fig. 12(*b*). In either case the dowel across the joint acts as a short beam, restrained at its two ends, which undergoes differential settlement. If its end conditions are identical, this beam will deflect in a symmetrical reversed curve and will show a point of contra-flexure midway between the slabs. However, since the end conditions are unlike, one end being bonded and the other end being unbonded (and, therefore, slightly loose in the concrete), it is qualitatively justifiable to assume that the moment at the sliding end is only 50% or less of the moment at the bonded end.

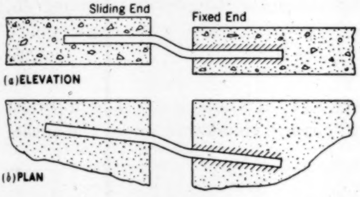

Fig. 12.—Distortion from Settlement and Misalignment

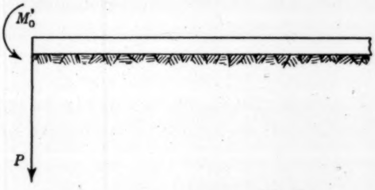

Fig. 13.—Bar on an Elastic Foundation

The general load condition, then, of a misaligned dowel bar is similar in most respects to the problem of any bar or beam of indefinite length, supported on an elastic foundation, as shown in Fig. 13. This problem has been analyzed by Professor S. Timoshenko[15] who found the deflection at the end of the bar to be:

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[14] Director of Civ. Eng., and Dean of the Graduate Div., Armour Inst. of Technology, Chicago, Ill.

[15] "Applied Elasticity," by S. Timoshenko and J. M. Lessels, Westinghouse Technical Night School Press, pp. 133–153.

$$y_0 = \frac{P + \beta\, M_0}{2\, \beta^3\, E\, I} \quad \text{.........................(18)}$$

in which,

$$\beta = \sqrt[4]{\frac{K\, b}{4\, E\, I}} \quad \text{.........................(19)}$$

In Equations (18) and (19), E, I, and b are, respectively, the modulus of elasticity, the moment of inertia, and the breadth or diameter of the bar; K is the foundation modulus, in pounds per square inch per inch of settlement. Hence, K is essentially an indeterminate factor in the case of a loaded dowel bar since it should combine the resistance of the slab with that of the sub-grade. For a dowel bar misaligned in the horizontal plane, K apparently becomes the same as the modulus of elasticity of the concrete.

It is possible to avoid a discussion of the factor, K, in one study by investigating the general equation for the deflection of the bar,

$$y = \frac{e^{-\beta\, x}}{2\, \beta^3\, E\, I} \left[P \cos \beta\, x + \beta\, M_0 \left(\cos \beta\, x - \sin \beta\, x\right)\right] \text{......(20a)}$$

which will be simplified temporarily by dropping the term involving M_0,

$$y = \frac{e^{-\beta\, x}}{2\, \beta^3\, E\, I} \left(P \cos \beta\, x\right) \text{...................(20b)}$$

Equation (20*b*) is plotted in Fig. 14 in terms of the angle, $\beta\, x$. The important considerations are that the positive or downward deflection under the applied load exists over a length, a, which is one-third the length to the second point of zero deflection.

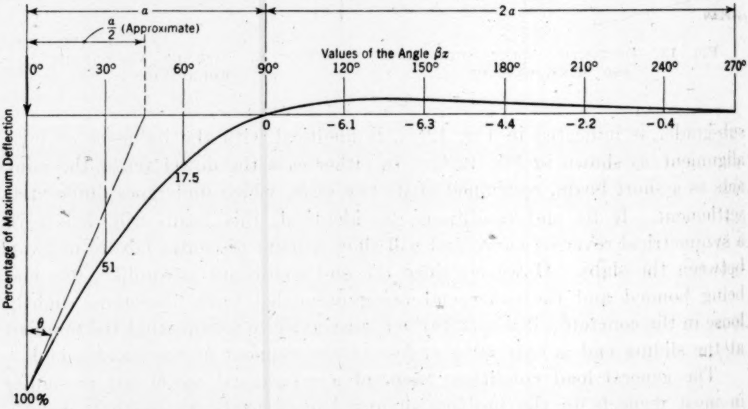

FIG. 14.—DEFLECTION DIAGRAM FOR A BAR ON AN ELASTIC SUPPORTING MEDIUM

From a study of Fig. 14 it seems reasonably evident that any dowel bar must act at some stage of its life in such a manner that only bearing pressure under about one-third its embedded length is available to resist the dowel shear, and that over this length the bearing pressure reduces in nearly a parabolic line from a maximum value to zero. The dowel moment, M_0, will act to increase the severity of the dowel pressure, but, in so far as the general shape of the deflection curve is concerned, the influence of M_0 will not be important.

Example of Misalignment Study.—If the concrete of the slab has an ultimate compressive strength of 3 000 lb per sq in., and if the dowels are $\frac{3}{4}$-in. round bars, 24 in. long, the ultimate shearing capacity per dowel before the concrete begins to spall slightly, probably, is not greater than $V = \frac{3\,000}{3}\left(\frac{12}{3} \times \frac{3}{4}\right) = 3\,000$ lb. This conclusion is based upon the form of the deflection curve of Fig. 14. The authors found that the alignment error in a 6-in. pavement could be 1 in. without causing spalling even when the joint was opened to a width of 0.75 in. An error of 1 in. in 22 in. amounts to 0.03 in. in a length of 0.75 in.

If the misalignment to be considered is in the horizontal plane of the pavement, the elastic compression of the concrete due to the bar pressure can be computed from Equation (18) when the value of K is the modulus of elasticity of the concrete. This modulus is assumed herein at the low value of 1 500 000 to account in a crude manner for deformations at stresses near the ultimate. Hence, by Equation (19) $\beta = \sqrt[4]{\frac{1\,500\,000 \times 0.75}{4 \times 30\,000\,000 \times 0.049 \times 0.75^4}}$ $= 0.88$. In order to obtain an average condition (joint open 0.75 in.), M_0 may be taken at $0.375 \times 3\,000$; and, by Equation (18),

$$y_0 = \frac{3\,000 + 3\,000 \times 0.375 \times 0.88}{2 \times 0.88^3 \times 30\,000\,000 \times 0.049 \times 0.75^4} = 0.0062 \text{ in.}$$

As shown by the exaggerated picture, Fig. 15, the misalignment is accounted for by three distortions:

(*a*) The bending of the bar as a fixed-end beam in the gap between slabs (this contribution is small, amounting to only 0.0002 in.);

(*b*) The movement, y, permitted by the elastic and inelastic deformation of the concrete; and,

(*c*) The movement, $\theta\, x$, that is permitted by the slope taken by the bar across the open joint.

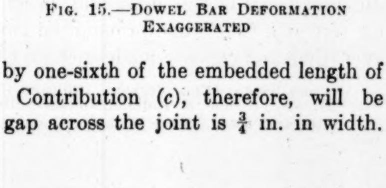

FIG. 15.—DOWEL BAR DEFORMATION EXAGGERATED

Contribution (*b*) evidently is twice the value of 0.0062 computed by Equation (18), or 0.0124 in. Angle θ can be approximated from Fig. 15 as the deflection divided by one-sixth of the embedded length of bar or $0.0062 \div 2 = 0.0031$ radian. Contribution (*c*), therefore, will be $0.0031 \times 0.75 = 0.0023$ in. when the gap across the joint is $\frac{3}{4}$ in. in width.

Hence, the total misalignment deformation would become 0.0124 + 0.0023 + 0.0002 = 0.015 in.

Conclusion.—Since this deflection is only about one-half the movement that the authors consider permissible, it appears that the action of the test slabs could be explained on one of the following bases:

(1) That the bars did not fit tightly into the slabs and, therefore, were able to "give" slightly (such dowels are of questionable value in transferring load across a joint, which is their major function since an unsupported interior edge is a line of great weakness);

(2) There may have been some undetected crushing or even a slight spalling which would prove a source of ultimate failure if slippage should be repeated many times; and,

(3) Looseness or "give" at one slab would produce a possible maximum moment in the bar of 3 000 × 0.75 = 2 270 in-lb. This moment would stress a $\frac{3}{4}$-in. round bar to 55 000 lb per sq in. If the bars used are of ordinary mild steel, they may, therefore, be deformed permanently.

The writer calls attention to the approximations involved in the computations presented herein. Much can be learned by such calculations, but they should be taken as significant qualitatively rather than quantitatively. From them the writer draws the general conclusion that the authors' specifications for permissible misalignment may be overly generous if proper dowel action is expected during the entire life of the pavement. Since a practical construction procedure could be devised to eliminate inaccuracy in excess of $\frac{1}{4}$ in. in alignment, this value could be set as a practical standard of workmanship. The importance of this requirement will be evident when it is understood that the stresses accompanying dowel action induced by loads, by heaving of the sub-grade, and by misalignment, may be additive.

Arthur R. Smith,[16] M. Am. Soc. C. E., and **Sanford W. Benham,**[17] Assoc. M. Am. Soc. C. E. (by letter).—In the discussions of this paper attention has been called to some very important considerations in dowel-bar installations and their functioning as a load-transfer device across joints in concrete pavement.

It was appropriately suggested that, in view of the results of the experimental work to determine permissible error, the recommended specification for accuracy was too liberal, particularly since the joints in the experimental slabs were opened and closed with no vertical loads, simulating wheel loads, at or near the joint. If the slabs had been loaded during the tests, it is quite likely that different results would have been obtained since the effect of wheel load and some directions of dowel-bar error are additive. In outlining the program of tests it was the original intention to conduct the tests in this manner on different types of sub-grade and, in addition, to construct experimental slabs for test at a time when pronounced curling was present at the joints. However, time and expense made such an elaborate program impossible. Because of the need for a specification covering accuracy of installation, to correct careless construction practice, the study was necessarily limited to a single phase of the subject; that is, to misalignment alone.

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[16] Cons. Engr. (Eng. Service), Chicago, Ill.; formerly Engr. of Materials and Tests, State Highway Comm. of Indiana, Indianapolis, Ind.

[17] Research Engr., State Highway Comm., Bureau of Materials and Tests, Indianapolis, Ind.

A dowel bar stresses the concrete in which it is embedded because its axis is being bent, due to such causes as traffic load transfer, curling of the pavement at joints, unequal heaving due to frost action, or binding when the joint opens or closes (due to misalignment). With the exception of misalignment all the other causes which tend to bend the bar, thereby damaging the concrete, are entirely independent of the accuracy with which the bar is installed; that is, a misaligned bar across a joint that has not changed in width will transfer wheel loads as effectively as a perfectly installed bar. The coating of bituminous paint (approximate thickness, 0.020 in.) to prevent bond provides a clearance which, although undesirable in load transfer, is of great advantage in permitting some movement of a joint with misaligned bars before the bars begin to bind in the concrete. Although clearance is undesirable as regards load transfer because the bar does not act until the loaded slab has deflected and because it also increases the funneling action of the bar, clearance must be had because, without it, it is not possible to install the bars and hold them in position during the placing and finishing of the concrete with sufficient precision to permit movement without cracking.

The joints in the experimental slabs were opened and closed at the rate of 0.1 in. per min. In practice, movement is very slow. The pavement has attained considerable age before 1-in. expansion joints have closed 0.75 in. (to a width of 0.25 in.). During this period funneling has probably occurred around the dowel bar providing clearance which, with that created by the paint and oil, is sufficient to permit closing of the joint without distorting the bar enough to crack the concrete. This is a possible explanation for the apparently satisfactory condition of the installations shown in Table 1, in some of which the bars are in extremely faulty positions. These installations are now (1938) three years old and all the joints are at present in apparently satisfactory condition.

For these reasons it was believed that a specification for accuracy of installation more rigid than that recommended would add very little to the satisfactory functioning of the joints; certainly not enough to warrant additional cost. Because of the already high cost of joints it is undesirable to specify additional refinements unless it appears rather certain that improved structural efficiency will be obtained. The recommended specification for accuracy, if it had been in effect during the construction season of 1935, would have required the contractor to replace the joints identified in Table 1 as Tests Nos. 1, 10, 11, and 13. Field tests conducted during the following season, when the specification was in effect, showed that the bars were being installed with a degree of precision considerably better than that obtained during the previous year, indicating that the specification resulted in an improvement in construction practice rather than an encouragement of carelessness, as has been suggested.

The question has been raised concerning the tension produced in the pavement due to the resistance to movement of misaligned bars when the pavement is contracting. As was stated in the paper the greatest resistance to the opening of a joint was 3 000 lb per bar which creates 50 lb per sq in. tensile stress in 5-in. pavement in addition to the tensile stress caused by sub-grade friction. The observed loads to open and close joints included, of course, both dowel-bar and sub-grade resistance but the experimental slabs were so short in comparison with slab lengths in actual pavements that sub-grade friction can be ignored. In this instance the dowel-bar error was maximum (1.5 in.). To open the corresponding slab in which the bars were installed perfectly required 2 100 lb per bar, or 35 lb per sq in. tension in the slab. Thus, reducing the dowel-bar error from 1.5 in. to zero (perfect installation) reduced the tensile stress in the pavement 15 lb per sq in. It is believed that this reduction in tensile stress is not great enough to add appreciably to the structural efficiency of the joint; certainly not enough to justify an increase in cost of construction.

It is impossible to state definitely whether or not there was a progressive destructive action during the successive cycles of opening and closing the joints. After the appearance of the first crack or cracks, additional cracks occurred during the remaining cycles of movement in very few of the test slabs, due probably to the partial relief of stress in the slab. This observation is subject to a certain amount of modification since, in some instances, cracks were not detected until after the second or third cycle of movement. It is possible that they were caused by the first movement but did not open wide enough to be seen until after additional cycles. The loads required to open and close joints decreased slightly and rather uniformly with successive cycles of testing which leads to the belief that continuing the tests beyond ten cycles was not necessary.

It has been suggested that vertical errors are more serious than horizontal errors since it would seem that the pavement would offer less resistance to failure of the type shown to the right in Fig. 9 than to the formation of a vertical crack throughout the depth of the pavement as would be caused by a bar having horizontal misalignment. This seems to be a reasonable opinion although failures were about equally divided between the two directions of error.

It is hoped that the interest displayed in this subject is an indication of increased attention to the perfection of load-transfer devices across joints in concrete pavement.

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TRANSACTIONS

Paper No. 2003

ESSENTIAL CONSIDERATIONS IN THE STABILIZATION OF SOIL[1]

BY C. A. HOGENTOGLER,[2] ASSOC. M. AM. SOC. C. E., AND
E. A. WILLIS,[3] ESQ.

WITH DISCUSSION BY MESSRS. JACOB FELD, DONALD M. BURMISTER, AND C. A. HOGENTOGLER AND E. A. WILLIS

SYNOPSIS

All soil mixtures commonly used for road surfaces and speedways are highly stable at some moisture content. This is equally true of cohesionless beach sands, friable glacial silts, and highly plastic clays. Hence, every possible combination of sand, silt, and clay may serve satisfactorily as aggregate in stabilized soil, provided the right binder is used to cement the soil particles together. Furthermore, the moisture which binds soil mixtures into masses stable enough to withstand the abrasion produced by racing motor cars must have surface tension and viscosity greater than those of free water. On these two facts the entire theory of soil stabilization is based.

The underlying principles involved in soil stabilization and the possible means of its accomplishment are discussed without attempting to evaluate the practical aspects or the relative values of the various methods. Particularly stressed is the application of the collodial phenomena of adsorption and base exchange (21)[4] as they affect: (*a*) Particles of soil, sand, crushed rock, gravel, slag, etc., coated with films of air, water, soluble chemicals, and binders not soluble in water; (*b*) the relative adhesion between solids and films; and (*c*) the effect of the chemical composition of aggregate and binders and the ions on the surfaces of the solid particles.

Construction methods, details of design, and methods of test have been discussed generally in another paper (1*a*) and will not be included herein. Reports listed in the Bibliography (which forms an Appendix

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[1] Presented at the meeting of the Highway Division, Birmingham, Ala., October 17, 1935, and published in June, 1937, *Proceedings*.

[2] Highway Engr., Div. of Tests and Research, U. S. Bureau of Public Roads, Washington, D. C.

[3] Associate Highway Engr., U. S. Bureau of Public Roads, Washington, D. C.

[4] Numbers in parentheses refer to the Bibliography in the Appendix.

to the paper) furnish more detailed information on the manner of applying the various treatments and making required tests as follows: Graded soil mixtures (2); calcium chloride (3); sodium chloride (4); asphalt emulsion (5), (6); asphalt, sub-oiling method (7); asphalt, mixing method (6), (8); tar (9), (10); Portland cement (11), (12); sulfite liquor (13); molasses (13), (14); calcium silicate (15); electro-chemical treatment (16); application of heat (17); mechanical analysis (18), (19); routine soil tests (18), (19); and Proctor tests (20).

ADSORPTION

Some soil particles have high attraction for air and, under proper conditions, become coated with air films; others have high attraction for moisture and become coated with moisture films. Dried soil clods retain moisture enough to bind the particles into hard and, in some instances, enormously strong masses. On the other hand, air is often adsorbed on soil solids so strongly after a period of drought that drops of rain will roll along on dust without wetting it.

The power that plastic clays possess of retaining their plasticity when mixed with sand or other non-plastic material, is caused by adsorption of one solid by another. The clay is not distributed uniformly through the pores or interstices of the coarse particles, but most of it forms a coating on the non-plastic material and many of the pores remain unoccupied even if there is more than sufficient clay to fill them.

For water or other binder to wet soil particles coated with air, it must displace the air, which means that the particles must attract the liquid more strongly than they attract the air. In like manner clay adsorbs different liquids to different extents; that is, its power of adsorption is selective and under suitable conditions one liquid will displace another in contact with the clay.

Bancroft (21), (22) calls attention to thicknesses of adsorbed moisture films which have been found on different materials as follows (in millionths of an inch): On glass of one kind, 5.1; on glass of another kind, 3.1; on sand (10 mesh), 11.0; on sand (60 mesh), 4.5; and on plastic wet clay, 2.0. The stationary film on copper was observed to be 0.000150 to 0.000225 in. when water flowed rapidly and about three times as much when it flowed more slowly over the copper. The air film between two pieces of glass compressed at 7 grams per sq cm (0.099 lb per sq in.), was found to be 0.000120 in. thick.

FILM MOISTURE IN SOIL

In completely saturated soil, every particle is covered with a coating of film moisture attracted to it more strongly than by the force of gravity. Until enough water is added to the soil to satisfy the condition of equilibrium between attraction of gravity and of the soil particle for water, all the moisture is arranged around the particles as films, with the interstices between the films filled with air.

It is impossible to remove all the film moisture from soils by enormous centrifugal forces exerted by powerful centrifuges. Therefore, it is concluded that much of the film moisture is held on the soil particles by adhesion equal to, or greater than, the enormous forces that fail to remove it. As a result of this great adhesion, film moisture is more viscous or glue-like than water in bulk.

The chemical composition of the soil particles, the nature of ions adsorbed on their surfaces, the moisture content, degree of compaction, and temperature of the soil control the thicknesses of moisture films, which vary widely.

Chemical composition of soil is indicated by the ratio of silica to the combined iron and aluminum oxides. This ratio is termed the silica sesquioxide ratio and is designated by the symbol, $\frac{SiO_2}{R_2O_3}$. Clays high in silica, termed "podsols," consist principally of the highly water-adsorbent, scale-like particles productive of high plasticity and shrinkage. Soils high in iron and alumina, termed "laterites," consist more of the bulky or spherical particles which do not attract water so strongly.

A colloid which holds hydrogen adsorbed on its surface is termed a hydrogen-ionized or H-colloid. One with calcium adsorbed on its surface is termed a calcium-ionized or Ca-colloid. If a substance like hydrated lime, $Ca(OH)_2$, is leached through soil containing H-colloids, the calcium replaces the hydrogen to form Ca-colloids, and the hydrogen thus released combines with the $(OH)_2$ to form water. Such exchange of ions is termed "base exchange."

The potassium ion is representative of those ions which undergo small volume change when alternately wetted and dried. It has a true diameter of 0.000000009 in. and in suspension becomes associated with 16 molecules of water, thus attaining an apparent diameter of 0.000000021 in. The lithium ion is representative of those ions which undergo great volume change when alternately wetted and dried. It has a true diameter of 0.000000006 in. and becomes associated with more than 120 molecules of water thus attaining an apparent diameter of 0.000000040 in. Between potassium and lithium other of the more common metallic ions can be arranged in the order of their attraction for water (36). Therefore, a potassium clay would be expected to undergo the least, and a lithium clay the greatest, volume change under climatic variations. Change of ions on the surfaces of the clay and colloidal particles effects a change in shrinkage, swell, and like properties on which the stability of the soil depends.

The concentration of the hydrogen ions determines the relative acidity of alkalinity. It is indicated by the pH-value which may be defined as the reciprocal of the logarithm of the grams of ionized hydrogen per liter of solution or suspension. The lower the pH-value is below 7.0, the greater is the acidity of the liquid and the greater is its base-exchange capacity.

The moisture content of a clay soil controls the thicknesses of film on the colloids. This, in turn, determines the density to which a particular pressure will compact the soil.

The relation between dry-weight density per cubic foot of samples compressed at equal pressures, to the moisture contents in percentage of the total soil and moisture volumes, at which the samples were compressed, was found by C. A. Hogentogler, Jr. (28) to consist of a series of straight lines with different slopes, as shown by the full lines in Fig. 1. The broken line in Fig. 1 shows the relation the density would have to moisture content if the samples contained no air. At any density, the difference in moisture contents indicated by the two lines represents the percentage of moisture, by volume, required to replace the contained air.

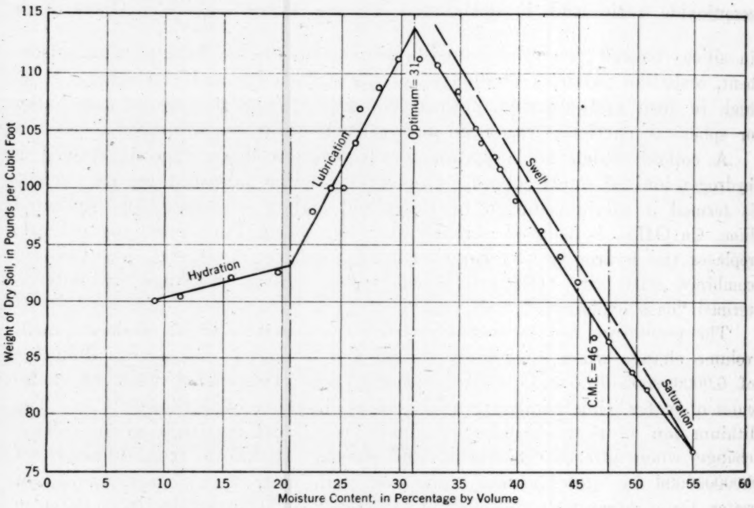

FIG. 1.—RELATION BETWEEN DENSITY AND MOISTURE CONTENT.

The moisture contents at which the straight lines intersect, indicate the limits of four distinct stages of wetting which the compressed samples undergo before their pores become completely filled with water. Wetting up to a moisture content of 20.7% (Fig. 1) may be termed the state of hydration, during which part of the contained water is absorbed by the soil particles, and the remainder is adsorbed on their surfaces in the form of cohesive films.

Moisture contents ranging from 20.7 to 31.1% indicate the stage of lubrication. Part of the contained moisture now acts as a lubricant to facilitate the re-arrangement of particles being compacted without, however, excluding all the air. Water in excess of 31.1% causes the soil mass to swell, although the air content is not further appreciably decreased until a moisture content of 47.7% is reached. Moisture contents between 47.7

and 54.3% represent the stage of saturation, during which practically all the air is displaced and the soil becomes truly saturated.

The moisture content (31.1% in this case) at which maximum density is attained is the optimum suggested by R. R. Proctor, M. Am. Soc. C. E. (20) for use in connection with the construction of earth embankments and dams.

Temperature is an influencing factor because in cold weather the adsorbed films are in general thicker than in warmer weather (1*b*).

Effect of Film Character on Properties of Soil

The free water, or pore water, is that which fills the interstices remaining between the outer surfaces of the adsorbed moisture films and not the pores between the surfaces of the soil particles as such. The flow of water through soil likewise occurs between the films adsorbed on the particles. As a result the speed at which water percolates through soil, as well as the moisture contents indicative of some specific state of stability of soil, varies wherever the thicknesses of the adsorbed films change, in some cases, when the viscosity of the free water changes.

Winterkorn (23), (24), (25) has shown that, due to the change of the kind of adsorbed ions on the particles of one type of soil, the liquid limit varied between 39 and 57, the plasticity index between 16 and 40, and the percentage of colloids between 11 and 28. The moisture content increase, indicating the swell of the samples in the Terzaghi compression device when the pressure was reduced from 3.2 kg per sq cm to 0, varied between 1 and 5 per cent.

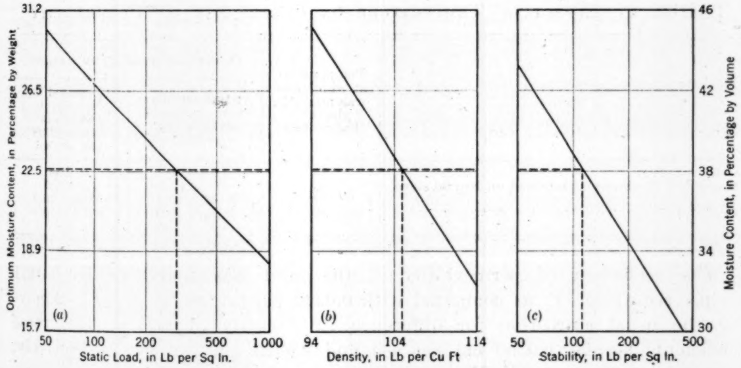

Fig. 2.—Control Curves.

The optimum moisture content (mentioned previously) at which maximum density is attained is not a constant for a particular soil, but varies as the compacting pressure is changed. The relations of optimum moisture content to compacting pressures, attained densities, and stabilities, as indicated by the Proctor plasticity needle for one soil investigated at George Washington University, are shown in Fig. 2.

Change of temperature alone, when other conditions are constant, causes considerable variation in the indicated properties furnished by tests. The test data furnished by A. M. Wintermyer, Assistant Highway Engineer, United States Bureau of Public Roads, and shown in Table 1 disclose no effect of temperature on the liquid limit of Sample S-9616, which is pumice having no plasticity.

TABLE 1.—EFFECT OF TEMPERATURE ON PLASTICITY TEST

Sample No.	Tested at 40° F			Tested at 90° F		
	Liquid limit	Plastic limit	Plasticity index	Liquid limit	Plastic limit	Plasticity index
S-9616	40	..	0	40	..	0
S-9617	57	30	27	49	23	26
S-9618	36	24	12	32	23	9

The liquid limits and plastic limits of both plastic soils (Samples S-9617 and S-9618, Table 1) were higher when the tests were made at the lower temperature. However, the corresponding variations in the plasticity indexes, which are most used for field control in the construction of stabilized roads, are relatively small.

The results of tests performed by Messrs. R. C. Thoreen and F. A. Robeson, of the U. S. Bureau of Public Roads, and shown in Table 2, reveal the relative influence of both viscosity and adsorbed film thickness on coefficients of permeability.

TABLE 2.—EFFECT OF TEMPERATURE ON COEFFICIENT OF PERMEABILITY

Soil	Temperature at which test was made, in degrees Fahrenheit	Coefficient of Permeability	
		At test temperature, in centimeters per second	Corrected for viscosity to 68° F, in centimeters per second
Sand	85	0.0481	0.0390
Passing No. 60 sieve and retained on No. 80 sieve	85	0.0222	0.0375
Clay soil	72 50	6.47×10^{-8} 3.49×10^{-8}	6.13×10^{-8} 4.54×10^{-8}

The coefficient of permeability of the sand was found to be 0.0481 cm per sec at 85° F, as compared with 0.0222 cm per sec at 35° F. Applying the usual correction for difference in viscosity of water (33), these coefficients become 0.0390 cm per sec and 0.0375 cm per sec, respectively, at a temperature of 68° F.

The coefficient of permeability of the clay soil was found to be 6.47×10^{-8} cm per sec when tested at 72° F and 3.49×10^{-8} cm per sec when tested at 50° F. These coefficients, when corrected for viscosity of water at 68° F, become 6.13×10^{-8} cm per sec and 4.54×10^{-8} cm per sec. Permeability tests made on samples of a similar clay at various voids ratios indicate that the decrease in permeability due to the greater film thicknesses at the lower temperatures, shown in Table 2, is equivalent to that produced by reducing the voids ratio from 0.97 to 0.89, which would indicate an increase in the effective volume of the solid particles of about 4 per cent.

The effect of temperature variation on the stability of soil is illustrated by tests on a clay soil compacted in the Proctor cylinder and penetrated with the plasticity needle (20). The data showed (1*f*) that the stability was 1 400 lb per sq in. when compacted at 42° F; 960 lb per sq in. when the temperature of the sample was raised to 130° F; 1 490 lb per sq in. when the temperature was then reduced to 33° F; and, finally, 1 100 lb per sq in. when the temperature was again raised to 118° F.

Types of Soil Stabilization

In the application of information now available on soil stabilization the methods to be used depend largely on the availability of required materials. In some locations, such as the southeastern part of the United States, Nature provided deposits of sand clay and top-soil having the grading and character required in the best of road soils.

In some locations, binder soils and aggregates are available for producing mixtures having the properties of the best of the naturally good soils. In other locations, materials are available for the graduations required in roads, but the binder soil may be of inferior quality. Finally, there are locations where there is deficiency of the aggregate required for properly graded mixtures.

Studies of the best of natural road soils indicate that the design requirement of stable soil mixtures should include the following:

(1) The aggregate should be hard and durable enough to resist weathering, traffic abrasion, and crushing. Sound, tough particles or fragments of gravel, stone, slag, or combinations of them, crushed to the proper size, should prove suitable. Certain types of shales and similar materials that break and weather rapidly when alternately frozen and thawed, or wetted and dried, should not be used.

(2) The soil fines should be of a character such as to provide graded mixtures with the proper balance of capillarity and adhesion without risk of detrimental volume change. It is particularly essential that the fines do not swell enough in the presence of moisture to cause the clay to become a lubricant instead of a binder.

(3) When local materials are available for the proper proportioning of aggregate and binder soil, but the natural clay does not have the binder value required in highly stable road surfaces, a number of admixtures may be used singly or in some combination. There are, first, the chemical salts, calcium chloride, sodium chloride, and magnesium chloride; and second, the waste products of industry, such as sulfite liquor from the manufacture of wood pulp, "blackstrap" from the molasses industry, and huge accumulations of waste sizes of the mineral aggregate industry.

(4) When only fine or poorly graded materials are available, the asphaltic, tar, and Portland cement binders may be utilized to provide stable base courses to be covered with bituminous surfacings. In isolated cases heat treatment has been used.

In the stabilization of fills the Proctor method or some modification is receiving considerable attention and for foundation soils, injection of chemicals and electro-chemical treatment furnish possibilities.

ADMIXTURES AFFECT SOILS IN A NUMBER OF WAYS

Among the ways that admixtures may assist in stabilizing soils are the following:

(1) By absorbing moistures from the air and reducing evaporation from the road surface, some chemicals may serve to keep the roads in a damp condition. This reduces the dust nuisance and facilitates the compaction of the surfaces under traffic.

(2) By electro-chemical action, chemicals and the waste products may serve to increase the density and stability of road mixtures.

(3) By chemical action, the same materials may combine with each other, or with certain of the natural clay constituents, and thus form water insoluble cements of high binding value.

(4) The Portland cement and bituminous materials serve the double purpose of binders and for eliminating the destructive water-absorbent properties of the clay binders.

It is difficult to distinguish between the electro-chemical and the purely chemical actions of admixtures for increasing the stability of soils. However, singly or in some combination they seem to explain not only the performances of soil roads in service, but also suggest relatively simple procedures for future construction.

As primes and fillers, certain soaps, etc., and admixtures of hydrated lime, and certain types of stone dust seem admirably suited for increasing the adhesion of bituminous binders for certain soils.

In addition to their deliquescent and water retentive actions, the chloride salts, by electrolytic action, may cause the compacted mixture to attain densities greater than untreated soils, and they minimize the volume change of the binder which is productive of disintegration of the untreated surfaces with changing moisture conditions.

Mixtures of acidic and neutral materials seem to attain greater stability than that of either one separately. Limestone dust and certain slags seem admirably suited for use as pre-treatments, or with other admixtures to neutralize acid soils. This may be especially desired in the case of siliceous clays. In some base courses now (1937) being contructed, an acid clay is mixed with non-acidic limestone aggregate.

In connection with the formation of natural colloidal cements two types of either rock powders or clays must be considered: Those which on soaking in water or watery solutions soften to the extent that they become, to ever so slight a degree, glue-like, and those materials which do not. To a small extent many of the sandstone, trap, and limestone powders are thus made glue-like by water alone and more so by water-soluble inorganic salts (13).

Experiments have shown that the cementing value of mixtures of the two types of materials (those which soften and those which do not) is considerably greater than the binding value of either material alone. Mr. A. S. Cushman and Prévost Hubbard, Affiliate, Am. Soc. C. E., found (13): (*a*) That by combining a granite with a cementing value of 3 and a limestone with a cementing value of 27, they obtained a cementing value of 110 for the mixture; (*b*) that a granite powder which had a cementing value of 10 when mixed with water alone, had a cementing value of 44 when mixed with lime water; and (*c*) that an amorphous chert which had a cementing value of 6 when mixed with water alone, had a cementing value of 22 when mixed with 4% of calcium oxide and water, and more than 2 000 when mixed with 13% of calcium oxide.

As a result it has been found possible to construct low type surfaces of only waste limestone or slag screenings treated with the chloride salt solutions. With granite screenings the addition of a small quantity of limestone also has been found to be beneficial.

ADMIXTURES OF PORTLAND CEMENT AND BITUMINOUS MATERIALS

Efforts to stabilize fine-grained soils by admixtures of lime and cement materials were made in Iowa and South Dakota as early as 1924 and in Ohio several years later. The results of this early work were not particularly promising, but they should not be considered as indicating the possibilities of such treatments, because the requirements of the thorough distribution of the admixture, the high degree of compaction, and the protective surface treatment now deemed necessary, were not recognized in the earlier work.

More recently (11), (12) research on the use of Portland cement for stabilizing soil bases has been performed in South Carolina by the State Highway Department. The purpose of this work has been to develop a base material that could be constructed at less cost than that required to provide one of properly graded sand clay or top-soil.

The results from these experiments were so promising that a number of similar roads are being constructed in other States. Experimental base courses constructed with available soil materials and the various tars (9), asphaltic emulsions (5), and cut-backs, and oils (6, 8) show similar promise.

USE OF OTHER ADHESIVES

Waste Sulfite Liquor.—Products prepared from the crude liquor have been marked from time to time under various trade names, for use as binding mediums in road construction. As early as 1910 Mr. Hubbard (13) published results obtained by the U. S. Bureau of Public Roads, showing the increase in compressive strength obtained by treating various rock powders with products prepared from waste sulfite liquors. However, the binding base of the adhesives tested was soluble in water, and, as a result, it was found that frequent rains tended to destroy the bond and remove the lignin from the road surface. In order to offset this difficulty attempts were made, at that time, to waterproof or make insoluble the residual base without destroying its binding value by admixtures of asphaltic road oils and deliquescent chemicals, such as calcium chloride.

In 1936 waste sulfite liquor, marketed under a trade name, was used for the treatment of gravel surfaces in New Jersey and experimented with by the State Highway Department of Washington for use primarily as a dust palliative, although the possibilities for its use as a stabilizing agent, especially in well-graded mixtures, have not been overlooked.

Molasses Residues.—A thick, sirupy liquid by-product obtained in the manufacture of sugar from sugar cane is known as "blackstrap" which, when treated with quick lime, forms compounds of high binding value known as calcium sucrates. Blackstrap, in combination with oil and lime, was applied as an experiment by the U. S. Bureau of Public Roads on a road at Newton, Mass., during the summer of 1908 (13), and a molasses-lime mixture was applied as a surface treatment on a section of the Bradley Lane Experimental Road, Chevy Chase, Md., in December, 1911 (29). Within a year, however, rains removed all traces of the treatment, causing the road to take on the appearance of an adjoining section which had been maintained as untreated macadam.

In January, 1936 (14), attention was called to a practice in India in the olden days, when a kind of coarse sugar, made by evaporation from the sap of palm trees, was added to lime to produce an improved quality of mortar. With this as a background, a short length of road surface formed of lime kankar was treated and the results were encouraging. (Kankar is a kind of limestone usually occurring as nodules in top-soil, used for making lime and in building roads in India.) In order to prevent the molasses from being washed away during rains, slaked lime was mixed with it, the claim being that the addition of burnt lime to molasses produces tri-calcium sucrate, which is insoluble in water.

Calcium Humate.—When lime is added to a soil containing humus (24), the calcium replaces the acidic hydrogen in humic acid, forming the more stable so-called calcium humate or neutral humus.

Hilgard (30) points out that there are several humates (of lime, magnesia, and iron) which assist in producing and maintaining crumb structure. When fresh, these substances are colloidal (jelly-like), like clay itself, but, unlike the latter, when once dried, they do not resume their plastic form by wetting. The crumbs thus formed are, therefore, quite permanent and contribute to the looseness of uncompacted soils rich in humus. One part of lime humate is said to be equal in cementing power to eleven parts of clay.

Calcium Silicate.—The insoluble calcium silicate binder is produced by treating soils first with sodium silicate followed by the application of calcium chloride according to the general principle used in the old English method of manufacturing artificial stone (31). Its use in soil stabilization has been principally in connection with increasing the stability of granular soils in connection with the construction of buildings and other structures (15).

TREATMENT WITHOUT ADMIXTURES

Electro-Chemical Treatment.—In a method utilizing electricity for stabilizing soils, described by Endell and Hoffmann (16), metal electrodes are introduced into the soil at appropriate distances, aluminum always being used for the anode and copper for the cathode. Direct current is then allowed to pass between electrodes until the soil is hardened. Experiments were described in which a clay specimen had been brought into its container with an almost liquid consistency, the water content being about 80 per cent. After treatment for some time with a current of 8 to 14 amperes, at 300 to 500 volts, the clay had become so hard that an iron rod, 1 sq cm in cross-section, failed to penetrate it when placed under a load of 10 kg (22 lb). Specimens cut from the side of this block nearest the aluminum electrode neither disintegrated nor swelled during several months of immersion in water.

Application of Heat.—The essential features of this method of treatment are illustrated by a method used in the so-called "black-soil" areas in Australia, in localities where the haul for gravel or crushed rock is excessive (17). A slow-moving, down-draft furnace with a speed of about 10 ft per hr was developed at Sydney, New South Wales, for baking the soil in place and converting it into a brick-like material. The machine (see Fig. 3) is wood-fired, of the air-gas producer type, on a chassis on road wheels, and is propelled under its own power.

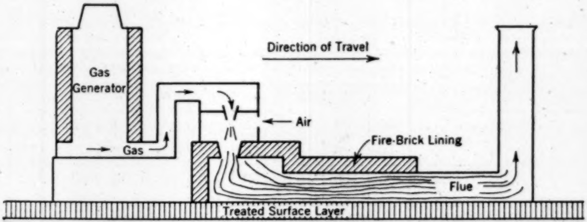

FIG. 3.—DIAGRAM OF FURNACE FOR HEATING SOIL IN PLACE.

Prior to heating, the road is graded to the desired cross-section and the strip to be treated is scarified to a depth of about 4 in. As the machine advances over a particular area, the soil is first subjected to the exhaust gases and then gradually increasing temperatures until the full intensity of the heat of the gases issuing from the generator is reached. Work proceeds continuously day and night, a machine 6 ft wide completing about 500 ft of roadway 18 ft wide per week. The effect of the treatment is noticed to a depth of about 6 in.

When traversed by roller the treated clods are broken down fairly readily. The clay binder is applied to the broken material, the road is shaped up and rolled again, and is then ready to be opened to traffic.

EFFECT OF GRADING AND ADMIXTURES ON DENSITY

The effects of grading and admixtures on stabilized soils compacted by the Proctor method (20) have been described in some detail elsewhere (1), the gradings and physical properties of red clay, Iredell, Arlington, and Manor soils being shown in tabular form (1*e*).

Grading.—The effect of the different gradings on the densities obtained at equal compactive efforts and fairly constant temperatures has been discussed in the earlier study (1*f*) (1*g*).

The red clay (Sample 1, Fig. 4, and Table 3) contains 68% of clay, 20% of silt, and 12% of sand. At the optimum moisture content of 16.7%, the dry weight was 113.0 lb per cu ft as compared with a dry weight of 129.8 lb per cu ft, at 9.6% optimum moisture content for the excellent soil mortar, Sample No. 6.

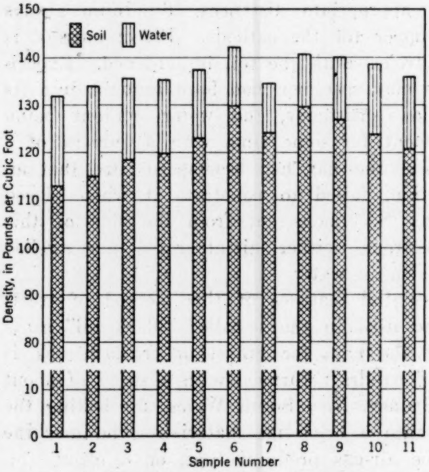

FIG. 4.—EFFECT OF GRADING ON DENSITIES OF COMPACTED SAMPLES.

TABLE 3.—GRADING OF COMPACTED SAMPLES (SEE FIG. 4).

Classification	GRADING (PERCENTAGES), FOR SAMPLE NOS.:										
	1	2	3	4	5	6	7	8	9	10	11
Clay	68	34	21	17	13	8	0	18	18	18	18
Silt	20	60	37	30	23	15	13	5	5	5	5
Sand	12	6	42	53	64	77	87	77*	77†	77‡	77§
Total	100	100	100	100	100	100	100	100	100	100	100

* 24% retained on a No. 20 sieve. † Nothing retained on a No. 20 sieve. ‡ Nothing retained on a No. 40 sieve. § 1% retained on a No. 60 sieve.

Samples Nos. 2, 3, 4, and 5, Table 3, were obtained by adding silt and sand to the red clay represented by Sample No. 1. An attempt was made to obtain proportions representative of the different fractions of Sample No. 6. Thus, Sample No. 5 represents approximately the grading of the fraction of material of Sample No. 6 that passes the No. 40 sieve; Sample No. 4 represents the fraction that passes the No. 60 sieve; Sample No. 3 represents the fraction that passes the No. 100 sieve; and Sample No. 2 represents the fraction that passes the No. 270 sieve.

Fig. 4, with Table 3, shows that as the total sand is decreased from that of Sample No. 6 to that of Sample No. 2, the weight of the compacted mixtures decreases and the optimum moisture contents increase. In Sample No. 7 an attempt was made to approximate the grading of the fraction of Sample No. 6 that was larger than 0.005 mm. As would be expected in the absence of the clay fraction, the optimum moisture content is smaller than that of Sample No. 6, being only 8.5 per cent. The weight of dry soil at maximum density, however, is only 124.1 lb per cu ft, which is 5.7 lb per cu ft less than that of the excellently graded material (Sample No. 6).

In the four samples, Nos. 8, 9, 10, and 11, Table 3, the sand, silt, and clay fractions are constant, but the grading of the sand fraction is varied. The maximum density of Sample No. 8 (with 5% of silt and 18% of clay, and approximately the same sand gradation as Sample No. 6) is slightly lower than that of Sample No. 6. As the coarser sand fractions are eliminated in Samples Nos 9, 10, and 11, and the mixtures approach a more nearly uniform grain size, their densities decrease.

Electrolytes.—In Fig. 5(*a*) are shown the effects of different electrolytes on the densities of compacted samples of the Arlington, Manor, and the Iredell soils. Using the Arlington samples for illustration, it will be noted that admixtures of sodium hypo-sulfite, sodium chloride, ferric chloride, and calcium chloride, effect an increase in weight of dry soil per cubic foot over that of the natural soil with water alone, whereas the weight of the sample treated with sodium silicate is less than that of the sample without admixtures.

Fillers and Adhesives.—In Fig 5 (*b*) are shown the effects produced by other admixtures on the densities of different samples. The fact that films of water-soluble adhesives might be thicker than those of water has no significance with respect to the relative stability of soils treated with the different materials.

TESTS USED IN SOIL STABILIZATION

In practice, the suitability of graded mixtures is determined by means of sieve analysis of the aggregate retained on the No. 10 sieve, combined sieve and hydrometer analysis of the fraction passing the No. 10 sieve, and the Atterberg plasticity tests performed on the fraction of the material passing the No. 40 sieve, an effort being made to combine materials in such quantities that the resulting mixture will conform to the requirements of stabilized road surfaces. To this end, specifications have been prepared on the basis of grading and the plasticity index.

However, any chance combination of two materials that falls within the limits of the specifications may not be the best possible combination. As has been shown by Miller (32), the specifications may call for a plasticity index of between 4 and 12, but for two given materials (aggregate and binder), there is one combination having a definite plasticity index at which highest density may be obtained.

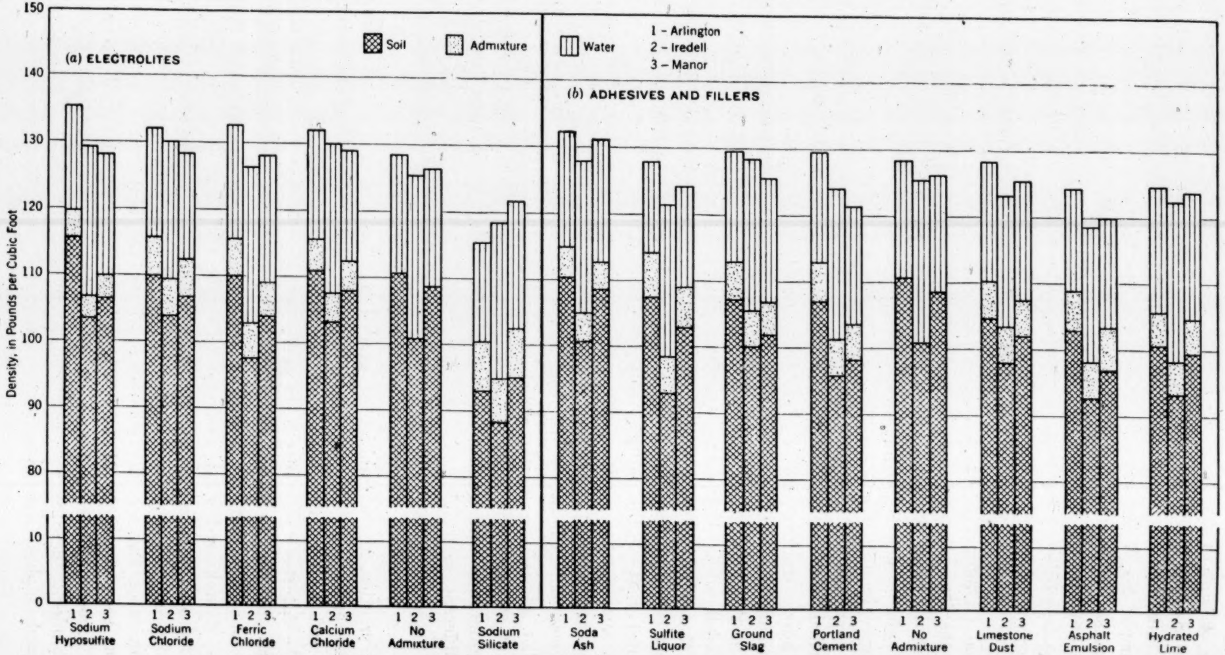

FIG. 5.—EFFECT OF VARIOUS ADMIXTURES ON THE DENSITIES OF COMPACTED SAMPLES.

The curve in Fig. 6 shows the maximum densities obtained in the Proctor test on five mixtures of Potomac River sand and Arlington loam. It indicates that the maximum density will be obtained by combining 30% of the Arlington loam with 70% of the sand. The plasticity index of this mixture is 6 and should be used for control in the field.

For the design of soil base courses in which asphaltic emulsions are to be used, C. L. McKesson, M. Am. Soc. C. E., advocates the use of special tests described elsewhere (5). The moisture content and the quantity of binder required in soil base courses stabilized with Portland cement, tar, and certain asphalts are determined by means of the Proctor tests.

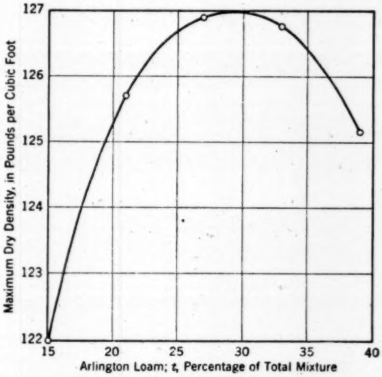

FIG. 6.—MAXIMUM DRY DENSITIES OF COMPACTED SAMPLES.

The Proctor tests are used extensively in connection with the construction of earth dams and embankments. However, it is believed that a modification of these tests, which furnishes data, such as those shown in Fig. 2, allows more latitude of construction procedure. For the compactive pressure to be provided by a given type of equipment, the corresponding optimum moisture content, resulting density, and plasticity needle readings are shown by the curves. Likewise, for any desired embankment density, the equivalent pressure that must be furnished by the rolling equipment, is disclosed.

To illustrate, let it be assumed that rolling equipment, exerting a compactive effort equivalent to a static pressure of 300 lb, is to be used. Then, the soil (Fig. 2) should be compacted at a moisture content of 22.9% of the weight of dried soil; the resulting density should be 104.4 lb per cu ft; and the plasticity needle reading, 117 lb per sq in. If, on the other hand, the embankment is to be constructed at an optimum moisture content equal to, say, the plastic limit (26%), the soil should be compacted at an equivalent static pressure of 130 lb per sq in. The resulting density is 99.2 lb per cu ft, and the plasticity needle reading, 69 lb per sq in.

The slopes of the density-moisture content curves may be quite different, as shown in Fig. 7. Assuming that the moisture content can be controlled in the field within a range of 5%, a variation of 2.5% above and below the optimum would be expected if the control value was set at the optimum moisture content. According to Fig. 7(*a*), densities from 98.4 lb per cu ft to 100.6 lb per cu ft would be expected from this particular soil. The density-moisture content relationship shown in Fig. 7(*b*) has a steeper slope than the one shown in Fig. 7(*a*), although the indicated maximum densities are the same. A variation of 2.5% above and below the optimum moisture content would result in densities between 94.35 lb per cu ft and 100.6 lb per cu ft. By setting the control moisture somewhat above the optimum in the case of the soil with a density-moisture content relationship, such as that shown in Fig. 7(*b*), it is possible to reduce the variation in density. Thus, using moisture contents from 2.5% below to 2.5% above 26.2%, the resulting densities would be between 97.4 lb per cu ft and 100.6 lb per cu ft.

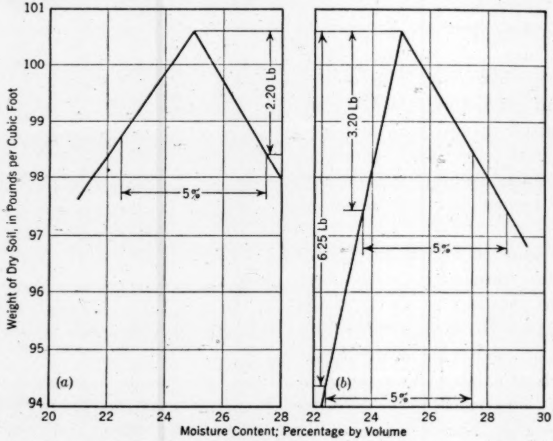

FIG. 7.—EFFECT OF MOISTURE CONTENT VARIATION ON DRY WEIGHT OF COMPACTED SOILS.

CONSTRUCTION VARIABLES

In the mixing of solid binders, such as clay and Portland cement, with aggregate, advantage is taken of the adsorption phenomenon that for thorough distribution it is only necessary for the materials to be brought in close enough association to permit the dry powders to become attached to and coat the aggregate.

Bituminous materials, in contrast, coat the soil more readily after the air films have been removed by wetting the particles. Therefore, water in various quantities is used to obtain the required distribution of tars and asphalts in soil mixtures.

The materials have been mixed by harrowing, blading, etc., by portable plants on the roads, and by means of plants set up at the source of supply of the soils or mineral aggregates.

Asphaltic materials used as stabilizers in Jackson County, Missouri (7), were introduced at desired depths below the surface of the loose road mix by means of a specially constructed "sub-oiler" (1*d*). It is essentially a tooth scarifier, having an oil line attached to the back of each tooth and running nearly to its point.

Graded mixtures have been compacted by rollers supplemented by traffic on roads and by rollers, trucks, etc., in fills. Generally, when the Proctor method of control is used, the prongs of a sheep's-foot or tamping type roller, or equipment that produces a similar effect, are required to penetrate the unconsolidated material and compact the layer from bottom upward, in order that the layer of fine material may be consolidated uniformly throughout its thickness without stratification. Compaction by vibrating the soil is another method being investigated.

SUMMARY

A review of the development of soil stabilization discloses the fact that the principal means followed in practice until several years ago involved the use of granular materials. With the increasing demand for farm-to-market roads the use of water-retentive chemicals was found to be a valuable supplement in the construction and stabilization of properly graded road mixtures. Attention was next directed toward the use of water-insoluble binders, including bituminous materials and Portland cement, for stabilizing poorly graded and fine-grained soils. Data now available indicate that:

1.—Soil stabilization involves film phenomena in one or more of the following steps: (*a*) Removal of the air film by wetting the soil particles; (*b*) making the liquid films stronger and more lasting by use of deliquescent and water-retentive electrolytes; (*c*) use of waste aggregate materials to provide the proper neutrality of the mixtures; and (*d*) replacement of the liquid or air films with bituminous materials, Portland cement, and other insoluble binders in the stabilization of fine or poorly graded soils.

2.—Admixtures which have been found unsatisfactory for treating road surfaces subjected to abrasion by traffic and action of the climatic elements may provide benefit when used in base courses protected from these influences by impervious bituminous surfacings. The use of adhesives is not expected or intended to render the soil sufficiently hard or tough to resist the abrasive action of traffic, but simply to render it resistant to water from capillarity and thus to retain the same stability that the soil had at the density obtained during construction.

3.—Use of natural soil with the thinner films, or chemical solutions to reduce the thickness of the films and thus provide greater density, furnishes three distinct benefits to soil structures in service. The thinner the films the smaller is the quantity of free water that can be released as the soil becomes warmer. Consequently, soils with chemically thinned films retain greater uniformity of stability under changing temperatures.

Reduced film thickness causes soil to attain high density with less compactive effort. This means lower cost for compacting fills, earth dams, etc., and less time required under equal traffic conditions for road mixtures to become stable. The latter is especially important in base-course construction where the highest density is desired in the shortest possible time, in order to prevent additional and, perhaps non-uniform compaction of the base course, after the wearing surface is applied.

The freezing point lowers as the thickness of the film diminishes. Therefore, the denser the soil at relatively low moisture content, the less likelihood is there of the films freezing, even if they consist of water alone. When the contained chemicals also depress the freezing point, a double provision against damage to stabilized soil due to frost is obtained.

CONCLUSION

In conclusion, it should be emphasized that soil stabilization is as yet in its infancy. Laboratory tests are helpful for disclosing the various possibilities of attaining stability of soils, but their value is limited and the constants furnished by them are truly constant only for the conditions of temperature, etc., at which the tests were made.

The plasticity tests disclose primarily only the binder properties of clay due to reversible phenomena. They do not throw light on the presence of colloidal cements that are likely to develop as the soils dry out during compaction under traffic.

The Proctor tests afford a simple and promising means of determining optimum binder contents and degrees of compaction required for the attainment of given densities. The optimum density to which a given soil should be compacted for greatest stability in embankments is not as yet known. It has been suggested that there is a theoretical optimum for a soil in dam construction beyond which tamping may become injurious (35). Meager data obtained in the laboratories of the U. S. Bureau of Public Roads seem to indicate the possibility of a drop in stability after the density of a sample at constant moisture content has been increased beyond a certain limit.

How much the different fine-grained soils will swell, over long periods of time, after being compacted to high density during the construction of embankments, is also a moot question. Generally, the pressure on the soil, at a given location, exerted by the weight of the embankment above it, is considerably less than the pressure at which the soil was compacted. Only colloidal glue action could prevent such soils from swelling. If swelling were likely to occur to considerable degree there would seem to be no reason for compacting the soil with a pressure much greater than that produced by the weight of the embankment the soil is to support.

The fact that some soils are particularly sensitive to small changes in moisture content indicates that from a practical standpoint they are undesirable for embankment construction. Furthermore, such soils when used as binders in stabilized road mixtures will re-act quickly to variations in moisture content with consequent changes in stability and cementing properties. The basic relationships between soil particles and surrounding moisture films are applicable to both embankments and stabilized soil roads, and any tests revealing these relationships are valuable aids in the design and construction of either type of structure.

Unfortunately, tests are not as yet available for disclosing the effects of climatic variables occurring during long periods of time on specially compacted and specially treated soils. As a result much of the desired information on the lasting effects of the base exchange, chemical actions, and especially produced film conditions must await the results of observations on stabilized soil roads and structures in service.

APPENDIX

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DISCUSSION

Jacob Feld,[5] M. Am. Soc. C. E. (by letter).—A fine summary of soil stabilization is presented in this paper which covers especially the effect of the colloids and moisture films. The real complexity of the subject can only be visualized by a complete definition of the word, "soil," as used in the engineering sense. Such a definition does not yet exist, and an agreement on the meaning of the word would be a great aid in the classification of materials and tests concerning soils. In 1924, the writer[6] enumerated the major constituents of soils and their respective contributions to soil properties as well as the effect on the colloidal constituents of various chemicals and electrolytes.

The farm technologist defines a soil as a porous mass of hard framework plastered over with a jelly containing chemically active matter, plant foods, and unstable organic compounds, the pores of which contain air and water. The farmer classes soils as heavy (clayey) or light (sandy), referring to the predominance of the mineral constituent.

The geologist distinguishes between (1) primitive soils formed by the mechanical loosening and chemical decomposition of rocks (or a combination of both); (2) derived soils formed by the mixture of primitive soils with possible additional weathering, due to water or wind acting as collecting agents; and (3) decayed vegetation mixed with small percentages of minerals forming humus, peat bogs, or similar highly organic soils.

It is quite well agreed that studies on pure sand, pure clay, uniformly grained granular substances, powdered rocks, or any laboratory combinations of these items, with or without water, can help but little, if any, in the solution of a field problem; and the impossibility of reproducing temperature changes, moisture distribution, and other variables, to say nothing of the difficulty of obtaining true soil samples, makes tests on actual soil samples of doubtful value. The foregoing statement would be true even in a uniform soil (a hypothetical and probably non-existent material).

Unfortunately, the problem has been attacked from the microscopic instead of the macroscopic point of view. It is so easy to study the leaves without even becoming aware of the existence of the tree. In the writer's opinion, much more valuable, as well as more rapidly usable, results will be obtained when an agreement is reached to classify soils on the basis of a few quantitatively measured physical properties, found by tests as simple as those now used for testing cement. Standards can then be determined for certain well known classes of soils, such as the list of "Important American Soils" issued by the Department of Agriculture.[7] Without such an agreement, tests on soils made by different men can never be compared, and the accumulation of test data will only be used by those who desire to show agreement between some chosen isolated tests and a "pet" theory.

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[5] Cons. Engr., New York, N. Y.

[6] Progress Report of the Special Committee to Codify Present Practice on the Bearing Value of Soils for Foundations, etc.: Discussion by Jacob Feld, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., January, 1924, p. 114.

[7] Year Book, U. S. Dept. of Agriculture, 1911, pp. 223–236.

Donald M. Burmister,[8] Assoc. M. Am. Soc. C. E. (by letter).—An excellent and comprehensive summary of soil stabilization, particularly the important physico-chemical surface phenomena, has been presented in this paper. The authors have shown that the grading and proportions of the fine and coarse materials, admixtures and chemical treatment, and temperature and film character have an important effect on the density, optimum moisture content, and stability of compacted soils.

The primary purpose of such investigations is to build a fund of knowledge on soil stabilization, so that the experiences and knowledge gained may be of practical value, and serve as a guide in solving such problems. In order to accomplish this purpose more completely, soil behavior must be translated into terms of soil character, which means that the soils in each case must be described, accurately and systematically, on the basis of a few quantitatively measured physical properties, so that they can be identified unmistakably, and classified. This would make possible comparisons and reliable correlations with soils from other localities.

In addition to the basic principles of soil compaction established by Mr. Proctor (see heading, "Bibliography," Reference (20)[9]), and the principles emphasized by the authors, there are a number of important physical relations which the writer believes are fundamental, because they bring into a more unified and consistent pattern many of the physical factors governing the behavior of soils. Five of these relations are as follows:

(1) There is a general tendency for a decrease in the maximum density and an increase in the optimum moisture content with increasing fineness of the soil, the degree of fineness being designated by a mean grain size or a size factor[10] which correlations show to be a practical and useful measure of fineness.

(2) The important influence of the grading of soils and the proportions of mixtures on density (that is, the grading-density relations[11]) can best be expressed in terms of the type of grading curve and the range of particle sizes,[10] there being consistent relationships at the optimum condition of compaction even to the finest materials.

(3) The plastic characteristics of the fine fraction of the soil, expressed in terms of the liquid and plastic limits, and the plasticity index relations, give a valuable clue to the physical character of the soil, because the plastic characteristics are influenced by the same physical factors that determine density and stability—namely, wetting characteristics, plasticity, mineralogical nature, binder, or cohesive qualities, etc.

(4) The proportion and the characteristics of the coarse fractions of the soil, particularly the wetting characteristics and the grading-density relations,[11] have an important influence on the moisture-density relations and on the stability of the whole soil. If the proportion of coarse materials is large, the fine fraction acts as a filler and binder, and the character of the coarse material predominates. If the soil is predominantly fine, the moisture-density relations and

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[8] Asst. Prof., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[9] See p. 1182.

[10] *Proceedings*, Am. Soc. C. E., February, 1938, p. 397.

[11] *Loc. cit.*, p. 365.

stability are determined almost entirely by the plastic characteristics of the fine fraction of the soil.

(5) There appears to be a fairly narrow range of clay content (finer than 0.005 mm) necessary to obtain both maximum density and stability, particularly for stabilized earth roads, where permanent stability is essential in both the wet and dry condition. Where there is a deficiency of clay (that is, less than about 5%) and where the soil is non-plastic, with a plasticity index less than about 4, the density at the optimum moisture content is materially less, and the soil has insufficient binder qualities. The optimum moisture content may be less, but there is a larger quantity of entrapped air. Clay content in excess of 35% to 40% and a plasticity index greater than about 15 to 20 are undesirable from the standpoint of stability of rolled-earth dams and fills, although the compacted soil may be very dense. The requirements for stabilized earth road soils, of necessity, must be defined more closely because of the extreme service conditions.

A study of compaction tests on different soils and an analysis of the compaction test data from a great many sources[12] show that these fundamental relations and those stated by the authors form the basis for correctly interpreting and evaluating the influence of many physical characteristics of soils on their density and stability. The writer wishes to suggest the following simple, systematic method of presentation and analysis of the data given by the authors in Fig. 5 and Table 3, which is intended to reveal and focus attention on the characteristic features of the soils under investigation, or soils from any given locality, so that the influence of their individual similarities, or differences, and qualities can be learned; and so that the inter-relationship of all physical factors can be studied in an attempt to trace their influence on density and stability. Although general statistical relationship is valuable, it is believed that the emphasis should be placed on the particular character of the individual soils. Differences in physical characteristics then become of equal importance with similarities of soils as reflected in density.

In Fig. 8, the mean grain size or size factor,[10] as a significant physical characteristic of soils, is used as the plotting argument. All related data for each soil are then plotted on a vertical line through the mean grain size in the different component parts of the diagram. This method of plotting serves to focus attention on the particular characteristics of the soils, and the confusion of superimposing a large number of grading curves is eliminated.

Size Characteristics.—The distribution of grain sizes is indicated for each soil by projecting and plotting certain soil fractions as, for example, percentages finer by weight than 5.0, 0.5 (approximately No. 40 sieve), 0.05, and 0.005 mm on to a vertical line through the respective size factors or mean grain sizes. A closer sub-division such as log 6 or log 2 intervals might have been chosen. The distribution of these fractions, or grouping (bunching above or below the 50% size) is characteristic and serves to indicate both the type of grading curve, which is designated by letter in Fig. 8, and the range of particle sizes.[10]

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[12] "The Physical Characteristics of Soils, with Special Reference to Earth Structure," by D. M. Burmister, Assoc. M. Am. Soc. C. E., *Bulletin No. 6*, Research Laboratory, Dept. of Civ. Eng., Columbia Univ., June, 1938.

A broken line traced through the respective fractions shows clearly the tendency in these soils for increasing clay content and decreasing coarse material with increasing fineness. The peculiarities of any particular soil with respect to the distribution of grain sizes are now indicated by the departures from the general trend of these fractions.

It is to be noted that for the soils in Group 1 the range of particle sizes decreases with decrease in mean grain size, and the type of grading curve changes from the wide-range, coarse *F*-type to the narrower range *F*-types (almost the *D*-type) and, finally, to the fine *E*-type. Soils in Group (2) have a constant proportion of fine material and vary only in the coarseness and

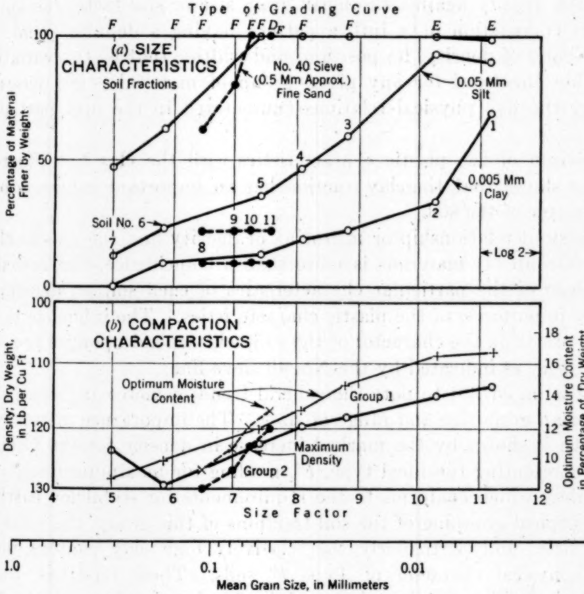

FIG. 8.—THE GRADING-MOISTURE-DENSITY RELATIONS OF FINE-GRAINED SOILS

range of the coarse fraction, and they change from the typical *F*-type to the D_F-type.

Compaction Characteristics.—These characteristics of the soils are now reflected in the maximum density and optimum moisture content. First, there is a decrease in density with decreasing mean grain size; and, second, this decrease is also associated with a decrease in the range of grain sizes and a change in the type of grading curve for these soils. The importance of the range of grain sizes is shown in Group (2), where change in the range of the coarse fraction alone causes a marked decrease in density. The decrease in density of Material No. 7, due to deficiency of clay content (or decrease in the range in the fine sizes), is very noticeable, but the optimum moisture content is also less, showing that there is a larger quantity of entrapped air.

However, the size characteristics alone are insufficient to describe and identify soils. The plastic characteristics give much more valuable and specific information on their true character. Fig. 8, therefore, is extended for the analysis of certain groups of soils in Fig. 9 to include the plastic characteristics of the fine fractions of the soil passing the No. 40 sieve (0.5 mm, approximately), which are defined in terms of the liquid limit and the plasticity index.

A study of the density relations at the optimum condition discloses consistent relationships with the specific characteristics of each soil. This method of plotting the density against the mean grain size or size factor, as an argument, brings compaction data into a pattern having a definite trend. The slope of the band of density, its position, and width—that is, the variation of density within the band for any given group of materials—are determined primarily by the five physical relations enumerated in the first part of this discussion.

A comparison of the plastic characteristics with the clay line (finer than −0.005 mm) shows that the clay fraction has an important influence on the general character of the soil.

The consistent relationship or mirroring of density and the plastic characteristics for Group (4) materials is more than a coincidence, but reveals the direct influence of the particular characteristics of each soil on density, and indicates the importance of the plastic characteristics. The influence that the fine fraction exerts on the character of the soil, will depend upon its proportion of the whole soil, as indicated by the No. 40 sieve line.

Again, it is important to note the general trend of increasing density with increasing mean grain size and range of sizes. The importance of the type of grading curve is shown by the marked increase in density for the Group (6) materials, representing the ideal type, *F′*, grading for maximum density—the Talbot grading, which conforms to the requirements for stabilized earth-road soils. The typical grouping of the soil fractions of this group, the wide range of particle sizes, and particularly the relatively high clay content indicate clearly the physical character of Type *F′* soils. These relations, together with the relatively high plasticity index of the fine fraction (passing the No. 40 sieve), represent the ideal requirements that must be satisfied in order to obtain maximum density. In general, where the plasticity index is lower than the trend the density is lower, and where it is higher, the density is greater, an inverse relationship, which is quite consistent.

Equally consistent relations are indicated for the Iredell, Arlington, and Manor soil of Group (3), which the authors used in their investigations (see Fig. 5). In view of the fact that the plastic characteristics reflect, consistently, the influence of the physical factors of the fine fraction that determines density and stability, the specific influence of admixtures, electrolytes, and chemical treatment, except bituminous materials, may be determined and correlated by their specific effects on the liquid limits and plasticity indices of the particular soils, because these substances affect primarily the wetting characteristics,

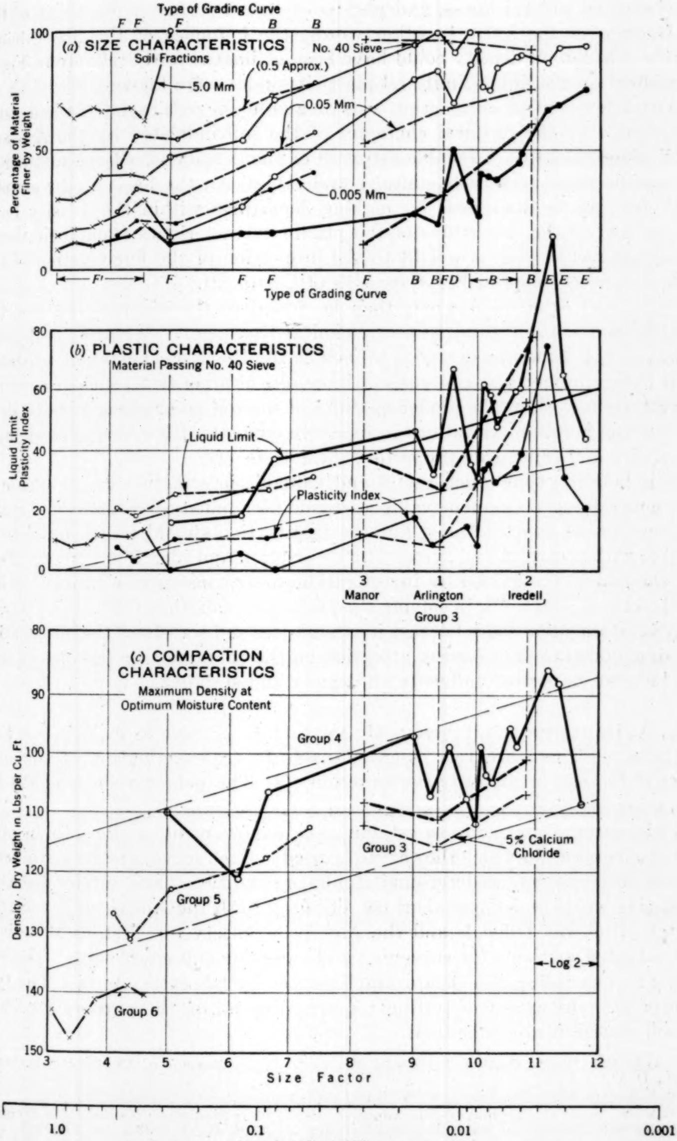

FIG. 9.—THE COMPACTION CHARACTERISTICS OF SOILS SHOWING THE INFLUENCE OF SOIL CHARACTER ON DENSITY

film character and thickness, and physico-chemical surface phenomena of the fine fraction of the soil. In other words, the influence of the 5% calcium chloride solution on density could have been estimated fairly reliably in Fig. 9 if the effect on the liquid limit and plasticity index were known.

With a few control compaction tests on a group of soils to define the density band, and with the physical character of the soil indicated by the size and plastic characteristics, a reliable estimate of the density of a large number of soils can be made more economically and rapidly on the basis of the simpler liquid and plastic limit tests by making departures within the density band proportional to the departures of the plastic characteristics from their trend, and by giving the proper weight to the proportion of the fine fraction of the whole soil, as revealed in Fig. 8 for soils of Group (2).

Additional sub-divisions may then be added to the diagram showing the quantity and kind of admixture or chemical treatment. Where it is deemed advisable, Fig. 9 may be extended to include the Proctor penetration resistance at the optimum moisture content, or the results of other tests, such as crushing strength tests to show the binder qualities of the soil and slaking tests to show how the soil reacts to variations in moisture content with consequent changes in stability and cementing properties, shear tests, etc.

It is believed that much useful and valuable information can be made of more general and permanent value as a guide for rolled-earth dams and stabilized earth-road constructions; but the information should be as complete as possible with a careful and systematic description and identification of soils so that the experience gained by investigation or by construction in one locality may be of practical value in solving the problems in another. The soil diagram affords certain advantages because it summarizes and correlates the information in a single diagram and focuses attention on the characteristic features of soils that have an important influence on density and stability.

C. A. HOGENTOGLER,[13] ASSOC. M. AM. SOC. C. E., AND E. A. WILLIS,[14] ESQ. (by letter).—There are two essential parts to an examination of soil as a material for use in highway construction: (1) The field survey, and (2) the laboratory analysis.

Field surveys are made to establish the environment of the soils in place or in the locations in which they are to be used. Such surveys include studies of climate, topography, underground drainage conditions, and service behavior of existing roads under prevalent conditions. Both the American Association of State Highway Officials and the American Society for Testing Materials[15] have adopted methods for surveying and sampling sub-grade soils. The surveying and sampling of soils are supplemented by laboratory tests. The type of tests and the extent of testing required depend on the purpose for which the soil materials are to be used.

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[14] Associate Highway Engr., U. S. Bureau of Public Roads, Washington, D. C.

[15] A. A. S. H. O. Method T-86, Adopted, 1934; Standard Specifications for Highway Materials and Methods of Sampling and Testing, A. A. S. H. O., 1935, pp. 213–221; also, A. S. T. M. Designation: D 420–35T, pp. 940–949, *Proceedings*, 38th Annual Meeting, Vol. 35, 1935.

In the paper, emphasis was placed on the character and properties of the soil materials as revealed by laboratory tests. It was assumed that satisfactory field conditions would be accomplished.

Highway engineers have been accustomed to select aggregates almost exclusively on the basis of their hardness, durability, and grading. These aggregates are mixed with Portland cement and bituminous binders with the expectancy of equal serviceability, this being possible because the designs utilize average strengths of groups of materials instead of the maximum strengths of the best mixtures individually; but in soil stabilization engineers strive for the maximum strengths of individual mixtures of soil aggregate and binder. They study films of air, moisture, chemical solutions, and insoluble binders; their relative adhesion to soil particles; and how their strengths and thicknesses are influenced by traffic, climate, and admixtures. Both Mr. Feld and Professor Burmister have indicated the desirability of a generally accepted system of soil classification for engineering purposes. Professor Burmister has described in his excellent discussion the multiplicity of factors which must be considered in any such grouping.

The authors are convinced that the practical method of attack is to group soils on the basis of performance for each particular engineering use. Simple tests may then be used to identify the individual members of each performance group.

Four groupings of soil materials based on performance when used for a particular purpose in highway construction have been adopted. They are: (1) A grouping for sub-grade materials; (2) a grouping for embankment materials; (3) a grouping for graded materials for use in base or surface courses; and (4) a grouping for fine-grained soils stabilized with Portland cement or bituminous material. On the basis of the test data, the soil samples are classified according to these groups and studied further in an effort to improve on the test result limits tentatively established.

The general grouping for sub-grade soil materials in which eight groups are recognized is widely used by State highway departments. The tests used for the identification of these materials are: (*a*) Mechanical analysis; (*b*) liquid limit; (*c*) plastic limit; (*d*) shrinkage limit; (*e*) field moisture equivalent; and (*f*) in some cases only, the centrifuge moisture equivalent.

A grouping for embankment materials has been adopted as a standard specification for the American Association of State Highway Officials.[16] The specification includes six groups for embankments less than 10 ft in height, and five groups for embankments greater than 10 ft in height. The tests used for identification in these groupings are: Liquid limit; plastic limit; and maximum weight, per cubic foot, as determined by the compaction tests.

The grouping for graded materials for surface and base course construction has been adopted as a standard for the American Association of State Highway Officials.[17] Seven groups—three for surfacing materials and four for base course materials—are included. These materials are identified by the liquid limit and plastic limit tests, and the mechanical analysis.

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[16] Standard Specification M 57–38, A. A. S. H. O.

[17] Standard Specifications M 61–38 and M 56–38, A. A. S. H. O.

The grouping for fine-grained soils stabilized with Portland cement and bituminous materials has been submitted to the American Association of State Highway Officials in part. Two groups of materials, differentiated in the case of stabilization with Portland cement by wetting and drying, and freezing and thawing tests, are recognized. The compaction test is used to determine the quantity of water to add during construction.

Results of check tests conducted during the nine years, 1929–1938, by the Bureau of Public Roads in co-operation with the various State highway testing laboratories show that tests for identifying the members of the various soil groups have been developed to an entirely satisfactory status.

The widespread usage of these groupings has indicated that they are of practical value and that the employment of specifications based on them are workable and produce results which are more satisfactory than those previously attained.

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SOIL REACTIONS IN RELATION TO FOUNDATIONS ON PILES

By R. M. Miller,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Hibbert M. Hill, George P. Stowitts, George A. McKay, Chandler Davis, Harry E. Sawtell, J. Stuart Crandall, and R. M. Miller.

Synopsis

Technical literature dealing with foundation problems contains voluminous reports on the properties of clay, silt, and mud, and the reactions of these soils to loads transmitted through round or square plates of varying sizes. There has also been as much written on the properties and reactions of sand and other permeable soils under various conditions of loading. Seldom, however, does one find a clear analysis of the behavior of a combination of many soils, a condition that is encountered so frequently in the construction of foundations. Even less frequently does one discover a publication that is written for the practical use of the designer and the construction engineer who are seldom equipped for this highly specialized branch of the profession. It appears possible that if more attention were directed to the application of theory rather than to theory itself fewer foundation failures would result. The attempt has been made in this paper, therefore, to use the construction field as a source of information, and to correlate the collected data on pile foundations under varying soil conditions so that the reasons for success and failure become evident.

The data presented herein may tend to show that the "science" of pile foundations can be called a science only when the piles are driven to rock, gravel, hardpan, or other substantial sub-stratum. Very little is known of skin-friction values or of the distribution of loads to the subsoils and, therefore, not much is known about the correct spacing of piles. It will be evident that in soils of clayish characteristics, and even in saturated sands, the dynamic pile-driving formulas are only a poor indication of the carrying capacity of a single pile; and that the carrying capacity of a cluster or mat of piles seldom equals the value of a single pile multiplied by the number contained within the cluster.

Note.—Published in June, 1937, *Proceedings*.

[1] Project Engr., PWA, Cincinnati, Ohio.

AVERAGE PRACTICE

Records collated from 250 pile-driving projects, well distributed over the United States, form the basis of this paper. The analysis is interesting in that it reveals the variety of soil combinations extant and the average treatment, by practicing engineers, of this complicated problem. As recorded from the boring data the soils were of such an infinite variety of mixtures and in such an infinite variety of proportions that only three examples will be noted: (1) 5 ft of sawdust, old paving blocks, and general refuse, on 25 ft of very soft and slushy mud, changing gradually to 5 ft of firmer blue mud on sand; (2) 6 ft of sand and clay, on firm sand and silt, on coarse sand and quicksand, on medium coarse gravel; and (3) 20 ft of hydraulic sand-fill, on 25 ft of mud, on 5 ft of peat, on marl. During the construction of the foundations for the Laurel Homes Housing Project, Cincinnati, Ohio, in an area of 35 by 150 ft, loam top-soil was found on soils which may be described by the terms, clay, "bull liver," and quicksand. Within the same area were found twelve cesspools and one spring flowing strongly during the drought of the summer of 1936. Imagination cannot picture a mathematical treatment of such a problem. With no apologies, it should be stated that such inexact terms as quicksand, clay, the dirt movers' "bull liver", etc., are used throughout this paper as describing the soils more clearly to the minds of most engineers and contractors than the correct technical terms. No piles were driven for the Laurel Homes Project.

The driving of piles in the widely varying soils indicated by the foregoing, develops reactions as diverse as the soils themselves. In some instances a heave as much as 6 ft resulted, and, in others, settlement followed. Added resistance to driving after rest resulted in some cases and lessened resistance to driving after rest in others. Frequently, shorter piles were required as more were driven in a cluster, due to the consolidation of the surrounding soil, and at times longer piles were required as driving progressed.

The solution of this problem may be demonstrated by average data computed from the aforementioned records. If the assumption is made that the design load must be carried by the soil below the pile-points, and if the average design loads are divided by the area represented by the pile-spacing, comparative loads per square foot on the several types of subsoils can be ascertained. It is understood, of course, that whereas only average values, are being compared concentrations of loads may often occur beneath pile-points greatly in excess of the average.

From Table 1, it is evident that, in spite of errors in assumptions, most of the serious settlements of foundations on piles in America should occur on projects where the pile-points rest in clay or other plastic subsoils. It is also evident that average practice is at times wasteful where the pile-points rest on firmer sub-strata. It would appear, therefore, that where piles have been driven in plastic soils, the average design load per pile has been too much, or the spacing of the piles has been too small, for the

the average design load. It would also appear that where piles have been driven to rock or other hard strata advantage should be taken of the possibilities, such as increasing pile diameter, pile spacing, the design load per pile, and the unit load on the hard sub-strata. In addition, it must be apparent that end-bearing piles driven to solid sub-strata present comparatively little difficulty.

TABLE 1.—BEARING VALUES FOR PILE FOUNDATIONS

Soil beneath pile-points * (1)	Design Load, in Tons per Pile			Average load upon subsoil, in tons per square foot (5)	Assumed safe load, in tons per square foot † (6)
	From: (2)	To: (3)	Average (4)		
Soft clay	12	35	25.0	2.8	1.0q
Sand	6	45	28.4	3.2	4.0‡
Gravel	14	40	28.7	3.2	6.0
Firm clay	20	39	29.5	3.3	3.0q
Rock	18	70	36.4	4.0	30–100.0§

* Average pile spacing equals 3 ft. † Building Code of Cincinnati, Ohio. ‡ Average for fine clean sand and coarse compact sand of Cincinnati Building Code. § Building Code of Boston, Mass.

Examination of the records indicates that the results are due in many instances to incorrect, incomplete, or a total lack of, sub-soil data, to misinterpretation of the results of preliminary investigation, and often to too much dependence placed upon standards of pile loading and spacing.

SOILS

Research and experience have brought to light soil characteristics that serve as a basis for the understanding of their reactions during the driving of piles. However, variations in the water content or imperceptible quantities of other soils in combination change the results so that at times predictions as to reactions are embarrassing.

In general, it may be said that: (1) Soils, to-day, are considered as solids definitely elastic in some degree; (2) compressibility varies from sand and gravel at the one extreme to gumbo and peat at the other; (3) sand under load compresses almost at once, whereas clay of low permeability from which water can only escape slowly under pressure, requires a long time for permanent deformation; and (4) undisturbed clay recovers considerable of the depth of its settlement when the load is removed, depending on the duration of application of the load.

THE BEHAVIOR OF IMPERVIOUS SOILS

The phenomenon of the compression and rebound of soils has been illustrated so many times by investigators, by means of compression and rebound curves, that additional curves are superfluous. It will be instructive, therefore, to examine a fairly common phenomenon that may serve to illustrate the behavior of the more impervious soils.

One of the railways in the East was built through red and yellow clay, with miles of clay cuts and fills. Large carloads of coal are hauled by heavy locomotives, producing heavy wheel loads. Observation of the track under traffic indicated that the clay sub-grade was originally very elastic, the deflection of the rails under the passing wheels being considerable, and the rebound being apparently equal to the deflection. For years after construction was completed, more and more ballast was added, and the track was lifted time and again, until, finally, for valuation purposes, the ballast was measured, and it was found that the road bed immediately under the ballast had assumed the shape of a ditch, 3 ft or more deep in places. This would mean that the repeated application of loads over a long period of time gradually compressed the clay until, after many load applications, permanent deformation was reached. Some evidence was discovered of side movement of the soil into cut ditches and of mixing of ballast and sub-grade.

The time required for driving of piles, however, is usually a matter of minutes and the application of comparatively few blows. If a clayish or impervious soil is considered as elastic, it should compress momentarily with the blow of the pile-hammer and rebound at once; or if the reaction is viewed as the displacement and flow of a plastic material under pressure the result in either case is movement upward or in the direction of the least resistance. This is known as "heave", and is less in soils containing larger proportions of sand or water. If the soil happens to be sand, the surface is more likely to subside than to heave. If the sub-soils are alternate beds of clay and sand, both heaving and subsidence may occur. No definite rules can be stated, but it is possible, from the examples that follow, to trace some similarity in the reactions of soils.

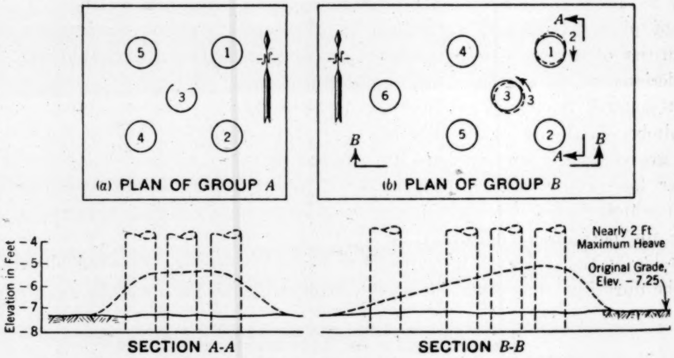

FIG. 1.—ARRANGEMENT OF PILES AND ORDER OF DIVISION.

Example 1.—During the spring of 1932, some interesting pile tests were made on the site of one of the larger Government Buildings in Washington, D. C. (3)[2] Two groups of piles were driven: Group A, of five piles, having an average length of 20 ft; and Group B, of six piles, having an average length of 13.6 ft. The piles in each group were spaced at 2.5-ft centers, and the two groups were separated by about 200 ft. All the piles were of straight shaft, uncased, compressed concrete, with a diameter of 15 in., and all but one (see Fig. 1, Pile 6, Group B), had a bulb or pedestal at the bottom.

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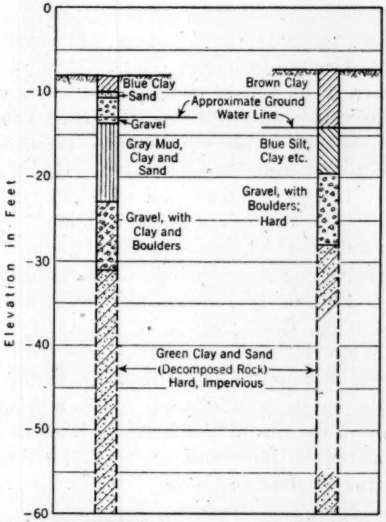
[2] The numerals in parentheses denote references to the Bibliography in the Appendix; thus: (2(*e*)) refers to Item No. 2, November, 1933, p. 1461.

From Table 2 it is evident that the soil had consolidated around the piles of Group A as the driving progressed, and the piles, therefore, had to be shortened. The ground was thrown vertically upward during the early part of the penetration of each pile; and yet, the maximum heave was only 0.5 ft. Fig. 2(*a*) shows that the strata penetrated consisted of a mixture of permeable and impermeable materials, the reactions of which were slightly on the side of the impermeable materials. Table 2(*b*) shows that, in Group B, the soil had not tightened up sufficiently to cause shortening of the piles as the driving progressed. The heave amounted to a maximum of 2 ft, equal to approximately 70% of the displacement of the piles in the clay and silt. It is evident, therefore, that the soil around Group B was of the clayish order (see Fig. 2(*b*)), as shown by its heaving reaction and small amount of consolidation. It is also evident that the soft concrete of the closely driven piles was more likely to become distorted in this group. This proved true; when the piles of Group B were excavated to the gravel stratum, 1 ft above final penetration, Piles Nos. 3, 4, and 5 had a tendency toward a

TABLE 2.—PILE-DRIVING DATA, EXAMPLE 1 (NO. 1 STEAM HAMMER; STROKE, 3 FEET)

Description	PILE NOS.					
	1	2	3	4	5	6
(*a*) GROUP A*						
Length, in feet	26	22	18	18	17	....
Blows per inch, in last foot of driving	5	5	7	5	7	....
(*b*) GROUP B						
Length, in feet	14	14	13	13	13	14
Blows per inch, in last foot of driving	7	6	6	6	7	6
Creep (in inches), in direction of arrow	2	......	3	......	......	....

* See Fig. 1.


0
-10
-20
-30
-40
-50
-60
Elevation in Feet
Blue Clay
Sand
Gravel
Gray Mud, Clay and Sand
Gravel, with Clay and Boulders
Brown Clay
Approximate Ground Water Line
Blue Silt, Clay etc.
Gravel, with Boulders; Hard
Green Clay and Sand (Decomposed Rock) Hard, Impervious

(*a*) GROUP *A* (*b*) GROUP *B*

FIG. 2.—BORING DATA; EXAMPLE 1.

smaller sectional area in the clay just above the gravel and a slight lateral deflection, in the direction of least resistance, outward and away from the most highly compressed soil.

Example 2.—On the construction of the Union Terminal, at Cincinnati, Ohio, a large proportion of the pile foundations were in clays and combinations of clays, in which the heave at times amounted to as much as 3 ft. Many of the footings were supported by encased, cast-in-place, concrete piles, placed by driving a pipe or casing containing a core. On withdrawing the core, a corrugated iron pipe was placed in the casing and filled with concrete, after which the outer casing was withdrawn, leaving an annular space of 1 in. around the corrugated iron pipe. On one occasion the corrugated iron pipe was ordered removed after the casing had been withdrawn. Due to unavoidable delay, the orders were not followed until a few hours when it was found that the clay had gripped the corrugated iron pipe so tightly that it could not be removed without destroying it, illustrating perhaps the rebound of elastic material when the pressure is removed. For the same purpose Example 3 is given.

Example 3.—At Potomac Park, Washington, D. C., three test piles were driven in 1934. One was a composite pile with a steel-encased concrete action section, 16 ft long, and a lower wooden section, 30 ft long. The second, driven 15 ft away, was a steel-encased, cast-in-place, concrete pile, 53 feet long, and similar to those mentioned in Example 2. The third test pile was a standard, wooden, peeled pile, 52 ft long. They were each driven through 10 ft of hydraulic fill, 30 ft of Potomac River mud, and then, at 2 in. per blow for 3 ft and 1 in. per blow for 1 ft, tightening up to four blows per inch through the next 5 ft.

The steel-encased, concrete pile had been provided with a steel center bar, 1 in. square, so that the pile could be subjected to an upward pull. On removing the casing, the usual 1-in. annular space was left. Static load tests were made on the three piles, so that several days elapsed before a pull test could be undertaken. Hydraulic jacks were used in pulling, which applied an upward pull of 47 500 lb upon the 1-in. center bar before it parted. Deducting the weight of the pile, a net pull of 40 000 ft had been applied, which was equivalent to a frictional resistance of at least 243 lb per sq ft, giving an indication of the grip upon return of a very plastic soil.

Clayish or fine-grained soils usually heave in some degree during driving, depending upon the water content and mixture with other soils. After rest they regrip the pile with added resistances, at times amounting to as much as 2.5 times in 12 hr, and even 4 times in 24 hr. Whether the cause is attributed to the rebound of an elastic soil against the pile or to the readjustment of the internal structure of the soil after remoulding is not as important as the facts themselves. Either explanation may be true at different times.

Reactions of More Pervious Soils

The addition of sand or coarse-grained mud to soil combinations changes the results in a surprising fashion. Charles Terzaghi, M. Am. Soc. C. E., refers (1(*b*)) to the driving of piles in fine-grained saturated sand, and notes that the surface of the sand subsided 1 ft due to the driving. Victor A. Endersby M. Am. Soc. C. E. (1(*c*)) describes the driving of piles in sand to satisfactory resistance on one of his projects. The piles were allowed to rest until the following day. When driving was resumed, it was discovered that they had lost 40% of their bearing power.

Example 4.—During the construction of the extension of Concrete Pier No. 314, Navy Yard, Charleston, S. C., under the supervision of the United States Bureau of Yards and Docks (5*a*), a number of 18-in., square, wooden, test piles were driven through 10 ft of silt and into stiff blue mud, apparently quite coarse-grained. At from 43 ft to 57 ft the test piles developed a resistance to driving of 40 tons, soon thereafter "fetching up" at virtual refusal. After a rest of from 3 hr to 2 days, driving was resumed, showing a marked decrease in resistance. The decreases in specific cases were from 43 to 21 tons, 45 to 32 tons, 47 to 39 tons, and 39 tons to 25 tons.

Example 5.—The following, to much the same purpose, is reported by Lieut. J. N. Laycock, U. S. Corps of Engineers (4*b*): Four timber test piles 18 in. square and 80 ft long, were driven at Balboa, Canal Zone, through soft silt to − 28 ft to − 43 ft; then through fairly compact and uniform blue clay, the blue clay extending well below the pile-points. With each of the four test piles, driving was suspended several times, the delays varying from a few minutes to overnight, and in every case of suspended driving, there occurred a loss of resistance. These losses varied from 3 000 lb for Pile No. 1, for a 15-min delay, to 48 000 lb out of 90 000 lb for Pile No. 4 for an overnight delay. Such losses follow so closely those of the test piles driven at the Charleston Navy Yard that both soil combinations must have been very coarse-grained, comparing in reaction to a saturated sponge placed suddenly under compression.

Example 6.—At the Naval Supply Depot, United States Naval Operating Base, San Diego, Calif. (5*b*), a number of concrete and wooden test piles were driven through 7 ft of soft plastic mud, carrying a small quantity of sand and shells. Beneath the mud lay strata of sandy clay and sand. A wooden test pile, 20 in. by 20 in. square, was driven to virtual refusal. The pile was allowed to rest 24 hr and driving was resumed. In the first few inches of re-driving there was a decrease in resistance, but the pile was again driven to refusal, after the driving had progressed 1 ft. It should be noted that Examples 4 and 5 refer to coarse-grained muds or more permeable materials. Example 6 refers to permeable sand and shells and impermeable mud, in which sufficient sand was present for the combination to act in a similar manner but in a lesser degree.

During the same tests at San Diego, an 18-in., precast, concrete pile was pulled, and brought up a natural shoe of compacted material on the point of the pile. The report states (5*b*) that "it consisted apparently of samples of the various materials through which the pile had passed, and was as hard as, and had the appearance of, soft rock." This phenomenon is not uncommon, as it has been noted many times in the driving and pulling of test piles in sand and shells. The possible explanation is that a permeable saturated soil such as sand under pressure, loses its water content rapidly, and is consolidated quickly. Piles in saturated sand will drive to virtual refusal, quickly forcing out the water in the sand immediately surrounding the pile-points and forming a bulb of solid material. The water in the sand surrounding the consolidated bulb having a means of ready escape, the pressure will be quickly dissipated during rest. Following rest, the pile and its sand bulb having no longer high pressure to work against, will drive with lessened resistance. Continued driving, however, accompanied by further consolidation, at length, brings the piles to final refusal.

Example 7.—Resort has been had to the artificial consolidation of soils for the support of structures since the dawn of history. The Egyptian sand pile has been, and still is, used successfully for this purpose in many parts of the world. This pile consists in driving a hole in the ground, filling it with sand, and tamping the sand to refusal. This process provides a satisfactory foundation, provided the soil can be consolidated. For example, into the sub-soils beneath the Federal Legislative Palace, in the City of Mexico, Mexico (6), consisting of an extremely unreliable mixture of volcanic ejections and alluvial matter, were driven 150 000 sand piles, 8 in. in diameter on 20-in. centers. The displacement of the piles could have caused a heave of 5 ft, if no consolidation had occurred. As the heave averaged only 1 ft, however, consolidation in the amount of four-fifths of the displacement had taken place. However, when water cannot quickly be lost from fine-grained impermeable soils under quickly applied pressure, very little if any consolidation occurs. Increase in soil pressures, due to the driving of piles, may be temporary, whereas consolidation of the soil is of permanent value in increasing bearing power. It must be obvious, therefore, that the driving of piles in moist to wet soils that cannot be consolidated for a long time leads to uncertain results.

Skin Friction

Perhaps no factor in the design of pile foundations has been as fruitful of foundation troubles as skin-friction values and their relation to the problem. Offering no ready-made solution or convenient tabulation that will apply to all conditions, the attempt will be made to indicate by illustration the wide range of values that may be secured and the reasons why the assumptions are of such small use.

In the absence of better information, engineers have been forced to use the best data available. The friction between pile and soil has often been computed from the results of static load tests, deducting assumed point resistances with no allowance made for temporary soil pressures.

Much better than the average of published data are the values shown in Table 3. It can be seen that: Either the soil terms are not clear; the point-resistance deductions may have been in error; temporary soil pressures may have affected the result; or, perhaps, the surfaces of the piles themselves have been important factors; for example, compare Items Nos. 1 and 2 of Table 3, both of which are for soft blue clay.

Example 8.—Exhaustive tests were conducted by the Whangpoo Conservancy Board during the Shanghai Harbor investigation in China, in 1920–1921 (8). The friction value (in pounds per square foot) of blue clay at that time was reported variously, as follows:

Timber pile, 10 in. by 30 ft	340
Timber pile, 20 ft long	360
Round Douglas fir pile, 45 ft long	650
Rough-surfaced concrete pedestal pile, 33 ft long	650

The report also indicates a slight decrease in skin friction for the longer piles and an increase of 10% in friction values during the subsequent twelve months.

TABLE 3.—VALUES OF SKIN FRICTION

Item No.	Soil	Friction value, in pounds per square foot	Point resistance deducted	Location
1	Stiff blue clay	1 850	No	Hull. England
2	Soft blue clay	740	Yes	Portland. Me.
3	Soft muddy clay	370	Yes	Tunis, Algeria
4	Mud. sand, and clay	370	Yes	Proctorville, La.
5	Soft clay	295	No	Rhine Valley, Germany
6	Soft mud	130	No	Rhine Valley, Germany
7	Mud	130	Yes	17th Street and North River, New York, N. Y.
8	Silty microscopic sand	130 to 900	No	Shanghai. China
9	Fluid mud	95	Yes *	Aguis Creek, Va.

* 15 per cent.

Example 9.—In 1933, in connection with the Sewerage and Sewage Disposal Plant program in Columbus, Ohio (*9a*), it was decided to drive precast concrete piles and to test for uplift. Eight piles, 15 ft and 20 ft long, were driven through loam, clay, sand, and gravel. Ground-water level was quite near the surface. The eight piles were pulled with the following results:

Piles	Depth below the ground, in feet	Frictional resistance, in pounds per square foot
1	11	538
2, 3, and 4	14	557
5, 6, 7, and 8	19	613
Average of eight piles	16	582

Evidently, the skin friction had been increased with depth in contrast to the results reported by the Whangpoo Conservancy Board (see Example 8).

Example 10.—Pull tests were also made for the Water Supply and Filtration Plant, at Fort Wayne, Ind. (*4c*), 4 600 conical concrete piles, averaging 16.8 ft in length being driven through dark yellow clay, fine sand, and coarse sand. Later, five piles were subjected to pull tests with hydraulic jacks. The friction value developed by these pull tests, averaged 755 lb per sq ft for steel on clay, fine sand, and coarse sand.

Example 11.—The sinking of caissons in the underpinning of the Naval Aircraft Building, at the League Island Navy Yard, Philadelphia, Pa., also developed data contrary to that furnished by the tests at Shanghai as to the relation of friction values to length of piles. Records were kept by the U. S. Bureau of Yards and Docks of the pressures necessary to move the caissons at various depths. The caissons penetrated silt and sand to gravel and cobble at 50 ft to 60 ft. Making allowance for the upward pressure of the air, it was found that the pressure necessary to move the caissons plotted in nearly a straight line for the upper depths and then tended to a flattened parabola for the lower depths. The friction values increased from 100 lb per sq ft at a 15-ft depth to 305 lb per sq ft at a 60-ft depth.

Example 12.—To illustrate the relative skin-friction values of rough and smooth pile surfaces, it may be interesting to compare the results of the load test on Pile No. 6 of Group B, Example 1, with those made on the precast concrete piles that ultimately became a part of the building foundation. The precast piles were $13\frac{3}{4}$ in. in diameter and cylindrical in shape. Eleven of them scattered in the vicinity of Group B were test loaded. These precast piles (of 12.6-ft average length, for a settlement of $\frac{5}{64}$ in.) were supporting loads averaging 31 tons. An exact comparison cannot be made with the settlements of $\frac{3}{32}$ in. used in connection with the piles in Group B. However, $\frac{5}{64}$ in. is close enough for reasonable comparison. Since the point-bearing area of the precast pile was approximately equal to that of the bulbless Pile No. 6, Group B, it would appear that the friction values of the precast piles were equal to $\frac{81}{50}$ times that of the rough-surfaced bulbless pile in the same soil.

For the same purpose it should be added that a part of the Potomac Park tests (Example 3) consisted in test loading both the wood pile and the corrugated encased concrete pile described. The wood pile failed at a test load of 32 tons whereas the concrete pile stood successfully a test load of 45 tons, indicating that the surface and perhaps the shape of the pile are important factors.

Summary.—Skin friction cannot be assumed as a factor that in itself supports a structure, but rather as the means of spreading the load to the strata below the pile-points. Knowledge of friction values, therefore, is of use in determining the bearing value of the pile and surrounding soil and whether or not the soil below the pile-points is overloaded due to concentration.

Friction values may be affected by variation in the soils, combinations of several soils, variation in water content, the length and surface of the pile, and the time that has elapsed since driving. It must be evident, therefore, that any tabulation of friction values that is intended to cover all possible conditions must be accepted with reservations. Dependable results can be secured by the driving of test piles of the type selected and by pull testing, at a lapse of time. This delay will be discussed herein under the heading, "Load-Carrying Capacity and Static Load Tests." The results are equally valuable when, as occasionally happens, the shearing strength of the soil, and not the pile surface, is the vital factor because if the former is the case the pile surface must be sufficiently rough. If the pile surface is the vital factor a pile with a rougher surface can be selected.

LOAD-CARRYING CAPACITY AND STATIC LOAD TESTS

There appears to be general agreement that the dynamic pile-driving formulas give only approximations of the carrying capacities of piles under static loads when driven in fine-grained impervious soils, but that the formulas are reasonably accurate when the piles are driven in sand or other pervious soils. As to the dependence generally placed upon the results of static load tests upon single piles, the results thus obtained are unreliable indications of what may be expected of the group or mat, of which the single pile forms a part. This applies, however, only to piles driven on close centers or in fine-grained impervious clayish soils. To illustrate how static load tests on single piles may mislead the engineer, a few examples will suffice.

Example 13.—The piles described as Group A in Example 1, were subjected to a final static load test of 250 tons, and Pile No. 5, of Group B, was loaded to 70 tons (see Fig. 1). Group A was loaded to 125 tons for 24 hr. The load was then increased to 250 tons, and allowed to remain for 5 days. The settlement was $\frac{5}{32}$ in. Pile No. 5, Group B, was test loaded to 25 tons for 24 hr, to 50 tons for another 24 hr, to 62 tons for 36 hr, and to 70 tons for 24 hr, with a total settlement of $\frac{1}{8}$ in. (see Fig 3). At a settlement of $\frac{5}{32}$ in., the piles of Group A were carrying an average of 38 tons each, and Pile No. 5, Group B, was carrying 62 tons.

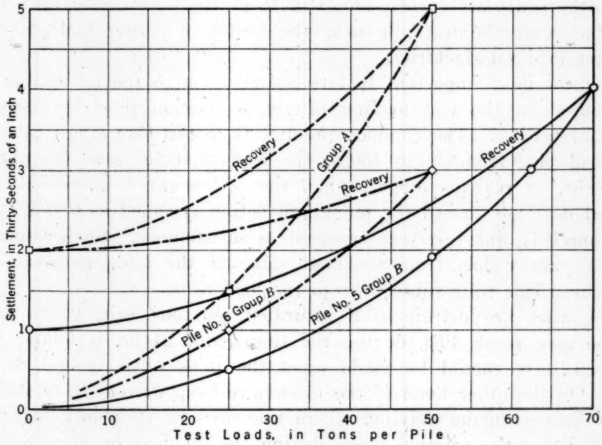


FIG. 3.—COMPARATIVE LOAD-SETTLEMENT CURVES; TESTS OF PILES SHOWN ON FIG. 1.

Comparative static load tests between single piles and groups of piles are very rare, due to the expense of making the tests and the speed usually demanded. In the absence of additional test results, the engineer must use the best information available bearing on the subject.

Example 14.—In 1918 and 1919, concrete piles to the number of 411, averaging 17.7 ft in length, of the compressed concrete pedestal type, were driven for an industrial building of the Texas Oil Company, at Providence, R. I. There are no records of the borings taken. The driving record indicates that the piles should develop, according to the *Engineering News* formula, from 41 tons to 83 tons, the average being 53 tons each. The structure was designed for a final loading on the piles of 30 tons each. Static load tests of 45 tons were applied for 24 hr on one pile of a group and on an isolated pile. The test results showed practically no settlement. There seems to have been no trouble with the piles themselves, but check levels taken on the completed structure since 1925 showed a settlement of from 1.75 in. to 3.75 in.

Example 15.—For the grain elevator at Portland, Ore. (4*d*), fourteen wooden test piles were driven, well distributed over the construction area. These test piles as well as the permanent wooden piles were driven to a resistance of 25 tons (as determined by the *Engineering News* formula) through soil of considerably fluidity, to — 41 ft, the elevation of the pile-points. One test pile at the southeast corner of the Storage Annex was satisfactorily load-tested to 40 tons. The foundation consisted of a 3-ft concrete mat cast upon the wooden piles, which were spaced 2.5 ft center to center. During construction, marked settlement was noted, and when the load per pile, under the Storage Annex, had reached 9 tons, the building had settled 1.96 ft at the northeast corner, 0.76 ft at the southwest corner, 1.4 ft at the southeast corner, and 1.72 ft at the northeast corner, in spite of the fact that the test pile near the southeast corner had supported a 40-ton test load satisfactorily.

One of the most important factors contributing to the misleading results often secured in the test loading of single friction piles is disregard of the time of testing. The common practice is to test-load single piles to the design load plus from 50 to 100%, the loads resting upon the piles from 24 to 48 hr, or perhaps longer, when the settlement is measured. If the settlement does not exceed the maximum (often specified as 0.01 in. per ton of load applied), the tests are accepted as satisfactory. Such results, however, may vary widely from the final value of the piles, as great changes often occur in the soils subsequent to the load tests.

If the piles are driven in a saturated pervious soil, where losses in resistance may reach 40% during the subsequent 24 hr, it follows that no static load tests should be made upon the piles until several days have elapsed. On the other hand if the piles have been driven in an impervious soil that heaves during driving, where increases in resistance may amount to several times the original value during the 30 days or more that follow driving, it seems useless to make static load tests upon the pile during that time. The problem, however, is not quite as simple as the foregoing implies, because piles often penetrate several types of soils, in which both "heave" and consolidation may occur. As boring data are not always sufficiently accurate guides, it would seem best when there is uncertainty to consider the soils as impervious and to test-load accordingly. In addition, if the test loading of single piles were replaced by a group test loading, beneficial results would follow.

SPACING FOR FRICTION PILES

The correct spacing of piles is of importance for two reasons: (1) For cast-in-place concrete piles too close spacing in some soils is likely to cause distortion in the partly set concrete of contiguous piles if the driving is carried in rows continuously; and (2) for friction piles the correct spacing of a group or mat allows for the maximum load-carrying capacity of each pile. Under such conditions the pile and the surrounding soil are acting as a unit.

Numerous examples could be given of the distortion of cast-in-place concrete piles driven in different soils and with different spacing, if the information would be of any value. There appears to be no basis, however, for the derivation of formulas to cover the proper spacing of piles in relation to length, diameter, and soil conditions, in order to prevent distortion. However the simple device of driving alternate rows and reforming over the same ground to complete the unfinished rows is successful in preventing distortion and cracking if sufficient time has elapsed.

Reference has been made herein to the work of the Whangpoo Conservancy Board (8) in soil investigation and pile tests. Mr. H. F. Meyer, commenting on the Shanghai findings states (9):

> "As soon as two piles are so near each other that the two volumes of influence cut into each other, the question becomes more complicated. The size of the affected prism of soil is so large that, in all ordinary cases under a footing or a foundation, they will overlap. The soil will thus be under greater pressure than when only a single pile is involved, the angle of friction will be smaller and the skin friction of the piles will decrease. By spacing the piles nearer than the spacing as given by the formula (developed by the Whangpoo Conservancy Board) no additional safety against settlement is gained, and the builder who has carefully test loaded every single pile and found that they would sink only ¼ inch for a load of 400 lb per square foot skin friction on their surface, will, to his astonishment, observe that his whole densely piled building settled more than a foot when completed, in spite of the fact that the piles are not nearly loaded up to their 'full capacity'. The distance from center to center of the friction piles driven next to each other, should never be less than half the circumference of one pile, if the whole friction area is to be utilized; the two piles, taken as one unit, will then present the same friction surface as would each of the two piles individually added together. If there are more than two piles, the circumference of the whole cluster of piles, should be bigger than or equal to the sum of the circumference of the individual piles in the cluster. The same should apply to any part of a cluster. Thus, in case of a rectangular foundation with equal distances between piles, and all piles square, and of the same size with sides 12″, the length of the entire foundation being 'A' and the width of same 'B', the distance 'Z' between piles will thus become: A and $B = 10'$; $Z = 4.5'$: A and $B = 100'$; $Z = 11.1'$; $A = 100'$; $B = 40'$; $Z = 8.7'$."

Such spacing is apparently intended for use only in the plastic clay to be found in Shanghai. Even in that soil, it appears to be sketchy indeed to engineers in the United States. On the other hand, the average spacing for friction piles used in the United States as discussed under the heading, "Average Practice", would appear to be the other extreme.

The transmittal of the load in part through friction tends to spread it upon the sub-soil and, therefore, has an important relation to pile-spacing and bearing power. If the spacing of the piles is such that the distribution of the load through skin friction to the sub-soil is uniform, the result is a "block load", and the piles and the soil surrounding them are acting as a unit. Obviously, the pile which (through side thrust or the roughness of the surface) transmits the largest percentage of the load to the surrounding sub-soil, will develop the least concentration of load under the point. Such piles, therefore, may be over-spaced and still act as a unit with the soil.

If the characteristics of the different soil strata that lie beneath the site are known, Boussinesq's analysis can be used with variations for the determination of the distribution of pressures to the sub-soils. Having solved for the pressure distribution on the crucial stratum, the proper spacing of the piles can be computed for uniform distribution of pressure. Although this practice would be a great improvement over some of the "hit-or-miss" methods, it is believed that in the hands of the inexperienced foundation engineer an actual test by means of driven and test-loaded groups of piles would be a more reliable method for securing this information.

Any group test intended for use in effecting uniform settlement in a pile foundation must cover such different conditions as changes of soil on the site and the difference in size and shape of the pile foundations.

Whether one or more group tests are made, the first step is a thorough exploration of the sub-soils by the use of core borings. From this information approximate pile lengths and the type of pile to be used can be selected. A test group can then be driven on centers preferably of two pile diameters and arranged so that the same loading platform can be used for the test loading of piles on centers of two diameters, four diameters, and for single piles.

To allow for the readjustment of the internal structure of the soil, the piles should rest after driving from a few to thirty days, depending upon the character of the soil. If, after the rest period, a single pile is test-loaded by increments, followed by the test load of the piles on four diameters and, finally, by the test loading of the piles on two diameters with the settlements recorded as the increments of load are applied, data may be had that will indicate the carrying capacities of the piles of a mat, the diameters, and the spacing for an assumed settlement.

The fact that so many foundation settlements are dish-shaped indicates that the spacing between the piles toward the perimeter may be farther apart than the spacing toward the center of the structure. Without doubt an answer will be found to this part of the problem similar to that used in the design of the foundations (6) for the Federal Legislative Palace, in the City of Mexico (Example 7), in which Boussinesq's suggestions were followed in the plotting of an ellipse, the ordinates of which represent soil pressures at any given point.

Experience indicates that the expense of preliminary work for the analysis of soils, pile types, and economical spacing, will usually effect a sufficient saving in the first and later costs, to more than justify the expense. This is particularly true of large foundation projects. It should be added, furthermore, that by using satisfactory load tests, the dynamic pile-driving formulas can be useful in selecting pile lengths and in comparing varying soil conditions. They need no longer be the "yardstick" for measuring bearing capacities, upon which so much has always depended.

SOME PILE FOUNDATION SETTLEMENTS AND THE CAUSES

The intention in selecting examples of settlement of structures on pile foundations has been to present the case fairly from available records. It must be freely admitted, however, that all foundation settlements are not due to the use of friction piles in plastic soils, as might be inferred from the following examples.

End bearing piles driven to hard sub-strata act as columns and may be limited only to the bearing capacity of the sub-strata and the pile itself. However, at times the weakest link may be the column, because some of the piles may be over-driven, or cast-in-plate concrete piles may, at times, be deformed or even pinched off. Local settlement on a pile or cluster may occur, but the records of such settlement are difficult to secure. Besides, injury to piles in placing is usually due to poor planning and supervision and is avoidable.

The following statement would appear to be a fair interpretation of the fundamental thought of the publications of Edmund A. Prentis and Lazarus White, Members, Am. Soc. C. E. (11): "It would seem elementary to state that each horizontal stratum has to support the total load above, including the overlying strata and the structures erected thereon." In the case of a pile foundation it is equally true that the soil strata below the pile-points must support the loads placed upon them, or settlement, and possibly failure, will follow. This principle is so fundamental and so clearly sensible that even the layman should be able to understand it. It would appear from the examples that follow, however, that this simple axiom is not generally recognized.

Most of the foundation settlements that will be described, fall into one main group, the causes and conditions of which are nearly identical. Perhaps the most common has followed the making of new land by filling in an old lake, swamp, or slough with a very soft bottom. Often the conditions that existed before filling, have been completely forgotten. When, years afterward, it is proposed to erect a structure upon the filled land, exploratory borings may or may not be taken. If piles are decided upon and are not driven through the compressible stratum at the bottom, or if spread-footings are used, settlement often results corresponding to the depth and compressibility of the soft stratum below the points of application of the loads. Later, when a complete survey of sub-surface conditions is made, the cause of the trouble is only too apparent. With this in mind, note the recurrence time after time of compressible sub-soils and particularly peat in the following examples.

Example 16.—For a great steel structure built not so many years ago, a creosoted timber bulkhead was driven in a bay. An hydraulic dredge filled the area contained within the bulkhead with sand to a depth of approximately 20 ft. About 500 ft away, a structure somewhat similar to the one proposed had been in operation for a number of years, and no appreciable settlement had occurred. Apparently, soil conditions on the site of the proposed structure were identical with those of the older structure and, therefore, no preliminary soil explorations were made. As a part of the inshore foundations, pedestal footings were planned, each to rest on twenty-one wooden piles, 40 ft long, carrying 15 tons each. These piles were to be driven into the hydraulic sand fill mentioned.

The pile-driving began, and apparently satisfactory results were being secured, as measured by the *Engineering News* formula. The engineer in charge, however, felt something to be wrong and increased the number of piles and the lengths from 40 to 45 ft, then to 50 ft, and, finally, to 60 ft. The concrete for the pedestal foundations was poured, and shortly before the time for erecting the steel upon the pedestals, the tops were checked for elevation. It was seen at once that instead of bush-hammering, some of the pedestal tops were an inch or more below elevation, indicating settlement. Upon rechecking, it was found that during the following weeks, the same pedestals, even without load, had continued to settle at an alarming rate.

Test borings were taken, the results of which will be seen in Fig. 4. The cause of the trouble was obvious at once. The heavy sand fill had been placed upon sand, underlaid by very compressible soils. The piles were driven through the sand fill and the original sand, and developed the required resistance. However, inasmuch as the piles did not penetrate through the compressible soils and into the substantial strata beneath them, the added weight of the new fill caused settlement and the piles, of course, settled with the fill.

The steel superstructure was erected, although the pedestals continued to settle, compensation being made by means of driving steel wedges and placing plates under the column shoes. The settlements of the several foundations will be seen by referring to Fig. 5. However, after some months devoted to a close study of the problem, it became evident that the increments of settlement were decreasing and that, without corrective measures, settlement would finally cease.

If the plotting of the soil strata (Fig. 4) can be assumed as correct between the points at which test borings were taken, and if the settlement is assumed to be proportional to the thickness and compressibility of the strata beneath the pile-points, very little, if any, settlement might be anticipated in Foundations *S*5, *S*10, *S*11, *N*5, and *N*11. By the same reasoning the maximum settlement might be predicted in Foundation *N*10, diminishing in the following order: *S*7, *S*6, *N*9, *S*8, *S*9, *N*7, *N*6, and *N*8, which is very close to what occurred.

It should be added that any type of pile that did not reach the stratum described as "sandy marl", under the compressible peat, sand and peat, sand and mud, etc., would have proved as ineffective as did the wooden piles. Had the test borings been taken before construction, longer wooden piles would

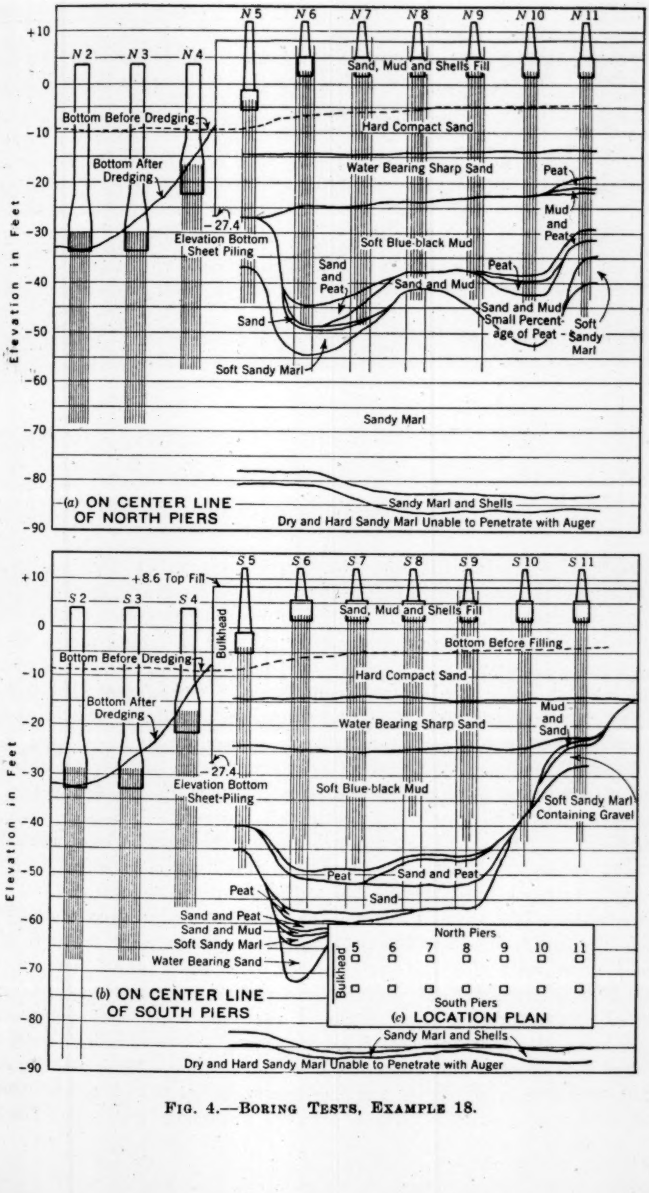

Elevation in Feet
+10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
N 2
N 3
N 4
N 5
N 6
N 7
N 8
N 9
N 10
N 11
Sand, Mud and Shells Fill
Bottom Before Dredging
Hard Compact Sand
Bottom After Dredging
Water Bearing Sharp Sand
Peat
Mud and Peats
-27.4
Elevation Bottom Sheet Piling
Soft Blue-black Mud
Sand and Peat
Peat
Sand and Mud
Sand
Sand and Mud Small Percentage of Peat
Soft Sandy Marl
Soft Sandy Marl
Sandy Marl
(a) ON CENTER LINE OF NORTH PIERS
Sandy Marl and Shells
Dry and Hard Sandy Marl Unable to Penetrate with Auger
S 2
S 3
S 4
S 5
S 6
S 7
S 8
S 9
S 10
S 11
+8.6 Top Fill
Bulkhead
Sand, Mud and Shells Fill
Bottom Before Filling
Bottom Before Dredging
Hard Compact Sand
Bottom After Dredging
Mud and Sand
Water Bearing Sharp Sand
-27.4
Elevation Bottom Sheet-Piling
Soft Blue-black Mud
Soft Sandy Marl Containing Gravel
Peat
Sand and Peat
Sand
Peat
Sand and Peat
Sand and Mud
Soft Sandy Marl
Water Bearing Sand
North Piers
5
6
7
8
9
10
11
Bulkhead
South Piers
(c) LOCATION PLAN
(b) ON CENTER LINE OF SOUTH PIERS
Sandy Marl and Shells
Dry and Hard Sandy Marl Unable to Penetrate with Auger

FIG. 4.—BORING TESTS, EXAMPLE 18.

have been used at an additional cost that would have been small compared to the several millions of dollars spent on the project.

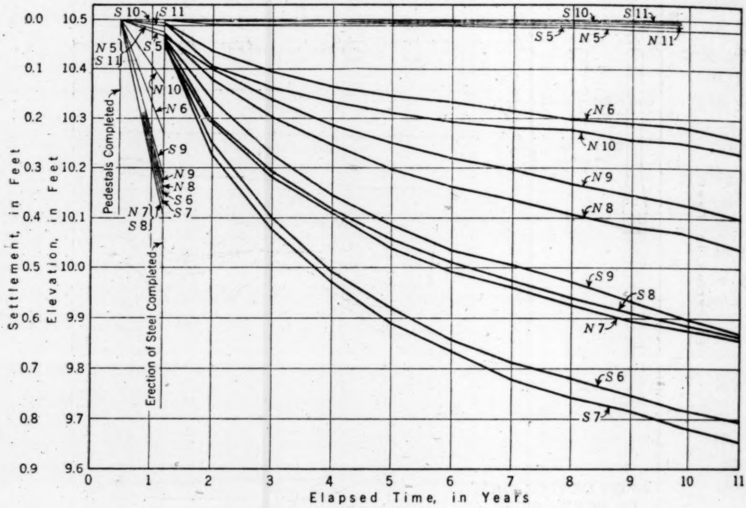
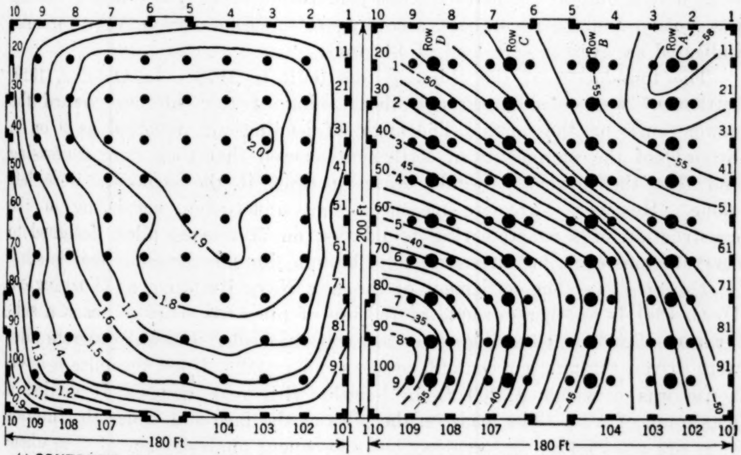


FIG. 5.—SETTLEMENT HISTORY, EXAMPLE 18.

Example 17.—During the rush construction period of 1917, an 11-story concrete warehouse was erected in the Brooklyn, N. Y., Navy Yard (5*c*). Due to the speed demanded, there was no time for sub-surface exploration. Piles were driven for the foundation from 10 ft to 34 ft in length into a fill over an old marsh. During the erection of the building, serious settlement occurred, which continued after completion. Subsequent investigation showed a stratum of peat 1 ft to 7 ft in thickness, at a depth of 30 ft to 40 ft below the surface, subjected to a load of approximately 35 lb per sq in. Tests on sample cubes of the peat gave a compression of 35% for this load. As such a condition is obviously disastrous, a part of the foundation was underpinned with steel pipe piles, filled with concrete. It is scarcely necessary to emphasize that had the times been normal, test borings would have been taken before construction, and piles would have been selected of sufficient length to reach good bearing below the compressible peat.

Example 18.—The original borings for the 12-story building of the Westinghouse Company, in Philadelphia, Pa. (4*e*) showed 8 ft to 10 ft of fill underlaid by uniform clay and sand to rock at 45 ft. Conical concrete piles, 25 to 30 ft long, were driven. Before the building was completed, settlement was noted. Investigation then disclosed that the soils were composed of 27 ft of loose fill, 4 ft of peat, 1 ft of gravel, 15 ft of silted peat, 8 ft of silt, and 2 ft of sand and gravel on hard mica schist. However, the strata were not uniform in thickness and the compressible peat lay mostly under the east side of the building, where 4 in. of settlement occurred. The west side of the building did not settle, again demonstrating that the thickness of the compressible sub-soils corresponds to the settlement. There appears to be but one answer to such a problem—underpinning. It appears certain, however, that if dependable test borings had been taken before construction, and the piles driven to the same bearing reached by the underpinning, no settlement would have occurred, and considerable money would have been saved.

Example 19.—At the League Island Navy Yard, in Philadelphia (5*d*), during the rush of the World War, two projects were constructed on concrete piles under fairly similar conditions as to soil. In one case, conical, cast-in-place concrete piles were used, and, in the other, the straight shaft, compressed, uncased, cast-in-place concrete pile with a bulb at the bottom. This Navy Yard is situated at the west end of the Horse Shoe Bend of the Delaware River. The eastern part of the reservation has been built up by an alluvial deposit of river silt and organic matter overlying an earlier deposit of coarse sand, gravel, and cobble. The sub-soil formations under the two structures, a concrete storehouse and a 175-ft chimney for the Naval Aircraft Factory, are described as follows: For the storehouse six borings showed silt to a depth from 35 ft to 55 ft below mean low water. Sand and gravel were found under the silt, sloping from a high point at the northwest corner of the building to the low point at the southeast corner (see Fig. 6(*a*)). For the stack, the sub-soils correspond roughly, except that the compressible silt is deeper under the stack than under the storehouse.


10 9 8 7 6 5 4 3 2 1
20 30 40 50 60 70 80 90 100
11 21 31 41 51 61 71 81 91
110 109 108 107 104 103 102 101
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
180 Ft
200 Ft
10 9 8 7 6 5 4 3 2 1
Row D
Row C
Row B
Row A
1 2 3 4 5 6 7 8 9
20 30 40 50 60 70 80 90 100
11 21 31 41 51 61 71 81 91
110 109 108 107 104 103 102 101
60
55
50
45
40
35
180 Ft

(*a*) CONTOURS OF TOTAL SETTLEMENT, TO FEB. 1922 (CONTOUR INTERVAL, 0.1 FT)

(*b*) CAISSON IDENTIFICATION AND ANTICIPATED BOTTOM CONTOURS

FIG. 6.—AIRCRAFT STOREHOUSE, U. S. NAVY YARD, PHILADELPHIA, PA.

The storehouse is a six-story concrete building, divided into ten bays, east and west, and nine bays north and south. Under the column footings, cast-in-place conical concrete piles, 30 ft long, were driven, designed to carry 30 tons each. In driving these piles, only a small number were driven to a resistance corresponding to that required by the design. It was discovered, however, that the resistance increased considerably after rest, and as this factor has been the salvation of many projects under apparently similar conditions, construction was continued.

In June, 1918, load tests on one pier, equivalent to that required by the design, were made. The pier settled 3 in. in 45 days, but the increments of settlement decreased rapidly after the first week. Check levels were taken weekly on all the columns during construction, which showed decreasing increments of settlement, even though the column loads were increasing as construction progressed.

Several months after completion, settlement cracks appeared in each floor. In January, 1919, the total settlement varied from 6 in. to 16 in., corresponding to the depth of the compressible silt below the pile-points, modified by the characteristic cup-shaped settlement. Note the similarity of the contours of Fig. 6(*a*) relating to settlement and Fig. 6(*b*) referring to depth of compressible material.

The storehouse was subsequently underpinned successfully, but, of course, at considerable cost. Had war conditions not existed, and had there been an ample supply, wooden piles of sufficient length to reach into the sand and gravel undoubtedly would have been used.

Example 20.—During the same period the 175-ft chimney was constructed. The foundation rested upon 64 cast-in-place, uncased concrete piles, 36 ft long, with a bulb at the bottom. The penetration of the piles during driving was steady, and the resistances, according to the *Engineering News* formula, indicated an average safe load of 130 tons.

The foundation of the chimney was built in 1918. In October, 1919, settlement was detected through the leaning of the chimney toward the power house on the north. The angle of leaning was reported as 9 on 1. Records of the increments of settlement showed that they were decreasing and that the movement would cease in time, if the structure were left alone. However, corrective measures were undertaken, consisting of the construction of a concrete ring foundation on 71 wooden piles, designed to carry 15 tons each, but driven to solid bottom through the compressible silt.

Example 21.—The settlement of the new Navy Building, at Washington, D. C. (5*e*) is another example of the use of piles too short to reach a substratum of sufficient bearing power to carry the loads. Conical concrete piles were driven through a recent fill over an old marsh. Upon the piles a head-house was constructed 200 ft in length, with rear wings. It settled a maximum of 4 in. The loads on the piles at the face and under the interior piers are reported as conservative, but subsequent analysis showed that where the wings meet the head-house, the piles under the piers were overloaded. The piers did not settle most, however. The greatest settlement occurred in the area of the greatest depth of fill where the settling of the fill itself carried down with it, the piles and the structure.

Example 22.—In 1905 an armory was built in Minneapolis, Minn. (4*f*). The site originally had been swampy ground, reclaimed by filling with sand

and gravel, so that, at the time construction began, the soils from the surface downward consisted of 15 to 20 ft of sand and gravel fill, 6 in. to 41 ft of mud on sand and gravel, with ground-water level at 1.5 ft below the pile cut-offs. Wooden piles 30 to 40 ft long were driven for the building, 189 ft by 218 ft, and some of them passed through the sand and gravel fill and through the mud into the hard sand and gravel sub-stratum. This was the case at the southeast corner. At the northwest corner, however, although the piles were driven through the sand and gravel fill and into the mud stratum, there was perhaps 10 ft or more of mud between the pile-points and the solid sand and gravel. A fairly good picture of sub-soil conditions may be had from the settlements that occurred over a period of several years. The southeast corner settled not at all, the southwest corner, 3 in., the northeast corner, 0.5 in., and the northwest corner, 37.5 in. As such unequal settlements are disastrous, in 1913 the west side of the building was condemned. It is probable that if test borings had been taken before construction began, and some extra money spent for longer piles, nothing would ever have been heard of the settling of the Minneapolis Armory.

Example 23.—The factory buildings of the Jurgens Oil Mill Works between the River Maas and the Dike at Zwyndrecht, Holland, were constructed on made land, consisting of sand fill, hydraulically placed. In 1915, the Jurgens Margarine Company (12) decided to erect on the site an oil mill to contain heavy machinery, tanks, etc. Since it was evident that piles would be necessary, test piles, 20 m (65.6 ft) long, were driven. The piles were driven to a resistance of 30 blows for the last 30 cm (11.8 in.), or theoretically, to a static load equivalent to 50 tons per pile. Inasmuch as the piles were to be loaded to only 5 tons each, there was apparently a factor of safety of 10. Creosoted wooden piles, 20 m (65.6 ft) long were driven. In fact, it appears from the accounts that longer piles could not have been placed under continuous driving, as they crushed at the heads under the driving resistance of 30 blows for the last 30 cm (11.8 in.). The soils through which the piles were driven were saturated and consisted of 4.5 m (14.8 ft) of sand fill, 13 m (42.7 ft) of peat and some clay, and 5 m (16.7 ft) of fine sand on coarse sand and gravel. From this, it may be seen that the pile-points reached into, but did not penetrate, the stratum of fine saturated sand, and fell short of the substantial stratum of coarse sand and gravel. Had the piles been allowed a period of rest and had driving been resumed, it appears probable that longer piles might have been driven farther and into the sand and gravel.

In 1916, the factory was completed, and by 1920 the Oil Hardening Building had settled so badly that collapse was threatened. Near the center, the structure had settled 70 cm (27.6 in.). For most of the building the load per pile did not exceed 3 tons although, under the tower and tank, the loads per pile were as great as 18 tons. The external loading on the piles, however, must have had little to do with the settlement, as the maximum did not occur where the piles carried the heaviest external loads. In fact, the piles under an extension of the floor outside the building, carrying almost no loads, settled in the same manner.

The engineering report contained the statement that the hydraulic sand fill, with its high friction value, was settling into the bed of 17 m (55.8 ft) of compressible peat and clay. By its grip upon the piles the fill was pushing them through the peat of comparatively low friction value, and thrusting the points farther into the bed of 5 m (16.4 ft) of saturated fine sand which was incapable of carrying the loads. The load of the hydraulic sand fill was estimated at 15 tons per pile, which still would have left an ample factor of safety, had the dynamic pile-driving formula been of any value.

CONCLUSION

In the past, the design of pile foundations in plastic soils has, in the main, followed standards that do not take into consideration all of the many factors involved in these complicated problems. Although most of the serious settlements of foundations on piles have taken place in plastic soils even there the results have, in the large majority of cases, been surprisingly successful. At times, however, foundation settlements occur that are very expensive to the owner and are no credit to either the architect or contractor.

Even when the best of core borings have been taken, classification and predictions as to reactions of soils on sight by the average engineer is extremely difficult, if not impossible. Soils into which piles are driven range from those that heave and do not consolidate easily to those that subside and consolidate almost at once. Within this broad range occur mixtures and combinations in infinite variety, the reactions of which are further complicated by variations in the water content. Physical analyses of course, can develop the nature of the soils and the proper classification can be made, from which experience can usually predict the result. For the average or inexperienced foundation engineer, however, a more reliable answer can be had from a combination of core borings and pile-driving and loading tests in the field.

The engineer who depends upon some convenient tabulation of friction values, who expects a static test load upon a single pile multiplied by the number of piles in his foundation to equal the total load-carrying capacity, who relies upon the results of static load tests made too soon after driving, or who places complete dependence upon the *Engineering News* formula under all conditions, may be not a little surprised, at times, at the results.

Standard pile spacing as accepted in the United States is usually too close for the loads applied upon them when the piles are driven in plastic soils. If the proper spacing is used corresponding to the length, size, shape, roughness of the pile surface, load, and the soil characteristics, greater loads can be placed upon the piles with less settlement. Mathematical analyses for pile spacing and the distribution of loads to sub-strata would undoubtedly be an improvement over the usual practices; but group test loadings approximating the conditions of construction and final soil consolidation should prove more dependable. When this is done, the dynamic formulas may be regulated to a position of secondary importance.

Had a sufficient number of borings been taken over the sites, and had the borings been accompanied by clear descriptions of the sub-soils, it is reasonably certain that only a few of the foundation settlements described in this paper would have occurred, since the piles in most cases would have been driven to virtual refusal upon satisfactory sub-strata. However, where the piles have been driven short of sub-strata on the theory that friction piles would support a structure in spite of overloaded compressible strata beneath, it can only be stated that such a saving in first cost is very expensive. If any of the compressible strata are suspected of being peat, or combinations of peat, and the peat bed is of appreciable thickness, disregard of such a warning is to invite disaster.

Reliable core borings should always be made in sufficient number to disclose the nature of the sub-soils beneath the entire site. When it is discovered that that strata upon which the pile-points are to rest are perhaps insufficient to carry the loads to be placed upon them, thorough analyses and pile-driving and loading tests should be made. In a word, convenient tabulations of values or standards should be viewed with distrust, because each pile foundation presents a new problem. No two are exactly alike.

Acknowledgments

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APPENDIX

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DISCUSSION

Hibbert M. Hill,[3] Assoc. M. Am. Soc. C. E. (by letter).—A very interesting series of examples is presented in this paper and the author's deductions from his study of them are exceptionally clear cut. After reading the paper, however, the writer has an impression that Mr. Miller makes it appear that more weight can be given to pile-driving and loading tests than is actually the case. Such tests are of value for specific purposes (as, for example, to determine whether a pile will develop frictional resistance sufficient to transfer its load to the surrounding soil, or, in the case of piles reaching a hard stratum through soft material, to test the strength of the piles as columns), but in the usual case it is impossible to test load an area comparable in size to the foundation. Therefore, in the usual case, the results of test loadings, in plastic materials especially, cannot be relied upon to indicate directly the settlements to be expected in the finished structure.[4]

The author places great emphasis, and rightly, on a thorough exploration of the underground by borings. It is not a flattering commentary on the state of the art that this point requires emphasis. The writer has had experience involving the design and construction of eight locks and dams on the Upper Mississippi River. These structures were founded on piles in undergrounds containing sand, silt, and clay, complicated in some cases by rock at depths of about 15 ft. Borings by various methods, driven test piles, and loaded piles were used to explore these foundations. The period of construction of the project as a whole was of sufficient length to provide settlement data for comparison with the test results from some of the structures before the final structure was completed. At the termination of this experience, the writer summarized in his notebook what he had learned about foundations. The red letter entry in this notebook is:

> "By far the most important foundation information is a thorough knowledge of the material in the underground to a depth of about twice the width of the foundation; to greater depths if soft beds might exist at those depths. For all explorations use a method which will obtain a whole sample of the material, from ground surface to bottom of boring, and where compressible materials are encountered, obtain undisturbed samples for laboratory shear and consolidation tests."

An expert, it is true, can forecast future behavior quite accurately by inspection of undisturbed samples, but the expert will reserve his opinion until he has the laboratory results before him. Only the inexpert are courageous enough to proceed, where compressible materials are involved, on the basis of inspection of ordinary boring samples.

George P. Stowitts,[5] M. Am. Soc. C. E. (by letter).—This discussion of the behavior of pile foundations in mixed soils, with its wealth of example and data, seems to the writer to make for clarification of certain basic considerations in the design of pile foundations.

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[3] Supt., St. Anthony Falls Water Power Co., Minneapolis, Minn.

[4] For a discussion of this subject see, "The Science of Foundations," by Charles Terzaghi, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), pp. 288–291

[5] Regional Engr., PWA, New York, N. Y. Mr. Stowitts died March 5, 1938.

In mixed soils, piles must still be primarily point bearing or primarily friction bearing. Where there is within reach an underlying stratum of sufficient bearing capacity, the nature of the superimposed strata becomes relatively unimportant, and the case may be considered as point bearing; it becomes entirely proper to resort to a spacing as close as driving conditions permit. It is then important that the piles be of as nearly a uniform cross-section as possible. Where there is no such underlying stratum within reach, which will carry the load of a normal footing within its own area, the piles must be considered as friction piles. A friction pile is one which depends on the cone of pressure to reduce the unit load at and below its point to a value capable of being supported by the available strata. If these cones of pressure intersect and overlap too much there is a waste corresponding to the overlap. Thus, in the case of friction piles, the spacing is the controlling factor.

Although the spacing called for by the Whangpoo formula (see heading "Spacing for Friction Piles" in the paper) is extreme, none the less it is based on sound logic. Spacings of 2.5 ft or 3.0 ft, perfectly correct with point-bearing piles, must usually be very wasteful when used with friction piles; 3.5 ft to 5.0 ft would represent the most usual practical answer. This will usually call for a larger foundation area; but not always, as the unit value of the piles will be higher and more uniform.

It seems to the writer that Mr. Miller has drawn the wrong conclusion from the dish-shaped settlement (see "Spacing for Friction Piles"). These necessarily reflect an overloading of the underlying sub-stratum; and, consequently, are more affected by the size of the foundation than by the exact number of piles. The interior piles may well be given a fairly generous spacing; crowding in more piles will have no effect one way or the other. Close spacing in the outer ring will often have the further effect of establishing a cleavage plane, so that it becomes difficult to get the full value of the supporting soil outside the pile cluster. Where one of these settlements has occurred it can be ascribed more often to the foundation being too small rather than to its having too few piles.

Where there are many piles under a footing, much can be done to make their value high and uniform by beginning to drive in the center and working out toward the edge. This throws the heaving and compacting outward where one will be relatively harmless, the other helpful. Starting at the edge makes the piles more and more difficult to drive, needlessly increases the trouble with heaving, and decreases the capacity of the piles.

All the foregoing comment involves essentially, a friction-pile problem; such phenomena can scarcely happen where the piles are entirely end bearing. It may be worth noting in this connection that tapered piles, regardless of their material, are at their best as friction piles. In an end-bearing pile, the useful column material (for load carrying as distinguished from driving) cannot be much more than double the lower half. In heaving ground it is important that the pile, as driven, have abundant tensile strength; that is to say, that cast-in-place concrete (one of the best possible materials in compressible soils) is out of place in an incompressible soil.

Much can be done to control heaving by the sequence of the pile-driving. Often there is one side of a foundation where heaving will be relatively harmless. By starting the driving on the side with the greatest resistance (the one against the bank, if there is one), and progressing back and forth over the long rows, the heaving can be mostly thrown away from the driving to where it will make the minimum of trouble. This takes more cribbing, and more moving of the driver, but will often save several times this extra cost.

Loss of bearing capacity after driving often occurs, but so does gain. Along the New Jersey shore, opposite about 50th and 65th Streets in New York City, there is a peat deposit as deep in places as 210 ft; 85-ft wooden pier piles driven with a 3 900-lb drop hammer must be watched closely lest they go below the cut-off. They will often be going down 3.5 in. at the last blow with a 10-ft drop; but the next day these same piles can scarcely be started with the same hammer. These piles are loaded successfully to 15 tons, despite the alarmingly small value given by the *Engineering News* formula. It has seemed to the writer that the nearer one comes to a pure friction pile, the less reliable are the results given by this formula.

GEORGE A. MCKAY,[6] M. AM. SOC. C. E. (by letter).—An important phase of foundation engineering which has received insufficient consideration, is treated in the paper by Mr. Miller. In contrast with the more exact determination of strength of the various parts of the more familiar superstructures, the fixing of the size and character of foundations has too often been a matter of rough approximation based on the limited information available as to the action of somewhat comparable constructions in somewhat comparable soils. If the structure stood, it was evidence of sound judgment, and if it settled, there was generally a self-satisfying excuse as to unknown, unusual, and unforeseeable subsurface conditions which, when investigated, were given but little publicity.

Although improvement in practice has been steady, there is, and there always will be, a long way to go before engineers can approach the confidence with which other better known and more easily studied elements of structural engineering are treated. In dealing not with pile strengths or pile formulas, but with soil behavior under pile loads, this paper marks a step toward focusing attention on this vital but comparatively neglected factor in safe construction.

There was a time, and not so many years ago, when the average engineer, in the course of designing a medium heavy structure on somewhat questionable soil, would cease to worry over possible serious settlement when he decided to use pile footings rather than spread footings. Such a decision, he felt, eliminated the uncertainties as to soil settlements from spread footings under load and rendered the design exact enough to fall safely within the factor of safety used in standard pile formulas, which with easy calculation convert a penetration from a dynamic blow into a static safe bearing value. Following such a decision, the remaining foundation design was easy. The estimated bearing value per pile, divided into the total load, gave the number of piles, and the size, shape, and depth of the foundation followed after consideration of other allied economic and easily analyzed conditions.

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[6] Capt. (CEC), U. S. Navy (*Retired*), Miami, Fla.

In general, two vital considerations were overlooked, and both fall within the scope of the paper and are illustrated by examples given therein. One was the limit which could be placed on the supporting power of the soil itself to carry the loads passing either from end bearing of the pile-points or from skin friction to the soil surrounding the pile. A false feeling as to the security of pile foundations had gradually developed from observed and published examples applicable to isolated pile behaviors, such as heavy loads supported by pier piles driven on, say, 8-ft to 15-ft spacing in soft mud where such piles safely carried much heavier loads than the standard pile formula indicated would be safe. Engineers knew that piles in the process of being driven were often, in some manner, lubricated, and that, in general, after rest the skin friction increased, and the piles showed higher resistance and increased bearing strength when driving was resumed. They knew that piles received support from end bearing and from skin friction, but they did not know, with certainty, what proportion of resistance arises from each pile, nor what changes in these proportions occur after rest and under load. These changes follow the slow return to equilibrium of the materials displaced by driving and the slow adjustment of increased water content pressures within the contracted voids in the disturbed and loaded areas.

The important point which was not stressed early enough was that it added nothing to the supporting power of a pile foundation to drive additional piles after the area affected had had inserted in it a sufficient number of piles to produce soil settlement from superimposed loads. Numerous examples of settlement trouble can be traced to this cause.

The second point was a lack of appreciation that under certain conditions the soil surrounding a pile from which support was expected, in cases where such soil was compacted or settled from other causes, would re-act through skin friction on the pile and overload it and carry it down. Ordinarily, settlements from this cause are entirely distinct from the superimposed load of the structures which the pile foundations are designed to support. They may be caused by compression and gradual settlement of new fill through which the piles are driven. One serious case arose from the placing of a heavy fill surrounding a concrete stadium which was properly enough designed, with sufficient concrete piles to carry, safely, the calculated loads from the structure, but these piles were unable to carry the added load arising from the settlement of soil compressed by the much greater load from a fill placed outside of, and around, the structure.

Somewhat more difficult to foresee was the settlement of batter piles driven to resist the outward movement of a quay wall constructed in 30 ft of water in a heavily silt-laden tidal stream. In this case the sheet-piles for retaining the fill below and to the rear of the relieving platform were driven 40 ft back of the face of the wall, thus exposing the batter piles under the relieving platform to the accumulation of silt deposits from the stream. Routine and frequent dredging along the face of the wall would leave banks of soft mud under the wall held by the piles against rapid sloughing down into the stream. This added load from skin friction on the batter piles produced settlement of the piles, and instead of resisting the outward movement of the wall they actually assisted in pulling it forward so that years later, after the wall had moved out-board several feet and was being reconstructed for other reasons, excavation showed open joints between the batter-pile heads and caps, with about an inch of drift-pin exposed.

It is by pointing out cases in which current practices are faulty that their limitations can be better visualized and appreciated. More literature along such lines will be helpful. Much light can be thrown on this problem by engineers who are fortunate enough to combine with their work of design and construction other duties of maintenance, alteration, and repair of old structures. The exposure of parts incident to reconstruction provides an opportunity for examination of members subjected to the test of years and often reveals conditions, the study of which demonstrates the accuracy or error of past standard practices. It is the accumulation of such facts which gives an engineer a background of experience leading to safer construction, and inasmuch as specific concrete examples are more easily visualized and remembered than abstract rules, young engineers, by the perusal of such examples, can more quickly gather from the experience of others that knowledge of what to avoid and what limitations must be kept in mind which insures safe and permanent construction.

The examples cited in the paper and the lessons drawn therefrom are useful; they focus attention on the need for such subsurface exploration of soils around and below the piles as the importance of the structure warrants, and also the need for recording permanently such data for future use by others. In the case of less costly constructions where the expense of subsurface exploration is not warranted, they help to accentuate the importance of spreading pile spacings and analyzing possible future physical changes in surrounding conditions so as to avoid excessive or irregular settlements of soil in contact with, or below, the piles, and thereby assist in safe design.

The paper alludes to the possibility, in some cases, of securing savings through reduction in the number of piles where pile-points reach hardpan or rock. In any consideration of such savings careful attention should be given to the advisability of increasing the size of the pile-point in the end bearing and the possibility that such points may have been damaged in driving. It is also of interest in this connection to appreciate that foundation engineers know less than they should of the permissible increase in safe compression unit values and in end-bearing values for piles driven in firm soil where the pile particles are restrained against the lateral expansion and disruption incident to the application of compressive loads. As this side restraint increases with the depth to which the piles penetrate, the safe unit load for compression and the resistance to crushing in wood and concrete must increase materially, depending, of course, on the character of the soil penetrated. Such increases in permissible safe compression are undoubtedly considerable in sandy or firm soil and possibly sufficient where piles have moderate tapers to permit the carrying capacity to be determined by the safe bearing on the fibers at the top of pile. Scientific investigation along this line would probably bring into closer agreement present practices in permissible loads on piles as compared with small caissons sunk to rock or hardpan bearing.

Present practices are conservative as far as piles driven to firm bearing strata are concerned. Not much trouble is encountered on pile foundations in sand or gravel where these materials are confined against possible future lateral movement. It is the compressible silts, mud, clay, and peat strata either below the piles or surrounding them, which should be investigated more thoroughly and studied cautiously.

Following thorough subsurface exploration much can be accomplished in certain soil combinations through the more scientific study of soil reactions and in the placing of piles to take economic advantage, safely, of any favorable factors which may be present. By securing a better distribution of loads to soil through a greater use of batter-piles, by lagging piles to secure controlled lengths and greater bearing at predetermined elevations, and, in certain cases involving lateral reactions, a more intelligent use of tension piles, a closer approach to the more exact methods used for the design of superstructures may be secured. A case illustrative of possible savings through use of lagging on piles exists at the U. S. Training Base on San Clemente Island, in California, where it is desired to extend a steel-pile pier to deeper water. The piles pass through successive layers of softer rock and sandy soil before hard rock is encountered. If carried to bed-rock, the steel piles at the outer end would be long and heavy; still it would not be safe to stop the narrow steel pile-points in any of the upper, softer, rock strata. Test piles show the strata to be fairly uniform and dipping toward the sea, and it is quite feasible to lag the piles with wooden fillers and secure safe bearing on one of the higher layers of softer rock. Somewhat similar would be the conditions encountered in coral where carefully planned use of lagging might reduce to reasonable figures what otherwise would be prohibitive costs of construction. In softer soils lagging, under certain conditions, would avoid the use of long composite piles. Such lagging, however, is feasible only for cases such as in pier construction, etc., where piles are single and not closely spaced in clusters, and not where the piles without lagging carry all the load the soil would support. In other cases, lagging will increase the effectiveness and value of piles subjected to tension.

The attainment of the objective of improving pile foundation design practice so as to bring it closer to the more exact methods used for superstructure design, however, is being retarded by the uncertainties surrounding and affecting the interpretation of standard pile-driving formulas which seldom take into consideration such vital and important factors as the thickness of, or material used for, cushions, the effect of jetting, speed of driving, and vibrations from hammer or soil irregularities, and from methods of holding a pile, etc. In time, some of these uncertain elements will be better investigated and better understood. A simple contrivance developed by Commander W. Mack Angas (CEC), U. S. N., M. Am. Soc. C. E. (used experimentally in the driving of a 20 by 20-in. concrete test pile for a Navy pier at the Supply Depot in San Diego, Calif., in 1927), secured such remarkably accurate data as to promise definite progress in the investigation of certain factors.

This apparatus consisted of a revolving cylinder, 15 in. in diameter and 18 in. high, mounted on a platform adjacent to the concrete test pile being driven, and a pencil inserted in a socket near the head of the pile and bearing against a small spring at the base of the socket. This pencil traced a line on a sheet of paper mounted on the revolving cylinder surface, with the result that a record was secured of the exact movement of the head of the pile for each successive blow of the hammer. Time increments could be read to less than one-hundredth part of a second. With a similar cylinder, 6 ft long, the exact behavior and velocities and rebound of the steam hammer were recorded and could be read in equally fine time increments for all parts of the stroke. The graphs secured showed clearly the considerable rebound of the concrete pile-head to the shock of the hammer blow and, by proving the temporary occurrence of tension in the pile, explained certain horizontal cracks in piles driven through hard coral on a previous pier constructed at Pearl Harbor, Hawaii. It was the intention to use the apparatus in the study of the effect of variable cushions commonly used in pile-driving apparatus, but this was prevented by lack of time, money, and opportunity for further experimentation. However, the possibility of accurate analysis was established, and some day this interesting and important element will be investigated further. Any company or engineer in a position to carry this study further, either in analyzing hammer actions or efficiencies or pile behaviors can undoubtedly secure from the Bureau of Yards and Docks, U. S. Navy Department, Washington, D. C., details of the apparatus, and information and charts covering the data secured in San Diego in 1927.[7]

CHANDLER DAVIS,[8] M. AM. SOC. C. E. (by letter).—Quite properly, Mr. Miller warns designing engineers not to depend on "some convenient tabulation of friction values." A thorough study of the underlying soils is required. This is especially true in New York Harbor, where the most varying and peculiar conditions are encountered.

The late G. S. Greene, Jr., M. Am. Soc. C. E., Engineer-in-Chief of the Dock Department of the City of New York, confronted with these difficulties, designed his well known floating wall.[9] However, each authorized section was thoroughly studied before work was begun and the final design of the quay wall determined. Table 4 contains some of the observations made over the area of the West 23d Street, North River Section. This section extends from 10 ft south of the north side of West 23d Street to the north side of West 30th Street, about 1 944 ft.

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[7] *Bulletin 36*, Public Works, U. S. Navy Dept.

[8] Cons. Engr., New York, N. Y.

TABLE 4.—CHARACTER OF BOTTOM AS DEVELOPED OVER THE WEST TWENTY-THIRD STREET SECTION OF THE NORTH RIVER, AT NEW YORK, N. Y.

Location	AVERAGE DEPTH, IN FEET, BELOW MEAN LOW WATER								
	Water		Dock Mud		Compact Mud		Blue Clay*		Rock Bottom
	From:	To:	From:	To:	From:	To:	From:	To:	At
South, 1 238.2 ft.........	0	12	12	64	64	148	148	168	168
North, 615 142.0 ft.......	0	16	16	76	76	144	144	153	153

* With gravel, sand, and shells.

Table 5(*a*) shows the penetration per blow at various depths at the foot of West 26th and 27th Streets, North River. In this case all the piles were straight, free of bark, and equipped with a cast-iron shoe. Blows were struck with a 1.5-ton hammer, falling 8 ft.

As shown in Table 5(*b*), the piles tested are part of the completed pier at the foot of West 26th Street, North River. At the time of the observations the pier had been completed about one year. All the piles tested were in the interior, each supporting about 16 tons; they were about 75 ft long, and the points were about 65 ft below mean low water. According to Table 5(*a*) the penetration per blow at this depth is about 6 in. and, by the *Engineering News* formula, the safe load is 3 tons per pile. The piles, however, were loaded to 35 tons per pile without damage to the pier. They are true friction piles and depend on the skin friction to support the working loads.

Whenever a large ship is breasted into the slip around an outer corner a perceptible movement of the pier is observed. The piles move and bend under the pressure of the vessel, acting as long columns of indefinite length. About 7 ft of their upper ends are braced securely and are fixed both laterally of the pier as well as longitudinally, and they act as one unit. At which point below the mud-line the lower ends are held immovable is indeterminate. The distance varies; each time the pier moves the mud grip is broken and the column length is increased by an unknown amount. However, as time goes by, the river bottom under the pier is built up; material in suspension in the river water is gradually deposited; the increased weight on the dock mud compresses it and renders it more dense, and thus the area over which the skin friction acts is increased and the pier becomes more stable. The writer removed a pier at the foot of Broad Street, East River, where the surface of the mud under the pier was level with mean low water. This deposit was of a very stiff consistency.

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[9] See reference to paper by the late S. W. Hoag, M. Am. Soc. C. E., based on a statement in American Civil Engineers Pocket Book, Second Edition, 1913, p. 608.

TABLE 5.—Driving Data, Piles at the Foot of West Twenty-Sixth Street and West Twenty-Seventh Street, North River, at New York, N. Y.

Pile No.	(a) Computed* Safe Load, Piles at the Foot of West 26th Street and West 27th Street						(b) Bearing Value of Piles not Driven to "Refusal"; New Pier No. 56, at the Foot of West 26th Street, North River, at New York, N. Y.					
	Penetrations, in Inches per Blow, at the Following Depths, of the Point, in Feet, Below Mean Low Water:						Diameters, in Inches		Pile in Contact with Mud		Load per pile, in pounds (11)	Resistance in mud, in pounds per square foot (12)
	60 (1)	65 (2)	70 (3)	75 (4)	80 (5)	85 (6)	Head† (7)	At the mud line (8)	Surface area, in square feet (9)	Depth, in feet (10)		
1	1.0	..	..	..	..	..	13.87	10.10	83.97	39.91	59 900	714
2	0.7	..	..	..	..	..	16.00	11.03	85.60	38.51	59 900	699
3	1.0	0.6	0.6	0.5	0.4	0.3	13.25	9.67	80.37	39.20	67 500	839
4	0.6	0.6	0.6	0.5	..	..	13.75	9.96	83.37	39.91	68 075	816
5	1.1	0.8	0.7	0.6	..	..	14.13‡	11.00	88.50	36.31	67 500	763
6	1.3	1.0	..	..	..	..	13.25	9.81	85.45	41.31	70 550	826
7	0.7	0.5	0.6	0.3	..	..	12.25	9.34	87.50	43.61	68 075	778
8	1.7	0.6	0.6	0.4	0.4	0.3	16.50	11.62	96.82	42.00	70 550	729
9	..	..	..	..	..	..	15.25	10.65	86.95	39.97	70 550	811
10	..	..	..	..	..	..	15.50‡	11.83	97.30	37.51	70 550	725
11	..	..	..	..	..	..	16.75	11.70	93.37	40.51	70 550	755
12	..	..	..	..	..	..	14.75	10.72	93.88	42.91	70 550	751
13	..	..	..	..	..	..	12.62	9.50	82.98	40.91	68 125	821
14	..	..	..	..	..	..	16.00	11.46	95.32	41.71	67 500	708
Average	0.9	0.7	0.6	0.5	0.4	0.3	....	....	....	....	....	767§
Average safe load, in tons*	2.0	2.6	3.0	3.4	4.0	5.2	....	....	....	....	....	....

* Computed by the *Engineering-News* formula.
† Diameter of point is uniformly 6 in., except as noted.
‡ Diameter of point, 8 in.
§ Point resistance included.

The foregoing are a few of the numerous observations made in New York Harbor on piles, all of which were not driven to refusal but are supporting their working loads. The pile-driving formulas are not applicable in these cases.

The observations and study of the river bottom in New York Harbor made by the Dock Department, covers more than 65 yr. The thorough study made by Mr. Miller of soil reactions in many countries bears out the foregoing findings and deductions. He further stresses the necessity of a thorough investigation of the subsoil and a complete understanding of the facts obtained; that is, Mr. Miller emphasizes the fact that experience is an important factor in such studies, particularly in the case of piles that are not driven to "refusal", and are friction piles.

Harry E. Sawtell,[10] M. Am. Soc. C. E. (by letter).—In the Synopsis the author states his purpose "to use the construction field as a source of information, and to correlate the collected data on pile foundations under varying soil conditions so that the reasons for success and failure become evident."

The success of his attempt, when citing different examples of construction, depends largely on the accuracy with which the conditions that surround each piece of construction are reported. This accuracy is not always present. The writer has elsewhere called attention to a case in which inaccurate assumptions led the investigators into some curious errors.[11]

Mr. Miller discusses soil reactions due to driving piles into clay, but does not cite examples of the remoulding effect on clay. The writer does not think that the results of actual construction or any published experiments can justify any assumption that driving piles into soft or medium clay will completely or largely remould it. In and around Boston, Mass., it has been the practice to support foundations by driving piles into clay, and the comparatively few cases of bad building settlements no doubt are caused by the slow but sure compacting of underlying clay beds due to the additional loads placed upon them.

Driving piles into clay cannot be compared to remoulding a clay sample in the laboratory. In fact, the usual close spacing of piles gives an area of soil around each pile of about 5.5 sq ft, and assuming a pile well above the point to be 7 in. in diameter, the pile is found to occupy less than 5% of this area.

To add an example to Mr. Miller's citations showing "Soil Reactions to Foundations on Piles," the writer wishes to cite the soil conditions, assumptions for design, and some results of the foundation design for the Eastman Laboratory, a Chemical Research Building for the Massachusetts Institute of Technology, built in 1931.

Soil.—This building is supported on piles which were driven through about 20-ft of low-grade fill (silt) and which stopped in a good sand bed under the northerly half of the structure; but for the southerly part the piles were driven through a loose sand bed, an average of 3 ft thick, and about 23 ft into a stratum of medium blue clay. Under the tips of the piles the clay was about 47 ft thick where boulder clay is reached. The clay has some small quantity of sand in it, in the form of layers or lenses, but the layers are seldom continuous.

Design of Pile Supports.—The basic idea of the design was to spread the load reaching the plastic (medium) clay over as great an area as possible, in order to impose upon the clay as small an additional loading per square foot as possible. The piles were spaced so that the pressure bulbs (having a small diameter in clay) did not overlap. The piles were designed to carry small loads and were deeply embedded, in order to transfer the load to the clay with a surface frictional value of about 300 lb per sq ft.

Load Tests.—To develop the value of skin friction certain piles were loaded covering a period of 200 hr. One pile was driven through a stratum of loose sand and into 22.7 ft of clay; and after being allowed to set, it was given a test load of 40 000 lb applied slowly, without causing failure. This pile (one of a group) settled less than $\frac{5}{16}$ in. and recovered to a total settlement of $\frac{3}{16}$ in. when the load was removed.

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[10] Structural Engr., Care, Charles T. Main, Inc., Boston, Mass.

[11] *Proceedings,* Am. Soc. C. E., November, 1933, p. 1467, *et seq.*

The original condition of the clay, evidently, was not changed by driving to such an extent as to destroy the elasticity and grip on the piles. The recovery at unloading was somewhat more than one-third the maximum settlement under the load of 40 000 lb. The working load is about 14 000 lb per pile in this foundation.

Surface Friction.—At the foregoing loadings, the pile showed a surface friction resistance equal to about 900 lb per sq ft for the total test load, and about 300 lb under the proposed working load.

Tests to determine the probable frictional value of a pile in soil under load show whether or not the actual working load upon a pile can be transferred to the surrounding soil safely. This is correct whether or not the pile is a part of a group. The effect of group action under certain loads is largely a question of what settlement occurs under the total loading of the group over a long period of time in which the moisture content of the clay may be pressed out into the surrounding soil. The question of how much settlement is permissible must be answered by other considerations, and must be suited to the surroundings, connecting buildings, or other structures.

To demonstrate, in another manner, the effect of driving piles into this kind of clay and the possible deformation and remoulding of the clay, a series of re-driving tests were made upon piles driven for the support of this Laboratory.

Re-Driving Pile Values.—Re-driving values, after hours of rest varying from 0.5 hr to 32 hr, have been reported in detail elsewhere.[12] The same formula and a single-acting steam hammer were used in both driving and re-driving and, therefore, the results were comparable. The clay was a so-called "medium" grade, but was inclined toward the soft-medium. No evidence of remoulding of clay was shown; nor was the elasticity of the soil destroyed as would be the case if the clay was remoulded much when the piles were driven.

Action of Clay During Driving of Piles.—Another demonstration of the action of clay soil, at this same building, when driving piles, was observed, namely, one end of this Eastman Laboratory (Building 6, Fig. 7) joined one of the other buildings (Building 2) of the group, also supported on piles driven into the soft-medium clay. The piles under Building 2 were 45 ft long, whereas those under the new Laboratory are 50 ft long, with the tips of the new piles 5 ft deeper than the old piles.

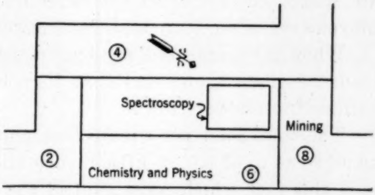

FIG. 7.—KEY PLAN, NEW EASTMAN LABORATORY.

After driving the new piles adjacent to the old ones, it was found that the wall and near-by corridor of Building 2 had been raised $\frac{3}{16}$ in. After two years had elapsed Building 2 settled back to its original level and has remained there.

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[12] *Journal*, Boston Soc. of Civ. Engrs., October, 1933, Table 1, p. 166.

The ground surface heaved around the new piles during the driving, but equalled less than one-half the displacement of the piles.

The lifting action and the displacement of soil around the piles indicate that the clay around the new piles was not appreciably remoulded but resisted compression to such an extent as to exert a positive pressure on the clay around the new, and especially around the old, piles under Building 2. It was evident that the clay had sufficient stability and density to transmit the upward component of the compression force to the old piles and to raise the building slightly.

This favorable action cannot take place except when piles are properly spaced, deeply embedded, and have a large reserve value of surface friction.

Pile Formulas.—There appears to be no one pile formula suited to all, or even to any two types or combinations of, soils. The engineer will use many times, therefore, the formula best known and recorded for its deviations and limitations when applied to the largest combination or types of soil.

No formula is used by the writer as a rule, but occasionally, the *Engineering-News* formula is favored as a yardstick or gage only, and seldom to obtain correct pile values directly. No other formula is so well known in regard to its faults and limitations, and, therefore, so safe when used as a gage.

Elastic Properties.—All the writer's research in soils seems to confirm the repeated statement that soils are somewhat similar to "weak solids" having elastic properties, and must be considered as such in foundation design.

Economic pile design is one in which just enough piles are used to distribute the load uniformly and properly to the highest stratum capable of carrying the total load without producing an unwarranted settlement.

Each pile corresponds to a small isolated footing or unit and has its own bulb of distributed pressure. Therefore, the most economical spacing of piles is one in which the pressure bulbs are far enough apart to prevent overlapping of the substantial pressure lines, thus using the entire area to good advantage and producing a fairly uniform pressure on the soil under such piles. If piles are spaced too far apart the subsoil pressure is uneven and permits upward movements of moisture and clay between piles where there is no pressure.

When piles are used in medium or soft clay, it is imperative that small loads be given them, and that they be given deep embedment to keep the surface friction very low.

The total load per square foot under the entire pile group must be kept small so as to obtain as little deformation or future settlement as possible, and it is this fact which must control the design of foundations on compressible soil rather than any result of a single pile-load test.

Examples of Settlement.—The material beneath the Laboratory has been described herein under the heading, "Soil." The design of the superstructure was based upon the idea that the unit loading on the deep bed of clay would be about the same (about 1 500 lb per sq ft additional pressure) whether piles were embedded in the clay or in the sand bed above it. This basic idea was used in the design of the original buildings as it was known at that time (1914) that the actual settlements would result from the consolidation of the clay bed below all fill or sand which might be found above it.

The settlements of this building have been recorded occasionally since its erection in 1931 and now (1937) amount to about 1.25 in. at the southerly end and about 2.25 in. at the other end, but with a somewhat greater settlement at the center of about 2.5 in. A greater settlement was anticipated at the center of the building and was allowed for by cambering the building.

The ends of the building were butted up to, but had no physical connection with, older buildings beneath which it was assumed that the subsoil was consolidated to a greater extent than would be the case in the center away from the old buildings (see Fig. 7).

The older buildings which surround this new Laboratory were built from 1914 to 1916 and the soil and foundations were similar to those described herein for the new Laboratory. These older buildings have settled almost exactly 6 in., one having shown no further settlement since 1927, and the others have had no appreciable settlement since 1932. It is anticipated that some time in the future, these buildings, which appear to have ceased settling, will show some slight additional settlement due to the small but continued consolidation of the clay.

The interesting point regarding these examples is that, from the very first, settlement was anticipated, and the foundations were designed so that the settlement would occur in the clay by consolidation and not at the piles or as a result of driving piles.

The proof of the soundness of this design is that under a number of the buildings of this group where a part of the piles were stopped in a sand bed above the clay, and a part were driven deeply into the clay (the sand did not extend under all the full length of each building), the settlements under both ends of the buildings were found to be almost exactly the same.

The author's "Conclusion" is quite proper and justified, as it clearly states the case as developed from actual work done. The writer agrees with most of the conclusions, but wishes to contend that, when designing foundations, engineers should insist on having authority to make more thorough soil investigations than is customary. The comparatively small extra cost of making more explorations, including the taking of "undisturbed" samples by boring machines or open pits, should not stand in the way of obtaining all the information possible, especially if the soil is stratified or includes compressible material. Such thoroughness not only results in a safer foundation for a structure, but frequently saves more than the extra cost of complete exploration.

A case in point was the foundation for the Baltimore (Md.) Plant of the American Sugar Refining Company. A sufficient number of borings were taken to permit a closely spaced contour map being made, showing the surface of the sub-stratum which was capable of supporting all the piles under this group of large heavily loaded buildings. This map was used by the contractors when ordering piles and when driving them, and they estimated their savings at many thousands of dollars in less waste of piles and in smaller cost for the routine handling of the piles on the job.

The writer wishes to emphasize the positive danger of using prepared tables either for driving values of piles or of following pile-driving formulas, or of using values showing safe friction per square foot of surfaces of piles for different kinds of soils. These values should be determined by investigation and study for each job.

Such a study should include certain developments brought forward by the science of soil mechanics, but caution should be exercised to use only those which have been substantiated and proved to be applicable.

J. STUART CRANDALL,[13] M. AM. SOC. C. E. (by letter).—Many sources of data are cited by the author and these should be helpful in permitting more logical design of foundations, and particularly pile foundations. The accumulation and careful interpretation of such information are vital to a proper checking of the assumptions now made or to be made in theoretical analysis. Many of the apparently antagonistic results can be interpreted readily with present knowledge of soil mechanics. Some require further research.

The elapse of time under the dynamic load, during the driving of piles, is very short as compared with the period of actual static load. Therefore, during driving the only data of value are those which give some measure of the behavior of the foundation or any part of it under static load. In a pile foundation:

(1) Each pile must have adequate strength to resist the stress developed in driving (usually several times the static load);

(2) Each pile must have adequate resistance against movement with relation to the soil; and,

(3) The load on the soil, created by the entire group of piles, must not cause dangerous or excessive differential settlement.

Requirement (1) depends on the size and the materials comprising the pile and on the actual stress caused by driving. Much over-driving of wooden piles has been due to the use of drop hammers and the application of the *Engineering News* formula in determining so-called safe loads, which may have a factor of safety of 12, or more, thus stressing the piles beyond the ultimate strength of the material. Requirement (2) is measured by a dynamic pile formula, or by a load test; and Requirement (3) depends on the net weight of the structure and the nature of the underlying supporting soil. As a maximum over the entire area, this imposed load is the weight of the structure per square unit, and this unit load can be reduced only if piles permit spreading the load over a larger area and provided that no loads are imposed by adjacent structures, present or future. As the author intimates, there is no doubt that many foundation difficulties have been due to blind reliance on Requirement (2) and complete neglect of the others. Too frequently this second requirement has been determined by the *Engineering News* formula, the primary merit of which is that the uninitiated can work out a result, however far it may be from reality.

One of the most logical driving formulas is that of Alfred Hiley[14] which is in current use in England:

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[13] Pres. and Chf. Engr., Crandall Dry Dock Engrs., Inc., Cambridge, Mass.

[14] "Pile-Driving Calculations," by Alfred Hiley, *The Structural Engineer*, London, England, Vol. VIII, July and August, 1930.

$$R_d = \frac{e\,W\,h}{s + k} \times \frac{W + n^2\,W_p}{W + W_p} \quad \text{(1)}$$

in which R_d = dynamic resistance to driving; e = efficiency of the hammer; W = weight of the striking parts of a hammer; h = the drop, or length of stroke; s = penetration of pile per blow; k = one-half the elastic rebound of the pile-head after blow; W_p = weight of pile; and n = Newton's coefficient, or coefficient of restitution. Inasmuch as for the materials in contact the values of n are usually small, the writer proposed[15] eliminating the term, $n^2\,W_p$, and has suggested the formula:

$$R_d = \frac{e\,W\,h}{s + k} \times \frac{W}{W + W_p} \quad \text{(2)}$$

If the actual resistance may be determined accurately, a factor of safety of 3 should be adequate. Applying such a factor with the usual efficiency of hammer (namely, 0.75 for a drop hammer overhauling the fall and drum; and 0.90 for a single-acting steam hammer), measuring drop, in feet, and penetration, in inches, the allowable load is:

For a drop hammer,

$$R_a = \frac{3\,W\,h}{s + k} \times \frac{W}{W + W_p} \quad \text{(3a)}$$

and for a single-acting steam hammer,

$$R_a = \frac{3.6\,W\,h}{s + k} \times \frac{W}{W + W_p} \quad \text{(3b)}$$

These formulas appear in the proposed new Boston (Mass.) Building Code.

In Equations (3) the term, k, is a measure of the energy loss due to elastic compression of pile and soil. Evidently, for any given pile, soil condition, and hammer, there is a certain height of fall below which the pile does not move, h_0, which is also a measure of this energy loss due to elastic compression. Based on this fact the writer has suggested the following formula[16]:

$$R_d = \frac{W\,(h - h_0)}{s} \times \frac{W}{W + W_p} \quad \text{(4a)}$$

Making the same assumptions as for Equation (3a):

$$R_a = \frac{3\,W\,(h - h_0)}{s} \times \frac{W}{W + W_p} \quad \text{(4b)}$$

in which h_0 is the maximum drop of the hammer below which there is no penetration. The value of h_0 for any particular case may be determined readily in the field. Measuring the penetration accurately for, say, four different heights of fall, plotting the penetration as abscissas and the drop as ordinates, these points should fall on a straight line whose intercept with the Y-axis gives the value of h_0. If the points do not fall in a straight line the test should be repeated. As h_0 varies as the square of the dynamic resistance, therefore, these determinations should be made when at the probable desired resistance.

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[15] *Journal*, Boston Soc. of Civ. Engrs., May, 1931, p. 176.

[16] *Loc. cit.*, Equation (14), p. 181.

Furthermore, the determination of h_0 permits a verification of the theoretical calculations of the values of k, because,

$$k = \frac{h_0 s}{h - h_0} \qquad (5)$$

The value of h_0 was determined by the writer, at Boulogne-sur-Mer, France, in 1933,[17] for 35-ft greenheart piles driven into sand with a 3 080-lb hammer for a safe load of 25 metric tons (factor of safety = 6), giving a value of 1 ft. Applying these data in Equation (5) gave a value of k equal to 0.06 in. as compared with a theoretical value of 0.12 in. Recently, a similar determination was made of an oak pile driven at the Massachusetts Institute of Technology.[18] The pile was 26 ft long, driven into sand with a 3 000-lb hammer for a safe load of 22.5 short tons (factor of safety = 3). The value of h_0 was determined as 2.2 ft, giving, by Equation (5), a value of k equal to 0.35 in. as compared with a theoretical value of 0.30 in.

A sound driving formula can give only the resistance to rapid penetration or, in other words, the load on the pile during driving. This is of value in determining the driving stresses in a pile. Furthermore, in soils where the resistance to rapid penetration is the same as that to slow penetration, such formula can give the resistance of a pile to movement in the soil under static load. This is true of sands, gravels, and such permeable incohesive soils, but is not true for such soils as clays and silts.

In clays and silts the remolding and disturbance of the soil during driving tends to reduce the frictional resistance which in most cases again sets up after an elapse of time at even a greater value, although on rare occasions the opposite occurs. In these soils there is no known relation between dynamic and static resistance. Therefore, it would appear that the logical procedure in such cases is to base the resistance to movement in the soil on the frictional resistance. An equation for this condition would take the form:

$$R_a = \frac{A_s f}{3} \qquad (6)$$

in which A_s = the area of contact surface; and f = "skin" friction per unit area. In Table 3 the author gives certain values[19] for friction which apparently are taken from the writer's paper. It should be pointed out that the soil at Hull, England (Item No. 1) was not soft clay, but actually stiff blue clay which makes the tabulated value of 1 850 lb per sq ft appear more reasonable. Furthermore, the writer verified certain data of pile-pulling tests at Boulogne-sur-Mer, France, where a value equal to, or greater than, 1 800 lb per sq ft was found for stiff clay. In some tests the friction per unit area decreased with depth; in others, it increased, and then became constant. Therefore, friction tests should be made for each locality and for the lengths of pile used unless other tests are available for the same soil and locality. Even then, corroboration by test is advisable for any important foundation.

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[17] Four piles in railway dry dock designed and constructed for Service des Ponts et Chaussées.

[18] By Stone & Webster Eng. Corp., October, 1937, Pile Z 240, New Architectural Bldg., Cambridge, Mass.

[19] *Journal*, Boston Soc. C. E., May, 1931, Table 4, p. 185.

To this point the writer's remarks have been concerned with the stress in individual piles and the resistance of individual piles to movement in the soil. The results of driving formulas and of load tests can give no indications of the action of a group of piles under static load. The load from a single pile stresses the soil over a fairly wide area. With a group of piles in close proximity these zones of stress overlap, causing more important stress concentrations at greater depths. The writer cannot agree with Mr. Miller regarding the value of load tests on a group of piles. Admitting that such tests on single piles can give no measure of the settlement of an entire pile foundation, particularly where underlaid by compressible soils (because the load caused by a single pile is distributed over considerable area so that the time-settlement effect is infinitesimal), the same is true of load tests of small groups of piles forming part of a larger foundation, although to a different degree. The settlement for the group would be greater than for a single pile, but it would not be comparable to that of the entire foundation. Assuming a structure of infinite extent, the additional load on the soil, whether or not piles are used, would be equal to the weight of the structure per square unit, less the weight of any excavation. If the weight of material excavated is equal to the weight of the structure there would be no additional load and, hence, no comparative settlement. On the other hand, if there is a net load increase and the structure is underlaid by such compressive soils as clay and silt, then there will be settlement. The magnitude of this settlement can be predicted only after an accurate exploration of the soil and a settlement analysis based on undisturbed samples.

The author has cited cases in which the soil has "heaved," others of subsidence, and, again, cases of no evident movement. This action is a result of the soil structure. The voids of clays and silts are usually filled with water and, being relatively impermeable, cannot be compressed during the short period of pile-driving, so that the soil must be displaced equivalent to the volume of the embedded piles, resulting in "heaving." In compact sands and gravels, piles cannot be driven without displacing the soil, causing "heaving" either by the driving process alone or with the aid of jetting. In loose sands, gravels, and fills, the action of driving tends to re-arrange the soil particles into a more compact structure. This may result in some subsidence of the surface, which would depend on the degree of volume change by reason of the reduction in voids. Where layers of several kinds of soil are penetrated, a combination of these effects may take place.

In cases where piles are driven through incompletely consolidated soil to a bearing in a more resistant one, there are certain special conditions to be considered. The imposed load on such a pile would be transferred through friction in the upper layer, and largely through end bearing in the lower one. However, as the upper layer continued to consolidate under its own weight, eventually it would cause an additional load on the pile through friction, so that the load carried by the point would amount to the superimposed load on the pile plus the total frictional load imposed by this consolidating layer, and thus cause settlement. This has been observed in Holland and is a factor in design.[20]

The profession has given much attention to refinement in the design of structures, all based on certain assumptions as to rigidity of foundations, while all too frequently the foundations are only superficially considered and empirically designed; yet the settlement of these foundations may completely vitiate the assumptions in the super-structure design. It seems to be particularly this point which Mr. Miller desires to emphasize and which merits serious thought. The mere driving of piles to some resistance does not, by itself, constitute a solution. The question of foundation design and its limitations should be the first consideration, after which definite conditions affecting super-structure design can be logically determined.

R. M. MILLER,[21] M. AM. SOC. C. E. (by letter).—One of the most interesting experiences that can come to the engineer is to have repeated contacts with engineers and architects of every professional status, who have piling foundation problems to solve. Perhaps no other work has been given as little actual study by the mass of engineers and architects and in no other field are mistakes repeated so persistently. The very general attitude seems to be that brains are needed to design and build a structure but that almost any one can drive piles into the ground, and that once driven the matter is settled for all time. The courage of ignorance has been so general that the writer was tempted and wrote "Soil Reactions in Relation to Foundations on Piles" in the hope that he might emphasize in simple terms, and in readable form, why some of the commonest mistakes occur. If he has made a few of the most dearly cherished fallacies seem questionable, something may have been accomplished. The discussion of this paper has been most considerate although by no means in entire agreement.

Mr. Hill feels that the writer gives more weight to pile-driving and loading tests than he should. Perhaps the thought was not well expressed. In substance, it should have been much as follows: In impervious soils or combinations of pervious and impervious soils, the commonly used "yardstick," the dynamic pile-driving formulas are of but small value whereas the test loading of single piles is not much better; nor is the test loading of single piles, a cluster or mat in such soils, if done too quickly after driving. If, however, a cluster is test-loaded after a sufficient lapse of time to allow for some re-arrangement of the soil particles, even if the cluster is not of an area comparable in size to that of the foundations, the results will more nearly approximate those of the foundations than those indicated by the dynamic formulas, single-pile load tests, or load-testing the cluster, or mat, immediately after driving. Test-loading a group as suggested should offer an improvement over the usual methods, but not the complete answer.

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[20] *Proceedings*, Conference on Soil Mechanics and Foundations, 1936.

[21] Project Engr., PWA, Cincinnati, Ohio.

The writer wishes to concur particularly in the closing sentence of Mr. Hill's discussion which states that "only the inexpert are courageous enough to proceed, where compressible materials are involved, on the basis of inspection of ordinary boring samples."

Mr. Stowitts advanced the thought that the order in which piles are driven is an important factor in the final result. Such matters appear to be lost sight of in favor of apparently more important considerations; but, if mishandled, they may become of as much importance as any other factor of the problem. The sub-soils underlying any two projects are never exactly alike: In one, it may be advisable to drive alternate runs of piles and return a second time over the same ground; in another, as suggested by Mr. Stowitts, it may be well to begin driving at the center and work outward toward the edges; and in another, it may be best to work uphill rather than down, or *vice versa*. In any case, careful consideration should be given to the control of heave and to lateral movement. Broken, cracked, or pinched piles have resulted from this cause in sufficient number to warrant thoughtful planning.

The writer agrees with Mr. Crandall that the dynamic pile-driving formulas should be used only where piles are driven in sands, gravels, and such incohesive soils, and also that his modification of Hiley's formula offers more dependable results than can be had from the generally used *Engineering News* formula. On the other hand, he feels with Mr. McKay that the unknown effect of the use of the several kinds of driving cushions of different thicknesses and different materials should be more thoroughly examined. That this factor affects the results in considerable but unknown degree cannot be doubted, and, yet, how many engineers give the matter consideration?

The use of Equation (6) for the resistance to movement of a pile through clays, silts, etc., appears logical provided the friction value of the soil and the type of pile are known within reasonable limits. Mr. Crandall states that "friction tests should be made for each locality and for the lengths of pile used unless other tests are available for the same soil and locality."

Sub-soil formations may lie fairly uniformly under a project site, or they may change radically within a comparatively small area. If the former is found to be the case one friction test should be sufficient; but if the latter is true the number of friction tests should be governed by the conditions. As an example, refer to the writer's description of the sub-soils underlying one of the buildings of the Laurel Homes Housing Project, in Cincinnati (see heading "Average Practice"). In the space given as 35 by 150 the use of Equation (6) would insure equal settlement, or no settlement, only if at least six or perhaps more friction tests were made. However, as equipment installations represent a large part of the cost of such tests a few added tests would affect the cost very little.

Mr. Sawtell states that the writer has cited no examples of the effect of the remoulding of clay during pile-driving upon subsequent settlement. This is quite correct, due to his inability definitely to connect effect to this cause. In fact, he agrees with Mr. Sawtell that "driving piles into clay cannot be compared to remoulding a clay sample in the laboratory," and that "he [Mr. Sawtell] does not think that the results of actual construction or any published experiments can justify any assumption that driving piles into soft or medium clay will completely or largely remould it." In fact, the writer's observations incline him to the belief that, although some remoulding of the clay surrounding a pile undoubtedly occurs during driving, the soil particles re-adjust themselves after driving so that, in time, the effect of any remoulding disappears. Furthermore "the slow but sure compacting of underlying clay beds" is a characteristic of this type of soil and furnished a more logical reason for settlement than the remoulding theory.

Although insistence should come from engineers that more thorough soil investigations be made than is customary, it should go further than that. So long as contracts for borings or open-pit explorations are let through competitive bidding to the low bidder, with little regard for his knowledge and experience, so long will much of the foundation work be based on inexact information.

Occasionally, the contractor's superintendent or foreman in charge of such preliminary investigation is competent to secure representative samples and to classify soils; but many times this is apparently not the case, judging from results. It has been the writer's experience, so many times, to work with boring records that subsequent construction has proved to be inaccurate that he looks with suspicion upon any records of soil exploration unless he is sure that competent men have been in charge. Results indicate that too much care can scarcely be taken to secure sufficient and accurate preliminary information.

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Paper No. 2005

MEASUREMENT OF DÉBRIS-LADEN STREAM FLOW WITH CRITICAL-DEPTH FLUMES

By H. G. Wilm,[1] Esq., John S. Cotton,[2] Esq., and
H. C. Storey,[3] Esq.

With Discussion by Messrs. R. L. Parshall, Martin A. Mason, Edwin S. Fuller, Harold K. Palmer and Fred D. Bowlus, Harry F. Blaney, R. L. Stoker, J. C. Stevens and H. G. Wilm, John S. Cotton, and H. C. Storey.

Synopsis

Heavy burdens of erosion débris in streams of the San Dimas Experimental Forest, in California, have been found to cause substantial errors in measurements of discharge at some of the gaging stations. Stream-flow records are kept at a number of points throughout the Experimental Forest. Furthermore, there are two sets of triplicate water-sheds, each shed having an area somewhat less than 100 acres, immediately below each of which is located a Parshall flume, a V-notch weir, a small reservoir, and a flat-crested weir over the dam forming the reservoir. The flow from these water-sheds is first measured through either the Parshall flumes or V-notch weirs, according to the volume of flow. Its débris content can then be measured in the reservoir, and the clear water can be measured as it passes over the flat-crested weir of the dam.

However, there are numerous other points on the water-sheds below which the entire dependence is placed on the Parshall flumes or V-notch weirs for the determination of stream flow. Due to the presence of large quantities of eroded material in all heavy flows, the results at these points have been somewhat erratic. This was due largely to deposition of débris on the level approach to the throats of the Parshall flumes during the falling stages of the stream flow, and to the filling of the weir boxes with silt which soon put the V-notch weirs "out of commission."

The experiments described in this paper were conducted with the consultation and under the supervision of E. W. Kramer, M. Am. Soc. C. E., as Regional Engineer (California Region, United States Forest Service) now (1937) Regional Director (Federal Power Commission, San Francisco, Calif.) and

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Note.—Published in September, 1937, *Proceedings*.

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under the authority of Professor E. I. Kotok, Director, and C. J. Kraebel, Project Leader, of the California Forest and Range Experiment Station (United States Forest Service). The objective was to improve the present gaging stations so that they would yield reasonably accurate results with bed-load material present in the stream measured. Included in the study were also tests of the Parshall flume as constructed in the Experimental Forest, to check its accuracy with clear and loaded flows.

Introduction

The California Forest and Range Experiment Station has instituted an intensive program of investigations for the purpose of solving problems concerning the inter-relationship of mountain vegetation and water, with the objective of developing methods of water-shed management which should provide a maximum yield of usable water with a minimum of soil erosion. These studies are largely concentrated in the San Dimas Experimental Forest, a chaparral-covered mountain area of 17 000 acres in the San Gabriel Mountains, approximately 30 miles east of Los Angeles, Calif.

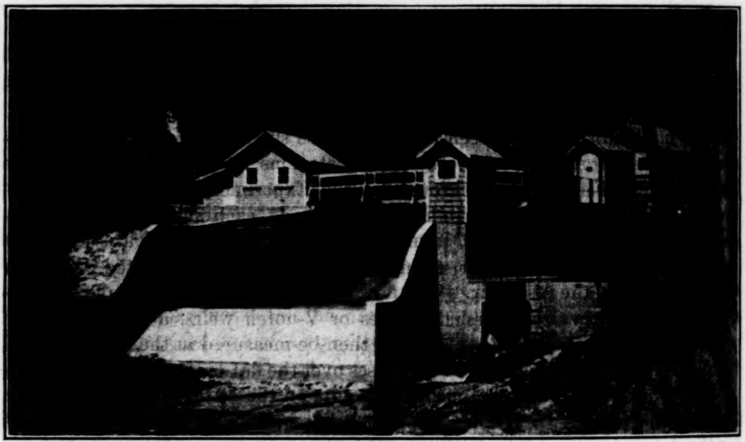

Fig. 1.—Large Gaging Station, San Dimas Experimental Forest; Parshall Flume and 90-Degree V-Notch Weir

In connection with studies of other phases of the problems, analyses have been made of rainfall-run-off ratios in seventeen drainage basins of various areas, from 0.06 sq mile to 14.1 sq miles.

Rainfall averages for these water-sheds, obtained by means of a system of nearly 400 standard rain gages and 15 intensity gages, are known to be accurate within 5 to 10% for the large areas, and from 3 to 5% for the small, intensively studied water-sheds. To date, however, stream-flow records have been of greatly inferior accuracy, due to the presence of quantities of eroded material in all storm peak flows. Deposition of this material results in erratic and undependable volume measurements in the gaging stations.

Sixteen of the seventeen gaging stations completed at present (1937) were constructed during the course of a single dry season from designs based on the best existing practice. (To the writers' knowledge, no control flume or weir had been designed previous to this time, which would operate as well as the Parshall flume in measuring silt-laden flows.) In each case a Parshall flume,[4] of throat width (3 ft to 30 ft), sufficient to handle the largest probable flows, was built to operate in conjunction with a 90° V-notch weir, intended to measure small continuous flows. Measurement of water stages in each is made by a continuous time-stage recorder.

In the design of Parshall flumes for the Experimental Forest, several minor modifications had been introduced, in order to increase their capacity and improve their efficiency in measuring silt-laden flows. The rising floor section down stream from the throat (see Fig. 1) was replaced by a downward sloping floor, the converging sections were lengthened, and the walls were increased in height.

Storm flows during the first rainy season following construction soon demonstrated the inadequacy of this type of control in measuring flows containing a substantial bed load. Even small post-storm streams carried enough sand and gravel to fill weir boxes quickly, and to build up a deposit on the flume floors so that they no longer functioned as critical depth meters. It is believed that the flumes should still measure large flows with reasonable accuracy, as the effect of bed load would constitute a much smaller percentage of the total flow; but as the stations stand at present they fail to give a satisfactory measurement of small and intermediate débris-laden flows, which form the run-off resulting from most storms.

Below the gaging stations, at each of the smallest, intensively studied water-sheds, is a cement-lined débris reservoir. The water depths measured in this basin and over the broad-crested weir spillway afford an efficient check on stream-flow measurement, in some cases furnishing the only reliable data on flow from the water-sheds studied.

PURPOSE OF THE INVESTIGATION

At the request of the Regional Engineer, the experiments described in this paper were set up in March, 1935, with two major objectives: (1) To test the effect of construction modifications on the rating of Parshall flumes in the San Dimas area; and, (2) to adapt the present stations to measurement of flows of all sizes, either by improving the débris-transporting characteristics of the Parshall flume, or, failing this, by attempting to develop a control flume that would function satisfactorily under a wide range of conditions and might be used in conjunction with the present stations to measure flows of intermediate volume.

EXPERIMENTAL EQUIPMENT

Through the courtesy of the San Dimas Water Company and the Los Angeles County Flood Control District, water from the San Dimas (flood-

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[4] "The Improved Venturi Flume," by R. L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 841.

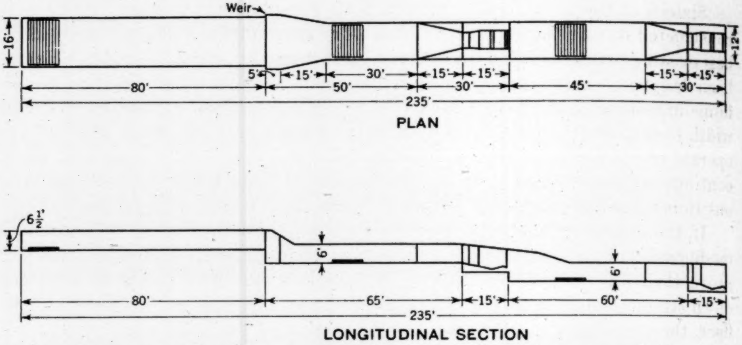

Fig. 2.—Plan and Section of San Dimas Experimental Flume, Showing Position of Sharp-Edged Control Weir, Experimental Flumes, and Raft Baffles

control) Reservoir was made available for experimental purposes, in any quantities desired up to 250 cu ft per sec.

Below the outlet gates in this dam was constructed a water-tight timber flume, lined with tar-paper, 16 ft wide, 6.5 ft deep, and 80 ft long, beyond the end of a short concrete channel leading from the sluice-gates. At the end of this section of flume was installed a 16-ft sharp-edged, steel suppressed weir, set level with a maximum variation of ± 0.001 ft. By the installation of additional bulkheads and short vertical sharp edges, this weir could be shortened to form a contracted weir of any desired length.

Beyond the weir the structure was continued, narrowing to a width of 12 ft, for 155 ft. At the weir, the floor of this section is 4.5 ft below that of the

Fig. 3.—General View of Flume as Seen from Top of San Dimas Dam

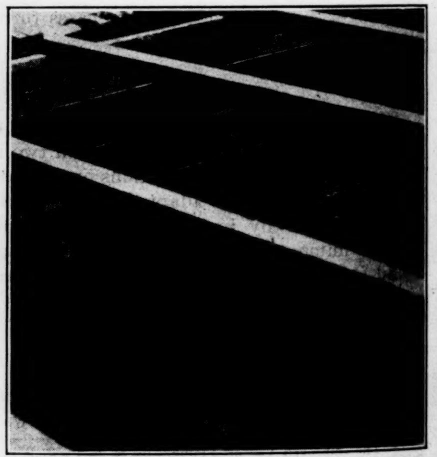

Fig. 4.—Wall of 5-Foot Parshall Flume, Showing Arrangement of Metal Strips Used in Plotting Water Surface Curves

weir channel; and a second drop of 6.8 ft occurs in the middle of the lower structure. Experimental flumes were installed here and at the down-stream end of the structure. Figs. 2 and 3 show the design and a general view of the entire installation.

Water depths over the weir and at both experimental controls were measured in 1-ft iron stilling-wells, by means of milliammeter point gages designed by the Staff of the Experiment Station, and accurate within ± 0.001 ft. In addition, water-surface curves were plotted by the use of metal strips tacked to the walls of the flumes being tested, at horizontal intervals of 1 ft and at vertical intervals of 0.1 ft (see Fig. 4).

CONTROL FLUMES STUDIED

In every test made, at least two flumes were rated simultaneously at the two stations constructed in the large test flume. Tests made with various experimental structures in the San Dimas test flume, listed in chronological order, and noting the length of weir used as a control for each test, are as follows:

I.—Tests of Modified Parshall Flume.—

(*A*) Five-foot flumes (rating with clear water), with 16-ft suppressed weir as control, using flows from 14.08 cu ft per sec to 234.5 cu ft per sec: (1) Same design as that adopted in the Experimental Forest gaging stations, with level floor in converging section; and (2) same design as that adopted in Experimental Forest gaging stations, except that the floor of the converging section was given a 5% slope.

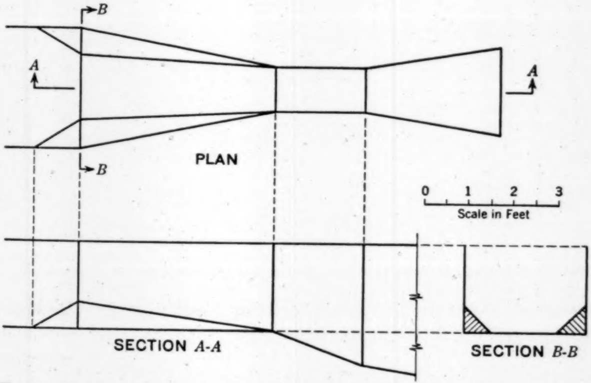

FIG. 5.—PARSHALL DESIGN, 45-DEGREE FILLETS IN A 1-FOOT PARSHALL FLUME

(*B*) One-foot flumes with 3-ft contracted weir as control, using flows from 0.11 cu ft per sec to 16.0 cu ft per sec: (1) Rating with clear water: (*a*) Level floor in converging section; 45° wing-walls extended from up-stream end of flume to walls of 12-ft channel; (*b*) floor of converging section on 5% slope; (*c*) floor of converging section on 3% slope; (*d*) level approach floor, with fillets installed on this floor from the throat back to the up-stream end of the flume, and set at an angle of 45° from the floor to the converging side walls (Section *B–B*, Fig. 5). This fillet design was suggested by Mr. Parshall[5] for the purpose of improving the efficiency with silt-laden flows; and (*e*) level floor in converging section, wing-walls extended parallel to each other from the up-stream end of the flume for 10 ft, instead of diverging as 45° wing-walls. (2) Rating with débris-laden flows: Same installations as in Item *B*–1.

(*C*) Six-inch flume, with a 3-ft contracted weir as control, using flows from 0.20 cu ft per sec to 2.71 cu ft per sec. The purpose of this test was to ascertain the effect on the rating formula of entrance conditions radically different from standard. This flume was installed in the side of the main structure below the lower 1-ft flume, and with its axis at right angles to that of the main structure. The result was considerable turbulence and an abrupt change in flow direction.

(*D*) Three-foot flumes with an 8-ft contracted weir as control, using flows from 0.62 cu ft per sec to 53.52 cu ft per sec: (1) Level floor, but with a series of low fillets along the floor. These fillets were triangular in cross-section, tapering in height from 4 in. at the up-stream end of the flume, to 1 in. at a point 1 ft up stream from the flume throat. Sloping ramps were constructed on the ends of each fillet.

II.—Experiments with Other Critical-Depth Flumes.—In the course of the studies, a new type of measuring flume was developed, which functions as a broad-crested weir in which water depths are measured at a point in the flume down stream of the section in which critical depths occur. In Fig. 6, for example, b_f denotes width of measuring flume; b_a = width of approach channel $= 3b_f$; h_a = height of structure in the approach channel $= \frac{\text{maximum flow}}{5\,b_f}$; L = length of the measuring flume $= r + 2h_a$; and r = radius of the transition $= b_f$. The object of this design is to provide for measurement of flow at critical or super-critical velocities, sufficient to transport débris rapidly through the flume and to keep its floor scoured clean. This flume differs in function, design,

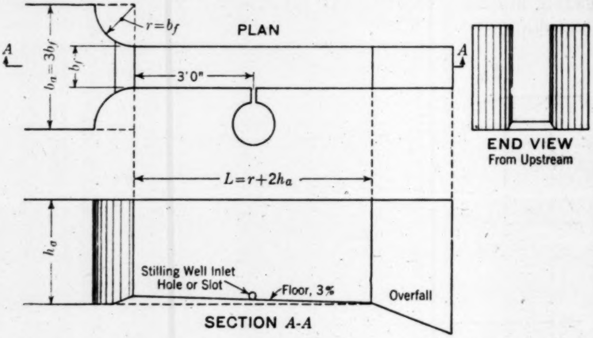

FIG. 6.—GENERAL DESIGN FOR SAN DIMAS RECTANGULAR CRITICAL-DEPTH FLUME

and point of depth measurement from the Venturi flume described by H. K. Palmer and F. D. Bowlus, Members, Am. Soc. C. E.[6] The meter described by Messrs. Palmer and Bowlus would not function satisfactorily for measuring flows containing bed-load, as measurements made up stream of the "critical" section must necessarily be affected by changes in approach velocity, caused by deposition of transported material in the back-water zone above the flume's entrance transition. Because of the necessity of measuring loaded flows, no conventional broad-crested weir could be used, as its contraction would soon become obliterated by accumulated débris. Therefore, a standard flume transition was constructed in the new flume to replace the bottom contraction.

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[5] Memorandum concerning the effect of fillets placed against the walls of the converging section of a 1-ft Parshall measuring flume, by Ralph L. Parshall, U. S. Bureau of Agricultural Engineering, Fort Collins, Colo., March, 1935.

The transition is the cylinder quadrant (see Fig. 7), described[7] by Fred C. Scobey, M. Am. Soc. C. E., in 1933, with a radius of curvature equal to the width of the flume. This type of transition was adopted because of its simplicity and relatively short length; and because, according to Mr. Scobey's experiments, the head loss through it should, in most cases, be no greater than in more complex transitions, such as the reversed curve described[8] by Julian Hinds, M. Am. Soc. C. E., in 1928.

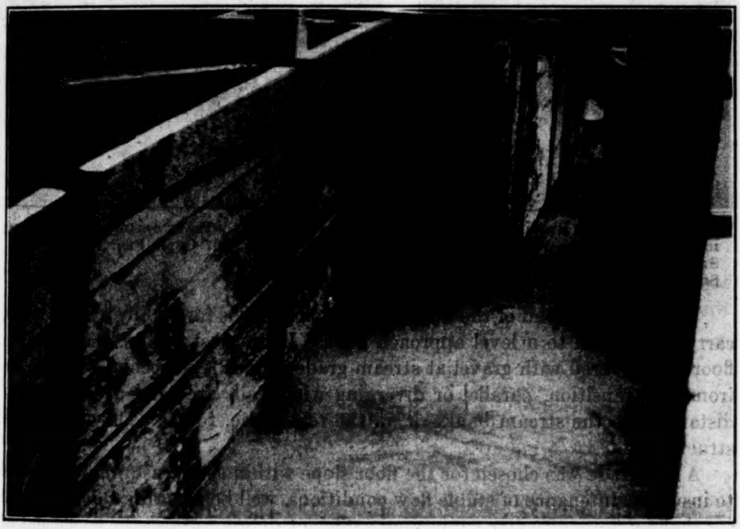

FIG. 7.—VIEW FACING DOWN STREAM; 1-FOOT RECTANGULAR, CRITICAL-DEPTH FLUME; FLOW, 2.71 CUBIC FEET PER SECOND

Below this transition the measuring flumes tested were either rectangular, as shown in Fig. 8, or trapezoidal in cross-section, with a floor laid on a 3% grade. The trapezoidal flumes had a wall slope of 1 on 0.25 (see Fig. 9), the object of this cross-section having been to increase capacity while retaining accuracy for low flows, and to attempt to hold critical depths at points close to the "crest," or to the down-stream end of the entrance transition.

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[6] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1195.

[7] "The Flow of Water in Flumes," by Fred C. Scobey, *Technical Bulletin No. 393*, U. S. Dept. of Agriculture, December, 1933.

[8] "The Hydraulic Design of Flume and Siphon Transition," by Julian Hinds, *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 1435–1439.

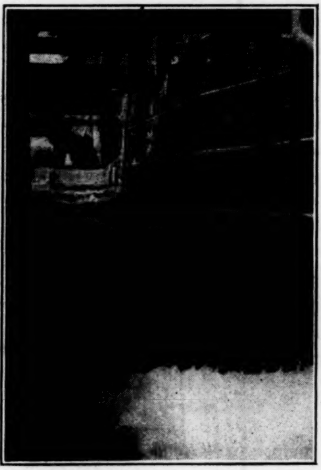

FIG. 8.—THREE-FOOT RECTANGULAR CRITICAL-DEPTH FLUME; VIEW FACING UP STREAM; FLOW, 12.68 CUBIC FEET PER SECOND

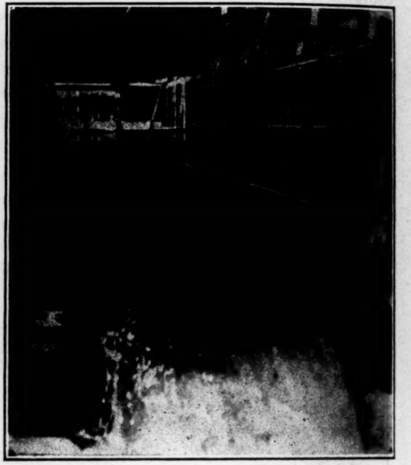

FIG. 9.—TWO-FOOT TRAPEZOIDAL CRITICAL-DEPTH FLUME; FLOW, 22.97 CUBIC FEET PER SECOND

The up-stream end of the 3% floor in each flume is formed by a short ramp, carrying over on to a level approach floor. In the test flumes, the approach floor was covered with gravel at stream grade (about 3 per cent). Up stream from the transition, parallel or diverging wing-walls are carried back a short distance into the stream banks or, in the test flume, to the walls of the main structure.

A 3% grade was chosen for the floor slope within the measuring flume so as to insure maintenance of stable flow conditions, well below critical depths, even for shallow flows of water containing sufficient débris to raise the coefficient of hydraulic friction of the flume as high as $n = 0.025$. Although this allowance may seem liberal, it was considered necessary because of present inadequate knowledge of the quantitative influences of transported material on flow characteristics in flumes. The only disadvantageous effect of such a floor slope lies in its increasing velocities which tend to reduce the sensitivity of the instrument to small changes in flow. This effect may be minimized, however, by locating the stilling-well inlet reasonably close to the section in which critical depths occur. Furthermore, any such reduction in depth variation per unit of volume variation is largely compensated in practice by the facts that any error in measurement of head causes only approximately 1.3 times that error in computed volume; and that velocity of approach has no effect, theoretically, upon the flume's rating.

Preliminary ratings have been made for three sizes of flume in both the rectangular and trapezoidal designs: 1, 2, and 3-ft widths in the former, and

0.5, 1, and 2-ft bottom widths in the latter. The 1-ft flumes were rated by 3-ft and 8-ft weirs; the 8-ft weir was used also for the larger widths of the flume. A 1-ft rectangular flume with a level floor was also tested, in order to verify the similarity in discharge behavior between this type of control flume and a broad-crested weir.

Experimental Procedure

In each test run, gate-openings in the dam were controlled by a staff member of the Los Angeles County Flood Control District, who was in continuous communication with a co-ordinator at the test-flume structure. To start the run, the gate was opened enough to fill all the basins in the test-flume structure, and then lowered to a small fixed opening. When the flow had reached stability, as indicated by constant point-gage readings at the lowest experimental flume, a signal was given and six to ten depth readings were made by separate observers at each point-gage, noting times of reading to the nearest $\frac{1}{4}$ min. Other observers plotted the water-surface curves in each flume being rated, and recorded the water depths in the channels up stream. Staff gage readings were also taken at the weir as a check on the point-gage, and current-meter velocities for the larger flows were measured to check the weir rating. After the completion of point-gage readings for each stabilized flow, the gate-opening was increased and the entire operation repeated until enough different volumes had been measured to insure the procurement of reliable rating curves. For the 6-in. Parshall flume only seven points could be taken; but for each of the others ten to fifteen separate volumes were measured.

Complete stability of flow through the gate was insured by the large head of water available behind the dam. Formulas used for the control weirs were:

For the 16-ft weir (Bazin's formula):

$$Q = \left(0.405 + \frac{0.00984}{H}\right)\left(1 + 0.55\left[\frac{H}{Z+H}\right]^2\right) L\,H\,\sqrt{2gH} \;\ldots\ldots (1)$$

and, for 8-ft and 3-ft contracted weirs:

$$Q = 3.33\,(L - 0.2H)\,H^{1.5} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which Z = the height of the crest above the bottom of the channel. Although no volumetric or other completely reliable rating could be made of these weirs, their accuracy is believed to be well within limits required for the study (± 2% to 3%), as demonstrated by their close correspondence with volumes computed from current-meter measurements, and by the agreement of observed discharges with points determined by formula.

In testing the behavior of débris-laden flows in the small Parshall and rectangular San Dimas flumes, a stream-bed mixture of gravel and sand was dumped as rapidly and continuously as possible by six men with two wheelbarrows into the channels just up stream from the flumes tested. In each case, water depths in the flumes were measured first with a stabilized flow of clear water; then, with the same rate of flow, débris was added as rapidly as possible, and the depths were remeasured.

Later, in testing the trapezoidal San Dimas flumes, bed-load material was added from large bunkers constructed above the flume structure just up stream from each flume. It was found difficult, however, to feed the débris uniformly from the bunkers, because the material became packed.

In computing results, all point-gage readings for each stabilized flow were averaged and checked with the corresponding staff-gage reading or the water-surface curve, and rating curves and formulas were obtained from the resulting average points. Variation of individual readings from their average seldom exceeded 0.01 ft.

Computations were made of velocities and energy gradients within each flume, in order to determine energy losses attributable to the transition and to friction and turbulence within the flume. The results were not satisfactory, probably owing to centrifugal effects in curved flow. However, as nearly as could be determined, losses through the transition and flume appear to be small.

All calculations were made with a 50-in. slide-rule and an electric calculator. Computed discharges obtained by "least squares" were checked against a straight line drawn through logarithmically plotted, observed discharges.

RESULTS

Rating curves and formulas were computed for the flumes tested, and compared with Parshall's ratings of his flumes. The significant results of the experiment are shown in Tables 1 to 3, and summarized as follows:

Modified Parshall Flumes with Level Floor in Converging Section.—

(*a*) The logarithms of all rating points fit a straight line closely, with deviations which exceed 5% only in the 6-in. flume.

(*b*) For a 6-in. flume,

$$Q = 2.13\, H_a^{1.47} \qquad (3)$$

Increasing the length of the entrance section of the Parshall flume and depressing the floor of the outlet section have little effect, if any, on the discharge characteristics of the flume.

(*c*) Flow in Parshall flumes, at depths as great as twice those originally rated, conformed with no increased error to computed rating formulas.

Alterations of the Parshall Flume.—The rating with clear water may be described as follows:

(1) A 3% to 5% slope in the floor of the converging section resulted in shooting flow at low stages. As depths were increased a hydraulic jump passed through the flume. All larger quantities flowed at depths greater than the critical, giving smooth rating curves with discharges for all heads considerably higher than standard.

(2) Introduction of fillets of either design resulted in discharges somewhat in excess of standard ratings. In the 3-ft flume with multiple fillets, low flows occurred at shooting velocities, and at all stages a slight drop was perceptible at the up-stream end of the fillets.

Effect of Débris on the Altered Flumes.—Excellent transportation of bed-load resulted as long as shooting velocities occurred. At higher stages, however, with normally slow velocities, débris movement was greatly reduced. The introduction of fillets somewhat improved the transportation of fine material, but failed to be effective for bed-load ranging above fine gravel in coarseness.

When bed-load was present in the flow, observed discharges were erratically in excess of normal values for the flumes at all stages tested (see Table 1).

TABLE 1.—EFFECT OF DÉBRIS-LADEN FLOWS UPON RATING OF MEASURING FLUMES

Reading No.	Observed head, at still well inlet, in feet	RATE OF FLOW, Q, IN CUBIC FEET PER SECOND: Observed*	Computed*	Deviation (Column (2) − Column (3))	Percentage deviation	Reading No.	Observed head, at still well inlet, in feet	RATE OF FLOW, Q, IN CUBIC FEET PER SECOND: Observed*	Computed*	Deviation (Column (2) − Column (3))	Percentage deviation
	(1)	(2)	(3)	(4)	(5)		(1)	(2)	(3)	(4)	(5)
a) ONE-FOOT PARSHALL FLUME, 3% FLOOR, IN CONVERGING SECTION						(*c*) THREE-FOOT PARSHALL FLUME WITH MULTIPLE FILLETS					
10	0.646	2.75	2.45	+0.30	+12.2	15*a*	0.966	13.73	11.85	+1.88	+15.9
7	0.885	3.83	3.94	−0.11	− 2.8	14*a*	1.131	16.11	15.15	+0.96	+ 6.3
8	1.295	12.10	6.67	+5.43	+81	13*a*	1.388	21.33	20.75	+0.58	+ 2.8
9	1.456	9.76	7.83	+1.93	+25						
(*b*) ONE-FOOT SAN DIMAS CRITICAL-DEPTH FLUME, 3% FLOOR						(*d*) THREE-FOOT SAN DIMAS CRITICAL-DEPTH FLUME, 3% GRADE					
17	0.693	3.89	3.90	−0.01	− 0.3	15*a*	0.683	13.73	13.80	−0.07	− 0.5
19	1.341	9.37	9.37	0.00	0.0†	14*a*	0.780	16.11	16.32	−0.21	− 1.3
20	1.645	12.11	12.31	−0.20	− 1.6†	13*a*	0.968	21.33	21.41	−0.08	− 0.4

* Observed head is for flows containing bed-load material; observed discharge is based on control-weir measurements; and computed discharge is based on San Dimas ratings for clear water. † Channel above flume built up to 25% with bed-load material.

These findings support field observations made at Experimental Forest gaging stations, where it was found that bed-loads are transported through Parshall flumes only at high discharges. Low flows carrying loads cause deposits of sand and gravel on the approach floors of all these flumes, and prohibit the use of notch weirs by similar deposition. These facts result in serious errors and gaps in flow records.

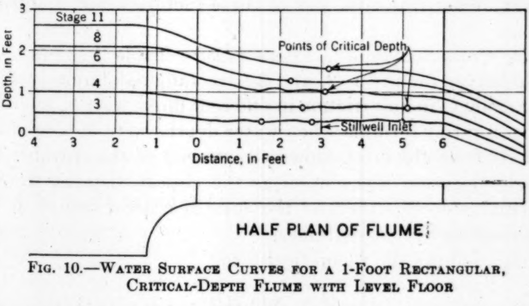

FIG. 10.—WATER SURFACE CURVES FOR A 1-FOOT RECTANGULAR, CRITICAL-DEPTH FLUME WITH LEVEL FLOOR

High-Velocity Flumes (San Dimas Design): Rectangular Cross-Section, Level Flume Floor.—Surface waves occurred in this flume (see Fig. 10), because a level floor does not supply gravitational energy to replace that lost in friction; therefore, the depths fluctuated at about the critical, within the flume. The resulting points lie on either side of a curve represented by the formula,

$$Q = \sqrt{g}\, H^{\frac{3}{2}} \qquad (4)$$

which is the theoretical formula for a broad-crested weir. The rating is unsatisfactory, as deviations of individual points from the curve are as great as 15% (see Fig. 11). Furthermore, addition of débris immediately caused the flow to jump above the critical depth down stream of the transition.

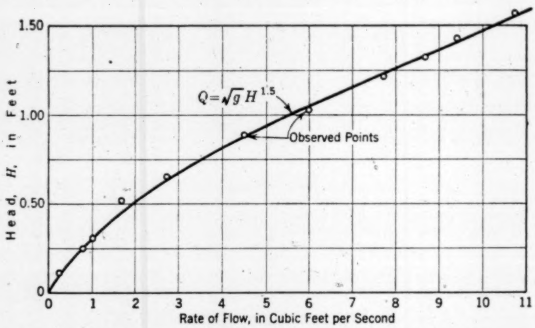

FIG. 11.—RATING CURVE FOR A 1-FOOT RECTANGULAR, CRITICAL-DEPTH FLUME WITH LEVEL FLOOR

As shown in Fig. 10, critical depths lay at points near the middle of the flume, where relatively parallel flow occurred. For the measurement of clear flows it might be feasible to use a flume similar to the San Dimas design, but with its floor set at a critical slope, so that parallel flow would be established at the critical depth and a rational formula might be applied. However, flow in such a flume would be very sensitive to small changes in floor friction or to the addition of silt or detritus, and any of these factors would result in unstable flow conditions.

Rectangular Cross-Section, 3% Flume Floor.—Table 2 shows observed and computed discharges for the 1, 2, and 3-ft rectangular flumes. With a small correction for changes in unit friction in different flume widths, these flumes gave the same general rating formula when water depths were measured at the same distance (3.0 ft) from the crest (down-stream end of the entrance transition). This piezometer location was chosen as the closest distance from the crest, which still remains down stream of the zone of critical depths at the highest water-stages measured in these flumes.

This formula follows the theoretical form,

$$Q = \sqrt{g}\, C B H^{\frac{3}{2}} \qquad (5)$$

in which C = an empirical constant to correct for the effects of floor slope and of energy losses in the flume. Based on present tentative ratings for flumes in

TABLE 2.—COMPARISON OF COMPUTED WITH OBSERVED DISCHARGES IN 1, 2, AND 3-FOOT RECTANGULAR SAN DIMAS FLUMES

Reading No.	Observed head, at still well inlet, in feet	RATE OF FLOW, Q, IN CUBIC FEET PER SECOND			Percentage deviation
		Observed	Computed	Deviation (Column (2) − Column (3))	
	(1)	(2)	(3)	(4)	(5)
	(a) ONE-FOOT FLUME: (EQUATION (7*a*))				
2	0.175	0.67	0.64	−0.03	−4.48
3	0.245	0.96	0.99	+0.03	+3.15
4	0.533	2.71	2.76	+0.05	+1.85
5	0.773	4.54	4.52	−0.02	−0.44
6	0.955	5.98	5.98	0.00	0.00
7	1.154	7.66	7.68	+0.02	+0.26
8	1.260	8.61	8.62	+0.01	+0.12
9	1.346	9.39	9.40	+0.01	+0.11
10	1.480	10.74	10.66	−0.08	−0.74
11	1.599	11.85	11.81	−0.04	−0.34
	(b) TWO-FOOT FLUME: (EQUATION (7*b*))				
3	0.188	1.53	1.55	+0.02	+1.31
2	0.438	4.54	4.55	+0.01	+0.22
4	0.759	9.27	9.17	−0.10	+1.08
	(b) TWO-FOOT FLUME: (EQUATION (7*b*)) (*Continued*)				
5	0.925	11.80	11.82	+0.02	+0.17
6	1.242	17.28	17.20	−0.08	−0.46
7	1.533	22.38	22.51	+0.13	+0.58
8	1.741	26.20	26.49	+0.29	+1.11
9	2.262	36.49	37.00	+0.51	+1.40
	(c) THREE-FOOT FLUME: (EQUATION (7*c*))				
3	0.131	1.53	1.57	+0.04	+2.61
2	0.315	4.54	4.72	+0.18	+3.96
1	0.523	8.94	8.88	−0.06	−0.67
4	0.541	9.27	9.25	−0.02	−0.22
5	0.660	11.80	11.86	+0.06	+0.51
6	0.891	17.28	17.23	−0.05	−0.29
7	1.105	22.38	22.53	+0.15	+0.67
8	1.256	26.20	26.43	+0.23	+0.88
9	1.638	36.49	36.80	+0.31	+0.85
10	2.145	51.43	51.44	+0.01	+0.02

1-ft to 3-ft widths, the general formula is as follows:

$$Q = \frac{1.120\ B^{0.040}}{H^n}\ BH^{1.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which $n = 0.179\ B^{0.320}$. Supplying the value for B in 1, 2 and 3-ft flume widths, simplified expressions of Equation (5) are:

For the 1-ft flume:

$$Q = 6.35H^{1.321} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7a)$$

for the 2-ft flume:

$$Q = 13.05H^{1.277} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7b)$$

and, for the 3-ft flume:

$$Q = 19.90H^{1.245} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7c)$$

The corresponding rating curves are plotted in Fig. 12.

On the basis of computed deviations based on stilling-well readings, it is considered conservative to believe that this type of control flume should be at least as accurate for field measurements, especially of débris-laden flows, as any standard measuring device now in use. However, final conclusions as to its accuracy and the applicability of the general formula must be reserved, pending final rating of the flume in smaller and larger sizes.

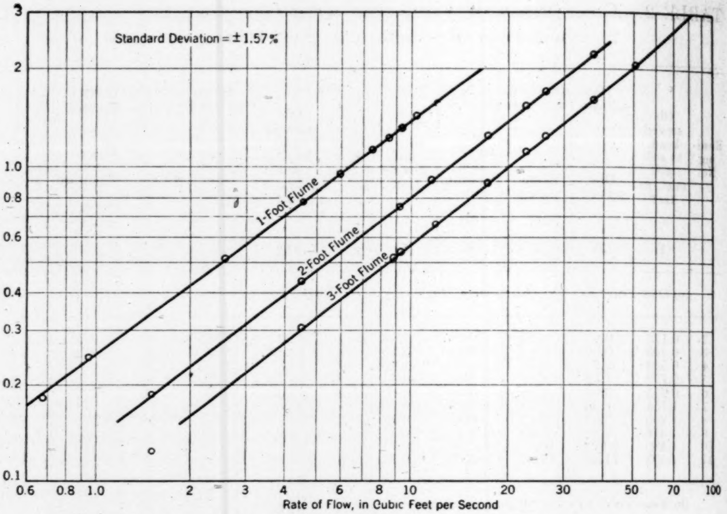

FIG. 12.—RATING CURVES FOR RECTANGULAR, CRITICAL-DEPTH FLUMES

Computed critical depths lie on the curved water surfaces (see Fig. 13) down stream of the transition. A line drawn through these depths is relatively straight, but lies at an angle of approximately 30° from the vertical. These results are similar to the findings of J. G. Woodburn,[9] Assoc. M. Am. Soc. C. E., when he tested broad-crested weirs with a crest slope of 2.6 per cent. In the San

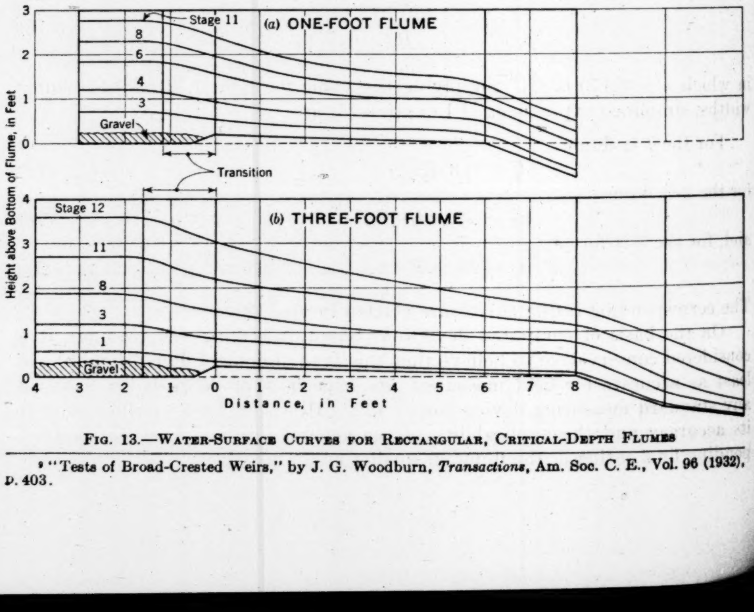

FIG. 13.—WATER-SURFACE CURVES FOR RECTANGULAR, CRITICAL-DEPTH FLUMES

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[9] "Tests of Broad-Crested Weirs," by J. G. Woodburn, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 403.

Dimas flume, true critical depths, as indicated by the inversion of water-surface curvature (Fig. 13), actually lie closer than computed to a line vertically above the down-stream end of the entrance transition. These locations were roughly verified by computation of the true critical depths, attempting to correct approximately for centrifugal action in curved flow.

The remaining slope from the vertical of a line through true critical depths may be due to shortness of the intake transition. It is believed that critical depths would remain closer to the "crest" if the transition were more gradual. This belief is borne out by the locations of critical depths in the Parshall flume, and on broad-crested weirs with gently sloping ramp approaches, as tested by A. R. Webb, M. Am. Soc. C. E.[10]

The addition of as much débris as could be fed continuously to these flumes had very slight effect on their ratings (see Table 1). All grades of material from sand to angular stones as large as 10 in. in diameter passed through rapidly, with no deposition on the floor. A 4-ft San Dimas flume has been in operation during the rainy seasons of 1935–36 and 1936–37 in a stream in the Experimental Forest which carries bed-loads estimated at quantities up to 10% of the flow. This flume has remained clean of débris deposits and has shown uniform flow characteristics throughout this period. A 2-ft flume, similarly installed, has given excellent records for one season, in a stream carrying heavy loads of material eroded from road fills.

Flow in the experimental flumes was not affected by velocity of approach or by back-water causing a jump within the flume, as long as the jump remained down stream of the stilling-well inlet. In one test, the stream bed up stream of the flume was built up with gravel to a 25% grade. Although there were shooting velocities in this channel section, a jump occurred above the transition, a hydraulic drop within the cylindrical entrance, and measured heads in the flume were identical with those recorded under standard conditions. A hydraulic jump caused by obstacles held in the flume exit also had no effect on depths at the stilling-well inlet.

Trapezoidal Cross-Section, 3% Flume Floor.—These flumes, tested in bottom widths of 0.5, 1.0, and 2.0 ft, exhibited characteristics similar to those of the rectangular flume, in that flow within the flumes remained well below critical depths at all stages measured, and the addition of bed-load apparently had little or no effect upon their rating. However, unstable flow conditions, expressed in surface waves, persisted in this shape of flume at all stages and in all three flume widths. The result was undesirable variation of observed flow from computed flow values. In Table 3, the equations for the 6-in., 1-ft, and 2-ft flumes were:

For the 6-in. flume,

$$Q = 4.40H^{1.53} \qquad (8a)$$

for the 1-ft flume,

$$Q = 9.60H^{1.45} \qquad (8b)$$

and, for the 2-ft flume,

$$Q = 15.10H^{1.45} \qquad (8c)$$

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[10] "Supplemental Tests—Weirs with Aprons Inclined Up Stream and Down Stream," by A. R. Webb, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 408–416.

These fluctuations may have been caused by over-abruptness of the conical entrance transition.

Critical depths deviated in this type of flume just as far from the vertical as in the rectangular flume.

TABLE 3.—COMPARISON OF COMPUTED WITH OBSERVED DISCHARGES FOR 6-INCH, 1-FOOT, AND 2-FOOT CRITICAL-DEPTH FLUMES: TRAPEZOIDAL CROSS-SECTION

Reading No.	Observed head, at still well inlet, in feet	RATE OF FLOW, Q, IN CUBIC FEET PER SECOND			Percentage deviation
		Observed	Computed	Deviation (Column (2) − Column (3))	
	(1)	(2)	(3)	(4)	(5)
	(*a*) SIX-INCH FLUME: (EQUATION (8*a*))				
3	0.326	0.802	0.792	−0.010	−1.2
2	0.481	1.389	1.434	+0.045	+3.2
1	0.932	3.918	3.951	+0.033	+0.8
4	1.509	8.488	8.272	−0.216	−2.5
	(*b*) ONE-FOOT FLUME: (EQUATION (8*b*))				
3	0.172	0.802	0.749	−0.05	−6.6
2	0.266	1.389	1.411	+0.02	+1.4
1	0.559	3.918	4.137	+0.22	+5.6
4	0.947	8.488	8.880	+0.39	+4.6
5	1.041	9.86	10.18	+0.32	+3.2
6	1.112	10.98	11.19	+0.21	+1.9
7	1.269	13.73	13.56	−0.17	−1.2
8	1.464	17.24	16.68	−0.56	−3.2
	(*b*) ONE-FOOT FLUME: (EQUATION (8*b*)) (*Continued*)				
9	1.777	22.97	22.10	−0.87	−3.8
10	2.177	31.26	29.60	−1.66	−5.3
	(*c*) TWO-FOOT FLUME: (EQUATION (8*c*))				
3	0.125	0.802	0.740	−0.06	−7.5
2	0.192	1.389	1.374	−0.02	−1.4
1	0.390	3.918	3.850	−0.07	−1.8
12	0.480	5.290	5.210	−0.08	−1.5
4	0.682	8.488	8.668	+0.18	+2.1
5	0.762	9.86	10.18	+0.32	+3.2
6	0.816	10.98	11.25	+0.27	+2.5
7	0.944	13.73	13.89	+0.16	+1.2
8	1.094	17.24	17.20	−0.04	−0.2
9	1.324	22.97	22.70	−0.27	−1.2
10	1.631	31.26	30.70	−0.56	−1.8
11	2.490	58.09	56.70	−1.39	−2.4

SUMMARY OF CONCLUSIONS

Conclusions drawn from these experiments may be summarized briefly as follows:

(1) Construction modifications used in Parshall flumes on the San Dimas area have a negligible effect on their rating. The results indicate that the Parshall flume gives accurate measurements of clear water or flows containing loads of fine material, and is reasonably accurate for measuring large flows with bed-load.

(2) Presence of bed-load substantially influences the rating of Parshall flumes at low stages, although the resulting errors at high flows appear to be small.

(3) No alterations introduced into the Parshall flume succeeded in materially improving its accuracy in measuring small and intermediate flows containing bed-load.

(4) A new design of critical-depth flume was developed, which appears to give accurate measurements of discharge and to be unaffected by velocity of

approach or presence of bed-load. This flume functions as a broad-crested weir in which a cylinder-quadrant transition is used in place of the conventional contraction. Depth measurements are made down stream of the "critical" section, where rapid flow occurs. A tentative rating has been established for three widths and a general formula computed.

(5) Use of this type of flume in conjunction with the present San Dimas stations should make possible the measurement, with reasonable accuracy, of small and intermediate loaded flows which at present fail to be measured because of débris deposition in weir boxes and in the Parshall flumes.

DISCUSSION

R. L. Parshall,[11] Assoc. M. Am. Soc. C. E. (by letter).—The authors have developed an interesting measuring device for the special purpose of measuring flood flow in steep channels where the streams carry heavy bed-loads and high percentages of suspended material.

The name applied to this new measuring flume, perhaps, is somewhat misleading. The flow passes through the section of critical depth at a point approximating the up-stream ends of the parallel walls of the structure. The section, or point of critical depth, varies with the rate of discharge and, as stated by the authors, is not at a definite place. The gage section, as chosen, is considerably down stream from the point of critical depth, and, therefore, the observed values of the H_a head are always less than the critical. It is probable that a better index point for determining the head would be at a point slightly up stream from the upper end of the parallel section where greater depths would exist. For larger values of the head, thus observed, the error in depth measurement then would be less effective in the final computed discharge. This point no doubt has been investigated.

The authors have studied the accuracy of the Parshall measuring flume for their special purposes and report that if the floor of the converging section is made level and the length of this section of the structure is increased, little or no effect will appear in the discharge characteristic of the flume. However, when the floor is sloped to 5% in the direction of flow, the indicated discharge is less than the actual flow. This would be an obvious result because the control is moved up stream from the throat to a point at or near the upper end of the converging section. Moreover, the depth at the standard H_a point would be less because of the higher velocity of the water passing this section. This alteration materially changes the characteristics of flow, and the law of discharge for the standard setting is not applicable, particularly for the shallower depths of flow.

Sufficient data are not presented by the authors in their discussion of results of testing the Parshall measuring flume, under modified settings, to permit complete analysis of their conclusions, "as shown in Tables 1 to 3." The writer finds only one table dealing with the modified flume where the floor in the converging section has been sloped downward. The discharge formula, Equation (3), as stated for the flow through the 6-in. flume, is apparently not for a standard setting. At the hydraulic laboratory at Fort Collins, Colo., this particular size of flume was calibrated where the law of discharge was found to be:

$$Q = 2.06\, H_a^{1.58} \quad \ldots\ldots (9)$$

based on twenty-three observations for discharges ranging from 0.05 to 2.42 cu ft per sec. In this series there were two tests in which the deviation approached 6 per cent. In one case the H_a head was only 0.093 ft, and for this shallow depth it is believed the control at the throat was ineffective; however, the actual difference between the observed and computed discharges was only 0.003 cu ft per sec. For practical purposes this difference is not significant. For the other test in question, the H_a head was 0.217 ft, with a difference in the observed and computed discharges of 0.01 cu ft per sec. Another test in this series with the H_a head at 0.204 ft indicated a difference of only 0.001 cu ft per sec. These slight variations in discharge, for low heads, are thought to be due to the approaching limit of control at the throat for shallow depths. For the remaining twenty tests the mean deviation between the observed and computed discharges was 0.8 per cent.

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[11] Senior Irrig. Engr., Bureau of Agricultural Eng., U. S. Dept. of Agriculture, Fort Collins, Colo.

An inspection of the data in Table 1 discloses that the 3% slope of the floor of the converging section, in the direction of flow, materially affects the indicated rate of discharge through the flume when based on a standard setting. For the very limited data presented concerning the four tests on the 1-ft Parshall measuring flume, having a sloping floor and carrying a débris-laden flow, the following relation has been found to exist: The gage point, H_a, is practically 3 ft up stream from the crest. At a 3% slope this point is 0.089 ft higher than the crest. When this increment of 0.089 is added to the observed H_a head the standard discharge for these corrected heads agrees reasonably well with the computed discharge shown in Table 1 as based on the San Dimas calibrations.

The development of a measuring device to meet extreme adverse conditions of heavy bed-load carried in the stream, has not yet been fully accomplished. The authors have intimated strongly that the Parshall measuring flume, when operated under the field conditions imposed on the experimental San Dimas Forest area, has failed to register the exact rate of flood-flow discharge. Experience in the field with weirs of various types, as well as the ordinary rating flume, has demonstrated that neither of these devices could be used successfully to measure flood flows such as were encountered in this experimental forest area. There were experimental data, both in the laboratory and in the field, in evidence prior to the recommendation of this type of measuring device for the San Dimas area, which indicated that the Parshall measuring flume did possess characteristics favorable in meeting such adverse conditions of measuring débris-laden stream flow. In the Fort Collins Laboratory numerous observations have been made to ascertain the effect of sand and gravel on the indicated rate of discharge where such débris was carried to, and passed through, the flume. No material effect was noted in the indicated rate of flow because of the fact that any bed-load brought by velocity of approach to the upper end of the structure was readily moved on through it, owing to the convergence of the flow and the increasing velocity through that part of the structure in which the control and measurement of the discharge was effected.

For a number of years, prior to 1937, at the east portal of the Laramie-Poudre Tunnel, 60 miles west of Fort Collins, a 6-ft Parshall flume was operated to measure the flow through this tunnel. The flume operated under adverse conditions as follows: First, the stream from the portal of this tunnel was abruptly turned 90° into a relatively short channel in which the measuring flume was placed; and, second, the excessive quantity of heavy material carried out of the tunnel was moved down and through the flume. Numerous current-meter check measurements had been made of the discharge by official hydrographers and others, which showed the rate of flow as indicated by the flume to be within reasonable approaches of accuracy. In Fig. 14 is shown the condition of flow through this 6-ft measuring flume on July 2, 1932. On July 10, a current meter gaging of the discharge was made of essentially the same flow and degree of gravel deposit, with the thread of the stream flowing along one side of the channel. For this gaging, the mean H_a gage height was found to be 1.18 ft, the computed discharge 31.25 cu ft per sec, and the metered discharge, 32.6 cu ft per sec, with a resulting deviation of about 4 per cent. Under such adverse conditions extreme accuracy cannot be expected.

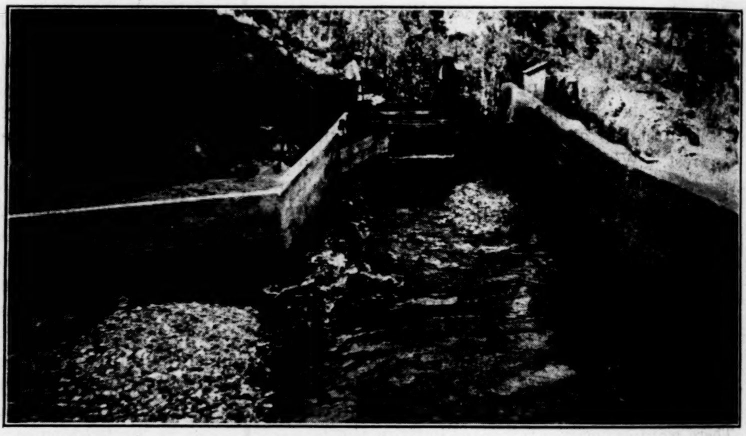

FIG. 14.—CONDITION OF FLOW THROUGH 6-FOOT PARSHALL MEASURING FLUME AT EAST PORTAL OF LARAMIE-POUDRE TUNNEL WHEN MEASURED AND COMPUTED DISCHARGES WERE IN REASONABLE AGREEMENT.

The Parshall measuring flume has been accepted throughout the irrigated Western States as a practical and reliable means of gaging the rate of flow of water diverted from streams and reservoirs under officially decreed rights. It is estimated that there are about 2 500 flumes in operation in Colorado at this time. This measuring flume is widespread in its use and is serving a useful purpose in this, and many foreign, countries for measuring both irrigation and power supplies, in sanitary engineering, and in industrial plants.

The first practical installation of this type of measuring device was a 10-ft framed structure built in the Las Animas Consolidated Ditch, near Las Animas, Colo., in 1926. The successful operation of this first flume resulted in the construction of other larger flumes in the Arkansas Valley in Colorado. These flumes were of reinforced concrete, and ranged in size from 15 to 40-ft throat widths. Prior to the construction of the 40-ft reinforced concrete flume in the Fort Lyon Canal, at La Junta, Colo., the discharge formula,

$$Q = 150\, H_a^{1.6} \qquad (10)$$

was set up for the purpose of designing this structure. After this flume was put into operation (November, 1928), current meter gagings of the discharge were made covering a period of several months. These gagings were made principally by the official State Hydrographer; however, some of the observations were made by the writer and others. This series of meter measurements, which consisted of twenty-three gagings, was found by comparison with the discharge, computed by Equation (10), to be as follows: Twelve indicated a plus deviation and eleven, a minus deviation; for nine of these comparisons, the difference was less than 1% and the maximum deviation for any one of the twenty-three observations was 2.5 per cent. For all flows measured, the submergence was well below the limit of interference. These check measurements of the discharge ranged from about 130 to 1 460 cu ft per sec. A very large number of current meter check measurements of the discharge through other Parshall measuring flumes of various sizes show close agreement between the observed and computed rate of flow.

The authors' report of tests on the newly developed critical-depth measuring flume, under laboratory settings, where the range of flow varied from less than 1 cu ft per sec to more than 50 cu ft per sec, shows marked consistency in the accurate measurement of flow. The practical operation of the flume in the field, for both large and small sizes, however, may indicate some shortcomings, especially where it is required to measure a flow of a few cubic feet per second through a structure having a capacity of more than 1 000 cu ft per sec. Experience with the Parshall measuring flume on the San Dimas Experimental Forest appears to indicate that better results would be obtained by having greater depths and less width of throat.

The writer is much interested in the development of a practical measuring device for the special purpose of gaging flood flows which carry an excessive quantity of bed-load. The authors' investigation of such a new device, to meet these requirements, is encouraging when viewed from the standpoint of laboratory results. There appears evidence, however, that from the standpoint of field experience this new critical-flow measuring flume has not been entirely successful.

MARTIN A. MASON,[12] JUN. AM. SOC. C. E. (by letter).—The accurate measurement of the discharge of streams carrying silt and solid materials has great interest for the hydraulic engineers of Europe, particularly those in Switzerland and the French Alps. Most of the streams in that section are of the nature of mountain torrents, transporting considerable solid material, and are usually characterized by very turbulent flow at high velocity. Because of the intensive power development of the streams the accurate measurement of their discharges is of more than ordinary importance. More or less satisfactory discharge determinations are generally made by the use of current meters at either a natural or an artificial control section; the use of weirs, Parshall flumes, Venturi flumes, and similar devices being precluded, except in unusual cases, on account of their lack of reliability when measuring silt-laden flows. The need for a simple, reliable, and accurate means which may replace current meter methods in the measurement of such flows is recognized; and it is to be hoped that the complete San Dimas investigations will represent, at least, a first step toward the development of such a method.

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[12] Scientific Aid and Junior Mech. Engr., National Bureau of Standards, Washington, D. C.; Freeman Scholar, 1937, Grenoble, France.

A particularly difficult problem in silt-laden flow measurement was recently presented by the need of gaging the La Sésia torrent, a small mountain stream near Le Sautet Dam in the Préalpes of France. The normal flow of about 2 m^3 per sec (70.63 cu ft per sec), with velocities above the critical, carries solid materials, ranging in size from fine sand to boulders 30 cm or 35 cm (12 in. to 14 in.) in diameter, in quantities varying from 10% to 20%, by volume, of the discharge. In times of flood or melting snow this value may become as high as 50 per cent. If possible, it was desired to establish a gaging flume in the stream which would permit a reliable, accurate determination of the discharge under varying flow régimes.

The lack of information on the performance of such devices under these conditions led to co-operative studies being made at the hydraulic laboratories of the School for Hydraulic Engineers and of Ateliers Neyret-Beylier and Piccard-Pictet, at Grenoble, France, with the purpose of developing a Venturi or a Parshall flume which would function satisfactorily under the unfavorable conditions obtaining in the La Sésia.

These studies did not succeed in the development of a flume of either type which was entirely suitable. It was found at once that the Parshall flume was practically valueless due to the deposition of some of the transported solids, particularly the larger stones and boulders, wherever shooting velocities did not exist. It is interesting to note that exactly the same result was obtained at San Dimas in the tests of the altered Parshall flumes with débris-laden flow. A rectangular flume with converging side walls, designed on the principle that the super-critical velocities obtained would be sufficiently high to carry all the transported solids through the measuring section without deposition, was also unsatisfactory. It was found impossible to attain velocities high enough to keep the flume clean at all stages of flow.

The solution finally arrived at, although it is admittedly crude and susceptible of improvement, is nevertheless of interest for its ingenuity and entirely different approach to the problem from that adopted at San Dimas. A rectangular wooden flume about 13 m long, 3 m wide, and 1.5 m deep (42.7 ft by 9.8 ft by 4.9 ft), was substituted for the natural stream channel in a short section where the slope was relatively small (2 cm per m or 0.24 in. per ft). The stream was led into and away from the flume by rock-bundle transitions. A movable gate was installed at the down-stream end of the flume.

The technique used in making a gaging is as follows: With the flume gate normally open and the stream flowing freely through the flume a deposit of the larger solids carried is formed due to the reduction in velocity in the flume brought about by the reduced slope of the bed. When it is desired to make a measurement of normal flow (that is, no flood conditions) the flume gate is partly lowered, and the increased velocity under the gate scours out the deposited material. With the velocities existing in normal flow this operation requires about 10 min. When the flume is clean, as determined by sounding-rod observations, the gate is completely closed and the measurement begun.

Velocity measurements are taken with current meters at a number of points dependent upon the conditions of flow. No trouble is experienced with deposits forming in the flume during the short time required to make the meter measurements because, with the gate closed, there is a reduction of velocity in the flume causing, in turn, a jump to form some distance up stream. The solid material, with the exception of the finer sand, is thus deposited in the stream bed in the vicinity of the jump, only the now relatively clear water passing through the flume and over the gate. With the reduced velocity obtained and with the measuring section located about 10 m (32.8 ft) down stream of the flume entrance the flow conditions are very favorable to accurate velocity measurements. Upon completion of the gaging the gate is opened, and again free flow through the flume, at a velocity above the critical, takes place. The increased velocity in the stream bed above the flume entrance scours the deposit formed there by the hydraulic jump during the gaging, leaving only the largest of the boulders that have lodged there. These are removed manually from time to time as required.

In times of flood or of melting snow, when the solid content of the stream may become as high as 40 to 50%, by volume, the gaging is made with the flume gate open, correction being made for the deposit in the flume as determined by sounding-rod observations. The error in the measurement of the flow at such times is of the order of 10%, whereas for normal flow conditions the error does not exceed about 5 per cent.

EDWIN S. FULLER,[13] ESQ. (by letter).—Every one engaged in collecting flow data on streams in which the regimen varies with almost every change in rate of flow, should be grateful that an organization of the character of the California Forest and Range Experiment Station should have concerned itself with the task of attempting to develop a stable rating device for use on such streams. Those who have had much field experience in attempting to stabilize the conditions of flow in flood channels can appreciate the difficulties under which the authors of this paper have been, and are, laboring.

When the rating flume studies were inaugurated in 1935 at the San Dimas Experimental Forest, the writer was privileged to witness some of the earlier tests. His first impression upon seeing the various types of rating flumes that had been constructed, especially those in the testing laboratory down stream from San Dimas Dam, was that these structures would form excellent controlling sections for gages located a short distance up stream from the entrance to the rating flume, and that there might be a splendid chance to compare the ratings of gages so located with those within the rating flumes themselves.

Since readings on staff gages in the approach channels up stream from the rating flumes were included in the comprehensive data gathered during the tests, the publication of these data has been awaited with considerable interest, and great disappointment was felt when it was found that data covering water surface heights in the approach channels were not included. However, the determinations of water surface elevations in the approach channels were later furnished to the writer and will be used in this discussion.

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[13] Hydr. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

It can be demonstrated that for a stable controlling section of rectangular shape, the rating curve for a gate situated a short distance up stream from the "control" section will have the form,

$$Q = K\,b\,H^u \qquad (11)$$

in which u is an exponent equal to 1.5; Q is the discharge, in cubic feet per second; b is the breadth of the rectangular "control" section, in feet; H is the height of the water surface, in feet, above the lip of the controlling section; and K is a constant depending only on longitudinal channel conditions. Such a rating curve, of course, can be defined from a single determination, at any stage of the discharge.

Applying the water surface elevations in the approach channel to the discharges listed in Table 2, rating curves for the 1-ft, 2-ft, and 3-ft rectangular, San Dimas type, rating flumes, were constructed on rectangular co-ordinates. The relation between the ordinates and the sub-tangents for these curves shows that in each case the points can be fitted by curves of the form of Equation (11), if $K = 2.80$ and $u = 1.5$, the deviation in all cases being well within the range of observational errors of the gage readings, which were made on gages marked on the walls of the approach channel, without any provision for stilling. This constant value of the coefficient and of the exponent offers much more promise of accurate extrapolation for the application to larger rating flumes than Equations (7), in which both the coefficients and the exponents vary with the size of the flume. Expressed in the same manner as Equation (11) formulas for a gage 3 ft down stream from the transition may be written from the following:

Width of flume, in feet	K	u
1	6.35	1.321
2	6.52	1.277
3	6.62	1.245

The extrapolation of these formulas up to widths of 50 ft or more, such as may readily be required, may easily introduce serious errors.

One of the causes for this observed variation in coefficient and exponent, of course, is that the stilling-well connections in the three sizes tested were at relatively different distances down stream from the flume entrance, this distance being 3 ft in each of the flumes tested. Had the stilling-well connection been made 3 ft below the entrance on the 1-ft flume, 6 ft on the 2-ft flume and 9 ft on the 3-ft flume, making each flume a model of the others, it is possible that the curve formulas might have shown better agreement for the three sizes.

In the three small rectangular flumes originally tested at the San Dimas Experimental Forest, the approach channel had a flat grade and was three times as wide as the rating flume itself. In all the runs reported by the authors the flow was at depths greater than critical in the approach channel and at depths less than critical in the lower end of the rating flume, passing through critical depth at some point between the transition section and the stilling-well connection section. Obviously, if the rating flumes were long enough normal flow at less than critical depth would eventually be established at some distance below the transition section, this distance depending on the rate of flow. For very low flows in the 1-ft flume, normal flow would be established in the rating flume at, or up stream from, the stilling-well connection. In such a case the ordinary formulas for uniform flow would apply. If the rating curve for a rectangular flume of uniform cross-section and slope is computed by the Manning formula, it will be found to be fitted closely by the formula,

$$Q = K_1 b H^{1.5} \dots\dots\dots\dots\dots\dots\dots\dots (12)$$

which, except for the coefficient, is the same as Equation (11) applying to gage heights observed in the approach channel. This coefficient will be found to depend only on the grade and roughness of the channel, wherever normal flow exists.

It seems, therefore, that the rating curve for a gage located 3 ft down stream from the lower end of the entrance transition section cannot be expressed as a simple parabolic function for the entire range of gage heights. It follows that the method of least squares, involving the assumption of expressions having constant exponents, cannot safely be used in computing the rating curve for a gage located in that part of a stream where the flow is accelerating. This point was illustrated recently in rating a 10-ft flume of the San Dimas type in actual use, when it was found that the discharges, when plotted against gage heights in the rating flume on logarithmic paper, could be fitted by a curved line. The extension of this curved line to higher stages than covered by the discharge measurements showed a wide departure from the straight line derived by the method of least squares applied to the discharge measurements.

The statement is made by the authors that the new design is "unaffected by velocity of approach" (Conclusion (4)), referring to the San Dimas type of rating flume. This statement must be incorrect, since the velocity of approach has a marked effect on the entrance losses and on the rate of acceleration in the rating flume at the stilling-well connection. If the approach velocity were zero, the velocity at the stilling-well connection would be in the zone of acceleration from zero velocity to normal velocity for a flume having a 3% grade. It should be expected, therefore, that a higher velocity of approach, such as would result from decreasing the width of the approach channel, would cause greater velocities at the stilling-well connection for a given rate of flow. This would mean smaller depths and larger discharge coefficients for the rating flume. That this is true has been demonstrated by recent experiments in which the width of the approach channel has been reduced.

Since the velocity of approach has a marked effect on the rating of the San Dimas type of rating flume, it follows that changes in the grade of the approach channel may cause changes in the flume rating. The experiments at the San Dimas Experimental Forest were made with an approach channel having a flat grade. Under these conditions the flow in the approach channel was at depths greater than critical during all the runs reported. In the ordinary flood channels in which some form of rating flume or artificial rating control is required if reasonably stable ratings are desired, the normal slope which has been established by the stream itself is commonly as great as 6% or 8%, the grade depending on some relation between the size of the stream and the nature of the débris being transported by the stream. In such a flood channel the velocities, even at low stages, are usually super-critical, and the depth of flow may be considerably less than critical.

If such a flood channel should be obstructed by a rating flume of the San Dimas type in which the width of the rating flume is one-third that of the approach channel, a hydraulic jump would probably occur at some point above the rating flume entrance, at all except low stages. Under these conditions, for clear-water flow, the rating of the flume might be expected to approximate that determined by the experiments reported by the authors, if the stilling-well connection is situated at a distance down stream from the transition bearing the same relation to 3 ft that the width of the flume bears to the width of the experimental flume whose rating curve is used.

However, with débris-laden stream flow, the conditions at the entrance to the rating flume should be expected to change radically after the construction of such an obstruction as this rating flume would constitute. In such a case any débris picked up by the stream in its upper reaches by the natural processes would be transported by the stream to the point of formation of the hydraulic jump. At this point a typical débris dune would be deposited, which would travel up stream or down stream in the manner typical of traveling débris dunes in flood channels. Eventually, as the supply of added débris became less than the transporting capacity of the stream, the dune would move down stream to the entrance of the rating flume, and would establish itself there with a down-stream face having the normal slope, depending on the relation between the rate of flow and the nature of the débris.

The establishment of such a débris dune at the entrance to the rating flume would tend to cause a change in entrance velocity similar to that caused by narrowing the approach channel, and a corresponding increase in rating-flume discharge coefficient should be expected.

In the experiments at the San Dimas Experimental Forest, débris was introduced by men with wheel-barrows, and at no time while the experiments were being observed by the writer did the dune fully establish itself at the entrance to the rating flume. Under the conditions of those experiments a small quantity of débris was cut away from the edges of the dune by the stream and carried through the flume, which kept itself clean. At no time, however, was the quantity of débris being transported through the rating flume more than a very small proportion of the débris-carrying capacity of the stream in the rating flume. Under these conditions it is not surprising that no serious change in flume rating was observed. To conclude from these experiments that, under natural conditions, débris would have no serious effect on flume rating is by no means safe.

In the case of the 10-ft San Dimas type of rating flume a small landslide occurred a short distance up stream some time after the flume was installed. Together with material dumped in the stream bed during the construction of the flume, this landslide formed a débris dune which was gradually moved down stream by the subsequent small flows of clear water. At the time a test was conducted for the purpose of determining a rating curve for the 10-ft flume the dune occupied a position about 25 ft up stream from the rating-flume entrance. The comparatively large flows used in the early part of this test moved the débris dune rather rapidly down stream until it occupied a position just above the rating-flume entrance. In this position it had rather a marked effect on the nature of the flow through the flume, as it caused the water to enter at an angle with the center line, resulting in cross-currents; and, at times, water "piled up" against the side of the flume opposite the stilling-well connection. Under these conditions it was not surprising to find variations of 10% or more in successive determinations of the stage-discharge relation. At the time the tests were concluded a small quantity of débris had begun to be discharged through the rating flume. At no time during the test was additional débris being added to the stream by further landslides above the rating flume; nor was the quantity of débris being carried through the flume, any appreciable percentage of the flow of clear water.

For the purpose of studying the probable behavior of this 10-ft flume under conditions of larger continuous landslides up stream from the rating flume, the writer constructed a small model and made qualitative studies of its behavior. In these studies the débris used was in the form of fine wind-blown beach sand taken from the back of a large sand dune about one-half mile back from the ocean shore. It consisted of irregularly shaped grains varying in size from about 0.001 in. to 0.020 in. and had a considerable degree of dimensional similitude with the actual débris in the prototype.

In the earlier part of his model tests the writer duplicated as closely as possible the conditions during the period leading up to, and including, the aforementioned rating tests. During this part of the model tests the appearances of conditions in the rating flume were about as previously observed in the prototype.

In the second part of the model tests a large volume of additional débris was introduced in the form of landslides some distance up stream from the rating flume. Under these conditions the approach channel gradually established itself at a uniform grade of about 8% leading down to the rating-flume entrance. This part of the test was concluded when the up-stream face of the débris dune reached the edge of the landslides. During the entire period of this part of the model tests some débris was rolling down the down-stream face of the dune and out through the rating flume, but at no time did the débris passing through the rating flume exceed 1% or 2% of the volume of clear-water flow, and the rating flume remained clean.

In the third part of the model tests a large volume of additional débris was introduced in the form of landslides occurring on top of the down-stream face of the large débris dune formed in the second part of the test, but still at a considerable distance up stream from the rating flume. Under these conditions the toe of the dune moved down into the rating flume to a point down stream from the stilling-well connection and the volume of débris passing through the rating flume increased to about 20% or 25% of the volume of clear-water flow. As the flow of water was decreased to zero, débris remained deposited in the rating flume, and the stilling-well was put out of commission. During this entire part of the model tests the rating flume obviously failed to function as a stable rating device.

These preliminary model tests indicated that the San Dimas type of rating flume is far from being the answer to the engineer's prayer for a stable rating device for shifting flood channels. Further quantitative tests are necessary to determine, first, how far this type of rating flume is from being the desired answer; and second, to find out, if possible, what type of rating device is best adapted to the conditions that must be met in the flood-control and flood-measurement fields. Experiments along this line should be based on the assumption that the best type of rating device will be that which least disturbs the balanced conditions that Mother Nature herself establishes in flood channels.

The most stable natural "controls" for a gage on a débris-laden stream are those in which, at all stages, there is a very slight increase in water-surface slope a short distance down stream from the gage. At such gaging stations there is a very definite relation between the exponent in the rating curve formula and the shape of the cross-section of the stream at the point where the change in slope of the water surface occurs.

The writer gratefully acknowledges the assistance he has received from members of the Los Angeles County Flood Control District, and from the authors themselves, in the preparation of this discussion. Unpublished water-surface elevations in the approach channels were made available through the courtesy of Professor Kotok.

HAROLD K. PALMER[14] AND FRED D. BOWLUS,[15] MEMBERS, AM. SOC. C. E. (by letter).—Débris-laden streams may be divided into three classes: (1) Muddy water in which the débris consists of fine silt, ordinarily considered as that passing a 200-mesh sieve, distributed uniformly throughout the entire cross-section; (2) water carrying a suspended load of coarser sandy material, the bulk of which is carried along in the lower part of the stream; and (3) water transporting rocks which roll and slip along the bed.

Any clear-water measuring device will give satisfactory measurement of the Class (1) stream. The authors state that with a level throat the addition of débris caused the critical point to move down stream, but that with a throat built on a 3% grade the critical point did not move when débris was added. In a throat with a level floor the acceleration of the water is caused by the difference in pressure due to the fall in the water surface. If there is a suspended bed-load which increases the specific gravity of the lower strata, but not of the surface, the force acting on the lower strata will be the same whether or not there is a bed-load. However, the weight of the débris-laden water being greater, this force must act for a greater distance to produce the same velocity. When the throat floor is on a grade, gravity accelerates the heavy water as much as the light water, but since the acceleration is caused by the slope of the bottom and the slope of the surface, the critical point may still be displaced somewhat by the addition of the débris.

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[15] Res. Engr., Los Angeles County Sanitation Dists., Los Angeles, Calif.

All attempts in the past to make use of critical-depth measurements have failed because of the difficulty of locating the point of critical depth. Near critical velocity there is no appreciable change in energy for small changes in depth, therefore, the water surface is very unstable and the resulting waves interfere with the measurements. Placing the flume on a 3% slope, as the authors have done, causes it to avoid the resulting waves, since the water quickly attains a greater velocity, resulting in a concave water surface with a single point of critical depth. Since this is not at the same position in the throat for all flows, the stilling-well will show critical depth for only a small range of flow. Other flows will require an empirical formula which will apply only to one ratio of channel and throat width.

It is unfortunate that the authors had no way of measuring the quantity of débris added to the stream while making the experiments at San Dimas, because, neglecting other effects, it should at least add its volume to the flow. If the débris consisted of silt uniformly distributed throughout the stream, its only effect on measurements would be to indicate the combined volume of water and silt. If it carried a bed-load in the lower strata, its apparent roughness coefficient in a smooth channel might be slightly greater than for clear water, and it would accelerate more slowly in the throat. If, in addition to the débris carried in suspension, rocks were rolled on the bottom, there would be a further retarding of the flow, with increased uncertainty.

If the change in flow in the throat, due to the addition of débris, is materially different from the change in flow in a smooth channel, a correction for bed-load might be found from a second measurement in a smooth channel about 50 or 100 ft below, after the water surface indicated a stable flow. If the bed-load does not increase the apparent roughness coefficient this second measurement would be useless.

Whatever form the throat and transition may take, the method requires a point of critical velocity which, in turn, is dependent upon a change from mild to steep flow. The mild flow may be produced by means of a jump above the transition if the channel of approach is on a steep grade to prevent deposits of débris, but proximity to a jump is not conducive to accurate measurements. However, considering the past failure to make a successful meter which depended on critical depth measurements, it seems likely that even near a jump a measuring point above the throat would give better results than one near the point of critical depth. This might require a longer transition than the authors suggest, but the writers believe it would be worth trying.

The paper reveals two interesting and important results which have already been established: First, the exact shape of the transition to a Venturi flume is unimportant as long as it does not waste energy; and second, the Parshall flume can be used to greater depths than those covered by previous experiments. This will permit its use in many new places and increase the range for any one flume by using a deep and narrow throat instead of a wide shallow one.

The need for high velocities to prevent débris deposits would require a narrower approach channel than three times the width of the throat. If no measurements are made above the throat it would seem that the channel should be only wide enough to insure critical velocity in the throat at all flows. Probably 25% greater than the throat width would suffice.

As the authors state, they have made only a start and much experimenting will be needed to reach a final solution, and when that solution is found it is probable that the results will be only approximate. Even so, approximate results will be better than those now available, and the problem is well worth solving.

HARRY F. BLANEY,[16] M. AM. SOC. C. E. (by letter).—Results of experiments on the San Dimas flume are presented in an excellent manner in this paper. The complex problem of measuring intermittent débris-laden streams having wide variations in flow has interposed difficulties in mountain watershed studies in Southern California for many years.

Since the Parshall flume was included in the studies made by the authors, perhaps it would be well to call attention to the fact that this flume was not originally designed to measure débris-laden streams, such as exist in the mountain water-sheds, and that heretofore it has been used principally in the valley areas. For many years Parshall measuring flumes of various sizes have been used successfully in California under less extreme conditions than those in the San Dimas Experimental Forest.

The application of flumes to measurements in mountain canyons involves problems not ordinarily encountered in valley areas. The stream gradient is steep (often 10%, or more) and the water tends to cut a narrow channel, travel at a relatively high velocity, and carry a large bed-load as well as a considerable volume of suspended material.

A large flume, for example, placed directly in a stream channel where the grade is as steep as 10% may have a rating curve quite different from the standard calibration. It may pass much less water than that given for the lowest gage heights in the standard tables. The reason for this is that a small stream of water entering the center of a wide flume at high velocity tends to proceed through the center without changing its cross-section greatly, leaving dead water along each side of the wide flume. At high stages, the flume may pass more water than is indicated by the gage height in the standard tables, because of the high velocity of approach across the entire width of the flume. Large flumes, when used under such extreme conditions, should be rated in place.

In connection with some experimental studies in 1931, the writer was confronted with the necessity of measuring stream flows varying from 0.1 cu ft per sec to more than 20 cu ft per sec in mountain areas.[17] In order to measure large winter flows and also to obtain an accurate record of small summer flows, a combination flume such as that shown in Fig. 15 was found desirable. This combination consists of two Parshall measuring flumes, one 2 ft and one 3 in. wide, arranged so that both large and small flows pass through the converging section of the 2-ft flume. The small flows are by-passed from the dip in the large flume into a basin above the 3-in. flume and thence through this latter flume, while the greater part of the large flows continues on through the 2-ft flume. A record of the larger discharges is obtained by a recorder operated by a float in a stilling-well connected to the larger flume, and a record of the smaller discharges is obtained by a flow recorder operated by a float in a stilling-well connected to the smaller flume. Some overlap is provided, so there is a small range during which a record may be obtained from both flumes. Two separate recorders may be used, or one duplex recorder is sufficient if it is desired to record gage heights only. Several combination flumes of this type were installed in the mountain sections of Coldwater Canyon and Devil Canyon, near San Bernardino, Calif. These flumes gave satisfactory results for the few years they were operated.

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[16] Irrig. Engr., Bureau of Agricultural Eng., U. S. Dept. of Agriculture, Los Angeles, Calif.

[17] "Water Losses Under Natural Conditions from Wet Areas in Southern California," California State *Bulletin No. 44*, Div. of Water Resources, 1933, p. 88.

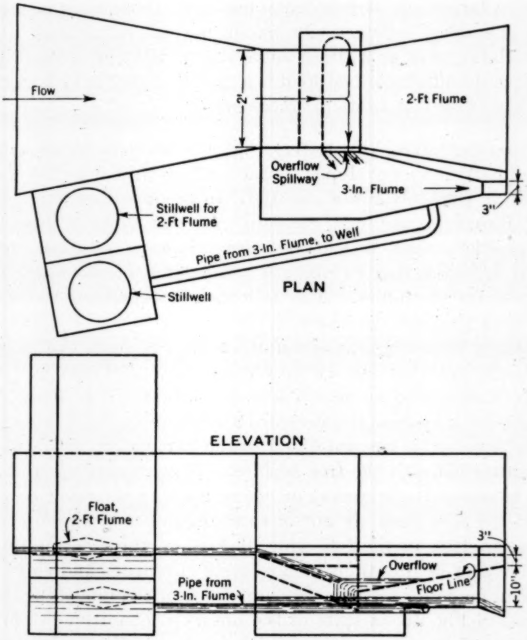
Flow
2'
2-Ft Flume
Overflow Spillway
3-In. Flume
3"
Stillwell for 2-Ft Flume
Pipe from 3-In. Flume, to Well
PLAN
Stillwell
ELEVATION
Float, 2-Ft Flume
3"
Overflow
Floor Line
10"
Pipe from 3-In. Flume

FIG. 15.—COMBINATION PARSHALL FLUME FOR MEASUREMENT OF WATER AT BOTH HIGH AND LOW STAGES

There are denuded water-sheds in Southern California where no type of flume will measure the débris-laden stream flow accurately. This was demonstrated by the run-off that occurred in the La Crescenta-Montrose area on January 1, 1934.[18]

In 1928, the writer was one of several who suggested replacing a 30-in. rectangular weir with a 12-in. Parshall measuring flume, after the weir had proved to be very unsatisfactory in measuring the run-off from a 44-acre experimental denuded water-shed near San Bernardino. The flume operated satisfactorily under normal conditions but filled with débris when excessive run-off occurred. Some time later it was replaced with a 12-in. San Dimas flume. In February, 1936, this flume was also filled with débris after a heavy rain storm. A larger San Dimas flume has now (1938) been installed, and it is apparently operating satisfactorily under normal conditions. Whether the present installation will or will not solve the problem of measuring flows that carry excessive quantities of bed-load is a question that may be answered when it is put to the same test as its predecessors.

R. L. STOKER,[19] ESQ. (by letter).—Model tests of various modifications of the San Dimas Flume developed by the authors were begun at the University of California at Berkeley, Calif., in 1937, in co-operation with the California Forest and Range Experiment Station. As this type of flume presents an analytical problem that should be susceptible to solution by established methods,[20] it is interesting to make a comparison of the observed and computed water-surface profiles. The following comparison is for the case of clear-water flow through the flume.

The ordinary equations for non-uniform flow in open channels are derived on the assumptions that steady flow exists and that the flow lines possess negligible curvature and slope. These assumptions regarding the flow lines make it possible to express the pressure anywhere in the flow as being greater than atmospheric by an amount equal to the unit weight of the fluid, times the vertical distance beneath the free surface. However, in open channels where the section changes size or shape, or where the bed slope varies abruptly, it is evident that the flow lines are not in conformance with the foregoing assumptions. Nevertheless, in the following discussion, the computations are based on the simplifying assumptions of negligible curvature and slope of flow lines and have been made for a particular run of one of the models of the San Dimas Flume shown in Fig. 16, in spite of the observed conditions shown in Fig. 17.

The particular flume was coated with sand of uniform diameter of approximately 0.02 in. The value of n in Manning's equation was found to be 0.0148 from the formula,[21]

$$n = 0.043\, D^{1/6} \quad \text{(13)}$$

in which D is the mean sand grain diameter, in feet. Various experimenters have obtained approximately this value of n. For example:

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[18] "Flood in La Cañada Valley, California, January 1, 1934," by Harold C. Troxell, Assoc. M. Am. Soc. C. E., and John Q. Peterson, *Water Supply Paper 796-C*, U. S. Dept. of the Interior, 1937.

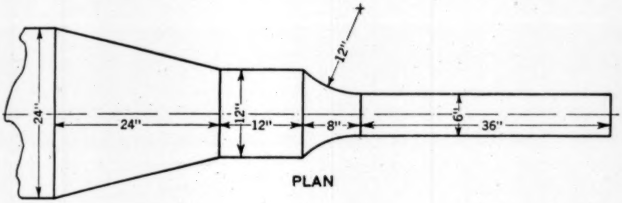
[19] Instructor in Mech. Eng., Univ. of California, Berkeley, Calif.

[20] "Hydraulics of Open Channels," by B. A. Bakhmeteff, M. Am. Soc. C. E.; and "Applied Fluid Mechanics," by M. P. O'Brien and G. H. Hickox, Associate Members, Am. Soc. C. E.

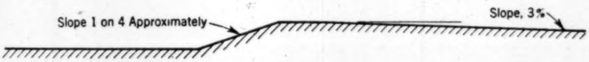
[21] "Laboratory Investigation of Flume Traction and Transportation," by Y. L. Chang, *Proceedings*, Am. Soc. C. E., November, 1937, pp. 1701–1739.

Authority	Manning's n
Strickler[22]	0.039 $D^{1/6}$
O'Brien[23]	0.049 $D^{1/6}$
Chang[21]	0.043 $D^{1/6}$

It is easily shown that the section at the beginning of the part of the flume that is 6 in. wide should act as the control section because the 3% bed slope is substantially greater than the critical slope for the given channel roughness and discharge. The discharge was 0.927 cu ft per sec for the particular run considered herein. Thus, in the computations the critical depth was assumed to occur at the beginning of the 6-in. width, and the back-water was computed up stream and down stream from this section by the specific energy method. The foregoing value of Manning's n was used in determining the energy loss. A comparison of the "observed," "computed," and "frictionless" water surface profiles is shown in Fig. 17.



PLAN



BED PROFILE

FIG. 16

It is interesting to note that although no contraction loss was assumed to occur in the flume transition, the frictionless and observed profiles are at very small variance in the up-stream section of the flume. In closed conduits the entrance loss due to a well-rounded transition is of the order of 2% of the increase in kinetic energy, which appears to be approximately the case in this open channel transition. According to much of the literature, a contraction loss of 10% of the kinetic energy increase is commonly assumed in designing transitions.

The observed and computed profiles are in fairly good agreement except in the immediate vicinity of the control. This would be expected, however, as it is here that the flow lines have their greatest slope, and this influence was neglected by the original assumptions. Some divergence is seen to occur at the free discharge end of the flume, but this is explainable in part because no allowance was made in the computed profile for the draw-down at discharge which is measurable 5 or 6 in. back from the brink.

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[22] "Geschwindigkeitformeln und Raughigkeitzahlen," by A. Strickler, *Mitteilungen des Amtes Wasserwirtschaft*, Bern, Switzerland, 1923.

[23] "The Vertical Distribution of Velocity in Wide Rivers," by M. P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1937.

The good agreement between theory and experiment demonstrated by the example in this discussion indicates the value of applying the same method to all the data presented by the authors. Substantiation of the theoretical method in this manner makes interpolation possible in other designs, other locations of pressure taps, and variations in surface roughness.

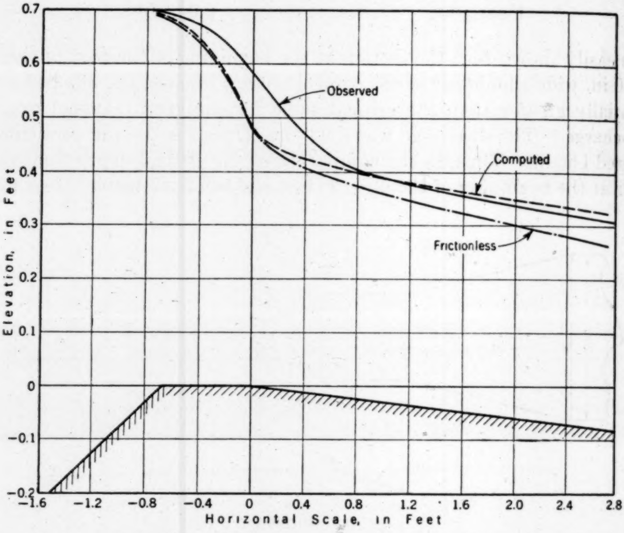

FIG. 17.—COMPARISON OF OBSERVED AND COMPUTED WATER SURFACE PROFILES; $Q = 0.927$ CUBIC FOOT PER SECOND

J. C. STEVENS,[24] M. AM. SOC. C. E. (by letter).—The authors of this paper offer a solution to a perplexing problem wherever the conditions permit the installations indicated. A prime requisite is an abundant drop at the structure. The profiles of Figs. 10 and 13 could not be maintained if the tail-water should cause a hydraulic jump on the bed of the flume and thus deposit much of the débris. A definite drop of substantial proportions below the end of the flume seems to be essential.

It would be interesting to determine some of the characteristics of the hydraulic jump when heavily ladened with débris. With coarse material this may be impossible but with finer sands some data might be secured.

It must be remembered that the Parshall flume was not designed for any such service as it has been subjected to here and it must not therefore be condemned as a measuring device. Its chief field of usefulness is in irrigation systems where, at most, sand or silt and not cobbles are encountered, and also where it is necessary to sacrifice a minimum amount of head. For such service the Parshall flume gives remarkably accurate and consistent results.

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[24] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

Since the paper was written, the San Dimas Experimental Forest has experienced some floods that have doubtless given these devices a real test. It is to be hoped the authors will take the Engineering Profession into their confidence in discussing the effect of these floods on their measuring equipment.

H. G. WILM,[25] ESQ., JOHN S. COTTON,[26] ESQ., AND H. C. STOREY,[27] ESQ. (by letter).—The writers appreciate the interest taken by members of the Engineering Profession in the San Dimas flume, and are particularly grateful for the numerous constructive criticisms.

Two points regarding this measuring flume should be made clear, perhaps, as several discussions indicate some misconstruction of the writers' claims for it. In the first place, it has not been intended that this flume should supplant the well-known and widely used Parshall flume. The latter, as testified by its widespread adoption, is ideally adapted to use in measuring streams of relatively low gradient, in which conservation of available head is of prime importance and in which any transported material is fine enough to be moved by the low velocities characteristic of these streams and of the Parshall flume.

Stream velocities in mountain channels, on the other hand, are the result of relatively high gradients, and, ordinarily, are greater than critical. It is known that débris transportation is a function of some relatively high power of velocity. Therefore, in measurement of high-velocity streams it would seem scarcely logical to expect accurate results from any measuring device that is dependent upon the maintenance of sub-critical velocities in the measuring section.

The writers' experiments have been directed entirely toward the development of a flume characterized by super-critical velocities, but with a "critical" section close enough to the piezometer orifice to provide stability of rating. It is their belief that a flume of the San Dimas type satisfies these requirements. A long series of carefully controlled model studies, using a 6-in model of 1-ft, 2-ft, and 3-ft San Dimas flumes, as well as field observations since the original experiments, have demonstrated that the final design for this flume provides a remarkably consistent and accurate relation of pressure to discharge, very low transition losses, and an excellent degree of stability in rating even when it is subjected to variations in approach velocities or back-water conditions. For about 330 runs, computed discharges showed an average deviation of ± 1% from observed discharges.

As a second point, the writers have made no claim that a flume of the San Dimas type has unlimited capacity to measure flows containing unusually high débris contents. Flow velocities in this flume do not exceed 12 or 13 ft per sec even at high stages, and the load-transporting capacity, therefore, has this definite limit. Furthermore, it is doubtless possible that high concentrations of load affect hydraulic characteristics of flow. Under such conditions it would be too much to expect that any standard rating would continue to apply.

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[25] Associate Silviculturist, California Forest and Range Experiment Station, U. S. Forest Service, Glendora, Calif.

[26] Engr., Federal Power Comm., San Francisco, Calif.

[27] Junior Geologist, California Forest and Range Experiment Station, U. S. Forest Service, Berkeley, Calif.

The writers' claim is simply this: That the San Dimas flume greatly extends the range of flow that can be measured with a reliable degree of accuracy in mountain streams.

Mr. Parshall and Messrs. Palmer and Bowlus suggest that a measuring point taken just up stream of the "critical" section would be better than that chosen by the writers. At the point suggested there is a considerable degree of flow curvature with its accompanying centrifugal forces, so that depth and pressure measurements deviate from each other, and the rating should be correspondingly undependable.

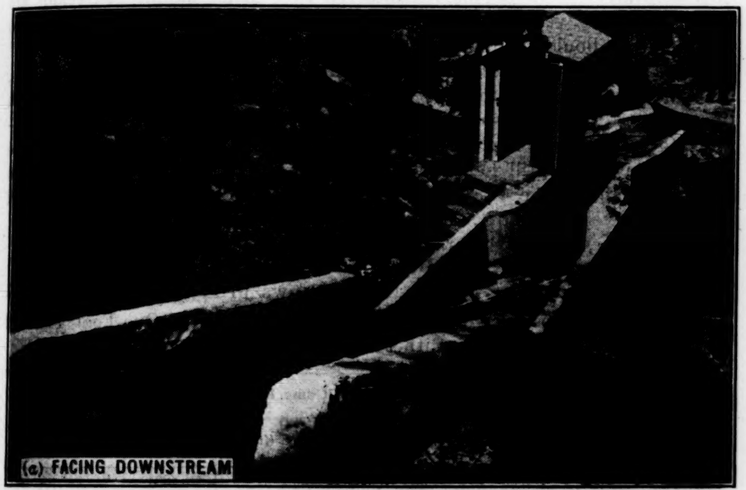
Mr. Parshall cites a good case of the Parshall flume having given accurate measurement of loaded flows. Accuracy of measurement depends largely upon movement of load at a velocity close to that of normal flow through the flume. Accordingly, the usability of any flume is determined by the relation between normal flow velocities in the flume and in the stream above, and by the concentration and "particle" size of the transported material.

Sixteen Parshall flumes installed by the U. S. Forest Service on the San Dimas Experimental Forest have been observed for four to five seasons and rated as well as can be expected under the difficult conditions imposed by loaded flows. As nearly as can be ascertained, all these flumes have produced discharges for any given depth considerably in excess of the standard rating. Although the over-rating percentage varies considerably with changing conditions of approach channel and bed deposits, it averages approximately 30 to 40 per cent. The reason for this fact is the deposition of bed-load within the flume proper to a gradient approximating that of the approach channel. Therefore, approach velocities continue through the entire flume, depths (even including the deposit) are lower than standard, and no critical section is present. Table 4 gives a comparison of standard discharge values and approximate actual discharges in a 30-ft and a 10-ft Parshall flume during the

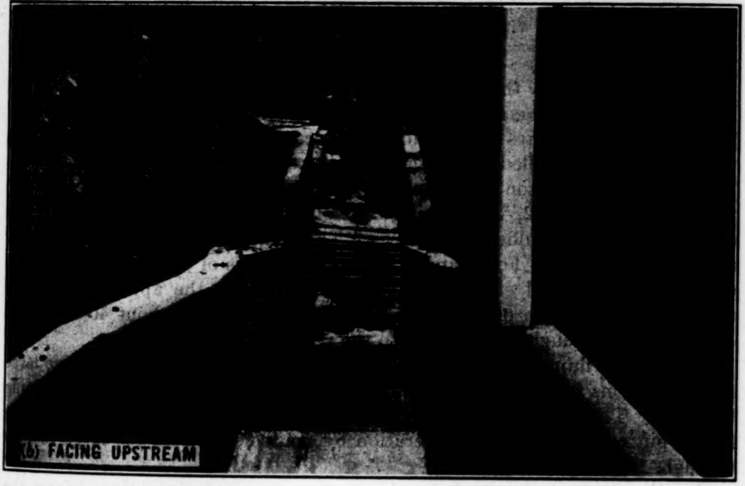
TABLE 4.—A COMPARISON OF STANDARD RATING WITH OBSERVED DISCHARGES FOR FLUMES INSTALLED IN STEEP STREAM CHANNELS

Values of H_a, in feet	DISCHARGE, Q, IN CUBIC FEET PER SECOND			Percentage deviation (4)	DISCHARGE, Q, IN CUBIC FEET PER SECOND			Percentage deviation (8)
	Standard Parshall rating (1)	Observed (2)	Difference (3)		Standard Parshall rating (5)	Observed (6)	Difference (7)	
	(a) THIRTY-FOOT PARSHALL FLUME				(b) TEN-FOOT PARSHALL FLUME			
0.3	17.7	28.8	12.1	+72	5.8	6.6	0.8	+14
0.5	37.3	62.0	24.7	+66	13.0	15.5	2.5	+19
0.7	64.0	103.0	39.0	+61	22.5	27.0	4.7	+21
1.0	113.2	176.0	62.8	+55	39.4	49.0	9.6	+24
1.5	216.5	320.0	103.5	+48	75.4	96.0	20.6	+27
2.0	343.0	490.0	147.0	+43	119.4	154.0	34.6	+29
2.5	....	....	....	....	170.6	225.0	54.4	+32
3.0	656.0	900.0	244.0	+37	228.5	300.0	71.5	+31
4.0	1 039.0	1 390.0	351.0	+34	....	....	....	....
5.0	1 486.0	1 920.0	434.0	+29	....	....	....	....
6.0	1 989.0	2 520.0	531.0	+27	....	....	....	....

great storm of February 27 to March 3, 1938, in which all flumes on the Experimental Forest were finally overtopped.



(a) FACING DOWNSTREAM



(b) FACING UPSTREAM

FIG. 18.—THREE-FOOT SAN DIMAS TYPE OF FLUME, SAN DIMAS EXPERIMENTAL FOREST, MARCH 18, 1938

Mr. Blaney states that Parshall flumes in natural channels should be rated in place. This statement is certainly true, as demonstrated by the writers' experience, although it means that under such conditions the flume becomes no more than a rather costly control section. The writers, and doubtless other mountain hydrographers, would like to learn an accurate method of making such a rating. It should be remembered that many mountain streams are characterized by very flashy peak flows containing all kinds of débris, up to and including boulders, limbs, and even occasionally whole trees.

Mr. Parshall states that by the addition of 0.089 ft to the observed H_a-heads, "the standard discharge for these corrected heads agrees reasonably well with the computed discharge shown in Table 1 as based on the San Dimas calibrations." In Table 5, the writers have made the suggested change, and the resulting deviations still do not seem reasonable in magnitude. This correction would be valid if the values in question were measurements of total head (pressure + velocity head), and if friction and other losses were neglected; but the measurements involved are of pressure alone.

TABLE 5.—EFFECT OF DÉBRIS-LADEN FLOWS UPON MEASURING FLUMES
(Comparison of Observed and Computed Discharges in One-Foot Parshall Flume with Floor on 3% Grade)

Observed values of H_a, in feet	$H_a + 0.089$ ft	DISCHARGE, IN CUBIC FEET PER SECOND			Percentage deviation
		Observed	Computed (from $H_a + 0.089$ ft)	Deviation	
0.646	0.735	2.75	2.51	−0.24	−8.7
0.885	0.974	3.83	3.84	+0.01	+0.3
1.295	1.384	12.10	6.55	−5.55	−45.8
1.456	1.545	9.76	7.08	−2.68	−27.5

Mr. Stevens asks that the writers describe the behavior of San Dimas Experimental Forest gaging stations during the storm ending March 3, 1938. That of Parshall flumes has already been described. San Dimas flumes, 3 ft wide, had been installed in six small gaging stations at the beginning of the 1937–1938 rainy season. These stations behaved perfectly up to the final peak flow of March 2, when four of the six were inundated and buried in a deposit of brush, boulders, and trees. The remaining two gave a complete record of the storm. For example, in the two views of Fig. 18, note the lack of deposited material either within the flume or in the approach channel, although in the storm of February 27 to March 3, 1938, about 400 cu yd of débris passed through this flume into the reservoir below.

San Dimas flumes, 2 ft and 3 ft wide, had also been installed in nine large gaging stations for the measurement of relatively low storm flows containing bed-load. These flumes were overtopped repeatedly, and most of them were finally put completely out of operation by boulders or trees too large to pass through.

During the major part of the storm, flow records were obtained from the adjoining large Parshall flumes by visual observation of velocities and recording of depths. Two of the 3-ft San Dimas flumes, over each of which about 2 000 to 2 200 cu ft per sec of peak flow had passed, cleaned themselves as the flow dropped and automatically went into operation at the end of the storm (see

Fig. 19). These two flumes, although badly battered, simply happened not to have been clogged by boulders or trees.

Mr. Mason describes an original, and apparently effective, technique for the gaging of very heavily loaded stream flow. Such a method would doubtless be the only possible method of measuring flows loaded beyond the range of a flume of the San Dimas type. Incidentally, Mr. Mason's observations verify the writers' conclusions as to the inadequacy of Parshall flumes in the measurement of high-velocity, heavily loaded stream flow.

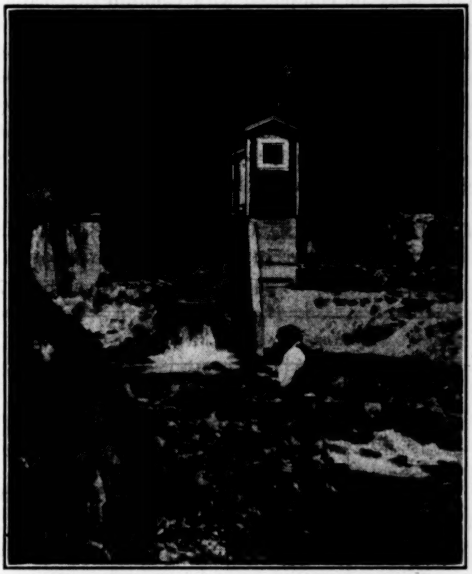


FIG. 19.—THREE-FOOT SAN DIMAS FLUME IN THE DOWN-STREAM END OF 15-FOOT PARSHALL FLUME; VIEW FACING UP STREAM, TWO WEEKS AFTER STORM OF FEBRUARY 27 TO MARCH 3, 1938

Mr. Fuller outlines a useful and valuable method for gaging streams when a stable section is available and cost requirements prevent the use of a critical-depth flume. His method can be nicely demonstrated as valid for stable sections. However, the measurement of depths up stream of such sections is believed by the writers to be subject to inaccuracies, due to changing bed deposits with changes in stream velocity at various stages. The San Dimas type of flume, on the other hand, provides a stable measuring section, characterized by high flow velocities and controlled by a "critical" section up stream and a free over-fall below the point of measurement.

Attention is drawn by Mr. Fuller to the variation of coefficients and exponents in the San Dimas flume with variations in flume width. He fails to note that this variation is an exact function of width, caused doubtless by variations in energy losses due to width variation. Under frictionless conditions discharge would be expected to show an exact relation to flume width.

Incidentally, the San Dimas flume might be considered simply as a broad-crested weir, in which depths are measured directly on the crest instead of up stream. In this case, it is easily seen that successively greater widths of weir do not have to be considered geometric models of each other as to total length and point of pressure measurement, in order to produce the same discharge per unit width.

According to Mr. Fuller, rating points for a 10-ft San Dimas flume installed by the Los Angeles County Flood Control District, formed a curved line on a log-log paper. Therefore, a least-squares formula could not be fitted to these data. Table 6 contains rating data kindly supplied by the Los Angeles County Flood Control District for this flume, based on several accepted hydrographic methods. These methods show some variation among themselves, due to the difficult conditions of measurement and the volume of transported material. They may be compared with Column (5), Table 6, however, showing values computed by extrapolation of data from the San Dimas model flume. The computed flows agree excellently with observed values obtained by Pitot-tube traverses which, of the various methods used, should provide the most reliable data. The observed values by Pitot tube can be fitted just as well by a straight line as by one which is slightly curved, and there would appear to be no more reason for charging the apparent slight curvature to the flume's characteristics than to errors of measurement.

TABLE 6.—PACOIMA CREEK BELOW PACOIMA DAM; SUMMARY OF RESULTS, TEST OF SAN DIMAS TYPE OF RATING FLUME

(Date of Test, November 17, 1937)

RATE OF FLOW, Q, IN CUBIC FEET PER SECOND					Mean gage heights, in feet †
By Pitot Tube			By current meter at 0.6 of depth	By formula, $Q = 84\,h^{1.16}$	
By vertical velocity curves	By 0.2 and 0.8 of depth	By 0.6 of depth			
(1)	(2)	(3)	(4)	(5)	(6)
39.4 39.2	40.4 40.2	40.9 40.3	37.2	42.0	0.55
151	155	154	139	149	1.64
222	224	221	201	195	2.07
197	199	204	186	194	2.06
101 *	102	102	96.0	94.8	1.11
31.1 *	31.5	32.0	30.2	31.6	0.43
108 *	110	110	101	108	1.24
122 *	124	123	111	122	1.38

* Pitot-tube measurements, each with thirteen vertical velocity curves, not computed.

† By tape in well; stilling-well connection, 4.5 ft down stream from transition.

Mr. Fuller states that "velocity of approach has a marked effect on the rating of the San Dimas type of rating flume." Theoretically, velocity of approach should have no effect, although deviation of practice from theory makes it also true that marked changes in approach velocity may have, not "marked," but some, effect on the flume rating. This phenomenon is logically caused by displacement of the location of critical depths, as a result of varying approach velocities.

In the San Dimas flume, however, depths (and pressures) at the measuring point are controlled not only by the location of the critical depth, but also by the over-fall down stream and by the floor slope of the flume. Accordingly, they are affected much less by horizontal displacement of the critical depth, resulting from increased approach velocities, than are depths up stream of the critical section. This fact is well illustrated by Mr. Stoker's analysis, showing a comparison of theoretical and actual water surfaces. It will be noted that, although the critical depth actually lies considerably down stream of its theoretical location (neglecting centrifugal effects), computed and actual water surfaces agree very closely at the piezometer location (1.5 ft in Fig. 16).

Mr. Fuller states that the San Dimas experiments were made "with an approach channel having a flat grade." On the contrary, floor gradients as steep as 4% were used for the approach channel, and, in one run, the channel was built up with heavy gravel to a 25% grade. These alterations had no measurable effect on pressures measured at the piezometer point. As another test of the effect of approach conditions, in the 6-in. model a standard obstruction was introduced into the approach channel during each experimental run. This obstruction consisted of a metal plate, about two-thirds of the channel width, placed at one side perpendicular to the direction of flow. It caused a violently changed flow direction and increased the approach velocities. The resulting effect on discharge, based on the average of right and left piezometer readings, did not exceed 4% of the true volumes.

It is true that, in these model studies, flumes having an approach channel narrower than in the original design showed higher discharge coefficients. It is probable, however, that this result was caused more by an improved and longer entrance transition than by increased velocities in the approach channel.

Mr. Fuller concludes his discussion with a description of qualitative studies made with a small model of the 10-ft San Dimas flume. The model (0.1 ft wide) had a geometric scale ratio of 1 : 100 in dimensions, and 1 : 100 000 in discharge. The relative friction was probably unknown. The writers' feel that too much credence should not be placed in results from such a small-scale model, especially in studies of débris movement, the hydraulic characteristics of which are little known. In any case, as stated previously, the writers recognize that any measuring device has limitations and do not claim the San Dimas flume to be capable of measuring landslides.

Messrs. Palmer and Bowlus present a neat analysis of flow phenomena in a critical depth flume. As indicated in their discussion, variations in approach velocity or bed-load content should be expected to cause some horizontal displacement of the location of critical depths. Such displacement has been demonstrated (see Mr. Stoker's discussion and the writers' comments on it) to have little effect upon pressure measurements at the standard piezometer location.

Messrs. Palmer and Bowlus comment on the advisability of a narrower approach channel than three times the flume width. As a result of the writers' model studies in co-operation with the University of California, a final design has been adopted in which the approach channel is twice the width of the flume proper, rather than three times. The entrance transition has been kept cylindrical in form, but with a radius twice the flume width instead of equal to it. This flume has very satisfactory flow characteristics, with little turbulence and entrance losses computed at less than 2 per cent. A sketch of the present design is presented by Mr. Stoker in Fig. 17.

The writers are personally acquainted with the tandem Parshall flume installation described by Mr. Blaney. In the stream in question, very little bed-load is transported except by unusually high peak flows. In the senior writer's experience, the installation functioned quite adequately—except in measurement of such high flows—as long as the approach channel was kept shoveled out to provide reduced velocities of approach.

The 3-ft San Dimas flume mentioned by Mr. Blaney functioned perfectly throughout the great storm of March, 1938. Although a heavy load of sand and gravel was carried through this flume into a small reservoir below, at no time were deposits formed in the flume, and it was perfectly clean at the end of the storm.

The question has been frequently raised as to actual contents of transported material in flows measured through San Dimas flumes. These contents are necessarily unknown as the writers are acquainted with no means of determining them accurately; they have been estimated, however, at values as great as 15 to 20% of solids.

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WATER TRANSPORTATION VERSUS RAIL TRANSPORTATION
A SYMPOSIUM

PAGE

The Value of Water Transportation.
By RUFUS W. PUTNAM, M. AM. SOC. C. E. 1280

The High Cost of Inland Water Transportation.
By S. L. WONSON, M. AM. SOC. C. E. 1296

WITH DISCUSSION BY MESSRS. GEORGE HARTLEY, W. D. FAUCETTE AND J. E. WILLOUGHBY, C. D. BORDELON, J. E. GOODRICH, H. R. FAISON, RUFUS W. PUTNAM, AND S. L. WONSON

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THE VALUE OF WATER TRANSPORTATION

By Rufus W. Putnam,[1] M. Am. Soc. C. E.

Synopsis

The completion, within the past decade, of the slack-water improvement of the Ohio River, the connecting of the Great Lakes to the Mississippi River *via* the Illinois Waterway, and the near completion of the Lower Missouri and Upper Mississippi projects give rise to a series of questions for which approximate answers might be found. When this 3 500-mile system of trunk inland waterways is completed what will have been the cost to the public of these facilities for commerce and what will be a fair approximation of the annual fixed charges and of maintenance and operation costs? To what extent and in what manner has this business of water transportation developed? How much traffic will have to be developed in order to reimburse the public treasury for the expenditures made?

These and other questions of like application the writer endeavors to answer in an effort to satisfy his own mind as to whether there is any value to water transportation.

Historical

The regional plan of the Mississippi Valley, as it is known to-day, is the direct result of the necessity of using its rivers to transport persons and goods during the early period of exploration and development. Access to the great domains which the early explorer claimed in the name of his sovereign, was had by way of the national water routes then existing; while the development of that rich territory was almost entirely dependent upon the use of these routes by the traders. Thus, water transportation had an early and continuing influence upon the political and economic growth of a large portion of the area lying between the Allegheny and the Rocky Mountains.

Before the introduction of the steam engine as a means of propelling vessels, the rivers of the Middle West were navigated first by canoe, and, later, by keel boats, flat boats, and a few sailing vessels. The commerce of these early days was largely a down-stream movement because, with the keel boats and the flat boats, down-stream movements were made with the current and up-stream journeys by man power. Cargoes were shipped from Pittsburgh, Pa., for instance, to New Orleans, La., and there the boats were taken apart and sold for what they would bring as lumber. Sailing vessels never were a success for river navigation in those early days because the channels were not buoyed or otherwise marked, and groundings and more serious disasters were too frequent to warrant any extended use of this type of vessel.

A few years after the introduction of steam as a means of propulsion (about 1810) the boats in use soon reached a degree of efficiency and reliability that permitted their being operated economically for up-stream as well as for down-stream traffic. The river settlements then began to increase in number and size; trade between various parts of the Mississippi Valley and the Gulf thrived; and New Orleans became the focal point of most of the domestic and foreign trade of the Middle West. By 1860, for example, there were about 3 600 arrivals and departures of river "packets" each year at that port, with more than 750 vessels of that type in commission on the Mississippi and Ohio Rivers.

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While water transportation was thus in its prime and had already determined the plan of development of the Mississippi Valley, rail transportation was still in its infancy. This new method of transportation found the framework of the economic structure of the country fairly well outlined when it became strong enough to become effective; it is only natural, therefore, to find that the first railroads were initially feeders to river ports as well as to seaports. As a result, the river communities received a further impetus in their development which was reflected in increased commerce on the rivers and also in greater stability of the regional plan for the interior, based on the rivers themselves.

When the rail lines between the Middle West and the Atlantic seaboard were completed traffic which formerly was routed by river to New Orleans and thence by vessel to export trade and to Eastern ports was diverted from the river routes to the direct rail routes to the East, and what began as a battle between two groups of ports resulted in serious injury to water transportation in the interior. Shortly thereafter came the completion of rail lines located parallel to the river lines; the latter were faced with a competition that they could not meet; and their general demise was then only a matter of time.

During the greater part of this period of thriving river traffic, the physical improvements to the inland water routes of the Valley were limited to projects for canal construction, undertaken by some of the States, with some Federal assistance, and some canals built by private undertaking. The main arteries of commerce, the rivers, were left as Nature made them. Many sections of the rivers, therefore, were impassable during low water; channels were unmarked; and snags were a constant menace. Apparently, it was not until after rail transportation had gained the upper hand in the struggle for carrying the commerce of the Mississippi Valley that a sincere effort to avoid the inevitable was made on the part of interests which had considerable investment at stake. It is somewhat significant that the first comprehensive undertakings for the improvement of the major inland water routes were not begun until the decade of 1870 to 1880, when projects for the betterment of navigation conditions on the Mississippi, Missouri, and Ohio Rivers were adopted by the Congress of the United States.

Even had they been promptly and effectively carried to completion, these improvement programs were begun too late to save the "packet" system of river transportation. Outmoded and obsolete, it had no place in the modern system of moving goods and passengers, and was due for death regardless of what might have been done to prolong its life. Nevertheless, although this particular method of transportation became practically extinct, there must have been something of the nature of John Brown in it for "its soul went marching on." Generations of men had made their living on the rivers, or were in some other ways directly dependent upon them; and they not only preserved the faith but they were determined that, somehow or other, river transportation would be returned to them if only the program of extensive improvement of navigation conditions could be completed. In this manner the river projects, launched well after the decline in the "packet" business had begun, were intermittently promoted until a new impetus came along and provided enough additional energy to bring them to their present state of near completion.

Navigation interests desirous of securing improvements on the Mississippi and Missouri Rivers were aided greatly in their efforts to obtain Federal appropriations for river works by two influences only partly related to water transportation. The great damages resulting from floods focused national attention on the general improvement of these streams from an entirely different angle, and, in addition, there was a desire to stabilize the banks which were quite subject to erosion during even intermediate and low-water stages. The alliance of the three interests involved in the control of these rivers has proved sufficient in recent years to insure the prompt execution and early completion of a program designed to serve them all.

The betterment to navigation conditions which has resulted from these programs of improvement to the rivers in the Mississippi Valley has permitted an efficient development of methods of water transportation which are entirely different from those of the packet-boat days. Passenger traffic (except for the excursion variety) has disappeared entirely. Freight traffic, which is now greater in volume and value than it was during the days when water transportation was the chief reliance of the commercial interests of the Mississippi Valley, is handled almost exclusively in large tows consisting of a number of barges adapted to carry the particular commodities which they are engaged in transporting, and pushed by towboats of ample power to handle up-stream as well as down-stream traffic. Naval architecture and marine engineering have made great progress with respect to the design of equipment suited to the various requirements of the water-transportation service. It is quite evident, therefore, that in treating the subject of present-day inland-water transportation in the Mississippi Valley one is dealing with facilities and equipment wholly different from those in use when the basis for the commercial and industrial framework of the interior was laid during the first half of the Nineteenth Century.

EXISTING WATER-TRANSPORTATION FACILITIES

Routes.—Of the 15 000 miles of rivers in the Mississippi Valley, said to be susceptible of improvement, only the main north and south and east and west arteries are under consideration in this paper. These arteries include the Mississippi River below Minneapolis, Minn., the Illinois Waterway from Chicago, Ill., to the Mississippi; the Ohio River; and the Missouri River below Kansas City. These comprise about 3 500 miles of completed or projected 9-ft channels. It is believed that all the more important factors involved in the problems affecting inland water transportation will be developed by thus limiting the scope of discussion. Improvements and operations on the tributaries are not sufficiently different from those on the main routes to justify their inclusion. A notable exception is to be found in the case of the improvement of the Tennessee River System by the Tennessee Valley Authority, but the basis of that project, as it is now (1937) being undertaken, is so fundamentally different that it is in an entirely separate category from that into which the projects herein described naturally fall.

The total expenditures chargeable to capital improvements for navigation on these main routes amounts roughly to about $550 000 000 to date (1937); and it appears that approximately $100 000 000 will be required to pay for the works necessary to complete these projects. The cost of maintenance and operation has been estimated to amount to $10 000 000 per yr.

Terminals.—With the exception of the Great Lakes cities of Milwaukee, Wis., Detroit, Mich., Toledo, Ohio, Cleveland, Ohio, Buffalo, N. Y., and the inland City of Indianapolis, Ind., all the great cities of the Middle West are situated on this trunk system of inland waterways. As a result extensive facilities have been provided at many places of importance for the transfer and storage of freight handled over these navigation routes. Pittsburgh has extensive equipment for unloading coal from the Monongahela River; its steel mills have ample arrangements for loading steel on to barges destined for points on the Lower Ohio and Mississippi Rivers. Coal-loading points have been established at Colona, Pa., Huntington, W. Va., and Ashland, Ky., on the Ohio River, and terminals for the receipt of water-borne coal have been provided at Cincinnati, Ohio, Louisville, Ky., and Evansville, Ind. Facilities for handling steel, pipe, petroleum products, building materials, and miscellaneous cargo are available at some of the aforementioned points, as well as at Paducah, Ky., and at Cairo, Ill.

On the Lower Mississippi River, equipment is available at all the river cities of importance for the handling of water-borne commerce of all kinds including, in addition to those mentioned, cotton, sugar, sisal, bauxite, bulk cement, molasses, and grain. On the Illinois Waterway System, only recently completed, facilities are available for the loading of grain at many points, of miscellaneous cargo at Peoria, Ill., and at Chicago, Ill., and of coal at Havana, Ill. On the Upper Mississippi River municipal terminals have been provided at Davenport, Iowa, Rock Island, Ill., Dubuque, Iowa, and St. Paul and Minneapolis, Minn.; and Kansas City has recently provided itself with terminal equipment to serve the barges navigating the Lower Missouri River.

Substantial investments have been made at all these terminal points and at many others. The writer has no knowledge of any cumulative estimate of the cost of all these facilities; the mere recital of them should be sufficient, however, to give an indication of the comparative magnitude of some of the elements of the subject under consideration.

Transport Equipment.—A study of the latest published Government report on the subject shows that the commercial fleet in active operation during 1935, on the main inland water routes under consideration in this paper, consisted of about 540 towboats and 2 340 barges. The aggregate installed horsepower in the towboats approximated 176 000; the carrying capacity of the barges totaled about 1 600 000 tons. Details as to size and type of towboats appear in Table 1. The replacement cost of this fleet (allowing $200 per 1 hp for towboats and $20 per ton for barges) approximates $67 000 000. More than 1 700 of the barges are built of steel.

TABLE 1.—Classification of Towing Equipment, Mississippi Waterways

Size (indicated horse power)	Steam						Motor					
	Steel Hull			Wood Hull			Steel Hull			Wood Hull		
	No.	Indicated Horse Power		No.	Indicated Horse Power		No.	Indicated Horse Power		No.	Indicated Horse Power	
		Total	Average		Total	Average		Total	Average		Total	Average
More than 2 500	2	5 400	2 700	....	....	....	1	2 600	2 600	....	....	....
2 000–2 499	9	18 785	2 087	....	....	....	....	....	....	....	....	....
1 500–1 999	9	15 600	1 733	1	1 500	1 500	....	....	....	....	....	....
1 000–1 499	10	12 070	1 207	2	2 400	1 200	6	6 360	1 060	....	....	....
500–999	52	36 620	705	17	10 445	614	8	5 325	665	....	....	....
300–499	19	7 245	381	29	10 760	371	10	3 670	367	....	....	....
200–299	7	1 470	210	24	5 820	242	15	3 440	229	7	1 895	271
100–199	9	1 300	144	28	3 825	137	29	3 715	163	42	5 425	129
Less than 100	1	95	95	13	880	68	20	1 230	62	169	7 850	46
Total	118	98 775	837	114	35 630	311	89	26 340	296	218	15 170	70

In 1935, there were twelve operators, each having in active service one or more towboats with a total of at least 2 000 hp and barge fleets of 10 000 tons or greater carrying capacity. These operators had 87 towboats with an aggregate installed horse-power of about 85 000 and 937 barges with a total carrying capacity of 911 000 tons. About one-half the total equipment in active service was thus operated by twelve major transportation companies. The towboats in use by these companies had propulsion equipment averaging almost 1 000 hp and the barges in use averaged nearly 1 000 tons in carrying capacity. It is appropriate, therefore, to assume the use of units of these sizes, or greater, in a study of modern inland waterway transportation costs.

As with other methods of transportation the cargo carriers are adapted to the kinds of commodities carried. Barges for carrying coal have been largely standardized, and are built to dimensions best suited for easy passage through the locks on the waterways they are required to traverse. These are open top barges and are suitable for the transportation of any other type of dry bulk cargo which does not have to be protected from the elements. Sand, gravel, and crushed stone are usually carried on deck barges, to permit draining if wet, and also because the greater weight per unit of volume of this class of material results in the requirement of less cargo space to obtain project draft. Special tankers have been developed for the transportation of liquids of different kinds, although in some cases liquid cargo is carried in side and bottom tanks built into barges primarily designed for carrying miscellaneous cargo.

One of the barge lines which transports more miscellaneous cargo on inland rivers than any other operator has been prominent in the development of the modern type of barge for the transportation of freight of that character. These barges have been standardized in two sizes. For operations on the Lower

Mississippi River, where a full 9-ft channel is available, barges designed to carry 2 000 tons are used extensively. (Each barge carries 2 000 tons of freight with a draft of 8 ft. The average running time for a tow, including all stops, is 8 days from St. Louis to New Orleans, and 15 days on the return trip.) The hull of the barge may be used for grain or for miscellaneous cargo; the cargo house which is built on top of the hull is also used for package freight of various kinds. Hatches and side doors have been standardized as to dimensions and as to spacing, and correspond closely with the door spacing at the terminals. Barges in this trade on the Upper Mississippi River and into Chicago (where vertical clearances are controlling) are similar in design, but built on a smaller scale.

Self-propelled barges are somewhat rare on the Mississippi System of waterways; quite contrary to practice on the canals and other waterways in the East. One barge line had three in service as express boats for several years, operating between St. Louis and New Orleans. They were self-contained units carrying about 1 600 tons of high-class freight on a draft of 7 ft. Although they made fast time in navigating, too much time was lost at the terminals, with the result that they were taken out of service in 1935.

Traffic.—Without going into details it should be sufficient for the purposes of this paper to state that the traffic carried on each component of this main system of inland waterways has been much greater in recent years than during the period when water transportation was the only means of transporting large quantities of freight over long distances. Furthermore, there has been substantial growth of traffic since the projects for the improvement of these routes have been nearing completion. Excluding the traffic on the tributaries, the total water-borne commerce on this system in 1915 approximated 15 000 000 tons; it now (1937) totals about 30 000 000 tons per yr. A large proportion of this movement is over a comparatively short haul and is composed of coal and building material on the Ohio River. The average haul has been gradually increasing, but whether this will continue is problematical, as an important factor in this tendency has been the rapid development of long-distance traffic, during the past 10 to 15 yr, over the Mississippi River routes.

WATER-TRANSPORTATION COSTS

There has been sufficient experience with reference to the cost of constructing and operating water-transportation equipment since about 1920 to make it possible to reduce this experience to useful data for those interested in the subject.

Equipment Costs.—Although there appears to be a wide variety in costs of towboats this is due largely to differences in size or class of operating machinery. The more modern type of steam towboats will be found to vary in cost, roughly, as follows:

Range of indicated horse power	Cost, in dollars per indicated horse power
500 to 1 000	250
1 000 to 1 500	225
1 500 to 2 000	200

Towboats using Diesel engines are usually slightly higher in cost. The foregoing data apply to installations with direct drive; if electrical drive is provided, the costs are higher. There is also a wide range in the costs of barges, but they may be grouped, roughly, as follows:

Type of barge	Cost, in dollars per ton of cargo capacity
Hopper type	15 to 20
Deck barges	18 to 25
Package freighters	30 to 40

Fixed Charges.—An approximate idea of the fixed charges which are usually applied to barge operations may be gained by reference to Table 2.

TABLE 2.—FIXED CHARGES IN BARGE OPERATION, AS PERCENTAGES OF THE TOTAL COST

Description	Towboats	Barges	
		Bulkers	Package freighters
Interest	6	6	6
Depreciation	5	6	4
Insurance	3	2	2
Taxes	2	2	2
Annual overhaul	2	2	2
Total fixed charge*	18	18	16

* Percentages of total cost.

Operating Costs.—Such costs include the items of wages, food, fuel, lubrication, repairs to towboats, repairs to barges, superintendence, and miscellaneous.

The wage item depends upon the size of the crew employed and also the locality in which the operations are conducted. One barge line pays an average of about $3 per man per day. Its operations extend throughout the entire stretch of the Mississippi River below St. Paul, and the crews on the towboats vary in number between 20 and 30 men. In the Pittsburgh District wages have averaged as high as $6 per man per day. The operators in that vicinity, however, have been able to use fewer men in the crews (normally, about twenty men on a towboat with a capacity of 1 500 ihp). Rates on the Upper Illinois River vary between $4 and $5 per man per day, and the crews more nearly correspond in number with those in the Pittsburgh District.

Subsistence costs vary between $0.55 per man per day for the barge line mentioned to as high as $1.10 per man per day in the Chicago District. Pittsburgh rates are in the vicinity of $0.75 per man per day. These rates depend upon the number of men employed and upon the availability of good markets.

Fuel costs vary in different parts of the country. In the Pittsburgh District coal is generally cheaper than fuel oil, whereas on the Mississippi and Illinois Rivers fuel oil has the advantage. Fuel oil requirements are best expressed in ton-miles of cargo transported per barrel of fuel. For steam towboats this factor averages about 2 600 ton-miles per bbl of fuel for oil-burning steamers and about 4 800 ton-miles per bbl of fuel for Diesel engines.

These performance data apply to operations in still water where ample depths are available.

Lubricating costs for steam towboats average approximately 3 mills per ihp for each full day of service. For towboats equipped with Diesel engines the cost of lubrication is about 8 mills per ihp per day for a two-cycle engine and about 4 mills per ihp per day for a four-cycle engine. These values are based on the use of a centrifuge for cleaning the oil and a price of $0.60 per gal.

Part of the total cost of repairs to towboats is included as a fixed charge under the caption, "Annual Overhaul." In addition, repairs throughout the season for steam towboats cost on an average of $0.03 per ihp per day and, in the case of Diesel engine installation, this cost is about $0.04 per ihp per day.

The cost of repairs to barges depends largely upon the service for which they are used. If used for handling bulk cargo, which is unloaded by heavy machinery, the total annual costs are comparatively higher than in the case of package-freight barges on which lighter materials are unloaded by hand. These two services represent the extremes, and repair costs will be found to vary, therefore, between 1% and 5% of the first cost of the barges.

Costs of superintendence vary with the character of service and the territory covered. A check of the various operations indicates that this item will amount annually to from 1% to 2% of the cost of the fleet. Miscellaneous charges include deck supplies, laundry, waste, and other small items and are found equal to $0.01 per ihp per day of service.

Performance.—A study of performance statistics of a large number of steam towboat operations on the Mississippi River gives the ton-miles per horsepower-hour, up stream, as 10; and down stream, 17. These values will be found to vary with the project depth and with the quantity of free water under the barges. If the barges are "hugging" the bottom, there is considerable drag, which is reflected in decreased speed for the same fuel consumption. For instance, in still water, performance factors would vary with the depth of water under the barges, as follows:

Depth of water under the barges, in feet	Performance, in ton-miles per horse-power-hour
5	15
2	12

Transportation Costs Consolidated.—Table 3 contains the estimate of costs for the several items involved in connection with the operation of a typical tow of 5 000 tons propelled at the average rate of $5\frac{1}{2}$ miles per hr in still water. Costs are estimated for different load factors applying to the propelling units and also for several load factors applying to conditions under which barges might be utilized. Table 4 gives the cost of transportation for self-propelled barges of three different sizes and under varying conditions as to load factor. The costs for the 1 600-ton unit were compiled from records of actual operations; those for the 2 400 and 3 200-ton units were computed for purposes of comparison.

Representative values have been used for unit costs of fuel and lubricating oil, and for wages and subsistence, and a suitable allowance has been made wherever necessary to correct for partial use of equipment. Referring to

TABLE 3.—COSTS OF WATER TRANSPORTATION BY STEAM TOWS

(Capacity, 5 000 tons; 1 500 indicated horse power; and speed of 5.5 miles per hour in still water)

Description	Load factor of barges (percentage)	ANNUAL COST, IN THOUSANDS OF DOLLARS*								Millions of ton-miles per year	Unit cost, in mills per ton-mile
		First cost of unit	Fixed charge	Bargemen's wages	Wages* and food on towboat	Maintenance and repair of barges	Fuel, lubrication, and maintenance and repair of towboats	Superintendence and miscellaneous	Total		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(*a*) TOWBOAT LOAD FACTOR, 80 PER CENT.											
Bulk cargo......	20	700	126.0	9.0	32	8.0†	70.5	9.0	254.5	158.4	1.613
	40	500	90.0	4.5	32	6.6†	70.5	8.0	211.6	158.4	1.336
	60	440	79.2	3.6	32	5.3†	70.5	7.0	197.6	158.4	1.248
Package freight ..	20	900	150.0	9.0	32	6.0‡	70.5	12.0	279.5	158.4	1.765
	40	600	102.0	4.5	32	5.5‡	70.5	11.0	225.5	158.4	1.425
	60	510	87.6	3.6	32	5.0‡	70.5	10.0	208.7	158.4	1.318
(*b*) TOWBOAT LOAD FACTOR, 70 PER CENT.											
Bulk cargo......	20	620	111.6	7.2	32	6.4	63.5§	8.1§	228.8	138.6	1.649
	40	460	82.8	3.6	32	5.6	63.5§	7.2§	194.7	138.6	1.404
	60	420	75.6	2.7	32	4.8	63.5§	6.3§	184.9	138.6	1.333
Package freight ..	20	780	130.8	7.2	32	4.8	63.5§	10.8§	249.1	138.6	1.797
	40	540	92.4	3.6	32	4.5	63.5§	9.9§	205.9	138.6	1.484
	60	480	82.8	2.7	32	4.2	63.5§	9.0§	194.2	138.6	1.400
(*c*) TOWBOAT LOAD FACTOR, 50 PER CENT.											
Bulk cargo......	20	580	104.4	6.3	32	5.6	49.4‖	6.3‖	204.0	99.0	2.060
	40	440	79.2	3.6	32	4.0	49.4‖	5.6‖	173.8	99.0	1.756
Package freight ..	20	720	121.2	6.3	32	4.2	49.4‖	8.4‖	221.5	99.0	2.238
	40	510	87.6	3.6	32	3.5	49.4‖	7.7‖	183.8	99.0	1.856
(*d*) TOWBOAT LOAD FACTOR, 30 PER CENT.											
Bulk cargo......	20	460	82.8	3.6	32	3.2	35.4¶	4.5¶	161.5	59.4	2.718
Package freight ..	20	540	92.4	3.6	32	2.4	35.4¶	6.0¶	171.8	59.4	2.892

* 300 days in commission. † 2% to 4% of Column (3). ‡ 1% to 3% of Column (3). § 90% of the corresponding values in Table 3(*a*). ‖ 70% of the corresponding values in Table 3(*a*). ¶ 50% of the corresponding values in Table 3(*a*).

Table 3(*a*), Column (3), the first cost includes: Towboats @ $300 000; bulk cargo barges @ $20 000; and package-freight barges @ $30 000. In Column (4), the annual fixed charge includes: Towboats and bulk cargo barges @ 18%, and package-freight barges @ 16 per cent. Towboat wages (Column (6), Table 3(*a*)), are based on an average rate of $1 350 per yr per man; and, bargemen's wages (one man for each two barges) on a rate of $900 per yr. The assumed food allowance for the towboat crew (Column (6)) is $250 per yr per man. Fuel and lubrication costs (Column (8), Table 3(*a*)) amount to $36 per yr per ihp, with the cost of maintenance and repair at $11 per yr per ihp. The costs in Column (9), Table 3(*a*), are based on an assumption of $20 to $30 per day for bulk carriers and $30 to $40 per day for package freighters. Table 3(*a*) is of no final use in itself but may be used in the process of computing water transportation costs if the necessary load factors are known. Unit costs are shown in Fig. 1.

TABLE 4.—COSTS OF WATER TRANSPORTATION, SELF-PROPELLED BARGES
(Speed in still water, 8 miles per hour)

Size, in tons	Indicated horse power	ANNUAL* COST, IN THOUSANDS OF DOLLARS										Millions of ton-miles per year	Unit cost, in mills per ton-mile
		First cost of unit	Annual fixed charges @16%	Wages†	Commisary‡	Lubrication§	Fuel‖	Maintenance and repair¶	Superintendence	Miscellaneous**	Total		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
						(*a*) LOAD FACTOR, 80 PER CENT.							
1 600	800	208	33.3	24.3	4.5	0.8	28.0	7.2	6.0	2.4	106.5	73.8	1.443
2 400	950	262	41.9	24.3	4.5	1.0	33.2	8.6	6.0	2.9	122.2	110.6	1.106
3 200	1 100	316	50.6	24.3	4.5	1.1	38.5	9.9	7.5	3.3	139.7	147.5	0.947
						(*b*) LOAD FACTOR, 70 PER CENT.							
1 600	800	208	33.3	24.3	4.5	0.7	25.2	6.5	8.4		102.9	64.5	1.596
2 400	950	262	41.9	24.3	4.5	0.9	29.8	7.7	8.9		118.0	96.8	1.219
3 200	1 100	316	50.6	24.3	4.5	1.0	34.7	8.9	10.8		134.7	129.0	1.043
						(*c*) LOAD FACTOR, 60 PER CENT.							
1 600	800	208	62.1			28.8			8.4		99.3	55.3	1.794
2 400	950	262	70.7			34.1			8.9		113.7	82.9	1.372
3 200	1 100	316	79.4			39.6			10.8		129.8	110.6	1.173
						(*d*) LOAD FACTOR, 50 PER CENT.							
1 600	800	208	62.1			25.2			8.4		95.7	46.1	2.077
2 400	950	262	70.7			29.9			8.9		109.4	69.1	1.583
3 200	1 100	316	79.4			34.7			10.8		124.8	92.2	1.356
						(*e*) LOAD FACTOR, 40 PER CENT.							
1 600	800	208	62.1			21.6			8.4		92.1	36.9	2.500
2 400	950	262	70.7			25.6			8.9		105.2	55.3	1.903
3 200	1 100	316	79.4			29.7			10.8		119.9	73.7	1.625

* 300 days in commission. † At $1 350 per yr per man; 18 members in the crew. ‡ At $250 per yr per man. § At $1.00 per yr per ihp. ‖ At $35 per ihp. ¶ At $9 per ihp. ** At $3 per ihp.

Load Factors.—A load factor in water transportation might be said to be the ratio between the total number of ton-miles resulting from one season's operation of a unit and the theoretically possible total movement. Therefore, to determine a load factor for a towboat it is first necessary to determine the total number of hours during which the towboat will be in commission. Multiply this by the number of tons in a capacity tow and by the still-water speed, in miles per hour; the result gives the theoretical total tonnage possible for the number of hours during the year in which the towboat is in commission. The actual performance will be measured by the number of hours the towboat is in line service, deductions having been made for terminal and mechanical delays and also for channel delays, such as groundings, time lost through passing through bridges or locks, etc. Another item that must be used to reduce the load factor for a towboat is the average loading of carriers as compared with the theoretical capacity load. The final item for consideration is the character of the cargo movement, itself; if it is a balanced movement no further adjustment in load factor is required; if the movement predominates in one direction correction must be made accordingly.

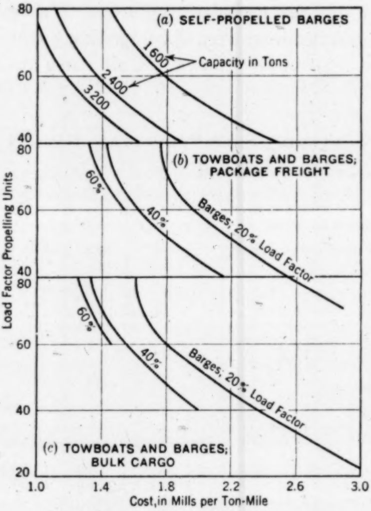

Fig. 1.—Unit Costs of Water Transportation

In the case of barges a similar procedure should be followed in determining the load factor for the operation under consideration. It will be seen that a very thorough knowledge of operating conditions must be available before these load factors may be determined intelligently.

In Justification

Preliminary.—Many notable papers have been published in an attempt to prove economic justification for many of the waterway projects that have been undertaken in the United States. Possibly as many contributions have been made on behalf of rail transportation. The waterway papers have been discussed and criticized by railroad advocates, while the railway point of view has been argued against by those favoring the waterways. One would think that all points for both sides of the controversy had been covered time and time again; that there was nothing more to be said, and that nothing could be gained by prolonging the argument; and yet the strife appears to continue; waterways are accused of "robbing" the taxpayer and of taking traffic away from their competitors, whereas the railroads are alleged to be engaged in the nefarious practice of reducing rates wherever water competition exists and making up the loss at the expense of their customers located where the advantages of water competition are not available. Of course, this attitude is understandable when it is realized that each side is attempting to protect its own interests; but the results are not necessarily along the lines of better understanding or of constructive accomplishments for the public at large.

Tangible Economic Values.—The main water transportation routes in the Mississippi Valley will have cost the United States at the rate of about $186 000 per mile of length when completed. Maintenance, including the operation of the locks and movable dams, will cost at the annual rate of approximately $3 000 per mile. The capacity of these water highways for carrying freight will vary. The open rivers will be able to carry practically an unlimited tonnage, but the canalized sections will be limited by the rapidity with which the carriers may pass through the locks of highest lift in those sections. For instance, the traffic from the Lower Mississippi River and the Missouri River to and from the Upper Mississippi and the Illinois Waterway System will be limited by the capacity of the lock on the Mississippi River at Alton, Ill.; through traffic between Chicago and the Valley may not exceed the capacity of the lock at Lockport, Ill., at the lower end of the Chicago Drainage Canal; and the capacity of the Upper Ohio River will be determined by that of the locks at the dams below Pittsburgh. With single locks at these three critical locations, as built or as proposed, the limits will approximate between 20 000 000 and 30 000 000 tons per yr, respectively. The construction of twin locks at these places would result in more than doubling the capacity of those sections of the waterways at a cost which would be comparatively small.

By way of comparison, it is understood that modern railroad construction costs in the Mississippi Valley vary between $50 000 and $70 000 per mile of single track, and that maintenance of way and structures cost on the average of from $2 000 to $3 000 per mile annually under heavy traffic. Furthermore, it is the understanding that the annual traffic capacity of a single-line railroad varies between 20 000 000 and 30 000 000 tons, depending on the class of freight which predominates. It is thus apparent that inland-waterway construction is much less flexible than is the case with railroad construction and, without regard to capacity, it costs more to build and to maintain. Furthermore, railroad distances between given points are generally shorter than those by water, which is an additional credit to railroad transportation when construction and maintenance costs are under consideration.

Conditions vary so much as to terminal operations that it is difficult to draw any general conclusions. Usually, rail terminals are located on more expensive property than water terminals so that total fixed charges are comparatively high; but since they are likely to have a higher use factor there is probably little difference between unit fixed charges. With operating costs conditions should favor the railroads because such transfer operations are generally conducted at one elevation, whereas at the river terminals there is a wide fluctuation between low and high-water stages which requires a considerable vertical movement of cargoes. Where joint rail and water transport operations are involved additional transfer operations are required, and as regards the waterway traffic in the Middle West, these transfers are always at the cost of the waterway carrier. This condition is somewhat different from the methods in vogue at several seaboard rail terminals where such costs are absorbed by the rail carriers.

As to comparative costs of the two methods of transportation, there should be no further cause for misunderstanding. Modern methods of inland water transport have been successfully in operation for a sufficient length of time, and enough data have been assembled on the subject so that fairly definite conclusions may be reached. First costs and fixed and maintenance charges for equipment for both types of transportation facilities on comparable operations are readily ascertainable, and actual operating costs for a variety of traffic conditions are known. Why argument continues along these lines is a question.

A study of various authorities on the subject indicates that for railroad operation in the Middle West the cost of maintaining equipment plus fuel, supplies, and wages, averages very close to 4 mills per ton-mile. Interest on investment in equipment and its depreciation (which are dependent upon the load factor when unit costs are under consideration) vary from about $0.0006 per ton-mile for a load factor of 100% to as much as $0.0015 per ton-mile when the load factor is 40 per cent. Total unit costs for this range of load factors, therefore, lie between $0.0045 and $0.0055 per ton-mile. Comparable costs for water transportation, as demonstrated previously, are from $0.00125 to $0.003 per ton-mile. The latter should be corrected for circuity as the water distances are generally longer than those by rail. Between points on the Ohio River this factor averages about 1.4; between Chicago and St. Louis, it is about 1.2; on the Lower Mississippi River, it approximates 1.5; and up stream from St. Louis the factor is 1.1.

Comparison of Costs.—It is now possible to make a rough comparison to determine the net tangible cost to the country at large of supporting traffic on the main inland waterways of the Mississippi Valley. If it is assumed that all the existing waterway traffic was carried by railroads serving the same points and allowing a cost of $50 000 per mile for the roadway, a capital investment of about $175 000 000 would be involved compared with the present cost of the water highways of $550 000 000. The annual traffic to be handled is about 5 500 000 000 ton-miles *via* water routes, the equivalent of about 4 000 000 000 ton-miles by rail. Comparative costs would then be as shown in Table 5(*a*). It is apparent, therefore, that the country at large is paying more for its transportation in the amount of about $22 000 000 per yr on account of the present use of the main inland waterway routes in the Mississippi Valley.

TABLE 5.—COMPARISON OF COSTS, IN MILLIONS OF DOLLARS

Description	(*a*) WATER TRAFFIC CARRIED BY RAILROADS SERVING THE SAME POINTS		(*b*) COMPARISON ON THE BASIS OF PRESENT WATERWAY IMPROVEMENTS		(*c*) COMPARISON IF WATERWAY TRAFFIC EQUALS THE CAPACITY OF A SINGLE-TRACK RAILROAD	
(1)	Rail (2)	Water (3)	Rail (4)	Water (5)	Rail (6)	Water (7)
Interest	17.5	44.0	35	52	17.5	52.0
Maintenance of way and structures	4.0	8.0	36	15	25.0	12.5
Operating charges	22.0	13.75	162	100	112.5	78.8
Total	43.5	65.75	233	167	155.0	143.3

When improvements under way are completed these water transportation routes could easily accommodate an annual traffic of at least 50 000 000 000 ton-miles and with the addition of duplicate locks at critical locations, this capacity could be doubled. If a comparison is made on the basis of the capacity of the present waterway improvement projects (in which case the first cost of railroad roadway to carry it would be twice the sum given previously) costs involved would approximate those in Table 5(*b*). In this case the comparison favors water transportation. If the water traffic totaled 35 000 000 000 ton-miles the corresponding railroad tonnage would be about 25 000 000 000, approximately the capacity of a single-line road. Under such conditions the comparison in Table 5(*c*) would apply. The future tangible economic justification of this large expenditure for waterway development, therefore, lies in the ability of industry and commerce in the Middle West to develop and support water traffic approximating 35 000 000 ton-miles per yr.

It is to be noted that these comparisons take no account of rates charged for either rail or water transportation. Rates are notoriously artificial as far as the railroads are concerned and where they are in competition with water they bear little relation to the cost of services performed. Water rates, on the other hand, are generally as close below the corresponding rail rates as the traffic will bear. In any event it is the costs that determine the net direct effect on the national wealth, and it is only when rates become so low as to be destructive of healthy competition or so high as to prevent industrial development that they affect a drain on national resources.

The foregoing comparisons of costs of transportation were intended to be made in a way that would avoid controversial questions. The results show the tax on the nation's pocket-book of the existing uncompleted system of waterways as reflected in transportation costs. Questions of subsidy to water carriers are thus eliminated; nor is consideration given to the subsidy enjoyed by rail lines paralleling the water routes paid for by shippers located where they cannot use them.

In one respect only are the comparisons incorrect; that is, with regard to the costs of the transportation routes themselves. For the water routes, the total costs result from a study of the records of actual expenditures by the Federal Government since the inception of the projects. These costs, therefore, are not replacement costs, but the accumulation of expenditures from sporadic appropriations made over a long period of years, a substantial proportion of which has been undoubtedly wasted by reason of the uncompleted nature of the construction works. For the rail routes replacement costs are used and, therefore, they make no allowance for wasteful expenditures of the past which are the experience of many great enterprises. There is no accurate method of arriving at a replacement cost of the waterway system; nor is there any way of telling how much has been spent to bring any given part of the railroad system of the country to its present efficiency and condition. In this respect, when the conclusion is drawn that inland water transportation over these particular routes costs the nation approximately $22 000 000 per yr, the rail carriers are given a substantial advantage.

Intangible Values.—In a discussion of the intangible effects of a major program of public improvements as involved in the development of water-transportation routes in the Mississippi Valley, it is necessary at the beginning to take as broad a point of view as possible. The investigator is concerned neither with the immediate effect on small communities nor with individual enterprises. Water carriers as well as rail carriers are only a means to an end, and that end is best expressed by the statement that the people still are engaged in building a nation.

Another point to be kept in mind is that most of the construction operations in connection with the improvement of these routes have been completed and the expenditures already made. The sum of $550 000 000 has been expended, 2 500 miles of channel finished, and about $100 000 000 will be required to complete the remaining 1 000 miles of channel to project dimensions. Arguments as to the wisdom of works already completed are somewhat academic as the money expended cannot be recovered without putting these projects to work. It would seem more to the point for all concerned to join efforts to insure as large a return on the previous investment as possible, whether it was good or bad.

There is little question but that the old "packet" boat was a controlling influence in the formation and growth of the "plan" for the Middle West. As a result, this "plan" was based on the great rivers, and as many as have been the changes in industrial development and in the forms of transportation which serve both commerce and industry, the fundamental elements of that "plan" still persist. The present problem, therefore, is simply to determine whether that "plan" has sufficient merit to justify its perpetuation as against any other that might be advanced.

All the great nations of the world have been developed on the premise that the sea is the base of the system of transportation. The world ports achieved their pre-eminence because they served as focal points for the lines of transportation to the interior and as gateways for commerce over the free routes of the high seas to domestic as well as foreign destinations. The controlling elements in this situation are: (1) That ocean transport was, and still is, cheaper than land carriage; and (2)—which is possibly of greater importance—that the water highways are free and dedicated to the unrestricted use of all.

By close analogy the interior of a great country may be developed to best advantage by preserving for the use of the general public the free water highways of its great rivers even at some expense, if necessary. The principle involved is identical. In the writer's opinion the price of about $22 000 000 per yr is a small one for this country to pay if, in making this investment, it accomplishes the multiple purpose of salvaging large expenditures made in previous years, of preventing a violent readjustment in the distribution of industries and population, and of preserving a group of minor "sea bases" located on a secondary coast line 7 000 miles in length.

There is another important set of intangible values involved in this question which can be emphasized to best advantage by quoting a short part of a statement[2] by the late C. E. Grunsky, Past-President, Am. Soc. C. E.:

"In his opening paragraph the author [the late William Murray Black, M. Am. Soc. C. E.] presents the conclusion that a new waterway line of transportation should not be established unless the need for it can be shown and unless 'if established, it will produce an annual saving in the cost of transportation greater than the interest on construction plus maintenance and operating costs.' There is another element which should sometimes be considered when the advisability of such a line of transportation is being studied, and by virtue of which many a project may be found advisable, despite the fact that it cannot, perhaps, for many years in the future, produce revenue to meet in full the interest charges against it. This element is the project's contribution to general prosperity as measured in terms of population, of business, and of property values. It becomes a factor because the expenditure of funds in its construction, and the utilization of the facilities which it provides when it is completed, bring increase of population; and to the extent that the growth in population of the zone coming under the influence of the improvement is thus accelerated, to the same extent will land values increase and wealth be created, which may fairly be weighed against the cost of the improvement when the advisability thereof is under consideration. Society, in other words, may reap material advantage from many an enterprise that would be condemned by the stockholder who weighs only the cost of the service against the prospective revenue."

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[2] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 553.

The application of the principle stressed in this quotation to the advisability of completing and utilizing the main water-transportation routes of the Mississippi Valley should not encounter serious opposition.

THE HIGH COST OF INLAND WATER TRANSPORTATION

By S. L. Wonson,[3] M. Am. Soc. C. E.

Synopsis

Due to its historical background, water transportation in the Mississippi Basin, in earlier years, impressed itself strongly into the national consciousness. After the decline of river traffic, following the fixation of traffic routes and the development of railways, having certain elements of inherent superiority in satisfying traffic needs, this early impression resulted in a public demand for the continuing development of river transportation at public expense.

This demand was largely based upon the supposed necessity of waterways to supplement inadequate facilities of the railways and to act as regulators of their rates and practices. Under present and foreseeable future conditions, these reasons have lost such validity as they may have had, and the justification of waterway development in the Mississippi Valley at the expense of the taxpayers now rests upon the test of relative economy of water and rail transportation, taking into account all elements of cost.

The rivers of the Valley are not of the class of waterways freely navigable in a state of Nature; they require large expenditures for channel improvement. The major elements in the cost to the public of transportation on these rivers are: The annual charges pertaining to Federal expenditures for improvement, and the service charges of the river carriers. Certain other elements of cost, such as those pertaining to bridges and river terminals, are not now known in total.

When the two major elements are reduced to a ton-mile basis, for comparison with the total cost to the public of rail transportation, which is closely known, it is found that they generally exceed the latter, in many cases to a marked degree, and that any impression as to superior economy of river transportation in the Valley is based upon a misapprehension.

Introduction

The question of waterway improvement for transportation purposes, particularly in the great interior basins of the United States, has been long and actively discussed, before this Society and elsewhere, and it would appear that every pertinent viewpoint has been adequately covered. The continued activity of the question, however, would seem to indicate that a realistic appraisal, from the viewpoint of the public interest, has not yet acquired general accept-

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ance, and that river improvement for transportation purposes has been and is being over-emphasized, to the detriment of the primary function of these streams, namely, that of drainage channels.

VIEWPOINTS

The rivers of the Mississippi Basin exercised an essential function in the national life from the time of the first explorers in the Sixteenth Century, through the years of the pioneers and settlers, and well into the period of settled trade and commerce that opened with the Nineteenth Century. As long as the muscles of men and animals were the only motive power available, river transportation held an inherent superiority in efficiency over any form of land transportation.

Consequently, there were impressed into the national consciousness certain feelings as to the superiority and virtue of river transportation as such, and as to the necessity of its continuance, in the future as in the past, as an essential part of the national life. It is a characteristic of such general impressions, upon other subjects as well as this, to persist for some time after their factual basis has passed, and to be evidenced, in discussion of the particular subject, by emphasis upon the emotional, rather than the realistic, viewpoint. The Mississippi Valley Committee of the Federal Public Works Administration, for example, writes of the waterways in the Valley, "they have a romantic and traditional interest which cannot be disregarded."[4]

DECLINE OF THE RIVERS

The function of the railroads as freight carriers did not develop immediately upon their first construction. Like the first river steamboats, the first railroads were primarily passenger carriers. An examination of the railroad map of 1840 will disclose the complementary relation of the early railroads to the waterways. West of the Alleghenies the first railroads were mostly short lines running to the Great Lakes and to various rivers. The same situation is marked on the railroad map of 1850. The map of 1860 shows a different picture. The shift of traffic from river to rail had begun, following the great change in direction of traffic initiated by the opening of the Erie Canal. The growth of the country was such, however, that neither of these causes produced an immediate decline in Mississippi River commerce; the year 1860, for example, exceeded all previous records in number of steamboat arrivals and value of goods received by river at New Orleans, La.

Professor Frank H. Dixon cites[5] two causes, independent of rail competition, for the rapid decline in Mississippi traffic after 1860: (*a*) The Civil War, which closed the lower river to commerce for several years; and (*b*) conditions at the mouth of the river which, after the introduction of steam navigation, seriously obstructed the access of ocean carriers to the Port of New Orleans, and which were not remedied until the completion of the Eads jetties in 1877.

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[4] Rept., Mississippi Val. Committee, PWA, October 1, 1934, p. 4.

[5] "A Traffic History of the Mississippi River System," by Frank Haigh Dixon, *Document No. 11*, U. S. National Waterways Commission, 1909, Govt. Printing Office, Washington, D. C

NATURAL DISADVANTAGES

In comparison with the service which the rail carriers were in a position to offer, some of the natural disadvantages of the rivers of the Valley as transportation routes may be mentioned as:

(1) Rigidity of location, preventing a universal and flexible service, requiring use of land transportation to serve interior points and involving additional time and expense for transshipment;

(2) In many cases the river courses lie across, rather than along, the major currents of traffic movement;

(3) Circuity, or greater distance between given points;

(4) Channel conditions, imposing limitations upon speed of movement and upon use of economical types of craft, and only to be partly remedied by large expenditures;

(5) Large variations in river level, requiring additional elements of dock and wharf facilities for handling goods between river bank and river craft; and,

(6) Closure of northern waterways during the winter season.

The rail carriers, in turn, held an inherent superiority over the rivers in transportation efficiency. This situation seems not to have been as clear prior to 1870 as it is in retrospect. Professor Dixon makes the interesting statement that:

> "* * * one of the causes assigned for the building of cars by shippers was the fear of the railways that the restoration of river business after the war would have such a serious effect upon their business that it would be unwise for them to make the necessary outlay themselves."

RIVERS NEEDED TO SUPPLEMENT RAILROADS

The shift of traffic from river to rail, and the rapid growth of the country and of its transportation needs after the Civil War, caused the railroad plant to be overtaxed at times and the situation of the waterways from the viewpoint of the public interest became the concern of various official committees and commissions, as well as of many private organizations and individuals. All these bodies based the need for inland waterway transportation upon the periodic inadequacy of the railroad plant as then functioning, and the national necessity for supplementary and additional transportation facilities.

The sudden and large increase in the nation's commerce and transportation needs during the World War again taxed the railroad plant to its limit, and, together with the taking over, by the United States, of transportation on certain waterways in 1918, apparently contributed to the feeling that land transportation could not be considered adequate for the nation's needs; that it must be supplemented by inland waterway development.

As the march of events and the passing of an era came to discount the correctness of this conclusion, emphasis seems to have shifted to a superior economy claimed for inland waterway transportation.

ECONOMY OF RIVER TRANSPORTATION

From the viewpoint of economy, waterways fall at once into two distinct classes: First, the oceans, the wide and deep estuaries of great rivers, and certain great lakes (including, in some cases, relatively short connecting channels); and second, rivers, such as the rivers of the Mississippi Basin, which have the natural disadvantages previously mentioned and which require large expenditures to permit navigation by a restricted type of craft. Waterways of the first class present no great limitation upon efficient type, size, and operation of the cargo carrier, and constitute transportation routes superior in economy to any present form of land transportation.

From the viewpoint of transportation economy no analogy can follow from one class of waterway to the other, unless there be considered and included the cost of creating and preserving such conditions of the rivers as navigation upon them may require.

Of the total costs of river improvement only a part is recorded so as to be available, namely, that part representing the direct expenditures of the United States and, in some cases, of political subdivisions. Substantial additional costs, such as those involved in the construction and reconstruction of highway and railroad bridges, combine to make up the true but unknown total. A finding as to the various elements of true total cost pertaining to public improvement work, including river improvement, was adopted by the American Railway Engineering Association in 1935[6] and is also stated in the 1935 report of the Committee of the Engineering-Economics and Finance Division on Principles to Control Governmental Expenditures for Public Works.[7]

Rivers as Regulators of the Railroads

Accompanying the thought that waterway development is necessary to provide additional transportation facilities, has been the viewpoint that improved waterways are natural competitors of the railroads and effective regulators of their rates. This theory was advanced as early as 1874 by the Windom Select Committee on Transportation Routes to the Seaboard of the United States Senate. The more clear-sighted commentators, however, have recognized the fallacy of this view.

In 1909, Professor Dixon stated that "if the purpose is to reduce railway rates, there are more direct and less costly methods of accomplishing this result." In its first report of 1910, the National Waterways Commission pointed out that expenditures on waterway improvement for the purpose of regulating railroads or reducing rates were not justified by either reason or experience.

The scope of railroad regulation has been so extended and strengthened in recent years as to justify the conclusion of these two authorities. The subject is treated by the National Transportation Committee, which, in its report of 1933 to the President, states, in part:

> "The development of regulation and of new methods of transport make it unnecessary for Government further to create and foster competition with or among railroads as a defense against monopoly. That is an expensive and ineffective attempt to do indirectly what Government has shown its ability to do directly. Regulation is sufficient."

Regulation of the railroads "has been practiced long enough and sufficiently extended to prove that it dominates competition or any other influence as the governing law of railroad practice. To the extent, that the monopoly inherent in the railroad franchise was a menace, it is of the utmost importance to recognize that current railroad regulation safely controls it."

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[6] *Proceedings*, A. R. E. A., Vol. 36 (1935) p. 238; also, *Bulletin 371*, November, 1934, p. 238.

[7] *Proceedings*, Am. Soc. C. E., February, 1936, p. 219.

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"In so far as government policies have been designed, by Federal intervention, to create and maintain competition with or among railroads as a defense against monopoly, they should be abandoned as wasteful and unnecessary. Regulation is sufficient."

The Public Necessity

In view of the foregoing, an attempt to appraise the fundamentals of the situation may be in order. The public necessity requires a satisfactory transportation service. A satisfactory service is one that is, to the highest practicable degree, adequate, efficient, and economical. Although these characteristics are more or less interdependent, they may be commented upon separately.

There seems to be general agreement that the railroad plant is adequate for the transportation needs it is called upon to meet and that it will continue to be so for some time to come. The total revenue freight transported by Class I rail carriers of the United States in 1929 and 1935, is shown in Table 6, Columns (2) and (3). The transportation performance of the major rivers of the Mississippi Basin—the Ohio, Missouri, and Mississippi—covering river traffic between Pittsburgh, Pa., Minneapolis, Minn., Sioux City, Iowa, and New Orleans, in 1935 was as shown for comparison in Column (4), Table 6; the figures for waterways include materials used in river improvement work as well as materials rafted.

TABLE 6.—Comparison of Revenue Freight Transported

Revenue freight transported:	1929	1935	
	Railways	Railways	Waterways
(1)	(2)	(3)	(4)
Millions of tons	2 452	1 427	41
Millions of ton-miles	447 322	282 037	5 487

In a broad sense, the railroads serving the Valley have a reserve capacity at least proportionately equal to that of the railroads of the United States. Considering that the railroad plant was not overtaxed in 1929, and that its productive efficiency has been increasing and will continue to increase, it would appear that the construction, maintenance, and operation of waterways in the Valley is no longer justified upon the broad ground that other agencies of transportation are, or will be, inadequate in capacity to serve the public need.

From the viewpoint of efficiency, the rail carriers have certain inherent or natural advantages, such as a superior universality and flexibility of service, and the power to render continuous transportation without delay or expense for transfer at intermediate points. Under an equality of conditions the relative efficiency of two or more types of transportation service will be clearly revealed by the test of public use. These conditions are such as would cause each type to offer a service reflecting its own natural advantages or disadvantages, as well as any superior or inferior degree of management, and are more or less as follows: (1) Substantial equality in respect of beneficial, restrictive, or punitive legislation and regulation; and (2) a system of service charges covering the total cost of the service.

Under such conditions there is little doubt as to the direction in which the verdict of public use would point. It is well known, however, that such equality of conditions does not now exist. The rail carriers are subject to a great mass of restrictive legislation and regulation, originally purporting to be in the public interest and now considered by many to extend far into the field of management. On the other hand, the river carriers are largely free to pursue such policies and practices as may seem to them appropriate, and are further enabled, by acts of Government, to offer their service for a charge which does not cover its total cost by a substantial margin; in other words, the users of the service are paying a part of the cost and the taxpayers are paying the remainder.

Even under the present inequality of conditions, the superior efficiency of rail transportation, based both upon its natural advantages and also upon alert and progressive management within the limits imposed by regulation, is generally conceded. Definite expression of this judgment may be found in voluminous shipper testimony before the Interstate Commerce Commission in one of its current general proceedings.[8]

Relative Costs

The third major requisite of transportation is that it be economical. Economy is necessarily relative, and certain approximately known elements of cost of river and rail transportation may properly be compared.

Broadly, the total cost of rail transportation to the public may be measured by the charge for the service, the freight revenue. In 1935 the average charge, or revenue, per ton-mile of revenue freight transported by the Class I rail carriers of the United States was 9.9 mills, of which 0.7 mill was directly refunded to the public in the form of taxes, thus reducing the average total charge to 9.2 mills per ton-mile. Substantially these figures apply to the railroads serving the Valley. For convenience, the rounded unit of 1% per ton-mile may be kept in mind as the outside total cost to the public of rail transportation.

The rivers of the Valley are of that class of waterways requiring large expenditures for navigation, and these expenditures do not appear in the accounts of the river carriers. Neither the costs of these carriers nor their charges for service are a measure of the total cost to the public of transportation on these rivers. This total cost includes at least the following elements: (1) Expenditures upon river improvement for purposes of navigation (*a*) by the United States; (*b*) by States and political subdivisions; and (*c*) incidental costs to public and private interests; (2) public expenditure on river terminals; and (3) service charges or costs of river carriers.

Expenditures by the Federal Government

The portion of river transportation cost paid by the Federal taxpayers, when reduced to the ton-mile unit, differs widely as between the rivers of the

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[8] I. C. C. Docket No. 26 712, Rail and Barge Joint Rates.

Mississippi Basin. In the following paragraphs these values are given for the major rivers, the Ohio, the Upper Mississippi, the Missouri, and the Lower Mississippi, and for a few of the tributaries. The data are derived from the Annual Reports of the Chief of Engineers, U. S. Army, and are for the year 1935, which is the latest for which ton-mile data are available. Interest at $3\frac{1}{2}\%$ and current maintenance costs are used, and no allowance is made for amortization. To produce unit costs comparable with those of rail carriage, a circuity factor is used, representing the excess of river distance over rail distance.

In order that the discussion may include every possible factor favorable to river transportation, a number of findings are quoted from the report of the Mississippi Valley Committee of the Public Works Administration, which seems to be the latest comprehensive and independent study of the subject. This Committee, stated by the Administration to be "a group of the nation's leading scientists and technicians," devoted a year to the preparation of a report upon the use and control of water in the Mississippi Basin, which was made public in December, 1934. The membership of the Committee included the Chief of Engineers, U. S. Army, and its attitude was one wholly sympathetic with river values. In considering the following data it should be remembered that transportation on these rivers is no new or untried experiment; it is older than rail transportation—as old as commerce in the Valley.

Ohio River.—During the period of the Revolution considerable trade developed between Ohio River settlements and New Orleans and *via* New Orleans to the Atlantic seaboard. A regular packet service of keel-boats was established on the river in 1794. The course of the river is largely parallel to the main currents of traffic; it flows through a section highly industrialized for many years and in which bulk commodities are found in great quantities. Tonnage on the Ohio exceeds that on the Mississippi from Minneapolis to New Orleans.

Notwithstanding these favorable factors, Federal expenditures on the Ohio River amount to 5.5 mills per ton-mile of river traffic. To make the total cost as low as the total for rail transportation would require that all the other costs of river transportation previously mentioned aggregate not more than 3.7 mills per ton-mile. It seems improbable that such is the case.

The Monongahela River should be mentioned as a case exceptionally favorable to low-cost river carriage. Its tonnage, but not its ton-mileage, has been larger than that of the Ohio; in 1935 it was about 86% of the latter. Disregarding circuity, which is slight, the Federal contribution to cost of transportation on this river is 1.3 mills per ton-mile; 85% of the tonnage is coal, moved down stream from mines along the river.

If it be asked, what interests have benefited from the expenditure of about $185 000 000 of Federal money on the Ohio alone, the Mississippi Valley Committee of the U. S. Public Works Administration answers:

> "Private carriers—largely coal and steel companies—now transport more than 95% of the total commerce on the Ohio River."
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> * * * * * * * *
>
> "Practically all of this traffic is handled by private carriers who pay no tolls. In other words, under present policy, the construction costs and the operation and maintenance costs of navigation projects are at Federal expense, and no

part of either of these costs is directly returned to the Government. The Committee believes that this policy should be modified."

Mississippi River Above the Ohio.—The first steamboat is said to have ascended to Fort Snelling, near St. Paul. Minn., in 1813. During the era of active settlement in the Northwest, beginning about 1845, the river carried a considerable commerce. Following the extension of railroads into the Trans-Mississippi region, traffic left the river rapidly because of its natural disabilities, such as the condition of the channel, its closure during the winter, and chiefly its location across the direction of traffic.

The contribution of the Federal taxpayers to the cost of river transportation on the main stem between Minneapolis and the mouth of the Ohio is 2 cts per ton-mile and maintenance alone amounts to 0.6 ct per ton-mile. One-quarter of the total tonnage is material for river improvement.

The Committee finds that as of June, 1934, expenditures on the Upper Mississippi System, plus expenditures required to complete the present project, exceed $217 000 000, exclusive of $40 000 000 for maintenance and operation; the Committee remarks:

"It is not possible by any calculations of business accounting to discover an economic justification for the vast expenditures on the projected improvement of these waterways; especially from the prevailing viewpoint of self-liquidation, but also even from the viewpoint of complete coverage of cost of maintenance and operation."

Missouri River.—Although the first steamboat ascended the Missouri River in 1819, its commerce has always been relatively insignificant. Its natural channel conditions are particularly unfavorable for navigation and its location up stream from Kansas City is equally unfavorable for the needs of traffic.

Disregarding circuity and any expenditure on the Fort Peck project, the cost to the Federal taxpayers of river transportation between Sioux City and the mouth is 22 cts per ton-mile of total tonnage and the maintenance cost alone is 4 cts per ton-mile. In 1929, more than 99% of the total tonnage was material for river improvement work. In later years, the percentage is simply stated as "large"; however, in 1935, rafted piling, stone, and sand made up 84% of the total tonnage.

The Committee finds that the total construction cost of improvements under way will be about $250 000 000 and characterizes this expenditure as having "very doubtful justification." Considering that $3\frac{1}{2}$% interest alone would require a ton-mileage of about forty-three times the present (in itself largely composed of Government materials), in order to reduce this interest cost to 1 ct per ton-mile, the characterization may be pronounced conservative.

As to the possibility that Missouri River traffic will ever be such as to justify this expenditure, or any substantial part of it, the Committee comments:

"Assuming that the more optimistic estimates of future traffic on the river will be realized, the savings to shippers which would result from free operation of the waterway would exceed by only a small margin the annual charge to be borne by the public for maintenance and interest on the investment. *** Most current estimates of prospective tonnage are exceedingly liberal, and if the present traffic on the section below Kansas City may be accepted as in any degree indicative of future prospects, the probability of those estimates ever being realized, even on a free waterway, is very slight."

To complete the picture, it should be mentioned that the construction period of the project has produced considerable business for the constructors, the producers of materials, and the transportation agencies, including the rail carriers.

As to the tributaries of the Missouri, the Committee finds that for the 5-yr period, 1928–1932, excluding non-recurring tonnage for construction of the Bagnell Dam, river traffic on the Osage cost the Government $3.50 per ton-mile for maintenance alone.

Mississippi River, Cairo, Ill., to New Orleans, La.—On the Mississippi below its three great branches one would expect to find, if at all, the great national waterway, carrying a large volume of traffic at a cost less than the possible range of any transportation cost by land. What are the facts?

Up to June 30, 1935, the total cost of construction and maintenance work done by the Mississippi River Commission above the Head of Passes was about $417 000 000, of which a relatively small amount pertains to the river above Cairo, Ill. The 1935 maintenance was $5 177 000. In recent years these figures include expenditures for flood control which are not segregated from expenditures for navigation. However, the Committee finds that the cost of improving this section of the river (Cairo to New Orleans) for navigation, including the amount appropriated for this purpose under the Flood Control Act of 1928, may be estimated conservatively as $190 000 000 and that the average annual maintenance is at least $2 700 000.

The Committee also finds that the average annual traffic for the 5-yr period, 1928–1932, exclusive of material used in Government work and of the coastwise and foreign traffic below Baton Rouge, La., was about 1 681 000 000 ton-miles and that, after allowing for circuity, the total Government subsidy provided to shippers by river would be about 9 mills per ton-mile.

Disregarding any expenditures subsequent to those considered by the Committee, a similar calculation for the ton-mileage of 1935 shows a subsidy of 7.2 mills per ton-mile.

The addition of costs pertaining to river terminals and the costs or service charges of river carriers will obviously produce a total ton-mile cost on this section of the river greater than that of rail haul.

The Committee comments that "the matter seems to merit more thorough and comprehensive consideration than it previously has received."

Tributaries of the Lower Mississippi.—These tributaries present a picture unfavorable to the economy of river transportation. Table 7 shows, for 1935, on the more important tributaries, the dominant tonnage, the total Federal subsidy per ton-mile, and that part of the subsidy represented by the year's maintenance cost. Circuity is disregarded.

Major Rivers Consolidated.—More broadly comparable with the substantially uniform unit cost of rail transportation, is the consolidated Federal subsidy to transportation on the aforementioned four major rivers. For the year 1935, the annual charge pertaining to Government expenditures on these rivers is

TABLE 7.—DOMINANT TONNAGE AND FEDERAL SUBSIDIES, ON TRIBUTARIES OF THE LOWER MISSISSIPPI RIVER

River	Character of tonnage	Total subsidy, in cents per ton-mile	Required for maintenance only, in cents per ton-mile
St. Francis	96% logs and piling rafted	1.7	1.3
White	98% sand, gravel, and logs	2.3	1.8
Arkansas	99% sand, gravel, and logs	9.6	4.7
Red	60% logs	45.7	7.6
Ouachita and Black	36% logs, gravel, and riprap	2.8	1.7
Yazoo and Big Sunflower	45% sand, gravel, and logs	13.3	3.4

$29 000 000 or, allowing for circuity, 10.1 mills per ton-mile of commercial river traffic.

OTHER EXPENDITURES

From time to time States and agencies of the States have made considerable expenditures on improvement of waterways in the Valley. In the case of highway and railroad bridges over these waterways, the additional expenditures necessary to provide the type of structure and the horizontal and vertical clearances imposed in the interest of navigation have been considerable, and there is a continuing element of cost in lifting traffic to the elevation of many fixed bridges. A number of cities in the Mississippi Valley have provided terminals, at their own cost, for the use of river carriers, and the results of these expenditures have not been satisfactory to the municipal taxpayers in all cases.

At present, there appears to be no comprehensive and authoritative compilation of these other expenditures on any of the major waterways of the Valley. Although the total would doubtless increase, perceptibly, the unit cost of river transportation derived from the cost of channel improvement, it is probably small in comparison with Federal expenditures.

REVENUE OF WATER CARRIERS

The Inland Waterways Corporation, owned and controlled by the United States, is the dominant common carrier by water in the Valley. Operating the Federal Barge Lines with modern and efficient equipment, its freight revenue is presumably an approximate measure of that part of the cost of river transportation represented by the necessary charge for carrier service. In 1935, this freight revenue was 3.85 mills per ton-mile.

It appears, therefore, that 3.5 or perhaps 4 mills per ton-mile should be added to the costs previously mentioned as representing the necessary revenue of the common carrier by river. The costs of private and contract carriers are doubtless less, although the cost of private terminals should be included; but, under present conditions, the provision at public expense of a free waterway to these carriers seems open to serious question.

Again, the circuity factor should be applied to the ton-mile cost or revenue of the river carrier, in order to make it comparable with the cost of rail transportation. On the Lower Mississippi, for example, the two factors—Federal expenditures and common carrier revenue—produce a ton-mile cost to the public of 12 or 13 mills as against a total cost of 9 or 10 mills for rail carriage.

Any consideration of the feasibility of common-carrier traffic on the river by the use of private capital should include the experience of barge lines operating between St. Louis, Mo., and New Orleans, from 1875 and 1903, as given by Professor Dixon.

The equipment appears to have been up to date for the time. The barges had a capacity of from 50 000 to 60 000 bushels of grain and the lines carried general merchandise as well. Fuel was carried for the round trip and prompt and reliable schedules were planned. One shipment in 1880 included: 6 757 bbl of flour, meal, and grits; 6 710 sacks of corn, oats, and bran; 1 500 packages of meat and lard; 150 bales of hay; and 24 992 bushels of bulk corn. In 1887, four lines were operating 16 towboats and 120 barges and were later consolidated into one company.

For the seventeen years, 1887 to 1903, inclusive, the published rates on grain from St. Louis to Liverpool, England, by river *via* New Orleans, were from 5 to 9 cts per bushel lower than those *via* rail to New York, N. Y., and, in the latter year, were 6 cts lower. Notwithstanding this differential, the movement of bulk grain by river showed a general decline after 1890, and by 1903, the barge lines seem to have passed out of the picture as an unprofitable investment.

At present, grain is not an important part of river tonnage. In 1935, after eliminating the tonnage in foreign and coastwise commerce on the lower river, wheat, corn, rice, and all other grains form the following percentages of the total tonnage: Ohio, 0.05; Upper Mississippi, 4; Missouri, 0.8; and Lower Mississippi, 1.6.

INTANGIBLES

The suggestion is sometimes made, and at first glance appears pertinent, that consideration of expenditures that have already been made on the waterways is "academic," apparently using the term in a somewhat disparaging sense. In effect, however, this suggestion is the same as stating that present and future action should ignore the experience of the past.

It is also suggested, at first with apparent force, that although the existing investment in waterways may not have been wholly wise, this investment should be preserved or "salvaged" by providing for its continued use. It is generally admitted that the United States has a very great surplus of transportation facilities. In so far as transportation efficiency is in the public interest, the use of, and the addition to, the less efficient agency is a drain upon the nation's resources and a check upon its total productivity. The real present question is not as to the salvage of an investment; it is as to the preference for the smaller and less efficiently functioning investment over the larger and more efficient.

No agency of transportation, rail, water, or any other, regardless of investment, should be preserved and fostered simply because of its past performance. If and when it becomes relatively inefficient, outmoded, and obsolete, the public interest requires that it be retired from use as rapidly as may be practicable. From the viewpoint of past performance, however, whatever it may be considered to be worth, there is something to be said for the rail carriers. In the words of the Mississippi Valley Committee:

> "A nation with our territorial extent, population, culture, and vital dependence of one section upon another, is unique. It has been made possible only by an elaborate and adequately supported railroad system. Therefore, any move, however desirable it may be in itself, which unduly infringes upon the serviceability of railroads, is to be deprecated."

SUMMARY

Considerations of "romance and tradition," the emotional viewpoint, although based upon an essential part in the national life formerly played by the waterways of the Mississippi Valley, is no longer a sound basis for expenditures upon the improvement and maintenance of these waterways for transportation purposes. The justification of these expenditures in the public interest has been based upon the supposed necessity of transportation facilities additional and supplementary to an inadequate capacity of the rail carriers, and also upon the assumed necessity of improved rivers as competitors of the railroads and regulators of their rates. These considerations may or may not have been valid in the past. To-day, they are no longer relevant; the public interest is based upon the relative economy of such available agencies of transportation as are capable of rendering a satisfactory service.

The total unit cost of river transportation in the Valley, including that part of the cost paid by the taxpayers, is, in general, greater than the total unit cost of rail transportation. Due to the fixity of river location, any benefits of river improvement paid for from the National Treasury tend to accrue, not to the nation as a whole, but to special or localized industries and localities.

The nation's credit is of the highest and is expected to remain so. Consequently, public expenditures will have to be repaid, with interest, by taxation. Expenditures which are an exercise of a power of Government, rather than an exercise of one of its basic and essential functions, should rest upon a basis of economic justification. Otherwise, they constitute a levy upon the nation's total consumption of goods and services.

CONCLUSIONS

The application of the foregoing to the matter of waterway improvement in the Valley would appear to prompt some such suggestions as the following:

(1) Projects upon which the cost of Federal maintenance exceeds the cost of using other transportation facilities, should, if such other facilities are available and adequate, be retired forthwith, or be turned over to such local interests as may be able to operate and maintain them on an economic basis.

(2) Projects upon which the cost of Federal maintenance is less than the cost of using other available facilities should, in justice to the taxpayers who provided and now support them, carry such service charges to the users as may be equitable in each individual case.

(3) Additional expenditures should not be made except upon a basis of economic justification; a definite showing, after consideration of all costs

involved and a realistic appraisal of traffic to be expected, that a satisfactory transportation service will be produced at a total cost less than that of existing agencies.

In the words of the Mississippi Valley Committee:

"It is believed that this phase of the subject has in some instances in the past been too lightly considered—if considered at all. Economic justification should be definitely established early in the study of any proposed river or canal improvement."

DISCUSSION

George Hartley,[9] Esq. (by letter).—An important factor mentioned in the introduction of the paper (see "Historical") was excluded from the remainder of the text.

Major Putnam calls attention to the fact that, in the past, those who have wished to secure improvements for navigation on the Mississippi and Missouri Rivers have been aided greatly in securing Federal appropriations for river works: (*a*) Because great flood damage focused national attention on river improvement from an entirely different viewpoint; and (*b*) because "there was a desire to stabilize the banks which were quite subject to erosion * * *." No further references to either flood control or erosion are made in the paper, and, in a later statement, when evaluating the cost of capital improvements, no specific mention is made as to what proportion of the capital improvements may be assigned to flood control and erosion. For example, under the heading, "Existing Water Transportation Facilities: Routes," the author states: "The total expenditures chargeable to capital improvements for navigation on these main routes amounts roughly to about $550 000 000 to date (1937)." The specific reference to navigation at once infers, by reference to Major Putnam's previous statement, that some allowance may have been made in an attempt to proportion specific charges between navigation and the two factors, flood control and erosion. In the event that the total cost, as given, includes the expenditure for flood control and erosion, the capital cost utilized as a comparative economic study later in the paper could very well be reduced by an amount equal to the cost of these improvements, and the resulting costs would place the waterway at once in a much more favorable position when viewed from an economic standpoint relative to railroad transportation.

No attempt has been made to evaluate the capital charge as well as the operation charges on bridges erected over these rivers to correct for increased cost of construction due to navigation requirements. Mr. Wonson makes the following statement (see "Other Expenditures") with respect to this feature:

> "In the case of highway and railroad bridges over these waterways, the additional expenditures necessary to provide the type of structure and the horizontal and vertical clearances imposed in the interest of navigation have been considerable, and there is a continuing element of cost in lifting traffic to the elevation of many fixed bridges."

The charges referred to by Mr. Wonson are directly chargeable in a comparative analysis to the capital cost of the waterway. In most instances, were it not for the navigation requirement, all the bridges constructed over the part of the rivers considered in the Symposium could be reduced a distance of approximately 50 ft and still provide more than sufficient elevation to clear the highest high water. This reduction in height might result in a saving of approximately 50% of the capital cost of construction. Furthermore, the reduction, if allowed, would result in a program of new bridge construction over these rivers, and to consider, economically, the advantages potentially released thereby, would be incalculable.

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[9] With Robinson & Steinman, Cons. Engrs., New York, N. Y.

Bridges have always been at a disadvantage when navigation requirements were the controlling economic factors. No attempt is ever made by the U. S. War Department to determine, economically, the relative value of the proposed bridge with respect to the vessels requiring increased clearance. These decisions are always in favor of water-borne traffic, and the War Department has always upheld any reasonable objection to reduction in clearance requirements. A bridge costing millions of dollars may never be constructed if some yachtsman using the waterway should require a clearance that would increase the cost of the structure to such an extent that the project would have to be abandoned.

W. D. FAUCETTE,[10] AND J. E. WILLOUGHBY,[11] MEMBERS, AM. SOC. C. E. (by letter).—The region included in Mr. Wonson's paper is a land area encompassing the Mississippi River and its tributaries. The truth, as disclosed by Mr. Wonson, is sound for general application to all inland river and canal transportation in the United States. That truth applies whether the relative land area to length of river and canal is great (as for the great interior land area drained by the Mississippi), or of less extent, as for the peninsula of Florida. This peninsula has a form such that no producing area is more than 75 miles from the coast. The annual rainfall is large, and there are many rivers from the interior to the sea. The rivers are drainage channels now flowing in submerged valleys, the filling of which was the result of the recent subsidence of the peninsula. Florida has a long shore line with many harbor indentations, and much Federal money has been spent on recommendation of the Chief of Engineers, U. S. Army, to develop a river traffic for Florida products to the cities along the Atlantic and Gulf Coasts, the river to be used to the seashore, and then the Atlantic Ocean or the Gulf of Mexico. This peculiar condition makes for minimum river haul and maximum ocean haul, as compared with the area discussed by Mr. Wonson. The inland waterways in Florida that have been created into a system at Federal cost aggregate: Thirty projects, including 1 869 miles, and with a 1935 traffic of 57 233 000 ton-miles. Federal expenditure, including cost of construction equipment, to June 30, 1936, has been $31 836 000. The average annual maintenance for the 5-yr period ending in 1936 was $425 000. Service at 3½% on the total expenditure of $31 836 000 is $1 111 250 per yr. The total annual maintenance and service charges are $1 536 250. The cost to the Federal Government in 1936 was 27 mills per ton-mile, not including the cost of vessel operation, or service on capital invested in vessels, wharves, etc. This ton-mile cost of 27 mills will be increased as compared with railway and highway transportation, due to the circuity of the water route. The four principal railways in the peninsula of Florida in 1935 handled a freight traffic of 4 337 243 000 ton-miles at a cost of 10 mills per ton-mile, which includes the total cost to the user—service on capital, maintenance of railroad and equipment, and cost of transportation.

The high cost of inland water transportation in the Florida Peninsula is demonstrated as follows:

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[10] Chf. Engr., Seabord A. L. Ry. System, Norfolk, Va.

[11] Chf. Engr., A. C. L. R. R., Wilmington, N. C.

Description	Cost, in mills per ton-mile
Inland Water Transportation:	
Service on total of permanent-way expenditures to 1936	20
Maintenance of permanent way (5-yr average, ending 1936)	7
Total paid by taxpayers	27
Estimated cost of transportation, paid by user	4
Aggregate cost for inland transportation	31
as compared with inland rail transportation:	
Service on total expenditures for permanent way and equipment, cost of maintenance of permanent way and equipment in 1936, and cost of transportation	10

It is of interest to note that the service charge on the cost of permanent way of a water carrier for freight service is 20 mills per ton-mile. The service charge on the cost of permanent way for rail transportation, when allocated as between freight and passenger traffic, develops only 2 mills per ton-mile for the Florida Peninsula.

In addition to the low cost per ton-mile of rail transportation as indicated in the foregoing list the four principal railways in Peninsular Florida paid taxes for the year, 1936, to the State, and other civic units in Florida, equal to $2 698 401, which is equal to 0.6 mill per ton-mile on freight traffic carried, leaving a net cost of inland rail transportation, as compared with inland water transportation, of 9.4 mills for rail against 31 mills for water.

Mr. Wonson touches upon the question of waterways having rigidity of location, which prevents universal and flexible service; and, in his "Summary," he shows that improvement when made, tends to accrue not to the nation as a whole, but to special and localized industries and localities.

This prompts the suggestion that, when it is being considered, inland water transportation should also comprehend consideration of the service features in connection with water-borne traffic. In the final analysis the question of service of transportation to the nation, is inextricably interwoven with considerations of economics. Probably at no time during the past 10 or 20 yr has the matter of transportation service entered so largely into problems pertaining to commerce and industry. This transportation, of course, means moving one's goods from where they are to where they are to be sold; and delivering them, with reasonable and suitable promptness, in a condition such that they can be sold. In order to serve the greatest number and maximum requirements for all classes of commerce and industry, of course, the complete transportation instrumentality should not be limited and rigid in its location and should not be circumscribed by being adaptable to only certain classes of industrial products, which narrows the field of usefulness. On the contrary, railroad facilities as they stand, offer and are able, in substance, to carry all classes and character of goods and commercial products with speed, safety, protection, and responsibility, and (when all factors of cost are properly considered and taken into account) economically, as compared to water transportation. The railroads permeate and serve the entire nation. They are not rigidly held to any one geographical location. They are performing transportation to a nation, highly developed and still developing. Therefore, in the public interest and in the expenditure of public funds, consideration must be given to the fact that rigid location for water transportation narrows it to certain commodities and is not a broad instrumentality that finds itself easily justified. The facts are that when goods and commerce are to be moved, it is service and quality and expedition of that service that is sought—a fact which cannot be ignored.

C. D. **Bordelon**,[12] Esq. (by letter).—The authors of the papers comprising the Symposium—otherwise of generally divergent views—agree that inland waterway transportation was essential to the very life of the Middle West and the Ohio Valley at least until about the 1860's. Before that time there was virtually no other means of mass transportation of goods and people.

Even at the middle of the Nineteenth Century there were no railroads west of the Mississippi River; St. Louis, Mo., Cairo, Ill., Chicago, Ill., and New Orleans, La., had no lines of railroad whatever until about 1852, and the important cities of New Orleans, St. Louis, Chicago, Cincinnati, Ohio, Pittsburgh, Pa., and the "Twin Cities" (Minneapolis and St. Paul, Minn.) were connected only by watercourses. It is small wonder then, that the inland waterways were regarded at that time as of such great value to the life and development of the nation.

The authors are also in agreement that the subsequent extension of rail lines to include all the vast territory tributary to these main rivers brought about the general demise of river transportation. These rivers were no longer indispensable to economic existence.

The essential differences between Major Putnam and Mr. Wonson may fairly be said to be whether there is at this time any economic justification for the further expenditure of public funds to create and maintain inland waterways whose use is free to private purposes as well as to operations for profit; and whether inland waterway costs should include interest on funds spent in creating and maintaining the channels. An appraisal of these differing points of view, as well as of governmental policy in respect to the inland waterways, may be aided by a review of certain facts probably, in the main, forgotten.

Beginning this brief discussion with the period in which the railways had virtually supplanted inland water carriers, and at which time also the capacity of the rail lines was occasionally over-taxed, it is found that the organized movement for a revival of inland water transportation began in the early 1890's and crystallized in 1895. During that year the International Waterways Convention was held in Cleveland, Ohio, followed five or six years later by the First National Rivers and Harbors Congress, in Baltimore, Md. The assembling of these two conventions, joined with the work of various commercial associations, stimulated new interest in waterway transportation. In 1902, Congress passed a Rivers and Harbors Act providing for the establishment of a

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[12] Asst. Freight Traffic Mgr., Mo. Pac. Lines, St. Louis, Mo.

Board of Army Engineers to review, from an engineering standpoint, all recommendations for waterway projects. In 1903, the people of New York voted $101 000 000 for the improvement of the Erie Canal.

Many conventions and congresses were held in 1907, and in that year, acting in part on the belief he had voiced in his Memphis, Tenn., speech and as a part of his general conservation program, President Theodore Roosevelt appointed the Inland Waterways Commission with the then Senator Theodore E. Burton, of Ohio, as Chairman. In the preliminary report of that Committee, in 1908, it was stated:

"While the railways of mainland United States have been notably efficient in extending and promoting the production and commerce of the country, it is clear that at seasons recurring with increasing frequency they are unable to keep pace with production or to meet the requirements of transportation."

The Commission further declared that, although navigation of the inland waterways had declined with an increase in rail transportation during the latter decades of the Nineteenth Century, it was clear that the time had arrived for restoring and developing such inland navigation and water transportation as upon expert examination appeared to confer a benefit commensurate with the cost, to be utilized as a necessary adjunct to rail transportation.

"While trustworthy estimates cannot be made without further data," said the Committee, "it is reasonable to anticipate that congestion of interstate commerce can be obviated in a large measure by judicious improvement of waterways adapted to barge and boat traffic, at a figure much less than estimated by competent authorities for so increasing railway facilities as to meet present needs."

The Commission recommended that the President, with the advice and consent of the Senate, appoint a National Waterways Commission to bring into co-ordination the Corps of Engineers of the Army, the Bureau of Soils, the Forest Service, and other branches of public service in so far as their work related to inland waterways, for the purpose of making further investigation of various kinds.

The National Waterways Commission, composed of twelve members of the Senate and House of Representatives, came into being as the result of an Act of Congress, dated March 3, 1909. The duty imposed upon this Commission by statute was to investigate questions pertaining to water transportation and the improvement of waterways, and to make recommendation to Congress.

In its preliminary report in 1910 this Commission declared that although there had been a tendency to improve waterways for the purpose of lowering railway rates, it could not endorse this as a desirable policy to adopt; that such a policy rested, in the first instance, upon the transparent fallacy that the railways constitute an intrenched and uncontrollable monopoly which could not be reached by legislation or other orderly legal methods; and that the only way in which to compel them to lower rates is by the expenditure of large sums of money.

It recommended that a definite policy be adopted regarding the relations between railways and waterways. Such a policy, the Commission stated, should be based on the idea that the purpose of improving waterways was to secure additional means of transportation, and not to reduce rates or regulate the railways. Experience had demonstrated, according to the Commission's report, that waterways could not be relied upon as the great cheapeners and regulators of railway rates, which they once were supposed to be. The report also contains the statement that no European country used its waterways to control railway rates, that function being accomplished by the proper administrative bureau of the Government, and that the waterways were improved with the idea of securing additional means of transportation.

In its final report, in 1912, it was said that some of the economic factors theretofore unfavorable to the development of water transportation were gradually changing. Population was increasing rapidly. The average increase for the decade preceding 1912 had been more than 1 590 000 per year. Industrial development of the country was also making rapid strides, according to the report, and the value of agricultural products had doubled in the preceding ten years.

The Commission found that the movement of traffic had increased entirely out of proportion to the increase in transportation facilities. During the period, 1900 to 1910, according to the report, the railway mileage increased about 36%; the number of locomotives increased 56%; the number of freight cars, 56%; while the ton-miles of service performed increased 80 per cent. Previous to that time, the development of transportation facilities had increased more rapidly than the demand for them, and the railways made special efforts to secure traffic of any kind for their idle equipment, even on very low rates. With the rapid expansion which took place between 1904 and 1907, however, all the surplus capacity of the railways was soon exhausted, and a traffic congestion ensued.

The car shortages of 1906–1907 caused widespread complaint and dissatisfaction, and increased the belief that the railroads were inadequate to meet the needs of the country's commerce. Herbert Quick,[13] one of the most ardent advocates of inland waterway development, declared that the railroads could not handle the nation's traffic. He referred to the car shortage in 1906 and 1907 as establishing this fact, and stated:

> "The best railway description of the situation is that we have been trying to force a three-inch stream of commerce through a two-inch pipe of railways; that we need over 75,000 to 120,000 miles of new track, and so many new cars and engines that on the whole there is not enough iron in the country to meet their needs, not labor enough to make and install the new equipment and track, and not enough money to pay for the transactions."

This thought persisted during the years prior to the World War.

In the "Census of Transportation by Water," a special report of the Census Bureau for 1916, it is stated that, although the development of inland waterways had not continued in the preceding decade to the extent desirable, further development was necessary to meet the demands of existing commerce, to prevent the recurrence of congestion at the ports and to permit water-borne commerce among the States.

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[13] "American Inland Waterways," by Herbert Quick (1909), p. 75.

The World War brought the situation to a head. The movement of traffic which it caused undoubtedly over-taxed the then existing railroad plant. Water transportation was partly taken over by the Government as a means of transport supply and to supplement existing transportation services in sections where abnormal production prevailed.

It is of more than passing interest to observe that the entry of the Federal Government into the water-transportation business was due to the exigencies of war. In the Annual Report of the Hon. William G. McAdoo, Director General of Railroads, to the President in 1918, the former stated that he had appointed a committee to make a prompt investigation and to suggest a definite plan for the additional use of internal waterways so as to relieve or supplement the railways under the conditions caused by war.

The Annual Report of the Chief of Inland Coastwise Waterways Service to the Secretary of War for the fiscal year 1920 stated that the particular purpose of the Committee on Inland Water Transportation, and that to which its efforts were mainly addressed, was the study of the feasibility of utilizing the waterways for the relief of war-time freight congestion of the railroads, which was then beginning to be keenly felt; and, that the necessities of the circumstances required the adoption of a plan that would give immediate relief to shippers and the public by moving a fixed quantity of water-borne freight to the relief of other facilities. He stated further that as rail-transportation facilities became yearly less able to handle, alone, the entire traffic business of the Mississippi Valley, and as annual periods of rolling-stock shortage brought increasing losses to the mercantile communities of the Central West, active interest began to be directed to the interior river system.

Corroborating this thought, Maj. Gen. T. Q. Ashburn, in his testimony in March, 1935, before the Joint Committee of the Senate and House considering the bill[14] to regulate water carriers, stated in part: "Please remember that the rehabilitation of our interior waterway commerce began with the crying need of our interior communicative system for more and better means of supplying the needs of our troops at home and overseas."

The experience of the war led to the deep-seated belief in many quarters that the railroads would never be adequate for the transportation needs of the country and that they would have to be supplemented by some other agency, which later was considered to be the waterways. This belief was repeatedly expressed by those who investigated inland waterway development or who championed the inland waterway movement. For example, James E. Smith, President of the Mississippi Valley Waterways Association, stated in 1919:[15]

> "The time has come when the people of our whole country are insisting upon the Government adopting a broad, comprehensive plan for the improvement and use of all of our important navigable rivers as transportation highways.
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> " * * * The people of our country know that our railroads have not kept pace with the growth of our commerce, and that its further growth and expansion and the full development of our country's resources are absolutely dependent upon the use of our navigable waterways * * *."

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[14] S. Doc. No. 1632, H. R. Doc. No. 5379, 74th Cong., 1st Session, March, 1935.

[15] *National Waterways Magazine*, March, 1919.

At the same time Jackson Johnson, President of the St. Louis Chamber of Commerce, made the following statement:

"Heretofore the Mississippi Valley has not had sufficient outlet to encourage intensive production. Agricultural, industrial, commercial and mining resources of the Valley are so varied and so unlimited, that it was not surprising that commerce, growing out of these resources, was expanding beyond the capacity of the railroads to handle, even before the war added to their burdens; and now with the return to peace conditions, the transportation outlet by rail is not encouraging. It cannot be for several decades to come, even at inestimable expense. There has been no railroad development at all in the last two years. In fact, there is less trackage now than there was two years ago, and yet there is an industrial condition that demands a greater outlet. We must find a market for the overplus of manufacture and agriculture, which was developed by the intensified war production."

The statement by A. W. Mackie, Manager of the Mississippi Section, Mississippi-Warrior Waterways, follows:

"Wonderful as has been the development of railroad transportation in the United States, commercial development employing that transportation has been equally marvellous, and we now face a situation more serious to business progress than is generally recognized. That situation is one of continued rapid development of industry and production without adequate expansion and development of rail facilities. Proof of this statement is offered every time there is a heavy crop movement on, as in the recent war emergency, a heavy demand upon transportation. The railroads alone cannot provide the necessary transportation facilities and service. We must, then, employ the proven capacity of our improved inland waters."

During the consideration by the House of Representatives of the original Inland Waterways Act, Mr. E. E. Denison, who was on the Committee, and more or less in charge of the bill, emphasized the supposed inability of the railroads to carry the nation's commerce.[16] To the same effect are the following conclusions in the 1922 report of the International Joint Commission on the St. Lawrence Waterway:

(1) The then existing means of transportation between the tributary area in the United States and the seaboard were altogether inadequate, and the railroads had not kept pace with the needs of the country.

(2) Although war conditions had something to do with the dislocation of railway traffic in the United States and, although various other factors should be taken into account, such as the congestion of traffic at certain critical points between the West and the Atlantic Seaboard, and the abnormal demand for cars at certain times of the year, the fundamental difficulty lay rather in the phenomenal growth of the population and industry throughout the Middle Western and Western States, a growth with which the railroads had failed to keep pace.

(3) The solution of the problem called for the utilization of every practicable means of communication, and particularly of the natural waterway extending from the Atlantic into the very heart of the continent.

That the belief of the Department of Commerce that the further and continued improvement of inland waterways was justified was based at least in part on inadequate rail transportation facilities, and this is clearly demonstrated in the statement of the Director of the Bureau of Foreign and Domestic Commerce in submitting, in June, 1923, to the then Secretary of Commerce, a survey of inland water transportation in the United States. The Director stated that, although interest in the development of internal water routes had never been entirely lacking in recent years, particularly those during and immediately following the World War, there had been a definite focusing of attention on rivers and canals as a means of supplementing the periodically inadequate transport system. He said that it was the opinion in many quarters that the railroads would be unable in the future to increase their capacity to the extent demanded by a rapidly expanding commerce, and that it would be necessary to rely to a greater extent than formerly on water routes for the carriage of both bulk and package freight.

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[16] *Congressional Record*, Vol. 65, Pt. 9, p. 8720.

Even after the inauguration of the program for the rehabilitation and expansion of the national railroad system, the notion of the permanent inadequacy of the railroads continued. Expressions of this so-called inadequacy are found in the hearings before the Committee on Interstate and Foreign Commerce, House of Representatives.[17] The so-called Denison Act, which resulted, gave rise to the Court proceedings relating to the Commission's power to prescribe, without a hearing, through routes and joint rates between the railways and inland water carriers.

Later, and in 1930, Mr. Theodore Brent, in an address before the Thirty-Sixth Annual Convention of the Ohio Valley Improvement Association, said that during the last 75 or 80 yr, the American public had given its attention almost exclusively to the development of railroads as a means of transportation. "Since the beginning of the Twentieth Century, however, there had been rude awakenings," said Mr. Brent. Furthermore, one need but read the elaborate and emergency provisions of the Interstate Commerce Law, dealing with congestions and break-downs of service, and remember the conditions that caused their enactment, to realize that the railroads had marked limitations, and that these reflections had brought studious people to renew their pleas for the development of the inland water resources as auxiliary means of transportation.

Senator Joseph E. Ransdell, in an address at the same Convention, stated that shortly after the National Rivers and Harbors Congress was formed in 1906, he traveled throughout the United States, everywhere talking waterways, but that his voice was like one in the wilderness; that no interest was taken in waterways, but that gradually, as the result of educational work of the Rivers and Harbors Congress, of the annual conventions which were being held, and of the many speeches made in the United States by the field directors and agents, some public sentiment was developed.

The divergence of views of these two waterways spokesmen is remarkable. Mr. Brent referred to the pleas for inland waterway development as an auxiliary means of transportation, and Senator Ransdell deplored the lack of public sentiment which manifested itself, in some degree, only after many years of intensive educational work. It may be said that the need for new and additional transportation facilities lent great strength to public demand and served largely to secure the appropriations of generous sums of money for the development of the inland waterways of the United States.

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[17] H. R. Doc. No. 10710 as amended by H. R. Doc. No. 13512, 70th Cong., 2d Session.

It is proper to conclude, therefore, that the periods of greatest interest in inland waterway transportation can be traced generally to those of railway congestion. The great majority of the people, apparently, have been indifferent to the question, except on those occasions in the past when railroad facilities and service have proved inadequate.

No one will deny that the existing railway plant is capable of handling much more tonnage than is available, or in reasonable prospect. The peak traffic of 1929, amounting to more than 447 billion revenue ton-miles, was handled with no car shortage. This compares with the traffic volume of 410 billion revenue ton-miles in 1920 during which year there was a reported maximum shortage of 147 309 cars.

The potential capacity of the railways is only the result of a plan of expansion adopted shortly after the end of Federal control. Early in 1923, railway executives agreed upon this program, looking toward the rehabilitation of facilities, and to an increase in efficiency and economy in operation. The entire program rested upon the basis of investment of large sums of new money in the industry. During the succeeding eight years, until the end of 1930, the program of heavy annual expenditures for additions and betterments to plants was consistently advanced. During that 8-yr period nearly 6.75 billions of dollars were so expended. The program had a twofold purpose: First, to supply the country with adequate rail transportation; and, second, to assure the public that the transportation would be efficiently and economically rendered, from the standpoint both of the user and the producer of the service. The program still continues.

In the report of Federal Co-Ordinator of Transportation, Joseph B. Eastman,[18] of March 10, 1934, the statement is made that from 1920 to 1932 the net total expenditures for new transportation facilities amounted to $19 110 000 000, or substantially the same as the railways' investment in 1920. Included in that sum is an investment of $423 613 000 in pipe lines for the transportation of petroleum products; $12 500 000 000 for highways and streets; and $604 000 000 of Federal money for the improvement of rivers and harbors.

With this great increase in transportation facilities and the fact that the physical expansion of the nation is largely accomplished, it is apparent that there is little if any justification for the continued spending of millions of dollars yearly on the inland waterways as supplements to land transportation facilities.

Despite the conviction that water transportation was necessary to supplement the railways, it has generally been considered by responsible authorities that the cost necessary for the creation and maintenance of waterways is a factor to be given consideration. The report of the Windom-Select Committee on Transportation Routes to the Seaboard, in 1874, stated that, after a most careful consideration of the merits of various proposed improvements, taking into account the cost and other factors, the Committee came to the conclusion that certain water routes would be feasible and advantageous. The following is quoted from the report: "The Committee believes that the water route suggested should constitute free highways of commerce, subject only to such tolls as may be necessary for maintenance and repairs." To quote further from the report: "If, however, Congress shall deem it expedient to require them to provide interest on the cost of construction and the means for ultimate redemption of the principal, the whole improvement will involve only a loan of Government credit."

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[18] Senate Doc. No. 152, 73d Cong., 2d Session.

The preliminary report of the Inland Waterways Commission, in 1908, contains the statement that inland waterway transportation should be restored wherever, upon expert examination, it appeared that a benefit commensurate with the cost would result; and, that the practicability of any waterway project depended not only on local and general demands of commerce, but upon various factors, including physical and economic considerations, entering into or tending to counterbalance the cost.

"The cost of facilities for carrying freight," quoting from the Commission's report, "whether borne by the Federal Government or by private capital is a burden upon the resources of the country. While the tendency of water improvements to lower freight rates is an important element to be considered, the fundamental criterion should be whether a railway or a waterway, whether constructed or improved, will be a profitable investment of capital." In its final report, submitted in 1912, the National Waterways Commission reached the conclusion that if inland waterways cannot afford cheaper transportation than railroads, the sums spent for navigation could be more profitably utilized for increasing railway mileage and efficiency, and that in determining the cost of transportation by water each ton should be charged with its proportion of the cost of improvement even if, in actual practice, this expense is borne by the Government.

The Commission found that a frequent method of demonstrating that water transportation is cheaper than rail is by comparing the average cost per ton-mile on some waterway with that for all the railways in the United States but that no such exact comparisons as these can be made for the reason that the rates compared do not include the same elements of cost. It made the definite statement that any comparison of transportation cost by rail and by water would be of value only when the two rates include similar elements of expense. The Commission held that whether water traffic should be charged with all the expenses incurred for aiding navigation, or whether these should be met, in whole or in part, by taxation, is a question of public policy.

As to differentials in favor of the water carriers, the National Waterways Commission declared that the main problem is whether the cost of transportation by a water route is sufficiently cheaper than that by rail to warrant the granting of a certain differential in its favor.

The Department of Commerce takes the position that, although the Government has authorized the expenditure of many millions of dollars to provide the most improved type of equipment for the experiment it has undertaken, and the benefits of its experience will be available to private capital, it may be that the experiment will not prove remunerative if all factors of cost are taken into consideration, including interest on the sums spent in constructing the channels; and, furthermore, that for services on channels already made navigable, or on canals already built, the returns need cover only current operating expenses and necessary maintenance of channel, terminals, and floating equipment. The Department further stated that, although the expenses for these waterways are borne by the Government, they are a drain on the economic resources of the country.

Recognition that the "savings" in transportation costs accruing to the users of barge service are made up by the taxes paid by the general public, is found in the Annual Report of the Chief of Inland and Coastwise Waterways Service, for 1923, wherein it is stated that there should be a free interchange of freight with the railroads, on the ground that otherwise it would be unfair discrimination "to tax the public at large" for the benefit of a small section of the country.

In an address at Louisville, Ky., on October 23, 1929, President Herbert Hoover, Hon. M. Am. Soc. C. E., stated that "as a general and broad policy, I favor modernizing of every part of our waterways which will show economic justification."

The Board of Engineers for Rivers and Harbors, of the U. S. War Department, in co-operation with the United States Shipping Board, deals with the subject in these words:[19]

> "The primary measure of the economic value of any waterway system is the savings in transportation costs which it affords. Usually, this saving is shown by comparing the water transportation cost with the rates for moving the same goods between the same points by rail. Included in this cost of water transportation should be the charges for interest on capital investment in waterway improvements plus annual cost of operation and maintenance."

In an address before the Thirty-Sixth Annual Convention of the Ohio Valley Improvement Association, in 1930, C. O. Sherrill, M. Am. Soc. C. E., in discussing certain data relating to Ohio River operations, stated that they were based on the real cost of transportation by water, including the interest charges on the costs of the improvement, as well as operation and maintenance costs.

The United States Chamber of Commerce submitted to the so-called Shannon Committee, investigating Government competition with private enterprise, a number of principles, among which is found No. IV, reading as follows:

> "Grants of public money, whether as gifts or as loans and regardless of the conditions attached, to particular forms of business enterprise, or for the special advantage of particular forms of business enterprise in their competition with other lawful forms, cause unfair damages to private citizens in their lawful pursuits and this damage cannot be offset either by pretended or realized benefits to other citizens or classes of citizens."

The report of the Shannon Committee,[20] states, among other things, that one of the facts which has been demonstrated in the investigation is that transportation by barges of the Inland Waterways Corporation is not cheap transportation. To quote the words of the Committee: "When a shipper ships by railroad, the rate which is charged by the railroad is the total charge for the transportation of material." The report further stated that the Barge Line pays nothing for taxes, makes no attempt to earn a return on its investment, and pays nothing for the use of the stream or to reimburse the Government for its expenditures for construction and maintenance. It also declared that part of the transportation cost represented by the maintenance of roadway, interest on investment, and taxes, which is paid by the shipper when he ships by rail, is borne by the taxpayer when the materials move by the Federal Barge Line, and that this burden which is borne by the taxpayer is much greater than the difference between the railroad freight rate and the Barge Line freight rate.

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[19] "Transportation on the Ohio River System" (Interim Report), Library of Congress Card 27-26771.

[20] H. R. Doc. No. 1985, 72d Cong., 2d Session.

The National Transportation Committee, in its report of February 13, 1933, reached the conclusion that Government assumption of all or part of the costs of inefficient competing transport as a defense against monopoly is no longer warranted and should be abandoned, and that, as a general principle, inland waterways should bear all costs of amortization, interest, maintenance, and operation of the facilities for their navigation. The report concluded that the studies of the Committee showed no economic benefits to the country commensurate with the money spent on inland waterways; that these costs bore heavily on the taxpayer as a direct burden. The Committee found it difficult to justify the wasteful outpouring of hundreds of millions of dollars for results so barren of economic returns.

It will be recalled that the National Transportation Committee was composed of the Hon. Calvin Coolidge, the Hon. Alfred E. Smith, and Messrs. Bernard M. Baruch, Clark Howell, and Alexander Legge.

In an address before the National Rivers and Harbors Congress, at Washington, D. C., on April 30, 1934, Joseph B. Eastman, Federal Co-Ordinator of Transportation, stated:

"The point is that the cost of construction of the new waterway and the annual cost of its maintenance are as much a part of the real cost to the public of the new transportation facilities as are the cost of construction and the cost of maintenance of the steamship or barges which operate over the waterway. The water carriers do not bear the expense of constructing or maintaining the waterway under our present practice; nevertheless, it is there—it cannot be escaped and, in one way or another, it is a burden upon the people of the country."

Similar views were expressed by Mr. Eastman in an address before the American Life Convention, at Chicago, on October 10, 1934, and before the Convention of the Mississippi Valley Association, at St. Louis, on November 26, 1934.

The report of the Mississippi Valley Committee of the Public Works Administration, which Committee was composed, according to the Department of the Interior, of a group of the nation's leading scientists and technicians, had the following to say on this point:

"A system of accounting is also required through which the capital investment, as well as maintenance and operating costs, on any given going navigation project can be currently determined * * *. The nation undoubtedly needs more definite and disinterested data to show the relative costs, taking in all factors, of transporting commodities by inland waterways in comparison with other media of transportation. Factual data are needed on this subject to establish justification or non-justification for either the building or continuation of any waterways program whether it be extensive or limited."

To summarize: (*a*) As recently as ten years ago justification for inland waterway development at public expense rested almost entirely on the supposed inadequacy of the railways; (*b*) the railway plant is now, and has been for many years, capable of handling much more tonnage than is reasonably in sight—much more than was moved in the peak year of 1929, and without car shortage or delay; (*c*) overwhelming expert opinion holds that the cost of providing and maintaining these waterways is a drain on the resources of the country; and (*d*), there has been a tremendous expansion in the transportation facilities of all kinds since 1920, so that to-day there are far more ways to move goods than there are goods to be moved.

In presenting this discussion, the writer does not have in mind so much a rebuttal of the conclusions or theories presented in the Symposium as the need for emphasizing, in this connection, the background and history of waterway development in the United States. In the earlier days, there was an absolute necessity for waterways and in the consideration of new projects it was the very definite view of various authorities, including governmental bodies, that costs incurred in creating and maintaining inland waterways were a fixed burden upon the people—one that must be borne by them through taxation.

If the writer has interpreted Major Putnam's theory correctly (especially as given in the text following Table 5) he has not given much weight to this fact. In common with all advocates of further inland waterway developments he ignores entirely the undisputed fact that land transportation (that is, rail, motor, or pipe line) is now fully capable of handling all the commerce of this country. This is a condition, of course, that did not exist even as late as 75 yr ago. In other words, the reasons that prompted and justified the creation of waterways at public expense, to move the Nation's goods, have disappeared completely.

J. E. GOODRICH,[21] ESQ. (by letter).—Much information of interest is contained in the papers of this Symposium. Fully realizing his incapacity to present any meritorious critical discussion of the opinions of experts in the matter, the writer requests that his comments be considered in the nature of inquiry.

Major Putnam has defined one important characteristic of the movement of commerce on an inland waterway—that is, the load factor of the floating equipment engaged therein. It is to be hoped that, in his closing discussion, he will present data as to actual existing load factors based on his definition. It may be found that they are quite low. For instance, he indicates that the commercial fleet in active operation in 1935 on the waters under consideration in his paper, included towboats with an approximate horse-power of 176 000 and barges with a carrying capacity of 1 600 000 tons and, furthermore, under his comparison of costs, that the annual traffic to be handled is about 5 500 million ton-miles. Using these data, it appears that the actual average load factor of the towboats is about 24% and of the barges about 8.7 per cent. It is possible that the foregoing is an improper derivation, or that after the waterways have been fully developed, the average operating load factor may be somewhat improved. At any rate, before a reliable comparison between water and rail costs can be made, a reasonable determination of the probable load factor is essential.

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[21] Senior Engr., with U. S. Div. Engr., St. Louis, Mo. Elected an Assoc. M. Am. Soc. C. E., on June 13, 1938.

It appears that the main purpose of comparisons between the costs of transportation by land and inland waterways is to determine whether additional waterway improvements should be undertaken. Obviously, an investigation for that purpose should take into account the extent to which existing transportation facilities are adequate. Reduced to a simple case in which a waterway under consideration for improvement is to serve an area now adequately served by rail, should not the comparison be based on the total cost of transportation in the area served by the two systems before and after the waterway is improved and placed in service? Before the waterway is put into service the costs include the fixed and operating charges of the railroad alone. When the waterway is in service the total cost includes the fixed and operating charges of the waterway and the carriers thereon plus the same fixed charges on the railroad as existed previously, as well as operating charges on such freight as is left to the railroad. The only way the railroads could avoid these fixed charges (which persumably the public must pay) would be to abandon the service. Even if this were permissible, it would entail a destruction of capital which, "in the long run," would be paid for by the public. If this view is entitled to consideration, it would appear necessary to include in the balance sheet of water *versus* rail transportation an evaluation of the public loss due to rendering the existing rail facilities less economically efficient.

It may be contended that by reducing the cost of certain commercial movements, water facilities would stimulate industry so that the traffic diverted from the rail line would be restored from other sources. Industry, however, is affected by taxes as well as by direct transportation costs. If the waterway improvement increases the tax bill to a greater extent than it reduces the direct transportation bill, it is difficult to foresee a resulting general stimulation of industry so as to bring about increased commerce in the country as a whole. On the other hand, a waterway which results in a substantial reduction in combined transportation and tax costs may be an actual benefit to the railroads as well as to other interests. In investigating the merits of a waterway to serve an area already adequately served, it should not be assumed that benefits of this character will arise until it has been shown from other considerations that the waterway is warranted and will actually reduce total costs. It should be shown that the horse exists before it is assumed that the cart will be drawn. Thereafter, these factors, too, should be taken into account and thoroughly analyzed in determining the extent to which the waterway improvements are worth while.

A knowledge of relative cost without consideration of the relative value of the service obtained is not sufficient to indicate the worth of an investment.

It is generally recognized that waterway and rail transportation are not equally expeditious, flexible, and dependable, and these inequalities should enter into the economic study. Perhaps the best indications available at this time of shippers' views as to these inequalities are the differentials between rail and barge line rates necessary to place the two types of service on a competitive basis where common carriers are involved. It is possible that shippers' views in this respect may be modified as the waterways become more adequately improved and their services better established.

Economic comparisons of rail and waterway transportation costs are frequently made in which the interest on the cost of the waterway is computed at a lower rate than that applicable to railroads. Such methods seem improper if the purpose is to show the inherent superiority of waterways aside from the advantage that they possess due to low-cost Federal financing, but proper if the purpose of the comparison is confined to determining the most economical medium under actual present methods of securing capital.

If economic comparisons were limited to reductions in the cost of freight movements, and the foregoing ideas were given application, some proposed inland waterway improvements might not be found so meritorious as to warrant the degree of enthusiasm often afforded them. However, waterways may be beneficial to a substantial volume of pleasure-boat navigation. In some degree waterway improvements, through increase in low stream flows, or the concentration of head, may facilitate the production of power. They may result in flood-control benefits by the storage of flood water, stabilization of banks, improvement of the hydraulic characteristics of streams at flood stages, and the supply of vessels for rescue work, although these benefits may be offset somewhat by the stimulation of objectionable and inadequately controlled concentration of industry in the flood-plains of streams. The availability, in operating condition, of ordinarily excess transportation facilities to meet unanticipated emergencies, is apparently of some benefit. Under certain conditions the undertaking of waterway improvements which approach the border line of economic feasibility may be warranted because it will afford productive employment of labor which, otherwise, would receive non-productive relief funds. Without involving sentimentalism, these items may properly be given consideration in an economic comparison of rail and waterway transportation.

H. R. FAISON,[22] M. AM. SOC. C. E. (by letter).—Opportunity to take part in the discussion of water transportation costs in an impartial technical publication is welcomed by the writer. After all, finding the value of improvements for navigation is a technical problem. Most of the publicity accorded this important topic in the commercial press is too partisan in viewpoint to serve the man who is seeking facts. The "picture" is too likely to be colored, perhaps unconsciously, by the interest of some particular section, industry, or type of carrier; or distorted by an array of selected statistics which illuminate only one side. As a consequence some theories as to the cost of water transportation, not strictly in accord with sound economics, have apparently gained wide popular acceptance even among engineers. In spite of Mr. Wonson's obvious sincerity, some of these theories leading to unsound conclusions seem to have crept into his otherwise able and fair treatment of the subject. With his basic principles the writer is in complete accord.

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[22] Senior Engr., U. S. War Dept., Washington, D. C.

The writer does not advocate providing or maintaining, by artificial stimulants, any route or means of transportation that proves uneconomical. There would be no point in trying to hold traffic on the rivers to keep water service alive if no general public benefit resulted. He would make an even broader application of this principle to cover outmoded and uneconomical transportation routes and service of all kinds in which the public has a stake. Differences with Mr. Wonson arise, not as to principles, but as to methods of appraisal and bases of comparison between the several types of transportation. It seems incumbent upon engineers confronted with waterway problems for solution in the interest of the people as a whole to exercise particular care in adopting the proper "yardstick." Because there are well known inherent draw-backs to barge transportation, it cannot soundly be assumed that all inland waterways are bad investments. Averages obtained by grouping good and bad projects together are obviously valueless. It is frequently found in the case of individual improvements that favorable factors, fully recognized and evaluated, outweigh all natural disadvantages.

Circuity.—Prominent among such disadvantages stressed by Mr. Wonson is the fact that river routes are generally longer than competing rail routes. Like most of the other elements of comparison, circuity in the case of individual waterways can usually be measured and given a definite value. Mr. Wonson attempts to do just that, but he derives his circuity factors from a comparison of river mileage with the shortest rail distance, ignoring competing rail routes. He uses these factors to raise the actual unit cost of freight movement by water to a higher sum for comparison with rail cost. This procedure gives no consideration to the fact that the great preponderance of rail freight also travels by circuitous routes. The Federal Coordinator of Transportation finds that the average rail carload moves 11% farther than would be necessary by a direct line in common use.[23] As a result of their analysis of freight movements by rail between river ports, the barge lines have petitioned the Interstate Commerce Commission to recognize officially the rail circuity factor of 20% in excess of short line mileage in the Mississippi and Ohio Valleys.[24] In cases of competition with water routes involving relief from the long-and-short-haul requirements of the Transportation Act, it is not uncommon to authorize rail routes as much as 50% longer than the short line. It should be remembered that the circuity of some of the principal inland water routes is less than that. The canalized Monongahela River is actually shorter, as a matter of fact, than competing rail lines, but the fact is dismissed in Mr. Wonson's analysis as negligible. Obviously, the circuity factors in the paper under discussion must be reduced to conform to the facts, if the reader is to arrive at the correct answer.

Amortized Expenditures.—It was not found possible to reconcile Mr. Wonson's distribution of costs among the several interests affected with published statistics. Apparently he charges such items as bank protection, and, in fact, all the money ever spent on inland streams except flood control allotments, to existing navigation facilities. According to his theory these charges must be borne by existing water lines on their present tonnage or improvements should be abandoned. In this situation the engineer must face the fact, squarely, that the millions spent for purposes other than navigation, and for navigation improvements which have been outmoded and abandoned or superseded by modern projects, are not involved in any conceivable way with his problem.

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[23] Freight Traffic Rept., Paragraph 71.

[24] Interstate Commerce Commission, Docket No. 26712.

Only a small part of the open channel work done on the Ohio and Warrior Rivers, for example, contributed in any manner to the present efficiency of these waterways, but it served its purpose for many years and its cost was amortized by contemporary river commerce. At most, only the cost of useful work necessary for present navigation has any practical bearing. On the Warrior River, canalization was begun in the Nineties and completed just prior to the serious congestion of the Nation's transportation system incident to the World War. Justification of expenditures providing this alternative freight route between an essential producing area and tidewater would scarcely have been seriously questioned at that time. Such waterways cannot be duplicated over-night, and naturally it is impossible to expand and contract permanent investment already made, to synchronize with fluctuating traffic volume and business cycles. In this connection it is far from clear just what end would be served with respect to all these past expenditures if the waterways were to be abandoned now as recommended in Mr. Wonson's paper.

Interchange of Traffic.—Mr. Wonson's analysis indicates that benefits due to the improvement of inland rivers are restricted to communities along their banks. The evident intent of all legislation having to do with water transportation is to secure the widest possible distribution of benefits paid for by the general public. The clause of the act establishing rail-water differential routes and rates is obviously designed to extend to "dry land" territory the same scale of savings enjoyed by ports along the waterways. Barge lines have in fact instituted a broad system of traffic interchange with railroads and motor truck lines extending far into the riverless regions of the country. The effect would doubtless be felt more generally were it not for the openly avowed unwillingness of the railroads to co-operate. As a group, their opposition has ranged from challenging the validity of the legislation in the Courts and opposing each proposed extension of the system, to the current proceeding before the Interstate Commerce Commission aimed at cancellation of all routes and differentials through which localities inaccessible by water already enjoy the benefits of barge service.[24] It is desired to emphasize at this point that the writer does not mean to imply that there is anything fundamentally wrong with this perfectly natural attitude of rail and water competitors toward each other, their interests being involved primarily in the profitable operation of their individual properties. The engineer must recognize, however, that these interests cannot be expected to coincide with each other nor with the interest of the country at large.

Distribution of Benefits.—For reasons obscure to the writer, Mr. Wonson seems to hold that savings resulting from common carrier water service may be counted as public benefits, whereas savings from contract or private water carriage are not public benefits. Congress does not attempt to assess the benefits resulting from the various types of river improvement to any particular region or group of beneficiaries, except where the effects are local in character. The waterways are made open to all, and because of the competitive element it is impracticable for the producer, carrier, or distributor of goods to monopolize the benefits. The writer has long recognized the futility of trying to trace savings from river improvement to their ultimate recipients, or to devise any system of charges that would distribute the burden more equitably than the present one in which the public pays for its waterway and the shipper pays for transportation on it. He agrees with Mr. Wonson that improvement for navigation is not warranted if the potential aggregate of such costs is greater than by other available means of transport. He differs from him in the method of measuring these costs, and the significance of the results. The engineer's problem is not to identify the beneficiaries, but to determine whether there is, in fact, a net economic saving to be distributed.

Tonnage Estimates.—Mr. Wonson attaches great significance to the relative public use of rail and water facilities. On the other hand, his estimates of the volume of water commerce that can reasonably be expected are based on the comparatively small tonnage which the boat lines thus far have been able to develop and hold in the face of competitive rail rate depression. It is necessary to bear in mind that inland river improvements comprise a system that is still under construction and far from complete. Records show[25] that the growth of water commerce is in the direction of longer and longer hauls as the individual waterways are progressively extended and connected into a system. The period for building up permanent water traffic involving the gradual development of storage warehouses, grain elevators, cotton compresses, industrial plants, and auxiliary services at the water side without sacrificing existing facilities, is a period of evolution reasonably expected to extend over a considerable number of years. It is believed that the greatest impetus toward building up a representative volume of tonnage on waterways will result from the eventual adjustment of freight rates to conform more closely than they now do to the cost of the transportation service. That this trend is officially favored is evidenced by numerous decisions of official regulatory agencies; for example, a concurring opinion of a member of the Interstate Commerce Commission in the Petroleum Products case,[26] reads in part as follows:

> "I am much inclined to believe that a system of rates for all agencies based on cost plus a reasonable profit under normal traffic conditions would allow them all to perform their appropriate functions, and in the end be better for all concerned, including the country and its industries, than any system of competitive rates."

Mr. Wonson's insistence that freight charges measure the cost of the service, and his attack on waterways as tending to favor regions adjacent to rivers at the expense of more inaccessible territory would lead the reader to infer that rail freight charges are commensurate with cost, and that rail carriers do not tax riverless regions with high rates in order to support low rates along the waterways. The writer agrees that when rates are made proportionate to cost of operation, it is reasonable to expect an appropriate distribution of traffic among the several forms of transportation, to each falling the share best adapted to its particular kind of service. Direct comparisons of tonnage would then be significant, and total relative ton-mile costs would offer conclusive evidence of the degree of justification of river improvements. That ideal condition is more remote from the rail rate structure, however, than from the scales prevailing on water.

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[25] See Tables, Annual Repts., Chf. of Engrs., U. S. Army, 1927 to 1936.

[26] 194—ICC—31.

In its profound effect on relative public use, it is necessary to recognize the practically universal railroad policy of depressing rates to profitless levels when confronted with competition from water lines. Such rates not only fail to cover fixed charges but usually cover only a part of operating and maintenance expense.[27] The expectation of this result on the part of shippers undoubtedly gives rise to much of the organized agitation for improvement of inland streams, and introduces the most troublesome single factor into the problem. The writer joins Mr. Wonson in earnestly deploring this use of waterways to regulate rail rates, but the depression of rates to meet water competition, of course, is a wholly voluntary act on the part of the railroads. It is clear that freight recaptured by this means from economical routes and diverted to uneconomical routes, causes the public to forego the real savings made available by water. When thus deprived of profitable tonnage the barge lines sustain material loss which is totally uncompensated by profit to the rail carrier or real benefit to the shipper.

The Added Traffic Theory.—For the most part the competitive depression of rail rates is confined to traffic between ports served by waterways, the rates between intermediate and interior points being held up on the "dry land" level. In addition to this non-competitive traffic between intermediate points, the railroads normally handle between ports many commodities not adapted to barge movement. Under these circumstances the depression of rail rates to competitive levels on commodity movements adapted to water theoretically, at least, leaves the bulk of rail traffic to carry the expense burden. Obviously, the total revenue collected by either type of carrier must exceed its total expenses "in the long run" if it is to prosper. Thus, the water carriers would be completely at the mercy of rail competition were it not for the fact that under normal conditions barge lines can handle adapted commodities at much less cost than rail carriers.

The "added traffic" theory invoked by the railroads to meet this situation is based on the premise that once a carrier is provided with adequate equipment and terminals, and is in operation, with supervisory staffs employed, and a sufficient volume of traffic moving at normal rates to pay the overhead, the out-of-pocket cost of handling any additional freight that may be secured is but a small part of the full cost of maintenance and operation. For the creation of new traffic there is nothing fundamentally unsound in this theory, provided always that rates are so fixed that the extra tonnage brings in as much money as is spent to move it. It is when carriers begin to view each other's business as potential added traffic, and to prey upon each other's basic tonnage by means of out-of-pocket rates, that the theory breaks down and the public eventually has to make good an uncompensated loss. The raided competitors certainly cannot afford to submit cheerfully to "subtracted" traffic, nor subscribe to the theory that their revenues and expenses are not affected thereby. When carried to the extreme now characterizing the aggressive campaign against water transportation, almost all traffic is rapidly taking on the status of added traffic, and revenue with which to pay full operating, maintenance, and fixed charges, not to mention dividends, is becoming progressively scarcer among the railways adopting this policy.[28]

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[27] ICC Stats., 3 352—1933.

Practical Illustration.—The most typical case brought to the notice of the writer was decided July 23, 1937,[29] and involves the shipment of about 100 000 tons of molasses annually from the New Orleans area to Peoria, Ill., on the Illinois River. When this movement began the rail rate was $6.00 per ton, which over the short rail line of 840 miles would yield 7.1 mills per ton-mile. The industry purchased a towboat and tank barge equipment and found that it could transport its own raw material for $2.60 per ton, or at 2.0 mills per ton-mile for a water distance of 1 270 miles. The initial floating plant being found inadequate, the shipper was negotiating for the purchase of an additional fleet when the railroads depressed their rate to $3.50 per ton. This reduction was sufficient to hold a substantial part of the business for the railroads and to bring about the cancellation of orders for new barge equipment. The reduced rate yields 4.2 mills per ton-mile gross. This rate compared with the 9.9 mills average rail rate used by Mr. Wonson would indicate that the molasses business to Peoria is not "pulling its weight" on the railroad but that the river loses tonnage it could carry profitably. The rates on molasses to intermediate dry-land points, of course, are held up to the old high level and, together with other noncompetitive traffic, are doubtless carrying the added burden. This typical case is used to emphasize the fact that each departure from the long-and-short haul regulations of which there are hundreds, tends to divert a substantial block of tonnage from the waterways where it could be handled economically, and where it would serve to decrease the average unit costs of water movement.

Effect of Rate Cutting.—No single type of carrier holds a vested right in any particular traffic. As already stated there would be no point in trying to hold traffic on the rivers to keep water transportation alive if no net benefit resulted. However, water transportation is clearly entitled to a chance to justify itself, before the public can pass judgment intelligently. There is no logic in recapturing tonnage from water by means of profitless rail rates, and then pointing to the lack of water-borne tonnage as significant. Out-of-pocket rail rates hold the volume of barge tonnage far below that which would normally seek that form of carriage, and which river carriers have justifiably equipped themselves to handle. Naturally unit costs of water line operation rise in response to such depletion, and the cost per ton to the Federal Government of waterway construction is unduly inflated. It will be readily understood that, in use of the term, "inflated," and all other expressions of difference of opinion with Mr.

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[28] See testimony of the rail representatives in Ex Parte 123, ICC, December, 1937.

[29] 223—ICC—221.

Wonson throughout this discussion, there is no thought of imputing any conscious or deliberate bias. The writer believes that the angles stressed herein have been overlooked, and that their full consideration will reverse some of Mr. Wonson's conclusions. He is not concerned with regulation of the practices of the carriers, but with due recognition of their effect on cost factors.

Summary.—Long-range analysis of the future of river commerce must contemplate, in all reason, a closer approach to the cost basis of fixing transportation charges than now prevails. The present system is leading toward bankruptcy for the carriers and a collapse of the transportation structure. Doubtless there is a present surplus of transportation capacity in certain areas and the least efficient and economical carriers, whether rail or water, must inevitably perish, to the advantage of the general situation. It seems scarcely reasonable to the writer, however, to hold that the internal waterway system is a liability rather than an asset to the country. Eliminating the cost of improvements for other purposes such as prevention of soil erosion and the cost of outmoded and superseded improvements, placing the remaining costs on a factual basis, and using the low operating costs of going barge lines, completely changes the picture in comparison between rail and water carriage.

It is firmly believed that traffic, if permitted freely to seek the best adapted routes, will flow in greatly increased volume to the important links in the waterway system, resulting in total ton-mile costs so reduced as to compare favorably with the most economical competitive service. Lest it appear that the points discussed herein are trivial in their bearing, the reader is asked to make a hasty computation of the results of applying Mr. Wonson's method of appraisal. A 50% drop in water-borne tonnage as a result of rate cutting, means a 100% inflation of the cost of Federal improvement, computed on the ton-mile basis, in addition to the rise in unit costs of the boat lines. Adding extraneous costs, and those for superseded improvements, using cost ratios of projects still under construction and not yet ready for traffic, employing excessive circuity factors, all err in the same direction, combining to depress still further the water-borne tonnage figures and inflate unit costs. If taken at their face value, these factors would nullify every real economy the most worthy waterway project is capable of effecting; and yet it is this depleted tonnage and these inflated costs that are cited as proof of the failure of river improvement to justify itself as a national policy.

RUFUS W. PUTNAM,[30] M. AM. SOC. C. E. (by letter).—In preparing one of the two basic papers for this Symposium, it was hoped that the main points at issue in this question of rail *versus* water transportation would be set up in such a manner that in the subsequent discussions there would be avoided rehearsals of the many minor subjects which in the past have been responsible for the apparent confusion of thought obscuring the few facts and principles involved. The almost complete absence of direct discussion or criticism of the paper itself leads the writer to the conclusion that it failed in its purpose and that he inadvertently so expressed himself as to stir up a "hornet's nest" of indirect comment.

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[30] Pres., Maritime Eng. Corporation, Chicago, Ill.

It has been held that the public necessity requires a satisfactory transportation service—one which is adequate, efficient, and economical. A general statement, such as this, necessarily becomes relative only when applied to specific cases. It would not do to condemn river transportation because of the closure of northern waterways during the winter season, or on account of circuity, or because of rigidity, any more than it would be correct to condemn transportation on the Great Lakes for the same reasons. From the point of view of the user these considerations affect only his over-all transportation costs, which include the cost of storage of requirements during the closed winter season.

No general plea for the necessity of waterway improvement should be entertained these days on account of the inadequacy of rail service. Even in times of national emergency, although the waterways themselves might have excess capacity available for use, the floating and terminal equipment might not have; therefore the value of this means of transportation, as a reserve against inadequacy of other forms, is not great.

On the tangible side, the entire question is one of economics; but, unfortunately, this seems to be a very difficult question when it comes to obtaining any degree of agreement as to the factors involved. In the first place whatever method of comparison is used it would be best to apply similar, if not identical, tests to rail as well as to water transportation. It is believed that the use of the method of comparing freight rates is generally recognized as incorrect. Rates of water carriers do not carry the investment, maintenance, and operating costs of the projects; those of rail carriers in some cases are no more than, if as great as, out-of-pocket costs, whereas, average rail rates are known to be insufficient to carry fixed charges on the entire railroad investment of the country. Rail rates should not be compared with water costs, even if the latter include fixed charges on the waterway improvement plus maintenance costs (see heading "In Justification: Comparison of Costs," in the writer's paper). The only solution apparent to the writer seems to be to attempt to compare rail costs with water costs, and, at best, such a procedure can result only in an approximation.

As to out-of-pocket operating costs of the carriers there appear to be no particular difficulties except the reluctance of water carriers to reveal their confidential data. Maintenance-of-way costs for the railroads and maintenance costs for waterways are readily available. The principal obstacle seems to be in reaching even a reasonable approximation of what should be the capital cost in each case.

If it is assumed that capital cost is to be measured in terms of expenditures to produce these transportation agencies, difficulties are encountered in either case. For the waterways, it is possible to go back and sum up the expenditures for "New Work" for any project. Where flood control and bank protection are not involved this is comparatively simple, but where they do form a part of the improvement, segregation is not so easy. An approximation is necessary and this the writer made in his paper in arriving at a 1937 cost of $550 000 000 for the system under discussion (refer also to Mr. Hartley's discussion). This estimate did not take into consideration the invisible costs of bridges which, in a more accurate determination, should be included. Merely for purposes of illustration, the writer has roughly estimated that the extra bridge costs involved on the 850 miles of the Mississippi River above Cairo, on account of greater clearances, and the provision and operation of draw-spans (capitalized), would approximate $16 500 000. The item is substantial but not large in comparison with the totals involved.

The writer's estimate made no deductions for the cost of early river improvements, long since superseded, and which had very little value as parts of the more modern projects now under way or recently completed (see Mr. Faison's discussion). A more accurate determination of costs would make allowance for these substantial items of expense.

To measure the cost of any part of the railroad system of the United States in a similar manner is next to impossible. Calculation of the actual sum of money expended on capital construction would not give the answer as there are costs of very substantial proportions which the investing public have paid during the lifetime of the railroads for which there are no visible, tangible returns. Several of the railroads in the Mississippi Valley have gone through the experience of alternate periods of prosperity, receivership, sale, and reorganization. Securities have been scaled down and in some instances "wiped out"; the cost of any such losses should be included as they represent expenditures by the public made for the purpose of providing transportation.

The use of the cost-of-reproduction method as a basis of comparison is not believed to be correctly applicable in this case. To do so would be to supplant fact with theory, and theoretical returns on theoretical investments cannot be used for the payment of taxes, interest, or dividends. As far as waterways are concerned, it would be necessary to calculate the cost of reproduction on the basis of an assumed rate of construction which is next to impossible to predict with irregular Congressional appropriations. Assumptions or theories would have to be applied to determine rights-of-way costs, which are substantial factors in railroad costs.

The writer made use of this method in his computation of railroad transportation costs merely because he was unable to determine the capital cost of the railroad system in the Mississippi Valley on the basis of past expenditures. His results, therefore, are only an approximation and should be so considered.

It is noted that Mr. Goodrich is of the opinion that the load factor of the floating equipment on the inland waterway system is an important factor, and he expresses the hope that the writer would present data as to actual existing load factors in the closing discussion. Such data are quite confidential so that only generalities may be given. One large operator has a towboat load factor of about 21% with a barge load factor of only 6.4%; this case represents about the minimum. Large coal operations on the Ohio River and the Monongahela River are conducted with towboat load factors of about 40% and barge load factors of 25 per cent. Only where an operator has a fairly well balanced two-way movement of bulk commodities do the load factors exceed these values.

Much of the confusion with which these questions are attempted to be solved, and most of the difficulties that exist without confusion, are borne of the effort to make a scientific comparison of two activities which cannot be reduced to terms with a common denominator. An approach to a satisfactory determination might be made by a committee of engineers skilled in this subject, but accord through published papers and discussions seems to be hopeless.

S. L. WONSON,[31] M. AM. SOC. C. E. (by letter).—The writer desires to express appreciation of the discussions of his paper and of the various considerations of interest and value that have been presented, both pro and con.

To the extent that the discussions present a viewpoint opposed to that of the paper, it would appear that a rebuttal would involve substantially a repetition of facts and conclusions already expressed.

There is one collateral consideration that seems worthy of mention, namely, that the natural and primary function of a river is that of a drainage channel, and that works for the improvement of navigation, or for other purposes, should not be allowed to impair the drainage function, or to impede any necessary improvement of it.

Of the force of this consideration, the disaster of 1937 along the Ohio River is a grim reminder. Along another major stream of the Mississippi Basin, upon which extensive works for the improvement of navigation are being constructed, there are communities in apprehension of the effects of a major flood following even the present stage of those improvements.

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PRE-STRESSED REINFORCED CONCRETE AND ITS POSSIBILITIES FOR BRIDGE CONSTRUCTION

By Ivan A. Rosov,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Charles S. Whitney, Karl W. Lemcke, Dean F. Peterson, Jr., M. Hirschthal, A. Floris, A. A. Eremin, and Ivan A. Rosov

Synopsis

This paper contains a brief citation of uses made of pre-stressed steel reinforcement in the past and the development of formulas for the solution of special cases. The symbols are defined where they first occur and are summarized for reference in the Appendix.

Introduction

It is customary to consider that the complex, non-homogeneous nature of reinforced concrete is its greatest disadvantage. It is impossible to utilize the strength of steel and of concrete, in the same structural member, with the same degree of economy. In parts of a structure where compression predominates the steel is always understressed, and when tension predominates the designer nearly always assumes that the steel resists all of it. The tensile stress in concrete often exceeds the ultimate, and the resulting cracks are a particularly undesirable characteristic of reinforced concrete. Although such cracks may not immediately endanger the stability of the structure (since the steel resists all the tension), they permit the structure to be destroyed gradually—the concrete by chemical reaction and the steel by corrosion.

On the other hand, the very difference between the two materials can be utilized to the advantage of reinforced concrete structures over all-steel structures. The reinforcement can be designed so as to confine the deformation of concrete within reasonable limits, or, conversely, to produce stresses in the concrete that will act in the proper direction. If it is desired to counterbalance some of the deformations in steel that are expected to occur under the action of working loads, a preliminary stress may be imposed upon the concrete.

This idea of restraining the effects of stress in either material has been accepted as a possibility since reinforced concrete was first introduced, and

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Note.—Published in September, 1937, *Proceedings*.

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several structures have been built according to this principle. Only in later years, however, have designers begun an extensive and far-reaching study of all the possibilities for the purpose of utilizing it consistently in structural design and construction. One method of restraining deformations may be illustrated by the case of a concrete column with spiral reinforcement, which, for all practical purposes, is the same as a steel "envelope" that prevents the member from bulging laterally under axial loads. The same effect can be produced by steel reinforcement arranged transversely in a structural member so that its bond with the concrete will serve to establish resistance to deformation in that direction. In this case the external restraining forces exerted by the "envelopes" are replaced by the internal forces in the steel. The workability of this principle has been proved by the results of tests[2] conducted by M. Caquot and P. Brice. These investigators found that any type of transverse reinforcement increases the ultimate compressive strength of concrete in so far as it tends to retard lateral bulging due to axial pressure. Similarly, E. Freyssinet[3] has made concrete that resisted, safely, the lateral bulging caused by axial pressures of 3 550 lb per sq in. He found also that the best type of reinforcement for this purpose is two series of parallel wires set in planes normal to the axis of the loads and tied by semi-circular steel bars.

Another general method of changing deformations of either material in a desired manner may be described broadly under the term, "pre-stressing." A brilliant demonstration of the advantages of this idea is afforded by experience gained in constructing the Albert Louppe Bridge, in France,[4] and the bridge across the Rogue River, in the United States.[5] Pressure exerted by means of jacks at key points along the arches had the effect of eliminating the shrinkage stresses and this decreased, materially, the dimensions of the arches.

The principle of pre-stressing was used by A. Mesnager and T. Vevriers in 1930,[6] and, in this case, was applied both to the concrete and to the steel separately. The arches of the bridge[6] at Vesinet, France, are reinforced by steel tubes, thick enough to constitute a considerable percentage of the cross-sectional area. Ordinarily, shrinkage stresses would have stressed the steel to 12 000 lb per sq in., but, by exerting a pull on the tubes and, simultaneously, a pressure on the concrete within the tubes, the shrinkage stresses were canceled. In other words, when the concrete had shrunk, the preliminary stresses in the steel tubes had disappeared, and the full strength of the metal was utilized to resist loading.

The next progressive step would be to increase the preliminary stresses sufficiently to have a residual stress even after the shrinkage stresses have canceled it partly. This would have the effect of introducing known internal stresses in a structure at previously (and precisely) determined points.

An example of the manner in which this principle is applied to reservoirs was published[7] in 1933. The circular concrete wall of the reservoir is fitted with a series of circumferential rings, and these rings finally are covered with a layer of concrete, leaving the connecting turnbuckles revealed. As the water rises in the reservoir tensile stresses develop in the concrete, which are counterbalanced by the pressure introduced in the steel hoops by means of the turnbuckle. In this manner the compression may be made to balance the tension in the concrete, thus safeguarding the structure against the effects of cracking and leaking.

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[2] *Structural Engineer*, June, 1931.

[3] *Memoires* de la Société des Ingénieurs Civils de France, July-August, 1930.

[4] *Le Génie Civil*, October 4, 1930.

[5] *Engineering News-Record*, 1931, No. 26.

[6] *Le Génie Civil*, January 17, 1931; also, *Engineering and Contracting*, August, 1931.

[7] *Engineering News-Record*, February 16, 1933.

This type of structure, resembling ordinary wooden tanks in principle, has recently been used quite extensively,[8] and with good results. The bond between the concrete and the steel is destroyed by coating the latter with a special asphaltic compound and the circumferential rods slip freely while strained by turnbuckles.[9] The same idea has been used for pipes, with equal success.

In Europe, the practice of pre-stressing does not involve destroying the bond, which conserves the character of the reinforced concrete as a structural unit and distributes the effect of the pre-stressing uniformly all over the length of the structure. In this manner, Professor F. Emperger applied the principle of the pre-stressing to precast pipes, and E. Freyssinet to pipes, posts, foundations etc.

It should be possible to prevent deformation in many other kinds of structures, by pre-stressing the materials of which they are composed. E. Freyssinet has stated his conclusion that reinforced concrete bridges of spans as great as 3 300 ft are possible in this manner. The development of structures of this kind may not be the problem of most immediate concern, but in ordinary engineering practice there is a wide field for economy and improvement in design by a proper application of the pre-stressing principle.

PRE-STRESSED BRIDGE STRUCTURES

Only simple slabs, beams, and girders are treated in this paper, because they are the structural elements that lend themselves most easily to pre-stressing, and in which the results are obvious and most interesting. The reinforcement along the tension face is straight and can be stretched by means of a simple device before the concrete is poured. After the concrete has set the tension is removed from the reinforcement, which then tends to return to its normal length. The result is that, through the bond between the reinforcement and the concrete, an internal force is created which, in turn, resists the deformation due to the application of external forces when the structural member is put to its final use. With respect to the concrete alone, of course, the bond in the steel constitutes an external force, and the amount of this force that remains after the concrete has set may be computed if the relation, $n = \frac{E_s}{E_c}$, is known.

The effect of pre-stressing is opposite to that of the final loading (see Fig. 1). Obviously, it is possible to pre-stress a structural member in such a way that the working load would never produce stresses great enough to cancel the internal stresses entirely. In that case, the cross-section of the beam (see Fig. 1(*c*)) would be entirely in compression.

Since there is no tension in the beam, any cracks that tend to form as a result of shrinkage, or other causes, are automatically closed by the pressure of the steel reinforcement. The net effect is to eliminate the most serious causes of deterioration in concrete, namely, penetration of water or gas and the rusting of the steel reinforcement. Furthermore, the entire cross-section of the concrete becomes effective in compression. Under such ideal conditions the dimensions of reinforced concrete members may sometimes be decreased as much as 50 per cent. The dead load is much more important in concrete structures than in steel frame structures. For larger spans the weight of a bridge, for example, increases so rapidly as to prohibit the use of reinforced concrete. By pre-stressing the reinforcement, as suggested herein, thus reducing the sectional dimensions appreciably, the limiting span for reinforced concrete structures can be increased considerably.

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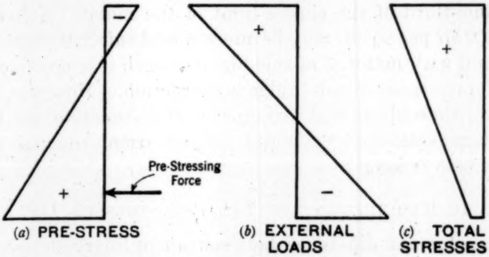
[8] *Engineering News-Record*, April 16, 1936.

[9] "The Preload System of Concrete Construction," by J. M. Crom.



(a) PRE-STRESS (b) EXTERNAL LOADS (c) TOTAL STRESSES

FIG. 1.—STRESSES IN A PRE-STRESSED BEAM

The foregoing ideas, as applied to the introduction of initial stresses in a beam, are not new. As early as in 1907 Lund and Koenen[10] proposed placing the tensile steel in tension before placing the concrete. Bending tests with beams submitted to an initial tension of 8 500 lb per sq in. showed that the first cracks appeared when the loads exceeded by about 50% those corresponding to beams made in the ordinary manner; but the efficacy of the beams decreased greatly as time passed.

Some other attempts also failed because the preliminary stresses in the steel were not sufficient to create the desired results. Steel that has a yield point from 33 000 to 40 000 lb per sq in., ordinarily has an assumed allowable stress from 16 000 to 20 000 lb per sq in., so that the bars cannot be stressed to more than 10 000 lb per sq in. In some cases this degree of pre-stressing could not even offset the effect of accidental errors. Even the largest permissible stress imposed on the reinforcement, in such cases, would require too much steel in order to create internal stresses in the concrete sufficient to afford a saving in material. In later years, however, steel has been developed with an ultimate strength of as much as 140 000 lb per sq in., with yield point stresses as great as 110 000 lb per sq in., and the prices for this high-strength steel differ only a trifle from those of ordinary structural steel. By hardening and annealing a cold-drawn steel, and then cold drawing it again, it is possible to raise the yield point to from 185 000 to 198 000 lb per sq in.

In steel structures the alloys that are brittle and lack plasticity are avoided. Steel members with large and irregular sections can be damaged during the cooling if the alloys are too brittle. In concrete structures it is the concrete that provides the plastic element to the structure. On the other hand, small cross-sectional dimensions of reinforcement bars prevent damage, when such bars are cooled.

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[10] *Le Génie Civil*, No. 9, 1936.

E. Freyssinet pre-stresses reinforcement so as to impose tension to between one-half and one-third of the elastic limit of the metal. A rod with a yield point of 120 000 lb per sq in. may be pre-stressed safely to as much as 80 000 lb per sq in., and with material of this high strength it is possible to utilize the advantages of pre-stressing for bridge construction. However, as far as the writer is aware, the only actual attempt in this direction has been the construction of a large-scale model of a girder by a German concern which, in 1936, tested a model 65.5 ft long.

Basic Requirement of the Pre-Stressed Design

The most important difference between an ordinary design of reinforced concrete and the "pre-stressed" design is that in the latter all the concrete in any cross-section may be kept in compression.

If external forces produce tensile stresses, they are treated merely as negative stresses, serving to decrease the compression created by the pre-stressed reinforcement. The entire cross-section of the concrete is considered effective in compression. The imposition of initial stresses should be considered in three successive stages, as follows:

Stage (1).—Before concrete is poured an external tensile force is applied mechanically to the reinforcement. This force may be termed the "preliminary" tension. After concrete has been poured, the effect of shrinkage is to restore the length of the deformed bar to a certain extent, thus releasing a part of the preliminary tension. On the other hand, the concrete is not permitted to shrink to its fullest extent, and bond stresses are created between the steel and the concrete, which have the effect of introducing an eccentric external force. Stresses from this source are not great, compared with those due to external loading, and they are not considered in ordinary design. No consideration is needed, and for the pre-stressed beam, the period in which the concrete is setting is the only time in which it is under slight tension.

Stage (2).—After the concrete has set, the preliminary tensile force is removed from the reinforcement, which then tends to shorten to its original length and is prevented from doing so only by the concrete; in this process, the shrinkage stresses are eliminated, leaving a residual eccentric force to act against the external forces that may be applied to the beam later. This eccentric force produces a direct pressure and a bending moment in the beam. If the bottom reinforcement is pre-stressed the effect is to introduce tension at the top, which, in turn, is offset by the dead weight of the beam; but before the initial pull is released from the reinforcement, this dead load is supported by forms and timbering. Upon release the beam bends upward, becoming loosened from the forms, and rests on its ends. In this stage the stresses in the beam are a combination of pre-stresses, f_p, and dead load stresses, f_D.

In the design of a pre-stressed reinforced concrete beam, the aim is to eliminate net tension in the entire cross-section and to confine the compression within allowable limits. Expressed algebraically, the following conditions must be satisfied at Stage (2):

(*A*) At the top of the beam:

$$f = f_{pt} + f_{Dt} > 0$$

(*B*) At the bottom of the beam:

$$f = f_{pb} + f_{Db} < f_{ac}$$

in which f = the combined or net unit stress; f_t = the unit stress at the top; f_b = the unit stress at the bottom; and f_{ac} = the allowable unit stress in concrete. In other words, the initial tension at the top of the beam should be absorbed by the compression due to dead load, and the initial compression exerted by the pre-stressed steel on the concrete at the bottom of the beam should be reduced to within the allowable value, f_{ac}, for concrete, by the dead load.

Stage (3).—The final stage is that in which the beam carries the full live load; the beam having been pre-stressed to offset the effects of dead load, it must have an excess of pre-stress to cancel the stresses produced by live loads. For this purpose the following conditions must be satisfied:

(*C*) At the top of the beam:

$$f = f_{pt} + f_{Dt} + f_{Lt} < f_{ac}$$

(*D*) At the bottom of the beam:

$$f = f_{pb} + f_{Db} + f_{Lb} > 0$$

(*E*) In the steel at the bottom of the beam:

$$f_s = f_{ps} + f_{Ds} + f_{Ls} < f_{as}$$

in which f_L = the unit stress due to live load; and, f_s = the unit stress in the steel.

To take full advantage of the methods described herein, the beam should be loaded temporarily up to the full dead loads before the preliminary stress in the reinforcement is released. Otherwise, the pre-stress in the reinforcement would be kept too low, limited by Conditions (*A*) and (*B*).

In analyzing the stresses in the reinforcement, two distinct factors must be taken into account: (*a*) The preliminary stress; and (*b*) the "designed pre-stress," f_{ps}, remaining after the partial loss of the "preliminary stress." This loss is due to the shrinkage, to the plastic flow, and to the compressive deformations of the surrounding concrete.

It is important to determine the "designed pre-stress" exactly. The loss of the preliminary stress, due to the shrinkage, depends on the quality of the concrete. For ordinary concrete the shortening may exceed 0.0004. This is equivalent to a decrease of 12 000 lb per sq in. in the tension of the steel. The shrinkage of compact concrete is much less.

The plastic flow should also be considered because the structure will be subject to permanent pressure. After 1 yr the plastic flow for ordinary conditions can reach a value equal to $0.13 \times \sqrt{365} \times 0.000001$, or 0.0000009 per lb per sq in. of stress.[11] The final flow after 5 yr will exceed this amount by not more than 30 to 40 per cent. Thus, if the permanent pressure in the concrete is 500 lb per sq in., the shortening of the concrete under plastic flow may reach $0.0000009 \times 1.4 \times 500 = 0.0006$, with a corresponding decrease of 18 000 lb per sq in. in the pre-stress.

A change in the pre-stress will be produced by the pre-stress itself while it is in the process of compressing the concrete. The reinforcement follows the shortening of the compressed concrete and a fraction of its stress is released. If the compression of the concrete around the bars is 500 lb per sq in., the decrease in the tension of the steel will be about 5 000 to 7 500 lb per sq in.

Thus, in time, the "preliminary stress" may lose as much as $12\,000 + 18\,000 + 5\,000 = 35\,000$ lb per sq in. and only the remaining tension in the steel should be taken into account for the design. This remainder is the "designed pre-stress." However, the structure should also be checked to meet the conditions of the first days, with a partial shrinkage and no plastic flow, to be sure that this temporary excess of the prestress will not produce an undesirable effect.

The stresses due to dead and live load will be governed by a factor of safety of 2. Ordinarily, these stresses are small compared to the designed pre-stress and range from 8 000 to 12 000 lb per sq in. Even with a factor of safety they cannot usually restore the loss in the preliminary stress.

Thus, the "designed pre-stress" may be fixed safely by deducting from 30 000 to 40 000 lb per sq in. from the yield-point value of any given steel. The "preliminary stress" should be from 5 000 to 10 000 lb per sq in. less than the yield point. The excess required for the "preliminary stress" above the "designed pre-stress" should be estimated as exactly as possible for a given structure.

As to the concrete, an ultimate strength of 2 000 to 2 500 lb per sq in. in compression is assumed in the examples of this paper. However, if the advantages of pre-stressing are to be fully utilized, special high-strength grades of concrete should be manufactured. Concrete of the highest strength can be produced most readily at a fixed central manufacturing plant, and, consequently, pre-cast beams and slabs can utilize the pre-stressing principle to the greatest advantage. Properties desirable in concrete are that: (1) It deforms at a fairly constant rate; (2) its total shrinkage is relatively slight; and (3) it possesses high strength and quick-hardening properties. The best method of obtaining these qualities is to manufacture a concrete with a minimum quantity of water and to insure that the product is placed carefully, vibrating the forms during the process. E. Freyssinet utilizes specially "treated" concrete, that is, concrete that is vibrated, pressed, and heated during setting. This improves the properties of the concrete to a remarkable extent; in tests conducted by M. Freyssinet the ultimate strength of the concrete made under pressure attained a breaking stress of 14 000 lb per sq in., and more, and a tensile stress of 150 lb per sq in. The "treated" concrete is compact, subject to only slight shrinkage, resists repeated stresses, and is suitable for pre-stressing.

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[11] *Journal*, Am. Concrete Inst., November, 1936.

In all ordinary structures no factor of safety should be assumed; if the actual working stresses exceed the computed stresses, the resultant tension will be relatively slight and will be eliminated subsequently as the structure is restored to a normal state of equilibrium. In special cases, where it is necessary that the design be particularly conservative, allowable working stresses due to live load may be increased by some factor of safety.

THEORY OF PRE-STRESSED RECTANGULAR BEAMS

In a simple pre-stressed reinforced concrete beam there are bending moments, M, shearing forces, V, and the horizontal force, T, introduced through the reinforcement, to be considered. All the concrete in any cross-section is considered as acting in compression. The cross-section of steel may be replaced, for purposes of computation, by the equivalent concrete area as in ordinary reinforced concrete design.

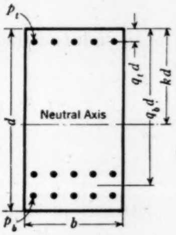

FIG. 2

Referring to Fig. 2, let $p =$ the percentage of steel in the entire cross-section of a beam (p_t referring to the steel at the top of the beam and p_b to the steel at the bottom); $d =$ depth of a beam; and $b =$ width of a beam. Furthermore, let the distance from the top of the beam to the centroid of the top reinforcement be $q_t\, d$, and to the centroid of the bottom reinforcement, $q_b\, d$. The distance down to the neutral axis of the section is $k\, d$, and the effective area of the beam is:

$$A = b\,d + (n-1)(p_t + p_b)\,b\,d = b\,d\,[1 + (n-1)(p_t + p_b)] = K\,b\,d \quad \text{. . (1)}$$

Taking moments about the top of the beam:

$$A\,k\,d = \frac{b\,d^2}{2} + (n-1)(p_t\,q_t\,d + p_b\,q_b\,d)\,b\,d \quad \text{. . . . . (2)}$$

from which,

$$k = \frac{\frac{1}{2} + (n-1)(p_t\,q_t + p_b\,q_b)}{K} \quad \text{. (3)}$$

The moment of inertia of the section about the neutral axis, after simplifying, is:

$$I + bd^3 \left\{ \frac{1}{12} + \left(\frac{1}{2} - k\right)^2 + (n-1)[p_t(k - q_t)^2 + p_b(q_b - k)^2] \right\} = C\,b\,d^3 \quad \text{. . . . . (4)}$$

Finally, the stresses due to the working loads are:

$$f_t = \frac{M}{I}\,k\,d = \frac{k\,M}{C\,b\,d^2} \quad \text{. . . . . (5)}$$

$$f_b = \frac{M}{I}\,(d - k\,d) = -\frac{(1-k)\,M}{C\,b\,d^2} \quad \text{. . . . . (6)}$$

and,

$$f_s = -\frac{M\,n}{I}\,(q_b\,d - k\,d) = -\frac{M\,n(q_b - k)}{C\,b\,d^2} \quad \text{. . . . . (7)}$$

If there is no top reinforcement, the expressions for k and C become correspondingly simplified. In the case of a slab of unit width, $b = 1$ in Equations (5), (6), and (7).

The pre-stresses are determined in a similar manner. Thus, subtracting the area of the bottom reinforcement, the effective area resisting the pre-stresses is:

$$A_1 = b\,d + (n-1)\,p_t\,b\,d - p_b\,b\,d = b\,d\,K_1 \ldots\ldots\ldots\ldots (8)$$

Taking moments about the top of the beam, and simplifying:

$$k_1 = \frac{1}{K_1}\left[\frac{1}{2} + (n-1)\,p_t\,q_t - p_b\,q_b\right] \ldots\ldots\ldots\ldots (9)$$

The moment of inertia of the section about the neutral axis, after simplifying, is:

$$I_1 = b\,d^3\left[\frac{1}{12} + \left(\frac{1}{2} - k_1\right)^2 + (n-1)\,p_t\,(k_1 - q_t)^2 - p_b(q_b - k_1)^2\right] = C_1\,b\,d^3 \ldots\ldots\ldots\ldots (10)$$

Stresses produced by the eccentric force, P, consist of uniformly distributed compression, $\frac{P}{A_1}$, and the bending stress, which varies from $-\frac{P\,d(q_b - k_1)}{I_1}\,k_1\,d$, at the top to $+\frac{P\,d(q_b - k_1)}{I_1}\,(d - k_1\,d)$, at the bottom. The area of the bottom reinforcement is $p_b\,b\,d$, and the value of the pre-stressing force is,

$$P = -\,f_{ps}\,p_b\,b\,d \ldots\ldots\ldots\ldots (11)$$

Consequently, the pre-stress at the top of the beam is expressed by:

$$f_{pt} = P\left(\frac{1}{A_1} - \frac{q_b - k_1}{I_1}\,d^2\,k_1\right) = -\,f_{ps}\,p_b\left(\frac{1}{K_1} - \frac{q_b - k_1}{C_1}\,k_1\right) \ldots (12)$$

and the pre-stress at the bottom is:

$$\begin{aligned} f_{pb} &= P\left[\frac{1}{A_1} + \frac{(q_b - k_1)(1 - k_1)\,d^2}{I_1}\right] \\ &= -\,f_{ps}\,p_b\left[\frac{1}{K_1} + \frac{1}{C_1}(q_b - k_1)(1 - k_1)\right] \ldots\ldots\ldots\ldots (13) \end{aligned}$$

If there is no top reinforcement, the factors, C_1, K_1, and k_1 in Equations (9), (10), (12), and (13) become:

$$C_1' = 0.083 + (0.5 - k_1)^2 - P_b(q_b - k_1)^2 \ldots\ldots\ldots\ldots (14)$$

$$k_1' = \frac{1}{K_1}\left(\frac{1}{2} - p_b\,q_b\right) \ldots\ldots\ldots\ldots (15)$$

and

$$K_1' = 1 - p_b \ldots\ldots\ldots\ldots (16)$$

As before, b = unity in the case of a slab. As an approximation, the effect of subtracting the area of the bottom steel may be neglected, with the following results:

$$C_1'' = 0.083 + (0.5 - k_1)^2 + (n - 1)\, p_t (k_1 - q_t)^2 \ldots\ldots\ldots (17)$$

$$k_1'' = \frac{1}{K_1''}\left[\frac{1}{2} + (n - 1)\, p_t\, q_t\right] \ldots\ldots\ldots (18)$$

and

$$K_1'' = 1 + (n - 1)\, p_t \ldots\ldots\ldots (19)$$

If the member has no top reinforcement, $p_t = 0$, and Equations (17), (18), and (19), are simplified accordingly. Substituting these simplified values in Equations (12) and (13):

$$f_{pt} = 2\, f_{ps}\, p\, (3\, q - 2) \ldots\ldots\ldots (20)$$

and

$$f_{pb} = -\, 2 f_{ps}\, p\, (3\, q - 1) \ldots\ldots\ldots (21)$$

As the design of pre-stressed beams and slabs is confined mostly to cases in which there is reinforcement on one side of the beam only, and as the effect of ignoring the steel area is slight, Equations (20) and (21) will be the formulas most commonly used.

The bending capacity of the pre-stressed beam can be determined from Conditions (*C*) and (*D*). For the most part, the first condition governs, that is,

$$\frac{k}{C}\,\frac{M}{b\,d^2} - 2 f_{ps}\, p\, (3q - 2) \gtreqless f_{ac}$$

Hence,

$$M \gtreqless [f_{ac} + 2\, f_{ps}\, p\, (3q - 2)]\, \frac{C}{k}\, b\, d^2$$

The bending capacity of the conventional reinforced concrete beam is,

$$M' = f_{ac}\, \frac{k'\, j\, b\, (qd)^2}{2}$$

and the ratio,

$$\frac{M}{M'} \gtreqless \left[1 + 2\, \frac{f_{ps}}{f_{ac}}\, p\, (3q - 2)\right] \frac{C}{k}\, \frac{2}{k'\, j\, q^2} \ldots\ldots\ldots (22)$$

Assuming that $f_{ps} = 45\,000$ lb per sq in.; $f_{ac} = 800$ lb per sq in.; $p = 0.009$; $k' j = 0.347$; $q = 0.9$; $n = 15$; and $\frac{C}{k} = 0.186$, $\frac{M}{M'} \gtreqless 2.26$. That is, the bending capacity of the pre-stressed beam is greater about twice that of an ordinary beam.

BOND

Bond stresses between the reinforcement and the concrete develop only when the stress in the steel changes in value from one point to the other, as when the external forces vary. The preliminary stress introduced in the reinforcement is constant from one end to the other. Considering only the bending moment due to the external load, the bond stress between two sections, a distance, Δx, apart is:

$$u\, \Sigma o\, \Delta x = \Delta f_s\, A_s \ldots\ldots\ldots (23)$$

in which Σo is the total perimeter of the steel bars, and Δf_s is the change in the steel stress due to the bending moment. As a limiting value,

$$u = \frac{A_s}{\Sigma o} \frac{d f_s}{dx} \quad \text{(24)}$$

In the conventional design, the appearance of cracks must be anticipated, in which case all the tension is carried by the steel, to balance the compression in the concrete at the top of the section. In that case, $f_s = \frac{M}{A_s \, j \, d_1}$, and,

$$u_1 = \frac{1}{\Sigma o \, j \, d} \frac{dM}{dx} = \frac{V}{j \, d_1 \, \Sigma o} \quad \text{(25)}$$

which is the well-known expression for bond. When the steel has been properly pre-stressed, however, the concrete over the entire section is always under compression.

The tension induced by the external loads, through the bending moment, is distributed over the concrete area as well as over the steel, with the result that the steel stress, and, consequently, the bond stress (which are proportional to Δf_s) are decreased considerably.

Referring to Equation (7) for the value of f_s, the bond is expressed by,

$$u_2 = \frac{A_s \, n \, (q - k)}{\Sigma o \, C \, d^2 \, b} \frac{dM}{dx} = V \frac{A_s \, n \, (q - k)}{\Sigma o \, C \, d^2 \, b} \quad \text{(26)}$$

and the ratio of bond values for the pre-stressed design and the conventional design is:

$$\frac{u_2}{u_1} = \frac{j \, d_1 \, A_s \, n \, (q - k)}{C \, b \, d^2} \quad \text{(27)}$$

Substituting for A_s its equivalent, $p \, d \, b$ and $d_1 = qd$, and simplifying:

$$\frac{u_2}{u_1} = \frac{j \, q \, p \, n \, (q - k)}{C} \quad \text{(28)}$$

which is always less than one. In Example 1 it proves to be 0.53, which indicates that, for this case, the bond stress to be designed for in the pre-stressed beam is only 53% as severe as in the conventional design. In fact, in most cases of pre-stressed beams, the bond to be provided is relatively weak and need not be considered. If it is desired to check the bond stresses the following formula may be used:

$$u = V \frac{p \, n(q - b)}{C \, \Sigma o \, d} \quad \text{(29)}$$

Toward the end of a beam the reinforcement is stressed in a manner similar to that in the vicinity of tension cracks in the conventional design. Theoretically, the bond stress is infinitely great across a crack but, actually, the steel slips, thus gradually transferring excessive stresses to the concrete at that point. As is known, concrete is plastic enough to "flow" under applied load as in the case of columns, the effect being to "repair" changes in the relation between particles, thus avoiding rupture. This phenomenon has not been studied adequately, but it is extremely important in problems involving the design of reinforced concrete.

In the case of pre-stressed concrete, by which preliminary forces are introduced in the reinforcement, there is a section of the steel at both ends, beyond the point at which the forces were applied, where the induced stresses are equal to zero. Theoretically, the transition between zero pre-stress and full design pre-stress occurs over a very short length of bar, and over this length the bond is infinitely great. As a matter of fact, the distance over which this transition occurs is appreciable because of the fact that the bars slip. E. Freyssinet, who conducted tests with preliminary stresses as high as 256 000 lb per sq in., does not report any difficulties in dealing with this problem. At the joint meeting of the Institution of Structural Engineers and the Société des Ingénieurs Civils de France in 1937, he explained that adhesion is not a question of the concrete sticking to the steel, but that it is a wedging action under the effect of the transversal deformations of the concrete, similar to the gripping of a bar in a wedge clamp. Its intensity and its efficacy depend on the quality of the concrete and the packing of the concrete around the bars. In the case of "treated" concrete, very restricted lengths (approximately 10 or 20 diameters) are sufficient to assure anchorage, provided there is a sufficient thickness of concrete and that the surface of the metal is not smooth in the zone of anchoring.[12] In particular cases, the use of vertical end anchor-plates can be recommended. If the reinforcement is designed to pass through such plates so that the ends are fastened securely at the outside surfaces, the preliminary stress at the end will then be divided between bond and the direct pressure against the concrete at the end section.

DIAGONAL STRESSES

In conventional reinforced concrete design, diagonal stresses are of great importance and a complicated system of special reinforcement is usually designed to offset them. The value of these stresses is expressed as follows:

$$f_d = \frac{1}{2} f_l \pm \sqrt{\frac{1}{4} f_l^2 + v^2} \qquad \text{.................... (30)}$$

in which f_l and v are the longitudinal stress and the shear, respectively, produced by the bending moment at a given section. If f_l is tension, the diagonal stresses at the point will vary from maximum tension, which will sometimes be considerably greater than the computed longitudinal stress, to a maximum compression which will be comparatively small. The longitudinal stress, f_l, decreases from the center of the beam to the support and from the bottom of the beam to the neutral axis, but combined with the effect of the shear, v, this stress can produce diagonal tension great enough to be unsafe. Consequently, diagonal reinforcement is designed to meet this condition. In the pre-stressed beam, conditions are entirely different. The longitudinal stress is always compressive, and the second term of Equation (30) gives usually only a negligible excess above the first term.

For an approximate estimation, the second term may be replaced by the two first terms of the corresponding series because, for the prevailing values of

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[12] *Structural Engineer*, May, 1937.

f_l and v, the series will be converging. Then,

$$f_d = \frac{1}{2} f_l - \left(\frac{1}{2} f_l + \frac{v}{f_l} \right) = - \frac{v}{f_l} \quad \text{(31)}$$

In the pre-stressed design f_l varies from f_{ac} at the top to zero or some positive value at the bottom, making at the neutral axis about $\frac{1}{2} f_{ac}$. For $f_l = 300$ lb per sq in. and $v = 150$ lb per sq in., the diagonal tension will be only $- \frac{150}{300} = - \frac{1}{2}$ lb per sq in., which is absurd.

Thus, the diagonal stresses need not be considered. They eliminate the bending of the longitudinal bars and considerably simplify the reinforcement. The use of stirrups, however, is desirable, because the pre-stressed beam is under permanent compression and the stirrups will play the rôle of hoops in columns. Correspondingly, stirrups should form rings restraining the effect of the bulging in concrete and should be placed along the entire span of the beam.

If a particular structure requires absolutely no tension in the concrete, the stirrups near the support may be pre-stressed as well.

DEFLECTION

The pre-stressing design introduces a direct pressure and new bending moment in addition to the bending moment of loading. The direct pressure uniformly distributed over a transverse section does not affect the deflection. The "pre-stress" bending moment acts in the direction opposite the loading and induces an upward deflection. Therefore, it may be stated that the pre-stress decreases the deflection of a beam.

In the customary balanced design, with the unit stress at the top of the concrete, f_{ac}, and at the reinforcement, f_{as}, the unit length of the top fiber will decrease by $a = \frac{f_{ac}}{E_c}$ and the unit length of the reinforcement will increase by $b = \frac{f_{as}}{E_s}$. This tends to bend the beam and the radius of curvature, r, referred to the top of the beam, will be computed from the equation,

$$\frac{r + q\,d}{r} = \frac{1 + b}{1 - a} \quad \text{(31}a\text{)}$$

in which $q\,d$ is the distance between the top fiber and the reinforcement.

Transformation of Equation (31*a*) gives,

$$r = q\,d \frac{1 - a}{a + b} \quad \text{(31}b\text{)}$$

In the "pre-stress" design, the unit length of the top fiber will decrease by a, as a maximum, and the bottom fiber will decrease too, or, in the extreme case, will not change. As before,

$$\frac{r_1 + d}{r_1} = \frac{1}{1 - a} \quad \text{(31}c\text{)}$$

Hence,

$$r_1 = d\,\frac{1-a}{a} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31d)$$

The ratio of both radii is,

$$\frac{r_1}{r} = d\,\frac{1-a}{a} \times \frac{a+b}{q\,d(1-a)} = \frac{a+b}{qa}$$

Assuming that $f_{ac} = 800$ lb per sq in.; $f_{as} = 20\,000$ lb per sq in.; $E_c = 2\,000\,000$ lb per sq in.; and, $E_s = 15\,E_c$; $q = 0.9$; $a = 0.0004$; $b = 0.00067$; and $\frac{r_1}{r} = 2.95$. On the other hand, the pre-stressed beam can carry about twice its normal load. If equal loads are considered, the deflection for the pre-stressed beam will be from five to six times less than for the conventional beam.

Peculiar Form of Pre-Stressed Beams

To eliminate the tensile stresses completely, Conditions (A) to (D) must be satisfied at every point along the span. If the section of the beam is designed in such a manner that these conditions are satisfied for the center of the span, it may be unsatisfactory for other points of the span where the outside bending moment is less than the maximum. Indeed at the top of the beam, the compression due to dead load (which must absorb the tension created by the pre-stressing) decreases to zero at the support where the outside bending moment is zero. Under such conditions, there could be a residual preliminary tension at the top of the beam near the support where Condition (A) would not be satisfied. Similarly, residual preliminary compression along the bottom of a beam may not have been adjusted to the allowable limit for compression with the result that Condition (B) would not be satisfied.

This difficulty can be overcome easily by varying the section. The bending moment due to dead load varies along the span according to the parabolic law, $x(1-x)$. To satisfy Condition (A), the pre-stress, f_{pt}, should be made to vary according to the same law by changing p or q, or both, in Equation (20). This is accomplished by increasing the width or the height of the beam. The reinforcement under preliminary tension is fixed in position in cross-sectional area and cannot be utilized to adjust the pre-stressed condition.

For simple beams and slabs the best method is to increase the depth, without changing the position of the steel. In this case, the depth, d_x, of a section at a distance, x, from the support, is determined theoretically by the expression (see Equation (20)):

$$2\,f_{ps}\,p\,\frac{d}{d_x}\left(3\,\frac{d}{d_x} - 2\right) = x(1-x)2\,f_{ps}\,p\left(3\,\frac{d}{d_x} - 2\right) \ldots\ldots (32)$$

or,

$$\frac{d}{d_x}\left(3\,\frac{d_b}{d_x} - 2\right) = x(1-x)\left(3\,\frac{d_b}{d} - 2\right) \ldots\ldots\ldots\ldots (33)$$

in which $d_b = qd$ is the permanent distance from the top of the beam to the steel. For the ends of the beam, where $x = 0$, or $x = 1$, Equation (33) reduces to $3\frac{d_q}{h_0} - 1 = 0$; or,

$$d_0 = \frac{3}{2} d_b \qquad (34)$$

Equation (34) is the algebraic expression for the well-known rule that when an eccentric force is applied at the third point of a section, the stresses at the farthest edge of that section are equal to 0. Condition (*B*) is usually satisfied when the depth is determined by Equation (34). The addition to the depth of a beam decreases rapidly from the supports and for $x = \frac{1}{4} l$, this addition is quite small. It is probably better to select a beam of the type shown in Fig. 3 rather than the parabolic shape dictated by the strict theory.

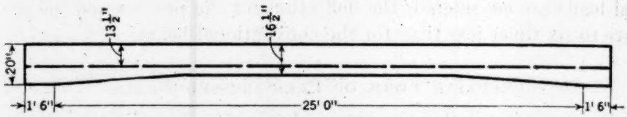

FIG. 3.—PRE-STRESSED SLAB BRIDGE

A second solution consists in lowering the top of the beam so that at the support the reinforcement is at a depth of $\frac{2}{3}\, d$ from the top. The distance from the bottom of the beam to the reinforcement is then represented by $d - q\,d$, and the depth of the beam at the support should be,

$$d' = 3\,d\,(1 - q) \qquad (35)$$

In the case of beams and slabs, Equation (35) yields values for depth which are too small, and difficulties are encountered due to high compressive pre-stresses and shearing stresses near the support. In the case of large girders, however, this solution of the problem is worth considering.

Continuous beams and rigid frames present the same problem to an even greater degree because in this case negative moments occur over the supports. The pre-stressing must absorb tension at the top and compression at the bottom of the beam. For this purpose the depth of the beam over the support must be increased to $d_b < \frac{2}{3}\, d_0$ or, even if the negative moments are large, to $d_b < \frac{1}{2}\, d_0$. In the latter case the sign of the "pre-stressed" moment changes and its effect upon the beam reverses.

Consequently, for the continuous beams and rigid frames the middle section is designed as for a simple beam, and the values of d_b and d are computed according to the foregoing equations. At the point of inflection, d increases to the value of $\frac{3}{2}\, d_b$. At the supports, the section is designed for negative moment and the same formulas are simply reversed, changing the top and bottom. In this case, the lowering of the top of the beam toward the support can be of great use because it augments the relative eccentricity of the pre-stress.

If the span of a bridge is great and if rectangular beams are not economical, I-shaped girders should be used. In the conventional reinforced concrete design such girders are not used because the bottom flange would be useless inasmuch as it would tend to crack regardless of its dimensions. Consequently, the T-beam section has been adopted; but with pre-stressing the I-shaped girder becomes as convenient and as economical in reinforced concrete design as it is in structural steel design.

The foregoing theory of the pre-stressed beam is easily applied to the I-beam shape. Final formulas and conditions for the design are the same and only the coefficients, K, k, and C, differ, according to the special shape of sections.

The method of meeting the change in the outside moments by varying the depth of the beam (that is, the eccentricity of the pre-stress), is identical with that described herein.

There are cases in which the principle of pre-stressing cannot be applied because it is impossible to select a value for the preliminary stress that will satisfy all the required conditions at the same time. This is true when the live load is considerably greater than the dead load, such as in the case of floor-slabs. The maximum ratio between live load and dead load for purposes of pre-stressing can be determined as follows: By Condition (D) the minimum pre-stress at the bottom required to absorb the maximum tension due to the loading is expressed by,

$$f_{bp} = -f_{bD} - f_{bL} \dots\dots\dots\dots \text{(36)}$$

Since f_{bL} is equal to $f_{bD}\,\dfrac{L}{D}$, Equation (36) can be written in the form:

$$\frac{f_{bp}}{f_{bD}} = -\left(1 + \frac{L}{D}\right) \dots\dots\dots\dots \text{(37)}$$

On the other hand, by Condition (A), the tensile pre-stress at the top must be absorbed by the compression due to dead load. Consequently, $f_{pt} = -f_{Dt}$; or, $\dfrac{f_{pt}}{f_{tD}} = -1$; from which,

$$\frac{f_{bp}\,f_{tD}}{f_{bD}\,f_{pt}} = 1 + \frac{L}{D} \dots\dots\dots\dots \text{(38)}$$

The relation, $\dfrac{f_{bp}}{f_{tp}}$, between the pre-stress at the bottom and at the top, by Equations (20) and (21), becomes,

$$\frac{f_{bp}}{f_{tp}} = -\frac{3\,q - 1}{3\,q - 2} \dots\dots\dots\dots \text{(39)}$$

and the relation between the stresses at the top and at the bottom, due to dead load, is,

$$\frac{f_{tD}}{f_{bD}} = -\frac{k}{1 - k} \dots\dots\dots\dots \text{(40)}$$

Multiplying Equations (39) and (40):

$$\frac{f_{bp}\,f_{tD}}{f_{tp}\,f_{bD}} = \frac{3\,q - 1}{3\,q - 2}\,\frac{k}{1 - k} \dots\dots\dots\dots \text{(41)}$$

Finally, by combining Equations (38) and (41) and solving for the ratio, $\dfrac{L}{D}$:

$$\frac{L}{D} = \frac{k\,(3\,q - 1)}{(1 - k)\,(3\,q - 2)} - 1 \dots\dots\dots\dots \text{(42)}$$

In other words, if the ratio between live load and dead load exceeds the value represented by the right-hand member of Equation (42), the preliminary stresses in the concrete cannot absorb the maximum tension at the bottom, and, at the same time, the stresses to be absorbed at the top by the minimum compression. Therefore, it is impossible to design the beam for complete elimination of tension in the concrete. At the values of $k = 0.55$, $q = 0.9$, and $\frac{L}{D} \leqq 2$.

The following illustrative problem will demonstrate the tremendous saving and improvement in design possible by introducing preliminary stresses.

Example 1.—To demonstrate the manner in which pre-stressing can influence the dimensions of a reinforced concrete structure, consider a pre-cast, slab bridge. At present, the excessive weight of pre-cast slabs limits their use to comparatively small spans, and any method of decreasing the thickness of such slabs would permit increasing the span length; for example, consider a bridge[13] with a clear span of 25 ft composed of separate units 29 ft long and 16 ft wide. Each slab is pre-cast in a central plant, transported to the site, and placed in position by cranes. For a slab 28 in. thick, the effective span may be assumed equal to $25 + 2.33 = 27.33$ ft. The dead load is 415 lb per sq ft, and for an element 1 ft wide the bending moment due to dead load is,

$$M_D = \frac{415 \times 27.33^2}{8} = 38\,600 \text{ ft-lb}$$

The live load consists of a 20-ton truck with one pair of wheels, arranged as shown in Fig. 4, for maximum live load moment. The value of this moment,

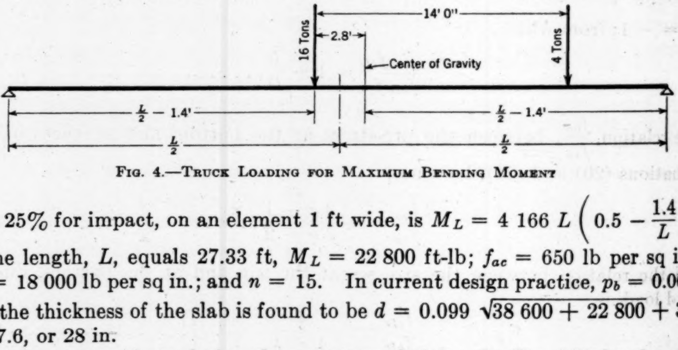

FIG. 4.—TRUCK LOADING FOR MAXIMUM BENDING MOMENT

plus 25% for impact, on an element 1 ft wide, is $M_L = 4\,166\, L \left(0.5 - \frac{1.4}{L}\right)$. If the length, L, equals 27.33 ft, $M_L = 22\,800$ ft-lb; $f_{ac} = 650$ lb per sq in.; $f_{as} = 18\,000$ lb per sq in.; and $n = 15$. In current design practice, $p_b = 0.065$, and the thickness of the slab is found to be $d = 0.099 \sqrt{38\,600 + 22\,800} + 3.0 = 27.6$, or 28 in.

The next step is to design the pre-stressed slab. In Fig. 5(*b*) let $d = 16$, $p_b = 0.0086$, and $f_{as} = 45\,000$ lb per sq in. The effective span length, L, is $25 + 1.33 = 26.33$ ft; the dead load is $1.33 \times 150 + 65 = 265$ lb per sq ft; and the bending moments are computed to be, respectively, $M_D = 22\,900$ ft-lb; and $M_L = 21\,700$ ft-lb. Since there is no top reinforcement, $p_t = 0$ and $q_t = 0$. Furthermore, $q_b = \frac{13.5}{16} = 0.844$; and by Equation (3), $k = 0.537$ and C (in

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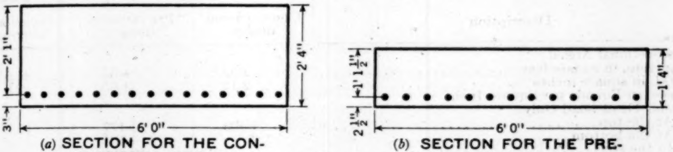
[13] "What the D. L. & W. R. R. Is Doing in Concrete Design", by M. Hirschthal, M. Am. Soc. C. E. *Railway Review*, October 9 and 16, 1926.

Equation (4)) equals $0.0833 + 0.037^2 + 14 \times 0.0086 \times (0.844 - 0.537)^2 = 0.096$. By Equations (5), (6), and (7), the stresses are computed to be (in pounds per square inch): $f_{tD} = 500$; $f_{bD} = -430$; $f_{sD} = -4\,290$; $f_{tL} = +473$; $f_{bL} = -408$; and, $f_{sL} = -4\,060$. By Equations (12) to (16), the stresses in the slab due to introducing preliminary stress in the steel are: $D_1 = 0.9914$; $k_1 = 0.497$; $C_1 = 0.0843$; $f_{tp} = -0.00885\,f_{ps}$; and $f_{bp} = -0.0265\,f_{ps}$. These five values must coincide with Conditions (*A*), (*B*), (*C*), (*D*), and (*E*), and the best method of checking the selected section is, first, to determine, from Conditions (*C*) and (*D*), what "designed pre-stress" in the reinforcement would impose the required stress in the concrete under full working load; and then, by Condition (*E*), to make sure that this preliminary stress is not excessive for steel. By Condition (*C*): $F_t'' = +0.00885\,f_{ps} + 500 + 473 \leqq 650$, and, therefore, $f_{ps} \leqq -36\,500$ lb per sq in.; by Condition (*D*): $F_b'' = -0.0265\,f_{ps} - 430 - 408 \geqq 0$, and, therefore, $f_{ps} \leqq -31\,600$ lb per sq in.; and, by Condition (*E*): $-F_s'' = -(f_{ps} - 4\,290 - 4\,060) \leqq +45\,000$, and, therefore, $f_{ps} \geqq -36\,650$ lb per sq in. The similarity between the optimum values of the "designed pre-stress" to be imposed in the reinforcement, in order to utilize concrete (see Condition (*C*)) and steel (Condition (*E*)) to the best advantage, indicates that the section is well balanced in this respect. Conditions (*A*) and (*B*), for dead load only, will be usually satisfied if Conditions (*C*) and (*D*), for full loading, are satisfied. Nevertheless, all these conditions should be checked independently.



(*a*) SECTION FOR THE CONVENTIONAL DESIGN

(*b*) SECTION FOR THE PRE-STRESSED DESIGN

FIG. 5.—COMPARISON BETWEEN THE CONVENTIONAL AND THE PRE-STRESSED DESIGNS

From Conditions (*C*) and (*E*) it is possible to state that $-36\,650 \leqq f_{ps} \leqq -36\,500$ lb per sq in.; and, assuming that $f_{ps} = -36\,500$ lb per sq in., the stresses due to dead load are (see Conditions (*A*) and (*B*)): $F_t' = +0.00885 \times (-36\,500) + 500 = +177$ lb per sq in.; and $F_b' = -0.0265 \times (-36\,500) - 430 = +538$ lb per sq in. It is unnecessary to compute the value of F_s' because the reinforcement is always stressed more under full loading.

Under maximum live load the corresponding stresses are: $F_t'' = +177 + 473 = 650$ lb per sq in.; $F_b'' = +538 - 408 = +130$ lb per sq in.; and $F_s'' = -36\,500 - 4\,290 - 4\,060 = -44\,850$ lb per sq in.

It will be noted that the design is especially well balanced. Tension can never occur in the concrete and the compression stresses are a minimum (+ 177 lb per sq in. at the top, and + 130 lb per sq in. at the bottom). This margin should be sufficient to absorb any accidental inaccuracies introduced by computations or assumptions, before tension stresses would appear in the concrete. Before the live load is applied, the reinforcement is under a tension of 40 790 lb per sq in. The live load adds only 4 060 lb per sq in., an unimportant increase of 10 per cent.

The advantages of pre-stressing, compared with the conventional design, are demonstrated in Table 1. The analysis indicates that the depth of the slab should be increased from the center toward the supports, where it reaches a maximum value of $d_0 = \frac{3}{2}\, q_b\, d = \frac{3}{2} \times 13.5 = 20.25$ in. At the supports the stresses in the concrete are zero at the top and, at the bottom:

TABLE 1.—ADVANTAGES OF PRELIMINARY STRESSES

Description	Conventional design	Pre-stressed design	Percentage, decrease
Cross-Sectional Areas:			
Concrete, in square feet	2.33	1.33	41.5
Steel, in square inches	2.12	1.65	22.0
Stresses, in Pounds per Square Inch:			
Under Dead Load Only:			
At the top	+409	+177	....
At the bottom	*	+538	....
In the steel		−40 290	....
Under Full Load:	−11 300		
At the top	+650	+650	....
At the bottom	*	+130	....
In the steel	−18000	−44 850	....
Percentage increase in steel stress under live load	59	10	....

* Tension cracks.

$$-2\,\frac{P}{A} = \frac{2\, f_{ps}\, p_b\, bd}{b\, d_0} = 2 \times 36\,500 \times 0.0086 \times \frac{16}{20.25} = 498 \text{ lb per sq in.}$$

Instead of computing the theoretical curve of the bottom of the slab, or beam, it is simpler to design sections for several points along the span. In the present case (see Fig. 5), the depths are found to be as follows:

x, as a ratio of:	d, in inches
$0.05\, l$	19.0
$0.1\, l$	17.8
$0.2\, l$	16.9

The total weight of the pre-stressed slab unit is about 19 tons, 37% less than for the conventional design.

Example 2.—Consider the same pre-cast slab (Fig. 5*b*) as in Example 1, except that it has double reinforcement. Let $d = 13$ in.; $p_t = 0.012$; and $p_b = 0.0105$. The dead load is $1.08 \times 150 + 65 = 227$ lb per sq ft; $M_D = \frac{1}{8} \times 227 \times 26.08^2 = 19.200$ lb-ft; and, $M_L = 4\,166 \times 26.08 \left(0.5 - \frac{1.4}{26.08}\right)^2 = 21.500$ lb-ft. By Equations (3) and (4), $k = 0.495$ and $C = 0.1208$; and by Equations (5), (6), and (7), $f_t = +\,0.0242\, M$; $f_b = -\,0.0247\, M$; and $f_s = -\,0.258\, M$.

Stresses due to external loads are (in pounds per square inch) computed as: $f_{tD} = +\,465$; $f_{bD} = -\,475$; $f_{sD} = -\,4\,950$; $f_{tL} = +\,520$; $f_{bL} = -\,531$; and $f_{sL} = -\,5\,550$. By Equations (8), (9), and (10), $K_1 = 1.157$, $k_1 = 0.447$, and $C_1 = 0.0988$. By Equations (12) and (13), the pre-stresses are: $f_{pt} = +\,0.00993\, f_{sp}$ and $f_{pb} = -\,0.0325\, f_{sp}$. On the other hand, the pre-stress in the steel required by Conditions (*C*), (*D*), and (*E*) (in pounds per square inch), is:

$$F_t'' = +\,0.00993\, f_{sp} + 465 + 520 \leq +\,650; \quad \text{or,} \quad f_{sp} \leq -\,33\,700$$

$$F_b'' = -\,0.0325\, f_{sp} - 475 - 531 \geq 0; \quad \text{or,} \quad f_{sp} \leq -\,32\,700$$

and

$$F_s'' = -\,(f_{sp} - 4\,950 - 5\,550) \leq +\,45\,000; \quad \text{or,} \quad f_{sp} \geq -\,34\,500$$

which indicates that f may be selected between − 33 700 and − 34 500 lb per sq in. Assuming that $f_{sp} = -\,34\,000$ lb per sq in.:

$$F_t' = +\,0.00993 \times (-\,34\,000) + 465 = +\,127$$

and,

$$F_b' = -\,0.0325 \times (-\,34\,000) - 475 = +\,631$$

For full loading:

$$F_t'' = +\,127 + 520 = +\,647 \text{ lb per sq in.}$$

$$F_b'' = +\,631 - 531 = +\,100 \text{ lb per sq in.}$$

and,

$$F_s'' = -\,34\,000 - 4\,950 - 5\,550 = -\,44\,500$$

Thus, all conditions are satisfied. The cross-section of concrete required is only 1.08 sq ft which is less than 50% of the area required by conventional design. The cross-section of steel is 3.51 sq in. which is 66% greater than conventional design. Such a design may be recommended only if the thickness or the weight of the bridge must be decreased in order to meet local conditions.

The slabs can be made even thinner by introducing a concrete of higher strength; for example, if the allowable stress in the concrete can be increased to 1 500 lb per sq in., the pre-stressed slab in Problem 1 will be 10 in. thick, with a steel ratio of $d = 0.021$.

TESTS ON PRE-STRESSED CONCRETE

To test the practicability of introducing preliminary stresses in reinforced concrete, E. Freyssinet constructed beams reinforced symmetrically with different kinds of steel bars and wires, including piano wire with an elastic limit of 284 000 lb per sq in. Initial tensile forces were applied such as to produce stresses from 99 600 to 256 000 lb per sq in. Mortar was placed in small portions and vibrated; samples were tested by bending; and the results were very favorable. Rupture occurred at loads much greater than were theoretically computed. In some cases the samples failed in the steel and, in some cases, in the concrete; but in the latter cases, too, it was the steel that produced the rupture by exceeding its yield-point stress. The deflections were small, being almost always only about one-fifth of the deflection observed in conventional design. What is of a special importance as applied to reinforced concrete, the deformations were completely reversible. No cracks appeared until tensile stresses due to working loads exceeded the initial compression stresses by from 280 to 430 lb per sq in.

In 1935, E. Freyssinet conducted reversed bending tests[14] on a post 40 ft long, fixed for a length of 6 ft 7 in. from the base. Repeated loads were applied from

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[14] *Memoires* de la Société des Ingénieurs Civils de France, September-October, 1935.

+ 1 000 to − 1 000 lb at the rate of eight times per minute. A controlling post of the conventional design, heavily reinforced with 286 lb of steel, was fissured after a few hundred deflections and broke after a few thousand deflections. A "pre-stressed" post of the "treated" concrete with only 110 lb of steel was subjected to 500 000 deflections without any alteration.

In 1934, W. K. Hatt, M. Am. Soc. C. E., tested a series of 8 by 14-in. beams, 13.5 ft long, reinforced with two rods 0.816 in. in diameter. These beams were loaded to produce bending moments from zero to 260 000 in-lb.[15] The bond between the rod and the concrete had been destroyed with an asphaltic coating and, after setting, the reinforcement was pre-stressed to loads of 8 372, 11 151, and 15 372 lb per sq in. Stresses in the concrete and in the steel were measured exactly. A beam of the same section, but not pre-stressed, was also tested.

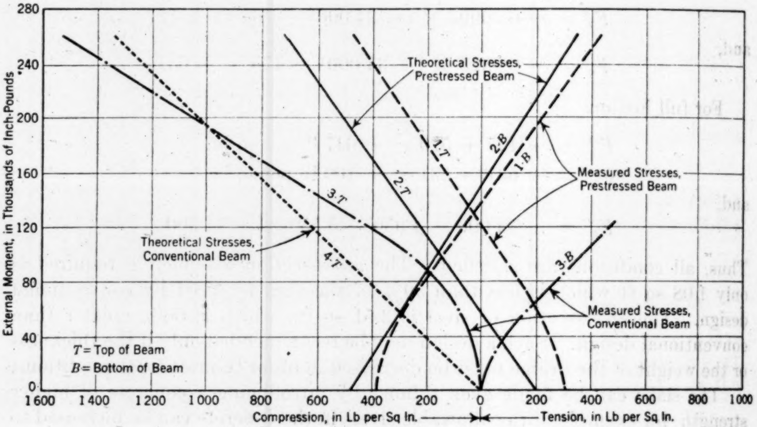

Fig. 6.—Unit Concrete Stresses

Figs. 6 and 7 show the unit stresses at the top and at the bottom of the concrete and in the steel: (1) Measured exactly, in tested pre-stressed beams; (2) computed by the theory of the pre-stressed beam; and (3) measured exactly, in a tested conventional beam. The computed stresses at the bottom of the concrete agree well with tests. Some greater difference is observed for the top of the concrete where the actual effect of pre-stressing proved to be greater than the computed value. Measured deflections follow strictly theoretical conclusions. The pre-stressing first produced upward bending. Under the bending moment of 100 000 in-lb, the beam regained its initial form. The conventional beam deflected under the same load as much as 0.035 in. Then the load was increased so as to produce a unit stress of 1 050 lb per sq in. in the concrete in the conventional beam; and the deflection measured in the conventional beam was 4.4 times that in the pre-stressed beam.

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[15] *Engineering News-Record*, September 13, 1934.

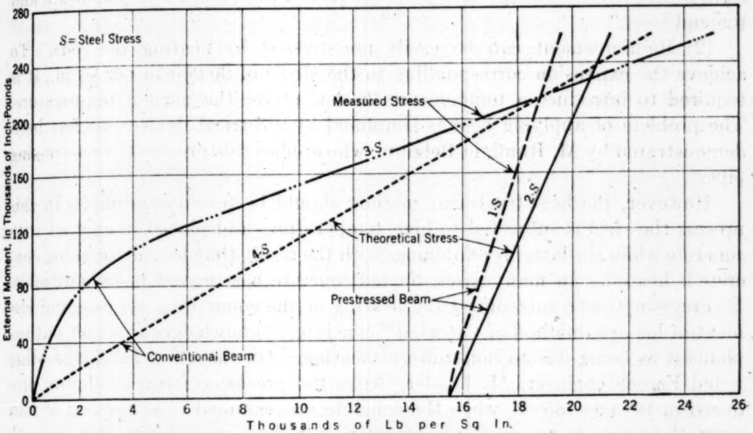

FIG. 7.—UNIT STEEL STRESSES

A most interesting test was conducted in 1936, in Germany.[16] A model girder, 65.5 ft long, varying in depth between 33 in. and 46 in. was used. Both the horizontal and the vertical reinforcement was pre-stressed to 70 000 lb per sq in. The working unit stress in the concrete was 2 056 lb per sq in. in compression and, at the steel, 78 000 lb per sq in. The girder weighed only 150 lb per lin ft, and carried a load of 940 lb per lin ft. Under repeated loading the girder acted like a perfectly elastic body and disclosed no signs of fatigue. Reinforced with steel having an elastic limit of 140 000 lb per sq in., the girder had a factor of safety of 2 in the steel and of about 4 in the concrete. The deflection was unusually small.

The details of this experiment have not been made public, but, on the basis of this test, it is concluded that a girder of 327 ft span, with an average depth equal to 0.05 *L*, could be built economically. Such a girder would carry 600 tons of its own weight and 300 tons of useful load. The deflection would be only one-third of that in a steel structure of the same dimensions.

METHODS OF PRE-STRESSING THE REINFORCEMENT

Three methods can be used to pre-stress the material required:

(1) Mechanical devices of various forms can be devised. Ends of the reinforcement bars can be hooked and connected to the special anchor-beams by means of these hooks, intermediate hooked bars and clamps being fixed on the anchors. One of the anchor-beams is subjected to the action of jacks held against a strong abutment. In this manner the reinforcement bars are stretched to a predetermined value. Devices of this type are suitable for the manufacturing of precast structures in a concrete yard where supports can be built permanently.

For field work, it may be preferable to use the forms as supports, strengthening them with end blocks and struts and carrying loads from one end of the form to the other. In this case, jacks are placed between the anchor-beam and the end block.

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[16] *Concrete and Constructional Engineering*, April, 1936.

(2) Reinforcement can be easily pre-stressed by heating the rods. To achieve the expansion corresponding to the stress of 90 000 lb per sq in., it is required to introduce a temperature 260° C above the normal temperature. The problem of applying heat is simplified by electrical devices, as has been demonstrated by M. Ruml, in Belgium, who applies this process to pre-stressing pipes.

However, the heat-treatment method should be tested carefully. In this process the steel is subjected to high temperature, and placed in contact with concrete while the latter is hardening, with the result that the surrounding concrete is heated. In many cases, heated concrete has proved to be unreliable. E. Freyssinet, who introduced the heating of the concrete as an essential element of his new method of "treated" concrete, acknowledges this fact but explains it as being due to non-uniform heating. On the other hand, the other noted French engineer, M. Lossier,[17] cites the precast electric insulators produced in 1913 at 185° F, when the concrete was rammed. At the end of four years they were in an excellent condition, but three years later their strength was reduced much below that of ordinary insulators. It appears possible that initial heating is injurious to chemical actions over long periods of time. If it results in destroying the bond between the steel and the concrete even "in the long run," heat cannot be used for pre-stressing.

(3) An ingenious method of "self-prestressing" was used by E. Freyssinet for piles in combination with his "treated" concrete. Piles were manufactured in place in sections between two cylindrical moulds. The external mould consisted of half-collars which were fixed with screw clamps. The internal mould is made of a steel tube surrounded by an envelope of india rubber with a cotton armor. The space between moulds is closed by an annular plate with holes for rods and for placing the concrete, and rods are clamped on the top of the plate. The concrete is poured between the moulds and vibrated by external vibrators. The excess water escapes through joints between the collars of the external mould. Then the vibration is stopped and a hydraulic pressure of 285 lb per sq in. is exerted in the internal mould between the steel tube and the rubber envelope. This pressure is transmitted to the concrete and through it to the top closing plate because, immediately after vibration, the concrete acts as a liquid. The plate rises and stretches the rods that are fixed to its top, almost to their elastic limit. The pressure is maintained for 20 min, the water leaks freely through the joints, and the reinforcement is pre-stressed as required.

CONCLUSIONS

From studies made by the writer and described in this paper, six broad conclusions seem to be justified, as follows:

1.—In reinforced concrete design, there are great advantages in increasing or decreasing, artificially, the stresses in one or the other materials involved. One of the methods consists in introducing preliminary tension in the reinforcement and tests by E. Freyssinet and others show that this method is practicable.

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[17] *Concrete and Constructional Engineering*, August, 1936.

2.—Invariably the results of introducing pre-stressed reinforcement in beams and girders are favorable when the preliminary stresses can be designed so as to counterbalance the stresses produced by dead loads and live loads.

3.—Pre-stressed designs may result in exclusively great savings of material or they may permit considerable increases in maximum span length. In either case, the structures are more reliable than in the case of conventional design because the tensile stresses in concrete are completely canceled and the compressive stresses that occur over the entire cross-section tend to prevent the occurrence of cracks.

4.—The arrangement of the reinforcement is simplified considerably because the diagonal tension is appreciably less than in the conventional design, and in continuous beams, reinforcement is not necessary at the top.

5.—In order to gain the greatest advantages from the principle of prestressing, it is important that the reinforcement have a high yield-point stress and that special devices for applying the preliminary stresses be available. The cost involved in both cases is not serious.

6.—The effect of introducing the pre-stressed design into reinforced concrete will be to develop much lighter, more reliable, cheaper, and simpler structures, thus increasing the field of usefulness of this material considerably.

APPENDIX

NOTATION

The following symbols, which conform to the Symbols for Mechanics, Structural Engineering, and Testing Materials[18] approved by the American Standards Association in 1932, are offered for the guidance of readers and discussers of this paper:

A = effective area; A_1 = effective area that resists pre-stresses.
a = a subscript denoting "allowable".
b = breadth, or width, of a beam; as a subscript, b denotes "bottom".
C = a constant.
c = a subscript denoting "concrete".
D = dead load; as a subscript, D, denotes "dead load".
d = depth of a beam; dx = depth, d, at a distance, x, from the left support,
E = modulus of elasticity.
f = net unit stress; f_c = unit stress in concrete; f_s = unit stress in steel; $+ f_p$ = unit pre-stress (initial compression); $- f_p$ = unit pre-stress (initial tension); f_D = dead load stress; f_t = unit stress at the top of a beam; f_b = unit stress at the bottom of a beam; f_a = allowable unit stress; f_L = unit stress due to live load.
I = moment of inertia of the effective area; I_1 = moment of inertia that resists pre-stresses.
j = ratio of lever arm of the resisting couple, to the distance between the outer compressive fiber and the point of application of the resultant tensile stress.

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[18] A. S. A.—Z10a—1932.

K = a substitution factor = $1 + (n - 1)(p_t + p_b)$; K' = the substitution factor for no top reinforcement = $1 - p_b$.
k = ratio of distance from the neutral axis to the top fiber of a reinforced concrete beam, to the depth of the beam.
L = live load; as a subscript, L denotes "live load".
l = length; as a subscript, l denotes "longitudinal" stress.
M = moment.
n = ratio, $\frac{E_s}{E_c}$.
P = a concentrated load; a "designed pre-stress" force applied to a member.
p = percentage of steel in the entire cross-section of a beam; p_t = referred to the top steel; p_b = referred to the bottom steel.
q = ratio of distance from the center of the reinforcement to the top fiber of the beam, to the depth of the beam.
s = a subscript, denoting "steel".
t = a subscript, denoting "top".
u = unit bond stress.
V = total shear.
v = unit shear stress.

DISCUSSION

Charles S. Whitney,[19] **M. Am. Soc. C. E. (by letter).**—The pre-stressing of reinforcing steel is of considerable advantage in the case of some special uses of concrete and has very interesting possibilities. The writer believes that Mr. Rosov is over-enthusiastic regarding the general application of the method to concrete structures. It may be that the author emphasizes its theoretical advantages without full consideration of the practical limitations.

Concrete is a very imperfectly elastic material, and any attempt to estimate the actual stresses in a pre-stressed member under loading conditions is certain to result in a very rough approximation. Pre-stressing improves the behavior of concrete under load by reducing the cracking, but because of the large strains which occur before failure, pre-stressing will have practically no effect on the ultimate strength of the member. Therefore any increase in allowable load due to pre-stressing will reduce the factor of safety against ultimate failure. No doubt this is warranted in some types of structures because of an increase in the useful strength limit.

The difference between the two slabs indicated in Fig. 5 cannot be justified by pre-stressing the steel, but is largely due to the inadequacy of the conventional design method used.

Applying the formulas recently proposed by the writer,[20] and assuming the concrete cylinder strength, $f_c' = 2\,000$ lb per sq in., and the steel elastic limit, $f_s = 40\,000$ lb per sq in., the ultimate bending strength of the conventional design (Fig. 5*a*) per foot width of slab, expressed by[20]:

$$M = b\,d^2\,p\,f_s\left(1 - \frac{p\,f_s}{1.7\,f_c'}\right) \qquad (43)$$

is equal to 1 945 000 in-lb. For balanced reinforcement[20]:

$$p = 0.456\,\frac{f_c'}{f_s} \qquad (44)$$

or $p = 0.0228$; and the ultimate moment, as expressed by[20]:

$$M = \frac{b\,d^2\,f_c'}{3} \qquad (45)$$

will be $12 \times 25 \times 25 \times \frac{2\,000}{3} = 5\,000\,000$ in-lb. The author's conventional design, therefore, has a factor of safety of $\frac{1\,945\,000}{12 \times 61\,400} = 2.64$ against steel failure and a factor of safety of $\frac{5\,000\,000}{12 \times 61\,400} = 6.8$ against concrete failure.

According to the author's statements an allowable unit stress of 35 000 lb per sq in. in the steel for the pre-stressed design would require an elastic limit of about 90 000 lb per sq in. Assuming this value of f_s and applying the writer's formulas to the design shown in Fig. 5(*b*), the percentage of steel required for balanced reinforcement would be (see Equation (44)), $0.456 \frac{2\,000}{90\,000} = 0.0101$. The author's design provides: $p = \frac{1.65}{12 \times 13.5} = 0.0102$.

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[19] Cons. Engr., Milwaukee, Wis.

[20] "Design of Reinforced Concrete Members Under Flexure or Combined Flexure and Direct Compression," by Charles S. Whitney, *Journal*, Am. Concrete Inst., March–April and September–October, 1937, Equations (7), (8), and (6), respectively.

The ultimate strength by Equation (45) is $12 \times 13.5 \times 13.5 \times \frac{2\,000}{3} = 1\,458\,000$ lb per sq in. The factor of safety of the author's design against ultimate failure is, therefore, $\frac{1\,458\,000}{12 \times 44\,600} = 2.72$, whether or not the steel is pre-stressed. The pre-stressing will reduce the amount of cracking in the concrete, but that can also be reduced by using more steel, which could then have a lower yield point and which would provide lower bond stresses. For ordinary structures the pre-stressing procedure does not appear advantageous.

Karl W. Lemcke,[21] Assoc. M. Am. Soc. C. E. (by letter).—Pre-stressed reinforced concrete is somewhat similar to the jacking of moments into truss members in order to nullify or greatly decrease objectionable secondary stresses. In each case the capacity of the member involved is increased without adding to its section. The author has presented an interesting and timely paper.

Although considerable progress has been made, the writer believes that reinforced concrete as a structural material is capable of much further development. While American engineers are generally in the forefront of progress, in the matter of conceiving long-span concrete bridges they seem to be behind European engineers. As against the 460-ft reinforced concrete arch span of the George Westinghouse Bridge, at Pittsburgh, Pa., the French have the 612-ft arches of the Plougastel Bridge, near Brest. Freyssinet, the designer of the latter structure, has very ably stated the reasons why greater progress has not been made in long-span concrete construction.[22]

Referring to pre-stressed concrete design, Mr. Rosov states in effect that, for long spans an I-shaped girder should be used, but that in conventional concrete design such girders are not used because the bottom flange would be useless, due to its tendency to crack. One such concrete girder bridge of conventional design and I-section was designed and built across Salt Creek in Humboldt County, California.[23] Reinforced concrete was chosen in preference to structural steel because it was felt that, due to the heavy fogs and moisture at the site, structural steel would be subject to great corrosion. The concrete design was also cheaper. The bridge was a two-span continuous one, 284 ft long. The girders were 12 ft deep, the top flanges, 2 ft wide, the bottom flanges, 1.5 ft wide, and the web varied in thickness from 7 in. to 18 in. As this structure was built about 1920 it would be interesting to know how it has withstood the elements, and whether the bottom flanges have cracked.

The author gives some interesting comparisons of designs for a highway loading and a 25-ft clear span, referring also to the conventional designs of the Delaware, Lackawanna and Western Railroad, as of 1926. In this connection the reinforced concrete grade-crossing designs of the Canadian National Railways might be of interest.[24] Among other types, single spans as long as 60 ft, both precast and poured in place, have been developed to carry Cooper's E-60 live load. It is stated that economies of from 30% to 55% are shown by these structures over those built in the past; if designed of pre-stressed reinforced concrete they would probably show a still greater saving.

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[21] With Robinson & Steinman, New York, N. Y.

[22] *Civil Engineering,* February, 1932, p. 96.

[23] *Engineering News-Record,* February 26, 1920, p. 427.

It seems to the writer that the 10-in. slab thickness mentioned for a 25-ft clear span is rather shallow even for pre-stressed reinforced concrete construction, and might produce excessive deflections; also that 2 000-lb to 2 500-lb concrete was a rather low grade to use, although the author may have taken it purposely.

The advantages of having no bends in the steel reinforcement would probably be offset by the increased cost of forms, due to having to increase the depth or width of the members.

On jobs that are poured in the field the pre-stressing of the reinforcement might be quite expensive due to the high wages of labor, but this cost would probably be reduced materially after the operation had been standardized.

In conclusion, the writer believes that Mr. Rosov's paper should call to the attention of the profession the advantages and economies of pre-stressed reinforced concrete, and should also further its development.

DEAN F. PETERSON, JR.,[25] JUN. AM. SOC. C. E. (by letter).—The method of analysis to be used in beams containing pre-stressed steel has been presented clearly in this paper, and Mr. Rosov has opened for discussion a subject, which, pending further mathematical and experimental investigation, may lead to some appreciable economy in reinforced concrete design.

The writer does not agree with Mr. Rosov's analysis of the principal stresses. Applying Equation (31) with $v = 150$ and $f_l = -300$, he determines f_d as equal to $-\frac{1}{2}$. However, if these values are substituted in Equation (30), from which the author derives Equation (31), $f_d = -150 \pm \sqrt{150^2 + 150^2}$; or -362 and $+61.5$ in which $+$ denotes tension.

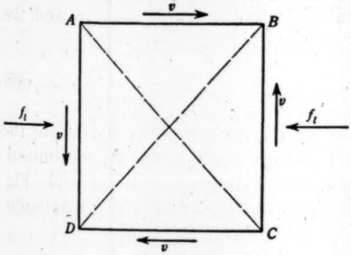

FIG. 8.

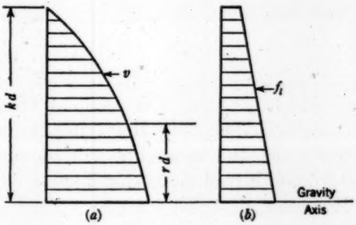

FIG. 9.

Analysis of Diagonal Tension.—Consider a longitudinal stress that is always compressive for pre-stressed beams. The stress on Plane *BD* (see Fig. 8) is minimum tension or maximum compression, and that on Plane *AC* is maximum tension or minimum compression. The former case is not important since, as Mr. Rosov states, the maximum value of f_l does not occur at a point of maximum unit shear stress, v, and the diagonal compression will never be much in excess of the allowable. In the case in which the principal stress occurs on Plane AC, f_d will always have a tensile value if shear exists. Writing,

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[24] *Engineering News-Record*, July 19, 1934, p. 65.

[25] Salt Lake City, Utah.

$$f_d = -\frac{1}{2} f_l + \frac{1}{2} f_l \left(1 + \frac{v^2}{\frac{1}{4} f_l^2} \right)^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

which may be expanded to,

$$f_d = -\frac{1}{2} f_l + \frac{1}{2} f_l \left(1 + \frac{2\, v^2}{f_l^2} - \frac{2\, v^4}{f_l^4} + \cdots \right) \ldots\ldots\ldots\ldots (47)$$

The series in Equation (47) is convergent only if $\frac{4\, v^2}{f_l^2} < 1$, or if $v < \frac{1}{2} f_l$. For values of v greater than, or equal to, $\frac{1}{2} f_l$, the series does not define the radical. For the first approximation,

$$f_d = \frac{v^2}{f_l} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

instead of $\frac{v}{f_l}$ as in Equation (31). As an example choose $v = 100$ and $f_l = 400$; then, $f_d = \frac{100 \times 100}{400} = 25$ lb per sq in. If the series is carried to the second approximation, $f_d = 23.44$. The value of f_d is between the first and second approximations, since the series is alternating. Equation (30) gives $f_d = 23.6$. The purpose of the foregoing comment is merely to show that the assumption expressed by Equation (31) is not valid. There is no gain in using Equation (48) for purposes of computation. Appreciable diagonal tension may occur near the end of the pre-stressed beam, although not to the extent that it would occur on ordinary beams of the same depth.

Assuming the section to be entirely in compression, the unit shear, v, may be determined at any point by using the transformed section and the formula,

$$v = \frac{V\, Q}{I\, b} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

in which Q = the first moment about the gravity axis of that portion of the transformed cross-section lying above the point for which v is to be determined. Since the end moments will be small the stress diagram will resemble Fig. 9(*b*), and it is obvious that a value of r will occur in which f_d will be a maximum. However, if,

$$\frac{d\left(\frac{v^2}{f_l}\right)}{dr} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

is taken, assuming a parabolic variation of v and a straight-line variation of f_l, a fifth-degree equation in r results, and it is probable that it would be easier to investigate three or four points on the section for v than to attempt to find r for each section.

Effect of Shrinkage and Plastic Flow.—It is also to be emphasized that the designed pre-stress is not the initial tension imparted to the bar. The effect of shrinkage will be to reduce the pre-stress directly, with no effect on the concrete, except to close the cracks and release the tension. The reduction in f_{ps} due to shrinkage is $m\,E_s$ in which m is the coefficient of shrinkage. The elastic deformation of the beam will further reduce pre-stress. The change in length of the concrete fibers, ΔL_c, must equal the change in length of the steel, ΔL_s, in any unit length of beam; but

$$\Delta L_c = \delta_1 + \delta_2 \qquad (51)$$

in which δ_1 and δ_2 = changes in length due to rotation (moment), and, shortening (direct stress), respectively. Furthermore,

$$M_{ps} = T_1 (q_b - k_1)\, d \qquad (52a)$$

and,

$$\theta = \frac{1}{R} = \frac{M}{E\,I} = \frac{T_1\,(q_b - k_1)\,d}{E_c\,C_1\,b\,d^3} \qquad (52b)$$

in which, T_1 is the pre-stressing force remaining on the section; but,

$$\theta = \frac{\delta_1}{(q_b - k_1)\,d} \qquad (53)$$

from which,

$$\delta_1 = \frac{T_1\,(q_b - k_1)^2}{E_c\,C_1\,b\,d} = \frac{A_s\,f_{ps1}\,(q_b - k_1)^2}{E_c\,C_1\,b\,d} \qquad (54a)$$

and,

$$\delta_2 = \frac{T_1}{E_c\,K_1\,b\,d} = \frac{A_s\,f_{ps1}}{K_1\,E_c\,b\,d} \qquad (54b)$$

in which f_{ps1} is the unit pre-stress acting on the section. Finally,

$$\Delta L_s = \frac{f_{ps0} - m\,E_s - f_{ps1}}{E_s} \qquad (55)$$

in which f_{ps0} is the unit pre-stress imparted to the reinforcement before placing the concrete. Equating $(\delta_1 + \delta_2) = \Delta L_c = \Delta L_s$, and reducing,

$$f_{ps1} = \frac{f_{ps0} - m\,E_s}{1 + p\,n\left(\dfrac{(q_b - k_1)^2}{C_1} + \dfrac{1}{K_1}\right)} \qquad (56)$$

is the value of f_{ps1} before the effect of plastic flow has occurred. Since the member is under compression from the effect of the pre-stressing force the fibers adjacent to the steel will tend to shorten due to plastic flow. This effect will occur progressively with time. The ultimate plastic flow modulus is approximately 1 000 000, which, when combined with E_c, will give a secant modulus, N, representing the combined effect of flow and elasticity of $N = 750\,000$, and a ratio of $\frac{E_s}{N}$, n', of 40. The dead loads will also have an effect on the designed pre-stress due to flow. The longitudinal shortening of the member will tend to release the designed pre-stress, and the sagging of the member will tend to increase it. The deformation of the unit length of fiber at the steel due to the dead load will be,

$$\delta = \frac{M_D\,(q_b - k_p)}{N_c\,C_p\,b\,d^2} \qquad \text{(57)}$$

in which C_p is based on n'. Equating the deformation of the concrete to the deformation of the steel,

$$\frac{f_{ps0} - E_s\,m - f_{ps2}}{E_s} = \frac{A_s\,f_{ps2}}{N_c\,b\,d}\left(\frac{(q_b - k_{1p})^2}{C_{1p}} + \frac{1}{K_{1p}}\right) - \frac{(q_b - k_p)\,M_D}{N_c\,C_p\,b\,d^2} \qquad \text{(58a)}$$

and,

$$f_{ps2} = \frac{f_{ps0} - m\,E_s + \dfrac{M_D\,(q_b - k_p)\,n'}{C_p\,b\,d^2}}{1 + p\,n'\left(\dfrac{(q_b - k_{1p})^2}{C_{1p}} + \dfrac{1}{K_{1p}}\right)} \qquad \text{(58b)}$$

in which f_{ps2} is the residual stress remaining after the effect of shrinkage, elastic, and ultimate plastic deformations have occurred, and the beam is carrying the total dead load moment. The beam at this stage still behaves elastically as far as the external loads are concerned, the effect of the plastic deformation being entirely represented in the pre-stress. The tension in the steel due to dead loads, the beam behaving elastically, is $\dfrac{M_D\,(q_b - k)\,n}{C\,b\,d^2}$. The pre-stress is then:

$$f_{ps1} = \frac{f_{psc} - m\,E_s + \dfrac{M_D\,(q_b - k_p)\,n'}{C_p\,b\,d^2}}{1 + p\,n'\left(\dfrac{(q_b - k_{1p})^2}{C_{1p}} + \dfrac{1}{K_{1p}}\right)} - \frac{M_D\,(q_b - k)\,n}{C\,b\,d^2} \qquad \text{(59)}$$

As an example of the possible effects of plastic flow Example 1 of the paper is investigated (see Fig. 5). The values of the section functions are given by Mr. Rosov. The plastic functions are $k_p = 0.586$, and $C_p = 0.113$. K_{1p}, k_{1p}, and C_{1p} are equal to K_1, k_1, and C_1, respectively. A designed pre-stress of $-$ 36 500 lb per sq in. is to be used. Substituting the proper values in Equation (56), assuming $m = 0.0004$, and making the necessary algebraic transpositions, $f_{ps0} = 60\,000$ lb per sq in. After ultimate plastic deformation has occurred, the effective tensile pre-stress in the steel is found by Equation (59):

$$f_{ps1} = \frac{60\,000 - 12\,000 + \dfrac{(0.844 - 0.586)\,(22\,900)\,(12)\,(40)}{(0.113)\,(12)\,(16)\,(16)}}{1 + 0.0086\,(40)\left(\dfrac{0.347^2}{0.0843} + \dfrac{1}{0.9914}\right)}$$

$$- \frac{22\,900\,(0.307)\,(12)\,(15)}{0.096\,(16)\,(16)\,(12)} = 26\,300 \text{ lb per sq in.}$$

Then, $f_{tp} = -0.00885\ (26\,300) = -230$ lb per sq in.; and, $f_{bp} = 0.0265\ (26\,300) = +700$ lb per sq in. For dead loads: $F_t' = -230 + 500 = 270$ lb per sq in.; $F_b' = 700 - 430 = 270$ lb per sq in.; and, for live loads: $F_t'' = 270 + 478 = 748$ lb per sq in.; and, $F_b'' = 270 - 408 = -162$ lb per sq in. At this condition, cracks would occur, and it is evident that the compression area would no longer extend over the face of the section and the foregoing analysis would be invalid. The condition tends to approach that of an ordinary beam, somewhat "under-designed." It is to be noted that the effect of plastic deformations due to dead load is opposite to the effect of plastic deformations due to pre-stressing. The case investigated is designed for a dead load greater than 50% of the total load. In cases where the dead load is less significant, the effect of reducing the pre-stress due to flow will be even greater.

Spirally Reinforced Columns.—Another application of the possibility of using a designed pre-stress to advantage might occur in the case of spirally reinforced columns. Shrinkage of the columns will induce compressive stresses of from 12 000 to 18 000 lb per sq in. in the longitudinal steel, with zero stress in the concrete. This compressive stress in the steel will be further increased by plastic flow of the concrete under dead load, with the result that when full-load conditions for the column are reached, the steel may be stressed from 20 000 to 30 000 lb per sq in.[26] This is borne out by the fact that test columns usually fail when the longitudinal reinforcement reaches its yield point and bulges. By pre-stressing the steel in tension, more of the stress would be transferred to the concrete, or, at least, the stresses in the steel due to shrinkage and the flow could be neutralized. It would be desirable, of course, to introduce a greater percentage of spiral reinforcement to restrain the additional deformations of the concrete. This is a matter for thorough investigation by experiment.

In conclusion, it seems to the writer that, although stress control due to induced pre-stress in beams may be a means of saving in design, the effect of deterioration of the pre-stress by plastic flow should be considered and an attempt made to evaluate it. Further investigation of the specific effect of flow on pre-stressed beams is needed. Although the particular beam investigated by this discussion apparently develops a rather unfavorable condition due to plastic flow it may be possible, using high-strength steel, to design beams of greater economy by using the principle of pre-stress, the limiting condition being that the stresses before and after plastic flow has occurred will be within the allowable limit. It is also concluded that appreciable diagonal tension may develop in spite of pre-stress, and that its occurrence should be fully investigated, especially since the unit shear will increase as the depth of the beam decreases.

M. Hirschthal,[27] M. Am. Soc. C. E. (by letter).—The pre-stressing of reinforced concrete in bridge construction, as suggested by Mr. Rosov, is very interesting, but it is to be feared that it is purely academic, particularly for members that are cast in place. To the writer, the most interesting angle is that referring to the elimination of shrinkage stresses in slabs and beams by means of pre-stressing.

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[26] "Principles of Reinforced Concrete Construction," by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, Chapter VII, 1932.

[27] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

In this connection, it may be interesting to note that the writer has requested the personnel of several university experiment stations to make tests on a form of pre-stressing, with special application to pavement slabs. The shrinkage (or temperature) reinforcement of a concrete slab (similar to a concrete pavement slab) is to project beyond the slab ends and be restrained from responding to normal shrinkage action which puts this steel in compression and the concrete in tension, thus causing the shrinkage cracks. The action resulting from the foregoing restraint would tend to put the steel reinforcement in tension and the concrete in compression—the normal action of reinforced concrete. In practice, two adjoining slabs would have the projecting reinforcement bars interlock.

However, in bridge work that is cast in place, the assumption that the reinforcement is free at the construction joints is a false assumption, in the main, because practically all construction joints are specified to be made at sections of minimum shear. This is invariably the point of maximum moment where the bars are certainly continuous through the joint because even the necessary laps in bars are excluded at such points or sections in careful design.

In the author's method of operation at Stage (2) when the tensile force is removed, there is a great probability of high concentration of bond stresses at the ends of the beams or slabs, and although Mr. Rosov notes that Freyssinet reported no difficulty in this connection, the 150-lb tensile strength for the very high-strength concrete indicates otherwise.

Moreover, pre-stressing is applicable only to a material assumed to be homogeneous and elastic. Although every designer specifies that concrete shall be of a composition and grading such as to result in a "homogeneous mass" when thoroughly mixed, the writer feels that the results, even with the best of care, fall far short of homogeneity. Any material of this character cannot be perfectly elastic even within the limits of working stresses. When the stresses approach the ultimate, the material may begin to fail from strains characteristic of a composite material such as concrete. The failure may result from fracture of the coarse aggregate, from fracture of the matrix that binds it, or by loss of adhesion or bond between the aggregate and the matrix. This type of material is not an ideal one for the application of pre-stressing. The material (concrete) is very different from mortar, on the action of which, under tests by Freyssinet, the author bases his assumption. Mr. Rosov cites Freyssinet's tests of treated concrete, in which a strength of 14 000 lb per sq in. in compression was developed, but the tensile strength was as low as 150 lb per sq in. Such concrete would not be very effective in bond and, when used with 45 000-lb per sq in. working stresses in steel reinforcement, would require an inordinate amount of embedment of the steel, or provision against the heavy bond stress developed on release of the "pre-stress."

The author's analysis is confusing at times; assuming the concrete section as taking tension at one time, then assuming the entire section to be in compression;

taking into consideration the reinforcement, and then dropping it. If the final assumptions in the paper are made at the beginning, Equations (20) and (21) could be derived directly instead of by means of the intervening steps. Mr. Rosov does not apply his test (*D*) for stresses at the bottom of the beam after having completed that of Condition (*C*). Incidentally, at the beginning of the derivation of the formulas the author assumes the entire concrete section to be in compression; and yet he indicates the neutral axis in the proximity of the center, implying it to be the gravity axis and so computing his moments of inertia of the section.

On the subject of economy, the author seems to be too optimistic as to the cost of special steels. He would find that a specification calling for steel reinforcement suitable for a working stress of 45 000 lb per sq in. would result in quotations that could scarcely be considered "only slightly" higher than, say, standard intermediate grade reinforcement which is usually specified.

In the author's comparison of slabs, he selects one, citing the writer's series of articles on pre-cast bridge slabs, published in 1926.[13] Some of these structures were designed as early as 1909, and the structure selected by Mr. Rosov is purely hypothetical since the overhead highway bridges there described consist of pre-cast T-beam units with clear spans varying from 30 ft $8\frac{1}{2}$ in. to 35 ft 6 in.; and corresponding depths of 2 ft 6 in. and 2 ft 8 in. These bridges were designed in 1913, when a concrete having a 6-bag cement content (the 1 : 2 : 4 variety) was computed as having a working stress of 650 lb per sq in. Had the author made a comparison with an actual design in the articles referred to and selected a railroad bridge at Lackawanna Place, Millburn, N. J., with pre-cast slabs continuous over a center support and having a maximum length of 60 ft $3\frac{1}{2}$ in., with maximum spans of 28 ft 0 in., from the center support to the inner edge of the abutment, such comparison would not have been so favorable to the "pre-stressed design." (In effect, a pre-stressed design is based on the assumption that pre-stressing leaves the entire concrete section above the tensile reinforcement in compression much after the assumption made by Charles S. Whitney, M. Am. Soc. C. E., in 1936,[28] without the requirement of pre-stressing.) The slabs have a depth at the center of 2 ft 4 in., and 2 ft 2 in. at the edges (thus providing drainage). They are subjected to a maximum bending moment of 104 400 ft-lb per ft of width for live load and impact as against the 21 700 ft-lb selected by the author for his slab—a moment more than $4\frac{1}{2}$ times as great.

In recent years, many methods have been followed to reduce the slab depths. Among them are the use of compressive reinforcement, continuity, end restraint (practically the same effect as pre-stressing), the use of higher-strength concrete, two-way, or multi-way, reinforcement combined with continuity, and end restraint. A girderless flat-slab bridge, carrying railroad loadings for spans about 30 ft, center to center of columns, required a slab thickness of only 2 ft.

Another comparison that would not have been so favorable to pre-stressing, would be the use for the moment in the author's slab, of a working stress of 45 000 lb per sq in. in the reinforcement in tension and the $0.4 f_c'$ allowable for a concrete with a 6-bag cement content—a concrete with a water-cement ratio of 6 gal per bag. Under these conditions, the ultimate strength is 3 500 lb, or $f_c = 1\,400$ lb per sq in., and $f_s = 45\,000$. Then:

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[28] *Journal*, Am. Concrete Inst., 1936.

$$k = \frac{12 \times 1\,400}{45\,000 + 12 \times 1\,400} = 0.272;$$

$$j = 0.909;$$

$$K = \frac{0.272 \times 0.909 \times 1\,400}{2} = 172;$$

$$d = \sqrt{\frac{44\,600}{172}} = 16 + 2.5 = 18.5 \text{ in.}$$

(which gives a somewhat higher dead load moment);

$$A_s = \frac{44\,600}{0.909 \times 16 \times 45\,000} = 0.82;$$

and, $p = 0.00425$. If 4 500-lb concrete were used, the value of f_c would be 1 800, and,

$$k = \frac{12 \times 800}{45\,000 + 21\,600} = 0.324;$$

$$j = 0.892;$$

$$K = \frac{0.324 \times 0.892 \times 1\,800}{2} = 260;$$

$$d = \sqrt{\frac{44\,600}{260}} = \pm\, 13 \text{ in.},$$

or 16 in. total; and $p = 0.006$. (For a value of $n = 1$, d becomes 13.5 in., giving a total depth of 16 in., which is the same as in the author's example.)

Aside from these considerations, the practical difficulties encountered on a structure in connection with pre-stressing would be almost insuperable, and probably only in a casting yard could such installation be considered.

Although the writer has cited his objection to the use of pre-stressing in this particular case (bridge construction), he realizes the value of such treatment for structures such as reservoirs, tanks, and poles. This paper will have accomplished a valuable service if it leads to research activities, so that possibly shrinkage stresses may be counteracted in concrete—an undoubted incipient cause of deterioration and final failure of reinforced concrete.

A. **Floris,**[29] **Esq.** (by letter).—The problems of pre-stressing steel in reinforced concrete structures are treated in this interesting paper. In the methods described therein the pre-stressing is done by mechanical devices or by heat. However, in certain cases, the same results can be achieved quite naturally, without external influences.

In reinforced concrete trusses, for instance, the steel of the compression members can be embedded in concrete during construction, but the steel of the tension members is left exposed. After the forms are removed and the structure is under full load, the tension steel is covered with concrete.

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[29] Dipl.-Ing., Los Angeles, Calif.

The advantage of this method, due to Dr. Ulrich Finsterwalder,[30] is that cracks, otherwise unavoidable in the tension members, are almost eliminated and the appreciable secondary stresses are reduced considerably, because of the flexibility of the exposed reinforcement in the tension members. In this manner the angular changes at the joints, caused by the weight of the structure, take place almost without restraint.

A. A. EREMIN,[31] ASSOC. M. AM. SOC. C. E. (by letter).—The possibility of pre-stressing concrete is important in the economic design of reinforced concrete structures. However, the methods of pre-stressing are still very little known.

An interesting method of pre-stressing a reinforced concrete tunnel lining was used in 1930 in the outlet tunnels of the dam across Dordogne River, in France.[32] At their lower ends these tunnels were bored through the ground formation with rock of inferior strength which was incapable of resisting the hydraulic pressure produced by a water head of 335 ft. Designed according to common practice, without pre-stressing, the tunnels would have required a concrete lining 3 ft 3 in. thick, which would have made the cost prohibitive. M. Mary developed a method of pre-stressing which reduced the lining to a thickness of only about 12 in. The arrangement is shown in Fig. 10. The concrete lining, with an inside diameter of 14 ft 5 in., is reinforced with steel cables placed 19 in. between centers. The ultimate strength of the cables was 242 tons. The computed maximum stress, produced by the hydraulic static pressure, was 30 tons per cable, and the maximum stress developed by hammer action was about 10 tons per cable. The cables were pre-stressed to 152 tons. which was considered sufficient to resist internal hydraulic pressure in the tunnel and to reduce

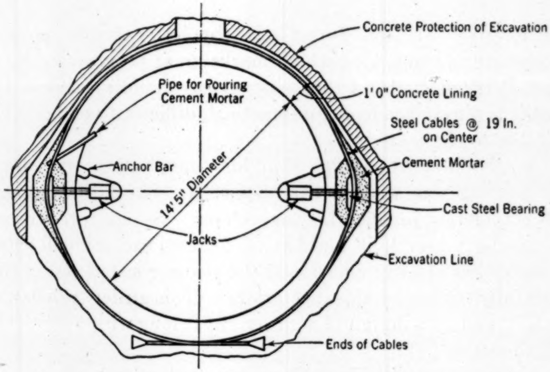
Concrete Protection of Excavation
Pipe for Pouring Cement Mortar
1'0" Concrete Lining
Steel Cables @ 19 In. on Center
Anchor Bar
Cement Mortar
14'5" Diameter
Cast Steel Bearing
Jacks
Excavation Line
Ends of Cables

FIG. 10.—METHOD OF PRE-STRESSING A TUNNEL LINING

shrinkage cracks in concrete. A small coefficient of safety was used because design assumptions were successfully verified by testing a full-sized model concrete cylinder 20 ft long. The cables were pre-stressed by hydraulic jacks placed at the horizontal diameter of the tunnel, in such a manner that the original circular form of the cables was changed to an elliptical one. The forces from the jacks were transmitted to the concrete lining through steel bar anchors. After pre-stressing, a cement mortar was poured around the cables and bearing plates. The jacks were removed when the cement mortar had set sufficiently to reach the prescribed strength.

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[30] "Untersuchungen über Knicksicherheit, die elastische Verformung und das Kriechen des Betons bei Bogenbrücken," von Dr.-Ing. Fr. Dischinger, *Der Bauingenieur*, September 3, 1937, p. 551.

[31] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[32] *Annales des Ponts et Chaussees*, April, 1936, p. 423.

In developing the method of pre-stressing considerable attention was given to reducing the bond stresses between the steel cables and the concrete. Bond stresses generally increase in proportion to the increased stresses in the cables during pre-stressing, and are affected by the change in length of the cables as shown by Equation (23). Special provisions for reducing the bond stresses in a concrete lining were found necessary. Heavy asphalt coating of the cables was not considered sufficient; and, furthermore, asphalt reduces the resisting strength of concrete. Finally, it was decided to place the cables in steel pipes permitting them to slide freely. The design and construction of the outlet tunnels at Dordogne Dam provide a valuable demonstration of the possibilities of pre-stressing reinforced concrete and serve to support the author's conclusions.

IVAN A. ROSOV,[33] M. AM. SOC. C. E. (by letter).—The pre-stressing of reinforced concrete is a new idea which marks great progress in this branch of engineering. It is only natural that a new idea should meet objection on the grounds that it is purely academic, impractical, difficult to embody in practice, and not suitable to the material.

The writer should acknowledge that his contribution was almost purely theoretical, when it was written early in 1934. Since that time, however, practice has completely justified his suggestions. Several pre-stressed bridges and roof trusses have been built and have developed exceptionally good properties. The writer has already mentioned the girder constructed in Germany in 1936,[16] which fully confirmed all expectations. This girder (which was actually tested) was followed by a design of a girder 198 ft long with a depth of only one-sixteenth of the span.

The Saale Bridge, at Alsleben,[34] Germany, with a span of 223 ft, is an example of a pre-stressed bowstring bridge. The success of this structure encouraged the construction of many other pre-stressed arched structures—especially hangars with spans longer than 320 ft.[35] Trusses are represented by hangars of the Caquot type constructed in France,[36] in which pre-stressing is done in a manner similar to the method described by Mr. Floris. For the most advanced design one should mention the Gerber (girder) Bridge, "Drei Rose," at Basle, Switzerland. Pre-stressing permitted increasing the span of this bridge to 410 ft, which is extremely unusual for a simply supported concrete structure. It is seen that pre-stressed bridges and roof trusses are already well established, at least in Europe. They can no longer be considered "academic."

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[33] Asst. Engr., New York City Tunnel Authority, New York, N. Y.

[34] *Beton und Eisen*, 1932.

[35] International Assoc. for Bridge and Structural Eng., 1936.

[36] *Travaux* (*Science et Industrie*), 1936.

As to the practical difficulties of pre-stressed construction, stated by Mr. Hirschthal as being "almost insuperable," the writer has already outlined the methods involved. Some of them are very simple, as is the lengthening of the reinforcement by means of electrical heating. The method described by Mr. Floris is even simpler and requires only that the compression members be poured first and the tension members later. In M. Freyssinet's method, which embodies the "treatment"[37] of the concrete, the pre-stress is obtained as a by-product of the latter procedure. This method was used under the difficult conditions presented by a narrow foundation at the Marine Station, at Havre, France. The work of fabricating and sinking the pre-stressed piles attained a speed of 25 ft per working day. Plant equipment was almost automatic and, as is stated by M. Freyssinet, "expenditure in material and labor was much less than for the ordinary process."[38]

Possibly the most complicated was the case of the aforementioned girder in which the degree of pre-stressing varied along the span; the stirrups were also pre-stressed and the concrete was "treated."[16] Even in this case the only additional equipment required, beyond the customary compressors, jacks, and mixers, was a stretching beam and a 5-ft section of a special steel form. To complete the operation for one 5-ft section (which included pouring the concrete, vibrating, hydraulic pressing, and heating) required only 2 hr. The entire girder, consisting of twelve sections, required 60 hr.

This case, and the foregoing list of structures already built, indicate that "insuperable" difficulties do not exist and that progress in methods of pre-stressing is rapid. The new "sursulfaté" cement developed in France is very promising.[39] This cement expands after hardening, the expansion reaching 0.003 in. per in., and the reinforcement, if embedded, would resist it creating an initial compression in the concrete.

Mr. Hirschthal states that the writer was too optimistic regarding the cost of special steels. The extra cost is actually small if proper methods are used to produce such steels. For bars $\frac{5}{8}$ in., or less, in diameter, M. Freyssinet used a mechanism, that he developed, which unrolled bunches of steel, welded the ends, and stretched the steel in one operation. At the cost of a "few centimes per kilogram," straight endless rods (which were tested at every point, including welds) with yield points of 114 000 lb per sq in. were obtained. The general cost for pre-stressed structures, as a rule, is considerably less. Mr. Eremin should be commended for his striking example of a tunnel lining that would be prohibitive in cost unless pre-stressing were used.

The writer cannot agree with those who state that pre-stressing is not adaptable to concrete because that material is not homogeneous and elastic. This general defect handicaps any concrete structure, those that are subjected to tension being affected much more severely than others. In the latter case the bond between particles is decreased (often entirely destroyed), and will not be restored if the tension is removed. This makes concrete unsuitable for repeated stresses which involve a wide range of tensile stress. On the other hand, if concrete is compressed, the particles approach one another, the bond is increased, and releasing it merely restores the concrete to its initial condition.

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[37] *Memoires et Compte Rendu des Travaux* de la Société des Ingenieurs civils de France, September–October, 1935.

[38] Rept. to Joint Meeting of Inst. of Structural Engrs. and Société des Ingenieurs civils de France, March, 1936.

[39] *Bulletin Technique* de la Suisse Romande, January, 1937.

Concrete can even heal minute cracks by chemical conversion of calcium hydroxide to calcium carbonate,[40] and permanent compression aids this process. Freyssinet describes a case in which a pre-stressed pipe subjected to an internal pressure, ten times greater than normal, at last "failed," resulting in seepage and the appearance of fissures. However, after the head had been reduced, the fissures closed at once, and a few weeks later, the concrete having sealed the fissures, the pipe was as good as before.[16]

Pre-stressing eliminates tension entirely. Professor Dischinger states that a "reinforced concrete body subjected to compression only, has, like natural stone, almost unlimited life."[35] Mr. Hirschthal's reference to the lack of homogeneity and elasticity in the concrete should be of concern to designers of customary structures much more than to designers of pre-stressed structures.

The writer's method of analysis inspired Mr. Peterson's very interesting contribution containing a more detailed calculation of the loss in pre-stressing. However, when Mr. Peterson refers to the writer's example, and subtracts from the designing pre-stress the loss due to the plastic flow, he did not notice that this had already been done. In the text of the paper preceding the heading, "Theory of Pre-Stressed Rectangular Beams," the writer considered possible losses, including the plastic flow, estimated them as 35 000 lb per sq in., and noted that this loss should be subtracted from the "preliminary pre-stress" to give the "designing pre-stress." The preliminary pre-stress in the writer's example is 36 500 + 35 000 = 71 500 lb per sq in., not 60 000 lb per sq in., as Mr. Peterson assumes. The example then remains intact. It is gratifying that Mr. Peterson's calculation for the loss gives 60 000 − 26 300 = 33 700 lb per sq in., which is almost identical with the writer's estimate of 35 000 lb per sq in. Plastic flow, however, has not been studied sufficiently, particularly for "treated" concrete. Such a study would be valuable in connection with pre-stressing. For this kind of concrete Mr. K. Lenk accepts the entire loss as only 21 000 lb per sq in.[41]

For diagonal tension, Equation (31) should be rewritten as $f_d = -\frac{v^2}{f}$. Tensile stresses exist although they are much smaller than for conventional design. In order to cancel them entirely the stirrups are also pre-stressed. In this manner, compression is introduced along two perpendicular axes and the problem of diagonal stresses is eliminated.

Replying to Mr. Hirschthal, the pre-stressed section is compressed at any point. This is the basic principle of the design. Bending moments acting over the section produce compression on one side of the section and tension on the other side. However, the tensile stresses are absorbed by counteracting compression and do not develop cracks. The area of the section remains constant, unlike the conventional case in which, after cracking, the section is decreased and the stresses are redistributed. The stability of the section permits the use of the law of superimposition and the separate consideration of the direct pressure and the bending moments produced either by loads, or by pre-stressing. This fact was ignored by Mr. Hirschthal. However, it is common practice to consider combined direct stresses and bending in the case when the entire section is under compression. If the writer assumed the concrete section as taking tension at one time, and then assuming the entire section to be in compression, it was because he considers superimposed bending moments separately in the former case and all stresses together in the latter case. The writer also "assumes the entire concrete section to be in compression; and yet he indicates the neutral axis in the proximity of the center, implying it to be the gravity axis and so computing his moments of inertia." This was because the neutral axis is considered for the bending moments only and, if the section is not cracked, it is the gravity axis and, of course, the moment of inertia should be computed about this axis.

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[40] *Concrete*, April, 1937.

[41] *Beton und Eisen*, May 20, 1937.

Mr. Hirschthal considers "confusing" the fact that the writer takes the reinforcement into consideration and then drops it. The writer "drops" the reinforcement in two cases: When he considers the action of outside load, he takes the full section, including the reinforcement; but when he considers the action of the pre-stressing, he subtracts the area of the reinforcement, because the pre-stress is the force produced by the steel and is acting on the concrete in the form of bond between the two. The second case is that in which the writer passes from the general design with both top and bottom reinforcement to a design with bottom reinforcement only. He "drops" the top reinforcement, making $p = 0$. In either case, it could not be done otherwise.

Mr. Hirschthal wants Equations (20) and (21) derived directly. The writer selected the most logical method. He considered the general case and then passed to particular cases, equating some quantities to zero. This is the common method of solving problems. Equations (20) and (21) represent only a special case when $p = 0$ and the reinforcement is small. Certainly, every special case can be derived directly but there seems to be no need of complicating the problem with each separate solution.

Mr. Hirschthal's last remark is that the writer does not apply Test (*D*) to his examples. The writer applied this test (see text following Fig. 5 and Table 1) and found that Test (*D*) does not control, because higher pre-stressing is required by Test (*C*). Therefore, only the latter was considered.

Mr. Whitney prefers to compare the pre-stressed design with his method. The writer made his comparison with a commonly used method because it was the only way to reveal clearly the advantages of pre-stressing. Recently, many new methods of designing reinforced concrete have been offered by Professor Emperger,[42] Professor Steuerman,[43] Dr. Gebauer,[44] Professor Saliger,[45] Mr. Whitney, and others. New methods differ widely from conventional design and are objected to by many concrete authorities.[46] This is not the proper place for discussing them. Personally, the writer is inclined to accept Mr. Whitney's method as logical and well adjusted to tests. However, his formulas cannot be used for pre-stress design without revision. They are based on the assumption that $\frac{a}{d} = 0.59$, which is taken from the tests of Messrs. Slater and Lyse,[47] Mr. Glanoille,[48] U. S. Geological Survey,[49] and Messrs. Humphrey and Losse.[50] In all these tests the steel had a yield point of 38 000 to 58 000 lb per sq in. Only the tests conducted by Messrs. Slater and Lyse considered steel with a yield point of 64 800 lb per sq in., but in this case also the steel was not stressed beyond 40 000 lb per sq in. No empirical assumption may be extended beyond the limits of tests. Mr. Whitney's formulas cannot be applied to stresses in steel as high as 90 000 lb per sq in. The writer believes that such stresses in the steel would induce cracks much wider and longer than those in conventional structures; therefore, the ratio, $\frac{a}{d}$, will be smaller, the "compression block" will decrease, and the breaking point for a conventional design will be much lower than that calculated by Mr. Whitney's method. On the other hand, if the reinforcement is pre-stressed, the outside moment should overcome: (1) The resisting moment due to pre-stressing; and (2) compression at the bottom due to the direct pressure from pre-stressing, before cracks would appear—a stage that occurs at the beginning in conventional design. This will delay the breaking of the concrete considerably. Since the stress in the steel increases only 90 000 − 45 000 = 45 000 lb per sq in. from the first tension in the concrete to the breaking point, the cracking will not be greater than for a design with ordinary steel, and the compression block will be about the same. It would be very interesting to see Mr. Whitney's method developed to include a design involving steel with high yield point and based upon pre-stressing. However, until this is done, it will be impossible to use this method in appraising pre-stressed structures.

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[42] *Beton und Eisen*, 1933, Hefte 3 und 4.

[43] *Loc. cit.*, July 20, 1935.

[44] *Loc. cit.*, 1936, Heft 2.

[45] Mitteilungen über Versuche d. Oesterr. Eisen Beton Ausschlusses.

Some discussers would replace pre-stressing by the customary methods of reducing slab depths, such as by continuity, end restraint, the use of stronger concrete, etc.; but pre-stressing is entirely compatible with these methods; it is based on an entirely different principle and can be combined with each of these methods to great advantage in design. As Mr. Lemcke suggests, the writer purposely used low concrete stresses and simple structures as examples so as not to obscure the advantages of pre-stressing by other elements. More complicated examples were eliminated from the paper as an unnecessary contribution to its length. Among examples that might have been cited was an alternate design of a continuous bridge with a span of 30 ft. Contrary to the opinion expressed by Mr. Hirschthal the comparison was as favorable as for simple structures. The volume of concrete was reduced 20%, the decrease in the depth at the center was 40%, and the reinforcement was straight and simple. Another example included a pre-stressed I-girder as an alternate to a T-beam design for the Delaware, Lackawanna and Western Railroad. The construction proved to be much more economical and expedient, saving 47% in concrete and 42% in steel.

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[46] *Concrete and Constructional Engineering,* 1937 (Discussions).

[47] *Journal,* Am. Concrete Inst., June, 1930.

[48] *Concrete and Constructional Engineering,* March, 1937.

[49] *Journal,* Am. Concrete Inst., 1920.

[50] *Technological Paper No. 2,* National Bureau of Standards, 1912.

The writer takes the opportunity, presented by Mr. Hirschthal's design of a non-pre-stressed slab with high-strength concrete and high yield-point steel, to demonstrate once more the effect of the pre-stressing.

In general, the use of steel stresses as high as 45 000 lb per sq in. would create important defects in the conventional slab. The concrete cracks when it cannot follow the uniform elongation of the reinforcement. Any increase in the elongation of the steel involves a much greater increase in cracking. At present, working stresses in the reinforcement are not exceeding 27 000 lb per sq in.[51] Raising them 70% would produce a badly cracked structure. The deflection also depends on the elongation of the bottom fibers. It is doubtful whether the construction of such a shaky and cracked slab would be allowed.

Moreover, the saving is not large enough in proportion to the quality of the concrete, because the ratio, $n = \frac{E_s}{E_c}$, decreases considerably. Mr. Hirschthal assumed $n = 12$, but this value does not correspond to that for high-strength concretes. Usually n is taken as $\frac{30\,000}{f_c'}$, in which f_c' is the crushing strength of concrete.[52] In England, n is assumed as $\frac{40\,000}{f_{cs}'}$ in which f_{cs}' is the "cube strength" of concrete.[53] Since f_{cs}' is $1\frac{1}{3}$ to $1\frac{1}{2}$ times f_c', the European method gives almost the same value of n as is accepted in the United States. Hence, in Mr. Hirschthal's first example, n should be taken as $\frac{30\,000}{3\,500} = 8.6$ instead of 12. With this correction, and taking into account the additional dead load, the depth of the slab should be as follows: $k = \frac{8.6 \times 1\,400}{45\,000 + 8.6 \times 1\,400} = 0.211$; $j = 0.930$; $K = \frac{0.211 \times 0.930 \times 1\,400}{2} = 137.5$; $d = \sqrt{\frac{50\,600}{137.5}} = 19$; and, $D = 19 + 2.5 = 21.5$ in. compared to 27.5 in. in the design with plain concrete and steel. The saving is not so great as to warrant sacrificing the stability of the structure.

Now, consider the pre-stressed slab. The depth of the slab is assumed as 10.5 in.; $p = 0.0155$; $n = 8.6$; and $q = \frac{8}{10.5} = 0.761$. From Equation (19), $K = 1 + 7.6 \times 0.0155 = 1.116$; from Equation (18), $k = \frac{1}{1.116} \times (\frac{1}{2} + 7.6 \times 0.0155 \times 0.761) = 0.527$; and, from Equation (17), $C = 0.083 + \overline{0.027}^2 + 7.6 \times 0.0155 \times \overline{0.234}^2 = 0.090$. The dead load is $0.875 \times 150 + 65 = 196$ lb per sq ft; $M_D = 22\,900 \times \frac{196}{265} = 16\,950$ ft-lb; $M_L = 21\,700$ ft-lb; and the total, $M = 38\,650$ ft-lb.

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[51] *Concrete and Constructional Engineering*, September, 1933

[52] New York City Building Code.

[53] *Concrete and Constructional Engineering*, January, 1937.

Stresses due to loading, from Equations (5), (6), and (7), are:

$$f_t' = \frac{38\,650 \times 0.527}{0.090 \times 12 \times \overline{10.5^2}} = 1\,702 \text{ lb per sq in.};$$

$$f_b = -\,1\,702 \times \frac{0.473}{0.527} = -\,1\,535 \text{ lb per sq in.};$$

and,

$$f_s = -\,\frac{38\,650 \times 8.6 \times 0.234}{0.090 \times 12 \times \overline{10.5^2}} = -\,6\,620 \text{ lb per sq in.}$$

Stresses due to pre-stressing, from Equations (20) and (21), are: $f_{pt} = 2 \times 0.0155 \times (3 \times 0.761 - 2) \times f_{ps} = 0.00877\ f_{ps}$; and, $f_{pb} = -\,2 \times 0.0155 \times (3 \times 0.761 - 1) \times f_{ps} = -\,0.0398\, f_{ps}$.

Conditions to determine the pre-stress required, are:

(*C*) $F_t = +\,0.00877\ f_{ps} + 1\,702 \leq 1\,400$; hence, $f_{ps} \leq -\,34\,400$ lb per sq in.

(*D*) $F_b = -\,0.0398\, f_{ps} - 1\,535 \geq 0$; hence, $f_{ps} \leq -\,38\,400$ lb per sq in.

and,

(*E*) $F_s = -\,(f_{ps} - 6\,620) \leq +\,45\,000$; hence $f_{ps} \geq -\,38\,400$ lb per sq. in.

Conditions (*D*) and (*E*) are controlling; therefore: $-\,38\,400 \leq f_{ps} \leq -\,38\,400$ and the final stresses are as follows: $F_t = 1\,702 - 336 = +\,1\,366$ lb per sq in. $\leq +\,1\,400$; $F_b = -\,1\,535 + 1\,535 = 0$; and, $F_s = -\,38\,400 - 6\,620 = -\,45\,000$ lb per sq in.

Compared to Mr. Hirschthal's design the pre-stressed slab: (*a*) Saves about 51% of concrete; (*b*) eliminates tension and cracking completely; and (*c*) decreases the deflection to a small amount because the steel stresses vary only within the narrow limit of 6 620 lb per sq in.

It is clear that pre-stressing is justified by results achieved by practice; it is not costly; and it results in exceptionally economical structures. Mr. Lenk who observed the German tests with the pre-stressed girder adds to this, the statement that, "the complete lack of cracks and a surface as hard as glass makes the pre-stressed concrete resistant to air, gases, and humidity. Its water-tightness is unrivaled. The expense of maintaining pre-stressed concrete structures is considerably less than other types."[54]

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[54] *Beton und Eisen*, May 20, 1937. (Quotation cited is a translation by the writer.)

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2008

PRACTICAL APPLICATION OF SOIL MECHANICS

A SYMPOSIUM

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LEVEES IN THE LOWER MISSISSIPPI VALLEY

BY SPENCER J. BUCHANAN,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

A levee may be defined as an earth embankment constructed along the margin of a stream to restrain its flow within a desired course during flood stages. The system of levees, as it exists to-day in the Lower Mississippi Valley, is one of Man's greatest works, involving approximately 761 000 000 cu yd of material throughout its length of 1 615 miles. The type used is similar in many respects to modern earth dams. In fact, it is natural that there should be a close similarity because a levee is merely an earth dam of great length, designed to function during relatively short intervals, after long lapses of time. This paper is concerned with the general features considered in the design of a levee and the use of the relatively new tool, Soil Mechanics, in solving unusual problems arising in connection with levee design.

INTRODUCTION

The first levee on record, to be constructed in the Lower Mississippi Valley, was built in the vicinity of New Orleans, La., by De la Tour under the orders of the founder of the city, Sieur de Bienville. The date of this work was between 1717 and 1727. It was a modest start as compared to the present system, but it proved so successful that, in 1743, the French Colonial Government ordered all land owners in that vicinity to "follow suit" and gave them one year to complete their units under penalty of forfeiture of their holdings immediately to the Crown. With the development of the Valley grew a haphazard system of levees which provided uncertain protection from the floods of the river. The resultant confusion, together with growth of national interest in the matter, resulted in the creation of the Mississippi River Commission by an Act of Congress in 1879, for the co-ordination of the efforts of the local interests and for the administration of the work of the National Government in controlling and improving the river.

The growth of the system under the administration of this Commission has been tremendous and its efforts have been marked with success as indicated by the satisfactory confinement of the 1929 and 1937 floods. The extent of the system at present is shown by Fig. 1. Accompanying this growth there has been a corresponding reduction in the area of the floodway and natural storage basins; greater flood stages have resulted, requiring higher levees. The average height at this time is approximately 22 ft. However, this height is as great as 50 ft at isolated points where the line crosses lakes or bayous. As the growth in size progressed, the experience of those directing the work broadened. The weaknesses in design became apparent under actual flood conditions and sufficient revision was made to insure stability.

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During the past few years the methods of levee construction have changed, the value of property protected has been enhanced, and the demand for designs complying with strict engineering practices has increased. The results of these changes have been many. For example, the change in construction methods makes it possible to complete the new and larger levees in a fraction of the time formerly required with mules and wagons or with men and wheel-barrows. Consequently, the foundations are loaded more rapidly and with heavier loads, and the chances of their failure are increased. When the circumstances are such that stable foundations are evident, and material is available with which sufficient experience has been accumulated, designs based on experience have been adequate. However, when the limits of the location are restricted, and it is necessary to construct a unit across an old lake bed or on some poor foundation, then the use of the new tool, Soil Mechanics, serves a need of long standing. The use of this branch of civil engineering permits design to be founded on a strength-*versus*-stress basis.

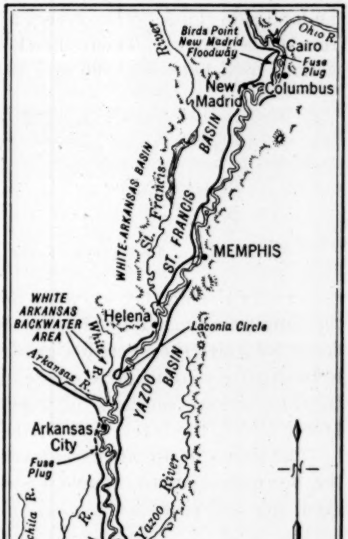
Ohio R.
Birds Point New Madrid Floodway
Cairo
Fuse Plug
New Madrid
Columbus
St. Francis River
WHITE-ARKANSAS BASIN
ST. FRANCIS BASIN
MEMPHIS
WHITE ARKANSAS BACKWATER AREA
Helena
Laconia Circle
White R.
Arkansas R.
YAZOO BASIN
Arkansas City
Fuse Plug
Yazoo River
N
Ouachita R.
Boeuf R.
TENSAS BASIN
Vicksburg
Black R.
Natchez
Red R.
Old River
Fuse Plugs
Bonnet Carre Spillway (Controlled)
Scale in Miles
0
50
Atchafalaya Spillway
ATCHAFALAYA BASIN
BATON ROUGE
Lake Pontchartrain
NEW ORLEANS
Grand Lake
GULF OF MEXICO

FIG. 1.—LEVEE SYSTEM, MISSISSIPPI RIVER, 1936

ESSENTIAL FEATURES OF DESIGN

The ultimate design of a levee requires consideration of the following essential features: Side slopes, foundation, and control of seepage. The first two of these features are of the utmost importance because, more than anything else, the structures must be stable for any conditions. The control of seepage is important and justifies discussion because of its effect on the stability of the structure. However, the volume of water lost is of little importance as its conservation is rarely desired in a flood control system. Furthermore, the normal drainage system of the areas adjacent to levees is generally more than adequate for the disposal of water accumulating through seepage.

Side Slopes.—The side slopes adopted for the standard sections of the levees over the period from 1882 to date, are shown by Fig. 2. The elevation of the maximum probable flood is that contemplated by the Flood Control Project authorized by Act of May, 1928. It may be noted that, as the size of the structures has increased, there has been a gradual reduction of the inclination of the land-side slope. The river-side slope has remained practically unchanged from the beginning. The broken land-side slope, as indicated for the banquette sections used between 1896 and 1914, was found to be undesirable in certain respects for the high levees on the Lower River. The steep land-side slope above the banquette proved unstable in some instances when confining the maximum floods for which they were designed. This unstable condition was probably due to the hydrodynamic forces created by the seepage, acting on the stepped slopes. In many instances slides or general sloughing actually occurred. For the Lower River, the levee section was altered to give a straight land-side slope.

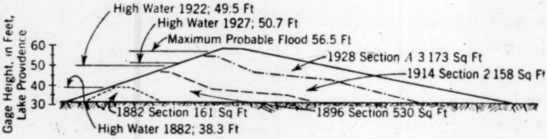

FIG. 2.—STANDARD LEVEE SECTIONS, LOWER MISSISSIPPI RIVER

The slopes in use at present vary somewhat with the material available in the borrow-pit areas. Table 1 shows this variation for the basic materials, sand, silt, and clay.

TABLE 1.—SLOPES FOR CLAY, SILT, AND SAND

Section	Predominating material	Crown width, in feet	River-side slope*	Land-side slope* containing assumed topmost flow line of following slope
A	Clay (75%)	10	1 on 3.0	1 on 6.0
B	Silt	10	1 on 3.5	1 on 6.5
C	Sand (75%)	12	1 on 5.0	1 on 8.0

* Topmost flow line assumed to spring from river-side slope, 1 ft below crown of structure.

The stability of these standard sections has been investigated, by the method described subsequently, and for normal conditions was found to vary between 1.5 to 3.5. This factor of safety is considerably greater than the average for earth structures of the levee type. The *B* section (Table 1) predominates throughout the Mississippi Valley. It has been found that the use of clay, of the richness requiring the *A* section, is generally undesirable. Consequently, the *A* section is rarely used. The foregoing sections have been developed over a long period of time and are based upon experience gained with the soil existing in the Valley. They should not be considered applicable to other localities where the material available may be entirely different although bearing the same local classification.

For situations in which new levees necessitate the use of material with which there has been little experience, the application of soil mechanics makes it possible to develop a levee design, based on the strength of the material, to give any factor of safety desired. The method of design depends on the nature of the material, on whether it is cohesionless, as a sand, or cohesive, as a silt and clay. For the slopes of structures formed of sands or like materials, the inclination must not be greater than the angle of internal friction. Their failure is by sloughing along the surface of the slope when this criterion is exceeded rather than in the manner described subsequently that applies to cohesive silts and clays. The stability of slopes formed of cohesive materials depends upon the shearing strength of the material along a deep-seated cylindrical surface. This cylindrical surface generally extends from a point just back of the crown of the slope, down through the section, and through the toe of the structure. The segment of the slope lying above this arc, in failing, appears to slide downward, rotating about the center of the arc. The method of locating the approximate position of this center of rotation for various slopes as established by W. Fellenius[2] and amplified by Charles Terzaghi,[3] M. Am. Soc. C. E., has been found to be satisfactory. Evidence of this type of failure in shear due to the over-stressing of the material has been observed in innumerable cases, particularly in levees constructed of material too wet for use.

The steps taken in the design of a slope, neglecting the effect of seepage on its stability, are relatively simple and much in accord with the prevailing practice. They are as follows: (1) The approximate position of the center of rotation for the arc of failure is determined; (2) the part of the slope of unit thickness, above this surface, determined by the arc of failure, is subdivided into vertical segments (the number—usually ten or twelve—depends upon the size of the structure); and (3) the gravitational force acting on each segment is determined and resolved into its normal and tangential components acting along the arc at the vertical projection of the center of gravity of the segment on the arc of failure.

Summation of the tangential components gives the force, T, tending to produce displacement. The force resisting this tendency is the summation of the frictional and cohesive forces acting along the arc. The total frictional force is determined by adding the normal components and multiplying the sum, N, by the tangent of the angle of internal friction, ϕ. The cohesive force is determined by multiplying the length of the arc, l, by the unit cohesive strength, c, of the material. The cohesive strength is obtained by a shear test performed upon a specimen of the material, prepared in a condition to simulate the most adverse situation expected. Cohesive strength is considered to be entirely independent of the pressure, depending solely upon the material and its condition. Thus, the stability of a slope or its factor of safety against sliding, may be determined by the expression:

$$\text{Factor of safety} = \frac{N \tan \phi + c\, l}{T} \dots\dots\dots\dots\dots (1)$$

The foregoing procedure is repeated until the center of rotation and radius of curvature are found, that give a minimum value of the factor of safety for the slope in question. The corresponding arc is then accepted as the critical surface upon which failure is most likely to occur. Several slopes are analyzed by this method and the one having the desired factor is selected.

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[2] "Jordstatiska beräkningar med friktion och kohesion för cirkular cylindriska glidytor", W. Fellenius, Kungl. Väg- och Vallenbyggnadskårens 75-årsskrift, pp. 79–127, Stockholm, 1926.

[3] "The Mechanics of Shear Failures on Clay Slopes and the Creep of Retaining Walls", Charles Terzaghi, *Public Roads*, Vol. 10, No. 10, December, 1929, pp. 177–192.

A factor of unity for the stability of the side slopes indicates a balance of the forces. However, a slight deviation of less than unity can not be taken as a direct indication that failure is imminent. Values of less than 0.75 may be taken as indicative of an unsatisfactory situation and values greater than 1.25 can be taken as assurance of stability. Although this method of analysis may have its limitations, it does provide an excellent yardstick whereby analysis may be made of a situation about which engineers have had previously only rule-of-thumb methods to serve as guides.

Another element enters into the stability of the land-side slope after seepage through the structure has developed, or for the river-side slope following the passing of a flood and the removal of the impounded water. Failures have been observed of the river-side slopes of levees under just such conditions. The hydrodynamic force created by the seepage water, either flowing into or out of the section, produces an overturning moment in the part of the slope lying above the arc of failure. For example, considering a levee through which flow has fully developed, the flow and equipotential lines being arranged as shown by Fig. 3, the loss of head, h_f, occurring through the part above the arc of failure, $b\,d\,e\,f$, is due to the friction overcome by this seepage water in flowing through the material. The net effect of this loss of head is considered as a force acting in the general direction of the tangential component tending to produce failure and should rightfully be considered in such an analysis. (In Fig. 3, F_f is the force tending to cause displacement in the direction of flow, equal to h_f times unit weight of water times area surface, fb, of unit thickness.

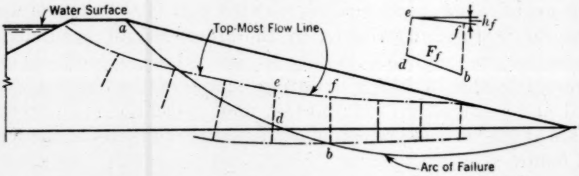

FIG. 3.—HYDRODYNAMIC EFFECT OF SEEPAGE ON STABILITY OF SLOPES

Although further research must be completed before definite conclusions may be safely drawn, a tentative method of analysis considering the effect of seepage water upon the stability of side slopes was advanced in a memorandum of the U. S. Waterways Experiment Station[4] this year. This analysis is similar to that reported by E. Meyer-Peter[5] and concurred in by D. W. Taylor, Assoc. M. Am. Soc. C. E.

Foundations.—It has been stated that the prime requisite of a levee was stability under all conditions. After the stability of the side slopes has been determined, the next essential feature to be considered is the foundation. It is not unusual that a setback unit of a levee, required by some channel change or improvement such as a cut-off, is constructed in one season or during a few months just prior to the annual flood period. It is then that a reliable method of design to assure stability is fully appreciated.

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[4] A Report of Experiments and Investigations to Determine the Efficacy of Sub-Levees and Berms in the Control of Seepage, *Technical Memorandum No. 101-2*, July 1, 1937.

[5] "Beitrag zur Berechnung der Standsicherheit von Erddämmen", von E. Meyer-Peter, H. Favre, and R. Müller, *Schweizerische Bauzeitung*, Vol. 108, No. 4, July 25, 1936, pp. 35–37.

In the past, one method[6] of dealing with a weak foundation where the lateral flow of an unconsolidated stratum of plastic material could be expected, was to build up the section and await the anticipated failure. After failure occurred, the placement of dirt was renewed, and the process continued until all the soft material was either squeezed out or until the levee had become stable for other reasons. This procedure required time and plenty of resources. An empirical method was developed about 1933 for the design of berms, which proved to be a very good remedy for this problem. This work was done by T. A. Middlebrooks, Assoc. M. Am. Soc. C. E., and was based entirely upon experience with such problems. His method of analysis had its limitations and has been supplanted by one described in the following paragraph.

In this case, again, the methods developed in the field of soil mechanics can be brought into play and the structures designed so that the stresses created by the structure will not exceed the strength of the foundations. The method of analysis contributed by Leo Jürgenson[7] makes it possible to compute the stresses created by this type of structure. Either the quick direct shear test, or the squeeze test, on specimens from undisturbed samples, can be used to obtain the ultimate strength of the material *in situ*. With these data it is a relatively simple matter to determine the stability of a foundation or the necessary adjustment of the design to give the factor of safety desired.

An application of this method of analysis to three full-scale levee structures is interesting and worthy of mention. This investigation was made in May, 1935, for the purpose of corroborating methods used in the design of the foundations of numerous earth dams in the Muskingum Valley, in Ohio. Of the three units, founded on similar material, one had been found stable and the other two had experienced complete failure, due to the over-stressing of their foundations. The first of these units was constructed after 1930 and the latter two prior to that date. The results of this application are illustrated in Table 2.

The evidence indicated by the analysis of the first two units appears to justify confidence in this method. Although that for the third, Milliken Bend, unit appears to fail in proving the desired point, it was the only one of the group whose original cross-section departed appreciably from the triangular shape. The analysis applies to triangular cross-sections. A comparison of the computed values of shear with those measured in models by photo-elasticity shows a variation of 1%, 2%, and 30% for the units in the order shown (Table 2). Thus, it is indicated that this method of analysis has its limitations when applied to a section deviating from the true triangular shape. Further use of this method of analysis on other projects has been made since 1935 and to date (1937) no difficulties have arisen. The structures have been satisfactory in every respect when the basic assumptions have been met.

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[6] "The Improvement of the Lower Mississippi River for Flood Control and Navigation", by D. O. Elliot, *Bulletin*, U. S. Waterways Experiment Station, May 1, 1932, Vol. 2, p. 177.

[7] "The Application of Theories of Elasticity and Plasticity to Foundation Problems," by Dr. Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXI, No. 3, July, 1934, pp. 206–241.

Settlement is an important factor in levee construction, as it is in building construction. However, the effect is quite different, for little damage, if any, is experienced by a levee through its settlement. The principal benefit from the predetermination of this factor is in the economy effected. A reliable estimate permits the gross grade to be adjusted at the time of initial construction to allow for the settlement. Thus, no subsequent enlargements are necessary to maintain the desired net section or grade. It has not been unusual for these structures when placed across old lake beds or bad swampy areas to experience 3 to 5 ft of settlement. Of course, allowance is made at the time of construction for the normal shrinkage of the fill. However, when in addition to this shrinkage a settlement occurs, the section must be restored. The cost per unit of fill for the enlargements is naturally greater than the initial unit cost. In addition, emergency protection may be required during a major flood. This emergency protection is in the form of sand bags or mud boxes placed in depressions caused by settlement, and is very expensive.

TABLE 2.—SAFETY FACTORS FOR THREE LEVEES

Item No.	Name of unit	Basic Dimensions, in Feet			Increase in pressure caused by structure, in tons per square foot	Maximum stress, in tons per square foot	Measured strength, in tons per square foot	Factor of safety
		Height	Base width	Depth of underlying soft stratum				
1	Halpino Levee*	36	432	27.0	1.8	0.113	0.152	1.34
2	Duckport Levee	30	270†	32.5	1.5	0.183	0.165	0.90†
3	Milliken Bend Levee	37	270	27.0	1.8	0.313	0.163	0.53‡

* No difficulty was experienced with this levee.

† After this levee failed the section was successfully reconstructed to a new base width of 347 ft, changing the factor of safety to 1.16.

‡ Structure failed, as was to be expected. The structure was successfully reconstructed to a new base width of 416 ft, changing the factor of safety to 0.80.

The detection of portions of a new unit in which excessive settlement may be expected is not a difficult matter. The site of a proposed unit as well as the borrow-pit areas are explored in a routine manner by borings, prior to the completion of the initial design. Adequate samples are taken during this exploration and subjected to tests as described later in this paper to check the general condition of the foundation, and to detect both the extent and depth of strata of cohesive material containing an apparent excess of water. Analysis of the evident weak spots by the Terzaghi theory of consolidation, combined with data from the consolidation tests of a few well-selected undisturbed samples, is sufficient to form the basis of an adjusted gross grade. The method for determining the settlement of a levee is illustrated subsequently in the solution of a typical problem.

Control of Seepage.—Seepage through the levees in the Lower Mississippi Valley, and its control, have formed the bases for the design of the standard sections previously described. It has been assumed, on the basis of past experience, that the topmost flow line of seepage through a section traveled on slopes of 1 on 6 to 1 on 8, depending on the material. The position of the point of entrance on the river side is fixed by the free-board desired, usually 1 ft of free-board being allowed for the super-flood anticipated. It was further assumed that the seepage through the structures would develop fully during the flood period. In so far as the records to the present time show, the duration of this period has not exceeded 120 days, while, of course, the flood waters would remain at the assumed free-board height for a small fraction, say, 15% of the time.

The shape of the section has been made to contain the topmost flow lines just mentioned. During an inspection of some of the levee units in the vicinity of Memphis, Tenn., the writer observed no evidence of serious seepage coming through the levee section. Although difficulties were experienced with the seepage through the foundations at some locations, this can not be attributed to any weakness in the design of the section. Pervious strata frequently exist in the foundations of levees and are the source of some trouble of local nature. Control of this excessive seepage is achieved by the use of sub-levees, or by placing a ring of sand bags so as to isolate the small local spots. Either of these means of control develops a negative hydrostatic head neutralizing or reducing to a harmless quantity the water leaking through the foundation. Thought is now being devoted to methods of sealing such pervious foundation strata so that this undesirable condition may be remedied.

Problems involving structures founded on pervious material are analyzed with the aid of models constructed in a glass-sided flume. The conditions under which these experiments have been conducted were arranged to simulate various positions of ground-water. The data from these tests are used to estimate both the position of the topmost flow line in, and the quantity of flow through, the section.

Structures founded on impervious strata may now be analyzed by methods such as one developed by Glennon Gilboy,[8] Assoc. M. Am. Soc. C. E. This problem is usually complicated by the fact that no selection of material is exercised in the construction of a levee. Hence, the usual assumption that the structure is a homogeneous mass is not justified. The state to which the material in the structure may be compacted is a variable that will affect the flow through the section. This variable can be judged only by experience gained through observation of structures recently completed; a limited number of observations and tests have been made. The details of this type of analysis are treated in the solution of a typical levee problem outlined subsequently in this paper.

The rate of development of seepage through a section of levee is a matter of importance. The maximum period of flood as estimated previously is 120 days, and it is only for a short part of this time that the maximum flood prevails. The rate of seepage development has been a matter of conjecture. Some interesting data have been obtained through observations of the development of the flow through an earth dam at the United States Waterways Experiment Station, at Vicksburg, Miss. These data throw some light on the subject. The dam is 35 ft high, with side slopes of 1 on 3.7 for both up-stream and down-stream faces. It is constructed of loess, a remarkably homogeneous material typical of the Vicksburg area, and is founded upon the same material. The loess, while unlike in some respects to the alluvial soils in the Valley, has, nevertheless, permeability characteristics similar to a great proportion of the material used in the levees, that is, the silts requiring the standard *B* section. The material was placed by trucks in layers 6 in. thick, and compacted only by the repeated passage of these vehicles. No exact control was exercised over the compacting operation. The elevation of the impounded water in the reservoir has been maintained at 164.5 ft (mean Gulf level) since May, 1935. The dam was constructed in February and March, 1935, with the spillway crest at Elevation 165.0 (mean Gulf level). A duplicate set of observation wells were placed in the structure at the time of its construction. The locations are shown by Fig. 4. Tests showed that the desired compaction was obtained by the constant passage of the trucks over the spread material. The results of the observations are shown by Fig. 5, which indicates that ten months were required for the full development of the seepage through this structure.

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[8] "Hydraulic-Fill Dams," by Glennon Gilboy, 1st Congress of Large Dams, Vol. 4, Rept. No. 46, pp. 231-267.

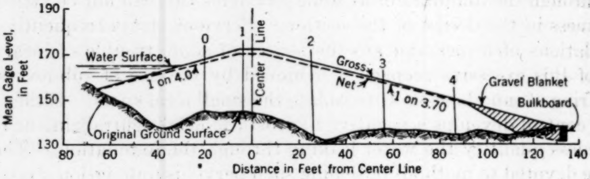

FIG. 4.—LOCATION OF OBSERVATION WELLS, STATION 0 + 66

The results of these observations show that seepage development is exceedingly slow, and tend to indicate that for sections formed of relatively impervious materials, silts, or finer, the flow does not develop during the flood period. Consequently, sections designed to comply only with the requirements of stability are satisfactory. In the past, numerous instances were reported in which the seepage through the standard banquette section did develop, outcropping at the upper edge of the banquette, and causing damage. The path of flow through the present section is two or three times as long and, consequently, requires a greater time to develop.

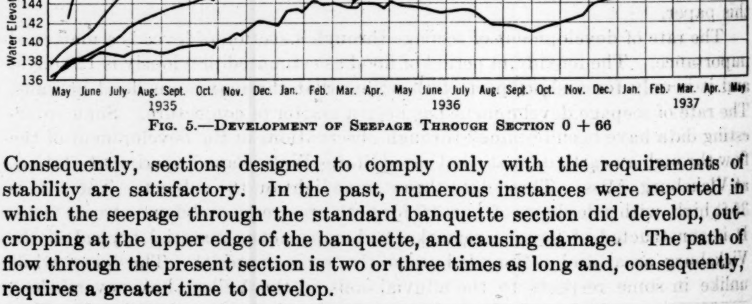

FIG. 5.—DEVELOPMENT OF SEEPAGE THROUGH SECTION 0 + 66

For the situation of pervious structures, formed of sand, the seepage may develop during the flood period. However, a unique method for controlling the seepage through this type of levee has been applied in some instances. This control is provided by means of impervious clay blankets, 3 ft thick, placed on the river-side slope and extended through pervious surface strata into an extensive impervious stratum in the foundation. Information as to the effectiveness of this control is provided by observations made by the U. S. Engineer District Office, in Vicksburg, of the pervious levee closing the gap formed by the famous Mounds Landing Crevasse that occurred during the 1927 flood. These observations were made before and after the placement of the clay blanket, that is, in 1933 and 1937; the crests of the floods of these years were within 0.5 ft of the same elevation at this unit. The seepage, as collected and measured by a system of weirs at the land-side toe for a part of the levee, 920 ft long, was 0.3 cu ft per sec in 1933 and 0.06 cu ft per sec in 1937. The atmospheric conditions during the 1933 flood were not the same as those that existed in 1937; that is, the precipitation in 1937 was the greater. Consequently, part of the measured seepage of 0.06 cu ft per sec may be attributed to drainage of rain water.

The Design of a Levee Unit

The design of levees, as accomplished through the application of soil mechanics, is basically the same as that used for earth dams about which much has been written recently. However, the attack of this problem becomes unique through consideration of several essential features: Location of levee, source of materials, placement of materials, and control of seepage.

Unfavorable locations along a line, as far as foundation conditions are concerned, cannot be avoided as readily as by shifting a dam up stream or down stream a short distance. Deviation from the shortest practicable route, to avoid crossing filled sloughs or lakes, increases the cost of the structures tremendously. Furthermore, the local interests, who are naturally desirous of obtaining protection for a maximum amount of their land, contribute toward the cost of the levees. Consequently, their desires must be given consideration and complied with in so far as is possible. Formerly, to simplify the enormous task when extensive programs have been undertaken, sections, based upon empirical designs, have been standardized with ample allowance for local "sore" spots in the foundations, both as regards unstable conditions and excessively pervious materials. Now, it is possible, through the application of soil mechanics, to alter the design to fit the conditions encountered.

The material adjacent to the river-side toe of the levee must be used for its construction, whether it is sand or clay. The principal reason for its use is the economy effected. The use of the material directly opposite the section minimizes the distance it must be transported, eliminating over-haul. The borrow-pits, being on the river side, are thus located in areas where land values are low and the borrow material is cheap. For the enlargement of sections, this results in even greater savings than for the original construction as the borrow areas have already been adjusted. In general, any attempt toward the selection of material would be fruitless due to the supply, of what is generally reckoned as suitable according to current practice in earth dam design, not being equal to the demand. In some regions, especially the lower part of the river, that at hand is just as suitable as any within about a mile. This situation makes it necessary to design the structures on the basis of the weakest of available materials. Although this often requires large sections with flat slopes when compared to some dams recently designed and constructed, the problem is simplified and solved at a low cost.

It is important that no borrow-pits or excavations be made adjacent to the levees on the land-side to expose pervious strata extending through to the river side. Exposure of such strata invites trouble from increased under-seepage, certainly; and from destruction of the levee structure due to piping, possibly. Although, in some instances, borrow-pits have been made on the land side, they are the source of never-ending trouble.

Control over the placement of the material in levees, in so far as its condition and state of compaction are concerned, has not been exercised as is the practice in the construction of earth dams. Most of the present system had been completed, or was under construction, at a time when little was known regarding the compaction of fills, antedating 1930. This date (1930) apparently marks the actual beginning of efforts to control compaction. To encourage low bids and costs, it has been necessary to minimize the restrictions on methods of construction. If the compaction of the material had been undertaken, it has been estimated that a direct increase in cost of about 4 to 6 cts per yd would have resulted, causing an increase in the cost of the structure of as much as 70% in some cases. Such an increase has not appeared to be justified for structures designed by empirical methods. Then, too, no selectivity of material has been exercised in the past, and, as the compacting of materials depends to some extent upon their uniformity, efforts toward control of condition and state would not have been marked with great success. Large allowances have been made for the shrinkage of the structures, as follows:

Equipment for construction:	Percentage, shrinkage allowance
Cableway (locally termed "tower machine")	25
Dragline	25
Tractor (wagons, tractrucks, or trucks)	15
Hydraulic fill	8

Uncertainty as to the condition and state of compaction of the material, and assumptions as to conditions and placement that need not be considered generally in earth dam construction, make the design of these structures difficult. For instance, the cohesive strength, as used for the design of side slopes, is usually neglected completely when it is known that draglines or cableways are to be used for the construction of a unit.

The control of seepage through the levee sections has been the prominent feature of their design. This has not been due to a demand for the conservation of water, as is generally the case for dams, but because of the manner in which they have been constructed. The application of the criteria in current use has resulted in designs which have proved satisfactory from the standpoint of stability. It has been shown that most of the material for their construction is silt or a similar soil which is considered as fairly impervious; also that the duration of the flood periods are relatively short as compared to the periods of high reservoir stages for dams. Consequently, the opinion that full seepage through the sections may not develop seems justified. Through the application of soil mechanics it is possible to attack the problem of design more directly; that is, to determine the stability of the structure on the basis of the strength of the materials used and, in addition, to estimate the seepage and its effect on the stability of the slopes.

The design of a levee, whether based upon empirical or stress-*versus*-strength analysis, depends upon the nature of both the material to be used for its construction and the foundation medium. Consequently, the first step is to explore the materials in the borrow-pit areas by means of borings, so that definite information concerning the available material for construction will be at hand. The material to form the foundation for the structure must be explored in a similar manner. The current practice is to make a system of preliminary borings along lines parallel to the route of the proposed levee, so that the lines divide the area into three parts. The borings are made in pairs on these lines, 1 000 ft apart. The depth of these borings is not great, but it is sufficient to explore the volume of material required for the structure adjacent to their location. In addition, borings are made at 1 000-ft intervals along the center line of the proposed levee, directly opposite those in the borrow-pits, to explore the foundation medium for sufficient depth to assure the non-existence of unconsolidated and apparent unstable material. When such a condition is indicated by the preliminary borings, or the features of the terrain, additional borings are made to trace adequately the extent of such weak formation. The spacing may be reduced to intervals of 100 ft.

The borings for the exploration of the borrow-pit areas and foundations are generally 3 or 4 in. in diameter. They are made with earth augers operated by hand. It has been found that this means of exploration is practicable, permitting the use of unskilled labor and requiring very simple equipment. The samples of the material are obtained directly from the augers as the work progresses and preserved in pint fruit jars which are properly labeled for identification in the classification laboratory.

For the situations requiring special consideration or where no experience has been had with the materials, a primary system of borings is made to supplant the preliminary system just described. The purpose is to provide samples for analysis of the strength of the materials in the borrow-pits and foundations. These samples are obtained from 6-in. borings, using special equipment designed for obtaining undisturbed samples. These large borings are also made, in most cases, by hand-operated augers. A casing of suitable size is used in this work to prevent the caving of the materials penetrated and to exclude the ground-water; thus the materials are sampled "in the dry". The device used for obtaining the undisturbed samples was developed under the auspices of the former Committee on Earths and Foundations of the Society by Professor Gilboy and the writer. The details of this device are shown by Fig. 6. Complete descriptions of the technique of boring and sampling, for both the general and undisturbed samples, are contained in other publications[9] of the writer.

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[9] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., *Paper A-23*, by S. J. Buchanan, Vol. 2, p. 72–81, June, 1936; "Technique of Soil Testing", by S. J. Buchanan, *Civil Engineering*, Vol. 7, No. 8, August, 1937.

The samples of material obtained by the preliminary exploration are subjected to classification tests, both as to kind of material and general condition. An experienced technician, trained in this work, first classifies the material by visual inspection. The samples are then grouped and such mechanical analyses and water content determinations as are necessary are made of representative members of these groups. Whenever a question arises as to the correctness of visual classification, additional tests are made to dispel any uncertainty. The Atterberg limits, liquid and plastic, are made of a few selected samples so that an index of their general condition, *in situ*, may be had. Thus, the general natures of the materials available for construction, as well as of those in the foundation, are established.

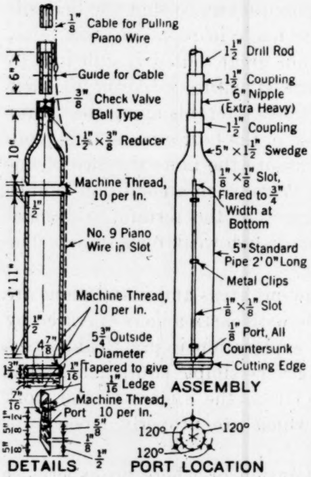
1/8" Cable for Pulling Piano Wire
6"
Guide for Cable
3/8" Check Valve Ball Type
10"
1 1/2" × 3/8" Reducer
Machine Thread, 10 per In.
1/2"
No. 9 Piano Wire in Slot
1' 11"
Machine Thread, 10 per In.
1/2"
4 7/8"
5 3/4" Outside Diameter
1 3/4"
1/16"
Tapered to give 1/16" Ledge
7/16"
Machine Thread, 10 per In.
Port
1"/2
5"/8
5"/8
5/8"
1/8"
1/2"
DETAILS
1 1/2" Drill Rod
1 1/2" Coupling
6" Nipple (Extra Heavy)
1 1/2" Coupling
5" × 1 1/2" Swedge
1/8" × 1/8" Slot, Flared to 3/4" Width at Bottom
5" Standard Pipe 2' 0" Long
Metal Clips
1/8" × 1/8" Slot
1/8" Port, All Countersunk
Cutting Edge
ASSEMBLY
120°
120°
120°
PORT LOCATION

FIG. 6.—SAMPLING TUBE

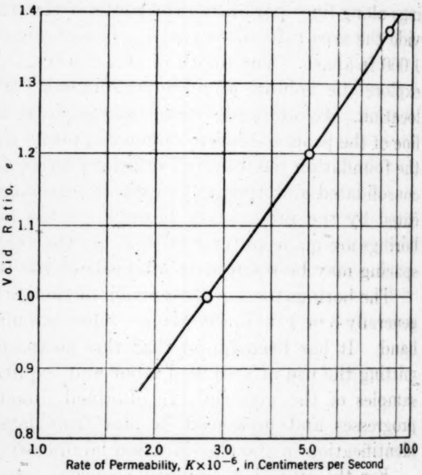

FIG. 7.—PERMEABILITY DIAGRAM

The strengths of the materials are next determined by tests of a representative specimen. The delayed shear test for ascertaining the angle of internal friction, ϕ, and cohesive strength, c, as required for the design of the side slopes of the structure, are made of a representative specimen of the borrow-pit materials. Complete descriptions of this, and subsequent tests mentioned, are available elsewhere.[10] The maximum side slopes of the structure are thus established. The strength of the foundation is determined by quick, and squeeze, shear tests of undisturbed samples of the cohesive materials in this medium. Then, the combined consolidation-permeability test is made so that the settlement and under-seepage can be estimated. The final shape of the structure is established, on the basis of information obtained from the latter group of tests.

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[10] "Laboratory Procedure in Testing Soils and Sediments", *Bulletin*, U. S. Waterways Experiment Station, November 1, 1936.

Basic Data.—The physical features of the problem to be considered are:

Elevation of natural ground surface at the site, in feet (mean Gulf level)	347
Elevation of net grade or crown, in feet (mean Gulf level)	377
Net height of structure, in feet	30
Free-board for maximum designed flood, in feet	1.0
Percentage allowance for shrinkage (construction by tractors)	15
Gross height, in feet, considering shrinkage (30 × 1.15)	34.5
Thickness of plastic zone in foundation, in feet	23
Factor of safety desired for side slopes	1.5
Factor of safety desired for foundation	1.25

The borrow-pit material is a clayey silt, and other characteristics are as follows:

Percentage of water content (natural state on dry weight basis)	31.5
Void-ratio (natural state on dry weight basis)	1.20
Absolute specific gravity	2.68
Unit weight, in pounds per cubic foot, when placed (judged by experience)	100
Angle of internal friction, ϕ (test result), in degrees	19.4
Cohesive strength, *c*, in tons per square foot	0.13
Permeability, in centimeters per second (result of tests)	5×10^{-6}

It is evident from the inspection of the conditions indicated by the log of boring, Fig. 8, that the only questionable material, in so far as strength is concerned, is the clay between Elevations 344 and 321. The data obtained from this material are shown as follows: The strength of this material, *in situ* (quick shear tests), equals 0.155 ton per sq ft. Its consolidation and permeability characteristics are as shown in Fig. 9.

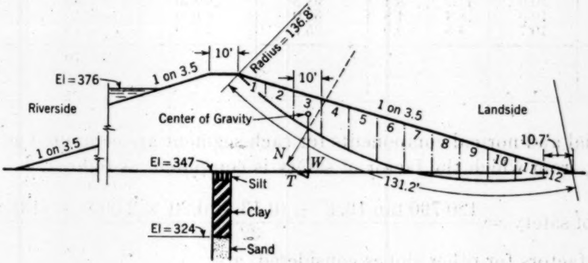

FIG. 8

Design of Side Slopes.—In the computations for a slope of 1 on 3.5, the position of the center of rotation and the radius of critical arc are both determined by methods referred to previously. These data are shown by Fig. 8. The weight of the material in the structure is assumed to be 100 lb per cu ft. It is assumed that 20% of the cohesive strength of material will be active. The

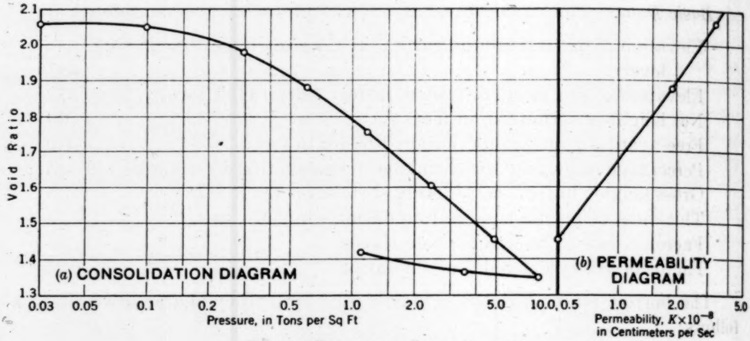

FIG. 9.—FOUNDATION INVESTIGATION

TABLE 3.—COMPUTATION OF TANGENTIAL AND NORMAL COMPONENTS

Segment No. (see Fig. 8)	Width, in feet	HEIGHT, IN FEET		Area, in square feet	Gravitational force, in pounds	Tangential component, in pounds	Normal component, in pounds
		Left	Right				
1	10.0	0.0	5.7	28.5	2 850	1 980	2 350
2	10.0	5.7	9.8	77.5	7 750	4 320	6 350
3	10.0	9.8	12.8	113.0	11 300	5 640	9 820
4	10.0	12.8	14.6	137.0	13 700	5 750	12 400
5	10.0	14.6	15.6	151.0	15 100	5 460	14 120
6	10.0	15.6	15.8	157.0	15 700	4 600	15 200
7	10.0	15.8	15.0	154.0	15 400	3 230	15 000
8	10.0	15.0	13.6	143.0	14 300	2 050	14 290
9	10.0	13.6	11.3	124.5	12 450	785	12 430
10	10.0	11.3	8.4	98.5	9 850	−80	9 850
11	10.0	8.4	4.8	66.0	6 600	−550	6 550
12	10.7	4.8	0.0	25.7	2 570	−315	2 430
$\Sigma\Delta T$	....	..	..	....	....	32 870	....
$\Sigma\Delta N$	....	..	..	....	....	....	120 790

tangential and normal components for each segment are computed as shown in Table 3, from which the factor of safety is computed, as follows:

$$\text{Factor of safety} = \frac{120\,790 \tan 19.4° + (0.13 \times 0.20 \times 2\,000) \times (131.2)}{32\,870} = 1.51$$

The factors for other slopes considered, are:

Slope	Factor of safety
1 : 3	1.32
1 : 3.5	1.51
1 : 4	1.70

Foundations.—Probable settlement is estimated in the following steps:

(1) The increase in vertical pressure on the foundation, in tons per square foot $\left(\frac{34.5 \times 100}{2\,000}\right)$ = ... 1.725

(2) The pressure, in tons per square foot, by which the clay stratum has been previously consolidated (estimated from Fig. 9(*a*)) . 0.315

(3) Void-ratio corresponding to initial conditions.................. 2.04

(4) Void-ratio corresponding to final condition..................... 1.64

(5) Thickness of compressible stratum, in feet...................... 23

(6) Total settlement, in feet[11] $= h \frac{e_i - e_f}{1 + e_i} = \frac{23\ (2.04 - 1.64)}{1 + 2.04}$...... 3.0

(7) New gross height, in feet, considering allowance for settlement (= 34.5 + 3.0) 37.5

A recheck of the settlement, using the new gross height (37.5) gives a new value of 3.1 ft, which is sufficiently correct for the purpose; and a recheck of the side slopes shows the factors of safety for the new condition practically unchanged.

The stability of the foundation is determined as follows:

(*a*) Vertical pressure, P, in tons per square foot, created at center of structure $\left(= \frac{37.5 \times 100}{2\,000} \right)$........................... 1.875

(*b*) Thickness, d_a, in feet, of plastic zone in foundation (see Fig. 10). . 23.0

(*c*) Measured strength, in tons per square foot of foundation (test result)... 0.155

(*d*) Allowable working strength of foundation material, in tons per square foot, to give a factor of safety of 1.25 $\left(= \frac{0.155}{1.25} \right)$ . . 0.124

(*e*) Base width of structure required, b_b, in feet,

$\left(= \frac{P\,da}{\text{Allowable stress}} = \frac{1.875 \times 23}{0.124} \right)$.................. 348

(*f*) Ratio of horizontal to vertical side slope required, allowing for crown width of 10 ft and height of 37.5 ft $\left(= \frac{348 - 10}{2 \times 37.5} \right)$...... 1 : 4.5

Side slopes of 1 : 4.5 are necessary because of the foundation condition, superseding the 1 : 3.5 as required for the material placed in the structure proper. Had the standard side slope for the *B* section, river-side slope, 1 : 3.5 (see Table 1), been used, the factor of safety of the foundation would have been 0.98.

Seepage.—The factors involved in determining the height of emergence of the topmost flow line above the base of the levee (using basic equations advanced by Professor Gilboy) are: H = head of water to be impounded; α = angular inclination of land-side slope; b_s = horizontal distance from land-side toe of section to vertical projection of intersection of river-side slope with water surface at a height, H, above the base of the structure; and $m\,H$ = the vertical distance above the foundation of the outlet point of the topmost flow line.

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[11] Progress Rept. of Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 788.

These factors are connected by the following implicit function (see Fig. 10 for values):

$$\frac{2b_s}{H} = m \cot \alpha + \frac{\sqrt{1 - m^2 \sin^2 \alpha}}{m \sin \alpha} - m \sin \alpha \log_e \frac{1 + \sqrt{1 - m^2 \sin^2 \alpha}}{m\,(1 + \cos \alpha)} \ldots (2)$$

The solution of Equation (2) for the value of m, using known values for the factors, b, H, and α, establishes a value of 0.34. The height of emergence above the base is equal to $m\,H = 0.34 \times 29.0 = 9.9$ ft.

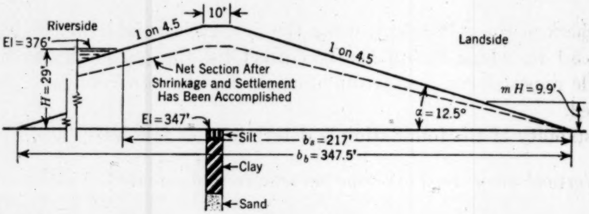

FIG. 10

The quantity of seepage per foot length of levee per second is expressed by:

$$Q = K\,m\,H \sin \alpha \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting the proper values in Equation (3): $Q = \frac{5 \times 10^{-6}}{2.54 \times 12} \times 9.9 \times 0.217 = 0.35 \times 10^{-6}$ cu ft per sec per ft of levee, or 0.11 cu ft per min per mile of levee, indicating that little concern is warranted regarding the quantity of seepage to be encountered.

Conclusions

The present levee system for the Lower Mississippi Valley was designed and constructed at a time when the science of soil mechanics was "in its swaddling clothes". The system as a whole has proved satisfactory under the test of actual use. However, the empirical methods of design were based upon long experience with such structures, the heights of which have increased as have the consequent stresses developed. For subsequent improvements, such as further enlargements or changes in locations that may have become necessary, economical designs may be predicated upon a strength-*versus*-stress basis, to better advantage. Time and funds are no longer necessary to develop empirical designs. The fund of knowledge available permits the problem to be attacked by this new method which has already "won its fighting pants" in other fields.

The net effect of applying knowledge gained through soil mechanics appears to be toward the elimination of such practices as actually producing failures of foundations and later repairing them. Also, it is indicated that the size of the sections may be reduced safely through steepening of the present standard land-side slopes. For new units involving materials with which there has been no experience, savings in time and cost are possible through the application of soil mechanics.

Acknowledgments

In addition to the acknowledgments and references embodied in the text, the writer wishes to thank the Director of the U. S. Waterways Experiment Station, Paul W. Thompson, Lieutenant, Corps of Engineers, U. S. Army, Jun. Am. Soc. C. E., for his co-operation in making this paper possible. Much valuable assistance was given by the staffs of the Mississippi River Commission and the U. S. Engineer District Office, of Vicksburg, Miss., for which the writer is grateful. He wishes also to acknowledge the assistance of members of his laboratory staff, in particular the following: William L. Wells, Jun. Am. Soc. C. E., Robert M. German, Junior Engineer, and William J. Rowland, Chief Technician, in the preparation of the paper.

QUABBIN DIKE BUILT BY HYDRAULIC-FILL METHODS

By Stanley M. Dore,[12] M. Am. Soc. C. E.

Synopsis

The thorough investigations of materials available for earth dam construction furnish not only the proper information by which to decide the type, dimensions, and cost of the earth dam to be built, but they furnish sufficient data to enable contractors to bid intelligently. Such bidding tends toward more economical and satisfactory construction.

The proper supervision of the construction of a full hydraulic-fill dam is of primary importance. The engineer strives to have the structure built safely and economically, at the same time securing the best quality, the materials available and the difficulty of securing and placing them being carefully considered. The control of construction operations by the engineer to obtain satisfactory results is influenced to a large extent by laboratory investigations and tests of samples from the borrow-pits and from the shoulders and the core of the dam.

The design of a hydraulic-fill dam and the supervision of construction, are influenced considerably by the rate of consolidation of the core material that will occur during and after the sluicing operations. As far as is known there is no existing method of computing this rate of consolidation during construction, other than the semi-empirical method presented herein which was developed from data accumulated on the Quabbin Dike. This method may be used for estimating core consolidation on another project when certain laboratory test data pertaining to that project are available.

Introduction

The Quabbin Reservoir[13] of the Boston, Mass., Metropolitan Water District (capacity, 415 000 000 000 gal), will be formed by a main dam and a dike. Both are full hydraulic earth dams, the former being 2 640 ft long, 170 ft high above the old river bed, and containing more than 4 000 000 cu yd of embankment above the original surface, and the latter being 2 140 ft long, 135 ft high, and containing 2 500 000 cu yd. A typical cross-section of the Quabbin Dike is shown in Fig. 11.

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[12] Associate Civ. Engr., Met. Dist. Water Supply Comm. of Massachusetts, Boston, Mass.

[13] See "Boston's New Water Supply," by Frank E. Winsor, M. Am. Soc. C. E., *Civil Engineering*, June, 1934, p. 283; "Boston Metropolitan Water Supply Extension," by Karl R. Kennison, M. Am. Soc. C. E., *Journal*, New England Water Works Assoc., Vol. XLVIII, No. 2; "Surface and Sub-Surface Investigations, Quabbin Dams and Aqueduct," *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 680; and, *Engineering News-Record*, June 18, 1936.

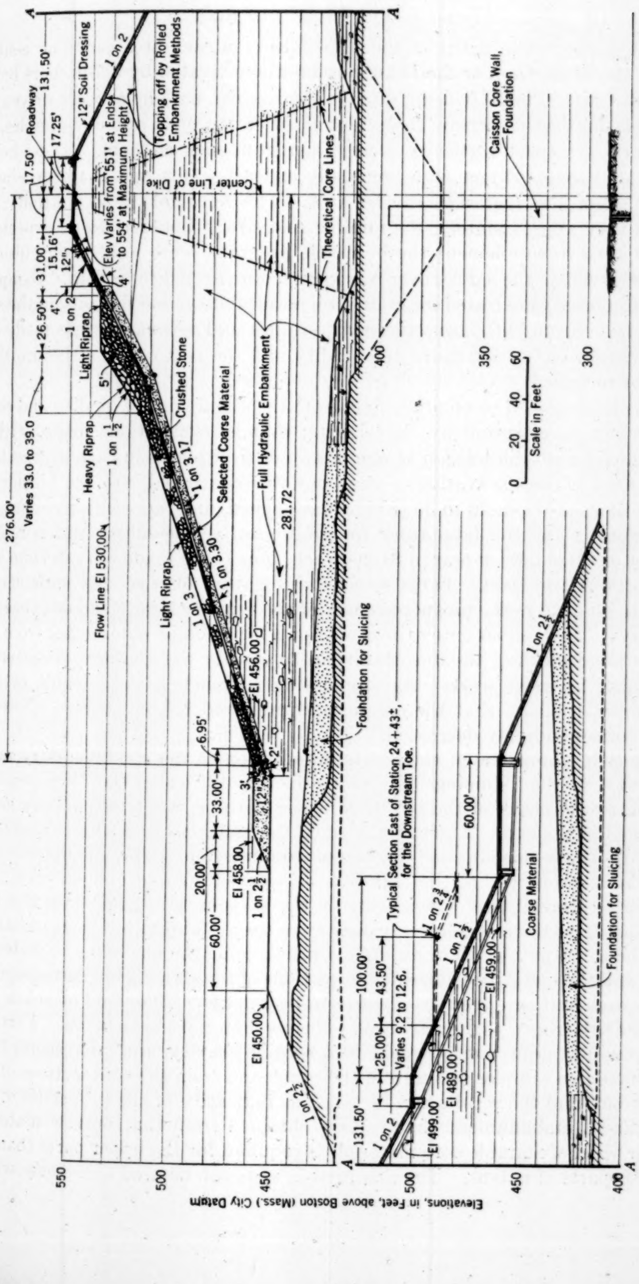

A
A
Roadway
131.50'
12" Soil Dressing
1 on 2
17.25'
17.50'
Topping off by Rolled Embankment Methods
Elev Varies from 551' at Ends to 554' at Maximum Height
Center Line of Dike
Theoretical Core Lines
Caisson Core Wall, Foundation
31.00'
15.16'
12"
4'
4'
26.50'
1 on 2
Light Riprap
5'
Crushed Stone
Varies 33.0 to 39.0
Heavy Riprap
1 on 1½
Selected Coarse Material
1 on 3.17
Full Hydraulic Embankment
276.00'
281.72
Flow Line El 530.00
1 on 3.30
Light Riprap
1 on 3
El 456.00
Foundation for Sluicing
6.95'
2'
3'
12"
33.00'
20.00'
El 458.00
1 on 2½
60.00'
El 450.00
1 on 2½
550
500
450
400
350
300
Elevations, in Feet, above Boston (Mass.) City Datum
0
20
40
60
Scale in Feet
Typical Section East of Station 24+43±, for the Downstream Toe.
100.00'
43.50'
25.00'
Varies 9.2 to 12.6
131.50'
1 on 2
El 499.00
El 489.00
El 459.00
1 on 2¾
1 on 2½
60.00'
Coarse Material
1 on 2½
Foundation for Sluicing
500
450
400
A

FIG. 11.—TYPICAL CROSS-SECTION OF DIKE EMBANKMENT, QUABBIN RESERVOIR

Borrow-Pit Investigations

The quantity and quality of the over-burden in each area easily accessible to either the Main Dam or the Dike were first investigated by 2.5-in. bore holes and by test pits, 6 to 10 ft deep, dug by hand. The bore holes were driven to reasonable depths to ascertain the extent of the earth cover, and dry samples, at least every 5 ft apart vertically, were taken from each of them. These holes were located frequently enough (generally at least 400 ft apart) to serve as a good index of the materials in the area. The bore-hole investigation also revealed very approximately, the number and size of boulders to be expected. The test pits, to supplement the bore-hole program, were located on hillsides and in areas where the earth cover was known or expected to be thin. Samples from both places were tested for grain size, permeability, and porosity in the soil laboratory. From these investigations, studies and estimates were made for each borrow area, to determine the quality and the cost of the structure that should be obtained.

Areas that seemed to be promising, from the standpoint of quality and cost, were further investigated by shovel cuts; that is, excavations made, with a power shovel, into the hillside at one or two critical places in each prospective borrow area. The excavation of each cut was continued into the hillside or slope until a face from 20 to 30 ft high was opened, and representative samples were tested in the soil laboratory for grain size, permeability, and porosity. The cost of these cuts averaged $1 200 each, four being made for the Dike and ten for the Main Dam. Large cuts of this nature offer a very satisfactory exhibit to indicate to the prospective bidder the type of materials to be expected. They pay for themselves many times over on a large job by eliminating from the bidder's mind much of the uncertainty as to exactly the qualities of materials with which he must work. By eliminating as much as practicable of this uncertainty, the sum that the careful and preferred bidder adds to offset his risk, is correspondingly decreased.

Materials from some of these cuts, in addition to the soil laboratory tests, were tested in the "sluicing bin" shown in Fig. 12, where they were washed down on to the beach of the bin with nozzle streams, the finer particles going into a pool at the end. The test subjected the materials to working conditions similar to those encountered in actual construction. The qualitative and quantitative properties of the materials considered for hydraulic-fill construction, can be studied more extensively on the basis of tests made with this bin. The results give definite relative values as to the suitability of any material for use in the construction. For example, by measuring the quantities of materials sluiced and the quantities deposited in sections of the sluicing bin corresponding to the beach and core sections of the dam, estimates of the yield of beach and core materials from a particular class of borrow material can be made. Furthermore, porosity, grain size, permeability, specific gravity, and microscopic tests can be made on samples taken from the beach and from the core sections of the sluicing bin, and the results can be taken as indicative of the properties of the hydraulic-fill embankment that will be obtained from that borrow material. A larger yield of suitable core materials is required for the upper parts than for the lower parts of a dam. The sluicing-bin tests can be used to indicate which

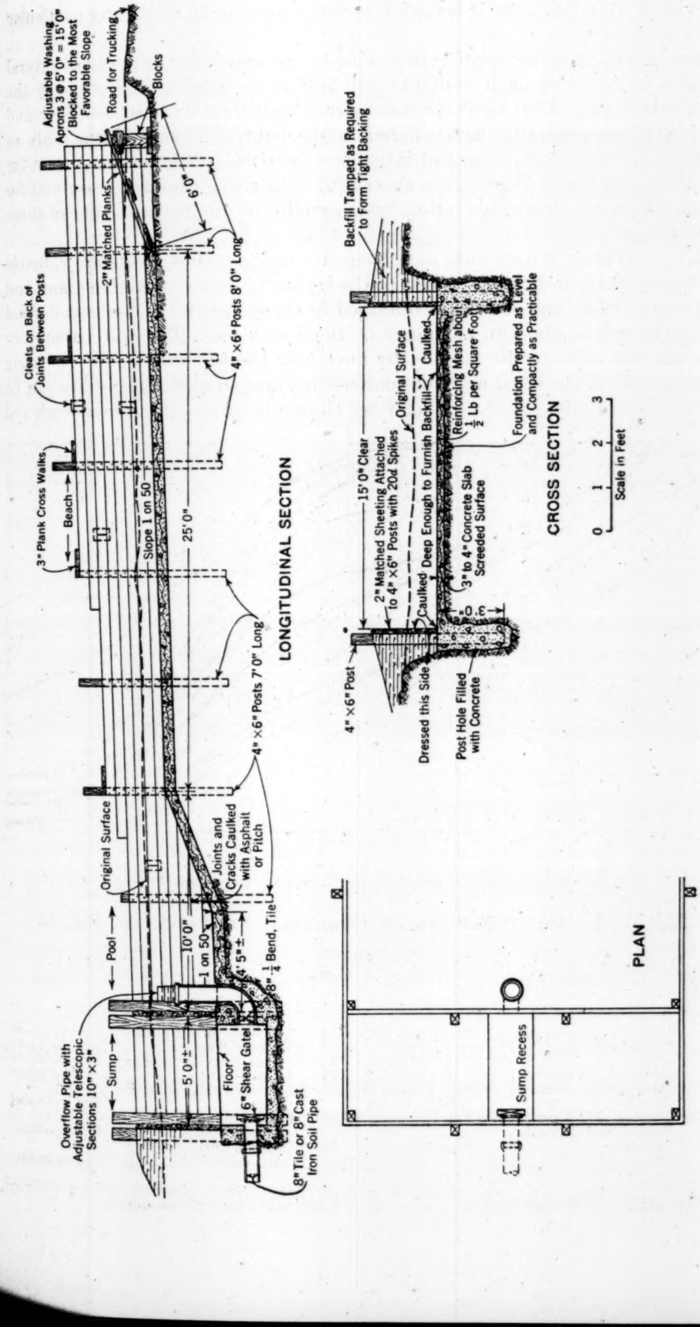

Overflow Pipe with Adjustable Telescopic Sections 10"×3"
8" Tile or 8" Cast Iron Soil Pipe
← Sump →
← Pool →
← Beach →
5' 0"±
Floor
6" Shear Gate
10' 0"
1 on 50
4' 5"±
8" 1/4 Bend, Tile
Joints and Cracks Caulked with Asphalt or Pitch
Original Surface
4"×6" Posts 7' 0" Long
3" Plank Cross Walks
Slope 1 on 50
25' 0"
Cleats on Back of Joints Between Posts
2" Matched Planks
4"×6" Posts 8' 0" Long
6' 0"
Adjustable Washing Aprons 3 @ 5' 0" = 15' 0" Blocked to the Most Favorable Slope
Road for Trucking
Blocks
LONGITUDINAL SECTION
4"×6" Post
Dressed this Side
Post Hole Filled with Concrete
Caulked
3' 0"
15' 0" Clear
2" Matched Sheeting Attached to 4"×6" Posts with 20d Spikes
Deep Enough to Furnish Backfill
Original Surface
Caulked
3" to 4" Concrete Slab Screeded Surface
Reinforcing Mesh about 1/2 Lb per Square Foot
Foundation Prepared as Level and Compactly as Practicable
Backfill Tamped as Required to Form Tight Backing
CROSS SECTION
0
1
2
3
Scale in Feet
Sump Recess
PLAN

Fig. 12.—Sluicing Bin Test

material or which mixture of materials is the proper one to use in any particular height.

The results of these investigations aided appreciably in the study of several methods of building each embankment and in estimating the costs of the various layouts. The methods that seemed satisfactory and feasible, and practicable economically, were offered to the bidders as suggestions, only to aid them in tendering a fair and intelligent proposal. After the contract for each dam had been awarded, the successful bidder was required to submit for approval his own plan of operation, which might, or might not, be one of those suggested.

The samples obtained from bore holes, test pits, or shovel cuts are valuable in influencing the design of the dam or the layout of the construction plant, and the proper design and the prices obtained in the construction contract depend to a large extent upon an adequacy of these samples. The Soil Laboratory enhances the value of the samples by analyzing the properties and presenting the results in a classified and summarized manner so that the results can be used easily and intelligently. Such work eliminates much of the guesswork and

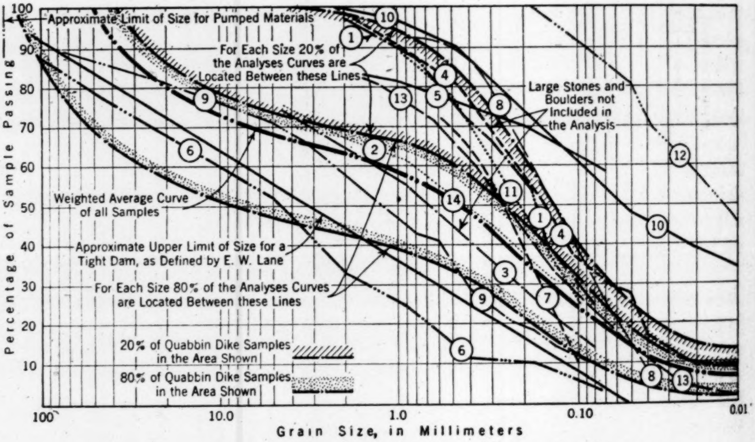


FIG. 13.—COMPARISON OF BORROW-PIT MATERIALS, IDENTIFIED IN TABLE 4

TABLE 4.—IDENTIFICATION OF HYDRAULIC-FILL DAMS IN FIG. 13

Curve No.*	Dam	Type	Curve No.*	Dam	Type
1	Henshaw, California	Full hydraulic	8	Linville, North Carolina	Semi-hydraulic
2	Swinging Bridge,† New York	Full hydraulic	9	Tieton,† Washington	Semi-hydraulic
3	Dwinnell, California	Full hydraulic	10	Saluda,† South Carolina	Semi-hydraulic‡
4	Davis Bridge, Vermont	Semi-hydraulic	11	Magic, Idaho	Semi-hydraulic
5	Somerset, Vermont	Semi-hydraulic	12	Alexander, Hawaiian Islands	Full hydraulic§
6	Conconnully, Washington	Full hydraulic	13	Cobble Mountain, Massachusetts	Full hydraulic
7	Paddy Creek,† North Carolina	Semi-hydraulic	14	Quabbin Dike, Massachusetts	Full hydraulic

* See Fig. 13. † Average of tests. ‡ Minor slip. § Serious slip during construction.

difference of opinion that accompanies the visual examination. If soil laboratory results and the classification of samples for various dams already built are available, comparative studies can be made of the materials available for construction and of those used elsewhere.

Fig. 13, with Table 4, shows a comparison of grain-size analyses curves for borrow-pit materials used in Quabbin Dike with those for some other large dams. A "full hydraulic" dam is an earth dam built by transporting the material on to the dam by pipes or flumes, in suspension in water and depositing it in the dam by water, a separation of sizes taking place during deposition. A "semi-hydraulic" dam is similar to "full hydraulic" except that the material is transported on to the dam in some other manner, usually in a "dry" state in cars, trucks, or wagons.

Control During Construction

To control the deposition of material in a hydraulic-fill dam is a complex process, and many features must be properly co-ordinated to obtain the best results. The analysis of "hog-box" samples and of core and beach samples at regular intervals is a valuable part of this control. Periodic tests for grain size, permeability, and porosity give indications that the structure is, or is not, being built satisfactorily, and they reveal the properties of all parts of the structure.

By the analysis of the hog-box samples the builder is enabled to anticipate the borrowing operations, and the engineer is enabled to regulate them properly so that poor construction conditions and results may be avoided. These analyses indicate, directly, to both builder and engineer, any lack of "coarses" for the shoulders or of "fines" for the core, any presence of too large a quantity of any one size, and other unsatisfactory size conditions. Any conditions of size deficiency or excess can be remedied or corrected as soon as it is indicated by the laboratory analyses, by borrowing materials of differently tested characteristics from other parts of the borrow area.

The analyses of core samples, taken at frequent intervals from the dam itself, indicate definitely the existence of any coarse tongue or intrusion into the core or fine deposit in the shoulders so that corrective measures may be taken before construction advances too far. By co-ordinating the result of rod-pipe soundings made along the edges of the core and pool with the size analyses made from the materials there, an inspector can learn to judge the quality of the core material by the resistance offered to the penetration of his rod. Thus, he can feel any coarse tongue that projects into the core. With routine testing the grain-size test acts as a constant check on his judgment. If these tests are made reasonably close (say, every 100 ft along the edges of the pool) and are supplemented by the rod-sounding work, unsatisfactory conditions can be ascertained for correction.

Glacial sands and gravels usually have such high internal frictional resistances, that shear tests are not necessary to the design, because a conservative cross-section obtained by using reasonable up-stream and down-stream slopes normally has ample volume and weight to withhold the pressures from the core during and, of course, after construction. For some types of materials, such as those obtained from alluvial deposits, shear tests should be used extensively to determine the qualities of the shoulders of the dam or the foundations under it, and in such cases the results will control many features of the design. However, an occasional shear test for any type of material is a valuable check of the qualities already judged suitable by visual inspection. In the case of control during construction, shear tests on the core material of a hydraulic-fill dam have a definite utility, as the frictional value of the core increases with the consolidation. If, from tests, the shear is determined for various degrees of consolidation of the core material, the safe rate of progress of construction and the rate at which the width of the core can be decreased safely can be controlled more intelligently. At Quabbin Dike the shearing resistance of the core materials, even a few days after being deposited, was sufficiently large so that it was unnecessary to limit the rate of construction or the rate of decreasing the core width.

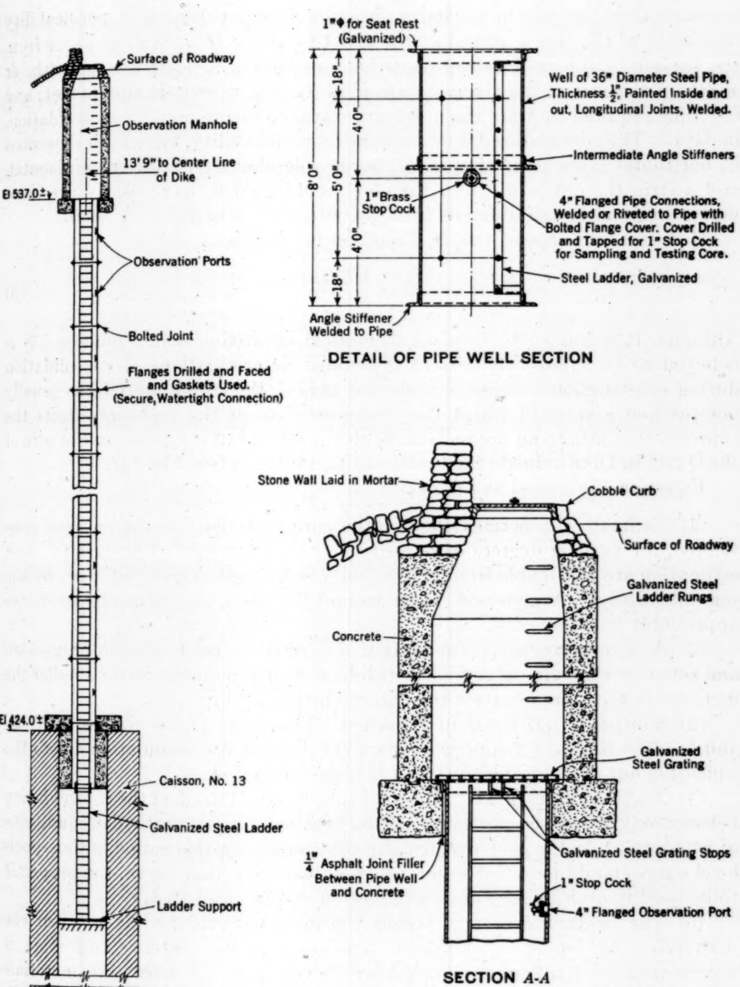
Core samples should be analyzed for moisture contents (or porosity) at frequent intervals to determine the rate at which the core consolidates during construction. These specimens can be obtained by sinking a pipe well into the core and taking samples, at various elevations, in as near an undisturbed condition as practicable. Although the core materials may vary in properties over a wide range, the results of the moisture content determinations, in conjunction with grain-size analyses curves, will show a definite trend of the rate of consolidation.

At Quabbin Dike a steel pipe of 0.5-in. plate, 36 in. in diameter, was built vertically in the core during the construction (see Fig. 14) and this well was equipped with ports through which samples can be taken at various elevations. If desired the samples can be taken several feet away from the observation well by the use of long tubes. Samples from the ports of this well were taken at regular intervals (approximately one month) and tested for grain size and porosity. The results indicate that certain definite laws operate to control the rate at which the core materials consolidate during construction.

In the very first stages of the consolidation considerable water is released from the part of the core near the bottom of the pool through vertical springs into the pool; but for parts of the core covered substantially, the drainage is mostly horizontal. The edges of the core consolidate more rapidly than the center for two reasons: First, they are nearer the coarse, and free-draining, parts of the dam, and the water squeezed out of the pores is quickly carried away; and second, they usually contain the coarser of the core deposits which will yield its water relatively quickly because the permeability coefficients are correspondingly higher.

Disregarding this more rapid consolidation of the edges, and assuming that there is only one consolidation rate for the core over its entire width, for each increment of height, the degree of consolidation during construction can be expressed by an equation involving the quantity of water squeezed from the core material, to effect a certain consolidation, as follows:

$$Q = K \frac{H}{0.25\,b} A\,t \qquad \text{(4)}$$


Surface of Roadway
Observation Manhole
13' 9" to Center Line of Dike
El 537.0±
Observation Ports
Bolted Joint
Flanges Drilled and Faced and Gaskets Used. (Secure, Watertight Connection)
El 424.0±
Caisson, No. 13
Galvanized Steel Ladder
Ladder Support
1"Φ for Seat Rest (Galvanized)
18"
4'0"
5'0"
4'0"
18"
8'0"
Well of 36" Diameter Steel Pipe, Thickness 1/2", Painted Inside and out, Longitudinal Joints, Welded.
Intermediate Angle Stiffeners
1" Brass Stop Cock
4" Flanged Pipe Connections, Welded or Riveted to Pipe with Bolted Flange Cover. Cover Drilled and Tapped for 1" Stop Cock for Sampling and Testing Core.
Steel Ladder, Galvanized
Angle Stiffener Welded to Pipe
Stone Wall Laid in Mortar
Cobble Curb
Surface of Roadway
Galvanized Steel Ladder Rungs
Concrete
Galvanized Steel Grating
Galvanized Steel Grating Stops
1/4" Asphalt Joint Filler Between Pipe Well and Concrete
1" Stop Cock
4" Flanged Observation Port
0
4
8
12
16
20
Scale in Feet

DETAIL OF PIPE WELL SECTION

SECTION *A-A*

VERTICAL SECTION

FIG. 14.—DETAILS AND SECTIONS OF OBSERVATION WELL

in which Q = the rate of discharge, in cubic feet per day; K = permeability coefficient of the core material at the given location; H = head of water from the surface of the pool to the elevation considered, in feet; b = breadth, or width, of the core at the given elevation, in feet; A = area, in square feet; and t = time required to drain the necessary water to secure a given consolidation, in days. The core material is not of uniform permeability, but as it is deposited in horizontal layers, the changes in permeable characteristics are horizontal, and a strip the entire width of the core would tend to have nearly the same permeability although different from the adjacent strip above or below.

If A = a unit area of 1 sq ft, Equation (4) becomes,

$$Q = \frac{4\,K\,H\,t}{b} \qquad (5)$$

Although it is known to be a mathematical approximation, Equation (5) is believed to be sufficiently correct in making determinations of consolidation during construction because, during that period, the core material has usually not reached a state in which the compressibility of the material affects the consolidation rate to an appreciable degree. Observations made on the core of the Quabbin Dike indicate the practicability of its use (see Fig. 15).

Equation (5) is used as follows:

(1) Estimate the permeability of the core material, for the location considered, for varying degrees of porosity or water content. The data for such estimation are obtainable from permeability tests on core material made by the soil laboratory. A constant value cannot be used, as permeability varies appreciably with porosity.

(2) Assume increments of reduction of porosity as consolidation progresses, and solve for the value of ΔQ corresponding to such reductions (the smaller the increments the more accurate the determination).

(3) Substitute ΔQ for Q in Equation (5) and the proper values of K, H, and b. The time, Δt, required to effect the increment consolidation due to the squeezing out of the water, ΔQ, can then be computed.

(4) Add the values of Δt. The total time required to effect the consolidation between any two given porosities can be obtained for the particular conditions of pool level. As the pool level usually changes during this period, a time-pool-level curve should be plotted and, progressively from that curve, the proper H to be used in each increment determination can be estimated.

(5) The beginning of any period considered should be taken when the material is no longer in complete suspension in the water. This state is represented by a different porosity for coarser than for finer core materials (about 65%, 60%, and 55% for materials containing 30%, 20%, and 10% less than the 0.01 mm size, respectively).

Fig. 15 shows a series of curves of the time of consolidation plotted against porosities for materials of various characteristics found in the core of Quabbin Dike. The curves were computed theoretically by means of Equation (5). The circles indicate observed porosities of samples taken from the core, and the line and arrows indicate the difference between the theoretical and the observed

The differences between the two are minor in the upper parts of the diagram, and the use of Equation (5) should be limited to construction conditions in which the core materials are in a highly saturated state, and in which the degree of consolidation has not progressed far enough to permit great support of the overlying load to be carried by the solids of the core.

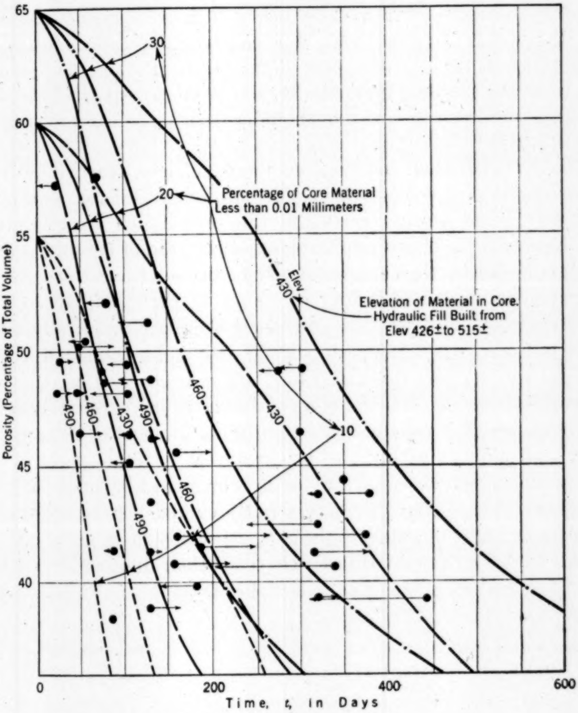


FIG. 15.—CONSOLIDATION DIAGRAM

The load above any volume of core materials is transmitted through it in two ways: Through the water, which is compressed and squeezed out as fast as the permeability of the core will permit; and, after sufficient relief by drainage of such excess water, through the solid particles of the core. The transition from the condition of full load carried by the water to load carried almost entirely by the core solids is usually an extremely gradual one because of the slow drainage of the tight core materials. However, the average construction condition is such that the entire load is not transmitted through the water. This state is one that exists when the solids are suspended in the water and occurs only in the materials in or near the bottom of the pool. This suspended state is quickly relieved by vertical drainage into the pool itself. The core materials are then in a state such that the solids are not suspended, but rest lightly upon each other, receiving some light and partial support through the solids beneath. It is this condition, or a similar one, which exists in the core as long as the pool is maintained full of water for the subsequent construction operations above; and Equation (5) is usable for just such conditions. This is the reason why H is the head of water. The excess weight of the liquid core over that of the water is neglected because of the partial support the solids obtain from each other and from those below in their loose and unstable state.

Thus, the writer believes that this construction consolidation formula (Equation (5)) can be used on other dams for regulating the speed of construction, when the shoulder materials are not as good as at Quabbin; or it may be used in making less conservative shoulder designs, in those instances, than are at present in use.

After the pool is drained and the excess water is slowly carried away, there is a slow transition into an entirely different condition in which support of the load above is largely, if not entirely, carried by the core solids. For this latter state, of course, Equation (5) is not applicable, and the rate of consolidation then is appreciably affected by the compressibility of the solid matter in the core.

However, for a silt or rock-flour core such as that at Quabbin where the percentage of true clay is very low and where there are practically no colloids, the texture of the core is a fine-grained solid one; and a condition in which the core particles will support the entire load above is reached relatively quickly after the pool is drained. Instead of a great number of years before most of the consolidation is secured, tests indicate that probably most of the core is almost completely consolidated within two or three years. Some parts of the core, such as those which have material 30% or 35% finer than 0.01 mm, will probably settle for many years before reaching a fairly compact state approaching the limiting or stable state; but most of the core, especially those parts with material coarser than 20% finer than 0.01 mm, will reach their limiting state much more quickly. Observations made on samples taken from the core indicate that such periods are approximately as follows:

Percentage of core material finer than 0.01 mm	Time required for consolidation to reach its limiting state, in years
10	1 to 2
15	1.5 to 3
20	3 to 5

In each case t is measured from the time the pool is first drained of water. Comparisons made with the consolidation formula[14] introduced by Glennon Gilboy, Assoc. M. Am. Soc. C. E., indicate that much longer periods would be expected but the result of tests of the core materials of Quabbin Dike leads the writer to believe that the rate of consolidation of solid-grained core materials of specific gravity of 2.6 to 2.7 similar to those in Quabbin Dike will be much more rapid than Professor Gilboy's formula indicates.

The consistency of core materials in a hydraulic fill when a fairly stable state of consolidation is reached, has been the subject of much discussion. Observations made to date (1937) on the core indicate that, for material of a size 25% finer than 0.01 mm, or coarser, the stable state will be at least that of a stiff putty. Even the finer portions will be in a putty-like state, but softer; and there will be no such condition as a permanent liquid or semi-liquid core condition.

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[14] "Mechanics of Hydraulic Fill Dams," by Glennon Gilboy, Assoc. M. Am. Soc. C. E., *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 187.

The impression that the lower parts of the core consolidate more quickly, due to the pressures from above, is not entirely true, because the width of the core is narrower near the top and consolidation occurs quickly at low pressures because the excess water can soon reach the permeable shoulders. Observa-

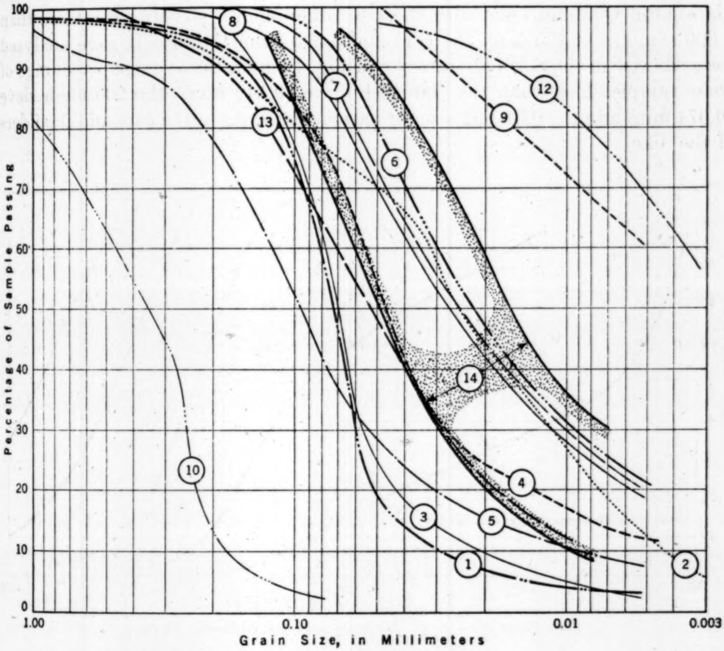

FIG. 16.—COMPARISON OF CORE MATERIALS, IDENTIFIED BY TABLE 5

TABLE 5.—IDENTIFICATION OF HYDRAULIC-FILL DAMS IN FIG. 16

Curve No.*	Dam	Type	Curve No.*	Dam	Type
1	Henshaw, California	Full hydraulic	8	Huffman, Ohio	Full hydraulic
2	Swinging Bridge, New York	Full hydraulic	9	Calaveras, California	Full hydraulic and semi-hydraulic
3	Dwinnell, California	Full hydraulic			
4	Davis Bridge, Vermont	Semi-hydraulic	10	Soft Maple, New York	Full hydraulic
5	Bridgewater, North Carolina	....	12	Alexander, Hawaiian Islands	Full hydraulic
6	Germantown, Ohio	Full hydraulic	13	Cobble Mountain, Massachusetts	Full hydraulic
7	Lockington, Ohio	Full hydraulic	14	Quabbin Dike, Massachusetts	Full hydraulic

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* See Fig. 16.

tions to determine the porosities of core samples indicate that after the pool has been drained, the rate of consolidation is fairly uniform throughout the core and that the pressure due to the depth does not then cause faster consolidation in the lower sections probably because, with the greater width, a relatively longer time is required to carry away the excess water even if it is under a greater pressure.

The qualities of the hydraulic fill obtained from the borrow-pit materials indicated in Fig. 13 are summarized in diagrammatic form in Figs. 16 (with Table 5), 17, and 18. Fig. 16 with Table 5 shows also a comparison of the core material of Quabbin Dike with those of other similar structures. More than 8 000 samples (4 500 from the core and 3 500 from the beach) were analyzed for grain size in the Soil Laboratory and Fig. 19 gives the average percentage of these samples finer than the 100-mesh sieve (0.149 mm), the 200-mesh sieve (0.074 mm) and the 0.01-mm size for various locations in the core and shoulders of the dike.

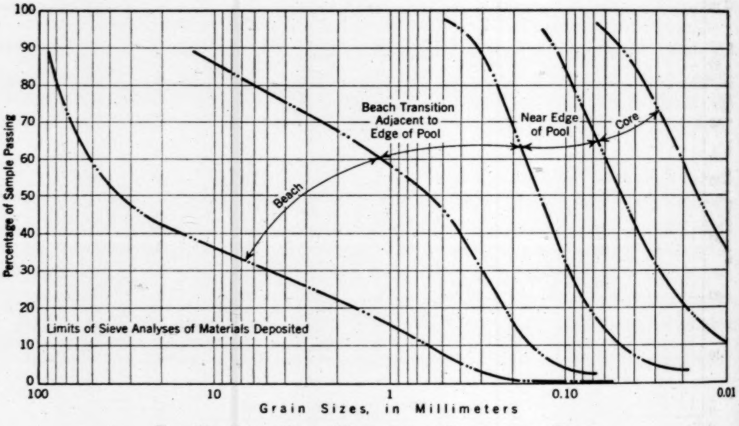

FIG. 17.—GRAIN SIZES; MATERIALS IN BEACHES AND CORE

SEEPAGE ESTIMATES

The quantities of water that will seep through the earth core of Quabbin Dike will be extremely small, but estimates were made to determine reasonable requirements for water-tightness of the core materials and reasonable dimensions to which to build the core, the qualities of the available borrow materials considered. The flow net method[15] was used in these estimates. It consists of establishing the location of the seepage flow lines, from them determining the equipotential loss-of-head lines, and computing the seepage from the latter in terms of the assumed permeabilities of the core and beach materials. The result is expressed in terms of *K*, the coefficient of permeability of the core materials; and by substituting the values of *K* that are considered as reasonable in the result, seepages for different qualities of cores can be secured. The locations of the flow lines were made mathematically by "cut and try" methods, and were checked experimentally by use of a laboratory model similar in some general respects to that described in 1934 by Hibbert M. Hill, Assoc. M. Am. Soc. C. E.[16]

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[15] See 1er Congrès des Grand Barrages-Scandinavie, Vol. III and IV Juin-Juillet, 1933.

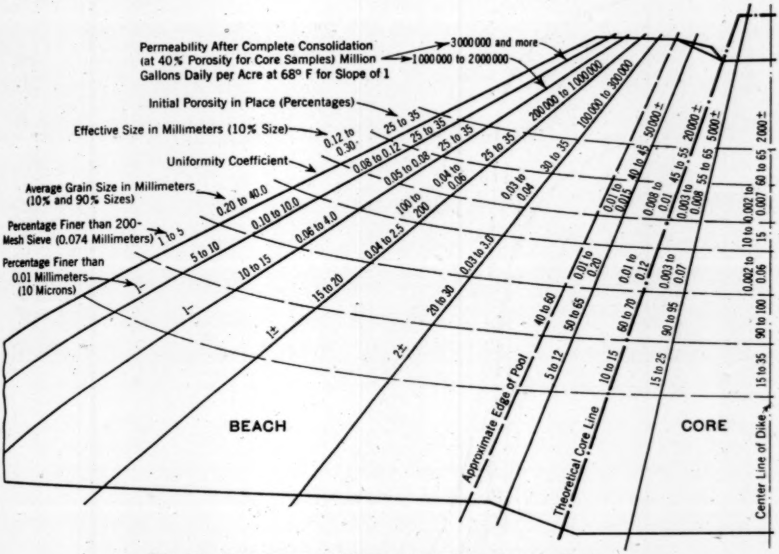
Permeability After Complete Consolidation (at 40% Porosity for Core Samples) Million Gallons Daily per Acre at 68° F for Slope of 1
3000000 and more
1000000 to 2000000
200000 to 1000000
100000 to 300000
50000±
20000±
5000±
2000±
Initial Porosity in Place (Percentages)
25 to 35
25 to 35
25 to 35
25 to 35
30 to 35
40 to 45
45 to 55
55 to 65
60 to 65
Effective Size in Millimeters (10% Size)
0.12 to 0.30
0.08 to 0.12
0.05 to 0.08
0.04 to 0.06
0.03 to 0.04
0.01 to 0.015
0.008 to 0.01
0.003 to 0.008
0.002 to 0.007
Uniformity Coefficient
100 to 200
10 to 15
Average Grain Size in Millimeters (10% and 90% Sizes)
0.20 to 40.0
0.10 to 10.0
0.06 to 4.0
0.04 to 2.5
0.03 to 3.0
0.01 to 0.20
0.01 to 0.12
0.003 to 0.07
0.002 to 0.06
Percentage Finer than 200-Mesh Sieve (0.074 Millimeters)
1 to 5
5 to 10
10 to 15
15 to 20
20 to 30
40 to 60
50 to 65
60 to 70
90 to 95
90 to 100
Percentage Finer than 0.01 Millimeters (10 Microns)
1−
1−
1±
2±
5 to 12
10 to 15
15 to 25
15 to 35
BEACH
CORE
Approximate Edge of Pool
Theoretical Core Line
Center Line of Dike

FIG. 18.—PROPERTIES OF MATERIALS IN BEACHES AND CORE

This model was a small water-tight wooden tank with a heavy plate-glass front (see Figs. 20 and 21). As the beach materials are many times as permeable as the core, the slope of the flow lines up stream and down stream from the core are exceedingly flat; and a satisfactory check on the location of the flow lines can be obtained by finding their position in and through the core itself. For this reason the sections tested were those of the core only. Several sections of core were considered, and their patterns were cut from aluminum plates about 0.1 in. thick.

When tested, the plate was set next to the glass front, and held at a uniformly short distance from it by metal shims. It was then backed up by a wooden, water-tight bulkhead which divided the tank into two parts. Water to the equivalent reservoir level was maintained in one-half the tank and to the equivalent tail-water level in the other half.

The first tests showed that laminar or stream-line flow was obtained throughout when the spacing between the glass front and the core plate was 0.006 in., or less, although a spacing of 0.012 in. could be used with only a small amount of turbulent flow occurring near the caisson cut-off. For greater spacing the area of turbulence increased in size. As the seepage of water through soil is laminar or stream-line flow, the spacing during testing was limited to less than 0.006 in.

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[16] *Civil Engineering*, January, 1934, p. 32.

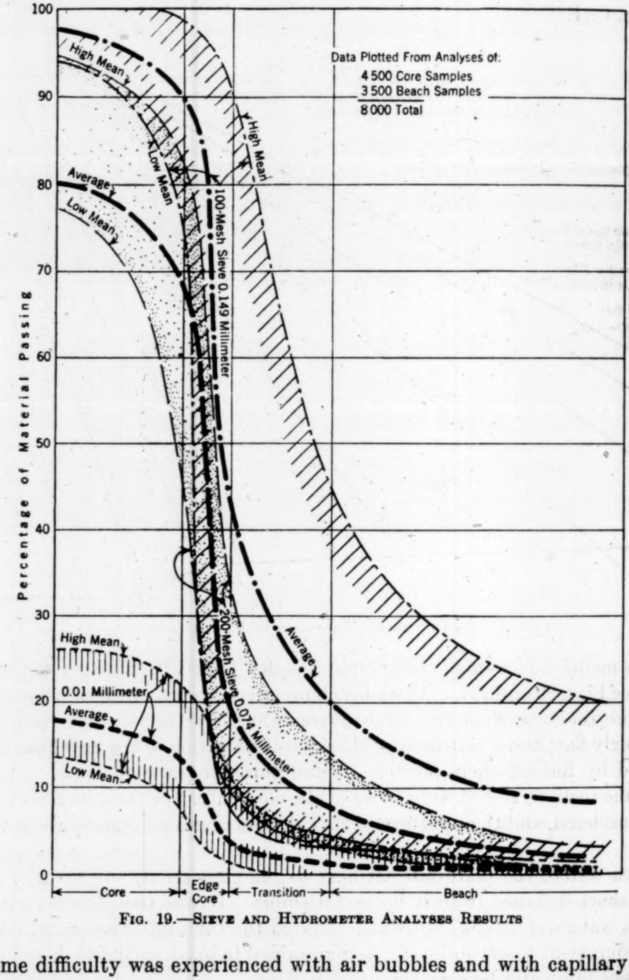

FIG. 19.—SIEVE AND HYDROMETER ANALYSES RESULTS

Some difficulty was experienced with air bubbles and with capillary action near the top of the core section. The former was controlled, to some extent, by running the water until the temperature of the tank was the same as that of the water. The latter was eliminated by use of a metal shim, shaped experimentally to reduce the effect of capillarity on the flow lines without depressing or otherwise unduly affecting their location. This shim was placed between the plates at the top of the core near the water level and prevented any water from being raised into the space between the plate and the glass above the upper normal laminar flow line, which was determined experimentally.

As Figs. 20 and 21 indicate, a dye was introduced into the water through small openings just up stream from the core plate. This dye would travel slowly across the core plate along the seepage lines, and the locations of these lines could thus be photographed. They checked with the mathematically determined locations in an extremely accurate manner. Turbulent flow was easily ascertained by observing the action of the dye in traveling across the plate.

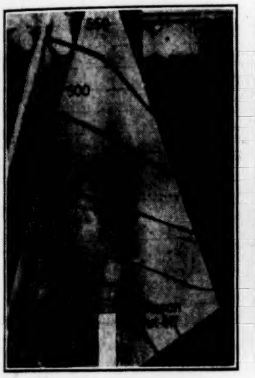

FIG. 20

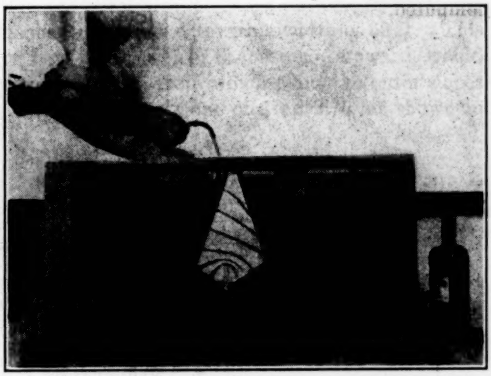

FIG. 21

The core section finally chosen was 100 ft wide at an elevation of 120 ft below the full reservoir level, with side slopes of approximately 3 on 1. Increasing this width of 100 ft to 120 ft would have reduced the seepage only about 7%, decreasing it to 60 ft would have increased the seepage about 33 per cent. Because of the very small seepages considered, the width chosen was determined principally by the more important factor, the probable minimum yield of suitable core material from the available borrow materials.

STABILITY DURING CONSTRUCTION

Estimates of stability of the Quabbin Dike were made for many assumptions of conditions. These estimates led to the development of a nomographic chart (see Fig. 22) for solving in a short time the rather cumbersome Gilboy formula:[14]

$$\sqrt{r_w} = \frac{(C_s - A_c)\sqrt{1 + B^2} + \sqrt{C_s - A_c}\sqrt{C_s - B}\sqrt{1 + A_c^2}}{(1 + C_s^2) - (C_s - A_c)(C - B)} \quad \ldots\ldots(6)$$

in which r_w = the ratio of the unit weight of core material to the unit weight of the shoulder material; A_c = co-tangent of the angle of the core slope with the horizontal; B = co-tangent of the angle of internal friction of the shoulder material; and C_s = co-tangent of the angle of the outer slope, with the horizontal.

If the dimensions of the core and beach sections and the unit weight of the core and shoulder materials are known or assumed, the internal friction or shear value necessary for stability can be estimated quickly from the chart; or, if the qualities of the shoulder and core materials are assumed, the outer slopes of the dam required for cores of assumed or required dimensions can be computed.

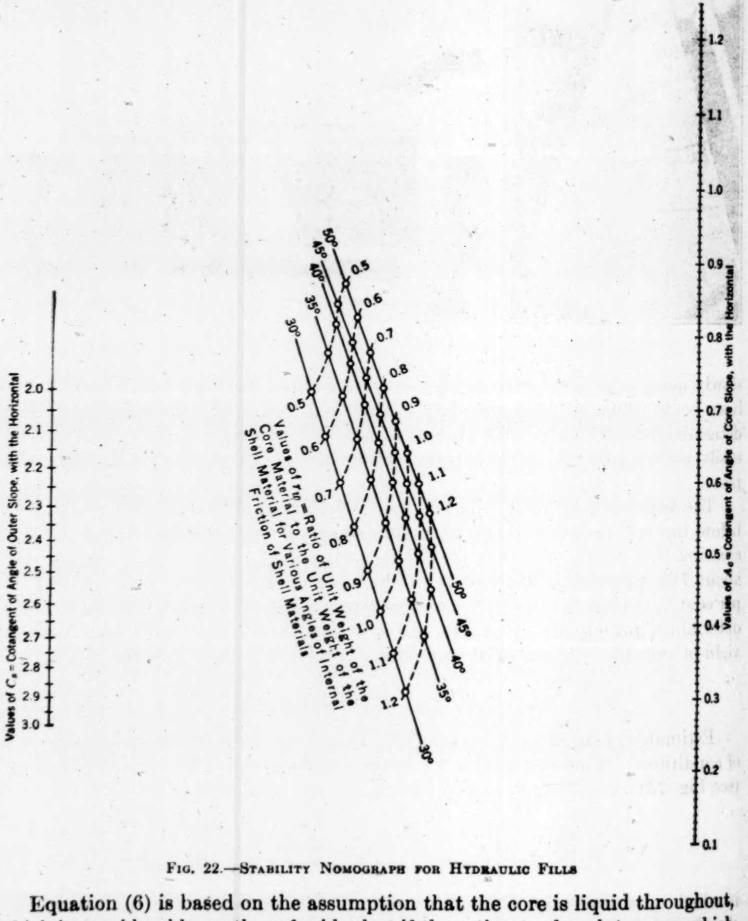

FIG. 22.—STABILITY NOMOGRAPH FOR HYDRAULIC FILLS

Equation (6) is based on the assumption that the core is liquid throughout, which is considerably on the safe side; but if the estimator has data upon which to base an assumption that only some fraction of the full liquid pressure is being developed for the conditions in mind, Fig. 22 can still be used by multiplying the ratio, r_w, directly by the percentage of the full liquid pressure that is existent, and using the corrected value of r_w in the graphical solution.

It will be noted that A_c and C_s are the usual slope constants, representing the ratio of horizontal run to vertical rise. Thus, an outer slope of 1 on 2.5 corresponds to a value of C_s equal to 2.5. Values of C_s are indicated on the scale on the left side of Fig. 22; values of A_c are on the scale of the right side.

The procedure for using Fig. 22 is as follows: Assume that the width of core at Elevation 410 was 100 ft and that the hydraulic-filling has progressed to the point at which the core has reached Elevation 450 with a width of 69 ft. This corresponds to a core slope of 1 on 0.387, which, if continued upward, would intersect the center line at Elevation 540.5, from which point the outer slopes are closely approximated by a line having a slope of 1 on 2.76, as shown in Fig. 23.

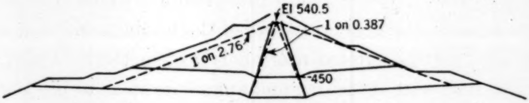

Fig. 23

Assume that the ratio of unit weight of core to unit weight of shell is 1.1 and that the angle of internal friction of the shoulder material is 45 degrees. It is desired to determine whether the section is stable.

A straight-edge is placed on 2.76 on the scale at the left and on 1.1 on the 45° line in the center of Fig. 22. This intersects the right-hand scale at 0.7 and indicates that for the conditions assumed the section is stable, since the values of A_c in the right-hand scale are maximum values for stability. Conversely, the values for C_s at the left are minimum values.

The beach materials at Quabbin Dike in the outer portions of the shoulders weigh as much as 135 lb per cu ft and average probably about 120 to 125 lb per cu ft. The core materials weighed about 100 lb per cu ft when first deposited and from 110 to 125 lb per cu ft when the dam was completed. For stability based on the assumption of full liquid pressure from the core, an internal friction angle of less than 30° in the shoulder materials was required. These shoulder materials had a tested friction angle of more than 45 degrees.

Acknowledgments

The writer wishes to acknowledge the aid of many members of the Engineering Department of the Metropolitan District Water Supply Commission, of which Frank E. Winsor, M. Am. Soc. C. E., is Chief Engineer, and Karl R. Kennison, M. Am. Soc. C. E., is Assistant Chief Engineer, in the preparation of data used in connection with this paper. He is particularly indebted to Clarence R. Day, Assoc. M. Am. Soc. C. E., in connection with work on the seepage model, Coleman C. McCully, Assoc. M. Am. Soc. C. E., in connection with work on the stability diagram, and Jerome L. Spurr, Assoc. M. Am. Soc. C. E., in connection with soil laboratory sampling and testing of materials.

STABILITY OF EMBANKMENT FOUNDATIONS

By B. K. Hough, Jr.,[17] Jun. Am. Soc. C. E.

Synopsis

The analyses made in connection with the design of two large rock-fill embankments proposed as part of the now discontinued Passamaquoddy Tidal Power Project, included certain unusual studies of stress distribution in clay foundations and the character of embankment settlement resulting from foundation over-stress. The report of these studies, which is presented herein, is intended to describe the general nature of the findings and to draw attention to a possible need for the extension of analytical methods (which, at present, are chiefly concerned with designing embankments so as to prevent over-stress of the clay type of foundations) to new methods in which a condition of over-stress is accepted for economic reasons, and the extent of the resulting embankment settlement is computed as the basis for cost estimates.

Introduction

In the field of Soil Mechanics, which itself is a new and specialized branch of Civil Engineering, a certain degree of internal specialization already seems to be in progress. One of the most recent manifestations of this process is the rapid growth of a group the members of which are chiefly concerned with the problems involved in the construction of large earth or rock-fill embankments.

One general and obvious requirement confronting an engineer in designing an embankment is that, after completion, the proposed embankment shall be stable. This involves, first, a determination of the physical characteristics (notably shearing strength) of the foundation and embankment material; then, a calculation of the stresses created within the embankment and extending from the embankment down into the foundations; and, finally, a judicious comparison of strength with stress in the light of past experience and a full realization of the limitations of the analytical methods and the test data which he had used.

Both experience and theory now seem to indicate that analyses for stability vary widely in character and complexity with the type of material encountered in the underground, and the nature (that is, both the material and the design) of the embankment. The soil over-burden at a dam site is generally either predominantly cohesive or cohesionless, and most embankments, with particular reference to their so-called impervious cores, may be classified in the same manner. The four possible combinations of conditions resulting from such classification represent four fairly distinct types of problems, each of which requires a certain degree of special treatment. Additional complications are introduced by the various methods of building embankments, such as rolled or hydraulic fill, and if rolled, whether compacted to a high or low relative density. The scope of this paper is limited to a discussion of certain studies which have relation to a definitely cohesionless uncompacted embankment founded on a relatively uniform, cohesive foundation. Methods of laboratory testing, model studies, and analysis of data performed in the U. S. Engineers Soils Laboratory for the Passamaquoddy Project are described, since it is believed that certain experiments performed there may point to a new conception of the action of embankment foundations. The work done at Eastport, Me., on this problem was predominantly of a qualitative nature, and it is hoped that the findings as reported herein will stimulate further and more detailed studies.

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[17] Engr., U. S. Engr. Office, Ithaca, N. Y.

FOUNDATION EXPLORATION

In leading up to the subject of this paper, the following description of the drilling operations in the foundations is given as a matter of general interest. In Eastport, the problem of foundation investigation for the two largest dams presented unusual difficulty. Known, respectively, as the Eastport and Lubec Dams, these structures were intended to close two channels or passages between the ocean and the waters of Cobscook Bay, the high level pool of the proposed power development. The unusual difficulty lay in the fact that each of these channels was approximately 3 500 ft wide, that in each channel the tidal range had a magnitude of about 26 ft, and that the resulting flow of water occurred with a velocity sometimes as great as 6 ft per sec. The soil over-burden to be explored lay at the bottom of these channels under depths of water as great as 100 ft, and from a somewhat limited investigation previously conducted, was believed to be predominantly a deposit of soft marine clay. In the exploration described herein, one drill hole was actually driven to Elevation −274.5 before rock was encountered.

Drilling for exploratory purposes, and to obtain undisturbed soil samples suitable in size and character for compression and shear tests, was conducted from two large derrick-boats, one operating in each channel. To protect the 6-in. drill casing from the force of the tidal currents and to provide a stable support for the drill platform, an extra heavy 21.5-in. casing or spud was first erected in sections over the sides of these boats. All guys to the top of the spud and all anchor lines to the boats were led to hand winches, constantly attended so that the spud could be kept vertical and the position of the boat maintained at all times, despite the rise and fall of the floating plant with the tide. The bottom of the spud was allowed to penetrate the over-burden sufficiently to give it lateral stability, while the weight of the spud and drilling equipment was carried by a large, circular plate clamped to the spud a short distance above its lower end so that it had bearing against the surface of the over-burden. After erection of the spud, the drill rig was mounted on the platform provided at the top and drilling was conducted in normal fashion in a 6-in. casing inside. A general view of the barge in actual operation in the Eastport Channel is shown in Fig. 24. In this work the importance of obtaining undisturbed soil samples for accurate foundation studies was realized and close inspection of the condition of all samples was made in the laboratory. Some comment is made subsequently on the effect of disturbance of samples, as indicated by the test data obtained in the Eastport Laboratory.

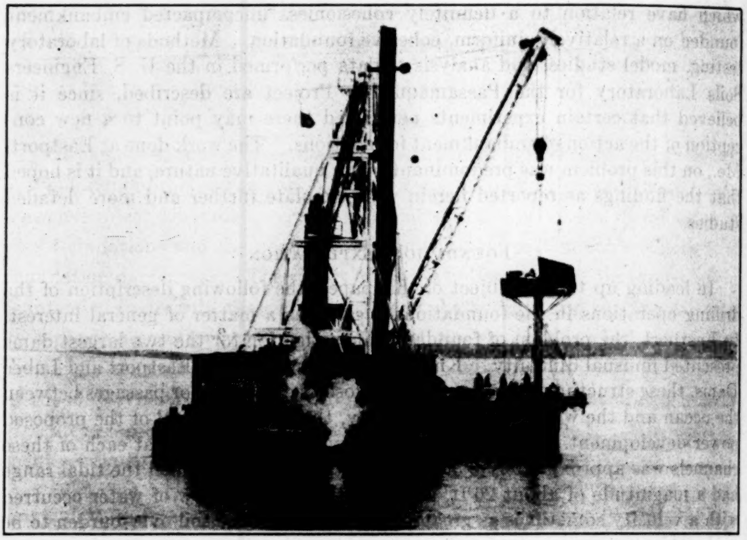

Fig. 24.—Deep-Water Drilling, Passamaquoddy Tidal Power Project; 160-Foot Speed in 120 Feet of Water

Laboratory Testing

Routine soil testing on this project was performed by methods which in general character are similar to those in use in the Soil Mechanics Laboratory of the Massachusetts Institute of Technology, Cambridge, Mass. The equipment used for performing the shear and consolidation tests was the same in principle

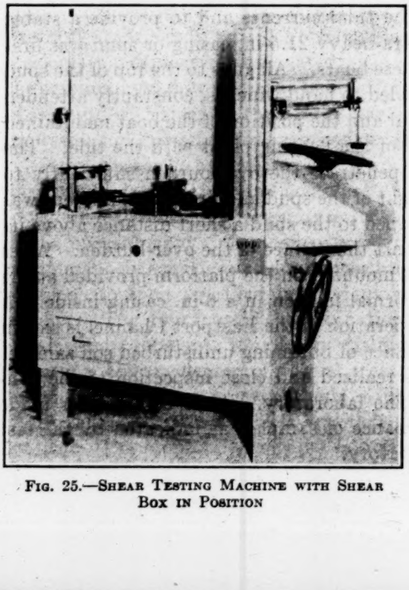

Fig. 25.—Shear Testing Machine with Shear Box in Position

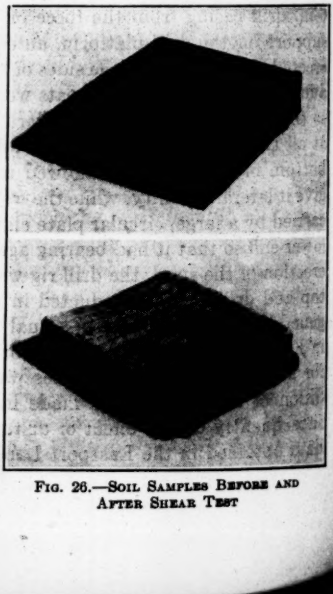

Fig. 26.—Soil Samples Before and After Shear Test

as that designed by Glennon Gilboy, Assoc. M. Am. Soc. C. E., for the U. S. Engineer Laboratory, in Zanesville, Ohio.[18] The shear machine, shown in Fig. 25, is operated in such a manner that the rate of horizontal strain of the sample during the test is constant. The particular rate chosen for standard tests on clay samples of the type shown in Fig. 26 was that which was found to give the most conservative value of shearing strength for the type of material tested. A rate of strain commonly used was 0.05 in. per min. In testing cohesive samples, a relatively light vertical load was used, in general equal to about one-fourth the horizontal pull. No effort was made to determine an apparent angle of internal friction for clay samples and no increase in natural shearing strength was allowed as a result of possible consolidation during the construction of the embankments. The shear tests were run simply to determine the natural shearing strength of the deposit of cohesive material at the site of the proposed structures.

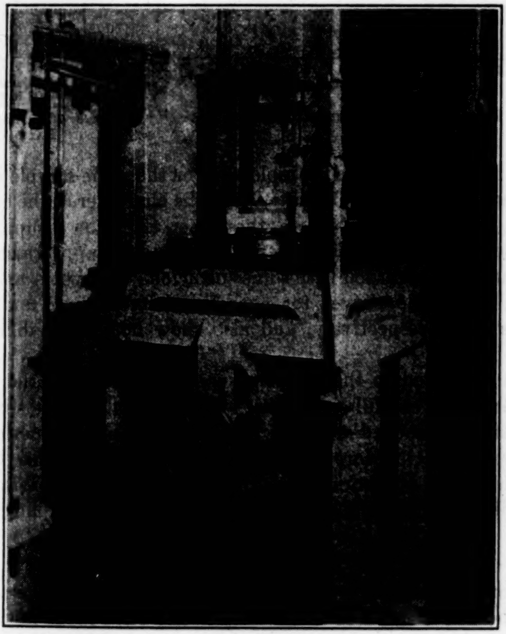

FIG. 27.—VIEW OF CONSOLIDATION MACHINE

At this point it seems advisable to acknowledge the fact that most of the statements made in the foregoing paragraph touch on highly controversial subjects. In making these statements, it is hoped rather than feared, that they will provoke discussion and constructive criticism. Consolidation tests were performed with the equipment shown in Fig. 27, and the method used for plotting test data was that developed[19] by Donald W. Taylor, Assoc. M. Am. Soc. C. E.

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[18] *Engineering News-Record*, May 21, 1936.

EFFECT OF SAMPLE DISTURBANCE

Correlation of the data from shear and consolidation tests was found to give an indication of the degree of disturbance suffered by individual samples during the drilling operations. Fig. 28 shows stress-strain curves, *A* and *B*, respectively, for an undisturbed sample and for the same sample retested after complete remoulding. It will be noted that the character of the curves is quite different. It was decided, therefore, that when a test on a supposedly undisturbed sample yielded data which when plotted had the appearance of Curve *B*, the sample should be regarded with some suspicion. Curves *A* and *B* in Fig. 29 are pressure, void-ratio curves for consolidation tests on undisturbed and remoulded samples, respectively, and also show noticeably different characteristics.

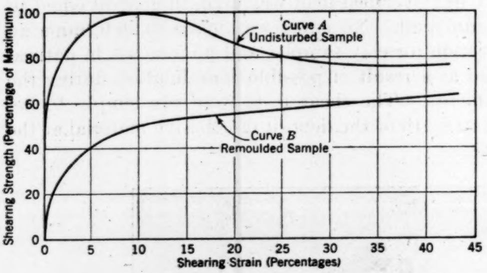

FIG. 28.—STRESS STRAIN SHEAR CURVES

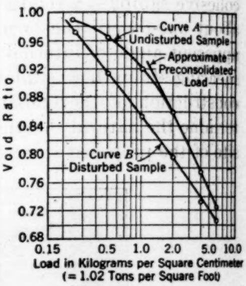

FIG. 29.—PRESSURE, VOID-RATIO CURVES

Attention is invited to the foregoing considerations and the suggestion is made that through a careful examination of test data in some similar manner, a judgment may be reached as to the value of the sampling methods in use on any particular project. More attention in the future than has been given in the past to this detail of laboratory testing is believed to be advisable and may lead eventually to the development of more dependable sampling technique.

SUMMARY OF TEST DATA

Correlation of drilling and testing records made possible the preparation of the profiles shown in Figs. 30 and 31. The information given in these diagrams may be amplified by the statement that, in general, the shearing strength of the clay in the Eastport Channel tends to increase with depth, whereas the reverse condition exists in the Lubec Channel. The average values in both cases range between 0.1 and 0.2 ton per sq ft, although in the Eastport Channel lower values predominate and in the Lubec Channel a few values as high as 1.0 ton per sq ft were found. The average water content for samples from both channels is about 30% of the weight of solids. The general variation of water content with depth was found to be in agreement with the variation of shearing strength, since in the Eastport Channel it decreases with depth and in the Lubec Channel it increases. A mechanical analysis of the material from both channels showed that it had a colloidal content of about 20% and about equal percentages of clay and silt. Its specific gravity is approximately 2.75.

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[19] "Improving Soil Testing Methods", by Glennon Gilboy, Assoc. M. Am. Soc. C. E., *Engineering News-Record*, May 21, 1936.

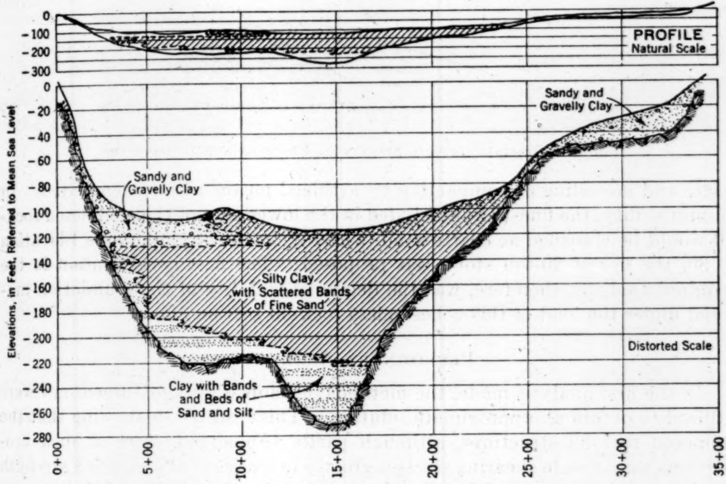
0
-100
-200
-300
PROFILE
Natural Scale
Elevations, in Feet, Referred to Mean Sea Level
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
-220
-240
-260
-280
Sandy and Gravelly Clay
Sandy and Gravelly Clay
Silty Clay with Scattered Bands of Fine Sand
Clay with Bands and Beds of Sand and Silt
Distorted Scale
0+00
5+00
10+00
15+00
20+00
25+00
30+00
35+00

FIG. 30.—PROFILE ALONG CENTER LINE OF DAM, EASTPORT CHANNEL

PROPOSED STRUCTURE

On this material it was proposed to build a structure of the type shown in Fig. 32. It was intended to construct the rock-fill section of these dams first as a separate operation in order to effect closure of the channels and, after this

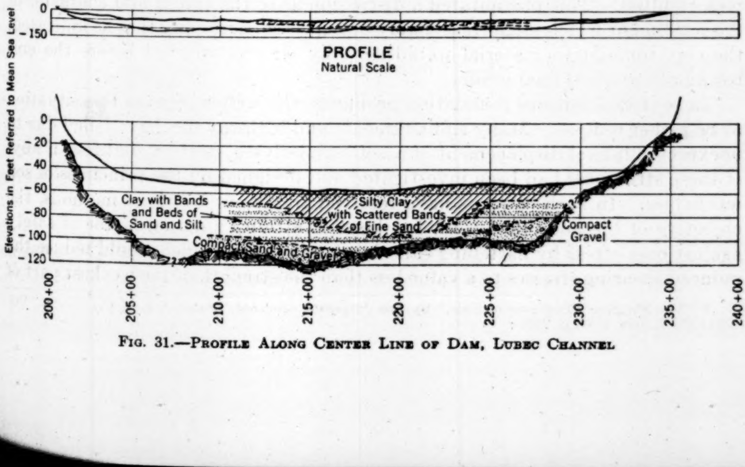
0
-150
PROFILE
Natural Scale
Elevations in Feet Referred to Mean Sea Level
0
-20
-40
-60
-80
-100
-120
Clay with Bands and Beds of Sand and Silt
Silty Clay with Scattered Bands of Fine Sand
Compact Gravel
Compact Sand and Gravel
200+00
205+00
210+00
215+00
220+00
225+00
230+00
235+00
240+00

FIG. 31.—PROFILE ALONG CENTER LINE OF DAM, LUBEC CHANNEL

step, to seal the dams with a relatively impervious soil blanket placed on the "up-stream" face. It should now be evident that the previous reference to a definitely cohesionless embankment is justifiable in connection with these structures. Nearly all the material in these dams would be placed in flowing water and no rolling or compacting by artificial means was contemplated. In all probability, the finest sizes included in the lower part of the completed rock-fill would be classified as coarse sand, whereas, in the section above Elevation − 30, the use of 20-ton stone was proposed. The first consideration in the required analysis, therefore, was the degree of stability of the foundation material under the load of this cohesionless rock-fill section alone.

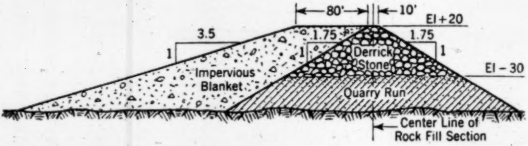

FIG. 32.—ORIGINAL TYPICAL SECTION OF EASTPORT AND LUBEC DAMS

PRELIMINARY ANALYSIS

In the first analysis made, the methods developed by Leo Jurgenson[20] were utilized to obtain an approximate solution. This resulted in showing that the proposed rock-fill structures, although partly submerged, were of such proportions as to create shearing stresses greatly in excess of the shearing strength of the foundation material. Comment on the applicability of Jurgenson's method to this particular problem is deferred to the latter part of this paper. Considerable importance, however, was attached to the preliminary finding that in order to achieve stability of the proposed rock-fill, the base width would have to be increased from five to ten times. The additional quantity of fill material required for this alternate design was found to be so considerable that the possibility presented itself of effecting an economy by building the embankment with the relatively steep slopes originally planned and letting failure and displacement of the underground take place, even if eventual settlement to rock resulted. This precipitated a discussion as to the extent and character of the settlement which might reasonably be expected, assuming that as indicated, the clay foundation material would actually be over-stressed before the embankment reached final grade.

In contemporaneous foundation problems, the writer believes this situation to be rather unique. Many embankments and highway fills have failed due to unexpected lateral displacement of a soft, underlying stratum, but few, if any, of these structures had been investigated and designed by the principles of soil mechanics. In such cases as have received treatment by these methods, the objective of the analytical work has been to attain an actual factor of safety against over-stress by flattening side slopes to a degree which would reduce the induced shearing stresses to a value less than the strength of the weakest part of the foundation. A definite lack of precedent was found, therefore, on procedure for an embankment analysis in which failure and at least partial displacement of the underground was actually contemplated, although as an exception to this statement the special case of settling highway fills by dynamiting soft underlying material may be cited.

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[20] "The Shearing Resistance of Soils," by Leo Jurgenson, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXI, No. 3, July 1934, p. 242.

MODEL STUDIES

The available information was so limited as to the type or extent of settlement to be expected, that some experimentation was considered necessary as a preliminary to deciding on a method of attacking the problem. Already set up in the laboratory was a simple polariscope (shown diagrammatically in Fig. 33) which had been used previously to study the distribution of shearing stresses in gelatin models by photo-elastic methods. By using a weak pour of gelatin and gradually increasing the height of the model dam over that previously used, the gelatin was finally made to fail under the load, a procedure quite different from that followed in ordinary work with this model, when failure of the gelatin was studiously avoided. Settlement of the model embankment resulted, of the type shown in Fig. 34. This view was photographed through the analyzer and besides showing the shape and outline of the embankment section after failure, it also shows a number of stress bands in the gelatin beyond the toes, where evidently the gelatin was not overstressed. Interesting though this last point may be to some, the particular object of introducing Fig. 34 is to show the character of deformation that occurred under the embankment. It is quite clear that, in this case, settlement was not maximum under the center line, although numerous predictions to this effect had been made, but rather it is apparent that maximum settlement occurred at points on either side of the center line, about half-way out toward the toes. This experiment was repeated

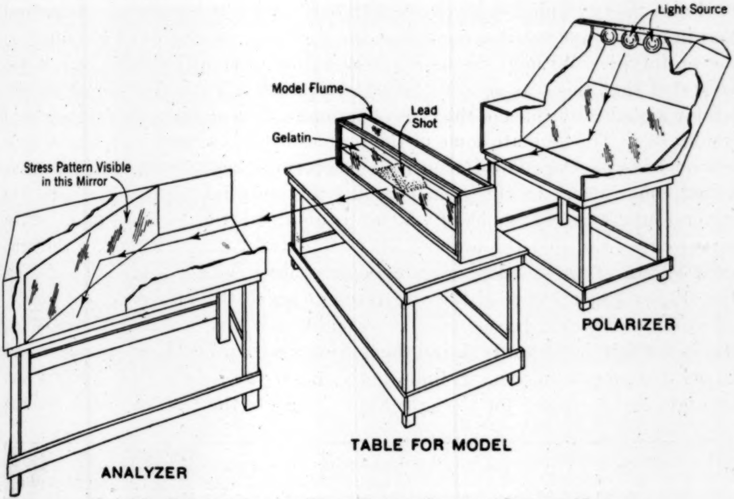

FIG. 33.—POLARIMETER FOR PHOTO-ELASTIC STUDY OF GELATIN MODEL

a number of times under a fairly wide range of controlling conditions until it was established as being typical. From this work, the type of settlement obtained was termed "heart shaped", due to its resemblance to the lower part of an inverted heart.

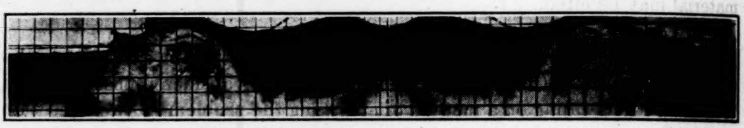

FIG. 34.—SETTLEMENT OF MODEL EMBANKMENT

From the beginning of the work with this model, it was clearly understood that in many ways it failed to achieve the necessary similitude to conditions in the prototype. Without discussing in detail its obvious shortcomings, it will be stated that this was simply the first and easiest method available for obtaining a qualitative idea of the type of action to be expected under the assumed conditions. To eliminate some of the sources of dissatisfaction, a second type of model was developed for the purpose of securing a closer approximation of the actual conditions. In this second model the material used to represent the underground was clay, much of which was obtained from samples taken at the actual dam sites, and the proportions of the model were changed so that the axial length of the section of dam being studied was considerably greater. Fig. 35 is a general view of the apparatus in which this study was conducted. The preliminary test procedure was to build up a bed of uniform, remoulded clay in the rather shallow tank and then, using irregular lead slugs, to construct a model dam across the center, following as nearly as possible the construction program contemplated for the prototype. During the test, movement of the

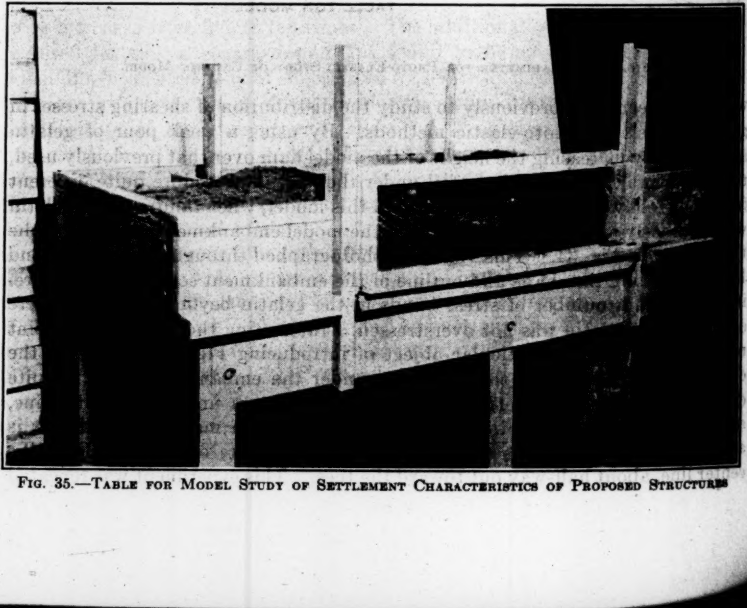

FIG. 35.—TABLE FOR MODEL STUDY OF SETTLEMENT CHARACTERISTICS OF PROPOSED STRUCTURES

clay under the embankment load could be watched through a window in one side of the tank.

To facilitate this observation, small white reference points were inserted between the clay and the window at the intersections of a grid system, prior to loading. Successive positions of each point were marked on the glass so that at the end of each test their individual paths were determined. At the center of the model, in a section perpendicular to the axis of the dam, a series of vertical reference lines was inserted. The deformation of these lines could not be observed during the test, of course, but were exposed afterward by removing the embankment and cutting the model along the mid-section, a view of such a section being shown in Fig. 36. The originally vertical reference lines may be seen clearly, and the profile of the clay is shown to have assumed the previously mentioned "heart shape". Also, plainly visible, are the so-called "mud wave" formations on either side of the embankment.

This confirmation of the results previously obtained, although still of a qualitative nature, was very encouraging. In this connection, attention is invited to the experience of A. Casagrande, Assoc. M. Am. Soc. C. E.[21] and Professor Gilboy,[22] who noted a similar type of failure under embankments on soft foundation material. Sufficient confidence in the accuracy of the general character of this work was gained from this accumulation of evidence to proceed with the development of a quantitative method of analysis for use in solving the particular problem previously outlined.

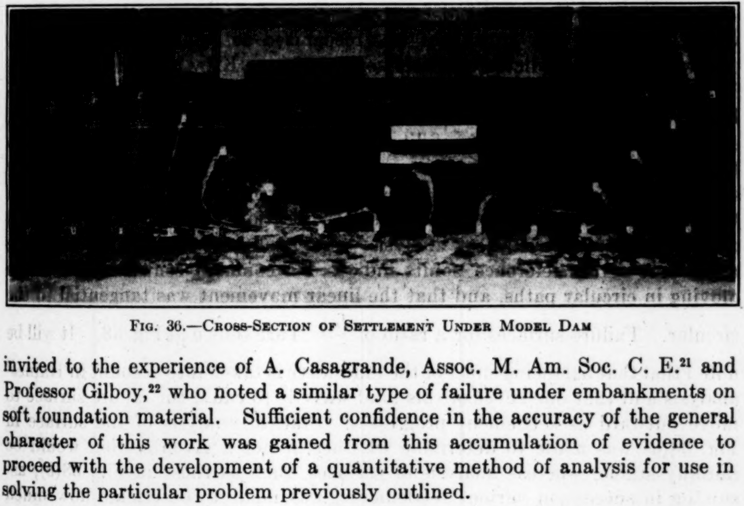

FIG. 36.—CROSS-SECTION OF SETTLEMENT UNDER MODEL DAM

HAINES METHOD

Continued experimentation with the clay model and careful observation of its action during test, led to the development of a method of stress analysis before failure, and a method of determining the extent of settlement after failure, of proposed embankments. The details of this analytical method were developed by R. M. Haines, Jun. Am. Soc. C. E.

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[21] "The Shearing Resistance of Soils", by Leo Jurgenson, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXI, No. 3, July, 1934; Discussion by A. Casagrande, p. 976.

[22] "Stability of Embankment Foundations", by Glennon Gilboy, Assoc. M. Am. Soc. C. E., *Transactions*, Third World Power Conference, Washington, D. C., 1936.

A characteristic set of records for the clay-model testing is shown in Fig. 37. Inspection of this diagram led to the conclusion that movement of the reference points was largely circular under the dam and became linear under the mud waves. This type of movement was recognized as being similar in general character to that predicted by Professor Dr. H. von Krey, of Berlin, Germany, for the movement of plastic material under long footings. Development of the Haines method proceeded from this observation to the task of making an analysis of the conditions of equilibrium along a failure surface also similar to that defined by Krey, but in this case located by trial and error in a manner somewhat resembling the work of K. E. Petterson, member of the Swedish Geo-Technical Commission in 1914–1922. Location of the failure surface and development of the statical analysis were facilitated by finding that the movement of the reference points indicated a common center for all points moving in circular paths, and that the linear movement was tangential to the circular. Failure surfaces for a ratio of $\frac{L}{a} = 4$ are shown in Fig. 38. It will be noted that that balancing effect of the mud wave is taken into account in Haines' analysis and that one effect of this mud wave is to cause the failure surface to move outward as settlement progresses. Statical analysis of the surface in Fig. 38(*a*) was made to determine whether or not a given section would be initially stable, whereas analysis of surfaces, such as those in Fig. 38(*b*), assuming in succession various settlements, was made to determine how much settlement could be expected when initial stability was not indicated. (In Fig. 38(*b*), Line *BEO* is the shear plane for 50% settlement; Line *AEO* is the shear plane for 75% settlement; Line *BDF* is the mud wave for 50% settlement; and, Line *ACF* is the mud wave for 75% settlement.) The same kind of study was also made for trapezoidal embankment sections in order to make possible analysis of various construction stages.

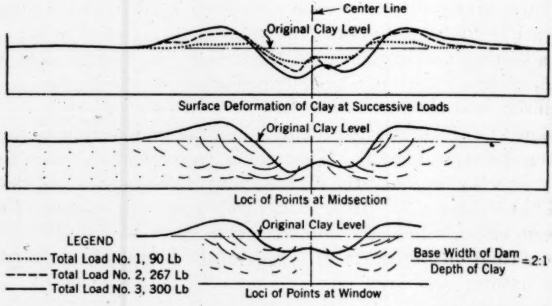
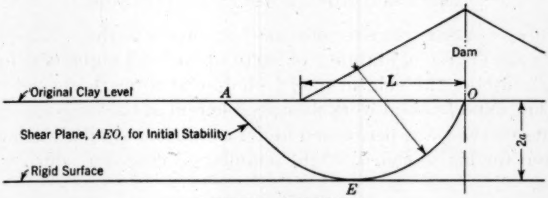
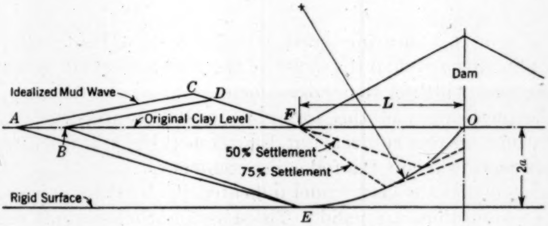

Center Line
Original Clay Level
Surface Deformation of Clay at Successive Loads
Original Clay Level
Loci of Points at Midsection
Original Clay Level
LEGEND
Total Load No. 1, 90 Lb
Total Load No. 2, 267 Lb
Total Load No. 3, 300 Lb
Base Width of Dam / Depth of Clay = 2:1
Loci of Points at Window

FIG. 37.—MODEL STUDIES OF SETTLEMENT AND SHEAR FAILURE CHARACTERISTICS

The types of analysis indicated in Fig. 38 were planned on the assumption that the half section of embankment tended to rotate about the center of the circular sliding surface and thereby caused a rotational movement of the foundation material included within this surface and an upward movement involving passive pressure of the material along the straight part of the failure surface. By equating the moments of the forces tending to cause motion with those tending to resist it, an expression was obtained relating the intensity of loading at the center line of the embankment with the cohesion or inherent shearing strength required for a factor of safety of unity. This expression can be simplified to the form,


Dam
L
Original Clay Level
A
O
Shear Plane, AEO, for Initial Stability
2a
Rigid Surface
E

(a) INITIAL STABILITY


Dam
Idealized Mud Wave
C
D
L
A
F
O
Original Clay Level
B
50% Settlement
75% Settlement
2a
Rigid Surface
E

(b) EXTENT OF SETTLEMENT

FIG. 38.—SURFACES INVESTIGATED FOR EXTENT OF SETTLEMENT

$$p = K\, s_c \qquad (7)$$

in which K is a variable with values as shown in Fig. 39.

For the case in which initial stability is not indicated and it is desired to compute the probable amount of settlement of the embankment into the foundation material, the following formula, was derived:

$$p = K\, s_c + K_i\, a\, w \qquad (8)$$

Fig. 40 is a set of curves for the solution of Equation (8) for the assumption of 50% and 75% settlement. It will be noted that solutions for various stages of construction are indicated from 25% of the final height of the embankment to the full height or the triangular section.

Solution of settlement problems by the Haines method as indicated by Figs. 39 and 40 involves a process of trial and error. If initial stability is not indicated, a solution is obtained for stability at 50% settlement. If this does not indicate stability, 75% settlement is assumed, and if stability is still not indicated, the assumption is made that the material will settle to rock.

No greater refinement is warranted by the assumptions involved in developing this method. One consideration, for example, is the extent to which the shearing strength of the foundation decreases as failure occurs. It is possible that additional investigation, however, may eventually modify the approximate nature of this method.

Discussion of Analytical Methods

In the foregoing remarks reference has been made to three different methods of analyzing the degree of stability of earth or rock-fill embankments, namely, Jurgenson's method, the gelatin model-photo-elastic method, and the Haines' method. The experimental work described herein as the basis for development of the Haines method was performed under conditions similar to those assumed by Jurgenson for his so-called "rigid boundary" case, for which he proposed the formula,

$$s = \frac{p\,a}{L} \qquad \text{(9)}$$

in which s = principal shearing stress; p = intensity of load at the center line of the embankment; a = half the depth of the plastic foundation material; and, L = half the base width of the embankment.

Since the publication of this formula[20] in 1934, Jurgenson has presented other discussions on this subject, but Equation (9) is still in current use to a sufficient extent to warrant the following comment.

It is believed that the clay model indicated the limit to which some of Mr. Jurgenson's assumptions are valid. These assumptions include neglect of the restraining effect of the foundation material beyond the toes of the embankment, and the supposition that the surface between embankment and foundation remains plane during loading. It can be seen almost intuitively that variation in the ratio of $\frac{a}{L}$ will vary the extent to which these simplifying assumptions affect the applicability of Equation (9) to embankment analysis. The work with the clay model indicates that, for ratios of $\frac{a}{L} = 0.1$, Jurgenson's method and Haines' method are in substantial agreement as to the magnitude of the shearing stress induced by a given intensity of embankment loading. For higher values of the ratio, however, a considerable divergence between the two methods was noted. It was also observed that when the depth term, a, is small compared with L, the condition of simultaneous shear failure on an infinite number of surfaces in the plastic material is approached more closely than when a is comparatively large. In the latter case, evidence was obtained to indicate that shear failure occurs at a single point and, therefore, in giving the intensity of stress at this point, Jurgenson's formula (Equation (9)) does not indicate the full extent to which the foundation can carry load, due to the redistribution of stress following point failure. As against this so-called "point failure stress", Haines' method is believed to show the total resistance of the underground, or what might be termed the "total failure stress".

The argument just presented applies also in the case of the previously mentioned gelatin model-photo-elastic method. This method, except when

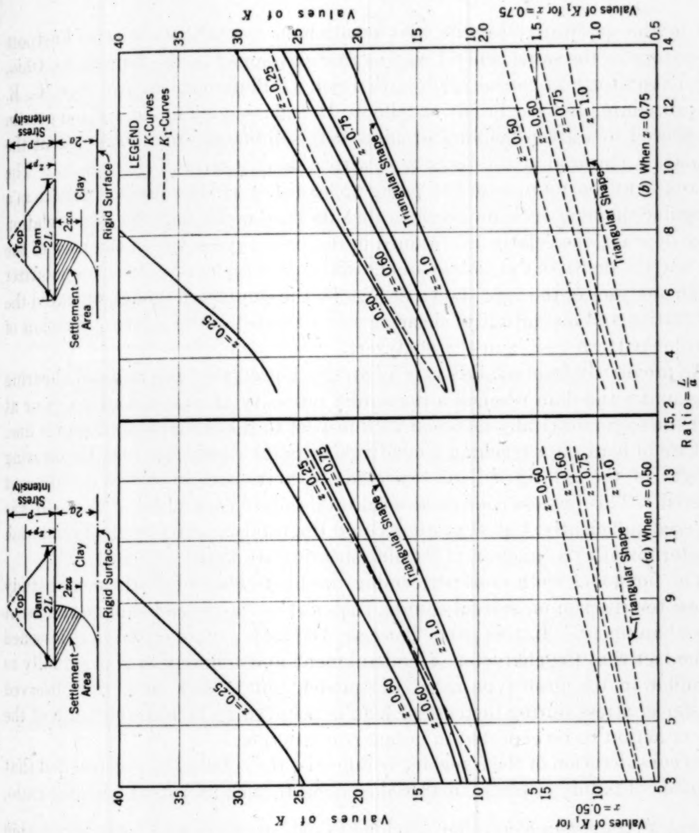


FIG. 40.—CURVE FOR SOLUTION OF SETTLEMENT FORMULA, EQUATION (8)

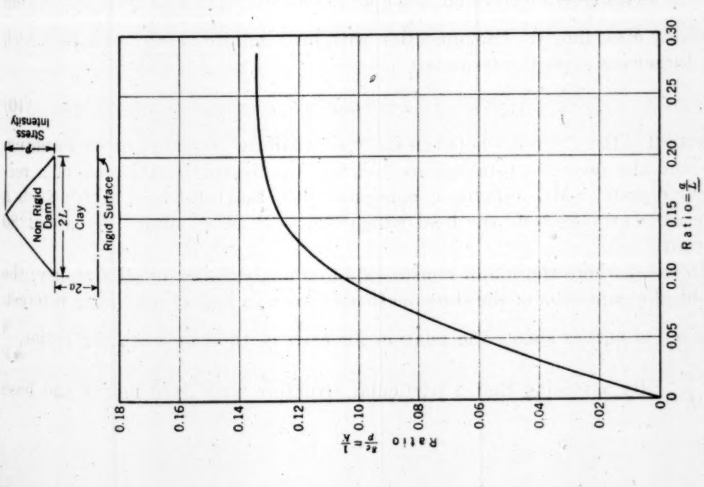


FIG. 39.—CURVE FOR SOLUTION OF INITIAL STABILITY FORMULA, EQUATION (7)

used in the qualitative studies first mentioned, was utilized in the Eastport Laboratory in the same general way as that developed in the Zanesville, Ohio, Soils Laboratory, in consultation with Fred. L. Plummer, M. Am. Soc. C. E. The procedure used was briefly as follows: In a glass-sided flume, melted gelatin was poured to a given height and allowed to solidify in order to represent the foundation material of the prototype with a doubly refracting medium. The embankment was represented by piling up lead shot on the solidified gelatin in a triangular shape of such proportions that its base width had the same relation to the depth of the gelatin as obtained in the prototype between the base of the dam and the depth of the plastic foundation. When placed between a polarizer and an analyzer of the type shown in Fig. 33, the completed model indicated the distribution and magnitude of shearing stress created in the gelatin by means of the color pattern visible in the analyzer.

As previously implied, this type of study showed that the induced shearing stress under the dam reached a maximum intensity at one central point or at two points symmetrically disposed with respect to the embankment center line. Increase of load after reaching a condition of point stress equal to the shearing strength of the gelatin led (as expected) to a transfer of stress to adjacent material. This process continued until total failure took place, if the load was increased sufficiently, but at no time could the total capacity of the foundation be determined as a function of its unit shearing strength.

The foregoing method of performing the photo-elastic-gelatin model study is open to criticism on several counts, such as the use of lead shot to represent the embankment. In this particular case, the force of this criticism is lessened by the fact that the character of embankment and foundation was actually as dissimilar in the prototype as in the model, but in any case, the observed transfer of stress during increase of load is believed to be characteristic of the type of action to be expected in actual construction.

In consideration of the foregoing comments, the conclusion was reached that the mode of failure of plastic material changes to such an extent with the ratio, $\frac{a}{L}$, that a straight-line equation, such as Equation (9), can be used only within relatively close limits. In connection with his "infinite depth" case for which Mr. Jurgenson gives the formula,

$$s = 0.256\,p \qquad (10)$$

the extent of the difference between the "point failure" stress given by Equation (10) and the desired "total failure" stress is indicated by the case of a rectangular plate. Mr. Jurgenson computes that the total load which such a plate can carry exceeds the load which causes "point failure" by about 80 per cent.

In cases where the major consideration is analysis for initial stability, the graphical comparison of the three methods shown in Fig. 41 may be of interest. This set of curves shows the relation for each method between the ratios, $\frac{s}{p}$ and $\frac{a}{L}$. By assuming that a particular structure with fixed height and base width is being considered, this plot gives, directly, the variation of the stress induced by such a structure if built on clay deposits of different depths. Jurgenson's "rigid boundary" case plots as a 45° line, of course, until it intersects his "infinite depth" case at a value of $\frac{a}{L} = 0.256$. From this intersection the curve is a horizontal line. The work with the gelatin model plots as a curve showing a more gradual transition from the "rigid boundary" case to the "infinite depth" case. The curve for Haines' method of analyzing the stability of triangular embankment sections may be seen to follow Jurgenson's "rigid boundary" case quite closely up to the ratio, $\frac{a}{L} = 0.10$. Beyond $\frac{a}{L} = 0.15$, it deviates abruptly, however, and flattens out into what amounts to an "infinite depth" case at a much lower value of $\frac{s}{p}$. Since Haines' method is intended to give the "total failure" stress throughout, this is not surprising, however. If it is admitted that Jurgenson's "infinite depth" case has a factor of safety in the order of magnitude of 1.8, as indicated by the example of the rigid rectangular plate, a correction could be made, as shown by the dotted line. It is very interesting and significant to note how close this line is to that for the Haines method.

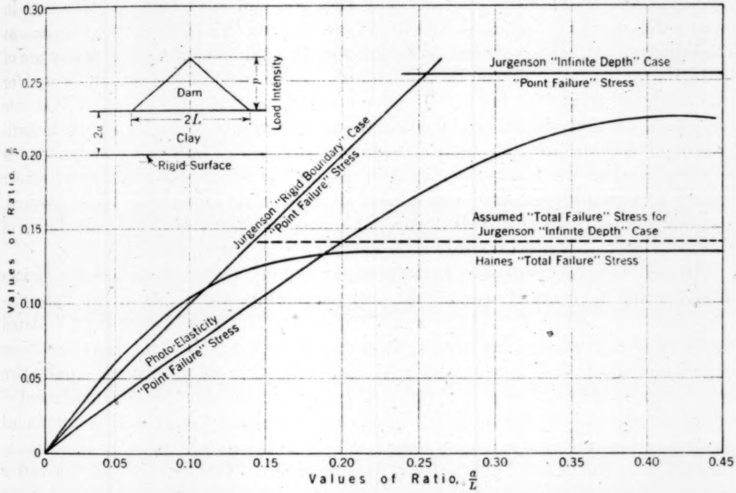


FIG. 41.—LOAD, SHEARING STRESS CURVES FOR TRIANGULAR EMBANKMENTS; $\frac{L}{a} = 4$.

In the field in which Mr. Jurgenson's method and the photo-elastic—gelatin-model method have previously served, it is believed that the Haines method offers the advantage of being more general in the sense that it is not confined to use in cases in which $\frac{a}{L} = 0.1$ and that it is more consistent in the sense that it always gives the "total failure" stress as previously defined.

The particular claim made for the Haines method, however, is that it can be extended beyond the field of usefulness of the other types. It is believed that work similar to that reported herein may attract the attention of engineers to the possibility of effecting economy in earth construction work by allowing artificial fills to sink into relatively soft and shallow foundations, rather than trying to attain stability by flattening the side slopes of such structures. In cases where a real economy is indicated by adopting this plan, as was the case at Passamaquoddy, the methods of Jurgenson and the gelatin-model study are of little, if any, value. The method developed by Haines, however, can be used for just such cases and as far as the writer knows, is the only method available for complete analysis for proposed earth embankments, since by its use investigation of the stability of such structures during all stages of construction is possible, and prediction of the extent of settlement which will occur due to shear failure of the underground may be made if such settlement is to be allowed. Estimate of settlement of this nature, of course, is essential to the calculation of the total cost of structures in which subsidence is anticipated.

It will be observed that in this paper much of the detail of the Haines analysis has been omitted although full particulars are given in the Passamaquoddy Tidal Power Development "Report of Soils Laboratory" dated September 1, 1936. This is due to a realization of the fact that the basic experimentation was performed with the objective of obtaining qualitative results applicable to one rather unusual type of problem. Since curtailment of activity on the Passamaquoddy Project took effect before all the intended studies of this nature were completed, it is considered advisable at this time to present only the general character of this research. It is hoped that this rather non-technical presentation will stimulate discussion and further study leading eventually to a complete and satisfactory solution of the indicated problem.

SUMMARY

In connection with cohesionless, uncompacted earth or rock-fill embankments founded on cohesive soil, model studies indicate that embankment settlement resulting from shear failure and lateral displacement of the foundation material will take the form of the so-called "heart shape". By citing the specific case of the two large dams proposed for the Passamaquoddy Project, attention is invited to the possibility in some instances of effecting economy in embankment design by allowing over-stress of foundations to occur rather than flattening side slopes in an effort to prevent over-stress to such an extent that excess material is required.

The need for development of a comprehensive method of stress and settlement analysis for earth and rock-fill dams is indicated by a discussion of some available methods, and the so-called Haines method is outlined in general terms as a lead to further study and consideration of this subject.

ACKNOWLEDGMENTS

The investigations described herein were pursued as part of the study of the Passamaquoddy Tidal Power Project, under the direction of the Eastport, Me., District of the U. S. Corps of Engineers; Maj. Gen. E. M. Markham, U. S. Army, Chief of Engineers, U. S. Army, M. Am. Soc. C. E.; Lieut. Col. Philip B. Fleming, U. S. Corps of Engineers, was District Engineer, with Captain Hugh J. Casey, U. S. Corps of Engineers, M. Am. Soc. C. E., as Chief of the Engineering Division; John Sweeney, M. Am. Soc. C. E., was in charge of field exploration; and the writer, assisted by R. M. Haines, Jun. Am. Soc. C. E., conducted the Soils Laboratory studies.

SETTLEMENT OF STRUCTURES IN EUROPE AND METHODS OF OBSERVATIONS

By Charles Terzaghi,[23] M. Am. Soc. C. E.

Synopsis

The methods and results of settlement observations on several structures in Europe, selected to illustrate accurately, the behavior of structures on various types of compressible ground, are described in this paper. Particularly, emphasis is given to the observations of settlements of buildings on pile foundations, and the comparison between the settlement of individual piles under load tests with that of the entire pile foundation.

The form of the settlement diagrams with ground conditions and loadings is recommended as a pattern to be followed. The instruments and bench-marks used for settlement observations are described, but other instruments and methods of observation will suggest themselves to engineers. However, such methods should permit the observation of very small settlements as they are valuable in computing future settlements.

The observations are the result of studies extending over a 10-yr period; they were supervised by the writer and supported by contributions made by the Committee of the Society on Earths and Foundations.

Purpose of Investigations

The design of shallow foundations is governed by the condition that the greatest pressure exerted on the soil should not exceed a certain value, called the allowable soil pressure. If the structure is to rest on a pile foundation, the load per pile should not exceed the "safe load." Both types of foundations are subject to settlement. If the loaded stratum consists exclusively of sand or gravel, the settlement ceases within a few weeks or months after construction is finished. In this case the settlement can be expressed by a single set of recorded data or by a contour map showing the lines of equal settlement representing the subsidence of the loaded area after the settlement stopped. On the other hand, if the settlement is due to an increase of the load on a bed of clay, it can be recorded only by time-settlement curves, or by means of a set of contour maps showing the settlement at different times after construction was finished.

At present, there is no method of predicting the settlement of buildings on a sand or gravel foundation and the prospects for discovering such a method are

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Note.—The Committee of the Society on Earths and Foundations, selected the settlement of structures as one of several for study and research. This paper, reporting an investigation sponsored by the Committee, was submitted to the Committee by its author, and the Committee has recommended its publication as a part of this Symposium.

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very slight. No method of computing settlement can be accepted unless its validity is demonstrated by numerous settlement records obtained under very different practical conditions and no such collection of records has ever been presented. Current procedures for predicting the settlement of buildings on clay foundations are more promising; and, yet, according to reports,[24] the application of these methods requires elaborate investigations and is limited to important structures. In every other case the engineer must estimate the settlements on the basis of previous experience. Such experience can be acquired only by systematic settlement observations. Considering the present state of knowledge in the field of foundations, the settlement observations represent by far the most important type of research. The results of theoretical and of laboratory investigations in this field cannot claim more than an academic value until the importance of the errors have been investigated thoroughly and repeatedly. Any proposal for the practical application of a new method should include not only a detailed statement of several settlement forecasts, but also the results of adequate settlement observations, including the magnitude, the distribution, and the time rate of the settlement for the full-sized structures.

METHOD OF OBSERVATION

In his earlier attempts to collect settlement data, the writer met with the following difficulties: (*a*) Most of the reference points (which consisted of bolts embedded in the masonry) disappeared during or immediately after construction; (*b*) in buildings with numerous partition walls the measurements within the building were difficult and not sufficiently accurate; and finally (*c*) the number of observation points was not great enough to permit the reliable construction of curves of equal settlement. In order to eliminate these difficulties the writer devised a new type of reference point shown in Fig. 42. It consists of a short piece of pipe, which is entirely embedded in the masonry (Fig. 42(*a*)). Normally, the opening of this pipe is closed by means of a brass plug, the outer surface being flush with the wall. In order to make a settlement observation, the plug is removed and temporarily replaced by a cylindrical piece such as that shown in Fig. 42(*b*). The observations are made not with a level, but by means of the device shown in Fig. 42(*c*) which consists of two glass tubes connected by a rubber hose. The entire system is filled with water, and in each tube the position of the water level is measured by means of the micrometer screw shown in Fig. 42(*d*). The vertical distances, Z_0, in Fig. 42(*c*) are always the same for both tubes, and the distances, Z_1 and Z_2, are measured by means of the micrometer screw. Hence, the difference between the elevation of the two reference points is equal to $\Delta Z = Z_1 - Z_2$. A careful survey of the errors of observation performed by Professor H. Löschner,[25] in Brunn, Austria, has shown that the error does not exceed 0.05 mm, or 0.002 in.

In order to obtain a reliable conception of the distribution of the settlement over the area occupied by the building, at least one reference point should be established for each 200 sq ft of this area. At least, one-third of all the points should be within the building. All the recent settlement data embodied in the following paragraphs were obtained by means of the device shown in Fig. 42.

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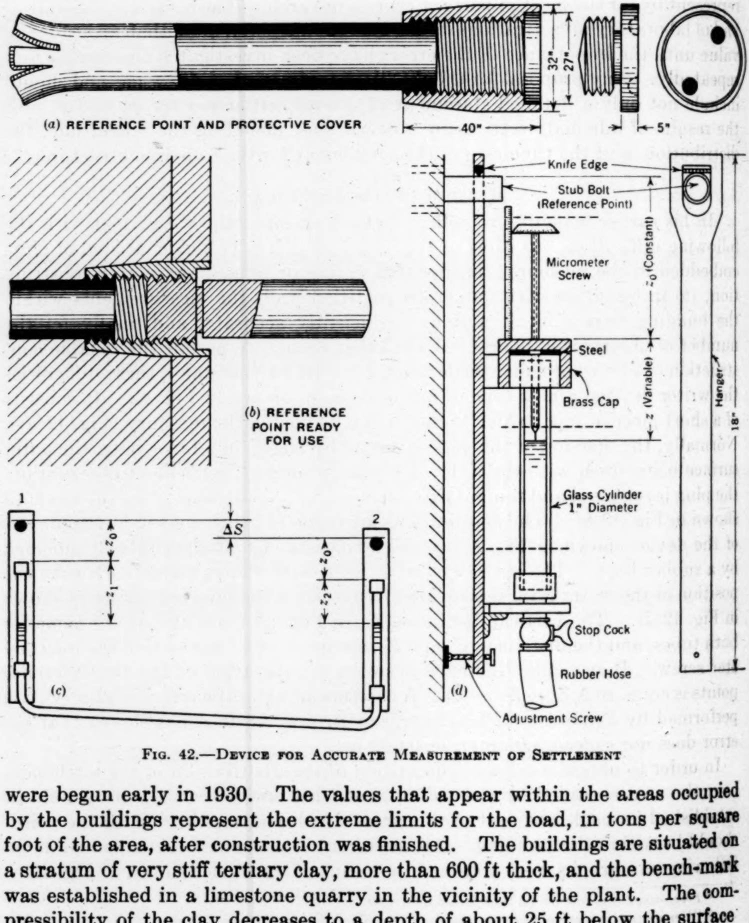
[24] *Proceedings*, Conference on Soil Mechanics, 1936.

[25] "Genauigkeitsuntersuchung zur Messung von Setzungen nach dem Verfahren von Professor Terzaghi," by H. Löschner, *Zeitschrift für Instrumentenkunde*, Vol. 56, July, 1936, Heft 4.

THE BENCH-MARK

In order to obtain reliable information as to the downward movement of a structure, the levels must be referred to a bench-mark that does not move. Usually, this bench-mark is established on some existing building at a distance of 100 or 200 ft from the structure under observation. If no reliable settlement record is available for this building, at least two bench-marks should be established on buildings, located on different sides of the structure under observation.

Fig. 43 shows the curves of equal settlement of a group of factory buildings for the year 1934. Construction was finished in 1929 and the observations were begun early in 1930. The values that appear within the areas occupied by the buildings represent the extreme limits for the load, in tons per square foot of the area, after construction was finished. The buildings are situated on a stratum of very stiff tertiary clay, more than 600 ft thick, and the bench-mark was established in a limestone quarry in the vicinity of the plant. The compressibility of the clay decreases to a depth of about 25 ft below the surface



(*a*) REFERENCE POINT AND PROTECTIVE COVER

(*b*) REFERENCE POINT READY FOR USE

32"
27"
40"
5"
1
2
ΔS
z_0
z_1
z_0
z_2
(c)
(d)
Knife Edge
Stub Bolt (Reference Point)
Micrometer Screw
z_0 (Constant)
Steel
Brass Cap
z (Variable)
Hanger
18"
Glass Cylinder 1" Diameter
Stop Cock
Rubber Hose
Adjustment Screw

FIG. 42.—DEVICE FOR ACCURATE MEASUREMENT OF SETTLEMENT

and the distance between the individual buildings is considerable. Nevertheless, the settlement extends over the entire area occupied by the plant. The contour interval in Fig. 43 is 5 and 10 mm, the equivalent values, in inches, being as follows:

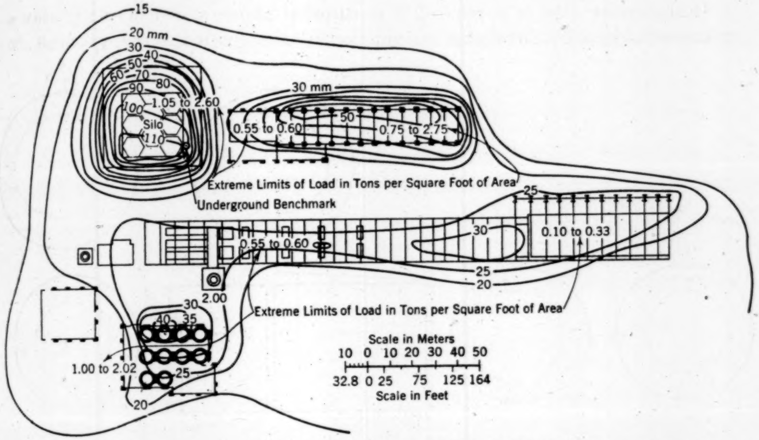

15
20 mm
30
40
50
60
70
80
90
100
110
1.05 to 2.60
Silo
30 mm
50
0.55 to 0.60
0.75 to 2.75
Extreme Limits of Load in Tons per Square Foot of Area
Underground Benchmark
25
30
0.55 to 0.60
0.10 to 0.33
25
20
2.00
30
40
35
Extreme Limits of Load in Tons per Square Foot of Area
1.00 to 2.02
25
20
Scale in Meters
10 0 10 20 30 40 50
32.8 0 25 75 125 164
Scale in Feet

FIG. 43.—OBSERVED CURVES OF EQUAL SETTLEMENT AFTER 4.5 YEARS; GROUP OF FACTORY BUILDINGS CONSTRUCTED IN 1929 (OBSERVATIONS BEGAN IN 1930)

Millimeters	Inches	Millimeters	Inches
15	0.59	60	2.36
20	0.79	70	2.76
25	0.98	80	3.15
30	1.18	90	3.54
40	1.57	100	3.94
50	1.97	110	4.33

Fig. 44 represents the settlement of a group of oil tanks.[26] At each point of the circumference of the buildings the radial width of the shaded area represents the settlement. The diagram shows plainly the influence of the weight of each structure on the settlement of adjacent structures, particularly the units toward the right. The 1932 Progress Report of the Committee on Earths and Foundations contains a brief mention of this case, including a section through the soil.[27]

Due to the inevitable subsidence in the vicinity of the loaded area, the short distance between the bench-mark (Fig. 42) should never be nearer the building under observation than twice the width of the building. If this distance exceeds 100 ft, it is advisable to establish intermediate points of observation. Each set of levels should be carried back to the point where it began and the settlement record should contain the ultimate error. With few exceptions, the observations made by the writer using the device shown in Fig. 42 have been made in rather inaccessible cellars with many partition walls; and yet the ultimate error has never exceeded 0.02 in.

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[26] "Kritische Betrachtung von Flach-und-Pfahlgründungen," von W. Loos, Berlin, Julius Springer, 1932.

[27] *Proceedings*, Am. Soc. C. E., May, 1933, p. 777.

If the entire city or a part of it is situated above a bed of silt or clay a progressive, general, irregular subsidence of the loaded territory must be expected. In Cambridge, Mass., the entire area on both sides of Massachusetts Avenue, settled from 1869 to 1909 (a period of 40 yr) through distances ranging between zero and 2 ft.[28] A similar general subsidence was found to have occurred in San Francisco, Calif., in the district adjoining Market Street. From 1923 to 1928, the greatest subsidence was more than 0.2 ft.[29] There is but little doubt, that a similar general and irregular subsidence occurs in every city built upon beds of clay or silt. If such a general subsidence occurs the foregoing procedure yields data that inform the engineer merely as to the difference between the general downward movement and the downward movement of the newly constructed building. In order to obtain the actual settlement it would be necessary to establish a bench-mark at the bottom of a drill hole which extends through the entire system of unconsolidated strata to bed-rock. Such a bench-mark was established in Cambridge, Mass., at a depth of about 125 ft below the surface. Such bench-marks would also facilitate the collection of data concerning the general subsidence of the area occupied by a city. It should be the duty of municipal engineering departments to keep records of such movements.

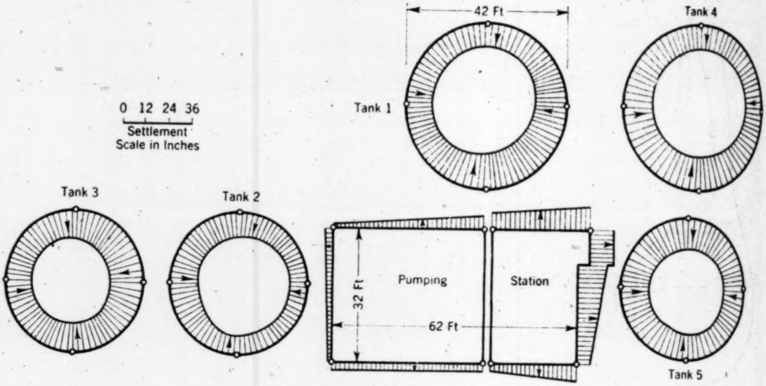

FIG. 44.—SETTLEMENT OF A GROUP OF OIL TANKS

SETTLEMENT OF PILE FOUNDATIONS

Observations made in Vienna, Austria, refer exclusively to foundations on conical, cast-in-place piles with a length of about 20 ft. They are arranged in two or three rows beneath continuous footings. The top-soil stratum usually consists of a loose, loamy, artificial fill and the piles get their support in a stratum of firm sand and gravel or very stiff clay of great thickness. The geological conditions practically exclude the presence of soft strata at any depth below the points of the piles. Before the settlement observations were made, many experienced contractors of the city believed that the settlement of the pile foundations was approximately equal to that produced by a loading test on an individual pile. According to this view, if all the piles are assigned equal loads, the settlement of the entire foundation should be uniform. Observations made during the past few years have destroyed both illusions. Figs. 45, 46, and 47(*a*) illustrate an example of such observation. The building consists of brick walls with continuous footings, 40 in. wide, supported by two rows of conical piles. Each pile carries a load of about 24 tons. The upper curve in Fig. 46(*b*) shows the result of a loading test on an individual pile, located at Point *P* in Fig. 47(*a*). Under a load of 24 tons a conical pile in Vienna very seldom settles more than a fraction of a millimeter during the loading test, and the loading test on the pile at Point *P* merely confirmed this empirical rule; and yet the settlement of the entire pile foundation was equal to more than forty times the settlement of the individual pile carrying the same load during the loading test. It was by no means uniform, although each pile carried the same load. The lower curve in Fig. 46(*b*) shows the settlement of a point, H_g, immediately above the location of the test pile. Fig. 46(*a*) contains the time-load diagram and the time-settlement curve for three typical points. Fig. 47(*a*) shows the curves of equal settlement during a period of six weeks and Fig. 47(*b*) the same curves eleven weeks after the first observations were made.

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[28] "Boston Foundations," by J. R. Worcester, M. Am. Soc. C. E., *Journal*, Boston Soc. of Civ. Engrs., January, 1914.

[29] "Subsidence and the Foundation Problem in San Francisco," San Francisco Section, Am. Soc. C. E., September, 1932.

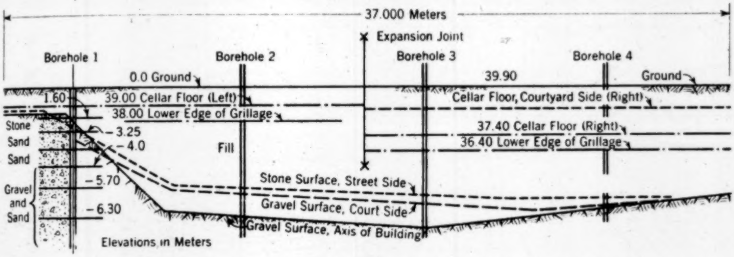

37.000 Meters
Expansion Joint
Borehole 1
Borehole 2
Borehole 3
Borehole 4
0.0 Ground
39.90
Ground
1.60
39.00 Cellar Floor (Left)
38.00 Lower Edge of Grillage
Cellar Floor, Courtyard Side (Right)
Stone
Sand
Sand
-3.25
-4.0
Fill
37.40 Cellar Floor (Right)
36.40 Lower Edge of Grillage
Gravel and Sand
-5.70
-6.30
Stone Surface, Street Side
Gravel Surface, Court Side
Gravel Surface, Axis of Building
Elevations in Meters

FIG. 45.—SOIL CONDITION AT SITE OF A BRICK BUILDING, IN VIENNA, AUSTRIA

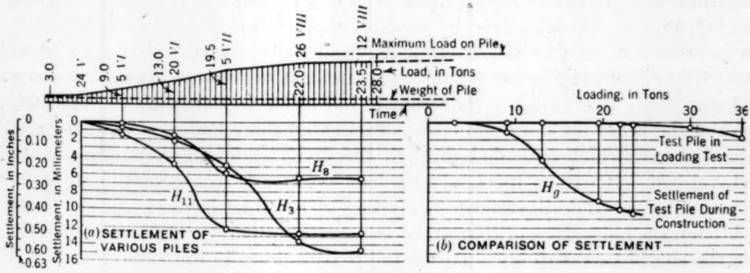
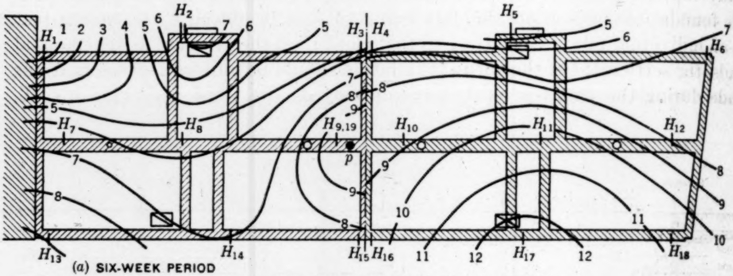


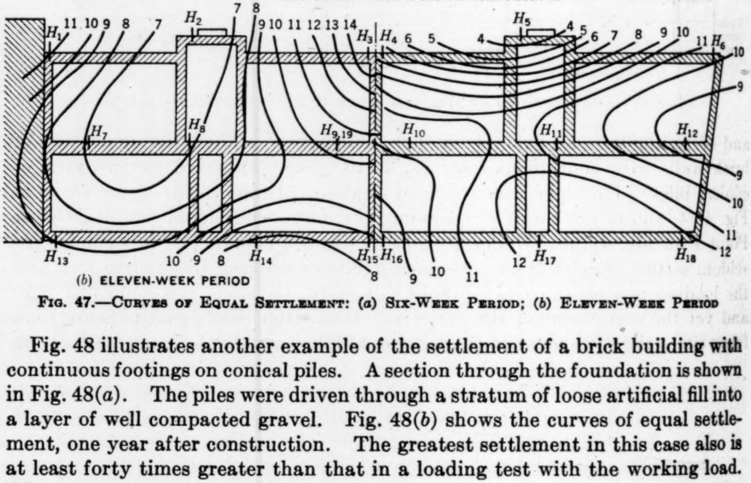
FIG. 46.—TIME SETTLEMENT CURVES

Since the time-settlement curves for various parts of the building are different the shape of the curves of equal settlement changes although the distribution of the load over the loaded area remains practically unaltered.


H_1
1
2
3
4
5
6
H_2
7
6
5
H_3
H_4
H_5
5
6
7
H_6
7
8
8
9
5
H_7
H_8
$H_{9,19}$
p
H_{10}
H_{11}
H_{12}
7
9
8
9
8
9
10
10
11
8
9
10
H_{13}
H_{14}
H_{15}
H_{16}
11
12
H_{17}
12
H_{18}

(a) SIX-WEEK PERIOD

Note: Contour Interval is One Millimeter 0.0394 In.

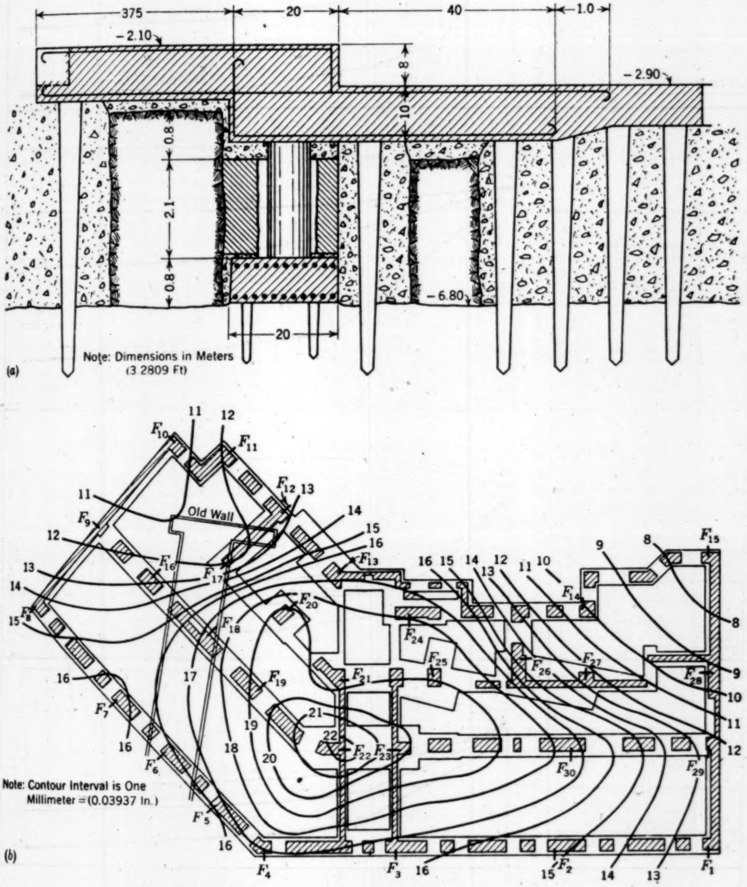

H_1
11
10
9
8
7
H_2
7
8
9
10
11
12
13
14
H_3
H_4
6
5
4
H_5
4
5
6
7
8
9
10
11
H_6
10
9
H_7
H_8
$H_{9,19}$
H_{10}
H_{11}
H_{12}
9
10
11
12
H_{13}
10
9
8
H_{14}
H_{15}
H_{16}
8
9
10
11
12
H_{17}
H_{18}

(b) ELEVEN-WEEK PERIOD

FIG. 47.—CURVES OF EQUAL SETTLEMENT: (a) SIX-WEEK PERIOD; (b) ELEVEN-WEEK PERIOD

Fig. 48 illustrates another example of the settlement of a brick building with continuous footings on conical piles. A section through the foundation is shown in Fig. 48(*a*). The piles were driven through a stratum of loose artificial fill into a layer of well compacted gravel. Fig. 48(*b*) shows the curves of equal settlement, one year after construction. The greatest settlement in this case also is at least forty times greater than that in a loading test with the working load. When the cellar for the building was excavated thick bodies of masonry were encountered which extended through the artificial fill to the surface of the gravel bed. If part of the new walls had been made to rest on top of the old ones, rupture would have been inevitable. Therefore, it was necessary to cut through the old walls and to provide an elastic support over the entire length of the continuous footings. This detail is also shown in Fig. 48(*a*).

Soil mechanics has made engineers realize that the settlement phenomena shown in Figs. 45 to 48 must be expected and that no simple relation can possibly exist between the result of the loading test on the individual pile and the settlement of the entire pile foundation, although the piles get their bearing in a firm stratum of gravel. At the same time, there is very little hope of finding a procedure for predicting the settlement of pile foundations of this particular type. Hence, an estimate can only be based on experience to be obtained by systematic observations on foundations supported by piles, with different lengths, driven to refusal in different types of strata. No general conclusions can be derived from two or three sets of observations. Thus, in both cases, represented by Figs. 45 to 48, the settlements ended not later than about two

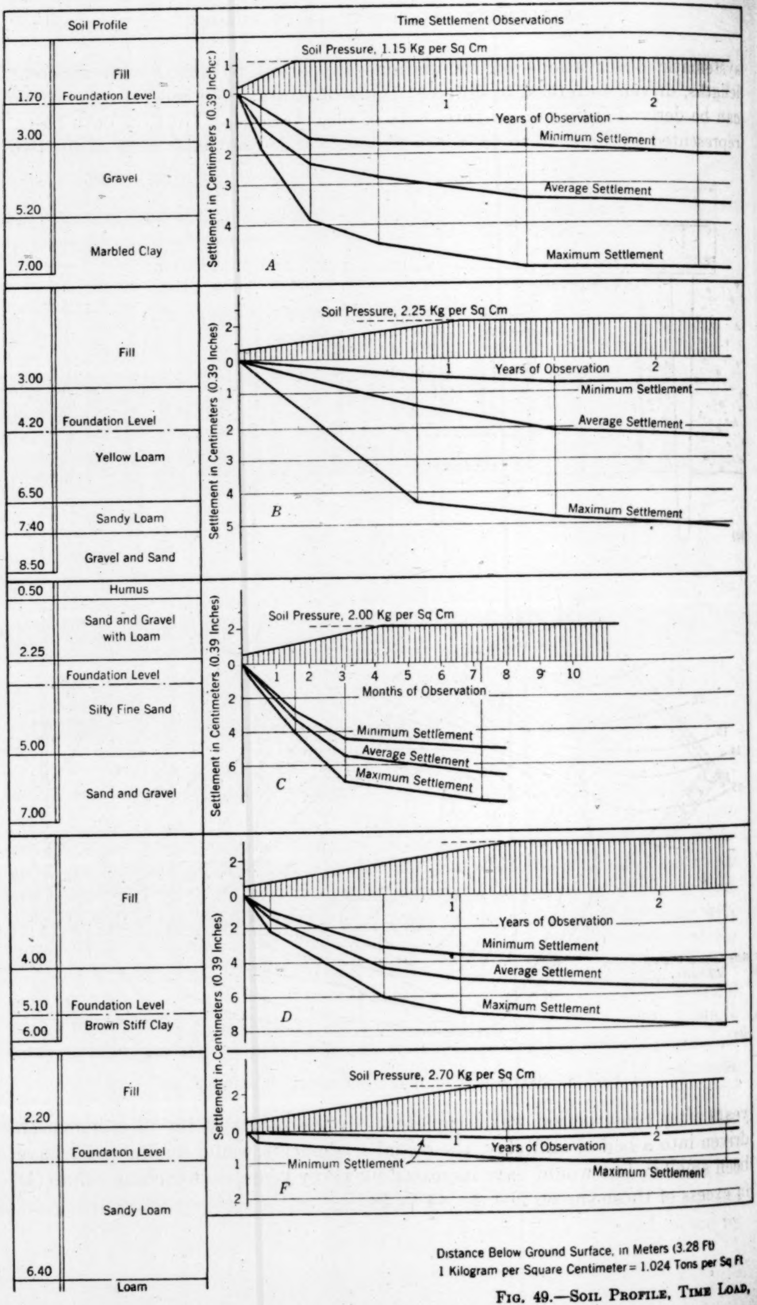

375
20
40
1.0
– 2.10
– 2.90
8
10
0.8
2.1
0.8
– 6.80
20
(a)
Old Wall
F_1
F_2
F_3
F_4
F_5
F_6
F_7
F_8
F_9
F_{10}
F_{11}
F_{12}
F_{13}
F_{14}
F_{15}
F_{16}
F_{17}
F_{18}
F_{19}
F_{20}
F_{21}
F_{22}
F_{23}
F_{24}
F_{25}
F_{26}
F_{27}
F_{28}
F_{29}
F_{30}
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
(b)

Note: Dimensions in Meters (3.2809 Ft)

Note: Contour Interval is One Millimeter = (0.03937 In.)

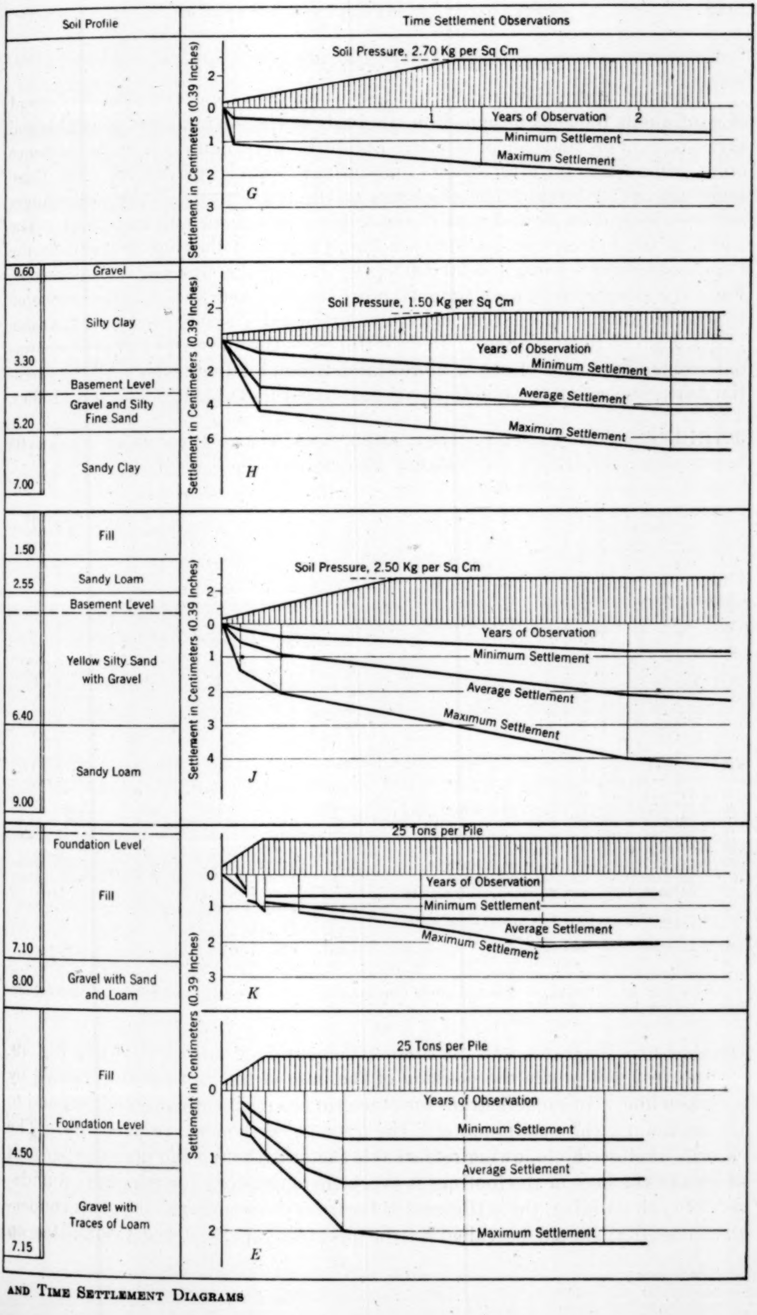
FIG. 48.—SETTLEMENT OF BRICK BUILDING ON CONICAL PILES

years after construction was completed. If the points of the piles had been driven into a bed of stiff clay, the initial settlements would undoubtedly have been smaller, but would have increased for many years, approaching values far in excess of those shown in Figs. 45 to 48.



Distance Below Ground Surface, in Meters (3.28 Ft)
1 Kilogram per Square Centimeter = 1.024 Tons per Sq Ft

FIG. 49.—SOIL PROFILE, TIME LOAD,



AND TIME SETTLEMENT DIAGRAMS

Settlement of Continuous Footings

It seemed interesting to collect at least some data concerning the settlement of continuous footings designed on the basis of the customary "allowable soil pressure." For that purpose eight buildings were equipped with reference points immediately after their foundations were constructed. All these buildings are of brick. The thickness of the lower parts of the walls ranges between 20 and 30 in. and that of the upper part between 15 and 20 in. The width of the footings ranges between 2 and 9 ft, and they are covered by the buildings between 4 000 and 20 000 sq ft. One of the structures (Building *A*, Fig. 49) is supported by a continuous slab, extending over the entire area covered by the building. The larger structures are usually subdivided by expansion joints, from 50 to 80 ft apart. In doubtful cases the allowable soil pressure was determined by means of one or more standardized loading tests. All the essential data concerning the buildings are assembled in Table 6. The soil profiles, the time-load diagrams, and the time-settlement diagrams are shown in Fig. 49.

TABLE 6.—Soil Profile, Time Load, and Time-Settlement Diagrams (Brick Buildings)

Building	Settlement, in Inches: Maximum	Settlement, in Inches: Minimum	Settlement, in Inches: Average	Number of observation points	Type of soil *	Soil pressure, in tons per square foot	Total area covered by building, in square feet	Load per unit of total area, in tons per square foot	Allowable Soil Pressure Determined by Loading Tests, in Tons per Square Foot: From:	Allowable Soil Pressure Determined by Loading Tests, in Tons per Square Foot: To:	Width of Foundations, in Inches: Minimum	Width of Foundations, in Inches: Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
A	2.11	0.71	1.26	60	Fill	1.07	20 300	1.07	1.1	1.9	‡	....
B	2.11	0.30	0.99	27	Sand with loam	2.09	15 300	0.84	2.3		53.2§	90.6
C	2.76	1.73	2.16	21	Silty fine sand with loam	1.86	3 910	0.84	1.4	1.5	50.0§	88.5
D	2.34	1.87	2.05	48	Loam and clay	2.51	12 100	1.41	2.3		49.2§	74.8
E	0.87	0.28	0.55	31	Gravel fill	25†	5 360	1.02	..	..	||	....
F	0.51	0.12	0.36	33	Sand with loam	2.32	10 200	0.90	3.7	4.7	47.3§	98.5
G	0.83	0.30	0.55	43	Sand with loam	2.51	6 480	0.93	3.7	4.7	43.4§	67.0
H	2.64	1.06	1.77	40	Marbled, stiff, plastic clay	1.63	16 640	0.96	2.3	3.7	47.3§	82.6
J	1.77	0.36	0.99	180	Loam, and loam with gravel	2.32	91 100	0.84	..	..	§	....
K	0.96	0.26	0.55	19	Gravel fill	25†	3 720	0.93	..	..	||	....

* See Fig. 49. † Buildings *E* and *K* are in tons per pile. ‡ Reinforced concrete slab. § Strip foundation. || Piles driven to gravel.

In every soil profile the position of the base of the footings is indicated by a broken line. In each diagram the three time-settlement curves correspond to the minimum, the maximum, and the average settlement, respectively. The records disclose the following remarkable facts: Although the pressure per unit of area of the base of the footings is everywhere the same for the entire foundation of each building, the settlement is far from being uniform. In this connection attention should be called to the progress report of the Committee on

Earths and Foundations[30] which demonstrates that the settlement produced by a uniform load on a circular area is also far from being uniform. The ratio between the largest and the smallest settlement of the buildings listed in Table 6 ranges from 1.25 to about 7.00. The "allowable soil pressure" for the buildings listed in Table 6 was selected on the basis of a standardized procedure and the thickness of the highly compressible strata never exceeds about 20 ft. Nevertheless, the average, settlement ranged between the wide limits of 0.25 in. and more than 2 in.

In Germany, the foundations for bridge piers are also based on standardized "allowable soil pressures"; and yet since the bridges were built in very different parts of a large country, the thickness of the compressible strata and, as a consequence, the settlement of the structures, varies between considerably wider limits than it does in Vienna. Table 7 contains an abstract of data that have been collected by L. Casagrande.[31] The settlement ranges between almost zero and more than 3 ft. In general, the settlements of the foundations on sand and on boulder clay, were very small and stopped a short time after construction was completed, whereas the important settlement of foundations on clay and silt continued and must be expected to increase for many years to come.

TABLE 7.—SETTLEMENT OF BRIDGE PIERS IN GERMANY

Number of structures (1)	Description of soil (2)	Average Soil Pressure, in Tons per Square Foot		Settlement, in Inches	
		From: (3)	To: (4)	From: (5)	To: (6)
9	Silt	1.1	2.0	8	40
8	Clay, loess, loam, etc.	1.1	2.6	2	8
31	Boulder clay, sand or gravel with clay	2.5	4.0	0	0.8
21	Sand, gravel	1.5	3.0	0	0.4

The preceding statements have demonstrated that the limitation of soil pressure to the so-called "allowable" values by no means involves the limitation of the settlement to a definite value. Experience in estimating settlement can only be acquired by means of reference points and levels.

DIFFERENTIAL SETTLEMENT

If a building settles uniformly, no matter how much, the stresses in the members of the structure are not affected. Therefore, it was interesting to investigate the differential settlement of continuous footings which were designed on the basis of equal loads per pile, or of a uniform soil pressure. In Fig. 50(*a*), Curve 1 represents the settlement of the straight front wall of the building discussed in connection with Figs. 45 to 47; Curve 2, the settlement of the rear wall; and Curve 3, that of the middle wall.

Fig. 50(*b*) shows the distortion of the walls of the building represented by Fig. 48. Curve 1 corresponds to the front wall of the building, Curve 2, to the outside wall toward the court, and Curve 3 refers to the wall that runs parallel to the front wall through the interior of the building. In both cases (Figs. 50(*a*) and 50(*b*)) the continuous footings were supported by almost equidistant piles in such a fashion that every pile carried practically the same load.

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[30] *Proceedings*, Am. Soc. C. E., May, 1933, p. 812, Case F.

[31] "Setzungsbeobachtungen an Brückenbauten der Reichsautobahnen," von L. Casagrande, *Proceedings*, International Assoc. for Bridge and Structural Eng., Berlin, 1936.

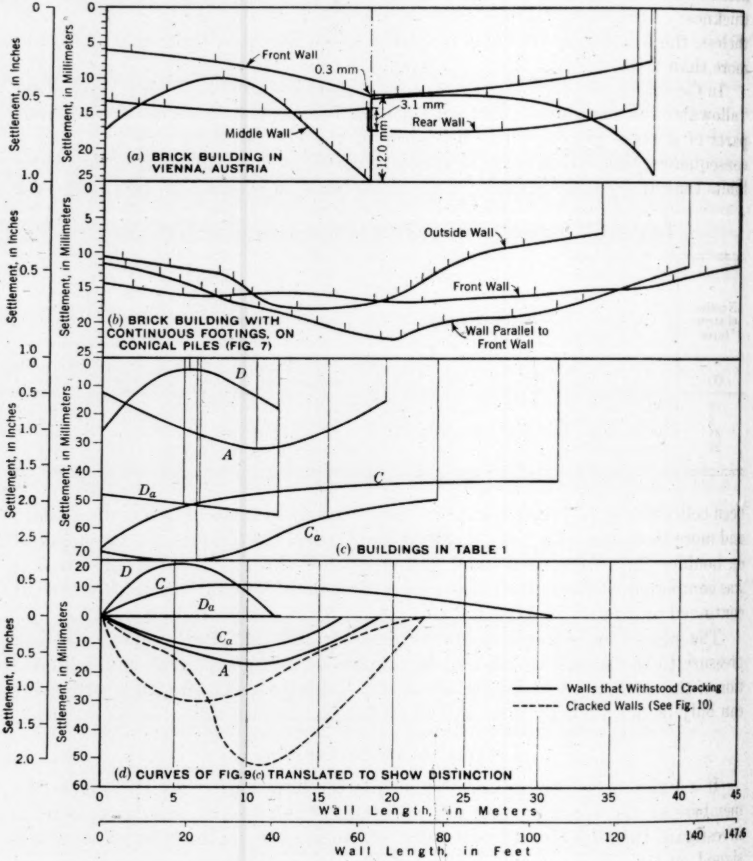

FIG. 50.—STUDY OF WALL SETTLEMENT

Fig. 50(*c*) shows the settlement of the continuous footings of the walls of the buildings listed in Table 6 and of several other buildings. These footings are supported by the natural ground without the assistance of piles, and they were designed in such a fashion that the pressure per unit of area of the footing is uniform throughout. Fig. 50(*d*) was obtained from Fig. 50(*c*) by translating the two ends of each curve up to the horizontal axis. As a consequence, Fig.

50(*d*) shows the distortion of the walls produced by differential settlement. All the walls represented by full black lines withstood the deformation without cracking. As a supplement to this diagram, two dotted curves were introduced. They represent the distortion of two of the walls of the building shown in Fig. 51 which is supported by a massive concrete slab, 4 ft thick. The distortion of these walls is greater than a brick wall can stand, as evidenced by the shearing cracks.

The facts represented in Fig. 50 lead to several interesting conclusions. Contrary to what many engineers still believe, settlement is never uniform, although the load per pile or the load per unit of area of the soil is the same everywhere. In none of the diagrams (except Fig. 48) do the curves of equal settlement have any resemblance to what one would expect from the theory of the settlement of loads supported by a homogeneous medium. Therefore, the differential settlement of the buildings represented by the diagrams is essentially due to local variations in the compressibility of the loaded strata. The importance of these variations cannot possibly be appraised by preliminary investigations of any kind, at a reasonable expense. Hence, an estimate of the differential settlement must be based on previous experience and this experience can only be acquired by systematic settlement observations such as those described herein. However, experience has shown that the erratic character of differential settlement is limited to those cases in which the thickness of the highly compressible strata does not exceed 20 to 25 ft. The greater the thickness of the highly compressible strata, the smaller becomes the difference between the real differential settlement and the settlement corresponding to a uniform and homogeneous medium.

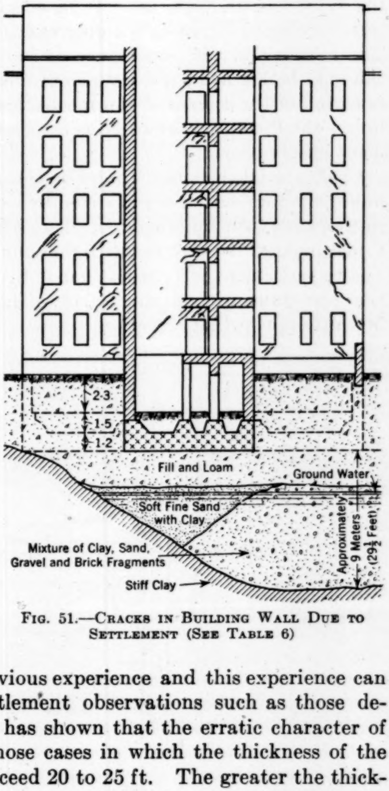
2·3
1·5
1·2
Fill and Loam
Ground Water
Soft Fine Sand with Clay
Mixture of Clay, Sand, Gravel and Brick Fragments
Stiff Clay
Approximately 9 Meters (29½ Feet)

FIG. 51.—CRACKS IN BUILDING WALL DUE TO SETTLEMENT (SEE TABLE 6)

For a given degree of non-uniformity of the subsoil, the importance of the differential settlement decreases with increasing stiffness of the walls. Thus, in the buildings discussed in connection with Figs. 45 and 48, the front wall (lower boundary of the area occupied by the building) is straight and the corresponding distortion, shown by Curve 1 to Fig. 50(*a*) is insignificant. On the other hand, the rigidity of the rear wall (upper boundary of the building lot in Figs. 45 and 48) is reduced by several corners. Therefore, the differential settlement shown by Curve 2 in Fig. 50(*a*), is far more important. A comparison of Figs. 48 and 50(*b*) indicates similar relations between the rigidity of the walls and the importance of the differential settlement. The settlement of the stiff wall, 1, is far more uniform than that of Wall 3, which, in plan, represents a broken line.

Any theoretical computation of settlements is based on the assumption that the subsoil is uniform, at least in every horizontal direction. The non-uniformity always increases the differential settlement and its detrimental effects. The nature and the importance of the difference between theory and reality can only be learned from settlement observations. Therefore, a practical application of the science of soil mechanics to settlement problems is not feasible unless the theoretical research is associated with, and supplemented by, settlement observations.

Differential settlement must be considered inevitable for every foundation, unless the foundation is supported by solid rock. The effect of the differential settlement on the building depends to a large extent on the type of construction. Consequently, it is necessary to find out by careful inspection of buildings with known settlement records how much distortion the different types of construction can stand without any harm. The data shown in Fig. 50(*d*) represent a first attempt in that direction.

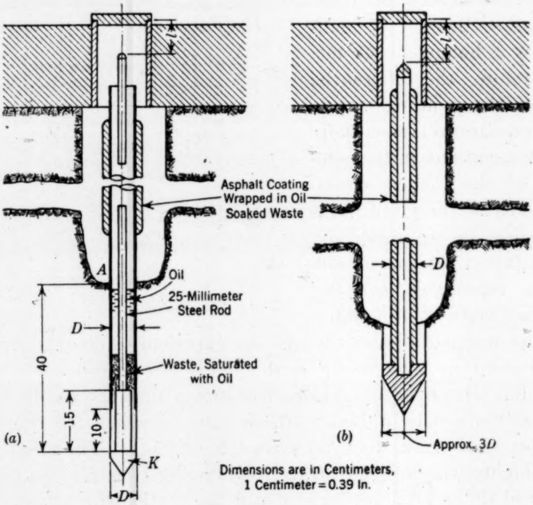
l
Asphalt Coating Wrapped in Oil-Soaked Waste
A
Oil
25-Millimeter Steel Rod
D
40
Waste, Saturated with Oil
15
10
(a)
K
D
l
D
(b)
Approx. 3D
Dimensions are in Centimeters, 1 Centimeter = 0.39 In.

FIG. 52.—UNDERGROUND BENCH-MARK USED IN DETECTING THE COMPRESSION OF STRATA AT VARIOUS DEPTHS

UNDERGROUND BENCH-MARKS

If it is desired to keep the settlement of a structure within specified limits, the foundation must be carried at least to a certain minimum depth which depends, among other factors, on the nature and the thickness of the soil strata beneath the building site. Similarly, if the settlement of an existing building is to be stopped by underpinning, such underpinning must be extended to some level below the seat of the settlement. The science of soil mechanics has furnished a set of theoretical rules for determining the distribution of the settlement over a vertical section through the loaded soil; and yet no such rules can be accepted for practical application on a broad scale until they have been confirmed by the results of observations on existing structures. Such observations can only be made by means of underground bench-marks; that is, by means of reference points established at the bottom of drill holes, at different depths below the surface of the ground.

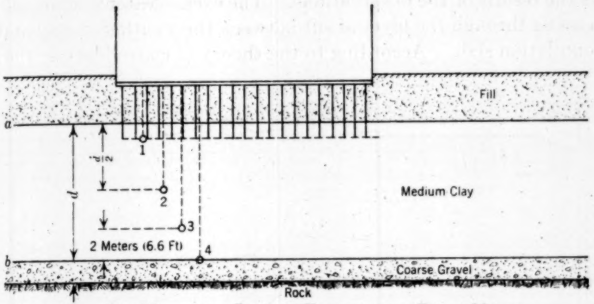

FIG. 53.—METHOD OF PLACING UNDERGROUND BENCH-MARKS

A description of reference points of the type was presented by the writer[32] in 1930. Figs. 52 and 53 describe these reference points and show how to establish them for the purpose of ascertaining the seat of the settlement of a pile foundation above a bed of medium clay. A method for constructing such bench-marks in very soft clay was described in 1936 and used in the City of Mexico, Mexico, by José A. Cuevas.[33] An underground bench-mark similar to that shown in Fig. 52(*b*) was established beneath one of the corners of the building marked "Silo" in Fig. 43, at a depth of 15 ft beneath the base of the foundation. As shown in Fig. 54, the building rests on a reinforced concrete slab strengthened by ribs. To a depth of about 8 ft below the base of the slab the soil consists of a yellowish, weathered, rather compressible clay. Beneath it

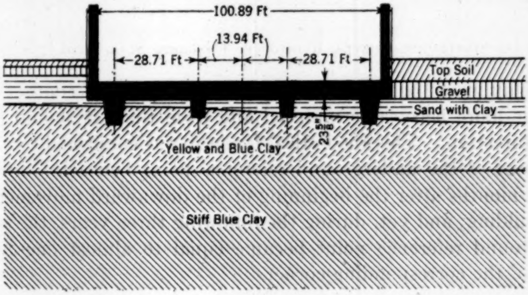

FIG. 54.—REINFORCED CONCRETE SLAB WITH RIBS, OF BUILDING (SEE FIG. 43)

is a very thick bed of stiff, blue clay. The underground reference point served the purpose of determining the extent to which the compression of the stiff blue clay was responsible for producing the total settlement of the structure. Fig. 55 shows the results of the observations. The excess water is squeezed from the clay, escaping through the layer of silt between the weathered clay and the base of the foundation slab. According to the theory of consolidation, the compression of the clay would be expected to begin at the point of contact between the clay and the more permeable material and from there it should proceed in a downward direction. This theoretical forecast was confirmed by the fact that the settlement of the underground reference point beneath Corner 44 began about eight months after the beginning of the downward movement of Corner 44 of the building; and yet, during the following three years, the settlement of the underground reference point became equal to almost one-half the total settlement of the corresponding corner.

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[32] "Die Tragfähigkeit von Pfahlgründungen," von Charles Terzaghi, *Die Technik*, 1930, Heft 31 und 34.

[33] *Proceedings*, International Conference on Soil Mechanics, Paper No. 5, Vol. 1, Cambridge, Mass., 1936.

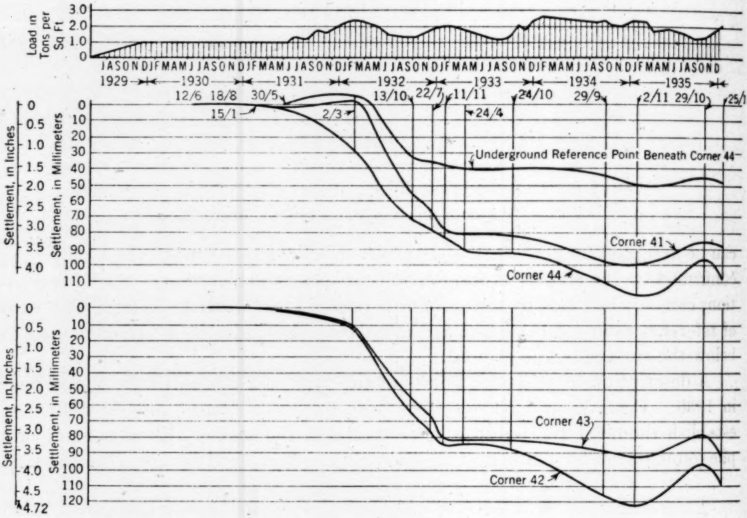

FIG. 55.—LOAD SETTLEMENT CURVES, REINFORCED CONCRETE SLAB, WITH RIBS (SEE FIG. 54)

SUGGESTION CONCERNING STANDARDIZATION

No method of soil investigation and of settlement computation can be accepted for practical use until the degree of accuracy of the results has been determined by experience. Such experience can be secured only by reliable and adequate settlement observations. Therefore, the first and the most urgent task in the establishment of a science of soil mechanics consists in standardizing the procedure for making settlement observations.

DISCUSSION

George E. Beggs,[34] M. L. Enger,[35] R. J. Fogg,[36] Harry T. Immerman,[37] D. P. Krynine,[38] F. A. Marston,[39] George Passwell,[40] Ralph R. Proctor,[41] Charles Terzaghi,[42] and Lazarus White,[43] Members, Am. Soc. C. E., and Glennon Gilboy,[44] Assoc. M. Am. Soc. C. E. (by letter).—Realizing that it is most important to obtain exact settlement data for existing structures, together with the underground conditions at the various sites, the Committee of the Society on Earths and Foundations has been financing, for the past years, an extended investigation by Charles Terzaghi, M. Am. Soc. C. E., mostly in Europe and, in part, also in the United States. These investigations have been valuable as a check on existing theories. In some cases quite close agreement has been found between computed and observed settlement, by the use of the Boussinesq equations to obtain internal pressures below the footing, and Professor Terzaghi's equations of consolidation as determined by compression based on undisturbed samples.

Professor Terzaghi has devised a graphic form of settlement curve and a diagrammatic method indicating conditions in the foundation and underground which is compact and complete. He calls the compressible strata beneath the settling structure the " seat of settlement " and the shape assumed by the base of the structure " the settlement trough ". He calls attention to the over-emphasis previously made on the pressure at the base of the footings; and he demonstrates by observation and calculation that most of the observed settlements are due to compression of the unconsolidated materials within the mass of the supporting underground; and that the " seat of settlement " may extend hundreds of feet in depth.[45]

At the request of the writers, Professor Terzaghi has kindly prepared a summary of his work on settlements with particular attention to the methods of obtaining accurate observations, and giving typical cases with a non-mathematical discussion.[46] Particular attention is called to the failure to obtain uniform settlements, even when uniform loading of walls, etc., was closely approximated, both in the case of pile foundations and spread footings.

The writers believe that this contribution is of great value. They hope that it will stimulate discussion, and are particularly desirous that other cases of observed settlements be submitted, with complete data.

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[36] Cons. Engr., Washington, D. C.

[37] Chf. Engr., Spencer, White & Prentis, Inc., New York, N. Y.

[38] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

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[41] Field Engr., Dept. of Water and Power, City of Los Angeles, Los Angeles, Calif.

[42] Dr. Ing.; Prof., Technische Hochschule, Vienna, Austria.

[43] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

[44] Assoc. Prof., Soil Mechanics, Mass. Inst. Tech., Cambridge, Mass.

[45] *The Structural Engineer*, London, England, March, 1935.

[46] See p. 1432.

S. C. Hollister,[47] M. Am. Soc. C. E. (by letter).—Mr. Hough is to be congratulated on the development of the model tests, reported in his paper, by means of which he was able to determine qualitatively the nature of settlement of the rock fill and the underlying clay bed.

Several points occur to the writer in regard to the determination of the safety of such a construction. The first of these relates to the fact that the rupture surface lies almost entirely within the bed material. This condition is characteristic when the material in the embankment has a high shear strength in comparison to the shear strength of the foundation material.

The second point refers to the stability of such a structure. When the rock fill has been completed and movement of the clay ceases, there exists at that time a factor of safety of one. This means that slight additional loading of the fill would cause a resumption of flow in the clay bed. If the rock fill came to rest in part upon the rock stratum below the clay, stability would be greatly increased. Furthermore, if the mud wave were counterweighted, stability again would be increased.

The "heart-shaped" failure surface, now that it is noticeable by the tests, is discernible from the theory of elasticity. A complete solution of the problem shown in Fig. 38, in which the foundation bed is loaded not only vertically but tangentially, is much needed.

Theodore T. Knappen,[48] M. Am. Soc. C. E. (by letter).—In the light of his personal experience with levee construction in this region, the writer desires to congratulate Mr. Buchanan on his accurate presentation of the Mississippi River levee problem. The soundness of the old design methods is amply demonstrated by the success of the Mississippi levee system in the 1937 flood, just as the soundness of the work of the Old Master Builders is demonstrated by the many ancient buildings that are standing to-day. Precisely as modern structural design has modified building practice, so will the rational methods of soil mechanics, under the leadership and guidance of experts, modify levee construction and gradually transform it from an art to a science.

Although it is true that most of the work on main line levees has been completed, there is still much to be done to provide an adequate factor of safety at the weak places in the existing system and to construct the tributary extensions. It is hoped that there will be sufficient infiltration of the principles expounded by Mr. Buchanan into the various design staffs to insure the application of rational design methods to the work yet to be done. Their best application is in the hands of these engineers who have, for years, carried on the design activities on the river. Familiar as they are with the ground conditions, they can apply the rational design methods with the conservatism that is necessary where protection of life and property is involved.

The ideal condition would permit wide variations in design resulting in a uniform factor of safety. Practical conditions and limitations of knowledge and its dissemination make the attainment of this condition a remote object to be approached gradually. The writer's comments are directed at the elucidation of a few of the factors discussed with the aim of clarifying some of the basic principles involved.

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[47] Dean, Coll. of Eng., Cornell Univ., Ithaca, N. Y.

[48] Associate, Parsons, Klapp, Brinckerhoff, and Douglas, New York, N. Y.

First, examine the standard Mississippi River sections described as *A*, *B*, and *C* in Table 1. The *A* section is for a material containing 75% or more of clay, defined on the river as material finer than 0.005 mm. Although this section has never been discarded it was early learned by the field engineers that levees, even of moderate height (15 to 20 ft), built of material in this classification, were not stable with slopes as steep as 1 on 3. On the other hand, such materials are so impervious that seepage never reaches the down-stream slope except through pervious foundations. Actually, with such materials both slopes should be the same, assuming foundations of similar material.

Now, it will be noted that a river-side slope of 1 on 5 is provided for the *C* sections. On analysis most of these materials will be stable with a slope of about 1 on 2, but in actual experience wave wash has proved so troublesome with the usual fine cohesionless material of this class that this slope is a practical necessity unless slope paving is used. Most materials in this class are sufficiently pervious to establish the normal seepage gradient during a flood. To provide stability without under-drainage the down-stream slope is, properly, quite flat. Greater economy and stability might be obtained by the use of drainage provisions, largely through selection of available materials.

Most of the levees are actually of the intermediate or *B* classification and the levee dimensions indicated are usually more than adequate; but they have limited or inadequate safety factors in a few cases, particularly as one limit or the other is approached.

Mr. Buchanan has discussed the method used in foundation design where a comparison of stress versus strength is used. He has indicated the limitations of this method. More emphasis might well be placed on the photo-elastic method of stress analyses in particularly difficult cases.

In analyzing embankment stability Mr. Buchanan has used the sliding-surface method. The writer is inclined to believe that the inaccuracies of this method are such that it would be better to develop the stress-strength relationship in a manner similar to foundation design practice. The photo-elastic method offers one approach to this problem. Incidentally, where embankment and foundation materials are both of questionable strength the two analyses should not be separated but should be treated as one problem.

It has been the practice for a number of years to provide for the installation of settlement gages (locally known as "crow's feet") as a means of measuring foundation settlement for payment purposes. These gages offer an excellent means of detecting incipient foundation failure. Where foundation investigations have been made in advance an estimate of the rate of consolidation can be made and compared with actual settlement. If excessive settlement develops, incipient failure is indicated. In such a case immediate revision of the design to prevent failure should be made. Even where a foundation analysis has not been made the behavior of these gages offers a good indication of trouble to an experienced inspector.

A common indication of incipient foundation failure is an upheaval of the ground surface beyond the embankment toes. Prior to failure this movement usually is small; but it is frequently visible to the eye and can be detected in most cases by instrumental measurements. By the establishment of hubs in the critical reaches in advance of construction a check may be maintained on the foundation condition. Likewise, embankment slides are indicated in advance by the bulging of embankment slopes. Instrumental checks can detect such conditions in sufficient time to permit corrective measures to be taken.

It is realized that many of the construction methods permitted on levee work do not lend themselves to the application of these methods of detection of probable failures. Some limitation on the method of construction in such cases may be advisable. Based on present-day hauling methods and costs there is no reason to believe that these limitations will result in materially increased costs.

With respect to the discussion of levee construction methods in the paper, it might be argued that an increase in cost of 4 cts to 6 cts per cu yd for compaction would be more than offset by economies in section design. However, it should be remembered that most of the levee work was executed before present-day knowledge of soil mechanics had advanced to the point where it could evaluate the improvement. It is the writer's opinion that the general solution was sound for the time that it was made, but that in the future it should be modified in the light of recently acquired knowledge.

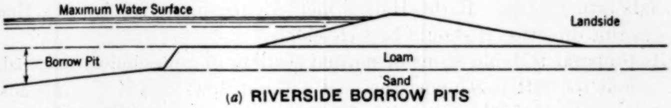
One of the most troublesome problems in levee planning is borrow-pit location. Mr. Buchanan points out that considerations of economy require that the material be obtained opposite, and as close as possible to, the proposed levee. He further states (see heading, "The Design of a Levee Unit") that,

> "It is important that no borrow-pits or excavations be made adjacent to the levees on the land-side to expose pervious strata extending through to the river side. Exposure of such strata invites trouble from increased underseepage, certainly; and from destruction of the levee structure due to piping, possibly. Although in some instances, borrow-pits have been made on the land side, they are the source of never-ending trouble."

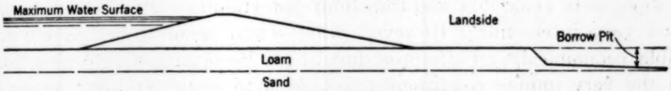
This policy has resulted in the use of river-side pits almost exclusively in recent years. These pits are separated generally from the levee toe by a 40-ft berm. At the edge of the berm they are 3 ft deep and deepened from 2 ft to 4 ft in every hundred depending on the specification in each case. Below the mouth of the Red River slopes of 10 ft in 100 are permitted. One of the greatest fields for improvement in levee design lies in the application of the principles of soil mechanics to borrow-pit location and design in connection with the hydraulic design of levees.

The Mississippi levees are built on, and of, alluvial soils. This means that almost every combination of sands, loams, and clays may be encountered. In some cases deep foundations of relatively impervious loam or clay exist. In such cases the only restrictions on borrow-pit depth need be stability and future enlargement considerations. More frequently the foundations are stratified, making a careful analysis of the problem most important. Often there is a layer of loam overlying a deep sand stratum as in Fig. 56(*a*). In this case the river-side borrow-pit cuts through the loam exposing the sand, which may be one hundred times as pervious as the loam. Here the head acting through the pervious sand on the loam stratum causes boils or piping at the land-side toe of the levee. This offers an explanation for many of the worst troubles on the levee system. In time the borrow-pit will silt up in the small annual floods and the trouble tends to become less in succeeding large floods.

Now, reverse this condition and put the borrow-pit on the land side as in Fig. 56(*b*), taking care to excavate to the sand. The loss of head then occurs to a large extent in downward flow on the river side where it can do no harm.



(*a*) RIVERSIDE BORROW PITS



(*b*) LANDSIDE BORROW PITS

FIG. 56.—TYPICAL LEVEE CROSS-SECTION

In this case the land-side pit is preferable. Now, land-side specifications only permit a deepening of 1 ft in 100; so the pit in this case would have a thin covering of loam over the sand. The result would be boils in the pit which would look bad although they would not be serious.

There are, of course, many other combinations. One would be the case of the pervious sand foundation overlying an impervious stratum like the case cited at Mound's Landing, where the treatment of a clay blanket extending to the impervious stratum was excellent. There are many other known weak spots in the system where a soil mechanics analysis should develop a solution whether it be by cut-off, blankets, or drainage. The important consideration is that Mr. Buchanan's studies should be extended to the solution of these problems. The present rigid borrow-pit requirements should be modified by individual design studies based on the exploration borings described in the paper.

L. F. HARZA,[49] M. AM. SOC. C. E. (by letter).—The need of a reliable basis of stability analysis for foundations under earth embankments is very great indeed. Much has been written and much research has been conducted applicable to selection of suitable embankment materials and their compaction to optimum density and "stability."

All this refinement in the embankment itself, however, amounts to little unless the foundation or underground is equally stable, or unless means exist for interpreting laboratory tests of foundation material into degree of stability under conditions of loading. Experience at Zanesville, Ohio, and elsewhere clearly indicates that knowledge and technique in building artificial embankments has progressed far beyond knowledge of whether a foundation will properly support the embankment when built. In general, the foundation must be accepted as it exists, and it is likely to be by far the weaker and more uncertain element of the structure, subject to much greater settlement and seepage.

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[49] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

Any foundation settlement which tends to cause cracks or fissures in the superstructure invites failure in the embankment. Data and methods of analysis should eventually become available for indicating the degree of flexibility of embankment sections and profiles for accommodating themselves to foundation settlement without cracking and for predicting such settlement from laboratory tests. If the Haines method, recommended by Mr. Hough, tends in this direction it should be welcomed.

His proposal to build dams of normal section, of cohesionless material, on foundations too soft to support the load without flow, in lieu of dams of such flat slopes as to avoid flow and thus limit distortion to vertical compression, is indeed very interesting. However, what might seem to be an obviously feasible, economical, and safe procedure at the Passamaquoddy project might, with the very unique conditions which seem to justify it there, never find another similar application.

It is scarcely conceivable that the new fill, in adjusting itself to the outflow of the soft foundation, could settle without cracks and fissures. Moreover, where the new fill is required to sag in profile, the bottom also tending to spread with outflow of soft foundation, invisible fissures in the bottom are probable, resulting in arching cavities in the fill. No cohesionless material is so fluid as to readjust itself without initial arching voids.

In the Passamaquoddy submerged fill, water could be allowed to flow back and forth through the pervious fill with each change of the tide until the structure is thoroughly settled and these cracks thus closed before applying the less pervious blanket. Failure at this stage could not occur. In a normal fill above water level, however, the final necessary readjustment of the fill to settlement could probably not be accomplished without filling the reservoir. Any distortion sufficient to accomplish such readjustment would seriously endanger the safety of the fill during the readjustment period if the reservoir were thus filled.

To be successful, the scheme would seem to be dependent upon conditions in which anticipated settlement would be practically completed during construction. It is doubtful whether this could be done except in a submerged dam where water would be available for settlement without unbalanced pressure as at Passamaquoddy.

EDWARD ADAMS RICHARDSON,[50] ESQ. (by letter).—This discussion of the paper by Mr. Hough will touch upon three subjects: (1) The proper stress to choose for the limiting shear in the case of plastic materials; (2) factors affecting the computation of the factor of safety of the foundation; and (3) methods for improving the design of such foundations as suggested by the tests and investigations.

In one of his model tests, the author utilized "a bed of uniform, remolded clay" to represent the foundation material. Such remolded clay exhibits materially lower shear resistance than the undisturbed material. It seems improper to base the design of a foundation on the strength of the fully remolded material rather than on that of the undisturbed material.

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[50] Associated with Bethlehem Steel Co., Bethlehem, Pa.

Considerable kneading is required to convert the original clay body to the uniform, fully remolded condition. This is true even in the case of very soft clay. This amount of kneading appears to be greatly in excess of that which may be found in the most stressed elements of material beneath the foundation. Most of the material that flows plastically can scarcely be said to be subjected to appreciable kneading action. Such action is measured by the relative movement or strain of adjacent particles of clay; not by the distance moved by the most traveled point of the plastic body as measured relative to a fixed point on the boundary. That being the case, it would appear that the proper value of the limiting shear stress for the plastic material should be closely that of the undisturbed material. In view of the considerable differences in this limiting stress, it becomes a matter of great importance to choose the value most nearly approximating actual conditions if an economical structure is to be secured.

The factor of safety is computed by the Haines analysis through an adaptation of the method presented by K. E. Petterson, a member of the Swedish Geo-Technical Commission. Due references were made to the investigations by Jurgenson[51] and Professor Dr. H. von Krey. In using similar methods, as in the case of sloping banks, it has been customary to apply loads to the surfaces of the rotating segment and then to assume that such added loads increase the friction of the sector on the surface of sliding, or at least contribute a friction force along this surface. In the case of dams, the friction force thus calculated may be considerable. It should be noted that a plastic clay segment does not develop any friction force due to such loads until time for consolidation has elapsed. As this time may be long (perhaps nearly as long as the life of the structure) such friction forces should be omitted from the initial stability calculations. It will be recalled that any load added to clay is initially carried entirely by hydrostatic pressure in the contained water and not through particle contacts; therefore, no friction force can be developed by the added load.

An approximation to the curved part of the surface of failure may be obtained in the following manner: The equations for the shearing stresses involved are those for an elastic body of indefinite extent loaded at a point of the surface, and were derived from the corresponding case given by Mr. John Prescott.[52] Such a derivation yields not only the p_z of the Boussinesq theory, but also p_x, p_y, and the shearing stresses.

For reference purposes, the formulas for all stress components at (x, y, z) are given herein, as Equations (11) and (12). Only Equation (11*c*) for p_z is commonly given. Fig. 57 may be used as a diagram, with OY perpendicular to the X–Z plane. Poisson's ratio is $\frac{1}{m}$. Direct stress components are p_x, p_y, and p_z parallel to OX, OY, and OZ, respectively. Shear stress components are s_1, s_2, and s_3, the axes of torque being likewise parallel to OX, OY, and OZ, respectively. That is, the stresses for s, producing a torque component about an axis parallel to OX, may be considered to lie in a plane, containing (x, y, z), which is parallel to the Y–Z plane. The equations are:

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[51] "The Shearing Resistance of Soils," by Leo Jurgenson, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXI, No. 3, July, 1934, p. 242.

[52] "Applied Elasticity," by John Prescott, Chapter XIX, p. 623, Longmans, Green & Co., London, 1924.

$$r = \sqrt{x^2 + y^2 + z^2} \quad \text{(11a)}$$

$$p_x = \frac{P}{2\pi}\left[\frac{r^5 - 2 r^3 y^2 + r^2 z^3 - 2 r^2 x^2 z - 6 r x^2 z^2 - 3 x^2 z^3}{r^5 (r + z)^2} - \frac{2}{m}\,\frac{r^5 - 2 r^3 y^2 + r^2 z^3 + r^2 x^2 z}{r^5 (r + z)^2}\right] \quad \text{(11b)}$$

$$p_y = \frac{P}{2\pi}\left[\frac{r^5 - 2 r^3 x^2 + r^2 z^3 - 2 r^2 y^2 z - 6 r y^2 z^2 - 3 y^2 z^3}{r^5 (r + z)^2} - \frac{2}{m}\,\frac{r^5 - 2 r^3 x^2 + r^2 z^3 + r^2 y^2 z}{r^5 (r + z)^2}\right] \quad \text{(11c)}$$

$$p_z = \frac{P}{2\pi}\left[-\frac{3 z^3}{r^5}\right] \quad \text{(11d)}$$

$$s_1 = \frac{P}{2\pi}\left[-\frac{3 y z^2}{r^5}\right] \quad \text{(11e)}$$

$$s_2 = \frac{P}{2\pi}\left[-\frac{3 x z^2}{r^5}\right] \quad \text{(11f)}$$

and,

$$s_3 = \frac{P}{2\pi}\left[\frac{(2 r^3 - 2 r^2 z - 6 r z^2 - 3 z^3)\, x y}{r^5 (r + z)^2} - \frac{2}{m}\,\frac{(2 r^3 + r^2 z)\, x y}{r^5 (r + z)^2}\right] \quad \text{(11g)}$$

When $y = 0$, or OX is so chosen that the X–Z plane contains r, the case of Fig. 57 is obtained. It should be obvious that the symmetry of point loading is such that all points having the co-ordinates, r, α, will have the same stress components on the plane, X–Z, chosen to contain r; hence Fig. 57 is a cross-section upon which the stress system is fully characterized for the entire body. The corresponding equations are:

$$r = \sqrt{x^2 + z^2} \quad \text{(12a)}$$

$$p_x = \frac{P}{2\pi}\left[\frac{r^5 + r^2 z^3 - 2 r^2 x^2 z - 6 r x^2 z^2 - 3 x^2 z^3}{r^5 (r + z)^2} - \frac{2}{m}\,\frac{r^5 + r^2 z^3 + r^2 x^2 z}{r^5 (r + z)^2}\right] \quad \text{(12b)}$$

$$p_y = \frac{P}{2\pi}\left[\frac{r^5 - 2 r^3 x^2 + r^2 z^3}{r^5 (r + z)^2} - \frac{2}{m}\,\frac{r^5 - 2 r^3 x^2 + r^2 z^3}{r^5 (r + z)^2}\right] \quad \text{(12c)}$$

$$p_z = \frac{P}{2\pi}\left[-\frac{3 z^3}{r^5}\right] \quad \text{(12d)}$$

$$s_1 = 0 \qquad (12e)$$

$$s_2 = \frac{P}{2\,\pi}\left[-\frac{3\,x\,z^2}{r^5}\right] \qquad (12f)$$

and,

$$s_3 = 0 \qquad (12g)$$

By combining point loads and their stresses, it is known that solutions approximating the present case may be obtained. The stresses in the soil will be closely approximated only when the entire soil mass behaves elastically under the applied load and the earth is of indefinitely great depth. The approximation may be good in those cases in which at least a portion of the earth is subjected to shearing stresses beyond the yield point of the material.

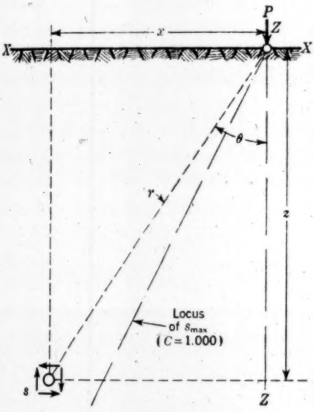

FIG. 57.—LINE OF SEPARATION, ELASTIC AND PLASTIC REGIONS UNDER A POINT LOAD.

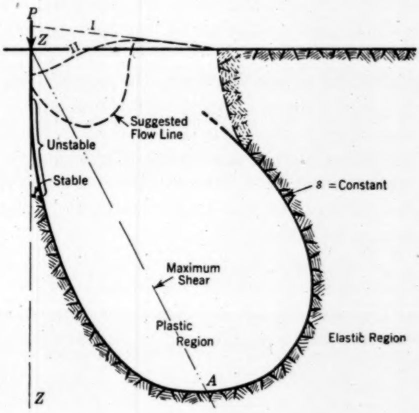

FIG. 58.

In such cases, one may determine a surface of shearing for the limiting shear values from point to point. This surface will set the boundary approximately between the elastically strained material and that which is in the plastic condition. Whether plastic flow does or does not occur will depend upon other factors, such as the restraint opposed to the direct pressures of the plastic portion of the mass. The character of an iso-shear line for a point loading will be seen in Fig. 58. The boundary is determined from a set of these lines for a series of *s*-values. As drawn, however, the boundary is determined by one *s*-value.

When the elastic body is a solid of great cohesive strength relative to the force of sliding friction, the boundary between the elastic and the plastic regions will approximate very closely to the *s*-curve having the value of the cohesive or shearing strength of the material. Treating Fig. 58 as representative of such a material, Curve II suggests the character of surface deformation for elastic behavior, whereas Curve I suggests that for plastic flow when the elastic limit is exceeded. These curves are purely diagrammatic.

If, however, the elastic body is more like a soil body in having a cohesive strength which may be small compared with frictional resistance throughout the stressed region, the procedure is more complicated. Friction increases with soil depth and depends upon the slope of the sliding surface, whereas cohesion remains substantially unchanged, or changes slightly. It is necessary to assign, to the soil, shearing resistance stresses, s_r, appropriate to the z-values investigated. It is also necessary to construct a set of constant (iso-) shear curves for the stresses due to the load, with equal increments in s between adjacent curves, the set to cover a sufficient range of values. The boundary surface is then obtained by choosing a z-value, noting the s_r-value considered appropriate, and plotting this value where the s-curves give its equal. Proceeding in this manner, a number of points may be located and the curve drawn. The s_r-values should now be checked. It is necessary to measure the normal force on the curve, compute the friction force tangential to it, and add the cohesive force. If the s_r-values, so computed for the curve as drawn, check the assumed values, the curve is really the probable surface of failure. If not, a second approximation must be made. It will be observed that this procedure is similar to that of Haines, but differs in that Haines approaches the problem from the standpoint of choosing, by trial, the curve yielding the lowest factor of safety. Whereas the curve to choose is virtually unknown in the Haines method, the stress method enables one to approximate the probable surface of failure.

The procedure used in drawing the iso-shear lines, or lines of constant shearing stress, is based on Equation (12*f*). It may be shown by differentiation of Equation (11*f*), holding z constant, that the maximum shearing stress, s_2, occurs when $x = 0.5\,z$. The value of this maximum stress, s_2, is denoted by s_m and,

$$s_m = \frac{0.1366\,P}{z^2} \quad \text{(13a)}$$

The value of the shearing stress for any other value of x, z remaining constant, may be found from s_m by using a factor, k;

$$s = k\,s_m \quad \text{(13b)}$$

By dividing s_2 by s_m, k may be obtained in terms of $\frac{x}{z}$ alone; therefore, k applies to any level, z, provided $\frac{x}{z}$ is the same:

$$k = \frac{25\,\sqrt{5}}{16}\,\frac{x/z}{\{1 + (x/z)^2\}^{5/2}} \quad \text{(14)}$$

In Fig. 57 the line of maximum shear stresses is shown making the angle, θ, with OZ such that $\frac{x}{z}$, or $\tan\theta_5 = 0.5000$. The right circular cone with such lines for elements is the locus of such maximum shear stresses, s_m, for the point load, P.

In Fig. 59, k has been plotted in terms of $\frac{x}{z}$. Certain other lines numbered n from 0 to 10 cross the chart parallel to the $\frac{x}{z}$-axis. These are located so that,

$$k = 0.01\, n^2 \qquad (15)$$

Suppose the curve, s, is to be drawn. The first point may be located by substituting s in Equation (13a) and solving for z. This point will lie at the z-value thus found and at $x = 0.5\, z$. Divide this z-distance into ten equal parts along OZ, pass horizontal lines through the points of division, and number them, n, from 0 at the surface to 10 at the first line found. Now,

$$s_{mn} = \frac{100\, s^2}{n^2} \qquad (16)$$

Hence, k at Level n must have the value of $0.01\, n^2$. The lines, n, in Fig. 59 give the $\frac{x}{z}$-values at the n-level where they intersect the k-curve, thus enabling one to determine the two x-values at each n-level. It merely remains to draw the s-curve through the points so found, and repeat for such other s-curves as may be required for the set. More exact values of k are given in Table 8.

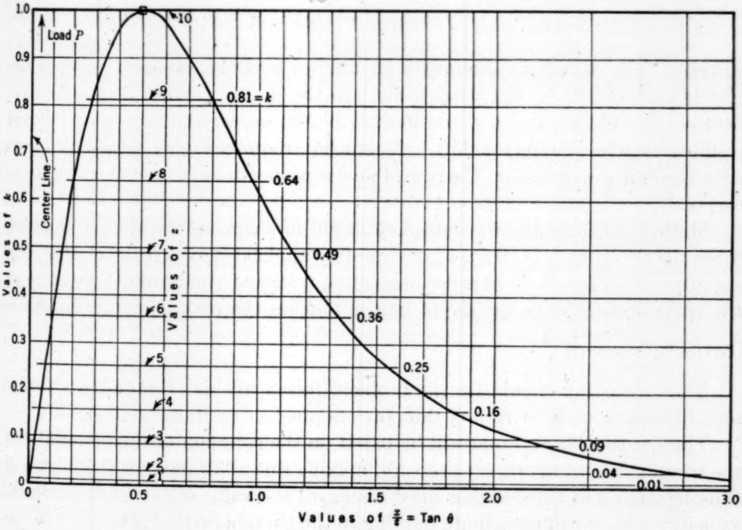

FIG. 59.—RADIAL VARIATION OF SHEAR ON THE X–Z PLANE.

If a distributed load is involved, it becomes necessary to combine stress components at each of a set of points, and thereby build up s_2-values on appropriate planes. In the case of a line loading of indefinite extent, this might be a center plane perpendicular to the line. Integration is easy for s_2. Such planes experience a principal shearing stress and, therefore, are critical.

In passing, it should be noted that a flowing plastic body may have higher shear stresses within it than the limiting shear stress for the material. Furthermore, a plastic body, restrained from flowing, cannot support a greater shear stress. That part of the stress system producing the limiting stress in the elastic body remains unchanged in the plastic body. The excess stresses are purely direct in the case of the non-flowing plastic body. This shift in the character of the direct stresses tends to modify the shape of the surface bounding the plastic region, because the surface computed in Fig. 58 involves the assumption of pure elastic stressing, and direct pressures on the corresponding surface. The shift in the boundary should not be great, however. When the plastic material flows, the stress system in the plastic portion is intermediate between the elastic and the non-flowing plastic cases, and, therefore, should result in even less boundary shift. Plastic flow requires higher shearing stresses than the limiting ones since the rate of flow is proportional to the shearing stress excess.

TABLE 8.—VALUES OF k, FOR SUBSTITUTION FOR EQUATION (12)

Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)	Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)	Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)	Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)
0.0	0.0000	0.6	0.9764	1.2	0.4508	1.8	0.1699
0.1	0.3408	0.7	0.9025	1.3	0.3827	1.9	0.1455
0.2	0.6335	0.8	0.8115	1.4	0.3245	2.0	0.1250
0.3	0.8450	0.9	0.7134	1.5	0.2752	2.4	0.0706
0.4	0.9643	1.0	0.6176	1.6	0.2338	2.8	0.0421
0.5	1.0000	1.1	0.5293	1.7	0.1990	3.2	0.0264

Methods such as the foregoing are useful in checking the Haines analysis. Since the elastic equations are approximately applicable even if the elastic body consists of two layers of differing properties, it should yield useful results up to the point at which the surface of failure touches the rock boundary for some particular value of $\frac{a}{L}$.

This discussion should tie the problem raised by Mr. Hough more firmly into the entire body of results thus far obtained in earth mechanics.

The limiting shear value will be based on the cohesion of the material and the friction due to the direct pressure, making due allowance for the degree of consolidation, but the designer must disregard the value of the load, P, which, as before stated, can contribute nothing to the friction force. The force of this argument may better be appreciated by reference to Fig. 60(a), which shows the variation of the coefficient of friction, based on the pressure, p, when both friction and cohesion are considered, provided the clay is considered fully consolidated for the actual value of p used. Fig. 60(b) applies if the clay is not fully consolidated. This curve represents the solution between the apparent coefficient of friction (based on the full applied load) and the degree of consolidation under that load. For all practical purposes the plot is a straight line. A sudden application of pressure is equivalent to a sudden reduction in the degree of consolidation. There have been a number of cases in which the disregard of such principles has resulted in failure. The coefficient of friction enables one to calculate the limiting shear for the direct pressure acting.

This Symposium is related primarily to dams and other structures erected on the surface of soft materials with the idea of permitting the structures to sink to equilibrium. Curves such as Figs. 37 and 38, suggest that it may prove much more economical to sluice and dredge soft material into the position where the "mud waves" must form. The cross-section of these waves appears to be much less than that of the dam or other structural material which must be lost by the sinking action. With mud waves formed in place in excess of the sizes shown, the structures erected on the surface of the soft body should remain substantially at that surface without any sinking action. The question of protecting the "waves" from erosion exists in either case.

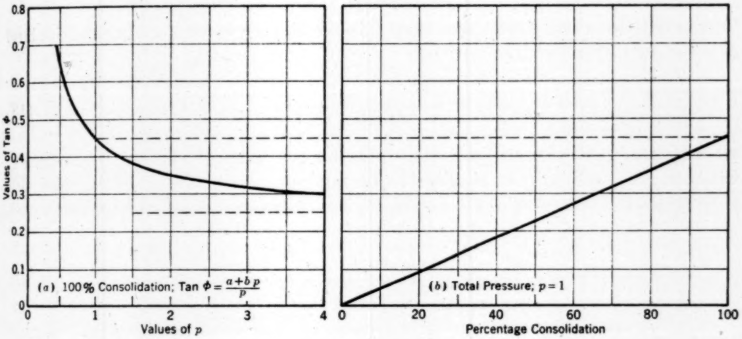

FIG. 60.—SHEAR IN CLAY.

RICHARDS M. STROHL,[53] M. AM. SOC. C. E. (by letter).—The application of soil mechanics to levee design, as described by Mr. Buchanan, is thoroughly helpful to those interested in levee problems.

In discussing the importance of seepage through the levee, reference is made to Fig. 10, in which the path of percolation is $\sqrt{29^2 + 217^2} \times 30.48 = 6\,675$ cm long. The time required for water to seep this distance at the rate, $K = 5 \times 10^6$ cm per sec, will be 15 450 days. As Mr. Buchanan states that the maximum high-water stages do not extend over a period of more than 20 days, it would appear that seepage could not be developed through such a levee during any flood of record.

Necessity for constructing a levee on a foundation similar to that shown in Fig. 61 frequently arises. Water from the river side readily finds access to the pervious sub-stratum. The quantity of percolation is found approximately by means of Professor Terzaghi's expression[54]:

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[53] Engr., U. S. Engr. Office, Memphis, Tenn.

$$Q = \frac{K\,h}{0.88 + \frac{L}{d}} \quad \text{(17a)}$$

which is the equivalent of Darcy's formula (published in Paris, 1858):

$$Q = K\,A\,i = K\,A\,\frac{(h_1 - h_2)}{L} \quad \text{(17b)}$$

The height to which water would rise above the pervious stratum on the land side for conditions of free flow is,

$$h_2 = h_1 - \frac{h\,L}{0.88\,d + L} \quad \text{(18)}$$

whereas, for static conditions, the uplift on the natural blanket is practically equal to h_1. Where it is possible to permit nominal flooding on the land side of the levee, the flow of water upward through the blanket can be controlled. By permitting discharge on the surface of the blanket, h_2 becomes equal to t, and the flow at the ground surface becomes,

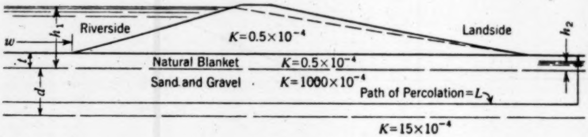

FIG. 61

$$Q = k\,d\,\frac{(h_1 - t)}{L} \quad \text{(19)}$$

per linear foot of the levee. This quantity will be rather small, and it appears more practical to control this flow than to try to prevent it.

The width, w, of the blanket on the river side of the levee is of importance in reducing percolation under the levee, Q being inversely proportional to L. It is the practice in levee building to secure the embankment material from borrow-pits in front of the levee, leaving a berm with a minimum width of 40 ft. Where the depth of the borrow-pit materially reduces the thickness, t, of the material blanket, it appears worth while to increase the berm width.

WILLIAM P. CREAGER,[55] M. AM. SOC. C. E. (by letter).—The theories of soil mechanics, as applied to the design of dams, are frequently quite complex. Mr. Buchanan has made an excellent summary of the fundamental principles involved; but, on account of the limitations in space for a paper of this kind, he has not been able to do more than give a hint of some of the extent of the procedures necessary for many cases. The only criticism which the writer can give regarding this paper is the lack of indication that, in some instances, the procedure involved is not as simple and direct as shown by the examples given. As examples of lack of simplicity in designs the writer gives two cases as follows:

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[54] Unpublished analysis of seepage, by Charles Terzaghi, M. Am. Soc. C. E.

[55] Cons. Engr., Buffalo, N. Y.

(1) The design of the slopes of any dam is not complete without the inclusion of seepage forces. This has been mentioned only briefly in this paper, and the impression has been given that the consideration of such forces must wait on "further research." A number of excellent papers, involving the consideration of such forces, were presented at the Second Congress on Large Dams held in Washington, D. C., in 1936. Mr. Buchanan's example is lacking in respect to this force.

(2) The stability of the foundations, as used in the example (see heading, "The Design of a Levee Unit"), is correct without modification, on the assumption that the foundation drains quickly. If, due to insufficient drainage time, the foundation is not completely consolidated, some of the weight is carried by the water in the foundation and part of the ultimate frictional resistance to shear is lacking. Therefore, the measured shearing strength as given by Item (*c*) of the example should be determined experimentally on the basis of the correct amount of partial consolidation when the dam is first completed.

Attention is also called to the following comments on the paper by Mr. Hough, regarding the necessary modification to foundation stability equations due to this condition.

Mr. Hough has presented an excellent paper which, among other interesting items, describes a proposed method of construction that is unique in the annals of dams. He has mentioned one limitation to photo-elastic model studies. The writer would like to mention another limitation.

In the case of a saturated foundation which has consolidated only partly during the construction of the dam, a part of the weight of the dam is carried by a direct loading of the soil of the foundation and the remainder is carried by the hydrostatic pressure of the pore water in the foundation. The hydrostatic pressure in the foundation, which is built up by, and equals, that part of the loading carried by the water in the foundation, acts in all directions, of course, and forces the surplus water to drain off in all directions with velocities proportional to the lengths of the various paths of percolation to the nearest point of relief.

Fig. 62(*a*) represents a pervious sand dam resting on a layer of silty clay which, in turn, is underlaid by pervious sand. The water in the silty clay layer drains almost directly upward and downward. Decrease the perviousness of the dam and the underlying sand layer, and some of the water in the silty clay layer begins to drain sidewise as well as upward and downward, as shown in Fig. 62(*b*). Should the dam and the underlying layer be absolutely impervious, all the drainage would be horizontal and diagonally upward to the valley bottom adjacent to the dam, as shown in Fig. 62(*c*).

Fig. 62(*d*) is an actual case of a concrete dam resting on a deep, lightly consolidated shale bed. Between the dam and the shale is to be placed an impervious coating of asphaltum to prevent drying out and slaking of the shale during construction. The drainage is downward at the center and horizontal or inclined upward at the edges of the base.

It will be noticed that, in all practical cases, some part of the drainage is horizontal and upward toward the sides of the foundation in the direction of failure if the foundation should prove weak.

The entire hydrostatic pressure in the foundation is balanced by the friction of the water in the direction of flow, the pressure decreasing to zero at the outlet of the seepage. Thus, the loading carried by the hydrostatic pressure is transferred to the soil, not as a vertical loading which can be simulated by the photo-elastic method, but in an infinite number of directions which cannot be simulated by present methods.

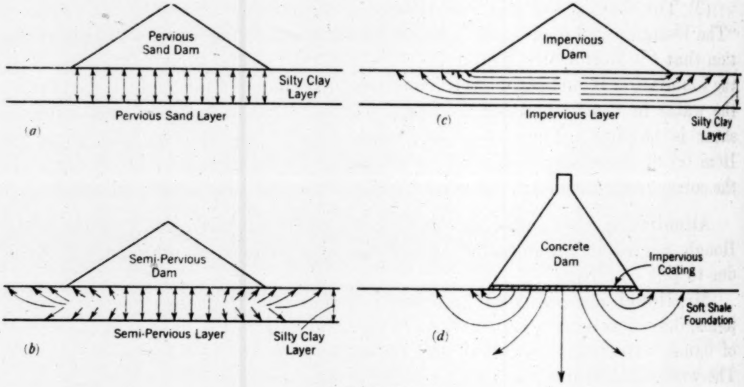

FIG. 62

Very obviously, a greater loading can be placed on a soil in a vertical downward direction than in a horizontal or diagonally upward direction, in the direction of failure. For this reason, the photo-elastic method and, incidentally, the commonly accepted theories and formulas are not applicable for the condition of partial consolidation of the foundation. Unfortunately, the writer does not have at present a solution for this special case. He presents it for the purpose of stimulating interest in the problem and as a thought to be considered in the interpretation of usual methods.

JACOB FELD,[56] M. AM. SOC. C. E. (by letter).—Seldom in engineering work is there an opportunity for full-scale research such as is available to the U. S. Waterways Experiment Station. Results cannot be expected immediately, but only from a continuous study over many years. The main problems are the determination of settlement and seepage, using variations of material combinations, not only at the end of and during construction operations, but the effect of age, receding flood levels, precipitation intensities, and unusually high and low temperatures. The problems are not simple, yet because of the fairly limited number of soils available for use, solutions covering a considerable range of time and weather conditions can be expected from a program of not more than 10 yr.

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[56] Cons. Engr., New York, N. Y.

It would be sound engineering as well as sound economics to design and contract for a series of test sections, using controlled soil mixtures and construction methods. In such a test group, attempts should be made to study the use of colloidal material from the river bottom as a temporary surface seal, to give added resistance to seepage until such time as the settlement of the fill reduces the seepage rate through the full section.

Systematic laboratory research can be of great aid in lessening the cost of full-scale tests. As far as seepage is concerned, such work as that of B. A. Bakhmeteff, M. Am. Soc. C. E., and N. V. Feodoroff[57] gives a clear picture of the problem as well as a mathematical solution of a number of conditions encountered in levee and dam design.

Little has been learned of the impervious region at the contact zone between layers of different grain size and shape. The contact lens or skin is denser, less pervious, and of greater bearing value than either of the adjacent materials. Such zones are often found in river beds, just below the shifting layer of sediment. Below this may be much more pervious strata. Cutting through this zone, whether on the land side or on the river side of a levee, in a borrow-pit, exposes the subsoil under the levee to excessive pressure. If the borrow-pit is on the land side, the added weight of the levee and the opening of the porous strata permit a ready squeezing out of the subsoil moisture with consequent rapid settlement and slumping of the fill. If the borrow-pit is on the river side, high water causes a sudden increase in upward pressure, which cannot be dissipated because of the impervious continuous layer on the land side.

In the example of design which Mr. Buchanan illustrates by Figs. 8, 9, 10, and Table 3, the assumption of a cylindrical surface of rupture starting at the top of the slope is not necessarily the true failure picture. More often, failure occurs by slippage of a dome section, with a vertical surface intersecting the original slope. In the design of the foundations, the assumption is made that the added weight is distributed uniformly, no allowance being made for the lateral flow of the subsoil. As has been proved by experiments, the maximum added pressure is not under the highest point of the fill. Item (*e*), (see heading, "Foundations") giving the necessary base width (348 ft), assumes that a uniform added base pressure results from the levee load over the entire width of the base, that there is no lateral distribution of loading below the base of the levee, and that the strength of the foundation material is not affected by the loading above. None of these assumptions is true. Uniform added base pressure would result only from a uniform depth of fill, unlimited in all directions. There would be no lateral distribution of loading below the base of the levee only if the material had a zero internal friction coefficient. The strength of the foundation material is affected by the superimposed loading because of the added internal resistance resulting from the confinement by such loading.

It is unfortunate that the sudden decision to abandon the Passamaquoddy Project ended the study of foundations for rock-fill dams, as reported by Mr.

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[57] *Journal of Applied Mechanics*, Vol. 4, September, 1937, pp. A97–100.

Hough. The report of the work done is of value for future studies, but the writer cannot agree that it has any value in regard to a solution of the foundation design for the proposed dams. The chief reason is that the later effect of the impervious blanket, a much greater weight than the rock-fill section, was not considered. Allowing the foundation material to re-adjust itself and build up resistance to support the rock-fill and then hope that the later addition of about 80 to 100 ft of impervious blanket fill on one slope of the rock-fill could also be carried by the original adjustment, is somewhat optimistic. References to the work of Carlo Barberis,[58] at Spezia, Italy, as well as to similar works[59] in Valparaiso, Chile, and also by Sakamoto and Takanishi[60] in Kobe, Japan, would certainly have been of assistance. In the foregoing three references, similar dams and wharves have been described built on mud or soft clay foundations by the aid of a "floating foundation" of fine sand pumped into place. The combination of mud and sand has proved a proper, safe, and economical solution of the problem studied by Mr. Hough. It may be of interest to mention that, in 1928, while making some studies for private finance groups for this same project, the writer made some designs and estimates for the dams required in connection with the Passamaquoddy Tidal Power Project, based on the work mentioned in the foregoing references. The results were much more economical than the sections illustrated by Mr. Hough.

From time to time, reports of, and papers sponsored by, the Committee of the Society on Earths and Foundations have appeared in *Proceedings*.[11] Except for this Symposium, the reports were optimistically heralding the advent of a new era in foundation engineering and the birth of a new soil science. The humility of approach in the 1937 reports, as so well exemplified by the paper by Professor Terzaghi, is refreshing; and it is well that the change of attitude has taken place. Too many practical and practising engineers were attempting to apply the published material to their work; and they were finding it of little help.

If the paper by Professor Terzaghi is to be taken as a criterion, the subject is in a new phase. Theoretical and laboratory investigations are relegated to second position; field investigations of completed structures must now be stressed so as to collect the necessary facts from which reliable theory may be deduced. This is opposite to the previously announced inductive theories which prophesied facts, such as settlements of unbuilt structures, and which too often required continuous adjustments to meet new data.

In order to be of value, complete descriptions of the structures must be presented with settlement readings. In this respect, the paper by Professor Terzaghi is somewhat remiss. For instance, in the example described with Fig. 45, the length, size, and spacing of the piles are not given. One is inclined to wonder why piles were used in that design. In Fig. 47(*b*), there appears to be a definite discontinuity in all settlement contours at the center division wall. An error in the H_3 reading seems to be one explanation of the impossible condition shown.

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[58] XVI International Cong. of Navigation, 1935.

[59] *Bulletin*, Navigation Congresses, January, 1927, p. 33.

[60] XIV International Congress of Navigation, 1926.

In Fig. 48, the maximum settlement is a trifle more than $\frac{3}{4}$ in. For the broken up design, with piles of unknown and varying lengths, such a value is not unexpected.

In Table 6, it should be carefully noted that all the soils listed, except that for Building *H*, are not the usual materials on which spread footings are placed. This fact explains the great variations in settlements for points on the same structure. Buildings founded on sand or clay mixtures with loam, with base pressures between 2 and 2.5 tons per sq ft, should show some peculiar settlements.

The writer agrees that no reasonable soil examination can show all the variations in soil structure for a complete foundation design. Where soil conditions are uniform, expected settlements are easily guessed. Where soil conditions are not uniform, it is necessary to design the footings as the sub-grades are exposed by excavation. In one large structure, north of Chicago, Ill., borings indicated that at least three different clays would be encountered at the expected base of the footings. After a careful study of all the clays encountered in that vicinity, and comparable loading tests in pits dug into the three different clays (differentiated by their colors), load-bearing values were determined to give equal settlements. The lack of time prevented load-test studies extending over more than a few weeks. Design tables were prepared for the three assumed load values for the complete range of column loads. All footings could be made symmetrical spread footings for single column loads, which simplified the problem. As each footing pit was dug to sub-grade, inspection was immediately made, the color determined, and design requirements chosen from the tables. No unusual or unequal settlements have been noticed.

Much foundation trouble arises from a common error of placing a structure on the side of a slope, exposed to seepage or drainage effects and lateral movements. The error is evident when the structure is placed on a sloping surface, such as the side of a ravine or river bed. If the gap is filled with rubbish, ashes, or the usual municipal rubbish fill, the error is not visible, but still exists. In many cases, the fill makes the matter worse, because its consolidation causes a lateral flow which further tends to tip or slide the building foundation piers or walls.

Y. L. CHANG,[61] ESQ. (by letter).—Engineers in China who are confronted with a similar, but more complex, problem when building regulating dikes on the Yellow River, will be much interested in the paper by Mr. Buchanan. Because of traditional methods of construction and the limited financial support given to such large engineering projects, all dikes along the channel of the river could be built and maintained only by using earth of uniform quality at the site. About one-fourth of the total area of the Yellow River basin consists of loess which is a very fine calcareous loam composed of an admixture of minute but considerably flaky particles of quartz and clay. This material is mostly an impermeable soil. The loess has a porosity exceeding 45%, enabling it to hold considerable moisture, and the fineness of the openings facilitates capillary action. Such material cannot be approved for dike construction; and, hence, the problem is reduced to a question of how to design the cross-section of the dike to obtain a degree of safety consistent with minimum cost. Mr. Buchanan certainly makes such an aim possible by a procedure so simple that every engineer can understand it.

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[61] Shanghai, China.

In the report of the Committee of Experts of the League of Nations,[62] five causes of the failure of the dikes on the Yellow River are given, the most important being substantially as follows:

> "A breach may be produced when the body of the dike is saturated with water, either by direct infiltration from the water face or by indirect infiltration through the more permeable layers at the base. The cohesive resistance of the earth is reduced, while at the same time hydraulic pressure, accompanied by seepage, occurs. Its effect is felt when the base of the inner side of the dike collapses. The dike itself is thus weakened, and a breach may result. This case arises especially when the high-water stage lasts a long time or when the dike is of inadequate construction or composition."

With this statement in mind, one can be assured that a failure of such dikes is mainly attributed to the infiltration of water through the dike itself and through the foundation beneath. The writer wishes to discuss Mr. Buchanan's paper as applied to conditions when the dike is subjected to infiltration through side slopes, and foundation, and as applied to the control of seepage.

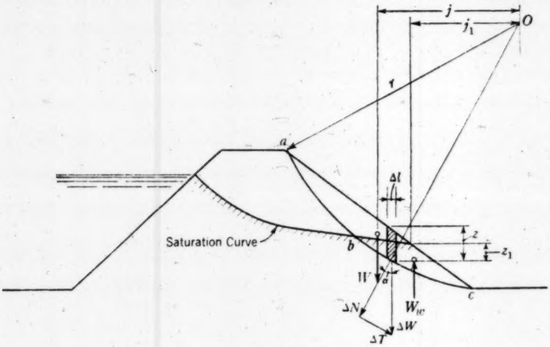

FIG. 63

Side Slopes.—Presumably, Equation (1), which expresses the factor of safety against sliding in terms of the shape of the dike and the frictional and cohesive forces of the material, is for a dry slope. For a dike subject to seepage, such as that shown in Fig. 63, Equation (1) can be written:

$$\text{Factor of safety} = \frac{c\,l + \tan\phi \sum_{0}^{n} \Delta l\,(w\,z - w_w\,z_1)\cos\alpha}{\sum_{0}^{n} \Delta l\,(w\,z - w_w\,z_1)\sin\alpha} \quad \ldots\ldots (20)$$

in which, ϕ, the angle of internal friction, is assumed to be the same for dry or wet material; w_w and w are the unit weights of water and soil, respectively;

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[62] Report by the Committee of Experts on Hydraulic and Road Questions in China, League of Nations, Geneva, 1936.

and l should be the arc length, $a\,b$. If the entire dike is assumed to be saturated, the term, $c\,l$, vanishes. A similar equation for the factor of safety against sliding can be expressed in the form of moment instead of force:

$$\text{Factor of safety} = \frac{r\left[c\,l + \tan\phi \sum_{0}^{n} \Delta l\,(w\,z - w_w\,z_1)\cos\alpha\right]}{W\,j - W_w\,j_1} \dots\dots (21)$$

in which W_w and W are, respectively, the total weights of water and soil within the arc of sliding, $a\,b\,c$. The smaller value determined by Equations (20) and (21) gives the safer criterion for stability.

Mr. Buchanan's treatment of the hydrodynamic effect of seepage on stability of slopes is especially creditable. In testing a model embankment of loess the writer has observed a slide near the toe despite the fact that the factor of safety, as computed by Equation (21), exceeded 1.78 for $c = 0$. The writer believes that the hydrodynamic force in a saturated slope plays an important part in starting a slide.

Foundations.—A breach may occur in the dike when its foundation is so saturated that the underlying material settles or is displaced. In this state, the foundation could be considered more plastic than elastic. Studies of such failures due to over-stressing the foundation have been presented by Mr. Hough. Attention should be called to the fact that there is a horizontal, as well as a vertical, load on the foundation when the dike is resisting the pressure of high water. For earth or rock-fill embankments this pressure can be transmitted to the foundation only in the form of a shearing force. In investigating the stability of dike foundations this horizontal hydrostatic load requires as careful study as the vertical dead load.

Control of Seepage.—Mr. Buchanan has studied the problem of the seepage through a dike with the aid of Professor Gilboy's formulas as given by Equations (2) and (3). The writer has compared this method with that suggested by Pavlovsky,[63] in the case of model dikes of Chinese loess, and has found that the latter coincides more nearly with experimental results. The latter also affords a simple procedure for designing the best section. A study of this method reveals that the permeability factor, K, cannot be constant in a newly built dike in which the seepage rate may be quite high.

The essential points demonstrated by this method are:

(1) A dike subjected to considerable infiltration over a long period of time is at its most dangerous stage; its stability is undoubtedly reduced by the hydrodynamic force of the water seeping through it.

(2) In estimating the shearing failure of the foundation of a dike the horizontal transmission of water pressure should not be neglected.

(3) Theoretically, the best method of reducing infiltration and lowering the saturation line of a dike is to increase the flatness of the wet surface.

(4) The orientation of flaky particles of a material by capillary flow probably plays quite an important part in the variation of permeability with time.

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[63] 1er Congrès des Grand Barrages—Scandinavie, Vol. II, 1933; see, also, "The Percolation of Water Through Earth Dams," by N. N. Pavlovsky; tr. by Andreas Luksch; U. S. Bureau of Reclamation, Denver, Colo.

Charles Senour,[64] Esq. (by letter).—In view of the interest in levee construction engendered by the fairly recent floods of New England and of the Ohio River Basin, Mr. Buchanan's paper is most timely. The latter flood (January, 1937), after reaching the Mississippi River, was carried to the Gulf between levees, and one aftermath of its successful passage has been a number of requests for information regarding standard practice in levee design and construction on the Lower Mississippi. This has been given gladly, of course, but with occasional misgivings as to its applicability, particularly in the department of design, to the problems of distant flood plains. The Mississippi River levee is the product of years of experience with the specialized problems of the lower alluvial valley. (Fig. 2 depicts its metamorphosis from the simple trapezoidal cross-section of 1882 through the several types of banquette section to the more satisfactory trapezoidal section of the present day.) There is one school of thought, and a rather large one, that holds to the view that the conditions surrounding levee construction make levee design unavoidably an empirical matter. Experience is invaluable, of course, but unless it is supplemented with a knowledge of the underlying principles that govern soil behavior, experience is distinctly at a disadvantage when it becomes necessary to design in a field not covered by prior operations. It is this inherent shortcoming that the paper by Mr. Buchanan seems particularly helpful in overcoming.

Until the comparatively recent past, there has been little to draw upon except experience. The science of soil mechanics is itself quite young, and its application to the field of levee design has lagged considerably behind its application to the related but, in many respects, dissimilar problems of earth dams. A levee is an earth dam, of course, but it is a very specialized form and the circumstances normally surrounding its construction are far less favorable to precision in design than are those which attend the building of a dam.

The main levees of the Lower Mississippi River are as a rule several hundred miles in length, and are tied into high ground (that is, they have an abutment) only at the up-stream end, the basin being left open at the lower end for drainage. The embankment skirts the river edge of a flood plain whose width may be 50 or 60 miles. This plain is composed of alluvium to great depths. Much of it has been worked over by the river, whose migrating meanders, cut-offs, and bank-caving and bank-building operations have produced a geologic hodgepodge of which the ingredients are clay, sand, and silt, with some vegetable matter. These ingredients occur singly and in an endless variety of combinations which follow no pattern and apparently recognize no law except that of change.

It is evident that one cannot very well haul upland earth to build his levee of homogeneous clay; nor can he afford to haul laterally from the possibly better soils of the flood-plain's distant hinterland. If he did so, furthermore, he still would have to found the levee upon the heterogeneous mass of which the area adjacent to the river is composed. The problem, therefore, is to build from the materials immediately at hand, and since these vary so erratically as to make selection seem rather impractical in a majority of cases, it has appeared advantageous to procure the material as close to the embankment as safety permits, so as to shorten the haul. The borrow-pit is usually on the river side, and is quite apt to encounter rather pervious materials after the surface strata are removed. Since these porous materials will be exposed to full hydrostatic head once they are uncovered, it is inadvisable to expose them very close to the embankment, and it is, of course, desirable to utilize as small a proportion of the more pervious materials in the construction as circumstances permit. Deep excavation close to embankment also invites foundation troubles. From the practical standpoint, furthermore, it is not safe to count on going very deep because of moisture.

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[64] Prin. Engr., Mississippi River Comm., Vicksburg, Miss.

With these conditions laid down, the best solution has seemed to be to construct from shallow side borrow-pits an embankment of sufficient bulk to compensate for any shortcomings with respect to homogeneity and compaction. When teams were the instruments of levee construction, compaction was automatic, but the size and quantity of construction rapidly expanded beyond the ability of teams to handle alone, and much of the system was completed or under way prior to the comparatively recent introduction of trucks and tractors as important tools. Much yardage has been placed by draglines equipped with buckets of capacities of 3 to 8 cu yd, and tower machines handling buckets of from 10 to 15 cu yd. Machinery of this type (largely inspired by the needs of levee construction) handles earth very efficiently within the limit of its reach, which for the tower machine, incidentally, is about 1 000 ft. Neither by the tower machine nor by the dragline is any deliberate attempt made to compact the fill.

No effort is made to control moisture content, in the sense of attaining the optimum. The susceptibility to wide seasonal variations in moisture of pits adjacent to a river whose range in stage is about 50 ft and the necessity for utilizing the working season to the best advantage had made precise moisture control appear unattainable. A limiting steepness of construction slopes (1 on 2.8) is often prescribed by the specification, especially when much clay is present. The contractor, however, is permitted to use fairly damp earth, subject to his own risk (and subject to warnings from the inspector when he appears to be taking chances). If a slide occurs from the use of material that is too wet, the contractor is required to cut out and remove the embankment to and beyond the sliding surface and to restore it with satisfactory material —all at his own expense. Since this is quite costly, it operates as a deterrent to the use of earth that is dangerously damp.

When the levee is built with hauling equipment it is constructed in layers, with an allowable thickness for vehicles hauling loads of 5 cu yd or more, of 5 ft, and for equipment hauling loads of less than 5 cu yd, 3 ft. When draglines (boom machines) or tower excavators are used, layer construction is not required. In all cases the contractor is required to build to a gross cross-section from 8% to 25% higher vertically at all points than the net cross-section, for which latter alone he receives payment. This surplus is designed primarily to compensate for the lack of compaction, the larger shrinkage allowances pertaining to those methods of construction which involve the least compaction. The shrinkage allowance seeks to insure a levee permanently to the grade desired even with some consolidation of foundation. It is, therefore, liberal.

A series of shrinkage observations made upon levees constructed in the Lower Mississippi Valley has been plotted in Fig. 64, in which the volume of

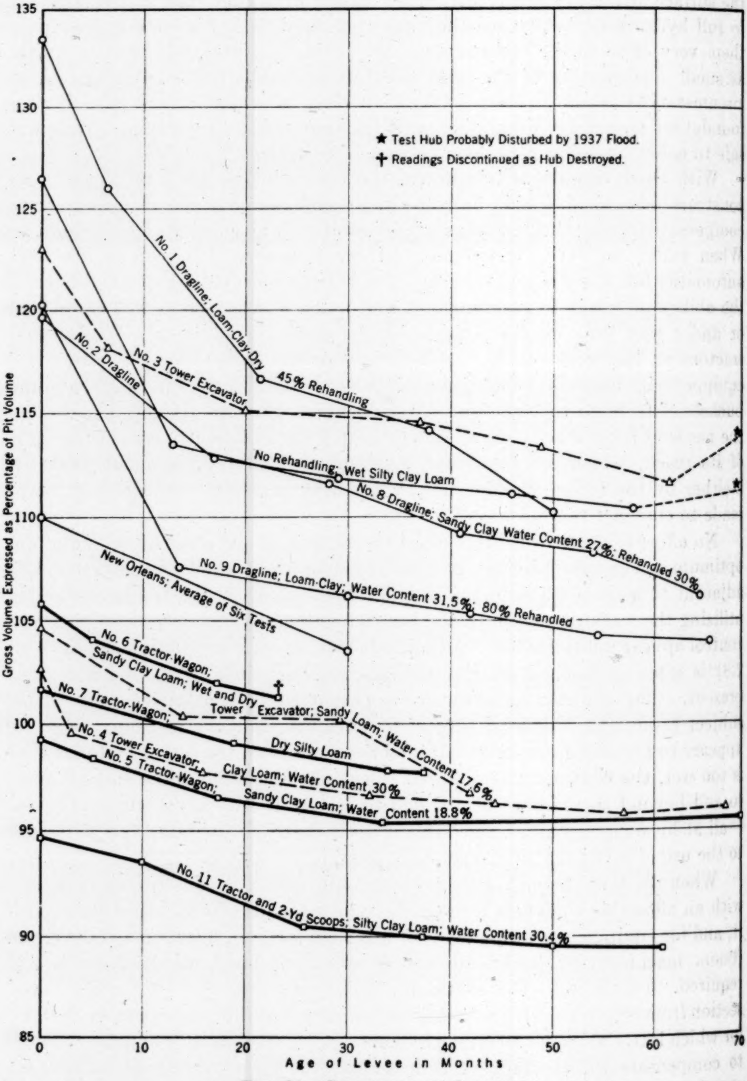

FIG. 64.—SHRINKAGE OF LEVEES

the embankment of various ages is expressed in terms of the volume of the borrow-pit from which it was constructed. Each curve represents a series of experiments on a single unit involving from 8 000 to 100 000 cu yd, except that labeled "New Orleans" which represents the unweighted mean of a group of six experiments on dragline-built units ranging from 160 000 to 660 000 cu yd in volume. In this latter case the results varied so little that individual plotting appeared confusing (from 1.05 to 1.15 borrow-pit volume at the time of completion, and from 1.08 to 0.998 at the end of 30 months).

The curves indicate a pronounced superiority in density of the tractor-built units over that of the dragline-built units, the tower excavators being represented in both groups. One method of construction with tower excavators is to "base in" the fill with one machine and mount a second, following machine upon the material so placed, to complete the cross-section. When this is done, the influence of the tremendous weight of the equipment, supplemented by its vibrating effect, apparently tends (in some soils, at least) to attain tractor densities in the fill.

On a big levee, tractors, draglines, and tower machines all may be engaged concurrently in placing different parts of the cross-section. The work is awarded by contract to the lowest responsible bidder, and there is no method of forecasting before the bids are opened who he may be or what method he will choose to adopt. Furthermore, the vagaries of weather and river stage may force him to modify his original plan from time to time, and each time he does so, the composition and density of the fill are changed somewhat.

Fig. 65 shows the theoretical shape of a standard borrow-pit. The depth depicted is the limiting value allowed by the specifications and the width, therefore, must be varied to the extent required by the river stage and water-table, which may necessitate much shallower borrow than that permitted by the specifications. Fig. 65 also shows a series of pit borings and groups of borings taken at the center line and the toe of a completed levee to illustrate the variability in composition of borrow, foundation, and embankment.

In general terms, levee design on the Lower Mississippi has concerned itself with compaction only from the standpoint of shrinkage allowance to insure a structure permanently to the grade and cross-section desired, and with moisture content only to the end that slides or sloughs shall not develop. As to character of materials, it recognizes three groups—sand, loam, and clay, as described in Table 1 of the paper by Mr. Buchanan. Most of the levees built are of the "loam" cross-section, "loam" being used to designate all mixtures containing less than 75% of either sand or clay. The loam cross-section has a base width equal to ten times the design head against the structure. It seeks ideally to prevent outcrop of seepage line on the rear slope by having the slope flat enough to contain the seepage line that will develop during the period of flood use. Of course, where the foundation is less pervious than the embankment, the seepage line will inevitably outcrop if the flood lasts long enough. Upon a stream such as the Mississippi it is essential that slopes be flat enough to be stable under such conditions.

In essence, the levee represents an embankment which relies upon great width rather than great density. It is feasible, physically, for the duty to which a levee is subject, to make the substitution, and in the case of the Lower Mississippi a consideration of local physical conditions and costs has caused

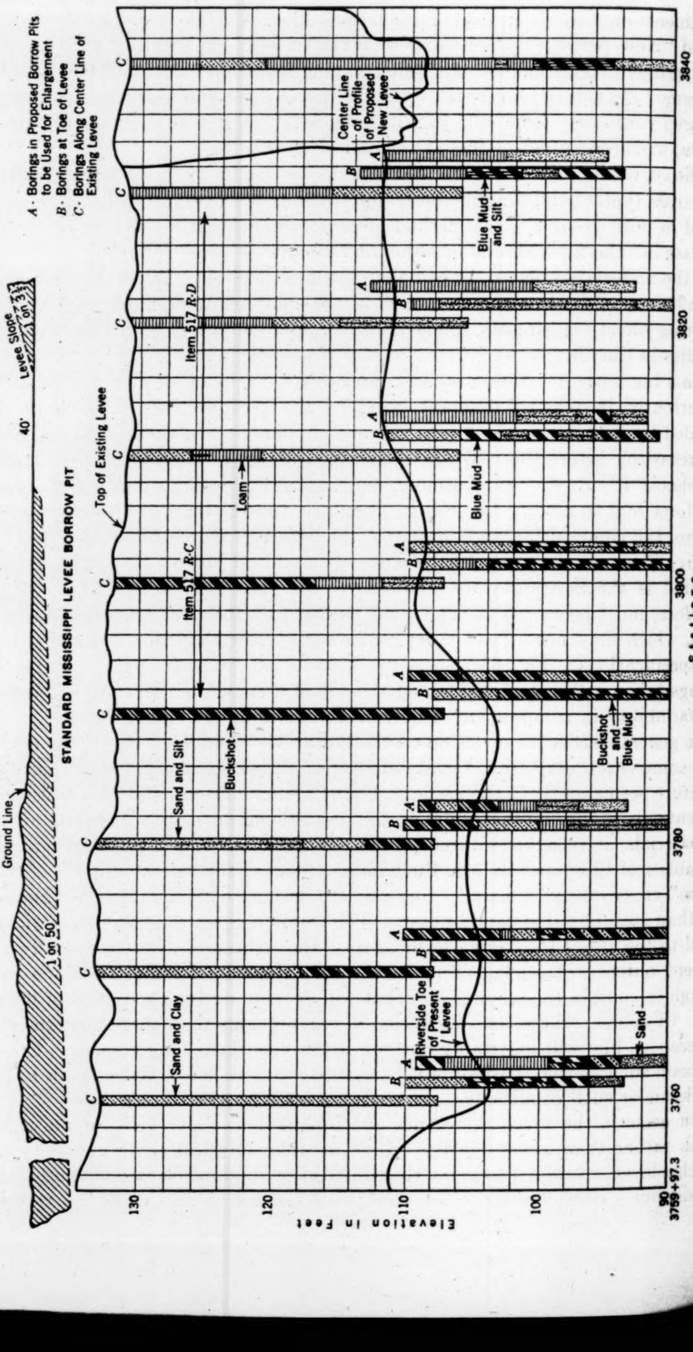

FIG. 65.—BORINGS BELOW GRAND LAKE

that to seem unquestionably the proper course. The average contract cost of the approximately 567 000 000 cu yd so placed from 1928 to June 30, 1936, has been about 18.75 cts. It cannot be doubted that proper compaction, if practicable, would permit some reduction of cross-section where the foundation was of a character to permit it. It should be borne in mind, however, that the structure rests upon a foundation which is alluvial to great depths, and that the passage of water beneath the levee may cause as much concern as the passage of seepage through it. The base should be comfortably wide as a rule.

A levee is built between floods and is always more or less of an emergency undertaking. During the height of the levee program, placement was at the rate of about 612 000 cu yd per day.

From the foregoing it is apparent that unless current construction practices are revised the design of levees by the principles of soil mechanics can scarcely achieve the exactness of earth-dam design. However, although practical difficulties are numerous, they do not appear insurmountable.

With the methods of construction normally permitted, for example, a density considerably less than that of the undisturbed borrow material may characterize the freshly built levee but it would seem that in designing an allowance could be made for this condition. In this connection it is well to remember that soils high in clay content, when fairly dry, tend toward lumpiness, particularly if dragline construction is utilized and the resultant percentage of voids in the fill is likely (temporarily) to be large, thus tending to offset the effect of the greater impermeability of the material itself. Curve 1, Fig. 64, illustrates such a case. This is a practical aspect of the matter which might prove troublesome if the width of cross-section were reduced too greatly on the basis of the impermeability displayed by borrow-pit samples. Obviously, the problem of design would be simplified if circumstances permitted compaction to be assumed or required. Where they do not so permit, caution is indicated in cases where the condition just described is likely to be encountered. Such levees improve rapidly with age but they must be ready and able to hold out floods the first year of their existence.

Study of borings in the borrow area will indicate the worst possible average composition that the levee can have. In deciding upon this feature and upon the questions raised by the preceding paragraph it would be well to consult some one familiar with the practical aspects of construction. It would also be well to consider, from knowledge of the local geology, the likelihood of encountering large enough deposits of any one unfavorable material to justify making that single type the sole basis of design.

The assumption of an impervious base should always insure an error on the side of safety in so far as seepage through the cross-section is concerned provided, of course, that the governing pit material is properly chosen and its observed permeability properly adjusted to compensate for lack of compaction. The partial or even complete disregard of cohesive strength in cases in which high water-content of cohesive borrow is at all likely should fully compensate for possible use of wet material in construction.

By familiarizing himself with local conditions, and adjusting his various allowances to meet them, the designer should be able to produce structures that will be both safe and economical. He should particularly guard against the temptation to produce too economical a structure, however, and, if possible, he should work "hand in hand" with the man of levee experience. Although the writer does not subscribe to the view that experience is the only guide in levee design, he is far from undervaluing it.

The designer cannot afford to ignore certain practical considerations not directly connected with soil mechanics. For example, levees should be maintained in sod, unless they are paved; hence the side slopes should be flat enough to permit the use of mowing machines, and to avoid excessive damage from rain and waves. If the structure is more than a few feet high the slopes for practical purposes should probably not be steeper than 3 horizontal to 1 vertical. In many cases, this may produce a section in excess of that indicated to be necessary by theoretical analysis.

The method of design outlined by Mr. Buchanan assumes an impervious foundation and a flood of great enough duration to develop fully the ultimate flow net. This is probably prudent on a stream such as the Lower Mississippi whose flood hydrograph may cover several months; but for lesser streams where the flood may gather and disappear in the course of a couple of weeks, or even days, it may result in over-design. The seepage behavior of the earth dam described by Mr. Buchanan indicates that for similar soils, similarly placed, the question of seepage can probably be omitted as a design criterion if the floods to be expected are brief, but other cases arise in which it seems unsafe to assume that the matter of seepage can be dismissed, yet in which the assumption of full development of seepage seems too pessimistic. It is felt that further research regarding the rate at which the flow net develops is needed to "round out" the equipment of the levee designer.

The main-line levees on the lower river were matured as to general design before the science of soil mechanics was sufficiently advanced to lend much aid, and, as emphasized by Mr. Buchanan, analysis of the adopted design does not indicate any theoretical need for its revision. No doubt, soil mechanics will play an increasingly important part in the design of levees now in prospect as adjuncts to the basic system.

It has also found a useful field in connection with the prevention or correction of foundation failures in the main levee system. Contrary to popular opinion, most of the levee line experiences no serious foundation trouble. There is a consolidation of base in practically all instances, but this is not harmful unless it becomes excessive.

Serious trouble often threatens, however, where the levee crosses (as it unavoidably must at times) an old lake or channel that has filled with sediment. In such cases special treatment by way of flattened slopes or false berms is necessary and, in the opinion of the writer, the science of soil mechanics offers by far the most rational if, indeed, not the only rational, approach to the mportant matter of their correct design.

DONALD M. BURMISTER,[65] ASSOC. M. AM. SOC. C. E. (by letter).—The Symposium reflects the recent and important advances which have been made in the practical applications of soil mechanics to the study and design of earth structures. This new technique should include not only the analysis of earth structures for stability but it should carry the entire study of such problems from the preliminary soil investigations to the results of experiences accumulated during the construction of the work. The preliminary soil investigations on the different borrow-pit soils, from which suitable materials can be excavated economically, have an important bearing not only on the construction but also upon the decision as to the type of dam to construct, the dimensions of the sections, and the selection of stable slopes.

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[65] Asst. Prof. of Civ. Eng., Columbia Univ., New York, N. Y.

The grain size analysis, to which Mr. Dore refers, has constituted an important soil test on every earth dam project, but its value has been somewhat limited because the information obtained was, primarily, of a qualitative nature. The writer has found that such curves can also be adapted to yield valuable information of a quantitative nature, for design and control purposes.

Grading Curves.—The size characteristics of soil may be defined in terms of three independent factors, which correlate many physical facts and bring them into a more unified and consistent pattern: (*a*) The degree of fineness (the position of the grading curve on the scale of fineness); (*b*) the type of grading curve (normal, skewed, or irregular distribution of sizes); (*c*), the range of particle sizes. In his studies, the writer has expressed the degree of fineness in terms of a mean grain size, which is defined by the length of the equivalent rectangular area and which is easily obtained by planimeter or by the summation of elementary areas. This mean is actually a weighted geometric mean and is the logical one to use in relation to a semi-logarithmic plotting of the grain size analysis.[66] The mean grain sizes for the average borrow-pit, beach, and core materials of the Quabbin Dike are given in Fig. 66.

A description of grading curves as to type, as shown in Fig. 66, furnishes a basis for interpreting, correctly, the influence of the distribution of particle sizes on density. Type *A* is a narrow-range material represented by a nearly vertical grading curve. Type *B* has a statistical normal distribution of grain sizes, or nearly so, as illustrated in Curves 3, 4, and 5 of the core materials of Quabbin Dike in Fig. 66. Type *E* is predominantly fine with a little coarse material and is skewed toward the fine fractions, as shown by Curve 2 of the beach material. Type *F* is predominantly coarse with some fine material and is skewed toward the coarse sizes as shown in Curve 1 of the beach material. Type *D* material, which is typical of 80% of the borrow-pit materials of the Quabbin Dike, is a mixture of two fractions, one very coarse, and the other quite fine. Such materials are always characterized by a hump in the grading curve and they are typical of certain soils in glaciated regions. It is interesting to note that the percentage corresponding to the mean size is not now an arbitrarily defined constant quantity, but varies with the type of grading curve in a characteristic fashion and in reality represents a skew factor, 45 to 55% being typical of the Type *B* grading curves, 55 to 65% of the Type *E*, and 35 to 45% of the Type *F* grading curves.

Soils may be alike in the foregoing important respects but may vary in the range of particle sizes. The writer, therefore, recommends that the size characteristics be defined finally in terms of the range of sizes, the designation of

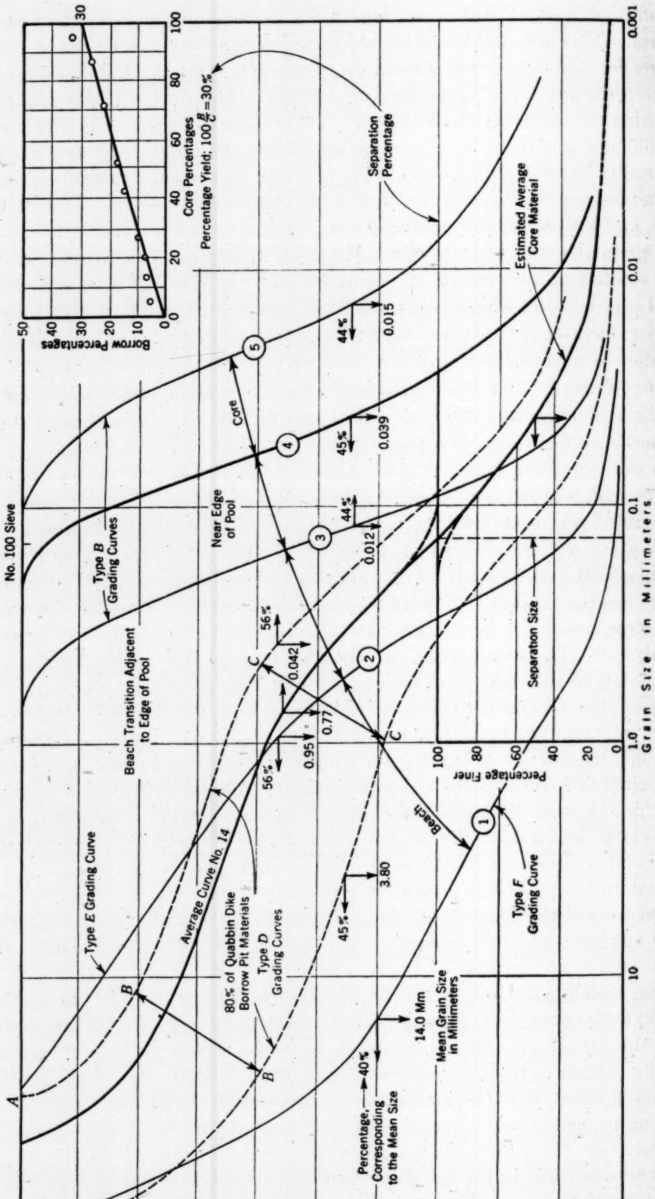

FIG. 66.—MATERIALS OF BORROW-PITS, BEACHES, AND CORE OF THE QUABBIN DIKE, SHOWING: (*a*) MEAN GRAIN SIZE; (*b*) CORRESPONDING PERCENTAGE; (*c*) TYPE OF GRADING CURVE; AND (*d*) ESTIMATION OF YIELD OF CORE MATERIAL

range to take in the entire curve being conveniently expressed as the ratio of the 95% to the 5% size. Thus, there is an important distinction between the shape of the grading curve and the range of sizes, which must not be confused. There is an interesting, characteristic, and progressive decrease in the range of sizes from the outer slope toward the center of the core pool.

Core Materials.—The sluicing-bin method described by Mr. Dore for estimating the yield and quality of the beach and core materials from a particular borrow-pit material or mixture of materials is a valuable one. The yield of core material may also be estimated by a simple analysis[66] of the aforementioned grading curves of borrow-pit materials, which, for eight dams, shows that there is quite a definite and complete separation point of materials by the hydraulic process at about the No. 100 sieve. The approximate percentage of yield of the average borrow-pit material of the Quabbin Dike is obtained in Fig. 66 by plotting percentages finer by weight of borrow material against those of the average core material (Curve 4) for a number of corresponding particle sizes, as shown in the insert.

The slope of the linear part of the curve defines the yield (30 per cent). The grading curve of the core material may then be fitted to the fine portion of the average borrow-pit material by proportional dividers, 30% becoming 100%, as shown. The yield and approximate separation size for nine dams are listed in Table 9. Thus, a study of curves of borrow-pit materials, for example, using the No. 100 sieve as the separation size, would indicate which materials to select or what mixtures to use in order to give the theoretical core width at any height, in the dam, as indicated in Fig. 18, as well as the approximate grain-size distribution of the average core material.

TABLE 9.—APPROXIMATE PERCENTAGE YIELD OF CORE MATERIAL

Item No.*	Name of dam (1)	Yield percentages (2)	Separation grain size, in millimeters (3)	Item No.*	Name of dam (1)	Yield percentages (2)	Separation grain size, in millimeters (3)
1	Henshaw†	41	0.16	10	Saluda‡	58	0.10
2	Swinging Bridge†	36	0.17	12	Alexander†	49	0.014
3	Dwinnell†	24	0.15	13	Cobble Mountain†	40	0.17
4	Davis Bridge‡	45	0.13	14	Quabbin Dike	30	0.14
9	Tieton‡	12	0.12	..	No. 100 sieve	..	0.148

* See Table 4. †Hydraulic-fill dams. ‡Semi-hydraulic fill dams.

A study of the data on core materials of existing dams in Fig. 16 reveals a number of significant facts: (1) The core materials of practically all existing dams are typically *B* Type: (2) all the core materials have about the same range of particle sizes, and the range is relatively narrow; and (3) the major distinguishing characteristics then are the degree of fineness as defined by the mean grain size and the clay content, which furnish a clue to the quality of the core material that can be washed out of any given borrow-pit material. It is important to note the characteristic gradation of the core material in Curves 3, 4, and 5 for the Quabbin Dike. However, physical factors other than grain-size distribution and fineness of the material passing the No. 100 sieve, may be of much greater importance. The liquid limit and plasticity index of the fine fraction furnish a more direct and useful index of the character and quality of the core material than can be obtained by the hydrometer test,[66] the practical limits being: Liquid limit, 15 to 40; and, plasticity index, 0 to 20. Values greater than these would indicate undesirable qualities in the core material and would indicate that difficulty might be experienced during, or subsequent to, construction; or it would dictate that fines should be wasted, where practicable.

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[66] For a full description of such grading curves, see "A Study of the Physical Characteristics of Soils, with Special Reference to Earth Structures," by D. M. Burmister, Assoc. M. Am. Soc. C. E., *Bulletin No. 6*, Research Laboratory, Dept. of Civ. Eng., Columbia Univ., June, 1938.

Beach Materials.—The quantity of beach material that can be obtained from a given borrow-pit material can be estimated from the yield on the No. 100 sieve. The quality of the beach material, which is the most important consideration for stability of the entire structure, is determined, primarily, by the "grading density relations" of the materials deposited in the different sections of the dam. Arthur Casagrande,[67] Assoc. M. Am. Soc. C. E., has shown that the density of a material has an important influence on the angle of friction and on the stability of materials placed in embankments. The density of a given soil will lie between the limits of some loose condition and some very dense condition, whether placed artificially in an embankment by some mechanical equipment, or by the hydraulic process. Investigations of the grading-density relations[66] of materials show that the distribution of particle sizes exerts a most important influence, as noted in Fig. 67.

The mean grain size of each type of material, as the range of particle sizes varies, is indicated on the grading curve. The loose and dense conditions for each material are then plotted directly below on the vertical line through this mean grain size. The conclusions on the grading-density relations are summarized as follows:

1.—Degree of Fineness.—For the same type of grading curve, range of particle sizes, and particle shape, the density decreases with fineness approximately according to the trend of the zone lines.

2.—Type of Grading Curve.—Other factors being equal, Type *F* materials are invariably the most dense, and approach the ideal grading for maximum density. Types *B* and *E* are of intermediate density; and Type *A* materials are always the least dense.

3.—Range of Particle Sizes.—The most dense condition, other factors being equal, is obtained by the material having the greatest range of particle sizes. The narrow-range Type *A* materials are always the least dense.

4.—The Spread Between the Loose and Dense States.—There is a characteristic spread between the loose and dense states for each material.

5.—Particle Shape.—The effect of angularity of particles is always to decrease density in both the loose and dense conditions.

Fig. 67 is a graphical expression of the fact that each material can exist in a certain loose condition in its natural state or in embankments and can be

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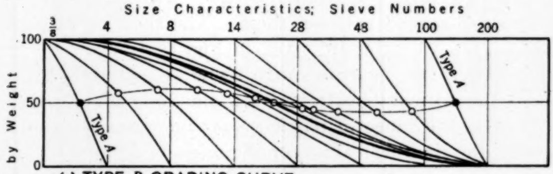
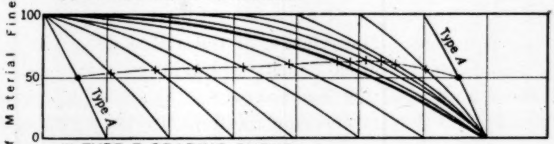
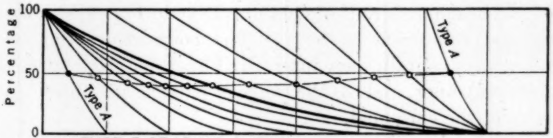
[67] "Characteristics of Cohesionless Soils Affecting the Stability of Slopes and Earth Fills," by Arthur Casagrande, *Journal*, Boston Soc. of Civ. Engrs., January, 1936.



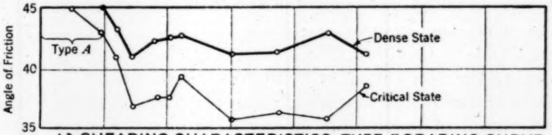
(a) TYPE *B* GRADING CURVE



(b) TYPE *E* GRADING CURVE



(c) TYPE *F* GRADING CURVE



(d) SHEARING CHARACTERISTICS; TYPE *F* GRADING CURVE

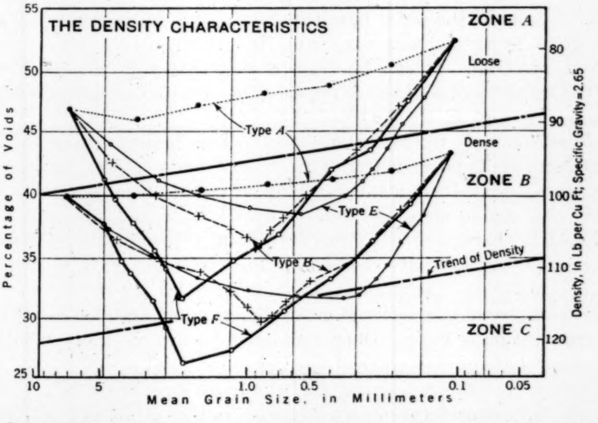


FIG. 67.—GRADING-DENSITY RELATIONS OF DRY GRANULAR MATERIALS, SHOWING EFFECT OF TYPE OF GRADING CURVE AND RANGE OF GRAIN SIZES

compacted only to a certain dense state by the particular method of placing, with a large or small spread between them. The shearing characteristics of Type *F* materials, for example, are given in Fig. 67 to show the influence of these factors. The coarse, narrow-range Type *A* materials are practically always at about the critical density[67] with a relatively high angle of friction, of about 45°, but as the range of particle sizes increases, density has an important influence, the limiting values for the angle of friction being about 37° for the critical density and about 42° for the dense state. These values appear to be independent of the degree of fineness and of the range of particle sizes, where the range increases. The relation of the actual density in place, of a given type of material to its critical density will indicate which angle of friction will control the stability of the embankment. Professor Casagrande[67] has shown that the critical density-shearing relations are very important factors in stability, particularly for certain fine-grained, saturated, cohesionless soils.

A study of the beach materials of Quabbin Dike in Fig. 66 reveals a number of interesting and significant facts: (1) The coarsest fractions, which are deposited first, always contain some intermixed finer material; (2) the materials become progressively finer; and (3) the range of particle sizes becomes narrower toward the edge of the core pool.

The question may be asked, whether the gradations of beach materials in Fig. 66 (Curves 1, 2, and 3) are typical of the hydraulic process or whether they are typical of what the hydraulic process will separate out of the Type *D* borrow-pit materials of Quabbin Dike. Keeping the foregoing three factors in mind and holding the No. 100 sieve point fixed as the lower limit of the range of particle sizes of the beach material, an upper limit is moved progressively along the 80% band of borrow-pit materials from the coarsest sizes in some proportion to the distance from the discharge point on the beach. From Point *A* to Point *B* the outer beach material must be typically of the coarse, wide range (*F* Type) and may vary from 20% to 50% of the bulk of the Type *D* borrow-pit material. It can be readily seen that this part of the curve is similar to Curve 1 and is of the *F* Type. The density for such material may be estimated from Fig. 67, by the trend of the zone lines, to be greater than 130 lb per cu ft as the tests by Mr. Dore show. This indicates that the hydraulic process places a Type *F* material in a very dense state. In the Quabbin Dike these materials form a substantial part of the entire beach.

When the coarse sizes are practically all deposited, the beach material must change in character from Point *B* to Point *C* to typically the finer *E* Type of material, having a density of about 115 to 120 lb per cu ft, and representing about 15% to 25% of the whole. Then, from Point *C* toward the edge of the pool the character of the material must change again and become typically the fine, narrow-range, *B* Type with an estimated density of 110 lb per cu ft, or less. It is evident that the character of the beach material is determined to a large extent by the type of grading curve of the borrow-pit material.

For stability considerations an angle of friction of 45° for the very coarse, Type *F* outer beach material, is to be expected, but as the materials grade into the finer *E* and *B* Types, the density not only decreases, but these finer materials may or may not be placed in a dense state by the hydraulic process and may even be less dense than the critical. The angle of friction may then be considerably less than 45°, and this would become an important consideration in the stability of the structure. However, in the Quabbin Dike an angle of friction of less than 30° was actually required in the beach material as a whole for stability, because the beach section was large.

It is believed that the information from soil investigations, together with the experiences accumulated in the construction of earth structures, when summarized, correlated, and made generally available, would be of permanent value as a guide for future earth dam construction. This means, however, the careful and complete description and identification of the materials used, and the determination of their density, shearing resistance in place, and behavior, so that the experiences gained in one locality may be of practical value in solving the problems in another.

DONALD W. TAYLOR,[68] ASSOC. M. AM. SOC. C. E. (by letter).—The civil engineer who has a general interest in soil mechanics will find much of value in the paper by Mr. Buchanan, which in addition touches upon a number of details of importance to those whose main interest is in this subject. Outstanding among such items is the effect of seepage forces upon stability of embankments. It may be noted that the effect of seepage may be handled by either of two methods which are closely related. It can be demonstrated that the vector sum of the total weight (both soil and water) of any mass of saturated soil and the resultant of all water pressures along its boundary is equal to the vector sum of the submerged weight of the mass and the resultant of all forces due to seepage within it. The same relation may be re-stated as follows: The resultant boundary water pressure equals the vector sum of the resultant seepage force and full hydrostatic uplift. Mr. Buchanan has suggested the summing of seepage forces throughout the mass whereas other investigators have preferred the summing of boundary pressures. A paper[69] by the writer presented in 1937, suggests an approximate method for simple embankments which involves no complicated summations.

One statement which may be misleading appears in this paper (see following Fig. 3), concerning the force, F_f. The statement as given is true only if the figure, *e f d b*, is square. Since in general the figure is not square, it should be stated that "F_f is the force tending to cause displacement in the direction of flow, equal to h_f times the unit weight of water, times the average area surface, $\frac{1}{2}(f\,b + e\,d)$, of unit thickness."

LEE H. JOHNSON, JR.,[70] JUN. AM. SOC. C. E. (by letter).—In a clear and straightforward manner, Mr. Buchanan has presented a general method of applying soil mechanics to the design of earth levees. He is to be commended for this important contribution to the small group of publications of scientific methods for the design of earth structures. He closes his paper with the remark: "For new units involving materials with which there has been no experience, savings in time and cost are possible through the application of soil mechanics." The writer wishes to cite an instance from his personal experience in which there has been such a saving in cost.

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[68] Research Assoc. in Soil Mechanics, Mass. Inst. Tech., Cambridge, Mass.

[69] "Stability of Earth Slopes," by Donald W. Taylor, *Journal*, Boston Soc. of Civ. Engrs., July, 1937.

[70] Acting Dean; Prof. of Civ. Eng., School of Eng., Univ. of Mississippi, University, Miss.

One of the problems with which the U. S. Engineer Office, at Mobile, Ala., has been faced is that of protecting the City of Rome, Ga., from recurrent floods on the Oostanaula and Etowah Rivers which unite in the heart of the city to form the Coosa River. These floods are of much shorter duration than the floods on the Mississippi River and on other large streams. A study of gage heights for floods that occurred during a period of fifty years shows that high water during a flood on these rivers lasts only several days.

A preliminary design of the protection works and incidental structures was prepared and submitted for approval in the form of a preliminary report. The protection works provided for in this report consisted mainly of levees with a land-side slope of 1 on 4 and a river-side slope of 1 on 3. These slopes were chosen on the basis of a general knowledge of the soil in the vicinity of the river banks at Rome, and the short duration of the high water.

The preliminary report was approved and the final design authorized. It became feasible to analyze the soil to be used in the construction of the levees. Borings were taken along those sections of the banks of the Oostanaula and Coosa Rivers, which are to be used as borrow-pits for the levee material. Representative samples were sent to the Soil Laboratory of the U. S. Waterways Experiment Station, at Vicksburg, Miss., for analysis. There, the samples were classified by means of mechanical analyses and such properties of the soil as angle of internal friction, cohesive strength, and coefficient of permeability were determined. A study of the stability of earth slopes composed of these materials, using the results of the laboratory tests, established the fact that the materials possessed sufficient shearing resistance to stand on a 1 on 3 slope with a satisfactory factor of safety. Furthermore, these materials were sufficiently impervious, by a very large margin of safety, to prevent flood waters from seeping through in the short time they were against the levees and threatening the stability of the land-side slope.

In view of these facts, the recommendation was made, and accepted, that the levees at Rome be built with a slope of 1 on 3 for both land-side and river-side slopes. This produced a saving of about 80 000 cu yd of earth and about $30 000, or more, in the cost of the levees. This example furnishes a striking illustration of the value of soil mechanics when applied to the design of levee units in new and unknown material.

The writer acknowledges his indebtedness to V. K. Wagner, Engineer in Charge of Design and Specifications, U. S. Engineer Office, Mobile, Ala., for making available and authorizing the use of this information.

GREGORY P. TSCHEBOTAREFF,[71] M. AM. SOC. C. E. (by letter).—It is interesting to note that three of the four papers of the Symposium treat the related subjects of levees, dikes, and embankments, and only one paper deals with foundations of bridges and buildings.

It would appear that this relationship reflects the actual situation. The efforts of most research scientists in the field of practical application of Soil

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[71] Asst. Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

Mechanics are directed toward solving problems of water-retaining works for the construction of which soil is used as an engineering material. The comparatively high total expenditure on such works, as well as the risk of particularly extensive damage to life and property in case of their failure, has led to greater research facilities being provided for this branch of the new science. The use of disturbed and recompacted soil material on actual constructions of the foregoing type facilitates the duplication of these field conditions in the laboratory. Furthermore, from the point of view of the organization of the necessary studies, this branch is not handicapped by the wealth of minor technical details and by obstacles created by professional and by misguided competing commercial interests, all of which are generally present when studies of foundations of structures are concerned.

A few years ago most scientists working in the field of Soil Mechanics published periodical appeals stressing the importance of full-scale settlement studies on buildings and bridges and deploring the difficulty of obtaining reliable data on the subject. Gradually, these appeals have diminished, apparently through sheer exhaustion of their authors, who have turned their attention to more well received subjects of study.

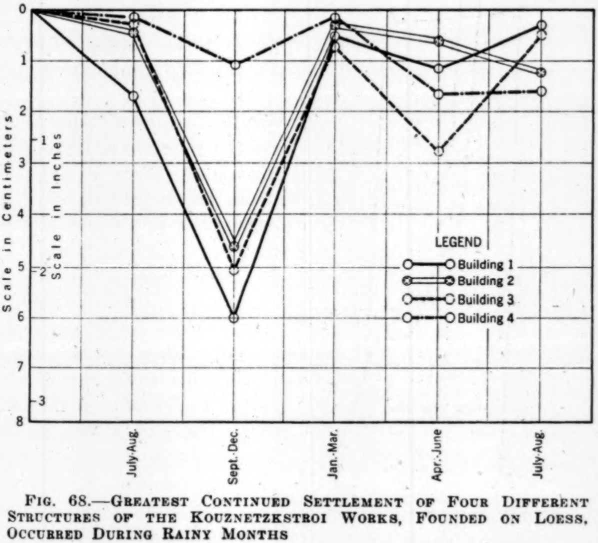

FIG. 68.—GREATEST CONTINUED SETTLEMENT OF FOUR DIFFERENT STRUCTURES OF THE KOUZNETZKSTROI WORKS, FOUNDED ON LOESS, OCCURRED DURING RAINY MONTHS

In view of this situation the paper by Professor Terzaghi is of more than outstanding interest. It outlines the knowledge recently acquired in Europe through the study of actual full-scale structures. It should be remembered in this connection that the proposal to make such systematic full-scale observations was first advanced in 1925 by Professor Terzaghi himself[72] for the purpose of the control, correction, and development of soil-testing methods; the necessity of such observations was stressed further in later publications of the Committee of the Society on Earths and Foundations in which he participated.[73] Professor Terzaghi not only made this proposal, but actually conducted such systematic observations and studies in Vienna and elsewhere. These studies and others which they have inspired (for instance, in Germany and in Egypt) form the only available source of information concerning the inter-related behavior of superstructure, foundation, and soil, as it actually occurs in Nature. Until the results of these observations became known, facts which are now generally recognized as such were contested vigorously by many engineers. The problems still awaiting solution in this field are likely to be clarified only by a continuation of similar systematic settlement studies in numerous other localities in different parts of the world, and by their correlation. Previously unsuspected facts are likely to emerge from the development of such observations in places where they had not been extensively made before.

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[72] "Erdbaumechanik," 1925, pp. 5, 386.

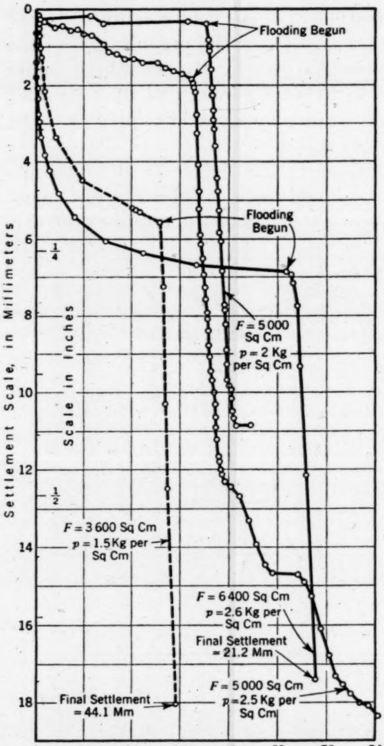

FIG. 69.—SETTLEMENTS OF TEST FOOTINGS ON LOESS SOILS FOUND TO INCREASE AFTER FLOODING OF SOIL SURFACE

Some such interesting examples are to be found in recent Russian publications. For instance, observations of structures founded on loess soils have revealed that the rate of settlement increased during the wet seasons—that is, in the autumn and in the spring.[74] Fig. 68 illustrates such observations on four different structures of the Kouznetzkstroi Works, in Siberia. It should be noted that this diagram does not show time-settlement curves in the usual manner of their presentation, but that the ordinates give the additional total settlement which occurred during periods of three months each. The large settlement during the rainy months was attributed to a breaking down of the porous structure of the loess in the upper moistened layers. Load tests (see Fig. 69), and other investigations in different parts of Russia showed that this was actually the case (to a varying degree for different types of loess) when the soil was loaded, but that preliminary wetting alone, not accompanied by the application of load to the soil surface, did not produce this break-down. Sand piles were used for structures on such loess soils with satisfactory results, as proved by full-scale settlement observations.

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[73] Progress Rept. of the Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 778.

[74] "Praktika Stroitelstva na Lessovidnih Gruntah," by Eng. I. M. Abeleff, *Onti*, Moscow, 1934 (in Russian).

The suggestion by Professor Terzaghi that the procedure for making settlement observations should be standardized is an important one and should be actively supported. The methods of observation he proposes should be adopted.

Several hundred reference-point leveling plugs of the type designed by Professor Terzaghi and illustrated in Fig. 42 were used for settlement studies in Cairo and in some provincial cities of Egypt, with which the writer was connected. These plugs were found to be very convenient and practicable when levelings were to be begun after the structure had reached the surface of the ground—that is, in most cases. A special method of leveling must be designed to meet different conditions only when a relatively considerable portion of the weight of the structure is beneath the ground level and when, therefore, a study of the early part of the time-settlement curve appears to be advisable from the very moment the foundation raft or the pile-capping is cast.

In Egypt the writer also had the opportunity of using the precision water-level system illustrated in Fig. 42. It was found to be invaluable for observations inside a building where the use of an optical instrument is impossible in most cases. The rubber hose of this water level could be guided easily around several corners of narrow corridors and small rooms or around machinery (in the case of factory buildings). Some precautions were required when this hose came to rest alternatively on floors exposed to the sun and in the shade. At first, the thermal expansion or contraction of the hose then caused slight fluctuations of the level of the water in the glass cylinder, which could produce errors in the readings unless these readings were carefully timed. It is probable that an increase in the diameter of these glass cylinders to at least three or four times the value of the diameter of the rubber hose would decrease the aforementioned fluctuations of the level of the water in the cylinders, and, therefore, would constitute a desirable minor improvement of this valuable instrument for special use in hot climates.

It is to be hoped that Professor Terzaghi's paper will stimulate interest in the United States for the development of systematic regional settlement studies of new structures erected on compressible deposits. Organized effort is required to obtain tangible results.

WILLIAM L. WELLS,[75] JUN. AM. SOC. C. E. (by letter).—Mr. Buchanan is to be congratulated on his presentation of the Mississippi River levee problem, and for his suggestions for improving the design of future developments by the application of a knowledge of soil behavior. Although, as a whole, the present sections have satisfactorily passed the test of actual usage, it is believed that better designs, for future extensions, can be effected by use of the rational methods offered by soil mechanics. The writer wishes to discuss briefly his observations of levees at various localities and certain studies which have been made by him. Each design feature will be discussed separately.

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[75] Asst. in Chg., Soils Section, U. S. Engr. Office, Memphis, Tenn.

Side Slopes.—The method of designing the side slopes is based on determination of soil constants and computation of the stabilities of various slopes of a given height by means of the so-called "method of slices." The position of the center of the critical failure surface must be determined by trial and error. A tedious graphical construction is required for each trial. The method advanced by D. W. Taylor, Assoc. M. Am. Soc. C. E., in 1937,[76] is a tremendous improvement over the graphical procedure. Mr. Taylor has determined the position of the center of the critical failure surface for different slopes and values of ϕ, the angle of internal friction of the soil in question. He has obtained a mathematical solution for the stability analysis and has shown his results in the form of nests of curves, by means of which a stability problem can be solved in a few minutes after values of soil constants have been determined.

Mr. Buchanan mentions briefly the possible effects of forces, created by seepage, upon the stability of the side slopes. A few studies made by the writer indicate that consideration of these forces is of the utmost importance. This is borne out by the results of Mr. Taylor's studies. These forces are created by the loss of potential in the direction of flow. The force acting upon a soil mass of a given volume is proportional to the hydraulic gradient of flow through that mass and acts in the direction of flow. Fig. 70 illustrates the general directions of seepage forces throughout the cross-section of a levee assuming that flow through the section is established during the high-water period.

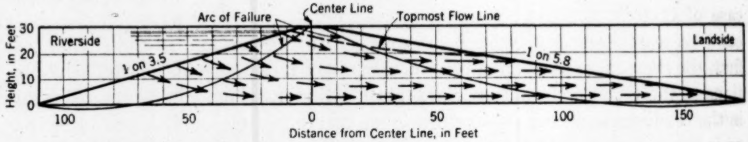

FIG. 70.—DIRECTION OF SEEPAGE FORCES THROUGHOUT CROSS-SECTION, STEADY SEEPAGE CASE

As can be noted, the effect of these forces is to increase the stability of the river-side slope and decrease the stability of the land-side slope. This condition of seepage, through the land-side slope, has been designated as the "steady seepage case" by Mr. Taylor. His studies and those made by the writer indicate that, for the land-side slope, this is the most dangerous condition that can occur. Consequently, the land-side slope should be designed so as to be stable when subject to this condition of seepage.

Fig. 71 illustrates the general directions of seepage forces throughout the cross-section immediately after the passage of a flood and consequent drawdown of impounded water.

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[76] "Stability of Earth Slopes," by Donald W. Taylor, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXIV, No. 3, July 3, 1937.

The effect of seepage forces in this case is to decrease the stability of both the river-side and land-side slopes, the former being affected to a greater extent than the latter. Mr. Taylor has designated this condition of seepage, through the river-side slope, as the "sudden drawdown case." The studies,[76] previously mentioned, indicate that, for the river-side slope, this is the most dangerous condition that can occur. Thus, the river-side slope should be designed so as to be stable when subject to this condition of seepage. These studies also indicate that the "sudden drawdown case" is more dangerous than the "steady seepage case" when the draw-down period is of less duration than the period required for development of flow through the section. This condition generally exists in the Mississippi Valley. If homogeneous material is assumed, it follows that the river-side slope should be of less inclination than the land-side slope in order to obtain the same degree of stability for both slopes. The river-side slope most commonly used for levees in the Lower Mississippi Valley is one having an inclination of 1 on 3.5 (*B* section). Experience has shown that, in the great majority of cases, this slope is stable when subject to flood conditions. Thus, it appears that, in so far as stability is concerned, a land-side slope having an inclination somewhat greater than 1 on 3.5 would be adequate.

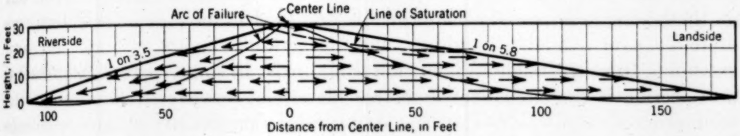

FIG. 71.—DIRECTION OF SEEPAGE FORCES THROUGHOUT CROSS-SECTION, SUDDEN DRAW-DOWN CASE

Control of Seepage.—As Mr. Buchanan states, seepage through the levees and its control have formed the bases for the design of the standard levee sections. Land-side slopes have been flattened in order to prevent out-cropping of seepage on this slope. It is interesting to note that in the case of pervious levees on relatively impervious foundations, model and other studies[77] indicate that a reduction in the inclination of the land-side slope results in a reduction in the inclination of the slope of the topmost flow line; however, the amount of seepage through the structure is decreased. For instance, assuming a levee 30 ft high, with a 10-ft crown, a river-side slope of 1 on 3.5 and a 1-ft free-board, the height of the point of emergence of the topmost flow line above the base, in the case of a land-side slope of 1 on 3.5, is 16.5 ft, whereas for a land-side slope of 1 on 5.8 (*B* section), it is 17.4 ft. The seepage passing through the structure is reduced about 35% by the reduction in slope. In the case of levees founded on relatively pervious foundations, the position of the topmost flow line appears to depend largely upon under-drainage conditions. If drainage conditions are such that a change in the inclination of the land-side slope would affect the position of the topmost flow line, it is probable that a reduction in the inclination of the land-side slope would cause a reduction in the inclination of the topmost flow line.

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[77] "Hydraulic-Fill Dams," by Glennon Gilboy, Assoc. M. Am. Soc. C. E., First Cong. of Large Dams, Vol. 4, Rept. No. 46, pp. 231–267.

A reduction in the inclination of the land-side slope causes an increase in the average length of the path of percolation; hence, there is an increase in the time required for the development of flow. Observations made during the 1937 flood in the Memphis (Tenn.) Engineer District showed, in general, no apparent seepage through the levees, except in the case of sand levees; hence, it is evident that the average length of the path of percolation and permeability of material for the *B* section are such that flow does not develop during a normal flood period. It is not surprising that, in some cases in the past, when steeper land-side slopes were used, a dangerous volume of seepage cropped out on the land-side slope. This is probably due to the fact that no selection was exercised over available materials, and, as a result, layers or lenses of sand extending from river side to land side would exist in some cases. At present, no selection is exercised over available materials; however, reduction in the inclination of the land-side slope has decreased the probability of the occurrence of continuous sand lenses.

For future extensions, it is believed that this purpose could be more economically accomplished by the use of steeper land-side slopes combined with underdrainage of these slopes largely accomplished through selection and proper placement of available materials. Should seepage develop through these sections during flood periods, its amount would be very small and it would not adversely affect the safety of the structures.

In the case of sand levees, the method used by the Vicksburg (Miss.) District for controlling excessive seepage seems to be excellent.

Under-Seepage.—Uncontrolled under-seepage, and resultant "sand boils," are by far the major causes of trouble along the levee system during flood periods. The fundamental cause of "sand boils" has been ably discussed[78] by L. F. Harza, M. Am. Soc. C. E. He demonstrates that "sand boils," or flotation, can occur only when the hydraulic gradient at the point of escape of under-seepage on the land side equals $(1 - P)(S - 1)$, in which P = the percentage of voids of the material, and S = the absolute specific gravity of the material. For all sands, the value of this expression is about 1.0.

The *B* section has a base width of 10 H in which H is the maximum design head against the structure. The base width is the shortest path for under-seepage and the average hydraulic gradient along this path is, therefore, $\frac{H}{10\,H}$, or 0.1. Under these circumstances, it would appear that "sand boils" could not occur unless foundation conditions, or the plan of construction were such as to create high escape gradients on the land side of the levee. A few typical "sand boil" areas have been inspected by the writer and, in general, the following has been found: Subsoil conditions consist of a shallow (5 ft to 10 ft) depth of relatively impervious top-soil under-laid by a deep stratum of sand. River-side borrow-pits have been excavated into this impervious top-soil, thus exposing the sand stratum directly to water. The impervious blanket is continuous on the land side. Under these conditions, the quantity of under-seepage is limited by the quantity that can pass up through the impervious land-side blanket and, as a result, the loss in head between the river-side pit and the bottom of the impervious land-side blanket is negligible. Then, the major part of the head on the structure is dissipated in upward flow through the land-side blanket. This would tend to lift and disrupt the blanket. A "sand boil" would occur at any weak point in the blanket, that is, through a crack, fissure, or hole due to these conditions and concentration of outflow at the weak point. The presence of high hydro-static pressures beneath the land-side blanket, as indicated by flowing wells, was noted by the writer during the 1937 flood in typical "sand boil" areas.

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[78] "Uplift and Seepage Under Dams on Sand," by L. F. Harza, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1352.

For future extensions, where these foundation conditions prevail, it is believed that these difficulties can be avoided by leaving the impervious blanket intact on the river side and allowing drainage of the under-lying sand on the land side by locating borrow-pits there. Under these conditions, practically all the head against the structure would be lost in flow downward through the river-side blanket, and, consequently, there would be no tendency for "sand boils" to develop on the land side. This procedure might increase the volume of under-seepage. However, the quantity would be small provided the river-side blanket were continuous for a reasonable distance (500 ft to 1 000 ft) away from the levee. Assuming a 30-ft head, an underlying stratum of coarse sand ($K = 500 \times 10^{-4}$ cm per sec) of 200 ft thickness, and a length of blanket of 800 ft, the volume of seepage would be about 0.0083 cu ft per sec per ft of levee. Since these unfavorable foundation conditions would prevail for only relatively short distances, the drainage system behind the levees would probably be more than adequate for the disposal of seepage.

The prevalent method of strengthening such weak places in the existing system is placement of a blanket over the affected area on the land side. At present, experiments with river-side blankets and seals are planned.

Stability of Foundations.—This phase of earth dam or levee design has been completely covered by Mr. Hough. Under the heading, "Laboratory Testing," he states that he made no effort to determine an apparent angle of internal friction for clay samples. The writer believes that no effort should be made to determine this factor for any type of material since the value obtained (quick shear tests) is meaningless in practically all cases and usually not on the safe side in the case of non-saturated silts when tested according to more or less standard procedure (vertical loads of 0.2, 0.4, and 0.6 ton per sq ft). This seems to be due to the fact that, in a given time of test, the percentage consolidation increases with an increase in intensity of vertical loading. Thus, the apparent angle of internal friction is often considerably greater than the true angle and serious error would result if extrapolated values are taken from the shear curve. Shear strength of material in place should either be determined from the results of quick shear tests performed at the least vertical loads practicable, or from the results of delayed or consolidated shear tests combined with a determination of pre-consolidation pressures.

Mr. Hough's model and analytical studies are most interesting. Fig. 41 summarizes the three well-known methods of determining shearing stress in the foundations of earth dams. Jurgenson's formula for the "rigid boundary" case cannot be expected to yield exact results since, in addition to the assumptions mentioned by Mr. Hough, vertical loading is assumed. The fallacy of this assumption has been discussed[79] by Glennon Gilboy, Assoc. M. Am. Soc. C. E. According to this formula (Equation (9)), the shearing stress induced in a given foundation by dams having the same side slopes but different heights, is constant. If the reaction, or total earth pressure, between the two halves of the dam is considered as the cause of shearing stresses, it can be demonstrated that the average shearing stress along the base of a dam, for the foregoing case, is proportional to the height of the dam. In computing earth pressures, if a hydrostatic pressure ratio, K, of 0.5 is assumed, it can be demonstrated that these stresses exceed Jurgenson's stresses when h is greater than $4\,a$. Thus, in the case of high dams on relatively shallow plastic foundations, Jurgenson's method, the photo-elastic method, or the Haines method would indicate small stress intensities, whereas actual stress intensities might be of considerable magnitude. In the same manner, it can be demonstrated that, for the "infinite depth" case, average shearing stresses along the base of the structure can exceed Jurgenson's values when L is less than h.

Since no assumptions are made in the case of the photo-elastic method, the results obtained by this method should be correct. Thus, it appears that the data from which the curve of Fig. 41 was plotted must have been obtained between the aforementioned limits.

None of these methods considers stresses induced in the foundation by water pressure acting on the reservoir side of the dam, nor the important stresses created by seepage or drainage forces in partly consolidated foundations. The effect of the former would probably be to increase shearing stresses in the landside foundation. The effect of the latter would depend largely upon direction of drainage; if horizontal, stresses would certainly be increased, possibly to a value, $p \times \frac{2\,a}{L}$ (assuming no consolidation at the end of a construction period, and a small ratio, $\frac{a}{L}$). The effect of these forces could be determined approximately by use of the Haines' method combined with flow nets.

The action of these forces explains the peculiar behavior of certain Mississippi River levees. These levees are impervious structures built on impervious soft foundations, generally overlaid by a 5-ft to 10-ft stiff clay blanket. In one case, heaving occurred 850 ft from the center line of the levee at a weak place in the overlying blanket. This behavior strongly suggests a fluid-like transmission of pressure. For future construction in such areas the placement of a drainage blanket over a part of the base seems to offer definite possibilities of relieving such conditions.

Valuable data concerning the behavior of soft foundations might be obtained by a continuation of Mr. Hough's studies. The factors discussed herein could be considered and their effects determined.

A. Streiff,[80] M. Am. Soc. C. E. (by letter).—There is an extraordinary difference in the flatness of slopes of the Mississippi River levees built by the

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[79] *Engineering News-Record*, February 10, 1938.

[80] Vice-Pres., Ambursen Eng. Corporation, New York, N. Y.

U. S. Army Engineers when compared with dikes abroad. Mr. Buchanan's instructive description is interesting and reveals this startling difference very clearly. In China,[81] Germany,[82] The Netherlands,[83, 84] France, Hungary, Italy, or other countries, dike slopes are much steeper, and volumes are much smaller. The Mississippi dikes are not higher than foreign dikes in several places.

In China, the Yellow River dikes have slopes of 1 on 2. In Cochin-China,[85] the river side has a 1 on 2, and the land side, a 1 on 3, slope. Steeper slopes also prevail in Germany and Denmark.[82, 86] In The Netherlands the inside slope, even on the most exposed sea dikes, is 1 on 1.75. A. Caland[83] states that the slope is determined mostly by the capacity of the bank to grow a good sod. The outer slope is determined by the location. Where exposed to heavy gales, the outer slope may be as flat as 1 on 12, but along the rivers this consideration does not exist and the slopes are 1 on 2.

It is interesting to note that another branch of the U. S. Government uses slopes of 1 on 2 for the dikes along the All-American Canal at the Imperial Dam. These slopes compare with 1 on 6 and 1 on 8 along the Mississippi River! The difference in yardage is enormous. Clay is said to have been unsatisfactory along the Mississippi. This can only be due to the fact that there apparently is no moisture control in the process of building. In The Netherlands, Germany, and Denmark, clay has been used extensively since Roman times. There are also many sand dikes, but these are always covered with clay. The present dikes through the open sea for the closure of the Zuider Zee are made of sand and boulder clay. The treatment described of Mounds Landing is very ancient practice in Dutch dikes. The writer has built many miles of clay dikes along the Saramacca Canal in Surinam, having slopes of 1 on $1\frac{3}{4}$ which proved quite satisfactory. The unsatisfactory experiences with clay along the Mississippi River can only be due to lack of moisture control.

The Dutch system, however, is more elaborate than the U. S. Army system along the Mississippi River, as described by Mr. Buchanan. A preliminary dike called "spekdam" is thrown up on the seashore. Behind this dike a ditch, called "verskade sloot," is dug deep enough to drain the borrow-pits. The borrow-pits are separated by strips, left in place, to prevent erosive currents along the dike. The ditch is drained or pumped. Clay is placed and rolled in convex layers draining to both sides. In this manner no trouble is experienced in obtaining satisfactory clay dikes.

On the great rivers of Cochin-China[85] the outer slope (1 on 2) was provided with a clay cover, rolled with corrugated rollers. The inner slopes are 1 on 3.

In the course of time the ancient art of dike building has been greatly perfected in those countries that were in need of it. Higher profiles are needed in many places, however, as Mr. Buchanan shows for the Mississippi River. In Europe, thirty-nine major catastrophes have occurred since the Twelfth

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[81] "Flood Problems in China," by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1405.

[82] "Wasserbaukunst," by Hagen, 1880.

[83] "Handleiding tot dyksbouw," by A. Caland, 1833.

[84] "Waterbouwkunde," by Storm Buysing, 1851.

[85] "Digues du Tonkin," by J. Gauthier, 1930.

[86] "Bau der Flussdeiche," by Paul Ehlers.

Century. The problem remains to be able to predict maximum flood heights. The art of building leaves little to be desired; but the means of protection are much more intricate than the single lines of dike shown by Mr. Buchanan. An entire network of dikes exists in The Netherlands, forming auxiliary lines of defense. There are "sleeper" dikes, "bosom" dikes, inner and outer, winter and summer, river, and sea dikes. In many places slopes are protected in an elaborate manner.

The expedient of flattening slopes, which Mr. Buchanan describes, to make the structure stable in soft ground, is an ancient one. Caland, an experienced dike builder, writing in 1833,[83] describes the building of dikes on soil into which a stick could be thrust 12 ft without effort. He states that the side upthrust can be counteracted by flattening the slopes, and describes a case that occurred in Middelburg Harbor, in 1816. The Dutch have another ancient and successful method of constructing dikes on very soft foundations. It consists of covering the dike seat in the center by a "zink stuk," or sink piece of fascines. This prevents the lateral escape of the dike material, and of the foundation.

Doubtless the great differences in design in Europe and the United States are due to differences in local conditions. In the low countries of Europe, river engineering is a special trade, just as coal mining or weaving. Fascine work has been developed to a specialty. Dikes 40 or 50 ft high have been constructed between two submerged dikes of fascines, made of sink pieces. There is a great variety of methods of slope protection. In that manner the Dutch are able to construct clay and sand dikes through a depth of 25 or 30 ft of open sea, subject to the force of storms. Such methods would neither be possible nor economical in the United States where, apparently, an inordinate flattening of slope has been found to be the best solution.

The method of computing these slopes, described by Mr. Buchanan, does not seem to the writer to offer any better guaranty for their stability than that obtainable by older methods. Unfortunately, the great advances in soil mechanics are confined to the laboratory. The writer disagrees with Mr. Buchanan and feels that, for the present at least, the application of soil mechanics has caused no visible progress in: (1) The art of foundation design; and (2) the methods for computing soil stability. The first has always been quite adequate, and computation methods are as approximate as they ever were.

The practical art of constructing earthwork has been entirely successful since ancient times. Feeder dams on the Grand Canal in China, built 500 yr ago on alluvial soil, are still in use.[81] In the Indian Province of Madras alone there are 43 000 native earth dams used for storage reservoirs, of a total length of 30 000 miles (more than nine times the length of the Mississippi dikes) containing 200 000 separate masonry outlet works. One of these, the Madduk Masur Dike, has stored 800 000 acre-ft every monsoon for 400 yr.[87] The entire Netherlands has been diked since the Middle Ages. Entire cities containing the heaviest medieval towers stand on ancient pile foundations in soft soil below sea level. Among the many dams constructed on soil foundations[88] in modern times may be mentioned the Alcona Dam, built on a fine quicksand foundation 100 ft deep under artesian pressure, and successfully holding 40-ft head since 1923. None of these works needed the modern soils laboratory.

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[87] "Ways and Works in India," by G. W. MacGeorge, 1894.

[88] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), pp. 1303–1307.

That the computation methods are as approximate as they ever were is admitted by the foremost workers in this field. Thus, Prandtl[89] states that his theory on plastics is no longer applicable if stress changes with deformation (which condition generally prevails in soils). Professors Terzaghi and Froehlich[90] as late as 1936 admit that "the investigations perforce have the character of approximations for the purpose of obtaining a rough guess as to the settlement expected [writer's translation]."

Mr. Buchanan states that earth dikes on impervious strata can "now" be analyzed by methods such as the one developed by Glennon Gilboy, Assoc. M. Am. Soc. C. E.;[91] but it appears that in this, and in another, paper Professor Gilboy only applies the methods of Rebhahn[92] and Coulomb[93] and indeed specifically mentions that fact. In one of these two papers[94, 91] Professor Gilboy resorts to doing by trial and error what Coulomb did in 1821 by differentiation. The soils laboratory cannot be credited with developing ancient methods that have simply survived to the present time.

The efforts to compute soil stability date from 1764[95] and before. Since Moeller's book on earth pressures was published[96] nothing has been added which is any more serviceable than was stated therein. Retaining walls everywhere have been built with success by these theories; the 90-ft reinforced concrete retaining wall at the Junction Dam on a silt foundation[97] ranks among the highest examples.

Mr. Buchanan applies the method of Fellenius to the dike slope. Fellenius, in his book, "Erdstatische Berechnungen," attempts to introduce shear or cohesion and friction separately. The latest edition of Emperger's "Handbook for Reinforced Concrete" states that this leads to contradictions. The method referred to by Mr. Buchanan concerns circular shearing planes. Such shearing planes are already to be found in Coulomb's original treatise (1821).[93] Coulomb preferred the plane shear surfaces, considering the entire wedge-shaped sliding earth mass as one piece for the simple reason that it lends itself more readily to differential calculus. Fellenius, by using circles, has to resort to a trial-and-error process in order to obtain the conditions for a minimum stability; but Fellenius' method suffers from many more approximations than the Coulomb method. He attempts to divide the slope segment in vertical slices which are supposed to be subject only to the weight and the friction at the base of the corresponding strip. This is definitely a gross approximation as each strip is also subject to lateral pressure on both sides, with opposite directions of friction, which are not equal and cannot be omitted in a rigorous solution. The bases of Strips 8, 9, 10, and 11 (Mr. Buchanan's Fig. 8) are subjected to more pressure than merely the weight of the strips. Coulomb, therefore, preferred to regard the sliding mass as one piece. A sub-division into strips without considering lateral pressures is a rough approximation that does not approach closer to the truth. If the vertical pressure is unknown, a division of resistances into shear and friction is also not possible. Engineers, therefore, are as far from achieving an exact solution with Fellenius' method as they were with Coulomb's solution. No one can say which method gives closer results. In addition, a laborious trial-and-error method must be used to obtain results as open to question as ever.

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[89] International Congress for Applied Mechanics, Delft, 1924.

[90] "Theorie des Setzung von Tonschichten," by Charles Terzaghi, M. Am. Soc. C. E., 1936.

[91] "Hydraulic Fill Dams," by Glennon Gilboy, Vol. 4, No. 46.

[92] "Theorie des Erddruckes," by Rebhahn, Vienna, 1871.

[93] "Theorie des Machines," by Coulomb, Paris, 1821.

[94] "Mechanics of Hydraulic Fill Dams," by Glennon Gilboy, *Journal*, Boston Soc. C. E., Vol. XXI, No. 3.

[95] "Recherches sur la construction des digues," by Bossut and Viallet, Paris, 1764.

[96] "Erddruck," by M. Moeller, 1902.

[97] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1321.

The Dutch method of building dikes on very soft foundations, by means of a sink-piece in the center of the dike, utilizes the upthrust in the center to hold the sink-piece while preventing the downward movement on both sides of the center by means of the strength of the sink-piece. It is self-evident that computing the strength of the foundation merely by multiplying the depth of the clay foundation with the vertical load per square foot and dividing this by the allowable shearing stress, in order to obtain the base width of the dike, can be classified only as a rule-of-thumb. The actual conditions are far too complicated to be computed accurately by this means. The writer uses Rebhahn's method in such a case. The shear plane proposed by Mr. Haines as reported by Mr. Hough in Fig. 38 can conveniently be rectified into two straight lines joining in the perpendicular through the toe of the dike. The active pressures on one side must balance the passive pressures on the other and can easily be obtained graphically.

Soil mechanics, at least to the present, has not visibly enriched the "tool box" of the practicing engineer. Nevertheless, continued research remains of the greatest importance in spite of the paucity of practical results. A sober, critical examination finds recent enthusiastic reviews subject to considerable discount; but tests such as those described in 1923, in the very complete paper[98] by Jacob Feld, M. Am. Soc. C. E., will always be of great value.

SPENCER J. BUCHANAN,[99] Assoc. M. AM. Soc. C. E. (by letter).—The discussions contributed to this paper, both as regards the number and their contents, are gratifying to the writer and are indicative of the general interest in the use of the relatively new tool—soil mechanics—in solving unusual problems arising in connection with levee design. It is natural that some should disagree with the principles advanced.

It is the desire of the writer to add the name of Arthur Casagrande, Assoc. M. Am. Soc. C. E., as one who contributed much toward the development of the device, the details of which are shown in Fig. 6, for obtaining undisturbed samples of soil.

Mr. Knappen's comments regarding the levees in the Lower Mississippi Valley are most apropos. The writer concurs in the opinion that the general solution in so far as the design of levees in the Lower Mississippi Valley was sound at the time it was developed and used; also that much may be gained by giving consideration to the modifications indicated by recently acquired knowledge.

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[98] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.

[99] Assoc. Engr., U. S. Waterways Experiment Station, Mississippi River Comm., Vicksburg, Miss.

Mr. Strohl's discussion regarding the application of soil mechanics to levee design and presentation of typical problems encountered in the Lower Mississippi Valley are noteworthy. The opinion voiced by the writer regarding the lack of development of seepage through a levee during a flood period is substantiated in the problem illustrated.

Mr. Creager's remarks regarding the limitations in space for a paper of this kind are appreciated. The design of levees, particularly in the Lower Mississippi Valley, involves consideration of practically every aspect advanced in the field of soil mechanics. Hence, to illustrate the general idea, very simple illustrations were used. It is true that in some instances the solutions of problems are not as simple and direct as indicated.

The restriction to mere mention of the effects of seepage forces on the design of slopes of levees or dams was deemed advisable inasmuch as at the time the paper was written, the writer felt (as indicated) that "further research" was needed. Some results are now available.

The stability of foundations as illustrated under the heading "The Design of a Levee Unit," is based on tests of specimens of the foundation medium in their undisturbed state. Effort is made to perform these tests so rapidly that no consolidation of the specimen occurs. Thus, the design is based not on the ultimate shearing strength that may eventually be developed upon complete consolidation, but upon the actual strength, in place, as nearly as may be ascertained.

The opportunity for full-scale research such as that mentioned by Mr. Feld is appreciated. However, as he states, it will be some time before the results of such studies can be reported. Much stress, of course, must be placed on field observations of full-scale structures as well as on a thorough investigation in the laboratory. Thus, the engineer should not veer to the "right" or to the "left" but follow a straight course. Mr. Feld is correct in stating that the surface of rupture developed during the failure of a slope is not always a cylindrical surface. However, it is the understanding of the writer, from discussion of this phenomenon with field engineers who have observed numerous such failures in the levees along the Lower Mississippi Valley, that cylindrical surfaces of rupture for these structures can be taken as typical. Each problem of this type must be considered separately. Attention is invited to the fact that the illustration regarding the design of a levee, in so far as the foundation is concerned, conforms closely with the basic assumptions as indicated by Mr. Jürgenson in his paper.[7]

Mr. Chang's presentation of the difficulties experienced in the use of loess for the levees in China is worthy of note. No complications have arisen in connection with the structure illustrated by Fig. 4 which is formed of loess.

The wealth of information contributed by Mr. Senour in his discussion of the design, construction, and maintenance of levees is most interesting. The writer concurs in the remark, "Experience is invaluable, of course but unless it is supplemented with a knowledge of the underlying principles that govern soil behavior, experience is distinctly at a disadvantage when it becomes necessary to design in a field not covered by prior operations." It is a thought that deserves serious consideration, particularly by those who maintain a pessimistic attitude regarding soil mechanics. Mr. Senour's descriptions of the equipment used for the construction of levees along the Lower Mississippi River should explain to many the difference in the design between the structures considered in the paper and those built elsewhere, particularly those constructed abroad. A hand-made levee, as compared with those constructed by machines of large capacity, would naturally result in structures different to some extent in shape and design.

Professor Taylor's treatment of the effect of seepage on the stability of side slopes, although it is only an approximate method for simple embankments, appears to be sound and meritorious. The statement following Fig. 3 concerning the force, F_f, is correct.

Mr. Wells' brief discussion of his observations of levees in various localities is interesting. The method of designing side slopes mentioned, as advanced by Professor Taylor, was not available at the time this paper was prepared. The writer concurs in the statement that this new method is an improvement over the graphical procedure. The basic paper appears to have served well one of the purposes intended by the writer in that it has brought into the open a discussion of general features such as those presented by Mr. Wells. Mr. Wells' comments regarding the plan of providing land-side drainage to assist in the solution of the problem of "sand boils" are worthy of consideration.

Mr. Streiff's comments are interesting in that they show how others have attacked the problem of levee design. It is natural that there should be considerable difference in the designs of levees in different parts of the world. Unfortunately, no information was given regarding the materials used and conditions encountered abroad. The writer does not agree with Mr. Streiff in his statement "for the present, at least, the application of soil mechanics has caused no visible progress." The interest manifested by the Engineering Profession both in the United States and abroad, involving foundations and structural stability of soils, appears more than to substantiate the writer's general attitude as expressed in the basic paper.

STANLEY M. DORE,[100] M. AM. SOC. C. E. (by letter).—The rating of soil grading curves or mechanical analysis curves into groups is particularly useful in connection with foundation or borrow-pit investigations and explorations. The method of rating described by Professor Burmister in his discussion of this Symposium and also in his discussion[101] of the paper by Frank B. Campbell, Assoc. M., Am. Soc. C. E., is similar in many respects to the Kendorco classification previously described[102] and later revised in the manner shown in Fig. 72 to include more classes of finer materials. The Kendorco system seems more simple, easier to apply, and more easily understood and used by engineers who are not soil laboratory technicians. Its usefulness has been tried under practical construction conditions by the Massachusetts Metropolitan District Water Supply Commission and by the Board of Water Supply of New York City.

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[100] Associate Civ. Engr., Met. Dist. Water Supply Comm. of Massachusetts, Boston, Mass.

[101] "Graphical Representation of the Mechanical Analyses of Soil," *Proceedings*, Am. Soc. C. E.. February, 1938, p. 397.

[102] "Permeability Determinations, Quabbin Dams," by Stanley M. Dore, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 690, Fig. 5.

A modification of this system has also been used by the U. S. Army Engineers in connection with their studies of flood-control dams and dikes for the Connecticut River and this modification was described in a discussion[103] of Mr. Campbell's paper by Frank E. Fahlquist, Assoc. M. Am. Soc. C. E., and Mr. Waldo I. Kenerson.

The use of mechanical analysis curves of borrow-pit materials for estimating the yield of core material that will occur in a hydraulic-fill embankment when that borrow-pit material is being used, has been made by the writer in a manner similar to that suggested by Professor Burmister. The writer has noted, however, that there may be a marked difference in the yield of core materials actually obtained in construction from that theoretically indicated by the use of the mechanical analysis curves in the manner suggested by Professor Burmister. Relatively large quantities of finer materials are trapped in the beaches and in the volumes between the beaches and the core. The theoretical estimate makes no allowance for this condition. Usually, the porosities of the core material are radically different from those of the borrow-pit materials, although, in many cases, the porosities in the beaches may not be dissimilar. In making any quantitative estimate, proper allowances for these differences in porosities must be considered. For these reasons, the writer does not accept the yield values given by Professor Burmister, for various hydraulic-fill dams, as correct. They represent neither yield in percentages by weight nor yield in

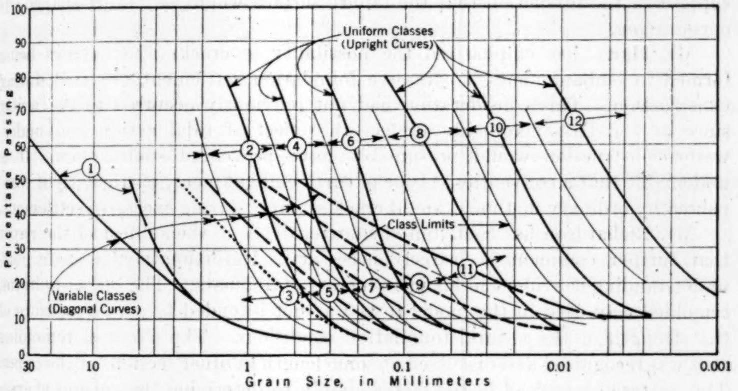

FIG. 72.—KENDORCO CLASSIFICATION

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[103] *Proceedings,* Am. Soc. C. E., May, 1938, p. 998.

percentages by volume, unless the volume of fines trapped in the beaches and elsewhere offsets the increase in volumes due to differences in porosities.

On the other hand, in the sluicing bin test, actual construction conditions are reproduced so that the yield results obtained should be representative.

The writer is in accord with many of the ideas expressed in Professor Burmister's discussion, in that the grading or the mechanical analysis curves can be used more widely to estimate, in advance, the quantity and quality of embankment construction to be expected, using certain borrow-pit materials.

BENJAMIN K. HOUGH, JR.,[104] ASSOC. M. AM. SOC. C. E. (by letter).—The several discussers of the paper have been helpful in making many valuable contributions to the subject. Dean Hollister has emphasized the gain in stability resulting from contact of the embankment with the rock surface underlying the plastic zone after complete displacement of the clay. His support of the prediction that the failure surface would be "heart-shaped" is encouraging.

Mr. Harza has emphasized the possibility of cracks and cavities being formed in embankments if excessive foundation settlement occurred during construction. This consideration had not previously occurred to the writer since at the Passamaquoddy project the effect of tidal action and underwater construction would presumably have prevented such action. It is undeniable that a conventional type of earth embankment, particularly if compacted by ordinary methods, would crack severely during excessive settlement.

Mr. Richardson has contributed so extensively to the subject of the paper that further comment is scarcely necessary. Unfortunately, a slight misunderstanding is evident in his introductory statements. The bed of uniform, remolded clay used in the model tests was not intended to give a measure of the strength of the natural foundation conditions. The effect of remolding clay was recognized and discussed at some length in other sections of the paper. This material was used in the model simply to determine the general characteristics of the failure surface or zone. Only qualitative results from the model studies were expected or obtained. Stress analysis was conducted analytically later with the model results as a guide.

Mr. Richardson's presentation of a method of stress determination based on the theory of elasticity is of considerable general value and interest, but the writer feels that in its present form it could not have been applied to the particular problem discussed in the paper. This problem was: (1) To design a section with sufficient base width to insure against foundation failure; (2) to determine how far an alternate, very steep-sided section would penetrate into the foundation as a result of foundation failure; and (3) to make an economic study of the initial and the alternate designs to determine their relative costs. For the first part of the problem, a method such as that described by Mr. Richardson could well have been utilized providing the effect of the rock surface or "rigid boundary" had been considered as it is in methods described by Messrs. Middlebrooks and Jürgenson. In the second part of the problem, the writer believes that these methods could not have been used at all.

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[104] Engr., U. S. Engr. Office, Ithaca, N. Y.

The apparent lack of suitable methods for determining settlement of embankments as a result of foundation failure was the primary reason for developing the Haines method.

The suggestion of Mr. Richardson that artificial mud waves be placed in the early stages of construction as a stabilizing expedient to prevent subsequent embankment settlement is noteworthy. This procedure might be adequate in certain cases but in the case of the Eastport (Me.) structures the initial sections (designed to prevent foundation failure) had base widths extending far beyond the mud waves anticipated for the alternate sections; and in these alternate sections final stability was dependent not only on the counterbalancing effect of the mud waves but also on displacement of soft clay by the much more stable embankment fill material, and decrease of the depth of the plastic layer. In some cases, computations indicated that this displacement would be so extensive as to result in penetration of fill material all the way to rock.

Mr. Creager has presented an argument, relative particularly to design by photo-elastic model studies, which should be given serious consideration in all future work of this nature. It is true that in these model studies, and in analysis by theories of elasticity, it is generally assumed either that embankment loads are transmitted entirely to the soil skeleton without regard to the pore water, or that a certain percentage of the load is transmitted to the soil and the remainder to the pore water; but in this latter case, insufficient consideration has been given to the direction in which the hydrostatic forces are dissipated. Analysis of the water forces at this time appears to be extremely complicated. However, it is felt that this consideration is an important one, not to be dismissed with the assumption that such forces are negligible, and it is hoped that Mr. Creager will continue his efforts toward developing a solution for this problem.

The writer takes issue with certain of the statements made by Mr. Feld in the paragraph which he devoted to the discussion of the writer's paper. There seems to be no justification for his statement that the impervious blanket, to be added after construction of the rock-fill, is of "much greater weight than the rock-fill section." Fig. 73 shows the relative proportions of blanket and rock-fill as considered in the studies reported in the paper. It is not denied that addition of the blanket would affect stability conditions, but this effect was analyzed and found to be insufficient to displace the rock-fill after the completion of its construction. The writer appreciates Mr. Feld's reference to the foundation treatment termed "floating foundation," consisting of fine

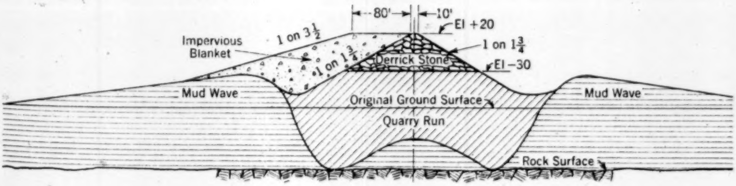

FIG. 73.

sand pumped into place. It is acknowledged that this should prove to be valuable in such applications as were discussed by the footnote references. The writer's studies, however, indicated that it would have been entirely inadequate in the case of the Eastport structures. In comparing the writer's conclusions with those of Mr. Feld, it is important to note that the foundation data available to the latter were limited to the results of a few, essentially unreliable wash-borings, whereas the writer had access to data from an extensive program of undisturbed sampling and a complete series of laboratory tests.

In preparing this final discussion the writer was fortunate in having had the assistance of Mr. Haines who was largely responsible for developing the method to which his name has been given. It is believed by both Mr. Haines and the writer that the problem discussed in the paper requires extensive additional study, and it is hoped that supplementary methods of analysis will be developed in the near future.

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DESIGN OF REINFORCED CONCRETE IN TORSION

BY PAUL ANDERSEN,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. C. W. DEANS, L. E. GRINTER, BRUCE G. JOHNSTON, DEAN PEABODY, JR., WALTER H. WHEELER, A. FLORIS, E. NEIL W. LANE, A. A. EREMIN, HAROLD E. WESSMAN, AND PAUL ANDERSEN

SYNOPSIS

In analyzing space structures torsional moments may be readily determined by the method of successive approximations.[2] For a given torsional moment an economical section is the square. If it is reinforced with 45° spirals such a section lends itself to mathematical investigation and a rational design procedure can be developed. The percentage of reinforcement in a rectangular section can also be approximated. For convenience of reference, a complete list of the letter symbols used in this paper is given in the Appendix.

DISTRIBUTION OF TORSIONAL MOMENTS

That the method of moment distribution can be applied to space structures[2] has been demonstrated by Hardy Cross, and N. D. Morgan, Members, Am. Soc. C. E. In order to find torsional moments in a three-dimensional frame it is necessary to supplement the "flexural" beam constants with "torsional" beam constants. Torsional stiffness is the moment which, applied at one end of a member, will produce unit rotation at this end, while the other end is fixed. It follows that the torsional stiffness of a member of length, L, is:

$$K_T = \frac{E_s\,T}{L} \quad \text{(1)}$$

in which E_s = the modulus of elasticity in shear; and T = the torsion factor which, for a circular section, is the polar moment of inertia and for a rectangular section, approximately:

$$T = \frac{b^3\,d^3}{3.58\,(b^2 + d^2)} \quad \text{(2)}$$

NOTE.—Published in October, 1937, *Proceedings*.

[1] Asst. Prof. of Structural Eng., Univ. of Minnesota, Minneapolis, Minn.

[2] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., John Wiley & Sons, N. Y., 1932, p. 117.

b being the width, and d, the depth of the member. The torsional carry-over factor is the ratio of $\frac{M_1}{M}$, in which M is an arbitrary moment applied at one end of the member, which is assumed as free to rotate, and M_1 is the resisting moment at the other end of the member, which is assumed as fixed against rotation. These two moments, of course, are always equal, and, therefore, the torsional carry-over factor is always unity.

Fixed end torsional moments are directly proportional to the distance from the point of application of the external moment to the opposite support. Thus, for a beam with a constant moment of inertia, the fixed end torsional moments are,

$$M_f = M \frac{n}{m + n} \qquad \text{(3a)}$$

and

$$M_t = M \frac{m}{m + n} \qquad \text{(3b)}$$

in which m and n are the distances from the applied moment to the fixed ends, respectively. If members subjected to torsion join members in flexure, the flexural stiffnesses of the latter are expressed by,

$$K_F = \frac{4\,E\,I}{L} \qquad \text{(4)}$$

If

$$E_s = \frac{E}{2.25} \qquad \text{(5)}$$

is substituted in Equation (1), the corresponding torsional stiffness of a member becomes,

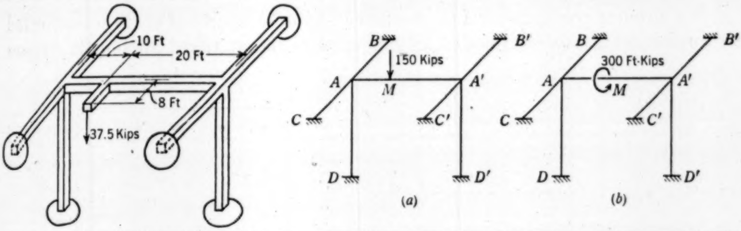
$$K_T = \frac{E\,T}{2.25\,L} = E \times \frac{1}{L} \times \frac{b^3\,d^3}{2.25 \times 3.58 \times (d^2+b^2)} \qquad \text{(6)}$$

If the relative stiffness of a member in flexure is expressed by $\frac{I}{L}$, the relative torsional stiffness will be $0.031 \times \frac{I}{L} \times \frac{b^3\,d^3}{b^2 + d^2}$.

The example in Table 1 will illustrate the procedure for distributing moments in a space structure due to an eccentrically loaded beam. The problem is conveniently solved by considering first the effects of the direct load on the center beam as in Table 1(a), and then the moments due to the torsional moment as in Table 1(b). For the sake of simplicity, side-sway is neglected even if there is some side-sway present, due to the eccentric load of 150 kips in Example (a), Table 1, deflecting Points A and A'. It is customary, in plane structures, to stop the process of moment distribution after a distribution, since the sum of the moments around a joint should equal zero. However, in a space structure this will not make the torsional moments in the members balance and the torsional moments (external and internal) in any one member should balance. It is just as correct, of course, to stop after having distributed the flexural moments as after having balanced the torsional moments. If the process is carried to the proper number of distributions it may be stopped after either, as the error would be insignificant.

It should be noted that the flexural and torsional moments in Table 1 can also be found by the slope-deflection method. The unknowns will be the rotations of Points A and A' in the two perpendicular planes; this makes a total of four rotations, which can be determined from the four equations expressing the condition that the two sets of moments at each joint, A and A', must balance.

TABLE 1.—MOMENT DISTRIBUTION IN SPACE STRUCTURES



Description	*AB*	*AD*	*AC*	*AM*	*A'M'*	*A'C'*	*A'D'*	*A'B'*	*AB*	*AD*	*AC*	*AM*	*A'M'*	*A'C'*	*A'D'*	*A'B'*
	(*a*) EFFECT OF DIRECT LOAD ON THE CENTER BEAM								(*b*) MOMENTS DUE TO THE TORSIONAL MOMENT							
Type of stress*	*T*	*F*	*T*	*F*	*F*	*T*	*F*	*T*	*F*	*F*	*F*	*T*	*T*	*F*	*F*	*F*
Stiffness ratio..	0.2	0.3	0.2	0.3	0.3	0.2	0.3	0.2	0.3	0.3	0.3	0.1	0.1	0.3	0.3	0.3
Carry-over factor.......	1.0	0.5	1.0	0.5	0.5	1.0	0.5	1.0	0.5	0.5	0.5	1.0	1.0	0.5	0.5	0.5
MOMENT DISTRIBUTION, IN FOOT-KIPS																
Fixed-end moments.......	..	..	..	167	−83	..	..	..	..	..	..	200	−100	..	..	..
First distribution.........	−33	−50	−33	−50	25	16	25	16	−60	−60	−60	−20	10	30	30	30
				12	−25							10	−20			
Second distribution.......	−2	−4	−2	−4	8	5	8	5	−3	−3	−3	−1	2	6	6	6
Final moments.	−35	−54	−35	125	−75	21	33	21	−63	−63	−63	189	−108	36	36	36

* *T* = torsion; *F* = flexure.

SHEARING STRESSES

The maximum unit torsional shearing stress in a rectangular section subjected to a torsional moment, M, is approximately,

$$v_m = \frac{M}{b\,d^2}\left(1.8 + \frac{3 \times d}{b}\right) \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For a given cross-sectional area this stress (which occurs at the middle of the long side, d) is a minimum when $b = d$; in this case it can be expressed by,

$$v_m = \frac{24\,M}{5\,b^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

For a rectangular section, the maximum unit vertical shearing stress due to a total vertical shear, V, equals,

$$v_m' = \frac{3}{2} \times \frac{V}{b\,d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and is dependent on the area of the rectangle.

As a rule, the bending moments that accompany heavy shears and torsional moments are comparatively small. The designer of concrete in torsion, therefore, is interested primarily in square sections or in sections that are nearly square. It should be noted that, in some cases, the beam that has a width greater than its depth may be still more economical than the square, because its maximum torsional shearing stress occurs where the vertical shearing stress is zero.

Reinforced Concrete in Torsion

If a circular concrete section, reinforced by a 45° spiral (Fig. 1), is subjected to torsion, and it is assumed that this reinforcement takes all tensile stresses greater than the permissible tension for plain concrete, it can be shown that the following relation exists[3]:

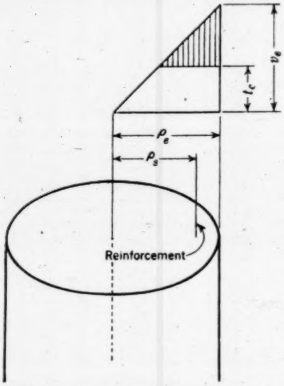


FIG. 1

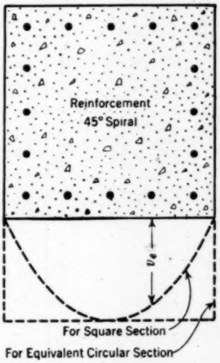


FIG. 2

$$N A_s = (v_e - s_c)^2 (3\,v_e^2 + 2\,v_e\,s_c + s_c^2)\,\frac{\pi \rho_e^3 \sqrt{2}}{12\,v_e^3\,\rho_s\,s_s} \qquad (10)$$

and for a square section (Fig. 2),

$$F\,N\,A_s = (v_e - s_c)^2 (3\,v_e^2 + 2\,v_e\,s_c + s_c^2)\,\frac{\pi \rho_e^3 \sqrt{2}}{12\,v_e^3\,\rho_s\,s_s} \qquad (11)$$

in which, N = number of bars, 45° to the axis, cut by a horizontal plane; v_e = shearing stress along the edge of a cylinder, produced by a twisting moment, M; s_c = permissible unit tensile stress for concrete; ρ_e = radial distance in a horizontal cutting plane, measured from the axis of a cylinder to the edge; ρ_s = radial distance to the steel reinforcement; s_s = permissible unit tensile stress for steel; and F = a reinforcement efficiency coefficient relating to the variation in tensile stress along the edge. The coefficient, F, is always less than one and will approach $F = \frac{2}{3}$ as the pitch of the spiral decreases, this being the ratio between the area of the parabola representing the variation of shearing stresses along the edge and the area of the rectangle expressing uniform tensile stress in all the steel rods of the spiral.

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[3] "Experiments with Concrete in Torsion," by Paul Andersen, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 949.

If c is the perimeter of the spiral and p the pitch, it is seen that,

$$N = \frac{c}{p\sqrt{2}} = \frac{8\,\rho_s}{p\sqrt{2}} \qquad (12)$$

With $F = \frac{2}{3}$, substitute Equation (12) in Equation (11) and transpose:

$$\frac{A_s}{p} = (v_e - s_c)^2(3\,v_e^2 + 2\,v_e\,s_c + s_c^2)\,\frac{\pi\rho_e^3}{32\,v_e^3\rho_s^2 s_s} \qquad (13)$$

For a given square subjected to a known torsional moment all quantities in Equation (13) are known except A_s, the cross-sectional area of one spiral rod, and p, the pitch of the spiral. The designer has the choice of selecting either one of these unknowns and then to find the corresponding value of the other.

The design of a reinforced concrete beam in torsion is shown in Fig. 3, in which the main reinforcement consists of five 1-in. round bars, top and bottom; the vertical stirrups, $\frac{1}{2}$-in. round bars; and the spirals, $\frac{1}{4}$-in. round bars. The maximum torsional moment in the beam occurs when the eccentric load on the beam is close to the column; the moment distribution for this case is indicated in Table 2. It should be noted that whereas the maximum vertical shear along the beam varies according to a curved line the maximum torsional moment will vary as a straight line.

The maximum vertical shearing stress is $v_m' = \frac{60\,000}{\frac{7}{8} \times 24 \times 21} = 136$ lb per sq in.; and the maximum torsional shearing stress is, $v_m = \frac{3 \times 400\,000}{5 \times 12^3} = 139$ lb per sq in.

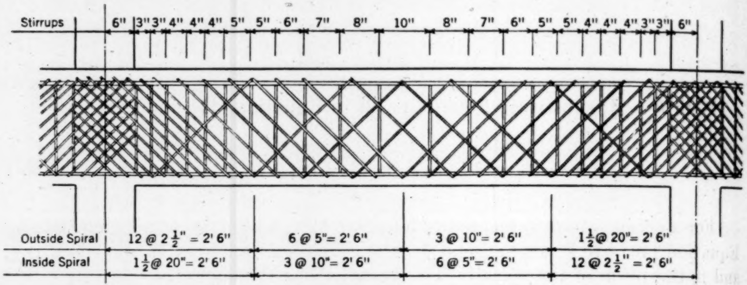
The beam should be designed first for vertical shear, adopting a lower allowable stress in the concrete, and then for torsion. Suppose that the total allowable shearing stress for concrete without web reinforcement is 90 lb per sq in.; it is now assumed that this stress can be divided into two parts in proportion to the magnitudes of the vertical and torsional shearing stresses: Of the vertical shear the concrete takes $\frac{136}{275} \times 90 = 44$ lb per sq in.; and, of the torsional shear, it takes $\frac{139}{275} \times 90 = 46$ lb per sq in. Applying Equation (13) to the part of the beam adjacent to the column support gives,

$$\frac{A_s}{p} = (139\text{–}46)^2(3 \times 139^2 + 2 \times 139 \times 46 + 46^2)$$

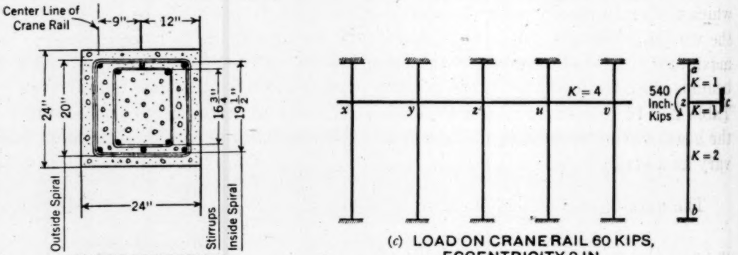
$$\times \frac{\pi \times 12^3}{32 \times 139^3 \times 10^2 \times 16\,000} = 0.0249$$

Using $\frac{1}{4}$-in. round rods for the spiral gives, $p = 1.97$ in.

A $2\frac{1}{2}$-in. horizontal spacing, as indicated on Fig. 3, will give a pitch somewhat less than required and, therefore, is on the safe side. Equation (13) is now used to compute the pitch, p, at the quarter-points and mid-span.


Stirrups
6" 3" 3" 4" 4" 4" 5" 5" 6" 7" 8" 10" 8" 7" 6" 5" 5" 4" 4" 4" 3" 3" 6"
Outside Spiral
Inside Spiral
12 @ 2 1/2" = 2' 6"
6 @ 5" = 2' 6"
3 @ 10" = 2' 6"
1 1/2 @ 20" = 2' 6"
1 1/2 @ 20" = 2' 6"
3 @ 10" = 2' 6"
6 @ 5" = 2' 6"
12 @ 2 1/2" = 2' 6"

(*a*) DETAILS OF CRANE GIRDER

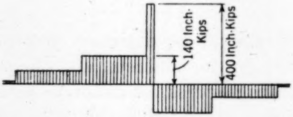

Center Line of Crane Rail
9"
12"
24"
20"
16 3/4"
19 1/2"
24"
Outside Spiral
Stirrups
Inside Spiral
K = 4
x
y
z
u
v
540 Inch-Kips
z
a
K = 1
K = 1
c
K = 2
b

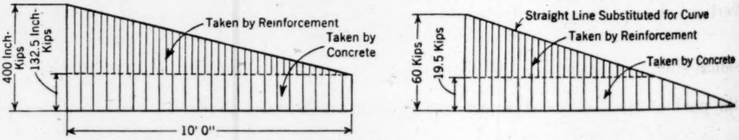
(*b*) CROSS SECTION OF BEAM

(*c*) LOAD ON CRANE RAIL 60 KIPS, ECCENTRICITY 9 IN.

(*d*) TORSIONAL MOMENTS IN GIRDER

Load at End *z* of *yz*





(*e*) MAXIMUM TORSIONAL MOMENTS

(*f*) MAXIMUM VERTICAL SHEAR

FIG. 3

THE RECTANGULAR SECTION

In the case of a rectangular section, A. A. Eremin, Assoc. M. Am. Soc. C. E., has suggested the use of formulas developed by the writer in 1934[4] with the exception that the value of ρ_e be taken as,

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[4] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 980.

$$\rho_e' = \sqrt[3]{\frac{2\,b^2 d}{\pi\left(3 + \dfrac{2.6}{0.45 + \dfrac{d}{b}}\right)}} \quad \text{.....................} (14)$$

in which ρ_e' = the radius of a circular section equivalent in strength to a rectangular section. To ρ_s (the distance from the center of the section to the spiral reinforcement) should be assigned the value of ρ_e' less the distance from the center of the spiral to the outside of the beam. The efficiency coefficient should also be somewhat reduced; in view of the lack of experimental data for rectangular sections, however, the writer suggests that the value of two-thirds be used also for rectangular beams, and that the same weight of spiral steel be placed on all four sides.

TABLE 2.—DISTRIBUTION OF MOMENTS, FIG. 3

Member	*xa*	*xb*	*xc*	*xy*	*yx*	*ya*	*yb*	*yc*	*yz*	*zy*	*za*
Stiffness	0.08	0.18	0.08	0.33	0.33	0.08	0.18	0.08	0.33	0.33	0.08
Carry-over factor	0.5	0.5	0.5	1	1	0.5	0.5	0.5	1	1	0.5
Fixed-end moment	..	..	..	..	..	..	..	..	..	..	540
First distribution	..	..	..	..	..	..	.. ←−180→	..	..	−180	−40
Second distribution	..	.. ←60→	..	..	60	14	32	14	60	.. ←120→	..
Third distribution	−5	−10	−5	−20	..	..	.. ←60→	..	..	−40	−9
Fourth distribution	..	..	..	.. 20	20	5	10	5	20	.. 20	..
Total	5	10	5	60	60	19	42	19	−140	−140	−49

Member	*zb*	*zc*	*zu*	*uz*	*ua*	*ub*	*uc*	*uv*	*vu*	*va*	*vb*	*vc*
Stiffness	0.18	0.08	0.33	0.33	0.08	0.18	0.08	0.33	0.33	0.08	0.18	0.08
Carry-over factor	0.5	0.5	1	1	0.5	0.5	0.5	1	1	0.5	0.5	0.5
Fixed-end moment	←540→		..	..	..	..	..	..	..	..	..	..
First distribution	−100	−40	−180	..	..	.. ←−180→	..	..	..	..	..	..
Second distribution	.. ←120→	..	..	60	14	32	14	60	..	..	.. ←60→	..
Third distribution	−22	−9	−40	..	..	.. ←60→	..	..	−20	−5	−10	−5
Fourth distribution	..	..	.. 20	20	5	10	5	20	.. 20	..	..	..
Total	−122	−49	−400	−140	19	42	19	60	60	−5	−10	−5

It would appear that in a rectangular section, designed in this manner, the reinforcement would be over-stressed along the center of the long side and under-stressed along the center of the short side. To the writer, however, it seems a reasonable assumption that, to some extent, the presence of equal quantities of spiral steel in both sides will re-distribute the shearing stresses to the advantage of the long side. For rectangular sections nearly square $\left(\frac{d}{b} \underset{<}{\equiv} 1.25\right)$, the writer believes the method outlined to be safe.

CONCLUSIONS

The paper outlines the various steps in the design of reinforced concrete in torsion. The Cross method provides a useful tool for determining torsional moments in a space structure.

The square section, which is a good torsion shape, can be reinforced effectively against torsional stresses by the 45° spiral. The quantity of reinforcement can be found by a procedure similar to the accepted practice for designing web reinforcement for vertical shear. This procedure can also be followed for rectangular beams.

APPENDIX

NOTATION

The following symbols, introduced in the paper, conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials" compiled by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932[5]:

A = area; A_s = cross-section area of steel bars;
b = breadth, or the short side of a rectangular section;
c = perimeter of a spiral;
d = depth, or the long side of a rectangular section;
E = modulus of elasticity; E_s = modulus of elasticity in shear;
F = a reinforcement efficiency factor that allows for surface tension taken by reinforcement;
I = moment of inertia;
K = stiffness; K_F = flexural stiffness; K_T = torsional stiffness;
L = length of a member;
M = bending moment; an arbitrary moment applied at one end of a member; M_f = fixed-end moment; M_t = torsional moment;
m = a distance from the fixed end of a beam (see, also, n)
N = number of bars, 45° to the axis, cut by a horizontal plane;
n = a distance from the fixed end of a beam (see, also, m);
p = pitch of a spiral;
s = unit stress; s_c = permissible unit tensile stress for concrete; s_s = permissive unit tensile stress for steel;

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[5] A. S. A.—Z10a—1932.

T = torsion factor = polar moment of inertia in a circular section;
V = total vertical shear;
v = unit shearing stress; v_m = maximum shearing stress; v_m' = maximum unit vertical shearing stress; v_e = shearing stress along the edge of a cylinder, produced by a twisting moment, M;
ρ = radial distance in a horizontal cutting plane, measured from the axis of a cylinder: ρ_e = radius to the edge; ρ_s = radial distance to the steel reinforcement; ρ' = radius of a circular section, equivalent in strength to a rectangular section.

DISCUSSION

C. W. Deans,[6] Esq. (by letter).—The interesting method presented by Professor Andersen is rational, and the resulting designs appear to be safe. The material in Table 2 could be presented in such a manner that the final results are found more readily by inspection. Table 3 demonstrates the writer's suggestion. The lines underscoring the balancing moments indicate that the joint moments are balanced at this stage. If the individual carry-over moments (see "C. M." in Table 3) are recorded as shown, the total unbalanced moments that are ready for distribution at each step are apparent. In the example solved, the moments in members meeting at Joints u and v are identical to corresponding members meeting at Joints x and y, respectively, and can be omitted.

TABLE 3.—Suggested Simplification of Table 2

Description*	Joint x†				Joint y†					Joint z†				
	xa	xb	xc	xy	yx	ya	yb	yc	yz	zy	za	zb	zc	zu
S. R.	0.08	0.18	0.08	0.33	0.33	0.08	0.18	0.08	0.33	0.33	0.08	0.18	0.08	0.33
C. F.	0.5	0.5	0.5	1.0	1.0	0.5	0.5	0.5	1.0	1.0	0.5	0.5	0.5	1.0
F. M.	..	..	..	..	..	..	..	..	..	..	..	..	..	+540
D.	..	..	..	..	..	..	..	..	..	−180	−40	−100	−40	−180
C. M.	..	..	..	..	..	..	..	..	−180	..	..	..	..	..
D.	..	..	..	..	+60	+14	+32	+14	+ 60	..	..	..	..	..
C. M.	..	..	..	+60	..	..	..	..	..	+ 60	..	..	..	+ 60
D.	−5	−10	−5	−20	..	..	..	..	..	− 40	− 9	− 22	− 9	− 40
C. M.	..	..	..	..	−20	..	..	..	− 40	..	..	..	..	..
D.	..	..	..	..	+20	+ 5	+10	+ 5	+ 20	..	..	..	..	..
C. M.	..	..	..	+20	..	..	..	..	..	+ 20	..	..	..	+ 20
Total	−5	−10	−5	+60	+60	+19	+42	+19	−140	−140	−49	−122	−49	+400

* S. R. = stiffness ratio; C. F. = carry-over factor; F. M. = fixed-end moment; D. = distribution; and, C. M. = carry-over moment. † See author's Fig. 3(*c*).

Attention should be called to Equation (12), in which p is defined, simply, as the pitch. It should be made clear that this is the value of the pitch at right angles to the spiral bars and not the distance between bars measured parallel to the axis of the spiral. The writer had some difficulty in checking the formula until he recognized this fact.

L. E. Grinter,[7] M. Am. Soc. C. E. (by letter).—The thought of studying a continuous framed structure as a space problem may seem a great refinement that strains all practical conceptions of design. However, the writer takes the position that no refinement of analysis is wasted that clarifies the picture of the action of the structure. Certainly it is a serious crudity to neglect entirely the torsional resistances of large beams and girders in the design of continuous frames. Whether a full space-frame study can be justified will depend largely upon the relative flexural and torsional resistances of the various members.

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[6] Estimator, Western Bridge Co., Ltd., Vancouver, B. C., Canada.

[7] Director of Civ. Eng., and Dean, Graduate Div., Armour Inst. of Technology, Chicago, Ill.

Distribution Factors.—The stiffness factor is proportional to $\frac{E\,I}{L}$ for a beam, whereas this factor is $\frac{E_s\,J}{L}$ for a circular torsional section, J, of course, being the polar moment of inertia. In this form the comparison is evident although the latter expression can be determined readily from an elementary study of virtual work. Since $\theta = \int \frac{T\,t'\,ds}{E_s\,J}$ and since both T and t', the real and virtual torque moments, will be constant and equal to unity, $\theta = \frac{L}{E_s\,J}$; and $\frac{1}{\theta}$, the stiffness factor, becomes $\frac{E_s\,J}{L}$. Obviously, the carry-over factor is unity since a torsional moment passes from one end to the other of a member without reduction.

Signs of Moments.—Several sign conventions would be possible in the study of continuity in space frames. However, there seems little justification in this case for any convention other than the one introduced by the writer for balancing moments in planar frames.[8] Although a sign convention based on fiber stresses (cantilever moment negative) may serve adequately for continuous beams and even passably for planar frames, it is fundamentally unsound for the study of space structures because of its inapplicability to torsional problems. The basic sign convention should be that a positive moment (either flexural moment or torque moment) is one which tends to rotate the adjacent joint clockwise. Fig. 4 illustrates this sign convention applied to torque moments.

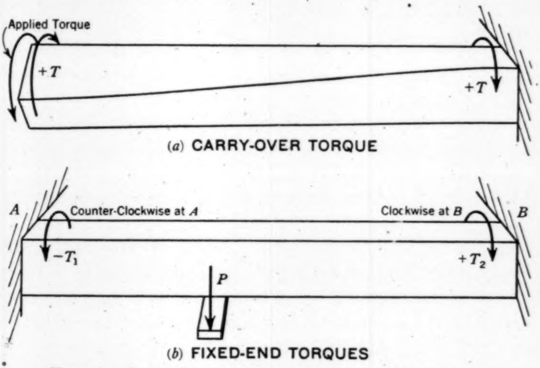

FIG. 4.—SIGN CONVENTION FOR TORQUE MOMENTS.

The sign of the carry-over factor based on the foregoing sign convention has the usual positive value for moment distribution. Furthermore, a positive torque moment must also be carried over with the same positive sign (see Fig. 4(*a*)). Hence, the common carry-over factor for bending moment, which is + 0.5, becomes + 1.0 for torque moment. The entire sign convention for planar frames, therefore, remains unchanged for solving space frames. No confusion is possible if one always considers only the action of the member on the joint, which is the fundamental conception used in classical truss analysis by the graphical method.

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[8] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 14.

Fixed-End Torque Moments.—As the author mentions, when an internal torque moment is applied, the fixed-end torque moments are supposed to be inversely proportional to the relative distances to the two ends of the member. For instance, if a torque moment, T, is applied to the member, AB, as shown in Fig. 5, the larger resisting moment occurs at the near end, A, and is equal to,

$$M = T\left(\frac{b}{a+b}\right) \qquad \text{(15)}$$

This will be evident from the fact that the rotation over the length, a, must be equal to the rotation over the length, b. Since the rotation is proportional to the acting torque moment and to the length, the relation expressed in Equation (15) follows for a homogeneous beam. For a haunched beam or one in which the torsional resistance is influenced by the addition of much steel for shear resistance, this simple relationship does not hold. Approximate fixed-end torque moments for a haunched beam might still be found by an integration. The writer calls attention to the fact that such fixed-end torque moments must be accepted as approximate since the action of vertical or diagonal shear reinforcement for torsional resistance is not predictable with mathematical accuracy. It is worth mention that torsional resistance or stiffness for reinforced concrete depends very largely upon the resistance of stirrups, the most unpredictable quantity in the analysis of reinforced concrete. In contrast, the flexural resistance or stiffness of a beam is dependent principally upon the action of tension steel, the most predictable factor in the analysis of reinforced concrete.

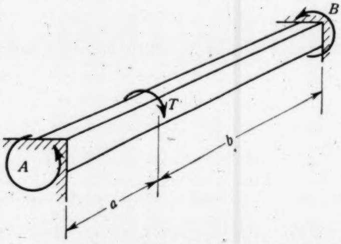

Fig. 5.—Fixed-End Torque Moments.

Arrangement of Computations.—The author appears to favor the use of tabulated coefficients for his analysis. More than a decade of experience with the method of balancing moments and with numerous similar balancing procedures seems to have convinced the originator of the method[9] and others who have used it widely, that such tabulation leads to no benefit. In fact, it eliminates the major reason for the choice of the method of balancing moments; that is, the close connection between the action of the structure and the arithmetic procedure of the computations. The use of a table of moments leads to a purely routine procedure and divorces the computations from the possibility of their interpretation as successive causes of the structural action. Fig. 6 was prepared to show the simplicity of arrangement of the computations at their proper places on a line perspective of the structure.

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[9] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 5.

These computations also illustrate the desirability of combining torque and flexural moments into one analysis although they must be entered in the separate groups designated in Fig. 6 as *T* and *F*. The writer's preference for this attack is simply that it is not possible to divorce these separate actions anyway because a flexural moment can turn into a torque moment in passing around a corner. Since torque cannot be studied separately from flexure in space frames, there seems little reason to separate the terms in the manner suggested by the author. The final point to notice in Fig. 6 is the separation of the flexural moments in the columns under the heading of *F* and *F′*. This designation is used to indicate that these flexural moments act in different planes and, hence, must be applied individually in design.

Accuracy in Balancing Moments.—It will be noticed that the final moments computed in Fig. 6 do not check exactly with those shown by the author in Table 1. It is of no moment which solution is more nearly exact since either would be more accurate than the standard tools of internal stress analysis for reinforced concrete sections. For instance, one flexural moment is computed as 75 kip-ft by the author and as 77 kip-ft by the writer. The variation of 3% is about the standard of accuracy that the writer considers desirable for the analysis of concrete framed structures. Naturally, it is understood that this value does not represent the probable inaccuracy inherent in any attempt to predict the true stresses in the actual structure.

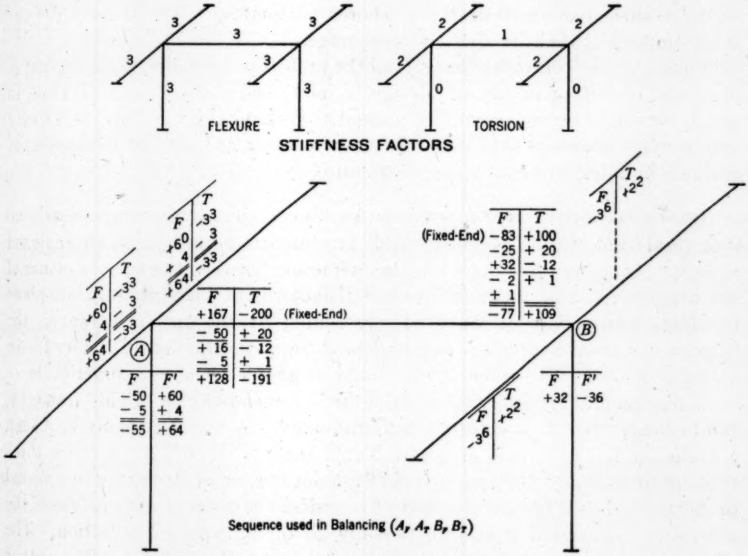
3
3
3
3
3
3
3
FLEXURE
2
1
2
2
2
0
0
TORSION
STIFFNESS FACTORS
F
T
(Fixed-End) −83 +100
−25 + 20
+32 − 12
− 2 + 1
+ 1
−77 +109
F T
+167 −200 (Fixed-End)
− 50 + 20
+ 16 − 12
− 5 + 1
+128 −191
F F′
−50 +60
− 5 + 4
−55 +64
F F′
+32 −36
A
B
T F −33 +60 3 4 −33 +64
T F +22 −36
Sequence used in Balancing (A_F A_T B_F B_T)

FIG. 6.—BALANCING MOMENTS FOR A SPACE FRAME.

Resistance of Reinforced Concrete in Torsion.—The author has clarified the necessary constants for investigating the stresses in a rectangular reinforced concrete section under torsion. This analysis is subject at least to the same questioning that in the past has been directed to the analysis of reinforced concrete beams. Actually, one should feel some question about the use of Equations (6) to (14) until tests have justified the author's procedure in the same manner that they have justified the ordinary theory of reinforced concrete beams. However, it is natural and proper that theory should precede full confirmation by tests.

One interesting feature of the design of a reinforced concrete crane girder for torsion, as indicated by the author, is the use of continuous diagonal stirrups. Since vertical stirrups and horizontal rods have been found quite adequate for resisting diagonal tension resulting from shear and flexure, there would seem to be no necessity for changing this proved and simple device when torsion is present. Failure is still by diagonal tension. The design criterion should still be to space hoops of any type in such a manner that every crack must cross at least one shear bar which itself is anchored adequately. The slope of the bar is less important than its area and anchorage.

Conclusion.—The author has served the profession by collecting in one paper many data on the analysis and design of reinforced concrete space frames, in which torsion is almost inevitably present. Perhaps this excellent paper may help convice designers that it is not too difficult, or at least not impossible, to design a building frame as a space structure.

Bruce G. Johnston,[10] Assoc. M. Am. Soc. C. E. (by letter).—Nearly all structural frames of the so-called "rigid" type are space frames; few are analyzed as such. In many cases there is little interaction between the various planes of the structure, and the analysis may be satisfactorily divided into separate plane problems. In frames of the type represented by the author, however, the three-dimensional analysis is essential because of the torsion introduced. In actual design direct torsional loads should be avoided wherever practicable.

Three general types of deflection occur in a member of a "rigid" frame—it can be bent, twisted, or deformed longitudinally. A complete analysis of such a statically indeterminate structure would include the effects of these three types of deflection. Only in "hybrid"[11] structures, or in structures of unusual proportions, does it become necessary to consider the effect on the analysis of the secondary interaction of two, or possibly all three, types of deflection. The effect on the design, or total deflection, may be another matter. The author has presented, in an orderly manner, the application of the Cross method of moment distribution to the analysis of the combined bending and twisting problem.

The writer wishes to point out certain variations which occur in extending the author's method to beams of non-uniform cross-section, or to cases in which structural steel beams are used instead of reinforced concrete. It is first necessary to discuss the basic torsion constant, denoted by the symbol, T, in the paper. In 1855, Saint Venant presented the solution of the torsion of a prism of rectangular cross-section.[12] In Fig. 7 the writer has shown the deviation of Equation (2) for the torsion constant from the solution of Saint Venant. It is seen that this formula is most nearly correct for the square section, and that an error of only 6% or less is introduced for ratios of long to short side as great as 4. If the ratio of the long to the short side is 1.6, or greater, as is the usual case for beams designed primarily for bending, another simple formula gives a closer approximation than Equation (2):

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[10] Asst. Prof. and Asst. Director, Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[11] See "The Relation of Analysis to Structural Design," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1931.

$$T = \frac{b^3 d}{3}\left(1 - 0.63\,\frac{b}{d}\right) \dots\dots\dots\dots\dots\dots (16)$$

The deviation of Equation (16) from the correct solution is also shown in Fig. 7. The greatest error is for the square section (about 12.3%) but it decreases rapidly to a negligible quantity as the ratio of side lengths increases. The writer introduces Equation (16) principally because it is in a form which is easily integrated for determining the torsional distribution factors for the important case of the haunched beam. The use of the exact solution is impractical since it is in the form of a series and, although Equation (2) can be integrated, it yields a very complicated expression.

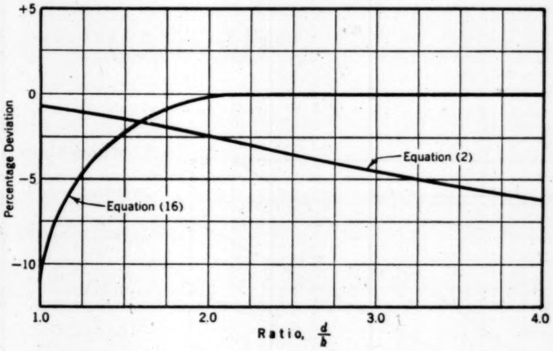

FIG. 7.—DEVIATION OF APPROXIMATE FORMULAS FOR TORSION FACTOR OF RECTANGULAR SECTION FROM EXACT SOLUTION

If a torsional moment is applied at any point of a straight beam, the author in Equations (3*a*) and (3*b*), has shown that the distribution to each end will be inversely proportional to the length of the segment. Such will not be the case for a beam haunched at one end, or at both ends, and this case will now be studied for the straight haunch. The additional notation required is given in Fig. 8, in which d_1 = least depth of haunch; d_2 = depth at wall; and, l_H = length of haunch. The angle of twist caused by a constant torsional moment, M, in an elemental length of haunched section is given by,

$$d\phi = \frac{M}{T\,E_s}\,dx \dots\dots\dots\dots\dots\dots (17)$$

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[12] "Torsion des Prismes," by Saint-Venant, Paris, 1855.

For the total length of the haunched element, therefore, the total angular twist equals,

$$\phi = \frac{M}{E_s} \int_0^{l_H} \frac{dx}{T} \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Assume that the breadth, b, is constant and that the torsion factor, T, at any point in the haunch is given by Equation (16). The depth, d, of the haunch at any point will be:

$$d = d_1 + \left(\frac{d_2 - d_1}{l_H} \right) x \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which x is the distance from the shallow end of the haunch. Making this substitution in Equations (16) and (18) the integration gives, for the average twist per unit length:

$$\frac{\phi}{L_H} = \frac{3\,M}{b^3\,E_s\,(d_2 - d_1)} \log \left(\frac{d_2 - 0.63\,b}{d_1 - 0.63\,b} \right) \ldots\ldots\ldots\ldots (20)$$

Designating by T_H the effective or average torsion constant over the length of the haunch,

$$T_H = \frac{M\,L_H}{\phi\,E_s} = \frac{b^3\,(d_2 - d_1)}{3 \log \left(\frac{d_2 - 0.63\,b}{d_1 - 0.63\,b} \right)} \ldots\ldots\ldots\ldots (21)$$

or,

$$T_H = C_H\,d_1{}^4 \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which C_H is a factor depending on two ratios, $\frac{d_1}{b}$ and $\frac{d_2}{d_1}$, with the restriction that $\frac{d_1}{b} > 1.2 \frac{d_2}{d_1} > 1.0$. Fig. 8 gives values of C_H for different ratios of $\frac{d_1}{b}$ and $\frac{d_2}{d_1}$.

The total angular twist between any point on the uniform section of a haunched beam and the end will be:

$$\phi = \frac{M}{E_s} \left(\frac{L_H}{T_H} + \frac{L_u}{T_u} \right) \ldots (23)$$

in which L_u and T_u are the length and torsion factors for the uniform segment.

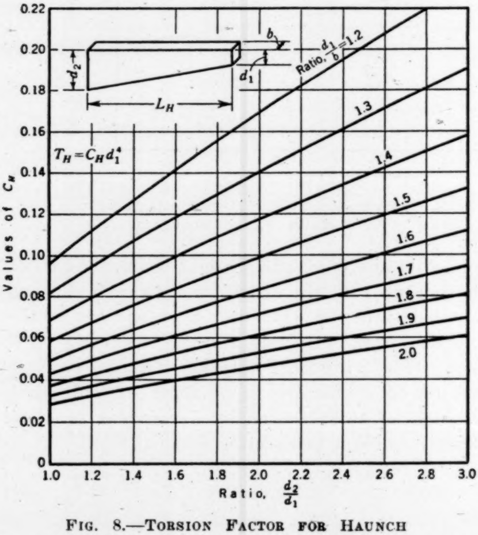

FIG. 8.—TORSION FACTOR FOR HAUNCH

The torsional stiffness for the combined segment will be,

$$K = \frac{1}{\dfrac{L_H}{T_H} + \dfrac{L_u}{T_u}} \quad \text{.......................(24)}$$

and the torsional moment will be distributed to either end in proportion to the torsional stiffness of that particular segment.

The application to an actual case will now be illustrated. Assume a concrete beam 12 in. wide and 20 ft long unsymmetrically haunched as shown in Fig. 9,

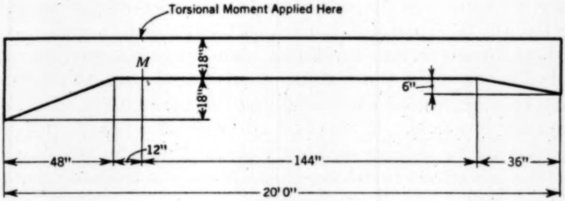

FIG. 9

with a torsional moment, M, applied 5 ft from the left end. For the haunch at the left end: $\frac{d_1}{b} = \frac{18}{12} = 1.5$; $\frac{d_2}{d_1} = \frac{36}{18} = 2.0$; and (see Equation (22)), $T_H = (0.985)\,(18)^4 = 10\,330$ in.^{4} Similarly, for the right haunch, $T_H = 7\,660$ in.^{4} For the uniform section, by Equation (16), $T_u = \frac{18 \times 12^3}{3}\left(1 - \frac{0.63 \times 12}{18}\right) = 6\,010$ in.^{4} Substituting in Equation (24), the torsional stiffnesses of the two parts of the beam are obtained: At the left, $K_L = \frac{1}{\dfrac{48}{10\,330} + \dfrac{12}{6\,010}} = 150.8$; and at the right, $K_R = \frac{1}{\dfrac{144}{6\,010} + \dfrac{36}{7\,660}} = 34.8$. From these values are obtained the distribution factors to give the fixed-end torsional moments, M_f, at the left and at the right ends: At the left end, $M_f = \frac{150.8}{150.8 + 34.8}\,M = 0.812\,M$; and at the right end, $M_f = \frac{34.8}{150.8 + 34.8}\,M = 0.188\,M$, instead of 0.75 and 0.25 for a beam of uniform cross-section.

The torsional stiffness of the entire beam may be obtained by similar computations or directly from K_L and K_R as follows:

$$K = \frac{K_L\,K_R}{K_L + K_R} = \frac{150.8 \times 34.8}{150.8 + 34.8} = 28.3$$

The torsional carry-over factor will always be 1 regardless of haunches, but the bending-moment carry-over and distribution factors will need to be computed by use of the column analogy, from the generalized slope deflection equations, or by reference to previously published diagrams. The analysis then proceeds in a manner analogous to that of the author.

In analyzing similar structures in which the members are steel I-beams instead of concrete, another factor affects the torsional distribution constants appreciably. The usual torsion factor is calculated on the assumptions that end sections are free to warp.[13] If the end sections are restrained from warping, there is a localized increase in torsional stiffness. This increase is insignificant in the case of a rectangular section, but in the case of the I-beam the prevention of the warping of the section at a fixed end has a considerable effect. P. W. Werner, Assoc. M. Am. Soc. C. E., has discussed[14] the general problem of a steel beam fixed at each end and subjected to a twisting moment at any point, which gives, directly, the torsional distribution factors for such a case.

Since these formulas are developed elsewhere, and require considerable routine calculations, only the results of an actual case will be given here. Assume an 8 by 8-in. wide-flanged beam at 67 lb per ft, 16 ft long, fixed at each end, with torsional moment, M, applied at the quarter-point. It is found that the distribution to the shorter segment is 0.818 M and to the longer segment, 0.182 M. The net effect, therefore, is similar to the haunching of a concrete beam, although for an entirely different reason.

The writer has discussed two cases in which the torsional distribution factors cannot be treated as simply as in the case of the rectangular beam of uniform cross-section which was used in the author's example. In some cases, the resulting variations will be negligible, but in other instances they will appreciably affect the analysis.

DEAN PEABODY, JR.,[15] M. AM. SOC. C. E. (by letter).—This paper marks a distinct advance into one of the unexplored areas of reinforced concrete design. The discussion of torsional stiffness and the distribution of the torsional moment in a space frame is of value to the designer.

For the stress analysis at a given section the author makes torsional computations independent of bending computations for fiber stress, shear stress, or diagonal tension. His design equation for the square section is based on the same line of reasoning used for diagonal tension computations and thus makes for consistent design. Equation (13) gives the useful result, $\frac{A_s}{p}$, which enables one to choose the spiral reinforcement immediately.

For a square section the maximum shear stresses occur at the center of each face. The illustrative computations for the problem of Fig. 3 apply to Particle A of a square section (Fig. 10), and the torsion reinforcement should be supplied at Point A for this section. At Particle C on the opposite side the shear stress, v_B, due to the vertical shear force, V, and the torsional shear stress, v_T, due to the torsion couple, M_T, are opposite in direction. For the beam of Fig. 3 the net shear stress at Point C equals 3 lb per sq in. (upward). Therefore, at Point C, no diagonal tension or torsion steel is needed but it is impractical, of course, for the steel to be omitted at C and supplied at A. Particle D has the same stress, v_T, as at A and Fig. 3 shows that an equal or greater percentage of spiral steel is supplied. Due to the slope of the spirals less torsion reinforcement may be supplied at Particle B than at Particle A but, since there is no shear stress, v_B, the full allowable shear stress of 90 lb per sq in. is permissible, although the spiral steel at A was computed for an allowable stress of 46 lb per sq in. The reduced number of spirals at B will be safe, except at sections where the shear force, V, is small. Maximum torsion usually occurs near the support where the shear forces are large. It would appear, then, that the torsional design for square sections is usually adequate when only the particle with maximum total shear stress is considered.

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[13] "Structural Beams in Torsion," by Inge Lyse, M. Am. Soc. C. E., and Bruce G. Johnston, Jun. Am. Soc. C. E. *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 857.

[14] *Loc. cit.*, p. 912.

[15] Assoc. Prof. of Structural Design, Mass. Inst. Tech., Cambridge, Mass.

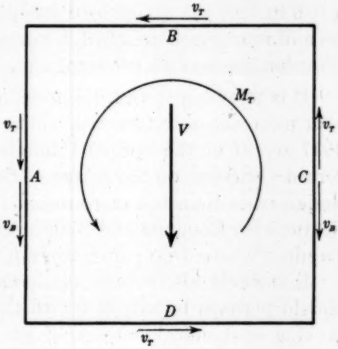

FIG. 10

No recommendation has been made for the allowable shear stresses, but the writer assumes that the customary values for diagonal tension will be used for the total allowable shear stress at Particle A.

The author apparently uses hoops or ties for the spiral steel. It is difficult to wire inclined hoops securely, and the writer believes the added cost of welding these hoops to the longitudinal steel is justified.

Square cross-sections will undoubtedly be chosen for members with large torsional moments, as the author states, but many wall beams and spandrel girders of rectangular shape are subjected to torsional moments. Still more are poured integrally with the slab and the design section is angle or tee-shaped. An authoritative discussion of the design of such sections must wait a comprehensive program of tests and inspection of existing structures.

WALTER H. WHEELER,[16] M. AM. SOC. C. E. (by letter).—The principles set forth in Professor Andersen's paper have important significance when applied to the determination of correct moments of resistance in flat slab structures. Numerous tests on flat slab floors have shown a consistently wide disagreement between the stresses determined by the commonly accepted beam-strip theory of analysis and the actual stresses in the structure under load. Various authorities have explained the discrepancy on the theory that the tensile strength of the concrete was responsible, whereas it would appear from the paper that a more logical explanation would be the torsional resistance of the slab.

Since the torsional resistance is based upon shearing strength and not upon tensile strength of the concrete it would seem logical and proper to give credit to this factor in the flat slab formulas. It does not seem logical to base the analysis of the stresses in a continuous plate supported on columns upon the theory of independent beams supported on columns without any connection between those beams, as is done in the generally accepted flat slab theory.

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[16] Designing and Cons. Engr., Minneapolis, Minn.

The writer wishes to call attention to the results[17] obtained in the test of the Western Newspaper Union Building, in Chicago, Ill., in 1916, in which four adjacent panels were loaded. The panels were 17 ft 5.5 in. by 19 ft 4.5 in., with a nominal slab thickness of 8.5 in. The actual slab thickness at the edge of the column capitals was 8.35 in. The diameter of the column capitals was 4 ft 6 in. and the design load was 250 lb per sq ft. The test load over the full area of four panels was 913 lb per sq ft and the minimum recovery of the slab after loading was 75 per cent.

It is to be hoped that the torsion theory will be developed for flat slabs and that more accurate formulas will result. It has seemed to the writer that the 1917 report of the Special Committee of the Society on Concrete and Reinforced Concrete, on the design of flat slabs,[18] came much closer to an accurate design basis than the more recent formulas of the Joint Committee on Specifications for Concrete and Reinforced Concrete particularly as to slab thickness required where drop panels are not used.[19]

If the only aim is to be conservative without regard to accuracy, designers should perhaps be satisfied with the present thickness formulas. If they are seeking economical designs which are reasonably accurate, these thickness formulas should be modified.

The progress report of the Joint Committee on Specifications for Concrete and Reinforced Concrete for 1937,[19] gives a basis of determining slab thickness which is reasonably accurate and then recommends that having found this reasonably accurate thickness, 20% be added to it. The Committee proposes adding 20% to the thickness that would be determined as proper according to the 1917 report on the design of flat slabs.[18] Following the Joint Committee's recommendation, the slabs in the Western Newspaper Union Building would be increased from 8.5 in. to approximately 10 in. thick for 3 000-lb concrete.

By the application of the torsion theory to this slab, a reasonable and ready explanation is found for its remarkable performance, and the necessity for increasing the thickness is not apparent.

Further data on thickness of flat slabs with capitals and without drop panels are given in a published report[19] by the writer.

A. Floris,[20] Esq. (by letter).—From the academic point of view, the author's treatment of rigid frames, as systems in space, is valuable and instructive. In practice, however, there may be some question. The solution of the problem, even as applied to the simplest frame, is too complicated to be practicable. Consequently, the application of the method to frames as ordinarily designed will be out of the question, especially if the problem is complicated by side-sway.

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[17] *Bulletin 106*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[18] Final Rept., Special Committee on Concrete and Reinforced Concrete, *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1135.

[19] Not yet published; see, also, "Comparison of Actual Flat Slab Designs with Code Requirements," *Engineering News-Record*, January 28, 1937, p. 132.

[20] Dipl.-Ing., Los Angeles, Calif.

If one considers that even a beam stressed by pure bending may be subjected to twisting moments by a beam connected to the same column, under a certain angle, it is evident that the proposed method is too theoretical to be practicable. For this reason, designers will continue to consider the analysis of building frames as a plane problem. In general, structural engineers should avoid direct torsion and, if this is not possible, they should at least minimize its effect by modifying the design.

The intent of this discussion is not to criticize the author's efforts, but to emphasize the need for keeping in mind the practical applicability of the theory. Professor Andersen deserves much credit in focusing the attention of engineers on the comparatively unexplored field of torsional resistance of rigid frames.

E. NEIL W. LANE,[21] JUN. AM. SOC. C. E. (by letter).—Due mainly to the desire for simplicity and a lack of adequate information, torsional stresses have been largely ignored in structural design. Generally, they are small in comparison with the flexural and shearing stresses, which makes a two-dimensional solution satisfactory for most structural problems.

Professor Andersen shows the application of the moment distribution method to the analysis of three-dimensional frames, but dismisses the solution by the slope deflection method with a general statement of procedure. Since certain methods are more adaptable to one type of problem than others, it is deemed advisable to show the application of the slope deflection method to a frame similar to that analyzed in Table 1.

The writer has assumed a length of 15 ft for the side members, and a modulus of elasticity of 3 000 000 lb per sq in. The position of the assumed 10-in. width of the members, relative to the assumed 24-in. depth, was such that the members, *A B*, *A C*, *A′ B′*, and *A′ C′*, were to take the greater proportion of the external torsion; and the frame, *DA A′D′*, was to take the greater proportion of the flexure.

Using stiffnesses, as in Equations (4) and (6), the following joint equations are obtained:

$$M_{f(AA')} + M_{f(A'D)} + M_{t(AB)} + M_{t(AC)} = 25.0\,\theta_A + 2.4\,\theta_{A'} - 1.0 = 0 \quad \text{(25a)}$$

$$M_{f(A'A)} + M_{f(A'D')} + M_{t(A'B')} + M_{t(A'C')} = 50.0\,\theta_{A'} + 4.8\,\theta_A + 1.0 = 0 \quad \text{(25b)}$$

$$M_{f(AB)} + M_{f(AC)} + M_{f(AD)} + M_{t(AM)} = 145.9\,\phi_A - 1.0 = 0 \quad \text{(25c)}$$

$$M_{f(A'B')} + M_{f(A'C')} + M_{f(A'D')} + M_{t(A'M)} = 265.4\,\phi_{A'} - 1.0 = 0 \quad \text{(25d)}$$

In solving, the rotations become $\theta_A = +0.0423$ radian, and $\theta_{A'} = -0.0242$ radian in the plane, *DA A′D′*; and $\phi_A = -0.00685$ radian, and $\phi_{A'} = -0.00367$ radian in the planes, *AB CD*, and *A′B′ C′D′*, respectively. Thus, a torsional moment of $+ 1.96 \times 10^6$ in.-lb and a flexural moment of $— 6.85 \times 10^6$ in.-lb are obtained at End *A* of the member, *A M A′*. Side-sway, although present, has been neglected to make this solution more comparable to the moment distribution method, as shown by the author.

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[21] Bridge Detailer, State Dept. of Roads and Irrig., Lincoln, Nebr.

It is hoped that Professor Andersen's worthy paper will interest others in the study of torsional analysis in connection with three-dimensional structural frameworks.

A. A. Eremin,[22] Assoc. M. Am. Soc. C. E. (by letter).—An interesting method of distributing torsion moments in rigid frames is developed in this paper. Professor Andersen assumed that the centers of joints in rigid frames remain at their original position at all variation of loading sustained by the frame. This assumption is generally used in computing torsion moments by the method of least work. It simplifies computation without serious error for practical purposes.

In the distribution of the bending moments in rigid frames a convenient rule for checking the computations was derived: The sum of bending moments at any joint in a rigid frame, at any loading, is equal to zero. There is no similar simple rule for checking computations of the torsion moments in rigid frames. However, a simplified approximate computation of torsion moments may be used for checking of the more exact computations.

The relation of the torsion moments at the ends of Member $A\ A'$ in the rigid frame shown in Table 1 may be expressed by the following equations:

$$M_A + M_A' = M \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

and,

$$K_{TA}\, M_A\, (1 - \Sigma\, K_A)\, m + K_{TA}'\, M_A'\, (1 - \Sigma\, K_A')\, n = 0 \ldots\ldots (27)$$

in which, M_A and M_A' are the torsion moments in the member, $A\ A'$, at the ends, A and A', respectively; K_{TA}, K_{TA}' are torsion stiffnesses in the member, $A\ A'$, at the ends, A and A', respectively; and $\Sigma\, K_A$, $\Sigma\, K_A'$ are the sum of the stiffness factors of the adjacent members at Ends A and A', respectively.

Solving Equations (26) and (27) simultaneously, the torsion moments, M_A and M_A', may be determined. The torsion moments in Member $A\ A'$, with the loading considered in Table 1 as computed by Equations (26) and (27), are $M_A = 200$ and $M_A' = 100$. These moments are the same as those computed for the fixed ends in Member $A\ A'$. Evidently, a slight degree of fixation at the ends of Member $A\ A'$, caused by the action of adjacent members on the distributed torsion moments, is due to their low flexural resistance and the symmetry of the frame. Evidently, the maximum error of computed torsion moments is only 8 per cent. This error is not excessive considering various approximate assumptions used in the more exact method. The distribution of torsion moments in a multiple-story, rigid frame may likewise be computed by Equations (26) and (27) by considering the frame to be composed of elementary frames similar to that shown in Table 1. Therefore Equations (26) and (27) may be used for approximate computations of torsion moments and for checking the distribution of torsion moments determined by the more exact method.

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[22] Associate Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

Harold E. Wessman,[23] M. Am. Soc. C. E. (by letter).—Two main elements of value are contained in this paper: First, it develops a logical procedure for the design of a reinforced concrete member, subject to a known torsional moment; and second, it focuses attention, indirectly, on a typical, but naive, procedure of designing a slab, beam, and girder floor. The writer wishes that more space had been devoted to the second element in order to emphasize the uncertainties existing in this problem of torsion as related to reinforced concrete structures. There is no doubt that the usual procedure of slicing a slab into a strip and designing the slab, the beam, and the girder as isolated units subject to forces and moments in one plane only has the virtue of convenience; but is it accurate enough for all cases? The paper may stimulate some new thinking about the question of accounting for torsion and treating a structure as a space frame.

It is not difficult to extend the theory of moment distribution in a plane to moment and torsion distribution in three dimensions, in order to get bending moments and twisting moments in the various units which make up a space frame. It is very difficult, however, to interpret the results in a monolithic, reinforced concrete floor system.

Several years ago the writer directed a graduate thesis[24] on the subject, "The Effect of Torsional Resistance of Girders on the Bending Moments in Connecting Beams." Varying ratios of torsion stiffness factors to bending stiffness factors were assumed and maximum moments then determined. For certain ratios of torsional stiffness to bending stiffness, beam spans were practically fixed at the ends. Moment coefficients in cases like these, even if the spans are equal and the live-dead ratio less than three, may be considerably in error if specified by code rules.

It is not difficult to establish torsion constants for isolated rectangular sections or for sections composed of an assembly of rectangles if they are assumed to be homogeneous, isotropic, and perfectly elastic; but, what are the boundary dimensions of a beam with a floor slab upon it? Where is the center of rotation of a girder which is part of a monolithic slab, beam, and girder floor? Will the slab tend to prevent any rotation of the beams and girders, thus making the torsion constant equal to infinity, and making all spans fixed at their terminals? How about the rotations of the ends of beams due to the deflection of the girder to which they connect, due to bending? These questions indicate the complexity of the problem. It may be that typical design procedure, even if it is admittedly naive, is the best that can be devised. This much can be said for usual design procedure: Definite paths of resistance are made strong enough to support the loads on the structure, even if the paths of resistance in the actual structure may not be exactly those which are prescribed in the analysis.

Paul Andersen,[25] Assoc. M. Am. Soc. C. E. (by letter).—The purpose of this paper was twofold: First, to present the procedure of determining torsional moments in reinforced concrete structures; and, second, to present a rational design method for reinforcing against torsion. The latter received scant attention in the discussion.

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[23] Prof. of Structural Eng., Coll. of Eng., New York Univ., New York, N. Y.

[24] Thesis submitted to the University of Iowa in 1935 by William Gallagher, in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

[25] Asst. Prof. of Structural Eng., Univ. of Minnesota, Minneapolis, Minn.

The practical applicability of the theory is questioned by Mr. Floris. The writer fully agrees that, in general, structural engineers should avoid direct torsion and, if that is not possible, they should at least minimize its effect by modifying the design. The crane girder, the balcony girder, and the beam curved in plan are examples of types of structural members in which torsion cannot be eliminated. Professor Wessman emphasizes the difficulty of finding torsional moments where these are modified by the presence of monolithic floor-slabs and connecting beams. If the torsion is due solely to deflection and the bending of connecting members, it is probably safe to ignore it and (as expressed by Professor Wessman) provide "definite paths of resistance strong enough to support the loads on the structure, even if the paths of resistance in the actual structure may not be exactly those which are prescribed in the analysis."

The writer favors the suggested simplification of Table 2 as presented by Mr. Deans. In the writer's opinion, the sign convention recommended by Professor Grinter is the one best suited for the study of continuity in space frames. Professor Grinter also calls attention to the fact that torsional fixed-end moments and carry-over factors for haunched beams must be found by integration. In this connection, the writer is pleased to acknowledge Mr. Johnston's valuable contribution to the discussion. Fig. 8 will enable the designer to compute torsion factors for haunched beams.

Mr. Lane shows how the illustrative example in Fig. 3 can be solved by the slope deflection method. This approach, together with Mr. Eremin's approximate method, offers the designer various methods of computing torsional moments. Professor Peabody indicates the main differences between torsional shear and what is generally known as vertical shear.

The writer does not agree with Mr. Wheeler's statement that torsional resistance is based upon shearing strength and not upon the tensile strength of the concrete. Torsion failures of concrete are diagonal tension failures and torsion reinforcement is tension reinforcement.

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ECONOMICS OF THE OHIO RIVER IMPROVEMENT

By C. L. Hall,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Eugene L. Grant, Fred Lavis, O. Slack Barrett, Edmund L. Daley and Forrest E. Byrns, J. E. Willoughby and W. D. Faucette, W. E. R. Covell, and C. L. Hall

Synopsis

Brief descriptions of the improved Ohio River are given in this paper, including methods used in navigating it, and the character of the freight borne upon it. The problem presented is then stated, namely, is the public compensated for the heavy national expenditures on the improvement? Commercial navigation costs are determined as accurately as possible, on a ton-mile basis, for each class of freight. Government costs are analyzed on a ton-mile basis, applicable to all classes of freight. The sum of these two costs is compared with rail rates for various commodities, both analytically and graphically. The conclusion is reached that the public has been compensated for its expenditures on the Ohio River improvement, and that this fact tends to become more and more evident every year.

Physical Situation

The Ohio River is formed by the confluence of the Allegheny and Monongahela Rivers, at Pittsburgh, Pa., and discharges into the Mississippi River at Cairo, Ill. The distance from Pittsburgh to the mouth, measured by water, is 981 miles. (The mile used in this paper is the statute mile of 5 280 ft, or 1 609.4 m. The ton is the short ton of 2 000 lb, or 907.18 kg.) The river is improved by four fixed dams, and forty-two movable (or navigable) dams. Two of the fixed dams, recently constructed, have movable crests.[2] At fixed dams, traffic always moves through the locks; at movable dams, it moves through the locks at low water. At medium and high stages, the movable dams are lowered, and traffic moves in the open river. The percentage of time during which

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Note.—Published in October, 1937, *Proceedings*.

[1] Col., Corps of Engrs., U. S. Army, Commanding 1st Engrs., U. S. Army, Fort DuPont, Delaware.

[2] Descriptions of the project are contained in the Annual Repts., Chf. of Engrs., U. S. Army, in Rept. for 1935, pp. 1043–1047.

open-river traffic is possible, varies from 28 at the up-stream movable dam, to 50 at the down-stream dam. At the Falls of the Ohio, at Louisville, Ky., there is a very high movable dam; and traffic moves through the locks there for 95% of the time. Fixed dams and the dam at Louisville are provided with twin locks, whereas the other movable dams have a single lock. The standard lock dimensions are 600 ft by 110 ft. The operation of the system has been described by the writer elsewhere.[3] The minimum depth of the channel is 9 ft, allowing a loaded draft of a little more than 8 ft. However, except at times of extreme low water, barges can be loaded to 9 ft.

Traffic is subject to interruption by high water, ice, and fog. Interruptions by high water at fixed dams are due to the passage of the water over the lock-gates, preventing their functioning. Technically, there are no interruptions by high water at movable dams, but, at very high stages, navigation becomes dangerous. Moreover, the economic functioning of terminals becomes impossible when the river is out of its banks. Ice in sufficient quantities makes navigation impossible. Ice at low water may also compel the lowering of movable dams, making the useful depth too small for economical navigation.[4] At certain seasons of the year, fog at night and in the early morning causes a suspension of traffic.

Conditions from year to year differ to such an extent that it cannot be said that there is any normal number of days during which traffic is interrupted. The median number of days, at Cincinnati, Ohio, about half-way down the river, for the years 1919 to 1935, inclusive, was as follows: By high water, less than one day; by ice, three days; and by low water following ice, less than one day. (These data were obtained from study of official records of the U. S. Engineer Office, at Cincinnati.) Traffic is tied up on account of fog about 80 hr per yr, but never for more than 12 hr at a time. (Little has been published on fog. The information used has been secured by the writer in private discussion with operators.)

The economics of Ohio River transportation are materially influenced by the improvement of five principal tributaries: The Allegheny and Monongahela Rivers, which form the Ohio, at Pittsburgh; the Kanawha River, which joins the Ohio, at Point Pleasant, W. Va. (Mile 265.7); the Cumberland River, at Smithland, Ky. (Mile 920.4); and the Tennessee River at Paducah, Ky. (Mile 934.5). All these tributaries have been at least partly improved by the construction of locks and dams, thereby permitting year-round navigation from and to the Ohio River. Of lesser importance are the other improved tributaries: Muskingum, Little Kanawha, Big Sandy, Kentucky, and Green Rivers.

History of the Improvement

The first appropriation for the improvement of the Ohio River was made by Congress in 1824. Allotments to the year 1874 were for an open-channel improvement, consisting of the construction of dikes, removal of snags and wrecks, and some dredging. This work permitted navigation at medium and high stages. During this period, the Government also purchased an interest in the Louisville and Portland Canal, which passes traffic around the Falls at Louisville. In 1874, a plan was proposed for canalizing the Ohio River by means of a system of movable dams, in order to secure a depth of 6 ft. The first dam constructed under this plan was completed in 1884, at Davis Island, about 5 miles down stream from Pittsburgh. Work under the plan of 1874 progressed slowly. From time to time, the plan was modified. In 1910, a project was adopted by Congress, which provided for the canalization of the river by a system of movable dams, which established a channel depth of 9 ft, substantially as subsequently executed. This project was completed[5] in 1929. Except for some early work on the Louisville and Portland Canal, the entire improvement has been in charge of the Corps of Engineers, United States Army.

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[3] Rept. No. 12, 1st Section, XVIth International Cong. of Navigation, Brussels, Belgium, 1935.

[4] *Loc. cit.*, p. 13; also, "Pack Ice and Movable Dams," by C. L. Hall, M. Am. Soc. C. E., *Military Engineer*, July–August, 1934, Vol. XXVI, No. 148, pp. 245–247.

NATURE OF TRAFFIC

Fig 1. shows, graphically, the traffic on the Ohio River for the calendar year 1934.[6] Although the tonnage differs from year to year, the general picture presented by Fig. 1 remains fairly constant, the principal alterations in recent years being the considerable increase in movement of petroleum products, and a decrease in the short-haul sand and gravel movements. The character of Ohio River commerce can probably be best explained by a discussion and amplification of the information contained in Fig. 1.

Steel is the most valuable cargo carried on the river, and the second most important in ton-mileage (26%). Manufactured steel is habitually loaded in barges at points in the Pittsburgh District. A small portion of it is discharged at Ohio River ports, but most of it is consigned to Lower Mississippi River towns. Practically all this steel is moved by contract carriers, or by private carriers owned by the steel companies, and is down-stream movement. There is a return cargo in the form of scrap steel, gasoline, sulfur, and fluorspar, which is collected at Lower Mississippi and Lower Ohio ports, for consignment to Upper Ohio River ports.

Coal forms the largest item, both in tonnage and in ton-mileage (42%), in Ohio River traffic. About 4 600 000[7] tons of coal pass out of the Monongahela every year. This coal is practically all unloaded and consumed at manufacturing plants in the Pittsburgh District. Almost none of it moves as far down stream as Wheeling (Mile 90.5). There is a movement from tipples in the pools of Locks Nos. 13 and 14 (Miles 96.1 to 114.0) in both directions, but principally up stream, to near-by manufacturing plants. In the stretch of river from below Wheeling to the mouth of the Kanawha River (Mile 265.7), practically no coal moves. A considerable tonnage of coal moves out of that river by water to Middle Ohio River plants. A much larger quantity moves from Huntington, W. Va., tipples (Mile 308.3). Some of this is discharged at plants in the Ironton-Portsmouth District (Miles 327.2 to 356.0). Much the greater proportion is unloaded at Cincinnati (Mile 470.2), and just below. A small quantity reaches Louisville (Mile 603.7). From Louisville to Caseyville, Ky. (Mile 871.2), a small quantity of coal moves on the river, principally from the Green River (Mile 784.2). From Caseyville to the mouth, there is a relatively small movement of coal consigned to Lower Ohio and Mississippi River towns. Practically all the coal moved on the river is transported by private carriers; those in the Pittsburgh District are generally owned by steel companies, and those in other parts of the river by coal companies. Except for the small tonnage which moves up stream from the pools of Locks Nos. 13 and 14, and on the Lower Ohio River, all coal moves down stream. There are no appreciable return cargoes on coal barges.

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[5] Annual Rept. of the Chf. of Engrs., U. S. Army, 1935, pp. 1043–1044; House Doc. No. 306, 74th Cong., 1st Session, pp. 134–137.

[6] From an official chart prepared from information submitted by carriers in accordance with law.

[7] Obtained by averaging the 1930–1934 tonnages of the Monongahela River, Annual Repts., Chf. of Engrs., U. S. Army, fiscal years, 1931–35, Pt. 2.

Petroleum products (11% in ton-mileage) move in both directions in special tank barges, normally from riparian storage tanks to riparian delivery tanks. However, the major movement is in the Lower Ohio River, from storage tanks along the Mississippi River and its other tributaries. These products are usually transported in private barges, but many of the barges are propelled by towboats engaged in contract or common carriage. About 75% of the movement (in ton-mileage) is up stream.

Sand and gravel (6% in ton-mileage) is dredged from sand and gravel bars in the river and moves in both directions in privately owned barges for comparatively short distances to delivery points.

Miscellaneous freight (13% in ton-mileage) moves in packet-boats, and also in barges owned by common and contract carriers, although some fluorspar from Lower Ohio River ports is carried up stream by private carriers owned by steel companies. Miscellaneous freight moves in both directions.

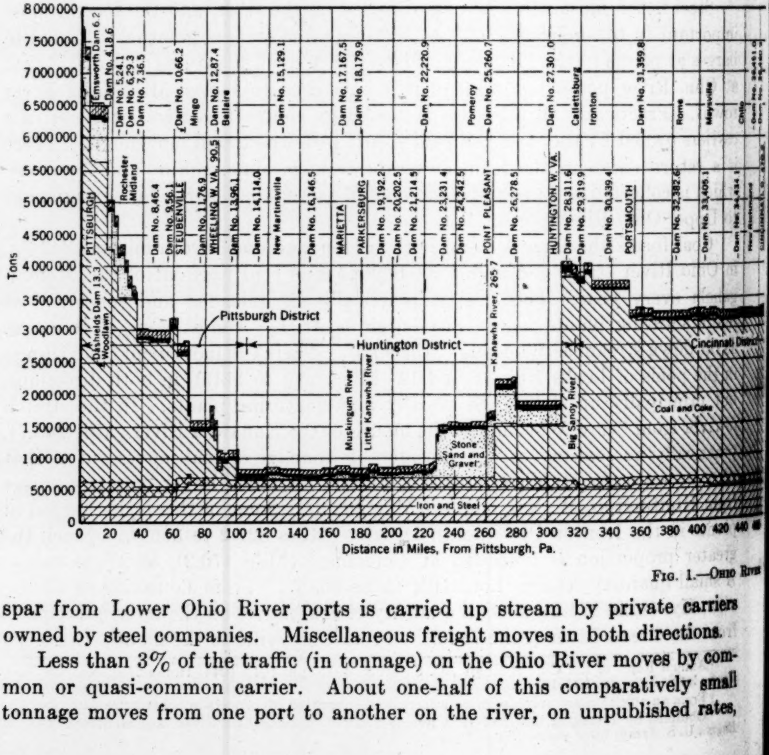
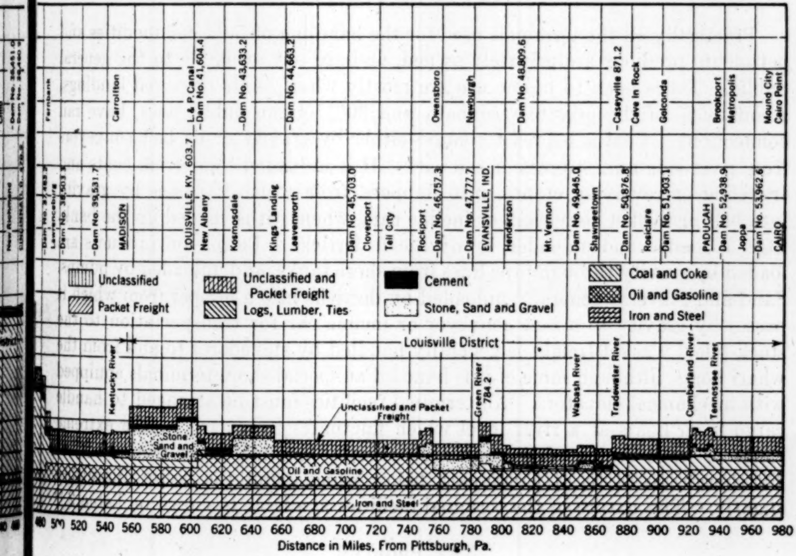


Fig. 1.—Ohio Riv

Less than 3% of the traffic (in tonnage) on the Ohio River moves by common or quasi-common carrier. About one-half of this comparatively small tonnage moves from one port to another on the river, on unpublished rates, or on rates frequently changed on short notice, and the remainder, the river-rail common-carrier traffic, moves on rates "established" in the railroad sense of the word.

Character of Boats

Bulk freight is habitually carried in steel barges normally about 26 ft by 175 ft, or in wooden barges about 26 ft by 135 ft. These carriers are propelled in fleets of from six to eighteen barges by stern-wheel or screw towboats, which invariably push rather than tow the barges. In the transportation of coal in wooden barges, tows of twenty-four barges are not infrequent. The towboats vary from 30 hp to 2 200 hp, with an average draft of from 4 to 6 ft. Package and perishable freight may be carried either in tows of closed barges (a method which is increasing in popularity among river operators), or on packet-boats, similar in type to the packets of the Nineteenth Century. Petroleum and other liquid products are carried in specially constructed tank barges. A few of the towboats are of the tunnel screw type. This type of towboat is becoming increasingly popular on Western rivers as is the Diesel engine instead of the steam engine. Many of the newer vessels on the Ohio River are of the Diesel, screw type.[8]



Rive onnage Graph, 1934

The size of the locks at Ohio River dams has been a controlling factor in the construction of barges. A tow of barges, 26 ft wide, four abreast (which is standard on the Ohio River), occupies a minimum of 104 ft in a lock 110 ft wide. In the purchase of new barge equipment, there is a trend toward the construction of barges 35 ft wide for assembly in tows of three abreast. The nature of the floating equipment using the river is shown in Table 1.[9]

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[8] The technique of inland navigation is described in the reports of the Towboat Board: H. R. Doc. No. 857, 63d Cong., 2d Session; H. R. Doc. No. 108, 67th Cong., 1st Sess.; and mimeographed report of June 17, 1929. Information on the character of vessels now using the Ohio River has been obtained from the U. S. Engineer Office, at Cincinnati, Ohio.

TABLE 1.—TRIPS AND DRAFTS OF VESSELS, OHIO RIVER, 1935

Draft, in feet	UP-BOUND TRAVEL				DOWN-BOUND TRAVEL			
	Steamers	Motor vessels	Barges	Total	Steamers	Motor vessels	Barges	Total
9	41	....	3 273	3 314	20	....	6 922	6 942
8	140	2	2 359	2 501	132	1	6 509	6 642
7	1 156	219	2 810	4 185	1 168	72	5 210	6 450
6	2 356	902	4 698	7 956	2 216	771	5 764	8 751
5	3 074	970	1 896	5 940	2 649	942	1 937	5 528
4	2 045	1 651	1 205	4 901	1 728	1 666	843	4 237
3	207	3 246	7 910	11 363	185	3 172	589	3 946
2	1	640	11 975	12 616	1	602	5 237	5 840
1	....	....	5 022	5 022	....	....	1 933	1 933
Total.............	9 020	7 630	41 148	57 798	8 099	7 226	34 944	50 269
Total net registered tonnage.........	2 095 863	579 533	25 713 164	28 388 560	1 780 229	347 583	21 339 623	23 467 435

CHARACTER OF TERMINALS

Practically all the terminals used for the handling of bulk commodities and petroleum products are privately owned, and are not available to the general public. Those open to public use are mostly wharf boats or paved landings. About 65% of the private terminals, and 30% of the public ones, have rail connections. Coal is loaded to barges either by gravity or by belt conveyors from river-side mine tipples or rail cars. It is unloaded by derrick-boats and traveling or revolving cranes, into hoppers, from which it passes by gravity into belt or bucket conveyors or incline cars, whence it is carried up the bank. Steel is loaded and unloaded by overhead derricks. Petroleum products are loaded by gravity through pipe lines from shore tanks, and unloaded by pumps. Sand and gravel is generally unloaded by derricks into a hopper from which it passes by gravity to a belt conveyor or incline car for transportation to the stock-pile. Packet freight is generally handled by stevedores to and from the wharf boats, although some of it is handled at special shore terminals equipped with mechanical elevators. All terminal facilities must be arranged to handle cargo from boats at a river level which fluctuates irregularly. The extreme range of stage during which traffic is normally handled varies from 16 ft, at Pittsburgh, to 44 ft, at Cincinnati, below which point the figure diminishes slightly. The nature of the terminal facilities available is shown in Table 2.[10]

THE PROBLEM

Freight moves on the Ohio River because it can be transported more cheaply by water carriers than by land carriers. It can be transported at low rates because engineering works have been built on the river at the expense of the taxpayer. Since no tolls are charged, the Treasury derives no direct return. The public obtains an indirect return in the reduced net cost of water-transported articles consumed. The problem is to compare the engineering costs of the improvement with the value of the public benefits. This Ohio River question, of course, is a part of a larger national inland waterways problem,[11] which is now arousing great interest among American engineers and economists. (For example, John S. Worley, M. Am. Soc. C. E., in 1936,[12] stated that "the operation on this [Ohio] river gives little encouragement that any justification will be found.") It is also receiving international attention, the International Congress of Navigation in 1935 having particularly requested its members to prepare reports on the economic value of improved inland waterways.[13]

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[9] From Rept. of Corps of Engrs., U. S. Army, Ohio Rivers, 1935.

[10] "Transportation in the Mississippi and Ohio Valleys," Transportation Series Nos. 2, Repts., Corps of Engrs., U. S. Army, 1929, p. 123.

TABLE 2.—TERMINAL FACILITIES (EXCEPT FOR MOTOR-BOATS) ON THE OHIO RIVER

Number	Type of freight	Shelter	Mechanical appliances	RAIL CONNECTIONS	
				With	Without
		(a) OPEN TO THE PUBLIC			
6	General	Paved wharf or landing	....	6	0
4	General	Warehouse	Crane	4	0
3	Coal, steel, and cement	....	Electric whirler or crane	3	0
1	General	Shed	....	..	..
12	General	Wharf boat	....	..	..
1	General	Movable warehouse and wharf building	....	..	..
1	General	Wharf boat and warehouse	....	..	..
10	General	Paved wharf or landing	....	..	..
2	Miscellaneous	Wharf boat	....	..	..
1	Miscellaneous	Rolling transit shed	....	..	..
41	Totals	....	....	..	..
		(b) NOT AVAILABLE TO THE GENERAL PUBLIC			
54	Petroleum products	....	Pipe line or pumps	27	27
78	Coal and coke	....	Clam-conveyor or tipple	42	36
8	Iron and steel	....	Crane	8	..
5	Cement	....	Pump, conveyor, crane, or hand trucks	3	2
57	Sand and gravel	....	Clam or conveyor	44	13
10	Coal, sand, and gravel	....	Clam or incline	8	2
4	Coal and steel	....	Crane or incline	4	..
1	Brick	....	Conveyor	1	..
24	General and bulk freight	....	Crane, incline, or slings	16	8
12	Building and repair of boats	....	Marine ways or floating docks	4	8
11	Logs and lumber	....	Crane or incline	10	1
5	Miscellaneous building materials	....	Crane, clam, or conveyor	4	1
6	Grain and feed	....	Tipple or conveyor	6	..
275	Totals	....	....	177	98

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[11] "The High Cost of Inland Water Transportation," by S. L. Wonson, M. Am. Soc. C. E. See p. 1296.

[12] "Regulating Transport," by John S. Worley, *Engineering News-Record*, July 9, 1936, p. 52.

[13] Resolutions of the XVIth International Cong. of Navigation, *Bulletin*, Permanent International Assoc. of Navigation Congresses, No. 20, p. 53.

PERIODS OF TIME COVERED

This paper, so far as its facts, as distinct from their analysis, are concerned, is a sequel to a report of 1926, by C. W. Kutz, M. Am. Soc. C. E., which was presented later to the public under the title of "The Relations of the Ohio River and Its Tributaries to Transportation in the United States."[14] No attempt has been made to restudy the investigations covering the period from 1905 to 1925 treated by General Kutz. However, improved collection of commercial statistics by the Engineer Department now permits the acquisition of information in regard to ton-mileage, as distinct from tonnage. When no such information was available, the segregation of credit for commerce moving on two or more rivers had to be made separately for each form of haul. Now, credit for commerce not moving on a single river can easily be determined from ton-mileage charts, after an average cost per ton-mile for the commodity has been established.

The commercial statistics are by calendar years, whereas Federal expenditures are reported by fiscal year (July 1 to June 30). This six-months time lag of benefits over costs will obviously be of no importance over a long period, and has been disregarded. Commercial statistics are closed as of December 31, 1934, and expenditures as of June 30, 1934, since it was convenient to establish limiting dates based on publications available at the time the study was initiated. (See heading, "Trends of Commerce" for a discussion of the most recent data.)

ASSUMPTIONS

As is necessary in all generalizations from data taken from sources of diversified origin, certain assumptions have been made in the statistical work of this study:

> Assumption (1).—The entire reduction of costs in transportation of freight, due to the existence of an improved waterway on the Ohio River, is passed on to the public in the form of lower prices of the goods transported, and hence becomes a public saving.

Assumption (1) complies with the principles of Nineteenth Century economics. When goods are manufactured at *A* and sold at *B*, the savings in transportation over the line, *A B*, are reflected in the lower price of goods at *B*. The assertion is undoubtedly true in a perfectly free economy. It may be doubted whether a perfectly free economy ever has existed, but it certainly does not exist now on the Ohio River. The two most important items of commerce—coal and steel—are manufactured, transported, and, to a large extent, distributed by firms which, on a small scale, correspond to what the Germans call "vertical trusts." Thus, the same firm, under different forms of capitalization, will own coal mines, steel plants, and steel yards, which are essentially jobbers' supply stations. They will also own towing companies for the transportation of coal and manufactured steel. Except over the comparatively short distances during which their raw materials move by common carrier, the manufacture, distribution, and sale of water-borne steel is handled under one corporate management. The same applies to coal companies, which own mines, tipples, transportation lines, and storage yards.

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[14] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1106–1122.

Free competition between the point of production and the point of transfer to the retailer does not exist. At the same time, there is great competition between corporate enterprises having more or less widely separated points of production, and almost adjacent points of distribution. This tends to set up free economic competition between the corporate enterprises, under which the economies due to water transportation tend to be passed on to the consumer. However, the extent to which this saving is transferred is not susceptible of accurate determination. Under certain circumstances it may become zero. The following testimony[15] of a coal operator is apposite:

> "The cost of the coal to our customers at the inland destinations is identically the same, regardless of whether the coal is shipped all-rail from the mines or ex-river from Cincinnati or Addyston."

The comparatively large industrial consumers, however, are undoubtedly able to secure reductions in cost of the delivered coal by bargaining. Where these consumers are utilities, and, to some extent, where they are not, this saving is passed on. Indeed, it would be surprising in a competitive market if savings in cost of transportation were not reflected in reduced retail costs. In course of time, as the number of firms technically fitted to engage in water transportation becomes greater, the assumption will become more and more true. It will thus eventually become accurate. As far as Assumption (1) is in error, the error is in favor of the waterway.

Assumption (2).—The reduction of cost in transportation of freight is equal to the rail cost of transporting the freight actually carried by water, minus the water cost of transporting that same freight.

In general, water-borne freight is some form of bulk commodity, habitually transported by rail where no water route is available. It is logical, therefore, to compare rail and water costs. The difficulty is that rail costs cannot be determined, and it is necessary to use rail rates. If one could use water rates, comparison would still be logical. However, except for the very limited common carrier traffic, water rates are as difficult of determination as rail costs. Therefore, it has been necessary to compare rail rates (in the general case), with water costs, adding to the latter a 6% return on the invested capital. The water costs consist of the line-haul cost of freight, plus the rail rate from the origin to one port and from the other port to the destination (if there has been any rail travel), together with a carefully determined terminal differential. Where the terminal differential is in favor of the waterway, as is apparently the case with liquid petroleum products, none has been included. Assumption (2) is reasonably accurate.

Assumption (3).—The reduction in the cost of ferriage over a waterway, caused by its improvement, is not a public saving.

Ferry transportation is obviously cheaper in a canalized river than in an open one. On the other hand, owing to the increased number of bridges, this form of traffic is of diminishing importance. There seems to have been no reduction in tolls as a consequence of canalization, so that no direct benefit has been passed on to the public. If there is any error in Assumption (3) it is against the waterway.

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[15] Testimony of H. E. Webster for the West Virginia Coal & Coke Corporation, Interstate Commerce Comm. Docket No. 25 933.

> Assumption (4).—The United States Engineer Department costs include the entire Federal cost of the improvement.

Certain expenditures of the Federal Lighthouse Service and of the Steamboat Inspection Service are obviously for the benefit of Ohio River transportation. It has proved impossible in practice to segregate these costs for each waterway, because the books of the Services do not show any such breakdown. The costs are so trifling when compared with those of the Engineer Department that it is not worth while to spend too much time on their study. Assumption (4) is favorable to the waterway.

> Assumption (5).—In comparing expenditures and benefits, no allowance has been made for taxes.

There is considerable claim that annual water costs should be charged with an arbitrary percentage of the existing value of the Federal improvement made in the stream. There would be something to say for this if it were applied generally to all carriers moving on public highways. Nobody, however, charges the cost of lighting an airway against an airline. Moreover, there is a countervailing tax benefit, of which no account is taken, the omission of which might be even more adverse to the waterway than an arbitrary tax allowance is beneficial. The carriers pay taxes on terminal properties which would have practically no value if the waterway were not improved. They pay taxes on floating plant which would not exist if there were no waterway. They pay income taxes. The first two forms of tax are presumably reflected in costs, but the income taxes are not. No determination can be made of them, because it would be necessary to set up a counter claim for the reduced income tax of competing forms of transportation. This would create a problem impossible of solution. It is not altogether unfair to state that the attempt to determine the taxable value of the waterway is about as difficult as that of determining the taxable value of Central Park in New York City. The entire value of the real estate abutting on that Park is based on its existence. A large part of the value of all real estate in the Ohio River basin is similarly based on the existence of the improvements. Under such circumstances, there is no rational method of assessing a charge for unpaid taxes against the waterway. There is no method of determining whether or not Assumption (5) is beneficial to the waterway.

To summarize, Assumptions (1) to (5), taken as a whole, may be regarded as favorable to the waterway, but the extent of this prejudice tends to diminish with time and, for all practical purposes, may eventually disappear.

Carrier Costs of Water-Borne Freight

The most difficult problem statistically in any study of water transportation is the determination of the actual costs incurred by the private or contract carrier, in moving bulk products concerning which he is not required to quote any rates to any one. By an analysis of figures furnished confidentially by various carriers in 1926, General Kutz prepared a graph of the costs of transporting coal by water.[16] Since that date, the movement of coal from the Kanawha River and from Huntington to Cincinnati, has become the greatest single factor in tonnage of any Ohio River movement, except the short haul immediately down stream from Pittsburgh. The private coal carriers in this area decline to furnish cost figures, even confidentially. It is not believed that the Pittsburgh coal carriers have been able to reduce their costs much below those shown in the Kutz study—that is, about 3 mills per ton-mile. The line costs of the Huntington-Cincinnati traffic seem to have been brought to a figure below this point. The best indications available to the writer are that the cost to the most economical carrier is now less than 2 mills per ton-mile. A hypothetical study by a prospective operator, which was shown to the writer, gives a still lower figure.

The steel carriers, although unwilling to be quoted individually, make no particular secret of the fact that the line cost of transporting steel for long hauls on the Ohio River is not more than 3 mills per ton-mile. A large contract carrier has submitted to the writer cost data which indicate that his operating cost for 1934, including a 5% return on the investment, was slightly less than 2 mills per ton-mile. A recent official report[17] cites 3 mills per ton-mile for Ohio River traffic in general, but this includes an arbitrary 20% for terminal costs. In computing the savings of the two all-important items of coal and steel, the writer has, in general, used line-haul costs of 2 and 3 mills per ton-mile for principal coal movements, and about 3 mills per ton-mile for steel. These figures are thus slightly higher than what might be termed the general opinion of the industry. The errors, if any, being in favor of decreased costs, are against the waterway. Although no statistical certainty can be reached, it seems probable that the errors against the waterway caused by assuming too high line-haul costs more than balance the errors in its favor caused by the five assumptions previously stated. The cost of hauling petroleum products is even more difficult to determine. The capital investment in the specialized barges (generally used) is so large a part of the total cost that density of traffic, or factor of use, really settles the matter of profit or loss. The operating costs, of course, are increased by the much greater proportion of up-stream traffic encountered in this trade. The writer has used values for savings in this connection which he regards as conservative, and which were based on quotations received from contract carriers. It is regretted that the confidential character of the basic figures obtained prevents a direct statement of the ton-mile cost upon which the savings given in Tables 3 and 4 were based.

The method of computing sand and gravel savings is sufficiently explained in the Appendix. The terminal costs of handling these products are so large, and the mileage of any particular lot is usually so small, that the line cost of transportation (independent of terminal charges) is not a controlling factor in determining the savings.

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[16] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1118.

[17] Not published.

TABLE 3.—ECONOMIC ANALYSIS OF OHIO RIVER IMPROVEMENT, 1905 TO 1925, INCLUSIVE

Year	COAL															SAND AND GRAVEL			STONE		
	Monongahela River to the Lower Ohio River			Monongahela River to the Upper Ohio River			Kanawha River			Huntington, W. Va.			Other Coal								
	Commerce, in thousands of tons (1)	Savings, in cents per ton (2)	Total savings, in thousands of dollars (3)	Commerce, in thousands of tons (4)	Savings, in mills per ton-mile (5)	Total savings, in thousands of dollars (6)	Commerce, in thousands of tons (7)	Savings, in cents per ton (8)	Total savings, in thousands of dollars (9)	Commerce, in thousands of tons (10)	Savings, in cents per ton (11)	Total savings, in thousands of dollars (12)	Commerce, in thousands of tons (13)	Savings, in cents per ton (14)	Total savings, in thousands of dollars (15)	Commerce, in thousands of tons (16)	Savings, in cents per ton (17)	Total savings, in thousands of dollars (18)	Commerce, in thousands of tons (19)	Savings, in cents per ton (20)	Total savings, in thousands of dollars (21)
1905	3 294	60	1 988	....	..	..	1 461	2	32	..	..	..	599	20	120	2 500	10	..	100	15	15
1906	3 225	60	1 919	....	..	..	1 176	3	35	61	20	12	409	20	82	800	10	80	100	15	15
1907	3 344	60	2 005	....	..	..	1 667	5	77	240	20	48	318	20	64	1 030	10	103	100	15	15
1908	2 406	63	1 505	....	..	..	965	5	52	326	20	65	300	20	60	950	10	95	100	15	15
1909	1 914	61	1 169	....	..	..	1 065	6	70	265	20	53	855	20	171	1 050	10	105	100	15	15
1910	1 657	60	996	117	6.4	13	1 248	7	89	386	20	77	357	20	71	980	10	98	546	15	82
1911	2 574	65	1 666	243	5.4	22	1 346	8	108	442	20	88	1 864	20	372	2 328	10	233	218	15	33
1912	1 731	64	1 110	262	5.8	26	1 276	9	112	339	20	68	1 301	20	260	1 184	10	118	24	15	3
1913	1 663	64	1 076	957	5.4	93	1 343	10	132	227	20	45	2 317	20	463	807	10	81	25	15	4
1914	1 099	64	709	804	5.5	80	1 053	10	102	336	20	67	937	20	187	1 831	10	183	193	15	29
1915	1 130	66	747	1 393	5.4	150	1 205	12	140	378	20	76	440	20	88	1 891	10	189	133	15	20
1916	653	68	448	1 237	4.0	99	1 381	12	172	643	20	129	....	..	..	2 011	11	220	30	17	5
1917	330	82	271	1 228	4.4	108	1 429	13	181	222	20	45	....	..	..	1 202	12	144	9	19	2
1918	....	..	....	1 883	6.2	233	1 092	24	264	36	20	7	795	20	159	1 688	13	219	20	21	4
1919	....	..	....	1 280	8.3	213	750	17	130	85	20	17	1 351	20	270	1 038	14	145	19	23	4
1920	....	..	....	1 735	8.3	318	1 323	10	132	528	60	317	3 064	20	613	1 217	15	182	23	25	6
1921	....	..	....	787	13.7	216	939	48	458	433	60	260	2 274	20	455	1 624	16	260	163	25	41
1922	....	..	....	1 292	13.2	393	712	37	266	234	40	93	1 954	20	390	896	17	152	10	25	2
1923	....	..	....	2 794	13.4	860	1 293	41	535	422	40	169	1 533	20	307	1 202	18	216	171	25	43
1924	....	..	....	2 490	13.4	900	648	44	287	514	40	205	2 160	20	432	3 747	19	712	261	25	65
1925	....	..	....	3 930	10.6	1 739	808	50	406	1 280	40	512	908	24	215	6 855	20	1 354	551	25	136
Total	**25 020**	..	**15 609**	**22 432**	..	**5 463**	**24 180**	..	**3 780**	**7 397**	..	**2 353**	**23 736**	..	**4 779**	**36 831**	..	**4 889**	**2 896**	..	**554**

TABLE 3.—(*Continued*)

Year	Logs, Railroad Ties, and Lumber			Iron and Steel			Oil and Gasoline			Packet Freight			Other Freight			Totals		
	Commerce, in thousands of tons (22)	Savings, in cents per ton (23)	Total savings, in thousands of dollars (24)	Commerce, in thousands of tons (25)	Savings, in cents per ton (26)	Total savings, in thousands of dollars (27)	Commerce, in thousands of tons (28)	Savings, in cents per ton (29)	Total savings, in thousands of dollars (30)	Commerce, in thousands of tons (31)	Savings, in cents per ton (32)	Total savings, in thousands of dollars (33)	Commerce, in thousands of tons (34)	Savings, in cents per ton (35)	Total savings, in thousands of dollars (36)	Commerce, in thousands of tons (37)	Savings, in cents per ton (38)	Total savings, in thousands of dollars (39)
1905	1 000	25	250	....	....	....	....	....	....	500	25	125	3 319	15	498	12 773	24	3 028
1906	1 000	25	250	....	....	....	....	....	....	500	25	125	3 538	15	531	10 809	27	3 049
1907	1 000	25	250	....	....	....	....	....	....	500	25	125	2 461	15	369	10 660	27	3 056
1908	1 000	25	250	....	....	....	....	....	....	500	25	125	1 490	15	223	8 037	28	2 390
1909	1 000	25	250	....	....	....	....	....	....	500	25	125	1 206	15	181	7 956	25	2 139
1910	1 013	25	253	126	50	63	1	50	0.5	500	25	125	3 831	15	575	10 760	21	2 443
1911	955	25	239	31	50	16	11	50	5.5	500	25	125	1 260	15	189	11 772	25	3 096
1912	271	25	68	10	50	5	24	50	12	500	25	125	76	15	11	6 999	26	1 919
1913	226	25	56	84	50	42	11	50	6	500	25	125	252	15	38	8 413	25	2 160
1914	667	25	167	39	50	19	4	50	2	500	25	125	127	15	19	7 589	21	1 690
1915	175	25	44	21	50	10	4	50	2	568	25	142	9	15	1	7 346	21	1 609
1916	70	25	17	7	60	4	6	60	3	412	30	124	4	18	1	6 455	18	1 222
1917	90	27	24	1	70	1	8	70	6	430	35	150	12	21	2	4 962	20	934
1918	190	29	55	22	80	18	21	80	17	423	40	169	3	24	1	6 171	20	1 146
1919	50	31	16	17	90	16	76	90	68	328	45	147	10	27	3	5 004	21	1 029
1920	393	33	130	355	100	355	46	100	46	664	50	332	34	30	10	9 382	25	2 441
1921	622	33	206	53	100	53	28	100	28	365	50	182	19	30	6	7 308	28	2 165
1922	275	33	91	316	100	316	31	100	30	539	50	270	33	30	10	6 292	33	2 014
1923	139	33	46	377	100	377	50	100	50	277	50	138	23	30	7	8 281	35	2 748
1924	104	33	34	550	100	550	82	100	82	220	50	110	91	30	27	10 867	33	3 405
1925	186	33	62	580	130	697	271	150	418	283	50	142	86	30	26	15 737	36	5 706
Total	10 426	..	2 758	2 589	....	2 542	674	....	776	9 509	..	3 156	17 884	..	2 728	183 573	27	49 389

TABLE 4.—ECONOMIC ANALYSIS OF THE OHIO RIVER IMPROVEMENT, 1926 TO 1934, INCLUSIVE

Year	COAL AND COKE								
	MONONGAHELA RIVER TO THE OHIO RIVER*				FROM THE KANAWHA RIVER				FROM HUNTINGTON, W. VA.
	Commerce		Savings		Commerce		Savings		Commerce
	In thousands of tons (1)	In thousands of ton-miles (2)	In mills per ton-mile (3)	Total, in thousands of dollars (4)	In thousands of tons (5)	In thousands of ton-miles (6)	In mills per ton-mile (7)	Total, in thousands of dollars (8)	In thousands of tons (9)
1926	5 735	172 159	10.6	1 825	765	156 925	2.1	329	1 689
1927	5 579	203 133	10.6	2 153	808	165 674	2.1	348	1 676
1928	5 756	234 829	10.6	2 489	962	197 117	2.1	414	1 753
1929	6 029	205 196	10.6	2 175	860	176 217	2.1	370	2 117
1930	6 445	215 113	10.6	2 280	791	162 115	2.1	340	1 952
1931	5 136	179 054	10.6	1 898	855	175 339	2.1	368	1 813
1932	3 915	150 580	10.6	1 596	903	185 098	2.1	389	1 846
1933	5 306	211 446	9.7	2 051	1 055	189 220	2.1	397	2 041
1934	6 106	211 516	9.7	2 052	848	162 435	2.1	341	2 321
Totals	50 007	1 783 026	....	18 519	7 847	1 570 140	....	3 296	17 208

TABLE 4.—(*Continued*)

Year	COAL AND COKE (*Continued*)							SAND AND GRAVEL		
	FROM HUNTINGTON, W. VA. (*Continued*)			OTHER COAL AND COKE				Commerce		Savings, in mills per ton-mile
	Commerce	Savings		Commerce		Savings				
	In thousands of ton-miles (10)	In mills per ton-mile (11)	Total, in thousands of dollars (12)	In thousands of tons (13)	In thousands of ton-miles (14)	In mills per ton-mile (15)	In thousands of dollars (16)	In thousands of tons (17)	In thousands of ton-miles (18)	(19)
1926	272 453	2.5	681	524	36 164	3.0	109	8 998	122 336	10.0
1927	282 856	2.8	792	453	31 331	4.0	125	9 187	137 889	10.0
1928	276 425	2.6	719	691	31 751	5.0	159	8 749	125 117	10.0
1929	336 720	4.1	1 380	745	39 279	6.0	236	8 709	125 070	10.0
1930	302 118	3.2	967	581	33 873	7.0	237	8 875	132 322	10.0
1931	282 276	2.8	790	715	33 884	8.0	271	6 526	132 664	10.0
1932	295 113	3.1	915	928	37 460	10.0	375	3 758	87 698	10.0
1933	305 247	3.3	1 007	1 576	58 192	12.0	698	3 355	66 314	10.0
1934	317 436	3.6	1 143	1 529	49 889	12.0	599	3 853	97 439	10.0
Totals	2 670 644	....	8 394	7 742	351 823	....	2 809	62 010	1 026 849	....

* Allegheny River coal included.

On account of the overwhelming importance of the four products, coal, steel, petroleum, and sand and gravel, it has not been deemed necessary to discuss herein the ton-mile cost of transporting the other products upon which the savings have been based. A discussion of each significant item is contained in the Appendix.

TABLE 4.—(*Continued*)

Year	SAND AND GRAVEL (*Continued*)	LOGS AND LUMBER				IRON AND STEEL				OIL AND GASOLINE
		Commerce		Savings		Commerce		Savings		
	Total savings, in thousands of dollars (20)	In thousands of tons (21)	In thousands of ton-miles (22)	In mills per ton-mile (23)	In thousands of dollars (24)	In thousands of tons (25)	In thousands of ton-miles (26)	In mills per ton-mile (27)	In thousands of dollars (28)	Commerce, in thousands of tons (29)
1926	1 223	395	9 002	12.0	108	575	238 746	6.5	1 552	318
1927	1 378	474	11 829	11.0	130	1 040	294 389	7.0	2 060	206
1928	1 251	394	11 483	10.0	115	1 413	350 557	7.6	2 664	323
1929	1 250	317	10 363	9.0	93	1 416	405 667	8.1	3 286	590
1930	1 323	336	7 110	8.0	57	1 481	407 780	8.2	3 344	482
1931	1 326	124	2 752	7.0	19	972	425 300	8.4	3 572	607
1932	877	95	2 442	7.0	17	610	289 401	7.0	2 026	964
1933	663	141	3 893	7.0	27	865	329 343	7.4	2 437	1 004
1934	974	167	5 642	7.0	39	844	473 196	8.8	4 164	1 272
Totals	10 265	2 443	64 516	....	605	9 216	3 214 379	....	25 105	5 766

TABLE 4.—(*Continued*)

Year	OIL AND GASOLINE (*Continued*)			STONE				CEMENT			
	Commerce, in thousands of ton-miles	Savings		Commerce		Savings		Commerce		Savings	
	(30)	In mills per ton-mile (31)	Total, in thousands of dollars (32)	In thousands of tons (33)	In thousands of ton-miles (34)	In mills per ton-mile (35)	Total, in thousands of dollars (36)	In thousands of tons (37)	In thousands of ton-miles (38)	In mills per ton-mile (39)	Total, in thousands of dollars (40)
1926	35 728	12.5	447	436	12 306	10.0	123	30	2 877	1.9	5
1927	24 238	12.5	303	387	11 088	10.0	111	49	5 363	2.7	14
1928	40 533	12.5	507	426	12 806	10.0	128	80	13 347	5.2	69
1929	74 575	12.5	932	365	12 220	10.0	122	116	14 874	5.7	85
1930	75 289	12.0	903	651	27 191	10.0	272	132	14 341	5.5	79
1931	95 920	12.0	1 151	557	15 005	10.0	150	152	17 698	6.6	117
1932	160 624	11.5	1 847	528	17 380	10.0	174	138	17 352	6.5	113
1933	194 465	10.5	2 042	239	8 333	10.0	83	176	12 416	4.9	61
1934	201 365	10.5	2 114	649	16 189	10.0	162	108	20 728	7.5	155
Totals	902 737	....	10 246	4 238	132 518	....	1 325	981	118 996	....	698

NET SAVINGS

In general, the costs on each of the more commonly used routes for each important commodity were determined by multiplying the ton-mileage moved by the average cost per ton-mile as estimated herein (see heading "Carrier Costs of Water-Borne Freight"), adding the terminal differential between rail and water transfers (except in the case of petroleum where it is in favor of the waterway), and also adding the actual rate for rail haul where (as in the general case) the commodity was hauled either to the river before transportation, from the river after transportation, or both. These costs were deducted from the comparative rail rates, and the gross savings established. From this

TABLE 4.—(*Continued*)

Year	MISCELLANEOUS FREIGHT				TOTALS			
	Commerce		Savings		Commerce		Savings	
	In thousands of tons (41)	In thousands of ton-miles (42)	In mills per ton-mile (43)	Total, in thousands of dollars (44)	In thousands of tons (45)	In thousands of ton-miles (46)	In mills per ton-mile (47)	Total, in thousands of dollars (48)
1926	289	42 932	3.3	142	19 754	1 101 628	5.9	6 544
1927	269	46 701	3.3	154	20 128	1 214 491	6.2	7 568
1928	388	50 720	3.2	162	20 935	1 344 685	6.5	8 677
1929	690	112 404	3.1	348	21 954	1 512 585	6.8	10 277
1930	611	96 676	3.0	290	22 337	1 473 928	6.8	10 092
1931	613	126 552	2.9	367	18 070	1 486 444	6.7	10 029
1932	632	149 081	2.8	417	14 317	1 392 228	6.3	8 746
1933	992	329 553	2.7	890	16 750	1 708 422	6.1	10 356
1934	939	228 088	2.7	616	18 636	1 783 924	6.9	12 359
Totals	5 423	1 182 707	....	3 386	172 881	13 018 335	6.5	84 648

value, the saving per ton-mile was easily obtained. In a small number of cases where published rates were available, these rates were used in lieu of costs. In general, since it was impracticable to obtain water rates or costs for each year separately, the savings for 1934 were calculated, and the average savings per ton-mile for the years, 1926 to 1934, were apportioned as explained in the Appendix. For the savings prior to 1926 the figures used by General Kutz in an official report were adopted. As explained previously, the break-down by ton-miles was impracticable for earlier years. The Appendix gives details of calculation for each of the more important items.

Official records (Annual Reports of the Chief of Engineers, for each fiscal year, Government Printing Office) give the tonnage and (in recent years) the ton-mileage of each commodity. After the unit saving per ton-mile for each commodity was determined, as shown in Table 3, simple arithmetic gave the commodity savings in dollars. These savings are summarized in Column (39), Table 3, and Column (48), Table 4. The values there given have been used as the net public benefit, with which the annual Federal costs must be compared (Column (15), Table 5).

GOVERNMENT COSTS

The determination of the Government costs is the simplest part of this study. Full data are available in the Annual Reports of the Chief of Engineers. The only explanation required refers to a few necessary assumptions in the statistics. The interest charge on Government money has been assumed as 4 per cent. This is high for recent years, but not excessively so. The interest during construction has been assumed at 2% for the year the money was spent; that is, it is assumed that costs were uniform throughout the year. The depreciation has been charged from the year of completion. In every case in which the lock has not been retired from service in less time, the life of the structure has been assumed as 40 yr. A straight-line depreciation has been assumed over that period. Where a lock has either been retired, or is due to be retired after less than 40 yr in service, the actual life of the lock, in years, has been determined, and a straight-line depreciation taken over that period.

In determining the remaining investment, or the surplus in the cumulated capital account (Columns (23) and (24), Table 5), depreciation of structures (Columns (14), Table 5) has been subtracted. This was done because the annual depreciation of the structures had already been added into the annual charges (Column (15), Table 5), thereby reducing the annual profit by the amount of the depreciation, which, in turn, reduced the net annual capital charges (Column (21), Table 5). Since the depreciation was thus added as a charge for the year, the remaining investment of the Government in the structures was obviously reduced by that amount. With these explanations, Table 5 should become perfectly clear.

Comparison

The graphical analysis in Fig. 2 shows the manner in which at first the annual charges (Column (15), Table 5) increased much more rapidly than the commercial savings (Column (16), Table 5), then how the savings overtook those charges until the onset of the depression, and then how rapidly these savings have advanced since 1932. Fig. 3 shows the manner in which the capital charges (Column (10), Table 5), as shown on the Government books, have, of course, tended to increase while the capital account (Columns (23) and (24), Table 5), from an investor's standpoint, assuming a public profit from savings, has tended recently to decrease. The relation of savings to expenses is shown analytically in Column (19), Table 5, in which the ratio, 1.0, of savings to expenditures means that the development has barely been

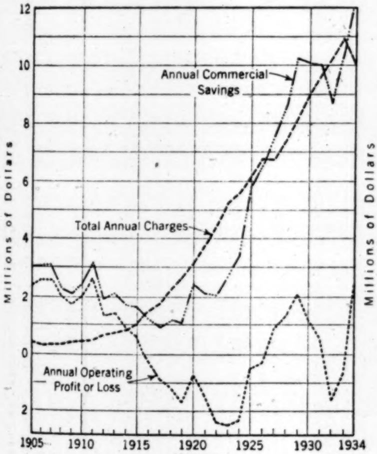

FIG. 2

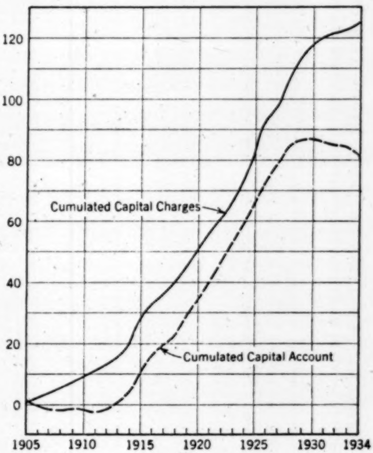

FIG. 3

TABLE 5.—ECONOMIC ANALYSIS OF OHIO RIVER IMPROVEMENT; FINANCIAL SUMMARY

Fiscal year	GOVERNMENT EXPENDITURES							CAPITAL ACCOUNT		
	Lock and dam construction (1)	Open river improvement (2)	Total new works (Columns (1) and (2)) (3)	Open river maintenance (4)	Operating and care (5)	Operating snagboats (6)	Total operation and maintenance (Columns (4), (5) and (6)) (7)	Interest on annual capital expenditures (2% of Column (3)) (8)	Total annual capital charges (Columns (3) and (8)) (9)	Cumulated capital charges (cumulation of Column (9)) (10)
Prior to 1905	....	....	....	....	....	....	....	....	....	....
1905	$ 905 738	$ 49 754	$ 955 492	$ 36 806	$ 195 901	$34 688	$ 267 395	$ 19 108	$ 974 600	$ 974 600
1906	1 101 754	72 278	1 174 032	15 437	154 354	33 094	202 885	23 480	1 197 512	2 172 112
1907	1 736 541	47 711	1 784 252	11 354	184 355	35 764	231 473	35 684	1 819 936	3 992 048
1908	1 224 586	90 253	1 314 839	17 415	181 814	37 063	236 292	26 296	1 341 135	5 333 183
1909	1 750 654	283 897	2 034 551	21 760	210 151	36 391	268 302	40 690	2 075 241	7 408 424
1910	1 147 489	133 920	1 281 409	11 508	211 108	34 497	257 113	25 628	1 307 037	8 715 461
1911	1 543 650	147 865	1 691 515	14 299	234 773	31 058	280 130	33 830	1 725 345	10 440 806
1912	1 991 513	290 481	2 281 994	15 279	334 886	31 348	381 513	45 638	2 327 632	12 768 438
1913	2 306 292	382 533	2 688 825	20 202	349 164	25 758	395 124	53 776	2 742 601	15 511 039
1914	4 267 384	261 297	4 528 681	18 545	330 894	36 449	385 888	90 572	4 619 253	20 130 292
1915	7 351 302	427 106	7 778 408	14 051	388 116	51 963	454 130	155 568	7 933 976	28 064 268
1916	4 981 651	177 914	5 159 565	80 408	429 024	36 143	545 575	103 190	5 262 755	33 327 023
1917	3 539 171	185 743	3 724 914	6 803	494 774	50 298	551 875	74 498	3 799 412	37 126 435
1918	3 128 749	25 310	3 154 059	174 917	609 422	44 745	829 084	63 080	3 217 139	40 343 574
1919	4 737 410	66 659	4 804 069	152 285	764 375	55 975	972 635	96 080	4 900 149	45 243 723
1920	5 151 353	29 096	5 180 449	152 247	864 987	32 201	1 049 435	103 608	5 284 057	50 527 780
1921	6 599 146	....	6 599 146	250 512	1 058 112	40 330	1 348 954	131 982	6 731 128	57 258 908
1922	4 459 681	....	4 459 681	276 116	1 100 862	33 049	1 410 027	89 192	4 548 873	61 807 781
1923	5 171 698	51 560	5 223 258	593 070	1 171 445	33 024	1 797 539	104 464	5 327 722	67 135 503
1924	5 747 989	372 788	6 120 777	309 632	1 420 574	57 864	1 788 070	122 414	6 243 191	73 378 694
1925	8 173 393	41 766	8 215 159	427 288	1 617 824	55 375	2 100 487	164 302	8 379 461	81 758 155
1926	8 464 716	....	8 464 716	359 492	1 744 551	47 263	2 151 306	169 294	8 634 010	90 392 165
1927	6 742 825	351 329	7 094 154	....	1 697 547	46 162	1 743 709	141 882	7 236 036	97 628 201
1928	9 618 807	410 135	10 028 942	....	2 096 828	35 121	2 131 949	200 578	10 229 520	107 857 721
1929	5 194 594	606 939	5 801 533	....	2 063 414	57 241	2 120 655	116 030	5 917 563	113 775 284
1930	1 236 476	3 052 438	4 288 914	301 454	2 273 100	45 792	2 620 346	85 778	4 374 692	118 149 976
1931	566 093	2 005 801	2 571 894	278 860	2 818 599	40 621	3 138 080	51 438	2 623 332	120 773 308
1932	979 379	−73 504	905 875	1 011 176	2 994 445	48 420	4 054 041	18 118	923 993	121 697 301
1933	1 224 260	−309 568	914 692	1 959 435	2 803 348	43 705	4 806 488	18 294	932 986	122 630 287
1934	1 474 834	290 727	1 765 561	1 574 972	2 200 894	49	3 775 915	35 311	1 800 872	124 431 159
Sub-total	**$112 519 128**	**$9 472 228**	**$121 991 356**	**$8 105 323**	**$32 999 641**	**$1 191 451**	**$42 296 415**	....	....	....
Before 1905	**9 313 472**	**4 024 560**	**13 338 032**	**1 500 932**	**2 624 084**	**402 878**	**4 527 894**	....	....	....
Total	**$121 832 600**	**$13 496 788**	**$135 329 388**	**$9 606 255**	**$35 623 725**	**$1 594 329**	**$46 824 309**	....	....	....

TABLE 5.—(*Continued*)

Fiscal year	Operating Account					Profit and Loss					Alterations in Capital Account			
							Annual				Net Annual Capital Changes		Cumulated Capital Account	
	Interest charged on investment (4% of Column (23), previous year) (11)	Interest charged on surplus (4% of Column (24), previous year) (12)	Expenditures for operation and maintenance (Column (7)) (13)	Depreciation* (14)	Total annual charges (Column (14) +Column (13) −Column (12) +Column (1)) (15)	Annual commercial savings (Column (39), Table 3, and Column (48), Table 4) (16)	Profit (Column (16) −Column (15)) (17)	Loss (Column (15) −Column (16)) (18)	Ratio of savings to expenditures (Column (16) ÷Column (15)) (19)	Percentage of annual return (Column (17) +Column (10), or Column (18) +Column (10)) (20)	Increase (Column (9) −Column (17) +Column (18)) (21)	Decrease (Column (17) −Column (9)) (22)	Remaining investment (Column (23) +Column (22) (21) −Column (14) −Column (24))† (23)	Surplus (Column (24) −Column (21) +Column (22) +Column (14) −Column (23))† (24)
Prior to 1905	$	$	$	$	$	$	$	$	....	......	$	$	$ 3 107 254	$ ......
1905	124 288	......	267 395	35 107	426 790	3 027 600	2 600 810	......	7.1	266.86	......	1 626 210	1 445 937	......
1906	57 836	......	202 885	35 107	295 828	3 049 300	2 753 472	......	10.3	126.76	......	1 555 960	......	145 130
1907	......	5 804	231 473	77 575	303 244	3 055 600	2 752 356	......	10.1	68.95	......	932 420	......	1 155 125
1908	......	46 204	236 292	114 821	304 909	2 389 900	2 084 991	......	7.8	39.09	......	743 856	......	2 013 802
1909	......	80 552	268 302	207 592	395 342	2 138 600	1 743 258	......	5.4	23.53	331 983	......	......	1 889 411
1910	......	75 576	257 113	207 592	389 129	2 442 700	2 053 571	......	6.3	23.56	......	746 534	......	2 843 537
1911	......	113 740	280 130	230 769	397 159	3 095 700	2 698 541	......	7.8	25.85	......	973 196	......	4 047 502
1912	......	161 900	381 513	362 024	581 637	1 919 200	1 337 563	......	3.3	10.48	990 069	......	......	3 419 457
1913	......	136 776	395 124	414 314	672 662	2 159 700	1 487 038	......	3.2	9.59	1 255 563	......	......	2 578 208
1914	......	103 128	385 888	414 314	697 074	1 690 300	993 226	......	2.4	4.93	3 626 027	......	633 505	......
1915	25 340	......	454 130	470 616	950 086	1 608 985	658 899	......	1.7	2.35	7 275 077	......	7 437 966	......
1916	297 516	......	545 575	525 645	1 368 736	1 222 000	......	146 736	0.9	−0.44	5 409 491	......	12 321 812	......
1917	492 872	......	551 875	612 723	1 657 470	933 500	......	723 970	0.6	−1.95	4 523 382	......	16 232 471	......
1918	649 296	......	829 084	725 508	2 203 888	1 145 700	......	1 058 188	0.5	−2.62	4 275 327	......	19 782 290	......
1919	791 288	......	972 635	790 042	2 553 965	1 028 700	......	1 525 265	0.4	−3.37	6 425 414	......	25 417 662	......
1920	1 016 704	......	1 049 435	974 469	3 040 608	2 440 831	......	599 777	0.8	−1.19	5 883 834	......	30 327 027	......
1921	1 213 080	......	1 348 954	974 469	3 536 503	2 165 215	......	1 371 288	0.6	−2.39	8 102 416	......	37 454 974	......
1922	1 498 196	......	1 410 027	1 335 127	4 243 350	2 014 439	......	2 228 911	0.5	−3.61	6 777 784	......	42 897 631	......
1923	1 715 904	......	1 797 539	1 539 941	5 053 384	2 747 756	......	2 305 628	0.5	−3.43	7 633 350	......	48 991 040	......
1924	1 959 640	......	1 788 070	1 629 822	5 377 532	3 405 063	......	1 972 469	0.6	−2.69	8 215 660	......	55 576 878	......
1925	2 223 072	......	2 100 487	1 701 248	6 024 807	5 705 827	......	318 980	0.9	−0.39	8 698 441	......	62 574 071	......
1926	2 502 960	......	2 151 306	1 950 257	6 604 523	6 544 000	......	60 523	1.0	−0.07	8 694 533	......	69 318 347	......
1927	2 772 732	......	1 743 709	2 024 037	6 540 478	7 568 000	1 027 522	......	1.2	1.05	6 208 514	......	73 502 824	......
1928	2 940 112	......	2 131 949	2 100 464	7 172 525	8 677 000	1 504 475	......	1.2	1.39	8 725 045	......	80 127 405	......
1929	3 205 096	......	2 120 655	2 641 243	7 966 994	10 277 000	2 310 006	......	1.3	2.03	3 607 557	......	81 093 719	......
1930	3 243 748	......	2 620 346	2 876 583	8 740 677	10 092 000	1 351 323	......	1.2	1.14	3 023 369	......	81 240 505	......
1931	3 249 620	......	3 138 080	2 899 950	9 287 650	10 029 000	741 350	......	1.1	0.61	1 881 982	......	80 222 537	......
1932	3 208 900	......	4 054 041	2 901 845	10 164 786	8 746 000	......	1 418 786	0.9	−1.17	2 342 779	......	79 663 471	......
1933	3 186 536	......	4 806 488	2 908 488	10 901 512	10 356 000	......	545 512	0.9	−0.44	1 478 498	......	78 233 481	......
1934	3 129 336	......	3 775 915	2 909 998	9 815 249	12 359 000	2 543 751	......	1.3	2.04	......	742 879	74 580 604	......

* Depreciation computed on the assumption of a 40-yr life for existing locks and dams. Where structures have been replaced, the actual life years were used.

† Columns (23) and (24) for previous year used in formulas for determining figure for Columns (23) and (24).

successful. A higher ratio indicates a profit. In considering these relations between commercial savings and public expenditures, it should not be forgotten that 4% interest has been charged against the entire remaining capital structure for every year. During the depression, railroad companies would have regarded themselves as fortunate, if they were able to pay 4% interest on their entire capital. Fig. 4 shows, in percentage form, the relation between the water-borne commerce and railroad commerce, using the commerce of each during the calendar year 1930 as a base. It can be seen that, after

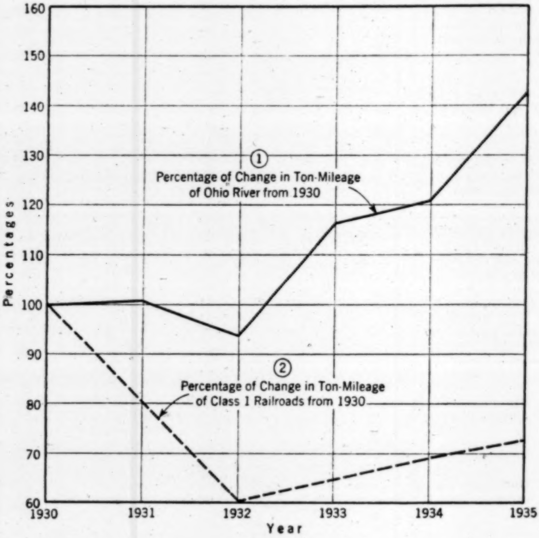

FIG. 4

TABLE 6.—RAIL AND WATER COSTS

Item No.	Commodity	COMMERCE		Percentage of total, excluding sand, gravel, stone, logs, and lumber	RAIL COST TO SHIPPER FROM CINCINNATI TO LOUISVILLE			WATER COST TO SHIPPER FROM CINCINNATI TO LOUISVILLE			
		In thousands of tons	In millions of ton-miles		In cents per ton	In mills per ton-mile over water distance	Proportion to sample ton, in mills	Assumed line cost, in mills per ton-mile	Terminal differential, in mills per ton-mile	Over-all cost, in mills per ton-mile	Proportion to sample ton, in mills
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	Coal and coke	10 805	741	44.5	149	11.2	5.0	2.3	2.2	4.5	2.0
2	Cement	108	21	1.3	364	27.3	0.4	5.4	6.7	12.1	0.2
3	Iron and steel	844	473	28.4	380	28.5	8.1	6.0	6.7	12.7	3.6
4	Oil and gasoline	1 272	201	12.1	380	28.5	3.4	6.4	0.0	6.4	0.8
5	Unclassified	939	228	13.7	428	32.1	4.4	..	..	20.0	2.7
6	Total	13 968	1 664	100.0	..	..	21.3	..	..	..	9.3

having been checked for a short time, in 1932 Ohio River commercial traffic recovered. (It is now (1937) more than 50% greater than it was in 1930.) In 1934 traffic of Class I railroads had not reached a figure equal to 75% of its 1930 figures. (It is now (1937) 12% less than in 1930.)

Table 6 has been prepared for the purpose of comparing the data on public savings by an entirely independent system of calculation. The following items supplement the data in Table 6:

(*a*)	Average haul of commodities on Ohio River (1934) in miles	119
(*b*)	Water distance, Cincinnati, Ohio, to Louisville, Ky., in miles	133.5
(*c*)	Total annual charges, 1934 (Column (15), Table 5)	$9 815 249
(*d*)	Total commerce, Ohio River, 1934 (Column (46), Table 4), in ton-miles	1 783 924 624
(*e*)	Government costs, in mills per ton-mile, 1934, $\left(\frac{\text{Item } (c)}{\text{Item } (d)}\right)$	5.5
(*f*)	Government cost plus water cost to shipper (Column (11), Table 6) = total water cost, in mills per ton-mile	14.8
(*g*)	Rail cost to shipper, reflected by quoted rates, in mills per ton-mile (Column (6), Table 6)	21.3
(*h*)	Saving, in mills per ton-mile, by water, on sample ton	6.5

The average haul was determined from the traffic data of the Ohio River, and two important points on the Ohio River (Louisville and Cincinnati) were selected as being about that distance apart. From the official figures of waterborne traffic, the proportion of each significant commodity in a representative ton was ascertained. Based on the rail rates (in carload lots) for these same commodities, it was possible, in the manner shown in Table 6, to determine the rail rate per ton-mile measured by water for a theoretical ton of representative cargo, moving in a carload lot. By similar methods, the water rate per ton-mile was determined for the same unit. Referring to Column (8), Table 6, the rate for coal and coke (Item No. 1) is that quoted by a water carrier. The rail rate on coal is somewhat higher than the average, but a reduction to 8 mills per ton would affect the final values only slightly. The rate for cement (Item No. 2) is that quoted by a water carrier for a 156-mile haul. Item No. 3 (iron and steel, Column (8)) was used in the 1934 study for the 164-mile haul. A large unit rate is used because a short haul is being assumed. The rate for oil and gasoline is that for an 118-mile haul. No terminal differential (see Column (9), Table 6) is assumed because of the more economical river loading. Item No. 5 in Column (10) is the common carrier average on four major commodities. Terminal handling charges are included in this rate. By dividing the annual expenditures by the total ton-mileage (Item (*e*)), the Government cost per ton-mile was also ascertained. As shown, the rail cost was found to be 21.3 mills per ton-mile, and the total combined water cost, 14.8 mills per ton-mile, giving an economic public benefit of 6.5 mills per ton-mile for water-borne traffic. Referring to Table 4, it will be seen, from examination of Column (47) for 1934, that the commercial saving by water on the average cargo was determined as 6.9 mills per ton-mile. The economic public benefit by the methods used in Table 4 would be 6.9 mills, less the Government cost of 5.5 mills, or 1.4 mills. Table 6 thus gives a result about five times as favorable to the waterway as Table 4.

Trends of Commerce

For reasons previously explained, commercial costs for the calendar years 1935 and 1936 have not been included in this study. Since its initiation, however, gross figures for commerce for those years have become available. They indicate an increase in tonnage of 9%, and of ton-mileage of 53 per cent. The greater increase in ton-mileage over tonnage is indicative of a trend which has been evident since 1926. In that year the average haul was 55.7 miles; in 1936, it was 108.7 miles. The reasons are obvious. Long-haul traffic (steel, petroleum products, etc.) has increased. Short-haul traffic, especially sand and gravel, has failed to increase proportionately, or has actually decreased. The increase in ton-mileage over 1934, which may be compared with the 9% increase in ton-mileage in the freight moved on Class I railroads for the same years[18] indicates clearly that, when there is any traffic, the Ohio River gets at least its share, and probably more.

At this point it might be well to take a general view of the commercial figures for the ten years since General Kutz presented his paper. They show clearly what has been noticed before, that no one is able to determine in advance the detailed trends of future commerce on a waterway. In that original study, through steel shipments (that is, steel from the Pittsburgh area to Mississippi River ports) were scarcely of enough importance to receive separate treatment. This movement has now (1937) become the most valuable commerce on the Ohio River; and its value seems destined to increase. More and more manufacturers are sending manufactured steel by water; and there is some evidence of an intent to transfer to the inland route steel which now moves from Pittsburgh to the Coast, and thence to the Gulf, coastwise. The movement of water-borne coal follows the general lines already forecast in 1926, except for the unpredicted activity around Wheeling, W. Va., although here, again, it appears that an industrial depression checks the movement of water-borne coal to a markedly smaller extent than it does rail-borne coal. In General Kutz' paper, the only mention of petroleum is in regard to delivery service from a refinery at Parkersburg, W. Va.[19] Petroleum movement has now become a large item in Ohio River traffic, and shows no signs as yet of approaching a saturation point. On the other hand, the common carrier traffic for which great things were hoped, has not developed proportionately. In spite of modern equipment and excellent management, the common-carrier towboat lines find the transportation of privately owned barges at contract rates absolutely essential for their support.

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[18] Figures were obtained by comparing those given in Annual Reports of the Interstate Commerce Commission.

[19] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1121.

The difficulties of common carriers on Western rivers are in considerable part due to the unbalanced character of the tonnage. On an open channel, the most profitable division of tonnage, from a strictly engineering standpoint, is 8 tons of down-stream traffic to 3 tons of up-stream traffic. For the calendar year 1935 the principal common carrier on the Ohio River informs the writer that his ratio was 21 tons down stream to 20 tons up stream. For the same year the report of the Inland Waterways Corporation indicates that, on the Mississippi River, tonnage was 29% down bound and 71% up bound. On the Upper Mississippi section it reached the extraordinary value of 18% down bound to 82% up bound.[20] From a strictly economic standpoint, the obvious remedy is to increase the up-bound rates and to decrease the down-bound rates until the traffic approaches a proper balance. However, a comparatively slight increase in the up-bound rates might divert traffic from the waterway altogether. Similarly, on rail-water movements, a substantial decrease in the down-stream rates might destroy the water carriers' share of the revenue entirely. Until this subject is adequately studied a remedy for the situation is not apparent. It is evident, however, that a profitable common carrier traffic awaits the discovery of the remedy.

The entire trend of Ohio River commerce is toward an increase—often an enormous increase—in the transportation of low-grade, long-haul, bulk products, in which the consumers' savings are only appreciable after a comparatively long period, and after time has been given for competing producers to engage in water transportation. Only to a very small extent have consumers' benefits become apparent in the immediate reduction in the prices of commodities, as a result of their more economical transportation by carriers entirely disinterested in either the manufacture or distribution of the articles transported. It can be asserted with confidence that the waterway reduces, materially, the proportion of the transportation cost contained in the gross cost of articles carried by water, either in their final state, or as raw materials. It is much more difficult to show, and from all indications it will always remain more difficult to show, that the waterway reduces in the same proportion the price which the consumer pays for the manufactured article.

It is evident at any rate that the pessimistic prophecies of 1926 have not proved true. It was then stated[21] by Frank H. Alfred, M. Am. Soc. C. E., that "it does not seem likely, however, that the completion of the improvement project will result in any considerable increase of river traffic." The data in Table 4 certainly prove the contrary.

The Special Board of Engineer Officers, in its report of December 15, 1906, which constitutes the basic plan for the present improvement of the Ohio River, makes a conservative prophecy, which might well be contrasted with the one by Mr. Alfred. The report states that the commercial saving on a 9-ft project with the then existing commerce would amount to $2 280 000 per year; and that "the ultimate annual saving *** would be several times the amount stated."[22] The figures for savings in recent years, Table 4, amount to several times $2 000 000 per year, and thus confirm the far-seeing estimate of 1906.

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[20] Rept. of Inland Waterways Corporation, 1935, p. 7, Govt. Printing Office, Washington, D. C., 1936.

[21] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1136.

Non-Statistical Considerations

Some commentators on transportation refer to the regulating effect of an improved waterway on the rates charged by other carriers. The writer has not been able to discover that the existing rail-rate structure discriminates between points having water competition and points that are without water competition.[23] On the other hand, the general rate structure of the Valley appears to be affected by the availability of the canalized Ohio River. One coal operator, not directly interested in water traffic, has recently made the statement that the cost of coal at Cincinnati would be $1.00 per ton higher if there was no river. Although this is mere opinion, it seems to coincide with the general opinion. However, the effect on rates is by no means the only effect of the improved waterway on the population of the region. Routes of communication, banking affiliations, the operations of jobbers, and all the other business relations which characterize the true "regions," in the normal sense of the term, have been built up on the assumption of the existence of the improved waterway. If the river were to disappear the population of the water-shed would begin to diminish with great rapidity. Freight rates, like other costs, would rise with the attenuation of the regional market.

Throughout this study no statistical comparison has been made in regard to passenger traffic. Such traffic is only to a very slight extent commercial. It is essentially travel for pleasure. Yet its general importance to the people of the Valley should not be overlooked. During the season of hot weather, boats of all sizes, from canoes to huge excursion boats, constitute a very important element of joy to the people. None of these boats could operate if the river were not improved.

Conclusions

From both an engineering and an economic standpoint, the canalization of the Ohio River has been successful. This success tends to become greater with time.

APPENDIX

The information in this Appendix supplies confirmatory details for the general information given under the heading "Net Savings."

Coal: Monongahela River to the Ohio River.—The savings per ton-mile for 1934 were 9.7 mills, and for 1925 they were 10.6 mills. As the reduction is believed to have been caused by river-rail movement of coal to Youngstown, Ohio, which was started in 1932, the reduced savings were used for the last two years only.

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[22] H. R. Doc. No. 492, 60th Cong., 1st Session, p. 36.

[23] A similar conclusion was reached by General Kutz, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1115.

Coal: From the Kanawha River.—In the absence of more accurate information for 1934, savings of 2.1 mills per ton-mile were used throughout.

Coal: From Huntington, W. Va.—The 1934 savings per ton were calculated as 49 cts, and for 1925 as 40 cts. There has obviously been a progressive increase in the saving on this traffic, and the difference in savings was distributed proportionately throughout the years involved.

Other Coal and Coke.—The 1934 savings were 38 cts per ton, and the 1925 savings were 24 cts per ton. It is reasonably clear that this change was due to an alteration in the character of coal movement. In 1934, 69% of the tonnage was from mines in the vicinity of Wheeling, W. Va., which scarcely produced any water-borne coal in 1926. The savings on the Western Kentucky coal, which is the other principal factor in this category, were very much less than those on the Wheeling coal. The savings were interpolated over the years 1926–1934 according to time. The change in the character of coal traffic has been progressive.

Sand and Gravel.—The savings per ton-mile decreased from 14.9 mills in 1925 to 10 mills in 1934. Apparently, the decrease in savings is due to a decrease in tonnage carried per barge employed. No terminal differential was charged on products moved directly to river points for use in repairing adjacent highways, or for the construction of river improvements, because it is more economical to unload a barge at the site of the work than to unload it from railroad cars, and then haul it in trucks from rail-heads. The decreased value of 10 mills per ton-mile is used throughout the period. Any error in this figure is against the waterway.

Logs and Lumber.—The 1925 savings were 12.2 mills per ton-mile. In 1934, they had decreased to 7 mills per ton-mile. The decrease in savings was adjusted proportionately according to time.

Iron and Steel.—The enormous increase in the savings on iron and steel is obviously due to the great increase in length of haul. In other words, the terminal differential chargeable per ton-mile has been greatly reduced. The saving per ton-mile has been proportioned on the basis of ton-mileage.

Oil and Gasoline.—There is no terminal differential on this product, or, if there is, it is in favor of the waterway. As the average haul increased, the savings per ton-mile were not increased. As up-stream traffic becomes a larger portion of the total haul, the saving per ton-mile decreases. There has been a slight decrease in savings per ton-mile since 1926, due to this latter fact. The decrease has been distributed according to the best information available on the change in character of traffic.

Stone.—On this comparatively small shipment, the figure for saving per ton in 1926 was used throughout.

Cement.—This commodity makes its first appearance at this time as a separate item. The savings, however, were estimated in an unpublished document at 1.9 mills per ton-mile in 1925. In 1934, calculations show the savings at 7.5 mills. The increase is due partly to the fact that no terminal differential is charged on cement delivered to riparian construction projects, and partly to a decided increase in volume of traffic. The savings for the years

1925–1934 have been proportioned on the basis of the ton-miles moved on the river.

Miscellaneous Freight.—The savings per ton increased from 45 cts in 1925 to 66 cts in 1934. The savings per ton-mile decreased from 3.3 to 2.7 mills in the same period. This curious result is due to the increase in the volume of river-rail commerce moving on published rates (with considerably reduced unit savings), combined with a considerable increase in length of haul. The present trend of movement began in 1931. The saving of 3.3 mills is used prior to that year, and the saving of 2.7 mills thereafter.

Terminal Differentials.—The differentials used were determined from the best evidence available. In some cases the sources were confidential. On coal the value generally used was 5 cts per ton for loading, and 15 cts per ton for unloading. On unmanufactured iron and steel it was 15 cts per ton for both loading and unloading. On manufactured steel it was 25 cts per ton for loading and (on an average) 75 cts per ton for unloading. Incidentally, the unloading differential for manufactured iron and steel represents the entire cost of unloading, as the ultimate consignee nearly always takes rail delivery anyway, and, thus, all the cost of unloading from barges is an extra charge. On cement, the loading differential used was 15 cts per ton, and the unloading charge 75 cts per ton (except when it was delivered for use in riparian structures). On freight moved by common carrier, of course, the terminal differential is included in the published rate.

DISCUSSION

Eugene L. Grant,[24] M. Am. Soc. C. E. (by letter).—The author is to be congratulated on his attempt to take the question of the economic justification for a high investment in the improvement of inland waterways out of the realm of conjecture and into the realm of measured facts, and for his careful statement of the assumptions of his study. Likewise, the Corps of Engineers should be commended for its records of river traffic which made possible such a factual study. However, a critical examination of the stated assumptions and of the calculations of Table 5 casts considerable doubt on the expressed conclusion that the Ohio River improvement has been an economic success.

The most important of the assumptions, of course, is Assumption (1) to the effect that all savings are passed on to the public in the form of lower prices. Even under the assumption of free competition of classical economic theory this is not necessarily the case. A saving in transportation costs to all the competing producers in a given industry would be passed on, but a saving to a limited number of producers might or might not be passed on, depending on their competitive positions—that is, on whether or not they are "marginal" producers in the classical economist's sense. Thus, even under free competition it would be likely that a considerable part of the saving in transportation costs from the Ohio River improvement would not be passed on. Under the well-known fixed-price structure of the steel industry, which has been in existence throughout the period of this study, it seems probable that, as far as this industry is concerned, none of the savings has been passed on; there has merely been a transfer of funds from the owners of the railroads that would have carried the traffic, had there been no river improvement, into the pockets of the owners of the steel companies.

With regard to Assumption (2), it may be noted that although the difference between the costs of water transportation and rail rates is a fair measure of the economic advantage to the shipper in instances where the commodity would move by rail if water transportation were not available, this is not the case with traffic that would not move at all if only rail transportation were available. Where traffic is created by the availability of cheap water transportation, the economic advantage to the shipper is the difference between a rate which would just serve to move the traffic (that is, "what the traffic will bear") and the cost of water haul. Although this is not measurable, it constitutes a reason for discounting, somewhat, the author's estimates of savings.

In connection with Assumption (3)—that the savings in the cost of ferriage have been neglected—it might be noted that the increased costs of highway and railway bridges due to the requirements of navigation have also been neglected.

Thus, it appears that the author's carefully stated assumptions favor the waterway to a much greater degree than he has indicated.

Table 5 is an example of a somewhat unconventional method of setting up an engineering economic study. It is in effect a compound amount calculation for all the disbursements and receipts (assuming "annual commercial savings" as receipts), with interest at 4 per cent. Such a calculation with respect to a terminated enterprise would tell whether the invested moneys had been recovered with a 4% return; however, its suitability for judging the economic success of the Ohio River improvement may be questioned.

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This is particularly the case because there are apparently two separate questions which should have been asked: (1) Did the moderate level of improvement of the early days (with \$13 000 000 invested before 1905), pay for itself in reduced transportation costs? and (2) does it appear as if the present canalization of the river (with its investment of \$135 000 000) is likely to pay?

From an inspection of Table 5, the historical picture seems to be somewhat as follows: In 1905, and for a few years thereafter, the investment in river improvement paid handsomely (not as much, however, as is implied by the 266% annual return shown for 1905 and the high returns for the years immediately succeeding; these high values resulted because the author failed to include the \$13 300 000 spent on capital improvements before 1905 as part of the investment on which the return was calculated; he also omitted depreciation on these improvements throughout his study). Then the competition of other forms of transportation apparently resulted in rapid obsolescence of the river improvements, and, by 1919, the commercial savings were barely sufficient to cover operation and maintenance costs, with no allowance for depreciation or return on investment. After this the increasingly high level of river improvement served to stimulate river traffic to the point of increasing the annual commercial savings to more than \$12 000 000 in 1934.

The author's plan of writing off subsequent investments against previous savings, and of assuming an interest income from calculated "surplus," has the tendency to show last year's savings as a return on next year's improvements. It should be clear that this is misleading—that the high commercial savings of the early years which resulted in the "surplus" in the years 1906 to 1912 have no relation at all to the question of whether the subsequent higher level of improvement will prove profitable.

Another objection to the use of the type of calculation involving the determination of a compound amount is the great importance of the assumed interest rate on the apparent conclusion. This dependence of the conclusion on the interest rate assumed is not always obvious either to the engineer making such calculations or to his audience. If, for instance, the author had used 6% as his assumed interest rate, his conclusion would have been that no profit existed in any year after 1915; this would have implied that the high level of improvement of the Ohio River was not an economic success.

The real question with regard to a proposed investment in capital goods—either in private enterprise or in public works—is whether the investment seems likely to be recovered during the economic life of the capital goods plus a return which is attractive in the light of the apparent risks involved. In private enterprise, it is customary not to make such investments unless the prospective rate of return is considerably greater than the cost of money; thus a concern with an average cost of capital of 7% might be likely to require a prospective return of 15% before undertaking a proposed investment. Where the possibility of obsolescence exists, the required safety factor may be much greater; a common requirement in the manufacturing industries is that a proposed investment in cost-reducing machinery must "pay for itself" in two or three years, or, in some cases, in one year.

Two considerations should be noted in connection with interest rates in economy studies regarding public works projects. The first is that when the Government collects taxes it is taking funds which might otherwise be productively invested by the taxpayers. It follows that public works so financed are not socially profitable unless they show capital recovery with a return at least as great as the return from the private investments which are displaced by the diversion of taxpayers' funds to public uses. This is a sound principle even if the difficulties of placing a money value on the benefits from public works make it difficult to calculate a rate of return in many instances; but where benefits are measured, as in the present instance, a project is not economically successful unless the return is as great as the average return obtainable by the investment of capital in private enterprise.

The second consideration in selecting an interest rate for economy studies for public works projects is that of the necessity of a safety factor. The high safety factors commonly used with regard to proposed cost-saving improvements in private enterprise have been mentioned. These safety factors are used because experience has shown that estimates of savings and of the economic lives of capital goods are likely to "go wrong." As errors in such estimates are not confined to private enterprise, it would seem reasonable that a safety factor should also be used in public works projects. This is particularly true in the present case, in which many arbitrary assumptions necessarily enter into the evaluation of benefits. It may also be noted that, although obsolescence does not operate as rapidly in public works as in many private enterprises, the history of the Ohio River navigation presented by the author shows a rapid obsolescence of the earlier moderate level of improvement.

The use of a 4% interest rate in this economy study is in effect the assumption that 4% is a satisfactory return on the Government's investment in the Ohio River improvement. In the light of the considerations which have just been mentioned it would seem as if the return should be considerably higher than this, before it can be said that the improvement has been economically justified.

For this reason it should be of interest to determine the apparent rate of return in each of the thirty years of record, making calculations which avoid any interest charges or any assumed interest rate. At first glance, it might appear that Column (20) of Table 5, gives the excess above, or deficiency below, a 4% return. This is not the case, however, because of the author's neglect of the investment prior to 1905 and because of the fact that the interest charge or credit has been based on the "cumulated capital account" rather than on the depreciated investment. Therefore, the writer has computed, in Table 7, the apparent profit or loss that would have been shown each year in the income statement of a private enterprise with receipts equal to the commercial savings of Column (16), Table 5, and operation and maintenance costs equal to the values shown in Column (7). In the absence of any information regarding the dates of the $13 000 000 investment prior to 1905, the depreciated investment at the beginning of 1905 has been assumed arbitrarily as $10 000 000 and depreciation has been charged on the investment on the basis of an assumed 40-yr life; the annual depreciation charges shown in Column (14), Table 5, are thus increased by $250 000. All values have been expressed to the nearest $10 000, as it does not seem that more significant places are justified by the character of the available data.

The record as shown from 1912 to 1934 in Column (5), Table 7, is not impressive; the rate of return is never great enough to be considered attractive in the light of the risks involved. It will be noted that this is so despite the fact that Table 7 gives the waterway considerably more than it is entitled to by assuming the revenue as 100% of the author's estimate of commercial savings. If, as suggested at the beginning of this discussion, much of the savings in transportation costs are not passed on to the public in the form of reduced prices, the "picture" becomes even more unfavorable to the waterway. For instance, if it is assumed that only 50% of the savings are passed on, there has been no profit in twenty-two of the last twenty-three years. It would seem, therefore, that even if weight is given to the "Non-Statistical Considerations" noted at the end of the paper, the Ohio River improvement can scarcely be considered an economic success.

TABLE 7.—ANNUAL PROFIT AND LOSS STATEMENT (IN MILLIONS OF DOLLARS) AND RATE OF RETURN ON DEPRECIATED INVESTMENT FOR OHIO RIVER IMPROVEMENT

Fiscal year	Gross income (1)	Annual expense operation, maintenance, and depreciation (2)	Net income Column (1) − Column (2) (3)	Plant investment less depreciation (4)	Percentage return on depreciated investment, Column (3) ÷ Column (4) (5)	Fiscal year	Gross income (1)	Annual expense operation, maintenance, and depreciation (2)	Net income Column (1) − Column (2) (3)	Plant investment less depreciation (4)	Percentage return on depreciated investment, Column (3) ÷ Column (4) (5)
1905...	3.03	0.56	2.47	10.69	23.1	1920..	2.44	2.27	0.17	50.33	0.3
1906...	3.05	0.49	2.56	11.59	22.1	1921..	2.17	2.57	−0.40	55.84	−0.7
1907...	3.06	0.56	2.50	13.08	19.1	1922..	2.01	3.00	−0.99	58.80	−1.7
1908...	2.39	0.60	1.79	14.06	12.7	1923..	2.75	3.59	−0.84	62.34	−1.3
1909...	2.14	0.73	1.41	15.68	9.0	1924..	3.41	3.67	−0.26	66.70	−0.4
1910...	2.44	0.72	1.72	16.53	10.4	1925..	5.71	4.05	1.66	73.13	2.3
1911...	3.10	0.76	2.34	17.77	13.2	1926..	6.54	4.35	2.19	79.56	2.8
1912...	1.92	0.99	0.93	19.49	4.8	1927..	7.57	4.01	3.56	84.53	4.2
1913...	2.16	1.06	1.10	21.57	5.1	1928..	8.68	4.48	4.20	92.41	4.5
1914...	1.69	1.05	0.64	25.53	2.5	1929..	10.28	5.01	5.27	95.44	5.5
1915...	1.61	1.17	0.44	32.74	1.3	1930..	10.09	5.75	4.34	96.68	4.5
1916...	1.22	1.31	−0.09	37.23	−0.2	1931..	10.03	6.29	3.74	96.15	3.9
1917...	0.93	1.41	−0.48	40.17	−1.2	1932..	8.75	7.20	1.55	93.93	1.7
1918...	1.15	1.81	−0.66	42.40	−1.6	1933..	10.36	7.97	2.39	91.70	2.6
1919...	1.03	2.01	−0.98	46.26	−2.1	1934..	12.36	6.94	5.42	90.34	6.0

FRED LAVIS,[25] M. AM. SOC. C. E. (by letter).—The members of the Society are to be congratulated on the paper by Colonel Hall and the very illuminating data he has presented in regard to transportation on the Ohio River and the costs thereof. They are an important addition to the available data on this subject.

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[25] Cons. Engr., New York, N. Y.

There are two points to be noted, however: First, his assumption that Government expenditures in the case of the improvements on the Ohio River are in the same class with those for ocean terminals and lighthouse service; and, second, that comparisons of costs of transportation between line terminals are a true criterion of comparison between rail and water-borne transportation.

The comparison with the provision of land beacons for the guidance of airplanes, while perhaps partly apropos, is not entirely so, because these beacons have to do with the carrying of the mails and the need of national defense to keep abreast of other nations in the development of aviation. They are also a necessary service for the actual military needs of to-day.

Improvements of harbors on the coast at the expense of the nation are justified because they affect the entire life of the nation and its people, unless they believe in, and can actually practice, an isolationist policy—which they neither do nor can. The coast line and harbors of the United States are the concern of the nation as a whole, not merely of those who live next to them, and, therefore, must be a charge against the Federal Government.

To take only two specific examples: The coffee on practically every breakfast table in the country, every morning, whether it be New York, Ohio, Virginia, Colorado, or anywhere else, must come in through an ocean terminal port. Automobiles made in Detroit, Mich., for export must be shipped out through an ocean terminal, etc. Ocean terminals really serve a national use.

It is quite understandable also that the lower reaches of some rivers can be included as part of the coast line, or as parts of, or extensions to, maritime harbors. The exact line of demarcation between coast line and inland waterways may be difficult to determine. Notwithstanding the long established legal authority of the Federal Government over all "navigable" waterways, it is difficult, however, to consider such improvements as those of the Ohio River other than local improvements or as beneficial to other than local interests. Colonel Hall's paper tends to confirm this.

Of course, the writer is not unmindful of the possible far-reaching effects of flood control, but that hardly affects the present discussion.

Comparisons of transportation costs between main line termini are not of much value; neither are ton-mile costs *per se* inasmuch as the river is almost sure to be longer than a parallel line of land transportation. The true criterion is the cost of transportation from producer to consumer. If coal is produced at a mine on the river, shot directly into barges, transported, and then consumed at another point on the river after direct transfer from barge to firebox, the river charge is an accurate measure of the cost of transportation, provided *pro rata* costs of construction and maintenance of river facilities are included.

Colonel Hall's paper indicates that these last named costs are not included, and he thinks their payment by the general public is offset by general public benefits. It is difficult to follow this reasoning, or to understand why a taxpayer in Portland, Me., Boston, Mass., New York, N. Y., Seattle, Wash., or San Francisco, Calif., should pay for these local benefits to people who happen to live alongside the improved channel of the Ohio River.

There is also to be considered the very strict regulation of rail rates and the limitation of return by Federal agencies on investments in railways which takes no account of this subsidized competition.

Leaving this subject, there still remains the question as to whether river costs (including the cost of the improvement) between termini are true measures of the transportation costs of the material carried on the river. Much of this material, it would seem, must have further charges for land transportation at either or both ends. A true comparison, therefore, can only be made by comparing all transportation charges, including loading and unloading en route, if any, between producer and consumer.

For any bulk freight such as that handled by river, this may be an important consideration. It might well be that the actual full cost of transportation of coal between off-river points might be less if it were handled from a railroad siding at a mine to a railroad siding in the plant where it is finally consumed.

There seems to be little doubt that the improvements on the Ohio River benefit somebody and provide low costs of transportation on some goods for some people. It does not, however, seem to be apparent from this paper, or from other available evidence, that this justifies taxation of others whose benefits are very small, or nothing at all. Moreover, the mere comparison of costs between main-line termini by one or more different methods of transportation is not a true measure of comparison of transportation costs.

O. Slack Barrett,[26] Esq. (by letter).—Complete and convincing as this paper seems to be, a careful study reveals the need of emphasis on several aspects of it.

History of the Ohio Valley Improvement.—Although the first dam was completed at Davis Island, in 1884 (as Colonel Hall states) there was no real canalization until years after 1895 when The Ohio Valley Improvement Association was organized. By the time the Panama Canal was opened to commercial navigation in August, 1914, only twelve dams had been completed on the Ohio River, and most of those between 1910 and 1914. The sum and substance of the matter is that these dams were located and built to facilitate navigation over some of the worst parts of the river, and served principally to expedite coal movements from Pennsylvania and West Virginia mines to markets, some of which were in the deep South. Even then, it will be noted, two factors seem to have been the motivating considerations, namely, cheaper transportation and wider markets.

In 1914, the war in Europe began at about the same time that navigation was opened in the Panama Canal. It is impossible to separate the effects of these two simultaneous events, but the four years of war demonstrated the necessity of adequate transportation facilities, and the Panama Canal, by shortening the commercial distance between the two coasts, increased the disadvantages suffered by industries in these inland valleys in competition with those situated at tide-water. The result was the popular demand for the completion of the improvement of the Ohio River, deepening and stabilizing channels in the Mississippi River, and the improvement of other waterways.

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[26] Pres., Ohio Valley Improvement Assoc., Cincinnati, Ohio.

About ten years after the close of the World War, and fifteen years after the opening of the Panama Canal, the canalized Ohio River was open for continuous navigation. In the mean time, oil had largely replaced coal in the Southern markets, and what was left of the coal movement amounted to a number of important isolated trades. It is significant, however, that the prospect of dependable channels in the Ohio and Mississippi Rivers attracted the attention of alert industrial managers to such an extent that the 1929 traffic (not counting that of the tributaries) amounted to 1 513 000 000 ton-miles, with coal accounting for 50%, and the growing iron and steel traffic accounting for about 27 per cent.

The year, 1929, will long be remembered as the beginning of the great depression. By 1932, the traffic decreased to 1 392 000 000 ton-miles, but in 1933, with the resumption of industrial activity in the United States, the traffic began to mount until 1936 when it attained 2 653 000 000 ton-miles, representing an increase of 75.5% over the traffic in 1929, with iron and steel accounting for almost 34 per cent.

Benefits.—Colonel Hall has definitely shown, by detailed figures, the most authoritative estimate of the actual savings afforded by the present use of the improved Ohio River. This might be considered as the direct public benefit. There has been some speculation as to whether the savings are enjoyed by the consumer or by the producer, by the shipper or by the consignee. Such speculation is interesting from an academic viewpoint as an effort to determine what part of the public derives the benefit. In fact, both the producer or the shipper and the consumer or the consignee are the public, and any benefits derived by either or by both are derived by the public.

In addition, there are the indirect benefits merely suggested in several places in the paper. These benefits cannot be measured, charted, and definitely analyzed, but they probably constitute the greater part of the economic value of the improvement. It has been freely admitted that the great steel industry could not exist in the United States without cheap water transportation. It is certain that the Pittsburgh industrial area is dependent upon it. No man would hazard the suggestion that the public is not benefited by such industrial activity. What public-spirited citizen of large vision would advocate concentration of all the major industrial plants on the seaboard? The improved Ohio River is a large contributing factor in retaining and attracting industry, thus contributing to a more balanced development of the nation.

It is a definitely established fact that the rate structure applicable to commodities which form the bulk of the freight on the rivers has been substantially lowered. The best evidence on this point can be found in the rates approved by the Interstate Commerce Commission in the last decade. The freight bill paid by citizens has been lowered substantially.

The net result of the improvement of the Ohio and Mississippi Rivers is that the purchaser has been able to buy, and the seller to sell, in wider markets. More opportunities have been afforded both buyer and seller, all to the direct and indirect benefit of the citizens of the nation. The benefits are increasing with the growing use of the river. These were the ends on which the improvement was predicated.

Edmund L. Daley,[27] **M. Am. Soc. C. E., and Forrest E. Byrns,**[28] **Assoc. M. Am. Soc. C. E.** (by letter).—Too often the discussion of the economics of river improvements reaches one of two fallacious conclusions:

(*a*) Internal waterways are anachronistic. They have outlived their usefulness in the development of the country. Their further improvement by the United States is, in all cases, a waste of Federal funds.

(*b*) Improved waterways constitute the panacea of all transportation ills. They provide the minimum in transportation costs and, in addition to the savings procured to the public through shipments over the waterways themselves, they have a beneficial effect on the rates charged by all other carriers.

Colonel Hall avoids either extreme position. Dispassionately, he has analyzed the economic factors involved in modern transportation on one particular waterway.

In a study of the Ohio River, derivation of tangible benefits is made easier because of the abundance of accurate data on the volume and types of river traffic. As in all studies of this kind, the greatest difficulty lies in determining the actual transportation costs to the shippers. The author follows a logical method of analyzing the complicated economic questions which must be considered.

All rivers are not worthy of improvement. Some rivers do have characteristics which constitute a *prima facie* case for economic river transportation. In the present day, river transportation is most useful in the movement of bulk commodities originating at points lying close to the river route and destined for points also close to the waterway. The Ohio River is especially well situated for manufacturers and shippers to take advantage of this condition.

Bulk commodities, such as coal and coke, iron and steel, oil and gasoline, stone, sand, and gravel, and cement, originate on or near the main river or its tributaries and move at low costs to points on or close to the river or connecting waterways. This is a natural and proper utilization of this national asset.

Colonel Hall analyzes the public benefits resulting from these low shipping costs. He limits the analysis to a discussion of transportation differentials and makes a careful comparison of the entire cost of the waterway shipping (including construction costs) with rail shipping. He finds that the total cost of water shipments on the Ohio River at the present time is less than the cost of rail shipments. In the past few years, traffic on the river has been increasing. The present trend indicates that both the tonnage and, more particularly, the ton-mileage will continue to increase. The conclusion reached is that from both an engineering and an economic standpoint, the canalization of the Ohio River has been successful and that this success tends to become greater with time.

The author is to be commended for the definite statement and discussion of the assumptions upon which much of his cost study is based. The most vulnerable point for criticism of this or any similar study is his Assumption (1). This is tacitly admitted by Colonel Hall, as well as the fact that any error in the assumption favors the waterway. An attempt in each individual case to determine the exact percentage of savings due to water shipments that is passed on to the public is clearly an impossibility. That the percentage is high and that, as Colonel Hall states, with the continued increase in the use of the waterway and the resulting keener competition, this percentage will continue to increase until eventually the original assumption will become accurate, is undoubtedly true.

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Presumably, opponents of river improvement will grant that as far as the Ohio River is concerned, if these computed definite actual benefits are in error, at least the error is not great enough to change the results very much in the other direction. Then one comes to the consideration of other intangible public benefits that are difficult of assessment. Although these intangible benefits were not considered by Colonel Hall, they are very real.

Up to the time of the Civil War, traffic that was moved for any distance was largely water-borne. The development of the United States, and the extension of its settlement from the eastern seaboard, followed naturally along the navigable waters. Many of the early canals and early river improvements that were in themselves largely unprofitable were warranted by the development of the country which resulted from them.

Since the Civil War, the transportation picture has changed. As canalboats were displaced by railroads and other more rapid means of transportation, commercial navigation on rivers and canals gradually assumed less and less importance.

Before this, due to the advantage of low-cost water transportation of bulk commodities, new basic industries had arisen. Large towns and cities had been built around these and related industries, with consequent economic benefit to the Federal and State Governments. Railroads have partaken of these benefits by reason of the high tariff freight which has been developed from new communities, such as Clairton, Aliquippa, and Ambridge, Pa., and Weirton and Follansbee, W. Va. These are very large benefits. New wealth was created. The monetary value of these benefits is positive but not definitely calculable.

Within the past few years manufacturers and shippers have begun to rediscover the advantages of shipping on the inland waterways. New equipment is being developed for the transportation of bulk commodities. Barges and self-propelled motor ships (especially tankers) are being designed particularly to fit the conditions existing on the more important inland waterways.

Time only will tell definitely whether the more recent improvements on the rivers of the United States, involving the expenditure of many millions of dollars, are in all cases economically justified. Colonel Hall's paper shows that the present commerce on the Ohio River and the resultant savings in transportation costs justify the large expenditures made for its improvement.

J. E. WILLOUGHBY[29] AND W. D. FAUCETTE,[30] MEMBERS, AM. SOC. C. E. (by letter).—The problem offered for discussion in this paper is stated by the author as follows (see heading: "The Problem"):

"Freight moves on the Ohio River because it can be transported more cheaply by water carriers than by land carriers. It can be transported at low rates because engineering works have been built on the river at the expense of the taxpayer. Since no tolls are charged, the Treasury derives no direct return. The public obtains an indirect return in the reduced net cost of water-transported articles consumed. The problem is to compare the engineering costs of the improvement with the value of the public benefits. * * *"

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[29] Chf. Engr., A. C. L. R. R., Wilmington, N. C.

[30] Chf. Engr., S. A. L. Ry. System, Norfolk, Va.

Assumption (1) is that the entire reduction of costs (the author means, not all the cost, but merely the boat operator's cost) in transportation of freight, due to the existence of an improved waterway on the Ohio River, is passed on to the public in the form of lower prices of the goods transported, and hence becomes a public saving.

His own discussion which follows indicates that this is a most violent assumption. In the report of the Mississippi Valley Committee of the Public Works Administration (PWA), submitted to Administrator Ickes on October 1, 1934, it was correctly stated: "Private carriers—largely coal and steel companies—now transport more than 95% of the total commerce on the Ohio River." Recently, there has developed a heavy volume of petroleum products moved in tank barges for the large oil companies.

As Fig. 1 shows, the really heavy tonnage of coal on the Ohio River is between Huntington, W. Va. (*via* the Kanawha River or by rail) and Cincinnati, Ohio, where some of it is re-shipped by rail. The author quotes testimony indicating that any difference between the rail-river-rail and all-rail costs is not passed on to the consumer, but that the cost to the consumer is identically the same. The writers know of other evidence before the Interstate Commerce Commission indicating that all such coal is sold on the basis of the all-rail rates. The great bulk of the coal consumed along the Ohio River and contiguous territory is moved by all-rail routes, and it may safely be stated that in the great majority of instances any alleged "saving" is simply "pocketed" by the seller.

It is well known that iron and steel, moving in heavy volume the length of the Ohio River, are generally sold on the basis of certain mill prices (usually Pittsburgh) plus the rail rates from there to their destinations. The alleged "saving" is retained by the shipper or producer.

It is certain that the price of gasoline to the consumer is not less because of the heavy volume of water-borne gasoline on the Ohio River. It is a matter of common knowledge that these prices closely reflect the price in the Mid-Continent Field plus the cost (usually rail rate) of transporting it to its destination.

It is useless to consider sand and gravel, moving in heavy volume by barge, because the value of river improvements and maintenance to such traffic is nil, if not a detriment.

Assumption (2) is that the reduction of cost in transportation of freight is equal to the rail cost of transporting the freight actually carried by water, minus the water cost of transporting that same freight.

The author complains because rail costs cannot be determined, and it is necessary, therefore, to use rail rates; but water rates are as difficult to determine as rail costs and, therefore, it is necessary to compare rail rates with water costs. In doing so, however, it must be obvious that like is not compared with like. As will be developed later the author's alleged "water costs" are nothing more than general averages plus so-called "terminal differentials" and there are available general averages of rail charges in the form of average ton-mile earnings which would have permitted of like-for-like comparisons.

The author's definition of cost in Assumption (2) is subject to criticism. Assumption (3) requires no comment, and Assumption (4) is admittedly incorrect. It is possible to obtain a break-down of the expenditures of the Federal Lighthouse Service and of the Steamboat Inspection Service sufficient even to approximate the costs chargeable to the Ohio River. Whatever they are, large or small, they should be included to secure the true picture.

Assumption (5) is that, in comparing expenditures and benefits, no allowance has been made for taxes. This is correct; but why not? One reason assigned by the author is that such a charge is not applied generally to all carriers moving on public highways. Although it may be true that motor carriers do not pay adequately for the use of the public highways, they do pay something for such use in the form of special gasoline or mileage taxes, whereas the water carriers pay nothing. Another assigned reason is that nobody charges the cost of lighting an airway against an air line. This is an irrelevant statement when the purpose is to compare rail costs with water costs.

He then argues that the water carriers pay taxes on terminal properties which would have practically no value if the waterways were not improved. Perhaps this is true, but he ignores the fact that such water terminal properties generally take the place of railroad terminal properties which are just as taxable. Then he states that they pay taxes on floating plants that would not exist if there were no waterways. Here, again, he ignores the fact that the floating plant takes out of commission, at the very least, an equivalent in railroad equipment which is taxable. The fact is that most of the water carriers incorporate in Delaware and thereby escape the payment of taxes anywhere on floating equipment. (In this connection see reports of the Mississippi Valley Barge Line Company and American Barge Line Company on file with the Interstate Commerce Commission.)

Furthermore, when the tax-free, improved, Ohio River waterway and the virtually tax-free barge companies operating over it engage in competition with the tax-paying railroads (paying in taxes approximately 7 cts of each gross revenue dollar), the result is not only to decrease the railroad volume and increase its unit costs, but it lessens the ability of the railroad to pay taxes needed for Government purposes, including public schools. The tax revenue loss thus created must of necessity be made up by heavier taxation on other property owners and taxpayers, as has been so well stated by Professor H. L. Lutz in "Public Finance," who stated further that the exemption of a utility from taxation means an exemption of the users of the service from taxation, to the extent of their use. In the present case, it means that a relatively few large steel, coal, and petroleum companies transporting 95% of the traffic on the Ohio River are receiving the benefit while the general taxpayers are paying the deficit. As bearing on taxes, the following quotation from Professor Lutz is apt:

"There appear to be good reasons for the inclusion of a charge against current earnings roughly equivalent to the local tax on a similar private property. In the first place, such a charge is a proper element of cost which the industry should bear, whether it is publicly or privately operated. Secondly, the imposition of such a charge would throw into clearer relief the actual incidence of the benefits of public as compared with private operation. It is not desirable in a democratic community that any limited class should continue to receive benefits which are being paid for by the remainder of the group, unless this class is defective in intelligence and earning power, or is subject to handicaps of health and strength which require special acknowledgment. A third reason for the inclusion of such a tax charge is found in the fact that by such a practice some part of the cost of government is diffused among those who are not readily reached by means of property taxes or other direct forms of revenue."

It is true that taxes are not actually paid on the improved waterway and scant taxes are paid by the boat operators; but when, from an economic and social standpoint, an attempt is made to measure the relative economics of waterway *versus* rail transportation, an allowance for taxes must be made, either by adding them to the waterway costs or by deducting them from the rail costs. Simply to ignore taxes as is done by the author is untenable.

Referring to that part of the paper headed "Carrier Costs of Water-Borne Freight": The basis for the approximate costs, as stated, is somewhat obscure. There is no way for the writers to check any of the alleged "savings," in cents per ton, or in thousands of dollars, appearing in Table 3. For the purpose of analysis the writers will have to be content to use the "line-haul costs of 2 and 3 mills per ton-mile for principal coal movements, and about 3 mills per ton-mile for steel." Although the so-called "terminal differentials" generally used by the author are stated at the end of the paper, it is difficult to make use of them, so they are largely disregarded herein.

In discussing Table 4, the author admits the overwhelming importance of the four products, coal, steel, petroleum, and sand and gravel.

Under the heading, "Net Savings," the author states that the costs on each of the more commonly used routes for each important commodity were determined by multiplying the ton-mileage moved by the average cost per ton-mile as estimated (for water-borne freight), adding the terminal differential between rail and water transfers (except petroleum), and also adding the actual rate for rail haul where the commodity was hauled either to the river, from the river, or both; these costs were deducted from the comparative rail rates, and the gross savings established, and from this value the savings per ton-mile were obtained.

Solely for the purpose of discussion, the writers accept the Government costs, in mills per ton-mile (1934), of 5.6 as stated. This is a water ton-mile cost to the Government. It is conservative to state that, due to the meandering of the inland rivers, the water distance, except for railways that follow the meandering river, is on the average at least 50% longer than the short-rail distance. It is perhaps true that the average movement by rail may be as much as 10% (the former Federal Co-Ordinator computed it at 11%) longer than the shortest route. Consequently, to obtain a fair comparison of water-line costs (Government and boat operator) per ton-mile with rail revenues per ton-mile, the former must be inflated 50% and the latter 10%, thus bringing both to a short rail-mile basis.

In this connection, it should again be stated that sand and gravel, now and long before the present improvements, is and was dredged from near-by sand bars and moves short distances to large cities. By no stretch of the imagination can sand and gravel secure a material benefit, if any at all, from the costly river improvements and maintenance. It has been said, truthfully, that the pools formed by the forty-nine dams in the Ohio River submerge and otherwise destroy (by failure to create) sand-bars from which the sand and gravel is dredged; and yet, the cost of the river improvements and maintenance is spread over all traffic, including this heavy tonnage of sand and gravel. Table 4 shows that of the total of 13 018 335 000 ton-miles in 1934, 1 026 849 000 ton-miles were represented by sand and gravel. It is likewise doubtful whether the improvements and maintenance were of any value to the log and lumber traffic, the ton-miles of which in 1934 were 64 516 000. Sand, gravel, logs, and lumber represented more than 8% of the total ton-miles. If eliminated, and if the Government cost were allocated to the traffic that benefits therefrom, obviously, the average Government cost would be in excess of 5.6 mills per water ton-mile. For the purpose of comparison, however, 5.6 mills will be used.

Although it is stated that the average Ohio River haul is 119 miles, clearly this is depressed because of the influence of the short-haul sand and gravel traffic. In any event, it is not a very important factor in this analysis.

The rail carriers that are principally competitive with the traffic moving over the Ohio River waterway, together with ton-mile revenues in 1934, actual and as inflated 10% to make them fairly comparable with absolute short-rail hauls, are as shown in Table 8.

TABLE 8.—REVENUES FOR RAIL HAUL FOR 1934

Railroad	REVENUES, IN MILLS PER TON-MILE	
	Actual	Inflated 10%
Chesapeake and Ohio	5.96	6.56
Norfolk and Western	6.62	7.28
Louisville and Nashville	8.18	9.00
Pennsylvania	9.15	10.06
Baltimore and Ohio	9.46	10.41

TABLE 9.—WATER TON-MILE COST

Operator	RATE, IN MILLS PER TON-MILE	
	Actual water ton-mile	Equivalent rail ton-mile
Government	5.6	8.4
Water operator	2.0	3.0
Total (water cost, 2¢)	7.6	11.4
Government	5.6	8.4
Water operator	3.0	4.5
Total (water cost, 3¢)	8.6	12.9

Each of these carriers transports a heavy volume of the same bulk commodities as are transported on the Ohio River, principally coal. Out of these earnings they must pay all costs, including taxes. With the exception of the Baltimore and Ohio Railroad all were prosperous, or fairly so. No one can deny that for comparative purposes they represent a fair selection to determine the relative true costs of moving traffic on the Ohio River compared with railroad haulage.

Ignoring entirely any factor of excess terminal costs of waterway over rail, and the exemption of water transportation from taxes, and merely using the average of 5.6 mills per water ton-mile, Government cost (also as inflated 50% to reflect an equivalent rail ton-mile) and water operator costs of both 2 and 3 cts (see heading, "Carrier Costs of Water-Borne Freight") per water ton-mile (also as inflated), the values shown in Table 9 result.

It will be observed that these partial waterway costs are greatly in excess of any of the foregoing rail lines. If average ton-mile costs are to be used for water transportation then it is eminently fair to use average ton-mile revenues for the railways; in fact, their use is decidedly favorable to the waterway transportation.

Fig. 1 shows a heavy tonnage of coal between Huntington and Cincinnati. This coal originates on the Chesapeake and Ohio Railway which line has established very low rates to Huntington for the movement beyond that point *via* river; it is unloaded through a tipple into barges at Huntington. Most of the coal then moves to a point just west of Cincinnati where it is unloaded, moved up an incline through a tipple and dumped into rail cars for movement to points in and about Cincinnati and points beyond. It is obvious that the barge movement between Huntington and Cincinnati is simply a substitute for Chesapeake and Ohio rail service between the same points. The Chesapeake and Ohio runs parallel to the Ohio River between these points, and both the river and the railroad are about 160 miles long. Movement *via* barge necessitates two intermediate terminal services which are avoided on all-rail movements.

Computing the barge movement at 2 mills per ton-mile, a low estimate (see author's discussion of Table 3), and loading at Huntington at 5 cts per ton, unloading at Cincinnati at 15 cts per ton (see "Terminal Differentials"), plus the Government cost of 5.6 mills per ton-mile, the total cost for 160 miles is $1.42 per ton.

The average ton-mile revenue of the Chesapeake and Ohio Railroad in 1934 was 5.96 mills. This includes all terminal expenses. This railroad had an average haul in 1934 of 284 miles. Although the substitute haul, in this case, is less, no additional origin terminal expense would have been necessary if handled through by the road, and the same would have been true if the coal had been consigned to points beyond Cincinnati. The average of 5.96 is conservative. It produces 95 cts per ton for the rail haul from Huntington to Cincinnati, as contrasted with a total cost of $1.42 per ton for the substitute river service. Manifestly, a movement *via* river is profitable to the transporter only because the taxpayers through the Federal Government contribute 90 cts of the true cost of such a movement.

It is probable that the alleged savings as determined by the author (Table 4) would not stand the test if it were possible to check his computations. In this connection note Table 6 and the author's discussion of Table 6. What he does here is to assume movements from Cincinnati proper to Louisville, Ky., proper (note this carefully) of:

Freight	Movement, in thousands of tons	Equivalent movement, in millions of ton-miles
Coal and coke	10 805	741
Cement	108	21
Iron and steel	844	473
Oil and gasoline	1 272	201
Unclassified	939	288

He then proceeds to determine the rail costs to a shipper per water ton-mile by using the local rail rates from Cincinnati to Louisville as contrasted with an assumed water cost per water ton-mile. A local rail movement of coal from Cincinnati to Louisville is almost impossible. No coal originates there and the rail rate used is virtually meaningless. The fictitious rail movement used represents nearly 50% of the total ton-miles used in the comparison, and greatly affects the showing. There is little likelihood of any appreciable movement of cement or iron and steel either by rail or by water locally, from Cincinnati to Louisville. The entire comparison partakes of the nature of aligning data in a direction favorable to the project and to produce a desired result.

By the method used, the author is able to produce an average rail cost to the shipper on the aforementioned traffic from Cincinnati to Louisville of 21.3 mills per water ton-mile as contrasted with a water cost of 14.9 mills per water ton-mile (cost to shipper 9.3 mills plus cost to Government 5.6 mills), and yet the average ton-mile earnings of the Louisville and Nashville Railroad Company and the Baltimore and Ohio Railroad Company (also operating between Cincinnati and Louisville) were only 8.18 and 9.46 mills, respectively.

Under the heading, "Non-Statistical Considerations," the author apparently attaches much value "to the regulating effect of an improved waterway on the rates charged by other carriers." This can only be predicated on an Interstate Commerce Commission so impotent as to permit rail carriers to maintain rates higher than are just and reasonable. Although it is true that, in many cases, water competition has depressed the rates of the rail lines, it must not be overlooked that: An efficient railway system serves the entire country; it serves points on the rivers as well as in the interior; it must have operating revenues sufficient to pay all expenses, taxes, and a reasonable return on investment; the people as a whole must of necessity support the railway plant by paying rates that will produce sufficient revenues in the aggregate; and that were it not for the necessity of reducing some rail rates to meet subsidized water competition which benefits a relatively small part of the general public, the transportation burden might be more equitably distributed. The writers cannot agree that the so-called "regulating effect of an improved waterway" furnishes any economic justification for the huge expenditures of the taxpayers on waterways. No Government function, on one hand, should assume that another Government agency is not alive to its duties.

If the author had chosen to balance alleged benefits of this kind against disadvantages, he might have mentioned the facts that: The 49 dams have created a series of 49 pools, or lakes, averaging 20 miles in length; in periods of slack-water the current is almost negligible; the sewage dumped in the river at the numerous large cities has made the Ohio almost an open sewer, with a stench so bad in summer as to be noticeable, and even chemicals cannot destroy the bad taste of the city water at such periods; and, due to these virtually stagnant pools, all the large cities along the Ohio River must eventually install sewage disposal plants at heavy cost.

W. E. R. Covell,[31] M. Am. Soc. C. E. (by letter).—An analysis of the economics of the Ohio River improvement was presented by C. W. Kutz,[32] M. Am. Soc. C. E., in 1926, who covered the period from 1905 to 1925, inclusive. It is true that improvements have been made in the collection of commercial statistics, which afford more complete data with respect to the movements of river commerce from origin to destination. However, intimate knowledge of the sources and treatment of the data which were presented in the initial paper, particularly with respect to the upper end of the Ohio River and its principal commercial head-water, the Monongahela River, with their long history of heavy traffic, lead to the belief that the economic status of the Ohio River improvement to the end of 1925, as accepted from General Kutz's paper, is reasonably adequate. Colonel Hall's paper, therefore, makes available a complete although composite analysis of the economics of the Ohio River improvement up to and including the calendar year 1934.

The author's economic analysis is predicated upon five basic assumptions, which he lists and discusses in the section dealing with this feature. It is believed that all the assumptions as made are tenable.

Under Assumption (1) it is considered that the entire reduction of costs in transportation of freight is eventually passed on to the consumer. A particular function of the Ohio River improvement is that it broadens and intensifies competition with respect to industries, such as iron and steel, situated both on the water and inland, with a favorable effect upon the prices the public must pay for the products of such industries. The general public also benefits directly in many other ways; for example, many electric utilities are situated on the river and receive their fuel supply *via* water. The cheaper cost of coal is reflected in cheaper rates to the consumers of electricity.

The river improvements have exerted a regulatory effect upon the rail-rate structure for coal in the Ohio River Basin by inhibiting a tendency toward higher prices. In this manner the savings in transportation effected by water are at least indirectly realized by the general public. General Kutz stated that his investigation indicated that, in general, the rail-rate structure for coal and steel movements in the Ohio River Basin did not discriminate between points having water competition and those without such competition. Nevertheless, certain exceptions were noted which are still applicable. Short-haul rail rates on coal in the region at the head of the Ohio River where coal movements on the rivers are very large, appear to be considerably less in the valleys where there is water competition than in the valleys without river improvements.

As indicated by the author in his discussion of Assumption (2), to present a rational analysis of the economics of the Ohio River improvement it is necessary to resort to rail rates as the basis for determining transportation savings by water. As long as the comparison is made completely from origin to destination (that is, not limited to line haul alone), the method serves the purpose adequately and, it is believed, equitably.

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[31] Lt.-Col., Corps of Engrs., U. S. Army; Dist. Engr., U. S. Engr. Office, Pittsburgh, Pa.

[32] "The Relation of the Ohio River and Its Tributaries to Transportation in the United States," by C. W. Kutz, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1106–1122.

The following difficulties attending the river-rail comparison may be cited: For long hauls involving the Ohio River, the rail distance between origin and destination is usually shorter, as stated by General Kutz in his paper. Moreover, more than one rail line or system may afford the same service, a matter which has a bearing upon capital investment in railroad facilities underlying the service. Undoubtedly, the revenue from the relatively high short-haul rail rates goes in some part toward making up possible deficiencies in revenue from long-haul rail service. The statement is also often made that a railroad system has no concern over the fact that the rates for certain short hauls may be practically the same as for much longer hauls, which may place certain areas at a competitive disadvantage with others. The average rail revenue per ton for the system is what counts. However, resort to average rail rates is manifestly not equitable in the comparison of the cost of the competing transportation services for the derivation of the savings, since only the rail rates applicable represent actual "out-of-pocket" costs to the shipper for the rail service between origin and destination.

The transportation savings as determined do not include those resulting from temporary or emergency freight rates which, during several recent years, were appreciable. The Association of American Railroads has petitioned the Interstate Commerce Commission for a flat increase of 15% in rail freight rates throughout the United States. If this increase or any substantial part of it should finally be granted, as seems possible at the present time, due to the financial straits of several of the railroad systems, a very material increase in mills per ton-mile will apply to the savings for the future Ohio River commerce even if the cost of haul on the Ohio River were also to increase as much as 15 per cent.

In Assumptions (3) and (4) Colonel Hall deals with general ferry traffic and the capital cost of the Ohio River improvement, respectively. The Ohio River, in the calendar year 1935, accounted for 1 160 639 tons of general ferry traffic; in 1936, such traffic mounted to 1 309 405 tons. Canalization of the Ohio River has permitted this traffic to be handled more safely and expediently. Although it is difficult to place a value on these benefits, they are positive and constitute a credit to the waterway improvement, which has not been included in the economic analysis. It is believed that there is no particular point in attempting to estimate and charge the waterway with the costs of the Federal Lighthouse Service and Steamboat Inspection Service on the Ohio River, since the costs are comparatively nominal.

Assumption (5) is as follows: "In comparing expenditures and benefits, no allowance has been made for taxes." This means that although the rail rates reflect taxes on the capital investment in railroad facilities, the Federal investment in the Ohio River improvement is not subjected to an equivalent charge in the economic analysis. There is some room for difference of opinion on this score. The basic question is whether or not the Federal Government would have been further ahead had the same funds gone into private investment on which taxes would accrue to the Government. It is believed that an investment of the Ohio River type does not necessarily result in a reduction of private investment of like amount. The improvement of the Ohio River has resulted through a subsidy on the part of the Federal Government to the Ohio River Valley which is rich in natural resources. The Federal Government, in its paternalistic attitude, has fostered the development of the Ohio Valley to such a degree that the river improvement has become a national asset. Every large city in the country is located on a navigable waterway. It is believed that the return in taxes resulting from the development of the Ohio River Valley made possible by the improvement of the river far outweighs any possible loss in taxes occasioned by the Federal investment in the improvement of the river.

There may be cited numerous indirect, practical benefits from the Ohio River improvement not mentioned by Colonel Hall, which are not susceptible to monetary valuation. The improved Ohio River has recreational features that are widely utilized. The navigation pools afford storage of water for industrial and, in some cases, domestic water supply. Pumping heads have been reduced. During periods of emergency, such as the World War, the water-transportation medium with its reserve capacity was extremely valuable to the country since the rail mediums were congested.

By his assumptions, Colonel Hall has indicated clearly the basis of the economic analysis. Whatever deficiencies obtain are of a minor nature only, and it is believed that they are more than offset by the intangible and indirect benefits.

Colonel Hall discusses the adequacy of the cost of transportation by water as adopted for the determination of the unit savings on the Ohio River. He states that the savings on the two all-important items of coal and steel are based upon line-haul costs of 2 and 3 mills per ton-mile for principal coal movements and about 3 mills per ton-mile for steel movements. An indication of the adequacy of these rates may be had from the following derivation of the cost of transportation for a coal haul, say, from Point Pleasant, W. Va., at the mouth of the Kanawha River, to the mouth of the Miami River below Cincinnati, Ohio.

Annual Cost of Coal-Towing Unit for Operation on the Ohio River (Tow = 10 Standard Barges, Each 26 × 175 × 11 Ft; and the Navigation Season Is a Full Year).—

Cost of Equipment:

Towboat, 1 200 hp, Diesel type	$325 000
Reserve towboat 1 per 10 active, charge	32 500
Barges in transit, 10 @ $14 000	140 000
Barges, 20 additional, at terminals (ratio 2 to 1)	280 000
Total	$777 500

Fixed Charges, Annual:

Interest @ 6% on $777 500	$ 46 650
Plant depreciation (20-yr life)	38 875
Insurance marine, 2½% average life value	9 720
Workmen's compensation insurance (2.28% of payroll)	775
Cargo and liability insurance	18 750
Taxes, 2% average life value	7 775
Total	$122 545

Maintenance and Annual Repairs:

Towboat, 1.1 @ $9 000	$ 9 900
Barges, 30 @ $500	15 000
Total	$ 24 900

Ship Supplies (1.1 Towboats):

Paint, rope, waste, lubricants, etc	$ 4 875
Ice, laundry, linen, etc	1 125
Total	$ 6 000

Payroll (First Watch):	
1 Master, 12 months @ $350	$ 4 200
1 Chief Engineer, 12 months @ $225	2 700
1 Striker Engineer, 12 months @ $125	1 500
1 Watchman, 12 months @ $140	1 680
2 Deck Hands, 12 months @ $110	2 640
Payroll (Second Watch):	
1 Pilot, 12 months @ $275	$ 3 300
1 Mate, 12 months @ $165	1 980
1 Second Engineer, 12 months @ $200	2 400
1 Striker Engineer, 12 months @ $125	1 500
2 Deck Hands, 12 months @ $110	2 640
1 Cook, 12 months @ $140	1 680
2 Chambermaids, 12 months @ $75	1 800
15 Sub-total	$28 020
Relief crews	6 000
Total	$34 020

Subsistence:

15 @ 80 cts per day for 365 days	$ 4 380
Relief crews	780
Total	$ 5 160

Fuel Oil:

1 200 hp @ 0.6 lb per hp-hr = 720 lb per hr,
or approximately 96 gal per hr
Assume 18 hr average use per day and 365 days per season:
96 × 18 × 365 = 630 720 gal per yr
630 720 gal @ 5 cts = $31 535

Summary of Cost:

Payroll	$ 34 020
Subsistence	5 160
Fuel oil	31 535
Maintenance and repairs	24 900
Ship supplies	6 000
Fixed charges	122 545
Payroll, subsistence, and fuel oil cost of reserve boat, 0.1 of $70 715 (arbitrarily added)	7 070
Total annual operating cost	$231 230
Overhead @ 5% (Fleet Superintendent, Assistants, etc.)	11 560
Total annual cost (rounded)	$242 790

Cost of Transportation of Coal on Ohio River from Mouth of Kanawha River to Mouth of Miami River (Tow = 10 Standard Coal Barges; 1 Ton = 2 000 Lb; and No Back-Haul Freight).—

Length of haul, in miles	226
Normal speed, in miles per hour	5
Number of locks in reach of river	12
Size of locks, in feet	110 by 600
Lockages required by tow per lock	1
Lockage time, average for system, hours per lock	0.5
Length of river reach assigned to operation, each lock, in miles	0.5
Loading of tow (down stream only), in tons	8 500
Round-Trip Time, in Hours:	
Assembling and landing time (2 × 2 hr)	4
Lockage time (24 locks at 30 min)	12
Pool time, $\frac{452 - 12}{5}$	88
Sub-total	104
Lost time, all causes, assumed at 35%	36
Total, round trip	140
Navigation season, gross, in hours	8 760
Number of round trips	63
Loading per round trip, in tons	8 500
Commerce per year, in tons	535 500
Haulage per year, in ton-miles	121 023 000
Annual cost, in dollars, per unit tow	242 790
Cost, in mills, per ton-mile	2

The following explanation is pertinent to the derivation of the cost of transportation of coal on the Ohio River as presented in the preceding data. The size of the tow is assumed to be 10 barges. However, the towboat is capable of handling a tow of greater size—up to 16 standard barges—which would require two lockings through each lock. It is assumed that all the traffic proceeds through all the twelve locks on the reach. However, during a considerable part of the year there is sufficient open-river stage to permit the traffic to proceed past the locks through the navigable passes of the dams with the wickets in a lowered position. An exception will be the new Gallipolis Locks and Dam, which will replace Lock and Dam No. 26 below the mouth of the Kanawha River, and Locks and Dams Nos. 24 and 25 above this point, and will require all traffic to proceed through the locks. An assumption of 35% of the normal operation time has been made for lost time to cover delays at locks and lost time due to fog, wind, ice, high water, and all other causes. This is probably conservative for steady coal movements such as obtain for this general reach of the river. It will be noted that no item of profit as such has been included in the derivation. However, the fixed charges include interest at 6% on the initial amount of investment in floating plant. Depreciation is charged on a simple life basis rather than on a sinking-fund basis with compound interest. It is believed that, considering all the aforementioned refining factors in actual practice, a

2-mill rate per ton-mile for coal haul is representative and as such affords an adequate margin of profit for contract-carrier service. Representative costs of transportation by water combined with terminal differentials that are usually in favor of rail haul, afford an equitable basis of comparing the cost of transportation *via* water with the parallel service by rail, from origin to destination.

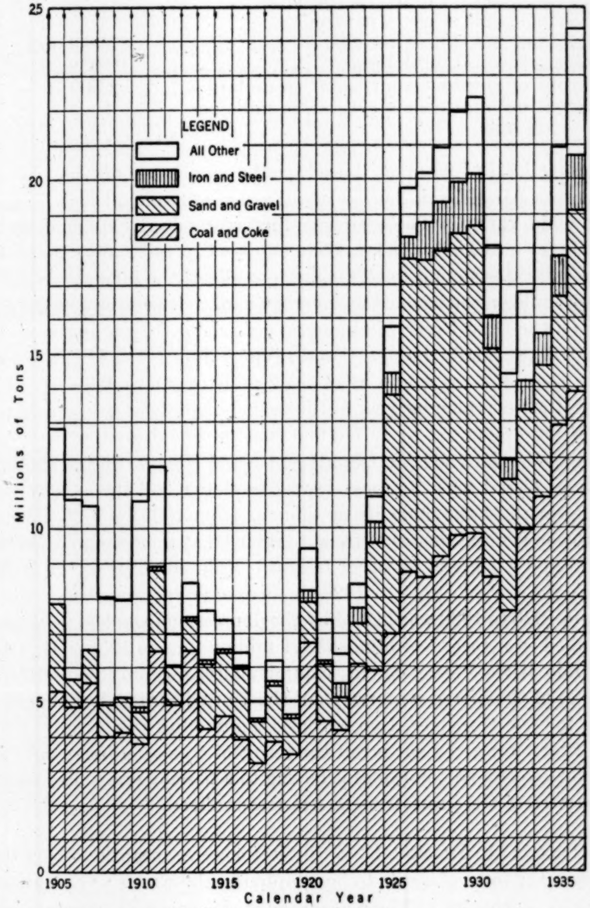

FIG. 5.—OHIO RIVER TONNAGE GRAPH, PITTSBURGH, PA., TO CAIRO, ILL. (1 TON = 2 000 POUNDS)

The profit and loss account for the economic analysis of the Ohio River, as made in the paper, is derived by subtracting the total annual commercial savings from the total annual charges. The following data for the Ohio River obtain for the calendar year 1934 (the latest year of the analysis):

Total freight, in tons	18 636 000
Total freight traffic, in ton-miles	1 783 924 000
Average haul, in miles	96
Savings, net, in mills per ton-mile	6.9
Savings, net, in cents per ton	66.5
Savings, net, total, in dollars	12 359 000
Charges, total, in dollars	10 058 293
Profit on improvement, total, in dollars	2 300 707
Profit on improvement, total, in mills per ton-mile	1.2

An average haul of 96 miles (based on the year 1934) on the Ohio River is equivalent to a parallel rail haul of about equal amount, at least in the upper end of the Ohio Valley. The corresponding average rail rate is about 15 mills per ton-mile for coal haul which takes a commodity rail rate. If the 15% flat increase in rail rates should finally come into effect, the increase would be 2.25 mills per ton-mile on the basis of the average haul. It is improbable that water-haul costs on the Ohio River will increase materially. If an increase of 10% is assumed for water haul, the additional savings due to the full increase, if applicable, in the rail rates would be about 2 mills per ton-mile.

In 1905, the first calendar year of the period of analysis, the freight traffic on the Ohio River amounted to 12 773 000 tons, although at that time only two locks and dams—No. 1 (Davis Island) and No. 6 (Beaver)—and the Louisville and Portland Canal with its lock, were completed and in operation. Fig. 5 shows the growth of the Ohio River tonnage from 1905 to 1936, inclusive, together with the general classes of freight traffic (that is, coal and coke, sand and gravel, iron and steel, and all other freight). The impetus to the growth that occurred in the period, 1924 to 1926, inclusive, is very marked. The steady growth from 1926 to 1930 was halted by the depression from which, however, the traffic climbed to a record of almost 24.4 million tons in 1936. It is apparent that the Ohio River improvement, commercially, not only constitutes an important part of the inland waterways of the United States, but that its field of usefulness for the transportation of basic commodities is steadily increasing. The Ohio River improvement is not the result of a passing fancy on the part of the Federal Government; nor does it belong to a passing era. The vision of the early advocates of the improvement has materialized.

C. L. HALL,[33] M. AM. SOC. C. E. (by letter).—Assumption (1) of the paper was stated in the form of a rule to be adopted in the writer's accounting for the Ohio River improvement. The inaccuracy of this rule was conceded, but it was presented as a condition that tended to become true in time. Professor Grant mentions one situation in which the assumption would not be true in a free economy, and hence it does not tend to become true in time, namely, if the producers favored by the waterway are marginal. As the principal producers of water-borne freight on the Ohio River are the great Pittsburgh steel companies and the Kanawha Valley coal mines, it may be conceded that they are not marginal. Professor Grant also shows that the full difference between rail and water costs cannot be claimed as a benefit if the traffic would not move by rail in the absence of water routes. This statement is correct; but it is moot because, except for a little river sand and gravel, all Ohio River traffic would have to move somehow—even if the waterway were to disappear from the face of the earth—unless the economic level of the region were to fall.

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[33] Col., Corps of Engrs., U. S. Army, Commanding 1st Engrs., U. S. Army, Fort Du Pont, Delaware.

In preparing the financial summary (Table 5) the writer assumed a balance at the end of 1904—with everything paid for at that time. In General Kutz's official unpublished report[34] he states that there is no doubt that the savings effected by the Ohio River during the period 1875–1904 greatly exceeded the expenditures. However, this assumption, although it may be entirely justified, gave an unduly rosy picture of the situation from 1905 to 1910. Incidentally, the cost of Ohio River improvement prior to 1905 is such a small share of the total expenditures that research into commerce before that date is unjustified. It may be added that comparisons of benefits prior to 1905 are of very slight value. The Chesapeake and Ohio Railroad, for instance, was not built until 1905. Before that time, therefore, the river was the normal method of carrying freight between river towns, and there is no base upon which comparisons of costs with competing carriers could be computed adequately.

The main point discussed by Professor Grant raises a question that extends far beyond the Ohio River. He declares that public works are socially unprofitable unless they give at least as great a return as the private investments displaced by the diversion of taxpayers' funds. In its literal form the principle is impossible of application because it would rule out, completely, structures like the Lincoln Memorial. What the statement would mean in practice is that public works competing with privately owned facilities should not be constructed unless they could produce revenues far greater than those accounted for by non-competing public works; that is, a lock and dam must show much greater benefits than a new post office. Professor Grant's complaint that profits during the period of regulation are not a proper charge against losses after canalization was begun, has a certain measure of justification. He establishes his case quite neatly in Table 7. However, the fact that there were profits for 1930–1933, when most other activities in the United States were losing money, is itself proof of a rather wise investment.

The writer must dissent from the view that the life of a prospective public investment should be calculated in commercial terms. Experimental commercial investments are only justified by large prospective profits. The Government, however, can afford to experiment (or pioneer) on a public project with an undoubtedly long engineering life, subject to the hazards of a shorter economic life. If the Government refuses to do this kind of experimenting, nobody will. Will the country be better off if such pioneering enterprises are never undertaken?

Mr. Lavis' distinction between internal and external waterways also could not be applied in practice. If it were made absolutely rigid the enormous public benefits caused by the improvement of the St. Mary's River, for instance, would never have been accomplished. If the distinction were not made absolutely rigid, it would assuredly be disowned by any American Government within the range of practical politics. Moreover, even on Mr. Lavis' principle, the secondary local benefits from channels to the ocean, obtained on streams such as the Upper Hudson River, would cause a subsidy to certain types of internal waterway traffic.

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[34] Rept. of June 30, 1926.

The writer does not believe that he has overlooked the advantages in distance of the usually shorter land route. In the preparation of Table 4 the comparison was between terminals, and the benefits were developed on a basis of ton-miles measured by water. The writer hoped that Table 6 would have made that point entirely clear.

In Mr. Lavis' discussion, as in some others, there seems to be a feeling that the costs given in the paper are based on distances between main-line termini. This is not true. The comparison was for rail rates between producer and consumer on the one hand, and net costs plus profit to the competing shipper (rail rates, water costs, and terminal charges) on the other. These values were reduced to ton-miles measured by water in order to give a unit by which typical studies could be applied generally; but the very methods used gave the all-rail carrier complete credit for his shorter mileage.

Mr. Barrett's opinion that there is no difference, in final result, between benefits derived by producers and those derived by a necessarily limited number of consumers has considerable logic behind it. It seems probable, however, that public funds will not be provided for waterway improvement unless it can be shown clearly that a direct financial saving is obtained by a large number of consumers. Of course, the benefits from a public route of transportation cannot be distributed uniformly over the entire nation; but they must be open to all the people of the region affected, and not merely to stockholders of great corporations.

The remarks of Colonel Daley and Mr. Byrns are much appreciated. The writer did not cover the general regional benefits that have accrued to upper river ports from the Ohio River improvement, because he could find no method of making a statistical comparison. The point that real new wealth has been produced by Ohio River development, however, is a vital one, and should have been discussed.

Messrs. Willoughby and Faucette attack the improvement of the Ohio River on two general grounds, as follows: (1) That the improvement, in so far as it does permit cheaper shipment, merely diverts money from rail carriers to water carriers; and (2) that if the Federal costs were charged against the shipper there would be no cheaper shipment. If the net total of goods moved in the United States were constant (that is, if the country were static), the first complaint would probably be valid. Along that line, every new ocean vessel (except a replacement for a scrapped one) would reduce the revenues of existing ocean carriers. The nation has always refused to make any such melancholy admission, however. It has always claimed that the reduction of distances in money or time caused by technical improvements in means of transit has increased total consumption, and total freight moved, by a far larger amount than the losses caused to pre-existing cargo carriers. Beyond dispute this was the effect of the substitution of railroads for mule-drawn wagons in the Middle West seventy years ago.

If true waterway costs are greater than rail costs, Messrs. Willoughby and Faucette are correct in their second complaint. The aim of the writer was to show that the costs of shipment by the Ohio River were, on the whole, less than those by competing carriers. His proof was based on certain carefully stated assumptions, followed by an investigation of all the data that he could procure. The assumptions are those used in the study of proposed waterways for many years. Perhaps more valid ones could be prepared, which would be concurred in by those responsible for authorizing public expenditures; but the writer ventures to doubt it. The only assumption criticized constructively is the fifth, in which the writer excluded, for accounting purposes, any allowance for unpaid taxes. The question of charging unpaid taxes as a constructive expense of Government enterprises is a very difficult one. If full interest and depreciation are charged, and if the Government "polices" the work with maintenance funds, it is difficult to show that taxes are an equitable charge against the property. On the other hand, there is a certain degree of statistical error in comparing tax-free carriers with taxed ones. Every one should welcome a rule to correct the rather unsatisfactory state in which the present situation leaves the student of governmental enterprises.

The discussers are wrong in believing that the Ohio River pools are not beneficial to sand-diggers. Sand is also more readily carried in pools of fixed minimum depth. From his experience as a District Engineer, the writer can assure them that the improvement does not prevent the formation of new bars.

Allowance has already been made for greater circuity of water routes. In the preparation of Table 4, as stated in replying to Mr. Lavis, the basic comparisons were between movements from origin to destination by different routes. No question of relative lengths of haul entered into this matter until a final benefit per ton per route was determined. This benefit was then distributed as commercial benefit per ton-mile measured by water, so that all rail advantage of shorter haul was already credited; for example, in the movement of steel pipe from Youngstown, Ohio, to Memphis, Tenn., the shipper's cost is:

By rail (tariff charges)		$14.60
By water:		
Rail rate to river terminal	$0.335	
Line cost of river movement; 1 188 miles at 3 mills per ton-mile	3.56	
Terminal charges	0.75	
Total water cost		$4.645
Commercial benefit		$9.955
Unit ton-mile commercial saving		$0.0084

It will be noted that this commercial saving, although calculated in mills per ton-mile measured by water as the only convenient method of generalization, gives the railroad full credit for its shorter haul.

As to the criticism of Table 6, the journey from Cincinnati, Ohio, to Louisville, Ky., was selected because these two river cities are separated by a distance of about the length of the average haul (119 miles). The object of the table was to check Table 4 by an independent calculation. The writer was astonished at his own results. Incidentally, the rail and water distances from Cincinnati to Louisville are about identical. It is believed that the criticism of this comparison between rail and water methods of moving a typical ton a typical mile would have been more useful if data had been given for making the same comparison over some other trip between major river ports.

Colonel Covell's very interesting discussion presents a highly valuable cost analysis of steamboat transportation. He shows that the values derived from confidential reports by the big river operators are inherently reasonable, and he thus supports the general accuracy of the statement of savings (see Table 4). The writer has made no claim that the improvement is justified, wholly or in part, by its regulatory effect on rail rates. Probably it is true, as Colonel Covell states, that the existence of the improvement has inhibited a general increase in coal rates; but it is believed that internal improvements constitute a rather expensive means of rate regulation. In that respect, the writer tends to agree with Messrs. Willoughby and Faucette.

During the progress of this discussion the commercial data for the calendar year 1937 have become available. They give a total commerce of 23 356 676 tons, an increase of 25% over 1934; and of 2 671 926 225 ton-miles, an increase of 50% over 1934. The average haul for 1937 was 114.39 miles.

Undoubtedly, the discussions have clarified the points upon which differences as to matters of principle are held. There is no certainty as to what profits to the public should be required from public works competing with privately owned ones. There is no certainty as to whether taxes not paid by the Government should nevertheless be arbitrarily added to public costs for purposes of comparing accounts. There is no certainty as to the minimum distribution of private advantages, secured from public works, required to establish such advantages as public benefits. These questions, of course, are political in their nature; but the political decision should follow a crystallization of engineering opinion. Needless to say the final decision should apply to all public works competing with commercial investments. It was the writer's aim to use this paper on the Ohio River as a case study of a completed competing project, which would bring out all disputes on principle, besides contributing to technical literature pertaining to the Ohio River. His aim seems to have been accomplished—at least in part.

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SOLUTION OF TRANSMISSION PROBLEMS OF A WATER SYSTEM

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With Discussion by Messrs. Lynn Perry, Charles M. Mower, Jr., Thomas R. Camp, Harold E. Babbitt, C. Maxwell Stanley, Warren E. Wilson, Harold K. Palmer, F. Knapp, Weston Gavett, and Ellwood H. Aldrich

Synopsis

Some of the problems commonly experienced in the design of the transmission and distribution elements of a water system are solved in this paper which presents a rational method involving an expansion of the graphical solution[2] first suggested by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E. A brief summary is given of the many variable and, from a practical standpoint, sometimes indeterminate, factors that must be considered in assembling the data upon which the design must be based. General cases are presented giving the solution of the more common problems encountered in design, with an illustration of the use of several of these general cases in the solution of a typical design problem.

Introduction

The design of the transmission and distribution facilities of a water system presents a complex and absorbing problem for one engaged in such a task. A mathematically accurate determination of the results to be expected in designing intricate grid systems or important reinforcements is impossible of attainment from a practical standpoint. When it is realized, however, that these elements of a water system commonly exceed 50% of the total value of such a system and that they materially influence the cost, as well as the adequacy of service, the reason becomes apparent why all the knowledge that is available should be utilized to secure an efficient design.

The opportunity to design a complete water transmission and distribution system of any magnitude is not often presented to an engineer. His work is usually concerned with providing extensions and reinforcements to existing

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Note.—Published in October, 1937, *Proceedings*.

[1] Associated with Reeves Newsom, Cons. Engr., New York, N. Y.

[2] *Journal*, New England Water Works Assoc., Vol. 7 (1892), p. 49.

systems. It is of primary importance for him, therefore, to determine the operating functions of the existing system as the basis for a determination of the necessity for, and the extent of, the most effective reinforcements.

Basic Data

Satisfactory service requires that a sufficient quantity of water shall be furnished to all parts of the system at pressures adequate for the requirements of the consumers under all conditions of their demands. To create such service proper reserves must be available in the form of pumping, transmission, and distribution capacity, often supplemented by storage, to meet all reasonable contingencies.

The more important basic data required by the designer before undertaking the solution of the hydraulics of a problem are: (*a*) The pressure and flow requirements under the varying rates to be expected in the respective parts of the system; (*b*) the topography of the service areas; (*c*) the pumping arrangements and pump characteristics in other than gravity systems; (*d*) the location, quantity, and elevation of storage; and (*e*) the condition of the pipe system in so far as the carrying capacities of its many elements are concerned.

Williams-Hazen Formula.—Many formulas have been developed to present the results of the flow of water in pipes. The Williams-Hazen formula has become more generally used than any other for water-works conditions because of the ease of its application with the aid of tables,[3] or by the use of a special slide-rule. It gives acceptable results provided the coefficient is carefully determined. This formula is:

$$V = C\,R^{0.63}\,S^{0.54}\,0.001^{-0.04} \qquad (1)$$

in which V = velocity of flow, in feet per second; C = coefficient, dependent on the interior conditions of the pipe (sometimes called the coefficient of roughness); R = the hydraulic radius or the area of the pipe divided by the wetted perimeter, in feet; and, $S = \frac{H}{L}$, the slope of the hydraulic gradient or the head loss, in feet per thousand feet. In a closed pipe of constant diameter, with flow under pressure, $R^{0.63}$ becomes a constant, so that,

$$V = C\,K\,S^{0.54} \qquad (2)$$

In other words the velocity, and, therefore, the quantity flowing in a given size of pipe, varies directly as the coefficient, C, and as the 0.54 power of the hydraulic gradient.

Therefore, several fundamental relations result: With C constant, V varies as $S^{0.54}$, or S varies as $V^{1/0.54}$; with V constant, S varies as $C^{1/0.54}$, or C varies as $S^{0.54}$; and, with S constant, V varies directly as C.

In using the flow tables based upon this formula, which were compiled by the late Gardner S. Williams and Allen Hazen, Members, Am. Soc. C. E.,[3] it is necessary to have some convenient means of interpolating between the standard flows, sizes of pipe, losses of head, and coefficients set forth therein. By plotting the standard flows as determined from the Williams-Hazen tables, or by slide-rule, on logarithmic scale, and, similarly, by plotting S against $S^{0.54}$ and $S^{1/0.54}$, as shown in Fig. 1, the interpolation and conversion of the varying factors of the formula are facilitated. With this formula as a base, it is possible to make all computations by the use of Fig. 1.

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[3] "Hydraulic Tables," by the late Gardner S. Williams and Allen Hazen, Members, Am. Soc. C. E., John Wiley & Sons.

SOLUTION OF GENERAL PROBLEMS

General.—Specific problems usually involve as a first step the simplification of intricate arrangements of many pipe lines and their combination into one or more equivalent lines. In making these combinations, several general cases arise in almost every problem. These will be illustrated by the following general cases: (I) Pipe connected in series; (II) pipe connected in parallel and in series; (III) "take-outs" and "put-ins"; (IV) compound storage; (V) pipe connected in parallel with intermediate cross-connections; and, (VI) pipe connected in grids.

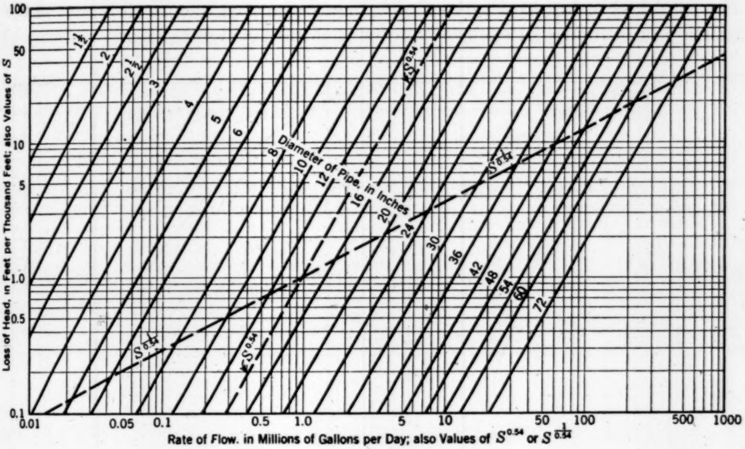


FIG. 1.—FRICTION LOSS BY EQUATION (1)

It is not the intent to imply that a graphical solution of the simpler combinations must be resorted to; neither is it always possible or necessary to take the time, or to make the effort, to achieve an accurate combination of many complex lines. Most combinations can be made by slide-rule computations without graphical help, but in certain cases much better progress can be made by using a graphical solution than is possible by trial and error, or by mathematical methods, and familiarity with the graphical representations of the simpler combinations is a prerequisite to an easy solution of the more complex systems.

The term, "hydraulic grade," is used in this paper to denote the piezometric pressure in the pipe or conduit at the point in question. Although in most of the cases discussed herein the coefficient has been assumed as 100, in order to permit the use of Fig. 1 without further computation, the special slide-rule permits solutions using actual coefficients with equal ease. It is also apparent that the graphical method described herein is applicable to other formulas expressing the results of flow of water in pipes.

Case I.—Pipe Connected in Series.—Fig. 2 is a graphical representation of the loss of head under varying rates of flow through a pipe line consisting of several pipes of different diameter connected in series. In such a line the total head loss for a given flow is a summation of the loss of head in each section for that flow. With abscissas representing the rate of flow and ordinates the loss of head under those rates of flow, curves are constructed for each section of pipe. The total loss of head for several pipe lines connected in series consists of the addition of ordinates of each section for each rate of flow.

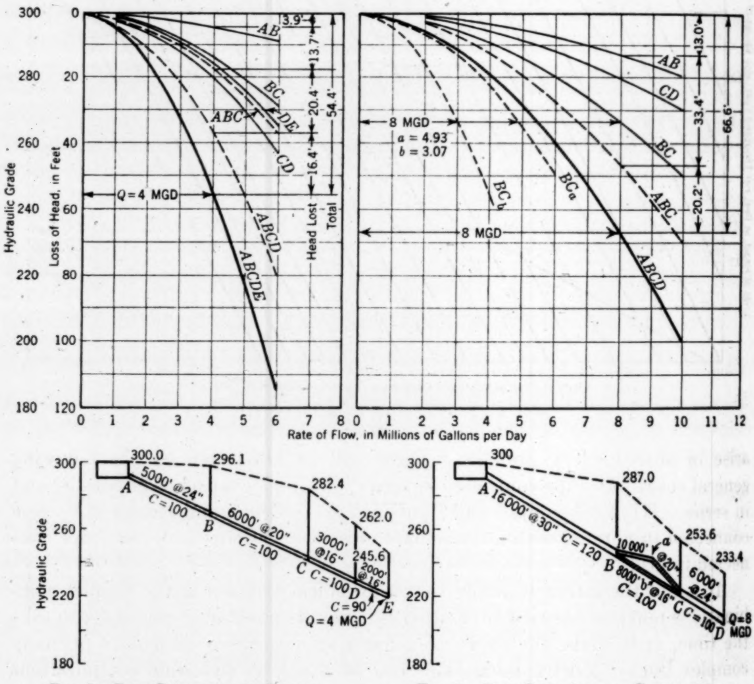


FIG. 2.—PIPE CONNECTED IN SERIES; CASE I

FIG. 3.—PIPE CONNECTED IN SERIES AND PARALLEL; CASE II

Case II.—Pipe Connected in Parallel and in Series.—Fig. 3 is a graphical representation of the loss of head under varying rates of flow through three sections of pipe, one of the sections consisting of two pipes connected in parallel. The graphical combination of pipe connected in parallel is effected by adding the abscissas of each pipe at each loss of head, since the loss of head through a given section consisting of several pipes connected in parallel is the same in each pipe and the total flow in the section is the sum of the flows in each pipe. Having determined the equivalent pipe curve for the section of pipes connected in parallel, the three sections of pipe connected in series are combined as illustrated in Case I.

Cases I and II are simple combinations of pipe easily solved with the aid of the hydraulic slide-rule. However, the graphical combination of such pipe arrangements greatly facilitates solutions in more complex combinations, involving consumption at points along the line, the introduction of water from storage or from other sources, and those combinations of pipe for which several factors are dependent upon each other.

The graphical solution of these more complex systems requires the moving of the quantity-head loss curves to represent the effect of variation in elevation of storage, the putting in or taking out of quantities of water, and the flow and pressure characteristics of centrifugal pumps. These solutions are facilitated by a graphical representation of the quantity-head loss relation which can be moved easily to represent the variations in conditions. Convenient transparent media for such graphs are prepared in the form of small rectangles about 4 in. by 6 in. in size, cut from thin celluloid, processed or coated on one side to aid the drawing of the quantity-head loss curves and their removal when no longer required. Careful trimming of the edges aids accurate placing in the proper position. They are sufficiently rigid to lie flat and stay in place, and their transparency renders possible the use of several cards superimposed one upon another.

Case III.—"Take-Outs" and "Put-Ins."—Fig. 4(*a*) is a graphical representation showing the taking out or putting in of a certain quantity of water from, or into, one of two mains connected in parallel. The quantity-head loss curve, *A E C*, representing the flow in Main *A E C*, is drawn upon the right of the *Y*-axis as shown. With no "take-out" or "put-in" at Point *B*, Curves *A B* and *B C*, representing the flow conditions in Mains *A B* and *B C*, are drawn upon the left of the *Y*-axis and combined as in any two pipes connected in series, as illustrated in Case I. A rate of flow of 8 mgd at Point *C* results in a loss of head of 59.8 ft, 4.3 mgd flowing through Main *A B C* and 3.7 mgd through Main *A E C*.

With a "take-out" at the rate of 4 mgd at Point *B*, Curve *A B* is moved to the right a distance equivalent to 4 mgd on the *X*-axis, as shown in Curve *A B''*, before combination with Curve *B C* to form the curve, $A\ B\ C^T$. In extending Curve $A\ B\ C^T$ to the right of the *Y*-axis, flow in Pipe *B C* reverses in direction, causing Curve *B C* to assume the position, *B C'*, which is used in combining the two curves for flows in Mains *A B* and *B C*. With a flow from Point *C* at the rate of 8 mgd and a "take-out" of 4 mgd at Point *B*, the loss of head from Points *A* to *C* is 84.5 ft, 4.5 mgd flowing through Main *A E C*, 7.5 mgd through Main *A B*, and 3.5 mgd through Main *B C*. By following vertically up from the intersection of this quantity with Curve $A\ B\ C^T$ to the intersection with Curve *A B''* the loss of head in Main *A B* is found to be 58.5 ft.

The intersection of Curve $A\ B\ C^T$ with the *Y*-axis gives the loss of head in the system at which reversal of flow in the main, *B C*, occurs. With any flow at Point *C* less than is given for this loss of head, the flow through Main *A E C* exceeds the quantity taken out at Point *C* and the remainder flows toward Point *B* to satisfy the "take-out" there. In other words, the scaled rate of flow between Curves $A\ B\ C^T$ and *A E C* represents the draft at Point *C*; that between $A\ B\ C^T$ and the *Y*-axis represents the flow back from Point *C* to Point *B* and the total flow through Main *A E C* is the sum of the two, or the scaled distance from Curve *A E C* to the *Y*-axis. The ordinate of the intersection of Curves $A\ B\ C^T$ and *A E C* measures the loss of head from Points *A* to *C*, 6.5 ft, at which there can be no flow out of Point *C* if the "take-out" of 4 mgd at Point *B* is satisfied.

If 4 mgd is assumed to be put into the system at Point *B*, Curve *A B* must be moved to the left of the *Y*-axis, opposite to its position for a "take-out", as Curve *A B′*, before combining with Curve *B C* to form Curve $A\ B\ C^P$. With an 8-mgd rate of flow at Point *C* and a "put-in" of 4 mgd at Point *B*, the loss from Points *A* to *C* is 47.3 ft, causing a flow through Main *A E C* of 3.3 mgd, through Main *A B* of 0.7 mgd, and through Main *B C* of 4.7 mgd. Following vertically up from the intersection of this total rate of flow with Curve $A\ B\ C^P$ to Curve *A B′*, the loss of head in Main *A B* is 1.2 ft and, similarly, in Main *B C* the loss is 46.1 ft. From these curves, the quantity-head loss relations in all pipes are established for flows at Point *C* under the three conditions assumed at Point *B*, namely, no "take-out" or "put-in", a "take-out" of 4 mgd, and a "put-in" of 4 mgd.

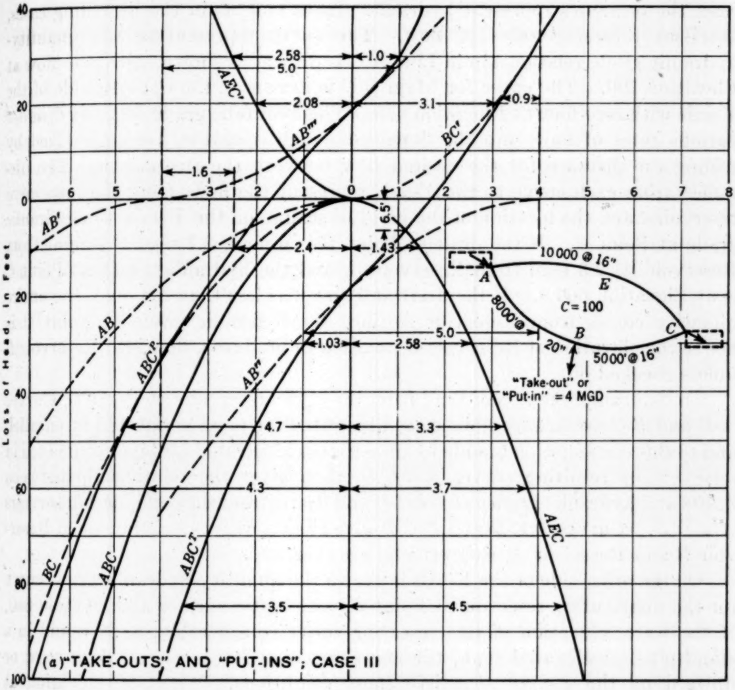
Another use of the curves in Fig. 4(*a*) can be made for reservoirs at Points *A* and *C*, capable of putting out or taking in the quantities of water required to balance the system without appreciable changes in elevations. A "take-out" of 4 mgd at Point *B*, when the elevation of Reservoir *C* is 30 ft below Reservoir *A*, causes a flow at the rate of 5 mgd from Points *A* to *B*, 1 mgd from Points *B* to *C*, and 2.58 mgd through Main *A E C*, or a total of 7.58 mgd flowing out of Reservoir *A* and 3.58 mgd flowing into Reservoir *C*. The hydraulic grade at Reservoir *C*, of course, is 30 ft below that at Reservoir *A*, and at Point *B* it is 27.5 ft below that at Point *A*, and 2.5 ft above that at Point *C*.

If Reservoir *C* is 20 ft above Reservoir *A*, under a similar "take-out" at Point *B*, there is a flow from Point *C* to Point *A* through Main *A E C* of 2.08 mgd, a flow of 3.1 mgd from Points *C* to *B*, and one of 0.9 mgd from Points *A* to *B*, or a total flow of 5.18 mgd leaving Reservoir *C* and of 1.18 mgd entering Reservoir *A*. The hydraulic grades are determined similarly as described.

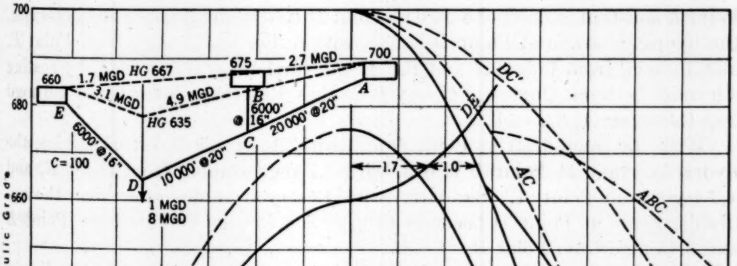
With a 4-mgd "put-in" at Point *B* and the elevation of Reservoir *C* 10 ft below Reservoir *A*, a flow of 1.43 mgd is caused from Point *A* to Point *C* in Main *A E C*, 1.6 mgd from Point *B* to Point *A*, and 2.4 mgd from Point *B* to Point *C*, or 3.83 mgd flowing into Reservoir *C* and 0.17 mgd into Reservoir *A*.

With a similar "put-in" at Point *B*, and Reservoir *C* 30 ft above Reservoir *A*, the resulting flow from Point *C* to Point *A* in Main *A E C* is 2.58 mgd, from Points *B* to *A*, 5 mgd, and from Points *C* to *B*, 1 mgd, or 7.58 mgd flowing into Reservoir *A* and 3.58 mgd flowing out of Reservoir *C*.

Case IV.—Compound Storage.—Fig. 4(*b*) is a graphical solution of the losses of head under varying rates of flow from a system of supply or distributing reservoirs. With three reservoirs as shown, two of the reservoirs and their transmission mains are first combined into one source. Points *A* and *B* represent two supply reservoirs or intakes not subject to appreciable variations in levels, one situated at Elevation 700, and the other at Elevation 675. In this



(a) "TAKE-OUTS" AND "PUT-INS"; CASE III



(b) COMPOUND STORAGE; CASE IV

FIG. 4

case, the ordinates represent hydraulic grades and, as in the preceding cases, abscissas represent rates of flow. The curve representing the quantity-hydraulic grade relationship in Line *A C* is drawn, as shown, with zero flow at Elevation 700. The curve for Main *B C* is drawn on the opposite side of the *Y*-axis with zero flow at Elevation 675. The hydraulic grade at Point *C* under various rates of flow, and the flows from each reservoir, are determined by scaling the distance for the desired flow between the two curves. The distances from each curve to the *Y*-axis represent the flows from the respective reservoirs, and the location of the point of scaling on the *Y*-axis, the hydraulic grade at Point *C*. If the draft at Point *C* is 6 mgd, 3.7 mgd is coming from Reservoir *A*, 2.3 mgd from Reservoir *B*, and the hydraulic grade at Point *C* is at Elevation 660.5. If the draft at Point *C* is less than 2.9 mgd, the entire quantity comes from Reservoir *A* since the hydraulic grade at Point *C* is above the elevation of Reservoir *B*, tending to produce a flow into Reservoir *B* unless checked off.

To determine the loss of head from these two reservoirs at Point *D*, Curves *A B* and *B C* are first combined, as illustrated for pipes connected in parallel, and to this combination is added Curve *C D* as illustrated for pipes connected in series. The resulting curve, *A B C D*, then gives the relationship between drafts and hydraulic grades at Point *D* and drafts from the combined Reservoirs *A* and *B*. Curves *A C D* and *B C D* give the conditions existing when Reservoir *B* and Reservoir *A*, respectively, are shut off.

On the other side of the *Y*-axis is drawn the quantity-hydraulic grade curve for the distribution reservoir at Point *E*, with water surface at Elevation 660. If the water elevation of this reservoir varies considerably, as it would in a stand-pipe, or elevated tank, the curve for Line *DE* is moved downward or upward on the *Y*-axis to satisfy these conditions. With the elevation at Point *E* constant, a draft of 8 mgd at Point *D*, and with all reservoirs in service, the hydraulic grade at Point *D* is 635, with a flow of 3.1 mgd from Point *E*, and 4.9 mgd from Points *A* and *B*. It can further be determined, by scaling 4.9 units between Curves *A C* and *B C*, that Reservoir *B* furnishes 1.6 mgd and Reservoir *A*, 3.3 mgd.

With the same draft at Point *D* and with Reservoir *B* out of service, the hydraulic grade at Point *D* is 623, with 3.7 mgd coming from Point *E*, and 4.3 mgd from Point *A*. Similarly, with Reservoir *A* out of service, the hydraulic grade at Point *D* becomes 616, with 4.15 mgd coming from Point *E* and 3.85 mgd from Point *B*.

Assuming that the draft at Point *D* is only 1 mgd with both supplies in service, the hydraulic grade at this point is above that at Point *E*, and there is a flow to the distribution reservoir. In such a case the flow from Point *A* and Point *B* is 2.7 mgd, but 1.7 mgd goes to Point *E*, 1 mgd is taken off at Point *D*, and the hydraulic grade at Point *D* is 667. As shown in Fig. 4(*b*) the curve for Main *D E* is reversed about the *Y*-axis for elevations above 660.

It is often necessary to determine the effect of a sustained but variable draft of a maximum day upon a distribution reservoir such as that at Point *E*. This is capable of determination from the curves. By setting up the hourly rates, a table can be compiled giving the drafts from or to each reservoir and the hydraulic grades at the desired points. This can be further complicated when the storage in any of the reservoirs varies in elevation by any appreciable extent.

A study such as this is particularly important in determining the size and elevation of storage. A sufficiently accurate determination can be made upon an hourly basis by converting the quantity drawn from a stand-pipe or tank to gallons, and from the diameter of the tank, converting this to drop in water elevation, in feet. During the next hour the curve must be moved down by the quantity of the drop. This will result in less water being drawn from the tank or the stand-pipe. This approximation can be continued, hour by hour, until, the peak loads past, or the lowering level in the tank (or more generally a combination of the two), the hydraulic grades produce a flow toward the tank and it begins to fill. In this manner curves can be drawn for almost any condition, giving the draft and hydraulic grade at a certain point, the flow from or to a stand-pipe or tank, the flows from the sources of supply, and the hydraulic grade, or its equivalent, the elevation of water in the tank.

Case V.—Pipes Connected in Parallel with Intermediate Cross-Connections.—Fig. 5(*a*) represents the flow conditions and losses of head through two pipes connected in parallel with an intermediate cross-connection (see Fig. 5(*b*)). The arrangement of the pipes and the construction of the individual quantity-head loss curves are self-explanatory. In all examples involving cross-connected pipe, the cross-connection is in effect a "take-out" of one pipe and a "put-in" to another of an unknown quantity, dependent upon the carrying capacities of the several pipes.

Since the hydraulic grades at the junction of Pipes (1) and (2) are the same, curves for these pipes are drawn, as shown, on each side of the Y-axis. It is unnecessary to have a definite flow through the system in order to convert it into an equivalent pipe; therefore, in Pipe (2) a loss of head of 30 ft is assumed. This produces a flow of 4.22 mgd through Pipe (2). The remaining unknowns are h_1, h_3, h_4, h_5, Q_1, Q_3, Q_4, and Q_5. It is known, however, that at all junction points the hydraulic grades are the same in each pipe, and the quantity flowing to and from each junction is in balance. Usually, by inspection, it is possible to determine the direction of flow in interior pipes, and curves for these pipes can be laid off, either to the right or to the left, with zero flow at the assumed head loss to the junction in question. In this case, from the lengths and sizes of pipe, it is evident that flow in Pipe (3) is from Point C to Point B, that a part of the flow in Pipe (2) is taken out at the cross-connection, leaving a smaller quantity to flow through Pipe (4). This "take-out" becomes a "put-in" at the other end of the cross-connection and is added to the flow in Pipe (1) to produce the flow in Pipe (5).

Curves (4) and (5) are traced on a movable card from Curves (4)′ and (5)′, which are set up anywhere. It is known that the losses of head to the ends of Pipes (4) and (5) are the same; therefore, they are drawn together as shown in Fig. 5(*a*). From inspection, it is seen that Curve (4) must intersect Curve (2) at Point *a*, at the same loss of head as the beginning of Curve (3). Similarly, Curve (5) must intersect Curve (1) at Point *b*. The card on which Curves (4) and (5) are drawn must be adjusted, therefore, so that Curve (4) intersects

Curve (2) at Point *a*, Curve (5) intersects Curve (1) at Point *b*, and the *Y*-axis of Curves (4) and (5) must intersect Curve (3) at a point, *c*, with the same loss of head and, therefore, in the same horizontal plane in Fig. 5(*a*), as Point *b*. Having adjusted the card in this manner, the following conditions have been met:

$$H = h_1 + h_5 = h_2 + h_4 = h_2 + h_3 + h_5 = h_1 - h_3 + h_4 \ldots\ldots \quad (3)$$

or, in feet, 72.8 = 32.4 + 40.4 = 30.0 + 42.8 = 30.0 + 2.4 + 40.4 = 32.4 − 2.4 + 42.8; and,

$$Q = Q_1 + Q_2 = Q_4 + Q_5 = Q_2 - Q_3 + Q_5 = Q_1 + Q_3 + Q_4 \ldots\ldots \quad (4)$$

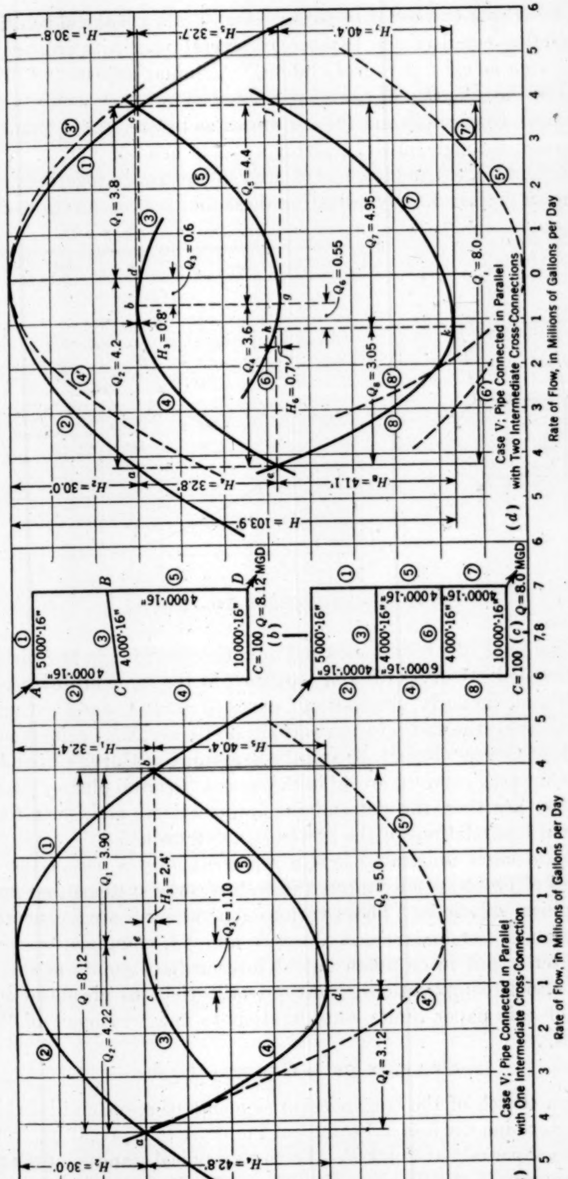
or, in million gallons daily, 8.12 = 3.9 + 4.22 = 3.12 + 5.0 = 4.22 − 1.1 + 5.0 = 3.9 + 1.1 + 3.12.

In Fig. 5(*b*) is shown a further step of the preceding problem with an additional interior connecting pipe. Similar procedure is adopted for its solution. Curves (1) and (2) are drawn as before and a loss of head of 30 ft is assumed in Pipe (2) giving a flow of 4.2 mgd. Since Pipes (3) and (4) begin at the same hydraulic grade, Pipes (5) and (6) and Pipes (7) and (8) end at the same grade, these sets of curves are drawn together, and there are three cards to be adjusted.

Curves (3) and (4) must start with the loss of head that occurs in Pipe (2), which, by assumption, is 30 ft. Similarly, from the condition of a balanced hydraulic grade at each junction, Curves (3) and (4), (5) and (6), and (7) and (8) must be adjusted so that the respective intersections, Points *a* to *k*, are as shown. Curve (3) can be drawn the same as in the previous example, but here it is as convenient to combine it with Curve (4) on the same movable card. There are many ways to arrange the curves for quantity-head loss. In these illustrations curves concave downward indicate flow away from the inlet, and curves concave upward indicate flow toward the outlet or flows from, or to, the apices of the curves. The interior curves may be considered either way as long as the fundamental relationships hold.

In these examples, the loss of head must be the same by any route connecting two points; and the quantity flowing through any straight section cut entirely through the system, must be the same, due regard being given to the direction of flow. The total head loss in the last system of pipes is 103.9 ft for a flow of 8 mgd. An equivalent pipe would be computed as follows: $\frac{103.9}{24.3}$ = 4 280 ft of 16-in. pipe, $C = 100$.

In the last illustration it is interesting to determine how much the cross-connecting pipes reduce the total loss of head. With two 16-in. pipes connected in parallel, one 20 000 ft, and the other 13 000 ft, long, eliminating the cross-connecting pipes, the equivalent pipe is 4 400 ft of 16-in. pipe, $C = 100$. The head loss is 24.3 × 4.4 = 106.9 ft, or the saving in head is only 3 ft. The conclusion is that pipes connecting points of approximately equal pressure are of little value in reducing the friction loss. As these pipes approach a parallel connection with the main pipes or connect points of greater pressure difference, their value in reducing losses or increasing flows becomes greater.



(a) Case V: Pipe Connected in Parallel with One Intermediate Cross-Connection

(d) Case V: Pipe Connected in Parallel with Two Intermediate Cross-Connections

FIG. 5

Case VI.—Pipes Connected in Grids.—Fig. 6 is a graphical solution of the pressure and flow conditions in a system of several pipes with cross-connections both ways, such as exist in a grid system. A similar adjustment of curves is required as in Case V, Fig. 5.

In this case, Curves (1) and (2) are drawn as before, and a quantity of 7.5 mgd is assumed through Line (2), giving a loss of head of 36.6 ft. Curves (3) and (10), (4) and (5), (6) and (9), and (7) and (8) are drawn together on movable cards as shown in Fig. 6. By adjusting these four sets of curves to obtain the proper intersections, the head losses and quantities flowing in the various pipes can be determined. Curves (3) and (10) start at the same ordinate as the end of Curve (2) and, similarly, the ends of Curves (3) and (4) and the beginning of Curves (6) and (9); the ends of Curves (9) and (10) and the beginning of Curve (8); the ends of Curves (5) and (6) and the beginning of Curve (7); and, finally, the ends of Curves (7) and (8) are all in the same horizontal planes, respectively. Points *a* to *m* are then the determining intersections, and, once found, the quantity-head loss relations of the system are determined.

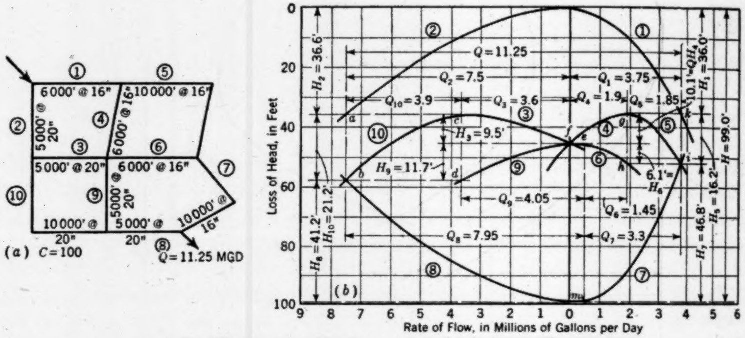


FIG. 6.—PIPE CONNECTED IN GRID, CASE VI

The general cases illustrated herein represent only a few of the simplest combinations of problems encountered in water-works systems that are adaptable to graphical solutions. These various general cases are frequently found in combinations. For instance, in Cases V and VI, complications are introduced if water is put in or taken out at intermediate points in the systems illustrated. Such complications, however, still yield to graphical solutions. The scope of this paper limits consideration to the treatment of the more general cases.

SOLUTION OF A GENERAL CASE

Fig. 7 is a sketch of the transmission elements of a system[4] in which it is desired to determine the loss of head from Points *A* to *X*, resulting from a fire flow of 1 500 gal per min at Point *X*. From a graphical standpoint the problem consists essentially in the simplification of certain pipe combinations to reduce the system into a problem similar to Case V; that is, pipes connected in parallel with an intermediate cross-connection.

---

[4] "Simplified Analysis of Flow in Water Distribution Systems," by J. J. Doland, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 117, No. 14, p. 475, Fig. 1, October 1, 1936.

To use the graphical method, it is necessary to have definite flows through the system, which may be determined, however, upon a percentage basis. Since the use of the hydraulic slide-rule simplifies the computations if flows are expressed in terms of million gallons per day, this basis is adopted herein and the fire flow of 1 500 gal per min, which forms 45% of the total, becomes 2.16 mgd of a total of 4.8 mgd. The various "take-outs" at points along the mains are shown in Fig. 7 in percentages, which are converted to million gallons per day in the computations.

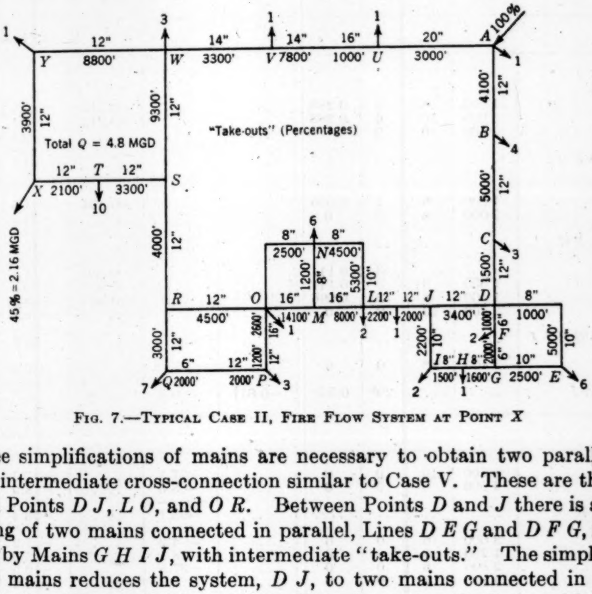

1
3
1
1
100%
12"
8800'
14"
3300'
14"
7800'
16"
1000'
20"
3000'
Y
W
V
U
A
1
3900'
12"
9300'
12"
4100'
12"
"Take-outs" (Percentages)
Total Q = 4.8 MGD
B
4
12"
T
12"
X
2100'
3300'
S
10
5000'
12"
45% = 2.16 MGD
6
8"
8"
2500'
N
4500'
4000'
12"
1200'
8"
5300'
10"
C
3
1500'
12"
R
12"
O
16"
16"
L
12"
12"
J
12"
D
8"
4500'
14100'
M
8000'
2200'
2000'
3400'
1000'
1000'
2600'
16"
1
2
1
2
6"
F
3000'
12"
6"
12"
1200'
12"
2200'
10"
I
8"
H
8"
2000'
6"
10"
5000'
10"
7
Q
2000'
2000'
P
3
1500'
1600'
G
2500'
E
6
2
1

FIG. 7.—TYPICAL CASE II, FIRE FLOW SYSTEM AT POINT *X*

Three simplifications of mains are necessary to obtain two parallel pipes with an intermediate cross-connection similar to Case V. These are the mains between Points *D J*, *L O*, and *O R*. Between Points *D* and *J* there is a system consisting of two mains connected in parallel, Lines *D E G* and *D F G*, followed in series by Mains *G H I J*, with intermediate "take-outs." The simplification of these mains reduces the system, *D J*, to two mains connected in parallel, which requires the application of Case II to determine the losses of head from Points *D* to *J* under varying rates of flow.

Since the flow in any section of the main system will be unknown until the final solution, the computations and graphs must provide for a range of flows sufficient to include the various possibilities, with provision for subsequent "take-outs." Table 1(*a*) and Fig. 8(*a*) represent the computations and graphical analyses to determine the head-capacity relationship of the mains between Points *D* and *J*. The compound mains between Points *L* and *O* are similar to Case V with an intermediate "take-out," and the head-capacity relationship is determined as previously described. The computations and graphical representations are given in Table 1(*b*) and Fig. 9. Similarly, the mains between Points *O* and *R*, two mains connected in parallel, with intermediate "take-outs," are solved as given in Table 1(*c*) and Fig. 8(*b*).

TABLE 1.—Loss of

Lines (see Fig. 7)	Dimensions		"Take-Out"		Quantities, in Million Gallons				
					−1.0		−0.5		−0.25
	Length, in feet	Diameter, in inches	In percentages	In million gallons daily	Quantity, in million gallons daily	Loss of head, in feet	Quantity, in million gallons daily	Loss of head, in feet	Quantity, in million gallons daily
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a) Loss of Head									
D G H I J									
D E G									
D E	1 000	8	6	0.288	....	....	−0.212	−0.9	0.038
	5 000	10	6	0.288	....	....	−0.212	−1.4	0.038
E G	2 500	10	0	0	....	....	....	−3.5	....
Total in *D E G*	....	..	..	....	....	....	....	−5.8	....
D F G									
D F	1 000	6	2	0.096	....	....	−0.404	−11.5	−0.154
F G	2 000	6	0	0	....	....	....	−34.0	....
Total in *D F G*	....	..	..	....	....	....	....	−45.5	....
D G	....	..	3	0.144	....	....	−0.356	− 1.1	−0.106
G H	1 600	8	3	0.144	....	....	−0.356	− 3.6	−0.106
H I	1 500	8	2	0.096	....	....	−0.404	− 4.2	−0.154
I J	2 200	10	0	0	....	....	....	− 3.1	....
Total in *D G H I J*	....	..	..	....	....	....	....	−12.0	....
D J	3 400	12	0	0	....	....	....	−2.0	....
Total, *D* to *J*	....	..	20*	0.96*	−0.04†	0.3	0.46†	2.5	0.71†
(b) Loss of Head									
L N	5 300	10	0	0	....	−27.1	....	− 7.5	....
	4 500	8	0	0	....	−68.0	....	−18.9	....
Total in *L N*	....	..	..	....	....	−95.1	....	−26.4	....
L M	8 000	16	0	0	....	− 4.2	....	− 1.2	....
M N	1 200	8	0	0	....	−18.1	....	− 5.0	....
N O	2 500	8	0	0	....	−37.8	....	−10.5	....
M O	14 100	16	0	0	....	− 7.3	....	− 2.0	....
Total, *L* to *O*‡	....	..	11‖	0.528‖	−0.472¶	− 1.4	0.028¶	0.6	0.278¶
(c) Loss of Head									
O P Q R									
O P	2 600	16	10	0.48	−0.52	− 0.4	−0.02	0.0	0.23
	1 200	12	10	0.48	−0.52	− 0.8	−0.02	0.0	0.23
P Q	2 000	12	7	0.336	−0.664	− 2.0	−0.164	−0.1	0.086
	2 000	6	7	0.336	−0.664	−57.5	−0.164	−4.2	0.086
Q R	3 000	12	0	0	....	− 6.3	....	−1.7	....
Total in *O P Q R*	....	..	..	....	....	−67.0	....	−6.0	....
O R	4 500	12	0	0	....	− 9.5	....	−2.6	....
Total, *O* to *R*	....	..	0^c	0^c	-1.0^d	− 3.6	-0.5^d	−0.6	-0.25^d

* "Take-outs" from Points *J* to *S*. † The rate of flow at Point *J*. ‡ Corresponding rate of flow
Points *O* to *S*. ¶ Rate of flow at Point *O*. [c] Corresponding rate of flow at Point *O* = 2.028 mgd.
of flow at Point *R*. For − 1.5 mgd, the flow at Point *R* = − 1.5 mgd and the loss of head at Point
Point *R* = 2.0 mgd.

HEAD IN MAINS

DAILY, FROM THE END OF THE LINE *A B . . . S*

−0.25	0		0.25		0.50		1.0		1.5	2.0
Loss of head, in feet (11)	Quantity, in million gallons daily (12)	Loss of head, in feet (13)	Quantity, in million gallons daily (14)	Loss of head, in feet (15)	Quantity, in million gallons daily (16)	Loss of head, in feet (17)	Quantity, in million gallons daily (18)	Loss of head, in feet (19)	Loss of head, in feet (20)	Loss of head, in feet (21)
IN MAINS *D* TO *J* (SEE FIG. 8(*a*))										
0.0	0.288	1.5	0.538	4.8	0.788	9.8	1.288	24.2	....	....
0.1	0.288	2.5	0.538	8.1	0.788	16.5	1.288	41.0	....	....
− 1.0	....	0.0	....	1.0	....	3.5	....	12.7	....	....
− 0.9	....	4.0	....	13.9	....	29.8	....	77.9	....	....
− 1.9	0.096	0.8	0.346	8.6	0.596	23.4	1.096	73.0	....	....
− 9.4	....	0.0	....	9.4	....	34.0	....	122.2	....	....
−11.3	....	0.8	....	18.0	....	57.4	....	195.2	....	....
1.0	0.144	5.2	0.394	12.0	0.644	20.7	1.144	46.0	....	....
− 0.4	0.144	0.7	0.394	4.3	0.644	10.7	1.144	31.0	....	....
− 0.7	0.096	0.3	0.346	3.2	0.596	8.7	1.096	27.0	....	....
− 0.9	....	0.0	....	0.9	....	3.1	....	11.2	....	....
− 1.0	....	6.2	....	20.4	....	43.2	....	115.2	....	....
− 0.5	....	0.0	....	0.5	....	2.0	....	7.1	....	....
4.2	0.96†	6.5	1.21†	9.1	1.46†	12.3	1.96†	19.7	28.5‡	38.0§
IN MAINS *L* TO *O* (SEE FIG. 9)										
− 2.1	....	0	....	2.1	....	7.5	....	27.1	57.5	....
− 5.2	....	0	....	5.2	....	18.9	....	68.0	144.3	....
− 7.3	....	0	....	7.3	....	26.4	....	95.1	201.8	....
− 0.3	....	0	....	0.3	....	1.2	....	4.2	8.8	15.0
− 1.4	....	0	....	1.4	....	5.0	....	18.1	38.5	66.0
− 2.9	....	0	....	2.9	....	10.5	....	37.8	80.3	137.5
− 0.6	....	0	....	0.6	....	2.0	....	7.3	15.5	26.4
1.9	0.528¶	4.0	0.778¶	6.3	1.028¶	9.5	1.528¶	18.0	29.0[a]	42.8[b]
IN MAINS *O* TO *R* (SEE FIG. 8(*b*))										
0.1	0.48	0.4	0.73	0.8	0.98	1.3	....	....	....	....
0.2	0.48	0.6	0.73	1.4	0.98	2.4	....	....	....	....
0.0	0.336	0.6	0.586	1.6	0.836	3.0	....	....	....	....
1.3	0.336	16.3	0.586	45.6	0.836	88.0	....	....	....	....
− 0.5	....	0	....	0.5	....	1.7	....	....	....	....
1.1	....	17.9	....	49.9	....	96.4	....	....	....	....
0.7	....	0.0	....	0.7	....	2.6	....	9.5	....	....
0.0	0.0[d]	0.7	0.25[d]	2.1	0.5[d]	4.4	1.0[d]	11.0	20[e]	31[f]

at Point *J* = 2.46 mgd. § Corresponding rate of flow at Point *J* = 2.96 mgd. ‖ "Take-outs" from

[b] Corresponding rate of flow at Point *O* = 2.528 mgd. [c] "Take-outs" from Points *R* to *S*. [d] The rate

R = − 8.7 ft. [e] Corresponding rate of flow at Point *R* = 1.5 mgd. [f] Corresponding rate of flow at

TABLE 2.—LOSS OF HEAD

Lines (see Fig. 7)	DIMENSIONS		"TAKE-OUT"		QUANTITIES, IN MILLION GALLONS									
					−1.0		−0.5		−0.25		0		0.25	
	Length, in feet	Diameter, in inches	In percentages	In million gallons daily	Quantity, in million gallons daily	Loss of head, in feet	Quantity, in million gallons daily	Loss of head, in feet	Quantity, in million gallons daily	Loss of head, in feet	Quantity, in million gallons daily	Loss of head, in feet	Quantity, in million gallons daily	Loss of head, in feet
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
(*a*) CIRCUIT														
A B	4 100	12	38	1.824	0.824	6.0	1.324	14.5	1.574	20.0	1.824	26.2	2.074	33.2
B C	5 000	12	34	1.632	0.632	4.5	1.132	13.2	1.382	19.0	1.632	26.0	1.882	33.8
C D	1 500	12	31	1.488	0.488	0.8	0.988	3.1	1.238	4.7	1.488	6.6	1.738	8.7
*D J**	..	..	20	0.96	−0.04	0.3	0.46	2.5	0.71	4.2	0.96	6.5	1.21	9.1
J K	2 000	12	20	0.96	−0.04	0.0	0.46	1.0	0.71	2.2	0.96	3.9	1.21	6.0
K L	2 200	12	19	0.912	−0.088	−0.1	0.412	0.9	0.662	2.2	0.912	3.9	1.162	6.1
L O†	..	..	11	0.528	−0.472	−1.4	0.028	0.6	0.278	1.9	0.528	4.0	0.778	6.3
O R‡	..	..	0	0	....	−3.6	....	−0.6	....	0.0	....	0.7	....	2.1
R S	4 000	12	0	0	....	−8.4	....	−2.3	....	−0.6	....	0.0	....	0.6
Total, Circuit *A B* to *S*	..	..	..	..	....	−1.9	....	32.9	....	53.6	....	77.8	....	105.9
(*b*) CIRCUIT														
A U	3 000	20	5	0.24	....	....	....	....	....	....	0.24	0.0	0.49	0.1
U Y	1 000	16	4	0.192	....	....	....	....	....	....	0.192	0.0	0.442	0.1
	7 800	14	4	0.192	....	....	....	....	....	....	0.192	0.4	0.442	1.7
Y W	3 300	14	3	0.144	....	....	....	....	....	....	0.144	0.1	0.394	0.6
Total, Circuit *A U* to *W*	..	..	..	....	....	....	....	....	....	....	....	0.5	....	2.5
(*c*) LINE														
S T	3 300	12	10	0.48	−0.52	−2.1	−0.02	0.0	0.23	0.5	0.48	1.8	0.73	3.9
T X	2 100	12	0	0	....	−4.4	....	−1.2	....	−0.3	....	0.0	....	0.3
Total, Line *S T X*	..	..	..	....	....	−6.5	....	−1.2	....	0.2	....	1.8	....	4.2
(*d*) LINE														
W Y	8 800	12	1	0.048	....	....	....	....	....	....	0.048	0.1	0.298	2.0
Y X	3 900	12	0	0	....	....	....	....	....	....	....	0.0	....	0.6
Total, Line *W Y X*	..	..	..	....	....	....	....	....	....	....	....	0.1	....	2.6
(*e*) LINE														
W S	9 300	12	0	0	....	....	....	....	....	....	....	0.0	....	1.5

* See Fig. 7. † See Fig. 8(*a*). ‡ See Fig. 9.

IN THE ENTIRE SYSTEM

DAILY, FROM THE END OF THE LINE

0.50		1.0		1.5		2.0		2.5		3.0	
Quantity, in million gallons daily (16)	Loss of head, in feet (17)	Quantity, in million gallons daily (18)	Loss of head, in feet (19)	Quantity, in million gallons daily (20)	Loss of head, in feet (21)	Quantity, in million gallons daily (22)	Loss of head, in feet (23)	Quantity, in million gallons daily (24)	Loss of head, in feet (25)	Quantity, in million gallons daily (26)	Loss of head, in feet (27)
AB TO *S*											
2.324	41.0	....	....	....	....	....	....	....	....	....	....
2.132	42.8	....	....	....	....	....	....	....	....	....	....
1.988	11.3	....	....	....	....	....	....	....	....	....	....
1.46	12.3	....	....	....	....	....	....	....	....	....	....
1.46	8.4	....	....	....	....	....	....	....	....	....	....
1.412	8.7	....	....	....	....	....	....	....	....	....	....
1.028	9.5	....	....	....	....	....	....	....	....	....	....
....	4.4	....	....	....	....	....	....	....	....	....	....
....	2.3	....	....	....	....	....	....	....	....	....	....
....	140.7	....	....	....	....	....	....	....	....	....	....
AU TO *W*											
0.74	0.3	1.24	0.8	1.74	1.5	2.24	2.3	2.74	3.4	3.24	4.6
0.692	0.3	1.192	0.7	1.692	1.4	2.192	2.2	2.692	3.2	3.192	4.4
0.692	3.9	1.192	10.6	1.692	20.4	2.192	33.0	2.692	48.4	3.192	66.4
0.644	1.4	1.144	4.2	1.644	8.2	2.144	13.4	2.644	19.8	3.144	27.2
....	5.9	....	16.3	....	31.5	....	50.9	....	74.8	....	102.6
STX											
0.98	6.6	1.48	14.3	1.98	24.5	2.48	37.3	....	....	....	....
....	1.2	....	4.4	....	9.4	....	16.0	....	....	....	....
....	7.8	....	18.7	....	33.9	....	53.3	....	....	....	....
WYX											
0.548	6.0	1.048	20.2	1.548	41.6	2.048	69.6	....	....	....	....
....	2.3	....	8.2	....	17.4	....	29.6	....	....	....	....
....	8.3	....	28.4	....	59.0	....	99.2	....	....	....	....
WS											
....	5.4	....	19.6	....	41.5	....	70.8	....	107.0	....	....

Having simplified the main, *A B . . . S*, the final computations and graph can be determined. Table 2 presents the computations upon which the curves in Fig. 10 are based. As previously described in Case V, for a total flow of 2.16 mgd at Point *X*, the loss of head is found to be 116.5 ft, and the quantities and losses of head in the various main elements of the system will be as shown in Table 3. In explanation of the method of taking care of "take-outs," and to allow for the additional quantities through Lines *A W* and *A S* required for

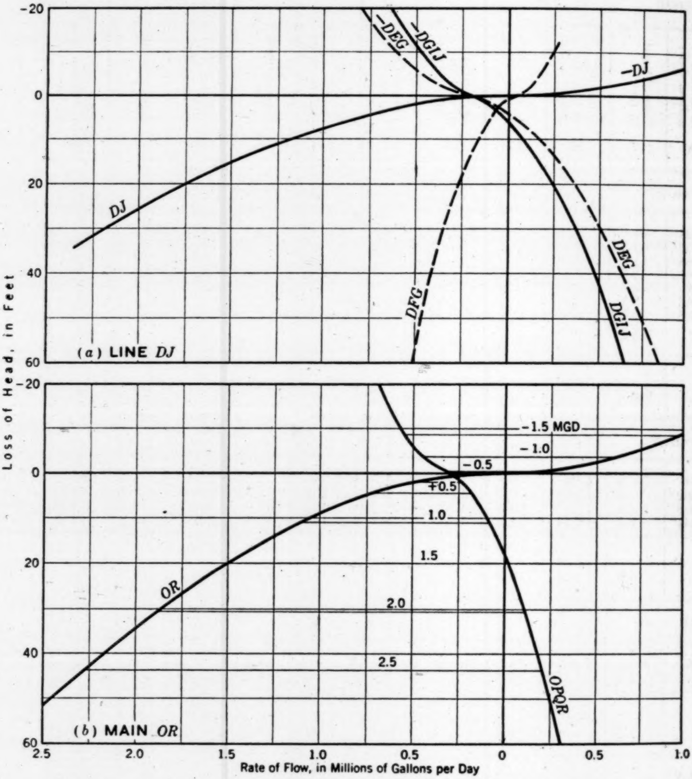


FIG. 8.—GRAPHS TO DETERMINE LOSS OF HEAD DUE TO FIRE FLOW AT POINT *X*

TABLE 3.—QUANTITIES AND LOSSES OF HEAD IN MAIN ELEMENTS

Description	Total	*A W*	*A S*	*W S*	*W X*	*S X*
Quantity, in million gallons daily	2.16	2.48	0.21	1.18	1.25	0.91
Loss of head, in feet	116.5	74.0	101.0	26.5	42.5	15.5

subsequent "take-outs," the curve for Line *A W* is moved to the right a distance equal to 1% of 4.8, or 0.048 mgd, and the curve for Line *A S* is moved to the left a distance equal to 10%, or 0.48 mgd, as shown in the dashed lines on Fig. 10.

The foregoing solution secures only one practical result—the loss of head in the system, under certain consumption requirements, with a definite fire flow at Point *C*. It is true that from this solution other flow and head losses can be computed by the $S^{0.54}$-relationship, but in this conversion the intermediate "take-outs" vary in the same ratio, a condition that occurs rarely in practical studies. The consumption in a system bears no relationship to fire flows, or to other extraordinary requirements. To secure the loss of head under any other fire flow with the same "take-outs," or the loss of head with the same fire flow

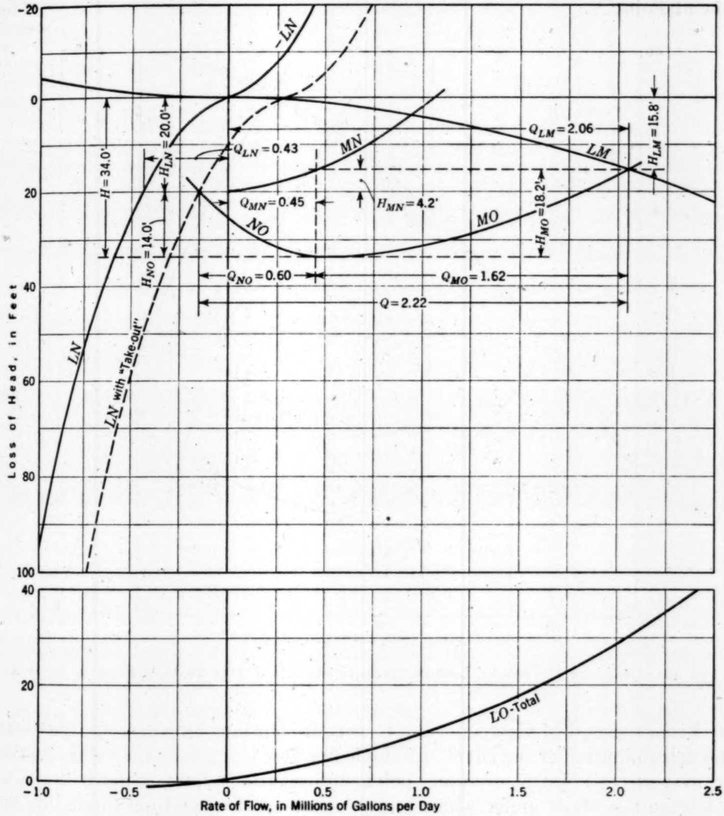


FIG. 9.—TYPICAL CASE II, LOSS OF HEAD, *L* TO *O*

under different consumption conditions, requires separate computations unless it is possible to construct a curve that will allow various flows to be taken through the system and the determination of losses of head without changing the quantities taken out. By obtaining various values of flow and corresponding head losses, a curve can be constructed from Fig. 10 which, the "take-outs" remaining the same, will give the head loss under any reasonable flow at Point *X*.

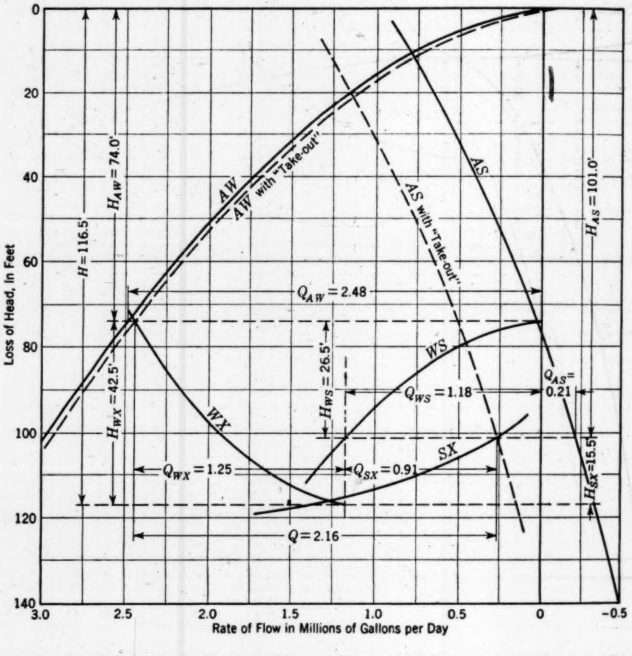


FIG. 10.—GRAPH TO DETERMINE LOSS OF HEAD IN MAIN *A X* DUE TO FIRE FLOW AT POINT *X*

From a practical viewpoint and to permit solutions for purposes other than the determination of the effect of a single fire flow (such as in the determination of pressures at Point *X*, or at any point, under various daily or hourly drafts, to calculate the heads under which pumps must operate, or to examine into the required capacity and height of storage), a slightly different analysis is of more value. For instance, let it be assumed that the rates of draft for ordinary domestic and industrial consumption vary from 1.5 to 3.0 mgd. If a series of capacity-head curves representing the loss of head under rates of 1.5, 2.0, 2.5, and 3.0 mgd are constructed, then, by interpolation, the loss of head under any flow and under varying rates of draft in the system can be secured directly.

To do this, four curves are constructed, representing losses of head through the system under rates of 1.5, 2.0, 2.5, and 3.0 mgd. In this analysis, the percentage "take-outs" are revised, as shown in Table 4, divorcing the normal consumption in the system from any extraordinary demands at Point *X*.

TABLE 4.—REVISION OF "TAKE-OUTS"

Point	"Take-out" (see Fig. 7), in percentages	REVISED "TAKE-OUT," PERCENTAGES		Point	"Take-out" (see Fig. 7), in percentages	REVISED "TAKE-OUT," PERCENTAGES		Point	"Take-out" (see Fig. 7), in percentages	REVISED "TAKE-OUT," PERCENTAGES	
		Theoretical	Value used			Theoretical	Value used			Theoretical	Value used
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
A	1	1.82	1	*K*	1	1.82	1	*U*	1	1.82	2
B	4	7.27	7	*L*	2	3.63	4	*V*	1	1.82	2
C	3	5.45	5	*N*	6	10.91	11	*W*	3	5.45	5
E	6	10.91	11	*O*	1	1.82	2	*Y*	1	1.82	2
F	2	3.63	4	*P*	3	5.45	5	*Z*	45	....	2
H	1	1.82	1	*Q*	7	12.73	13	Total, Points *A* to *Z*	100	100.00	100
I	2	3.63	4	*T*	10	18.20	18				

Fig. 11 shows the head-capacity curves of the main lines which form the basis for the 2.5-mgd rate curve of Fig. 12. The latter shows four curves which give the head-capacity relationship for any flow from Point *X* under rates of 1.5, 2.0, 2.5, and 3.0 mgd, distributed through the system on the percentages shown in Table 4. By scaling the distances between the curves and the *Y*-axis, the losses of head for a flow of 2.0 mgd from Point *X* are determined, as follows:

Normal consumption, in million gallons per day	Loss of head, in feet
1.5	73.6
2.0	87.8
2.5	102.5
3.0	120.0

It is interesting to note in Fig. 12 that by scaling a distance on each rate curve equal to the same percentage of the total, a curve is constructed that may be termed a percentage curve, which can be extended, by the $S^{0.54}$-relationship, to any value. For instance, in the typical example, Fig. 7, the normal consumption in the system is 2.64 mgd. The extraordinary fire demand of 2.16 mgd is 81.7% of the normal consumption. Therefore, if a point is determined upon each rate curve equal to 81.7% of the normal consumption, a percentage curve, Fig. 12, is secured which, when expanded by the $S^{0.54}$ ratio, passes through the point, $Q = 2.16$ mgd, $H = 116.5$ ft, and an interpolated 2.64-mgd rate curve—the same solution as previously determined.

At their intersection with the *Y*-axis, these curves also show the loss of head in the system with no flow at Point *X*. Abscissas to the left of the *Y*-axis, therefore, represent "put-ins" which are required at Point *X* to satisfy the requirements of draft in the system. Furthermore, with the extension of these curves to the *X*-axis the abscissas of the intersections would represent the quantities that must be introduced at Point *X* to satisfy the normal drafts

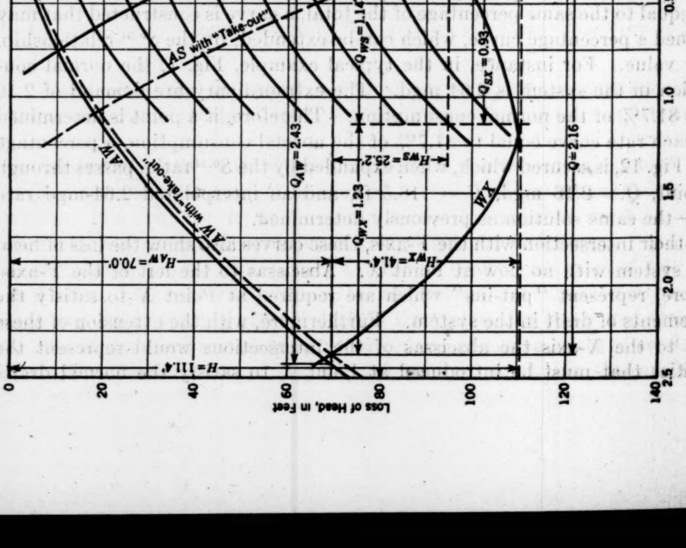


FIG. 11—TYPICAL CASE I; TOTAL LOSS OF HEAD; RATE, 2.5 MILLION GALLONS DAILY; VARYING DAILY DRAFT

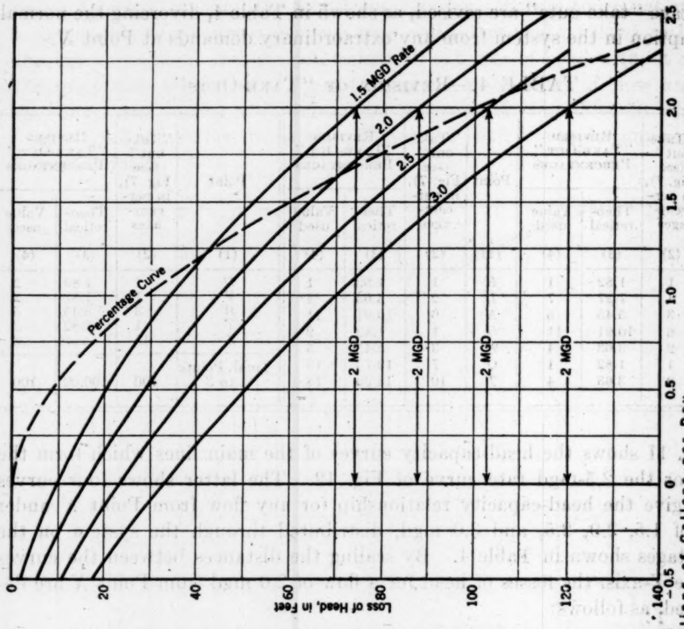


FIG. 12.—TYPICAL CASE I; LOSS OF HEAD IN LINE A X; VARYING DAILY DRAFT

and maintain Points *A* and *X* at the same hydraulic grade; that is, with no total loss of head between the two points.

From Fig. 12, it is possible to study the effect of storage at, or coming into, the system at Point *X*. For instance, if a curve is drawn on the left of the *Y*-axis to represent the loss of head in a main bringing water from storage, the hydraulic grades at Point *X* and at storage, and the flows from each source, can be obtained directly under varying rates of draft in the system. If the storage is variable in elevation, this curve will move, as required, up or down the *Y*-axis during the time interval being studied. In this manner the size and elevation of storage best suited for the requirements can be determined.

Having determined the losses of head in, and the quantities flowing through, the various elements, it is a simple matter to check back along the lines to determine the hydraulic grades at other points and to prepare a pressure contour map.

CONCLUSIONS

The graphical analysis of some of the common hydraulic problems arising in the design of the transmission and distribution elements of a water plant represents an approach to the solution of problems which, by their nature, contain so many variable factors as to cause most designers to hazard guesses upon the results which they hope to obtain, based upon premises which may be at wide variance with actual facts.

In spite of the great sums of money invested in these facilities, no element of a water system receives less attention in design than the transmission and distribution system. Proper consideration given to the correct interpretation of the data upon which the design of these elements must be based, and to a study of the hydraulics involved, in conjunction with field tests which are helpful in planning extensions or improvements to the system, will go far in providing adequate and efficient installations with a minimum outlay of capital.

DISCUSSION

Lynn Perry,[5] **M. Am. Soc. C. E. (by letter).**—The thorough manner in which the author has presented this subject certainly should be appreciated by the membership of this Society. For many years, the writer has solved problems of the nature herein presented by the time-worn analytic method. On many occasions, during this time, he has urged graphic methods for the solution of various types of problems. Such methods eliminate many errors, save time, and give a result sufficiently accurate for all practical purposes.

It appears that the upper parts of Figs. 2 and 3 show curves of parabolic form on rectilinear cross-section paper. If so, these curves would be straight lines when plotted on logarithmic cross-section paper and it seems as if some advantage might be gained from its use.

Frequently an old municipality having an inadequate water supply enlarges its supply by acquiring another, more abundant, at a higher elevation, and from an entirely new direction and source. A larger water main is likely to be installed, leading the new supply to the existing distributing system, sometimes under the direction of an engineer. In most such cases that have been brought to the writer's attention, the new line supplies the ordinary demand and overflows into the old intake. Only at periods of excessive demand is the old supply likely to be drawn down into the distributing system.

Charles M. Mower,[6] **Jr., M. Am. Soc. C. E. (by letter).**—The graphical approach to the problems of flow and loss of head in the elements of transmission and distribution systems is a method of analysis readily adaptable to a variety of problems. Mr. Aldrich has shown how the fundamental Freeman curves may be expanded to cope with the more complicated problems in water distribution which have too often been solved by guess.

With this, as with other office methods, however, there is the danger that the mathematical accuracy of the theory may hide inaccuracies in fundamental assumptions. Coefficients of carrying capacity, rates of flow, and rates of consumption, are often assumed, and the behavior of a pipe system is then analyzed by a flawless process of reasoning. However, the end result is in error because quantities that could, and should, be measured in the field were assumed.

Although one of the most usual assumptions is that $C = 100$ in the Williams-Hazen formula, a few loss-of-head tests on representative mains in a distribution system will show how far this assumption may be from the truth. Similarly, assumptions of flow, take-offs, or consumption may be too far from the truth to be of value, because these quantities are influenced by too many unknown factors. It is essential, therefore, to make actual field measurements instead of assumptions if the design of additions to a distribution system is to have a sound foundation.

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[6] With Glenn D. Holmes, Syracuse, N. Y.

The graphical method so clearly presented by Mr. Aldrich is of great value in determining how new mains and new storage facilities will affect a distribution or transmission system; but the analysis of flows and carrying capacities in the existing system should first be made by instrumental methods.

Thomas R. Camp,[7] M. Am. Soc. C. E. (by letter).—The adaptation of the graphical method of determining "equivalent pipes" to the solution of flow-distribution problems in complex networks, as presented in this paper, is not new. W. E. Howland,[8] Assoc. M. Am. Soc. C. E., has demonstrated that the Freeman method may be used for cross-overs and for simple networks similar to those in Fig. 6. The author seems to be the first, however, to demonstrate that accurate solutions may be obtained by this method for somewhat complicated water-distribution systems.

Three methods have now been shown to be practical for the analysis of flow in networks of a similar order of complexity to that of the network shown in Fig. 7. These methods are as follows:

(1) The "electric network analyzer" method[9];
(2) The Hardy Cross method[10]; and,
(3) The Freeman graphical method.

It should be pointed out that all these methods are "trial-and-error" methods of solving a large number of simultaneous equations. These equations are of three types, arising from three laws which may be stated as follows: (*a*) The algebraic sum of the rates of discharge toward any junction point is zero; (*b*) the algebraic sum of the head losses around any closed circuit is zero; and (*c*) for any pipe or system of pipes the head loss is directly proportional to some power of the discharge. The number of equations involved is so great that their direct solution algebraically is impracticable, even for simple networks. For example, in the single cross-over net of Fig. 5(*b*), eleven simultaneous equations are involved, five of which are exponential and the remainder linear.

Methods (1) and (2) may be classified as "controlled" methods; that is, the errors produced by successive trials become smaller. The Freeman method (Method (3)), however, appears not to be controlled. The solution is obtained by "juggling" the position of the curves that represent the elements in the system. The writer is at a loss to understand how one is to know in which direction to shift a curve when a large number of curves are involved. In the simple net of Fig. 6, consisting of two bays in each direction, four sets of curves must be adjusted. The distribution system represented by Fig. 7, although it involves a large number of elements, is less difficult to solve by the graphical method than the net of Fig. 6, because no stage of the solution of this system involves nets of more than one cross-over. It appears to the writer that a grid system consisting of numerous bays in each direction would not yield to solution by the graphical method except by chance.

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[7] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[8] "Expansion of the Freeman Method for the Solution of Pipe Flow Problems," by W. E. Howland, *Journal*, New England Water Works Assoc., December, 1934, p. 408.

[9] "Hydraulic Analysis of Water Distribution Systems by Means of an Electric Network Analyzer," by Thomas R. Camp, M. Am. Soc. C. E., and H. L. Hazen, *Journal*, New England Water Works Assoc., December, 1934, p. 383.

[10] "Analysis of Flow in Networks of Conduits or Conductors," by Hardy Cross, M. Am. Soc. C. E., *Bulletin No. 286*, Univ. of Illinois Eng. Experiment Station, November 13, 1936.

In Methods (2) and (3), the process consists of assuming a division of the flow at junctions and computing the corresponding head losses (or, conversely, of assuming the head losses and computing the corresponding flows). If the corresponding head losses do not conform to Law (*b*), an adjustment in the assumed flows is made and the process is repeated until the heads do conform reasonably well with this "head-loss summation" law. In Method (1), both Laws (*a*) and (*b*) are taken care of automatically. Since Ohm's electrical resistance law is not analogous to the hydraulic resistance law, adjustments must be made in the resistors representing the elements until the head-discharge (that is, voltage-current) relation conforms to Law (*c*). The number of corrections or adjustments required by both Method (1) and Method (2) will seldom exceed three. In both methods, with each adjustment, the errors in the assumed flows converge toward zero for nearly all the elements in a system. In the graphical method (3), one shift in the position of all the curves corresponds with one adjustment in the other two methods. A shift in the position of the curves, of course, is much more readily accomplished than is an adjustment by the other two methods; but a great many more than three shifts may be necessary and there appears to be no means of knowing that the shifts are being made in the right direction for a solution.

In comparing the three methods, the water-works engineer is interested in the relative lengths of time and amounts of equipment required. In the electric analyzer method, each element is represented by a resistance box. In preparing to solve a problem, the resistors must be connected to one another in a pattern similar to the water distribution system. "Put-ins" and "take-outs" are represented by leads from and to a battery or other source of electricity. Flow is represented by electric current and head loss by voltage drop. Hence, a scale ratio must be selected for conversion between the electric and hydraulic systems, and the proper range of resistance estimated for each element before the actual solution of a problem can be started. This preliminary work in Method (1) corresponds with the preparation of the curves representing the elements in Method (3). The analogous work in Method (2) consists only of computing a constant for each element which represents the relation of flow to lost head, all of which may be done on a slide-rule.

The solution of a problem by the electric analyzer method (1) consists of reading voltage and current for each element, and of adjusting the resistance so that the head (voltage) bears the proper relation with discharge (current). The readings and adjustments are quickly made. A solution by the Hardy Cross method (2) consists of assuming a division of flow at each junction, computing the corresponding heads, and then correcting the flows by an amount computed from the observed error in the heads. No equipment other than a slide-rule is necessary.

The time required to set up the electric analyzer for the problem represented by Fig. 7 was about 10 hr, and about 8 hr was required for the solution. The same problem solved completely by Method (2) occupied the writer for about 6 hr. J. J. Doland,[11] M. Am. Soc. C. E., states that in his solution of the same problem by the Hardy Cross method (2) only about 4 hr was required. It will be helpful if the author would give an estimate of the time required to solve this problem by the graphical method (3).

Harold E. Babbitt,[12] M. Am. Soc. C. E. (by letter).—Problems involving the hydraulics of flow in conduits in series, in parallel, or in a network, are commonly met in the study of water distribution systems. Problems involving the flow in conduits in series or in parallel, or in combinations of conduits in series and in parallel, have been subject for some time to accurate and practical solution by mathematical analysis but, until recently, no method has been available for the solution of problems involving cross-overs or networks; unless Mr. Freeman's graphical method[13] may be so considered. This method involves so much labor for the solution of a practical problem that it has not been widely used. The electrical analogy, introduced[14] in 1934 by Professor H. L. Hazen and Thomas R. Camp, M. Am. Soc. C. E., offers another method of solving such problems. It has not, however, been widely used, possibly because of the equipment required and its rather recent introduction. However, the author is not correct in his statement (see "Introduction") that "a mathematically accurate determination of the results to be expected in designing intricate grid systems or important reinforcements is impossible of attainment from a practical standpoint." Professor Cross' method for the analysis of flow in networks and conductors[15] makes it possible to solve such problems mathematically to any desired degree of precision, and results, using this method, are reported within 1% of accuracy by James J. Doland, M. Am. Soc. C. E. in his illustrative problem which the author has quoted.[4]

Until the publication of the Cross method problems involving the hydraulics of network distribution systems were most commonly solved by guesswork or by approximate methods whose results were little more than guesswork. The author's method is a simplification and an extension of Freeman's graphical method, which is applicable to some special conditions of flow in networks. It could probably be extended to include all possible problems in such networks.

Problems that may arise in a study of the hydraulics of a network distribution system may be illustrated on the simple cross-over diagram shown in Figs. 13(*a*) and 13(*b*), in which the diameters and lengths of all pipes are known. The principles are applicable to any number of pipes and cross-overs in the network.

Problems of Type I are illustrated in Fig. 13(*a*), in which all put-ins and take-outs are known, and it is desired to determine the distribution of flow in all the pipes and the pressures at various points in the pipe system. This can be solved by the Cross "Method of Balancing Heads."

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[11] *Engineering News-Record*, October 1, 1936, p. 475.

[12] Prof., San. Eng., Univ. of Illinois, Urbana, Ill.

[13] *Journal*, New England Water Works Assoc., Vol. 7, p. 49, 1892.

[14] "Hydraulic Analysis of Water Distribution Systems by Means of an Electrical Analyzer," *Journal*, New England Water Works Assoc., Vol. 48, p. 383, 1934.

[15] *Bulletin 286*, Eng. Experiment Station, Univ. of Illinois, 1936.

Problems of Type II are illustrated in Fig. 13(*b*), in which the pressures are known at all points of put-in and take-out, and it is desired to determine the rate and direction of flow in all pipes and the rate of all put-ins and take-outs, and the pressures at other selected points in the distribution system. This can be solved by the Cross "Method of Balancing Flows."

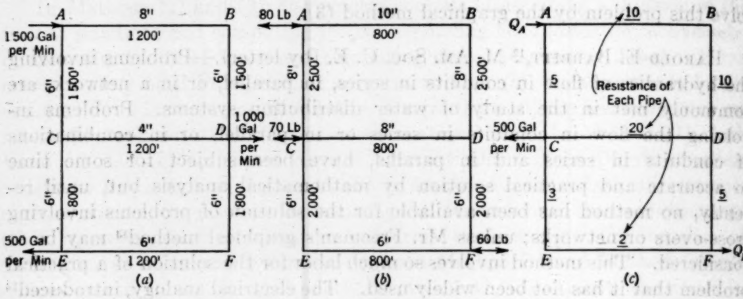
A
8"
B
1 500 Gal per Min
1 200'
6"
1 500'
6"
1 500'
C
4"
D
1 000 Gal per Min
1 200'
6"
1 800'
6"
1 800'
500 Gal per Min
E
6"
1 200'
F
(a)
80 Lb
A
10"
B
800'
8"
2 500'
8"
2 500'
70 Lb
C
8"
D
800'
6"
2 000'
6"
2 000'
60 Lb
E
6"
800'
F
(b)
Q_A
A
10
B
5
Resistance of Each Pipe
10
500 Gal per Min
C
20
D
3
5
2
E
F
Q_F
(c)

FIG. 13

Problems of Type III involve a combination of the conditions and unknowns in Types I and II.

The second type of problem is commonly met in water-works practice as, for example, where the pressures at a hydrant and at the pumping station, or at one or more distribution reservoirs, are known and it is desired to know the rate and direction of flow through the pipes in the distribution system. It is to be noted that where the Cross method is used for the solution of such a problem a change in one or more of the assumed pressures, put-ins, or take-

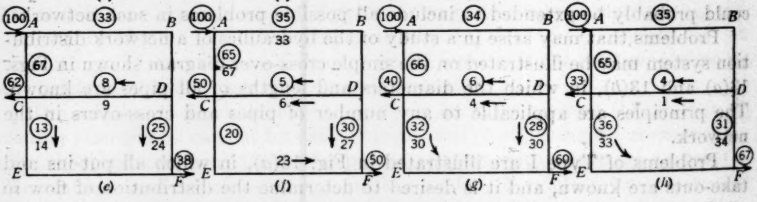
A
10
B
5
20
C
500 Gal per Min
3
5
2
E
F
(a)
(100)
(10)
38
(90)
62
10
(7)
10
(100)
(3)
31
28
(3)
31
28
B
D
C
E
F
0
(b)
(100)
(38)
31
(62)
69
(10)
13
(100)
(28)
21
18
B
D
C
E
F
0
(c)
(100)
(31)
32
(69)
68
(13)
(100)
(18)
19
B
D
C
E
F
0
(d)
(100)
A
(33)
B
(67)
(62)
(8)
9
C
D
(13)
14
(25)
24
(38)
E
F
(e)
(100)
A
(35)
B
33
(65)
67
(50)
(5)
6
C
D
(20)
(30)
27
23
(50)
E
F
(f)
(100)
A
(34)
B
(66)
(40)
(6)
4
C
D
(32)
30
(28)
30
(60)
E
F
(g)
(100)
A
(35)
B
(65)
(33)
(4)
1
C
D
(36)
33
(31)
34
(67)
E
F
(h)

FIG. 14.—CONDITIONS OF FLOW FOR SOLUTION IN TABLE 5

outs, will involve the complete reworking of the solution, whereas if there is no change of the pipes in the distribution system, the problem resulting from changes in the conditions can be solved relatively easily by the method suggested by the author.

TABLE 5.—COMPUTATIONS TO DETERMINE DISTRIBUTION OF FLOWS IN THE PIPES SHOWN IN FIG. 13(*c*), WITH TAKE-OUT AT POINT *C* = 500 GALLONS PER MINUTE, AND DIFFERENT TAKE-OUTS AT POINT *F*

Pipe* (1)	Resistance, K (2)	Flow, Q (percentages) (3)	KQ^n† (4)	KQ^{n+1} (5)	$\Sigma (KQ^{n+1})$ (6)	ΔQ (7)	Flow, Q (percentages) (3)	KQ^n (4)	KQ^{n+1} (5)	$\Sigma (KQ^{n+1})$ (6)	ΔQ (7)
$Q_F = 0$; FIRST TRIAL (SEE FIG. 14(*b*))							$Q_F = 305$; ONLY TRIAL (SEE FIG. 14(*e*))				
ABD	20	10	200	2 000			23	660	21 800		
DF	5	3	15	45	2 045		25	125	3 125	24 925	
AC	5	90	450	40 500			67	335	22 400		
CEF	5	3	15	45	40 455		13	65	845	23 245	
Σ			680 × 2 =	1 360		$\frac{38\,410}{1\,360} = (28)$		1 185 × 2 =	2 370		$\frac{1\,680}{2\,370} = (1)$
CD	20	7	140	2 800			8	160	1 280		
DF	5	31	155	775	2 025		25	125	3 125	1 845	
CEF	5	31	155	4 800	4 800		13	65	845	845	
Σ			450 × 2 =	900		$\frac{2\,775}{900} = (3)$		350 × 2 =	700		$\frac{1\,000}{700} = (1)$
$Q_F = 0$; SECOND TRIAL (SEE FIG. 14(*c*))							$Q_F = 500$; ONLY TRIAL (SEE FIG. 14(*f*))				
ABD	20	38	760	28 900			35	700	24 500		
DF	5	28	140	3 920	32 800		30	150	4 500	29 000	
AC	5	62	310	19 200			65	325	21 100		
CEF	5	28	140	3 920	15 280		20	100	2 000	23 100	
Σ			1 350 × 2 =	2 700		$\frac{17\,540}{2\,700} = (7)$		1 275 × 2 =	2 550		$\frac{5\,900}{2\,550} = (2)$
CD	20	10	200	2 000			5	100	500		
DF	5	21	105	2 210	210		28	140	3 930	3 430	
CEF	5	21	105	2 210	2 210		22	110	2 420	2 420	
Σ			410 × 2 =	820		$\frac{2\,420}{820} = (3)$		350 × 2 =	700		$\frac{1\,010}{700} = (1)$
$Q_F = 0$; THIRD TRIAL (SEE FIG. 14(*d*))							$Q_F = 750$; ONLY TRIAL (SEE FIG. 14(*g*))‡				
ABD	20	31	620	19 200			34	680	23 100		
DF	5	18	90	1 620	22 400		28	140	3 930	27 030	
AC	5	69	345	23 800			66	330	21 800		
CEF	5	18	90	1 620	22 180		32	160	5 130	26 930	
Σ			1 145 × 2 =	2 290		$\frac{220}{2\,290} = (1)$		1 310 × 2 =	2 620		$\frac{100}{2\,620} = (1)$
CD	20	13	260	3 380			6	120	720		
DF	5	19	95	1 800	3 380		28	140	3 920	3 200	
CEF	5	19	95	1 800	3 600		32	160	5 130	5 130	
Σ			350 × 2 =	700		$\frac{220}{700} = (1)$		420 × 2 =	840		$\frac{1\,930}{840} = (2)$

* See Fig. 14. † $n = 1$. ‡ Computation for $Q_F = 997$ (Fig. 14(*h*)) has not been included.

It is possible, however, that a number of problems, with successively slight changes in the conditions, might be solved by the Cross method, the results being plotted graphically with little or no more effort than would be involved in a series of solutions by the author's method. For example, let it be desired to determine the magnitude and direction of flow in each pipe in the simple cross-over system illustrated in Fig. 13(*c*). All the water enters at Point *A*; 500 gal per min is constantly taken out at Point *C*, and the take-out at the hydrant at Point *F* may vary between zero and 1 000 gal per min. The method suggested by Professor Doland has been used in this solution and, for simplicity, it has been assumed that $H = K\,Q^2$, in which *H* is the head loss in the conduit; *Q* is the rate of flow in the conduit; and *K* is a resistance factor dependent upon the length and diameter of the pipe. If the more commonly used Williams and Hazen formula were used here, the expression would be $H = K\,Q^{1.85}$. The computations involved in the solution are shown in Table 5 and Fig. 14, and the flows in each pipe, to the nearest 1% of accuracy, are entered in Table 6. These values can be plotted so that the rate of flow in any pipe, corresponding to any rate of flow from Point *F*, can be read from the curves. In Fig. 14, the underlined values denote relative resistances and the values in circles are the first assumptions of the percentage flow. The values beneath the circled numbers indicate corrections. The arrows in Table 6 indicate when the direction of flow in Fig. 13(*c*) is from left to right. When no arrow is shown the direction of flow is away from Point *A*.

TABLE 6.—RATES OF FLOW IN PIPES IN FIG. 13(*c*) FOR VARIOUS VALUES OF Q_F

(Values of *Q* Are Given in Gallons per Minute)

Pipe	Q_F = ZERO		Q_F = 305		Q_F = 500		Q_F = 750		Q_F = 997	
	Percentages	*Q*	Percentages	*Q*	Percentages	*Q*	Percentages	*Q*	Percentages	*Q*
ABD..	32	160	33	270	33	330	34	425	35	525
DF....	18	90	24	190	27	270	30	375	34	510
AC....	68	340	67	540	67	670	66	825	65	975
CEF...	19←	95←	14	110	23	230	32	400	34	510
CD....	13←	65←	9←	72←	6←	60←	4←	50←	1←	15←

Where added conditions are involved, such as storage reservoirs on the distribution system, or where the distribution system is on two or more levels, as on different floors of a multi-storied building, Professor Cross' method can be applied. There is no limit to the number of problems which may arise in the hydraulics of flow in a network distribution system. Any problem in this field can be solved by the Cross method and many problems can be solved by the author's method. Where both methods are applicable to the solution of a problem the choice will depend on the experience and the judgment of the computer.

Some confusion may result from the use, by the author, of the term, "hydraulic grade," to represent the piezometric head or the internal pressure at a point in a conduit. The term, "grade," is usually synonymous with "slope" rather than with "elevation" in most texts on hydraulics. The term, "pressure," or "head," would be more significant of the author's meaning.

C. Maxwell Stanley,[16] **Assoc. M. Am. Soc. C. E. (by letter).**—Being useful in the solution of the complex problems involved in the design of systems for the transmission and distribution of water, the method suggested by Mr. Aldrich is both interesting and valuable. Methods of design for such systems have had too little attention from water-works engineers, who have usually considered the design of a distribution system as something too complex for accurate computation, and yet too simple and practical to warrant much study. As a result, the art of designing transmission and distribution systems has developed slowly and has been out-distanced by the developments in supply, treatment, and conditioning of water. The distribution system is easily overlooked as it has usually grown bit by bit and seldom does the water-works man realize the accumulated investment that lies beneath the streets—out of sight and considered only when necessary.

The author's statement that the transmission and distribution "elements of a water system commonly exceed 50% of the total value" is conservative. Table 7, pertaining to distribution in several municipal water systems in Iowa, indicates an average of more nearly two-thirds and in only one case does the ratio drop below 50%, in spite of the varying size and types of supply studied.

TABLE 7.—Percentage of Total Investment in Distribution

System (1)	Population served, in thousands (2)	Supply (3)	Treatment* (4)	Percentage investment in distribution (5)	System (1)	Population served, in thousands (2)	Supply (3)	Treatment* (4)	Percentage investment in distribution (5)
A..	1.1	Deep wells	None	49.5	*G* .	17.0	Shallow wells	None	81.5
B..	1.5	Deep wells	None	78.3	*H* .	22.0	Deep wells	A, Ch	66.5
C..	4.1	Deep wells	None	84.5	*I*..	29.0	River	C, S, F, Ch, So	64.5
D..	6.6	Reservoirs	C, S, F, Ch	40.8	*J*..	42.0	Deep wells	Ch	64.5
E..	9.0	Impounding reservoirs	A, C, S, F, Ch	55.0	*K* .	48.0	Wells	A, S, Ch	66.0
F..	12.0	Shallow wells	Ch	58.2	*L* .	145.0	Infiltration galleries	Ch	69.0

* Key to symbols on treatment: A = aeration; C = coagulation; S = sedimentation; F = filtration; Ch = chlorination; and, So = softening.

The graphical solution outlined by the author seems to be much better suited to problems of transmission or distribution which involve not more than a few parallel circuits, than to complicated grids with numerous circuits or cross-connections. The solution of grids by the methods of Case VI is very ingenious, but is laborious and time-consuming as compared to the method devised by Hardy Cross,[17] M. Am. Soc. C. E. Indeed the example contained in the paper as a "Solution of a General Case" seems in itself proof that the Cross method is more quickly and easily applied to networks. The solution of this problem, as presented by J. J. Doland,[4] M. Am. Soc. C. E., is more concise and much quicker than the solution presented by the author. The graphical solution becomes too involved with a complicated grid. Furthermore, a grid or network is generally analyzed for a maximum condition, usually that of normal consumption plus a fire flow located at some particular point which, for the particular section of the system under consideration, will give the worst condition of pressure. Thus, usually one condition only is studied and the work required to prepare the curves for each section of pipe for the graphical method is excessive. This fact further tends to limit the usefulness of the method to transmission problems.

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[16] Vice-Pres., Young & Stanley, Inc., Muscatine, Iowa.

[17] "Analysis of Flow in Networks of Conduits or Conductors," by Hardy Cross, *Bulletin 286*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

The graphical method appears well adapted to problems with varying rates of flow. In such cases, the curves for different pipes and groups of pipes present a visual analysis of the happenings with varying flows. Direct mathematical computation, on the other hand, presents only the answer for a given condition and the effect of varying flow is not so easily visualized.

The author's method is exceptionally well adapted to problems involving storage (similar to Case IV) and those involving transmission mains, interconnecting sources of supply, pumping stations, and storage reservoirs or tanks. In such problems, the graphical picture of the characteristics of the system permits a simple showing of water elevations, friction losses, and quantities which is extremely helpful. It seems to the writer that it is for problems of this type that the graphical method is best suited and will find its widest application and use. The method also has a value in that it may be applied to small sections of a large system when it is desired to obtain a clearer "picture" of the relationship of flow, friction, and head with varying conditions. Often, a chart or graph as a supplement to computations affords a better understanding of the situation.

The foregoing comments apply to the general usefulness and application of the method. A few comments may also be made on some of the details.

In the list of "Basic Data" required there should be added under Group (*d*)—which deals with storage—the item of depth of storage. Such depth plays a large part in the value of a given storage volume. The effectiveness of a given total volume in an elevated tank, as compared to the value of the same volume in a stand-pipe, is a good example of the need for considering depth of storage as an important factor.

Conversion factors for various values of C in the Williams-Hazen formulas for application to the friction losses obtained from Fig. 1 ($C = 100$), are, as follows:

C	Correction factor	C	Correction factor
70	1.94	110	0.84
75	1.70	115	0.77
80	1.51	120	0.71
85	1.35	125	0.66
90	1.21	130	0.62
95	1.10	135	0.58
100	1.00	140	0.54
105	0.91	145	0.50
		150	0.47

Conversion factors may be more readily obtained from such a list than from the extra curves, in Fig. 1, showing the relationship of $S^{0.54}$ and $S^{\frac{1}{0.54}}$.

Warren E. Wilson,[18] Jun. Am. Soc. C. E. (by letter).—An interesting application of graphical methods to the solution of problems encountered in the design of water distribution systems, is presented in this paper. It is claimed for the method that in certain cases better progress can be made than with mathematical methods or by trial and error. It is not entirely clear which cases are in this category. Since the purpose of the graphical solution is avowedly to obtain a practical result in a short time it should be on the basis of this purpose that it be compared with other methods. Great accuracy is not required; nor can it be expected, since the basic assumptions one makes are necessarily merely approximations. Hence, any of the methods may be expected to give results that have the required degree of accuracy.

It appears that the preliminary process through which one must go, namely, that of plotting curves to suitable scales, necessarily consumes considerable time. In addition, one must adjust a number of curves to the proper positions to obtain the desired results. This adjustment is doubtless made with greater speed as one gains experience in the method, but must nevertheless take considerable time. The fact that considerable experience is required to use the graphical method to advantage then appears to be an undesirable feature of the method since it is possible to teach the Cross method to a group of students and develop in them sufficient skill to handle complex networks in a period of not more than a week. It would be difficult to develop the same proficiency in the graphical method in a similar period of time. From the standpoint of the professional engineer the need for an experienced designer in the manipulation of such a method is a disadvantage since the work could be delegated to an inexperienced person after giving him just sufficient instruction to enable him to handle the method of successive approximations as a routine set of calculations.

It is the writer's belief that if one acquaints himself fully with the fundamentals of the Hardy Cross method[4] and clears away any false notions he may have about a method of successive approximations in the question of accuracy, he can solve even the most complicated network problems in a comparatively short time. There can be no question that the preliminary preparation required in the Cross method is far less than that required for the graphical method, since it involves only the calculation of a constant for each section of pipe in the network. The actual work that is necessary in making the successive approximations is certainly no more difficult and requires very little time.

The writer would be interested in a statement of the particular types of problems in which the graphical method is believed to be superior to the Cross method.

Harold K. Palmer,[19] M. Am. Soc. C. E. (by letter).—The method of handling complex problems in water supply design, which is presented in this paper, well merits the careful study of all who have similar problems to solve, and the author deserves praise for the ingenious manner in which he has solved some of the worst of them. The writer has used the same method in handling pumping problems, which the author mentions, for several years and has found it a very convenient tool. The following comments are offered to supplement the author's handling of the subject and to emphasize the fact that it is a method which can be used for many purposes, depending largely upon the ingenuity of the engineer.

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[18] Asst. Prof. of Civ. Eng., School of Civ. Eng., Coll. of Eng., Tulane Univ., New Orleans, La.

[19] Chf. Draftsman, Los Angeles County Sanitation Dists., Los Angeles, Calif.

One fact that should be especially emphasized is that there is nothing obscure or complex about it. Instead of trying to solve algebraic formulas of the second or higher degree by computation the designer simply plots the curves, combines these curves, and the intersections give the solution desired. Thus, the solution of the problem is no more difficult than the drawing of the curves. It is only necessary to keep in mind the factors that produce loss of pressure and to apply them in the right places by simple methods. The principal way in which it differs from the ordinary computation is that the latter is performed for one pressure or one value of Q, and if the result is not correct another approximation must be made. In the author's method all values of pressure and Q within certain limits are considered and the solution is given at once by the intersection of certain lines.

Fig. 1 was prepared probably from the Williams-Hazen tables, which is also the writer's method of making similar drawings, but these tables give only standard cast-iron pipe sizes, and other sizes must be interpolated. This can be done very easily by the following method, which is applicable to any similar exponential formula. Fig. 15 is a simplification of Fig. 1 showing only a few of the lines to avoid confusion, and yet show the method. Marking the intersections of the line for 20-in. pipe with the 20 ordinate, the 30-in. pipe line with the 30 ordinate and the 60-in. pipe line with the 60 ordinate, it will be noted that all the intersections marked are on the same straight line, which can be called the interpolating line. Why this should be a straight line can be proved by any one mathematically inclined, but it is not required in this case, and it is easier to draw lines for two or three sizes of pipe and join the proper intersections than to compute the slope of the intersecting line. To interpolate any other size of pipe it is necessary only to note the intersection of the interpolating

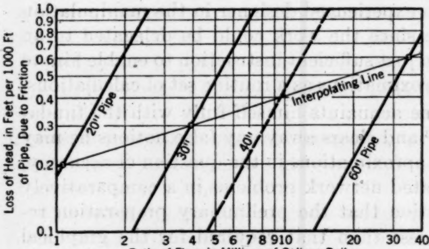


FIG. 15

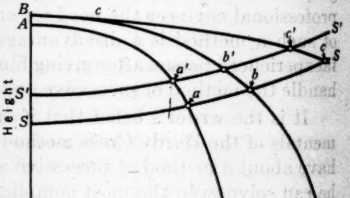


FIG. 16

line with an ordinate corresponding to the diameter required, and draw through this intersection a line parallel to the other pipe curves. Thus, any size of pipe can be interpolated without difficulty and in much less time than it takes to find the values in a table and plot them. Furthermore, a graph for metric sizes could be computed from the data for English sizes.

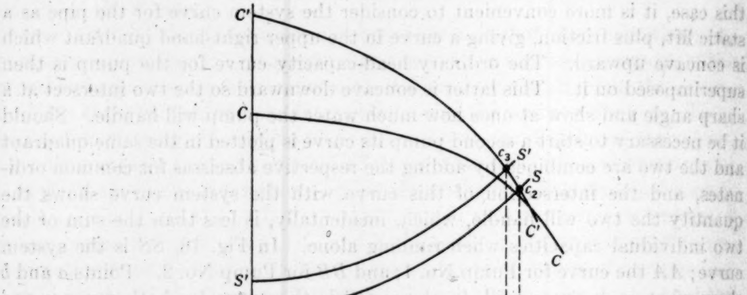
Several manufacturers of centrifugal pumps are using a method similar to that of the author to show the behavior of pumps on a given pipe system. In this case, it is more convenient to consider the system curve for the pipe as a static lift, plus friction, giving a curve in the upper right-hand quadrant which is concave upward. The ordinary head-capacity curve for the pump is then superimposed on it. This latter is concave downward so the two intersect at a sharp angle and show at once how much water the pump will handle. Should it be necessary to start a second pump its curve is plotted in the same quadrant and the two are combined by adding the respective abscissas for common ordinates, and the intersection of this curve with the system curve shows the quantity the two will handle, which, incidentally, is less than the sum of the two individual capacities when running alone. In Fig. 16, *SS* is the system curve; *AA* the curve for Pump No. 1; and *BB* for Pump No. 2. Points *a* and *b* show what each pump will do alone; *CC* is the curve for both pumps; and Point *c* gives the quantity of flow that the two will handle.

If the pumps are discharging into the bottom of a tall stand-pipe in which there is considerable fluctuation in the height to be pumped, it is advisable to draw another curve for the stand-pipe full, such as Curve *S′S′*, Fig. 16, in which the height, *SS′*, is the increased depth. Points *a′*, *b′*, and *c′* now show what the pumps will do as the stand-pipe fills.

Should the system consist of two or more pipes in parallel they can be added to the diagram in a similar manner and the saving in head, and increase in output of the pumps, is shown at once.

The foregoing method serves very well where the pumps are close together and the individual suction losses are small in comparison with the discharge head. It is obvious that if two 1 000-gpm pumps are running in parallel the friction back of the junction of the two is no more for both pumps running than it is for only one. In fact, it is rather less because each alone might pump 1 100 gpm and only 1 000 gpm when both are running.

For a comprehensive case consider two deep-well pumps, *A* and *B*, Fig. 17(*a*), separated several hundred feet, both feeding into the suction of a booster pump, *C*, near Pump *B*, which forces the water into a high stand-pipe far enough away to offer some appreciable pipe friction, and subject to an increase in depth with its equivalent increased pumping lift; also let the standing water surface in the two wells be at the same elevation but with different draw-down characteristics for each. To avoid too many overlapping lines in Fig. 17(*b*) the two well pumps are shown in different quadrants. The distance from the static water surface to the suction of the booster pump, *C*, is *OO′*. As water in Well *A* draws down, this lift increases along the line, *O′w′*. The suction losses and pipe friction to Pump *C* for Pump *A* are shown by Curve *Of′*, and the total lift and losses for Pump *A* are shown by *O′t′*. Arc *AA* is the head-capacity curve for this pump, and subtracting (vertically) *O′t′* from Curve *AA* gives Curve *aa* which represents the elevation at which Pump *A* will deliver various quantities of water to Pump *C*. A similar system is shown for Pump *B* in the right-hand quadrant, *ab* representing the elevation at which Pump *B* delivers water to Pump *C*. Adding *aa* and *ab* horizontally gives Curve *a–ab* which is the corresponding curve for the two running in parallel. Curves *SS* and *S′S′* represent the system curves for the stand-pipe empty and full, respectively, and *CC*



(*a*) PROFILE OF PIPE LINE

FIG. 17.—LOSS OF HEAD, IN FEET PER THOUSAND FEET OF PIPE, DUE TO FRICTION

is the head-capacity curve for Pump *C*. Adding (vertically, since the two are now in series) Curves *a–ab* and *CC* give Curve *C′C′*, the combination curve for all pumps, intersecting the system curves at Points c_2 and c_3, which represent the quantities pumped when the stand-pipe is empty and full. Lines drawn vertically from these points to Curve *a–ab*, and then horizontally to Curves *ab* and *aa* with vertical lines from these intersections to Curves *AA* and *AB*, show at Points A_2 and A_3, and at Points B_2 and B_3 how much each well pump is handling, and if it is working at its best efficiency.

The theory on which this method of attack is based is that the pump adds pressure to the water, which pressure is reduced at any point along the pipe by the static lift plus the friction. These respective losses are charged against each pump at the junction point where it is obvious that both must be exerting the same net pressure, and the combined flow will be the sum of the flows corresponding to this net pressure. Since there was only one booster pump and no complications it was not necessary to use the same method on it, although it could be done if so desired.

This method of analysis is very useful in investigating low-head complex pumping systems and easily solves some very difficult problems with remarkable accuracy.

F. KNAPP,[20] ESQ. (by letter).—In 1936, the writer was called upon to investigate the pressure and flow conditions of the water supply and fire protection system of an important Brazilian chemical plant where interruption of the flow of water would lead to disastrous results. It was especially desired to know the effect of shutting off, quickly, certain flow demands for production purposes. As a prerequisite to this extensive surge investigation (by the graphical method), the flow and the resulting head losses in the system for steady conditions had to be found, and a graphical method, first published by Spiess,[21] was used and extended for the requirements of this specific problem. As far as the author's Case V is concerned, the graphical solution as proposed is far simpler and more general than the method used by the writer, a combination of graphical and analytical procedures.

The author is to be commended heartily for his timely paper which is of considerable interest to all hydraulic engineers engaged in problems of water flow in pipes. It is only too true that the design of the transmission and distribution system receives little attention in spite of the great sums of money invested in such installations. The rational method proposed by the author makes it possible also to decide at once whether certain cross-connections are essential and of decided benefit. In the investigation made by the writer, considerable savings were effected by removing several long pipes connecting points of about equal, or slightly different, pressure.

At first glance, the paper seems more complicated than it really is. The discharge-head relations for the solution of the several fundamental cases are plotted, using tables or any other appropriate means. Furthermore, it is not necessary to make Table 1 as complete as the form presented by the author. It is sufficient and facilitates the work to record only the results of the simplified layout, such as, for example, "Total, *D* to *J*."

As far as the solution of the case presented in Fig. 12 is concerned, the author did not make it clear that, actually, four diagrams of the type shown in Fig. 10, drawn for the various flow demands in the system, are required; and only when the writer realized this point, did he understand the solution.

Except for this minor criticism, the paper deserves to be mentioned as essential to the art of hydraulics, and the writer hopes that its contents will come to the attention of authors of books on applied hydraulics, who too often do not pay enough attention to professional papers representing an advance in the art.

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[20] With São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[21] *Journal für Gasbeleuchtung und Wasserversorgung,* 1887, p. 513; also treated in "Hydromechanik der Druckrohrleitungen," by R. Winkel, München, 1919.

WESTON GAVETT,[22] ASSOC. M. AM. SOC. C. E. (by letter).—In so far as it demonstrates the application of Mr. Freeman's method to the solution of complicated pipe systems, the paper by Mr. Aldrich is most interesting. By showing the application of the graphical method, first to simple typical problems and then to a more complicated system, the author clearly demonstrates the method and advantage of this procedure. Its advantage is the use of natural co-ordinates for head and flow, permitting the graphical summation of these values.

Celluloid sheets described by the author are convenient and may be easily prepared by rubbing the surface of commercial celluloid with an ink eraser.

Although the author mentions the hydraulic slide-rule as an easier means of solving simple problems than the graphical method, he gives no details as to the slide-rule procedure so that the reader may compare the two methods. In stating that "a mathematically accurate determination * * * is impossible of attainment from a practical standpoint," he is correct. This statement, however, tends to encourage rather than dispel the current opinion that, until 1935, it was impossible to design a distribution system. To assume that for years water-pipe systems have been designed by guesswork is extreme.

The computation of the losses in a distribution system that may be checked by actual tests to an accuracy greater than the precision of the basic data should be sufficiently accurate for all except academic purposes. Such computations may be made and have been made with the hydraulic slide-rule at least since 1905.[23]

ELLWOOD H. ALDRICH,[24] M. AM. SOC. C. E.—The helpful comments offered in the discussion of this paper are appreciated. In many of the discussions, as well as in previous papers, the writer is surprised to note the general admission that practical solutions of these problems were impossible prior to the Camp-Hazen and Cross methods of analyses. The writer has used the graphical method presented in the paper over a period of more than ten years in many involved problems and has quite successfully checked his computations by the results obtained after the completion of several construction projects.

The discussions have been devoted largely to comparisons of the three methods of analyses, namely, the Camp-Hazen method, the Cross method, and the graphical method presented in the paper. In general, they have been restricted to the solution of distribution networks. In explanation, this paper was prepared and submitted prior to the publication of the Cross method. Typical Case II was substituted for a more general case in the final revision of the paper to secure a better comparison of the three methods of analyses.

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[22] San. and Hydr. Engr., Clyde Potts, New York, N. Y.

[23] "Hydraulic Tables," by Williams and Hazen, 1905.

[24] Associated with Reeves Newsom, Cons. Engr., New York, N. Y.

The discussions, therefore, have been concerned largely with subsequent developments in the solution of grid-system problems.

The method of analysis to be used in solving the various types of problems is only a means to an end. The choice of method, the Cross or the graphical (the Camp-Hazen method is essentially similar to the Cross method), will be governed principally by the purpose of the problem, its complexity, and the familiarity of the designer with the two methods. In intricate grid-system problems the Cross method more readily effects a simplification of the problem. In simple transmission studies, such as may be termed Typical Case II, and in storage and pumping studies, the graphical solution appears simpler and more adaptable to a portrayal of the varying conditions usually present in a problem. In most problems a combination of the two methods would appear desirable. As the intricacy of network problems increases, so does the labor of solution, by any method.

The variation of the Cross method, suggested by G. M. Fair,[25] M. Am. Soc. C. E., further simplifies the mechanics of the Cross method, permitting the use of the hydraulic slide-rule and making unnecessary the compilation of a pipe resistance table.

Professor Perry has pointed out that the use of logarithmic paper in the graphical method would result in straight lines instead of curves. This is true, but in the more complicated problems, involving "take-outs," "put-ins," varying elevations, and interconnecting pipe, the measurements of head and quantity are made more difficult, so that a rectilinear scale is to be preferred.

The value of Mr. Mowers' comments in stressing the field investigation of drafts and carrying capacities of pipe is very evident. When *C* varies from 100 to 80, the resulting friction losses are increased by 50 per cent. To one who has had experience in measuring coefficients and flows it is apparent how illogical, and even absurd, it is to attempt to make an office design within an accuracy of 1% or 2% unless the data upon which the design is based can be determined with reasonable accuracy. However, assumptions of draft, distribution of consumption, and carrying capacities of pipe are, and always will be, required when designing for future works. Judgment must be tempered with a knowledge of field variations when making these assumptions and a proper balance maintained between the desirable accuracy of the several parts of the problem.

Professor Camp has summarized some of the comparative advantages and disadvantages of the three methods of analyses as they are applicable solely to network problems. It appears that the electrical analogy method, because of the time and extensive equipment required for the solution of a simple problem, has no advantage over the Cross method. It is questionable whether an intricately connected grid system requires elaborate and time-consuming methods of analyses, except as an academic problem. In many cases a judicial combination of mains and "take-outs" will not materially affect the accuracy of the result. Typical Case II, as first solved by Professor Camp by the electrical analogy method, prompted this paper. When this example first appeared,[26] an answer to the problem was secured by the graphical method, within 2% of the accuracy of the final result, in less than 2 hr, using an hydraulic slide-rule and a piece of co-ordinate paper. This naturally prompted the query as to the necessity for the extensive equipment, time, and energy required for the solution of the problem by the electrical analogy method.

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[25] *Engineering News-Record*, March, 1938, p. 342.

In presenting the problem of Typical Case II more time was consumed in order to present the method of analysis clearly. Approximately four hours were required to secure the answer to the problem with the accuracy determined. In the actual design of such a system several short-cuts are possible to one familiar with the graphical method since the shape of the curves give a picture of the problem. It is immediately apparent that the loops *DJ*, *LO*, and *OR* (Fig. 7) can be approximately simplified with very little effect upon the final accuracy.

Irrespective of the fact that the statement: "A mathematically accurate determination * * * is impossible of attainment from a practical standpoint" was submitted before the writer knew of the Cross method (a statement to which Professor Babbitt takes exception as being incorrect), it is a fair statement in its broader sense when the variations in consumption rates and coefficients, which obtain in practical application, are considered. With certain assumed conditions, it is true that any of the methods of analyses can give a mathematically correct solution. The results secured in practical application, however, may be at wide variance with the solution due to inaccuracies of assumptions.

As illustrated by Professor Babbitt, the Cross method can be used, with graphical assistance, to cover varying conditions without repeating all computations.

Mr. Stanley has clearly presented the various fields in which each method of analysis might be the more applicable. In determining the influence of storage, as he points out, the depth—or rather, the effective range and elevation of storage—must be considered.

It is questionable whether (as intimated by Professor Wilson) the adaptability of a method of solving engineering problems should be measured by the speed with which students can be taught the mechanics of using it, unless, at the same time, they obtain a proper appreciation of the fundamental theory. Any short-cut method of solving the type of problem considered in the paper is too susceptible to misuse without an understanding of the underlying theory and conditions which actually affect the problem in practice.

With a knowledge of the use of the hydraulic slide-rule the plotting of the head-capacity curves is a relatively easy and quick process. The adjustment of the curves of an intricate network becomes more difficult as the intricacy increases and, unquestionably, in the more difficult problems of this nature the modified Cross method is to be preferred. In general, such problems are not so important to the practising engineer as those pertaining to storage, pumping, and transmission, wherein the flow and head conditions are constantly varying. It is in such types of problems that the ability to allow for the varying conditions by the simple process of moving a curve indicates an advantage of the graphical method. As a knowledge of this method of solution increases, the picture which the curves portray by their shape immediately indicates the conditions which are, and those which are not, important.

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[26] *Journal*, New England Water Works Assoc., December, 1934.

A further example of the use of graphics in the solution of pumping problems is clearly presented by Mr. Palmer. The scope of the paper was limited to the presentation of the basic cases. The writer, however, has used the graphical method almost exclusively in pumping studies involving the centrifugal pump, since the head-capacity curve of such a pump is similar to that of a pipe line and a combination of the two is easily effected.

It is true, as pointed out by Mr. Knapp, that the presentation of the cases in the paper are more complicated than they need be in the actual use of the graphical method, but it appeared desirable to present them in a comprehensive manner. In actual use many short-cuts are possible to an experienced designer.

To combine pipe connected in series and pipe connected in parallel, or combinations of the two, it is unnecessary to draw curves for each piece of pipe and combine them graphically. In answer to Mr. Gavett, series-connected pipe of any number, without take-outs or put-ins, can be combined directly by making an addition of the head losses in each pipe determined for each flow from the hydraulic slide-rule and plotting the composite pipe directly. In parallel-connected pipe, the head loss is first assumed, the quantities each pipe will carry under the assumed head loss is added, and the composite curve drawn. Cases I and II of the paper were included only to illustrate the fundamental theory.

The use of the graphical method of solution of some of the distribution and transmission problems of water supply alone or in combination with the Cross method evidently permits more accurate and economical solutions of such problems than have hitherto been generally possible.

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AERATION TANKS FOR ACTIVATED SLUDGE PLANTS

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With Discussion by Messrs. Norval E. Anderson, Weston Gavett, and S. W. Freese

Synopsis

The design of aeration tanks in which diffused air is used for aeration, as differentiated from tanks with mechanical, surface aeration apparatus, is treated in this paper. An endeavor has been made to present the bases of theory and practice governing the design of the principal elements of diffused-air aeration tanks. The derivation of practicable bases of design has been made possible by the wealth of research, experimental, and plant-operation data which have been made available during the past few years particularly through the work of such agencies as the Sanitary District of Chicago.

Introduction

The results of experiments and plant-operation data offer rational bases for the design of the elements of diffused-air aeration tanks for the activated sludge process. Experiments run in parallel, with one element varying as between two experimental units, and plants that are adjusted to optimum operating conditions, are particularly good design guides.

The required capacity of the tanks depends upon quantity of sewage, aeration period, and percentage of return sludge. The results at plants and experimental units treating sewage of widely varying strengths indicate that the minimum aeration period and optimum percentage of return sludge vary with the strength of the sewage being treated. Parallel experiments with aeration tanks, 10 ft and 15 ft deep, indicate that between these two depths the economic depth of tank may be determined by balancing the cost of compressing air against fixed charges on tank cost. Experiments at Chicago, Ill., show that a ratio of tank width to tank depth of 2 to 1 may be used safely.

Note.—Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936, and published in October, 1937, *Proceedings*.

[1] Cons. Engr. (Hawley, Freese & Nichols), Fort Worth, Tex.

Data from a number of plants indicate that, except possibly in the case of some unusual trade wastes, the quantity of air required is approximately 0.0045 cu ft per gal of sewage per ppm of 5-day bio-chemical oxygen demand (B.O.D.) removed over-all by plants using air-diffusion aeration tanks of the spiral-flow type. The "spiral-flow" type of tank requires less air than the "ridge and furrow" type, and the combination of air diffusion with mechanical agitation of the paddle-wheel type offers a further possibility of material savings in both air and total power required. The data from a number of plants indicate that the quantity of air required for "complete treatment" is independent of the degree of preliminary sedimentation and that plants with no preliminary sedimentation require no more air, in parts per million of over-all bio-chemical oxygen demand removal, than plants with preliminary sedimentation.

In the matter of tank details the use of deflectors placed longitudinally at corners of the aeration channels is now standard practice. Recent experiences indicate that diffusers with permeabilities of from 35 to 45 may be used safely, and that the size of the air bubbles does not differ appreciably from those diffused from plates of lesser permeability. The plates of higher permeability do not clog as readily, and they offer less resistance to the flow of air than plates of lower permeability (15±) which have been used in the past. Cement grout for jointing diffusion plates into plate containers is still common practice. The development of corrugated rubber gaskets with clips for holding the plates in place offers a possibility in overcoming the objection to cement grout joints which prevent the removal of plates intact.

HISTORICAL

In his classic "Modern Methods of Sewage Disposal," published in 1894. Mr. George E. Waring, Jr., devoted a chapter to "Filtration with Aeration," in which he reviewed the work done prior to 1894 in the matter of the aeration of sewage. Mr. Waring concluded the chapter with the following prophetic statements:

> "Probably if a mass of sewage could be held in a state of continual effervescence, due to injected air, for a sufficiently long time, its organic matter would be as completely oxidized as it would be in the best-regulated filtration; as it is, indeed, in a rapid river, after a certain period of flow. It is also conceivable that a deep and narrow channel, with an air-supply at its bottom, would purify a slow stream flowing through it for a practicable distance. It seems likely, however, that air introduced under pressure would be much more economically used in the pores of a coarse filter than in a freely moving body of liquid, through which it would rise in bubbles so rapidly as to come less completely in contact with the suspended and dissolved matters.
>
> "This whole subject is still in a speculative stage, and it is introduced here chiefly with a view to suggesting further investigation in an interesting and promising field."

The later development of the activated sludge process of sewage treatment and more recent experiences in the design of aeration tanks have confirmed the wisdom of these early statements.

Early experiments with the aeration of sewage prior to the development of the activated sludge process of sewage treatment may be conveniently reviewed in the first chapter of "The Activated Sludge Process," (1927), by Mr. A. J. Martin. In 1914 and 1915 Messrs. Edward Ardern and William T. Lockett described, in a series of three papers read before the Manchester (England) Section of the Society of Chemical Industry, their experiments which are generally credited with having originated the activated sludge process of sewage treatment. Messrs. Ardern and Lockett gave credit to Mr. C. J. Fowler for his suggestions which led to, and were followed in, the making of the activated sludge experiments.

The process was patented by Jones and Attwood, Limited, Stourbridge, England, in a series of patents taken out in 1914 and 1915 in England and later perfected in the United States. Messrs. Jones and Attwood designed the first activated sludge plant of any size (approximately 900 000 U. S. gal per day) in 1916, at Worcester, England, part of an existing sedimentation tank being converted into aeration and sludge settling tanks. The aeration tank consisted of a battery of five channels, 8 ft wide by 80 ft long, 17 to 18 ft deep (water depth), and with three vertical under-flow baffles in each channel. Air diffusion was of the transverse ridge and furrow type with rows of diffusers at 5-ft centers in the first channel and at 10-ft centers in the remaining channels. The ratio of air diffuser area to tank area was approximately 1 to 10. The sewage passed through the channels in a modified "endless channel" type of continuous flow, at the end of each channel a portion of the flow being carried back by induced circulation into the preceding channel.

Laboratory experiments at Manchester were followed by the installation of an activated sludge sewage treatment plant at the Withington Sewage Works of the Manchester Corporation, this plant being placed in operation in September, 1917. The Withington aeration tank was constructed by converting a part of an existing sedimentation tank, approximately 20 ft by 100 ft, into a series of five channels, 4 ft wide and 100 ft long, with 5.5-ft water depth over diffusers. The tank was of the series, "once through," continuous flow type. The floor was laid out in transverse ridges and furrows, the slopes of the ridges being at 35° with the horizontal, and the air being introduced through rows of diffusers placed in the furrows, on 6.75-ft centers. The diffuser area amounted to one-seventh of the aeration tank area or the water surface area. This plant successfully treated 440 000 gal (U. S.) of domestic sewage per day. At another Manchester Corporation plant, the Davyhulme Sewage Works, the "spiral-flow" type of aeration tank was first used, the plant being put in operation in 1921. The aeration channels at Davyhulme are 8 ft wide by 9 ft deep (water depth) with diffusers along one side of the channel at the base of the curved footings of division walls; the ratio of diffuser plates to tank area is approximately 1 to 18. The plant has a rated capacity of 1 200 000 gal (U. S.) per day.

During 1915 a number of experiments with the activated sludge process were initiated in the United States, notably at Champaign, Ill., Lawrence, Mass., Baltimore, Md., Milwaukee, Wis., Brooklyn, N. Y., Chicago, Ill., Houston, Tex., and Cleveland, Ohio.[2] Based upon the experimental work at Houston and Milwaukee, Mr. E. E. Sands designed two large activated sludge plants for Houston. The North Side Plant at Houston, with a nominal capacity of 10 000 000 gal per day, was placed in operation in May, 1917, and the South Side Plant, with a nominal capacity of 5 000 000 gal per day, was placed in operation in August, 1918. These two plants were the largest activated sludge plants in operation until 1925 when the Milwaukee and Indianapolis plants were placed in operation.

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[2] *Proceedings*, 1916 Convention, Am. Soc. of Municipal Improvements.

The Houston aeration tanks consist of four units at the North Side Plant and two units at the South Side Plant. Each aeration unit consists of two channels, one 18 ft wide and the other 9 ft wide, of a length of 280 ft and a water depth over diffuser plates of 9.75 ft. The bottoms of tanks are of the sawtooth ridge and furrow type with rows of plates at 5-ft centers, the individual plates being spaced so as to give a ratio of diffuser-plate area to tank area of 1 to 7. For several years the plants were operated as the continuous flow, "endless channel" type, with a ratio of incoming raw sewage to circulating mixed liquor of from one-third to two-thirds; however, the North Side Plant has since been enlarged and remodeled as a "once through" type with provision for the reaeration of return sludge.

The successful operation of the pioneer activated sludge sewage treatment plants, such as those at Manchester, Houston, and Worcester, along with the extensive experimental work at Chicago, Milwaukee, and Indianapolis, and elsewhere, not only firmly established the activated sludge process, but also gave a fairly rational basis for the design of aeration tanks and the other principal features of activated sludge plants.

Aeration Tank Capacity

The first step in the design of aeration tanks is to determine the required capacity. This capacity depends upon three factors: (1) The quantity of sewage to be treated; (2) the percentage of return activated sludge; and (3) the aeration period or time of detention of the mixed sewage and activated sludge liquor in the aeration tanks. Design data for aeration tanks have necessarily been related to the average daily sewage flow, and the writer has used that basis in considering the various factors involved. The average daily sewage flow is a practical basis of design for works treating the average domestic-industrial sewage collected in separate sewer systems. The ratio of maximum hourly flows to daily averages may vary considerably without disturbing the activated sludge process, due to the capability of the sludge to store "biological energy."

Other factors being constant, the length of the aeration period depends primarily upon the quality of the sewage to be treated and the degree of treatment required. Fig. 1 is a typical curve showing the relationship between the aeration period and the degree of purification, the latter being measured after clarification of the mixed liquor. For a particular sewage this curve will vary with the percentage of return activated sludge, mixed with the sewage, the character and biological activity of the sludge, the kind and intensity of aeration, the time and type of clarification, temperature, and other factors of less importance.

During the first stage of the aeration period the impurities in the sewage are rapidly coagulated and adsorbed by the activated sludge, and the rate of coagulation adsorption during this first stage is proportional to the percentage of biologically active sludge in the mixed liquor.[3] After the initial coagulation adsorption has taken place, the purification of sewage, measured in terms of bio-chemical oxygen demand, proceeds at the much slower and fairly uniform rate of oxygenation of the mixed liquor. During this second stage the presence of a large proportion of sludge probably retards the oxygenation of the sewage proper because of the relatively greater oxygen requirements of the sludge.[4] The oxygenation of the mixed liquor during both stages, and of the sewage during the second stage, no doubt proceeds in accordance with the law expressed by E. B. Phelps: "The rate of biochemical oxidation of organic matter is proportional to the remaining concentration of unoxidized substance, measured in terms of oxidizability." [5]

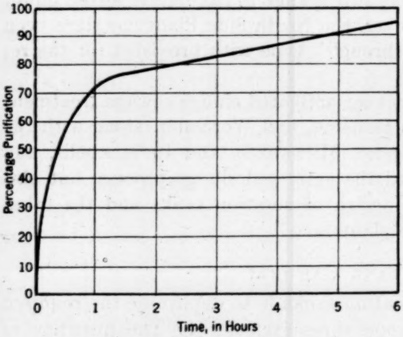

Fig. 1.—Relationship Between Aeration Period and Degree of Purification

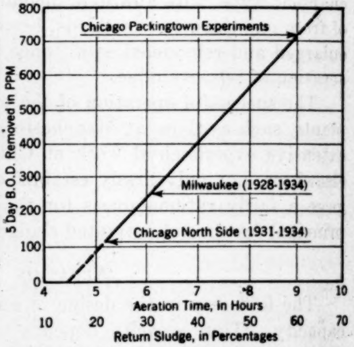

Fig. 2.—Relation Between 5-Day B.O.D. Removal and (1) Aeration Period, or (2) Percentage of Return Sludge

For "complete treatment" the minimum aeration period varies with the strength of the sewage. To show this relationship the writer has plotted on Fig. 2 the aeration period against the 5-day B.O.D. removed for sewages of widely differing strengths—that is, the weak North Side Chicago sewage, the comparatively strong Milwaukee sewage, and the very strong Chicago Packingtown sewage. Both the Chicago North Side and the Milwaukee plants were loaded to capacity and were operated so as to obtain the optimum results during the years included in the study. The Chicago Packingtown activated sludge experiments were conducted with the idea of obtaining optimum plant operation characteristics. In each case a high degree of purification was accomplished. The sludge is not re-aerated at Chicago and Milwaukee, and the data used from the Chicago Packingtown experiments cover only those in which the sludge was not re-aerated.

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[3] "Observations on Biological and Physical Properties of Activated Sludge and the Principles of Its Application," by F. W. Harris, T. Cockburn, and T. Anderson, *Proceedings*, The Assoc. of Managers of Sewage Disposal Works, 1926, p. 52 *et seq.*

[4] "The Oxygen Requirements of the Activated Sludge Process," by S. Grant, E. Hurwitz, and F. W. Mohlman, *Sewage Works Journal*, Vol. II, No. 2 (April, 1930), p. 228 *et seq.*

[5] "The Disinfection of Sewage and Sewage Filter Effluents," by E. B. Phelps, *Water Supply Paper 229*, U. S. Geological Survey, 1909, pp. 74–78.

The aeration-period curve shown in Fig. 2 is plotted from the data given in Table 1(*a*). In Fig. 2 the point plotted for the Chicago North Side Plant is a practicable minimum of aeration period for a weak sewage. The aeration period plotted for Milwaukee (6.3 hr) is the average total aeration time, of which 5.44 hr constitutes the time in the aeration tanks proper; however, a material part of the difference is accounted for by the time of detention of the mixed liquor in the mixing channels. In the large scale experiments conducted by the Sanitary District of Chicago with Packingtown sewage in 1916 and 1917, a period of 9 hr was found to be a practicable minimum aeration time without re-aeration of the sludge.

TABLE 1.—CHARACTERISTICS OF "COMPLETE TREATMENT" OF SEWAGES OF DIFFERENT STRENGTHS

Plant	Period	(*a*) AERATION PERIODS REQUIRED					(*b*) OPTIMUM PERCENTAGES OF RETURN SLUDGE		
		Five-Day Bio-Chemical Oxygen Demand				Aeration period, in hours	Return sludge (percentage of incoming sewage)	Solids in return sludge (percentage)	Volatile matter in sludge solids (percentage)
		Raw sewage, in parts per million	Effluent, in parts per million	Plant removal, in parts per million	Plant removal (percentages)				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Chicago, Ill., North Side	1931–1934	115.8	11.3	104.5	90.3	5.2	21.7	1.39	67.7
Milwaukee, Wis.	1928–1934	270.1	13.1	257.0	95.2	6.3	32.5	1.19	76.6¶
Chicago, Ill., Packingtown experiment*	1916–1917†	752.0‡	32.0§	720.0	96.0	9.0	60.0	0.85‖	62.6**

* "Report on Industrial Wastes from the Stockyards and Packingtown in Chicago" (Vol. II, 1921); The Sanitary District of Chicago. † 15 runs. ‡ 10-day B.O.D., 990 ppm. § 10-day B.O.D., 42 ppm. ‖ Minimum, 0.4, and maximum, 1.3. ¶ Average of 1933 and 1934. ** Minimum, 55, and maximum, 73.

The second important determination to be made in arriving at the aeration tank capacity is the percentage of return activated sludge. In the foregoing discussion of minimum aeration periods and, particularly in arriving at the aeration period curve shown in Fig. 2, it was assumed that the optimum percentage of sludge was being returned. The return of a large percentage of biologically active sludge will provide a quick coagulation adsorption of the impurities in the sewage, the time being inversely proportional to the percentage of sludge. However, after the initial coagulation adsorption has taken place, the larger percentages of return sludge seem to "rob" the sewage as the further oxygenation proceeds. Too small a percentage of return sludge (that is, less than that required to maintain biological activity) will require re-aeration of the sludge.

Without re-aeration of the sludge it appears, therefore, that the theoretical optimum percentage of return sludge is that percentage which will permit the required purification in a minimum of time and, at the same time, maintain an equilibrium of biological activity in the sludge. In general, the minimum percentage of biologically active return sludge will mean the minimum time for "complete" purification of the sewage. On the other hand, if this percentage of return sludge drops too low, the sewage will not remain biologically active.

As was the case with the aeration period and in the absence of experiments with the particular sewage, the best guide as to the optimum percentage of return sludge is the experience of plant operators, or of experimentation in plants where the percentage giving the best results has been determined.

As would be expected from theoretical considerations, the optimum percentage of return sludge found in the practical operation of activated sludge plants varies with the strength of the sewage in much the same manner as the aeration period. The optimum percentage of return sludge has been plotted against the 5-day B.O.D. removed for the same three sewages of widely differing strengths for which the aeration periods were plotted against 5-day B.O.D. removed. The data are such that the two curves may be plotted as an identical curve, as has been done in Fig. 2. The curve showing percentage of return sludge, in Fig. 2, is plotted from data in Table 1(*b*).

Although the curve for percentage of return sludge, shown in Fig. 2, does not take into consideration the percentage of solids in the activated return sludge, the determination of the optimum percentage of return sludge is usually related to the parts per million of solids maintained in the mixed sewage and activated sludge liquor.

In referring to the operation of the North Side Plant, in Chicago, treating a weak sewage with a 5-day B.O.D. of 116 ppm, Langdon Pearse, M. Am. Soc. C. E., states that it has been found practical to operate the plant with a smaller proportion of solids in the mixed liquor than the 3 000 ppm formerly assumed, and that by operating with approximately 2 000 ppm more flexibility is secured, with opportunity to store the increase of solids produced by storm flows. Mr. Pearse also found that some saving in air resulted and that it was practicable to cut the percentage of return sludge in volume, below 20.[6]

At the Milwaukee Plant with a sewage having a 5-day B.O.D. of 270 ppm, D. W. Townsend, M. Am. Soc. C. E., reports that it has been the custom in operating this plant to maintain a mixed liquor suspended solids content of approximately 3 000 ppm (0.3 of 1%), this solids content within reasonable working limits having been found to be about optimum for the general conditions encountered.[7]

The percentage of volatile matter in the sludge also affects the characteristics of the activated return sludge at least in so far as the rate of absorption of oxygen by the sludge is concerned.[4] However, few data are available to the writer as to how this affects the optimum percentage of return sludge.

The foregoing comments have been confined to the minimum practicable aeration period and optimum sludge return required for substantially complete treatment, as differentiated from partial treatment, and to the minimum time and optimum sludge return required without re-aeration of the sludge. Partial treatment and re-aeration of the sludge "go hand in hand." Re-aeration or re-activation of the sludge is required when the aeration period of the mixed liquor is insufficient to maintain either the biological activity or the oxygen requirements of the sludge, or both. With partial treatment, however, the demands on the sludge are so much greater and the required aeration period so much smaller that re-aeration of the sludge is required. Re-aeration is also required with either complete or partial treatment if the aeration period is kept at a minimum and the percentage of return sludge is too small, re-aeration being required because of the relatively heavy demands made on the sludge.

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[6] "Comparison of Original Design Basis for North Side Treatment Works with Present Accepted Practice," by Langdon Pearse, Report to Public Works Administration, Board of Review, February 19, 1934.

[7] "Five Years Operation of Milwaukee Treatment Plant," by Darwin W. Townsend, *Proceedings*, Am. Soc. of Municipal Improvements, Vol. 39 (1933–1934), p. 189.

Even with complete treatment of the sewage, re-aeration of the sludge may be economical in many cases, particularly in the case of very strong sewages or industrial wastes.

The Chicago Packingtown sewage required 9 hr of aeration without re-aeration of the sludge. With re-aeration the average aeration period in the mixed liquor tanks was 5.3 hr with 3.8 hr aeration in the re-aeration tanks, the weighted average being equivalent to a 6.6-hr period as compared to the 9-hr period without re-aeration. The settling period however had to be increased from 1.3 hr without re-aeration to 1.7 hr with re-aeration. The re-aeration experiments indicated a net saving of 20% of tank volume (aeration tanks and settling tanks).

Evidence as to the economy in tank volume required in the treatment of sewage of average strength, because of re-aeration of the sludge, is not so clear. Unless it is possible to conduct large-scale experiments with the particular sewage being studied, the safe procedure is to design aeration tanks on the basis of the minimum volume required without re-aeration, in line with the curve shown in Fig. 2, but to provide sufficient flexibility in the number of aeration tanks or channels such that varying periods of re-aeration may be tried. Return sludge connections can usually be provided so as to use several of the channels as re-aeration channels at a relatively small additional cost.

DEPTH OF TANK

The use of a depth of 15 ft or, in some cases, a few inches less, for aeration tanks is almost universal in the large activated sludge plants in the United States. A number of diffused air plants in Great Britain use depths from 5 ft to 10 ft; however, the recently built and comparatively large plant at Manchester (16 mgd) uses a 15-ft depth. The plants at Houston have a depth over the diffuser plates of 9.75 ft. There are also a number of smaller plants in the United States using diffuser tubes at relatively shallow depths (5 ft, and less), but with tanks as deep as 15 ft.

In England the use of depths shallower than 10 ft can be largely attributed to the fact that many of the activated sludge plants were converted from existing works of other types of treatment where the existing tanks were of the shallower depths. Where the existing tanks were deeper, as at Worcester, England (17 to 18 ft), and at Reading, England (23.5 ft), the remodeled aeration tanks are of the same depth.

The prevalence of the 15-ft depth in the United States can be attributed largely to experience at Milwaukee and Chicago, where large-scale experiments were conducted with aeration tanks 10 ft deep, and 15 ft deep, run in parallel. The 15-ft depth was adopted at both Milwaukee and Chicago for the large activated sludge plants. At Milwaukee, the tests of the tanks 10 ft and 15 ft deep were discussed by the late Harrison P. Eddy, Past-President, Am. Soc. C. E., as follows:[8]

"The comparative efficiency of treatment in tanks 10 and 15 ft. in depth, was made the subject of careful and prolonged study at the Milwaukee Testing Station. Tests carried out with the greatest care, under the personal direction of William R. Copeland, Affiliate, Am. Soc. C. E., Chief Chemist, showed that, taking all the facts into consideration, it appeared that, for the same quantity of free air per gallon of sewage treated, there was little difference in the work accomplished by the 10-ft and the 15-ft tanks. The 15-ft tanks have produced an effluent of somewhat greater stability, whereas the 10-ft tanks have accomplished a slightly greater reduction in suspended matter and bacteria. If one depth of tank has an advantage over the other in purification efficiency, it seems to be the 15-ft. depth.

"An important consideration in the selection of depth of tank is the power required for compressing the air sufficiently to overcome the hydrostatic pressure of the several depths of sewage under consideration. In the cases of the 10 and 15-ft. depths, the latter would require 50% more power than the former, disregarding the effect of friction in pipe lines and porous plates. This saving will generally be offset, partly at least, by the increased cost of the larger area of tanks of the lesser depth. At Milwaukee, this is important because of the cost of the bulkhead, the sheet steel enclosure, and the pile foundation. Another important factor there is the limited area available for the treatment plant, as it is felt that additional area could not be devoted to this purpose, on account of the requirements for harbor development."

At Chicago, the sewage treatment works on the Des Plaines River were designed for experimental purposes with three aeration tanks of a depth of 10 ft and one of a depth of 15 ft. The results of the experiments as between the two depths, using the same volume of air per gallon of sewage, were summed up by Mr. S. L. Tolman, of the Sanitary District of Chicago, as follows:[9] "The operating data do not indicate any great difference in purification between these two depths."

Aside from the economy of construction cost of deep tanks over shallow tanks, another consideration is as to whether or not any appreciable oxygenation is accomplished by the air bubbles as they rise through the greater depth of liquor in the deeper tanks. As indicated by the Milwaukee and Chicago experiments this additional oxygenation in the deeper tanks is not material. This is probably due to two reasons: First, the bubbles rising through the liquid are enveloped in a surface tension film, including some colloidal matter (which film interferes with the passage of oxygen into the liquid at least until the bubble breaks the surface of the liquid[10]); and, second, the oxygen requirement of the sewage amounts to only a small percentage (not more than 10) of the oxygen in the air required for mixing and for maintaining aerobic conditions.

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[8] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 864.

[9] "The Operation of the Des Plaines River Sewage Treatment Works and Small Plants of the Sanitary District of Chicago," *Journal*, Western Soc. of Engrs., Vol. XXXI, No. 7 (July, 1926).

[10] *Proceedings*, Royal Dublin Soc., 1920; *Journal*, Royal Sanitary Inst., October, 1926; and *Minutes of Proceedings*, Inst. C. E., Vol. CCXVII, pp. 136, 137, and 191.

In a series of experiments conducted by the writer in 1925, nitrogen and air (also oxygen and carbon dioxide) were used in parallel for agitation of the mixed liquor, the raw sewage, the percentage and quality of activated sludge, the volume of gas, the aeration period, the depth of liquor, and the clarification period being identical for each gas. The air and the inert gas—nitrogen—gave practically the same results as may be judged from Table 2, which gives the results of the experiments.[11] The results of these experiments indicate that a large part of the oxygenation occurs as the bubbles break the surface of the mixed liquor.

TABLE 2.—COMPARISON OF NITROGEN AND AIR BUBBLE AGITATION

Description	FIVE-DAY BIO-CHEMICAL OXYGEN DEMAND IN PARTS PER MILLION		
	Medium strength sewage, Run No. 5	Strong sewage, Run No. 8	Average, Runs Nos. 1 to 8
Raw sewage	180	600	365
Effluent, using air agitation	16	40	24
Effluent, using nitrogen agitation	20	36	24

The foregoing considerations, particularly the results of the experiments at Milwaukee and Chicago, indicate that within practicable limits the economic depth of tank can be determined by balancing the greater cost of compressing air for the deeper tank against the lesser fixed charges for the aeration tanks of the greater depth. This problem may be readily analyzed by plotting the various costs against the corresponding depths.

The economic limit of depth is probably 15 ft to 16 ft, except in very extreme cases where the cost of site or of foundations (or of both) is very high. At Milwaukee, for example, where a 15-ft depth was used, the aeration tanks were built on "made" land and required bearing piles to support them. A number of British plants operate quite economically with depths of tank from 5 ft to 10 ft. Factors that would tend to increase the depth are, as follows: (*a*) Comparatively weak sewage requiring a small amount of air; (*b*) cheap power; (*c*) high construction costs; and (*d*) costly land for site. The City of Chicago is a typical example of a case in which these factors prevail. As a corollary, factors that would tend to decrease the depth are, as follows: (1) Comparatively strong sewage; (2) costly power; (3) low construction costs; and (4) inexpensive land.

In an attempt to gain the advantages of both the deep tank and the shallow tank a number of plants have been built in which the air diffusers, usually diffusion tubes, were placed above the mid-depth point in the tank, in some cases quite near the surface. In an aeration tank of this type it is essential that sufficient motion be imparted to the liquor to mix the sludge with the sewage and to prevent deposition of solids, as well as to maintain aerobic conditions in the mixed liquor.

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[11] For a description of the experiments and analyses of oxygen consumed, dissolved oxygen, free ammonia, nitrates, and nitrites, as well as data covering other runs, see *Proceedings*, Texas Section, Am. Soc. C. E., Fall Meeting, 1925, pp. 13–36.

WIDTH OF TANK

Until quite recently, the ratio of width to depth of aeration tanks or channels has been maintained at less than 1.5, the British plants maintaining a ratio of approximately 1.0 and less. This was true of both the ridge and furrow and of the circulating or spiral-flow types of tanks. Inasmuch as the circulating or spiral-flow type of tank has largely superseded the ridge and furrow type due to the lesser air requirements of the circulating or spiral-flow type (as will be mentioned subsequently), this paper is confined to the latter type.

The new Cleveland Easterly Plant has a ratio of 1.8 (27-ft width by 15-ft water depth)[12] and the new Chicago Southwest Plant has a ratio of 2.1 (32.75-ft width by 15.5-ft water depth).[13]

The most careful observations available to the writer as to how wide the tank can be and still maintain good mixing and aeration are those made by the Sanitary District of Chicago at the North Side Plant. The observations were reported[6] as follows:

> "In the plant design, the batteries were divided up into 12 water-tight tanks, each of which was divided by a longitudinal baffle into 2 channels, 420 feet long and 16 ft. 1¾ inches average width. These channels were equipped with 2 rows of diffuser plates, running longitudinally. However, in the westerly tank, the dividing wall was omitted, giving a separate tank 33 feet 3½ inches average width. Experiments were carried out with channels 25 feet and 33 feet 3½ inches width to determine the efficiency of aeration with three longitudinal rows of diffuser plates.
>
> "Tests have shown that the effluent was equally as good with these widths as with the 16 ft. 1¾ inch width tank and no more air was required per gallon of sewage treated."

Based upon the operating data of the North Side Plant (0.4 cu ft of air per gal, 25% return sludge, and 4-hr aeration period), the use of air per foot of length for this particular channel may be assumed as between 4 and 5 cu ft per min. Inasmuch as the use of air at the North Side Chicago Plant, per gallon of mixed liquor per unit of time, is comparatively low, the foregoing observations indicate that a ratio of width to depth of as much as 2 is safe for a spiral-flow tank with diffusion plates along one side, unless the sewage is so weak as to require even less air than that at the Chicago North Side Plant.

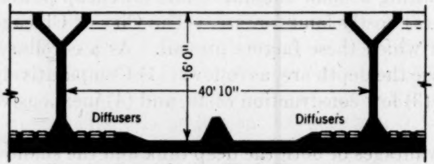

FIG. 3.—SAN ANTONIO (TEX.) EXPERIMENTAL AERATION TANK

At the treatment plant in San Antonio, Tex., the dividing wall was omitted between two aeration channels in order to observe the aeration in the wider channel. This channel has a ratio of width (40 ft 10 in.) to water depth over diffusion plates (14 ft 7.75 in.) of 2.8. The channel has a length of 150 ft, with three rows of diffuser plates longitudinally along each side of the channel and with an inverted V-shaped baffle, 3 ft high, longitudinally down the center of the channel. Fig. 3 shows a cross-section of the wide channel at San Antonio.

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[12] *Engineering News-Record*, Vol. 107, p. 1035.

[13] From Contract Plans, Southwest Sewage Treatment Works, The Sanitary District of Chicago.

During 1934 the use of air per foot of length of the channel averaged 8.5 cu ft per min, or 4.25 cu ft per min per ft of length of diffusion plate system at each side of the channel (three longitudinal rows of plates in each diffusion plate system).

The liquor surface in the wide experimental channel at San Antonio reveals no well-defined dividing line along the center of the tank between the two spiral flows. The spiral flows creep from one side to the other of the center line of the tank; this creeping varies constantly and it is not known whether this is helpful in the mixing-aeration process, or whether it tends to create dead spots. H. A. Hunter, Assoc. M. Am. Soc. C. E., and W. S. Stanley, Chief Chemist and Superintendent, Sewage Treatment Plant, San Antonio, Tex., have made tests recently with the wide experimental tank as compared with the 20-ft tanks with diffuser plates on one side only. The effluent from the wide unit showed a smaller 5-day B.O.D. (3.3 ppm, as compared with 9.3 ppm); however, the flow through the wide tank had been throttled down so much that there was a considerably longer detention time in the wide unit than in the 20-ft unit used in the comparison. Further tests will be required to determine whether or not the results obtained in the experimental double unit are the equal of those obtained in the single units.

In a letter dated January 6, 1936, L. C. Whittemore, M. Am. Soc. C. E., states that an experimental aeration tank similar to the one at San Antonio, except that it is 68 ft wide, has just been placed in operation at the Chicago Calumet Plant. As at San Antonio, the diffuser plates are placed along both sides of the channel. He states further:

> "While it is, of course, too early to obtain any quantitative results or form any opinions as to the success of this arrangement, I saw the tank in operation the day sewage was put in it. The dividing line where the two currents from the sides of the tank was rather wavy and irregular, but there appeared to be good velocities maintained across the top of the tank toward the center."

Not only will the quantitative results at Calumet and more comparable results at San Antonio from the operation of the wide experimental units at these plants be instructive, but it will also be interesting to know the results obtained by placing the air diffusers along the center line of the tanks as compared with placing them on each side of the tank.

These experiments indicate the possibility of eliminating baffle-walls, or at least of cutting them down to small ridges midway between rows of plates, by properly placing the longitudinal rows of systems of plates. If the spiral-flow systems can be maintained satisfactorily by the energy of the air alone, the only limiting factors as to width would be a ratio of length to width sufficient to avoid short circuiting through the tank and a sufficient number of tanks to provide flexibility of operation.

Until such experimental works prove otherwise, existing data indicate a limit of the ratio of width to depth of approximately 2 to 1 for channels with diffusion plates on one side. The ratios should be modified to some extent depending upon the quantity of air necessary in the particular case.

QUANTITY OF AIR

The introduction of air in an air diffusion type of activated sludge plant serves two purposes: First, it mixes the sewage with the coagulating-adsorbing activated sludge and maintains the contact between the particles of activated sludge and the impurities in the sewage; and, second, it maintains aerobic conditions for the oxygenation of the mixed sewage-sludge liquor, or the sludge (or both), such that the sludge, or at least the return sludge, will be in a state of equilibrium in so far as its biological activity is concerned. The meeting of the second condition (the introduction of sufficient air to maintain the activity of the sludge) will more than satisfy the first condition as to the mixing of the sludge with the sewage, except possibly in the case of partial treatment plants requiring re-aeration of the sludge.

The air required to maintain aerobic conditions in the mixed liquor, or sludge (or both), for the required aeration period is approximately ten times that required to meet the bio-chemical oxygen demand of the sewage in an efficiently operated air-diffusion plant. Even then, a large part of the oxygen demand is no doubt met by oxygen from the atmosphere rather than from the diffused air introduced into the sewage, the atmospheric oxygenation taking place as air bubbles break the surface of the liquid. The quantity of air necessary to maintain sufficiently aerobic conditions for the oxygenation of the sewage or sludge (or both) is directly proportional, within a fairly narrow range, to the bio-chemical oxygen demand removed in an efficiently operated air-diffusion plant.

TABLE 3.—AIR REQUIRED PER PARTS PER MILLION OF FIVE-DAY BIO-CHEMICAL OXYGEN DEMAND REMOVED

Plant	Length of record, in months	FIVE-DAY BIO-CHEMICAL OXYGEN DEMAND, IN PARTS PER MILLION			QUANTITY OF AIR	
		Raw sewage	Plant effluent	Over-all plant removal	In cubic feet per gallon of sewage	In cubic feet per gallon per parts per million. 5-day B.O.D. removed
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Chicago North Side (26 min preliminary sedimentation)	18	112	9	103	0.46	0.0045
Chicago North Side (preliminary sedimentation for 5.4 months only)	42	116	13	103	0.39	0.0038
Milwaukee (fine screens)	84	270	17	253	1.15	0.0045
San Antonio (41-min preliminary sedimentation)	61	196	15	181	0.97	0.0054
Indianapolis (fine screens)	96	287	35	252	1.13	0.0045
Chicago Packingtown	15 runs	752	32	720	3.06	0.0042

Table 3 shows the quantity of air required for a number of plants per gallon of sewage per part per million of bio-chemical oxygen demand removed. Table 3 also shows the nature of the preliminary treatment. Referring to Column (6), Table 3, the quantity of air used by the Milwaukee "ridge and furrow" type of aeration tanks (1.53 cu ft per gal) has been reduced by 25% (1.15) to obtain the comparison with the "spiral-flow" tanks used in the table. One-half the aeration tanks used in the Chicago Packingtown experiments were of the "ridge and furrow" type and air used in the experiments (3.5 cu ft per gal) has been reduced 12.5% to obtain (3.06) an approximation of the corresponding "spiral-flow" figure.

During the last twenty-four months of the Indianapolis record the plant was operated largely as a straight aeration plant rather than as an activated sludge plant. During this period (1933 and 1934) the plant reduction in parts per million of 5-day B.O.D. was from 256 to 94, making an over-all plant removal of 162 ppm, with a use of air of 0.64 cu ft per gal of sewage treated.

The quantity of free air required per part of bio-chemical oxygen demand removed over-all by a "complete treatment" air-diffusion plant seems to be largely unaffected by, and independent of, the aeration period, the percentage of return sludge, re-aeration of sludge, preliminary treatment by screens, or settling, or by depth of aeration tank. This assumes efficient plant operation and sufficient final clarification. The arrangement of the diffuser plates at the bottom of the aeration tanks, however, has a decided effect on the air required to maintain aerobic conditions in the mixed liquor. It has been demonstrated at a number of plants, particularly in experimental and full-scale units at Manchester, Chicago, and Indianapolis, that the "spiral-flow" type of tanks with the diffuser plates longitudinally along one side of the channels requires much less air to accomplish the same results than does the "ridge and furrow" type of aeration tank with the rows of diffuser plates cross-wise of the channels. The air saved by the "spiral-flow" type of tank over the "ridge and furrow" type approximates 25 per cent.[9] [14]

It is to be noted from Table 3 that the omission of preliminary treatment does not adversely affect the quantity of air required per part per million of 5-day B.O.D. removed by the plant as a whole. This is due no doubt to the fact that any solids removed by the preliminary treatment would settle out in any case in the final clarifiers and to the further fact that those solids are of such a nature that they will absorb a negligible quantity only, of oxygen as they pass through the aeration tanks.

The fact that Table 3 shows such uniform results with respect to the quantity of air required per part per million of 5-day B.O.D. removed by the over-all plant, in spite of the fact that a large percentage (30 to 40% in some cases) of the 5-day B.O.D. had been removed in the preliminary settling tanks in some of the plants, is attributable to the fact that the ratio to 5-day B.O.D. of suspended solids (and 5-day B.O.D. associated with those solids) is approximately the same for each of the plants for which comparable data were available. It is questionable whether the air consumption per part per million of bio-chemical oxygen demand removed, indicated in Table 3, would hold good for a strong industrial waste with a relatively small volume of settleable solids.

It is desirable to remove the grit from the sewage so that it will not settle in the aeration tanks or require too great a quantity of air to keep it in circulation. It has also been considered advisable from the standpoint of sludge disposal to settle out a considerable part of the solids in the preliminary tanks so as to obtain the higher concentration of solids (approximately 5%, or more) obtainable in the preliminary tanks as compared with the concentration obtainable in the final clarifiers (1% ±). However, there is some question as to whether the over-all concentration of solids from both the preliminary and the final tanks is any greater with the use of preliminary settling tanks. In a report to the Board of Review, Public Works Administration, for the West-Southwest Plant, Mr. Pearse stated,[6] as follows, with reference to preliminary settling tanks:

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[14] "Sewage Aeration by Diffused Air," by Frank C. Roe, *Sewage Works Journal*, Vol. V, No. 5 (September, 1933).

> "In the original design, a preliminary settling period of 30 minutes on 175 m.g.d. was thought advantageous to reduce the settling solids and cut the use of air, based on experience at Calumet. Operating results for 18 months have indicated, however, on the North Side Plant, that the preliminary tanks are not necessary from the sewage treatment standpoint, as there is practically no difference in the amount of air used. Further, there has been no need of their use as skimming tanks for floating material. However, in the dewatering of the sludge the mixture of some fresh sludge with the activated sludge appears to aid in obtaining a higher rate of solid cake per square foot from the vacuum filters."

From the data available, therefore, it seems safe in the design of aeration tanks for "complete treatment" plants (without paddle-wheels or other mechanical aids to aeration), treating sewage without any unusual trade wastes, to provide 0.005 cu ft of free air per gal of sewage (average flow) per ppm of 5-day B.O.D. to be removed from the raw sewage by the over-all plant. Sufficient flexibility should be provided in the number and capacity of air compressors to permit a reduction to 0.004 cu ft (or even lower) of free air, should the actual operation of the plant permit of this minimum usage of air.

Considered entirely as an agitation device, a modern air-diffusion type of plant has an efficiency from "wire to sewage" of between 60% and 70%, depending to some extent upon the size and type of air compressors used. In other words, the work done on the sewage after taking care of all motor, compressor, air-piping, and diffuser-plate losses, approximates 60% to 70% of the energy delivered to the motor in a modern plant. (See operating data for Chicago North Side Plant and San Antonio.) Although these efficiencies were computed for plants using electrical power, plants using turbine-driven compressors show approximately the same efficiencies for the corresponding operations.

However, the function of an aeration tank is not agitation, in itself, but the maintenance of contact between the activated sludge and the sewage, under aerobic conditions. One form of agitation with air diffusion may be much more effective than another form, as is shown by the improvement of the "spiral-flow" form of aeration over the "ridge and furrow" type. There is promise of further improvement by utilizing a combination of air-diffusion agitation with mechanical agitation, such as the combination being utilized by Karl Imhoff, M. Am. Soc. C. E., at the sewage treatment plant in Essen-Rellinghausen, Germany, in which a submerged paddle mechanism revolves on a horizontal shaft running the length of the aeration channel in a direction opposite to the motion imparted by the air-diffuser plates located longitudinally along the side of the channel as shown in Fig. 4. Dr. Imhoff reports that with diffused air from 0.8 to 1.12 cu ft per gal and 24 hp per mgd were needed, but that with paddles in operation 0.16 cu ft of air per gal and 8.4 hp per mgd were found to suffice.[15] Mr. F. W. Mohlman and Charles E. Wheeler, Jr., Assoc. M. Am. Soc. C. E., of the Sanitary District of Chicago, reported similar results obtained on large-scale experiments (1 500 000 gal per day) with combination air-diffusion and paddle-wheel agitation. They found that it would require 0.80 cu ft of air per gal, with air alone, and a 5-hr aeration period to produce an effluent comparable with that produced by the paddle-wheel plus air, with 0.20 cu ft of air per gal of sewage and a 5-hr period. The paddles required 6.7 hp and the air 6.4 hp, giving a total of 13.1 hp per million gallons. With air alone at 0.8 cu ft per gal, and a 5-hr period, the total requirement would be 26 hp per million gallons.[16]

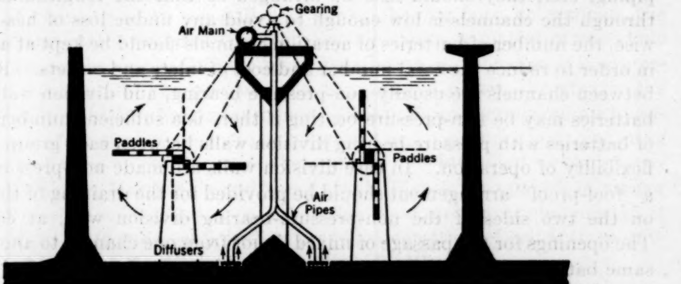

FIG. 4.—CROSS-SECTION OF AERATION TANK EQUIPPED WITH AIR DIFFUSERS AND PADDLE-WHEELS

AERATION TANK DETAILS

General Layout.—The laying out of the aeration tanks is entirely a matter of the most economical arrangement for the local conditions encountered. Plants of the air-diffusion type can be divided in general into two classes: (1) Those with an even number of channels in each aeration tank battery, arranged so that the inlet and outlet are at the same end of each tank battery of channels; and (2) those with an odd number of channels in each battery, arranged so that the inlet and outlet are at opposite ends of each battery. Conditions, any one of which may favor Class (2), are: (*a*) Incoming sewage main entering one side of plant site, with the effluent outfall out of plant site necessarily on opposite side; (*b*) re-aeration of return sludge, thus allowing return to the opposite side of the aeration tank battery through aeration channels; and (*c*) sludge return by pumping.

Conditions, any one of which may favor Class (1), are: (*d*) Plant size requiring "pipe gallery"; (*e*) incoming sewage main and effluent outfall entering and leaving the side of the plant site; and (*f*) sludge return by air lift.

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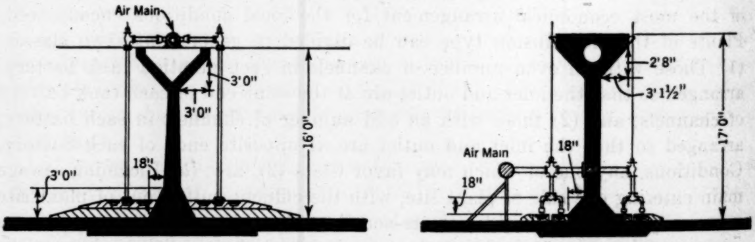
[15] "The Activated Sludge Process," by A. J. Martin, Macdonald and Evans, Lond., 1927, p. 252.

[16] "Activated Sludge Experiments at the Calumet Sewage Treatment Works," by F. W. Mohlman and C. E. Wheeler, Jr., Assoc. M. Am. Soc. C. E., *Sewage Works Journal*, Vol. II, No. 4 (October, 1930)

The aeration tanks should be divided into a sufficient number of batteries with pressure-bearing division walls between batteries to provide flexibility of operation and also to provide for the repair of diffuser plates, plate containers, piping, etc.; they should also be arranged so that the longitudinal velocity through the channels is low enough to avoid any undue loss of head. Otherwise, the number of batteries of aeration channels should be kept at a minimum in order to reduce the total number and cost of inlets and outlets. Baffle-walls between channels are usually non-pressure bearing, and division walls between batteries may be non-pressure bearing if there is a sufficient number of groups of batteries with pressure-bearing division walls between each group to provide flexibility of operation. In case division walls are made non-pressure bearing, a "fool-proof" arrangement should be provided for the draining of the batteries on the two sides of the non-pressure-bearing division wall, at equal rates. The openings for the passage of mixed liquor from one channel to another in the same battery should be large enough to avoid any appreciable head losses and should be at the bottom of the baffle-wall. The openings between channels in the new Southwest Sewage Treatment Works, at Chicago, are 10 ft high by 8 ft wide.

As previously stated, provision for reserving a part of the channels as sludge re-activation tanks can usually be made at a relatively small additional cost. This seems advisable in the case of strong sewages particularly in the absence of experimental plant data with the particular sewage to be treated. Aeration channels for mixing the return sludge with the sewage, influent channels into the different batteries, and effluent channels from the different batteries can often be made of substantially the same dimensions as the channels in the aeration tank batteries proper, usually at some saving in over-all plant cost and at no sacrifice in operating costs.

Section of Channels.—The capacity of tanks and the depth and width of channels have already been discussed. Since the development of the fact



(*a*) DALLAS, TEXAS, PLANT (DESIGN) (*b*) CHICAGO SOUTHWEST PLANT

FIG. 5.—TYPICAL WALL SECTION SHOWING AIR PIPING AND DEFLECTORS

that the spiral-flow type of tank or channel, with the diffusion plates along one side of the channel, saves considerable air over the ridge and furrow type of aeration channel with plates across the channel (see heading "Quantity of Air"), practically all new plants have utilized the spiral-flow type of tank. The main object to be attained in this type is to maintain the spiral flow with a minimum of dead spaces, eddying, and friction losses. As a result of considerable experimentation corner deflectors at an angle of approximately 45° with the vertical are generally used. Fig. 5 shows two typical examples of channel cross-sections with the dimensions of corner deflectors.

Diffuser Plates.—Tank dimensions, aeration period, percentage of return sludge, and quantity of air per gallon of sewage having been determined, the area of diffuser plates required is, within certain limits, a matter of economic balance between the cost of diffuser-plate installation and the cost of compressing air to take care of the pressure loss through the plates, the greater the number of plates the less the pressure loss. The practicable range of air quantity per square foot of plate seems to lie between 1 and 4 cu ft per min, operating difficulties being encountered outside these limits. As indicated by the experience of the Chicago North Side and San Antonio Plants, 4 to 6 cu ft of air per min per ft of channel will induce a circulatory flow motion in channels 15 ft deep and 33 to 40 ft wide, sufficient to prevent deposition of solids from grit-free mixed liquor (see heading "Aeration Tank Widths").

The size of air bubbles is important. Mr. Lockett reached the conclusion, as a result of some experiments with Manchester sewage, that diffused air is approximately three times as effective as air applied in the form of large bubbles such as obtain with plain tube aeration.[17] It was thought formerly that the size of bubble from diffuser plates varied inversely as the permeability of the diffuser plate, permeability being defined as that quantity of dry free air, in cubic feet per minute, which will pass through 1 sq ft of dry plate at 70° F, and 30 in. of mercury absolute, under a pressure equivalent to 2 in. of water. For this reason plates of a relatively low permeability (10 to 25) and with relatively high frictional resistance (8 to 12 in. of water with 2 cu ft per min per sq ft of wet plate) were used. It has been found, however, that permeabilities of between 30 and 40 and even as much as 50 are satisfactory, and that there is little difference in the size of bubbles diffused from plates of 15 permeability and those diffused from plates of 60 permeability. Mr. Arnold J. Beck, of the Sanitary District of Chicago, has found that the average bubble diameter diffused from forty permeability plates under normal operating conditions is 0.0813 in., having a volume of 0.00028 cu in.[18] The plates of the higher permeability not only have lower frictional resistances (4± in. of water), but also do not clog as readily as the plates of lower permeability.

Plate dimensions have been standardized at 12 in. by 12 in., with thicknesses from 1 to 1.5 in.; however, it is understood that plates 18 in. square are being considered for large installations.

The diffuser plates may be set in concrete or in metal plate containers, or they may be set directly into concrete air channels in the bottom of the aeration tanks. Plate containers holding as many as eight to ten individual plates are in common use. Whether plate containers are used, or whether the plates are set directly into channels formed at the bottom of the tanks, the bottoms of the aeration tanks should be poured flat and the containers or concrete channels built up in the bottoms of the tanks. Plate containers are favored on large installations; however, with careful workmanship in the forming and pouring of concrete plate channels, plates may be satisfactorily set into the bottoms of the tanks without the aid of plate containers.

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[17] "Activated Sludge Process: Aeration and Circulation," by William T. Lockett, *Proceedings*, Assoc. of Managers of Sewage Disposal Works, 1925, p. 94.

[18] Paper presented before the Eighth Annual Convention of the Central States Sewage Works Assoc., Urbana, Ill., October 25, 1935. (Not published.)

Plate containers have usually been of pre-cast concrete sometimes with non-rusting metal edges next to the plates, or with metal supports between the plates, or both. A cast-aluminum container, recently developed, is now being used. Cement grout is the generally accepted jointing material; however, the plates cannot be readily removed where a cement grout is used. The use of rubber gaskets and wedges with clamps for holding the plates in place offers possibilities in the direction of a jointing material which will allow easy removal of the plates. Rubber gaskets should be corrugated so as to obtain a good air seal which is not obtainable with a plain rubber gasket. A special rubber has been developed for use in sewage.

Diffusion tubes have been used on several smaller installations. These tubes have the advantages of being easy to remove and clean without draining the aeration tank and of utilizing the economy of deep tank construction and, at the same time, of being placed at relatively shallow depths, thereby making a saving in the cost of air compression. The disadvantages of diffuser tubes are the higher cost than the cost of plates for a given air capacity and the unevenness of air dispersion due to the fact that the air tends to escape through the top of the tube.

The air used in diffuser plates or tubes should be filtered so as to remove any foreign matter that would tend to clog the diffusers; air compressors, air piping, and plate containers should be of materials and construction such as not to permit oil, rust, or flaking from metal to get into the air stream.

Air Piping.—The main object to be attained in the design of air piping is a uniformity of air pressure over the entire aeration tank with due regard to economic balance between installation and compression costs. In general, this will mean over-sized pipe rather than the economic size of pipe computed strictly in accordance with the air-flow formula.[19] Losses in heads in valves are conveniently computed on the basis of the additional length of straight pipe to give an equivalent loss (see tables in compressed air handbooks). Although air-valves are almost universally placed on the feeder pipes to a group of diffuser plates, these valves should not be depended upon for close regulation of air pressure. Standard angle-valves are usually all that are required for feeder pipes to each plate container, or group of containers, although needle-valves have been found convenient in small plants.

In some large plants air mains are run longitudinally along the aeration channels 4 or 5 ft above the bottom of the channels, each main serving two channels with laterals through the baffle-walls or the division walls; generally, however, the mains are carried on top of the V-shaped baffle or division mains with each main serving two channels. The two methods are illustrated in Fig. 5. The question as to which system to use is largely a matter of cost in each particular case; however, the mains placed down into the channel have the disadvantage of creating eddies in the spiral flow of the liquor and of increasing the tank friction. In so far as possible the air piping should be designed and housed so as to conserve the heat in the air, inasmuch as the volume of the air delivered to the plates and the work done by it merely as a mixing agent is directly proportional to the absolute temperature of the air. Where the plate containers are poured in place on the bottom of the tanks and where the air channel beneath the plates is continuous and sufficiently large, the number of air-pipe laterals (down pipes) may be considerably reduced, or even eliminated, by making connections at each end of the air channel beneath the plates. The use of a continuous air channel also assists in the drainage of the air channels beneath the plates. Where a continuous channel is used the plates should be distributed uniformly along the channel with reference to permeability.

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[19] For a comparison of various formulas see "Air-Pressure Losses in Piping of Activated-Sludge Plants," by Henry L. McMillan, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 91, No. 5, August 2, 1923.

Piping may be of cast iron, galvanized iron, or steel, galvanized iron being preferred for small sizes (4 in., or less), cast iron for intermediate sizes (6 in. to 36 in.), and steel for larger sizes, although this is a matter of wide variation. Inasmuch as the air piping is subjected to differences in temperatures of as much as 200° F, ample expansion joints should be provided. In general, bell-and-spigot joints on cast-iron pipe and flexible couplings on steel pipe will provide for contraction and expansion.

Method of Returning Sludge.—The sludge should be returned uniformly from the clarifiers (that is, the same percentage should be returned) to each aeration tank battery. This may be obtained: (1) By mixing all the return sludge and raw sewage for the entire plant in a mixing channel before the mixed liquor goes into the aeration batteries; (2) by dividing the plant into a number of units which will accomplish the same result as under Method (1); or (3) by measuring or proportioning devices which will insure the correct ratios of sewage and sludge being drawn into each aeration battery from a common sewage-feed channel and a common return-sludge channel.

Many of the early activated sludge plants (Manchester, Worcester, Houston, and others) have air-lifts for returning sludge; however, most of the large plants built in the United States since the Houston plant have been equipped with centrifugal pumps for returning sludge. The personnel at the Chicago Sanitary District has obtained, by experimentation, a sufficiently efficient air-lift to meet its particular needs at the new Southwest Sewage Treatment Works and has incorporated sludge return air-lifts in the plans for the Southwest Treatment Plant. Again, this is a matter of cost, although the air-lifts no doubt accomplish some aeration. Although pumps have some advantage over air-lifts in the matter of efficiency, in a large plant, the cost of, and the head losses in, the piping or channels (or both), used in getting the sludge from the clarifiers to the pumps and back to the aeration tanks, may more than offset the efficiency advantage of the pumps. In a large plant the air-lifts may be placed between one or more aeration tank batteries and the clarifiers serving them so as to obtain a maximum of simplicity in sludge-return piping.

Measuring Devices.—However small or large the plant, the aeration tanks and appurtenances should be equipped with devices for measuring sewage flow, power, air, return and excess sludge, temperatures of air, sewage, and atmosphere, and air pressures. If the plant consists of more than one aeration tank battery, either measuring or proportioning devices should be provided for assuring an equable flow of sewage, return sludge, and air to each aeration tank. Measuring devices of the Venturi tube type have proved satisfactory for sewage, return sludge, and air. In many plants the return sludge is mixed with the raw sewage before subdividing the flow of mixed liquor into the individual aeration tank batteries, adjustable weirs being used either at the inlet or at the outlet of the aeration tanks to secure an equal distribution to the different tanks. Air-valves alone are usually depended upon to secure an even distribution of air.

CONCLUSIONS

The curve shown as Fig. 2 is offered as an attempt to arrive at a practical guide to be used in approximating the detention period and percentage of return sludge to be provided for in the design of aeration tanks. It is questionable whether the curve is applicable to activated sludge plants treating certain trade wastes with unusually high ratios of bio-chemical oxygen demand to suspended solids. The plant-operating data from which the curve was plotted indicate that the detention period and percentage of return sludge and, consequently, the capacity of the aeration tanks, should be largely proportional to the strength of the sewage to be treated. In the writer's opinion, this is in accordance with the theoretical considerations involved.

Once the capacity of the aeration tanks is determined, details of the tank design are largely a matter of the economic balancing of costs. Improvements in the design of aeration-tank accessories are being made continuously.

ACKNOWLEDGMENTS

Information to support the views offered in this paper was secured from various sources, for the most part, as cited by footnote references. However, special acknowledgment is due to L. C. Whittemore, M. Am. Soc. C. E., for data pertaining to the Chicago North Side Plant; Darwin W. Townsend, M. Am. Soc. C. E., for data observed at the Milwaukee Plant; and C. K. Calvert, Affiliate, Am. Soc. C. E., for data relative to the Sewage Treatment Plant at Indianapolis.

DISCUSSION

Norval E. Anderson,[20] M. Am. Soc. C. E. (by letter).—In his comprehensive presentation of the factors entering into the design of aeration tanks, the author notes the almost general acceptance of the 15-ft depth in the United States, but apparently is not convinced that this standardization is justified. The apparent inadequacy of data on the optimum depth of aeration tanks prompted the writer to undertake a rather extensive theoretical investigation of the question, the results of which were presented[21] in 1934. This theoretical investigation did not lead to any conclusive results. However, it is believed that a logical approach to the question was made and that some of the possibilities and limitations were evaluated approximately.

The method of approach was to set up three different theories of the possible relation between depth and air requirements, derive the necessary equations for each, assume basic operating conditions, and compute for each theory the relative air requirements for various depths.

The linear velocity theory is based on the assumption that for a given sewage, a certain minimum quantity of air (considerably less than present use) is required for oxidation, but that additional air is required to maintain a certain degree of agitation or velocity of circulation, and that this degree of agitation may be measured by the linear velocity of circulation. From this it would follow that the air required (within practical limits) is determined by the power that must be imparted to the sewage, in order to maintain a given velocity of circulation of the liquid in the tank. It can be demonstrated that on this basis the power-depth relation is a simple hyperbolic function of the form: $\text{Power per unit volume} = \frac{1}{\text{depth}}$.

The relative velocity theory, like the linear velocity theory, is based on the assumption that for a given sewage a certain minimum quantity of air is required for oxidation, but that additional air is required to maintain a certain degree of agitation or velocity of circulation. However, for this relative velocity theory, it is assumed that the necessary degree of agitation is measured by a velocity related to the tank depth. The relative velocity is assumed to be that which would give an equivalent hydraulic effect (the relation used in comparing hydraulic models), which is $V = \sqrt{\text{depth}}\, V'$. It can be demonstrated that to maintain this relative velocity, the power per unit of volume, neglecting distribution losses, remains the same for any depth.

In the contact theory the effectiveness of aeration may be measured by the area and time of contact between the air and the liquid in the tank. Furthermore, to obtain the same results from tanks of various depths, the unit of air contact area per unit of liquid volume must be kept the same. The air contact would be both that with the air bubbles and that with the atmosphere.

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[20] Engr. of Treatment Plant Design, The San. Dist. of Chicago, Chicago, Ill.

[21] "The Economical Depth of Aeration Tanks of the Diffused Air Type for the Activated Sludge Process of Sewage Treatment," by Norval E. Anderson. (Thesis presented in partial fulfillment of the requirements for the Professional Degree of Civil Engineer in the Graduate School of the Univ. of Illinois, 1934.)

In order to compute results in accordance with this contact theory, a number of approximations and assumptions (many of them controversial) must be made. These assumptions, all of which have some experimental basis, are as follows:

(1) Air forms bubbles of approximately the same size (about 0.01 ft) soon after emerging from the diffusers; and they remain this average size throughout the length of travel.

(2) The upward velocity of air bubbles relative to the liquid is approximately 0.8 ft per sec;

(3) The average velocity of circulation of the liquid in the path of travel in a 15-ft tank using 0.40 cu ft of air per gal of sewage, is approximately 1.7 ft per sec;

(4) The length of travel of air bubbles in the tank is approximately 1.2 times the depth of water over the diffusers; and,

(5) The atmospheric surfaces and the bubble surfaces are equally effective per unit of area, except as modified by Henry's law.

Assumption (5), although apparently contrary to the results of Mr. Freese's experiments, is supported by the experimental results of A. S. Parsons and H. Wilson[22]; G. J. Fowler and S. N. Chatterjee[23]; W. E. Adeney and H. G. Becker[24]; and H. G. Becker.[25] W. K. Lewis and W. G. Whitman[25] conclude that "gas bubbles * * * are particularly adapted to the absorption of the less soluble gases."

Operating data assumed as a basis for computing the comparisons are taken from approximate actual conditions at the North Side Treatment Works of The Sanitary District of Chicago, Chicago, Ill., representing rather efficient operation with a weak sewage. These assumptions are: Aeration tanks are of the circulating or spiral flow type; the water depth of the tank over the diffusers is 15 ft; the air used was 0.40 cu ft of free air per gal of sewage; the air distribution pressure loss was 1.30 lb per sq in.; the blower efficiency was 65% over-all; the average aeration period of sewage was 5 hr; and the average sludge return was 20% of raw sewage. It is further assumed that for any depth of tank the width may be approximately twice the depth.

The comparative air and power requirements for tanks of various depths, computed in accordance with the foregoing theories and assumptions, are given in Table 4.

The almost total lack of direct experimental evidence makes any discussion of the relative merits of the different theories unconvincing. However, there is some indirect evidence which seems pertinent.

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[22] "The Solution of Oxygen in Sewage," by A. S. Parsons and H. Wilson, *The Surveyor*, London, November 18, 1927, Vol. LXXII, No. 1896, pp. 490–494.

[23] "Note on the Aeration of Water," by Gilbert J. Fowler and S. N. Chatterjee, *The Surveyor*, London, August 12, 1927, Vol. LXXII, No. 1855, p. 139.

[24] "The Determination of the Rate of Solution of Atmospheric Nitrogen and Oxygen by Water," by W. E. Adeney and H. G. Becker, *Philosophical Magazine*, Series 6, April, 1920, Vol. 39, pp. 385–404.

[25] Absorption Symposium: "Principles of Gas Absorption," by W. K. Lewis and W. G. Whitman; "Mechanism of Absorption of Moderately Soluble Gases in Water," by H. G. Becker; and "Effect of Gas Velocity and Temperature on Rate of Absorption," by R. T. Haslam, R. L. Hershey, and R. H. Keen, *Industrial and Engineering Chemistry*, December, 1924, Vol. 16, No. 12.

Consideration of the factors involved in the computations for the contact theory indicate that if they were applied to aeration tanks of the ridge and furrow type, the results would show a greater air economy than for tanks of the circulating type, since the bubbles would be retained longer in the tank, due to the absence of the circulating velocity, whereas the surface contact would be the same. Operating experience has demonstrated the opposite. This points to the predominating influence of mixing or stirring the liquid.

The importance of stirring is emphasized in the Absorption Symposium,[25] previously cited, in which Messrs. Lewis and Whitman call attention to the remarkable agreement between the results of Becker and others as to the effect of stirring on the rate of absorption, and find that this rate increases as the 0.8 power of the rate of stirring. Further evidence to the same effect is offered by the experiments of The Sanitary District of Chicago and by Karl Imhoff, M. Am. Soc. C. E., at Essen-Rellinghausen, Germany, with submerged paddle-wheels to maintain circulation, thereby reducing the quantity of diffused air required.

TABLE 4.—COMPARATIVE AIR AND POWER REQUIREMENTS

Water depth over diffusers, in feet	FREE AIR REQUIRED, IN CUBIC FEET PER GALLON			BRAKE HORSE-POWER PER MILLION GALLONS PER DAY		
	Linear velocity theory	Relative velocity theory	Contact theory	Linear velocity theory	Relative velocity theory	Contact theory
5	3.26	1.09	0.63	48.8	16.3	9.5
10	0.87	0.58	0.48	20.4	13.6	11.2
15	0.40	0.40	0.40	12.4	12.4	12.4
20	0.23	0.31	0.36	8.9	11.9	13.6
25	0.15	0.26	0.32	6.9	11.5	14.6
30	0.11	0.22	0.30	5.7	11.4	15.6

Of the two velocity theories, the relative velocity theory seems much the more reasonable since it gives a measure of the stirring in relation to the mass of the liquid. Furthermore, any such pronounced change in air requirements with change in depth, as indicated by the linear velocity theory, would probably have been noticed before, even with the limited opportunity for comparison.

Again referring to the Absorption Symposium,[25] it is found that the rate of absorption is dependent upon the area of interface in relation to the volume of liquid, as well as upon the rate of stirring. Since the relative importance of these factors in an aeration tank is not known, it is possible that the true power-depth relation lies between the contact theory and the relative velocity theory, although the latter probably predominates, especially for a weak sewage.

In view of the foregoing comment it appears that there probably is not much difference in power requirements per million gallons per day of sewage treated over a practical range of depths, and that more consideration may be given to relative construction costs as affected by any particular site. If the foregoing is accepted, it would modify the author's conclusions regarding depth (see text following Table 2) in two respects: (1) It is not certain that there is an increase in the cost of power with increased depth; and (2) the economical depth may be considerably more than 15 ft under less than extreme conditions as applied to costly sites or foundations.

Weston Gavett,[26] Assoc. M. Am. Soc. C. E. (by letter).—The larger sewage treatment works generally have better facilities for research and investigation of plant problems than the smaller works although all experience the same difficulties. No doubt a large fund of valuable information has been accumulated at various plants that has not been published. Mr. Freese has done a useful service in presenting and interpreting data from some of the larger works.

Table 1 and Fig. 2 are interesting in indicating the optimum aeration time and percentage of return sludge for sewages of various strength as expressed by bio-chemical oxygen demand (B.O.D.). The direct relationship between optimum aeration time and percentage of sludge seemed surprising at first glance. The fact that 5-hr aeration time required 20% return sludge whereas 8-hr aeration time is definitely coupled with 50% sludge, appeared to contradict the opinion of Mr. F. W. Harris and his collaborators[27] that percentage sludge times aeration time is a constant for any given sewage. When it is realized that the values in Fig. 2 are minima for complete treatment without re-aeration of sludge, the data are not inconsistent with Harris' results. The curves in Fig. 2 may be expressed approximately in the formulas:

$$T = 4.5 + 0.00625\ (\text{B.O.D.}) \qquad (1a)$$

$$S = 15 + 0.0625\ (\text{B.O.D.}) \qquad (1b)$$

$$S = 10\,T - 30 \qquad (1c)$$

and,

$$T = 3 + \frac{S}{10} \qquad (1d)$$

in which T = aeration time, in hours; S = percentage return sludge; and, B.O.D. = 5-day bio-chemical oxygen demand, in parts per million. Mr. Harris expressed the relationship between time and percentage of sludge as:

$$S\,T = C \qquad (2)$$

in which C is a factor for a given strength of sewage. The percentage of sludge used by Mr. Harris is the percentage in the mixed liquor rather than that in return sludge as used in Fig. 2.

Assuming that C is a function of the B.O.D., the values in Fig. 2 indicate that $S\,T = 0.7$ B.O.D. to 0.8 B.O.D., except for low values of bio-chemical oxygen demand. For a sewage with a bio-chemical oxygen demand of, say, 400 ppm, Fig. 2 indicates the minimum time of aeration as 7.0 hr and the minimum percentage of returned sludge as 40. If, however, $S\,T = 0.7$ B.O.D., with the same 400 B.O.D. sewage, good results should be obtained with 10 hr aeration and 28% returned sludge or 6 hr aeration time and 47% sludge, although the need of re-aeration of sludge would be expected with an aeration time of less than 7 hr as shown by Fig. 2. A more general and useful expression would be obtained if the percentage of sludge were expressed (as done by Mr. Harris) as the percentage of sludge in mixed liquor after 1 hr of settling, or better as parts per million of suspended solids in the aeration tank. As the author states (see heading, "Aeration Tank Capacity"), "the percentage of return sludge, shown in Fig. 2, does not take into consideration the percentage of solids in the activated return sludge * * *." In view of the considerable variation in the concentration of solids in return sludge carried at various plants, relationships based on percentage return may be misleading unless based on sludges of similar concentration.

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[26] San. and Hydr. Engr., Clyde Potts, New York, N. Y.

[27] "The Activated Sludge Process," by A. J. Martin, Macdonald and Evans, 1927, p. 165.

Experience at five plants confirms the author's statement of the advantages of diffuser tubes. The writer questions the "unevenness of air dispersion due to the fact that the air tends to escape through the top of the tube" (see "Aeration Tank Details: Diffuser Plates"). A photograph of air diffusion from a tube which appeared in a trade publication showed most of the air escaping from the top of the tube, but the tube had little water over it. When a tube is submerged 6 ft in water, the effective pressure on the top of the tube equals 6 plus tube friction and on the bottom of the tube, 6 ft 3 in. plus tube friction. The difference is slight and should not cause great difference in air diffusion. The writer believes a further advantage of tubes is the efficient distribution of air through the tank contents. Experiments at Morristown, N. J., showed that with tubes 4 ft above the bottom in a 10-ft depth of sewage, best results were obtained when the tube was located away from the adjacent wall a distance equal to one-tenth the width of the tank. When near the wall much of the air appeared to escape along the wall without doing its share of useful work. When placed too far away from the wall the circulating velocity of the tank decreased. At the best location the air appeared to be well distributed through the sewage.

As with plates, a higher permeability was found advisable for tubes. At Mineola, N. Y., in 1928, and for Rockville Centre, N. Y., in 1929, tubes were specified with a permeability equivalent to a plate 1 ft sq with permeability of 14 to 16. Flushing by water was found useful in keeping tubes clean at Rockville Centre. Mr. Paul Molitor, Sr., cleaned tubes by a 40% nitric acid bath and heating at Chatham, N. Y.[28] Tubes at Bernardsville, N. J., were specified to pass 50 to 60 cu ft per min under 2-in. water pressure when dry, or approximately equivalent to 25 to 30 permeability for a 1-sq ft plate. At this plant only one-half the available aeration tanks are in use, so that tubes are changed once a year and allowed to rest every other year. No cleaning has been necessary since the plant was placed in operation in October, 1933. Perforated copper pipes installed at Morristown were replaced by diffuser tubes. The writer believes that the possibilities of perforated pipe have not been sufficiently investigated. When holes are perforated in the bottom of pipes which are placed above the bottom, the velocity of the rotating sewage has a sweeping effect that disperses the air bubbles effectively.

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[28] *Proceedings*, New Jersey Sewage Works Assoc., 1932, p. 26.

S. W. Freese,[29] M. Am. Soc. C. E. (by letter).—The discussion by Mr. Anderson, giving possible theoretical bases of determining the economic depth of aeration tanks, is a long step toward providing a rational solution of this problem. The difficulty of choosing between the various possible theories or combination of theories emphasizes the desirability and need of parallel experiments with aeration tanks of different depths, such as 5, 10, 15, and 20 ft. The results of such experiments, taken together with the possible theories outlined in the discussion, should provide a rational basis for determining the depth of aeration tanks. In the meantime, the data available tend to limit the designer to a 15-ft depth of tank.

An experimental determination of the weight to be given to "linear velocity," "relative velocity," "contact," and any other factor or factors affecting aeration, would be useful not only in determining depth, but in determining whether the air-diffusion type, the "mechanical" type, or a combination of the two, is the most economical method of aeration.

Mr. Gavett calls attention to the fact that Mr. F. W. Harris and his collaborators found[3, 27] that "percentage sludge times aeration time is a constant for any given sewage." This is largely true during the first or "coagulation-adsorption" stage rather than in the second or "oxygenation" stage of the aeration period for complete treatment plants. In the writer's opinion, the findings of Messrs. Grant, Hurwitz, and Mohlman[4] complement those of Messrs. Harris, Cockburn, and Anderson[3, 27] in affording a complete picture of the two stages.

The formulas for aeration time and percentage of return sludge which Mr. Gavett has developed provide useful guides for the determination of the capacity of aeration tanks for complete treatment plants of the air-diffusion type. As to the percentage of solids in the return sludge, it is to be noted from Table 1 that the Chicago North Side return sludge contained 1.39% of solids and the Milwaukee sludge contained 1.19 per cent. It is not believed that the solids in the return sludge of a well-operated plant treating domestic sewage will vary greatly from these latter values.

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[29] Cons. Engr. (Hawley, Freese & Nichols), Fort Worth, Tex.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2013

THE RELATION OF THE NATIONAL SOCIETY TO THE ENGINEERING PROFESSION

ADDRESS AT THE ANNUAL CONVENTION, SALT LAKE CITY, UTAH, JULY 20, 1938

BY HENRY EARLE RIGGS,[1] PRESIDENT, AM. SOC. C. E.

The long continued depression in the United States has created many grave and serious problems, not only for individuals and business firms and corporations but for social and professional organizations and groups. The profession of engineering has been greatly affected. From a peak of professional activity in 1930, with the highest scale of compensation for service that has ever prevailed, the profession plunged to the depth of depression in 1932 with many thousands of engineers out of work, and many apparently substantial and prosperous engineering firms out of business. In the years since then a great part of the work of the civil engineers has been in connection with Federal pump priming or relief work, some of it being major projects of large size and of fine character, but a large part of it being pure relief work of little professional interest and offering inadequate pay.

Any thorough investigation will compel admission of the fact that there is a great increase of engineering employment on government work, and an appalling lack of employment of engineers on privately financed corporation enterprises.

If a study of the profession were to be made to-day it would disclose the fact that large numbers of the older engineers, many of whom had believed that their investments assured a safe and comfortable old age, and several thousand younger ones who had purchased or built homes or made considerable investments on which obligations were outstanding, have had everything swept away and find themselves now wholly dependent on the continuance of steady employment.

There has been disillusionment and despair for many and there is doubt and uncertainty as to the future in the minds of many others. The student in the upper classes in college is asking whether engineering is a worth while pursuit. The engineering graduate who has found little or no engineering work in the last seven or eight years or who has been given no chance to assume any degree of professional responsibility has the same doubt.

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[1] Honorary Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

The whole ugly picture of depression—of the great volume of "made work," much of it without proper engineering supervision and all of it with underpayment of professional men; of the great labor unrest and of the excessive activity of labor unions; of the distinctly domineering character of some of these unions; of the extensive entry of government into business in competition with private capital; and of the fear that undeniably is nation-wide, that stifles individual initiative, and prevents individual investment in new projects—all this constitutes a picture that compels analysis of conditions in the engineering profession. What must be done to keep up the morale and to help to eliminate the abuses which many engineers have experienced?

In large measure this Society and the other great engineering societies founded in later years are responsible for the splendid development of the profession of engineering and for its gradual but sure recognition as one of the great and learned professions. It is in only comparatively recent years that full understanding has come of the importance of engineering to the nation and of the need for engineers not only to design and build plants but to maintain and operate them and to administer the higher executive offices in transportation, utility service, and manufacturing as well as in municipal, state, and national service.

It is to the great engineering societies that the discouraged engineer naturally looks for help in his time of need. Can the Founder Societies be of service? Is the American Society of Civil Engineers so organized that it can take the place of leadership when leadership is so much needed?

Any discussion of the relationship of the profession of engineering to these conditions of to-day should be prefaced by a review of the growth and development of the American Society of Civil Engineers which is probably typical of that which has taken place in all the older engineering societies. Such a review is found in the Annual Address of Arthur S. Tuttle, Past-President, Am. Soc. C. E., in 1935.[2] There is no need to repeat the history given by him. The three years that have passed since then have justified many of his prophecies. They have also brought new problems which call for consideration some of which were indicated by Louis C. Hill, Past-President Am. Soc. C. E., in his Annual Address in 1937.[3]

FUNDAMENTAL ORGANIC CHANGES IN THE CONSTITUTION

This address is an attempt to supplement and expand the discussion of these problems in the light of present-day conditions and to point out why and how the fundamental organic changes in the Constitution of our Society were brought about.

During the eighty-six years of the Society's existence it has changed its organic form and adjusted itself to meet the developing needs of the profession. Starting as a local society in the largest city of the United States it passed a membership of 500 in its twenty-third year—1875. The need of those earlier times was for improved technical work and the Society was established with the major objectives of bringing together engineers of the highest professional standing, of developing a professional *esprit de corps*, of raising standards of ethics and the practice of engineering, and of building up American technical literature in all branches through the recording of the accomplishments and researches of its members. The excellence of its technical work and its high standards of membership drew to it many engineers resident outside of the City of New York so that it grew to a membership of 1 000 in 1887, to 2 000 in 1897, and 3 000 in 1904.

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[2] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), pp. 1407–1425.

[3] *Loc. cit.*, Vol. 102 (1937), pp. 1173–1177.

The Constitution adopted March 4, 1891, undoubtedly represented the opinion of the then membership as to the form of government desirable at that time. It provided for no zones and no districts. The Board of Direction consisted of four Vice-Presidents and eighteen Directors at large and all living Past-Presidents. The Year Book of 1894 shows twelve Past-Presidents as members of the Board, making a total of thirty-five members. Nine members of the Board constituted a quorum at the time of the annual meeting and annual convention and five at all other meetings of the Board.

The Constitution provided that nominations for all offices should be made by a nominating committee. In 1887 five districts were established for the purpose of selecting this nominating committee at the annual business meeting. In 1891 the committee was increased to seven, and seven districts were established. There were two general business meetings of the Society a year, at the annual meeting and the annual convention, besides monthly business meetings at New York, at which thirty members constituted a quorum. There were no local sections, no student chapters, no technical divisions, no technical or research committees, and only three standing committees of the Board, on Finance, Library, and Publications.

It is obvious that a Board of Direction with a membership of ten or more from the New York metropolitan area, with a quorum of nine at the annual meeting and annual convention and five at other meetings, all of which were held in New York City, was largely governed by the representatives of the New York area, especially as non-resident Board members did not receive mileage allowance for attendance. Furthermore as the membership was classed as "resident" and "non-resident," the "resident" members paying $10 more per year as dues, there was ample ground for the desire of the "resident" members to control Society policies. All meetings of the Society, about twenty per year, excepting the annual conventions were held in New York City under the auspices of the national society. Of the twenty-seven annual conventions held prior to 1896, only three were held west of the Mississippi River, one at St. Louis, Mo., one at St. Paul, Minn., and one at Denver, Colo.

A growing recognition of the increase of non-resident members and of the need for a change in the form of organization of the Society to give them a proper voice in its affairs began to find expression in the middle nineties and led to the adoption of various amendments to the Constitution.

ESTABLISHMENT OF LOCAL SECTIONS

The establishment of Local Sections began with the recognition by the Board of the San Francisco Section in 1905. Members of the Society in St.

Louis had organized an association as early as 1888, with the result that in 1894 there were more corporate members in that city than in any one of forty-one states and territories of the Union, only New York, New Jersey, Massachusetts, Pennsylvania, Ohio, Illinois, and California having more members than the city of St. Louis. The second Local Section officially approved was the Colorado Section in 1909, and the third, the Georgia Section in 1912. In 1913 five more Local Sections were organized; in 1914, five, including the recognition of the old group at St. Louis; and twenty more were organized prior to October, 1921. Since then thirty Local Sections have been organized, making a present total of 63.

The creation of Local Sections was the fundamental change in organization which resulted in the Society becoming a truly national body. The bringing of the benefits of social and technical meetings to the members in all parts of the country has been a most important factor in the great growth of the Society and has made possible the radical changes in the Constitution which have been made in recent years. With twenty-one local groups established by 1918, holding meetings and discussing Society affairs, it naturally followed that the "non-resident" membership became more vocal and suggested the incorporation in the new Constitution of many ideas tending to a more democratic administration of Society business.

Also a marked change in the rate of increase in membership began with the establishment of Local Sections, in 1905, at which time there were a few more than 3 000 members. Membership reached 5 000 in 1909, 7 000 in 1913, and 10 000 in 1921. The growth in the sixteen years following the establishment of the first Local Section was twice as much as that of the preceding fifty-three years.

The Constitution of 1891 was amended eleven times between its adoption and the beginning of 1920, providing, among other things, a reduction in the number of Past-Presidents serving on the Board of Direction from all those living to the five who had last served the Society.

ESTABLISHMENT OF STUDENT CHAPTERS

The establishment of eight Student Chapters in 1920 and of thirty-one in 1921, followed by eighty more in succeeding years up to 1938, was a second radical organic change which had far-reaching results. The more than 5 500 students who were enrolled in these Student Chapters in the school year 1937–38 are not members of the Society, and a large majority of them probably will not enter as Juniors on graduation, but several hundred of the most promising leaders among them will take the easy way provided for entrance to the Junior grade immediately on graduation.

The benefit to the Society of this early contact with thousands of young men each year cannot be questioned. The results of the founding of Student Chapters have been to bring many practicing engineers into much closer touch with the colleges than would otherwise be possible, to give to college students a knowledge of the Society and its objectives, and to make the way into the Junior grade easy for college graduates. The Student Chapters form a great reservoir from which the Society will draw the larger part of its membership in the years to come.

In 1921 the Society had 458 Juniors, approximately 4½% of the total membership. In 1926 there were more than 1 000 Juniors, or 9½ per cent. In 1929, 2 000, or 15.4%, and, in 1933, we passed a membership of 3 000, or 20% of the total membership. On May 31, 1938, there was a Junior membership of 3 736, or 23.7% of the total membership on that date. This rapidly increasing number of Juniors and the steadily rising percentage of this group in proportion to the total membership is the direct result of the formation of Student Chapters. It indicates a continuous lowering of the average age of membership. It also indicates a steadily increasing percentage of young employee engineers in our membership. It creates a present problem but it also augurs well for the future of the Society if these young men can be put to work in and for the Society and can be given the maximum of benefit from their connection with it.

The Constitution adopted October 5, 1921, with the eight amendments which have been adopted to date, marks the final change from a local association to a national society. The democratic method of nomination and election by letter-ballot by members resident in each Zone or District insures a Board of Direction that is truly representative of each area. The provision for changes of District and Zone boundaries by the Board insures adjustment of representation in proportion as membership changes. The present Board with nineteen Directors, four Vice-Presidents, two Past-Presidents, and the President, not more than six of whom could possibly come from the New York metropolitan area, removes the last vestige of local control. The payment of mileage allowances sufficient to substantially cover the actual cost of attendance at meetings has resulted in the presence of from 80 to 100% of all members of the Board at all meetings of recent years.

Tables 1 and 2 present an analysis of the membership as of 1894, 1920, and 1938, three dates selected arbitrarily to show the classification and distribution of members (*a*) at the mid-life of the Society at a time when it was still essentially a local society although more than 60% of its members were "non-resident;" (*b*) in 1920 when many changes in the organic structure had been

TABLE 1.—SUMMARY SHOWING CHANGES IN THE CHARACTER OF SOCIETY MEMBERSHIP*

Grade of membership	1894		1920		1938	
	Number	Percentage of total	Number	Percentage of total	Number	Percentage of total
Honorary	9	0.53	3	0.03	25	0.16
Member	1 170	68.78	3 990	41.50	5 642	36 23
Associate	148	8.70	4 697	50.01	6 162	39.05
Corporate†	1 327	78.01	8 690	91.51	11 829	75.44
Juniors	255	14.99	506	5.40	3 664	23.53
Corporate and Juniors†	1 582	93.00	9 196	96.91	15 493	98.97
Corresponding	3	7.00	1	3.09	0	1.03
Affiliates (associates)	68		166		78	
Fellows	48		10		1	
Total	1 701	100.00	9 373	100.00	15 572	100.00

* Data from Year Books of 1894, 1920, and 1938. †Sub-totals.

made but before the establishment of Student Chapters had started to build up the Junior grade; and (*c*) at the present time.

TABLE 2.—CORPORATE MEMBERS* RESIDENT IN THE UNITED STATES, SHOWING CHANGES OF MEMBERSHIP WITHIN THE PRESENT ZONE

Zone	1894		1920		1938		
						PERCENTAGE OF TOTAL	
	Number (1)	Percentage of total (2)	Number (3)	Percentage of total (4)	Number (5)	Corporate members (6)	Total, all grades (7)
I	492	39.87	2 212	31.20	2 982	27.18	26.95
II	262	21.23	1 919	27.10	2 590	23.60	23.18
III	362	29.34	1 704	24.00	2 703	24.63	25.10
IV	118	9.56	1 255	17.70	2 698	24.59	24.77
Total	1 234	100.00	7 090	100.00	10 973	100.00	100.00

* Except Column (7), in which the present percentage of total members in the United States is given for comparison.

ESTABLISHMENT OF TECHNICAL DIVISIONS

A fourth fundamental change in the Society organization came with the establishment of the Highway, Irrigation, Power, and Sanitary Engineering Technical Divisions in June, 1922. These have been followed by the creation of eight other Divisions, and still others are in process of formation.

The net result of the establishment of Technical Divisions has been to turn a large part of the technical programs of the annual meetings, annual conventions, spring and fall meetings over to the Divisions, and, more recently, to put them in charge of research committees dealing with matters coming under their jurisdiction. In effect, to the Divisions have been given the direction of the important technical work of the Society, placing each specialized field of activity in the hands of men whose life work and principal interest is in that field, and insuring to each full opportunity to hold sessions at every meeting where local interest calls for them.

The Technical Divisions will undoubtedly encourage the development of a number of highly specialized fields of engineering, induce the membership to do more work of research, and increase the quality and quantity of publication in the various special fields.

NEW PROBLEMS

The foregoing discussion has briefly outlined the fundamental changes that have taken place during the past forty-five years. They are changes made necessary by the growth of membership, the increase in the number of special fields in Civil Engineering, the development of the United States which has resulted in a great increase of engineering work, in the number of members of the Society in the Central, Southern, and Western States, and by the granting to members non-resident in New York a voice in Society affairs proportionate to their membership. These adjustments indicate healthy growth in the

Society and prove its ability to keep up with changing conditions in the profession and nation.

The Board of Direction has been continually on the alert to improve the Society and encourage greater activity among its members. There is need for constant introspection, frequent examination of methods of administration to see that old and obsolete practices are not continued, that there is no waste of Society funds through duplication of effort or through the doing of unnecessary things, and to insure the doing of all new things which the Society may with propriety assume to be within its province.

There are now three major problems before the Board:

I.—Local Sections.—How can these agencies be made of the greatest possible benefit to the membership? What duties and functions may properly be turned over to the Local Sections?

II.—Junior Members.—How can these young men be best assimilated in the Society? What methods shall be adopted to enable them to get the maximum benefit from their connection with it?

III.—Technical Divisions.—What form of organization is best adapted to carrying on the specialized technical work of the Society and securing adequate publication of worthy papers?

The objective that must be kept in mind is *maximum benefit to the individual members of the Society.*

I.—Duties and Functions of Local Sections

With the entire area of the United States and its possessions now assigned to the sixty-three Sections, and with a formula for allocation of funds by the Society to the Sections which assures sufficient to care for all current expense of postage and mailing of notices of meetings and frequent bulletins to all members resident in the area, and to cover the cost of many of the activities of the smaller groups, it is expected that there will be increased interest on the part of many of the Sections and a greater incentive for many qualified engineers who have not joined the Society to do so and take part in its work.

(*a*).—*Legislation.*—The legislative problems of the profession in any state can best be solved by the combined effort of the various engineering groups in the state. Such matters as securing proper registration laws; insisting on complete enforcement of such laws; the encouragement of the adoption of laws providing for the merit system in civil service; the establishment of a proper classification of engineering employees and the provision of adequate salaries for each class; insistence on the payment of reasonable salaries for all engineering service; safeguarding public positions in engineering by securing the enactment of legal requirements that only qualified registered engineers shall hold them; and other legislation in the interest of the profession are all matters in which engineers resident in the state can do much more effective work if properly organized for joint effort than can be done by reliance upon any national society. The latter can aid as a medium of exchange of information and can give support to local effort, but the work of local societies only can be truly effective. There must be continued watchfulness on the part of the profession in matters of legislation. Not only must good legislation be supported, but that which is injurious to the profession of engineering must be opposed.

(*b*).—*Membership.*—The membership of any professional organization can be expected to increase only when the activity of the organization is pronouncedly along lines that are of distinct benefit to the profession. Under the economic conditions now existing, and especially in view of the labor unrest and the activity of the labor unions, the national engineering societies, particularly those of the civil engineering branch, cannot confine their activities solely to technical papers and discussions. The social and economic problems of the members, especially of the younger men just coming into the profession, must receive the earnest thought of all engineers.

The American Society of Civil Engineers has never made a campaign for membership. Its specifications for membership in all grades are clearly written. The requirements are higher than those of any other engineering society. There are hundreds of engineers in the United States, doubtless many in almost all of the States, whose experience fully meets the requirements of the specifications of the Society but who for one reason or another never have joined.

Many men are reluctant to apply for membership unless spoken to by some member. There is no sound reason why a membership committee in each Local Section should not make a list of all engineers in the area, examine their qualifications, and when a man is clearly eligible for membership in one of the grades, see that some member of the Section suggests to him the desirability of applying for that grade.

In case such a plan is followed the greatest of care must be exercised to see that the applicant asks only for the grade for which he is fully and clearly qualified. Border-line cases should not be encouraged to believe that the Society's Committee on Membership is careless or lax, because it is not, and the award of full membership only comes to those who are fully and abundantly qualified.

(*c*).—*Students and Juniors.*—The Local Section is expected to take an active interest in all Student Chapters in its area. The college authorities will almost invariably welcome the active interest of practicing engineers in matters of engineering education and in the employment of graduates in engineering work. Many Sections award as prizes membership in the Junior Grade, paying the entrance fee and, in many cases, one year's dues. Sometimes the prize is given for the best paper written by a senior in each of the colleges; other Sections make the award for outstanding ability based on excellence of school work, activity in campus affairs, and notable work in public speaking.

Some of the Sections pay part of the expense of students to Student Chapter Conferences and in other ways show an interest in and encouragement of the work of the students for the Society. Every Section should make this work with Student Chapters a prominent part of its program of activity.

When the student becomes a Junior in the Society on graduation he is entitled to much more of the interest of the Local Section than he received when in college. He now rates in the membership, has his name in the Year

Book, and receives all publications. Under the Society's formula for return of money to the Section he counts the same as any other member.

No matter what form of Junior organization may be adopted in the Section the individual Junior is entitled to the full consideration of officers and members as one of the future leaders and workers of the Section, and he should be trained for that work.

Where there is an association of Juniors in a Section area there should be close co-operation on the part of the Local Section officers. Such a group must be kept as an essential and integral part of the Section, given financial aid by the Section, and its members encouraged to attend all Section meetings.

This work with young men is very distinctly personal work dealing with matters of social, financial, and economic interest. It is work that can be done in a satisfactory manner only by the Local Sections. A national society cannot possibly do it. No single formula can be developed for it because of the wide differences of conditions in the different states and section areas.

(*d*).—*Co-Operation.*—There is present need not only for strong and active Local Sections of the Society but for the most cordial co-operation of the Sections with each other and with Sections of other national societies and with state and local groups within the state, to the end that the united engineering profession in each state may be prepared to act whenever action is needed. The standing of the American Society of Civil Engineers is such that our membership and our Local Sections may properly be looked to for leadership in this matter of co-operation. In several states the Sections are now the recognized leaders in all matters affecting the welfare of the profession and handle all matters in a highly efficient manner. It is the task of the Society to build up strong and active Local Sections in all the states and to encourage them to take leadership not only in technical papers, publication, and research, but in the general social and economic welfare not only of their own members but also of the whole civil engineering profession in the state.

II.—Responsibility of the Society to Its Junior Members

Junior membership appears not to have been particularly attractive until after the formation of Local Sections. The number of Juniors increased rapidly from about 250 in 1893 to around 800 in the years 1911 to 1913; then for some reason the number dropped to less than 500 in 1920, 1921, and 1922.

The importance of Junior membership to the Society can hardly be overestimated. The class of young men in college which is attracted by membership in Student Chapters will be found to be the able and promising students whose interest in technical engineering is marked and who expect to go into engineering work. Those who apply for Junior membership in the Society on graduation are generally those with the highest grades, real interest in the profession, and reasonable assurance of a job.

These young men need the encouragement of the older members of the Society after they enroll as Juniors. Employee engineers all of them—frequently married shortly after graduation—with a limited income but with a keen interest in technical problems— and a desire to get the most out of engineering associations—they can be greatly helped by friendly recognition on the part of older engineers and by being given the opportunity to do active work in the Section and to have a part in the technical discussions.

Where there is a group of fifteen or twenty or more in a single city some type of association of Juniors appears to be the desirable form of organization. It offers to the young men the opportunity to prepare papers, take an active part in discussion, and really get into worth-while technical activity in a large way. Such an association also makes possible intimate social contacts among themselves. Then when they attend the Section meeting they extend their acquaintance with older men and get the benefit of both the technical and social activity of the Section. Where this solution is adopted the Section should have a contact committee and the Juniors should be assigned to Section committee work and given duty wherever possible with the older group.

Where the group of Juniors in a Section is small it would appear that a desirable policy is to give Juniors an active part in the work of the Section by putting one on each committee, designating them as aides to the Section officers and giving them places on technical programs. Such a plan will tend to hold their interest and permit them to develop as working members. There are thirty-three states which have at present less than 50 Juniors and thirteen states with more than 50. Six of these states have from 100 to 200 Juniors and four have more than 200.

This problem of Junior membership is one for each Local Section to solve for itself, but the correct solution is of vital importance to the Society. It is fair to assume that the opportunity to take an active part in the work of an association of Juniors whenever one is formed, will attract many young men who otherwise would not be inclined to join a Section made up of older practicing or employer engineers.

The important thing is that these young men are just entering the profession. Their concepts of professional ethics, of high standards of practice, and of engineering loyalty are being formed. They are entitled to proper classification of position and to adequate pay and a right to prove themselves worthy of advancement. They constitute the group that is most subject to irregularity of employment and to exploitation by labor unions. The inspiration that they can gain from membership in the Society, from the acquaintance with old and successful engineers, and from the advice and counsel of the members of the Section will be of lasting value in the making of the great engineers of a few years hence.

We may as well face the fact that, like it or not, the national engineering societies must give more thought to the economic and social problems of their members. Engineering is a great profession and to keep it so its members must recognize their responsibility to the younger men who are entering it. The engineering society is in fact, and must recognize itself as being, the graduate school in which young engineers gain that knowledge of professional relations, ethical conduct, and ideals of good practice that can be secured nowhere else.

III.—TECHNICAL DIVISIONS AS FUNCTIONING AGENCIES

For the first seventy years of its existence the Society asked its members to prepare papers and to discuss them, and published those passed upon as most meritorious. It made no distinction as between the different fields of technical interest and it had no committees or groups of its members engaged in the encouragement of specialization in engineering practice. Any study of American engineering literature would be compelled to give to the Society credit for the greatest contribution to engineering knowledge and to the literature of the profession in America made by any organized group of engineers. These papers were on all kinds of subjects and no one field had a monopoly of publication.

The Society has done a magnificent job, but in the doing of it failed to recognize sufficiently the divergence of engineering interest and the growth of many new fields of specialization.

In its early days the railroad engineers were most active in the work of the Society. In 1893 the Society's Committee on Standard Rail Sections made its report and the "A.S.C.E. Standard Rail Section" was laid on hundreds of miles of railway track. Apparently, the difficulty of securing the appointment of desired committees and of getting prompt publication of reports and manuals caused this group of members to foster the activity of a non-professional organization of railroad men and to change its entire character so that it ultimately emerged as the American Railway Engineering Association.

This Society could never have done what that great and active Association has accomplished in the standardization of railway construction and maintenance practice, but one wonders whether a Railway or Transportation Technical Division, if it had been established forty years ago, might not have afforded a medium for individual discussion of railway problems that would have attracted hundreds of qualified engineers to our membership who have never joined because A.R.E.A. with its intensive committee work absorbs their time and interest. The A.R.E.A. does not afford opportunity for much individual expression of opinion, however, nor does it encourage the writing of papers by its members on topics of general interest in transportation.

The question may well be asked whether or not the other Founder Societies also have been remiss in not caring for specialized fields of technical interest much earlier than they did. Could not the work of such organizations as the American Society for Testing Materials, American Welding Society, American Society of Refrigerating Engineers, American Society of Heating and Ventilating Engineers, American Illuminating Society, and American Society for Metals, and other highly specialized technical groups, have been done just as well by wisely organized technical divisions of some of the Founder Societies? It would appear that some of these organizations could do just as much good work and do it even more effectively and economically as Divisions of the great engineering societies.

We now have twelve Technical Divisions and requests for more. There may be some overlapping and duplication in existing Divisions. There may be other agencies in the Society capable of doing some of the work which the proposed new Divisions contemplate undertaking. Certainly, there is need for the Committee on Technical Procedure to make a careful study of the Divisions with a view to possible improvement of the technical work of the Society.

Although the provisions of the By-Laws of the Society are apparently broad enough to meet all requirements, it appears to be possible to improve the organization of the Divisions. For example: We have a Highway Division. Might not a Transportation Division, with sections or sub-sections dealing with Railways, Highways, Highway Transport, and Waterways, each working independently in its own special field but keeping close contact with the other transportation interests, be more effective in finally helping to solve the great problem of internal transportation which confronts the nation?

It appears certain that the splendid work of our Sanitary Engineering Division is just as good as it would be if the same men were organized as an American Society of Sanitary Engineers, and the chances of just as wide publicity and publication ought to be available to them at much less cost than if organized as a separate society.

These problems call for careful study by the Board, the various committees, and by interested members. The past history of this Society makes it clear that they can be fairly and justly solved.

Technical papers and discussions at all conventions and regional meetings are under the supervision of the Divisions. Fact-finding and research committees dealing with any specialized field should be under and report to the officers of the Division in charge of that field. In other words, each Technical Division should be in effect a technical agency operating within the American Society of Civil Engineers, and making use of its organization, staff, and publications.

THE SOCIETY AS IT FUNCTIONS NATIONALLY

All matters of general Society policy, qualifications for membership, election and transfer of members, establishment of standards of ethics and practice, the general administration of the affairs and property of the Society, publications, relations with other societies, maintenance of all joint agencies through which the national societies are co-operating, all matters of national legislation, and all dealings with departments or bureaus of the federal government, are functions of the Board of Direction. Under no conditions should any group, either Local Section or Technical Division, undertake to speak for the Society, other than as an authorized spokesman on matters that have been determined by action of the Board of Direction.

The tendency of all changes in the organic structure of the Society in past years has been in the direction of one large, dignified national society with uniform and high standards of individual membership, having numerous state and metropolitan subdivisions, each one freely acting in matters of social and economic interest to the membership in its own area, and co-operating with other professional organizations within the area for the good of the engineering profession.

In the future as in the past it is probable that a great number, if not a majority, of the most important technical papers, those which are entitled to rank as monumental contributions to the advancement of technical knowledge, will be submitted by members of the Society who have devoted long study and research to some special subject and have given time and care to the preparation of the paper. Such studies rarely fit into the program of a meeting or convention. They are submitted to the Society for publication as a matter of long established custom rather than of special invitation.

The technical and social programs of each Local Section are under the control of the Section. The major technical programs of the quarterly meetings of the Society, each in charge of a Technical Division, and independent papers of members of the Division based upon or prompted by the research work of the Division, furnish a large part of the material published in *Civil Engineering*, with more highly technical papers appearing in *Proceedings* and *Transactions*. The high quality of the publications of the Society has been maintained for more than half a century and needs no comment.

With such an organization as has been described, with a loyal membership made up of the leading civil engineers in the country, uniformly strong in all the states, financially sound, and with well established and favorably known publications, the American Society of Civil Engineers is, as it should be, the one professional society which is in the best position to help the individual members of the civil engineering profession in difficult times like the present. It should assume leadership in many matters. It should by its example develop a spirit of co-operation among all the engineering organizations.

The wide differences in the fields of technical engineering make it unlikely that there will ever be one all-inclusive national organization of engineers. The other great national societies will inevitably gain in numbers and strength as other fields of engineering continue to increase in importance.

Every state and every large city has a goodly number of engineers and men who are intimately connected with engineering work and keenly interested in it, none of whom is eligible for membership in the national societies. This fact is responsible for the existence of the more than forty state and nearly one hundred and twenty-five local engineering societies, all of which number in their membership many who belong to this and the other national organizations.

Most of these state and local societies have a real service to perform, and none of them is in any sense a competitor of this Society. It is with these groups, as well as with Local Sections of other national societies, that our Local Sections must work and co-operate on all matters of legislation and welfare.

The most annoying problem of the present day is that of Unionization. With labor leaders active in the attempt to bring every employee group into some form of union organization, with political and official pressure being brought to bear in some states or cities to compel such affiliation, many young engineers and employees in sub-engineering positions need some professional society or group to turn to for advice and support.

The true answer is that Engineering is a profession and not a trade. The labor union or trade union does not solve the engineer's problems, and he has no rightful place in the unions because his professional obligation makes him the arbitrator or umpire between the owner and builder, or the public and the contractor on public works. Unless the engineer is a member of a professional society which can and will speak for him, and unless his society has the support and backing and hearty co-operation of the other societies of the profession in that state, he has little chance of having his argument listened to.

The time is apparently here when the engineers of the United States must take the position very definitely that engineering is a profession—yes—a learned profession reached only by years of study and training and experience in design, construction, or research, and that the obligation placed on the engineer by his profession to exercise independence of judgment, to be fair and impartial, honest and unbiased in all matters with which he deals is wholly inconsistent with any form of employee unionism. The obligation is just as binding on the engineer employee of the railroad, power company, city, state, or manufacturing plant as it is on the consulting engineer.

And if it be true that these problems face the profession to-day then it is equally true that the profession must speak through its societies. The societies can no longer devote themselves exclusively to matters of technical theory, construction methods and materials, or of design and research. They must give more thought to registration and the enforcement of registration laws; to the merit system in civil service; to proper classification of employees, and not only to the fixing of adequate salaries for each class, but to insisting that those salaries be paid. They will have to give thought to such matters as political discrimination against engineers, unjust discharge, and many other matters that involve the relationships of the engineer employee of city, state, or federal government.

The American Society of Civil Engineers has been doing several of these things during the past six or eight years. If present conditions persist this Society and all other engineering societies will have to do them in an organized way and to a much greater extent if the young engineer or the employee engineer is going to have his profession speak for him with the same degree of authority that the trade union speaks for its member.

This concept of Society activity is repugnant to many members of the profession. Perhaps the civil engineers need these things to a greater degree than others because they are to a much greater extent employed in public service. During recent years the American Society of Civil Engineers has acted in these matters through its Board of Direction and a few of the large metropolitan sections. If such things are to be undertaken successfully the Board of Direction must have the whole-hearted co-operation of all the Local Sections in carrying it out. The Board cannot do it alone. The Society and all of its sixty-three branches must co-operate actively.

It would appear to be self-evident that any engineering society to achieve real success must be primarily a technical society having as its objective the highest standards of technical, professional, and ethical practice. Furthermore, it should deal with all economic problems which are involved in engineering construction and should fearlessly express its opinion and assume leadership in the formulation of broad economic policies of state and national interest. There is no sound reason why such a society cannot and should not, through its Sections, extend its activities to include also all matters which have to do with the financial and economic well-being of its members. Failure to do so will result in the attempt to organize new societies with this objective and it is obvious that in times like these the long established and financially solvent Founder Societies are in much better position to accomplish definite results than any new group whose energies must be devoted largely to organizing and securing members.

The American Society of Civil Engineers has a record of eighty-six years of fine accomplishment. It has a loyal membership, several thousand of whom have been members for many years, a thousand who have loved the Society and supported it for a third of a century.

The standards of membership qualifications, of ethics, and of professional practice have always been of the highest, and every change in the past has been to raise these standards. They should never be lowered. The Society is aristocratic in the sense that it takes into its membership only those who have proved themselves to be of high personal and professional standing. The traditions of the Society must be kept.

It is respectfully submitted that it is such an organization, and only such, that should be looked to for leadership by the engineering profession; and this leadership must be in all matters that pertain to the social, economic, and political welfare of the Engineer as well as in his practice, ethics, and scientific perfection.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2014

STANDARD PRACTICE IN SEPARATE SLUDGE DIGESTION

FINAL REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON SLUDGE DIGESTION

INTRODUCTION

Satisfactory disposal of sludge is an important part of the treatment of sewage, from the standpoint of cost as well as of sanitation. It has long been a difficult problem, and many processes and combinations of processes have been experimented with and developed.

At present, primary sludge is usually digested in separate compartments or in the lower part of an Imhoff tank, dried on sand beds, and disposed of either as waste or as fertilizer. Mechanical dewatering is being used as an alternate to the sand-bed for large projects. Activated sludge is frequently dewatered, dried, and sold as fertilizer, but it, also, may be digested with primary sludge in separate tanks.

Although there is doubt as to the first use of the method known as separate digestion, several of the earliest sludge-treatment plants employed it. The late Rudolph Hering, M. Am. Soc. C. E.,[1] states that, in 1884 and 1885, separate digestion tanks were used in tests at the Pimlico Pumping Station, in London, England, under the direction of Mr. W. J. Dibdin. The method was again used about 1899, at the Lawrence (Mass.) Sewage Testing Station, and the results are discussed in the Annual Reports for that Station for 1899 and 1900.

In the first decade of the present century, Imhoff and Travis developed the 2-story tank. Most of the sludge-digestion plants installed during the second decade were of that type, but since about 1920 the trend has been decidedly toward the adoption of the separate system; at present, several hundred such plants are in operation in the United States.

Structures and equipment for separate digestion of sludge may account for from 20 to 35% of the total cost of the sewage treatment works. They include digestion tanks, storage tanks, pumping stations, drying beds, meters, boilers, buildings, etc. Ample capacity and good design as to detail of

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[1] *Engineering News*, August 14, 1913.

arrangement and of mechanical equipment are required, as a deficiency in these points not only hinders digestion, but may result in reducing the efficiency of sedimentation and other processes thus lowering the entire standard of plant operation.

Good design alone is not enough, however; even in the best plant, satisfactory operation depends upon the operator's knowledge of the basic principles involved, and upon his constant attention to the proper methods of testing and routine procedure.

Adoption of the separate digestion method and progress in sludge-treatment research have been so rapid that on the whole neither the principles of design nor the methods of operation have been thoroughly correlated. The purpose of this report, therefore, is to state the fundamental aspects of sludge digestion and to assemble in useful form worth-while information relating to the design and operation of separate sludge-treatment plants.

The outline and general content of this Final Report is the same as the Progress Report published in 1937[2], with several essential additions and corrections. The report begins with a description of sewage and of the end products of digestion—gas and digested sludge. Important environmental factors in the digestion process are then discussed. Next, the various types of sludge-digestion and gas-utilization structures and equipment are described, and methods for computing the required capacity of tanks and heating units are presented. The following section outlines the various proposals for disposing of sludge after digestion, and the report closes with a discussion of operating routine. Two Appendices are included, giving data on the operation of typical plants.

I.—CHARACTERISTICS OF SEWAGE AND OF THE END PRODUCTS OF DIGESTION

Sewage

The water-borne wastes of a modern city are made up of such a great variety of compounds that only a general discussion of sewage characteristics is practicable. Data on the composition of sewage have been secured from thirty cities representing a total contributing population of 4 760 000, and are summarized in Tables 1 and 2. It will be noted that the average daily flow per capita varies from 306 gal to 48 gal, and that a marked trade-waste contribution was reported by nine cities, while this factor was negligible in the others. Fifteen of the cities reported combined sewerage systems, and fourteen, separate systems.

Waste products are present in sewage as solids, colloids, and solutions. The solids, in turn, may be classified as volatile or organic, and fixed or mineral (sometimes called "inert"); or, with respect to their settleability, they may be graded from the readily settleable to the practically non-settleable. Colloids ordinarily undergo a marked reduction when sewage treatment is carried to a high degree of purification. The reduction, however, can not be accomplished by sedimentation unless that process is preceded by some special treatment adapted to that end. The materials in solution, also, are obviously not susceptible to removal by plain sedimentation, although certain of the soluble ingredients of the incoming sewage may stimulate sedimentation. (They may also retard or accelerate the digestion of the sludge.) Any changes that the dissolved wastes undergo in their passage through the settling tanks is due to bio-chemical agencies.

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[2] *Proceedings*, Am. Soc. C. E., January, 1937, p. 39.

One factor having an important bearing on the accuracy and comparability of analytical data is the state of comminution of the solids in the samples on which the data are based. A sample from sewage in which very little comminution has taken place may be too high or too low in solids,

TABLE 1.—CHARACTERISTICS OF SEWAGE OF TYPICAL CITIES OF THE UNITED STATES

(From Data Secured in 1930. Total Contributing Population, 4 760 000)

Item No.	City	Tributary population, in thousands (as of 1930)	Average sewage flow, in gallons per capita per day	Type of sewerage system‡	Trade waste content	Total Solids, in Parts per Million			Number of samples on which value is based§	Suspended Solids				
										Total Suspended Solids			Volatile Suspended Solids	
						Total	Volatile	Fixed		Actual, in parts per million	Parts per million adjusted to sewage flow of 100 gal per capita per day	Pounds per capita per day	Actual, in parts per million	Parts per million adjusted to sewage flow of 100 gal per capita per day
	(1)	2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	Akron, Ohio........	220	156	C	Yes	2 069	870	1 199	W	377	588	0.491	285	445
2	Alliance, Ohio.......	24	97	S	No	880	406	474	52	174	169	0.140	124	120
3	Aurora, Ill..........	47	122	C	..	909	...	...	...	167	...	......	125	...
4	Baltimore, Md......	735	82	S	No	.....	...	...	...	266	...	0.182	...	...
5	Brockton, Mass.....	54	51	S	Yes	671	383	288	W	245	125	0.104	198	101
6	Canton, Ohio.......	100	97	S	No	.....	...	...	...	288	279	0.233	...	...
7	Cleveland, Ohio....	...	...	C	Yes	678	...	...	...	184	...	......	...	...
8	Columbus, Ohio....	300	97	C	No	986	396	590	172	222	215	0.178	175	170
9	Dallas, Tex.........	273	73	S	..	1 334	605	729	150	342	250	0.208	258	188
10	Decatur, Ill.........	45	306	C	Yes	.....	...	...	365	200	612	0.510	100	306
11	Delaware, Ohio.....	10	48	S	No	.....	...	...	W	235	113	0.094	180	86
12	Elyria, Ohio........	21	71	C	..	666	297	369	250	224	159	0.133	171	121
13	Fitchburg, Mass....	40	58	C	No	398	200	198	...	140	81	0.068	88	51
14	Flint, Mich.........	130	75	S	..	860	484	376	365	225	169	0.141	168	126
15	Grand Rapids, Mich.	168	145	C	No	1 246	728	518	16	221	274	0.229	135	196
16	High Point, N. C...	25	97	S	Yes	738	...	...	365	224	217	0.181	...	...
17	Houston, Tex.......	...	...	..	..	......	...	...	...	246	...	......	...	...
18	Indianapolis, Ind...	328	140	C	Yes	1 040	...	...	365	319	447	0.373	...	...
19	Lincoln, Nebr.......	80	50	S	No	265	174	81	...	265	132	0.110	...	...
20	Madison, Wis.......	32	130	S	No	785	...	...	19	156	203	0.169	121	157
21	Marion, Ohio.......	23	70	S	No	1 261	...	...	W	238	167	0.139	180	126
22	Milwaukee, Wis....	700	121	C	Yes	1 021	471	550	615	328	397	0.331	218	264
23	Pasadena, Calif.....	115	74	S	No	...	...	...	W	290	215	0.179	247	183
24	Philadelphia, Pa....	175	188	C	No	.....	...	...	416	188	353	0.295	124	233
25	Pontiac, Mich.......	56	91	S	Yes	809	436	373	184	359	327	0.273	181	165
26	Rochester, N. Y.*...	298	169	C	..	615	310	305	365	141	238	0.199	99	167
27	Rochester, N. Y.†...	15	117	S	No	689	355	334	W	166	194	0.163	121	142
28	Springfield, Mass...	67	112	C	No	.....	...	...	300	186	208	0.174	116	130
29	Syracuse, N. Y.....	200	142	C	No	.....	...	...	365	218	310	0.258	...	...
30	Worcester, Mass....	190	92	C	Yes	785	321	464	M	251	231	0.193	154	142
31	Dayton, Ohio.......	...	92	..	..	....	...	...	...	202	186	0.155	167	154

* Irondequoit Works. † Brighton Works. ‡ C = combined, and S = separate systems. § W=Weekly composite and M=Monthly composite. In general, analyses were made on composite samples.

TABLE 1.—(*Continued*)

Item No.	Suspended Solids (*Continued*)		Settleable Solids						Five-Day Bio-Chemical Oxygen Demand at 20°C		Ether soluble matter, in parts per million	pH	Chlorine as chlorides, in parts per million	Immediate chlorine demand	N as NH_3, in parts per million
			In Cubic Centimeters per Liter		In Parts per Million										
	Volatile suspended solids, in pounds per capita per day	Fixed suspended solids, actual, in parts per million	Actual	Adjusted to sewage flow of 100 gal per capita per day	Actual	Adjusted to sewage flow of 100 gal. per capita per day	In pounds per capita per day	In percentage of total suspended solids	In parts per million	In pounds per capita per day					
	(15)	(16)	(17)	(18)	(19	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)
1	0.371	92	....	.....	305	476	0.397	81	146	0.190	....	7.2	717	25.5	8.5
2	0.100	42	5.2	5.1	...	.....	.....	..	193	0.156	....	...	72	....	17.8
3	....	....	3.5	.....	...	.....	.....	..	120	.....	....	7.6	156	....	.....
4	....	47	....	.....	164	135	0.112	62	235	0.209	....	...	.....	....	.....
5	0.084	....	....	.....	...	.....	.....	..	...	.....	....	6.9	85	....	37.9
6	....	....	5.1	4.9	224	217	0.181	78	202	0.163	....	7.3	79	....	.....
7	....	47	2.9	.....	...	.....	.....	..	163	.....	....	7.1	67	....	17.5
8	0.141	84	3.9	3.8	120	116	0.097	54	226	0.182	....	7.1	92	6.7	.....
9	0.157	100	9.1	6.7	...	.....	.....	..	445	0.271	....	7.5	.....	....	13.2
10	0.255	55	2.8	8.5	...	.....	.....	..	154	0.392	....	7.0	.....	9.5	8.0
11	0.074	53	5.3	2.5	...	.....	.....	..	344	0.138	....	8.0	.....	....	.....
12	0.101	52	3.5	2.5	...	.....	.....	..	171	0.101	....	...	53	....	16.3
13	0.043	57	....	.....	93	54	0.045	66	...	.....	....	...	43	....	10.5
14	0.105	86	5.0	3.7	...	.....	.....	..	280	0.175	48.0	7.6	99	12.0	7.9
15	0.163	....	4.4	.....	...	.....	.....	..	169	0.274	....	7.1	.....	....	.....
16	....	....	....	.....	...	.....	.....	..	358	0.289	....	...	.....	....	16.9
17	....	....	....	.....	...	.....	.....	..	193	.....	....	7.6	.....	....	18.2
18	....	....	....	.....	...	.....	.....	..	338	0.395	39.3	7.7	.....	7.9	.....
19	0.072	35	3.3	1.7	...	.....	.....	..	...	.....	....	...	.....	....	.....
20	0.131	58	6.5	8.5	...	.....	.....	..	216	0.234	....	7.4	.....	....	14.1
21	0.105	110	3.7	2.6	...	.....	.....	..	270	0.158	....	7.4	89	....	21.1
22	0.220	43	....	.....	...	.....	.....	..	269	0.272	52.4	7.3	.....	....	.....
23	0.152	64	9.1	6.7	...	.....	.....	..	257	0.159	....	7.2	.....	....	19.0
24	0.194	178	4.6	8.6	...	.....	.....	..	204	0.320	52.0	...	78	....	10.0
25	0.137	42	6.6	6.0	...	.....	.....	..	213	0.162	....	...	.....	....	.....
26	0.139	45	3.1	8.3	...	.....	.....	..	233	0.329	....	7.1	69	....	8.9
27	0.118	70	7.1	5.2	...	.....	.....	..	244	0.239	....	7.1	49	....	8.3
28	0.108	....	4.1	4.6	...	.....	.....	..	166	0.155	....	7.1	51	....	12.1
29	....	97	....	.....	148	210	0.175	68	...	.....	....	7.0	.....	....	.....
30	0.118	35	10.1	9.3	...	.....	.....	..	212	0.163	....	6.8	96	....	14.6
31	0.129	....	5.6	5.1	...	.....	.....	..	...	.....	....	...	.....	....	.....

depending on whether or not the coarse particles happen to have been included. More truly representative samples may be obtained in cases where the sewage has been subjected to pumping, or to flow at high velocity, or even simply to flow through a great length of sewer, for in such cases the suspended-matter content is likely to be more evenly distributed.

Other conditions influencing the reliability of data are: (1) Correctness of the measurement of flow and of the count of the contributing population; (2) the method of sampling and the number of samples taken; and (3) the accuracy of the analytical methods.

It will be noted that twenty-two cities reported settleable solids in cubic centimeters per liter, and six in parts per million. The latter method has the advantage of expressing the results as a definite quantity, but the data are not as valuable as they would be if this method were in general use, because the results cannot be compared with those expressed in cubic centimeters per liter.

TABLE 1.—(*Continued*)

Item No.	Free ammonia, in pounds per capita per day	ORGANIC NITROGEN					IRON AS FE, IN PARTS PER MILLION			COMPOSITION OF SLUDGE				
		Total		Suspended		Dissolved, in parts per million	Total	Suspended	Fixed	Specific gravity	Percentage, H_2O	pH	Percentage of volatile matter	Percentage of ether-soluble matter
		In parts per million	In pounds per capita per day	In parts per million	In pounds per capita per day									
	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)
1	0.011	21.0	0.027	8.8	0.011	12.2	....	....	....	1.08	89.9	6.5	51.1	10.6
2	0.014	18.8	0.010	6.4	0.005	6.4	....	....	....	1.02	95.0	7.4	57.0	.....
3	......	......	......	......	......	......	....	....	....	....	....	....	....	.....
4	......	......	......	......	......	......	....	....	....	....	93.9	6.7	61.2	.....
5	0.009	......	......	......	......	......	....	....	....	1.02	94.0	5.3	....	.....
6	......	......	......	......	......	......	....	....	....	1.03	94.0	6.7	65.0	.....
7	......	10.7	......	8.4	......	2.3	10.4	....	....	1.05	86.8	7.1	40.3	8.3
8	......	......	......	......	......	......	....	....	....	....	....	....	....	.....
9	0.008	29.0	0.018	......	......	......	....	....	....	....	95.0	6.9	....	.....
10	0.020	13.0	0.033	......	......	......	....	....	....	....	93.0	7.0	38.0	3.5
11	......	......	......	......	......	......	....	....	....	1.03	94.1	....	57.8	.....
12	0.010	7.4	0.004	3.7	0.002	3.7	....	....	....	1.04	92.7	7.2	42.0	.....
13	0.005	......	......	......	......	......	....	....	....	1.05	85.6	7.1	34.4	.....
14	0.005	28.0	0.017	......	......	......	....	....	....	....	92.8	6.6	48.3	10.1
15	......	......	......	......	......	......	....	....	....	....	95.1	7.4	67.7	.....
16	0.014	10.2	0.008	......	......	......	....	....	....	....	88.4	7.8	40.5	6.3
17	......	......	......	......	......	......	....	....	....	1.05	....	....	65.0	.....
18	......	28.3	0.039	......	......	......	6.4	....	....	1.07	80.0	....	45.0	3.1
19	......	......	......	......	......	......	....	....	....	....	89.2	....	....	.....
20	0.015	18.5	0.020	......	......	......	....	....	....	1.02	95.0	7.4	50.0	10.0
21	0.012	13.3	0.008	7.9	0.005	5.4	....	....	....	1.02	92.8	8.1	50.0	8.0
22	......	31.8	0.032	......	......	......	35.0	....	....	....	....	....	....	.....
23	0.012	20.8	0.013	......	......	......	....	....	....	....	....	....	....	.....
24	0.016	26.0	0.041	......	......	......	....	....	....	1.03	90.4	....	48.0	17.7
25	......	......	......	......	......	......	....	....	....	....	91.6	7.0	53.9	.....
26	0.012	......	......	......	......	......	....	....	....	....	92.8	6.6	49.6	11.6
27	0.008	......	......	......	......	......	....	....	....	....	88.7	6.6	46.9	8.3
28	0.011	19.6	0.013	......	......	......	....	....	....	....	90.4	7.0	40.7	.....
29	......	......	......	......	......	......	....	....	....	....	98.0	7.0	45.0	.....
30	0.011	13.3	0.010	6.7	0.005	6.6	47.2	24.0	23.2	1.03	93.3	6.8	48.8	.....
31	......	......	......	......	......	......	....	....	....	....	....	....	....	.....

TABLE 2.—RÉSUMÉ OF SEWAGE CHARACTERISTICS, BASED ON A SEWAGE FLOW OF 100 GALLONS PER CAPITA PER DAY

(Computed from Data in Table 1 for Typical Cities of the United States)

Item	Number of sewage works reporting	Contributing population, in thousands	PARTS PER MILLION			POUNDS PER CAPITA PER DAY		
			Maximum	Minimum	Average	Maximum	Minimum	Average
Total suspended solids	27	3 711	612	81	254	0.51	0.07	0.21
Volatile suspended solids	22	2 978	445	51	171	0.37	0.04	0.10
Settleable solids	19	2 144	9.3*	1.7*	5.4*	.....	.....	......
Settleable solids	6	1 595	476	54	20	0.40	0.04	0.17
Bio-chemical oxygen demand, 5 days, at 68° F	22	3 337	473	121	262	0.39	0.10	0.22
pH-value	23	3 437	8.0†	6.8†	7.3†	.....	.....	......
Immediate chlorine demand	6	1 213	39.8	6.5	.....	.....	.....	......
Nitrogen as free ammonia	18	1 937	34.5	6.1	14.3	0.020	0.005	0.012
Organic nitrogen	16	2 606	48.8	5.2	22.1	0.041	0.004	0.018

* Values are in cubic centimeters per liter. † Values are not parts per million.

Digested Sludge

The characteristics of digested sludge depend largely upon whether the contributing sewerage system is separate or combined, and upon the proportion of industrial wastes and the degree of treatment of the sewage. Sludge from combined sewers usually contains more mineral matter and less moisture than sludge from separate sewers. Abnormal quantities of oil, iron, or other chemicals may be due to industrial wastes. Fine screening, skimming, or removal of detritus in preliminary tanks will reduce the quantity of coarse suspended solids in the sludge.

Color is somewhat of an indicator of the degree of digestion, well-digested sludge being usually quite black or very dark gray. Color, however, depends somewhat upon local conditions—particularly upon the type and quantity of industrial wastes. The moisture content of digested sludge should be less than that of fresh sludge, although the reduction in some cases is not great. In general, well-digested sludge has the following characteristics:

Physical.—

(*a*) Texture: Uniform, black or dark colored, with no visual evidence of fresh sewage solids, such as feces, paper, or grease.

(*b*) Density: Thick; the moisture content of sludge on withdrawal from the tanks is generally not more than 90 per cent. (Sludge with as much as 95% moisture, however, may be thoroughly digested.)

(*c*) Odor: Faintly tarry; practically unobjectionable. After draining or drying, well-digested sludge should have an earthy or musty odor.

(*d*) Ability to drain on sand-beds: Good.

Chemical.—

(*a*) Mineral matter content: Substantially greater (expressed as a percentage) than that of the fresh sewage solids.

(*b*) Volatile matter content: 50%, or less than 50%, of the volatile matter content of fresh sludge.

(*c*) Organic content (of sludge removed from the drying beds): Not generally more than 50 per cent. The bio-chemical oxygen demand of this content should be substantially lower than that of the fresh solids.

(*d*) Hydrogen-ion concentration: pH usually between 7.0 and 7.6; that is on the alkaline side of neutrality.

It should be noted that the process of digestion reduces the value of the sludge for use as fertilizer by as much as two-thirds, because of the gasification of organic matter containing nitrogen.

Gas

Gas produced during the digestion of sludge consists principally of methane (CH_4) carbon dioxide, and nitrogen, but includes also small quantities of hydrogen, hydrogen sulfide, and other gases. Although the composition depends upon the character of the sewage and the stage to which digestion has progressed, the particular sources of the various constituents are not definitely known.

Table 3 shows the analyses of gases collected at various sewage works. The average composition, based on the data there given, is approximately 68% methane, 25% carbon dioxide, 5% nitrogen, and 2% hydrogen. In general, gas obtained at German installations contains more hydrogen and hydrogen sulfide than that produced in the United States.

TABLE 3.—AVERAGE ANALYSES AND HEAT VALUES OF GASES COLLECTED AT SEWAGE TREATMENT WORKS*

(Gas from Separate Digestion Tanks Unless Otherwise Noted)

Location	Date	Percentage by Volume							Heat value, in British thermal units per cubic foot
		CH_4	CO_2	N_2	O_2	H_2	H_2S	Other gases	
Ann Arbor, Mich.	1937	64.4	25.6	8.7	.....	.....	......	1.3[r]	672
Antigo, Wis	1930	62.0	31.4	3.4	0.6	2.6	......	.....	596[l]
Aurora, Ill.[a]	1930	51.8	32.3	13.4	0.4	0.0	Slight	2.1[i]	534[l]
Aurora, Ill.[b]	1930	48.2	28.9	19.8	0.8	0.0	Slight	2.2[i]	495[l]
Aurora, Ill.[t]	1936	61.6	31.8	2.5	0.5	3.6	.....[q]	.....	570
Baltimore, Md.	1930	.....	22.6–31	.....	.....	.....	......	.....	675–779[l,m]
Battle Creek, Mich.	1937	65.0	34.4	0.2	0.1	0.0	......	0.3[j]	590
Berlin, Germany (Wassmannsdorf)	1929–30	69.59	17.77	8.36	0.0	4.15	0.13	.....	719
Birmingham, England (Saltley)	1930	69.29	28.02	1.89	0.26	0.05	0.0	0.49	685[l]
Bochum, Germany	1929–30	64.2	22.1	7.1	.....	6.6	......	.....	687
Cedar Rapids, Iowa	1937	68.6	30.1	0.7	0.6	.....	......	.....	601
Chicago, Ill. (Calumet)	1926–27	76.6	14.7	8.2	0.5	.....	......	.....	780
Dayton, Ohio	1930	76.6	14.0[g]	9.0	0.4	0.0	Some[g]	.....	777
Decatur, Ill	.....	70–75	15–20	4–6	0	1–3	0–2	.....	700[l]
Elyria, Ohio[c]	1930	85.0	12.8	0.3	0.7	1.2	Not run	.....	900[l]
Elyria, Ohio[d]	1930	69.0	29.8	0.1	0.5	0.6	Not run	.....	740[l]
Essen-Frohnhausen, Germany[e]	1929–30	59.8	29.6	6.0	.....	4.6	......	.....	619
Essen-Frohnhausen, Germany[c]	1929–30	63.3	27.1	5.1	.....	4.5	......	.....	631
Essen-Nord, Germany	1929–30	69.0	20.8	7.0	.....	3.2	......	.....	715
Essen-Nordwest, Germany	1929–30	68.3	19.3	6.8	.....	5.6	......	.....	704
Essen-Rellinghausen, Germany	1930	68.3	28.3	3.4	0.1	.....	0.03–0.2	.....	732
Gelsenkirchen-Nord, Germany	1929–30	71.0	20.0	8.0	.....	1.0	......	.....	730
Grand Rapids, Mich	1931	65.30	31.80	1.34	0.10	1.12	......	0.34[j]	707[l]
Halle, Germany (Tafelwerder)[f]	1929–30	69.8	29.0	1.1	0.0	0.0	0.1	.....	787
Halle, Germany (Tafelwerder)[c]	1929–30	72.9	24.6	1.6	0.6	0.0	0.3	.....	.....
Hattingen, Germany	1930	76.0	16.1	2.3	.....	5.6	......	.....	818
High Point, N. C. (East Side)	1929–30	.....	28.0	.....	.....	.....	......	.....	683
Iserlohn, Germany	1930	72.6	23.1	4.2	0.1	.....	......	.....	.....
Kettwig, Germany	1929	75.8	14.4	9.6	0.0	.....	......	.....	815
Langendreer, Germany	1930	81.5	18.0	0.5	.....	.....	......	.....	876
Leipzig-Wahren, Germany	1930	70.5	27.0	1.9	.....	0.6	0.01	.....	742
Los Angeles, Calif. (Experimental Station)	1930	68.5	12.0	17.5[h]	2.0	0.0	0.0	.....	689
Munich, Germany (Grosslappen)	1930	77.0	16.0	7.0	.....	Trace	Trace	.....	753
Newark, N. Y.[s]	1935	67.7	27.7	4.1	0.2	0.0	......	0.3[j]	.....
Nürnberg, Germany (Nürnberg-Sud)	1930	.....	23.1	.....	0.7	.....	......	.....	722
Oberhausen, Germany	1929–30	66.4	23.0	6.4	.....	4.2	......	.....	697
Peoria, Ill.	1937	66.9	31.6	.....	0.0	.....	......	.....	609
Plainfield, N. J. (Joint)	.....	65.8	30.6	3.6	0.0	0.0	Slight	.....	.....
Springfield, Ill.	1937	59.6	31.8	.....	.....	3.0	5.6	.....	551
Stuttgart, Germany (Mühlhausen)	1930	70.6	23.2	1.45	0.2	4.475	0.075	.....	772
Toledo, Ohio[p]	1937	59.5	29.6	1.2	2.2	4.1	.....[o]	3.4[n]	632
Toronto, Ont., Canada (North Toronto)	1930	59.26	29.35	0.50	0.19	7.70	0.00	3.00[k]	622

* Values for German localities were converted from calories per cubic meter.

[a] Sample taken October 17. [b] Samples taken October 6. [c] Gas from Imhoff tanks. [d] Gas from separate digestion tanks. [e] Gas from Prüss digestion tank. [f] Gas from supplementary tanks. [g] H_2S included with CO_2 percentage. [h] Air included with N_2 percentage. [i] CO. [j] Illuminants. [k] 0.77% CO; 2.23% illuminants. [l] Calculated at sewage treatment works. [m] Calculation based on laboratory work relating to sludge digestion. [n] 0.4% illuminants; 3.0% CO_2. [o] 6 grams per 100 cu ft. [p] December 16, 1935. [q] 24 grams per 100 cu ft. [r] Ethene CH_3CH_3. [s] May 16, 1937. [t] Average for 5 yr. period.

TABLE 4.—DATA RELATING TO GAS COLLECTION AT SEWAGE TREATMENT WORKS*

Location	Period of Observation		Contributing population	Sewage flow, in million gallons daily	Suspended solids removed, in parts per million	Characteristics of Raw Sludge		Type of Digestion Tanks from Which Gas Was Collected		General Data on Digestion			Quantity of Gas Collected, in Cubic Feet			
	Year	Length of period				Volatile solids, percentage by weight of total solids	Moisture content (percentage)	Type	Heated	Temperature, in degrees Fahrenheit	pH	Time, in days	Average daily	Per million gallons daily Column (14) Column (5)	Per pound of volatile matter added (Col. (14) × 100) Col. (5) × Col. (6) × Col. (7) × 8.33	Per capita daily, Column (14) Column (4)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Ann Arbor, Mich.	1937	11 months	39 000	3.7	......	78.5	95.1	Separate	Yes	......	......	.....	52 000	14 000	....	1.33
Antigo, Wis.	1930	1 yr.	5 600	0.594	258[a]	....	95.5	Dorr	Yes	80.0	7.2	.....	7 240	12 000	....	1.29
Aurora, Ill.	1930	11 months	43 000	4.89	......	....	.....	Dorr	Yes	97.6	.....	180[a]	33 190	6 800	....	0.77
Aurora, Ill.	1936	12 months	47 000	7.1	......	71.6	96.2	Dorr	Yes	......	.....	.....	45 630	6 450	....	0.97
Baltimore, Md.	1930	1 yr.	12 000[a]	......	206	77.3	94.6	Separate (covered)	Yes	73.4	6.9	.....	5 130	....	....	0.43[e]
Battle Creek, Mich.	1936–37	14 months	40 000[i]	5.5	183	82.8	94.7	Dorr	Yes	105	7.6	.....	55 165	10 000	11.8	1.38
Berlin, Germany (Wassmannsdorf)	1929–30	1 yr.	435 762	17.0	278	75.7	95.7	Imhoff	No	62.5	7.3	60–90	186 740	11 000	6.3	0.43
Birmingham, England (Saltley)	1930	1 yr.	......	36.0	380	56.8	91.75	Separate (uncovered)[m]	No[f]	46.6	7.5	90[a]	128 000	3 600	2.0	......
Bochum, Germany	1929–30	1 yr	183 000	13.39	......	64.0	94.0	Imhoff	No	......	6.8	80	34 800	2 600	....	0.19
Cedar Rapids, Iowa	1936–37	1 yr.	55 000	7.9	309	77.0	95.2	Separate	Yes	98	.....	.....	143 100	18 100	9.4	2.60
Charlotte, N. C. (Sugar Creek)	.....	....	45 000	4.0	200[d]	....	95.0	Separate (uncovered)	No	......	7.0	120	......	....	....	0.5[h]
Chicago, Ill. (Des Plaines)	1927–28	1 yr.	1 000[a]	......	......	75.0	.....	Dorr	Yes	73.3	7.3	180	400	....	....	0.40[a]
Chicago, Ill. (Calumet)	1926–27	1 yr.	4 400	1.4	63	55.0	.....	Imhoff	No	58.0	6.3–7.0	150	1 930	1 400	4.8	0.44
Chico, Calif.	.....	....	10 000	1.0	......	....	.....	Dorr	Yes	70–85	6.8–7.2	.....	11 000[a]	11 000[a]	....	1.10[a]
Dayton, Ohio	1930	1 month	200 982	17.40	119	85.0	.....	Imhoff	No	63.0	6.9	90	67 200	3 900	4.6[a]	0.33
Decatur, Ill.	1929	1 yr.	112 000[b]	11.35	88	60.0	95–97[a]	Imhoff	No	70–85[g]	6.7	.....	49 000	4 300	9.8	0.44
De Kalb, Ill.	1930	3½ months	8 545	0.75	350	45.0	98.5	Dorr	Yes	90.0	8.0–8.2	.....	11 140	14 900	11.3	1.30
Elyria, Ohio	1930	7½ months	13 000[a]	0.9[a]	140[a, d]	85.0	...	Imhoff		......	......	.....	......	....	....	......
								Downes	Yes	78.8	7.1	45[a]	10 000	11 100	11.2[a]	0.77[a]
Essen-Frohnhausen, Germany	1929–30	1 yr.	26 000	1.19	......	70.0	95.0	Pruss (separate)	Yes	61.5	7.3	90	12 700	10 700	....	0.49
								Imhoff (separate)	No	55.9	7.3	170	5 400	4 500	....	0.21
Essen-Nord, Germany	1929–30	1 yr.	120 000	15.32[c]	......	59.0	92.0	Imhoff (separate) Pruss (separate)	No	60.8	7.2	100	48 700	3 200	....	0.41

---

* Quantities and temperatures for German localities were converted from cubic meters or degrees Centigrade.

[a] Approximate. [b] About 45 000 human population plus 67 000 population (equivalent of industrial wastes). [c] High iron content. [d] Mixture of primary and activated sludge. [e] Gas collected from portion only of contents. [f] Sludge which has been warmed by steam is added in water. [g] Winter and summer, respectively, due to warm industrial wastes. [h] Assumed value. Gas quantities not available due to leakage. [i] In addition, packing-house wastes were partly treated. [j] Cooling water from gas engines was used to heat two digestion chambers. [k] Caused by warm wastes from wool washery. [l] Not all the gas was metered. [m] Three Imhoff, two separate, one Neustadt, one Stuttgart, and one supplementary tank. [n] Sludge heated before being added to tanks. [o] Gas utilized. No record of gas wasted.

TABLE 4.—(*Continued*)*

Location	Period of Observation		Contributing population	Sewage flow, in million gallons daily	Suspended solids removed, in parts per million	Characteristics of Raw Sludge		Type of Digestion Tanks from Which Gas Was Collected		General Data on Digestion			Quantity of Gas Collected, in Cubic Feet			
	Year	Length of period				Volatile solids, percentage by weight of total solids	Moisture content (percentage)	Type	Heated	Temperature, in degrees Fahrenheit	pH	Time, in days	Average daily	Per million gallons daily Column (14) Column (5)	Per pound of volatile matter added (Col. (14) × 100) Col. (5) × Col. (6) × Col. (7) × 8.33	Per capita daily, Column (14) Column (4)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Essen-Nordwest, Germany	1929–30	1 yr.	50 000	......	......	58.0	95.0	Imhoff	No	59.0	7.0	110	9 740	....	....	0.19
Essen-Rellinghausen, Germany	1930	4 months	50 000	6.9	.....[d]	....	93.6	Imhoff (supplementary separate)	No	57.2–59.0	7.2–7.8	55	28 200	4 100	....	0.56
Fond du Lac, Wis.	1929–30	....	26 449	1.75	......	....	.....	Dorr	Yes	80.0	7.0	180	18 800	10 700	....	0.71
Fort Worth, Tex.	1930	7 months	148 500[c]	8.17	286[d]	76.0	97.25	Dorr	Yes	88.0	7.15	.....	85 250	10 400	5.8	0.57
Galesburg, Ill.	1935	1 yr.	25 000	3.06	......	59.6	92.7	Separate	Yes	82	.....	.....	14 079	4 600	12.7	0.56
Gelsenkirchen-Nord, Germany	1929–30	1 yr.	100 800	6.92	......	....	96.0	Imhoff (separate)	Partly	68.0	7.0	100	26 000	3 800	....	0.26
Geneva, N. Y.	1936	8 months	17 200	1.55	257	70.0	98.4	Harding	Yes	86	7.0	.....	12 003	7 800	5.2	0.70
Grand Rapids, Mich.	May, 1931	1 month	168 592	20.7	85	65.7	94.0	Dorr	Yes	84.0	7.75	20	159 980	7 700	16.6	0.95
Grand Rapids, Mich.	1935–36	1 yr.	176 000	28.2	78	67.1	94.6	Dorr	Yes	88	7.8	.....	129 665	4 598	6.4	0.74
Hagen, Germany	.....	....	90 000	7.9	......	....	.....	Imhoff (supplementary)	....	......	......	.....	26 500	3 400	....	0.29
Halle, Germany (Tafelwerder)	1929–30	1 yr.	190 000	6.9	340	67.0	96.0	Imhoff (supplementary)	.... Yes	...... 68.0–77.0	6.8–7.0 7.3	60–90 15	75 400	10 900	5.8	0.40
Hattingen, Germany	1930	7 months	17 000	0.7	246[d]	70.0	92.0	Imhoff with aerators	Yes	60.8	7.5–8.0	30	9 400 7 620	13 400 3 000	9.4 3.5	0.55 0.21
High Point, N. C. (East Side)	1929–30	13 months	36 745	2.51	160	64.8	93.8	Separate	Yes	77.0	7.0	90[a]	......	....	....	......
Iserlohn, Germany	1930	1 yr.	19 000	0.9	437[d]	....	92.0	Imhoff	No[j]	57.2	7.2	.....	7 940	8 800	....	0.42
Kettwig, Germany	1929	1 yr.	6 300	0.7	......	....	.....	Imhoff	No	62.6[h]	7.2–7.3	80	2 510	3 600	....	0.40
Langendreer, Germany	1930	1 yr.	30 000	2.1[c]	......	....	90–95	Imhoff	No	......	7.5	.....	2 840	1 400	....	0.10
Leipzig-Wahren, Germany	1930	1 yr.	19 000	0.66	.....[d]	67.0	94.0	Imhoff (with aerators)	No	......	7.2	60	6 500	9 900	....	0.34
Los Angeles, Calif. (Experimental Station)	1930	7 months	1 750	0.13	370[d]	70.0	95.0	Dorr	Yes	85.0	7.1	25	1 410	10 800	5.0	0.81
Middletown, N. Y.	.....	....	21 276	......	......	70.0[a]	.....	Downes	Yes	78–80	.....	.....	17 000[a]	....	....	0.80[a]
Munich, Germany (Grosslappen)	1930	1 yr.	570 000	70.74	......	....	92–93	Separate (uncovered)	No	52.5–54.3	7.2	99–100	266 000[e]	3 800	....	0.47[e]

TABLE 4.—(*Continued*)*

Location	Period of Observation		Con-tributing population	Sewage flow, in million gallons daily	Suspended solids removed, in parts per million	Characteristics of Raw Sludge		Type of Digestion Tanks from Which Gas Was Collected		General Data on Digestion			Quantity of Gas Collected, in Cubic Feet			
	Year	Length of period				Volatile solids, percentage by weight of total solids	Moisture content (percentage)	Type	Heated	Temperature, in degrees Fahrenheit	pH	Time, in days	Average daily	Per million gallons daily Column (14)	Per pound of volatile matter added (Col. (14) × 100)	Per capita daily, Column (14)
														Column (5)	Col. (5) × Col. (6) × Col. (7) × 8.33	Column (4)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Neuheim, Germany	1930	1 yr.	6 000	0.4	......	....	.....	Imhoff	No	50.5	7.4	.....	1 450	3 600	....	0.24
New Castle, Pa.	1930	1 yr.	48 674	4.31	......	....	.....	Imhoff	No	59.5	6.9	.....	25 400	5 900	....	0.52[l]
Nürnberg, Germany (Nurnberg-Süd)	1930	1 yr.	190 000	11.3	127	64.1	93.5	Imhoff	No	56.3	7.2	80[e]	46 400	4 100	6.1	0.24
Oberhausen, Germany	1929–30	1 yr.	30 000	1.24	......	65.0	93.0	Separate	No	59.2	7.4	90	11 760	9 500	....	0.39
Peoria, Ill.	1937	1 yr.	125 000	12.8	......	71.8	96.6	Dorr	Yes	......	.....	.....	190 600	15 300	....	1.57
Plainfield, N. J. (Joint)	.....	....	50 000	3.3	128	76.0	95.0	Downes	Yes	70.0	7.2–7.4	40	16 000[e]	4 900[e]	6.0[e]	0.32[e]
San Antonio, Tex.	1936	1 yr.	220 000	18.3	179	75.5	93.3	Dorr	Yes	......	.....	.....	......	....	....	....
Sioux Falls, S. Dak.	.....	....	......	......	......	....	.....	....	....	......	.....	.....	......	....	....	......
Springfield, Ill.	1930	....	71 864	5.0	......	....	.....	Downes	Yes	82.0	6.9	.....	50 000	10 000	....	0.70
Springfield, Ill.	1937	1 yr.	75 000	8.4	179	65.6	96.9	Downes	Yes	99	7.2	.....	55 100[o]	6 562	6.51	0.74
Sturgis, Mich.	1930	7 months	6 950	0.5	......	....	.....	Dorr	Yes	76.0	7.2	.....	3 740	7 500	....	0.54
Stuttgart, Germany (Mühlhausen)	1930	1 yr.	360 000	17.1	442	59.6	92.0	Miscellaneous[m]	No	59.0	7.74	120	132 000	7 700	3.5	0.37
Toledo, Ohio	1937	1 yr.	300 000	34.1	......	69.4	94.5	Separate	Yes	......	.....	.....	254 000	7 450	....	0.85
Toronto, Ont., Canada (North Toronto)	1930	6½ months	56 000	5.08	164[d]	77.9	96.5	Dorr	No[n]	82.0	7.2	40[e]	28 080	5 500	5.2	0.50
Velbert, Germany	1930	1 yr.	18 000	0.6	340	60.0	.....	Imhoff	No	52.0	7.5	180	4 890	8 100	4.8	0.27
Waco, Tex.	.....	....	52 848	3.5	235[d]	65.0	96.5	Dorr	Yes	108.0	7.5	40[e]	48 000	13 700	10.8	0.91
Waukesha, Wis.	1929–30	1 yr.	17 176	1.75	......	....	96.0	Dorr	Yes	74.0	7.4	180	12 000	6 900	....	0.70
Wetter, Germany	1929	1 yr.	6 000	0.2	......	....	.....	Imhoff	No	54.3	.....	.....	1 790	9 000	....	0.30

Heat value varies almost directly with the percentage of methane, although the hydrogen content, while small, has some effect. The heat value of gas from the installations reporting in Table 3, is grouped and averaged as follows:

Group	Average Btu per cubic foot
European	738
United States:	
Imhoff	789
Separate	639

In their early studies, Buswell and his associates assumed that the weight of gas produced was equal to the summation of the weights of protein, grease, cellulose, and crude fiber digested. Buswell later found that the higher fatty acids of grease produce more than their equivalent weight of gas, because more or less water enters into the reaction. The quantity depends upon the number of carbon atoms in the acid. Buswell and Pearson obtained 1.25 lb of gas per lb of volatile matter digested, at Champaign-Urbana, Ill.

Information relating to quantities of gas collected at various sewage works has been assembled in Table 4. It is apparent that no averages can be made which are of general applicability, because the characteristics of the local sewage are so different. The average of certain groups is indicated as follows for those plants listed in Table 4:

Group	Cubic feet of gas per capita per day
Germany:	
Unheated tanks	0.41
Heated tanks	0.53
United States (excluding Cedar Rapids):	
Imhoff	0.55
Plain sedimentation	0.76
Trickling filters	0.92
Activated sludge	0.91

At Antigo, Wis., Cedar Rapids, Iowa, Chicago, and DeKalb, Ill., the gas production is high, possibly because of the nature of the industrial wastes in the sewage.

II.—ENVIRONMENTAL FACTORS

Among the environmental factors that have an extremely important influence on the bio-chemical processes of sludge digestion may be mentioned the temperature at which digestion is carried on, the hydrogen-ion concentration and the percentage of return sludge (reaction and seeding), and the trade-waste content. Other environmental factors of less importance include pressure and density. All these factors are considered in the following paragraphs. Attention should first be called, however, to the fact that the properties of the influent to the sludge digestion equipment may themselves have a decided effect on digestion. These properties depend on the nature of the processes to which the influent has already been subjected. Thus, if the influent comes directly from primary settling tanks through which crude sewage flows, it will consist almost entirely of fresh, or primary, sludge. On the other hand, if primary settling tanks are operated as septic tanks, a partly digested sludge will result, that may be termed septic sludge; it will ordinarily be a mixture of relatively fresh and quite well-digested sludge, the proportion depending largely upon operating conditions. Still other general types of digestion-equipment influent are secondary sludge (from final or secondary settling tanks following filters) and activated sludge (from settling tanks following aeration tanks).

TEMPERATURE

Temperature has been found to have a considerable influence on the time required for sludge digestion, as it determines to a large extent the type of organism that predominates in the sludge. The importance of this factor has led to considerable investigation.

Until a few years ago, prior to the time of temperature control, sludge digestion was carried on at or near the temperature of the sewage, or from about 50°F to 70°F, and in the early investigations of temperature control, the optimum temperature was considered to be about 82°F. Recent investigations have indicated, however, that a temperature of 110°F to 130°F makes possible a still shorter digestion period. Digestion at these higher temperatures has been designated as thermophilic digestion. Less information is available on results at intermediate temperatures. There are not enough operating data from large-sized plants to justify a final conclusion as to the best and most economical temperature. Present information indicates that temperatures between 80°F and 95°F are desirable. Savings in cost and improvement in operating conditions with temperatures in excess of 100°F have not been established.

Some data on the relation between temperature and the time required for digestion are illustrated by Fig. 1. In addition to the effect of temperature on the time, and hence on the capacity, required to bring about a satisfactory reduction in volatile matter, the effect on quantity and rate of production of gas is also important. The rate of gas production appears to be greater at the higher temperatures, and Willem Rudolfs, M. Am. Soc. C. E., states that the total quantity per unit of volatile mater is also greater, although the evidence supporting such a conclusion is as yet limited.

The percentage of reduction in 108 days in the volatile content of sludge, unseeded but with pH-adjustment, at different temperatures, was stated by Rudolfs as follows:

Temperature: In degrees Centigrade	In degrees Fahrenheit	Percentage reduction in volatile matter
10	50.0	16
18	64.4	26
24	75.2	58
29.5	85.1	64
35	95.0	54

Heukelekian[3] states that with temperature control at 70° F, the digestion period is about 35 to 40 days, but he declares confidently that it is possible to draw ripe sludge 24 hr after the addition of the raw material, provided there is ripe active thermophilic sludge and the temperature is maintained at approximately 120° to 125°F. According to Heukelekian:

> "The sludge will be just as ripe as that drawn at the end of 35 to 40 days at 70°F. It will have an ash content of 45 percent, or more, a pH of 7.8 or more, and a B.O.D. [bio-chemical oxygen demand] per percent of organic matter of 1 500 ppm, or less."

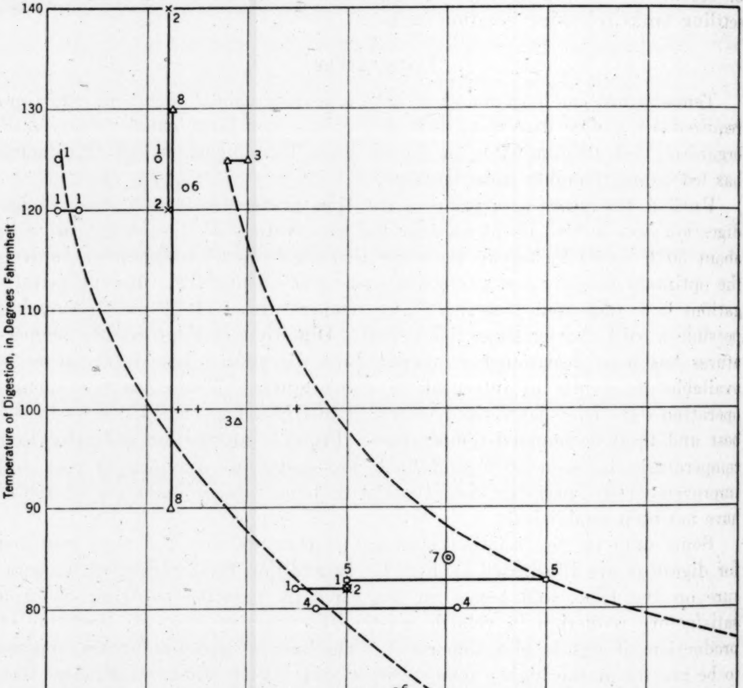

FIG. 1.—EFFECT OF TEMPERATURE ON SLUDGE DIGESTION: 1, HEUKELEKIAN (*Sewage Works Journal*, January, 1931); 2, NEW JERSEY SEWAGE EXPERIMENT STATION (AM. SOC. C. E., COMMITTEE, 1930); 3, RUDOLF AND HEUKELEKIAN (*Industrial and Engineering Chemistry*, January, 1936); 4, HEUKELEKIAN (*Sewage Works Journal*, July, 1930); 5, KEEFER AND KRATZ (*Sewage Works Journal*, July, 1930); 6, HEUKELEKIAN (*Sewage Works Journal*, April, 1930); 7, INDICATED AT GRAND RAPIDS, MICH.; 8, AURORA, ILL. (*Water-Works and Sewerage*, June, 1931)

In discussing the foregoing, Rudolfs states:

"If under ordinary 82°F temperature 50 to 60 percent of the volatile matter is decomposed in 25 to 30 days, about 70 to 75 percent is destroyed at

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[3] "Digestion of Solids between Thermophilic and Non-Thermophilic Range," *Sewage Works Journal*, September, 1933. (Heukelekian found little difference between digestion at 20°C (82°F) and 42°C (108°F).)

120°F in 1 to 3 days * * * if at 82°F the quantity of gas amounts to 6 cubic feet per pound of dry volatile matter, it will be about 7 cubic feet under thermophilic conditions."

At Aurora, Ill., digestion is reported to have been accomplished in 12.5 days at temperatures ranging from 90°F to 130°F.

Keefer and Kratz[4] found that, for Baltimore, Md., sludge, seeded with 50-day digested sludge in the ratio of 2 parts of raw to 1 of seeding sludge, digestion was more rapid at 37°C than at 27°C, but the difference in rate was not great.

In 1930 a committee of the Sanitary Engineering Division of the Society summarized the results obtained at the New Jersey Sewage Experiment Station, as follows:

Temperature in degrees Fahrenheit	Number of days required for digestion
55	120
68	42
82	30
120–140	12
170	11

The actual operating temperatures in several large sludge digestion tanks are shown in Table 5.

TABLE 5.—DIGESTION TANK TEMPERATURES, IN DEGREES FAHRENHEIT

Month, in 1933	Aurora, Ill.	Springfield, Ill.	Rockford, Ill.	Grand Rapids, Mich.	Peoria, Ill.	Erie, Pa. (1932)
January	89	73	80	83	89	67
February	83	75	81	87	89	70
March	82	81	82	84	89	71
April	75	80	86	83	90	67
May	74	80	92	88	90	....
June	78	85	92	91	87	78
July	85	94	88	94	89	81
August	89	93	90	90	86	79
September	90	94	91	89	86	81
October	88	95	92	89	91	81
November	87	....	87	85	89	75
December	90	88	93	85	90	71

Among the considerations that should be taken into account in selecting an operating temperature, and in designing digestion tanks, are:

(*a*) The balance between the cost of providing extra capacity for digestion at a lower temperature and that of extra equipment for heating and insulating at a higher temperature, due consideration being given to the value of the additional gas.

(*b*) The capacity of the selected type of tank for being heated and for retaining heat.

(*c*) The method of heating and rate of transmission of heat.

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[4] "Digesting Sludge at 37° C," *Sewage Works Journal*, March, 1933. (Keefer and Kratz found some advantage in digestion at 37° C (98.6° F) as compared to digesting at 28° C (82° F).

(*d*) The effect of higher water temperatures on the rate at which sludge cakes on heating coils.

(*e*) The effect on the heat balance of the concentration of solids in the tank sludge and in the fresh sludge.

(*f*) The heat lost with the sludge liquor and the possibility of some recovery by heat exchange.

(*g*) The effect of more active gas release on the area and depth of the tank and on the formation of scum.

(*h*) The need for and the relative cost of winter storage.

(*i*) The fact that the total size of sludge digestion and storage tanks is not directly proportional to the time required for digestion.

(*j*) The effect of the duration of low winter temperatures on the relative costs of operation at different temperatures.

Reaction and Seeding

Freshly settled sewage solids are normally alkaline or neutral. The pH-value may vary from 6.8 to 7.2 for solids from domestic sewage. Within the first 24 hr this reaction begins to change toward the acid side, and the longer the solids are held in a settling tank the more acid their reaction becomes. If no seeding sludge is present the solids undergo a period of acid fermentation before true digestion occurs. If suitable seeding sludge is present this acid phase is shortened or eliminated. There are, therefore, two courses of sludge digestion. Acid sludge is developed and digestion delayed when a digestion process, either Imhoff or separate, is started. After ripe sludge is developed this acid condition is restricted and digestion is greatly accelerated. If solids are held for many hours or days, either in sewers or in plain sedimentation tanks, acid sludge will be produced and discharged into the digestion tanks.

The formation of acid in unseeded sludge in process of digestion was studied in Germany 15 or 20 yr ago. Thumm and Reichle, in 1914, noted an acid reaction accompanying poor digestion, and recommended the addition of lime to the sewage.

The first study of the pH-value of fresh solids undergoing digestion was reported by Rudolfs in 1924. Solids collected from the Plainfield, N. J., sewage, which is from separate sewers, were digested at 20°C. Initially 6.4, the pH dropped to a minimum of 4.9 in 9 days, increased to 6.0 in 48 days, to 6.8 in 66 days, and to 7.0 in 120 days. In studies by Fair[5], of the digestion of fresh sewage solids from separate sewers at Brockton, Mass., the initial pH of 6.4 dropped to the very low value of 4.7 in 20 days, recovered to 6.4 in 35 days, to 6.8 in 60 days, and reached a maximum of 6.9 in 70 days. Keefer and Rudolfs report very prolonged acidity with unseeded Baltimore sludge from separate sewers. Solids from the primary settling tanks were pumped into a large open tank and allowed to digest without any seeding material. The pH initially was 6.1 and the volatile matter 65 per cent. The pH fluctuated between 5.5 and 7.3 for the first 200 days, then dropped to 5.6 in 300 days, and 5.2 in 400 days, and slowly rose to a maximum of 6.8 in 830 days. The prolonged acidity was probably due to lack of seeding material and to the low temperature in winter.

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[5] "Time and Rate of Sludge Digestion and Their Variation with Temperature," *Sewage Works Journal*, January, 1934. (Fair and Moore find four zones of temperature with optimum for non-thermophilic at 33°C (91°F).)

On the other hand, the sludges used in the experiments reported by Zack and Edwards, from combined sewers contributing to the Des Plaines Treatment Works of the Sanitary District of Chicago, did not produce high acidities even when unseeded. The fresh solids had a pH of 6.3 when incubated at 25°C, and this value rose to 7.0 in 13 days. The fresh activated sludge had an initial pH of 7.3, which decreased to 6.6 in 10 days and then quickly rose to 7.0 in 20 days. The volatile content of the fresh solids was 67%, and that of the activated sludge was 60 per cent.

The acidity produced by unseeded solids apparently increases greatly if the initial pH is low and the volatile matter high. From a practical standpoint this means that solids high in organic matter, from separate sewers tend to produce greater acidity than solids from combined sewers normally lower in organic matter. Proper seeding, therefore, is more necessary with highly organic solids.

Storage of solids in settling tanks is undesirable, since the acidity increases, and lengthens the period of digestion if seeding material is insufficient. Fair and Klein show that digestion of unseeded solids is greatly retarded if the fresh solids are held until high acidity develops. They state that "it seems desirable that the settling solids reach the digestion tanks as soon as possible."

Undoubtedly, therefore, it is desirable, in separate digestion, to remove the solids from the settling tank as promptly as possible. The production of acid sludge should be prevented throughout the entire digestion cycle; therefore, the initial solids should not be allowed to become acid if it is practicable to prevent it.

Sewage solids do become acid, however, when there is not enough ripe sludge available to prevent acid decomposition. This occurs (1) when digestion tanks are first started: (2) when ripe sludge is lost by improper operation; or (3) when acid industrial wastes are present. From the results of analyses of gas from various digestion tanks, Buswell concludes that a high CO_2 content of the gas may serve as an index of improper seeding conditions, as correlated with acidity shown by pH.

When ripe sludge is not available, lime is commonly used for neutralization of acidity. The experience of a number of plant operators as reported by Bachmann indicates that liming is a useful remedy under abnormal conditions, but hardly to be recommended, from a practical standpoint, for normal operation.

When well-digested sludge is mixed in proper amount with fresh sludge, the reaction of the mixture and the course of digestion are different from those of unseeded sewage solids. Dr. Imhoff states:

> "Basically different from acid fermentation is the second kind, that establishes itself in well-conditioned digestion tanks. It results in the formation of carbon dioxide, nitrogen and, above all, methane (marsh gas), the

latter constituting about 75 percent of the gas by volume. The pH-value consistently remains slightly above the neutral point. The process is odorless and is called alkaline or methane fermentation."

The low pH of unseeded solids is caused primarily by the formation of fatty acids that decompose very slowly, but that must be decomposed before digestion is complete. In the presence of well-digested sludge, these acids decompose more quickly, and, in addition, the reserve alkalinity or buffer action of the digested sludge neutralizes them to a varying degree, depending upon the composition of the inorganic solids in the digested sludge.

The proportion and the character of digesting sludge in the tank are important. Rudolfs found in his earlier work that, at 70°F, the daily additions of fresh organic solids should not exceed 2% of the weight (dry basis) of the organic matter in the digesting sludge. In a later work, he has shown that this ratio might be increased greatly (the ratio in general falls between 2 and 6%) by increasing the temperature and, if necessary, adding lime. The use of lime is not recommended as a routine procedure, however.

Ripe sludge used for seeding must obviously be neutral or slightly alkaline in order to provide the proper buffer action or to neutralize the acid reaction of raw sludge. Its moisture content, within normal limits, is immaterial, but it should be actively digesting sludge.

TRADE WASTE CONTENT

Trade or industrial wastes originate from a great variety of manufacturing processes and have widely differing characteristics. When such wastes are mixed with domestic sewage the quantity of sludge is increased, the extent of increase depending on the characteristics of the particular wastes. The effect of industrial wastes on the process of digestion may vary over a wide range from somewhat beneficial to very detrimental, depending on the characteristics of the industrial sewage, the pre-treatment it has received, and the relative quantities of industrial waste and domestic sewage.

The effects of various industrial wastes on sludge digestion are summarized in Table 6.

PRESSURE

The pressure on sewage solids undergoing digestion results mainly in common practice from the depth of the digestion tank. In one of the shallowest 2-story tanks (at Schenectady, N. Y.) the liquid depth is 13.8 ft, and in one of the deepest (at Rochester, N. Y.), it is 33.9 ft. Most separate digestion tanks have depths of 20 to 30 ft. The pressure may be changed by applying a negative pressure (vacuum) or a positive pressure to the surface of the digesting sludge.

The major possible effects of pressure on sludge digestion are: (*a*) The effect on the biological life and its activities; (*b*) the effects on the gas produced by digestion, as, for example, the rate of gas production, the solubility of the gas, the size of the bubbles, and their motion in the liquid; and (*c*) the effect on the density of the sludge.

Other possible effects, believed to be of minor importance, are the pressure on the viscosity and on the surface tension.

TABLE 6.—EFFECTS OF VARIOUS INDUSTRIAL WASTES ON SLUDGE DIGESTION

Source of wastes	Quantity of sludge produced	Characteristics of sludge	Remarks
Bleacheries........	Small............	Usually alkaline; may contain disinfectants.	The waste contains soaps and decoloring agents.
Canneries..........	Usually large (depends on material canned).	Contains both carbonaceous and nitrogenous organic matter; develops organic acids in digesting.	It is usually necessary to provide preliminary treatment of cannery wastes before mixing with domestic sewage. These wastes are likely to cause foaming and scum.
Creameries and dairies	Depends on treatment process.	Fats and sugars break down forming organic acid, delaying digestion and causing obnoxious odors.	Organic acids that are generated coagulate the casein, which rises to the top as scum that does not digest readily.
Corn-starch plants	Variable, depending on process at corn - products plant.	Contains residual SO_2 which breaks down during digestion to form H_2S.	There is a tendency for sludge to bulk in aeration tanks.
Coke, coal, gas, and creosoting plants	Small............	Contains phenols and other compounds having germicidal characteristics.	This sludge will not digest. Normal domestic sewage sludge will digest if this type of industrial waste does not exceed 2% by volume.
Dye-houses........	Small............	Usually acid, and likely to produce offensive odors.	Will not interrupt digestion if dilution is large.
Meat packing plants.	Large............	Organic matter of high nitrogen content. Also, fats, hair, and paunch manure.	Much of the packing-house sludge digests with difficulty, if at all. Treatment at the source is desirable before mixing with domestic sewage.
Oil (crude oil, fuel oil, and spent crank-case oil)	Variable..........	Relatively inert chemically.	Interferes with digestion by surrounding the sludge particles, preventing proper drainage, and forming undesirable scum.
Paper and pulp mills (strawboard waste)	Very large........	Contains fiber and chemical residues. Sulfite wastes are germicidal.	Sludge digests slowly if at all. Most of the solids in waste from paper mills have commercial value and may be recovered by use of "save-alls", or similar equipment.
Pickle. Liquor; Iron	Variable.	Strongly acid. If neutralized by alkalinity of the sewage or by lime, iron compounds precipitate and form sludge.	The precipitated iron compounds do not interfere with digestion of the domestic sewage solids. The iron floc may be quite bulky. Treatment at the source is desirable.
Tanneries..........	Large............	Contains considerable organic matter. Reaction may be acid or alkaline depending on the part of the process contributing the waste.	If properly neutralized, tannery wastes do not interfere materially with sludge digestion. Hair and lime residues do not digest.
Wool scouring.....	Medium...........	Has a very foul odor and contains dirt, grease, organic matter, soaps, and fiber.	When mixed with domestic sewage these wastes increase the bulk of the sludge to be digested and create masses of floating scum that do not digest readily.

Pressures likely to obtain in practical sludge digestion appear to have no effect on bacteria. There is some difference of opinion on the effect of very high pressures. Fischer states:

> "High pressure seems to be absolutely without influence upon bacteria. The processes of putrefaction and alcoholic fermentation go on without interruption under pressures of from 300 to 500 atmospheres, * * *."

Dr. F. Sierp[6] made experiments on the digestion of sludge at different depths. Samples containing one part of fresh sludge to two parts of water were placed in vessels at depths of 3.0, 1.0 and 0.15 m (9.8, 3.3 and 0.5 ft), respectively, and observed for 40 days. At the end of this period the sludge characteristics of the three samples were very similar.

Tests on the vacuum degasification of Imhoff tanks have been made by Steel and Zeller[7] at the Texas Agricultural and Mechanical College, College Station, Tex., on two Imhoff tanks treating the college sewage. The gas vents of one tank were placed under a vacuum of from 1.75 to 3.7 ft of water, generated by an automatically operated blower. Observations were made daily for five weeks of the change in H_2S, suspended matter, and bio-chemical oxygen demand of the sewage, and weekly for sixty days of the solid and ash content and the bio-chemical oxygen demand of the sludge. The authors concluded that degasification has no appreciable effect on the tank effluent nor on the digestion of the sludge.

The effect of pressure on the quantity of gas held in solution has also been considered. The quantity of gas that will be dissolved in a liquid at a given temperature is directly proportional to the absolute pressure, and where a number of gases are in contact with a liquid, the quantity of each that will dissolve is proportional to its respective partial pressure. Regardless of the pressure on the tank, however, the gas lost in the sludge supernatant will be less, in general, than 1.0% of the gas produced, provided only that the sludge supernatant is withdrawn from a point where the pressure is 18 in., or less, of water.

The effect of pressure on sludge digestion may be summarized as follows:

(1) Pressures likely to obtain in practice appear to have no effect on bacterial action causing sludge digestion.

(2) Pressures other than normal do not shorten the digestion time.

(3) Degasification under vacuum does not produce more gas nor does it have an appreciable effect on the tank effluent or on the digestion of the sludge.

(4) Pressures in the tank with reference to its effect on the quantity of gas passing into solution and drawn off in the sludge supernatant are of negligible influence.

(5) In general, depth tends to produce dense sludge, but density is not directly proportional to depth because of the influence of other factors.

MOISTURE CONTENT

There are ample data from operating sludge digestion tanks to show that the density of the sludge is greatest near the bottom of the tank as indicated on Fig. 16 and Table 16. The density of the sludge does not increase uniformly from top to bottom of the tank, and probably varies at different times at the same depth. The density of sludge as withdrawn from tanks of different depth appears to be very irregular and, within limits, a deep tank does not necessarily yield a sludge with a higher percentage of solids than does a shallow tank.

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[6] *Technische Gemeindeblatt*, Vol. XXIX, p. 267 (1927).

[7] *Sewage Works Journal*, Vol. 2, p. 29.

Experience indicates that the moisture content of sludge has less effect upon digestion than other factors. Fisher compared rates of digestion of raw sludges containing 6, 12, and 18% solids, and concluded that within these limits there was no retardation of rate of digestion in the more concentrated sludges. There appears to be some direct correlation between moisture and content of volatile matter, with indications that activated sludge may hold greater moisture than sedimentation tank sludge, for a given volatile content.

III.—TYPES OF DIGESTION TANKS

Tanks Built of Earth.—The earliest large-scale development of separate sludge digestion tanks was at Birmingham, England, in 1912. This installation followed approximately fifty years of efforts to eliminate the odor arising from crude sewage and, later, from septic tank effluent. It included a total of fifty tanks, of which thirty were constructed of embankments of engine ash, in some cases faced with concrete paving. The remaining twenty tanks were of brick masonry.

Birmingham has a population of about 1 000 000. The dry-weather sewage flow in 1929 was 38 400 000 gal per day, of which 34 800 000 gal was treated. Digestion is carried out in two stages, sludge when partly digested being pumped from primary tanks to secondary tanks, where the process is completed. The aggregate capacity of the 50 tanks is 340 000 cu yd, individual tanks varying from 1 200 to 17 000 cu yd. The older tanks are 12 to 14 ft deep; some of the newer ones are 20 ft deep.

At several places in the United States, earth tanks or lagoons have been used for handling undigested sludge, but their operation is not, in general, analogous to that of the tanks at Birmingham. However, at Indianapolis, Ind., "deep pit digestion" has been used for both settled and activated sludge. There are seven digestion pits or lagoons about 300 ft square, diked to give a depth of 7 to 10 ft. The pit capacity may amount to 14 cu ft per capita, or more.

Open Masonry Tanks Without Mechanical Equipment.—By far the greatest number of separate digestion tanks in this country are of masonry, and it is in connection with concrete tanks that the major improvements in design and operation have been made in recent years. Concrete tanks have many advantages over the less expensive earth tanks, the most important of which are permanence, space economy, and adaptability to the requirements of operation and structural design.

The earliest concrete tanks were open at the top and had no mechanical equipment. Usually, there was no attempt to control temperature, either by insulating the tanks with earth embankments or by heating the sludge. As a result, relatively high per capita capacities were required.

The sewage works at Baltimore, Md., is the first outstanding example of separate sludge digestion in the United States. Its first unit was built in

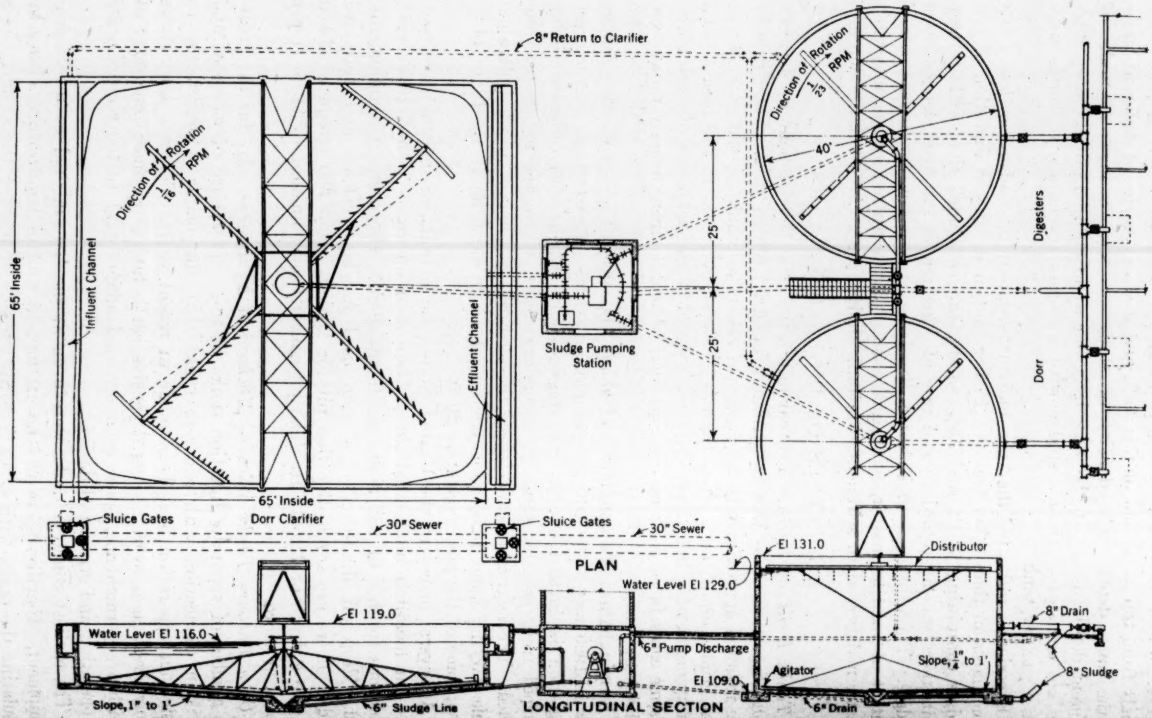

8" Return to Clarifier
Direction of Rotation $\frac{1}{23}$ RPM
40'
25'
25'
Digesters
Dorr
Beds
Sludge
Direction of Rotation $\frac{1}{16}$ RPM
65' Inside
Influent Channel
Effluent Channel
Sludge Pumping Station
65' Inside
Dorr Clarifier
Sluice Gates
30" Sewer
Sluice Gates
30" Sewer
PLAN
El 131.0
Distributor
Water Level El 129.0
El 119.0
Water Level El 116.0
6" Pump Discharge
8" Drain
Agitator
Slope, $\frac{1}{4}$" to 1'
8" Sludge
El 109.0
Slope, 1" to 1'
6" Sludge Line
LONGITUDINAL SECTION
6" Drain

FIG. 2.—OPEN DIGESTION TANKS WITH ROTARY STIRRING EQUIPMENT. SEDIMENTATION TANK AT LEFT

1909–1910, and consisted of three open, rectangular tanks, each 140 ft long, 103 ft wide, and varying from 13½ to 15½ ft in depth. Additional tanks have since been installed. In 1912 there was available a digestion-tank capacity of 11.60 cu ft per capita, and in 1930, with a tributary population of 735 000, the capacity was 3.7 cu ft per capita. In that year the sewage, domestic in character, amounted to 60 650 000 gal per day.

FIG. 3.—TYPICAL INSTALLATION OF OPEN SLUDGE DIGESTION TANKS WITH ROTARY STIRRING EQUIPMENT, LAKELAND, FLA.

Concrete tanks are used for storing digested sludge with covers at Grand Rapids, Mich., and open at Peoria and Rockford, Ill. Costing roughly only one-third as much as the covered, controlled tanks in which the sludge is digested, they provide a relatively cheap storage space where sludge may be held during the winter, when the open drying beds are not in use. The storage tanks at Peoria are about 130 ft in diameter; the floors are of light concrete and the walls, lined internally with steel plate, are of 12-in. reinforced concrete, to resist earth pressure. The tanks are not heated and require no particular attention.

Open Tanks with Rotary Stirring Equipment—Experiments with mechanically cleaned digestion tanks were begun at Rochester, N. Y., in 1921. At that plant the open digestion tank was equipped with a stirring device consisting of two sets of horizontal arms attached to a central vertical shaft supported by a bridge across the tank. The lower set of arms concentrated the sludge in the bottom of the tank and gradually moved it to an outlet pipe at the center, while the upper arms prevented the formation of scum and distributed the incoming sludge over the surface.

In 1923, the first full-scale municipal plant incorporating this type of equipment was built at Brownsville, Tex. The digestion tank is circular, 35 ft in diameter, and 16 ft deep at the center. The agitator is supported from a steel superstructure spanning the tank, and consists of a gear-driven vertical shaft with two sets of arms. The top arms, constructed of steel channels, are placed just above the normal level of the liquid in the tank. Raw sludge is introduced through an inlet well mounted at the top of the shaft and flows outward along the channels, dropping into the tank through a series of holes.

Short chains, hung from the upper arms, extend into the liquid a foot or more and serve to prevent the accumulation of scum. The power unit for driving the agitator is mounted on the truss.

The success of the Brownsville unit led to the building of a number of similarly equipped open tanks in all parts of the United States (see Table 7, and Figs. 2 and 3). Since about 1925, however, this type has yielded to the mechanically equipped, covered tank.

Tanks with Fixed Covers but Without Stirring Equipment.—The desirability of controlling odors and of collecting gas has doubtless been the chief cause of the rapid adoption of covered tanks in preference to open ones, although the closed units have other advantages as well. They are markedly superior in retaining heat, and, therefore, promote bacterial activity. Furthermore, by operating with the under side of the cover under a slight pressure, scum can be kept submerged; the digestion of floating solids is thus accelerated and the formation of a sometimes troublesome thick scum layer prevented.

Among the tanks with fixed covers should be mentioned the Kremer-Kusch design, (Fig. 4) developed in Germany and in use in a number of German cities. The largest plant making use of it was built in 1920 at Hildesheim, a city of about 60 000 population. The single example in the United States was built at Palmyra, N. J., in 1922.

TABLE 7.—OPEN SLUDGE DIGESTION TANKS WITH ROTARY STIRRING EQUIPMENT*

Ref. No.	Location	Date installed	Design population	TANK DIMENSIONS		Number of tanks	Volume of one tank, in cubic feet	Total tank volume, in cubic feet	Tank volume, in cubic feet per capita
				Diameter, in feet	Water depth, feet and inches				
1	Brownsville, Tex.	1923	15 000	35	15–0	1	14 400	14 400	1.0
2	Bartow, Fla	1924	4 000	26	15–0	1	7 950	7 950	2.0
3	Vero Beach, Fla	1925	4 000	30	16–8¼	1	11 800	11 800	3.0
4	Lakeland, Fla	1926	25 000	40	20–0	2	25 100	50 200	2.0
5	Fort Lauderdale, Fla	1926	30 000	50	17–0	2	33 400	66 800	2.2
6	Fort Myers, Fla	1927	8 000	55	15–0	1	35 600	35 600	4.4
7	Miami, Fla	1927	Experimental	20	13–0	1	....	....	....
8	Haines City, Fla	1927	3 000	26	13–9	1	7 300	7 300	2.4
9	Conway, Ark	1927	7 500	35	20–9	1	20 000	20 000	2.7
10	Lake City, Fla	1929	3 000	24	14–0	1	6 330	6 330	2.1
11	Carlsbad, Calif	1929	2 000	20	15–0	1	4 700	4 700	2.3
12	Witten (Ruhrverband) Germany	1929	......	65	....	1	....	....	....
13	Atwater, Calif	1930	3 600	22	15–0	1	5 700	5 700	1.6
14	Hartford, Wis	1924	5 500	38½	14	1	16 400	16 400	3.0
15	Sheboygan, Wis	1924	5 000	36	15	1	15 300	15 300	3.0
16	Kiel, Wis	1925	2 000	25	15	1	7 370	7 370	3.7
17	Winterset, Iowa	1926	4 000	30	14–9	1	10 400	10 400	2.6
18	Moberly, Mo	1927	10 000	30	23–0	1	16 200	16 200	1.6
19	Monroe, N. C	1927	7 000	35	16–0	1	15 400	15 400	2.2
20	Blackwell, Okla	1927	12 000	45	24–0	1	38 200	38 200	3.2
21	Salina, Kans	1927	17 000	45	21–6¾	1	34 400	34 400	2.0
22	Pratt, Kans	1927	5 200	30	21–0	1	14 900	14 900	2.9
23	Dunsmuir, Calif	1927	2 500	38	15–0	1	17 000	17 000	6.8[a]
24	Easton, Pa	1927	20 000	75	15–0	1	66 200	66 200	3.3
25	Roseville, Calif	1927	10 000	40	16–0	1	20 100	20 100	2.0
26	Gilroy, Calif	1927	4 000	50	24–6	1	48 100	48 100	12.0[a]
27	Moberly, Mo	1928	4 000	35	20–6	1	19 700	19 700	4.9
28	Aurora, Colo	1929	2 600	26	15–0	1	7 950	7 950	3.1
29	El Cajon, Calif	1929	1 500	18	12–0	1	3 050	3 050	2.0

* Installations Nos. 1 to 13 are without cover of any kind; Installations Nos. 14 to 29 are open-type tanks, but have covers of boards laid on I-beams spanning space between superstructure and tank walls.

[a] Large capacity to handle trade wastes discharged into sewerage system.

These tanks are a development of the old Kremer sedimentation tank by the addition of a Kusch sludge cylinder below the apex of the steep hopper bottom, and a Kusch sludge digestion tank adjacent to the sedimentation tank. Sludge is transferred from the bottom of the cylinder to the digestion tanks by hydrostatic head. Two chambers are provided, rapid digestion and gasification taking place in the first, which is comparatively large and shallow. When partly digested, the sludge is removed to the second chamber, smaller in area, but deeper than the first, where the digestion process is completed.

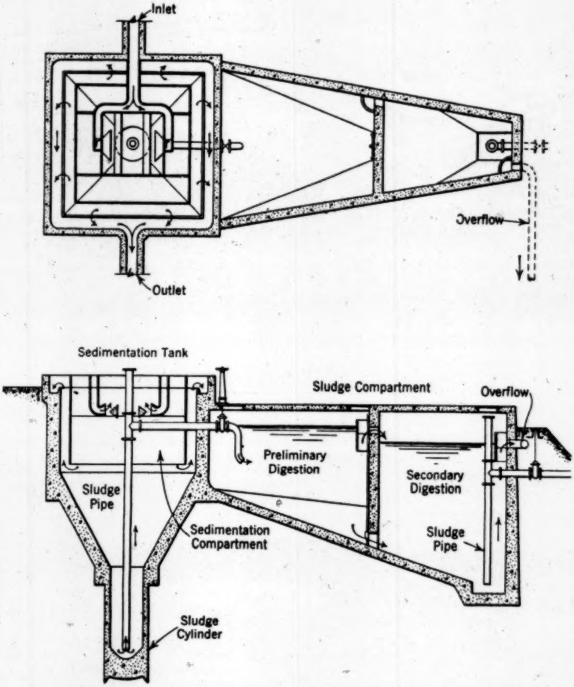

Inlet
Outlet
Overflow
Sedimentation Tank
Sludge Compartment
Overflow
Preliminary Digestion
Secondary Digestion
Sludge Pipe
Sedimentation Compartment
Sludge Pipe
Sludge Cylinder

FIG. 4.—KREMER-KUSCH SEDIMENTATION TANK WITH SEPARATE SLUDGE DIGESTION COMPARTMENT

Tark states that for a sedimentation tank 15 to 20 ft square, the usual diameter of the sludge cylinder is about 4½ ft, and that because the area of contact between the sludge in the cylinder and the sewage is small, the sludge may be retained in the cylinder for three or four days. The effect of this detention period and the depth of the cylinder is to secure a concentrated sludge with about 88% moisture.

Covered Tanks with Stirring Equipment.—Although a few covered tanks with stirring equipment are square in plan, by far the greater number are circular. The floor is cone-shaped, usually with a slope of about 1 to 48

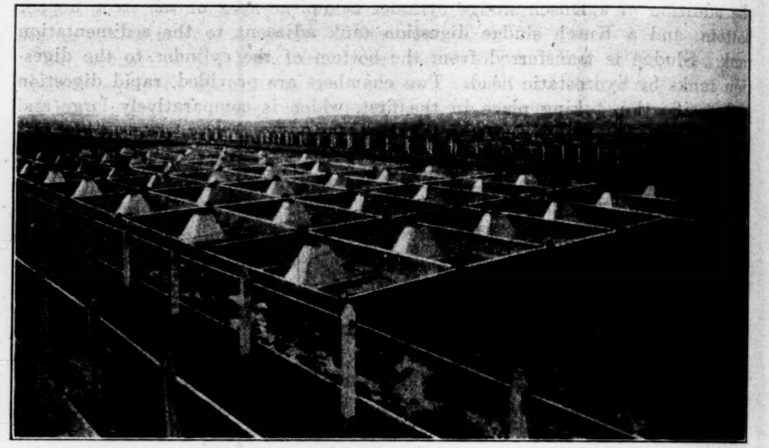


FIG. 5.—FLOATING CONCRETE COVERS ON SLUDGE TANKS, AT BIRMINGHAM, ENGLAND

FIG. 6.—TYPICAL INSTALLATION OF SLUDGE DIGESTION TANKS WITH FLOATING COVERS, AT ELYRIA, OHIO

toward a sump in the center. A pipe under the floor, terminating in the sump, permits the withdrawal of digested sludge. The roof is usually of concrete, and either slopes slightly upward toward the center or is dome-shaped. It is equipped with a gas dome, and with pipes for adding raw sludge and for carrying away the supernatant liquor. When the tank is in operation the surface of the sludge is usually up in the gas dome. The latter is equipped with a water-seal and arranged so that gas can be collected under a pressure sufficient to force it through the pipe lines and meters. The stirring device is similar, in general, to that used in the open tank. Table 8 gives data on plants using this type of equipment.

Tanks with Floating Covers.—Floating covers were first developed to permit the collection of gas at plants originally built with open tanks, but there is now at least one type of floating cover intended for use in new construction. Covers have been added to existing tanks, for example, at Plainfield, N. J., and Birmingham, England. The Plainfield covers are of wood; those at Birmingham are of concrete. The concrete covers (Fig. 5) are each 10 ft wide by 20 ft long, and are placed side by side, completely covering the tanks. At the center of each cover is a pyramidal gas dome, connected to the mains by flexible piping.

The floating cover designed for new construction is shown in Figs. 6 and 7. The framework consists of a system of radial trusses extending from

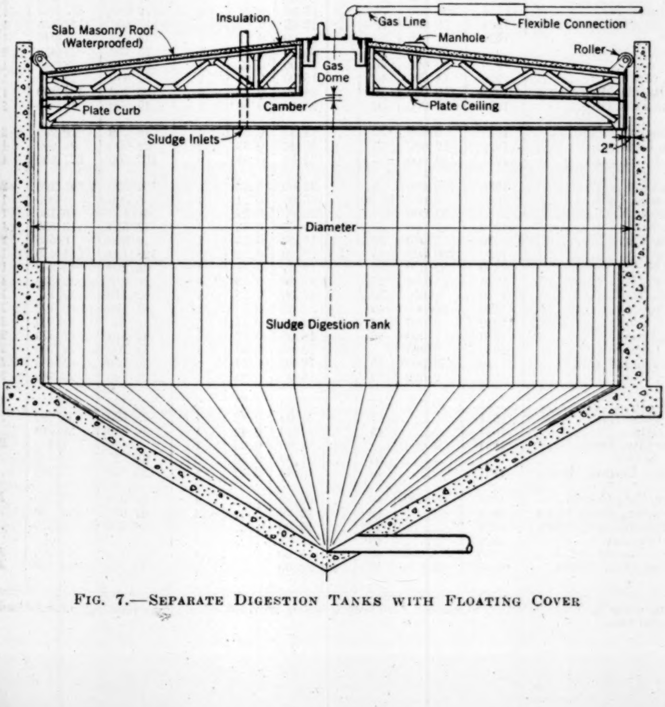
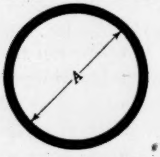
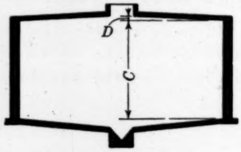

Slab Masonry Roof (Waterproofed)
Insulation
Gas Line
Flexible Connection
Manhole
Roller
Gas Dome
Plate Curb
Camber
Plate Ceiling
Sludge Inlets
2"
Diameter
Sludge Digestion Tank

FIG. 7.—SEPARATE DIGESTION TANKS WITH FLOATING COVER

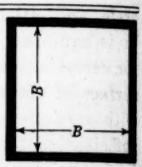
TABLE 8.—Covered Sludge Digestion Tanks with Rotary Type Stirring Equipment, Designed for Gas Collection



PLAN OF ROUND TANK



SECTIONAL ELEVATION



PLAN OF SQUARE TANK

Location	Date installed	Design population	Tank Dimensions				Number of tanks	Volume of one tank, in cubic feet	Total tank volume, in cubic feet	Tank volume, in cubic feet per capita
			A	B	C	D				
Antigo, Wis	1926	11 000	50	..	17.33	0.50	1	34 710	34 710	3.2
Sioux Falls, S. Dak	1926	53 000	85	..	25.00	0.83	3	145 200	435 600	8.2[a]
Storm Lake, Iowa	1927	5 000	30	..	23.00	0.33	1	16 380	16 380	3.3[b]
Sedalia, Mo	1927	21 000	35	..	21.00	0.58	2	20 460	40 920	2.0
Fond du Lac, Wis	1927	35 000	.....	50	16.00	0.67	2	40 000	80 000	2.3
Springfield, Mo	1928	47 500	65	..	27.17	0.67	1	91 650	91 650	1.9
Northampton, Pa	1928	10 000	50	..	15.00	0.50	1	30 130	30 130	3.0
Aurora, Ill	1928	65 000	.....	50	16.50	0.50	3	41 250	123 750	1.9
Waukesha, Wis	1928	27 000	50	..	18.50	0.50	1	37 000	37 000	1.4
Salem, Ohio	1928	15 000	50	..	20.00	0.50	1	39 950	39 950	2.7
Fort Worth, Tex	1928	225 000	75	..	25.00	0.75	3	112 660	337 990	1.0[c]
Topeka, Kans	1928	40 000	65	..	23.00	0.67	2	77 830	155 650	3.9
Red Bank, N. J	1928	16 000	45	..	20.00	0.42	1	32 330	32 330	2.0
Petoskey, Mich	1928	9 000	40	..	16.00	0.42	1	20 430	20 430	2.3
Liberal, Kans	1929	3 500	30	..	16.00	0.33	1	11 430	11 430	3.3
San Antonio, Tex	1929	300 000	75	..	25.00	0.75	4	112 660	450 650	1.5
Waco, Tex	1929	75 000	75	..	25.00	0.75	1	112 660	112 660	1.5
De Kalb, Ill	1929	14 000	40	..	18.00	0.42	1	22 940	22 940	1.6
Lombard, Ill	1929	10 000	40	..	20.00	0.42	1	25 450	25 450	2.5
Chico, Colo	1929	20 000	40	..	18.00	0.42	1	22 940	22 940	1.1
Sturgis, Mich	1929	14 000	40	..	18.00	0.42	1	22 940	22 940	1.6
Grand Rapids, Mich	1929	250 000	70	..	24.50	0.75	4	96 200	384 800	1.5
Klamath Falls, Ore	1929	25 000	50	..	25.00	0.50	1	49 800	49 800	2.0
Cheektowaga, N. Y	1929	5 000	30	..	14.33	0.33	1	10 250	10 250	2.0
Ottawa, Kans	1929	12 000	35	..	20.25	0.33	1	21 900	21 900	1.8
Peoria, Ill	1929	140 000	85	..	29.00	0.83	4	167 900	671 600	4.8[a]
Toledo, Ohio	1930	600 000	85	..	29.00	0.83	8	167 900	1,343 200	2.2
Los Angeles, Calif	1930	3 000	19.50	..	15.00	0.21	1	5 000	5 000	1.7[d]
Greensboro, N. C	1930	35 000	60	..	23.00	0.58	1	66 190	66 190	1.8
Sparks, Nev	1930	7 000	30	..	19.00	0.33	1	13 550	13 550	1.9
Horicon, Wis	1930	3 000	26	..	16.00	0.25	1	8 560	8 560	2.8
Erie, Pa	1930	200 000	85	..	22.00	0.83	3	128 180	384 530	1.9
Ithaca, N. Y	1930	32 000	55	..	18.67	0.58	1	45 260	45 260	1.4
Galesburg, Ill	1930	35 000	35	..	17.00	0.33	3	16 610	49 830	1.5
Freehold, N. J	1930	8 000	30	..	17.00	0.33	1	12 140	12 140	1.5
Reno, Nev	1930	25 000	60	..	23.00	0.58	1	66 190	66 190	2.5
Abilene, Kans	1930	6 000	33	..	22.00	0.33	1	18 800	18 800	3.0
Carroll, Iowa	1930	7 000	35	..	18.00	0.33	1	17 570	17 570	2.5
Spring Lake, N. J	1930	6 925	.....	35	12.00	0.33	1	14 700	14 700	2.1
Spartansburg, S. C	1930	30 000	45	..	19.00	0.42	2	30 740	61 480	2.0
Wilmar, Minn	1930	9 000	40	..	18.00	0.42	1	22 940	22 940	2.5
Lima, Ohio	1931	70 000	85	..	22.00	0.53	2	128 180	256 350	3.6
Tulsa, Okla	1931	......	45	..	24.00	0.42	1	38 690	38 690	....
Tulsa, Okla	1931	......	45	..	19.00	0.42	1	30 740	30 740	....
Boulder City, Nev	1931	3 000	.....	24	13.00	0.25	1	7 490	7 490	2.5
Huron, S. Dak	1931	......	50	..	21.00	0.50	2	41 910	83 820	....
Muskegon Heights, Mich	1931	......	40	..	18.00	0.42	1	22 940	22 940	....
Toronto, Ont., Canada	1928	50 000	40	..	20.50	....	4	26 050	104 200	2.1
Johannesburg, South Africa	1929	......	40	..	22.00	....	4	27 970	111 880	....
Johannesburg, South Africa	1929	......	28	..	15.00	....	1	9 360	9 360	....
Essen, Germany	1929	......	40	..	27.88	....	1	....	....	....
Mildura, Australia	1930	8 000	40	..	15.00	....	1	19 170	19 170	2.4
Kitchener, Ont., Canada	1930	40 000	65	..	22.00	....	2	74 500	149 000	3.7

[a] Large capacity to handle industrial wastes. [b] No heating system provided. [c] Including equivalent industrial load. [d] Experimental unit.

a gas dome at the center to a point about 2 in. from the tank wall. Attached to the lower chords is a gas-tight and water-tight diaphragm of steel plate, which floats on the surface of the liquid. On the upper chords rests an insulating roof of wood and water-proofed concrete At the outer ends of the trusses is a vertical curb of steel plate, extending from the top chords to below the lower chords and making a gas-tight and water-tight joint with the diaphragm. To insure free vertical movement, the cover is equipped with rollers that bear on the concrete walls of the tank. Statistics of sludge digestion tanks with floating covers are given in Table 9.

TABLE 9.—SLUDGE DIGESTION TANKS WITH FLOATING COVERS

Location	Date installed	Design population	Tank Dimensions		Number of Tanks		Volume of one tank, in cubic feet	Total Volume of Tanks[a]	
			Diameter, in feet	Side-wall depth, in feet	Floating covers	Total		In cubic feet	Cubic feet per capita
Elyria, Ohio	1928	36 000	50	....	2	2	39 300	78 600	2.2
Springfield, Ill.	1928	90 000	40	21.5	2	4	30 000	120 000	1.3
Durham, N. C. (North Side)	1933	30 000	60	....	3	3	60 000	180 000	6.0
Madison, Wis.	1934	52 500	70	18	2	..	75 000		
			75	19	2	4	92 500	335 000	6.4
Cedar Rapids, Iowa	1935	140 000	60	27	2	4	56 250	225 000	1.6
Chicago Heights, Ill.	1935	30 000	65	21.5	1	2	71 500	143 000	4.4
Coney Island, New York	1935	300 000	54.5	26.5	8	8	59 650	478 000	1.6
Medford, Ore.	1935	14 000	40	20	1	1	25 000	25 000	1.8
Winona, Minn.	1935	30 000	50	22.67	1	2	45 000	90 000	3.0
Appleton, Wis.	1936	32 500	70	23.25	4	4	90 500	362 000	11.1[c]
Cleveland, Ohio	1937	770 000	95[b]	....	12	12	167 000	2 000 000	2.6
Denver, Colo.	1937	315 000	85	30	4	4	192 500	770 000	2.4
Buffalo, N. Y.	1938	750 000	90	27.5	4	4	184 250	737 000	1.0

[a] Including both digestion and storage. [b] Hexagonal. [c] Industrial waste.

Multi-Stage Digestion.—Multi-stage digestion consists of a system of two or more sludge digestion tanks, each connected with the other for the progressive and automatic transfer of the sludge and gas during digestion.

The primary tank may be equipped with a fixed steel dome cover and with agitators for keeping the sludge in a homogeneous condition which is considered by some to produce digestion and gas generation at the optimum rate. The primary tank may be equipped with a hot-water heat exchanger or other equipment for heating the digesting sludge to the desired temperature. The secondary or tertiary tanks may also be equipped with steel covers designed to serve as gas-holders of varying capacities.

Settled solids from the sedimentation tank enter the first stage of digestion. This primary tank is at all times kept filled to a constant level with sludge at a proper temperature and in a homogeneous condition by the mixers. The gas produced during digestion is collected at one point on the dome roof and taken to the secondary tank for storage.

Approximately 90% of the digestion is said to be accomplished in the first stage and the sludge from the first tank is transferred to the secondary tank by hydrostatic head. In the second stage, quiescent conditions prevail which allow the solids to settle to the bottom of the tank and leave the supernatant liquor relatively clear for return to the treatment process. The settled sludge is withdrawn from the bottom for final disposal. The small quantity of gas produced in the final stages of digestion is collected in the gas-holder cover where it joins the gas produced in the primary stage.

There are at present about thirty cities in the United States that have multi-stage digestion tanks under construction.

SLUDGE PUMPING EQUIPMENT

Compressed-Air Lifts.—One of the simplest means of removing sludge from tanks is to introduce compressed air in the sludge discharge pipe. When this method is used, the sludge pipe is usually placed vertical, or nearly so, with its lower and open end near the bottom of the tank. The air pipe, perforated in its lower portion, is placed inside it.

Pneumatic Ejectors.—Although pneumatic ejectors are more commonly used for pumping sewage, they have also been installed in a number of places for pumping sludge. The body of the ejector is a cast-iron pot into which the sludge flows by gravity through a check-valve. When a predetermined

TABLE 10.—UTILIZATION AND SALE OF GAS AT SEWAGE TREATMENT WORKS

Location	Period Reported		Quantity, in Thousand Cubic Feet		
	Date	Length	Heating buildings	Heating sludge tanks	Operating gas engines
(1)	(2)	(3)	(4)	(5)	(6)
(a) Plants Reporting Quantities					
Aurora, Ill.	1936–37	14 months	4 920[aa]	....	8 250
Berlin, Germany (Wassmannsdorf)	1929–30	1 yr.	12 400[a]	....	31 800[c]
Birmingham, England (Saltley)	1930	1 yr.	....	....	46 750
Bochum, Germany	1929–30	1 yr.	106	....	....
Cedar Rapids, Iowa	1936	1 yr.	....	....	30 000
Chicago, Ill. (Des Plaines)	1927–28	1 yr.	....	145[b]	....
Chico, Calif.	....	1 day	....	2 to 3	....
Coney Island, New York	1937	1 yr.	....	....	36 000
Dayton, Ohio	Dec., 1930	1 month	434	....	....
Decatur, Ill.	....	1 day	5 to 25	....	....
Essen-Frohnhausen, Germany	1929–30	1 yr.	106	....	1 766[d]
Essen-Nord, Germany	1929	1 yr.	....	....	....
Essen-Nordwest, Germany	1929–30	1 yr.	....	....	....
Essen-Rellinghausen, Germany	1930	4 months	....	....	....
Fort Atkinson, Wis.	1936	1 yr.	....	....	2 790
Fort Worth, Tex.	1930	7 months	Some	6 820	....
Gelsenkirchen-Nord, Germany	1929–30	1 yr.	....	1 059	....
Hagen, Germany	1930	1 yr.	460	....	....
Halle, Germany (Tafelwerder)	1929–30	1 yr.	....	2 100	2 800
Hattingen, Germany	1930	7 months	....	....	1 314
Iserlohn, Germany	1930	1 yr.	71	....	2 380
Kettwig, Germany	1929	1 yr.	....	....	....
Langendreer, Germany	1929	1 yr.	18	....	....
Leipzig-Wahren, Germany	1930	1 yr.	26	....	883
Munich, Germany	1930	1 yr.	....	....	....
Nürnberg, Germany (Nürnberg-Süd)	1930	1 yr.	530	....	....
Oberhausen, Germany	1929–30	1 yr.	106	....	....
Palo Alto, Calif.	1936	1 yr.	655	....	4 370
Peoria, Ill.	1936	8 months	....	....	204 000
Springfield, Ill.	1937	1 yr.	....	....	20 750
Stuttgart, Germany (Mühlhausen)	1930	1 yr.	....	....	....
Velbert, Germany	1930	1 yr.	....	....	....
Waukesha, Wis.	1930	1 yr.	1 460	548	....
Zurich, Germany	....	1 yr.	....	....	....

TABLE 10.—(*Continued*)

Location	QUANTITY, IN THOUSAND CUBIC FEET (*Continued*)*			AMOUNT RECEIVED†	
	Incinerating screenings	Miscellaneous	Sold to local gas works	Total, in dollars	Dollars per thousand cubic feet ‡
(1)	(7)	(8)	(9)	(10)	(11)
(*a*) PLANTS REPORTING QUANTITIES (*Continued*)					
Aurora, Ill.	4 000	2 380	None	...	...
Berlin, Germany (Wassmannsdorf)	1 910	1 180[e]	None	...	...
Birmingham, England (Saltley)	...	...	None	...	...
Bochum, Germany	...	...	None	...	...
Cedar Rapids, Iowa	5 060	15 600[bb]	None	...	...
Chicago, Ill. (Des Plaines)	...	...	None	...	...
Chico, Calif.	...	...	None	...	...
Coney Island, New York	...	...	None	...	...
Dayton, Ohio	465	...	None	...	...
Decatur, Ill.	...	...	None	...	...
Essen-Frohnhausen, Germany	...	...	Electricity sold	191	...
Essen-Nord, Germany	...	138[f]	17 110	4 416	0.26
Essen-Nordwest, Germany	...	...	3 231	743	0.23
Essen-Rellinghausen, Germany	...	49[g]	3 390	780	0.23
Fort Atkinson, Wis.	...	...	None	...	...
Fort Worth, Tex.	2 693	...	None	...	...
Gelsenkirchen-Nord, Germany	...	...	None	...	...
Hagen, Germany	...	...	None	...	...
Halle, Germany (Tafelwerder)	...	...	22 600	5 730	0.25
Hattingen, Germany	...	...	None	...	...
Iserlohn, Germany	...	71	None	...	...
Kettwig, Germany	...	...	920	286	0.31
Langendreer, Germany	...	...	932	193	0.21
Leipzig-Wahren, Germany	...	...	None	...	...
Munich, Germany	...	...	97 110	29 838	0.31
Nürnberg, Germany (Nürnberg-Süd)	...	...	16 406	2 000[h]	0.12[i]
Oberhausen, Germany	...	...	None	...	...
Palo Alto, Calif.	...	3 759[cc]	None	...	...
Peoria, Ill.	...	66 000[bb]	None	...	...
Springfield, Ill.	...	2 600[dd]	None	...	...
Stuttgart, Germany (Mühlhausen)	...	...	48 200	26 047	0.54
Velbert, Germany	...	...	1 784	550	0.31
Waukesha, Wis.	...	...	None	...	...
Zurich, Germany	...	...	21 200	7 420	0.35

level is reached, air under pressure, from either a rotary blower or a pressure tank, enters at the top and forces the sludge through a second check-valve into the discharge pipe. Ejectors can be made to operate either automatically or manually.

Diaphragm Pumps.—These pumps are made in two different designs. The first, not generally used for pumping sludge, has a free discharge; the second, more common, is used where the sludge must be pumped against a head. The pump consists of a cast-iron pressure chamber with inlet and outlet ports and a flexible diaphragm, the periphery of which is securely fastened to the pump body. An up-and-down motion is imparted to the diaphragm, through shafting, by an electric motor or gasoline engine. The ports are usually equipped with rubber ball-valves and renewable rubber valve seats. An air-pressure chamber to smooth out the pulsations of the discharge is provided just outside the pump.

Reciprocating Pumps.—A reciprocating pump consists essentially of a cylinder equipped with a piston or plunger and suction and discharge valves. Both ball-valves and metal flap-valves are used. In order to facilitate valve cleaning, the pump-casing is usually provided with removable hand-hole

TABLE 10.—(*Continued*)

Location	Gas Was Used For:					Remarks
	Heating buildings	Heating sludge tanks	Operating gas engines	Incinerating screenings	Miscellaneous	
(12)	(13)	(14)	(15)	(16)	(17)	(18)
(*b*) Miscellaneous Plants						
Antigo, Wis	Yes	Yes	No	....	....	....
Baltimore, Md	No	Yes	No	....	....	....
Charlotte, N. C. (Sugar Creek)	Yes	No	Yes	....	....	....
DeKalb, Ill	Yes	Yes	No	....	....	....
Elyria, Ohio	No	Yes	No	....	....	....
Fond du Lac, Wis	Yes	Yes	No	....	Water-still	Received $20 per month from sale of distilled water.
Grand Rapids, Mich	Yes	Yes	No	....	....	....
Greensboro, N. C	Yes	Yes	No	....	....	....
High Point, N. C	No	Yes	No	....	....	....
Middletown, N. Y	....	Yes	....	....	....	....
Neheim, Germany	Yes[j]	No	No	....	....	....
New Castle, Pa	Yes	No	No	....	....	....
Plainfield, N. J. (Joint)	No	Yes	Yes	....	....	....
Sturgis, Mich	No	Yes	No	....	....	Excess blown off to furnace chimney
Toledo, Ohio (Bay View Park)	....	Proposed	....	....	....	....
Toronto, Ont., Canada (North Toronto)	No	Sludge	No	....	Laboratory	....
Waco, Tex	No	Yes	No	....	....	....
Wetter, Germany	No	No	No	....	....	....

[*] Quantities for German localities were converted from cubic meters. [†] Based on average value of Reich Mark in 1930 (23.87 cents). [‡] Column (10) ÷ Column (9). [§] Works where quantities were not available. None of the gas was sold. [aa] Also for heating sludge tanks. [a] Also for heating gas-holder. [bb] Wasted. [b] In addition, 113 300 cu. ft. of city gas was used. [c] 65-, 335-, and 750-hp engines for driving sewage pumps. [d] 50-hp gas engine-generator. [e] 830,000 cu. ft. for street lighting; remainder for cooking. [f] Bottled and sold for explosive in coal mine. Received $279. [g] Laboratory. [h] Amount for December was estimated. [i] Cost of laying main and erecting gas-holder was borne by gas-works. [j] Attendant's house; [cc] 3 540 000 cu. ft. wasted; 219 000 cu. ft. for airport lighting. [k] Also incinerating scum. [dd] Waste gas not metered.

plates held in place by wing-bolts. Valve passages should be amply large, with as few bends as possible and free from any projections that might collect fibrous or suspended materials. Where the pump is operating against moderate or high heads, an ample air chamber should be provided in the discharge main as close to the pump as possible.

Centrifugal Pumps.—Centrifugal pumps have been widely used for handling all kinds of sludge. Single and double-suction impellers, both open and enclosed, have been tried. The best adapted to sludge pumping is probably the single-suction, enclosed impeller, secured to the end of the pump shaft. With this arrangement the sludge cannot come in contact with the shaft, whereas in double-suction pumps the shaft is in direct contact with the sludge, and stringy materials frequently wrap around it and form obstructions. The pumps should operate at speeds not greater than 800 rpm, and preferably at 720 rpm, or less, in order to reduce the wear at the inlet of the impeller. One or more easily removable hand-hole plates in the pump-casing, through which obstructions can be removed, will minimize operating difficulties. Pumps with horizontal split cases have the advantage of being easily disassembled for inspection, cleaning, or repairs.

EQUIPMENT FOR COLLECTING AND UTILIZING GAS

General.—Although gas from digesting sludge has been utilized extensively only during the past four or five years, the practice is by no means new. Perhaps the first use made of it was at Exeter, England, where, in 1895, Cameron, the Borough Engineer, collected it and burned it in a street lamp.

In 1902, James, at the leper colony at Matunga, India, constructed a gas-tight cover over two compartments of the septic tank, provided a gas-holder, and used the gas for cooking, lighting the compound, and running a ½-hp engine. The first American installation utilizing sewage gas was made in 1915 at the Peachtree Sewage Works, at Atlanta, Ga. Collectors were set in the gas vents of the Imhoff tanks, and sufficient gas was obtained to supply the plant laboratory and the dwelling of the operator.

Sludge gas is now used extensively for heating sludge tanks and buildings, and operating gas engines. Miscellaneous uses include lighting and incineration of screenings. In Germany, where it has perhaps been applied to a greater variety of purposes than elsewhere, quantities of it are frequently sold to the local gas plants. Table 10 summarizes the data on utilization obtained from a number of plants in both Germany and the United States.

EQUIPMENT FOR COLLECTION OF GAS

Gas hoods for separate digestion tanks are located above openings at high points in the roof slab. Generally, they are provided with separate water-seals, to insure against loss of gas around the hood in case the level of the liquid in the tank is lowered. Scum grids are used infrequently. In a few installations the hood consists simply of a casting tightly fastened to the roof slab without any seal.

A typical circular gas hood suitable for digesters as large as 55 ft in diameter is illustrated in Fig. 8, and Fig. 9 shows a typical rectangular hood for use on larger tanks. Data on hoods at twenty-five plants are given in Table 11.

Explosions in covered tanks containing digesting sludge are a very real hazard, but one that may be avoided if air is prevented from mixing with the gas in explosive proportions. Floating covers furnish an adequate and entirely automatic means of excluding air, since they rise and fall with the change of liquid level, and maintain a constant pressure in the dome. For tanks with fixed covers, the dome may be connected to a gas-holder, so that whenever sludge or liquor is drawn from the tank, the void will be filled by gas flowing back from the holder instead of by air entering through the water-

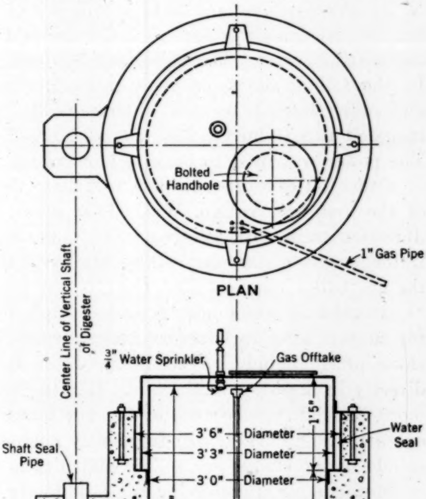


FIG. 8.—TYPICAL CIRCULAR GAS HOOD FOR SMALL DIGESTION TANKS, WITH DORR EQUIPMENT

seal. It should be noted that this method of protection fails if the gas-holder is allowed to become empty. Either the gas meters installed between dome and holder must be of a type that permits free flow of gas in both directions, or they must be equipped with a by-pass containing a check-valve.

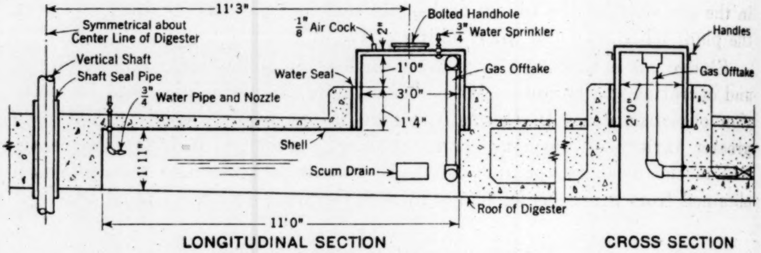

11'3"
Symmetrical about Center Line of Digester
Vertical Shaft
Shaft Seal Pipe
3/4" Water Pipe and Nozzle
1/8" Air Cock
2"
Bolted Handhole
3/4" Water Sprinkler
Water Seal
1'0"
3'0"
1'4"
Gas Offtake
Shell
1'11"
Scum Drain
Roof of Digester
11'0"
LONGITUDINAL SECTION
Handles
Gas Offtake
2'0"
CROSS SECTION

FIG. 9.—TYPICAL RECTANGULAR GAS HOOD FOR LARGE DIGESTION TANKS, WITH DORR EQUIPMENT; FIXED COVERS

SLUDGE-HEATING EQUIPMENT

Gas was first used for heating sludge, in February, 1926, at Plainfield, N. J. An earthen tank with a capacity of 25 000 cu ft was provided with a floating wooden cover for gas collection and a coil of iron pipe for circulating the heating water. The boiler was equipped for use of both gas and soft coal. In the fall of the same year, two additional sludge tanks were constructed and were heated by the sludge gases. The Plainfield installation also included two 15-hp gasoline engines, modified to use the sludge gas for fuel. The power was used to operate blowers for activating the sewage.

Other cities were not slow to follow the example set by Plainfield. One of the first was Antigo, Wis. That plant, built in 1927, included a covered digestion tank with heating coils, a gas boiler, a pump for circulating hot water, a gas meter, and other auxiliary devices. For a few months before the gas boiler was installed, the gas was used in a coal-burning steam heater.

At present, water heaters and steam boilers are both used to generate heat for maintaining satisfactory temperatures in sludge tanks. Boiler installations must include an auxiliary water heater, because steam is not used directly in digestion tank coils. Heating the water directly by gas is slightly more efficient, but because of the low temperature at which the water heaters operate, their burners are frequently corroded by the action of the hydrogen sulfide in the sludge gas. This difficulty is not encountered in the boilers.

Methods for transferring the heat to the sludge include:

(1) Circulation of heated water by means of a closed system through coils of pipe in the digestion tank.

(2) Discharge of heated water or heated sludge liquor directly into the contents of the tank.

(3) Pre-heating of the sludge before its discharge into the digestion tank.

(4) Circulation of sludge through a separate tank equipped with heating coils.

TABLE 11.—Data Relating to Gas Hoods for Separate Sludge Digestion Tanks

Location	Type of gas hood	Scum grid	Separate water-seal	Size in plan	Area, in square feet	Number per tank	Inside area of tank, in square feet	Ratio of hood area to tank area	Size of gas pipe, in inches
Antigo, Wis.	Circular	No[a]	Yes	3 ft. 0 in.	7.1	1	1 964	0.0036	1½
Aurora, Ill.	Circular	No	Yes	4 ft. 0 in.	12.6	1	2 500	0.005	1½
Baltimore, Md.	Sheet-iron, rectangular	No	No	2 ft. 6 in. by 7 ft. 0 in.	17.5	1	1 134	0.015	1½
Charlotte, N. C. (Sugar Creek)	Cast-iron flanged pipe	Cast-iron grating	No	2 ft. 0 in.	3.1	4	....	....	2
Chicago, Ill. (Des Plaines)	Galvanized sheet iron	No	No	....	6.0	2	314	0.038	1
Chicago, Calif.	Circular	No	Yes	3 ft. 0 in.	7.1	1	1 257	0.006	1¼
De Kalb, Ill.	Circular	No	Yes	3 ft. 0 in.	7.1	1	1 257	0.006	1½
Elyria, Ohio	Floating cover	No	Yes	....	....	1	1 964	....	1¼
Essen-Frohnhausen, Germany	Sheet-iron, circular	No	Yes	....	10.8[b]	1	1 614[b]	0.007	1½
Essen-Rellinghausen, Germany	Sheet metal	No	No	2 ft. 7½ in. sq.[b]	6.9	.....	....	....	2[b]
Fond du Lac, Wis.	Circular	No	Yes	4 ft. 0 in.	12.6	1	2 500	0.005	2
Fort Worth, Tex.	Rectangular	No	Yes	2 ft. 6 in. by 6 ft. 8 in.	16.7	2	4 418	0.008	2
Grand Rapids, Mich.	Rectangular	No	Yes	3 ft. 0 in. by 5 ft. 0 in.	15.0	2	3 848	0.008	3
Greensboro, N. C.	Rectangular	No	Yes	3 ft. 0 in. by 6 ft. 0 in.	18.0	2	2 827	0.013	1½
Halle, Germany (Tafelwerder)	Cast-iron	Wood	No	....	....	2	....	....	....
High Point, N. C.	Cast-iron flanged pipe	No	....	4 ft. 0 in.	12.6	3	363	0.104	2
Los Angeles, Calif. (Exp. Station)	Circular	No	No	2 ft. 0 in.	3.1	1	314	0.010	1
Munich, Germany (Grosslappen)	Sheet-iron	Wood	No	....	....	1	....	....	4[b]
Plainfield, N. J.	Steel, circular	No	No	5 ft. 0 in.	19.6	1	1 300	0.015	1
Springfield, Ill.	Floating cover	No	....	....	....	1	1 257	....	....
Sturgis, Mich.	Circular	No	Yes	4 ft. 6 in.	15.9	1	1 257	0.013	1½
Toledo, Ohio (Bay View Park)	Rectangular	No	Yes	2 ft. 5 in. by 6 ft. 0 in.	15.0	2	5 675	0.005	3
Toronto, Ont., Canada	Concrete, circular with bolted iron cover	No	No	3 ft. 0 in.	7.1	1	1 257	0.006	
Waco, Tex.	Rectangular	No	Yes	2 ft. 6 in. by 4 ft. 2 in.	10.4	2	4 418	0.005	3
Waukesha, Wis.	Circular	No	(Oil seal)	4 ft. 0 in.	12.6	1	1 964	0.006	

[a] Used for a while but found to be unsatisfactory

[b] Converted from metric units.

Digestion tanks are sometimes covered with earth to reduce heat losses; the earth should be kept dry, for it loses about one-half its efficiency if it becomes saturated. In some cases special insulating materials may be found valuable.

Any heating system requires considerable mechanical equipment and many regulating devices which need frequent checking and servicing.

Gas-Burning Power Equipment

One of the largest generating plants in the world driven by sludge gas is that at Birmingham, England. The first power unit—a 25-bhp gas engine—was installed by Mr. John D. Watson in 1921. The engine was used to operate a 5-in. centrifugal sludge pump with a capacity of 540 gal per min against a head of 36 ft. It consumed, per horse-power hour, 20 cu ft of gas, with a methane content ranging from 60 to 77 per cent. The digestion tank that furnished the gas had a sludge capacity of 2 160 cu ft.

Although the experimental unit showed several inherent weaknesses, and the rate of gas production fell short of expectations—especially during the winter—the engine worked successfully and produced sufficient power to drive the pump. As a result, a much larger engine (Fig. 10) was installed in 1927. It was rated at 150-bhp and drove a 100-kw alternator. A second engine, with an output of 400 bhp, was added in 1929, and another unit of the same size was installed a year later, making a total capacity of 950 bhp. Gas used in these engines in 1930 totaled 46 750 000 cu ft, an average of 128 000 cu ft per day.

Among sewage treatment works in the United States utilizing gas for power is the Sugar Creek Plant, at Charlotte, N. C. The installation includes two gasoline engines of standard design, except that the carburetors have been removed and a gas pipe has been attached to the intake manifold. In case it should be necessary to use gasoline, the carburetors can be re-attached in a short time. One of the units is a 75-hp, 4-cylinder engine, direct-connected to a centrifugal pump handling 3 000 gal of sewage per min, and the other is a 225-hp, 6-cylinder engine driving a centrifugal air-compressor with a capacity of 3 500 cu ft of air per min. It has been estimated that the saving in electric current would repay the cost of the two engines (approximately $10 500) in about three years. Fig. 11 shows a 300-hp engine at Peoria, Ill., operated by sludge gas.

A list of sludge gas engines and their horse-power installed in sewage treatment plants in the United States, to 1937, is as follows:[8]

Place	Horse-power	Place	Horse-power
Springfield, Ill.	157	Richmond, Ind.	120
Los Angeles, Calif.	200	Edwardsville, Ill.	15
Cedar Rapids, Iowa	300	Sheyboygan, Wis.	240
Janesville, Wis.	30	Green Bay, Wis.	210
Peoria, Ill.	835	Elmhurst, Ill.	30
Topeka, Kans.	360	Kokomo, Ind.	45
Coney Island, New York	900	Hutchinson, Kans.	90
Ann Arbor, Mich.	120	Kewanee, Ill.	30

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[8] *Proceedings*, Am. Soc. C. E., June, 1937, p. 1117.

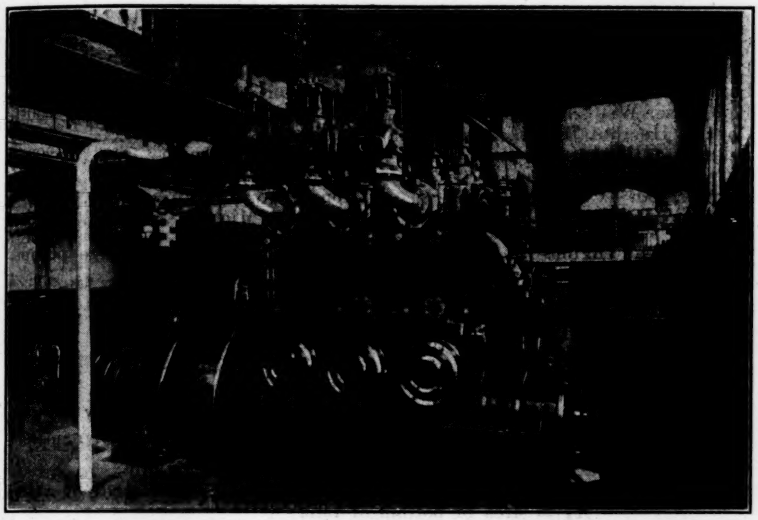

Fig. 10.—Engine Using Sludge Gas as Fuel, Birmingham, England

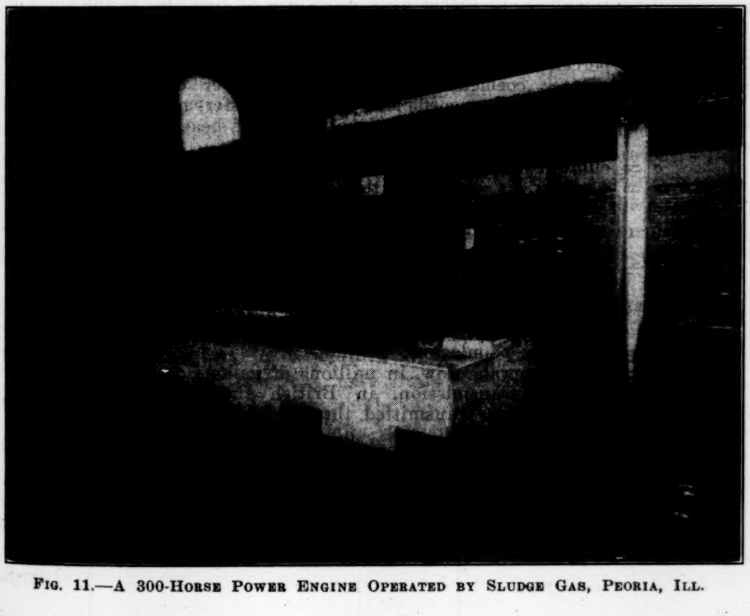

Fig. 11.—A 300-Horse Power Engine Operated by Sludge Gas, Peoria, Ill.

Place	Horse-power	Place	Horse-power
Wichita, Kans.	70	Durham, N. C.	300
Litchfield, Ill.	30	Fort Atkinson, Wis.	40
Dixon, Ill.	30	Rockville Center, N. Y.	...
La Crosse, Wis.	180	Grand Rapids, Mich.	50+
Chicago Heights, Ill.	125	Aurora, Ill.	140
Atlanta, Ga. (Intrenchment Creek)	300	District of Columbia	1 200
Atlanta, Ga. (Clayton Plant, approximate)	600	Madison, Wis.	230
		Monroe, Wis.	30
		Escanaba, Mich.	15

These gas engines are used for the operation of blowers, sewage pumps, and generators.

IV.—COMPUTATIONS OF TANK CAPACITY AND HEATING REQUIREMENTS

NOTATION

The notation introduced in this report conforms essentially to the "American Tentative Standard Symbols for Heat and Thermodynamics"[9] approved by the American Standards Association in 1931.

A = area, in square feet; A_R = area of roof; A_W = area of walls; A_B = area of bottom of tank;

a_t = the fraction of volatile solids digested in time t; a_0 = the fraction of volatile solids digested in time t_0;

B = theoretical required capacity of a sludge tank, in cubic feet;

b = thickness, in inches b_R = thickness of roof; b_W = thickness of walls; b_B = thickness of the bottom of the tank;

C = calorific value of gas, in British thermal units per cubic foot;

e = percentage efficiency of utilization;

f = surface coefficient of transmittance per degree Fahrenheit; f_i = surface coefficient of heat transfer, to the inner surface of tank; f_o = surface coefficient of heat transfer from the outer surface of the tank;

g = cubic feet of gas produced per pound of volatile solids digested;

K = a ratio; K_l = ratio of the volume of liquid sludge added daily to the volume of sludge in the tank; K_v = ratio of weight of volatile solids added daily to the weight of volatile solids in the tank;

k = thermal conductance of a homogeneous material 1 in. thick, in British thermal units per square foot per hour of area, per inch of thickness, per degree (Fahrenheit) difference of temperature, between the two surfaces of the material;

N = tributary population;

Q = rate of sewage flow, in millions of gallons daily;

q = thermal transmission, in British thermal units per hour; q_R = heat transmitted through the roof; q_W = heat transmitted through walls; q_B = heat transmitted through the bottom of the tank;

S = dry solids added daily to sludge tank in parts per million of sewage;

T = temperature; T_t = temperature of the sludge in the tank; T_o = temperature of the outside air, or earth; T_s = temperature of incoming sludge;

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[9] A.S.A. — Z10c — 1931.

t = time, in days; adopted period of digestion; t_0 = time required for substantially complete digestion (arbitrarily defined as the time required to reduce the volatile solids by 75%); t_1 = maximum interval between sludge drawings; t_2 = maximum time during which no digestion occurs;

U = over-all coefficient of heat transmittance, in British thermal units per square foot per hour, per degree (Fahrenheit) difference in temperature over-all; U_R = coefficient for heat transmittance through the roof; U_W = coefficient for heat transmittance through the walls; U_B = coefficient for heat transmittance through the bottom of the tank;

V = volatile fraction of solids;

W = weight, in pounds, of dry solids added daily;

w = percentage of water by weight; w_0 = percentage of water in entering sludge; w_1 = percentage of water in the withdrawn sludge; w_m = mean percentage of water in the sludge remaining in the tank $= \dfrac{w_0 + w_1}{2}$.

Tank Capacity Computations

General.—Capacities of from 1.5 to 3.5 cu ft per capita have been recommended, as the result of experience, for the sludge chambers of Imhoff tanks. In separate digestion tanks, however, less space is required, because the period of digestion is shortened by heating the sludge, and by thickening, agitation, or circulation. Moreover, there is no waste space such as that taken up by the sloping baffles in the lower story of the Imhoff tank. With the smaller, heated tanks, however, it must be borne in mind that the sludge must be drawn more frequently, and that the supernatant liquor is not cared for automatically.

It has been the practice to state the capacity of a sludge tank in cubic feet per capita, or cubic feet per million gallons daily of sewage. Neither of these units is altogether satisfactory, because of local differences in per capita quantities and in the strength of the sewage. The amount of solids in industrial wastes or in street wash from combined systems is a function of neither volume nor population. A better basis of design is the weight of solids in the sludge removed daily from the sewage. Consideration must also be given to the percentage of volatile matter in the solids, percentage digested, and, most important of all, the water content of the sludge.

Rational formulas have been developed for capacities of sludge tanks under several typical conditions, and diagrams have been prepared by the use of which capacities may be quickly obtained for widely varying conditions.

Case I.—Continuous Operation, with Daily Withdrawals.—Assume that a tank is in operation with a given weight of solids added daily and that an equal weight of solids disappears daily by digestion and withdrawal, so that the measurable sludge in the tank remains approximately at constant depth. Solids withdrawn include those passing out with the overflow of the supernatant liquor. The error in assuming a uniform rate of digestion from entry to withdrawal is on the side of safety.

The quantity of dry solids added on any one day will be reduced by digestion in the following period of t days by an amount, $a_t\ V\ W$ (see

Fig. 12). The average reduction in all daily additions made during the period will be $\frac{a_t V W}{2}$. The corresponding total reduction in weight will be $\frac{a_t V W t}{2}$, and the weight of solids remaining in the tank will be $W t \left(1 - \frac{a_t V}{2}\right)$.

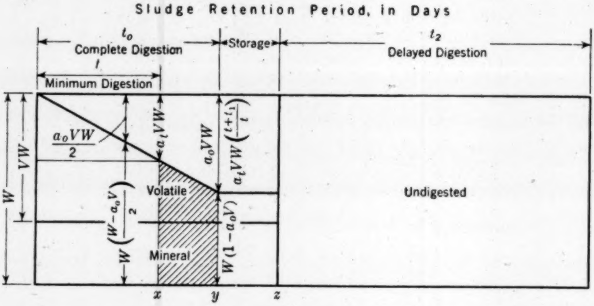
Sludge Retention Period, in Days
t_o
Complete Digestion
Storage
t_2
Delayed Digestion
t
Minimum Digestion
W
VW
$\frac{a_o VW}{2}$
$W\left(\frac{W - a_o V}{2}\right)$
$a_t VW$
Volatile
Mineral
$a_o VW$
$a_t VW\left(\frac{t}{t+t_1}\right)$
$W(1 - a_o V)$
Undigested
x
y
z

FIG. 12

In order that the quantity of solids in the tank may remain constant, the daily withdrawals (in pounds, dry basis) must be $W \left(1 - \frac{a_t V}{2}\right)$. Obviously, the weight of solids to be provided for in the tank is equal to t times the daily withdrawals. The theoretical total weight of solids and liquid is $\frac{W t \left(1 - \frac{a_t V}{2}\right)}{1 - w_m}$ which divided by the weight of a cubic foot of liquid (62.5 ± lb) gives the minimum theoretical required tank capacity, in cubic feet:

$$B = \frac{0.008\ (2 - a_t V)\ W t}{1 - w_m} \quad \text{.....................(1)}$$

The required capacity, in cubic feet per pound of dry solids added daily, is,

$$\frac{B}{W} = \frac{0.008\ (2 - a_t V)\ t}{1 - w_m} \quad \text{.....................(2)}$$

Capacity per capita of population may be expressed either as $\frac{B}{N}$, or as $\frac{S Q t (2 - a_t V)}{15 N (1 - w_m)}$.

Example 1.—Let $a_t = 0.50$; $V = \frac{2}{3}$; $t = 30$ days; $N = 100\,000$; $W = 10\,000$; $S = 120$; $Q = 10$; and $w_m = 0.925$ per cent. Then, by Equation (1), $\frac{B}{W} = 5.33$ cu ft per lb of solids added daily; and $B = 53\,000$ cu ft (net capacity). The net capacity per capita is 0.53 cu ft. Two tanks, 40 ft in diameter and 21.2 ft deep, will give the required volume. If the depth is increased 3 ft to allow for scum and supernatant, the gross volume will be 60 800 cu ft, or 0.608 cu ft per capita.

This solution is for a controlled tank under ideal conditions. A liberal factor of safety should be applied to the theoretical quantities, because the assumed conditions in any sewage are "subject to change without notice."

Case II.—Periodic Withdrawals, Incomplete Digestion.—Assume that a tank is in operation with a given weight of solids added daily, but that the withdrawals of sludge take place at intervals of t_1 days. Further, assume that every part of the mass of sludge removed from the tank at a drawing has undergone digestion for t days. If $t + t_1$ is not greater than t_0, the conditions are as indicated in Fig. 12, in which t_1 (max.) is represented by the horizontal distance between the ordinates at x and y, in the diagram. The weight of solids removed at said withdrawal is represented by the cross-hatched area and equals $W\ t_1\left[1-\frac{a_t V}{2}\left(1+\frac{t+t_1}{t}\right)\right]$. The maximum weight of solids in the tank before drawing, is $W\,(t+t_1)\left[1-\frac{a_t V}{2}\left(\frac{t+t_1}{t}\right)\right]$.

The required volume is,

$$B = \frac{W\,(t+t_1)\left[1-\frac{a_t V}{2}\left(\frac{t+t_1}{t}\right)\right]}{62.5\,(1-w_m)} \quad \ldots\ldots\ldots\ldots\ldots(3)$$

Example 2.—Substitute the following numerical values: $W = 10\,000$; $t = 30$; $t_1 = 15$; $a_t = 0.50$; $V = \frac{2}{3}$; $w_m = 0.925$, in Equation (3); and $B = 72\,000$ cu ft, net capacity.

Again: If $t + t_1$ is greater than t_0, the conditions are as indicated in Fig. 12, in which t_1 (max.) is represented by the horizontal distance between the ordinates at x and z, in the diagram. The maximum weight of solids in the tank, before drawing is $W\left[t_0\left(1-\frac{a_0 V}{2}\right)+(t+t_1-t_0)\,(1-a_0 V)\right]$, and the required volume is:

$$B = \frac{W}{62.5}\left[\frac{t_0\left(1-\frac{a_0 V}{2}\right)}{1-w_m}+\frac{(t_1+t-t_0)\,(1-a_0 V)}{1-w_1}\right] \quad \ldots\ldots(4)$$

Example 3.—Substitute the following numerical values: $W = 10\,000$; $t = 30$; $t_1 = 30$; $t_0 = 45$; $a_0 = 0.75$; $V = \frac{2}{3}$; $w_m = 0.925$; $w_1 = 0.90$, in Equation (4); and $B = 84\,000$ cu ft, net capacity.

Case III.—Periodic Withdrawals, Complete Digestion.—The situation is shown in Fig. 12, in which t_1 is the horizontal distance between the ordinates at y and z, in the diagram. The maximum weight of solids in the tank before drawing is $W\left[t_0\left(1-\frac{a_0 V}{2}\right)+t_1\,(1-a_0 V)\right]$ and the required volume is:

$$B = \frac{W}{62.5}\left[\frac{t_o\left(1 - \frac{a_o V}{2}\right)}{1 - w_m} + \frac{t_1\,(1 - a_o V)}{1 - w_1}\right] \ldots\ldots\ldots\ldots (5)$$

Example 4.—Substitute the following numerical values: $W = 10\,000$; $a_o = 0.75$; $V = \frac{2}{3}$; $t_o = 45$; $t_1 = 15$; $w_m = 0.925$; $w_1 = 0.90$, in Equation (5); and $B = 84\,000$ cu ft, net capacity.

It is to be noted that the identical results obtained in Examples (3) and (4) indicate that, under the assumed conditions, 75%, or complete, digestion may be reached when the intervals between drawings do not exceed 15 days, whereas 30-day intervals may result in only 50% digestion.

Case IV.—Unheated Tanks in Cold Climate.—In unheated tanks in a cold climate there will be periods during which digestion practically ceases. In such cases, capacity, in addition to that computed by Equation (4), will be required. If t_2 represents this period of suspended digestion, the additional capacity will amount to $\frac{W t_2}{62.5\,(1 - w_o)}$. Adding this to the right-hand member of Equation (4), a new expression is obtained for total capacity, as follows:

$$B = \frac{W}{62.5}\left[\frac{t_o\left(1 - \frac{a_o V}{2}\right)}{1 - w_m} + \frac{(t_1 + t - t_o)\,(1 - a_o V)}{1 - w_1} + \frac{t_2}{1 - w_o}\right] \ldots (6)$$

The expression, $t_1 + t_2$, represents the maximum interval between drawings. The point where t_1 stops and t_2 begins is not sharply defined, but may be estimated, within reasonable limits, from the temperature of the sewage. Digestion may be assumed to cease when the temperature of the sludge has lowered to 50° F.

It has been observed that early drawings from Imhoff tanks, following cold weather, give sludge of greater density than is indicated by $\frac{t_2}{1 - w_o}$. This has led to the assumption that undigested sludge is densified by long storage and the volume correspondingly reduced. With increase in temperature, however, rising gas will expand the sludge mass and cause "foaming" and overflow if ample capacity is not provided.

Example 5.—Let the various numerical values be assumed as in Example 3 and let $t_2 = 60$; and $w_o = 0.95$. By substitution in Equation (6), $B = 276\,000$ cu ft.

Case V.—Tank Capacity as Determined by Permissible Ratio of Daily Additions of Volatile Solids to Volatile Solids Remaining in Tank.—Experience has demonstrated that the ratio, K_v, of volatile solids added, daily, to the volatile solids in the tank, should not exceed about 2%, for uncontrolled (unheated) tanks, nor 5%, for controlled tanks. Capacity determined by the preceding Equations (1), (3), (4), or (6), should be checked to see that it meets this requirement. If digestion is assumed as progressing at a uniform rate, the ratio of weight of solids in the tank to their volatile content, may be expressed by $\dfrac{W\left(1 - \dfrac{a_t V}{2}\right)}{W V\left(1 - \dfrac{a_t}{2}\right)} = \dfrac{2 - a_t V}{V\,(2 - a_t)}$. In order not to exceed the ratio, K_v, the volatile solids in the tank at all times must be $\dfrac{V W}{K_v}$; hence, the total solids in the tank must be $\dfrac{V W\,(2 - a_t V)}{K_v V\,(2 - a_t)} = \dfrac{W\,(2 - a_t V)}{K_v\,(2 - a_t)}$. Taking into account the water content and dividing by 62.5, the required capacity is:

$$B = \frac{0.016\ W\ (2 - a_t V)}{(1 - w_m)\ K_v\ (2 - a_t)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The tank should not be drawn lower than the capacity indicated by Equation (7). Expressing the permissible quantity of influent, daily, in terms of its volumetric ratio to the quantity of sludge in the tank:

$$K_l = \frac{\text{Sludge added daily}}{\text{Sludge in the tank}} = \frac{0.016\ W}{1 - w_o} \div \frac{0.016\ W\ (2 - a_t V)}{(1 - w_m) K_v\ (2 - a_i)}$$

$$= K_v\ \frac{(1 - w_m)\ (2 - a_t)}{(1 - w_o)\ (2 - a_t\ V)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Example 6.—Substituting in Equation (7), the numerical values given in Example 1, $B = \dfrac{2\,370.37}{K_v}$. For an uncontrolled tank, $B = \dfrac{2\,370.37}{0.02}$ $= 118\,500$ cu ft, and for a controlled tank, $B = \dfrac{2\,370.37}{0.05} = 47\,400$ cu ft. Substituting numerical values in the last member of Equation (8), $K_l = 1.35\ K_v$. For uncontrolled tanks, $K_l = 1.35 \times 0.02 = 0.027$; and for controlled tanks, $K_l = 1.35 \times 0.05 = 0.0675$. $\left[\text{Check: } \dfrac{0.016 \times 10\,000}{0.05 \times 47\,400} = 0.0675.\right]$

It will be noted that the foregoing value of B, for the controlled tank (47 400 cu ft) is about one-ninth less than the value obtained in Case I, in which the period of digestion is assumed at thirty days. If the method presupposed in Case I were followed, namely, daily withdrawals, the former computation would stand and the capacity would be theoretically sufficient, providing the tank was never drawn down below a capacity of 47 400 cu ft, or eight-ninths of its computed net capacity.

In Case II, Equation (3), the net capacity was computed as 72 000 cu ft. In order always to leave 47 400 cu ft in the tank, it should never be drawn lower than 47 400 ÷ 72 000 = 65.8% of its net capacity. Similarly, in Equation (4) and in Case III, the limit of draw-down would be 47 400 ÷ 84 000 = 56.4% of the net capacity.

In Case IV, the unheated tank, during the period of delayed digestion, should not be drawn below $118\,500 \div 276\,000 = 42.9\%$ of its net capacity.

It will be observed that the ratios, K_v and K_l, have two applications, useful to the plant operator: (1) The fixing of the limits of sludge withdrawal; and (2) the transformation of the ratio of volatile solids, K_v, into terms of relative volume, K_l. In the latter determination there are two principal factors which affect the value of K_l, namely, the change in water content of the sludge and the percentage reduction in volatile solids. For example, the greater the reduction of the water content, in the tank, the greater the value of K_l. On the other hand, the greater the reduction in volatile solids, the lower the value of K_l.

Fig. 13 contains five diagrams: Fig. 13 (*a*) and (*b*) refer, respectively, to heated and unheated tanks. They are plotted from Equation (8) and exhibit the relations of K_v and K_l. Fig. 13 (*c*), (*d*), and (*e*) are based, respectively, on Equations (2), (4), and (6), from which, by using the constants employed, respectively, in Examples 1, 3, and 5, the following three abridged formulas have been deduced: $\frac{B}{W} = 3.2\ (2 - a_t\, V)$; $\frac{B}{W} = 7.2 \left(\frac{5}{3} - a_0\, V\right)$; and, $\frac{B}{W} = 7.2 \left(\frac{13}{3} - a_0\, V\right)$. Fig. 13 (*c*), (*d*), and (*e*) are plotted from these formulas with "Amount of Volatile Solids" and "Amount Digested" (both expressed in percentages) as variables. In each diagram, the quantity sought may be found on the curves or may be approximated closely by interpolation. For different assumptions of the time elements and water contents, variations may be made in the constants in the abridged formulas from which different, although similar, curves will result.

HEATING COMPUTATIONS

General.—The quantity of gas produced by the digestion of sludge amounts to from 0.5 to 1.0 cu ft per capita per day, or from 5 to 19 cu ft (average, 7.8 cu ft) per lb of volatile solids added daily to the tank. A more definite unit for use in computation, however, is the amount produced per pound of volatile solids digested. The general average for this value is about 12 cu ft. Calorific values of sludge gas are discussed in Section I.

When a tank is first warmed, the temperature of the entire mass of sludge must be raised, and a relatively large amount of heat will be required; but after the operating temperature has been reached, it is only necessary to make up the losses due to conduction and radiation and to heat the relatively small quantity of incoming sludge.

Heat losses through the roof, walls, and bottom of the tank must all be taken into account. The formula commonly used for determining heat loss through a wall of homogeneous material with air or gas on one side, maintained at a given temperature, T_1, and with the other side open to the air,

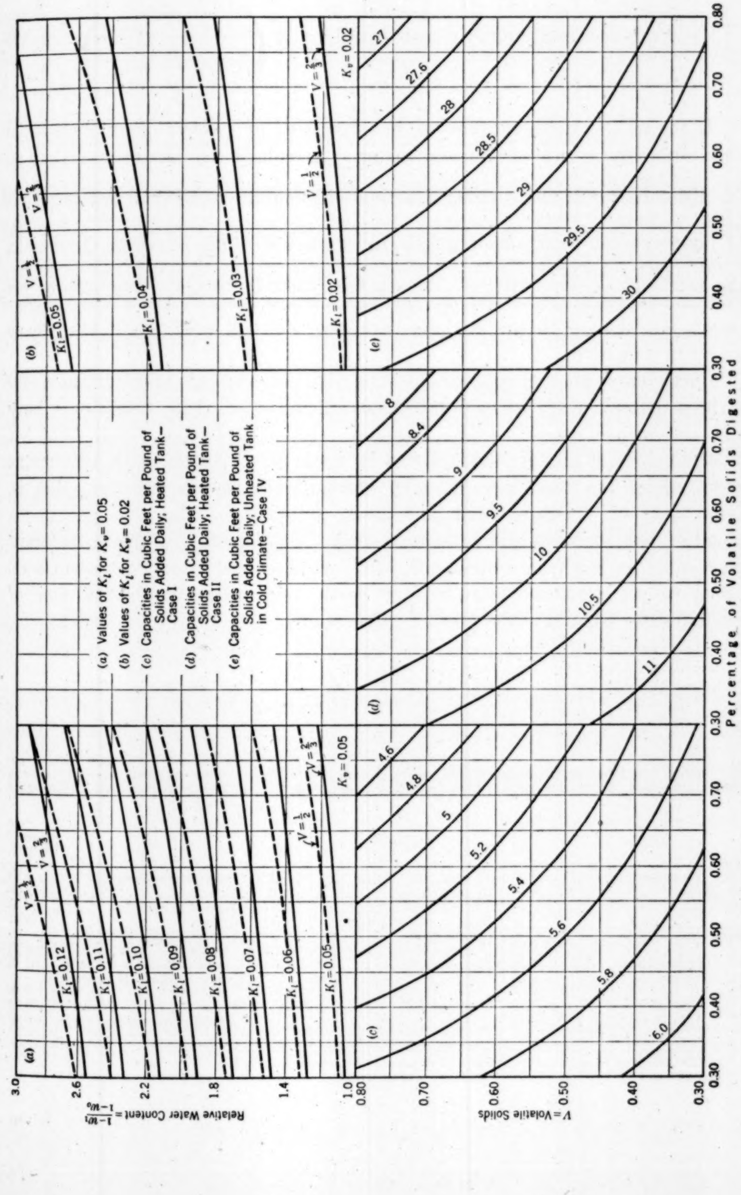

FIG. 13.—DIAGRAMS FOR DETERMINING THEORETICAL CAPACITY OF SLUDGE DIGESTION TANKS

or in contact with earth at a lower temperature, T_o, is:

$$\text{British thermal units per hour} = A\,U = A\left(\frac{T_i - T_o}{\dfrac{1}{f_i} + \dfrac{b}{k} + \dfrac{1}{f_o}}\right) \quad \ldots\ldots(9)$$

If the wall is not homogeneous, but composed of several layers of material with different values of k, each material is considered separately; that is, the denominator of the complex fraction will contain more than one term, $\frac{b}{k}$. If liquid rather than air or gas is in contact with the wall on the inside, the fraction, $\frac{1}{f_i}$, is omitted. The surface coefficients on either side of air spaces less than ½ in. in thickness are also omitted, assuming there are no convection currents.

Coefficients of thermal conductance and of surface transmittance for various materials are given[10] in Table 12. Most of these coefficients have been determined in air for heating and ventilating purposes and from hot pipes buried in the ground. There is need for experiments to determine losses under the conditions actually obtaining in sewage practice; that is, from tanks of concrete, wood, and steel, containing sludge and gas at the range of temperatures met in sewage work. Coefficients of surface transmittance into the air from hollow tile, steel, wood, concrete, and earth surfaces, losses into surrounding wet or dry earth, and losses caused by flow of ground-water, should all be given further study.

TABLE 12.—COEFFICIENTS OF THERMAL CONDUCTANCE AND SURFACE TRANSMITTANCE

(a) CONDUCTANCE THROUGH HOMOGENEOUS MATERIALS, 1 INCH THICK, IN BRITISH THERMAL UNITS PER SQUARE FOOT PER HOUR; COEFFICIENT k.				(b) TRANSMITTANCE FROM SURFACES EXPOSED TO STILL AIR*, IN BRITISH THERMAL UNITS PER SQUARE FOOT PER HOUR.	
Material	Coefficient	Material	Coefficient	Material	Coefficient*
White pine (across the grain)	0.78	Air spaces thicker than ½ in.	1.10	Smooth plaster surfaces	0.93
Virginia pine	0.96	Gypsum plaster	2.32	Concrete	1.30
Yellow pine	1.00	Dry brickwork	4.00	Wood	1.40
Maple	1.10 to 1.20	Damp or wet brickwork*	5.00	Brickwork	1.40
		Cement mortar	8.00	Average of many building materials	1.34
		Cinder concrete	5.20		
Wood (average value)	1.00	Stone concrete	8.30	Concrete exposed to dry earth	1.00
Cellular gypsum	0.59			Concrete exposed to water or wet earth	2.00
				Hollow tile (2-in. furring)†	1.18
				Tar and gravel roofing (¾-in.)†	3.67

* For air motion at 15 miles per hr., these coefficients should be multiplied by 3.
† Over-all value for non-homogeneous combinations of materials.

Example 7.—Let $W = 4\,000$ lb; $V = \frac{2}{3}$; $a_o = 0.75$; $w_o = 0.95$; $g = 12$ cu ft; $C = 700$ Btu per cu ft; $e = 60\%$; $T_i = 82°$ F; $T_s = 56°$ F (mean annual);

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[10] See 1931 Guide, Am. Soc. of Heating and Ventilating Engrs.

T_o in air, = 50° F (mean annual); T_o, in earth, = 60° F (mean annual) and T_o, for ground-water, = 54° F. The velocity of the air is assumed to be 15 miles per hr.

The dimensions of the digestion tank are as follows: Diameter, 40 ft; height, 24 ft, of which all but 3 ft, is banked with dry earth; roof, 8-in. concrete; walls 18-in. concrete; and floor, 12-in. concrete, located below the water-table. Gas is present between the sludge surface and the roof of the tank.

The heat available per hour is:

$$\frac{W\,V\,a_0\,g\,C\,e}{24} = \frac{4\,000 \times \frac{2}{3} \times 0.75 \times 12 \times 700 \times 0.60}{24} = 420\,000 \text{ Btu}$$

The heat required to bring the incoming sludge to the specified digestion temperature is:

$$\frac{W\,(T_t - T_s)}{24\,(1 - w_o)} = \frac{4\,000\,(82 - 56)}{24\,(1 - 0.95)} = 86\,666 \text{ Btu per hr}$$

The losses are found by substituting known values in Equation (9): $A_R\,U_R = 20\,600$ Btu per hr. and,

$$A_w\,U_w = (3 \times 40\pi\,U_1) + (21 \times 40\pi\,U_2) = 120\pi\,U_1 + 7\ U_2$$

in which U_1 refers to the part of the wall above the ground and U_2 to that below. Then,

$$A_w\,U_w = 377 \left[\frac{82 - 50}{\frac{18}{8.3} + \frac{1}{4.02}} + \frac{7\,(82 - 60)}{\frac{18}{8.3} + \frac{1}{1}} \right] = 23\,300 \text{ Btu per hr}$$

and,

$$A_r\,U_r = \frac{1\,257\,(82 - 54)}{\frac{12}{8.3} + \frac{1}{2}} = 18\,100 \text{ Btu per hr}$$

The total heat loss is $A_R\,U_R + A_W\,U_W + A_R\,U_R$, or 62 000 Btu per hr. Adding to this the amount required to heat the incoming sludge, the total heat required is found to be 148 667 Btu per hr, or 35.4% of the heat available from the gases of digestion.

These computations, it will be noted, were based on assumed mean annual temperatures for incoming sludge, air, earth, and ground-water. More heat will be required per hour in the winter, and less in the summer. A complete investigation, of course, would include computation of the requirements for both extremes

Although the cost of construction rather than the question of heat loss will usually dictate structural dimensions, it is of interest to note the effect of changes in those dimensions on the amount of heat lost. For example, a decrease of 6 in. in the thickness of the tank wall would raise the value of

A_wU_w to 28 900 Btu per hr—an increase of 24 per cent. On the other hand, if the thickness of the roof were doubled, $A_R U_R$ would become 13 380 Btu per hr, or 65% of the original value. It should be noted also that if in Example 7 the sludge had been in contact with the roof, the fraction, $\frac{1}{1.34}$, would have been omitted from the denominator in the expression for $A_R U_R$ and the heat loss would have been increased to 33 800 Btu per hr.

V.—METHODS OF SLUDGE DISPOSAL

Regardless of whether or not disposal is preceded by digestion, the problem of getting rid of sludge is an important one both to the designer and to the operator. Local conditions will determine the extent of the provisions necessary, and, together with the method of sewage treatment in use, must be taken into account in selecting the methods to be used. In general, the methods may be divided into (1) methods for disposing of liquid sludge; and (2) methods for disposing of dewatered sludge.

Liquid Sludge Disposal

The five main methods of disposing of liquid sludge are: (*a*) Transportation by scow to some suitably isolated point in deep water, in a lake, sea, or ocean; (*b*) transportation by tank wagon or truck to agricultural lands where the solid portions are eventually plowed under and become a source of plant food; (*c*) "lagooning," or intermittently flooding the sludge upon a favorably located tract of land, until deposited solids completely fill the area thus used; (*d*) broad irrigation; and (*e*), discharging into the flood waters of a stream, or by pipe line into the deep water of a lake or sea.

Method (*a*) has a limited applicability. Outstanding examples are large seaport cities. The City of London has used this method, and New York City will dispose of some undigested sludge from the Wards Island plant in this manner.

As the fertilizing value of liquid sludge is extremely small in proportion to its bulk, Method (*b*) is not promising unless local conditions are peculiarly favorable.

Method (*c*) is not final unless the deposited solids constitute a permanent fill of suitable character. The dried sludge may be hauled away by farmers for soil treatment, or by others for use on lawns, golf courses, parks, etc., so that the capacity of the lagooned area will be prolonged indefinitely Undigested sludge is lagooned in this manner at Indianapolis, Ind.

The use of Method (*d*), also, is possible in only a few localities, where the soil and the climate conditions are both favorable.

Method (*e*) has a limited application and its use is not likely to meet the approval of either laymen or public health officials.

Disposal of Dried Sludge

In a large majority of sewage works, the liquid tank sludge is dewatered before being finally disposed of. This may be done on drying beds or by several types of pressing or vacuum filtration equipment. In general, the drying beds produce a cake from 3 to 5 in. thick, and the filter presses and vacuum filtration equipment produce a cake less than 0.5 in. thick. Either cake can be transported readily.

There are six main methods of disposing of dried or partly dried sludge: (*a*) Sale or free disposal for use as fertilizer; (*b*) wasting; (*c*) use as fill; (*d*) burying; (*e*) incineration; and (*f*) transportation to sea by scow.

Method (a).—Sale or Free Disposal for Use as Fertilizer.—In the Municipal Index for 1931, twenty-seven cities and villages in the United States, representing a population of 639 000, reported that all their sewage sludge was sold as fertilizer, and thirteen additional cities, representing a population of 664 000, reported the sale of quantities varying from 10 to 95% of the total production. Data on the sale of sludge by some of these cities are given[11] in Table 13.

TABLE 13.—DATA ON SALE OF SLUDGE FOR FERTILIZER

City	1930 population	Type of plant	Percentage of total sludge sold	Price per ton
Arkansas:				
DeQueen	2 900	....	100	....
California:				
Los Angeles County Sanitary Districts	200 000	Activated sludge	100	$2.00
South Pasadena	13 724	Activated sludge	100	36.00
Alhambra	29 500	Activated sludge	100	25.25
Pasadena	123 000	Activated sludge	100	20.00
Pomona	20 804	Imhoff tanks and areation	100	6.00
San Bernardino	37 500	Imhoff tanks: Trickling filters	100	1 000 *
Indiana:				
Brazil	8 744	Imhoff tanks: Trickling filters	100	20.00 *
Huntington	13 420	Imhoff tanks	....	1.50
Kentucky:				
Fort Thomas	10 039	Imhoff tanks: Trickling filters	100	2.00
Michigan:				
Harbor Beach	1 892	Imhoff tanks: Sand filters	75	1.50
Pontiac	64 928	Imhoff tanks: Trickling filters	100	0.50
Missouri:				
Moberly	13 772	Activated sludge	60	1.50
Springfield	57 527	Imhoff tanks: Trickling filters	50	2.00 †
New Jersey:				
Audubon	8 904	Imhoff tanks: Trickling filters	Some	1.00
Sea Isle	850	Plain sedimentation	50	....
Trenton	123 356	Imhoff tanks	10	0.75
New York:				
East Rochester	6 627	Imhoff tanks: Trickling filters	50	0.50
Mt. Morris	3 238	Imhoff tanks	100	....
Rochester	328 132	Imhoff tanks: Trickling filters	95	0.25
Schenectady	95 692	Imhoff tanks: Trickling filters	100	0.25 ‡
North Carolina:				
Charlotte	82 675	Activated sludge	40	....
Winston-Salem	75 274	Sedimentation	100	0.00
Ohio:				
Celina	4 664	Imhoff tanks	10	....
Chardon	1 800	Imhoff tanks	50	....
Columbiana	2 484	Imhoff tanks: Trickling filters	100	1.00 ‡
Pennsylvania:				
Elizabethtown	3 940	Imhoff tanks: Trickling filters	100	1.00
South Carolina:				
Greenville	29 154	Septic tanks	16	....
Wisconsin:				
Madison	57 899	Imhoff tanks: Trickling filters	100	1.00
Monroe	5 013	Septic tanks: Sand filters	100	0.25

* Price per year. † Price per cubic yard. ‡ Price per load.

The most conspicuous example of the disposal of sewage sludge as fertilizer is Milwaukee, Wis. (not included in Table 13) where during 1928, 1929, and

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[11] See Municipal Index, 1931.

1930, 91 800 tons of sludge (Milorganite) was sold for $1 720 000—an average price of $18.75 per ton and an average production of approximately 1.0 ton of fertilizer per million gallons of sewage treated. The Milwaukee works include grit chambers, coarse screens, fine screens, activated sludge treatment, clarification, and sludge conditioning, filtration, and drying.

The major element contributing to the value of sludge as a fertilizer is nitrogen. Schaetzle[12] reported the nitrogen content (dry-residue basis) in Baltimore, Md., sludge to be as follows:

	Percentage
Imhoff tank sludge	2.75
Digestion tank sludge	2.45
Raw sludge	2.64
Final settling basin sludge	3.19

Keefer[13] reported the results of sludge-drying at Baltimore extending over a period from August 1, 1921, to June 1, 1924, during which time 7 448 tons of wet sludge containing 68% moisture were dried to 2 949 tons of a moisture content varying from 9.4% to 23.6% and having a content of nitrogen as ammonia of from 2.05% to 2.60% on the dry basis.

In general, it may be said that the sludge produced in an activated sludge plant is considerably richer in nitrogen than that produced in clarification-sludge digestion plants. The Milwaukee sludge, for example, averaged about 6.5% nitrogen as ammonia during 1928, 1929, and 1930.

Phosphoric acid and potash are usually present in sewage sludge in small quantities (less than 1%). To the present, the relatively low market price of these ingredients has removed them from serious consideration; however, in the future, they may be of value.

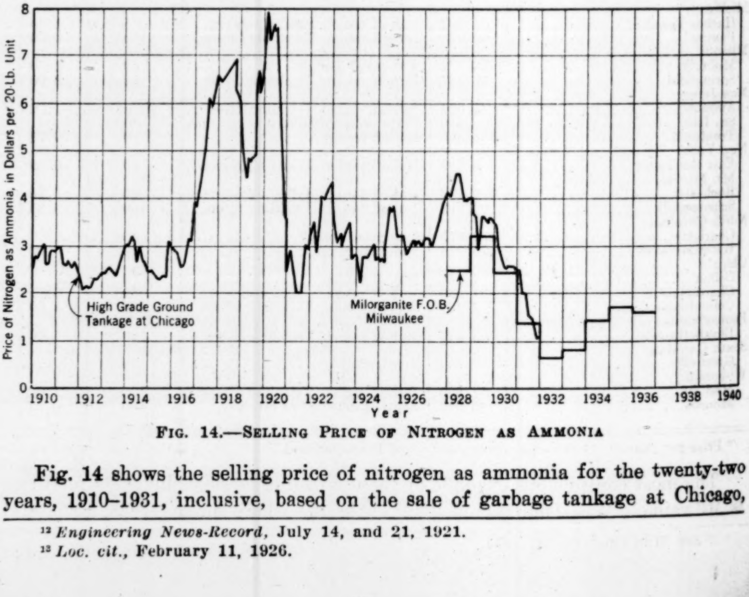

FIG. 14.—SELLING PRICE OF NITROGEN AS AMMONIA

Fig. 14 shows the selling price of nitrogen as ammonia for the twenty-two years, 1910–1931, inclusive, based on the sale of garbage tankage at Chicago,

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[12] *Engineering News-Record*, July 14, and 21, 1921.

[13] *Loc. cit.*, February 11, 1926.

Ill., and, for the nine years, 1928 to 1936 inclusive, based on the sale of sludge (Milorganite) at Milwaukee. The higher nitrogen percentage composition of Milorganite probably enhanced its unit price and, in addition, it is probable that more of it was sold direct to ultimate users. Excess activated sludge has been dried and prepared for the market at Pasadena, Calif., and at Houston, Tex. Digested sludge from open beds has been dried and sold for fertilizer for local use at Grand Rapids, Mich., Dayton, Ohio, and Rockford, Ill. The net revenue at these plants has been relatively small but, in some years, has been sufficient to more than meet operating expenses and thus to dispose of the sludge with an income instead of an expense.

Method (b).—Wasting.—In this method the air-dried sludge is transported from the drying beds to some convenient place where it is allowed to accumulate; no plan is prepared for its ultimate disposal.

Method (c).—Use as Fill.—In this case the air-dried sludge is transported to a borrow-pit, or some other excavated area. This method of disposal is limited by the capacity of the area which can be used economically for this purpose.

Method (d).—Burying.—In this method the dried sludge is thrown into trenches and covered with earth. This method is expensive, and is warranted only where a lack of isolation makes it desirable to remove all possible source of nuisance.

Method (e).—Incineration.—The sludge is burned in specially adapted furnaces or in open piles. If air-dried sludge is properly incinerated, nothing remains but the ash, which is inert and not a source of nuisance. In some cases, the sludge from a drying bed is collected in a pile on the bed itself and burned, leaving only the ash to be removed.

Method (f).—Transportation to Sea by Scow.—This method of disposal is limited to seaport cities, and it is a question of economy as to whether the extra cost of drying and the smaller bulk to be transported is preferable to the disposal of the larger bulk of liquid sludge in the same manner.

Summary.—In many instances, it will be found that in actual practice, two or more of these methods will be in use at the same place. For example, where sludge is wasted or where it is used as fill, portions of the accumulated material may be hauled away by private individuals for use as fertilizer.

The Municipal Index for 1931 reports the methods of sludge disposal of 398 cities and villages in the United States, as follows:

(1) Sale or use as fertilizer	210
(2) Wasting	68
(3) Use as fill	38
(4) Burying	23
(5) Discharge into a body of water	18
(6) Part used as fertilizer, remainder wasted	9
(7) Part used as fertilizer, remainder used as fill	9
(8) Incineration	7
(9) Lagooning	6
(10) Transportation by scow for disposal in ocean	3

(11) Part used as fertilizer, remainder incinerated	3
(12) Broad irrigation	1
(13) Part used as fertilizer, remainder lagooned	1
(14) Part used as fertilizer, remainder buried	1
(15) Part incinerated, remainder used for fill	1
Total reporting	398

In the foregoing fifteen methods of disposal, the first five comprise 90% of the total, as indicated in the following percentage comparison:

(1) Sale or use as fertilizer	52.8
(2) Wasting	17.1
(3) Use as fill	9.5
(4) Burying	5.8
(5) Discharge into body of water	4.5
Total	89.7

It is probable that some who reported that sludge was incinerated or buried had in mind screenings instead of sludge, and that others who reported it as discharged into a body of water were confusing it with plant effluent.

VI.—OPERATING ROUTINE OF SLUDGE DIGESTION

The entire process of sludge digestion should be carried on without creating objectionable odors; should leave the solid and liquid by-products in a favorable condition for further treatment or final disposal; should provide for recovery of by-products if desirable; and, finally, should be economical and dependable.

The basic design of two-story digestion tanks has been standardized to a large extent. As a consequence, the operating technique is also more or less standard. This has not been the case with separate digestion tanks, in which variations in basic features of design, and differences in the quality of sludge handled, have resulted in a wide variety of operating practice. Some of the following statements, therefore, may be subject to considerable modification as future data are accumulated and co-ordinated.

Placing Tanks in Operation

Experience has shown that digestion tanks can be put into successful operation without the use of seeding material, but the time required for building up an alkaline sludge will undoubtedly be longer, and more careful supervision of operation may be necessary, than if the tank is seeded initially. The quantity of seeding material introduced usually ranges from 20% to 40% of the tank capacity. Seeding is especially desirable if a tank is being put into operation in the winter, although even then it is not absolutely necessary for either the two-story or the separate digestion tank.

INTRODUCTION OF SLUDGE

After tanks are in operation the problem of maintaining a proper balance between the digested, partly digested, and fresh solids, is encountered. The quantity of seeding material in the digestion compartment should always be sufficient to counterbalance the acid produced by the incoming fresh solids. It is advisable to add fresh solids continuously, or at frequent intervals, and at rates not exceeding 3% to 5% of the solids in the tank. This proportion should probably be based upon volatile matter content rather than on a total solids basis (see Section IV).

USE OF LIME

The optimum pH-value, or reaction, for any particular plant must be determined by experimentation. Reaction control should not be necessary if the tanks are properly operated, although it may be necessary in cases where the unusual character of sewage or improper operation of tanks has destroyed the alkaline balance of the sludge.

The re-agent generally used in reaction control is lime hydrate, which not only increases alkalinity, but also coagulates the colloidal organic matter. The results obtained depend to a large extent upon the quantities of re-agent added, and the method of application or mixing.

STIRRING OF SLUDGE

There is considerable difference of opinion as to the advantages of stirring sludge. It does seem necessary or desirable, however, that some stirring be done, at least during intervals when raw sludge is being introduced into the tank. The amount of stirring that should accompany this process remains open to question. There is apparently little necessity for stirring in a 2-story tank, although some agitation may be necessary when sludge is being withdrawn. Stirring in separate digestion tanks has been practiced to varying degrees in many installations. The advantages usually claimed are that it sets free the entrained gas at a uniform rate, and promotes digestion by helping to maintain a favorable reaction, mixing any chemical re-agents that may be added, and assisting in distributing raw sludge through the tank.

COLLECTION, MEASUREMENT, AND STORAGE OF GAS

The operation of this phase of the sludge digestion process may be summarized, as follows:

1.—Individual and master meter readings should be recorded daily. Meters should be checked occasionally for accuracy.

2.—Gas lines, domes, etc., should be checked frequently and kept clear and free from obstructions at all times. Usually, ample sizes of pipes will insure against troubles of this nature.

3.—Pressures in the system should be checked and recorded daily, or more often.

4.—Water in seals should be maintained at proper levels, and kept clear of sludge and scum. Outside seals should be filled with oil or similar material during winter months to prevent freezing.

5.—Lights and fires should be kept away from digestion tanks and from the gas-collection system.

6.—Gas masks should be kept available for use in case of emergency.

REMOVAL OF SLUDGE

The removal of sludge depends upon the degree of digestion attained. In unheated tanks it is advisable to hold sludge in the tanks as long as possible in the spring in order to take care of the winter accumulation of partly digested sludge.

In the process of drawing, the sludge should be allowed to flow at a slow regular rate, in order to prevent the formation of channels and the withdrawal of partly digested sludge. Failure of sludge to flow may usually be remedied by agitation with water-jets, or by mechanical means.

REMOVAL OF SCUM

The accumulation of large quantities of scum, which consists of fibrous material, such as leaves, sticks, matches, and seeds, in the gas vents of 2-story tanks, or on the surface of separate digestion tanks, may cause odors, prevent the even and free discharge of gas, or clog the pipe lines. Heavy layers of scum may form only during the first few months of operation, or may persist for long periods without necessarily causing a reduction in tank efficiency.

When necessary, various methods are used for preventing excessive formations of scum, such as the installation of submerging racks, or the mechanical breaking up of the layer by hand or by stirring equipment with revolving arms, thus causing the scum to re-settle. In some cases, it has been found successful to flush with water or sludge liquor so that the liquid scum can be drawn off through pipe lines. Ordinarily, it will be necessary to remove scum only once or twice a year, if at all.

REMOVAL OF SUPERNATANT SLUDGE LIQUOR

The satisfactory disposal of the supernatant liquor which separates from the solids in separate digestion tanks, presents a difficult problem. This liquor is never clear, has a high oxygen demand, and is relatively high both in suspended and dissolved solids.

Draw-off lines for supernatant sludge liquor should be provided at different elevations in the tank. The various methods of disposing of the liquor include its return to the inlet of sedimentation tanks or to the digestion compartment of 2-story tanks, or its discharge to drying beds.

CONTROL OF FOAMING

The term, "foaming", is applied to a condition which may exist in either a 2-story tank, or a separate digestion tank, when large quantities of light, gas-lifted sludge rise in the gas vents or to the surface of the tanks. This condition is not only unsightly in appearance and productive of foul odors, but it interferes seriously with the efficiency of the plant.

Various methods have been suggested for the control of foaming, such as (*a*) drawing sludge; (*b*) hosing gas-vent areas by water under pressure; (*c*) resting the tank by putting it out of service; (*d*) circulation of supernatant liquor on to the surface of the material; (*e*) adding hydrated lime either to the raw sewage or, preferably, to sludge as it is pumped back into the gas vents; (*f*) treatment of the raw sewage with chlorine; and (*g*) the removal of all the sludge from the tank and the building up of a new sludge. The accepted practice seems to be the withdrawal of sludge from the tank and the maintenance of proper quantities of good seeding material.

CONTROL OF ODORS

The entire elimination of odors about a sewage treatment plant is impossible, but a thorough knowledge of the treatment processes and careful operation of the plant will usually make it possible to avoid odors about which justifiable complaint could be made. It is more effective to avoid conditions that produce odors than to control the odors themselves.

One of the most important single factors having a direct bearing upon the control of odors is that of general cleanliness. All accumulations of sewage solids and grease on the walks or walls of the tanks, or about the grounds, should be removed immediately. Except for well-digested sludge drawn upon the beds, or otherwise disposed of, no sludge should be left exposed to the atmosphere. The hydrogen sulfide odors which sometimes develop in sedimentation and sludge digestion tanks may be controlled rather effectively by the use of liquid chlorine.

TOOLS REQUIRED IN ROUTINE OPERATION

General Tools.—These include tools that may be used elsewhere about the plant, such as wheel-barrows, shovels, hoes, brooms, ladders, and valve keys.

Special Tools.—Tools required for 2-story or separate sludge digestion tanks include water hose and nozzle for flushing, rubber squeegees, or chain drag, for cleaning flow compartment walls, perforated metal or wire skimmer for removal of scum, pitcher pump and hose for determining sludge level, and sampling buckets for composite samples, and long-handled dippers for securing the small amounts.

ANALYSES AND RECORDS

Laboratory control and the keeping of records of laboratory analyses are important as a guide to proper operation. The records of solids introduced and removed from tanks are important because the efficiency of digestion processes is largely measured by the reduction of volatile water. Gas quantities and temperature records are important in studying rates and progress of digestion.

Every care should be taken to have the results of the analyses represent a true portrayal of what has been accomplished during the process of changing (fresh) solids to digested sludge[14]. Data that should be obtained in connection with sludge digestion tanks may be summarized as follows:

(1) General Data (recorded daily): Temperature of atmosphere, in degrees Fahrenheit; hot water circulated, in gallons per day; temperature of circulating water (incoming and outgoing); and temperature of sludge in tank.

(2) Incoming Sludge: Time of introducing sludge, in hours; volume added, in gallons; temperature of sludge, in degrees Fahrenheit; and analysis of sludge, daily, or for each volume added: (*a*) Percentage of total and dissolved solids; (*b*) percentage of ash and volatile matter; (*c*) pH-value and alkalinity; and (*d*) specific gravity or actual weight per gallon.

(3) Sludge Withdrawn: Volume withdrawn, in gallons; and analysis of sludge, daily[14] or for each volume withdrawn (same as for incoming sludge).

(4) Supernatant Sludge Liquor: Volume withdrawn, in gallons; analysis of supernatant, daily[15], or for each volume withdrawn: (*a*) Percentage of total and dissolved solids; (*b*) percentage of ash and volatile matter; and (*c*) pH-value and alkalinity.

(5) Scum (Recorded Monthly): Thickness, in inches; volume removed, in gallons; analysis of scum: (*a*) Percentage of moisture; (*b*) percentage of ash and volatile matter; (*c*) pH-value and alkalinity; and (*d*) specific gravity.

(6) Gas: Volume, in cubic feet (recorded daily); analysis of gas (weekly): (*a*) Heat value, in British thermal units; and (*b*) percentage of CO_2, N_2, CH_4, O_2, and H_2.

In sampling digested sludge, composite samples should be collected proportionate to the quantity of sludge pumped. The fresh solids should be sampled by suspending small buckets in the flow compartments of sedimentation tanks.

Analyses of sludge removed from primary settling tanks and sludge digestion tanks should include observations as to color, odor, and streaking on a glass plate; and determination of volume, percentage of moisture, percentage of volatile matter, pH-value, alkalinity, specific gravity, and occasionally grease and total nitrogen, and possibly bio-chemical oxygen demand.

It is of great importance, in the operation of digestion tanks of both types, to measure and record the depth of sludge in the tank, as well as the quantities added and removed. In the case of a 2-story tank, the information is important in order to keep the sludge line well below the slot.

Sludge may be measured with any of the following types of equipment:

1.—Pitcher pump with flexible hose. The suction end of the hose can be raised or lowered until the sludge line is found.

2.—Flat tray of fine wire mesh or perforated plate, attached to a chain.

3.—Bottles (with removable stoppers) which can be lowered by means of a pole to obtain samples at various depths.

All of the aforementioned methods are fairly reliable, but the pitcher pump and hose is the equipment most widely used.

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[14] "Standard Methods for the Examination of Water and Sewage," pub. by the Am. Public Health Assoc.

[15] Daily or sterilized weekly composites from daily samples.

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Respectfully submitted,

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APPENDIX I

OPERATING DATA FROM GRAND RAPIDS, MICHIGAN, APRIL, 1931, THROUGH JUNE, 1932

Through the courtesy of the City Officials at Grand Rapids, Mich., and of James R. Rumsey, M. Am. Soc. C. E., Superintendent of the Sewage Treatment Works, and N. G. Damoose, Chemist, the Committee was given the opportunity of making observations and collecting specific information relating to the operation of the sludge digestion tanks at Grand Rapids, from April 1, 1931, through June, 1932.

The sewage treatment plant at Grand Rapids comprises screening, sedimentation, and separate sludge digestion, and includes square settling tanks and traction-type equipment for removing sludge. There are four sludge digestion tanks and two sludge storage tanks. The digestion tanks are 70 ft in diameter and 24.5 ft deep (liquid side depth), and are equipped with tight covers, stirring devices, and heating coils. Three pumps are provided for handling raw sludge and two for digested sludge. Interconnections permit its transfer from one tank to another, although during the observation period there was practically no re-circulation of sludge between the tanks, and no seeding.

The rated capacity of the plant is 32.5 mgd, and the average flow during the period of observation was 24.4 mgd. Thus, the digestion tanks were not heavily loaded. The estimated connected population upon which all the following per capita computations are based is 168 000. The population equivalent, based on a 5-day bio-chemical oxygen demand of 0.168 lb per capita, is 180 000.

The City of Grand Rapids has a combined sewerage system. The sewage is relatively fresh, and the average flow is about 140 gal per capita per day. Although there are no unusual industrial sewages tributary to the treatment plant, the per capita quantity of suspended matter is relatively high, amounting to 0.230 lb per capita per day. A summary of typical analyses for the observation period is given in Table 14.

TABLE 14.—CHARACTERISTICS OF SEWAGE AT GRAND RAPIDS, MICHIGAN, DURING PERIOD OF OBSERVATION BY COMMITTEE

Period	Sewage flow, in million gallons daily	Sewage temperature, in degrees Fahrenheit	TOTAL SOLIDS			SUSPENDED SOLIDS			5-day biochemica oxygen demand	pH
			Total	Volatile, in parts per million	Percentage volatile	Total, in parts per million	Volatile, in parts per million	Percentage volatile		
1931:										
April	22.2	....	1 064	539	49.6	252	172	71.2	199	7.05
May	20.7	....	1 092	620	56.6	154	115	78.8	172	7.06
June	26.2	....	1 509	916	60.1	175	135	77.1	149	6.98
July	23.4	69.5	1 635	882	54.1	172	122	72.8	132	6.98
August	24.3	70.2	1 785	1 095	61.2	233	156	66.6	140	7.24
September	22.0	70.5	1 632	1 136	69.5	248	178	74.7	152	7.20
October	21.7	66.0	1 385	907	65.0	198	149	75.4	129	7.28
November	21.8	60.6	995	534	53.3	175	122	71.9	124	7.21
December	25.0	55.4	957	562	58.2	154	129	83.0	189	6.88
1932:										
January	23.1	50.2	933	467	50.4	148	91	65.4	188	6.97
February	25.5	50.2	1 098	569	51.8	213	139	65.3	203	7.33
March	27.6	49.0	1 095	643	58.8	187	136	72.7	264	7.32
April	26.8	52.2	1 049	569	54.1	184	129	73.3	151	7.25
May	29.2	59.3	1 105	619	56.0	168	116	68.9	184	7.04
June	26.3	67.3	1 366	875	64.1	206	142	69.1	168	7.12
Average	24.4	....	1 246	728	57.5	191	135	72.4	169	7.12

OPERATING ROUTINE

The sludge-collecting equipment in the primary settling tanks is operated continuously. At 8-hr intervals the valves on the sludge lines are opened and fresh sludge is pumped to each digestion tank, for a period of 10 to 30 min. This routine of operation results in fresh sludge with an average solids content of 4.9 per cent.

Space for the incoming sludge was made available either by the discharge of sludge supernatant, or by the withdrawal of digested sludge. The ordinary routine was to allow sludge supernatant to flow from the digestion tanks whenever fresh sludge was introduced, but to withdraw digested sludge only once in 24 hr. The supernatant was returned to the inlet of the primary settling tanks. There was no difficulty in maintaining a practically uniform temperature in the digestion tanks, the departure from the average during the delivery of sludge being negligible.

There is a sampling cock on the discharge of each of the five pumps. During pumping, a 1.5-pt sample was taken every 10 min and composited. The length of the pumping period was determined by the thinning of sludge at the sampling cock.

To facilitate uniform operation, a ratio of sludge withdrawn to sludge introduced was determined from time to time, in order to maintain the proper sludge depth in the tanks. Ratios ranging from 17.5% to 50% were applied.

DATA SECURED

During the period of observation, an effort was made to secure a complete record of the operation and behavior of the sludge digestion tanks. All the necessary analyses relating to incoming sludge, withdrawn sludge, sludge supernatant, scum, and gas were included and were made by the Laboratory Staff of the Grand Rapids treatment works on daily composites with few omissions. In addition, several sets of determinations were made to ascertain the characteristics of the sludge at depth intervals of 5 ft. It was impossible to get more than a few analyses of the scum and the sludge supernatant.

The data were averaged from daily analyses. A summary is given in Table 15. The percentage of volatile matter and of solids in the fresh and withdrawn sludges for each of the four tanks is shown on Fig. 15.

The sampling and analysis of sludge are subject to some error and, therefore, the results cannot be considered as precise. As a measure of accuracy, a computation of a solids balance was made.

All determinations of the solids content of sludge were made by evaporation, and the results, therefore, are in total solids rather than in suspended solids. The difference between the two is only a matter of 2% or 3%, depending upon the moisture content of the sludge.

For the computation of the solids balance, a period between June 15, 1931, and May 31, 1932, was taken, as these were the first and last dates on which analyses were made of the total solids content of the digestion tanks.

During the observation period there was but one unusual condition of any importance. Difficulty has been experienced in Grand Rapids with the dis-

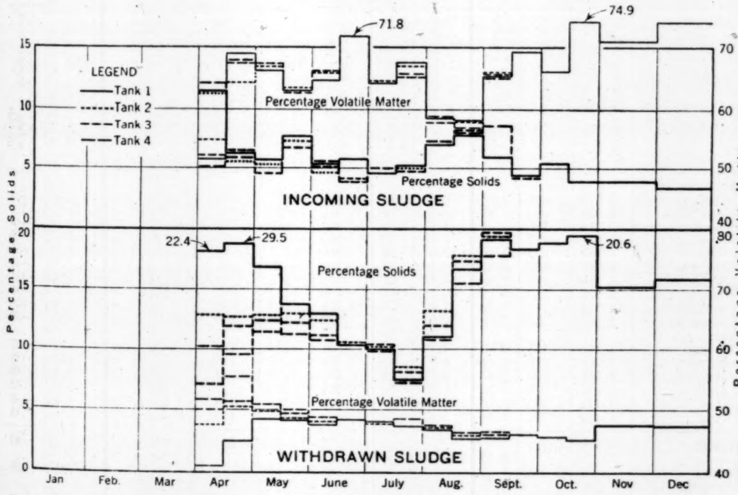

FIG. 15.—CHARACTERISTICS OF INCOMING SLUDGE AND WITHDRAWN SLUDGE FOR EACH OF THE FOUR DIGESTION TANKS AT GRAND RAPIDS, MICH.

TABLE 15.—SUMMARY OF DATA COLLECTED AT GRAND RAPIDS, MICHIGAN, SEWAGE TREATMENT PLANT

Period	INCOMING SLUDGE				SLUDGE WITHDRAWN				SLUDGE SUPERNATANT			
	Total Solids		Volatile Solids		Total Solids		Volatile Solids		Total Solids		Volatile Solids	
	Percentage	Im thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds
1931:												
April	8.8	817	59.0	482	14.6	534	44.1	247	0.5	74	61.0	39
May	6.3	455	65.8	300	13.0	330	49.5	164	0.5	50	61.0	30
June*	4.9	260	66.1	172	11.9	126	48.7	61	0.5	28	61.0	17
June†	....	368	71.9	265	10.4	104	48.3	50	0.5	27	61.0	16
July	4.9	608	65.7	400	9.0	160	47.9	77	0.5	49	61.1	30
August	7.6	1 005	58.7	590	14.5	508	46.6	235	0.5	60	61.1	36
September	5.5	949	67.9	645	18.8	430	46.1	198	0.5	38	61.1	23
October	4.7	667	70.4	470	19.8	611	45.8	280	0.5	39	63.8	25
November	4.0	518	70.7	366	15.3	840	47.8	402	0.5	27	57.7	16
December	3.6	470	74.5	350	16.1	308	47.5	146	0.5	37	57.7	21
1932:												
January	3.5	446	68.2	304	13.9	795	46.7	371	0.35	13	57.0	7
February	3.5	777	71.9	559	10.8	361	48.1	173	0.35	21	57.0	12
March	3.2	732	74.2	544	8.8	124	49.1	61	0.34	35	59.4	21
April	3.26	675	75.1	507	9.0	150	49.9	75	0.35	31	57.7	18
May	4.0	680	66.2	450	9.3	179	48.8	88	0.51	41	57.7	24
June	5.2	798	66.8	533	9.6	178	48.1	86	0.27	25	57.0	14
Totals‡	....	10 225	....	6 937	....	5 738	....	2 714	....	595	....	349
Average‡	4.9	....	67.7	....	12.9	....	47.3	....	0.44	...	58.6	...
Totals§	....	7 895	....	5 450	....	4 570	....	....	....	418	....	...
Average§	....	....	....	....	....	....	....	....	....	...	....	...

TABLE 15.—(*Continued*)

Period	GAS					SOLIDS BALANCE						
	Volume, in thousands of cubic feet (corrected to 30 inches at 60° F)	In thousands of pounds ‖	Cubic feet per capita per day (168 000 population)	Cubic feet per pound of incoming volatile matter	Percentage of incoming volatile matter reduced to gas	Total solids discharged, in thousands of pounds	Accumulation of total solids, in thousands of pounds	Volatile solids discharged in sludge and sludge supernatant, in thousands of pounds	Reduction in Volatile Solids		Total volatile solids discharged, in thousands of pounds	Accumulation of volatile solids, in thousands of pounds
									In thousands of pounds	Percentage		
1931:												
April	2 879	188	0.57	6.0	39	796	+21	286	196	41	474	+8
May	4 520	296	0.87	15.1	99	676	−221	194	106	35	490	−190
June*	2 114	138	0.79	11.0	80	292	−32	78	96	56	216	−44
June†	2 014	132	0.86	7.6	50	263	+105	66	199	75	198	+67
July	4 159	272	0.80	10.4	68	481	+127	107	293	73	379	+21
August	4 715	308	0.91	8.0	52	876	+139	271	319	54	579	+11
September	4 176	273	0.83	6.5	42	741	+208	221	424	66	494	+151
October	3 898	250	0.77	8.3	53	900	−233	305	165	35	555	−85
November	4 114	267	0.82	11.2	73	1 134	−616	418	−52	−14	685	−319
December	3 577	232	0.69	10.2	66	577	−107	168	182	52	400	−50
1932:												
January	2 348	153	0.45	7.7	50	961	−515	378	−74	−24	531	−227
February	2 960	192	0.61	5.3	34	574	+203	185	374	67	377	+182
March	4 519	294	0.87	8.3	54	453	+279	82	462	85	376	+168
April	4 198	273	0.83	8.3	54	454	+221	93	414	82	366	+141
May	4 134	269	0.80	9.2	60	489	+191	112	338	75	381	+69
June	4 454	290	0.88	8.4	54	493	+305	100	433	81	390	+143
Totals‡	58 779	3 827	....	....	..	10 160	+65	3 064	3 875	..	6 891	+46
Average‡	......	.....	0.77	8.5	55	.....	.....	....	.....	56	.....	.....
Totals§	......	2 915	....	....	..	7 903	−8	....	.....	..	5 321	+129
Average§	......	.....	....	....	..	.....	.....	....	.....	..	.....	.....

*June 1 to 16, inclusive. † June 17 to 30, inclusive. ‡ Entire 15-month period. §June 17, 1931, through May 31, 1932.
‖Assuming 65 lb. per 1 000 cu. ft.

posal of lime sludge from the water-softening plant. During April and August, 1931, some of it was discharged into the sewers. This sludge was almost entirely calcium carbonate, with a small quantity of magnesium hydrate. It had no caustic alkalinity. About 44% of the sludge was volatile. During the first period, the dry weight of lime sludge was about three times, and during the second period, about twice, the dry weight of suspended sewage solids. After the first period (ending on April 13, 1931) it required 36 days for the lime to work out of the plant. After the second period (ending on September 3, 1931) practically all the lime added during August had disappeared from the raw sludge by September 4.

Condition of Sludge in Tanks

Fig. 16 shows the percentage of solids in the sludge in each of the four tanks at various depths on the days when samples were taken and analyzed.

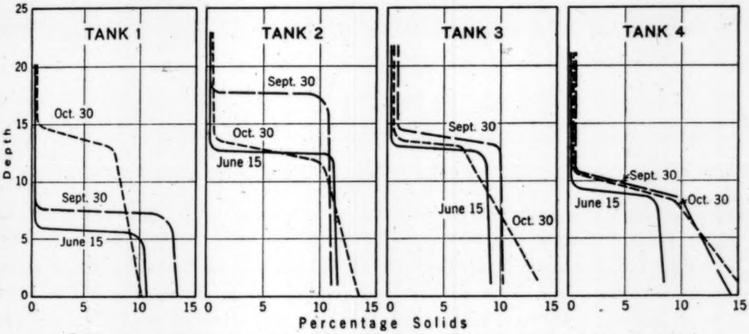

Fig. 16.—Percentage of Solids in Digestion Tanks at Grand Rapids, Mich.

The average percentage of solids for all four tanks was 4.7. The reason for this abnormally low value is that the sludge line itself was low—about at mid-depth in each tank. As indicated in Fig. 16, the tanks were nearly half full of sludge so that the average moisture content was abnormally high. This line was relatively definite, and the average percentage of solids below it, for the four tanks, was 8.9. The average solids content of the sludge as withdrawn, was 12.9.

There was not much variation in the pH-value between the top and bottom of the tanks, the change in general being from 7.0 at the top to 7.6 at and near the bottom. The volatile content of the tank solids decreased appreciably from top to bottom, as shown in Table 16.

Sludge Supernatant

The quantity of sludge supernatant was determined by computing the difference between the quantity of fresh sludge and the quantity withdrawn, as obtained from Venturi meter readings.

In general, the supernatant carried from 2 700 to 5 000 ppm of total suspended solids. The solids discharged from the tanks in sludge supernatant

TABLE 16.—VOLATILE CONTENT OF SOLIDS IN DIGESTION TANKS AT GRAND RAPIDS, MICHIGAN, AT VARIOUS DISTANCES FROM BOTTOM OF TANK

Distance from bottom, in feet	VOLATILE CONTENT OF SOLIDS, IN PERCENTAGES						Distance from bottom, in feet	VOLATILE CONTENT OF SOLIDS, IN PERCENTAGES					
	Oct. 30, 1931	Feb. 4, 1932	Mar. 1, 1932	Apr. 14, 1932	May 4, 1932	June 1, 1932		Oct. 30, 1931	Feb. 4, 1932	Mar. 1, 1932	Apr. 14, 1932	May 4, 1932	June 1, 1932
TANK No. 1							TANK No. 3						
2.5	47.4	48.4	46.5	49.6	48.4	49.4	2.5	47.4	47.1	48.6	48.9	50.0	46.7
7.5	48.3	49.3	47.6	49.3	51.2	49.8	7.5	47.8	47.2	55.0	48.7	49.3	46.8
12.5	49.3	55.4	57.3	50.9	52.2	53.8	12.5	49.8	55.0	57.6	54.0	51.5	52.8
17.5	66.7	60.3	58.2	61.5	60.4	54.7	17.5	62.4	58.3	55.2	53.4	56.4	52.9
TANK No. 2							TANK No. 4						
2.5	47.0	*	47.4	48.5	50.2	49.4	2.5	46.2	44.6	47.6	49.8	50.7	46.8
7.5	47.2	*	49.1	48.2	50.1	49.2	7.5	46.3	45.2	54.0	51.1	49.4	46.4
12.5	49.6	*	57.5	56.3	50.0	50.9	12.5	57.5	53.4	55.6	57.8	50.2	49.1
17.5	74.7	*	58.8	55.6	59.7	57.0	17.5	60.6	54.1	51.2	57.8	55.8	55.7

* Empty

for the entire period of observation amounted to about 5.8% of the total solids pumped into the tanks during the same period.

SCUM

There was a typical, black, somewhat fibrous scum, relatively thick, under the roof of each digestion tank during the observation period. The scum varied in thickness up to a maximum of about 3 ft. It gave no trouble, and no effort was made to remove it or to break it up. Moisture content and other pertinent data are given in Table 17. When two of the tanks were emptied (in January and May, 1932), it was found that the scum layer, upon reaching the bottom of the tank, could not be pumped out even after strong hosing. It was 1.5 to 2.0 ft thick and was of a heavy clay-like consistency matted with hair and sticks.

TABLE 17.—CHARACTERISTICS OF SCUM ON DIGESTION TANKS AT GRAND RAPIDS, MICHIGAN

Date	Tank No.	Moisture content	Percentage volatile	pH	Remarks
(*a*) FLOATING SCUM LAYER					
September, 1931..........	1	93.3	96.2	7.45	Average of four analyses
September, 1931..........	2	93.2	96.2	7.47	Average of four analyses
September, 1931..........	3	94.2	95.8	7.47	Average of four analyses
September, 1931..........	4	93.2	96.2	7.46	Average of four analyses
October, 1931............	1	95.37	95.3	7.0	....
(*b*) SCUM REMAINING AFTER TANK WAS EMPTIED					
January, 1932............	1	57.6	83.1	......	Composite of four samples
May, 1932................	2	66.2	80.3	......	Composite of four samples

TEMPERATURE

The temperature of the digestion tanks was observed continuously by recording thermometers of the Bristol type. An average temperature was

TABLE 18.—DIGESTION TANK TEMPERATURES AT GRAND RAPIDS, MICHIGAN

MONTH	TANK No. 1			TANK No. 2			TANK No. 3			TANK No. 4		
	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average
1931:												
April	100	61	80	87	75	84	88	73	83	87	70	80
May	86	82	84	87	84	85	86	82	84	86	83	84
June	85	82	84	88	85	86	86	84	85	86	85	85
July	87	85	86	88	85	87	87	84	86	88	85	86
August	86	86	86	90	85	86	88	84	86	88	85	87
September	90	85	87	88	84	86	86	84	85	88	84	86
October	90	83	86	89	83	85	87	82	85	89	83	86
November	87	85	86	87	85	86	87	85	86	87	85	86
December	91	83	86	91	83	86	91	83	86	91	83	86
1932:												
January	90	78	86	90	78	86	90	78	86	90	78	86
February	84	72	79	84	72	79	84	72	79	84	72	79
March	87	81	85	87	81	85	87	81	85	87	81	85
April	94	85	88	94	85	88	94	85	88	94	85	88
May	91	88	90	91	88	90	91	88	90	91	88	90
June	93	86	90	93	86	90	93	86	90	93	86	90

secured by placing a temperature element across each tank, rising from a point near the bottom at one corner to a point near the top at the corner diagonally opposite. Relatively uniform temperatures prevailed, as shown by Table 18.

TABLE 19.—CHARACTERISTICS OF GAS PRODUCED AT GRAND RAPIDS, MICHIGAN, SEWAGE TREATMENT PLANT

Period	British thermal unit, V_1	PERCENTAGES						
		CO_2	N_2	Illuminants	CH_4	O_2	CO	H_2
1931:								
April 1–16	661	29.4	6.45	0.18	59.2	0.11	0.05	4.7
16–30	705	31.1	2.40	0.30	65.3	0.07	0.0	1.0
May 1–16	692	31.8	2.3	0.30	63.6	0.10	0.0	2.0
16–31	735	31.7	0.6	0.30	67.4	0.10	0.0	0.0
June 1–16	714	30.1	3.1	0.20	66.0	0.60	0.0	0.0
16–30	686	30.4	4.3	0.30	65.0	0.00	0.0	0.0
July 1–16	683	30.8	4.3	0.20	63.2	0.10	0.0	1.5
16–31	683	29.9	5.9	0.10	62.2	0.10	0.0	2.0
August 1–16	682	28.5	7.4	0.10	63.7	0.10	0.0	0.2
16–31	653	28.9	6.8	0.10	59.7	0.10	0.0	4.4
September 1–16	654	29.8	1.8	0.40	57.1	0.20	0.0	10.7
16–30	664	31.0	1.9	0.60	58.8	0.10	0.2	7.4
October 1–16	733	31.3	0.6	0.20	68.6	0.10	0.0	0.0
16–30	727	31.7	0.0	0.10	68.1	0.20	0.0	0.0
November	725	31.7	0.4	0.2	67.6	0.2	0.0	0.0
December	729	31.6	0.1	0.2	68.0	0.2	0.0	0.0
1932:								
January	731	31.8	0.1	0.3	68.0	0.2	0.0	0.0
February	706	32.4	0.2	0.4	65.6	0.3	0.0	0.0
March	706	33.8	0.3	0.2	66.1	0.2	0.0	0.0
April	725	32.0	0.1	0.2	67.6	0.2	0.2	0.0
May	714	33.2	0.1	0.4	66.3	0.4	0.0	0.0
June	706	33.2	0.5	0.3	65.6	0.4	0.0	0.0

Gas Production

The metered quantities of gas are considered to be somewhat lower than the amount actually produced, because of occasional blowing of the gas seals and some clogging of the lines. The seals have recently been increased in depth, and changes have been made in the domes to eliminate some of these difficulties. Subsequent meter records indicate somewhat higher rates of production. For the purposes of this study, however, the records as obtained have been used—proper consideration, of course, being given to barometric pressure, moisture, and temperature. The quantitative results are shown in Table 15, and the composition and calorific value as indicated by various analyses throughout the period are shown in Table 19.

This gas has been used in the boilers for digestion-tank heating, without any difficulty, and in the laboratory, and for general heating about the plant.

Solids Balance

The data for the computation of solids balance were taken between June 16, 1931, and May 31, 1932, as these are the first and last dates on which analyses were made giving the total solids content of the digestion tanks. A summary of the solids balance follows:

(*A*)	Total weight of solids in digestion tanks on June 16, 1931 (as computed from the data in Fig. 15), in pounds	1 010 000
	Total solids introduced as fresh sludge from the primary settling tanks between June 16, 1931, and May 31, 1932, in pounds	7 895 000
	Total solids to be accounted for, in pounds	8 905 000
(*B*)	Total weight of solids in digestion tanks on June 1, 1932 (as computed from the data in Fig 15), in pounds	1 302 000
	Total solids removed from digestion tanks as sludge, in pounds	4 570 000
	Total solids withdrawn from tanks with sludge supernatant, in pounds	418 000
	Equivalent weight of gas as reported by the gas meters with a correction of 1.0% added for gas lost through water-seals (and representing the weight of solids volatilized), in pounds	2 944 000
	Total weight of solids accounted for, in pounds	9 234 000

In the foregoing list, the solids accounted for are greater than those to be accounted for, by 329 000 lb, or approximately 3.7 per cent. This is a close balance, considering the difficulties of sampling, of measuring sludge in meters, and of determining the percentage of volatile matter and the specific gravity of the sludge.

Computed Digestion Time

A digestion time was computed as the displacement period of solids withdrawn, by dividing the total weight of solids in all four tanks by the daily weight of all solids removed therefrom, whether as sludge, as sludge supernatant, or as gas. Table 20 gives these quantities and the displacement periods.

TABLE 20.—COMPUTATION OF DIGESTION TIME

Date of analysis	Weight of dry solids in four tanks, in pounds	Weight of dry solids removed per day*, in pounds	Displacement period, in days
1931:			
June 15	1 010 000	16 500	61
October 30	1 363 000	28 100	49
1932:			
March 1	701 000	14 900	47
May 4	863 000	16 100	54

* Average for the two months following the date of analysis of tank content.

All the sludge was well digested, the volatile matter being consistently less than 50 per cent. The time used for digestion, as indicated by these observations, is not more than 50 days and may be somewhat less.

APPENDIX II

OPERATING DATA FROM ELYRIA, OHIO, APRIL 1, 1931, THROUGH SEPTEMBER 30, 1932

For an 18-month period extending from April 1, 1931, to September 30, 1932, arrangements similar to those described in Appendix I were made for collecting information relative to the operation of separate sludge digestion tanks at Elyria, Ohio. The opportunity was afforded through the courtesy of the Hon. James A. Hewitt, Mayor, and the Hon. J. N. Eidt, Safety-Service Director, and the work was done through the faithful co-operation of James R. Collier, Superintendent and Chemist at the sewage treatment works.

The sewage works at Elyria consists of bar gratings, grit chambers, preliminary sedimentation tanks of the 2-story type, aeration tanks, activated-sludge settling tanks, separate sludge digestion tanks, and covered sludge-drying beds.

The two 2-story tanks, each 62 ft in length, 31½ ft in width, and with a water depth of 27 ft, provide a normal detention period in the sedimentation compartment of 60 min. They are equipped for gas collection.

There are two separate sludge-digestion tanks, each 50 ft in diameter, with conical bottoms, the total water depth being 34 ft, measured from the wall coping. Both tanks are equipped with floating covers with provision for gas collection. Each tank is heated by water circulated by a centrifugal pump through galvanized iron pipe, hung around the circumference of the tank 18 ft below its coping. A continuous record of the temperature of incoming and outgoing heating water is kept by recording thermometers. Sludge temperatures are measured, as desired, by thermometers suspended in the sludge. All gas is metered and two gas boilers are provided for heating the water used in the circulating system.

Between the two digestion tanks are a sludge-mixing and control chamber and a room housing the water-seals, a master gas meter, sludge-control valves, and an automatic gas-pressure regulator and excess gas escape. There is no provision for gas storage except in the digestion tank. Sludge is transferred from the activated-sludge settling tanks by gravity, and may be discharged either to the separate digestion tanks, the sludge-drying beds, or to the sludge suction well (whence it can be returned to the primary tanks, or to the digenstion tanks). Sludge mixing may be accomplished in the digestion tanks by drawing off at the bottom and pumping back to the top of the tank.

Sludge supernatant from each tank can be drawn off at either of two elevations and discharged directly to the sludge-drying beds, or pumped to the sewage as it enters the 2-story tanks, or to the sludge compartments of the 2-story tanks.

The plant is designed to serve a population of 36 000, with an average sewage flow of 100 gal per capita daily, although the estimated connected population at the time of the observations was 21 000. The design capacity was based upon a 60-day period of digestion, with heating for maintaining a temperature of 77° F.

The sewerage system, with the exception of that serving the older and more densely populated section of the city lying between the east and west forks of the Black River, is of the separate type. The sewage is relatively fresh and of normal strength, although there is a small percentage of pickling liquor wastes. The average volatile content of the suspended solids in the crude sewage is 64.2% and the percentage of volatile matter in the partly digested 2-story tank sludge introduced into the sludge-digestion tanks is 61.7 per cent. Summaries of sewage analyses and of the characteristics of the sludge removed from the 2-story tanks are contained in Tables 21 and 22.

TABLE 21.—Characteristics of Raw Sewage at Elyria, Ohio, During Period of Observation by Committee

Date	Sewage flow, in million gallons daily	Temperature of air, in degrees Fahrenheit	Temperature of sewage, in degrees Fahrenheit	Total Solids		Suspended Solids		Total iron as Fe, in parts per million	5-day biochemical oxygen demand	pH
				In parts per million	Percentage volatile	In parts per million	Percentage volatile			
1931:										
April	3 289	48	51	760	37.6	211	61.1	8.9	173	7.2
May	2 300	59	57	843	38.5	241	79.0	19.0	208	7.0
June	1 960	71	64	805	44.9	231	71.5	11.0	209	7.0
July	1 675	78	69	841	49.3	241	76.3	4.4	215	6.9
August	1 793	73	69	775	47.6	205	55.1	3.5	208	6.9
September	1 857	69	71	774	51.2	218	52.3	3.8	224	6.9
October	1 803	56	68	748	51.1	198	58.1	3.7	227	7.0
November	1 798	49	65	839	56.7	216	71.8	3.2	251	6.9
December	2 355	39	58	725	46.7	186	62.9	3.7	187	7.3
1932:										
January	3 837	39	53	717	44.5	155	61.3	5.4	119	7.1
February	2 839	36	50	785	44.2	194	64.9	8.8	190	7.2
March	2 660	32	49	819	45.3	221	61.5	4.8	192	7.2
April	3 066	46	49	815	41.7	180	66.7	8.1	200	7.2
May	2 618	62	53	806	42.8	197	61.4	7.5	262	7.1
June	1 770	72	63	911	50.2	305	64.3	6.4	320	7.1
July	1 851	74	68	740	48.9	194	62.9	6.6	285	7.0
August	1 558	72	70	688	58.0	200	60.5	4.6	281	6.9
September	1 695	65	71	681	62.8	212	65.6	4.1	251	6.9
Average	2 263	....	....	782	47.8	212	64.2	6.5	222	7.03

TABLE 22.—CHARACTERISTICS OF SLUDGE ADDED TO SLUDGE-DIGESTION TANKS AT ELYRIA, OHIO

(Sludge Drawn from Preliminary Sedimentation-Activated Sludge Settling Tanks)

Date	Volume added, in gallons	PERCENTAGE			pH	Alkalinity	Specific gravity
		Moisture	Ash	Volatile			
1931:							
April	433 160	96.80	35.56	64.44	6.2	1 100	1.010
May	275 184	95,38	40,04	59,96	6.2	1 350	1.011
June	275 184	94.70	37.49	62.51	6.2	1 510	1.008
July	323 596	95.31	39.32	60.68	6.2	1 630	1.010
August	363 090	96.18	43.61	56.39	6.2	1 210	1.008
September	397 488	96.85	39.07	60.93	6.3	970	1.007
October	229 312	97.50	38.06	61.94	6.4	837	1.008
November	305 760	97.27	37.02	62.98	6.4	1 066	1.008
December	295 528	96.52	34.68	65.32	6.3	1 275	1.008
1932:							
January	229 320	95.16	36.18	63.82	6.2	1 325	1.008
February	183 456	94.65	37.22	62.78	6.2	1 600	1.015
March	257 348	94.14	33.16	66.84	6.2	1 283	1.018
April	168 168	93.87	34.31	65.69	6.2	1 616	1.013
May	183 456	93.60	36.60	63.40	6.2	1 635	1.017
June	275 184	94.13	38.10	61.90	6.3	1 587	1.014
July	259 896	96.02	40.20	59.80	6.4	1 397	1.012
August	447 174	97.03	42.70	57.30	6.5	873	1.012
September	391 118	97.04	41.50	58.50	6.3	653	1.011

OPERATING ROUTINE

The 2-story settling tanks are operated in parallel and the direction of flow is reversed regularly at the beginning of each month. Scum is drawn from beneath the gas-collection domes not oftener than twice each month.

TABLE 23.—SUMMARY OF DATA COLLECTED AT ELYRIA, OHIO, SEWAGE TREATMENT PLANT

Period	SLUDGE TO DIGESTION TANKS				SLUDGE WITHDRAWN			
	Total Solids		Volatile Solids		Total Solids		Volatile Solids	
	Percentage	In pounds	Per centage	In pounds	Per centage	In pounds	Per centage	In pounds
1931:								
April	3.2	117 270	64.5	75 470	4.4	17 300	47.5	8 340
May	4.6	107 310	60.0	64 350	4.7	9 730	42.4	4 130
June	5.3	112 610	62.5	70 460	5.2	25 690	46.7	11 600
July	4.7	127 950	60.7	77 650	5.6	39 010	46.6	18 170
August	3.8	116 190	56.4	65 380	5.5	19 150	46.2	8 850
September	3.0	98 470	60.9	60 580	6.0	43 290	47.8	20 650
October	2.5	48 140	61.9	29 818	5.9	31 141	47.1	14 877
November	2.7	70 126	63.0	44 168	5.8	15 034	47.4	7 121
December	3.5	86 356	65.3	56 411	5.6	15 778	48.8	7 693
1932:								
January	4.8	93 281	63.8	59 536	4.3	3 962	49.4	1 957
February	5.4	82 999	62.8	52 109	4.6	11 959	47.6	5 694
March	5.9	127 848	66.8	85 458	5.6	24 773	47.9	11 875
April	6.1	87 057	65.7	57 190	5.2	20 892	48.9	10 216
May	6.4	99 475	63.4	63 071	5.3	32 978	47.7	15 732
June	5.9	136 402	61.9	84 437	5.7	24 521	47.0	11 527
July	4.0	87 336	59.8	52 226	6.1	31 824	48.4	15 391
August	3.0	111 999	57.3	64 173	5.8	34 882	48.2	16 821
September	3.0	97 614	58.5	57 106	5.9	36 317	46.4	16 862
Totals	...	1 808 433	....	1 119 593	...	438 231	....	207 306
Average	4.3	....	61.7	....	5.4	....	47.2	....

TABLE 23.—(*Continued*)

Period	SLUDGE SUPERNATANT				GAS				
	TOTAL SOLIDS		VOLATILE SOLIDS		Volatile corrected to 30-in. Baro.-60°F, in cubic feet	In pounds	Cubic feet per capita per day	Cubic feet per pound, incoming volatile matter	Percentage of incoming volatile matter reduced to gas
	Percentage	In pounds	Percentage	In pounds					
1931:									
April	0.3	5 900	48.2	2 840	363 620	23 590	0.57	4 99	32.5
May	0.4	7 070	48.4	3 410	320 190	21 000	0.49	5.15	33.7
June	0.2	2 660	47.8	1 270	323 960	20 940	0.52	5.76	37.3
July	0.9	17 580	47.9	8 360	339 200	21 910	0.53	4.39	28.3
August	0.8	14 230	48.5	6 910	334 020	21 490	0.51	5.45	35.2
September	0.2	9 070	43.8	4 210	326 060	20 940	0.52	8.59	55.2
October	0.3	7 294	45.7	3 332	340 593	21 764	0.52	11.42	73.0
November	0.9	16 605	46.0	7 639	295 681	18 983	0.47	6.69	43.0
December	0.5	11 030	46.9	5 173	308 449	19 772	0.47	5.47	35.0
1932:									
January	1.8	37 541	48.2	18 093	366 214	23 877	0.56	6.15	40.1
February	2.9	26 246	48.9	12 823	326 503	21 321	0.54	6.27	40.9
March	2.4	47 149	46.9	22 108	377 181	24 441	0.58	4.41	28.6
April	1.8	14 375	48.5	6 978	319 263	20 848	0.51	5.58	36.5
May	3.1	53 986	49.0	26 444	267 562	17 258	0.41	4.24	27.4
June	3.3	31 712	46.4	14 699	316 971	20 223	0.50	3.75	24.0
July	2.2	27 942	46.8	13 079	268 143	17 376	0.41	5.13	33.3
August	1.5	52 361	49.2	25 751	300 495	19 292	0.46	4.68	30.1
September	0.2	3 313	50.2	1 664	215 122	14 047	0.34	3.77	24.6
Totals	...	386 064	....	184 783	5 709 227	369 072	....	....	....
Average	1.3	.....	47.5	.....	....	.....	0.50	5.65	36.5

Sludge removed from the activated sludge-settling tanks is introduced continuously into the incoming sewage at the entrance to the 2-story tanks.

Once each week sludge supernatant is drawn from the separate digestion tanks and discharged into the sludge compartments of the 2-story tanks at points just above the top of the hoppers. Following the supernatant draw-off, if desired, ripe sludge is applied to the sludge-drying beds, and regularly each week, a charge of primary tank sludge is admitted to each digestion tank. After this, if mixing is desired, it is accomplished by drawing ripe sludge from the bottom of the tanks and pumping it into the fresh solids which have been charged at the top.

The total solids content of the digested sludge during the 18-month period of observation averaged only 5.3 per cent. This could undoubtedly be increased by a longer detention period in the digestion tanks, but due to the demand for dried sludge (which is sold at a fixed price per bed), sludge was withdrawn as soon as it was satisfactory for drying.

DATA SECURED

A summary of the data collected during the 18-month period of observation is given in Table 23.

Measurements of sludge added to, and removed from, the digestion tanks were made by noting the actual displacement in the tanks. The volume of sludge removed was also checked by observations of wet sludge applied to the sludge-drying beds.

SLUDGE SUPERNATANT

The sludge supernatant from the digestion tanks contained from slightly less than 5 000 to 33 000 ppm of total solids, with a volatile matter content of 47.5 per cent. The total solids discharged in the sludge supernatant amounted to about 21% of the total solids admitted to the digestion tanks.

Analyses of the supernatant were made at frequent intervals, with results shown in Table 24.

TABLE 24.—CHARACTERISTICS OF SLUDGE SUPERNATANT AT ELYRIA, OHIO

Date	Percentage			pH	Alkalinity	Specific gravity
	Moisture	Ash	Volatile			
Sept. 30, 1931:						
East tank	99.75	59.4	40.6	7.3	3 340	1.004
West tank	99.82	63.2	36.8	7.4	3 140	1.003
Oct. 15, 1931:						
East tank	99.58	51.7	48.3	7.4	2 860	1.005
West tank	99.47	50.2	49.8	7.3	2 600	1.006
Oct. 27, 1931:						
East tank	99.71	61.2	38.8	7.4	2 720	1.004
West tank	99.77	58.8	41.2	7.5	2 680	1.003
October	99.65	54.3	45.7	7.4	2 710	1.004
November	99.12	54.0	46.0	7.2	2 620	1.004
December	99.46	53.1	46.9	7.2	2 760	1.006
1932:						
January	98.16	51.8	48.2	7.4	2 980	1.009
February	97.09	51.1	48.9	7.3	3 600	1.014
March	97.55	53.1	46.9	7.5	3 870	1.014
April	98.22	51.5	48.5	7.5	4 290	1.007
May	96.90	51.0	49.0	7.5	4 560	1.009
June	96.72	53.6	46.4	7.4	4 730	1.007
July	97.75	53.2	46.8	7.4	4 630	1.006
August	98.46	50.8	49.2	7.4	3 280	1.007
September	99.81	49.8	50.2	7.3	2 350	1.003

GAS

The gas from the separate sludge digestion tanks, as well as that collected from the gas vents of the 2-story tanks, is metered through separate gas meters and is used for heating the water circulated in the digestion tanks. At present, all excess gas is burned and wasted. During the first part of the period considerable gas escaped through the water-seals before the covers had been finally adjusted to maintain a level position. It is estimated that the loss amounted to at least 10% of the total volume metered, and, therefore, the quantities have been corrected to include this allowance. (In addition, a quantity of gas amounting to about 10% of these volumes was collected from the primary settling tanks.) The characteristics of the gas are shown in Table 25.

The number of cubic feet of gas per pound of incoming volatile matter was undoubtedly lower than could have been obtained, as sludge was drawn to the drying beds sooner than would be the case ordinarily. The gas has an average calorific value of about 750 Btu per cu ft, and burns readily under the boilers. No trouble has been experienced with clogging of the gas lines and the discharge of gas has been fairly uniform.

SCUM

No attempt was made during the period of observation to remove or to break up the layer of scum in the digestion tanks. The thickness of the layer

TABLE 25.—CHARACTERISTICS OF GAS PRODUCED AT ELYRIA, OHIO, SEWAGE TREATMENT PLANT

Period	British thermal units per cubic foot	Percentage		
		CO_2	CH_4	O_2
1931				
April......	738	30.2	69.2	0.6
May.......	730	31.1	68.2	0.7
June.......	748	30.0	69.5	0.5
July.......	764	29.9	69.6	0.5
August....	744	29.6	69.9	0.5
September..	758	29.4	70.0	0.6
October....	752	28.9	70.6	0.5
1931 (*Continued*)				
November.	750	29.4	70.0	0.6
December.	764	29.2	70.3	0.5
1932				
January...	732	30.8	68.7	0.5
February..	731	30.8	68.6	0.6
March....	746	30.2	69.3	0.5
April.....	730	30.9	68.6	0.5
1932 (*Continued*)				
May......	742	29.9	69.7	0.4
June......	756	28.8	70.7	0.5
July......	738	30.2	69.3	0.5
August....	745	29.5	70.0	0.5
September.	744	30.8	68.6	0.6
....	...	....	....	...
....	...	....	....	...
....	...	....	....	...
....	...	....	....	...

varied from 10 in. to a maximum of 24 in. An average of several analyses showed it to have the following characteristics:

Moisture content	86.4%
Ash content	34.2%
Volatile matter	65.8%
pH-value	6.9%
Ether soluble (dry solids)..................	27.2%

TEMPERATURE

The temperature of the sludge in the digestion tank was maintained at an average of 78° F, and varied from a minimum of 74° to a maximum of 82 degrees. The average temperatures of the incoming and outgoing circulating water were 107° and 96° F, respectively.

SOLIDS BALANCE

No determination of the total solids content of the tanks was made at the end of the period of observation; hence, the solids balance can only show the difference between total incoming and discharged solids for the 18-month period. This computation is as follows:

Total weight of solids added to tanks........		1 808 433
Total weight of solids removed as sludge	438 231	
Total weight of solids withdrawn with supernatant	386 064	
Equivalent weight of gas, including allowance for loss..........	394 019	1 218 314
Solids unaccounted for....................		590 119

The solids unaccounted for, without adjustment for a possible difference in the amounts of solids in the tanks at the beginning and at the end of the test period, amount to 32.6% of the total solids added to the tanks. Undoubtedly, a large part of this unaccounted for loss is attributable to liquefaction during digestion.

PERIOD OF DIGESTION

If it is assumed that the period of digestion is represented by the actual time of displacement of the sludge in the digestion tanks, the average number of days required for digestion during the period of observation at Elyria would be: $548 \div \frac{5\,293\,422}{2\,(300\,540)} = 62.$

It will be noted that the volatile matter in the sludge removed from the digestion tanks was at all times less than 50 per cent. The sludge was odorless, of good quality, and thoroughly satisfactory for drying and final disposal.

MEMOIRS OF DECEASED MEMBERS

GEORGE HERNDON PEGRAM, Past-President and Hon. M. Am. Soc. C. E.[1]

Died December 23, 1937

George Herndon Pegram was born in Council Bluffs, Iowa, on December 29, 1855. His father, Benjamin Rush Pegram, son of Dr. John Coleman Pegram and Caroline Pegram, was born in Petersburg, Va., on April 28, 1828; and his mother, Mercy Adelaide Robbins, daughter of Abner Robbins and Eunice (Sears) Robbins, was born in Brewster, Mass., on September 5, 1828. The Pegrams of Virginia and the Sears of Massachusetts have long been prominent families of their respective sections of the country.

Mr. Pegram's father owned a bank and a store in Council Bluffs and a store in Denver, Colo., and also was engaged in freighting across the plains. At the time Mr. Pegram was born, Council Bluffs was a town in Indian country, crudely built, most of the houses being of wood with a few of brick. It was the headquarters of the Pottawottomies. Mr. Pegram's birthplace was a log house, one of the best of its time. The late General Grenville M. Dodge, Hon. M. Am. Soc. C. E., the noted railway builder, was a partner of Mr. Pegram's father from 1857 to 1861, when the Civil War broke out. Soon after the Pegrams moved to St. Louis, Mo., where the senior Pegram joined his brother in steam-boating on the Mississippi River. At one time they owned and operated six river steamers, some being the finest in service at that time.

Young George entered school when he was seven years old, but within a month suffered a severe attack of typhoid fever, and it was six months before he was strong enough to walk. In 1864, the family moved to New Orleans, La., where they remained for two years, returning to St. Louis in 1866.

The long convalescence from typhoid fever and the frequent changes in residence seriously retarded George Pegram's education, and he has stated that his progress was slow " because I could not seem to learn." In the fall of 1866, when he was eleven years old, he entered the lowest grade in the preparatory school of Washington University, at St. Louis, and finished his course in one year less than the scheduled time. His comment on this was, " I learned to work." He then entered the Polytechnic Department of Washington University and was graduated in 1877 with the degree of Civil Engineer. At the graduation exercises, Dr. Eliot, the Chancellor, remarked that Mr. Pegram had sustained the highest standard of any graduate.

Business was bad in 1877, and it was difficult to find a position; however, before the end of the year Mr. Pegram joined a survey party on the location of an extension of the Utah and Northern Railroad, from Franklin, Idaho, north through country in which war with the Nez Percez Indians was raging. The party was protected by a detachment of soldiers but no trouble occurred with the hostile Indians. This work was suspended at the end of the year.

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[1] Memoir prepared by a Committee appointed by the Board of Direction of the Society, consisting of Richard S. Buck, *Chairman*, F. W. Gardiner, and O. E. Hovey, Members, Am. Soc. C. E., and Arthur S. Tuttle, Past-President, Am. Soc. C. E.

In March, 1878, Mr. Pegram joined the staff of the late Colonel Charles Shaler Smith, M. Am. Soc. C. E., in St. Louis. Colonel Smith was a man of unusual ability, character, and personality, and one of the greatest bridge engineers developed in the United States up to that time. Association with him was a great privilege of which Mr. Pegram took full advantage. He remained with Colonel Smith until May, 1880, except for a period of six months when he was engaged in mercantile business in Leadville, Colo.

In May, 1880, Mr. Pegram was recommended by Colonel Smith to the position of Chief Engineer of the Edge Moor Iron Company, near Wilmington, Del. He was engaged shortly after and remained in that capacity until he resigned in 1886. His organization of the Engineering and Drafting Departments at Edge Moor was so excellent that it soon became recognized that service there constituted the best post-graduate course available to young men who wanted to follow Bridge and Structural Engineering. Edge Moor was the leader in the progressive development of the best shop equipment and practice, and fully one-half the Office Staff was made up of engineer graduates anxious to learn the best in preparation for an engineering career. Many well-known bridge and structural engineers acquired their first practical experience at Edge Moor under Mr. Pegram and his able successors.

Mr. Pegram had early decided that at thirty years of age he would "change his course of life, whatever it might then be." Accordingly, he resigned and left Edge Moor, on May 1, 1886. However, hoping he would return, the Company did not fill his place for some time.

He sailed for England early in May, 1886, and spent five months traveling in England, Wales, France, Switzerland, and Germany. Shortly after his return, he was recalled to Edge Moor to design a proposed elevated railroad in Philadelphia, Pa. On the completion of this work, he opened an office in New York, N. Y. He patented the "Pegram truss," which he had designed to simplify and cheapen truss-bridge construction, and first introduced it in 1889 in the construction of a 200-ft pin-connected railway span over the Verdigris River, in Indian Territory, for the Missouri Pacific Railway Company.

Closing his New York Office in 1889, Mr. Pegram became Consulting Engineer with the Missouri Pacific Railroad Company at St. Louis, having charge of new bridges, and also, under Mr. James W. Way, the Chief Engineer, the construction of new lines. The System then comprised about 7 500 miles of track. The new lines, built during Mr. Pegram's period of service were from Coffeyville, Kans., to Fort Smith, Ark.; Kansas City to Fort Scott, Kans.; and Monroe to Alexandria, La. The "Pegram truss" was used for all new bridges and several replacements on old lines. The bridge across the Arkansas River, at Fort Smith, consists of three spans of 200 ft each, and a span 350 ft in length, all Pegram trusses.

In 1891, the Terminal Association of St. Louis, formed of the seven railroads terminating in that city, decided to build a Union Station. Several plans were considered, and that proposed by Mr. Pegram was unanimously approved by a Committee composed of the Chief Engineers of the several railroad companies and by H. P. Taussig, M. Am. Soc. C. E., President of the Terminal Association. The train-house is 610 ft wide and 700 ft long, covering 32 tracks. The roof is carried by five spans of Pegram trusses, the upper chords forming a circular arc.

On February 1, 1893, Mr. Pegram became Chief Engineer of the Union Pacific System, then composed of about forty railroads having about 9 000 miles of track, only about 35 miles of which were double-tracked. He assumed this position under President S. H. H. Clark, with whom he had been associated while Mr. Clark was Vice-President and General Manager of the Missouri Pacific Railway Company. He remained in this position for five years, during which many new steel bridges were built, chiefly in Wyoming, Idaho, and Kansas. In most of these, Pegram trusses were used. The Town of Pegram, Idaho, was named in his honor.

In the fall of 1896, Mr. Pegram was called to New York City for a consultation with the late Jay Gould, who requested him to make an examination of the elevated railroad structures. Mr. Gould had been advised that the Sixth Avenue Line would have to be replaced. Mr. Pegram, on examination, reported in favor of making extensive reinforcements over a long period of time, on the completion of which he held that the structures would serve safely for many years. This report, doubtless, led to his being called to New York in 1898 to become Chief Engineer of the Manhattan Railway Company. In this capacity he was intimately associated with the development of transit facilities in New York City from 1898 until his death.

The development of electric traction prior to 1899 made possible a large increase in the carrying capacity of the existing elevated railroads by the use of this new motive power. It also made possible the use of subways for city transit. Increased transit facilities in the city were urgently needed and the construction of subways followed rapidly upon the development of this new motive power, and elevated extensions followed a little later.

When Mr. Pegram came to New York City in 1898 there were elevated railroads with a total of 167 single-track miles operated by steam locomotives, and these were the only rapid transit railroads operating in the city. They carried about 250 000 000 passengers per yr, with much congestion. In 1937, there were 780 single-track miles of elevated and subway railroads operating in the city, all with electric motive power, and carrying 1 900 000 000 passengers per yr.

During its early period this development was pioneering work, both in construction and equipment, to the success of which Mr. Pegram made a large contribution. Outstanding examples of this are the reinforcement of the existing elevated railways in Manhattan and The Bronx for electric traction; the construction of the 74th Street Power House—the largest and most efficient power-generating station in the world at that time; the construction of seven subsidiary sub-stations; extensions of the elevated lines and completion of third track and its equipment, the latter work being carried on without interruption of train operation; the construction and equipment of the first railroad tunnel under the East River, involving much difficult and novel treatment; the equipment of subway lines, comprising electrical road equipment, signaling, drainage, and ventilation, totaling 157 single-track miles; the construction of twelve subway power sub-stations; the complete construction of parts of the subway lines; the construction of subway platform extensions and a new deep-tunnel station, all without interruption of train operation. This work involved many unavoidable hazards and required much novel treatment and unceasing care.

During much of this work, Mr. Pegram was Chief Engineer of both the Operating Company and the Subway Construction Company, the latter Company as a subsidiary of the Interborough Rapid Transit Company constructing seven sections of the more difficult subway work, in particular, the connecting sections between the operating subways and the new work, where the safety of train operation was involved. In addition, from 1912 to 1924, Mr. Pegram was Chief Engineer of the New York Railways Company, operating a large part of the surface lines in Manhattan. All these activities were conducted in close co-operation with the State and city agencies, in which his ability as an engineer was supplemented by a genius for friendship, and the success of the co-operative effort, necessary for the accomplishment of the vast and complex engineering work of rapid-transit development, owed much to the genial atmosphere of mutual respect and friendliness, with which he was able to surround meetings of many and diverse personalities, with different points of view and different interests, and to bring them into effective co-operation.

George Herndon Pegram was a great Engineer, measured not only by volume, variety, and character of work done under his direction, but still more by the influence he exerted on those who participated with him in this work. He was a man of strong convictions, but this never impaired his sense of fairness nor his consideration of the viewpoints of others. He was modest—even diffident—and " never spoke out of his turn." When he spoke it was clearly and to the point. He treated with the same kindly courtesy all with whom he came in contact, from the highest to the lowest. Being very human, he sometimes lost his temper, but invariably followed this with some consoling or humorous remark that wiped out the sting of his words and restored good feeling. He found time to cultivate qualities as a man which brought to him a large circle of friends outside of, as well as within, the Engineering Profession. He had a keen sense of humor, and the rare faculty of enjoying a good joke on himself as well as on the other fellow. He was always welcome in any company, whether it was gathered together for professional discussion, or for good fellowship, being an appreciative listener as well as an entertaining talker.

His professional staff was ever loyal and devoted, knowing well he lived by the rule that true loyalty works downward as well as upward, extending all the way down the line. No one misses him more than those who have served under him.

He was peculiarly fortunate in retaining to the end of his long and eventful life a youthful interest in people and events. He was a great lover of music, Wagner being his favorite composer and grand opera his favorite entertainment. Another favorite avocation was sailing and deep-sea fishing.

Despite his conscientious absorption in his professional work, he was most devoted to his home and family. Although he was known to many as a great engineer, it was the privilege of comparatively few to appreciate fully his sterling personal character as son, brother, husband, father, and friend. In passing he left voids in many lives, never to be filled, but his good works, his generosity, his kindliness, and his good fellowship have left imprints on these lives that will endure for long down the ages.

Mr. Pegram was President of the Society during the period of the World War and conducted its affairs during that time of stress in such a manner that the Society emerged stronger than ever before. Under his guidance as President, the Society moved its headquarters from West 57th Street to the Engineering Societies Building, part of which it now occupies; and it joined with the other three large National Societies in the creation of Engineering Council and assisted in the formation of what is now United Engineering Trustees, Inc.—agencies providing the machinery for concerted effort in all matters concerning the profession.

When Honorary Membership was conferred upon Mr. Pegram at the Annual Meeting of the Society on January 20, 1932, he was addressed as follows: "The Honorary Membership now conferred upon you completes the list of rewards which this Society can give to members who have held its highest office and have rendered outstanding service to the profession and inferentially it should carry with it a recognition of the possession of human qualities which endear the man to those with whom he comes in contact."

It was characteristic of Mr. Pegram that, in attempting to respond, he broke down in tears.

Mr. Pegram served on a number of the important Committees of the Society. He was also Past-Trustee of Engineering Foundation; member of the American Society of Professional Engineers; member and Past-Director of the Engineers' Club of New York; member of the Virginia Society of New York; and Doctor of Laws of Washington University.

An examination of the *Transactions* shows that he presented his first paper to the Society in 1884, four years after his admission, and that this paper was followed by fifteen other papers and discussions, most of them relating to bridge and railroad subjects.

The death of George Herndon Pegram on December 23, 1937, brought to a close the long career of one of the most distinguished engineers in the United States. He was one whom engineers and others delighted to honor for his exceptional character and ability and his charming personality.

Mr. Pegram was married, at St. Louis, on September 8, 1897, to Jessie Merrilees Crawford, who survives him. He is also survived by his two daughters, Jean Forsyth Pegram and Mrs. Horace M. Buck; also by his brother, Frank R. Pegram and his sister, Mrs. Minnie (Pegram) Fabian.

Mr. Pegram was elected a Junior of the American Society of Civil Engineers on April 7, 1880; a Member on January 3, 1883; and an Honorary Member on October 5, 1931. He served as a Director of the Society from 1902 to 1904; as Vice-President in 1909 and 1910; and as President in 1917.

AMBROSE SWASEY, Hon. M. Am. Soc. C. E.[1]

DIED JUNE 15, 1937

Ambrose Swasey was born on December 19, 1846, near Exeter, N. H., the youngest child of Nathaniel and Abigail Swasey. At an early age his formal education (received in the elementary schools of Exeter) ceased, and he began his mechanical training as an Apprentice in the Exeter Machine Works, apparently following a natural bent in occupation. This, later, through his many awakened talents, developed into a wonderfully long and successful career.

From 1865 to 1869 Mr. Swasey remained with the Exeter Machine Works, leaving this Company to go, as a Master Machinist, to the well-known shops of Pratt and Whitney, at Hartford, Conn. While at Exeter he began his life-long association with Mr. Worcester R. Warner, who accompanied him to the works of Pratt and Whitney, also as a Master Machinist. In 1880, Mr. Swasey and Mr. Warner joined forces and fortunes through the formation of their own business in Chicago, Ill., under the name of Warner and Swasey, thus founding an organization destined to live many years and to develop a reputation widely and favorably known in the United States and in foreign countries. In 1881, the business was moved to Cleveland, Ohio, where the partners made their permanent homes and where the business is still conducted by their successors.

Mr. Swasey, early in his career, developed improved processes of manufacturing machine gears, and the first method of generating and cutting teeth of spur-gears entirely by mechanical processes. His engineering and technical accomplishments continued largely in connection with designing and building machines and instruments of precision both for commercial and scientific purposes. He either perfected, or made possible, through his expert and inspiring aid and direction in the work of the Warner and Swasey Company, the design and manufacture of high quality turret lathes, high-class astronomical instruments, and especially the construction and mounting of many notable telescopes. Such great telescopes include that at the Lick Observatory with its 36-in. lens, the Naval Observatory telescope, and the 40-in. Yerkes refracting telescope which remains the largest in the world. Several great reflecting telescopes also added to Mr. Swasey's fame; the 72-in. instrument of the Dominion Astrophysical Observatory, in British Columbia, the 69-in. telescope at Ohio Wesleyan University, and the most recent, the 82-in. reflector for the University of Texas. Of great importance was also the dividing engine, used in graduating with extreme accuracy the great circles necessary in astronomical observation of the stars, reported to be unexcelled in accuracy by any other dividing engine.

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[1] Memoir prepared by Robert Hoffmann, Hon. M. Am. Soc. C. E., and Ralph Budd, M. Am. Soc. C. E., largely from the article by W. W. Campbell, reprinted from the publications of the Astronomical Society of the Pacific, August, 1937; from the citation presented at the time Mr. Swasey was made an Honorary Member of the American Society of Civil Engineers, and at the time of presentation of the John Fritz Medal; also from the August, 1937, issue of *Mechanical Engineering*. Additional information may be obtained from these sources.

Although Mr. Swasey's name will always be remembered through his accomplishments in the field of science and the manufacturing successes of Warner and Swasey, he also showed willingness to devote much of his time, energy, and means to aid educational institutions, assisting civic organizations, and above all to promote science and the welfare of engineers and engineering. Among Mr. Swasey's many benefactions may be listed those to the following educational institutions: The Science Building for the University of Nanking, the Christian Association Building of Canton Christian College (now Lingman University, China), the Denison University Chapel and Observatory, the Colgate-Rochester Divinity School Library Building, the endowment of a Chair of Physics at Case School of Applied Science, and an observatory to the same institution, presented jointly by Mr. Swasey and Mr. Warner.

Especially appreciated by the entire Engineering Profession was Mr. Swasey's active interest in the welfare of engineers and in the promotion of work undertaken by them. His interest in securing the building occupied as the home of the four great engineering societies in New York, N. Y., and built through the generosity of Andrew Carnegie, will long be remembered. Since 1914, Mr. Swasey gave generously of his means and much of his time and support to establishing the Engineering Foundation, having for its objective "the furtherance of research work in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind."

It is eminently fitting that Mr. Swasey's unselfish devotion to the welfare of others should have been recognized and honored by many. In addition to honors from the educational institutions heretofore mentioned, there were also conferred the following: From the Government of France, Chevalier of the Legion of Honor in 1901, and Officer of the Legion of Honor in 1921; the Cleveland Chamber of Commerce Medal for Distinguished Public Service; the Gold Medal of the Franklin Institute; the Gold Medal of the American Society of Mechanical Engineers; Washington Award by the four National Engineering Societies, and the Western Society of Engineers made in 1934; in 1936, the Gold Medal established by President Hoover to be awarded in recognition of distinguished services rendered in the engineering field; and the John Fritz Gold Medal awarded by the Engineering Profession of America in 1924.

Mr. Swasey was a Charter Member, Past-President, and Honorary Member of the American Society of Mechanical Engineers; Past-President and Honorary Member of the Cleveland Engineering Society; Honorary Member of the American Institute of Electrical Engineers; Member of the American Astronomical Society; the Institute of Mechanical Engineers; the British Astronomical Society; Fellow of the Royal Astronomical Society; Member of the American Philosophical Society, and the National Academy of Sciences; and also a Trustee of Western Reserve University, Denison University, the University of Nanking, and of the Case School of Applied Science.

Although his early schooling may appear meager and limited, Mr. Swasey's life is another example showing how learning and culture may be developed through the school of experience and by individual effort, even if early book learning, as judged to-day, seems to have been largely lacking. Ambrose Swasey was a highly educated man measured by any standard—a fact well attested by the respect and recognition given his career and accomplishments by educational institutions. The degree of Doctor of Engineering was conferred upon him in 1905 by the Case School of Applied Science, at Cleveland, Ohio; the degree of Doctor of Science in 1910 by Denison University, at Granville, Ohio, in 1924, by the University of Pennsylvania, at Philadelphia, Pa., and in 1931, by Brown University, at Providence, R. I. The degree of Doctor of Laws was also conferred upon him by the University of California, in Berkeley, Calif., in 1924, by the University of Rochester, at Rochester, N. Y., in 1925, and by the University of New Hampshire, at Durham, N. H., in 1930.

The career of Ambrose Swasey was a long and eventful one, so many sided that his accomplishments, distinguishing him as a great man personally, cast reflected credit and honor on the many interests and organizations with which he was affiliated rather than to any single cause or interest. He belonged to many—to mankind. Rarely is the opportunity afforded and the necessary ability, willingness, and training given to one individual to do the things that fell to the lot of Mr. Swasey. Long life, success in business, a keen mind, an engaging personality, and impressive appearance, all contributed to make him and his career an example that engineers may well look upon as an ideal well worthy of aspiration.

Mr. Swasey was married in 1871 to Lavinia D. Marston, whose death preceded his by twenty-four years. There were no children.

Ambrose Swasey was elected an Honorary Member of the American Society of Civil Engineers on June 6, 1921.

ROBERT ADAM, M. Am. Soc. C. E.[1]

DIED APRIL 15, 1937

Robert Adam was born on February 21, 1866, at Paisley, Scotland, the son of Robert Adam—a member of a Border family. He was educated at Heriot-Watt College, in Edinburgh, Scotland, and from 1882 to 1885, served his Apprenticeship under Mr. A. S. Biggart. He received his training under Mr. R. E. Middleton, acting as Assistant to Mr. J. Phillips.

The following year, at the age of twenty, Mr. Adam engaged in his first engineering work which was in connection with the construction of the Firth of Forth Bridge in Scotland. He continued in this work for four years.

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[1] Memoir prepared from data supplied by the family and on file at the Headquarters of the Society.

In 1890, he joined the staff of Messrs. James Young and Company, Contractors, and worked on the Blane Valley Section of the Glasgow Water-Works (five miles of aqueduct in tunnel); and from 1891 to 1893 he was Engineer to the same firm on the Stobcross Section of the Glasgow Central Railway, and the Stobcross Section of the Lanarkshire and Dumbartonshire Railway. The work was principally tunneling and cut and cover.

As Assistant to the late Sir Ernest William Moir, M. Am. Soc. C. E., Mr. Adam served for two years as Engineer-in-Charge of Approaches on the Blackwall Tunnel.

In 1895 he joined the staff of Messrs. S. Pearson and Son Limited, Contractors, as Engineer during the construction of the Surrey Commercial Dock, in London, England. A year later he was transferred to the staff of the same firm in Mexico and served as Chief Assistant Engineer on the Vera Cruz Harbor Works, Drainage, and Water Supply of the City of Vera Cruz, under J. B. Body, M. Am. Soc. C. E., as Chief Engineer. He continued in this work until 1900, when he was appointed Engineer of the Salina Cruz Harbor Works, as well as of drainage and water supply.

From 1902 to 1905 Mr. Adam was Consulting Engineer to Messrs. S. Pearson and Son for the drainage and water supply of the Vera Cruz Harbor Works, the Coatzacoalcos Harbor Works, and the Salina Cruz Harbor Works, at a cost of about $50 000 000.

He opened his own office as Consulting Engineer in the City of Mexico, Mexico, in 1906, remaining there until 1912, when he returned to England and continued his practice. Because of his vast engineering knowledge, he was held in high esteem as a Consultant.

Mr. Adam was created an Officer of the British Empire for his special service during the World War. He was engaged by the War Office on the Railway Material Branch, and practically built up that section of war work, retiring from it only at the end of the war. He was a Member of the Institution of Civil Engineers, in London, England.

He is survived by his widow, Lena (Phillips) Adam, and by two daughters, Dr. L. C. Adam and Dr. F. A. Adam-Thomson.

Mr. Adam was elected a Member of the American Society of Civil Engineers on May 2, 1900.

WILLIAM AUGUSTIN AIKEN, M. Am. Soc. C. E.[1]

DIED OCTOBER 5, 1935

William Augustin Aiken was born August 29, 1851, at Baltimore, Md. Both his parents came of old Colonial stock. His paternal great-great-grandfather came from the south of Scotland toward the close of the Seventeenth Century, one of two brothers, of whom one settled in New England and the other in New York. A first cousin settled in South Carolina, whence the Aikens of that State.

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[1] Memoir prepared by W. D. Aiken, Assoc. M. Am. Soc. C. E.

His grandfather, John Aiken, was born in Poughkeepsie, N. Y., and was graduated from Yale University, at New Haven, Conn., in 1798. Later, he was admitted to the bar and practised law in Troy, N. Y. Through his paternal grandmother, who was a Cooke, he was connected with the Braesteds and Motts of New York.

His father, Dr. William E. A. Aiken, a distinguished Chemist and Toxicologist, was Professor of Chemistry in the Medical School of the University of Maryland, at Baltimore. He lectured there for fifty years, and was Professor Emeritus at the time of his death. His mother was Rebecca Josephine Nicholson whose ancestors came to the United States from Berwick-on-Tweed, England, at the beginning of the Eighteenth Century, settling first in Virginia, and, later, moving to Easton on the eastern shore of Maryland, where she was born.

The Nicholsons from the earliest to the present time have been Navy people. Mr. Aiken's grandfather, Commodore Joseph Nicholson, was in command of the American Naval defense of Baltimore in the War of 1812. His great-aunt was the second wife of Albert Gallatin, Secretary of the Treasury in Jefferson's Administration, and is buried in Trinity Churchyard, in New York City. Through his mother's family, William Augustin Aiken was closely connected with the Wilmer, Lardner, Lippincott, Biddle, and Rush families of Philadelphia, Pa.

Mr. Aiken attended the Jesuit College, Loyola, in Baltimore, and was graduated in 1869 with the degree of Bachelor of Arts. He then entered Rensselaer Polytechnic Institute, at Troy, as a Sophomore in the Class of 1872, and was graduated with the degree of Civil Engineer. It is of interest that his father was a special student in the first class to be graduated from the Institute.

After a year's preliminary experience with the Western Maryland and Pennsylvania Railroad Companies, Mr. Aiken went to California in 1873, and for six years had a varied experience, embracing field work on irrigation canals in the San Joaquin Valley, water supply investigations for the City of San Francisco, original surveys for the Golden Gate Park in that city, preliminary location and construction work on railroads throughout California, and, for three years, he was engaged in mining and metallurgical work in Nevada in association with the late William Starke Rosecrans, M. Am. Soc. C. E.

Returning East in 1879, Mr. Aiken associated himself with Colonel John G. Clark, who had been prominent as an Engineer in the Armies of the South during the Civil War. At this time the railroads running south from Washington, D. C., were being rehabilitated and Mr. Aiken had charge of the replacement of numerous bridges and was engaged in this work for several years.

Growing out of this experience he joined the Morse Bridge Company, of Youngstown, Ohio, in charge of its Chicago, Ill., Office, and for several years was engaged in contracting in the Northwest.

Upon the dissolution of the Morse Bridge Company, Mr. Aiken became associated with the Pittsburg Testing Laboratory and located at Athens, Pa.

As its Resident Engineer at the shops of the Union Bridge Company he had charge of the fabrication of the Poughkeepsie Bridge, as well as other important work, notably the manufacture of the eye-bars for the Hawksbury River Bridge, in Australia, which were forged from Scotch steel, shipped in bond, and later reshipped to Australia.

After some years of inspection work at Athens, Mr. Aiken accepted a position as General Manager and Chief Engineer of the Mt. Vernon Bridge Company, at Mr. Vernon, Ohio. He largely re-organized this highway bridge shop, installing facilities for heavier construction and, over a period of several years, built many bridges in the Middle West.

Resigning from this connection Mr. Aiken returned to his association with the Pittsburg Testing Laboratory as its Chicago Representative, and was active in its interests during the 1893 World's Fair in that city.

When the Pittsburg Testing Laboratory discontinued its Chicago Office, Mr. Aiken entered private practice and was engaged upon important construction, such as the United States Government bridge over the Mississippi River, at Rock Island, Ill., under Ralph Modjeski, M. Am. Soc. C. E., and the Duluth and Superior Bridge under the late Alfred Pancoast Boller, M. Am. Soc. C. E.

When the contract for the original subway in New York was finally signed early in 1900, the attention of the Chief Engineer, the late William Barclay Parsons, Hon. M. Am. Soc. C. E., was called, without Mr. Aiken's knowledge, to his experience with structural materials, with the result that he was subsequently appointed Division Engineer with the title of General Inspector of Material. Mr. Aiken organized the Department of Inspection of Material and for eight years it functioned efficiently and to the satisfaction of the Chief Engineer.

Resigning after eight years' service on the subway work, Mr. Aiken associated himself with the H. S. Spackman Engineering Company, of Philadelphia, and, as Vice-President, had charge of the Chemical and Physical Laboratories. It was a source of pride to Mr. Aiken that for several years a large amount of his work came directly through his former connection on the subway, from General Parsons and his associates.

When the United States entered the World War, Mr. Aiken had retired from the Spackman Engineering Company, and was prevented from obtaining a commission in the Engineer Corps by a serious injury to his hip due to a fall. He was obliged to content himself by doing "his bit" as a Testing Engineer in the United States Ordnance Department. He was located at several plants and, finally, at Pottstown, Pa., which plant was said to have been the only one, except Government Arsenals, from which no finished shells were rejected.

After the war he returned to his favorite work of material inspection and was engaged on several large projects through his association with General Parsons, such as dry dock construction, at Portsmouth, Va., Navy Yard; concrete warehouse construction, in Havana, Cuba; and a concrete dam across the Hudson River, at Glens Falls, N. Y., where for three years he had charge of the manufacture of concrete. Mr. Aiken considered his association with General Parsons on the original Rapid Transit work, and subsequently, as the happiest professional engagements of his career.

Owing to increasing physical debility due to an earlier injury Mr. Aiken was not active in business after about 1930.

He was a devout member of the Roman Catholic Church and his life conformed strictly to the requirements of his faith.

Mr. Aiken held membership for many years in the American Society for Testing Materials and was active in that Society, serving as Chairman of several committees and presenting numerous papers at its Annual Conventions.

In 1881, he was married to Jennie M. Drees, of Xenia, Ohio, whom he met in Columbia, S. C. He is survived by his widow, a son, W. D. Aiken, Assoc. M. Am. Soc. C. E.; a daughter, Mrs. Jean A. Reinke, and two grandchildren.

Mr. Aiken was elected a Member of the American Society of Civil Engineers on October 2, 1889.

WILLIAM BULLARD ALLEN, M. Am. Soc. C. E.[1]

DIED JANUARY 9, 1938

William Bullard Allen was born in Indianapolis, Ind., on March 12, 1863. He was the son of William Allen and Martha (Carter) Allen. He received his early education in the public schools in Indianapolis, and, later, attended Hanover College, at Hanover, Ind.

Mr. Allen began his engineering career in 1881, on surveys for, and the construction of, the Cincinnati and Georgia Railroad (later, a part of the Southern Railway); and for the following four years he was active on surveys and construction on the East Tennessee, Virginia, and Georgia Railroad, the Northern Alabama Railroad, and also on the railroad belonging to the Cahaba Coal Mining Company, between Woodstock and Blocton, Ala.

He then became, for a time, an Assistant Engineer with the Guatemala Northern Railroad Company, in Guatemala. On his return to the United States he was engaged in like capacity for the Kansas City, Memphis and Birmingham Railroad Company. Afterward, he was an Assistant Engineer in the Maintenance-of-Way Department of the Louisville and Nashville Railroad Company, between Decatur and Montgomery, Ala.

In 1888, Mr. Allen conducted a survey for the Mexican Government for a railroad from Jiménez to Balleza, Mexico, upon the completion of which, he entered the service of the Texas and Pacific Railway Company as an Assistant Engineer. From this position he was, successively, promoted to that of Division Engineer, Assistant Resident Engineer, and Resident Engineer, in which latter capacity he had charge of bridges, buildings, and water service for the entire System, which was then about 1 500 miles.

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[1] Memoir prepared by J. F. Coleman, Past-President and Hon. M. Am. Soc. C. E.

In May, 1892, Mr. Allen resigned the latter position to take charge of an extension of the Natchitoches Railroad, in Louisiana; and in May, 1893, he entered the service of the Tennessee Coal, Iron, and Railroad Company, in which service he remained until his retirement on October 1, 1932.

During this time, Mr. Allen was also Manager of the Land Department, in charge of all the Company's real estate until March, 1918. Subsequently, he was Manager of the Chickasaw Utilities Company and of the Chickasaw Land Company (subsidiaries of the Tennessee Coal, Iron, and Railroad Company). Later, he was made Agent in the Executive Department of the Tennessee Coal, Iron, and Railroad Company, Assistant to the President of the Ensley Land Company, Vice-President of the Birmingham Southern Railroad Company, and Chairman of the Executive Committee, Port of Birmingham Company.

He was married on November 3, 1902, to Lucy Beall, of Birmingham, Ala., who survives him. He is also survived by a sister, Mrs. Edward M. Hughes, and a niece, Mrs. N. E. Nichols.

In the course of his lifetime, Mr. Allen was never robust, and suffered almost constantly from ill health. His rather remarkable advancement in his profession was the result of an indomitable will, and a stubborn courage that refused to recognize or admit his bodily weakness.

He was a very reserved man, and slow to give his friendship. Notwithstanding which he had a host of friends, and wielded an influence upon the lives of others far beyond his own realization.

In his passing the Engineering Profession has lost a noble soul, a man of high ideals, and of unswerving integrity.

Mr. Allen was elected a Member of the American Society of Civil Engineers on February 3, 1904.

WILLIAM DELANO ARMSTRONG, M. Am. Soc. C. E.[1]

DIED NOVEMBER 5, 1937

William Delano Armstrong was born at Wailuku, Maui Island, Hawaii, on June 20, 1872. His father, Goodale Armstrong, was a member of one of the early pioneer families of California. He was a merchant in Marysville, Calif., after his migration overland from Cleveland, Ohio, in which city, Goodale Park still exists as a gift of the Goodale family. His mother, Lucia

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[1] Memoir prepared by Earle A. Burt, M. Am. Soc. C. E.

Peck, was the daughter of a missionary family from New England, who came to California *via* Cape Horn during the early period.

The early years of William Armstrong's life were spent in the Hawaiian Islands, where he received his elementary and preparatory education. For a period of two years he attended Oahu College, in Honolulu, as a student in Civil Engineering, after which he worked in an iron foundry in the same city for a period before entering the service of the Olinda Oil Company as its Honolulu Representative in California.

During 1901 and 1902, Mr. Armstrong was employed as Chief Draftsman in the office of the Los Angeles Railway Company, subsequently joining the Staff of the Pacific Electric Railway Company, as a Draftsman, with which he held the position of Chief Draftsman for the years from 1904 to 1907. At the end of this engagement, he became associated with the United States Reclamation Service and spent the years from 1907 to 1909 on work at the Granite Reef Project, in Arizona.

The formation of the Los Angeles County Highway Commission in 1907 and the subsequent Good Roads Bond Issue in 1908 created one of the pioneer highway organizations of the West, and in 1909, Mr. Armstrong joined the forces of the Highway Commission as Structural Designer, thus entering a field of endeavor which was to become his principal life work. In 1914, the work of the Highway Commission was delegated to the newly organized Los Angeles County Road Department, with which his services were continued in the positions of Senior Structural Draftsman, Bridge Draftsman, and Senior Bridge Draftsman, respectively, until 1925, at which time he was appointed Bridge Engineer, with the responsibility of directing the design and construction of all bridge and structural work of the Road Department. In this position he continued his work until the time of his death.

Mr. Armstrong's personality was of the type which has become known, in the popular mind, as typical of the engineer, in that his work was one of his paramount interests. On the other hand, to those who knew him well, his quiet, witty, gentlemanly disposition made his presence in any gathering a source of pleasure and enjoyment. The people of Los Angeles County who really knew his work will be, for many years, reminded of him as, from time to time, they pass over the innumerable structures into which his life and ambition have been incorporated. The public service, to which the best years of his life were dedicated, is benefited, and the esteem which public employees should warrant, is improved by the type of man which he exemplified.

In 1902, Mr. Armstrong was married, at Los Angeles, to Lillian Bovard, the daughter of Dr. Marion McKinley Bovard, and Jean (Allen) Bovard, pioneer of co-education at DePauw University, at Greencastle, Ind. He is survived by his widow and two brothers, Frank and Wade Armstrong.

He was a member of the Sons of the Revolution, and of Pentalpha Lodge, F. and A. M., of Los Angeles.

Mr. Armstrong was elected a Member of the American Society of Civil Engineers on June 10, 1929.

RICHARD I. DOWNING ASHBRIDGE, M. Am. Soc. C. E.[1]

DIED JULY 26, 1936

Richard I. Downing Ashbridge was born on September 22, 1861, at Ondawa Farm, near Downingtown, Pa. He was the son of Abram S. Ashbridge and Elizabeth (Downing) Ashbridge, both descendants of Colonial families. He was of the sixth generation from George G. Ashbridge, who represented the County of Chester in the Colonial Assembly for a period of thirty years, from 1743 to 1773. The County of Chester then comprised all of the State of Pennsylvania west of Philadelphia County and south of the Schuylkill River.

Mr. Ashbridge prepared for college at the Chester Valley Academy, in Downingtown. Later, he attended the University of Pennsylvania, at Philadelphia, Pa., from which he was graduated, in 1881, with the degree of Bachelor of Science in Civil Engineering. During one or more of his summer vacations, he was employed by the United States Geological Survey.

From June, 1881, to October of that year, Mr. Ashbridge was Topographer for the Central Virginia Mining Company. He then went West, and from November, 1881, through November, 1882, he was Assistant Engineer during the construction of the Little Falls and Dakota Branch of the Northern Pacific Railway. The next year (December, 1882, to December, 1883), he was Assistant Engineer in charge of two 12-mile Divisions of the Duluth and Iron Range Railroad, in Northern Minnesota. While engaged in this work, he lived under canvas a considerable part of the time, with the thermometer going as low as — 40° F.

Returning East, in April, 1884, Mr. Ashbridge took a position with the Department of Public Works in the Bureau of Surveys of the City of Philadelphia, and from then until April, 1888, he served in various capacities as Rodman, Leveler, and Assistant Engineer in charge of construction and location of a new intercepting sewerage system. For the next year, until June, 1889, he was Engineer in charge of the construction of the City Line Bridge, over the Schuylkill River.

From June, 1889, to January, 1890, he was employed as Assistant Engineer in charge of a section of the Ohio Extension of the Norfolk and Western Railroad. Mr. Ashbridge then became Engineer of the Roanoke Development Company, and, until August, 1890, was in charge of the laying out and erection of four bridges over the Roanoke River, and of grading and draining the town site and the construction of buildings.

In August, 1890, he returned to Philadelphia, and until September, 1894, was once more with the Bureau of Surveys of the Department of Public Works, this time as Engineer in charge of the location and construction of the Wissahickon Intercepting Sewer and its branches. From September, 1894, to November, 1895, he was in charge of the construction of the drainage system of the Pennsylvania Avenue (Philadelphia) Subway and Tunnel, embracing about seven miles of sewers built into the tunnel.

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[1] Memoir prepared by Whitney Ashbridge, Assoc. M. Am. Soc. C. E.

As Engineer in charge of the Hydrographic Corps, of the City of Philadelphia, he supervised the surveys for, and the excavation of, the ship channel in the Schuylkill River.

He was Assistant Engineer on lines and grades for the Pennsylvania Avenue Subway and Tunnel from August, 1896, to September, 1900. From that time until March, 1902, he served as Assistant Engineer on the improvement, extension, and filtration of the Philadelphia Water Supply. During this engagement Mr. Ashbridge assisted in the construction of the Belmont Water-Works, and the Torresdale Filter Plant, both of which are still in operation.

Becoming First Assistant Engineer of the Bureau of Filtration, City of Philadelphia, in March, 1902, he held that position for a year, leaving in 1903 to become Engineer for Stern and Silverman, of Philadelphia, in charge of the construction of various railway lines, including the following: In 1904, the New Jersey Short Line Railroad (later, a part of the Public Service Corporation of New Jersey); in 1906, the Atlantic City and Shore Railroad; in 1907, the Atlantic City and Ocean City Railroad; in 1911, the Chester and Philadelphia Railroad; and, in 1917, the Salem and Pennsgrove Traction Company.

For more than twenty-five years prior to his death, Mr. Ashbridge held the position of Chief Engineer of the Atlantic City and Shore Railroad Company.

He was the author of a widely used practical engineering book entitled "Civil Engineering Specifications and Contracts," which he published in 1914. He also wrote chapters on subjects with which he was familiar for various engineering textbooks.

At the time of his death, and for many years prior thereto, Mr. Ashbridge was a Vestryman of St. James Protestant Episcopal Church, in Downingtown.

A lover of music, painting, and flowers, his main interest, outside his profession, was floriculture. Most of his spare time during the spring and summer months was spent in the garden of his home at "Ondawa," just east of Downingtown. Although his particular hobby was the raising of flowers, he was interested in all kinds of horticulture. For many years, he was a member of the Engineers' Club of Philadelphia.

He is survived by four sisters, Mrs. J. D. Perkins, Mrs. C. S. Rutter, Mrs. Henry Ecroyd, and Eleanor Ashbridge; and by two brothers, A. S. Ashbridge, Jr., and Joseph S. Ashbridge.

Mr. Ashbridge was elected an Associate Member of the American Society of Civil Engineers on September 3, 1902, and a Member on April 30, 1907.

JOHN LELAND BECTON, M. Am. Soc. C. E.[1]

DIED JANUARY 8, 1938

John Leland Becton, son of George and Mollie Becton, was born at Goldsboro, N. C., on October 24, 1885. His technical education was obtained at the

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[1] Memoir prepared by a Committee of the North Carolina Section, consisting of W. C. Riddick, J. D. Spinks, and J. E. Willoughby, Members, Am. Soc. C. E.

North Carolina State College of Agriculture and Engineering, at Raleigh, N. C., from which he received the degrees of Bachelor of Engineering in 1908, and Civil Engineer in 1913.

Mr. Becton began his professional work in Wilmington, N. C., in a subordinate capacity in 1908; and, in 1911, he established there a general engineering practice. During the development and progress of that practice his work covered farm and drainage surveys; reclamation projects, such as prevail in the flat plains of Eastern North Carolina; industrial railways and industrial plants; the design and construction of bridges, docks, and terminals; water and sewerage systems; and other municipal projects.

Positions held by him in public service include: Deputy City Engineer, of Wilmington; Assistant Chief Engineer, of the Liberty Ship Building Corporation (a World War agency for building concrete ships); Member of the Board of Trustees for Greater University of North Carolina; President of the North Carolina State College Alumni Association; Member of Board of Engineering Registration for North Carolina; President of the Wilmington Chamber of Commerce; and Chairman of the Board of Stewards of Grace Methodist Episcopal Church South. A short time before his death, he was elected Lay Delegate to the 1938 General Conference of his Church.

Mr. Becton took an active part in the work of the North Carolina Society of Engineers, and the North Carolina Section of the Society, in both of which he held many official positions, including that of President. He held membership in Phi Kappa Phi (Honor Society), the Rotary Club, Cape Fear Club, Cape Fear Country Club, and the Carolina Yacht Club, all of Wilmington.

In the practice of his profession, Mr. Becton undertook only such work as could be done by himself, or by his assistants under his personal direction, thus insuring that the client would receive clear, seasoned, engineering judgment. He never sought clients; he was never without them. The monetary return from his practice was generous, and yet the return in the esteem of those who knew him and his worth was of greater moment to him than the professional fees received, and he will be remembered as a public-spirited, Christian citizen, and as a friend, long after the "gnawing tooth of time" has destroyed all records of his work as an engineer. He was never married.

Mr. Becton was elected an Associate Member of the American Society of Civil Engineers on December 2, 1914, and a Member on June 6, 1921.

CHARLES ROBERT BETTES, M. Am. Soc. C. E.[1]

DIED OCTOBER 29, 1937

Charles Robert Bettes, the son of Edwin Wareham and Eliza (Kinnear) Bettes, was born in Youngsville, Pa., on April 9, 1863. He received his early education in Titusville, Pa., later attending the Columbia Preparatory School and the School of Mines of Columbia University, in New York, N. Y.

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[1] Memoir prepared by Joseph Goodman, M. Am. Soc. C. E.

Mr. Bettes began his engineering career in 1881 in the New York Office of the construction firm of Charles B. Brush, Inc., where he was employed, until the end of 1893, as Assistant in Charge of Surveys and Construction, chief among which were the water supply structures, including dams, pump-houses, and stand-pipe at Union Corners, N. Y., for the New Rochelle (N. Y.) Water Company; Weehawken Reservoir No. 2, of 30 000 000 gal capacity; and the enlargement of the Queens County Water Company's plant at Far Rockaway, N. Y.

In January, 1894, he accepted the position of Resident Engineer with the Queens County Water Company, designing and constructing its high-service and auxiliary pumping stations, slow-sand iron removal filters, and extensions of its distribution system. In 1897, he was appointed Chief Engineer and, shortly thereafter, General Manager of this Company (which in 1925 became the Long Island Water Corporation), and continued in that capacity until his retirement on June 1, 1931. During most of this time he was also a member of the firm of Tribus and Massa, in New York City, specializing in water supply, sewerage, and sewage disposal, continuing his association with this firm until its dissolution upon the death of Louis Lincoln Tribus, M. Am. Soc. C. E., in 1930.

The Water Company served not only the Rockaways, constituting the Fifth Ward of Queens in New York City, but also about fifteen or more villages in the southwestern part of Nassau County adjoining the Queens County line, and its success was due in a large measure to the combination of technical ability, foresight, and scrupulous integrity possessed by its Chief Engineer and General Manager.

The co-operation of Mr. Bettes with the New York City representative in 1915 was in great part instrumental in the subsequent agreement reached—as regards the Water Company's plant lying within the city's limits—on the rates to be charged to consumers and the compensation to be made the Company for the fire protection furnished. It was one of the early and one of the few instances in the eastern section of the United States where compensation was based upon a fair return of the estimated capital invested in the service, together with a proper allowance for depreciation, taxes, maintenance, and operation, and was cited as a precedent in many subsequent proceedings of a like nature. The aforementioned agreement remained in force until 1931 when the City acquired that part of the Company's plant lying within the city limits.

During his long association with the Water Company, Mr. Bettes saw the areas that he served, which contained many extensive estates, subdivided and developed into the populous urban and suburban communities of the better type with which they have been replaced, and in these developments he took an active part. Despite the demands made upon him by the need of extending and improving the water service to meet the rapid growth of these communities, he found time to be a member of the Chamber of Commerce for the Borough of Queens, a Trustee of the Rockaway Savings Bank, a member of the Executive Committee of the Queens Branch of the Bank of Manhattan, and a Director of the Gibson Building Corporation, of Valley Stream, N. Y. He also took an active interest in the work of the American Water Works Association, having served as Trustee from 1924 to 1926, and as a member of its Executive Committee in 1929; he was also a member of the New England Water Works Association.

Mr. Bettes did not neglect the social and spiritual sides of life. He numbered among his friends many of the leading citizens of Nassau County; and few civic or philanthropic movements of merit were carried through without the benefit of his good judgment and advice. He was a Trustee of the Sage Memorial Presbyterian Church, of Far Rockaway, a Son of the American Revolution, and a member of many Masonic organizations. He was interested in the Boy Scout Movement from its inception; and, in 1937, for his service to the boyhood of America, the National Council, Boy Scouts of America, at Washington, D. C., bestowed upon him the Badge of the Silver Beaver, as a token of his work. Those qualities of tact and patience, courtesy, geniality, and kindliness, which are so greatly stressed in "scouting," were personal attributes which all who came in contact with Mr. Bettes will remember.

Surviving is his widow, the former Frances H. Murray, of Cooperstown, N. Y., to whom he was married in 1902.

Mr. Bettes was elected a Junior of the American Society of Civil Engineers on October 1, 1890; an Associate Member on June 7, 1893; and a Member on June 1, 1909.

JAMES BONNYMAN, M. Am. Soc. C. E.[1]

DIED APRIL 9, 1937

James Bonnyman was born in Lexington, Ky., on July 9, 1879. His parents, George and Sarah Ann (Toner) Bonnyman, had moved to Kentucky from Edinburgh, Scotland, a few years before his birth. He attended preparatory school and the University of Kentucky, at Lexington.

His first work was as a Chainman in Government engineering work near Chattanooga, Tenn. Soon thereafter he became connected, in a subordinate position under the Mining Engineer, with the Durham Coal and Coke Company, the only coal-mining property in Georgia, operating in Lookout Mountain near Chattanooga.

Two years later, Mr. Bonnyman became Assistant Engineer and Roadmaster for the Chattanooga, Rome and Southern Railroad Company (later, a part of the Central of Georgia Railroad Company.

In 1902, for the second time, he became connected with the Engineering Department of a coal mine, being employed as an Engineer and the first

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[1] Memoir prepared by Alexander Bonnyman, M. Am. Soc. C. E.

Superintendent of the Stearns Coal Company, then under initial development and to-day one of the largest and most important coal companies in Kentucky.

The offer of a position as Division Engineer in railroad work caused Mr. Bonnyman again to leave coal mining, and, from 1904 until 1906, he was Division Engineer in Georgia for railroad companies, principally on the Central of Georgia Railroad, Chattanooga Division, of which he had previously been Assistant Engineer and Roadmaster.

In 1906, at the age of twenty-seven years, James Bonnyman was employed to represent capitalists in New York, N. Y., and Boston, Mass., under the Presidency of Mr. H. M. Atkinson, of Atlanta, Ga., in supervising and consummating the delivery of a $6 000 000 purchase, in the Birmingham, Ala., District, of coal and iron property from the Tutwiller and Adler interests, including several going coal and iron mines and iron furnaces, with a twelve months' time limit to conclude the transaction.

This work was done by Mr. Bonnyman with the title of Treasurer of the Birmingham Coal and Iron Company. It was finished, and full delivery was made in the twelve months' contract period. He was then continued as Treasurer and Acting Manager of the Birmingham Coal and Iron Company.

He was soon made Vice-President and General Manager and continued as Treasurer until the panic of 1907 caught this new Company with several million dollars of unfinished development. As no money was available for continuing the development, and the promoters of this big enterprise could not continue its financing, the Company was forced into receivership, Mr. Bonnyman being first made Receiver and then Trustee in bankruptcy, representing the bondholders and the New York and Boston capitalists, a bank president in Birmingham being Co-Receiver representing local creditors.

In a comparatively short time Mr. Bonnyman and his associates were able to pay all the obligations in full and he, again, became Vice-President and Treasurer, the President under the re-organization being the head of a New York Trust Company.

He continued as the operating head of the Birmingham Coal and Iron Company until about 1912, when, under direction of his President and Directors, he concluded negotiations by which the Company was purchased on a very favorable basis by one of the larger coal and iron corporations, bringing its owners about $12 000 000.

After this achievement, Mr. Bonnyman, in 1912, at the age of thirty-three years, began promoting his own coal and iron mining companies. His first development was the Clinton Mining Company, operating in a 16-ft vein of red hematite ore in Red Mountain, above the City of Birmingham. Later, he organized the Brookside-Pratt Mining Company which developed several coal mines in the Birmingham District, being one of the most important commercial coal-mining companies in the district.

In 1916, Mr. Bonnyman sold the Clinton Mining Company of which he was the promoter and President to the Gulf States Steel Corporation. In 1922 he resigned as President and sold his interest in the Brookside-Pratt Mining Company, moving to Cincinnati, Ohio, to join interests with his older brother, the writer, in the operation and development of several coal companies in Virginia, Tennessee, and Kentucky. All these companies were consolidated, in 1926, under the name of the Blue Diamond Coal Company, and Mr. Bonnyman was made President of the Consolidated Company, which has more than \$6 000 000 invested and a capacity of 3 500 000 tons per yr.

Although James Bonnyman began his career as an Engineer and had connection with or supervision over engineering work all the forty years of his business life, his principal experience was as a financiar and an executive.

He was considered by all who knew him to be a man of great courage, force, and ability, combined with a gentle and kindly personality. Making true, loyal, and sincere friends, particularly with people working under him, was an outstanding characteristic. In religion, he was a Roman Catholic.

Mr. Bonnyman was married at Bainbridge, Ga., on December 14, 1905, to Allie Golden Cliette. He is survived by Mrs. Bonnyman and his children, James Bonnyman, Jr., and Mrs. Theodore Irving (Mary Golden Bonnyman) Reese.

He was a member of the Cincinnati Country Club and the Queen City Club, of Cincinnati.

Mr. Bonnyman was elected a Member of the American Society of Civil Engineers on April 2, 1912.

CHARLES BROSSMAN, M. Am. Soc. C. E.[1]

DIED JUNE 30, 1937

Charles Brossman, the son of Charles G. E. and Emily (Kohler) Brossman, was born in Philadelphia, Pa., on January 17, 1877. After his graduation from the Philadelphia Manual Training School, he entered the employ of Willyoung and Company, of Philadelphia, makers of scientific instruments and apparatus. His next work, with the Baldwin Locomotive Works, in Philadelphia, was interrupted by service in the Spanish-American War with the Puerto Rican Expedition as a member of the First Troop, Philadelphia City Cavalry.

In 1900, Mr. Brossman moved to Indianapolis, Ind., where he was engaged in the design of filtration plant and pumping equipment for the Indianapolis Water Company. At the same time, through association with the Dean and Shibley Company, he had a responsible part in the construction of the filtration plant in Washington, D. C.

Mr. Brossman, in 1904, entered the private practice of engineering and opened a Consulting Office in Indianapolis, with Mr. R. P. King, under the firm name of Brossman and King. This partnership was dissolved in 1907, and Mr. Brossman continued the consulting practice in his own name. His engineering work centered in the Middle West, notably in Michigan, Illinois, Iowa, Ohio, Kentucky, and Indiana.

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[1] Memoir prepared by H. S. Morse, M. Am. Soc. C. E.

As with most young practicing engineers, his earlier work had considerable variety, including design and supervision of construction of factory buildings, asphalt plants, street improvements, and heating and power plants. Gradually, however, as his reputation grew, he specialized in water supply and sewage disposal for cities, towns, and institutions. His career was stopped at its peak by his sudden and untimely death on June 30, 1937.

As is usual with successful engineers, Mr. Brossman's activities were extended in generous service to his profession. In addition to his membership in numerous technical and business organizations, he was Secretary of the Indiana Engineering Society for twenty years, and, later, was President of that organization. He served as President of the Indiana Section, American Society of Mechanical Engineers; Treasurer of the Indianapolis-Lafayette Section, American Institute of Electrical Engineers; President of the Scientech Club of Indianapolis; and President of the Indiana Sanitary and Water Supply Association (later, the Indiana Section of the American Water Works Association). At the time of his death, he was President of the Indiana Section, American Society of Civil Engineers.

Not satisfied with his contributions to the up-building of his profession, Mr. Brossman was the author of many articles in the public and technical press, in the publications of the organizations of which he was a member, and of numerous papers presented at engineering meetings. The Ohio Valley floods in January, 1937, added to his burden of engineering engagements, but he still found time to prepare articles for publication on flood-control measures in Indiana.

His interests were by no means confined to his professional work, but were carried on into the life of the community in which he made his home. At the time of his death, Mr. Brossman was Chairman of the Boys Work Committee of the Indianapolis Rotary, an incident in his busy life, but in itself indicative of his widespread interest in the people about him.

Socially, he was a genial companion, possessed of a rare sense of humor and full of sparkling fun. A man of unimpeachable integrity, his code of professional ethics was exceptionally high. His varied interests and his willingness to serve made him an unusually active and useful citizen, as well as an honor to his profession.

In the fall of 1903, he was married to Mary Edith Dean, of Indianapolis. His widow and three children, Charles Dean Brossman, Mrs. Francis (Brossman) Meyer, and Mrs. Frank (Brossman) Parish, survive him.

Mr. Brossman was elected a Member of the American Society of Civil Engineers on June 18, 1918.

MELVIN DAVID CASLER, M. Am. Soc. C. E.[1]

DIED JANUARY 27, 1937

Melvin David Casler was born at Bordentown, N. J., on September 23, 1881, the son of Melvin Lipe and Carrie (Fiske) Casler, of German ancestry.

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[1] Memoir prepared by Arthur V. Ruggles, M. Am. Soc. C. E.

Following his early education in the public schools of Fort Plain, N. Y., he entered Union College, at Schenectady, N. Y., in 1902, and was graduated in 1906 with the degree of Bachelor of Science in Civil Engineering.

Before entering college, Mr. Casler was employed from June, 1899, to January, 1900, as Machinist and Draftsman by the Marvin and Casler Company, of Canastota, N. Y., the firm of which his uncle, Herman Casler, was a member. From January, 1900, to May, 1902, he was engaged in office and factory management for S. E. Trumbull, of Johnstown, N. Y., and from May to September, 1902, was Engineering Draftsman for H. C. Allen, of Syracuse, N. Y.

Throughout his college course, Mr. Casler continued outside employment, during the college terms as well as the summer vacations. From September, 1902, to June, 1903, he assisted his father, Melvin L. Casler, of Fort Plain, N. Y., on surveying and drafting, and from June to September, 1903, he was in charge of surveys and maps, and drainage investigations for Eugene R. Smith, of Islip, N. Y. From September, 1903, to June, 1904, he was engaged on road and pavement drawings for C. W. Trumbull, of Schenectady, and from June to September, 1904, he was Chief of Party on drainage surveys and investigation of flood control for the New York State Court of Claims. Throughout his third year at Union College (September, 1904, to May, 1905) Mr. Casler was employed by that College as Engineering Instructor and the following summer (May to October, 1905), he was in charge of construction work costing $670 000 on Contract 3 of the New York State Barge Canal, at Fort Miller, N. Y. During his Senior College year, he was employed by the late Olin H. Landreth, M. Am. Soc. C. E., of Schenectady, on sewer work and maps, and also served as Instructor in Mathematics and Physics at the Schenectady High School.

After his graduation from Union College, he returned to the New York State Barge Canal, where, from July to November, 1906, he was in charge of surveys of alternative routes and studies of drainage problems and economical canal location, and from November, 1906, to May, 1907, on the design of locks and dams, and on problems of rainfall, run-off, and storage.

In May, 1907, Mr. Casler began a period of service which lasted until September, 1916, with the Board of Water Supply of the City of New York. Until March, 1910, he was Assistant Engineer Designer on structures for the Ashokan Reservoir, Catskill Aqueduct, and Hill View Reservoir. From March, 1910, to July, 1912, he was Assistant Section Engineer, and from July, 1912, to September, 1916, Section Engineer, in charge of construction, in Yonkers, N. Y., of Section 2, Hill View Division, which included Hill View Reservoir and appurtenant structures, costing $3 400 000.

He was employed from September, 1916, to June, 1918, as Designing Engineer by the Bronx River Parkway Commission, at Bronxville, N. Y., and from June to October, 1918, he was engaged on the installation of the H. L. Gantt Industrial Management Methods and Cost Accounting Systems, in the United States Naval Aircraft Factory, at Philadelphia, Pa., and other manufacturing plants.

From October, 1918, to November, 1919, he was with the National Conduit and Cable Company, of Hastings, N. Y., as Management Engineer, and from November, 1919, to August, 1921, as Production Manager for the Watson Truck Company, of Canastota, N. Y.

Mr. Casler was again in the employ of the City of New York from August, 1921, to June, 1924, serving as Assistant Engineer with the Board of Estimate and Apportionment in charge of a force of designers and draftsmen on the preparation of preliminary designs and estimates and contract drawings for the proposed freight and passenger subaqueous tunnel between Staten Island and Brooklyn, N. Y. Mr. Casler was in direct charge of the design of a considerable part of this tunnel and personally developed the theoretical stress analysis for the shield tunnel section.

From June, 1924, to September, 1929, he was Designing Engineer for the Hudson River Regulating Board, at Albany, N. Y. As such, he had charge of the preparation of plans and specifications for the Conklingville Dam and appurtenant structures. This dam was built to create the Sacandaga Reservoir on the river bearing that name, and serves both to regulate the flow of the Hudson River, to which the Sacandaga is tributary, and to develop power at the Conklingville Dam.

He was Senior Assistant Engineer, from September, 1929, to April, 1930, with the North Jersey District Water Supply Commission, at Newark, N. J., on studies, estimates, and a report on projects for additional water supply for the North Jersey District.

From April, 1930, until his death, Mr. Casler was Assistant Engineer with the Port of New York Authority on the design of the Lincoln Tunnel—the shield-driven vehicular tunnel now (1937) under construction under the Hudson River on line with West 39th Street, in New York City.

Mr. Casler was proficient in both the design and construction of engineering works and as engineer in charge carried through successfully important work of both kinds. He had unusual mathematical ability and most of the articles which he contributed to engineering publications were of a mathematical nature. A book which he published in 1915 was entitled "Simplified Reinforced Concrete Mathematics" His work was invariably noted for painstaking thoroughness, and his associates knew that any task which he undertook would be carried through faultlessly.

He was quiet and unassuming, at the same time genial and sincere. He was a man of the highest character and a true friend to many who knew him. Besides his professional activities, his interests were mainly in his family, his home in Mount Vernon, N. Y., and his summer camp near the Upper Hudson River. He was a member of the Madison Avenue Presbyterian Church, of Albany, N. Y., of the Masonic Lodge of Canastota, N. Y., and of Sigma Xi Fraternity.

In 1907, the year following his graduation from Union College, Mr. Casler was married to Edna Lucy Greene, of Fort Plain, N. Y. Mrs. Casler and an uncle, Herman Casler, of Canastota, N. Y., survive him.

Mr. Casler was elected an Associate Member of the American Society of Civil Engineers on September 5, 1911, and a Member on April 19, 1916.

GEORGE CHAMPE, M. Am. Soc. C. E.[1]

DIED AUGUST 24, 1937

George Champe was born at Dublin, Ind., on July 29, 1868, the eighth child of Dr. George Washington and Elizabeth (Nation) Champe. His father was descended from John Champe, a Revolutionary War veteran, whose daughter married George Washington's brother. His childhood was spent in the Village of Dublin, where he received his preliminary education at the Grade School, and his preparatory education at the Dublin High School. In 1888, he was graduated from Dublin High School with scholarship honors in mathematics. In 1889, at the age of 21, he was matriculated at Indiana State University, Bloomington, Ind., and was graduated from that institution in June, 1893, with the degree of Bachelor of Arts in Mathematics. As a student, he assisted in the planning of the campus of this young university. His college course was interrupted in 1890 for six months when he was engaged as a Computer for the Missouri River Commission.

Immediately upon Mr. Champe's graduation from Indiana State University, he became City Civil Engineer of Bloomington, and continued in that position until the spring of 1895, when he resigned to accept the position of Assistant City Civil Engineer of Indianapolis, Ind. He then took a term course in engineering subjects at Rose Polytechnic Institute, at Terre Haute, Ind., and also at Indiana University, returning in 1896 to Bloomington as City Civil Engineer and remaining in that capacity until June, 1900.

During his early years of engineering practice, George Champe often expressed a desire to locate in Toledo, Ohio. In the early part of 1900, he had occasion to make a business trip to Toledo, and he was so greatly impressed with the geographical facilities and transportation opportunities of the city that he resolved to accept a position with Carlin Brothers, builders of railroads, and locate there. Accordingly, in June, 1900, he and Mrs. Champe moved to Toledo, where he established a private consulting office. This was during the time of rapid expansion of the electric interurban railways, and his principal energies were directed into that field. He was associated with the design of many of the sixteen electric railways which radiated from the City of Toledo like the spokes of a wheel from the hub; in particular, he was largely responsible for the design of the Toledo and Western Railroad, the Toledo and Indiana Railway, and the Toledo, Fostoria and Findlay Railway. In the course of this railway work he made many friends in the cities and villages which were tapped by these carriers, and these friendships later were especially valuable because of the era of hard-surface pavement construction that was ushered in following the advance of the railways. More than 275 pavements in approximately 50 communities in Ohio, Michigan, and Indiana, to the value of more than \$3 000 000, were designed and supervised by him.

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[1] Memoir prepared by Carleton S. Finkbeiner, Assoc. M. Am. Soc. C. E.

In this period also, there was a rapid increase in the demand for water-works and sewerage facilities in the outlying municipalities, and George Champe was responsible for the design of more than twenty-five original water-works systems and numerous other water-works improvements, representing expenditures in excess of \$2 000 000. Sewerage systems, sewage-treatment plants, and some power plants, including hydro-electric ones, were built under his design, representing investments of more than \$1 000 000. Bridges also came under his planning, and he was Associate Engineer on the \$3 000 000 High-Level Bridge at Toledo.

During the last ten years of his life, George Champe became more and more identified with civic movements in his home city, becoming Chairman of the Lucas County Planning Commission, Vice-Chairman of the City Plan Commission, and member of the Port Commission and the Mosquito Elimination Committee. In 1931, he volunteered his service in the design of the airport runways at the Toledo Transcontinental Airport.

He was a member and Elder of the Collingwood Avenue Presbyterian Church, member of the Masonic Order, Rotary Club of Toledo, American Water Works Association, Toledo, and the Ohio Society of Professional Engineers. From 1933 to his death, he was a part-time Instructor at the University of Toledo in Sanitary Engineering.

The character of George Champe was firmly molded along the lines of absolute integrity and devotion to his profession. All phases of engineering struck a responsive chord in him because his interests were manifold. His many transactions with political sub-divisions had developed a very fine flair for diplomacy.

On November 17, 1897, Mr. Champe was united in marriage to Harriett May Noble, of Indianapolis. Surviving him, in addition to Mrs. Champe, are two daughters, Mrs. Dorothy C. Longnecker and Elizabeth Champe; and two sons, Willard and Carlton.

Mr. Champe was elected a Member of the American Society of Civil Engineers on December 4, 1922.

ARNOLD GOODWIN CHAPMAN, M. Am. Soc. C. E.[1]

DIED JUNE 3, 1937

Arnold Goodwin Chapman was born in Guilderland, N. Y., on July 23, 1884. He was the son of George Ashley and Ida (Goodman) Chapman. His preliminary education was obtained in the public schools in Albany, N. Y., on completion of which he entered Union College, at Schenectady, N. Y., from which he was graduated in 1906 with the degree of Bachelor of Engineering.

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[1] Memoir prepared by Dwight B. La Du, M. Am. Soc. C. E.

The greater part of Mr. Chapman's professional life was spent in public service. Immediately following his graduation from Union College he entered the Department of the State Engineer and Surveyor of the State of New York as a Chainman, where he served continuously until 1915. During this period he was promoted through the grades of Assistant Engineer, Assistant Deputy State Engineer, and Deputy State Engineer, which position he held from 1912 to 1915 under the late John A. Bensel, Past-President, Am. Soc. C. E. The scope of Mr. Chapman's engineering experience during these nine years was broad, including highway construction, and the many and varied problems which presented themselves in connection with the design and construction of the Barge Canal System, the greater part of which was built during this period.

Mr. Chapman retired as Deputy State Engineer in 1915, and entered private practice as a Consulting Engineer which he continued through 1922. While thus engaged, he was connected with many important engineering works involving the design and construction of flood-control projects, municipal construction projects, and various other engineering activities.

In 1923, he returned to the State service as Deputy State Engineer under Dwight B. La Du, M. Am. Soc. C. E., and occupied this position until the end of 1924. During 1925 and 1926 he was retained by the Department of Public Works of the State of New York, as Supervising Engineer in charge of the Hornell and Canisteo Flood Abatement Projects, and during 1927 as a general Engineering Consultant.

In 1928, Mr. Chapman was appointed Chief Engineer of the Albany Port District Commission and served as such until 1936. He prepared the comprehensive plan which was followed in the building of the Port and was in charge of the design and construction of the Port Terminal facilities, involving the dredging of nearly 4 000 000 cu yd of material, the reclaiming of 300 acres of land, the construction of dock walls extending more than 5 000 ft, the largest single unit grain elevator in the world, storage sheds, streets, sewers, railroad facilities, and, in fact, everything that goes to make up an inland port equipped to handle ocean-going traffic.

While his work in connection with the Barge Canal and the many other projects on which he was at one time engaged was noteworthy, the contribution he made as an Engineer to the Port of Albany was his outstanding engineering achievement, and his name will be inseparably linked with this development for all time.

In 1936, he resumed private practice as a Consulting Engineer in Albany, and was carrying on such practice at the time of his death. He was also serving as Manager of the New York State Waterways Association and as Executive Secretary of the New York State Society of Professional Engineers.

The professional societies with which he was affiliated, included the Albany Society of Civil Engineers, New York State Society of Professional Engineers (of which he was a Past-President), and the National Society of Professional Engineers. His college fraternities were Psi Upsilon and Sigma Xi. His clubs in Albany were Fort Orange, University (of which he was President at the time of his death), and the Albany Country Club. Mr.

Chapman was a Mason, Knight Templar, and Shriner, and a member of the First Presbyterian Church, of Albany.

On June 12, 1912, he was married to Marjorie A. Nead, who, with three children, Arnold G. Chapman, Jr., Marjorie Chapman, and Carol Chapman, survives him.

Due to his many years of outstanding public service, both to his State and City, Mr. Chapman had established a reputation as an Engineer which extended beyond the bounds of his own City and State. During the later years of his life, he gave unselfishly of his time and energy to the advancement of the standing of the engineer through his advocacy of licensing the practice of engineering and the fixing of minimum requirements which must be met before one may call himself an engineer.

The death of Arnold Goodwin Chapman marked the passing not only of a capable engineer, but also of a man who was beloved by all whose good fortune it had been to know him.

Mr. Chapman was elected a Junior of the American Society of Civil Engineers on July 1, 1909; an Associate Member on April 30, 1912; and a Member on January 20, 1920.

FRANK DAVID CHASE, M. Am. Soc. C. E.[1]

DIED JULY 23, 1937

Frank David Chase was born in Riverside, Ill., on August 2, 1878, the son of David Fletcher Chase and Emily Frances (Tabor) Chase. He was a descendant of old American families, and in the paternal line traced his ancestry from Thomas Chase, who came to America about 1638.

Frank David Chase received his early education at the Evanston (Ill.) Township High School, and was graduated in 1901 from the Massachusetts Institute of Technology, in Boston, Mass., with the degree of Bachelor of Science in Civil Engineering. After his graduation he spent four years in the Engineering Departments of the Minneapolis and St. Louis Railroad Company and the Chicago and Alton Railroad Company.

From 1905 to 1907, he was employed by the Leonard Construction Company as Superintendent on building work, and during the following six years he held the position of Architect, first for the Illinois Central Railroad Company and then for the Western Electric Company, with headquarters in Chicago Ill. In 1913, he organized his company of Frank D. Chase, Incorporated Engineers and Architects.

Soon after beginning his active career, he became impressed with the possibilities of industrial engineering, the design of manufacturing plants, and other large business and industrial structures, with a view to the greatest efficiency in production and the most economical utilization of space, and decided to become a Specialist in this field.

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[1] Memoir prepared by Harry H. Hadsall, M. Am. Soc. C. E.

As President of the Company, he directed its management, and under his personal leadership its business developed very rapidly. His services were retained by railroad companies, large industrial corporations, and newspapers throughout the United States, and the ideal utility of his designs, approved by the test of practical operation and experience, won him a notable reputation.

Mr. Chase designed, among others, the plants of the *St. Louis Star-Times,* in St. Louis, Mo., *The Oklahoman,* in Oklahoma City, Okla., and the *Milwaukee Journal,* in Milwaukee, Wis., the latter of which has been called one of the finest newspaper plants in America.

As Architect and Engineer of his Company he designed and constructed more foundries than any other consultant in the United States. He was also retained on plant construction in Europe, and executed a large industrial contract in Soviet Russia. He did not confine his efforts solely to industrial projects, however, and designed a number of mercantile buildings in the Loop District, in Chicago, including the 23-story building, at 100 West Monroe Street, the 16-story building, at 173 West Madison Street, and the 10-story Starck Building. He also designed the Chicago Memorial Hospital and the South Chicago Community Hospital.

Mr. Chase, who was active until his death, was an acknowledged authority in his chosen field. His professional standing was reflected in his election as President of the Western Society of Engineers, serving from June 1, 1931, to June 1, 1932. In 1933 and 1934 he was Executive Director and Chief Engineer of the Civil Works Administration for the State of Illinois, and Chief Engineer of the Illinois Relief Commission. In 1936, he served as Chairman of the Washington Award Commission, and presided at the presentation of the Award for that year to Charles Kettering, M. Am. Soc. C. E., the ceremonies being broadcast by radio.

Mr. Chase was formerly a member of the Engineers Club, of New York, N. Y., the Engineers Club, of Chicago, the Barrington Hills Country Club, and the Union League and the University Club, of Chicago. He enjoyed outdoor life and travel, and spent much of his spare time in the West, particularly in Arizona and New Mexico, where he frequently went on camping trips to study the relics of earlier Indian culture.

He was married, in 1927, to Cecile Chambers, of Chicago, who, with one daughter, Eloise, survives him. His brother, Al Chase, is the Real Estate Editor of *The Chicago Tribune.*

Mr. Chase was elected an Associate Member of the American Society of Civil Engineers on June 30, 1910, and a Member on April 2, 1912.

JOHN CARROLL CHASE, M. Am. Soc. C. E.[1]

DIED APRIL 15, 1936

John Carroll Chase was born on July 26, 1849, at Chester, a town in the southern part of New Hampshire, not far from Manchester, the only child of

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[1] Memoir prepared by C. Frank Allen, M. Am. Soc. C. E.

Charles and Caroline (Chase) Chase. He was of the eighth generation in descent on his father's side from Aquila Chase who settled in 1640 at Hampton, at that time a part of the Massachusetts Bay Colony. His mother also was descended in the eighth generation from the same Aquila Chase. A Chase family in Chesham, England, has in its records the name, Aquila, which caused some claim that the American family came from that town, but John Carroll Chase, himself an eminent authority in such matters, following researches in England, states that it "may be descended from the Chesham family, but our researches fail to show such a connection." The ancestors of John Carroll Chase soon moved from Hampton to Newbury, Mass., and, later, to Chester, where he was born.

Brought up on a farm, he attended the local district school and, later, Chester Academy. Afterward, he attended Pinkerton Academy, in Derry, N. H., from which he was graduated in 1869. After a year of teaching, he entered the Massachusetts Institute of Technology, at Boston, Mass., as a member of the Class of 1874, and took up the study of Civil Engineering. His grandfather, Benjamin Chase, had acquired a mastery of mathematics, geometry, trigonometry, surveying, navigation, and astronomy, his proficiency in the latter allowing him to predict eclipses well in advance. He also prepared a map of the township of "Old Chester." He is said to have possessed unusual mechanical skill and was much in demand as a millwright throughout neighboring towns in New Hampshire and Massachusetts. In 1871, after spending a year at the Massachusetts Institute of Technology, young Chase was married, and his studies at the Institute of Technology were discontinued, but his purpose to become a Civil Engineer was not quenched; his inheritance perhaps asserted itself.

His first employment in Civil Engineering was in the office of Joseph B. Sawyer, in Manchester. More importantly, for about four years, he served as an Assistant Engineer on the construction of the water-works for Manchester. Following this, he was Assistant Engineer on the construction of the conduit of the Sudbury River System of the Boston (Mass.) Water Supply, and, afterward, as Assistant Engineer on the elevated railway of New York, N. Y.

In 1880, failing to secure work in engineering, Mr. Chase filled a Civil Service position under the Federal Government and became Assistant Cashier in the Naval Office. Late in 1881, he resigned to accept the position of Superintendent and Engineer of the Clarendon Water-Works, at Wilmington, N. C., of which later he became Chief Engineer. During his term of service in Wilmington, he occupied as well the position of City Surveyor and also became a member of the Wilmington Board of Health. Mr. Chase was also, for four years, the Engineer Member of the State Board of Health and rendered valuable service in this capacity; this was in the early days of sanitation in North Carolina. His activities, however, were not confined to Wilmington, for during his residence there, he was engaged in general practice in North Carolina, South Carolina, and Georgia. During this time he designed and constructed the water-works and sewerage systems for Albany, Ga.

Although a Northern man in a Southern community during the reconstruction period, he promptly made many friends in Albany, and the fact that he was continuously acceptable was alike creditable to those with whom he became associated, as well as to Mr. Chase himself.

After more than twenty-four years in engineering activities, Mr. Chase resigned in 1897 to return to Derry to enter the employ of his uncle, Benjamin Chase, Jr., in a business which, in 1907, was incorporated as the Benjamin Chase Company, of which he was General Manager and Treasurer. Subsequently, he became President, an office which he held for more than twenty years prior to his death. The business is that of manufacturing nationally known specialties in wood, among them tree and pot labels used by nursery men and florists. Probably two-thirds of all such labels that have been used in the last thirty years in the United States and Canada, have been made by this Company. Its administration was successful and was continued by Mr. Chase as his main or sole business interest to the time of his death. However, the attention he found it necessary to give to it did not prevent him from giving much time to many interests of a varied sort.

While still in the South, before his return to Derry, Mr. Chase was elected a Trustee of Pinkerton Academy, being the first alumnus to be so chosen. Soon after his return to Derry, he became Secretary of the Board and Chairman of the Executive Committee of the Academy, and was recognized as its most influential member. He was also Secretary of the Alumni Association. He devoted considerable time to a study of its records and compiled many interesting data with relation to it. Mr. Chase was an active helper of the Congregational Church, at Derry, and the "mainspring" of the Men's Club. He was a Trustee and Treasurer of the Taylor Library of that city, and also President of the Nutfield Savings Bank. He was regarded as one of the leading citizens of Derry; undoubtedly, he was the most widely known.

Although the practice of his profession was laid aside, his interest in engineering persisted. Aside from continuance in the Society, he enrolled as a member of the Northeastern Section, and also maintained membership in the New England Water Works Association (of which he had become the senior member), and in the Boston Society of Civil Engineers. He was well-nigh a constant attendant at the dinners and meetings of these societies. He also attended many of the Annual Meetings and Conventions of the American Society of Civil Engineers. Equally, he attended the meetings of the Alumni Association of the Massachusetts Institute of Technology and of its Alumni Council. It is of interest to note that although he attended Technology only one year, he was the representative, on the Council, of his Class of 1874. He seemed to possess something of the "gang" instinct, a get-together habit, as appears more fully later, so that his attendance at these and other meetings could perhaps be regarded in his later years in the light of recreation. Due to his interest in engineering, he was an early member, as well as a Fellow, of the American Public Health Association.

In fraternal organizations, Mr. Chase was active in Free Masonry in which his memberships extended from Master Mason to Knight Templar, and he was elected to the highest offices in his Chapter, his Council, and his Commandery. He was Past Deputy Grand Commander and member of the Grand Commandery of Knights Templars of North Carolina and, frequently, went South to attend its Annual Conclave. He was a member and Past Patron of the Order of the Eastern Star. He held the highest office in the Supreme Council of the Royal Arcanum. He was a member, too, of the Independent Order of Odd Fellows for more than fifty years.

His clubs included the University, City, and Appalachian Clubs, of Boston, the Technology Faculty Club, of Cambridge, Mass., the Technology Club, of New York, and the University Club, of San Diego, Calif.

His interests in another direction led him to become a member of a number of historical and hereditary patriotic organizations, among them the Society of Colonial Wars in the Commonwealth of Massachusetts (Deputy Governor General), the Society of Colonial Wars in the State of New Hampshire, the Massachusetts Society of Sons of the American Revolution (Member of the Governing Board), the Bunker Hill Monument Association (Director), the Governors and Company of the Massachusetts Bay in New England, the New Hampshire Historical Society, the Piscataqua Pioneers (Vice-President), the Bostonian Society, and the Society for the Preservation of New England Antiquities.

Membership in these societies was quite in line with what appears to have been his main interest for many years, membership in the New England Historic Genealogical Society. He became a member in 1899, soon after his return from the South and later was made a life member. In 1921, he became its President, an office which he filled until his death. In this Society he also was a member of a newly established class designated as Pilgrim Tercentenary Member, receiving the certificate stamped No. 1. Three years after he became the President of this Society, on the wall of the stair hall of the New England Historic Genealogical Society's house, in Boston, Mr. Chase placed a tablet, surmounted by the representation of a ship of early Colonial days under full sail, and bearing this inscription: "Aquila Chase Mariner 1618 1670 Hampton N H 1640 Newbury Mass 1646 By old repute the first Pilot at the mouth of the Merrimack River Erected by his descendant John Carroll Chase 1924." To his genealogical work he brought the accurate and honest mind of a trained engineer. The last public function which he was able to attend was in January, 1936, when he presided as Deputy Governor of the Society of Colonial Wars of Massachusetts at a meeting of that Society.

Whether or not there was an element of cause and effect in it, it happened that coincident with his joining the New England Historic Genealogical Society, Mr. Chase, as one of the Founders, was instrumental in the formation of the Chase-Chace Family Association of which he became the first President. Later, in 1928, he published, with the co-operation of George Walter Chamberlain, a volume of 600 pages entitled "Seven Generations of Aquila and Thomas Chase," a book presenting a broad survey of the Chase family in America and his family history. In 1926, he published a book of about 550 pages entitled "History of Chester, New Hampshire, Including Auburn, a Supplement to the History of Old Chester," the latter having been written by his grandfather. He was also author of "The Descendants of William Chase." He was a frequent contributor to newspapers and society journals, of articles of both civic and professional quality. As late as March, 1935, in his capacity as President of the New England Historic Genealogical Society, he made an address on "Ninety Years of the Corporate Existence of the Society," which was duly published in the "Register" of the Society.

Mr. Chase was kindly by nature, helpful to others in many ways, and generous beyond his means, some thought, as he was not a wealthy man. Although unassuming in habit, unselfish, and not contentious, that Mr. Chase possessed high qualities of leadership is abundantly evident from the prominence accorded him in various societies and organizations of which he was a member. At meetings of such bodies, his presence was familiar and welcome. He readily made friends, young as well as old, and his friendships were many. His trips to the Pacific Coast became an annual circumstance rather than an event, and it is interesting to note that he maintained membership in the University Club of far-off San Diego. These trips were often in the interests of his Benjamin Chase Company at some convention of the nursery men of that section, when he was greeted as "Uncle John," or at times at meetings of Technology Alumni who were glad to secure fresh news from their Alma Mater.

His frequent transcontinental trips by varying lines of travel were often interrupted en route, giving him opportunity to know more of the United States and also to make contact with genealogical societies and individuals interested in genealogy in other States. It was not uncommon for him to go long distances out of his direct route in order to call on some friend whose acquaintance he had made in earlier travels.

As stated in a memoir [2] published by another Society,

> "The cosmopolitanism of this man greatly impressed me at the time his associates gave a dinner in honor of the eightieth anniversary of his birth. The response which it evoked in Wilmington, N. C., in San Diego, Calif., and in Vancouver, B. C., would have led one to believe that he was a resident of each of these communities, quite as much as of Derry, N. H., where his business interests lay, or of the adjoining town of Chester, where he was born, or of Brookline, where he spent his last eight winters. He had entered into the life of all these communities."

The same memoir, referring to his journey to Europe in 1932 in his eighty-fourth year, recites that,[3]

> "* * * on such journeys he visited the great engineering projects which might be in the making, observing them with a keen professional insight and interest. In this way be kept fully abreast of the progress of civil engineering, for he always remained an engineer at heart, even though engrossed in the problems of business or the preparation of his books and papers dealing with historical research."

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[2] *The New England Historical and Genealogical Register*, July, 1936, p. 221, Line 46, to p. 232, Line 4.

[3] *Loc. cit.*, p. 22, Line 34 to Line 39.

Quoting from another memoir,[4]

"New England to the core, by descent and temperament, Mr. Chase lived an abundant life, active, keen of mind, friendly, of broad cosmopolitan interests and of sterling character. His life may well have been guided by the Chase motto, 'Ne Cede Malis.'"

Mr. Chase was married in 1871 to Mary Lizzie Durgin, of West Newbury, Mass., who died in 1927, following a protracted illness. In 1928, he was married to Florence Anne Buchanan, of Brookline, Mass., who survives him. He is also survived by two daughters, Carolina Louise, married to Dr. Raffael Lorini, formerly of Coronado, Calif., and for many years a prominent physician there, but now retired; and Alice Durgin, married to Professor Samuel Cate Prescott, Dean of Science at the Massachusetts Institute of Technology.

Mr. Chase was elected a Member of the American Society of Civil Engineers on March 1, 1893.

MELVILLE FISK CLEMENTS, M. Am. Soc. C. E.[1]

DIED JUNE 8, 1937

Melville Fisk Clements was born at What Cheer, Iowa, on March 13, 1875, the son of Wilbur F. and Rebecca (Fair) Clements. After completing his early education in the public schools, he entered the University of Iowa, at Iowa City, Iowa, from which he was graduated, in 1899, with the degree of Bachelor of Science. Later, he did post-graduate work at George Washington University, at Washington, D. C. In 1904, he received the degree of Civil Engineer from the University of Iowa.

Previous to completing his course in the University of Iowa, Mr. Clements was employed for a short period as a Draftsman in the Topographer's Office, in the Post Office Department, at Washington, D. C. Following his graduation in 1899, he was employed as Assistant Engineer on Maintenance on the Burlington, Cedar Rapids and Northern Railway until 1902, when he joined the Engineering Department of the Chicago, Rock Island and Pacific Railway Company, where he served as Chief Draftsman in the Maintenance-of-Way Department, as a Designer in the Bridge Department, and as Assistant Division Engineer on Maintenance. In 1906, he joined the Clinton Bridge and Iron Company, at Clinton, Iowa, and was in charge of design and shop inspection.

In 1907, Mr. Clements began his connection with the Northern Pacific Railway Company, which continued until his death. His first work was in the Bridge Department as a Designer. Following this, he was Assistant Engineer on Construction for several years, handling many important projects, including heavy bridge construction, reconstruction of branch lines, tunnel and subway work, and a large grade-separation project, in Spokane,

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[4] *Journal*, New England Water Works Assoc., December, 1937, p. 481.

[1] Memoir prepared by M. W. Beach, Bridge Engr., N. P. Ry., St. Paul, Minn.

Wash. In 1916, he was made Bridge Engineer, with headquarters at St. Paul, Minn., which position he held at the time of his death.

For several years, Mr. Clements had been retained as Consulting Engineer on important deep foundation work. He invented the Sand Shell Island Method of sinking caissons, which consists essentially of the erection and sinking of a circular steel shell which is filled with sand to the water surface, and in which the ordinary practice of constructing and sinking the caisson is followed. This method was followed successfully under his direction on two important foundation projects, the Suisun Bay Bridge, on the Southern Pacific Railway, at Benecia, Calif., and the Huey P. Long Memorial Bridge, at New Orleans, La.

Throughout his life Mr. Clements was known and admired for his fairness, and for his friendly interest in the problems of others. He was endowed with a keen intellect, broad vision, and a pleasing personality. He was a man of sterling character and was liked by all who knew him.

He was a member of Sigma Xi, the St. Paul Engineers Club, the American Railway Engineering Association, the St. Paul Chapter of the American Interprofessional Institute, and the St. Paul Athletic Club. He was an Elder of the House of Hope Presbyterian Church, of St. Paul, Minn., in which he had a deep interest.

In 1900, he was married to Minnie Coover, who survives him; also surviving are his mother and two sisters.

Mr. Clements was elected a Member of the American Society of Civil Engineers on September 2, 1914.

CLARKE PELEG COLLINS, M. Am. Soc. C. E.[1]

Died September 19, 1937

Clarke Peleg Collins was born on December 14, 1868, at Narragansett, R. I., the son of Peleg Gardner and Mary Winslow (Hawkes) Collins. The family was a member of the Society of Friends. His forefathers, coming from England, settled in Lynn, Mass., in 1635.

Mr. Collins was graduated from Friends School (later Moses Brown School), at Providence, R. I., with the Class of 1887. Although he was able to attend only one year at Rensselaer Polytechnic Institute, at Troy, N. Y., he gained a thorough knowledge of his profession through constant study and practical experience. He completed the Civil Engineering Course and Mining Course of the International Correspondence School of Scranton, Pa. He also held a certificate of proficiency from the American School of Correspondence, of Chicago, Ill., in Hydraulic and Sanitary Engineering. He studied municipal and sanitary engineering under the late Samuel M. Gray, M. Am. Soc. C. E., Consulting Engineer, of Providence; hydraulic and sanitary engineering under E. J. McCaustland, M. Am. Soc. C. E., of Cornell University, at

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[1] Memoir prepared by Thomas S. Lang, M. Am. Soc. C. E.

Ithaca, N. Y.; mining engineering and geology under John Fulton, Consulting Mining Engineer and Geologist, of Johnstown, Pa.; and bridge engineering under Emil Goldstein, of the American Bridge Company, in Johnstown. In addition to technical courses studied under the direction of others, Mr. Collins averaged two hours a day in the study of technical subjects for a period of ten years after leaving Rensselaer Polytechnic Institute.

In 1888, Mr. Collins joined the Staff of Mr. Gray, then City Engineer of Providence, in the Park and Highway Department. He continued as an employee and student under this well known Engineer until July 28, 1890.

From July, 1890, to August, 1893, he was employed as Assistant Engineer in the Mining Engineering and Survey Department of the Cambria Iron Company, later the Bethlehem Steel Corporation, of Johnstown. He was placed in charge of outside surveys in connection with the mining properties of that Company, railroad maintenance, and extensions. He also assisted in the development of Westmont, an exclusive residential suburb of Johnstown, for the Westmont Land Improvement Company, a subsidiary of the Cambria Iron Company. This work included the grading and paving of streets, installation of sewers, water lines, and gas lines.

In 1893, depression came over the country and in August of that year, while on his honeymoon in Providence, Mr. Collins was notified that the Cambria Iron Company had closed. He immediately went to work in the City Engineer's Office, at Providence, in charge of the construction of a section of the improved sewerage system of that city.

Mr. Collins was recalled by the Cambria Iron Company in 1894, with which he remained until 1900. During the last two years with this Company he was engaged in the investigation of water supply for the Manufacturers Water Company and the Johnstown Water Company, both subsidiaries of the Cambria Iron Company. Following these investigations, he designed the No. 2 Dam on Mill Creek for the Johnstown Water Company. This dam with a capacity of 90 000 000 gal was built according to his plans. He also had charge of the construction of the Otto-Hoffman coke ovens for the Cambria Iron Company.

In 1900, Mr. Collins entered into private practice with offices at Johnstown, conducting business in civil, sanitary, and mining engineering. During his practice in Johnstown, he installed some of the largest mining operations in Western Pennsylvania. He acted as Chief Engineer in charge of making surveys of 40 000 acres of virgin coal in Somerset County, Pennsylvania, for Baltimore interests. Mr. Collins served as Chief Engineer of the Somerset Coal Company, later the Consolidation Coal Company, in charge of twenty-two mining operations in Somerset County. The large plant and development at Ralphton, Pa., was installed under his supervision for the Quemahoning Coal Company of Somerset, Pa.

From 1904 to 1906, he was retained by the United Coal Company, of Pittsburgh, Pa., in charge of the construction of the Jerome Branch Railroad from Hollsopple to Jerome, in Somerset County, as well as the construction of the plant at Jerome, including steel tipple, stone power-house, main and air shaft, 265 ft deep, and miners' houses and store. This plant was considered one of the most modern in Western Pennsylvania. For the Maryland Coal Company, he designed and installed the deep shaft and plant situated at St. Michael, Pa., and also placed in operation for this Company a number of other mines in Pennsylvania, Maryland, and West Virginia.

During his practice in Johnstown he also served as Consulting Engineer for the following: Haws Coal Company, Operators Coal Company, and Citizens Coal Company, all of Johnstown; Federal Coal Company, Black Lick Coal Mining Company, Commonwealth Coal Company, and Maryland Coal Company, of Philadelphia, Pa.; Quemahoning Coal Company, of Somerset; and United Coal Company, of Pittsburgh. This work consisted of examination and valuation of coal properties and reports on virgin coal fields. Reports were made on coal and mineral lands in Pennsylvania, West Virginia, and Canada.

He also executed the following work: Sewerage system for Lilly and Meyersdale, Pa.; drainage system for Hollidaysburg, Pa.; water-works and sewerage system for New Florence and Windber, Pa.

Mr. Collins laid out a number of subdivisions in the vicinity of Johnstown, the most important of which is the attractive Borough of Southmont, which adjoins Westmont and, like the latter, is an exclusive residential suburb of Johnstown.

In 1912, Mr. Collins left his private practice to become Special Engineer for the Berwind-White Coal Mining Company, in which capacity he designed treatment plants for water supply and the disposal of sewage from mining hamlets near Windber.

From 1914 to 1918, he was engaged by the City of Johnstown as Sanitary Engineer. His plans for a sewerage system and disposal works were approved by Mr. Gray, and the late Rudolph Hering, M. Am. Soc. C. E., Consulting Engineer of New York, N. Y. The system was designed to serve a population of 168 000, the estimated cost of which was more than $2 000 000. The work was constructed according to his plans.

From 1918 to 1920 Mr. Collins was in the service of the Government as Senior Engineer with the United States Shipping Board, Emergency Fleet Corporation, at Washington, D. C., and Philadelphia, in charge of sewerage, drainage, and sewage disposal for eighteen housing projects built to serve the merchant shipyards. These projects were situated along the Atlantic Coast from Maine to Florida and in the Central West. During this period he was also delegated to report on the valuation of 30 000 acres of coal land owned by the German Imperial Government, at Farmersville, Ill. This report was made to the late Morris Knowles, M. Am. Soc. C. E., of Pittsburgh, the engineer appointed by the Alien Custodian.

Mr. Collins moved to Clarksburg, W. Va., in 1920, where he was engaged in private practice until March, 1936. During this time he made reports and valuations of properties in Pennsylvania, Maryland, West Virginia, and Kentucky.

In July, 1920, he was engaged by the Mountain Water Supply Company, of Philadelphia, to study the pollution of its dam on Indian Creek, Fayette County, Pennsylvania, by mine drainage. This investigation of streams affected by mine drainage covered a total of 30 687 acres of mined-out areas situated in four counties of Western Pennsylvania. Diagrams were prepared showing the relation between the total flow of mine drainage, in gallons per day, and the total flow of the streams, in gallons per day, at the point where the mine drainage collected, the total sulfates in the mine drainage, and the total sulfates in the stream at the gaging and sampling station. This was the first authoritative report relating to the pollution of streams by mine drainage. Since that time extensive work has been done in the sealing of mines to prevent stream pollution in Pennsylvania, West Virginia, and other coal-producing States.

He was later employed by the Yough Hydro Electric Company to report on the valuation of coal under its property at Swallow Falls, Garrett County, Maryland.

Mr. Collins designed a sewage disposal plant for the Borough of Somerset, and reported on the sewerage system and sewage disposal plant for the City of Spencer, W. Va.

In March, 1927, he was retained by the City of Clarksburg to value the plant and holdings of the Clarksburg Light and Heat Company, for the city's use in presenting its case as protestants before the West Virginia Public Service Commission. He was also employed by the City of Weston, W. Va., in its case against the Weston Water Company, which was presented before the West Virginia Public Service Commission.

In June, 1930, he was retained by the United States Government, Department of Internal Revenue, to testify as to the value of the property of the Quemahoning Coal Company and other coal companies in Somerset County, Pennsylvania, as of 1903.

In 1933 and 1934 Mr. Collins prepared and submitted the necessary data for a $1 400 000 loan and grant from the Public Works Administration for a sewerage system and sewage disposal plant for the City of Clarksburg. This preliminary report was approved by the Public Works Administration, but the loan and grant were not accepted by the city.

In July, 1935, he was retained by the Pennsylvania Coal and Coke Company to testify in its case for reduced freight rates before the Interstate Commerce Commission, at Washington, D. C.

From March, 1936, until the time of his death, he was employed by the Public Works Administration and had just completed supervision of the installation of sewers at Mason and South Parkersburg, W. Va.

Mr. Collins was the author of a number of technical papers: "The Engineer on Development Work,"[2] which was prepared as a helpful hint to young engineers in developing virgin coal fields; "A Comparison of the Combined and Separate Systems of Sewerage;"[3] "The Drainage, Sewerage and Sewage Disposal of Emergency Housing Developments," read before the American Society of Municipal Improvements at its Convention in New Orleans, La., in November, 1919;[4] and "Pollution of Water Supplies by Coal Mine Drainage."[5] In addition, he also made a number of addresses before civic groups in Johnstown and Clarksburg.

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[2] *Coal Age*, March 9, 1912.

[3] *Municipal Engineering*, October, 1898.

[4] *Proceedings*, Am. Soc. of Municipal Improvements, 1919.

His hobby was writing, and he was the author of several novels, one of which, "In the Wake of the Storm," is a historical novel based on the life of Abraham Lincoln. It was written after several years of research.

As an engineer, Mr. Collins was extremely accurate, painstaking, and thoroughly dependable, possessing a very high conception of proper ethics in his chosen profession. The tireless energy with which he applied himself to his work won him an enviable reputation among his associates.

His philosophy of life is best expressed in a quotation taken from his journal, which he kept from early manhood:

> "A man's life, when weighed in God's balance, is not measured by the wealth he possesses, nor by the number of years he is permitted to live. His span of life may be short, yet the use he makes of the years allotted to him is the true measure of the man."

Mr. William Monroe Davis, of St. Petersburg, Fla., a lifelong friend of Mr. Collins, says of him:

> "From heredity, environment, and training he received all the elements of integrity, honor, and righteousness common to his Quaker ancestors. His life was full, complete, and well rounded out. 'Well done, good and faithful servant of mankind. Enter into thy everlasting reward.'"

In August, 1893, Mr. Collins was married to Leila Thoburn Coleman, of Johnstown, who died in June, 1908. From this union three children survive him: Leon Waterman Collins, Mary Winslow Collins (Mrs. Carl G. Rau), and Helen Thoburn Collins. In August, 1910, Mr. Collins was married to Jessie Ellen Coleman, a niece of his first wife, of Chicago. She and one daughter, Shirley Elizabeth Collins, survive him. He is also survived by one sister, Mrs. William A. Woodward.

He was a member of the American Society for Municipal Improvements (later, the American Public Works Association).

Mr. Collins was elected an Associate Member of the American Society of Civil Engineers on June 4, 1902, and a Member on January 18, 1916.

ELBRIDGE ROBBINS CONANT, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 24, 1937

Elbridge Robbins Conant was born at Acton, Mass., on September 23, 1865. He was the son of Luther and Celeste J. (Robbins) Conant, both of Acton. After finishing his studies at the local schools, he attended the Chauncy Hall School, of Boston, Mass., where he prepared for the Massachusetts Institute of Technology. He completed a four-year course in Civil Engineering at the Institute in 1889, as a Special Student.

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[5] *Engineering News-Record*, October 18, 1923.

[1] Memoir prepared from information on file at the Headquarters of the Society.

His early work, after leaving the Institute, included a year with the New York and New England Railroad Company, as Instrumentman; a year with the City of Newton, Mass., on the construction of a sewerage system, part of the time as Principal Inspector; and approximately a year with the civil engineering firm of Aspinwall and Lincoln, of Boston, on miscellaneous development projects.

In the fall of 1892, Mr. Conant went South to assume a position with the United States Engineer Department, Southeastern District, with headquarters at Savannah, Ga. He remained with the Department for twenty-one years. Starting as Surveyor or Inspector, he was made a Junior Engineer in 1902, Assistant Engineer in 1903, and from 1904 to 1912, he was Principal Assistant Engineer in full charge of the Sub-Engineer's Offices, at Brunswick, Ga., where he had immediate supervision of the Department's work for ports between Savannah and Fernandina, Fla., and on rivers entering the ocean between these ports. In 1912, his headquarters were transferred back to Savannah.

While with the Engineer Department, Mr. Conant was in immediate charge of important dredging operations and jetty construction at various points, and of the erection of fortifications at Tybee Island and Fernandina. His work also included extensive hydrographic surveys and a large amount of general planning of river and harbor improvement operations.

Mr. Conant left the Government Service in 1913 to become Chief Engineer and Purchasing Officer for the City of Savannah. During the six years that he held this position, he had charge of all public improvements, including the water-works system. He supervised a million-dollar extension of the City's sewerage system, a large amount of pavement construction, and the erection of a "high temperature incinerator plant" for the disposal of the City's garbage. On account of the large quantity of wet garbage, especially in the melon season, this plant involved certain special problems, but these were successfully solved. The operation of the plant attracted wide attention among sanitary engineers at the time.

After leaving the City's service in 1919, Mr. Conant was engaged for more than a year in general consulting work in and about Brunswick, Ga., specializing in paving and road construction. In 1920 he returned to the North. For a year, from March, 1921, to March, 1922, he was Town Manager of Mansfield, Mass., and, later, for a short time, Surveyor (City Engineer) for the Department of Public Highways, in Manchester, N. H. He retired from active work several years ago.

For several years Mr. Conant was a member of the American Society for Municipal Improvements and served as Vice-President of that organization in 1917–1918 and as President in 1918–1919. He was also Chairman of that Society's Committee on Street and Sidewalk Design. From 1924 to 1931, he was a member of the Boston Society of Civil Engineers, and, at one time, a member of the New England Water Works Association.

In 1895 Mr. Conant was married to Elfletah Launey, of Savannah, who survives him as do two daughters, Mrs. Edmund Francis Leland, Jr., and Mrs. Ralph S. Robinson, each of whom has three children. For several years prior to his death, Mr. Conant resided in Belmont, Mass., spending his winters in Florida.

His engineering work was constructive and sound and made its contribution to that reputation for reliability and integrity which is one of the chief assets of the profession. His activities brought him frequent notice in the local newspapers, and he received many expressions of commendation. At all times he had in mind the factor of cost and although receptive to new methods, he constantly aimed at efficiency and economy.

In temperament, Mr. Conant was fortunate, possessing an even and cheerful disposition. Courteous and kind to all, he was singularly free from petty animosities. He had a wide circle of friends, and he leaves a pleasant memory.

Mr. Conant was elected a Member of the American Society of Civil Engineers on March 13, 1917.

HUGH LINCOLN COOPER, M. Am. Soc. C. E.[1]

Died June 24, 1937

Hugh Lincoln Cooper was born in Sheldon, Houston County, Minn., on April 28, 1865, the elder son of George Washington and Nancy Marion (Parshall) Cooper, grandson of James Henry and Phoeba Lavinia (Bailey) Cooper, and great-grandson of James Henry Cooper, who came from England and settled in Montgomery County, Illinois. His early schooling was in Rushford, Minn., Grade and High Schools. At the age of seventeen, while working for a farmer, during a school vacation, he designed and built a timber highway bridge for his employer, across Money Creek which flowed through the farm. This bridge, with a span of 40 ft, and resting on stone piers, was in service for about forty years.

This early interest in engineering construction developed into a firm determination to become an engineer. In order to gain the necessary practical experience, Mr. Cooper had to leave home, after he was graduated from the Rushford High School in 1883, and seek employment in the field of his chosen profession. Employment of an engineering nature was not immediately available, however, and in order to support himself, he worked in a livery stable in Eau Claire, Wis., which was operated as a part of a piano firm for the delivery of pianos and organs. He soon became a Salesman for the firm, and successfully followed that calling until March, 1884, when he obtained employment with the Chicago, Milwaukee and St. Paul Railroad Company, as an Axman and Rodman in an engineering party assigned to a bridge-building project at Chippewa Falls, Wis. Mr. Cooper's intuitive engineering knowledge and ability enabled him to master the details of the bridge and its construction, so that when the Contractor failed and the Resident Engineer was unable to continue the work because of illness, he was given complete charge of the job and finished the bridge to the entire satisfaction of the Chief Engineer of the Railroad Company.

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[1] Memoir prepared by Valentine Ketcham, Esq., of Stamford, Conn., and Brig.-Gen. M. C. Tyler, Corps of Engrs., U. S. Army, M. Am. Soc. C. E.

This accomplishment made him decide to specialize in bridge building, and, in January, 1885, he entered the employ of the late Horace E. Horton, M. Am. Soc. C. E., Bridge Engineer, with offices in Rochester, Minn. After an intensive apprenticeship of several years Mr. Cooper was made Superintendent of erection of the High Bridge and the Wabash Street Bridge across the Mississippi River, in St. Paul, Minn. Upon the completion of this work in 1889, he was transferred to Chicago, Ill., as Superintendent and Assistant Chief Engineer of Mr. Horton's Chicago Bridge and Iron Works, where he remained for two years.

In 1891, he became the Manager of a Branch Office of the San Francisco Bridge Company, in Spokane, Wash., and built several railroad and highway bridges in that territory, among them a steel cantilever bridge 500 ft long for the Spokane Falls and Northern Railway Company.

In 1894, Mr. Cooper became interested in hydro-electric engineering—then in its infancy—believing that it offered more opportunities and advantages than bridge building. He made an examination for a small water-power development on the Missouri River, at Canyon Ferry, Mont., for the late John T. Fanning, M. Am. Soc. C. E., Consulting Engineer, located the site, and put in the foundations. By this time he had fully decided to devote his entire energies to hydro-electric engineering, and, therefore, abandoned his bridge-building career.

Wishing to learn more about this new science, he applied for work, in 1896, with Stillwell, Bierce, Smith-Bailey Company, of Dayton, Ohio, which firm manufactured water-wheels and built small hydro-electric plants in which to install them. He was told there was no work for him, but he explained that he wanted to learn about water-wheels and would gladly work without pay. His perseverance won him recognition on these terms and he worked without compensation for several months. Eventually, he was made Assistant Chief Engineer of the Company and designed or built (or both) hydro-electric plants at Ellenville, and Dolgeville, N. Y., and on the Island of Jamaica, British West Indies.

In 1898, Mr. Cooper joined the organization of the late Fred Stark Pearson, M. Am. Soc. C. E., in New York, N. Y., and built two granite dams for the 20 000-hp high-head hydro-electric station of the São Paulo Tramway, Light and Power Company, Limited, on the Tieté River, in Parnahiba, Brazil.

He returned to the United States in 1901, and for a short time was associated with the firm of Viele, Blackwell, and Buck, in New York City. He located, designed, and, later, built a hydro-electric plant for the Winnipeg General Power Company on the Winnipeg River, near Winnipeg, Man., Canada.

In 1902 and 1903, as Chief Engineer of the Mexican Light and Power Company, of Montreal, Que., Canada, he was engaged in the design and location of a high-head hydro-electric plant in the mountains of Mexico, at Salto Grande, on the Necaxa and Tenanga Rivers, for the street railways of the City of Mexico, Mexico.

Simultaneously, he located, designed, and constructed the 100 000-hp plant for the Electrical Development Company of Ontario, Limited, at Niagara Falls, Ont., Canada, above the Horseshoe Falls. The design included two daring and unique features—first, a wing dam extending into the Niagara River a short distance above the Falls, where the water attained a velocity of more than 20 miles per hr; and, second, a tail-race tunnel discharging directly behind the waters of the Horseshoe Falls. Both these features seemed fantastical and impossible of accomplishment, when first proposed, but both were successfully completed and the plant was put in operation in 1907.

In 1905, Mr. Cooper opened his own office for private practice, in New York City. In 1906, 1907, and 1908, he designed and for the greater part built the hydro-electric plant for the Pennsylvania Water and Power Company on the Susquehanna River, at McCalls Ferry, Pa.

In 1907, he visited Japan as a Consulting Engineer on the Lake Biwa project in that country. During this same year, he started the designing and financing of the low-head hydro-electric plant on the Mississippi River, at Keokuk, Iowa, which was started in 1910 and completed in 1913.

In 1913, Mr. Cooper was invited by the Egyptian Government to make a survey of a proposed power plant, which would be a part of the Assuan Dam on the River Nile, and the output of which was to be used in the manufacture of fertilizer. Plans were prepared and accepted for a plant capable of generating 200 000 hp, but the outbreak of the World War in 1914 caused the abandonment of the project.

In 1914, he again opened offices in New York City, where he continued a general consulting engineering practice.

When the United States entered the World War in April, 1917, he volunteered to serve, and, in May, was appointed a Major of Engineers. He was ordered to France where he arrived in July. During the preliminary stages of the organization of the engineering activities of the Service of Supply of the American Expeditionary Forces, Major Cooper was engaged on the duty of making investigations and plans for the development and use of base-port facilities and as Consultant to the Chief Engineer of the Expeditionary Forces on many of the difficult construction problems that had to be solved. He was promoted to the rank of Lieutenant-Colonel in October, 1917. On the recommendation of the Chief of Engineers of the Army, Lt.-Col. Cooper was ordered back to the United States in March, 1918, to begin the construction of the navigation and hydro-electric power development at Muscle Shoals, Ala., which was intended to furnish energy to the War Department Atmospheric Nitrogen Fixation Plant then being built as a war measure for the production of explosives. In April, 1918, he was promoted to Colonel of Engineers. In May, 1918, at the request of the Commanding General, A. E. F., Colonel Cooper was relieved from duty at Muscle Shoals and again ordered to France where he served as Section Engineer of Base Section No. 2 at

Bordeaux, supervising the construction of camps, hospitals, docks, railroads, and other engineering works in that area. For this service he was awarded a Meritorious Citation Certificate by the Commanding General of the American Expeditionary Forces for "Exceptionally Meritorious and Conspicuous Services as Engineer Officer, Base Section No. 2, A. E. F." By the French Government, he was awarded the Order of the University of Palms, grade of Officier d'Instruction Publique.

After the war, Colonel Cooper held a commission as Colonel, Engineer Officers' Reserve Corps, until he reached the age of 64 when he was transferred to the Auxiliary Reserve.

In May, 1920, he entered into an agreement with the Chief of Engineers, U. S. Army, who had been ordered to proceed with the construction of the Wilson Dam, at Muscle Shoals, which provided that Hugh L. Cooper and Company would act as Consulting Engineers, in the design and construction of this project. Under this contract, Colonel Cooper furnished the Chief of Engineers with general and detailed plans of the main dam, the power house, its hydraulic and electrical machinery, and the lock system for navigation, together with complete specifications for all the work. He also passed upon the acceptability of all foundations, and inspected all permanent work through a Resident Engineer and a Corps of Inspectors, terminating his contract with the completion of the project in 1926.

Between November, 1919, and 1926, he made the following important engineering studies and reports relating to the United States-International Waterways situation:

First.—A general investigation for the Frontier Corporation of the power and navigation potentialities of the St. Lawrence River, particularly in its international section.

Second.—A comprehensive report to the Hydro-Electric Commission of Ontario on the general design and cost of its Chippewa Generating Station.

Third.—A report, at the request of Herbert Hoover, Hon. M. Am. Soc. C. E., then Secretary of Commerce in President Coolidge's Cabinet, on the St. Lawrence problem as a whole, including the results of intensive studies as to restoring and perpetuating the scenic beauty of the Horseshoe Falls in the Niagara River.

In 1926, Colonel Cooper was chosen by a Government Commission of the Soviet Republic to go to Russia to study and report upon a projected power plant below the Kitchkas Rapids, in the Dnieper River, in the Ukraine. In February, 1927, he entered into a contract with the Soviet Government to design and supervise the construction of this 800 000-hp project which included a series of locks and required an expenditure of more than $100 000 000. This work was carried through to a most satisfactory and successful conclusion, and when the project was formally accepted in August, 1932, by the Soviet Commissariat of Heavy Industry, Colonel Cooper was awarded the Order of the Red Banner by the Government of the Soviet Republic, the highest civil honor that can be awarded to any non-citizen.

In 1928, the Egyptian Government appointed him a member of an International Commission of three engineers to pass upon the feasibility of raising the Assuan Dam on the River Nile. The survey was made, plans and specifications were prepared to raise the dam 30 ft, and the work was carried to a successful completion, providing water for the cultivation of 700 000 additional acres of land in the Nile Valley.

Colonel Cooper's last active work was as a Consulting Engineer for the Union Carbon and Carbide Company, on a high-head hydro-development on the Kanawha River in West Virginia. At the time of his death, he was working on plans for a huge development in Z-Canyon, on the Columbia River, in Washington, the electrical development of the Nile at Assuan, which had been recently revived, and a hydro-electrical development in Greece.

He had been a resident of Stamford, Conn., since 1901. He had been prominent in its civic, and active in its religious, life; he attended the First Congregational Church. He belonged to the Hubbard Heights Golf Club and Woodway Country Club, in Stamford; the New York Yacht Club and the Uptown Club, in New York City; The Congressional Country Club, in Washington, D. C.; Royal Societies Club, London, England; Engineers Club, of St. Louis, Mo.; Engineers Club, of Kansas City, Mo.; and the Harrisburg Club, of Harrisburg, Pa.

The following honorary degrees had been conferred upon him: Doctor of Engineering, by the Polytechnic Institute of São Paulo, at São Paulo, Brazil; Doctor of Laws, by Parsons College, Fairfield, Iowa; Doctor of Laws, by the University of Missouri, at Columbia, Mo.; and Doctor of Engineering, by Syracuse University, at Syracuse, N. Y.; also a degree as Minister of Public Instruction of the Republic of France, and the Beaux Arts.

He was a member of the American Society of Mechanical Engineers, the American Institute of Mining and Metallurgical Engineers, American Institute of Consulting Engineers, the Society of American Military Engineers, also the Institution of Civil Engineers, and the Institution of Water Engineers, of Great Britain.

Colonel Cooper was intensely interested in government and economics and wrote several monographs relating to these subjects from the engineer's point of view.

He had a striking and unique personality. His intuitive technical knowledge and his broad engineering experience commanded the respect of all, but he was admired not only for his eminence in his chosen profession, but for what he was himself. He was absolutely candid in all personal relations, frank and sincere in his judgments, outspoken in the expression of his convictions, and absolutely honest in everything that he did. He inspired men with his vigor and enthusiasm and friendliness; every one loved him and considered it a privilege to be associated with him. He possessed a rare combination of good common sense and genial humor and radiated cheerfulness and vitality in every circle in which he moved. His friends were always glad to be with him and enjoyed the impact of his vigorous and rugged personality. The twinkle in his eye, the chuckle in his voice, his pithy and pointed comments on life, endeared him to all and made him delightful company. He will ever be remembered as a gentleman of distinction, a citizen of sterling worth, a patriot whose love of country was courageous and sincere, and an engineer pre-eminent.

He was married at Rochester, Minn., on October 12, 1892, to Frances B. Graves. He is survived by his widow and two daughters, Mrs. Ralph H. Sheldon and Mrs. John R. Hardin, Jr., and a sister. A brother, Dexter Parshall Cooper, M. Am. Soc. C. E., died on February 2, 1938.

Colonel Cooper was elected a Member of the American Society of Civil Engineers on September 6, 1905.

ERLE LONG COPE, M. Am. Soc. C. E.[1]

Died March 19, 1937

Erle Long Cope was born on March 25, 1883, at Meridian, Calif., the son of Leroy Cope and Ida (Long) Cope. His father was for many years a much respected county official of Sutter County, California, where Erle's boyhood was spent. He was graduated from the High School, at Marysville, Calif., in 1902. Four years later he was graduated from the College of Civil Engineering of the University of California, at Berkeley, Calif.

During his college vacations Mr. Cope readily found employment with the County Surveyor of his home county. Immediately after his graduation he spent a short period with the Engineering Staff of the Southern Pacific Company, at Los Angeles, Calif., and then returned to San Francisco, Calif., where he was employed by the firm of Couchot and O'Shaughnessy, first in designing the structural features of buildings under the direction of the late M. C. Couchot, M. Am. Soc. C. E., and, thereafter, as Resident Engineer of the Dulzura Conduit of the Southern California Mountain Water Company in San Diego County, California, under the immediate direction of the late M. M. O'Shaughnessy, M. Am. Soc. C. E.

After the completion of this assignment Mr. Cope did some important structural designing for the firm of Couchot and Thurston (E. T. Thurston, M. Am. Soc. C. E.). In April, 1909, he became an Assistant Engineer with John Galen Howard, Architect, in which capacity he had opportunities to exercise his great ability in solving difficult problems of structural design. He also supervised the construction of some of the important buildings of the University of California. Thereafter, he did some work on water-power projects and valuations.

From September, 1912, until August, 1934, Mr. Cope maintained an office in San Francisco, part of the time with associates and part of the time in independent practice, specializing in Structural Engineering. During this period he carried on such varied work as the purchase of equipment, machinery, and materials for bridges and structures in the Dutch East Indies; the investigation of irrigation projects in California; the design of a number of pump houses for the Banta-Carbona Irrigation District; and the structural engineering of several buildings in San Francisco and neighboring cities, including the Mills Tower, the Stock Exchange Building, the Apparel Center Building, and others. He also designed and superintended the construction of bridges and concrete viaducts in Sutter County and many other smaller structures.

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[1] Memoir prepared by Walter L. Huber, and Thomas H. Means, Members, Am. Soc. C. E.

He was in charge of the Bureau of Building Inspection of the City of San Francisco from August, 1934, until his untimely death on March 19, 1937. His administration of this Bureau was characterized by impartiality and fairness and, as in other employment, he made no effort to spare himself. He was a prodigious worker. His direct and frank manner won for him many friends.

He is survived by his widow, Mrs. Gladys W. Cope, his mother, Mrs. Ida Cope, and a brother, Jesse.

He was a member (Director, 1932–33, and President, 1933) of the Structural Engineers Association of Northern California. He was also a member and President (1933) of the Structural Engineers Association of California, and a member of the Engineers' Club of San Francisco.

Mr. Cope was elected a Junior of the American Society of Civil Engineers on January 7, 1908; an Associate Member on December 6, 1910; and a Member on November 27, 1917. He was, throughout his membership, active in the San Francisco Section, and, at various times, was a member of a number of the Section's Committees.

THURSTON CARLYLE CULYER, M. Am. Soc. C. E.[1]

DIED DECEMBER 9, 1936

It is difficult to express in cold type the work and achievements of one who is gone, and to put the warmth of life into the memoir of a friend, making it alive to the associates who survive. The writers, who were friends of Mr. Culyer for more than fifty years, realize this, but they wish to pay a tribute of respect to the fine traits of his character, to his high sense of honor, and to the loyalty he showed to his work and to his friends. They know that he put into his work the best that was in him and they wish his devotion to it was better known.

Thurston Carlyle Culyer was born in the Washington Square section of New York, N. Y., on September 19, 1864, the son of John Y. and Phœbe A. (Matthias) Culyer. His paternal ancestry extended back to Thomas and Elizabeth Norton, of Norfolk, England. John Culyer, the grandson of Thomas Norton, came to the United States about 1829 and settled in New

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[1] Memoir prepared by Edward S. Larned, Esq., Boston, Mass., and Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E.

York City. His son—John Y. Culyer, the father of Thurston Carlyle Culyer—was a noted Landscape Architect and Engineer who served as an Assistant Engineer on the construction of Central Park, New York City, until the Civil War. He then became Assistant Secretary of the United States Sanitary Commission, and, later, joined the Engineering Branch of the Federal Military Forces and served in the defenses south of the Potomac River under the late Major-General John Gross Barnard, U. S. A., Hon. M. Am. Soc. C. E. After the war, John Y. Culyer was again employed as an Engineer in Central Park until 1872 when he became an Assistant Engineer, and, later, Chief Engineer and Superintendent, of Prospect Park, in Brooklyn, N. Y., and other parks in that city, remaining in this service until 1886. Subsequently, he was engaged in general and consulting practice as a Landscape Architect and Engineer, in which field he gained a wide and enviable reputation. For twenty-five years John Y. Culyer served as a member of the Board of Education of Brooklyn, and was active in the New York State National Guard, attaining the rank of Lieutenant Colonel. His son—Thurston—inherited his love of trees and of nature generally.

Thurston Carlyle Culyer obtained his preliminary education in the Friends Seminary, in Brooklyn, and then spent four years (1878 to 1882) at the Brooklyn Collegiate and Polytechnic Institute, later, the Polytechnic Institute of Brooklyn. Between September, 1882, and January, 1883, he was with the Drainage Construction Company, of Boston, Mass., on the building of 13 miles of sewer in Keene, N. H. On July 1, 1884, he was appointed a Chainman in the service of the Department of Public Works of New York City, under the late George W. Birdsall, Chief Engineer, and worked in the Croton water-shed on surveys for the new Croton Aqueduct and the Muscoot Dam. Later, Mr. Culyer was promoted, successively, to the positions of Rodman, Leveler, and Transitman with increasing responsibilities, and on September 20, 1892, to the position of Assistant Engineer in charge of all work for the Department in the Croton water-shed in Westchester and Putnam Counties.

When, in 1898, the several outlying Boroughs were consolidated into the Greater City of New York, the City departments were re-organized and the functions of the Department of Public Works were merged with those of the new Department of Water Supply, Gas, and Electricity, Mr. Culyer continued with the new Department's Bureau of Water Supply, his work including the design and construction of sanitary structures for the protection of the water supply. From March, 1910, until his retirement, he was in charge of all the reservoirs and aqueducts of the Croton and Bronx River Systems, including the reservoirs within the City limits—such as Jerome Park, High Bridge, and Central Park. In 1914, there was added to his duties for a time the operation and maintenance of that part of the new Catskill Aqueduct which extended from the Hudson River crossing at Storm King to the City line.

Some of his associates feel that Mr. Culyer's most important contribution to the service was his sanitation work which began in 1892 during a scourge of cholera in New York City. Previous to that time land acquisition in the water-shed was confined largely to the actual needs of the reservoirs built and the consequent road substitutions. Mr. Culyer's opinion had great weight in deciding the authorities to acquire about 3 000 acres for the sanitary protection of the streams and for the actual removal of the then existing gross sources of pollution. He was also instrumental in abating many minor sources of pollution existing elsewhere in the water-shed.

In 1893, he constructed a plant for the disposal of sewage of a part of the Village of Brewster, N. Y., which was unique in that it provided for the sterilization of sewage by electrically dissociating common salt as a source of chlorine. His interest in sterilization continued, and shortly after the experiments made by the late George A. Johnson, M. Am. Soc. C. E., at the Chicago (Ill.) stockyards, Mr. Culyer began the sterilization, with bleach powder, of the flow of the new Croton Aqueduct—probably the first large city in the United States to furnish this safeguard. Later, on the line of the aqueduct at Dunwoodie, N. Y., he built a sterilization plant designed to use this material accurately and, later, as knowledge of liquid chlorine became available, converted the Dunwoodie plant to its use. He was also responsible for the setting out of the large plantings of evergreen trees on the marginal lands about the reservoirs in the Croton water-shed, that does so much to enhance their beauty and attractiveness. His taste for such work was remarkable. He knew every phase of the work under his charge, and loved it and the beautiful country in which it was located.

On January 1, 1919, Mr. Culyer was retired from the service of the City which he had served so faithfully and well for thirty-five years—nearly all his working life. One of his associates in the Department writes:

> "His love for the trees, the flowers and streams made him one of those whose life work on the watershed was more a pleasure than a task, and in his contact with the countryside he was more the squire than the stern representative of the big city whose water supply he so carefully guarded. In his retirement, New York City lost a loyal and competent engineer who devoted himself to her interests to the exclusion of all others."

After his retirement, he became identified with the real estate firm of Pease and Elliman, of New York, as a Real Estate Appraiser, and served with this firm until a short time before his death. For a number of years he was afflicted with a serious heart trouble which was a source of constant concern to his friends and his devoted wife. He died of a sudden attack on the night of December 9, 1936.

Mr. Culyer was respected by, and possessed the sincere affection of, those who were privileged to know and be associated with him. That he was not better known to the Engineering Profession and to the community was due to his modesty and a disinclination to put himself forward. He had a high sense of honor and left behind a record of which his friends are proud.

He had vision and a good sense of humor. He possessed a rare ability for sketching. He was a Charter Member of the Municipal Engineers of the City of New York, and a member of the American Water Works Association.

He was married, on February 6, 1892, to Edith Mead Stocum, of Purdys, N. Y., who, with a daughter, Katharine Grey Culyer, and a sister, Marian Prescott Culyer, of Mount Kisco, N. Y., survives him.

Mr. Culyer was elected a Member of the American Society of Civil Engineers on June 4, 1913.

WILLIAM HERBERT CUSHMAN, M. Am. Soc. C. E.[1]

Died October 10, 1937

Members of the Society and his many other friends were greatly shocked by the accidental death, in an automobile accident, of William Herbert Cushman, Engineer of the Works Progress Administration, which occurred near Watertown, N. Y., on October 10, 1937.

William Herbert Cushman was born at Duxbury, Mass., on March 27, 1869, a son of Alden and Jane (Hatch) Cushman. He was a descendant of Robert Cushman, one of the founders of Plymouth Colony, in Massachusetts.

After High School, he attended a private academy, and, from 1887 to 1892, worked with E. W. Bowditch, a Civil Engineer, of Boston, Mass., on topographic surveys, sewer surveys, estimates, and construction. In 1892, Mr. Cushman was engaged by the Town of Lancaster, N. H., to supervise the construction of a water supply system; in 1893, he had charge of a water supply system at Sistersville, W. Va.; and from January to September, 1894, was associated with Hoge and White, City Engineers of Wheeling, W. Va. From February, 1895, to September, 1896, he was District Superintendent, Street Cleaning Department, of New York, N. Y., under Col. George E. Waring, Jr. During this time, he collected data and submitted a report leading up to the construction of the Barren Island Reduction Plant.

For five years, 1897 to 1902, Mr. Cushman was Resident Engineer in charge of construction of the 52 000-hp hydro-electric project on the St. Lawrence River, at Massena, N. Y., which was built by the St. Lawrence Power Company. The late John Bogart, M. Am. Soc. C. E., was Engineer-in-Chief, and Kincaid, Waller, and Manville, of London, England, were Associated Engineers on this project. The plant is now owned by the Aluminum Company of America.

In May, 1902, the principal construction work for the Massena development having been completed, Mr. Cushman continued his association with Colonel Bogart by making preliminary studies and design for a 12 000-hp hydro-electric development on the Chattahoochee River at Morgan Falls, about twenty miles north of Atlanta, Ga., on which Colonel Bogart was then Consultant. Mr. Cushman prepared the design and in August went to Morgan Falls and started the construction, remaining on the site until all but minor details were completed. This job was especially appealing to his energetic nature as the urgent desire to place the plant in operation permitted an unusual amount of authority over the contractor's operations being placed with the engineer. The construction was completed in 1904.

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[1] Memoir prepared by William T. Field, and Charles A. Pohl, Members, Am. Soc. C. E.

From August, 1904, to August, 1905, he was Resident Engineer on the Oswego Division of the New York State Barge Canal. As such, he had charge of all field work and studies connected with the contracts for two locks and the canal through the City of Fulton, N. Y., and had completed much of the field and design work for the remainder of the canal, along the Oswego River, when he left the State employ.

From August, 1905, to January, 1907, Mr. Cushman was Chief Engineer of a hydro-electric development on the Juniata River, near Huntingdon, Pa., for the Juniata Hydro-Electric Company. This project had a 6 000-hp hydro-electric plant and a 3 000-hp steam auxiliary which furnished light and power to Altoona, Pa., and neighboring towns.

In 1907, he was Engineer and Superintendent for W. J. Oliver and Company, Contractors for the Hales Bar Dam, across the Tennessee River, near Chattanooga.

In 1908, Mr. Cushman was Resident Engineer, representing Sellers and Rippey, Consulting Engineers, of Philadelphia, Pa., on a hydro-electric development on the Estacada River, about 25 miles southeast of Portland, Ore., for the Portland Railway, Light, and Power Company. Great difficulties from leaks, due to a porous foundation, were encountered on this work.[2]

From 1910 to 1913, he was engaged in private practice as a Consulting Engineer, making investigations, reports, and giving expert testimony on hydraulic power work. In 1913, he served as Engineer and Superintendent for James Stewart and Company, on the construction of the Barge Canal, at Scotia, N. Y., involving caisson work, and, in 1914 and 1915, he designed and built a hydraulic power plant on the Genesee River, at Mt. Morris, N. Y.

As head of the Hydraulic Construction Company, Mr. Cushman built a dam and reservoir for the City of Johnstown, N. Y., at Rockwood, N. Y. During the World War, this Company built a steam-electric plant for the New York Air-Brake Company, at Watertown, which Company was engaged in making munitions for the Federal Government. Several other buildings and structures were built by the Hydraulic Construction Company, during the period, 1915 to 1920.

From 1917, for a considerable period, Mr. Cushman was also associated with the firm of Jones and Hazel, at Watertown, in the investigation and appraisal of water-power projects and, in this connection, he appraised more than 150 hydraulic power installations. He was also retained during this time by the Federal Treasury Department to study and criticize reports pertaining to water-power development from all parts of the United States, and appeared many times as an expert witness in legal proceedings pertaining to water power.

In 1920, Mr. Cushman opened an office as a Consulting Engineer, in Watertown. He designed and supervised the construction of a 7 500-hp hydro-electric development for the St. Regis Paper Company, at Black River,

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[2] *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 447.

N. Y., and, from 1921 to 1924, was Consultant for the City of Watertown in the development of a 7 500-hp hydro-electric plant for that city.

In 1926 and 1927, he designed and supervised the construction of a 2 200-hp hydro-electric development for the Newton Falls Paper Company, at Newton Falls, N. Y.

When the Temporary Emergency Relief Administration was organized in 1934, Mr. Cushman was named Project Engineer and, later, County Engineer, under the Works Progress Administration, at Watertown.

He was essentially a positive individual. He was a dynamo of energy, always active. He always used direct methods of approach which might have been construed as lacking in tact. On account of his sincere directness and abhorrence of evasion or equivocation, he knew no other way. He sometimes was impatient over restraint, especially when he thought that it might properly have been avoided. He was as stalwart in character and physique as Plymouth Rock in the shadow of which he was reared.

As an engineer, Mr. Cushman was particularly gifted with common sense—not only in directing men, but in handling the forces of Nature as found in his chosen field of damming and controlling rivers.

He was a loyal friend, but he never allowed friendship to interfere with the administering of discipline if needed or in the best interest of any work under his charge. Many have said of him, "No man could deny his word, integrity, or ability."

On April 19, 1891, he was married to Ida M. Wanamaker, and there were two children, Eugene A. Cushman, who died in Texas, and Dorothy E. (Cushman) Littlewood, who, with Mrs. Cushman, survives him.

He was a member of the New York State Society of Professional Engineers, and of the Benevolent and Protective Order of Elks.

Mr. Cushman was elected a Member of the American Society of Civil Engineers on January 3, 1906.

HARRY JOCELYN DIGNUM, M. Am. Soc. C. E.[1]

DIED DECEMBER 26, 1936

Harry Jocelyn Dignum, who was born on June 26, 1878, in Falmouth, Jamaica, British West Indies, was one of the pioneer engineers who risked their lives in the early days of construction on the Panama Canal when American engineers began to change the Isthmus from a pest hole of disease to a place of comfortable homes and modern methods of living.

His birthplace, the seaport of Falmouth in the Parish of Trelawny, was one of the oldest sugar-producing areas in the West Indies. His father, Judge Andrew B. Dignum, Resident Magistrate of Trelawny, was an ardent sportsman, and the owner of a fine racing stable. It is not strange, therefore, that Harry Jocelyn Dignum was known for his love of fishing, shooting, riding, and polo, as well as for his strength of character, engineering ability, and as an authority on sugar production.

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[1] Memoir prepared by Norman J. Dignum, Esq., Miami, Fla.

In 1899, he was graduated from Jamaica College, and went to work for the Jamaican Government Railroad Company, building concrete tunnels, keeping costs, etc. After two years in this work he left the Island of Jamaica for Ecuador to take a position as Junior Engineer on the Guayaquil to Quito Railroad. After having risen to be Engineer in Charge of Camp and Construction on this railroad, Mr. Dignum returned to Jamaica with blackwater fever. Three of his senior engineers and sixty of the one hundred imported laborers had succumbed to the ravages of malarial and yellow fevers.

The following year, fully recovered from his illness, he returned to the service of the Jamaican Government Railroad Company as Engineer in Charge of Terminal Buildings and Construction. In 1905, he again left Jamaica, this time to work for the Isthmian Canal Commission as Instrumentman and Transitman, and, later, as Engineer in charge of surveys and sewer construction, dams, and water supplies. He returned to the Jamaican Government Railroad Company in 1908 as Superintendent of Construction and Maintenance of Railroad Tracks and Buildings.

In 1909, Mr. Dignum was appointed by the Government as Superintendent for Messrs. Nicholson and Corlette in charge of construction of Government buildings.

The following year, 1910, he again left Jamaica to go to Cuba where he was employed as Superintendent of Construction and Engineer of Surveys for Water Supplies by the United Fruit Company, in Preston. From 1914 to 1915 he was Resident Engineer of the Cuba Railroad Company and the Guantanamo Sugar Company on railroad location and construction.

Back with the United Fruit Company in 1916, he spent two years in Tela, Spanish Honduras, as Engineer in charge of bridge construction and Superintendent of general construction. Still with the United Fruit Company, he was transferred to Jamaica to take charge of topographical surveys. For the four years between 1918 and 1922 Mr. Dignum was General Superintendent of the Baragua Sugar Company in charge of railroad surveys and construction, general construction, and the design and construction of wharves and lighters. He also charted and supervised the dredging of ship channels, and was in charge of the transportation of sugar and cane.

From 1933 to 1935, Mr. Dignum was employed by the National City Bank of New York, N. Y., and the Royal Bank of Canada, as Assistant Manager in the readjustment and operation of their sugar mills in Cuba. In 1935, he was attached to the British Legation in Havana, Cuba, as Inspector of Immigration for the Jamaican Government.

He served as Assistant Manager of the Cayo Mambi Sugar Mill for the Atlantic Fruit Company from 1926 to 1930. In 1931 when the business depression forced many sugar companies into bankruptcy, Mr. Dignum returned to Jamaica where he was employed as Surveyor in the Surveyor General's Office, in Kingston, until illness forced his retirement in 1936.

Years of exposure, tropical fevers, and self-sacrificing work finally broke down an iron constitution, and deprived him of his cherished dream—a modern sugar mill on the River Martha Brae in his beloved Trelawny. On December 26, 1936, at the age of 58, a Master Mason, with thirty-eight years of experience in every line of civil engineering and at the peak of his capabilities, he died at his home in the suburb of Kingston, Jamaica. He is survived by his widow, Helen A. Dignum, two sons, Norman J. and Harry, and one daughter, Daphne.

Mr. Dignum was elected an Associate Member of the American Society of Civil Engineers on April 2, 1913, and a Member on April 4, 1922.

JOSEPH OSCAR ECKERSLEY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 19, 1930

Joseph Oscar Eckersley was born on September 12, 1873, in New York, N. Y. He received his preliminary education in the New York public schools, attended Columbia University from 1895 to 1897, and was graduated in 1899 from Cooper Union Institute, with the degree of Bachelor of Science. He continued his studies there for two years (1906–1908), and also acted as Night School Instructor in Mechanical and Structural Drawing and Machine Design, receiving his Civil Engineering degree in 1908.

Before his college days were over Mr. Eckersley had obtained some practical engineering experience. He had been employed from 1894 to 1895 as Rodman and Transitman in charge of municipal work for the City Engineer of Mount Vernon, N. Y.; as Transitman with J. A. Doolittle, Landscape Engineer, in charge of the design and layout of a masonry wall at Woodlawn Cemetery, New York City, from 1897 to 1899; as Engineer with Terry and Tench Construction Company, New York City, checking erection sheets for the Boston (Mass.) Elevated Railroad, the New York Rapid Transit Subway, a ten-story steel-frame building, and designing derricks, falseworks, etc., for the erection of structures. In 1900 he served in the Topographical Bureau in the Borough of The Bronx, New York City.

After qualifying in a civil service examination as a bridge draftsman, Mr. Eckersley was appointed in 1901, by the Department of Bridges (later, the Department of Plant and Structures), for the City of New York; and, in 1902, he became Assistant Engineer. The next year he served as Assistant Engineer on design and details of Blackwell's Island Cantilever (later, the Queensborough) Bridge, and on calculations and design for an eye-bar top chord in place of the existing cable for the Manhattan Bridge—both over the East River.

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[1] Memoir prepared by W. R. Bascome, M. Am. Soc. C. E., and Theodore Belzner, Affiliate, Am. Soc. C. E.

Mr. Eckersley was Resident Engineer-in-Charge of the construction of the University Heights Bridge over the Harlem River, including caisson and coffer-dam work, sewers, dock work, etc., the cost of which amounted to approximately $900 000. This bridge was completed in 1908.

As Assistant Engineer, Mr. Eckersley also had responsible charge of reviewing the design and check calculations for different types of loading for Blackwell's Island Bridge, and adjustments of the structure prepared by the late William H. Burr, Hon. M. Am. Soc. C. E., and the late Alfred P. Boller and H. W. Hodges, Members, Am. Soc. C. E., Consulting Engineers. In 1916, he made preliminary studies of layout and design for the Tri-Borough Bridge, connecting the Boroughs of Manhattan, The Bronx, and Queens, and studies for the Hendrick Hudson Memorial Bridge over the Harlem Ship Canal.

In January, 1916, Mr. Eckersley was appointed Engineer-in-Charge of the Division of Design, and supervised the design of the Eastchester Bridge, over Eastchester Creek, in The Bronx, a double-leaf bascule bridge with a span of 127 ft 8 in., and the Beach Channel and North Channel Bridges, on the line of Cross Bay Boulevard, Jamaica Bay, Queens. The latter are double-leaf bascule bridges with a span of 123 ft. He was also in charge of the design for the Unionport Bridge, over Westchester Creek, in The Bronx, a double-leaf bascule with a span of 75 ft, and the Roosevelt Avenue Bridge, over the Flushing River—one of the largest lift bridges in the United States—which is a double-leaf bascule bridge with a span of 212 ft. The Roosevelt Avenue Bridge has one roadway and two footwalks on the lower deck, and three tracks for rapid transit railway trains on the upper deck.

Mr. Eckersley was also in charge of the Division of Design when plans and specifications were prepared for ten housing stations or garages in the various Boroughs, for the Department of Sanitation, New York City, two destructor plants or incinerators, and the Central Motor Repair Shop, in New York City.

From August, 1924, to January, 1927, he was associated with the Department of Public Markets, in New York City, as Chief Engineer of the design, preparation of plans, specifications, and construction of The Bronx Terminal Market, and the designs for markets in Manhattan and Brooklyn.

In January, 1927, Mr. Eckersley returned to the Department of Plant and Structures where he remained until his death on November 19, 1930.

He is survived by his widow, Mrs. Mary (Connolly), Eckersley, and a son, Joseph O. Eckersley, Jr.

As a specialist in bridge engineering, his most important accomplishments were in the designs and calculations for the great East River Bridges.

Mr. Eckersley was a lover of music and frequently attended the opera. He had many friends, and all who knew him were proud of this acquaintance. He was a member of the American Mathematical Society, and the Masonic Fraternity.

An intimate view of his character and outstanding qualities as an engineer is expressed briefly in the following tribute made by the members of Mr. Eckersley's staff, and his many engineering associates and friends:

"Mr. Eckersley affectionately known as 'J. O. E.,' was a deep and penetrating thinker. He read mathematics in many languages.

"As a professional man he was active in the interchange of technical information and opinion and in the maintenance of his professional and ethical standards.

"As a man, he was wise, honest and conscientious, considerate of the interests and opinions of his fellow men, a devoted husband and father.

"He was a man of sterling character. His exceptional ability, and judgment, and spirit of fairness in dealing with those with whom he came in close contact, won their admiration and respect. Although mild in manner, he was firm in his decisions and insistent in carrying out work according to the best practice. His clear-sighted analysis of conditions enabled him to arrive quickly at the proper solution of a problem and to point out the fault or error in a proper design."

"Mr. Eckersley was possessed of a keen scientific mind, and a thorough knowledge of mathematics and mechanics, he was always interested in developing a theoretical perfect structure.

"His untimely demise has removed an Engineer who was devoted to his profession and whose genial nature has won him a host of friends, both professionally and socially.

"His professional work was marked by a high degree of integrity. His death is deeply deplored by all his associates by whom he was held in the highest esteem and who hold his memory in affectionate regard."

A tribute to Mr. Eckersley was made by Leon S. Moisseiff, M. Am. Soc. C. E., as follows:

"I have known Mr. Eckersley from the time he came into the Department. He worked in my division a long time and I had a great liking for him and considered him one of my friends.

"* * * he was much interested in mathematics and had acquired a large library in that line especially in its more abstract branches. He was a profound student of mathematics and also showed much interest in philosophy, related as these subjects are. He was a thorough engineer.

"As a man he was very loyal and dependable. He had a high sense of justice and always endeavored to follow his views in life.

"He was one of the few men who leave in the minds of the people associated with him an indelible impression of personality and character. His memory is dear to me."

D. B. Steinman, M. Am. Soc. C. E., writes:

"I have always held Mr. Eckersley in the very highest regard for his character, interests and professional ability."

Mr. Eckersley was elected a Member of the American Society of Civil Engineers on August 31, 1909.

ALFRED DOUGLAS FLINN, M. Am. Soc. C. E.[1]

Died March 14, 1937

The death on March 14, 1937, of Alfred D. Flinn, Director, Engineering Foundation, brought to an end a career of exceptional engineering accomplishment encompassing in the first half of his life water supply development and technical journalism and in the latter half administration of the properties and the research activities of the four National Engineering Societies.

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[1] Memoir prepared by Clinton L. Bogert, J. Vipond Davies, and Otis E. Hovey, Members, Am. Soc. C. E.

Alfred Douglas Flinn was the eldest child of Matthew Bonner and Sarah Jane (Jones) Flinn. He was born in New Berlin, Union Co., Pa., on August 4, 1869, in the old pre-Revolutionary homestead. His father was a practicing physician. He came of Irish-Scottish Presbyterian stock on his father's side, and among his ancestors on his mother's side were the early settlers of Connecticut. He was the eldest of six children of a busy country doctor who was often away from home over the country side on sick calls; during these absences, as recalled by his brother, Professor Frederick B. Flinn, Alfred D. Flinn "was a great comfort to his mother" and helped her to take care of the younger children. This willingness to lighten the responsibilities of others was one of Dr. Flinn's outstanding traits.

Another outstanding trait was his seriousness of mind, colored, however, with a prompt appreciation of humor. He only read the more serious books, and very rarely if ever attended the theatre. He was reading Shakespeare assiduously before he was eleven years old. At about this time in his boyhood, his next younger brother, to whom he was very much attached, died from injuries inflicted by another boy. After the brother's death, Alfred assumed a very serious and grown-up attitude toward life. His brother believes that this incident had a great deal to do with this attitude, and his lack of appreciation of the frivolities of life. His guiding motive was to improve himself and those around him.

Mr. Flinn started his schooling in Reedsville, Pa., but was graduated from the Providence, R. I., High School, whither his family had later moved. After graduating from High School, he went for a short period with the Boston and Albany Railroad Company in the Clerical Department, changing later to employment in the largest hardware company in Worcester, Mass. He remained in this work until 1889 when at twenty years of age, he was offered a junior partnership in the Company. This brought him to a crisis in his life, and he made the important choice of a career. He rejected a future career in the hardware business, for one in engineering.

He entered the Worcester Polytechnic Institute in the fall of 1889 and was graduated in 1893, with the degree of Bachelor of Science, in Civil Engineering, at which time he received the Salisbury Prize for high quality of work. His graduating thesis, "The Cippoletti Trapezoidal Weir," was based on large-scale experiments at the Holyoke Testing Flume. It was published[2] as a paper by the Society in 1894, and has been helpful in promoting the use of this means for measuring irrigation water in the United States.

After starting to work in Boston, Mass., Mr. Flinn took post-graduate evening courses at the Massachusetts Institute of Technology, and undertook home study, in mathematics, bridge design, and in other engineering subjects, and in science and literature. The notes which he made at this time were models of clarity and completeness. During the summer vacations, he was not idle, but found employment in engineering work with the City Engineer of Providence, R. I.

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[2] *Transactions*, Am. Soc. C. E., Vol. XXXII (1894), p. 9.

Upon graduation from college, Mr. Flinn's first employment was with the firm of Rice and Evans, Civil Engineers, of Boston, of which the late George S. Rice, M. Am. Soc. C. E., of New York Subway fame, was head. With this firm he worked for several months on early studies for rapid transit in Boston, embracing the so-called "Alley Route"; he also was engaged on surveys for a proposed large public park.

The period from the fall of 1893 to the end of 1917—more than 24 years —was devoted mostly to specialization in water supply engineering, with the exception of a 2-yr interlude for editorial work.

During the winter and spring of 1893–94, Mr. Flinn was with the Associated Factory Mutual Fire Insurance Companies, of Boston, on experimental work on sprinklers and the flow of water in pipes; and on factory surveys for fire prevention. He then went to the Boston City Engineer's Office, in the Waterworks Division, on experimental work on all forms of fire-fighting apparatus used by the City of Boston, also, on pumping engines and boilers, and the flow of water in large pipes. He also made studies for the design of cast-iron pipes, special castings, and valves, and routine surveys for field work on the construction of pipe lines, reservoirs, and pumping stations.

On the formation of the Massachusetts Metropolitan Water Works in 1895, Mr. Flinn became Section Engineer on surveys, construction, and studies within the Metropolitan Area, under the late Frederic P. Stearns, Past-President Am. Soc. C. E., as Chief Engineer. Later, he became Principal Assistant Engineer in charge of the preparation of designs and contracts, the writing of reports, and various investigations, including experimental studies of dams, and the effects of ice on dams and other structures. These works comprised the Wachusett and Sudbury Systems of additional supply for more than twenty communities, and included large dams, reservoirs, aqueducts, pipe lines, stand-pipes, and pumping stations. The most notable structures were the Wachusett Dam and Aqueduct, the Weston Aqueduct, the Sudbury Aqueduct, and the Chestnut Hill Pumping Station.

In October, 1902, Mr. Flinn moved from Boston to New York City, in the vicinity of which (and chiefly in Westchester County), he made his future home. From October, 1902 to August, 1904, he was Managing Editor of *Engineering Record,* giving special attention to water supply and municipal engineering.

For the next year (to August, 1905) he was with the Croton Aqueduct Commissioners as General Inspector, which was a meaningless civil service name for responsible charge of the New York Office. Under the late J. Waldo Smith, Hon. M. Am. Soc. C. E., as Chief Engineer, Mr. Flinn was engaged on the designs, specifications, and contracts for the completion of the New Croton Dam and Reservoir, the Muscoot Dam and Reservoir, the one basin of the Jerome Park Reservoir, and the Jerome Park Pumping Station.

He was also in charge of studies of rainfall and stream flow of the Croton water-shed and of possible reservoir sites, to the end of providing quickly additional water to meet the City's needs until a new source could be developed. This work included the preparation of designs, contracts, and estimates of cost for the Cross River and the Croton Falls Dams and Reservoirs. These works cost several million dollars.

In August, 1905, when Mr. Smith was called from the Aqueduct Commission to become Chief Engineer of the Board of Water Supply of New York City, he showed his appreciation of Mr. Flinn's peculiar talents for organization by taking him with him, and making him in 13 yr, successively, Division Engineer (8 months); Department Engineer of the all-important Headquarters Department (9 yr); and Deputy Chief Engineer for 3½ yr. During this time, on Mr. Smith's absence from the City, Mr. Flinn acted as Chief Engineer. The work of the Board during the years of Mr. Flinn's connection with it covered construction totaling $140 000 000, and included the building of the Catskill Aqueduct, 90 miles long, the Ashokan and Kensico Dams, and two minor dams and four reservoirs, and studies for the Schoharie diversion. During this period, the combined forces of the Board and all contractors engaged on the Catskill Aqueduct reached a peak of 25 000.

Mr. Flinn's first responsibility was to recruit the Engineering Force of several hundred men under civil service restrictions and delays; he personally interviewed a large proportion of the engineers after they had passed the civil service tests. As Department Engineer, he was instrumental in outlining surveys and field studies; he was responsible for the designs for practically all structures and equipment; for the preparation of contracts and specifications; for estimates of cost; for the writing of special and annual reports; for inspection of manufactured materials; for the selection of consulting architects and co-operation with them; for co-operation with general and special consulting engineers, geologists, landscape engineers, sanitation specialists, and, during the World War, with the military organizations guarding the water-works; for the establishment and management of a special laboratory for research and the testing of materials, and for co-operation with other organizations in special researches and tests relating to steels, brasses, cements, and other supplies; for the preliminary studies, experimental tests, and final design of the aerators for the entire flow in the Catskill Aqueduct; for water-softening plants, filtration plants, small sewage disposal works; and, lastly, for City Tunnel No. 1 and the connecting pipe lines.

Mr. Flinn was called on to recommend to Mr. Smith, and to the late John R. Freeman Past-President, and Hon. M. Am. Soc. C. E., F. P. Stearns, Past-President, Am. Soc. C. E., and W. H. Burr, M. Am. Soc. C. E., his superiors, important changes in the materials of water-works construction. Until 1905, metropolitan water systems had used massive masonry constructions, grade tunnels (the exception being the pressure tunnel of the New Croton Aqueduct under the Harlem River), brick lining for tunnels, and cast-iron pipe siphons. As a result of Mr. Flinn's recommendations the Board of Water Supply used rubble concrete and ashlar stone facing for the dams; pressure tunnels to a greater extent than had been used heretofore in New York; concrete lining for all tunnels; aqueducts of concrete without brick lining; steel pipe siphons with concrete linings and jackets; reinforced concrete siphons; removable reinforced concrete beams and slabs in gate-chambers; reinforced concrete tile for chamber roofs; architecturally attractive superstructures; and landscaping and sodding for the tunnel spoil banks. It was due largely to his sense of fitness that Kensico Dam received such notable architectural treatment, and that the grounds below the dam were so strikingly landscaped.

Mr. Flinn undertook special investigations of the causes of defects in concretes in several locations and of "season cracking" of brasses; supervised the studies of an improved flexible joint for large submerged pipe lines; of special devices for closing the access and unwatering shafts of the pressure tunnels, including a novel scheme for unwatering; of grouting of rock tunnels; of unusual methods of core drilling for exploration purposes; of methods of construction and materials for large earth dams; and of drainage galleries and other special features of masonry dams.

His duties included also a wide variety of administrative activities and contacts with public officials. He delivered the illustrated address descriptive of the Catskill Aqueduct, the reservoirs, and structures, as part of the ceremonies by the New York City Government at its celebration of the introduction of the Catskill water, in October, 1917.

The value of his services to the Board of Water Supply is expressed in the resolution on his death, which reads as follows:

> "*Whereas,* Alfred D. Flinn, formerly department engineer, Headquarters department, and deputy chief engineer of the Board of Water Supply, departed this life on March 14, 1937, now, therefore,
>
> "*Be it Resolved,* that the Board of Water Supply expresses its deep sorrow at the death of Mr. Flinn, whose services to the City were vitally important in organizing the engineering bureau at the inception of the Catskill project and in designing the works and directing their construction.
>
> "*Be It Further Resolved,* that this resolution be spread on the minutes of the Board of Water Supply and that a copy thereof be transmitted to Mrs. Flinn."

Near the end of his service with the Board of Water Supply, Mr. Flinn became a Member of the Board of Direction of the Society and served on the Board of Engineering Societies Library. This was at the time of important changes in the Library, and as Chairman of a Special Committee, he investigated the various systems of classifications used in libraries and presented a plan for the re-organization of the Library, and the unification of the book collections of the several Societies into one well-arranged library.

On January 1, 1918, Mr. Flinn resigned from the Board of Water Supply to take up the most congenial work of his life with the several joint organizations of the Engineering Societies. In this position he was eminently successful; and he worked in it happily and untiringly until his health failed. One of the members of the 1918 Board of the United Engineering Societies who served two terms in this formative period, writes:

> "Flinn came as a godsend to us. * * * In the early years before Flinn's advent, we were constantly in difficulties. * * * Flinn's coming in to those Boards made a very different picture of it. The business and ef-

forts of those joint Boards went along much more smoothly than in the earlier days, and the 'Foundation' and the other organizations benefited very much."

The primary purpose in engaging the same person to be the executive officer of two or more organizations simultaneously was economy. No one of the organizations participating in joint arrangements could then afford a separate establishment. Operations were simplified and the work was reduced. For the Engineering Foundation and the Division of Engineering of National Research Council, the arrangement assured harmony of program and avoidance of duplication in closely related activities.

From January 1, 1918, to the time that his health began to fail in 1934, Dr. Flinn was Secretary (Executive Officer) of the United Engineering Society and its successors, Engineering Foundation, Inc., and United Engineering Trustees, Inc. The position entailed supervision of the Engineering Societies Building; execution and records of actions of the Board of Trustees, including administration of trusts and other funds; and secretarial services to standing and special committees.

He was Secretary of The Engineering Foundation from December, 1917, and Secretary and Director from 1922, until his death. He initiated or aided in developing and financing Fatigue of Metals Research (in co-operation with National Research Council), the Arch Dam Investigation, Alloys of Iron Research, Education Research Committee, including the pamphlet "Engineering: A Career—A Culture", smaller researches, assistance to special research committees of the Founder Societies, and organizations co-operating with them. His duties involved relationships with industries, universities, governmental bureaus, National Research Council, American Engineering Council, Society for the Promotion of Engineering Education, and other bodies, in all parts of the United States, and correspondence with other countries. The work included writing, compiling, or editing annual and special reports for the Foundation and its Committees, the Research Narratives in leaflet and book forms, Directory of Hydraulic Laboratories in the United States, and other publications.

In co-operation with National Research Council he aided in establishing the Personnel Research Federation, the Highway Research Board, the American Bureau of Welding, and in conducting the Marine Piling Investigation (damage to structures by marine borers), and compiling the Directory of Industrial Research Laboratories in the United States.

He was Secretary of the Engineering Council from January, 1918, to the time of its disbanding on December 31, 1920, to make way for the Federated American Engineering Societies, later known as the American Engineering Council. As such he dealt with "questions of general interest to engineers and to the public", some of which were activities incident to the World War and reconstruction, classification and compensation of engineers, uniform State law for "Registration of Architects, Engineers and Land Surveyors", improvement of the Patent Office, Engineering Societies Employment Bureau, and the National Public Works Department. An office was maintained at Washington, D. C., from January, 1919, to the disbanding of the Council.

He was connected with the Division of Engineering, National Research Council, as First Vice-President Chairman, November 1, 1920, to December 8, 1921, and then as Chairman until February 15, 1923, when he resigned. His activities included co-operation with The Engineering Foundation, other technical and scientific bodies, universities, industries, and governmental bureaus; and researches related to testing of hardness of metals and other substances, uses of tellurium and selenium, highways, welding, moulding sand for foundries, marine piling, pulverizing (of material for technical processing), electrical insulation, high-speed tool steels, heat transmission, applications of psychology and psychiatry to industry, rock drill steels, magnetic analysis, and phosphorus and sulfur in steel.

Dr. Flinn was Assistant Secretary of the John Fritz Medal Fund Corporation from January 16, 1920, until he resigned in October, 1935. He was elected Secretary and Treasurer of Daniel Guggenheim Medal Fund, Incorporated, on February 29, 1928. He resigned as Treasurer on May 3, 1935, and as Secretary on May 6, 1936. He served temporarily other bodies whose routine work was committed to United Engineering Trustees, Inc., from time to time, and only gave up these duties as ill health required.

Dr. Flinn received many honors for his work in engineering. The University of Louvain, Belgium, conferred the honorary degree of Doctor of Applied Sciences on him at its 500th Anniversary celebration in 1927, and the Worcester (Mass.) Polytechnic Institute, from which he had received his degree of Bachelor of Science in 1893, made him an Honorary Doctor of Engineering in 1932. He was an Honorary Member of the Masaryk Academy of Prague, and held other Czecho-Slovakian decorations.

He held membership in many societies, notably:

American Society of Civil Engineers (Director, 1917–1919);
American Institute of Mining and Metallurgical Engineers;
American Association for the Advancement of Science (Fellow);
American Mathematical Society;
American Iron and Steel Institute;
Municipal Engineers of the City of New York (Past-President); Gold Medal for the best paper of year.
Sigma Xi Scientific Fraternity;
The Century Association;
The Engineers Club of New York;
Masaryk Academy, Prague; Honorary Foreign Member of Section of Engineering Sciences;
Knight of the Order of the White Lion, Republic of Czecho-Slovakia;
Personnel Research Federation (Past-President and an organizer);
Engineering Societies Library Board, representing the American Society of Civil Engineers;
Boston Society of Civil Engineers, Librarian (honorary) for several years; re-organized and cataloged the Library and enlarged its collection;
National Research Council: Member of Executive Board for five years; Assistant Secretary (complimentary) as Secretary of the Engineering Foundation; Vice-Chairman and then Chairman of Division of Engineering; Vice-Chairman of Research Information on Service;
American Society for Testing Materials;
New England Water Works Association.

Dr. Flinn had always been interested in writing, and had a unique concise style which was easily recognized even though his name did not head the article. His hobby was to omit the article, "the," wherever possible. He never signed a letter "Yours very truly" and often omitted the salutation. He considered middle initials superfluous.

In 1911, he was requested by McGraw Hill Book Company to undertake a reference book on water-works. In this work, he recruited Robert Spurr Weston, and Clinton L. Bogert, Members, Am. Soc. C. E., and the late John H. Gregory, M. Am. Soc. C. E. Professor Gregory withdrew during the first year, but the other three authors worked together for five years producing the first edition. "Waterworks Handbook" is now in its third edition, and is used extensively in the United States and other countries. With characteristic modesty, Dr. Flinn inscribed the "Preface" as a compilation—not an original contribution to literature.

He was justly proud of his originating and editing for twelve years the Research Narratives published by The Engineering Foundation, which were widely circulated and reprinted in this and other countries.

Dr. Flinn was always considerate of the working hours of his assistants, but not of himself; he was a most persistent worker. It was his habit, before ill-health came, to be at his desk at home at 5:00 A.M., working for two hours at writing and editing before breakfast. In his early years, he kept Sunday free of his life work, but in later years, much of his time on Sundays was devoted to his absorbing writing of reports, and editorial work.

He was a pleasing and instructive public speaker. For two years in the Nineties he gave courses on water-works and water supply and on sewerage to Senior Classes of Lawrence Scientific School, Harvard University. On invitation, he gave the Chester S. Lyman Memorial Lectures on Conservation of Water by Storage, to the Senior Class, at the Sheffield Scientific School, Yale University.

From his youth, he was interested in church work, and during his residence in New York City he was a member of the Fifth Avenue Baptist Church.

In 1900, Dr. Flinn was married to Mary Brownell Davis, of Boston, who survives him. During the past year he sensed the impending summons, and sold his spacious home in Park Hill, Yonkers, and gave away most of his books and furniture, and retired to Heathcote Inn, Scarsdale, N. Y., where he died. His distribution of his belongings in this orderly manner, by gift rather than by sale, before his death, was thoroughly characteristic of the man.

He was noted for his logical presentation of facts, accuracy of statement, and thoroughness of detail. This characteristic went to the extent of leaving a concise summary of his activities from which this memoir is largely compiled.

Alfred D. Flinn was a Christian gentleman dominated by the highest ideals. His unfailing courtesy and thoughtfulness of others were outstanding characteristics. He worked unselfishly for the advancement of Engineers and the Profession of Engineering. His influence toward these objectives has been great, and will be more fully appreciated as passing years yield a better comprehension of his accomplishments.

Dr. Flinn was elected an Associate Member of the American Society of Civil Engineers on December 6, 1899, and a Member on May 2, 1905. He served as a Director of the Society from 1917 to 1919.

SIFROY JOSEPH FORTIN, M. Am. Soc. C. E.[1]

DIED MAY 16, 1937

Sifroy Joseph Fortin, the son of Jules and Sophie (Roy) Fortin, was born at Saint Sebastien, Iberville County, Province of Quebec, Canada, on May 6, 1867. Mr. Fortin's ancestors left France and settled in Canada about 1650.

After his preparatory schooling, which he completed at the Plateau Commercial Academy, in Montreal, Que., Canada, Mr. Fortin enrolled in the Civil Engineering Course at Laval University, the Applied Science Faculty of which is now the "École Polytechnique." He obtained his degree from this institution in 1889, at the same time winning the Governor General's Medal for the highest standing in his Class.

On leaving college, Mr. Fortin's first position was that of Inspector on the construction of the Lachine Aqueduct, which supplies water to the City of Montreal. From February, to December, 1890, he was engaged as Draftsman with Livering and Garrigues, Civil Engineers and Contractors, of Philadelphia, Pa.

In December, 1890, he joined the Staff of the American Bridge Company and continued with that Company until January, 1892, working on the detailing, design, and estimating of structural steelwork for bridges and buildings. Mr. Fortin was then employed for a short period with the Riverside Bridge Works, in New Jersey, and, later, became Assistant to the late Charles O. Brown, M. Am. Soc. C. E., Consulting Engineer and Contractor, of Brooklyn, N. Y., where he had charge of the estimating and drafting-room and made the calculations, under Mr. Brown's supervision, for the following structures: Manhattan Life Building, 17 stories and tower (cost, $1 000 000); Empire Building, 22 stories (cost, $2 000 000); addition of 6 stories to Standard Oil Building (cost, $1 000 000); and a number of smaller structures, including an armory in New York, N. Y. During the latter period of his engagement with Mr. Brown, he helped him with the revision of the sections on steel, concrete, and foundations in the Building Code of the City of New York.

In 1899, Mr. Fortin joined the Milliken Brothers, Civil Engineers and Contractors, of New York City, with which firm he remained until 1914, a period of fifteen years. He represented this Company in various parts of the world, spending two years in Hawaii, two years in China, Japan, and The Philippines, and eleven years in Mexico.

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[1] Memoir prepared by Vernon R. Davies, Esq., Westmount, Montreal, Que., Canada.

In Hawaii he prepared the specifications, obtained the contracts, and supervised the erection of six large sugar refineries. His Company having next successfully negotiated contracts with the Russian Government for the steelwork in the arsenal, and dry dock and ship-repairing plant, at Port Arthur, China, Mr. Fortin proceeded to China where he supervised the completion of the design and took charge of the erection for the owners. The benefits to his Company's clients were of short duration because, about two years after Mr. Fortin's departure from China, the Russo-Japanese War broke out and the greater part of these works were destroyed by the bombardment of this city.

Upon leaving the Orient, Mr. Fortin next negotiated, for his firm, the contracts with the Mexican Federal Government for the steelwork in the following buildings: Post Office (estimated cost, $1 000 000); War and Navy Building ($1 800 000); The National Theatre ($4 000 000), not finished until later; and the Federal Legislative Palace or Parliament Building (estimated cost, $10 000 000), which was not completed. Mr. Fortin prepared the plans for, and supervised the steelwork in, these buildings.

During the period of his residence in Mexico as Representative of Milliken Brothers, his Company acted as Government engineers under President Porfirio Diaz's régime. The Mexican Government assigned to them the task of preparing plans and specifications for the steelwork and reinforced concrete, foundations, and other structural details for various public undertakings. Several collateral contracts covering fireproofing and roofing were also obtained and carried out for the Mexican Government. In addition to the Government's contracts, Mr. Fortin also obtained for his firm some large contracts from private individuals and corporations in Mexico for the building of car sheds, sugar refineries, bridges, and other structures.

The construction of the Parliament Buildings, in the City of Mexico, offered a special problem in foundations due to the nature of the sub-soil, which is a volcanic ash possessing a low bearing value. A system of foundations developed by Mr. Fortin was accepted by the Ministry of Transportation and Public Works and, later, formed the basis of a monograph entitled "Novel System of Foundations Used in Connection with the Federal Legislative Palace, Mexico City." [2]

The Revolution which broke out in Mexico in 1914 caused the suspension of all work upon which Mr. Fortin was engaged, and he returned to Canada to become Structural Engineer for the Public Works Department in Ottawa, Ont. This position he held until 1918, during which time he was engaged on the construction of railway and highway bridges, steel and wooden terminal freight sheds, dry-docks, wharves, dams, and hydraulic projects. The works undertaken included bridges at Kingston, Ont., Ottawa, and Riviere-des Quinze, Que.; dry-docks at Halifax, Nova Scotia; Levis, Que.; and Esquimalt, B. C.; sheds at Halifax, St. John, N. B., and Vancouver, B. C.; in addition to docks and dams throughout Canada.

Mr. Fortin left the Government Service in September, 1918, to become Deputy Director and Deputy Chief Engineer of the Public Works Department of the City of Montreal. His duties included the general supervision of all engineering projects undertaken by the City connected with the Roads Division, sewers, and municipal shops.

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[2] *Transactions*, Canadian Soc. of Civ. Engrs., Vol. III, 1916.

In April, 1922, he was called upon to form the Technical Commission and became its first Chairman. In the same year he was appointed to the Water Board. One of the functions of the Technical Commission was the development of a town planning scheme for the City, and Mr. Fortin incorporated his studies in this connection into a monograph entitled "Considerations on a Plan of Organization and on Functions of a Town Planning Commission for the Island of Montreal." [3]

In September, 1930, he became Consulting Engineer to the Executive Committee of the City of Montreal, and in the same year he revived the Technical Commission which had previously lapsed. This Commission was composed of seven specialists, with Mr. Fortin acting as Secretary. He carried on with these duties until his retirement in 1935, due to failing health. He died two years later at the age of 70 yr.

Mr. Fortin was a member of the Engineering Institute of Canada, the Corporation of Professional Engineers of Quebec, and the Association des Anciens Eleves de l'École Polytechnique. He was a life member of the Board of Governors of l'École Polytechnique, and a member of the Marlborough Golf and Country Club.

On June 14, 1894, he was married to Maria Graziella Lalonde, who survives him. He is also survived by an only child, Gaston L. Fortin, Civil Engineer.

Mr. Fortin's great diversity of interests won for him a wide circle of friends. He had a very intimate knowledge of the French and English languages and literatures and also a very thorough knowledge of Spanish. His principal hobby was the collection of records relating to the early history of the French in Canada, and particularly those records relating to the history of his own family, which he incorporated into a brochure which was printed a short time before he died.

Mr. Fortin was elected an Associate Member of the American Society of Civil Engineers on November 6, 1895, and a Member on December 6, 1904.

ABRAHAM GETMON GIDEON, M. Am. Soc. C. E.[1]

DIED DECEMBER 17, 1936

Abraham Getmon Gideon was born in Tula, Russia, on July 1, 1871, the son of Myron O. and Reba (Rubens) Getmon Gideon. When Mr. Gideon was a boy, railroad construction near the town of his birth aroused in him an interest in Engineering, which later took him to the United States. He was graduated from Cornell University, at Ithaca, N. Y., in Civil Engineering and also attended Law School for two years. His scholastic record was outstanding.

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[3] *Revue Trimestrielle,* and the *Engineering Journal of Canada,* 1924.

[1] Memoir prepared by Paul W. Mack, Assoc. M. Am. Soc. C. E.

Mr. Gideon began his engineering career in Little Falls, N. Y., under Stephen E. Babcock, City Engineer, in the branch (water-works) of engineering which he followed all his life.

From 1900 to 1902, he was Assistant Engineer and Engineer in charge of design, specifications, etc., of the drainage and sewerage systems of Havana, Cuba. His work there was the foundation of important subsequent work in the Philippine Islands.

He spent one year with Westinghouse, Church, Kerr, and Company, on the design and investigation of a hydro-electric plant on the Housatonic River, and from there he went to Manila, Philippine Islands, in 1905. For twenty-five years, he devoted his engineering ability to the problems of the water supply and sewerage of Manila. Mr. Gideon designed the storm drains and was Superintendent of Water Supply and Sewers, and, from 1910 to 1919, he was Chief Engineer of the Department of Sewers and Water-Works. In 1919, he served as Consultant to the Quartermaster, United States Army, on the water supply of the fortifications on Corregidor Island and designed the water supply system for Fort Stotensburg.

Mr. Gideon had the confidence of the late Governor General Leonard Wood, and during the World War acted as City Engineer of Manila. He saw the need of changing the municipal Water Department to a Metropolitan Water District. Governor General Wood approved of the change and appointed him to be the first Manager. His financial ability turned the annual deficit of the Water Department to a surplus of the Metropolitan Water District, and an enlarged supply was planned and financed. He saw this work well on the way to completion before his retirement.

Mr. Gideon was a Director of the Philippine National Bank and the Bank of the Philippine Islands, the Philippine Railroad Company, and the Manila Hotel. He was a member of the Army and Navy Club, the Casino Espanol, and a Thirty-second Degree Mason.

He is survived by his widow, Mrs. Edith Mary Gideon, and a son, David of Oakland, Calif.

Mr. Gideon was elected an Associate Member of the American Society of Civil Engineers on September 3, 1902 and a Member, on November 1, 1910.

WILLIAM JOHNSON HARAHAN, M. Am. Soc. C. E.[1]

Died December 14, 1937

William Johnson Harahan was born in Nashville, Tenn., on December 22, 1867, the son of James Theodore Harahan, late President of the Illinois Central Railroad Company, and Mary B. (Kehoe) Harahan. The family moved to New Orleans, La., soon after he was born, and it was there that he received his primary education, and was graduated from St. John's College, in 1885, at the age of seventeen. He had worked as shop apprentice, and in various other minor positions, for the Louisville and Nashville Railroad Company, during his college vacation periods, starting in 1881, when he was fourteen years old. Upon graduation he began working regularly for the Louisville and Nashville Railroad Company, and from that time until his death he was engaged continuously in railroad work.

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[1] Memoir prepared by a Committee of the Cleveland Section, consisting of William Michel and Leonard B. Allen, Members, Am. Soc. C. E.

Father and son spanned almost three-quarters of a century of railroading. For twenty-five years, they worked on the same railroads together, much of the time side by side. Almost the complete history of railroad transportation may be told in the span of years served by the Harahans. The elder Harahan started his career on the old Virginia Midland Railroad, later part of the Southern Railroad System, in 1864, in the most devastating period of the war between the States.

Mr. Harahan spent more than fifty-two years working with railroads. They were his work and his play. This devotion, and his outstanding ability, together with a natural heritage from his father, made him one of the greatest railroad men in the United States. He built several thousand miles of new railroads and additions to existing railroads, and was in charge of other large construction work for many years, including the building of a number of large bridges over important streams.

He worked in various minor capacities on the Louisville and Nashville Railroad, from 1885 to 1886, and from 1886 to 1889, he was in the Engineering Department of the same Railroad Company as Chainman, Transitman on Location, and Assistant Engineer.

In 1889, when he was twenty-two years of age, Mr. Harahan was appointed Division Engineer of the Cincinnati Division of the Chesapeake and Ohio Railway Company. One of the first tasks assigned to him in that capacity was the repair of the damage wrought by an Ohio River flood. That experience was to be of practical help to him in the recurrence of flood conditions on the Chesapeake and Ohio Railroad in later years, including the floods of 1936 and 1937.

In 1890, he received appointment as Superintendent of Structures of the Baltimore and Ohio Southwestern Railroad Company. In 1892, he became Roadmaster and Trainmaster of the Illinois Central Railroad Company, and progressed rapidly to be Superintendent, Chief Engineer, Assistant General Manager, and General Manager. In 1905 he was appointed Fourth Vice-President, in charge of Traffic and Engineering. In 1907, Mr. Harahan relinquished the Illinois Central Vice-Presidency and accepted appointment as Assistant to the President of the Erie Railroad Company, remaining in this position until January, 1911, when he was elected Vice-President in charge of Engineering. In September, 1912, Mr. Harahan was elected President of the Seaboard Air Line Railroad Company and served in this office until 1918. Between 1918 and 1920—the Federal Control period—he was Federal Manager of the Seaboard Air Line. From April to December, 1920, he served the Director General of Railroads as a Staff Officer. In December, 1920, he was elected to the Presidency of the Chesapeake and Ohio and Hocking Valley Railway Companies and served these roads in that capacity until May, 1929, when impaired health prompted him to give up this post to become Senior Vice-President of the two railroad companies, and also of the Pere Marquette Railroad Company. The late J. J. Bernet, who succeeded Mr. Harahan to the Presidency of the Chesapeake and Ohio and Hocking Valley Railway Companies in 1929, died in 1935. Improved in health by that time, Mr. Harahan was prevailed upon to resume the Presidency of the Chesapeake and Ohio Railroad Company, and, at the same time, take upon his shoulders the responsibilities of the same office for the New York, Chicago and St. Louis Railroad Company (Nickel Plate Road) and Pere Marquette Railroad Company. The Hocking Valley had lost its identity in 1930 when it was merged with the Chesapeake and Ohio Railroad Company.

In addition to being President and a Director of each of the three railroads at his death, Mr. Harahan was also a member of the Board of Directors of the Erie Railroad.

During his first tenure as President of the Chesapeake and Ohio and Hocking Valley Railway Companies, Mr. Harahan developed an executive personnel that was to become recognized as one of the most efficient in the railway field. He had a faculty of appraising ability. He liked to encourage talent and reward achievement. The organization as it is to-day is of his making, for Mr. Bernet, on assuming direction of the roads, was surprised at the proficiency he found in all departments and made no changes. What he thought of it was perhaps best summed up in his comment that, "the road is so well run it seems to run itself."

At the inception of Mr. Harahan's administration, the Chesapeake and Ohio Railway did not have a physical connection with its subsidiary, the Hocking Valley Railway, its outlet for coal moving to Great Lakes ports. This physical connection was supplied in 1927, through the construction, under Mr. Harahan, of a 63-mile double-track line between Greeg and Valley Crossing, from Columbus to Waverly in Southern Ohio, without grade crossings (with one exception), and with ideal ruling grades. The line was completed in ten months, an all-time record in American railroad construction.

Physically and financially, in the period from 1923 to 1929, the road underwent transformation and acquired a recognized status as one of the world's most efficient transportation machines. Its sphere in the marketing of coal, once narrowly circumscribed, was expanded East and West, North and Northwest, through affiliations that were provided with such related lines as the Nickel Plate Road, the Pere Marquette, and the Erie.

Mr. Harahan's instincts were scholarly. He read the Classics. He had a definite liking for history. Apart from a fondness for baseball, reading seemed to be his only hobby.

On the personal side, Mr. Harahan lived quietly and unpretentiously, and as a devoted husband and father. He was married twice. His first wife was Susannah M. Smith, of Nashville, Tenn., to whom he was married in 1890 and by whom he had twelve children. Following the death of his first wife, he was married on March 23, 1935, to Mrs. Elizabeth M. Smith. They resided at Wade Park Manor, Cleveland, Ohio.

Mr. Harahan's pride, and the source of his greatest happiness, was his large family of children, comprising his own by his first wife, along with the children of his widowed sister, whom he adopted, and some other orphaned children, not related to him, whom he reared. At one time, the children numbered as many as twenty-one. The dining rooms in the Harahan homes at Chicago, Ill., and at Norfolk, and Richmond, Va., before the children grew up, had to be quite extensive.

Besides his second wife, and her son by a former marriage, Thomas W. Smith, he is survived by eight children of his first marriage, including three sons: James Theodore II, William Johnson, Jr., and Joseph Patrick; and five daughters: Mrs. Louis Moquin and Mrs. Richard Markey, Catherine Agatha, Virginia Mary (Sister Catherine Virginia of the Sisters of Notre Dame), and Theodora Margaret Harahan. He was also survived by a brother, J. T. Harahan, Jr., and two sisters, Mrs. A. H. Effinger, and Mrs. Theodore H. Templeton.

A devout Churchman, Mr. Harahan interested himself actively in Catholic charities. He was Vice-President of the Society for the Propagation of the Faith of the Diocese of Richmond, Honorary Vice-President of the Catholic Conference on Family Life, Director of the National Council of Catholic Men, and a member of the Knights of Columbus.

He was a member of the American Railway Guild, Railroad Club of New York, Midday Club (New York), Union League (Chicago), the Country Club of Virginia, the Commonwealth Club, and the Westmoreland Club, of Richmond.

Mr. Harahan was elected an Associate Member of the American Society of Civil Engineers on October 3, 1894, and a Member, on October 7, 1903.

LEWIS MUHLENBERG HAUPT, M. Am. Soc. C. E.[1]

Died March 10, 1937

Lewis Muhlenberg Haupt was born in Gettysburg, Pa., on March 21, 1844, the son of General Herman Haupt, M. Am. Soc. C. E., Scientist and Engineer, and Builder of Military Roads and Bridges during the Civil War, and his wife, Anne Cecilia (Keller) Haupt.

He studied at the Lawrence Scientific School of Harvard University, at Boston, Mass.; and in the autumn of 1863 he was appointed by President Lincoln to the United States Military Academy, at West Point, N. Y. He was graduated, in 1867, with the rank of Second Lieutenant, United States Corps of Engineers.

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[1] Memoir prepared by Charles E. Billin, M. Am. Soc. C. E.

Lieutenant Haupt's first work in the Service was with a party conducting the triangulation of Lake Superior and the "on-shore" line survey and "off-shore" hydrography of the Lake and of St. Marys River. In 1868, he was appointed Engineer Officer of the Fifth Military District of Texas.

In 1869 Lieutenant Haupt resigned from the Army Engineer Corps to accept the position of Assistant Engineer and Topographer in charge of the surveys of Fairmount Park, in Philadelphia, Pa., in which position he collected data for an elaborate map, located and constructed drives, drains, and other engineering features of this extended pleasure ground.

He was intimately associated with Philadelphia and its civic developments. He soon became interested in the traffic problems of the City, current and future, putting forward the claims that the corner of 33d and Market Streets was the most suitable site for a Union Rail Terminal—a project for which the facilities now exist but which still awaits realization. As early as 1877 he proposed that diagonal streets be cut through from City Hall so as to facilitate the movement of highway traffic.

In 1872, Mr. Haupt was appointed to the Technical Staff of the University of Pennsylvania, at Philadelphia, Pa., as Instructor in Mathematics and Drawing. In 1873, he became Assistant Professor of Civil Engineering, and continued in that capacity until 1875, when he was advanced to be Professor of Civil Engineering, which position he held until 1892.

His extended experience gave him practical knowledge as an Instructor. He had the vision to stimulate earnestness and invention in his students and make them realize the importance of their work. His long summer vacations were spent in practical engineering work, thus giving him a constantly increasing store of experience to draw upon in his teaching. His students were often given the opportunity to help him in these undertakings.

In 1873, his second year at the University, Professor Haupt made the first survey for range lights on the Delaware River, the students working with him, thus bringing them into actual contact with the problems of the times and giving them a working efficiency unattainable by more abstract methods of instruction.

For five years he was also Assistant with the United States Coast and Geodetic Survey in charge of the triangulation of Pennsylvania. His students will remember how, in 1875, they worked with him in determining fixed points on the ground for accurate surveys for latitude, longitude, boundary lines, railroads, etc., and thus, through all the succeeding years of his connection with the University, he worked to make the Department of Civil Engineering rank among the first with those of the most eminent institutions in America. In 1883, the University of Pennsylvania conferred on Professor Haupt the Honorary Degree of Master of Science.

Beginning in 1893, he devoted his time quite fully to the subject of economy in transportation, giving especial attention to deepening the coastwise system of canals. In 1894, he was selected as Engineer in Charge, by the Canal Commission of Philadelphia, to make the surveys and report upon a ship canal between Philadelphia and the sea, across New Jersey. He was also chosen by the Provisional Committee of Pittsburgh, Pa., to report on a ship canal between that city and Lake Erie.

Professor Haupt was frequently consulted in matters pertaining to the economics of transportation, particularly the utility of deep-water canals, and was also called upon to take an active interest in some of the great enterprises of the Western Continent. In 1897 he was appointed by President McKinley to study the feasibility of a canal linking the Atlantic and Pacific Oceans, *via* Nicaragua. He advised such a site, but eventually the Panama site was selected; and, later, he was appointed to the Panama Canal Commission.

Although engrossed in these vast enterprises, he found time to write extensively on technical subjects. He was the author of a number of publications relating to the economies of transportation and the improvement of navigable channels by the utilization of the forces of Nature.

Professor Haupt devoted much time to the problems of beach and harbor protection, with special attention to jetties and to the automatic deepening of harbor entrances. He studied all the major ship canals in the United States and many abroad. In 1887, he patented the Reaction Breakwater, utilizing the forces of tide-water itself to clear and maintain the channel, a principle which has been widely adopted. Of equal utility was his later invention of the "hooked jetty" designed to check coastal erosion. For these inventions he received the highest award at the National Export Exposition, in Philadelphia, in 1899. Their great utility, economy, and efficiency were demonstrated at Aransas Pass, Texas, in 1895 to 1907 as a purely tidal inlet. His inventions were endorsed by experts and engineers throughout the country.

The Elliott Cresson Gold Medal and Premium of the Franklin Institute, in Philadelphia, the highest award within its gift, was presented to Professor Haupt in 1901 for "Improvements in Methods of Removing Ocean Bars." These inventions also won for him the Gold Medal of the Louisiana Purchase Exhibition in 1904.

In 1887, for his paper on the "Physical Phenomena of Harbor Entrances," and for his invention of a system of harbor improvements, he was awarded the Magellanic Premium by the American Philosophical Society—the highest acknowledgment that is in the power of that Society to bestow, and an honor which has been conferred only twice in the last forty-five years.

Combined with another development he used the installation of a double row of open pilings, which device has resulted in the recovery of many acres of beach front at Far Rockaway, Long Island, N. Y., and at many points along the New Jersey Coast.

The writer's acquaintance with Professor Haupt began in 1877 when he was unanimously elected the first President of the Engineers' Club of Philadelphia, at its organization meeting on December 17 of that year. On March 14, 1916, he was elected to Honorary Membership in the Club. The Club owns many of his papers on subjects relating to river and harbor improvements, such as the removal of Smith's Island in the Delaware River, directly opposite Philadelphia; inter-communication of cities; and others covering a wide range, for his pen was an active one.

Professor Haupt published many scientific works, including "Engineering Specifications and Contracts," "The Physical Phenomena of Harbor Entrances," "Canals and Their Economic Relation to Transportation," and "A Move for Better Roads."

He was a member of the American Philosophical Society, National Geographic Society, American Historical Society, American Association for the Advancement of Science, The Franklin Institute of Philadelphia, and The Engineers Club of Philadelphia.

Professor Haupt's personality was most attractive; he was energetic, always genial, and a great favorite with his students and his professional and social friends. Although it is conceded that it requires brains and ability to accomplish great things in life, still it is equally true that even brains and ability will not accomplish much without energy and opportunity.

His life was one of great achievement, backed by ability and pushed forward by energy. His record is an example, and should be an inspiration, to every young engineer.

On June 26, 1873, he was married to Isabella Christiana Cromwell, who died on December 21, 1912. He is survived by four daughters and one son.

Professor Haupt was elected a Member of the American Society of Civil Engineers on June 6, 1888.

WYATT SWIFT HAWKINS, M. Am. Soc. C. E.[1]

Died July 15, 1937

Wyatt Swift Hawkins was born on December 18, 1876, in Hannibal, Mo. He was the son of Judge Thomas W. Hawkins and Anna Belle (Newland) Hawkins. When he was quite young the family moved to Palmyra, Mo., where he attended the local schools, and Centenary College. Although he was not a graduate of the College, he completed practically the full course of study.

After leaving college, Mr. Hawkins went to what was then Indian Territory, later the State of Oklahoma, in the service of the United States Geological Survey and was employed as Field Assistant and Surveyor until that work was completed in 1899. He was then assigned to the Commission to the Five Civilized Tribes and was in charge of the allotment to the Creek and Seminole Indians. In 1906, he was transferred to the service of the Commission of Indian Affairs and made Chief Engineer of the Osage Indian Reservation having charge of the complete re-survey of the reservation.

Immediately, thereafter, he was appointed State Engineer of Oklahoma, under Governor Haskell, and served with him until the close of his administration. From 1908 until 1910, Mr. Hawkins was a Special Engineer with the Missouri, Kansas and Texas Railroad Company, on the design and estimate work connected with the raising of all the tracks to an elevation above high water. This work involved the expenditure of about $14 000 000. In 1910, he went into private consulting work in Oklahoma and Texas and engaged in municipal work and road and bridge design.

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[1] Memoir prepared by Davis K. Jackson, Esq., Jefferson City, Mo.

In 1913, he was called back to Missouri by Governor Major to take the position of State Highway Engineer and Deputy Commissioner of Missouri, and he served in this capacity throughout the Governor's term. In 1915, he returned to Durant, Okla., where he again entered the practice of engineering, acting as consultant on many municipal projects in that area.

While essentially a peace-loving man he offered his services to his country in the spring of 1917, and served as Captain with the 55th Regiment of Engineers until 1919, having fifteen months overseas duty. His time in the Army was devoted mainly to road and railway construction and this was especially true in regard to his service overseas.

At the conclusion of his period of Army service Mr. Hawkins again returned to Durant, and the private practice of engineering, specializing in municipal work. In 1922, he moved to Greenwood, Miss., where he was the Consultant on a program involving the expenditure of $850 000 for public improvements. In 1925, he moved to Gulfport, Miss., where he was the Engineer on the Harrison County road and bridge program, involving an expenditure of $3 176 000; the Biloxi, Miss., $335 000 paving program; the $140 000 general improvement works in Pass Christian, Miss.; and a $5 000 000 project at Gulf Hills, Miss., as well as many smaller projects in that area. He supervised more than $10 000 000 worth of work in Mississippi alone.

At the conclusion of this work in 1929, he received the appointment as an Engineer of Special Assignment with the Missouri State Highway Department. At this time, Missouri was just inaugurating its "Farm-to-Market" program, and Mr. Hawkins' position involved the selection of various routes over the State. He was especially well fitted for such work due to his wide experience in road design and construction. With the inception of the Federal Emergency Relief Administration he was placed in charge of all such projects being handled by the State Highway Department, and continued in charge when that work was transferred to the Works Progress Administration. In September, 1935, he was appointed to the post of Division Engineer of Division No. 3 of the State Highway Department, and returned to his old home in Hannibal. He served in this capacity until his death. His life was an example of complete devotion to his family, friends, and work.

Although ill during a greater part of the last year of his life, he seldom missed a day at the office to see that everything was running smoothly. It has been justly said that there was no man in the Highway Department who was more popular nor who had more friends. He took a special interest in his subordinates. An Elder in the Presbyterian Church, he was always active in its circles.

On May 17, 1898, he was married to May Colbert, of Oklahoma (at that time Indian Territory) who survives him, with three daughters, Mrs. Ragan B. Jones, Mrs. J. M. Garrison, and Mrs. Emory Lampkin. Two brothers, Robert N. Hawkins and Alfred L. Hawkins, also survive him.

Mr. Hawkins was elected a Member of the American Society of Civil Engineers, on June 6, 1927.

HARRY MONMOUTH HERBERT, M. Am. Soc. C. E.[1]

Died October 6, 1937

Harry Monmouth Herbert was born at Monmouth Battle Ground, Monmouth County, N. J., on January 31, 1857, the son of Major Jacob Van Wickle Herbert and Eliza Jane (Smock) Herbert.

He received the degree of Bachelor of Science from Rutgers University, at New Brunswick, N. J., in 1878, and the degree of Master of Science in 1881.

Mr. Herbert was Assistant Engineer, with the United States Army, under the late General John Newton, U. S. Army (*Retired*), Hon. M. Am. Soc. C. E., from 1878 to 1883, in which capacity he was employed on surveys at Elizabethport, and Raritan River, N. J., and the Gowanus Canal in Brooklyn, N. Y.; in charge of the dredging and diking on the Raritan River, and of dredging at Gowanus Bay and Buttermilk Channel, in New York Harbor.

From 1884 to 1885, he was Assistant Engineer on topographical surveys and on the location of railroad lines at and near Lake Hopatcong, Greenwood Lake, and Barnegat Beach, in New Jersey; from 1885 to 1886, he was Assistant Engineer on the location of the Pennsylvania, Slatington, and New England (later the Pennsylvania, Poughkeepsie, and Boston) Railroad, between Slatington, Pa., and Washingtonville, N. J., and also on the location of about fifteen miles of the Bloomsburg and Sullivan Railroad in the vicinity of Ganoga Lake.

From 1887 to 1893, Mr. Herbert was Resident Engineer for the Lehigh Valley Railroad Company. The following work was done under his supervision: Construction of the main line between South Plainfield, N. J., and Newark, N. J.; location and construction of the Perth Amboy and Raritan River Branch; monumenting the line and right of way from Newark and Perth Amboy to Easton, Pa.; and the rebuilding of nearly all the bridges on the Easton and Amboy Division.

In 1893, Mr. Herbert had general engineering offices at Bound Brook, N. J., and was Borough Engineer of Bound Brook from 1893 to 1895, during which time he designed and built the sewerage system there; designed and built the extension to the Roselle, N. J., sewers; designed a sewerage system for Lambertville, N. J., as well as a system for East and North Somerville, N. J., and a system of water supply for Middlebrook Heights, N. J.; designed a water supply for South Bound Brook, N. J.; and, with Louis Focht, Assoc. M. Am. Soc. C. E., designed a sewerage system for Flemington, N. J.

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[1] Memoir prepared by William H. Boardman, Esq., Philadelphia, Pa.

From December, 1896, to December, 1898, he was Chief Engineer of the New York and Philadelphia Traction Company, in which capacity he located and made plans and specifications for more than 25 miles of line.

From 1898 to 1900, Mr. Herbert was Director and Chief Engineer of the American Engineering Company, which Company built, equipped, and placed in operation the following electric railways: Newtown, Pa., Street Railway; The Elmira and Seneca Lake Railway; and a line between Battle Creek, Mich., and Kalamazoo, Mich., for the Michigan Traction Company.

As Contracting Engineer from 1901 to 1904, he built, equipped, and placed in operation the Philadelphia and Easton Railway, a high-speed electric railway between Doylestown, Pa., and Easton.

On April 12, 1906, Mr. Herbert was appointed a member of the State Sewerage Commission of New Jersey by Governor Stokes. The powers and duties lodged in the State Sewerage Commission were vested in the Board of Health of the State of New Jersey by an Act which became effective on April 16, 1908. Mr. Herbert was appointed Chief of the Division of Sewerage and Water Supplies of the State Board of Health on May 26, 1908.

He resigned from the aforementioned position on May 2, 1911; and on June 2, 1911, he was appointed Chief Engineer of the Camden, N. J., Water Department, where he remained until 1913. He then returned to Bound Brook and resumed private practice in hydraulic and sanitary fields. He was appointed Borough Engineer of Bound Brook in 1918 and served until 1925.

He was later appointed Borough Engineer of Seaside Heights, N. J., where he inaugurated many municipal improvements, including a sewerage system and sewage disposal works, and extensions and improvements to the water supply.

Mr. Herbert established a reputation as one of the foremost engineers in hydraulic and sanitary work, and was relentless in his fight to guard the public health. He was held in the highest esteem by his associates for his personal probity and business integrity.

Mr. Herbert was married in November, 1891, to Ella Tucker, of Bound Brook, who survives him, as does a daughter, Mrs. Norman J. Elmes; two grandchildren, Anita and Norman J. Elmes; and a sister, Emma S. Herbert.

He was a member of the following organizations: Society of American Military Engineers; American Water Works Association; American Public Health Association; and Past-President of the New Jersey Sanitary Association.

Mr. Herbert was elected a Member of the American Society of Civil Engineers on April 6, 1898.

NICHOLAS SNOWDEN HILL, JR., M. Am. Soc. C. E.[1]

Died October 18, 1936

Nicholas Snowden Hill, Jr., was born in Baltimore County, Maryland, on June 18, 1869, the son of Major Nicholas Snowden Hill and Mary (Cocke) Hill. He was graduated from Stevens Institute of Technology, at Hoboken, N. J., in 1892, with the degree of Mechanical Engineer.

Mr. Hill entered professional life as Mechanical Engineer for the South Side Elevated Railroad, in Chicago, Ill., in 1892. The following year, he returned to Baltimore, Md., to serve as Engineer-Secretary of the Baltimore Sewerage Commission, for which Commission he formulated a report which served as a basis for the existing sewerage system of Baltimore. Later, he was appointed Engineer of the Electrical Commission of Baltimore and designed an underground conduit system for the accommodation of all the electrical wiring of the city, the first comprehensive underground conduit system in the United States. Following this, Mr. Hill was made Chief Engineer of the Water Department of the City of Baltimore, and in this capacity, he re-organized the Department and designed and supervised extensive improvement to the water-works system.

In 1898, he left Baltimore to become Chief Engineer of the Charleston (S. C.) Consolidated Railway, Gas, and Electric Company and as such he was active in the consolidation of that Company's various properties and rebuilt and rehabilitated them.

After the completion of this work, Mr. Hill was appointed Chief Engineer of the Water Department of the City of New York, N. Y. As Executive of the Department he was in responsible charge of improvements amounting to several millions of dollars annually, and instituted a comprehensive investigation of the ground-water supplies of Long Island.

In 1903, Mr. Hill resigned as Chief Engineer of the Department to establish a private engineering practice, and continued as a Consulting Engineer with offices in New York City until his death. As such, he gave special attention to problems of water supply and sanitary engineering, and to public utility appraisals, being recognized as one of the foremost experts in the field of public utility valuations.

In his consulting engineering work he established an international reputation, and was identified with notable enterprises in the United States, Canada, Mexico, Cuba, and China. During his professional life he was retained by more than three hundred and fifty States, districts, counties, municipalities, and private corporations, and he appraised more than one hundred water companies and departments.

Some of the more notable projects and problems with which Mr. Hill was associated in his capacity as Consultant were: Member of a committee of three engineers to study and report on the pollution of harbors along the

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[1] Memoir prepared by George H. Buck, M. Am. Soc. C. E., and Charles F. Jost, Assoc. M. Am. Soc. C. E.

Atlantic seaboard; a report on the Bronx Valley Sewer Project for the Bronx Valley Sewer Commission, of Westchester County, New York; Chairman of the Jerome Park Filter Commission to consider the advisability of filtering the Croton Water Supply for New York City; Consulting Engineer for Jersey City, N. J., regarding the proper method of treating the Jersey City Water Supply; a report on additional water supply for the City of Baltimore; a comprehensive report on water supply, drainage, sewerage, and sewage treatment for Tampa, Fla., and environs, and the design and supervision of construction of new water supply system; collaboration on investigation and report upon available new water supplies for Regina, Saskatchewan, Canada, and surrounding communities; a survey and report on water supply resources and water supply conditions throughout Westchester County and the best means of developing and organizing a county controlled system; a complete and comprehensive report for the City of Albany, N. Y., on sources of water supply available and recommended development; Consulting Engineer to the New Jersey Water Policy Commission, including study and investigation of the best means of developing and conserving the water resources of the State, and a comprehensive survey of resources and requirements; a report on the development of a water supply for the Metropolitan District of Northern Ohio; testimony for the State of New Jersey in the Delaware diversion case before Special Master appointed by United States Supreme Court; and investigation and report on water supply and financial administration of the Shanghai, China, Water Works Company, Limited.

During the World War Mr. Hill was Supervising Engineer of Camp Merritt, New Jersey, and Designing and Supervising Engineer for the United States Housing Bureau in connection with its developments at Norfolk and Portsmouth, Va.

Among his outstanding designs in the field of water-works engineering is the Oradell Dam, of the Hackensack, N. J., Water Company. He was awarded the Thomas Fitch Rowland Prize of the Society in 1926 for his paper on the design of the Oradell Dam.[2]

Mr. Hill served as President of the Board of Water Commissioners of East Orange, N. J., from 1912 to 1920, and as President of the Consolidated Water Company of Suburban New York from 1921 to 1923. He was closely associated with the Hackensack Water Company, serving about fifty odd communities in Northern New Jersey. He acted as Consulting Engineer for this Company from 1910, as Director from 1922, and as President from 1926.

He was an Honorary Member and Past-President of the American Water Works Association, and donor of the Nicholas S. Hill, Jr., Cup; a former member of the Council of the American Institute of Consulting Engineers; Chairman and member of several important technical committees; and the author of numerous technical papers.

He was a member of the following technical societies: American Institute of Consulting Engineers (Member of Council; and Chairman, Committee on Professional Practice and Ethics); American Society of Mechanical Engineers (Membership Committee; and Standardization Committee and Sub-

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[2] "The Oradell Dam of the Hackensack Water Company," by Nicholas S. Hill, Jr., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1181.

Committees); American Society for Municipal Improvements (Chairman, Committee on Water-Works); American Water Works Association (Honorary Member; President 1915–1916; Chairman, Committees on Private Fire Protection; Electrolysis and Electrical Interference; and Water Quality and Water Treatment; Committee on Collection of Material and "Manual of Water Works Practice"; and Committee on Diven Memorial); American Society for Testing Materials (Committee on Cast Iron); American City Planning Institute; American Electrochemical Society; National Conference on City Planning; National Municipal League; International City Managers Association; American Committee on Electrolysis (Representative of American Water Works Association); American Committee on World Power Conference; Committee on Standardization of Pipe Fittings and Flanges; New England Water Works Association; Connecticut Society of Civil Engineers; New England Sewage Works Association; New York Sewage Works Association; New Jersey Sewage Works Association; Municipal Engineers of the City of New York; American Association for the Advancement of Science; and American Academy of Political and Social Science. He was also an Associate Member of the American Institute of Electrical Engineers, and a Fellow of the American Health Association (Committee for Public Health Engineering Section).

Mr. Hill was a member of the following non-technical organizations: American Child Hygiene Association; National Economic League; National Geographic Society; Metropolitan Museum of Art, New York City; Merchants Association of New York City (Committee on Sanitation); and Bergen County, New Jersey, Chamber of Commerce (Executive Committee).

He also devoted time to social duties and was a member of the following: Civitan Club (Governor); Citizens Guild of Washington's Boyhood Home; University Club, New York City; New York Railroad Club; Southern Maryland Society; Manhattan Club; Pequot Yacht Club; Old Colony Club; Brooklawn Country Club; Fairfield County Hunt Club; Longshore Beach and Country Club; Alumni Association of Stevens Institute of Technology; and Delta Tau Delta Fraternity.

Mr. Hill died in New York City on October 18, 1936. He is survived by his widow, Florence (Acheson) Hill; a sister, Mary (Hill) Mills; and two children by a former marriage, Nicholas Snowden Hill, III, and Isabelle (Hill) Guthrie.

Mr. Hill was elected a Member of the American Society of Civil Engineers on October 10, 1916.

ROBERT BRUCE HOADLEY, JR., M. Am. Soc. C. E.[1]

DIED DECEMBER 25, 1937

Robert Bruce Hoadley, Jr., was born at Camanche, Iowa, on March 22, 1879, the son of Robert B. Hoadley and Harriet (Rood) Hoadley. The family removed from Iowa to Binghamton, N. Y., where he attended the public schools. After his graduation from the Binghamton Central High School, he entered Union College, at Schenectady, N. Y., from which he was graduated, in 1902, with the degree of Bachelor of Engineering.

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[1] Memoir prepared by John A. Giles, Esq., Binghamton, N. Y.

During his summer vacation, previous to graduation, he was employed by the United States Geological Survey. In 1902, he entered the service of the Western Maryland Railway Company as Topographer, and continued as Resident Engineer and Locating Engineer on its lines in West Virginia and Pennsylvania until 1907.

In the first six months of 1907, Mr. Hoadley was employed by the Western Pacific Railway Company in charge of its Drafting Office on 400 miles of new construction in Nevada. In the latter part of the same year he was engaged by the Ely (Nev.) Townsite Company to take charge of a steam-turbine power development.

Returning to his home in Binghamton in 1908, he entered the service of the New York State Highway Department and was connected with this Department until 1917 as Assistant Engineer and First Assistant Engineer in charge of surveys, design, and construction on highways in Broome and Delaware Counties, New York.

Leaving the New York State service, after nine years with its Highway Department, Mr. Hoadley entered private practice, with an office in Binghamton, and from 1919 until his death, had continued under the partnership of Hoadley and Giles.

During his twenty years as a practicing engineer, his endeavors resulted in the design and successful construction and operation of many water and sewerage projects in the development of West Endicott and Johnson City for the Endicott Johnson Corporation, in neighboring villages in the territory of the Binghamton District, in adjacent counties of Pennsylvania, in subdivision work and in the planning of its necessary utilities in New York and Florida. He was also employed by the J. G. White Engineering Corporation for a period of one year, in 1927–1928, in a comprehensive sewerage and sewage disposal design for Bogota, the capital of Colombia.

His endeavors were not entirely confined to his private interests, for he served as a member of the City Planning Commission, Consultant for the Traffic Committee, and as a member of the Committee for the Rehabilitation of the Water-Works, in his own community—the City of Binghamton—for a period of several years.

His career was notable because of his interest in the younger members of the profession and his helpfulness in promoting their progress and success, his fairness and ethical dealings in his engineering contact with others, and a rare sense of humor and human understanding which endeared him to his many associates.

In 1909, Mr. Hoadley was married to Florence C. Jones, of Burlington, Vt., who died in 1929.

He was killed on Christmas Day, 1937, in an automobile accident in Jacksonville, Fla., where he had gone for a holiday visit to his daughter, Mrs.

Richard (Jane Hoadley) VanKleeck, who with two grand-children, and two sisters, Mrs. Agnes Tay, and Mrs. Eugenia Comstock, survives him.

Mr. Hoadley was elected a Member of the American Society of Civil Engineers on July 11, 1921.

RICHARD FREDERICK HOFFMARK, M. Am. Soc. C. E.[1]

Died November 29, 1937

Richard Frederick Hoffmark, the son of Simon Hoffmark and Marie (Baumgarten) Hoffmark, was born in Bremen, Germany, on April 7, 1882. Brought to America at an early age by his parents, who chose to settle in the Middle West, Mr. Hoffmark attended Purdue University, at Lafayette, Ind., where he was active in athletics, particularly football, and was graduated with the degree of Bachelor of Science in Civil Engineering in June, 1906.

On June 18, 1906, immediately after his graduation, Mr. Hoffmark was appointed a Rodman in the service of the Isthmian Canal Commission. He arrived in Empire, Canal Zone, and reported for duty on June 25, 1906. On October 1, 1906, he was appointed Levelman. He was transferred from the Culebra Division of the Isthmian Canal Commission to the Panama Railroad on June 28, 1907, and was promoted to Levelman II, on July 1, 1907, to Instrumentman on February 1, 1908, and was again promoted on November 1, 1908. He resigned from the service of the Panama Railroad on June 6, 1910. During this period Mr. Hoffmark received the groundwork of varied construction experience that was to stand him in such good stead in later years, for he also held positions as Foreman in the Bridge and Building Department and Supervisor of Construction in the Roadmaster Department of the Panama Railroad, and thus early gained wide, practical experience.

It was at this time that Mr. Hoffmark established a friendship that was to be lifelong, with Ralph Budd, M. Am. Soc. C. E., who was then Chief Engineer of the Panama Railroad.

In May, 1910, he returned to the United States to accept a position as Transitman and Resident Engineer on the construction of the Oregon Trunk Railroad in Central Oregon, under Mr. Budd, who was then Chief Engineer of the Oregon Trunk Railroad Company. Mr. Hoffmark worked directly under the late J. C. Baxter, M. Am. Soc. C. E., who was then Mr. Budd's Assistant.

On the completion of this assignment, Mr. Hoffmark, on June 28, 1911, was married to Helen Slifer, daughter of the late Hiram J. Slifer, M. Am. Soc. C. E. (who was formerly General Manager of the Panama Railroad, and later a Lieutenant Colonel with the 21st Engineers, United States Army).

Mr. Hoffmark left the United States immediately after his marriage to accept an appointment as District Superintendent of the Alaska Road

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[1] Memoir prepared by Natt McDougall, Esq., Portland, Ore.

Commission, for the Nome District, under the direct charge of Lieutenant, later, Colonel, G. E. Edgerton, Corps of Engineers, U. S. Army, M. Am. Soc. C. E. Mr. Hoffmark had charge of all general road and bridge building for the District, and the maintenance of the same. This included locating and staking trails in winter, for the use of dog teams, chiefly in carrying the mail, those on land being constructed with some effort for permanency. His work with the Alaska Road Commission was very valuable and was recognized by at least one promotion during his short period of service.

In November, 1912, Mr. Hoffmark made a connection that was to last for nearly twenty years, for at that time he joined the staff of Guthrie, McDougall and Company as a Field Superintendent in charge of the construction of railroads, tunnels, bridges, and irrigation canals.

In July, 1916, Mr. Hoffmark was called to the Western Office of the Company, which had become A. Guthrie and Company, Incorporated, to assume the position of Western General Superintendent, in Portland, Ore. The firm operated as contractors on all types of heavy construction projects, and Mr. Hoffmark's field of activity became the Far West in general and the Pacific Northwest in particular. His work was as broad in scope as the territory which he covered. For the next sixteen years he was the Contracting Executive responsible for the preparation of proposals and the construction of millions of dollars worth of all kinds of projects, including railroads, highways, tunnels, bridges, dams, irrigation canals, and buildings and structures of varied types and kinds, all of which contributed their share to his ever-broadening practical knowledge of construction in all its phases.

Notable among the projects that were constructed under Mr. Hoffmark's active direction during this period were: The West Okanagon Valley Irrigation project, in the State of Washington; a 4-mile section of the Hetch Hetchy water supply tunnels, in California; the Cushman Dam and Power-House for the City of Tacoma, Wash.; Echo Dam, near Ogden, Utah; the Leaburg hydro-electric project, in Oregon; several large paper mills; and many lesser construction jobs.

In November, 1925, A. Guthrie and Company, Incorporated, was awarded the contract for the construction of the new 8-mile Cascade Tunnel for the Great Northern Railway Company, 90 miles east of Seattle, Wash., with the provision that the tunnel was to be completed and ready for operation in three years. This was accepted as a challenge to the spirit of the force that was to do the work, and to "Dick" Hoffmark fell the vital task of leadership in the field. He moved directly to the site of operations and remained on the ground almost constantly during the organization period, which lasted more than a year, planning and directing the work which had to be done on a high-speed schedule never before equalled or even approached in the field of tunnel driving. New and untried methods were devised and put into operation, modern equipment which has since become standard, was developed for the work, and for three years almost to the day the crews kept up their incessant rounds of drilling, blasting, and mucking, imbued with that spirit of creative workmanship and forceful action so well exemplified by the man under whose active direction they were working.

The Cascade Tunnel being successfully completed in record time, Mr. Hoffmark was elected a Vice-President of A. Guthrie and Company, Incorporated, and, in 1928, he journeyed to the far-off Peruvian Andes to direct the work of his Company's forces on a large tunnel contract for the Cerro de Pasco Copper Corporation. This work also included a sizable railroad location project, and due to his varied experience, Mr. Hoffmark was exceptionally well fitted to handle the assignment.

The construction of the 100-mile extension of the Great Northern Railway, from Klamath Falls, Ore., to Bieber, Calif., was the last job handled by Mr. Hoffmark for A. Guthrie and Company, Incorporated, and upon its completion in the spring of 1932 he resigned his position with this firm and accepted an appointment as Assistant to the President of Carey, Baxter and Kennedy, of New York, N. Y. He had direct charge of the building of the Outside Bowl, on Long Island, for the Madison Square Garden Corporation.

In the spring of 1933 he was made General Manager for Woods Brothers Construction Company, of Lincoln, Nebr., and had general charge of its contracts, which consisted chiefly of the construction of pile dikes, revetments, dredging, and similar work on the Missouri and Mississippi Rivers. He held this position until his death at his home in Lincoln, on November 29, 1937, which was due to an illness of several months' duration.

As Engineer in the fullest sense of the word, " Dick " Hoffmark was noted as an unusually able student of construction in all its manifold phases. Endowed with rare analytical ability, he supplemented this with a knack for practicality in his work that was founded on a broad understanding of sound engineering fundamentals and wide experience. His love for his work and his fellow men placed him high in the esteem of those who knew and worked with him, and his passing is a source of regret to construction men of both high and low degree throughout the United States.

At the time of his death, he was a member of the Nebraska Engineering Society and the Engineers' Club, of Lincoln.

He is survived by his widow and one daughter, Helen Jessie Hoffmark.

Mr. Hoffmark was elected an Associate Member of the American Society of Civil Engineers on March 5, 1912, and a Member on December 16, 1929.

THOMAS HERBERT JACKSON, M. Am. Soc. C. E.[1]

Died April 8, 1937

Thomas Herbert Jackson, the son of Noah W. and Pauline (Adams) Jackson, was born at Westmeath, Ont., Canada, on January 18, 1874. He was descended from early Colonial stock, his forebears having settled in the Colonies prior to the Revolution. He was brought to the United States by his parents in 1880 and spent his boyhood on a farm near Montague, Mich., where he experienced early in life the physical toil attendant to farming. He attended Grammar School and High School, in Muskegon, Mich. When the local Congressman consulted the Principal of the High School concerning an appointment to the United States Military Academy, at West Point, N. Y., the Principal gave it as his opinion that young Jackson had the best mind in the school. He was selected for the appointment and entered the Military Academy as a Cadet on June 15, 1895. He was graduated Number Six in his Class and appointed a Second Lieutenant of Engineers on February 15, 1899.

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[1] Memoir prepared under the direction of Major General E. M. Markham, Chf. of Engrs., U. S. Army, M. Am. Soc. C. E.

Lieutenant Jackson's first assignment was as Assistant to the District Engineer, at Portland, Me. He completed the course at the Engineer School of Application, at Willets Point, N. Y., in 1902. In 1903, he went on foreign service in the Philippine Islands, where he was on duty as Engineer Officer, Department of the Visayas, at Iloilo, and performed duties pertinent to the establishment of the Military Post of Fort William McKinley, until August, 1905.

Returning to the United States, Lieutenant Jackson was stationed with the 2d Battalion of Engineers at Fort Leavenworth, Kansas, and having been promoted to the rank of Captain, he was appointed District Engineer of the Third San Francisco (Calif.) District, in 1907. During his 4-yr tour in San Francisco he formulated the plans for the Sacramento River Flood Control Project which is now (1937) nearing completion, at a cost of $50 000 000. The original plan for this project is still referred to as the "Jackson Plan." It provided for the utilization of a system of by-passes in addition to levees for the control of floods. In this respect, it is a prototype of the plan later adopted for control of floods on the Mississippi River. Subsequently, for the period 1911 to 1917, Major Jackson was assigned as District Engineer at Dallas, Tex., and at Wheeling, W. Va. He was promoted to the rank of Major in 1912.

Upon the entry of the United States into the World War, Major Jackson was assigned as Instructor of the Citizens' Training Camp, at Madison Barracks, New York. In August, 1917, as a Colonel of Engineers, he proceeded to France and, in March, 1918, was assigned as Section Engineer of the Advance Section, Services of Supply. After holding various important positions in the Services of Supply, he was assigned on January 6, 1919 as Engineer Supply Officer of the American Expeditionary Forces, at Paris, France, and, in July, was made Chief Engineer Officer of the American Expeditionary Forces. Colonel Jackson was awarded both the Purple Heart and the Distinguished Service Medal for exceptionally meritorious and distinguished services in these various positions. He was also an Officer of the Legion of Honor, France; Commander, Order of Leopold II, Belgium; and had received the Polish Order of Valiant.

Upon his return to the United States, he returned to the grade of Lieutenant Colonel and after a tour on supply duty in the Office of the Chief of Engineers, at Washington, D. C., he proceeded to Camp Grant, Illinois, where he demobilized the 318th Engineers. In 1921, after his promotion to the grade of Colonel, he was assigned as Instructor of the National Guard at Kansas City, Mo., but remained on this duty only a few months before proceeding to San Francisco as Corps Area Engineer. In 1926, Colonel Jackson was assigned as Division Engineer of the Pacific Division, which assignment was followed by a short tour in New Orleans, La., as Division Engineer of the Gulf Division. In 1928, he was appointed President of the Mississippi River Commission and Division Engineer of the Western Division, of the Engineer Department. He had the distinction of being the first Army Officer to be so appointed under the new plan of organization established in 1928, the appointment carrying with it the rank of Brigadier General.

It was in the position of President of the Mississippi River Commission that General Jackson rendered his most outstanding service to his country. He personally established the organization for doing the work on the Mississippi River provided for in the $300 000 000 program approved by Congress in May, 1928. About one-half this program was completed during his 4-yr tour as head of the Commission.

Upon completion of his assignment on the Mississippi, he was appointed Division Engineer, South Atlantic Division, which post he held for two years. General Jackson returned to San Francisco, in 1934, as Division Engineer, South Pacific Division, and President of the California Débris Commission. This was his third detail on the Débris Commission, and he became recognized as a high authority on all phases of hydraulic gold mining. He was retired on March 31, 1937, on account of disability incurred in line of duty, and died on April 8, 1937, at Tsingtao, China, while en route around the world on the S. S. *Reliance.*

He was married on August 24, 1912, at St. Louis, Mo., to Maude Edgar, who survives him.

General Jackson will long be remembered by the Junior Officers of the Corps of Engineers as one who took unfailing interest in their professional development. He was never too busy to spare the time to instruct a junior officer in some phase of his work, which would prepare him for responsible positions in the future.

In view of his broad experience and keen technical mind, General Jackson was regarded in the Corps of Engineers as its outstanding authority on flood control and organization matters.

His citation for the Distinguished Service Medal was as follows:

> "For exceptionally meritorious and distinguished services. As a member of the Division of Construction and Forestry, he displayed untiring energy and marked ability in the performance of his important duties. His stupendous task was fraught with numerous difficulties, which he overcame with noteworthy success, rendering services of signal worth to the American Expeditionary Forces."

In writing of his career, General Malin Craig, Chief of Staff, U. S. Army, concluded:

> "General Jackson was a loyal, able, and zealous officer, strong in convictions and forceful in action. He possessed splendid engineering and or-

ganizing ability, keen initiative, sound judgment, and the capacity for hard work. During his military career of more than forty-one years, he was outstanding in his efficiency and rendered invaluable service to his country during the trying times of war as well as in peace, and was signally honored in his profession. The many commendations of his superiors attest to the esteem in which he was held by his associates. His death is deeply regretted."

General Jackson was elected a Member of the American Society of Civil Engineers on July 15, 1929.

CHRISTIAN PETER JENSEN, M. Am. Soc. C. E.[1]

DIED MAY 5, 1937

Christian Peter Jensen was born in Summerville, Calif., on November 9, 1873. His parents, Martin and Adelheid (Jorgensen) Jensen, came to America from Denmark in 1870, going first to Ohio and, later, to California. They were among the first settlers of Central Colony, in Fresno County, and it was there that Mr. Jensen attended the public schools. After graduation from High School, he entered the Van der Naillen School of Engineering, in Oakland, Calif., from which he was graduated in 1895.

After a short engagement with the Sanborn Map Company, of San Francisco, Calif., Mr. Jensen returned to Fresno and entered the firm of I. Teilman, M. Am. Soc. C. E., Civil Engineer. Later, the two engineers became associates under the firm name of Teilman and Jensen, which association continued until 1906.

The practice of civil engineering in Central California at that time was wide and varied in its scope. Of particular interest was the engineering work done in connection with the location and construction of lumber flumes, logging railroads, mountain roads, and irrigation systems. At this period in the history of Fresno and the San Joaquin Valley, lumbering in the Sierra Nevada Mountains, at the eastern edge of the Valley, was much more of a major industry than it is at the present time. The product of the smaller mountain saw-mills was hauled to the Valley towns, about sixty miles, over rough, dusty, mountain roads, in great wagons drawn by 12 to 16-mule teams, but the larger mills found it economical to float their lumber to the Valley by swiftly flowing water in **V**-shaped timber flumes.

In the mountains these flumes clung to the sides of the canyons, crossing ravines and gullies on high trestles, and continuing on descending grade on trestles until they reached their destination in the planing mills and lumber yards of the San Joaquin Valley towns of Fresno, Madera, Clovis, and Sanger. The location, design, and construction of these flumes called for high engineering skill and judgment. In this period, also, there were still being constructed the large irrigation systems which transformed the dry plains of the San Joaquin Valley to the present fertile fields and vineyards.

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[1] Memoir prepared by a Committee of the San Francisco Section, consisting of Henry D. Dewell and Harold K. Fox, Members, Am. Soc. C. E.

Mr. Jensen had an important part in the engineering features of this early industrial and agricultural development of Fresno County and also of adjacent counties. In 1900, he was Assistant Engineer on the location survey and the construction of the 54-mile lumber flume for the Madera Sugar Pine Company, in Madera County; from 1901 to 1904, he was Chief Engineer of this Company, engaged in the location and construction of logging railroads and related structures; from 1902 to 1917, he was Consulting Engineer for the Madera Canal and Irrigation Company in investigations pertaining to duty of water, legal work, and construction of canals and siphons; from 1904 to 1909, he was Chief Engineer of the Settlers Branch Canal, Kings County, where he located and constructed 50 miles of canal and related structures, including a large weir across the Kings River; from 1910 to 1919, he was Chief Engineer of Reclamation District No. 1 604 on designs and construction of levees on the San Joaquin River; he was also, during this period, Chief Engineer for J. C. Forkner, of Fresno, in the subdivision and irrigation of 6 000 acres.

From 1909 to 1913, Mr. Jensen was City Engineer of Fresno; from 1912 to 1914, Consulting Engineer for the City of Bakersfield, Calif., and Engineer for the City of Dinuba, Calif., on the design and construction of sewers and sewage disposal systems; from 1912 to 1919, he was City Engineer of Clovis; from 1913 to 1917, City Engineer of Sanger; from 1913 to 1919, City Engineer of Reedley, Calif.; from 1913 to 1914, he also served as Engineer on the design and construction of water-works for the Town of Strathmore, Calif.; from 1913 to 1919, Chief Engineer of the Fresno Traction Company; and from 1915 to 1919, City Engineer for the Town of Fowler.

From May, 1917, to January, 1919, Mr. Jensen was associated with his brother, the late Andrew Martin Jensen,[2] M. Am. Soc. C. E., and Mr. J. A. Ross, under the firm name of Jensen, Jensen, and Ross, Civil Engineers.

In 1919 Mr. Jensen became County Surveyor of Fresno County and Chief Engineer of the Fresno County Highways, for the construction of highways under a $4 800 000 bond issue. Mr. Jensen developed the Jensenite Paving Process, which was widely used, and eliminated the payment of royalties to private paving concerns, thus saving the County thousands of dollars. The satisfactory service which Mr. Jensen rendered as County Surveyor is evidenced by the fact that he was returned to that office every four years and occupied the office until his death.

In January, 1896, Mr. Jensen was married to Isabelle Malcolm Baird, of Fresno. The two children of this marriage were Katharine and Donald. Six years later, in 1902, his wife passed away. In August, 1909, he was married to Bertha Franklin Coleman, at San José, Calif. The two children of this marriage were Elizabeth and Mary. He is survived by his widow, Mrs. Bertha Franklin Jensen, and by three of his children, Donald, Elizabeth, and Mary.

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[2] See Memoir, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1504.

Mr. Jensen was active both in professional and civic affairs. He was Past-President of the Engineers' Club of Fresno, of the Fresno Commercial Club, and of the Fresno Exchange Club. He was also an active member of the State Chamber of Commerce. He was Past Master of Los Palmas Lodge, No. 366, of the Free and Accepted Order of Masons; Past Commander of Fresno Commandery No. 29, Knights Templar; Past High Priest of Fresno Chapter No. 69, of the Royal Arch Masons; Past Patron of Raisina Chapter of the Eastern Star, and also of the Royal Order of Amaranth; and Past-President of the Fresno Shrine Club.

Mr. Jensen was elected an Associate Member of the American Society of Civil Engineers on January 31, 1911, and a Member on October 15, 1919.

JOSIAH RAYMOND JOHNSTON, M. Am. Soc. C. E.[1]

Died April 13, 1938

Josiah Raymond Johnston was born on February 13, 1890, in Ottawa, Kans., the son of Josiah K. and Nancy Jane Johnston. His education was begun in the Grade School, at Ottawa, and was continued in the Grade School and High School at Greenfield, Iowa. He was graduated, in 1913, from Iowa State College, at Ames, Iowa, with a degree of Bachelor of Science in Civil Engineering.

Mr. Johnston began his professional work in 1913 as Engineer on drainage in Murray County, Minnesota. Later, he was elected County Surveyor of Murray County, which office he held when the United States entered the World War in 1917.

In May, 1917, Mr. Johnston enrolled in the first Officers' Training Camp, at Fort Snelling, Minnesota, and in July he was commissioned as First Lieutenant, in the Corps of Engineers, United States Army. In October, 1918, he was promoted to the grade of Captain. He commanded Headquarters Company, 313th Engineers, 88th Division, in training at Camp Dodge, Iowa, and, later, in service with the 88th Division in France. He was honorably discharged from the Army as a Captain in August, 1919, his entire war service having been with the 313th Engineers.

From August, 1919, to April, 1923, he was engaged first as a Drainage Engineer in private practice in Southern Minnesota and, later, as an Engineer and as a Consulting Engineer on drainage, paving, and sewer work, with headquarters at Slayton, Minn.

In April, 1923, he accepted the position of Assistant Superintendent, and in December, 1936, was promoted to that of Superintendent, of the Bureau of Construction and Repair, Department of Public Works, at St. Paul, Minn. Except for the period from June, 1934, to February, 1936, Mr. Johnston's service with the Department of Public Works was continuous to the time of his death. From June, 1934, to February, 1936, he served as an Engineer Special Agent for the Division of Investigation, Federal Emergency Administration of Public Works. This work included engineering investigations of construction projects in Minnesota, North Dakota, South Dakota, Nebraska, Washington, Oregon, Idaho, and Montana. He resigned from the Public Works Administration in February, 1936, to return to city service in St. Paul.

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[1] Memoir prepared by William N. Carey and George M. Shepard, Members, Am. Soc. C. E., and George M. Garen, Assoc. M. Am. Soc. C. E.

During the thirteen and one-half years of Mr. Johnston's service with the Department of Public Works in the City of St. Paul, his work embraced many phases of municipal construction. He personally directed the construction of several difficult sewer and paving projects done by city forces. He also supervised the engineering inspection of contract work, sewers, pavements, street widening, grading, etc., totaling more than $10 000 000 in value. For several years, in addition to his other duties, he was responsible for the maintenance of all graded but unpaved streets and alleys in the city—about 450 miles in all. In connection with this work he became a leader in the development of low cost, asphaltic street-surfacing methods ranging from simple, road-oil treatments to various types of low cost, secondary type, asphaltic pavements. To him, more than to any one else, should be given the credit for transforming with little money, the unpaved streets and alleys of St. Paul from a condition where dust, ruts, holes, and mud all too often prevailed to one closely approaching the all-year-round smoothness and usefulness of permanently paved streets.

Mr. Johnston possessed, to a most unusual degree, the faculty for making friends. He was affectionately known to all by his Army nickname, "Josie." He was never too busy to listen to the troubles of others and help them if possible. People of all walks of life from State and City officials to the lowliest laborer who came in contact with him felt and quickly appreciated the real warmth and sincerity of his friendliness. He seemed, too, to have some secret password directly into the affections of little children and dogs. He enjoyed sports—hunting and golf particularly—and the society of men outdoors.

For many years before his death Mr. Johnston put much of his time and energy into the activities of the engineering organizations of his City and State. He held, in succession, all the offices of the St. Paul Engineers' Society from Secretary to President. For several years he was continuously a Director of the Minnesota Federation of Architectural and Engineering Societies and Secretary-Treasurer of that organization, which office he held at the time of his death. He was a member of the Merriam Park Presbyterian Church, of St. Paul, and a Thirty-Second Degree Mason. His college fraternity was Pi Kappa Alpha. He was an active member of the American Legion, a Past Commander of Highland Park Post of St. Paul, and was much interested in American Legion welfare activities.

On June 29, 1918, he was married to Louise Whitney, of Slayton, who survives him. They had no children.

Mr. Johnston was elected a Member of the American Society of Civil Engineers on April 12, 1926.

WINFRED MILLER KALLASCH, M. Am. Soc. C. E.[1]

DIED JUNE 8, 1937

Winfred Miller Kallasch, better known as "Bill" to his host of friends, was born on March 27, 1877, on a farm near Perry, Ill. His father, Edward Kallasch, lived on a farm, but was a cabinet maker by trade.

Winfred Kallasch was graduated, in 1902, from the University of Nebraska, at Lincoln, Nebr., with the degree of Civil Engineer. His first work after leaving the University was with the Chicago Great Western Railroad Company as Masonry Inspector of concrete arches and abutments. This work lasted only a short time, and in October, 1902, he went with the Illinois Central Railroad Company, in the Bridge and Building Department, as Masonry Inspector, being promoted later to the position of Resident Engineer on the construction of a 280-ft draw-bridge over the Yazoo River, at Yazoo, Miss. While with the Illinois Central, he designed and built a temporary lift-bridge for maintenance of traffic during construction, and was then made Assistant Resident Engineer on the Tennessee River Bridge. Following this engagement he was transferred to Memphis, Tenn., as Assistant Engineer in general charge of the construction of concrete arches, abutments, and bridges on Southern Lines. From February to November, 1907, he was with the Construction Department of the Illinois Central System as Resident Engineer on double-track at Memphis, this work including bridges, viaducts, and the re-arrangements of streets and sewers.

In November, 1907, Mr. Kallasch left the Illinois Central Railroad Company for a connection with the Leonard-Martin Construction Company, in the capacity of Superintendent of Construction of bridges, viaducts, and warehouses. For a short period, from December, 1908, to June, 1909, he was with the Missouri Pacific Railway Company as Resident Engineer on the reconstruction of the Arkansas River Bridge, near Helena, Ark., sinking two large piers by the pneumatic process and transferring the 300-ft steel span across the river.

In June, 1909, he returned to the Leonard Construction Company as General Superintendent of Construction on reinforced concrete for the Union Passenger Station, at Gary, Ind.; grade-crossing track elevation for the Illinois Central Railroad Company, at Chicago, Ill.; the Mulnomah Hotel, at Portland, Ore.; the plant of the Corn Products Refining Company, at Pekin, Ill.; the "Soo" Line Freight Terminal, at Chicago, consisting of a yard on an elevated reinforced concrete structure covering eleven city blocks, as well as buildings and other facilities for the rapid handling and storage of freight; and in charge of the construction and field engineering on a zinc ore-reducing and sulfuric acid plant for the Mineral Point Zinc Company, at Wheeling, W. Va.

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[1] Memoir prepared by H. B. Bushnell, Vice-Pres., Western-Austin Co., Aurora, Ill.

In July, 1914, Mr. Kallasch left the Leonard Construction Company to accept the position of Engineer and Executive Officer in charge of operations for the Illinois Hydraulic Stone and Construction Company, at Elgin, Ill. He had been interested in this Company since 1906, but until 1914 had devoted very little of his time to its operation. While with this Company he built concrete roads, municipal improvements, factories, buildings, and warehouses. From May, 1917, to April, 1918, he returned again to the Leonard Construction Company, in Chicago, as General Field Superintendent on the Donnelley Building and the Chicago District Station of the Sinclair Refining Company; the plant of the Inter-Ocean Refining Company, at McCook, Ill., and the Federal Rubber Company's plant, at Milwaukee, Wis. In April, 1918, he was appointed General Superintendent and Manager of Work on Explosives Plant "C", for the United States Government, at Nitro, W. Va., this plant costing approximately $7 000 000. After the completion of this war work, Mr. Kallasch remained with the Leonard Construction Company as General Field Superintendent in charge of the construction of large industrial plants until 1924, at which time he assumed the duties of Plant Superintendent and Manager of the Crown Electric Company, at St. Charles, Ill. In March, 1925, he returned to the Illinois Hydraulic Stone and Construction Company, at Elgin, as Vice-President, which position he held at the time of his death.

On December 5, 1914, Mr. Kallasch was married to Cleo Frances Burtis, of Hudson, Ill., and is survived by his widow, a brother, Ernest Kallasch, of Tacoma, Wash., a sister, Cora Kallasch, of Centralia, Wash., and a niece, Inez Carr, of Los Angeles, Calif.

He was a splendid engineer and constructor, a hard worker and an exceedingly thorough one. He lived an interesting and useful life, and had an active part in the building of many worth while projects.

Mr. Kallasch was elected an Associate Member of the American Society of Civil Engineers on May 6, 1914, and a Member on March 11, 1919.

FRANCIS HOWE KENDALL, M. Am. Soc. C. E.[1]

Died October 7, 1937

Born in Belmont, Mass., on August 23, 1869, Francis Howe Kendall retained his residence in that town throughout his life. His parents were Josiah Shattuck and Martha Helen (Wellington) Kendall, both of whom were descended from families that were among the first to settle in what was originally known as "Pequossette Plantation," later included in the Towns of Belmont and Watertown.

Josiah Kendall was a farmer, and, as might be expected from one of his ancestry, he had a high sense of personal obligation to his home town, and served it for many years in various capacities, such as Assessor, member of the School Committee, and Selectman. This feeling of duty to the community, which New Englanders like to consider as typical of the best of the founders, was an especially marked characteristic of Francis Kendall, who devoted practically all his mature life to public service, especially to the County of Middlesex, by which he was employed, and to the Town of Belmont, in which he lived.

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[1] Memoir prepared by Charles W. Sherman and Harry B. Burley, Members, Am. Soc. C. E.

Francis Kendall attended the public schools at Belmont. After graduation from the High School, in 1885, he decided to study Civil Engineering, and entered the Case School of Applied Science, at Cleveland, Ohio, where he remained for three years. In 1888, he entered the junior class at Massachusetts Institute of Technology, at Boston, Mass., and was graduated with the Class of 1890. During the following year he was Assistant to the Professor of Railroad Engineering at the Institute.

From June, 1891, to November, 1892, Mr. Kendall was Assistant Engineer with the Cleveland, Cincinnati, Chicago and St. Louis Railway Company, in Ohio. In 1892, he became Assistant Engineer under the late Charles Mills, then Engineer of Middlesex County, Massachusetts, and upon Mr. Mills' resignation Mr. Kendall succeeded him as County Engineer, on July 1, 1894, and held this position until his death.

As County Engineer, Mr. Kendall's duties were primarily to advise the County Commissioners on all technical matters relating to roads, dams, bridges, and buildings. In many cases the construction or reconstruction was by joint action of county, State, and town, and proper co-ordination of action was of the greatest importance. The highway system within Middlesex County was developed greatly during his term of service, and a large part of the improvement was accomplished as a result of his wise counsel, patience, and tact, as well as sound engineering judgment. Dams within a county are under the supervision of the County Commissioners, and under Mr. Kendall's advice the Commissioners required repairs and changes in such structures from time to time to effect a greater degree of safety. No such structure under his supervision ever failed with resulting loss of life. Bridges were rebuilt, repaired, or replaced, to provide adequate strength and, at the same time, to secure a pleasing appearance and avoid unreasonable expense. Mr. Kendall was not a bridge designer, but his advice and assistance resulted in obtaining structures which bear evidence of his good judgment. Plans of location, approaches, grading, sewers, and drains for county buildings, such as court houses, correctional institutions, hospitals, etc., were prepared by him and utilized in the construction, alteration, and repairs of such buildings.

Mr. Kendall served the County Commissioners as Secretary as well as Engineer, and, finding that a considerable knowledge of law was essential to his work, he took a correspondence course in law. This, and the experience in such legal matters as he acquired in the course of his duties, resulted in his becoming, as some lawyers have stated, probably the best posted man in the State on legal affairs relating to counties, although he never applied for admission to the Bar. In 1922, under legislative authority, he prepared a compilation of laws relating to counties in Massachusetts. His duties as

Secretary involved the preparation of estimates of receipts and expenditures submitted annually to the Legislature for the county budget; the preparation of bills for legislative action; the submission each year to the Clerk of Courts of a detailed order assessing the expenditures of maintaining the Middlesex County Sanitarium upon the communities making up the hospital district; and the preparation of legal briefs and returns relating to county highways and bridges.

The record of Mr. Kendall's service to the Town of Belmont is an impressive one, the more so because most of it was rendered as a civic duty, and without financial recompense. He served twelve years as a member of the Board of Water Commissioners, and ten years as Selectman. For years, he was a member of the "Warrant Committee"—the town's budget committee—and he also served on ten or twelve special committees. There was no citizen of the town whose opinion and judgment carried greater weight with the voters.

Besides his direct service to the town as a municipality, he gave much to its development as a "town of homes." He was one of a group that formed a real estate trust and developed an area with such restrictions and on such a plan that it would benefit the community, without regard to the question of gains to the investors. Later developments have been made along similar lines, to the great advantage of the town. Mr. Kendall served the Belmont Savings Bank as Trustee for more than thirty years, having been its President for fifteen years. He was also a Director of the Waverley Co-operative Bank.

Modest and somewhat retiring by nature, Mr. Kendall was not a man to push himself into public notice, or to claim the credit for accomplishments which resulted in large measure from his unselfish work. No monumental structures can be pointed out as examples of his skill and ability as an engineer; no great reform or notable achievement as a public official can be claimed as essentially his. His life was made up of a multitude of comparatively small accomplishments, the aggregate of which constitutes an inspiring example of what a man can do and be in his own community, through years of simple, upright, honorable living.

Mr. Kendall was married, in 1900, to Lilly Miller, who survives him.

He was a Member of the Boston Society of Civil Engineers, and the Massachusetts Highway Association.

Mr. Kendall was elected a Member of the American Society of Civil Engineers on October 1, 1926.

THOMAS HENRY KING, M. Am. Soc. C. E.[1]

Died November 8, 1933

Thomas Henry King was born in Baltimore, Md., on July 29, 1877, the son of Thomas Harmon King and Ann (Booz) King. His early education was received in the public schools and in Deichman's Preparatory School of Baltimore. In 1896, he entered Johns Hopkins University, at Baltimore, where he completed two years of study as a Special Student in Electrical Engineering.

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[1] Memoir prepared by Irving C. Hess, Assoc. M. Am. Soc. C. E.

In 1898, Mr. King entered the employ of a general contractor in Baltimore and served, successively, as Draftsman, Estimator, and Office Engineer until 1903, when he went into business for himself as a General Contractor in the same city. From 1909 to 1913, Mr. King was a member of the engineering firm of King and Barnes, of Baltimore, engaged in engineering design and superintendence of construction.

In 1913, Mr. King moved to San Diego, Calif., and for a year served as Chief Draftsman for D. L. Bissell, City Engineer for the Towns of La Mesa, El Cajon, and East San Diego, Calif. He next entered the employ of the City of San Diego as a Draftsman in the City Engineer's Office and worked on reinforced concrete design, etc., for a period of about one year. In 1915, he was employed as an Engineer for H. M. Byllesby and Company, engaged in valuation of the San Diego Consolidated Gas and Electric Company's properties, in San Diego.

Visualizing the future importance of water supply in Southern California, Mr. King decided, in 1916, to specialize in water supply and distribution, particularly in San Diego County. In 1916, he entered the service of the Volcan Land and Water Company, and from that time until his death he was prominently identified with water supply and distribution in San Diego County. He served, successively, as Chief Draftsman, Chief Office Engineer, and Designing Engineer for the Volcan Land and Water Company, and, when the San Dieguito Mutual Water Company was formed as an outgrowth of the Volcan Company, he served as Chief Office Engineer and Designer until 1919 for that Company. During this period the Lake Hodges System was constructed which included Lake Hodges Dam, an Eastwood multiple-arch dam, 133 ft high and 750 ft long; the San Dieguito Dam, also an Eastwood multiple-arch dam, 50 ft in height and 700 ft in length; 4½ miles of concrete conduit connecting Lake Hodges and San Dieguito Reservoirs; and about 10 miles of main distribution lines. This conduit had many unique features of design including a 42-in. reinforced concrete siphon and a pre-cast concrete trestle, both designed by Mr. King. At the time it was built, the precast concrete trestle attracted widespread attention.

From 1919 to 1922 he served as Chief Engineer of the Ed Fletcher Company, of San Diego, and in that capacity he was Engineer in charge of the Volcan Land and Water Company, as well as having occasional engineering duties in connection with the San Dieguito Mutual Water Company and the Cuyamaca Water Company. As Engineer for the Volcan Land and Water Company, he had charge of certain water supply studies covering practically all of San Diego County. It was during his association with the Cuyamaca Water Company that Murray Dam, an Eastwood multiple-arch dam, 112 ft high and 900 ft long, was constructed for the Cuyamaca Water Company.

From 1922, when the Volcan Water Company was dissolved, until the time of his death, Mr. King was engaged in the private practice of engineering, specializing in water supply and irrigation. However, he also designed and constructed, from 1927 to 1929, more than $2 000 000 worth of public improvement work, including municipal improvement, highways, and sewers. He was associated in some capacity at one time or another with every Irrigation District in San Diego County, and served as Chief Engineer of the San Dieguito Irrigation District, Lakeside Irrigation District, San Ysidro Irrigation District, Ramona Irrigation District, and La Mesa, Lemon Grove and Spring Valley Irrigation District, all located in San Diego County, and the Niland Water District, in Imperial County, California. As Chief Engineer, he designed and constructed the principal works of the following irrigation districts: San Dieguito, Lakeside, Ramona, and La Mesa, Lemon Grove and Spring Valley. The work ranged in cost from a $35 000 bond issue for the Lakeside Irrigation District to a $2 500 000 bond issue for the La Mesa, Lemon Grove and Spring Valley Irrigation District. In 1931, he designed and constructed the first unit of a water development on a 4 000-acre ranch on the Sweetwater River, in San Diego County, owned by Mr. J. W. Sefton.

It is generally recognized that no one person had a more thorough knowledge of the water supply problems in San Diego County than Mr. King. He made water supply investigations on the following streams and their tributaries in San Diego County: San Luis Rey River, San Dieguito River, Santa Ysabel River, Santa Maria Creek, San Diego River, Sweetwater River, Otay Creek, Cottonwood Creek, and the Tia Juana River. The decision of the City of San Diego to build El Capitan Dam instead of a dam at Mission Gorge No. 2 site, on the San Diego River, was due largely to studies of the river made by Mr. King.

His personal interest in public welfare was evidenced by active membership in the Rotary Club of La Mesa, and, later, at San Diego. Fraternally, he was a Mason. His principal hobby was yachting. For five consecutive years he served as a Director of the San Diego Yacht Club. Mr. King was highly esteemed by those who knew him as an upright citizen, a man of high ideals, a true friend, and a competent engineer. In his passing the Engineering Profession loses an honorable member and a strong character.

He was married in Baltimore, on October 30, 1901, to Mary Edna Bedford, who survives him.

Mr. King was elected an Associate Member of the American Society of Civil Engineers on June 19, 1922, and a Member on October 1, 1926.

WILLIAM ALFRED LAMB, M. Am. Soc. C. E.[1]

Died February 9, 1938

William Alfred Lamb was born on December 16, 1881, near Hillside, Colo. His father, Joseph Milton Lamb, born in North Carolina in 1836, was a veteran of the Union Army during the Civil War and was famous as an Indian fighter during early days in Colorado. He achieved particular renown as the slayer, in 1864, of Espinoza, the Mexican bandit, murderer and assassin. He was married to Elizabeth Cantonwine, who went to Colorado with her parents in the days of the ox-team prairie schooner.

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[1] Memoir prepared by A. H. Tuttle, Esq., Helena, Mont., and C. S. Heidel, M. Am. Soc. C. E., assisted by N. C. Grover, M. Am. Soc. C. E.

William Alfred Lamb attended grade schools at Hillside and was graduated from the Hillside High School in 1897. He entered the Colorado State College, at Fort Collins, Colo., in 1898, and was graduated in 1902 from the Civil and Irrigation Engineering Division with the degree of Bachelor of Science. He taught mathematics at the College for one year after his graduation.

He entered the United States Reclamation Service as Hydrographer, in 1903, at which time it was a branch of the United States Geological Survey. Over a period of six years, Mr. Lamb's work with the Geological Survey was widely diversified, although generally related to water, and took him into many States. In 1909, he was appointed District Engineer for the Helena District of the Water Resources Branch of the U. S. Geological Survey, and, thereafter, Helena, Mont., was his home. His duties as District Engineer included the execution, direction, and supervision of investigations and reports concerning surface water and its utilization, in Montana, North Dakota, and South Dakota. He became a recognized authority on the surface waters of those States and their utilization for all purposes. Although always a Federal official, he was the trusted adviser of the State Engineer and other State officials and citizens.

Mr. Lamb represented the United States in the division, between Canada and the United States, of the international waters of the St. Mary River and the Milk River and their tributaries. His pleasing personality, his unquestioned integrity, and his conscientious attention to official duties won for him the respect and confidence of all interested parties on both sides of the International Boundary. These qualities won him a host of friends throughout the United States and Canada who mourn his untimely passing.

On December 16, 1932, he was appointed by President Herbert Hoover as a Special Representative of the United States in negotiations between the States of Montana and Wyoming relating to a proposed compact for the division of the waters of the Yellowstone River.

A Joint Report by Mr. O. H. Hoover, representing Canada, and Mr. Lamb, representing the United States, on International Forecasting and Apportionment of Waters by the United States and Canada in the St. Mary and the Milk River Drainage Basins, based on the snow surveys on the International Boundary, was presented at the World's Conference on Snow Studies in Edinburgh, Scotland, in 1936.

Among those who feel the loss of Mr. Lamb most keenly are his associates in the office of the U. S. Geological Survey, in Helena. They knew him as a true friend taking a conscientious interest in their welfare.

He was a member of the Helena Lodge of the Benevolent and Protective Order of Elks; of King Solomon's Lodge No. 9, A. F. and A. M.; Past High

Priest of Helena Chapter No. 2, R. A. M.; Past T. I. Master of Helena Council No. 1, R. and S. M.; Past Commander, Helena Commandery No. 2, Knights Templars; a member of Helena Consistory, Thirty-Second Degree Scottish Rite; St. Peter's Conclave No. 8, Red Cross of Constantine; Algeria Shrine; and of the Order of Jesters.

Mr. Lamb was a delegate to the 1927 meeting of the American Legion at Paris, France, and to other national meetings.

He was a Director and Past-President of the Montana Club; a Past-President of the Helena Federal Business Association; and a member of the American Geophysical Union.

In 1918, Mr. Lamb volunteered as a private in the United States Army, and was assigned to the Officers Training Camp, at Fortress Monroe, Virginia. He was honorably discharged from the Army in 1919 with the rank of Captain of Coast Artillery in the Officers Reserve Corps. He maintained his interest in the Reserve Corps and was promoted to the rank of Major on April 4, 1925.

Surviving him are three sisters, Cora Lamb, Mrs. Flora McGlothlin, and Mrs. Martha Adams, and three brothers, C. A. Lamb, J. A. Lamb, and F. R. Lamb.

Mr. Lamb was elected a Junior of the American Society of Civil Engineers on November 5, 1907; an Associate Member on May 3, 1910; and a Member on April 21, 1920.

FRANK PERRY LARMON, M. Am. Soc. C. E.[1]

DIED JUNE 3, 1937

Frank Perry Larmon, the youngest child of John and Laura Frances (King) Larmon, was born on March 2, 1878, at Salem, N. Y. He was of Scotch, English, and French descent.

Hugh Learmouth (the name was later changed to Larmon), the first of the family to come to America, was a native of Falkirk, Scotland. He came to the United States in 1772, and, while yet a young man, was one of the pioneer settlers in Cambridge, N. Y. He married Catherine Laurier, of Dutchess County. Their son, Alexander, and Ruth (Corey) Larmon were the parents of John. Through the King ancestry, Mr. Larmon was a *Mayflower* descendant, tracing direct line to Francis Cook.

John Larmon moved to Cambridge, N. Y., in 1884, and Frank attended the Cambridge Union School, graduating with the Class of 1895. He then entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1900 with the degree of Civil Engineer.

Mr. Larmon began his engineering career in September, 1900, as Assistant Engineer of the Hudson River Water Power Company on the reconstruction of Spier Falls Dam, near Glens Falls, N. Y. This work was completed in 1902, and from February of that year, until February 1903, he acted as Assistant Superintendent on the reconstruction of the Street Railway System of Little Rock, Ark.; and afterward as Assistant Superintendent on the construction of conduits and the power plant of the Paxtang Electric Company, of Harrisburg, Pa.

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[1] Memoir prepared by Alex Larmon, Esq., Charleston, W. Va.

In September, 1903, he was made Assistant City Engineer, of Utica, N. Y., and as such was engaged on the design and construction of general municipal work. From June, 1904, to April, 1907, he was Assistant Engineer for the State of New York on the design and construction of highways and canals and in charge of Barge Canal contracts amounting to more than $2 000 000.

Mr. Larmon entered private practice in April, 1907, his work including the design and construction of a hydro-electric plant for Salem, N. Y., and the investigation and reports on claims against the State and City of New York for damages to riparian rights and property appropriated.

From January, 1909, to June, 1913, he was employed as Engineer for the Lovejoy Company, manufacturers of agricultural implements. Starting in July, 1913, he served as Assistant City Engineer, of Lowell, Mass., in charge of work for the Water Department. He then became Chief Engineer of the Consolidated Water Company, of Utica, on the design and construction of all new work, and the operation of the system. In this capacity, he was engaged in the construction of 5 miles of 24-in. supply main, a 15 000 000-gal distributing reservoir, 15 miles of distribution mains, a stand-pipe (50 ft by 30 ft), and three 24-in. pipe crossings under the Mohawk River and New York State Barge Canal in silt and quicksand.

In 1917, Mr. Larmon went to Cuba as Resident Engineer on the construction of the electric railway and ocean pier in Cienfuegos. Returning to the United States in 1918 he spent the next six years in Omaha, Nebr., as Chief Engineer and Acting Manager of the Metropolitan Utilities District. This work covered the construction and operation of the water, gas, and ice plants—the outstanding accomplishments being the design and construction of a 60 000 000-gal per day filter plant, the installation of 170 000 000-gal pumping equipment, construction of two 120-ton ice plants, with 30 000 tons of refrigerated storage, and rebuilding a gas plant.

Mr. Larmon spent the years from 1924 until his death in 1937 in water plant and sewage disposal work. He designed and constructed an 8 000 000 gal per day filter plant at Fort Pierce, Fla., which work also included the installation of a 1 150-hp Diesel engine with generator, and the revision of water rates. He undertook the valuation of the water plants at Wilkes-Barre, Pa., and Charleston, W. Va.; he investigated the design, construction, and operation of the principal water-works plants in Brazil, in South America; and for six years he was the Chief Engineer for the Midwestern Division of the Federal Water Service Company, including West Virginia, Ohio, and Illinois Water Service Companies.

Mr. Larmon was a born executive. He had a practical and scientific knowledge of engineering problems. Due to his many years of experience both in the United States and abroad, he was an authority on the design and operation of all types of water plants. He was modest about all his achievements. He was always the student—well informed, independent, and liberal in his views.

He had a charming personality and was courteous and gracious to all with whom he came in contact. He had a keen sense of humor which was always a source of pleasure to his friends. He was fond of sports, enjoyed hunting and fishing, and all kinds of outdoor life. His chief recreation was golf in which he was very proficient.

Mr. Larmon was a member of the American Water Works Association; Oriental Temple of the Mystic Shrine, at Troy, N. Y.; a Thirty-Second Degree Mason and a member of the Cambridge Valley Chapter, F. and A. M. He was also a member of the Presbyterian Church.

On June 28, 1904, he was married to Ruth May Beckerof, of Cambridge, N. Y., who, with two children, Alex Larmon, and Mary Larmon Short (Mrs. John Winchell), survives him, together with four grandchildren, Mary Lou and John Larmon Short, Charles and Peter Short; one brother, the Hon. Charles W. Larmon; and a niece, Frances Larmon.

Mr. Larmon was elected an Associate Member of the American Society of Civil Engineers on July 9, 1906, and a Member on June 24, 1916.

LASLEY LEE, M. Am. Soc. C. E.[1]

Died November 15, 1937

After an illness of only a few days duration, Lasley Lee died of pneumonia at his home in Columbus, Ohio. He was buried at Central College, Ohio, where his pioneer ancestors had settled shortly after the War of 1812.

Lasley Lee was born at Carbondale, Pa., on January 2, 1887, the son of the Rev. Charles and Myrtie Virginia (Lasley) Lee. He was educated in the schools of Carbondale and attended Lafayette College, at Easton, Pa. He was graduated from Massachusetts Institute of Technology, at Boston, Mass., with the degree of Bachelor of Science, in 1910.

In May, 1910, Mr. Lee accepted the position of Assistant Engineer in the City Engineer's Office, at Sioux City, Iowa, where he was employed on municipal work until August, 1911, when he resigned to accept the position of Junior Engineer in the Water Resources Branch of the United States Geological Survey, at San Francisco, Calif.

Mr. Lee was engaged in general stream-gaging work with the Geological Survey until May, 1914, after which he was connected with the investigation of the Hetch Hetchy Valley as a source of additional water supply for the

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[1] Memoir prepared by a Committee of the Central Ohio Section consisting of C. Vernon Youngquist, Assoc. M. Am. Soc. C. E., *Chairman,* and F. H. Waring and J. R. Shank, Members, Am. Soc. C. E.

City and County of San Francisco until 1916. Following his California assignment, he was detailed to the Washington, D. C., Office, in July, 1916, where he served until January, 1917, when he was transferred to the Tacoma, Wash., Office of the Survey. After serving in the Tacoma Office for four years, his exceptional ability was recognized by his appointment in July, 1921, as District Engineer of the newly established Ohio District.

The State of Ohio at that time was sorely lacking in stream-flow data. Recognizing this lack and the great need in the State for this information, Mr. Lee spent the remainder of his life intensively carrying on this work with the result that Ohio became one of the foremost of the States in the quality of its gaging stations and stream-flow records.

Mr. Lee's remarkable industry is best indicated by the fact that when he came to Ohio only two gaging stations were being maintained in that State in co-operation with the U. S. Geological Survey; this number he gradually increased to more than one hundred. The result was that the ability to deal intelligently with water problems was greatly improved in Ohio.

In addition to carrying on this intensive program of establishing gaging stations and collecting and compiling stream-flow data, Mr. Lee was always interested in improving and developing instruments and structures used in stream-gaging work. These developments and improvements were described and detailed in many pamphlets, of which he was the author, under the general title of "Equipment for River Measurements." These pamphlets received widespread use and are recognized as authoritative sources of information.

Mr. Lee was endowed with a mechanical genius which resulted in his great interest in improving instruments and equipment used for river-measurement work. In collaboration with other members of the Geological Survey and the Department of Industrial Engineering at Ohio State University, he developed and improved river-sounding reels, wire-weight gages, electric contact tape gages, stream line sounding weights, and many other instruments and appurtenances used in river-measurement work. These instruments are now in general use all over the United States and have resulted in decided improvements in stream gaging.

Probably the most spectacular mechanical developments that Mr. Lee supervised were the power-driven rigs for obtaining discharge measurements of the Mississippi River, at Vicksburg, Miss., and Memphis, Tenn. This equipment made possible the accurate measurement of quantities of water with a degree of ease and facility which never before was deemed possible.

As a member of the Committee of the Geological Survey for Experimental Work at the National Hydraulic Laboratory, Mr. Lee was active in the arrangement and supervision of the various laboratory projects relating to measurements of river discharge which have been set up by the Survey for investigations at the Laboratory. These projects include "Studies of Artificial Controls"; "Investigations of Current-Meter Performance in Measurements of the Velocity of Water in Shallow Depths"; "Calibration of a Portable Parshall Flume"; "Study of Intakes for Gage Wells"; and "Measurements of Discharge near Bridge Piers."

He was co-author of about thirty *Water Supply Papers* containing records of stream flow in Ohio; also co-author of *Water Supply Paper 486,* "Water Powers of the Cascade Range," Part IV, Wenatchee and Entiat Basins; and *Water Supply Paper 492,* "Summary of Hydrometric Data in Washington, 1878–1919."

Mr. Lee was a member of the Delta Tau Delta (social) Fraternity, Sigma Xi (honorary) Fraternity, and Masonic fraternities among which were Scioto Consistory, Thirty-Second Degree Masons, and Aladdin Temple, Ancient Arabic Order of Nobles of the Mystic Shrine. He was a member of the American Geophysical Union, the Faculty Club of Ohio State University, President of the Columbus, Ohio, Massachusetts Institute of Technology Alumni Association, and Past-President of the Central Ohio Section of the Society.

He was married on July 11, 1921, to Mable V. Arthur, of Seattle, Wash., who, with a son, David, survives him.

Mr. Lee was a descendant of pioneer ancestors, his great-grandfather having settled a short distance from Columbus on land received for service in the War of 1812, a part of which he donated for the site of Central College. Lasley Lee displayed all the rugged virtues of his pioneer ancestry.

He was a hydraulic engineer pre-eminent in the highly specialized work of his profession. He was an untiring worker, devoted to engineering and scientific ideals. Personally, he was a staunch and loyal friend loved and admired by his associates. His death is a severe loss to his friends, to all who had the privilege of knowing him, and to the Engineering Profession. The memory of his kindly and courteous nature and genial disposition will always be cherished by his former associates.

Mr. Lee was elected an Associate Member of the American Society of Civil Engineers on March 11, 1919, and a Member on April 18, 1927.

ATTILIO FELIX LIPARI, M. Am. Soc. C. E.[1]

DIED FEBRUARY 13, 1938

Attilio Felix Lipari was born in Bompietro, Palermo Province, Italy, on February 12, 1890, the son of Joseph and Maria (Miserendino) Lipari. His mother was active in educational work in Italy. From his parents he inherited the studious and painstaking analytical qualities of mind which were so marked in his later life.

His father came to the United States in 1896 and established himself here. Five years later his family joined him in New York, N. Y., and it was in the public schools of this city that Attilio Lipari continued his education. He was graduated from Stuyvesant High School; and, in 1909, he entered Columbia University, to study Civil Engineering. He was graduated from Columbia in 1913 with the degree of Bachelor of Science in Civil Engineering.

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[1] Memoir prepared by Rear-Admiral Frederic R. Harris, U. S. N. (*Retired*), M. Am. Soc. C. E.

Directly after his graduation, in July, Mr. Lipari was employed by the Standard Concrete Steel Company, Engineers and Contractors, as Assistant to the Chief Engineer—designing, estimating, and assisting in the field, on the erection of industrial buildings. He left this Company in January, 1917, to go with the New York Central Railroad Company in its New York City Office, on the design of structures and general drafting work, remaining there until April of the same year, when he became Assistant Engineer with the Octagonal Reinforced Floor Company on the design of reinforced concrete buildings.

Mr. Lipari was then employed by the Italian Commission which was in the United States, purchasing and inspecting war material, his work as an Assistant including the placing of orders, inspection, and shipment of ammunition and supplies. On September 10, 1917, he enlisted in the United States Army as a private, and was assigned by the War Department to duty with the Italian Purchasing Commission. On October 4, 1918, he was promoted to the rank of Second Lieutenant, from which he was honorably discharged on December 20, 1918. He was especially suited for this war work because of his thorough knowledge of the Italian language and his engineering training and experience.

After his discharge from the Army, Mr. Lipari immediately went with the J. G. White Engineering Corporation, of New York City, as Draftsman on the design of various industrial buildings, and continued with that firm until May, 1921. Two months later, he entered the employ of Mr. C. B. Comstock, as Structural Engineer on industrial plants and terminal work of various types, continuing in this position until May, 1925.

About that time Arthur L. Bobbs, Consulting Engineer of Chicago, Ill., became active in connection with a special type of design for hotel floors and similar building construction, and opened an office in New York City. Mr. Lipari went with him and was placed in charge of the structural design for the New York work. From January to May, 1927, he also did designing work with the General Engineering Management Corporation.

In May, 1927, Mr. Lipari was employed by the writer and Mr. Bobbs, who became associated as Consulting Engineers, the former having just been retired by the United States Navy Department. His work for Mr. Bobbs was in connection with the structural design of buildings and theaters, and as Assistant to the writer he received his first experience on water-front design work, principally graving and floating docks, so that in January, 1928, when Mr. Bobbs withdrew from the association, Mr. Lipari became the writer's Principal Assistant in charge of engineering design. His most important work in 1928 was the design of the graving dock for the Robins Dry Dock and Repair Company, in Erie Basin, Brooklyn, N. Y., the largest privately owned graving dock on the North Atlantic Coast. This work involved not only the design of the structure, but the planning of the actual methods and supervision of construction, as the owners had decided, after receiving bids, to undertake the work with their own force under the supervision and management of the Consulting Engineers.

This work was successfully completed in the latter part of 1929. During this same period Mr. Lipari was also in charge of the design work for the methods of construction of the W. G. Triest Contracting Company on two subways in the Borough of Queens, New York City, a notable piece of work being the special method of underpinning the Interborough Rapid Transit and Brooklyn Manhattan Transit Elevated Railroad structures, especially the approach to the Queensboro Bridge.

The engineering work of the writer was then taken over by Frederic R. Harris, Incorporated, Mr. Lipari becoming Vice-President in charge of design, and shortly thereafter Vice-President of Construction Management, Incorporated, a management of construction corporation. Among the important work handled by him was the investigation of the design of the foundations, pier, and bulkhead of the assembly plant for the Ford Motor Company, at Edgewater, N. J., of which Albert Kahn was the Architect; plant extension for the Todd Shipyards Corporation, in New York, Mobile, Ala., and New Orleans, La., including the design of a new ship repair plant for the same Company in Galveston, Tex.

He collaborated with the writer in connection with the preliminary investigations and reports on timber and steel floating dry docks for the Pacific Coast; on floating dry docks for various Gulf of Mexico locations, including one in Mexico; an investigation of the ship-building facilities of the United States for the Joint Conference of Ship Owners and Ship Repairers of the United States; an examination and report for the Port of Havana Docks Company, in Havana, Cuba; investigation of concrete foundation piles for the Department of Justice Building, Washington, D. C.; and various other activities.

In 1932, Mr. Lipari, as Vice-President of the Construction Management Corporation, designed the airplane plant of the Curtiss-Caproni Corporation, a subsidiary of the Curtiss-Wright Corporation, in Baltimore, Md., and also an airplane plant for the Curtiss-Wright Corporation, which, in modified form, was eventually constructed by that Corporation for the Chinese Government. He was also engaged on various water-front work and handling appliances, in connection with timber handling, including that of the J. C. Turner Lumber Company, at Irvington-on-Hudson, N. Y.

In 1936, Mr. Lipari went to Lima, Peru, and there, in conference with the Frederick Snare Corporation, Contractors for the new Port of Callao, investigated studies for a graving dock for the Republic, and prepared outline plans for such a dock, returning to New York in the fall of 1936, to take charge of the preparation of more detailed plans, and, eventually, under the writer, the supervision of the construction of this dock (Frederick Snare Corporation, Contractors), which at the time of his death was nearly completed.

During this same period, Mr. Lipari assisted and was active in the design of a new type of circular bottom floating dry dock, at Seattle, Wash., with a lifting capacity of 16 000 tons, which was successfully completed in 1937.

Shortly before his untimely death, Mr. Lipari had been engaged with the writer on special work for the United States Government in a review and examination of the plans for the new graving dock at Mare Island (California), working on the computations and design, and the method of construction involved; on a special type of graving-dock construction under consideration for use in the Hawaiian Islands; and on a general review of steel floating docks, with a view to recommending a type of construction for a structure of this kind of the largest capacity.

Since the end of the World War and with the Naval Disarmament Agreement, the United States Government had had no occasion to design or build additional graving docks or floating docks of the largest size, so that, consequently, on all these structures built or designed in the United States, Mr. Lipari had something to do with the design and was among the few experienced engineering experts on this type of work. Realizing his ability to serve his Government in case of an emergency requiring such assistance, all arrangements had been made for him to enter the Civil Engineer Corps of the United States Navy as a Lieutenant Commander in the Reserve. He modestly failed to take advantage of this opportunity, as, although not a pacifist, his retiring nature made him feel that even a reserve military title was something he did not want in time of peace, because, as he stated: "If I am here, and my adopted country needs me, I will then be available to them as a civilian or in any other capacity they want me."

Mr. Lipari was a devout Roman Catholic and a member of the congregation of St. Margaret's Church, in the Riverdale Section of The Bronx, New York; a member of the Holy Name Society; and unknown to many of his friends, was active in charitable work in The Bronx. He always believed in keeping his contributions and assistance to his fellow men a secret. He was modest and self-effacing, almost to the point of timidity. His circle of close friends was small, and he had few hobbies other than that of philately. On this subject he was an expert and one of the few authorities in the United States who knew the stamps and postal history of the Old Italian States.

He was highly interested in organizations and activities for the benefit of the Engineering Profession; was a registered Professional Engineer in New York; a member of the New York State Society of Professional Engineers; and, until shortly before his death, a member of the Society of American Military Engineers.

Mr. Lipari is survived by his father, Joseph (83 yr old); a brother, Dr. Angelo Lipari, head of the Italian Department of Yale University; and three sisters, Sarina, Angela, and Emma.

He was elected a Member of the American Society of Civil Engineers on March 10, 1930.

FRANK ALEXANDER McINNES, M. Am. Soc. C. E.[1]

DIED AUGUST 9, 1937

Frank Alexander McInnes, the son of John C. and Rachel J. (McBeath) McInnes, was born on November 27, 1856, in Fredericton, New Brunswick, Canada.

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[1] Memoir prepared by Frank A. Barbour, M. Am. Soc. C. E.

He obtained his early education in the public schools of Fredericton, and in the High School of that city he came under the stimulating influence of its principal, George A. Parkin, who later became internationally known as the Executive Head of the Rhodes Scholarship Fund. In 1874, Mr. McInnes entered the University of New Brunswick, at Fredericton, from which he was graduated with the degree of Bachelor of Arts.

His first engineering experience was obtained in various preliminary and location surveys in New Brunswick and Maine. Later, he was Engineer in Charge of a preliminary survey for a so-called Air Line Railway between Boston, Mass., and New York, N. Y.

In 1888, Mr. McInnes found employment in the Engineering Department of the City of Boston and—except for an interval from 1915 to 1918—he continued to serve that city in various capacities until his retirement in 1924. From 1891 to 1895 he was an Assistant Engineer in the construction of improved sewerage; in 1895, he became Division Engineer of the Sewer and Water Division of the Public Works Department, which position he held until 1915. In 1918, after the separation of the Sewer and Water Divisions, he was made Division Engineer of the Water Division, and continued as such until 1924. From 1915 to 1918, he was employed as Engineer in charge of various improvements of the water system of St. John, N. B., Canada. Noteworthy in his service to the City of Boston was the installation of a high-pressure fire system.

Mr. McInnes was elected President of the New England Water Works Association in 1914 and was awarded Honorary Membership in 1929.

On his retirement, in 1924, from the service of the City of Boston, he became associated with William R. Conard, Assoc. M. Am. Soc. C. E., of Burlington, N. J., whom he represented in New England, in the testing of cast-iron pipe and other structural material and equipment. After 1924, Mr. McInnes also rendered consulting service in various water supply problems.

His most notable personal characteristic was rugged integrity and independence. Physically vigorous throughout his life, he continued to drive his car and visit his office until the day of his death at the age of eighty years. This physical capacity and his fearlessness were well shown in 1875 by his rescue from drowning, under most difficult conditions, of his boyhood friend, Sir Charles G. D. Roberts, the Canadian poet, who in various published articles has made grateful reference to the incident. For this exploit Mr. McInnes was given a watch by the citizens of Fredericton.

Generous in thought and deed, fortunate in a keen sense of humor, Frank Alexander McInnes had a great capacity for making and retaining friends, and for many years he was one of the most respected and best loved men in the water-works field.

He was married on October 16, 1893, to Laura E. Yerxa. His daughter, Dr. Frances McInnes, a member of the Medical Staff of Smith College, Northampton, Mass., survives him.

Mr. McInnes was elected a Member of the American Society of Civil Engineers on January 18, 1908.

LUDWIG THEODORE MAENNER, M. Am. Soc. C. E.[1]

DIED APRIL 6, 1937

Ludwig Theodore Maenner was born on October 1, 1863, at Neustadt, Bavaria, Germany, the son of Theodor and Josephine (Heigl) Maenner. He received his early education in the schools of Neustadt and was graduated from the Technical High School, at Straubing, Germany.

He came to the United States in 1883, and was employed by the Great Northern Railroad Company, at St. Paul, Minn., from July, 1890, until March, 1903, as Draftsman and Chief Draftsman in charge of map work, yard design, estimates for authorities for expenditures, and all other drawing-room work, under the jurisdiction of the Chief Engineer.

Mr. Maenner was employed from March, 1903, to June, 1906, by the Chicago, Rock Island, and Pacific Railroad Company, at Chicago, Ill., as Chief Draftsman in the Office of the Chief Engineer. In this position he had charge of the preparation of all plans and specifications of work assigned to the Chief Engineer; he also handled all the drafting-room work for the Real Estate and Tax Department and the Maintenance-of-Way Department, at Chicago.

In June, 1906, he accepted the position of Chief Draftsman with the Isthmian Canal Commission, with headquarters at Culebra, Canal Zone, Panama. He installed a system for filing maps and records, and prepared forms for cost-keeping and excavation statements. He also prepared statistical statements, in graphic form, of various expenditures, supervised the preparation of topographic maps of the Canal Zone, and land maps in connection with lawsuits, boundary disputes, etc. He completed his work with the Canal Commission in August, 1909.

Mr. Maenner's service with the Missouri Pacific Railroad Company began on September 1, 1909 when he was employed as Chief Draftsman in the Maintenance-of-Way Department, with general supervision of the Drafting-Room. When the Maintenance-of-Way and Construction Departments were consolidated in 1912, he was appointed Chief Draftsman in the Office of the Chief Engineer with direct supervision over all general drafting work, maintenance of records, etc. In 1917 Mr. Maenner was promoted to the position of Office Engineer, in which capacity he served faithfully and conscientiously until September 30, 1933, when at the age of 70 he was retired.

He was a man of strong character, ambitious, energetic, a good organizer, thorough in all his undertakings, loyal to his friends and employers, and was devoted to his family.

Mr. Maenner was a lover of chess and was President of the Western Chess Association from 1930 to 1933, inclusive, and a Director in 1934 and 1935. He organized the Missouri Pacific Chess Club in 1923, and was its President continuously until his death. He was a member of the Masonic Order.

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[1] Memoir prepared by a Committee of the St. Louis Section, consisting of S. L. Wonson, M. Am. Soc. C. E., and William T. G. Topping, Esq., St. Louis, Mo.

He is survived by his widow, Mrs. Frieda Maenner, a son, Theodore H. Maenner, and two daughters, Mrs. Selma (Maenner) Huff and Mrs. Mabel (Maenner) Schwarz.

Mr. Maenner was elected a Member of the American Society of Civil Engineers on October 14, 1919.

FREDERIC MOLITOR, M. Am. Soc. C. E.[1]

DIED MARCH 12, 1938

Frederic Molitor was born in Detroit, Mich., on April 21, 1868, the son of Albert Molitor and Lucille I. (Goodell) Molitor.

Mr. Molitor was educated at Trinity Church School, New York, N. Y., and at Cornell University, in Ithaca, N. Y. On leaving Cornell he entered the service of the Central Railroad Company of New Jersey as Instrumentman and Draftsman. Later, he was employed by the Baltimore and Ohio Railroad Company advancing rapidly through the positions of Assistant Engineer, on the construction of the Kill van Kull Bridge; Principal Assistant Engineer, Kentucky Central Railroad Company, on the Huntington Bridge, at Cincinnati, Ohio; Engineer, Maintenance of Way, with the Chesapeake and Ohio Railway Company; and General Roadmaster, Louisville and Southern Railway Company. He constructed coal and freight piers for the Philadelphia and Reading Railway Company, and was in charge of construction of the Montauk Division, Long Island Railroad Company, in 1895, when he was appointed Chief Engineer of the Choctaw, Oklahoma and Gulf Railroad Company.

In the ensuing ten years Mr. Molitor built more than 1 000 miles of railroad in Askansas, Indian Territory, Kansas, and Texas, for the Choctaw and its Allied Companies (later incorporated in the Rock Island System), including many tunnels, heavy bridge work, and pneumatic foundations. As General Manager and Chief Engineer for the Midland Valley Railroad Company, he built and operated 350 miles of road in Arkansas, Oklahoma, and Kansas. A significant item in this work, showing Mr. Molitor's passion for direct thinking and accuracy in professional reporting, was the preparation of a Manual for Resident Engineers, which became a classic on Western railroads.

Mr. Molitor's broad experience and his ability as an Engineer and Administrator brought him to the attention of President Theodore Roosevelt, and, in 1905, he was appointed Supervising Railway Expert to the Philippine Government. He found the Islands with a railway system of 120 miles and left them, in 1907, with 550 miles.

In 1908, after a study of railroads and terminals in Europe, Mr. Molitor opened an office as Consulting Engineer in New York City, which he maintained until he retired from active practice in 1936. His services were sought by important financial houses for examinations of American railroads and South American development enterprises, and he was a trusted adviser of the United States Government. He was a member of the Special Panama Canal Committee and Chairman of the Committee of Valuation of Damages of the National Railways of Mexico.

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[1] Memoir prepared by J. P. Hallihan and William J. Shea, Members, Am. Soc. C. E.

Mr. Molitor took an active interest in preparatory military training. Returning from an examination of the Brazilian Railways, he was commissioned, in January, 1917, as Major, Engineer Reserves, United States Army; and in July, 1917, he was assigned to duty with the Director General, Bureau of Military Railways, Office of the Chief of Engineers, U. S. Army. Major Molitor was promoted to the rank of Lieutenant Colonel in December, 1917, and assigned to duty with the 22d Engineers, U. S. Army, on June 1, 1918. On July 28, 1918, Lieutenant-Colonel Molitor was transferred to the staff of the Chief Engineer, American Expeditionary Forces (A. E. F.). On September 14, 1918, he was promoted to rank of Colonel; and on September 30, 1918, was appointed Officer in Charge, Engineer Supply Section, A. E. F.

Following the Armistice, Colonel Molitor directed a complete inventory of Engineer Supplies in France at all ports and depots, returned to the United States on March 6, 1919, and was honorably discharged from the Army on March 14, 1919. His interest in National Defense caused him to remain in the Corps of Engineers Reserve, until his transfer to the Auxiliary List, Reserve Corps, on March 18, 1931. In recognition of his war-time service Colonel Molitor was cited by General Pershing for " distinguished and meritorious service," and awarded the " Ordre de l'Etoile Noire," by the French Government.

An engineering service of far-reaching importance was undertaken by Colonel Molitor as Chairman of the Board of Economics and Engineering, created by the National Association of Owners of Railroad Securities in 1921, under the direction of the late S. Davies Warfield, President. Selecting as associates the late John Findlay Wallace, Past-President, Am. Soc. C. E., John F. Stevens, Past-President and Hon. M. Am. Soc. C. E., Walter W. Colpitts and W. L. Darling, Members, Am. Soc. C. E., and Professor Lewis B. Stillwell, with a staff of experts, Colonel Molitor made an exhaustive investigation of the operating and economic position of the railroad systems of the United States. Recommendations of the Board from the facts developed resulted in betterments of operating practice acceptable to the Interstate Commerce Commission and led to a better understanding of the economic and operating effects of proposed consolidations.

These few items selected from a crowded lifetime as an Engineer serve to indicate the high professional standing of Colonel Molitor. As a man he was imbued with a clear sense of justice and of civic and national duty. He enjoyed the high regard of his professional associates and of the contractors on the work he directed, as well as the respect and confidence of the financial leaders he served as Consultant. He was a leader in measures for professional advancement, and an active member of several technical societies, among them the American Institute of Consulting Engineers, the Institution of Civil Engineers of Great Britain, the American Railway Engineering Association, the Society of Engineers and Architects of Mexico, and the Society of American Military Engineers.

He was a member of the Century Club of New York, the Military Order of the Loyal Legion, and the Military Order of the World War.

He was married in 1896 to Katherine Jeffries, who survives him.

Colonel Molitor was elected a Member of the American Society of Civil Engineers on November 4, 1896.

DANIEL EDWARD MORAN, M. Am. Soc. C. E.[1]

Died July 3, 1937

Daniel Edward Moran passed quietly to rest at his home in Mendham, N. J., on July 3, 1937. For several years he had suffered a heart ailment and had been losing in physical strength, although his mental faculties remained vigorous to the end. He was buried in the family plot at Mendham.

Daniel Edward Moran was born on April 12, 1864, at Orange, N. J., the son of Daniel Edward and Annie Augusta (Blake) Moran. From 1866 to 1884, he lived in Brooklyn, N. Y., and attended the local schools, the last being Brooklyn Polytechnic Institute, from 1874 to 1880. He entered the School of Mines at Columbia University, in New York, N. Y., in the fall of 1880, taking the Civil Engineering Course, and was graduated in June, 1884, with the degree of Civil Engineer.

During his years at Columbia, Mr. Moran interested himself in various college activities, becoming a member of his class football team, School of Mines Editor of the *Columbia Spectator,* an Editor of the '84 School of Mines Yearbook, and a contributor to the *School of Mines Quarterly.* He was a member of the Delta Psi Fraternity.

After his graduation, in 1884, Mr. Moran designed and superintended the construction of a country residence for his mother. His personal notes state that in the spring of 1885, he entered the employ of the Delameter Iron Works, West 14th Street, New York City, "as apprentice at 50c per day, working 59 hours a week." He continued in this work until the spring of 1887 when he was employed as Instrumentman on a survey for the extension of the Nevada and California Railroad, north of Reno, Nev., owned by his uncle, Charles Moran. He quit this job in the fall of 1887 and returned to New York City *via* San Francisco, Calif., and Portland, Ore. Concerning this return trip, he writes that he went from San Francisco to Portland by "the new R. R. being built * * * there being a 20-mile gap which was covered by a 4-horse stage coach of the Wild West type. The Canadian Pacific Transcontinental line had been put through in 1886 but was still a rather 'sporty' trip."

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[1] Memoir prepared by George L. Freeman, M. Am. Soc. C. E.

On his return, Mr. Moran had great difficulty in getting a position in New York City, but finally "found work at map making for Samuel McElroy for a few weeks in Brooklyn * * * and was fired because, as he said, I was getting more out of the maps than he was paid." For a short period, in 1887 and 1888, he was employed as Draftsman by the late William Barclay Parsons, Hon. M. Am. Soc. C. E., who then was "engaged in drawing plans for a proposed rapid-transit subway using the subgrade portions of private property, and only streets so far as necessary to cross from block to block."

Mr. Moran's long career as a Foundation Engineer began with his employment, early in 1889, by the late Charles Sooysmith, M. Am. Soc. C. E., a New York contractor who specialized in substructure work by the pneumatic, freezing, and other methods. Mr. Sooysmith first sent him to Iron Mountain, Mich., as Assistant Superintendent and Technical Observer on the construction of a mining shaft, for the Chapin Mining Company, to be constructed through a deep quicksand formation by the Poetsch-Sooysmith freezing method. A description of this work was afterward published by Mr. Moran.[2]

In the summer of 1889, the work at Iron Mountain completed, Mr. Moran was transferred to the position of Assistant Superintendent on the construction of the Deer Island Lighthouse at the entrance to the harbor at Boston, Mass. This structure was on an exposed reef extending south from the east end of Deer Island, its base being a circular shell of cast-iron plates founded on the reef, and backed up with concrete and brick masonry. Shortly after his arrival, he was placed in full charge of the work, which was completed before the end of the year despite bad weather conditions and the "dyspeptic army officer" in charge.

Late in 1889, Mr. Sooysmith had several pneumatic bridge foundation jobs in progress in the field, while designs for others were being developed in his office, and Mr. Moran was delegated to study designs and methods for this work, the study perhaps having been prompted by a serious accident which occurred about that time on a pneumatic job being constructed by Mr. Sooysmith at Needles, Calif. He visited a pneumatic job being constructed by Mr. Sooysmith at Wyoming, Pa., and interestingly records:

> "I found Christy (Supt.) laboriously breaking up rock, putting the pieces into sacks, hoisting the sacks two at a time into a common man lock, piling the lock as full as he could and operate, then locking out and rehandling the sacks to the dump. He used in hoisting the sacks a length of wire rope, which passed through a stuffing box in the head of the lock. I looked up all the data we had on the subject and started to design a safe and economical lock. Sooysmith told me not to waste time (his and mine) so I continued my studies out of office hours."

These studies led to the development of the first Moran air-lock, which Mr. Moran patented, and had constructed in 1892 by the Continental Iron Works, of Brooklyn, N. Y.

The late Alfred Pancoast Boller, M. Am. Soc. C. E., who was a specialist on drawbridges and a friend of Mr. Sooysmith, borrowed Mr. Moran in 1890 to design caissons for the McCombs Dam Bridge crossing the Harlem River at 155th Street, New York City. The main caisson was of steel construction, 19 ft wide by 100 ft long, and provided with two excavating shafts with locks located in the working chamber at the bottoms of the shafts, the soil to be removed through special buckets which formed a part of the lock arrangement. Later, Mr. Sooysmith was the successful bidder for the foundation work and, in 1892, placed Mr. Moran on the work as Night Superintendent. Both the lock buckets failed before the big caisson was sunk to its final depth, one becoming stuck at the bottom of the shaft, and the Moran lock, completed just in time, was secured to the top of the shaft and the bucket removed through it so that work could proceed.

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[2] *Columbia School of Mines Quarterly*, No. 3, Vol. XI.

This was the first use of the Moran air-lock, and but one lock of this original design was built. It was of the top-and-bottom-door type, the top opening being fitted with two hinged doors meeting at the center, where the hoist rope passed through, with a stuffing-box to reduce air leakage. This lock marked a distinct advance in the speed and economy of pneumatic caisson work, as the arrangement of top and bottom doors made it possible to lower empty buckets through the lock and down the shaft directly to the floor of the working chambers, and to hoist the loaded buckets without rehandling or delay. Subsequent refinements made by Mr. Moran in the details of the air-lock, including the change to a single top door, resulted in the present widely used Moran lock.

Prior to the completion of the foundation for the McCombs Dam Bridge, late in 1892, Mr. Moran was sent to Sioux City, Iowa, where Mr. Sooysmith was to build the pneumatic piers for the foundations of the Pacific Short Line Bridge across the Missouri River. The engineer's plans had left the design of the caissons entirely to the contractor, the work was all ready to proceed and Mr. Moran's job was to design the caissons in the shortest time possible so that the work would not be delayed. He remained in Sioux City until the summer of 1893, when he was sent back to New York City by Mr. Sooysmith and put in charge of the foundation work for the American Surety Company Building, at the corner of Broadway and Pine Street. This was the second building in New York, or in the world, to be supported by pneumatic caissons.

Of this work, Mr. Moran writes that,

> "Many architects and some Engineers still questioned the reliability of Concrete, so the piers above the steel caissons (working chambers) were built of brick from Hudson River points. At first the bricklayers insisted on building level and plumb, not realizing that a caisson base is level only by accident. In sinking, the caisson might drop from 2 to 4 ft suddenly; then all the bricklayers would jump off the caisson."

The underpinning of buildings adjacent to caisson foundation jobs was then, as now, frequently necessary and, at that time, such underpinning was practically a monopoly in the hands of one New York contractor. Mr. Moran tried to convince Mr. Sooysmith that he should include underpinning with his other substructure work but the latter replied "these absurdly high buildings requiring caisson foundations are overdone. They are operated at financial loss to the owners and no more will ever be built." Mr. Moran thereupon decided that he would become familiar with underpinning work and prepare for the construction of future high buildings.

Continuing with Mr. Sooysmith until 1897, Mr. Moran then became Special Partner and Foundation Engineer with John Monks and Son, of New York City, remaining with that firm until about 1900. During this period, he designed and superintended the construction of pneumatic caissons for the water-works plant at Cincinnati, Ohio. In 1900 and 1901 he superintended the construction of a power canal and the foundations for a pulp mill and power-house at Great Bend, N. Y.

In 1901, with Franklin Remington and Edwin S. Jarrett, M. Am. Soc. C. E., Mr. Moran organized the Foundation Company, becoming its Chief Engineer and a Vice-President and Director. He continued with the Foundation Company until 1910, during which time it became the foremost builder of deep and difficult foundations in the United States and Canada. Mr. Moran was responsible for the engineering of many of its important foundation works.

From 1910 until his death, Mr. Moran was in private practice in New York City as a Consulting Engineer, under the name of Daniel E. Moran from 1910 to 1919; Moran, Maurice and Proctor from 1920 to 1928; Moran and Proctor from 1929 to 1935; and Moran, Proctor and Freeman from January, 1936, until his death.

His engineering works include the foundations for a great many buildings in New York City and elsewhere, such as the Woolworth and Municipal Buildings, Federal Reserve Bank, Equitable, Morgan, New York Stock Exchange, Barclay-Vesey Telephone, Bank of America, Chase National Bank, Bank of Manhattan, City Bank Farmer's Trust Company, Irving Trust Company, and many other buildings in New York City; the Federal Building, in Cincinnati, Ohio, Federal Building, in Pittsburgh, Pa., and Royal Bank of Canada, and Aldred Building, in Montreal, Que., Canada.

Mr. Moran was engaged on many important bridge projects among which were the Philadelphia-Camden, at Philadelphia, Pa., Mid-Hudson, at Poughkeepsie, N. Y., Ambassador, at Detroit, Mich., Huey P. Long, over the Mississippi River, at New Orleans, La., and the Suisan Bay, in California. He was for years Foundation Consultant to the Port of New York Authority, advising on foundations for the George Washington, Triborough, Bayonne Arch, and other bridges, and the Midtown-Hudson River Tunnel. Finally, came his greatest achievement, the foundations for the San Francisco-Oakland Bay Bridge, made feasible by the "domed" type of caisson which he devised and patented for use on this work.

He was a pioneer in the study of the properties of soils, his experiments on sands and clays, with home-made testing apparatus, pre-dating those of present-day soil technicians. He was among the first to understand the nature of volume change in clay and silt soils under loads and to apply this knowledge to actual foundation problems.

Although he was a leader in the development of pneumatic foundation methods, Mr. Moran, in his later years, because of their increasing cost, steadily oppposed their use in any work until he was convinced that no cheaper or more expeditious type of construction was possible. Here, again, we find him pioneering in the elimination of the very methods of construction he had spent so many years to develop. Outstanding examples of this advance in his practice were the deep foundations for the City Bank Farmer's Trust Company and The First National Bank Buildings, built by open methods in down-town New York, on sites where twenty years ago pneumatic caisson foundations would have been considered necessary.

Mr. Moran received the honorary degrees of Master of Science and Doctor of Science from Columbia University. In 1934, the Columbia Engineering Schools Alumni Association, calling him the "Dean of American Foundation Engineers" made him an Honorary Member. A year later, Dr. Nicholas Murray Butler, President of Columbia University, bestowed on him the Gold Medal of the Class of 1889 "for Eminent Achievement."

In December, 1896, Mr. Moran was married to Sarah V. Kelly, of Glasgow, Scotland. He is survived by his widow, his sons, Daniel E., Jr., Archibald A., and Hugh B., and by his daughters, Mrs. George Frazer and Mrs. Carl Bricken.

At the time of his death, he was a member of the American Institute of Consulting Engineers, The American Society of Mechanical Engineers, the Franklin Institute of Philadelphia, Pa., The Newcomen Society, of London, England; and the Engineers' Club, Downtown Association, University Club, Columbia University Club, and The New York Zoological Society, of New York City.

Mr. Moran was elected an Associate Member of the American Society of Civil Engineers on June 3, 1891, and a Member on January 1, 1896.

JOSEPH S. MORRISON, M. Am. Soc. C. E.[1]

DIED NOVEMBER 30, 1937

Joseph S. Morrison was born in Hedrick, Iowa, on January 8, 1874. His parents were John Morrison and Martha (Doolittle) Morrison. John Morrison was a Civil War Veteran, and was well known in Iowa because of his service in county office and the State Legislature. Joseph S. Morrison attended High School, at Hedrick, and was graduated from Iowa State College, at Ames, Iowa, in 1897, with the degree of Bachelor of Science in Civil Engineering. He received his Civil Engineering degree at Iowa State College in 1908. For a short time in 1895, he taught in a rural school near Guttenberg, Iowa.

The engineering career of Mr. Morrison began in 1898 as a Rodman on the Iowa Central Railway. He secured employment with the Chicago and

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[1] Memoir prepared by W. J. Smith, Esq., Ottumwa, Iowa.

Northwestern Railway Company in 1899 as an Instrumentman, and during the next three years he was advanced to be Chief of Field Party on general surveys. He did instrument work for the Railway Company on the viaduct across the Des Moines River, west of Boone, Iowa, and made soundings and current observations for the double-track bridge over the Mississippi River, at Clinton, Iowa. He also worked a short time in the Bridge Designing Department of the Company, in Chicago, Ill. In 1903, he was engaged in construction for the Company in Nebraska and South Dakota. In 1904, he served as Assistant to the Division Engineer in Iowa and Minnesota, and was employed on maintenance and track work. In 1905, he served as Engineer in charge of revision work on yards and viaduct construction, at Long Pine, Nebr.

Mr. Morrison continued his service with the Chicago and Northwestern Railway Company as Assistant to the Division Engineer from 1906 to 1908, locating and constructing tracks in Michigan for the iron mines and logging camps. In 1909, he was in charge of a field party on the construction of the cut-off and viaduct at Valentine, Nebr. For the next three years, he had charge for the Company of the construction of 26 miles of track in Wisconsin and during this time, also supervised the construction of bridges across the Wisconsin and Yellow Rivers, and the building of terminal yards at Wyeville, Wis. In 1912, he had charge of the construction of 10 miles of track in Illinois. From February, 1913, to May, 1914, he was engaged on Federal valuation on the St. Louis, Peoria and Northwestern Railway.

Mr. Morrison did his last railway engineering work in 1914, and then entered the service of the Iowa Highway Commission. It was his good fortune to enter highway work at the time the extensive use of automobiles and trucks made necessary the building of a system of surfaced highways. He was appointed District Highway Engineer of the 5th District, with headquarters at Ottumwa, Iowa. In 1914, there were no surfaced highways in Southeastern Iowa outside the cities and towns. The present system of surfaced roads in the 5th District was built under his supervision. From 1914 to 1919, his duties, in general, were to advise the County Boards of Supervisors relative to the standards required by the State Highway Commission.

The construction of Federal Aid Highways began in 1919 and, at that time, Mr. Morrison began directing a force of assistants on construction work. He had an active part in locating lines, making surveys and plans, and supervising construction. At present (1938) there are 1 379 miles of primary roads in the District, and he was familiar with every structure. A few of the larger bridges built in the southeastern part of Iowa, during Mr. Morrison's service as District Engineer, are those over the Des Moines River, at Keokuk, Ottumwa, and Belle Fountain, Iowa, and two bridges over the Iowa River, at Columbus Junction, Iowa. In 1925, the State Highway Commission became responsible for the maintenance of primary roads. As District Engineer, Mr. Morrison had charge of this work in the 5th District. In 1925, the primary roads in the District were constructed of earth and gravel, and maintenance work consisted of smoothing the surface following rains. In the next few years, the requirements of road maintenance increased, and permanent garage buildings were erected by the State in each county, much equipment was purchased, and a force of 200 men was employed to keep the roads in good condition during all seasons.

On September 5, 1905, Mr. Morrison was married to Sophia Schott. During the time he was engaged in railway work they made their home in the localities near his work. Since 1914, however, they had lived in Ottumwa. He is survived by his widow and four children, John, George, and Alice, and Mrs. George Welz.

Mr. Morrison and his family have had a wide acquaintance with students and graduates of Iowa State College, and it is rather unusual to find a family in which so many members have had an interest in the same school. He had four sisters and one brother who attended Iowa State. He and Mrs. Morrison and three of their children are graduates, and their youngest son is a student at Iowa State College. His father was a Trustee of the college in the early Nineties.

His chief interests in life were his work and his family, and he devoted practically all his time to them. Most of his friendships were made with those who had contacts with him through his work. He enjoyed a good story to the extent of appreciating those told on himself. He was able to delegate authority to his subordinates and judge them on the results obtained. When he assigned a task he expected good work, but he did not insist on his own methods, or try to force his opinions on others. He encouraged young men to accept responsibilities and when mistakes were made he was willing to accept his just share of the fault. When it was necessary to criticize an employee he could do it thoroughly without offending. His employees liked to work under his supervision and were loyal to him.

Mr. Morrison was elected a Member of the American Society of Civil Engineers on August 31, 1915.

JAY JOHNSON MORROW, M. Am. Soc. C. E.[1]

DIED APRIL 16, 1937

Jay Johnson Morrow was born at Fairview, W. Va., on February 20, 1870. He was the son of James Elmore Morrow and Clara (Johnson) Morrow. His father was a graduate of Jefferson College, at Canonsburg, Pa. (later Washington and Jefferson College, Washington, Pa.), and a teacher and professor in a number of schools in West Virginia and Western Pennsylvania. James E. Morrow entered the Union Army during the Civil War and rose to the rank of Captain. In 1871, he became President of Marshall

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[1] Memoir prepared by Brig. Gen. Herbert Deakyne, U. S. Army (*Retired*), M. Am. Soc. C. E.

College at Huntington, W. Va., and, later, was a Teacher of Mathematics and Principal of Schools in Pittsburgh, Pa., and vicinity. He was a man of culture, fond of learning, a scholar in Latin and Mathematics, and had a great talent for teaching.

On the maternal side of the family, Clara (Johnson) Morrow was the daughter of John Jay Johnson, and possessed an acute and active mind combined with strong practical sense.

Jay Johnson Morrow was the second of eight children in this family. With this background, he grew up in an atmosphere that fostered his mental development. The family lived on the limited income of the father, but the home surroundings stimulated an interest in books and study, and a desire to excel in the tasks of the hour. Jay was graduated from the High School in Allegheny, Pa., at the age of 14, and then went to work as a Clerk and Bookkeeper at a modest salary. At 17 he secured an appointment to the United States Military Academy, at West Point, N. Y. He entered the Academy on June 16, 1887, and was graduated on June 12, 1891, standing No. 5 in a class of 65 members. He was assigned to the Corps of Engineers, United States Army, and served as a member of it until his retirement at his own request on August 5, 1922.

His first service after graduation from the Military Academy was at the Engineer School of Application at Willets Point, Long Island, N. Y., later, Fort Totten. Lieutenant Morrow completed the course at the Engineer School in 1894. In 1895 he returned to West Point as an Instructor in Practical Military Engineering, remaining on that duty until September, 1896. In addition to his duties as an Instructor, he was engaged in the construction of a new water-supply system for the Post of West Point. From September, 1896, to August 1898, he was on duty in the Washington, D. C., Engineer District and was in responsible charge of the construction of fortification works and submarine mining casemates at Fort Washington, Maryland.

Lieutenant Morrow was at the Military Academy again from August, 1898, to February, 1899, as Instructor in Civil and Military Engineering. He was then detailed on fortification construction in New York Harbor and was in charge of fortification work at Fort Wadsworth and Fort Hamilton. In August, 1899, he returned to West Point and from that time until March, 1901, was Instructor and Assistant Professor of Civil and Military Engineering. During this time, he made two surveys of the Post of West Point.

He was promoted to the rank of Captain in February, 1901, and from that time until June of the same year, was at Fort Totten, in command of a Company of Engineer Troops which he then took to the Philippine Islands. He remained in the Philippines until November, 1903, during which time he had charge of road construction and improvement, and of the design and construction of pile and masonry wharves at various points in the Islands.

Returning to the United States Captain Morrow was detailed as Assistant to the Engineer Commissioner of the District of Columbia. As such he had charge of the Surface Division, which included street, sidewalk, and alley pavements, highway construction and extension, surveys, street parking, and inspection of asphalt and cement. In May, 1907, he became the Engineer Commissioner of the District, and held this position until December, 1908, having been promoted to the grade of Major in January, 1908.

Major Morrow's next duty was in charge of the Washington, D. C., Engineer District and of the water supply of the City of Washington. This tour continued until March, 1910, when he was transferred to Portland, Ore., in charge of river and harbor work. He remained in charge of this important District until June, 1915, with a temporary interruption of three months—from December, 1912, to March, 1913—during which time he was Chairman of the Alaska Railroad Commission.

He was promoted to the rank of Lieutenant Colonel in March, 1915, and in June of that year he began his connection with the Panama Canal, first as Assistant to the Engineer of Maintenance and, later, as Engineer of Maintenance. This duty was interrupted by the entry of the United States into the World War. Colonel Morrow's first war duty was the command of the Fourth Engineers at Vancouver Barracks, Washington, and at Camp Greene, North Carolina. He had been promoted to the temporary rank of Colonel in August, 1917. In May, 1918, he took his regiment to France.

Shortly after reaching France, he was promoted to the rank of Brigadier General, and when the First American Army was organized there, he was detailed as its Chief Engineer. He proceeded to organize this important command to cover the many and varied services required to carry on the engineer work of an army in the field. He was Chief Engineer of the First Army during the engagements of Chateau Thierry, Vesle River, St. Mihiel, and Verdun-Argonne. General Morrow had a short period of duty as Chief Engineer of the Third Army, and was then made Deputy Chief Engineer of the American Expeditionary Forces in France. For his service during the war he received the French decoration of the Legion of Honor.

After the Armistice he returned to the United States and reverted to his regular rank of Lieutenant Colonel. He was in command of the Post and the Engineer School at Camp Humphreys (later, Fort Belvoir), Virginia, from January to May, 1919. Colonel Morrow then resumed the duty he had at the beginning of the war, that of Engineer of Maintenance of the Panama Canal. He continued in this position until March, 1921, when he was advanced to the responsible duty of Governor of the Panama Canal. In the meantime, he had been promoted in April, 1920, to the regular rank of Colonel. In August, 1922, at his own request, he was placed on the retired list of the U. S. Army with the rank of Colonel, after more than thirty-five years' service. This did not terminate his duty as Governor, which continued until October, 1924.

After his Panama Canal service he held another responsible position from March, 1925, to June, 1929, as the United States Member and Chairman of the Special Commission on Boundaries, Tacna-Arica Arbitration.

On June 21, 1930, in accordance with the Act of Congress of that date, he was advanced on the retired list to the grade of Brigadier General, because he had held that rank during the World War.

General Morrow was married on October 15, 1895, to Harriet M. Butler, the daughter of Brigadier General John G. Butler, U. S. Army. She died on April 24, 1935. They had no children.

Any account of the life of General Morrow would be incomplete without reference to his brother, the late Dwight W. Morrow. The latter was born January 11, 1873, and was thus about three years younger than General Morrow. He was anxious to obtain an education and hoped to succeed his brother as the Cadet at West Point from the Congressional District in which they lived. With this in view Dwight entered a competitive examination for the appointment and came out first, but was denied the appointment because his brother had already represented the District.

Dwight Morrow despaired for a time of obtaining an education, but after his brother's graduation from West Point in June, 1891, it was arranged that he should go to Amherst College, at Amherst, Mass. Out of his pay as a Second Lieutenant, Jay Morrow was able to save enough to lend his brother the funds that enabled him to go through college. His subsequent career as a member of an important law firm in New York City, as a partner in the banking house of J. P. Morgan and Company, Ambassador to Mexico, and United States Senator, justified the effort it required to finance his college course. Harold Nicolson, in his book on the life of Dwight Morrow, quotes General Morrow as saying of the money he advanced for Dwight's education: "But there is no one who can say that these were not the best investments that I ever made."

There was a strong bond of affection between General Morrow and Dwight's daughter, Anne Morrow. This dated from her infancy when she exhibited a decided liking for her uncle. Her marriage to Charles A. Lindbergh made her a public character of first importance. The tragedy of the kidnaping and death of her infant son was a source of great sorrow to General Morrow. One of the last things he did was to cross the Atlantic to pay a visit to her and Colonel Lindbergh, and he had just returned from this trip when he died.

During his later years General Morrow lived in Englewood, N. J., where his brother, Dwight, made his home. He took a great interest in West Point and in the reunions of his Class. He contributed much to the success of the Fortieth Reunion in 1931, and the Forty-Fifth Reunion in 1936 by arranging for the wives of his classmates to be guests during the reunion periods at the country estate of his nephew near West Point. He provided transportation so that they could attend the ceremonies at West Point in comfort.

General Morrow was a lover of the opera and is said to have attended 600 performances over a period of 40 years. The newspapers reported that he commuted from Panama and Chile to New York to attend the opera. This exaggeration of statement merely serves to emphasize his devotion to the art.

General Morrow was not only a capable engineer and an efficient administrator, but he was also fond of athletic sports. In his younger days he was a good tennis player and he also played football and baseball. He was well-rounded both mentally and physically. His mental grasp was broad and comprehensive. This was particularly noticeable in France during the

World War. Detailed as Chief Engineer of the First American Army created in France, he had to organize that service with its great variety of functions to be performed. Roads, railroads, water supply, electrical and mechanical work, camouflage, shelters, obstacles, demolitions, mining —these were some of the things to be looked after, with special troops for many of them. To organize such a service, to secure the Officer Personnel to administer it, and to handle the troops to do the work, made a task of the first magnitude.

The Governorship of the Panama Canal is also a position calling for high ability. It involves not only the engineering work of keeping the Canal in operation, but also many other tasks, such as relations with the Republic of Panama, with the military forces on the Isthmus, with the ships of all nations using the Canal, with labor, with the civilian population on the Isthmus, and with the dignitaries, American and foreign, continually moving through as passengers.

To handle successfully such duties as these requires alertness, breadth of view, and a well-balanced mind.

General Morrow was elected an Associate Member of the American Society of Civil Engineers on June 5, 1901, and a Member on March 1, 1904.

THOMAS SARSFIELD O'CONNELL, M. Am. Soc. C. E.[1]

DIED NOVEMBER 3, 1937

Thomas Sarsfield O'Connell was born in San Francisco, Calif., on July 14, 1888, the son of Thomas Sarsfield O'Connell and Gussie (Lovell) O'Connell. He was a lineal descendant of the same family as William, Cardinal O'Connell, and was a direct descendant of Patrick Sarsfield, famed Irish patriot, for whom he was christened. As a youth, he went to Arizona, where he attended preparatory school. His technical education was received at the University of Arizona, at Tucson, Ariz., and the United States Military Academy, at West Point, N. Y., where he was a member of the Class of 1911.

Mr. O'Connell's first practical experience in his profession was with the Sud Pacifico de Mexico. His career in the Arizona State Highway Department began in 1913 and was unbroken, except during the World War. In May, 1917, he was commissioned as First Lieutenant in the Army and served overseas with the 362d Infantry, 91st Division. After leaving the Army in May, 1919, he again entered the service of the Arizona State Highway Department as Locating and Construction Engineer, being promoted to District Engineer in 1924, which position he held until his appointment as State Engineer, on April 1, 1931.

As State Highway Engineer of Arizona, Mr. O'Connell made an enviable record as a progressive road builder. He was recognized by engineers throughout the nation as an authority on design, construction, and the financing of highways. He was a strong advocate of Federal aid for Western highways, and was instrumental in placing the problems of the Western States before Congress. His untiring efforts in obtaining a more equitable Federal aid for the Western States was a large factor in the development of the splendid highway system of the West. He was progressive—always ready to adopt new ideas that he considered sound.

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[1] Memoir prepared by a Committee of the Arizona Section, consisting of J. R. Van Horn, E. V. Miller, and Jane H. Rider, Assoc. Members, Am. Soc. C. E.

Mr. O'Connell helped to pioneer the modern low-cost highway surfaces, utilizing light asphaltic oils. Without this type of surface, the fine dustless roads which Arizona now has, would not have been economically possible. He did much to further the modern trend of highway design, such as roadside improvement, soil stabilization studies, prevention of soil erosion, divided highways, etc.

He took an active part in the affairs of his profession, being a Past-President of the Western Association of State Highway Officials, member of the Executive Committee of the American Association of State Highway Officials, as well as Chairman of the Committee on International Highway Relations of that organization. He was a member of the Board of Directors of the American Road Builders' Association.

Mr. O'Connell was considered by all who knew him as a man of great courage, force, and ability, combined with a courteous personality and an unswerving loyalty to his employees. The greatest compliment that can be paid him is exemplified in the love and respect borne him by the members of his profession.

He was married in March, 1930, to Inez Elmer, who survives him.

Mr. O'Connell was elected an Associate Member of the American Society of Civil Engineers on July 11, 1927, and a Member on October 14, 1930.

ROBERT VANCE ORBISON, M. Am. Soc. C. E.[1]

DIED OCTOBER 6, 1937

Robert Vance Orbison was born in Sidney, Ohio, on February 20, 1882, the son of David Rush and Emma Jane (Ewing) Orbison. The family traced its ancestry to good old Irish stock which, no doubt, accounts for the good nature and tireless energy which "Bob" Orbison evidenced throughout his life.

As a boy, he attended the public schools in his native town, and was graduated from the Sidney High School in 1900. He then attended the Utah State Agricultural College, at Logan, Utah, and took up the study of Civil Engineering.

Upon leaving college, Mr. Orbison entered the service of the Utah Sugar Company, at Garland, Utah. The Company, at that time, was engaged in reclaiming and developing a large area in Northern Utah. During the 3½ yr that Mr. Orbison remained with the Company, he advanced from Chainman to Field Engineer, and had an important part in locating and constructing an irrigation system, dams, and dividing the Company's holding into farms and town sites.

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[1] Memoir prepared by Morris S. Jones, M. Am. Soc. C. E.

Leaving Utah in 1905, Mr. Orbison entered the employ of the Yosemite Valley Short Line Railroad Company, which line was then being built through the precipitous gorge of the Merced River into the Yosemite Valley. This railroad was the first means of easy travel into this beautiful valley.

In 1907 he became associated with the Stone and Webster Company as a Traveling Civil Engineer, and for three years was engaged in the construction of electric street railways in various cities, in Texas.

This work being practically completed in 1910, Mr. Orbison moved to Los Angeles, Calif., where he secured a position with the Janss Investment Company, a large realty company then engaged in subdividing and marketing large tracts of land in and around Los Angeles. In addition to sub-division work and the installation of street improvements, Mr. Orbison had charge of the installation of several moderately large water distribution systems, including the design of the pumping plants and reservoir.

In May, 1912, he became associated with the City Engineer's Office of the City of Pasadena, Calif., becoming Chief Deputy a year later. In December, 1914, he was appointed City Engineer in charge of the office. At that time, the most important problem before the City was to find an adequate means of sewage disposal. The disposal works then in use were very much overloaded, and created such a nuisance that the City was in constant danger of facing damage suits.

In 1915, the activated sludge process for sewage disposal was in its infancy. Experimental plants had been established in a few places in the United States, most notably in Milwaukee, Wis., but the superiority of this method had not then been established. After a visit to the Milwaukee Plant, Mr. Orbison built an experimental plant and laboratory and conducted tests which led to his recommendation for the construction of an activated sludge plant. The Pasadena Sewage Disposal Plant, built as a joint project by the Cities of Pasadena, South Pasadena, Calif., and Alhambra, Calif., in accordance with Mr. Orbison's design, has been in continuous operation since 1924, with most satisfactory results.

In March, 1920, after an enviable record as City Engineer of Pasadena, Mr. Orbison was called to be the first City Manager of the neighboring community of South Pasadena. While in the service of South Pasadena, he acted in the dual capacity of City Manager and City Engineer. During the period he was in charge there, the largest program of municipal improvements in the history of the city was completed. The Municipal Water Department of South Pasadena was organized by Mr. Orbison to take over the privately owned water systems then supplying the major part of the city. He was also instrumental in securing a new source of water supply for the city.

In August, 1925, after having been in charge of the affairs of the City of South Pasadena for 5½ yr, he was called back to Pasadena, this time as City

Manager. At that time Pasadena was entering into a period of most rapid growth and development. Under his régime, the $4 000 000 Civic Center Project, which consisted of a new City Hall, Library, and Municipal Auditorium, was completed. Nearly $5 000 000 worth of street improvement work was done under his supervision, and $10 000 000 in bonds were voted for the construction of the San Gabriel Water Project which was to bring into Pasadena a supply of water from the San Gabriel River.

In 1931, Mr. Orbison tendered his resignation to the City Board of Directors. For a time after leaving the service of Pasadena, he entered the consulting field in sanitary engineering. With the passing of the Federal Housing Act in 1933, he went to Washington, D. C., in the capacity of Federal Housing Engineer. Two years later he returned from the nation's Capital to accept a position with the California State Government in the Veterans' Bureau, appraising and investigating loans to World War Veterans, which position he held at the time of his death.

Mr. Orbison passed away on October 6, 1937, at his home in South Pasadena, his death being due to a sudden heart attack. Although he had not been in the best of health for some years, his death was most unexpected and a great shock to his many friends in Southern California.

"Bob" Orbison was a kindly man, witty, and always good company. He was absolutely fair and had the respect and loyalty of all those who worked with him. He was a hard worker himself, an excellent executive, and could direct others easily and efficiently. His greatest interest was in sanitary engineering, on which subject he contributed many articles to technical publications.

He was married to Evelyn Alice Kanouse, in El Paso, Tex., on November 23, 1909. He is survived by his widow, a daughter, Phyllis Orbison, his mother, Mrs. Emma J. Orbison, and two brothers, Harold E. Orbison and James E. Orbison.

He was a Thirty-second Degree Mason and a Shriner. He was exceedingly active in the affairs of the Los Angeles Section of the Society. He served as Vice-President of the Section in 1927, and was elected President in 1928.

Mr. Orbison was elected a Member of the American Society of Civil Engineers, on August 28, 1922.

FRED BAILEY OREN, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 29, 1936

Fred Bailey Oren was born in Watseka, Ill., on September 16, 1880, the son of Asa Oren and Mary (Frazier) Oren. His father came of a Quaker family whose home was in a village near Wilmington, Ohio. His mother was a Watseka girl. The removal of the family to Englewood came when his father purchased a drug store in Chicago, Ill. The boy's Grammar School education was obtained at Watseka, but his High School work was taken at Englewood High School, in Chicago, from which he was graduated in 1897, at the age of seventeen years.

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[1] Memoir prepared by Winfred Dean Gerber, M. Am. Soc. C. E., as a Committee of the Central Illinois Section.

Mr. Oren entered upon his engineering career shortly after his graduation from High School, when he entered the employ of the Engineering Department of the Illinois Central Railroad Company as Rodman.

About 1902 he was made Resident Engineer on the construction of the Illinois Central Bridge over the Little Wabash River, near Watson, Ill., with headquarters at Effingham, Ill. In 1903, he was made Division Engineer, with headquarters at Water Valley, Miss., and, later, became Roadmaster with headquarters at Champaign, Ill.

While Roadmaster at Champaign, Mr. Oren decided to go into the contracting business and, in 1919, undertook the execution of his first contract, an 8-mile section of highway paving on Old Route No. 11 of the Illinois Highway System, near St. Elmo, Ill. The contracting firm then was known as Mautz and Oren.

Out of the partnership of Mautz and Oren, there was formed in 1923 a corporation, the officers of which were A. H. Mautz, President; W. P. Mautz, Vice-President; with Mr. Oren as Secretary-Treasurer. This firm continued in the contracting business so well started, specializing in all fields of highway construction for both the Illinois State Highway Department and for various County Highway Departments.

As a business man, Mr. Oren had the respect and admiration of all who knew him; and his firm ranked high with the State Highway Department.

On June 21, 1903, he was married to Aldora Forrest Jaycox, of Watson, Ill. After severing his connection with the Illinois Central Railroad Company, he established his residence in Effingham, Ill.

He was a great fisherman and thoroughly enjoyed his vacations at his winter home in Biloxi, Miss. It is said that he knew the fishing grounds of the Gulf and its tributaries as few other men did, and could always be counted upon to bring in a good catch.

Mr. Oren was elected a Member of the American Society of Civil Engineers on April 14, 1919.

RAYMOND STANTON PATTON, M. Am. Soc. C. E.[1]

Died November 25, 1937

Raymond Stanton Patton was born in Degraff, Ohio, on December 29, 1882, the son of Oliver and Ida M. (Cloninger) Patton. His early education was obtained in the public schools at Sidney, Ohio. He received his technical training at Adelbert College of Western Reserve University, at Cleveland, Ohio, from which he was graduated in June, 1904, with the degree of Bachelor of Philosophy.

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[1] Memoir prepared by J. H. Hawley, M. Am. Soc. C. E.

Within a month after his graduation Mr. Patton was appointed an officer in the Field Corps of the United States Coast and Geodetic Survey. He rose to the position of Director of the Survey in 1929, in which position he served until his death.

Mr. Patton's first field work began in August, 1904, on the Atlantic Coast where he served as a Junior Officer on the Coast and Geodetic Survey ship *Hydrographer,* engaged in Coast Pilot revision, and with a shore party carrying on topographic surveys in Virginia. This was followed in 1906 by assignment to duty on the Survey ship *Gedney,* operating in Southeastern Alaska.

In the spring of 1907, he began a 3-yr tour of duty in the Philippine Islands where, as a Junior Officer on the Survey ships *Romblon* and *Research,* he took part in surveys of various localities including Tanon Straits, the north coast of Negros Island, and the southeast coast of Luzon. For a part of this time he was also a member of several shore parties engaged in surveys of Mindanao, Bohol, and the Camiguin Islands.

During this period, conditions in the Philippine Islands were far different from those at present. The Insurrection was not far in the past and the Filipinos, especially in areas away from centers of population, were in a state of unrest, and in many cases were resentful toward Americans. Living conditions, affecting health, recreation, and even the ordinary comforts, were far from satisfactory. In this, as well as in his other early assignments, Mr. Patton's cheerfulness in enduring hardships and his zeal in the performance of his duties indicated the pioneering spirit and devotion to the interests of the Service which characterized his entire career.

Upon his return to the United States in 1910, he was engaged on various projects along the Atlantic Coast, including triangulation in Massachusetts and resurveys of Delaware Bay and Albemarle Sound until the summer of 1911 when, after a brief assignment as Executive Officer of the Survey ship *Bache,* on the Gulf Coast, he again went to Alaska as Executive Officer of the Survey ship *Patterson.* Later, in the same year, this ship operated on the Pacific Coast of the United States and in the western approaches to the Panama Canal. In 1912, Mr. Patton was given command of the Survey ship *Explorer,* and for the following three years was in charge of the work of that ship in Alaska, of which a survey of the approaches to the Kuskokwim River was especially important.

Returning to Washington in 1915, Captain Patton was placed in charge of the compilation of the volumes of the *Coast Pilot* published by the Coast and Geodetic Survey to provide mariners with information required for coastal navigation in addition to that shown on the nautical charts. In this capacity he had charge of the field and office work required for the periodic revision of the *Coast Pilot* and was the author of two volumes, the 1916 edition of the Alaska *Coast Pilot* for the region from Yakutat Bay to the Arctic Ocean, and the 1917 edition of the *Coast Pilot* for the Pacific Coast of the United States.

With the entry of the United States into the World War, Captain Patton was anxious to join the armed forces, and in September, 1917, together with a number of other officers of the Coast and Geodetic Survey, he was transferred by Executive Order to the Navy Department and served as a Lieutenant and, later, as a Lieutenant Commander in the Naval Reserve Force for the duration of the war.

After the conclusion of this service he returned to the Coast and Geodetic Survey and, in 1919, was appointed Chief of the Chart Division. It was during his tenure in this office that his ability as an organizer and executive began to have considerable effect on the progress of the Survey's work. For some time prior to his appointment to this position, it had been realized that the Bureau's service to maritime interests was impaired by the rather lengthy period required for the compilation of nautical charts following the completion of field surveys. This condition, however, had been regarded as inevitable on account of the careful and painstaking work which is always required when extreme accuracy is essential and the arduous nature of many of the processes involved.

Recognizing the need for improvement in this respect, Captain Patton immediately applied himself to the problem and met with marked success. Through a complete re-organization of the Division, the adoption of a comprehensive production schedule, and his constant encouragement of those under him in the development of more efficient methods and equipment, the period necessary for chart compilation was gradually reduced to about one-third of the time formerly required, with no impairment in quality.

In connection with his work in the Chart Division he became interested in beach erosion and devoted extensive study and research to this subject. In 1921, the New Jersey Board of Commerce and Navigation asked Secretary of Commerce Hoover to designate a representative of his Department to serve on an engineering advisory board created to study erosion problems in that State, for the purpose of devising better means of protecting the highly improved and valuable sections of the coast. Captain Patton was named by Secretary Hoover for that duty, and co-operated actively in the execution of the project. The studies made were embodied in part in two reports, one published in 1922 and one in 1924. These reports form the basis for an active participation by the State of New Jersey in the problem of shore protection.

In 1925, he was appointed a member of the Committee of the National Research Council on Shoreline Investigations and in the following year was named its Chairman. Knowledge of the importance to New Jersey of coast protection suggested to the Committee that a similar situation might exist in other States to such an extent as to constitute a national problem in which the Council would be justified in becoming interested. Inquiry along the Atlantic and Gulf Coasts revealed that almost every State was struggling with conditions similar to those in New Jersey, each State or community groping its way toward solutions independently of the others.

Since the National Research Council had neither the funds nor the personnel for a direct attack upon the problem, the obvious method was to link together all these isolated efforts into a united one. This was accomplished through the creation of the American Shore and Beach Preservation Association organized in December, 1926, through joint action by the Committee on Shoreline Investigations and the Governors of the Atlantic and Gulf Coast States. Captain Patton took a major part in its formation, was its Secretary-Treasurer until June, 1929, and a Director until his death. On account of his extensive knowledge of this subject and of the activities and records of the U. S. Coast and Geodetic Survey he was frequently called upon to serve as an expert witness in litigation concerning riparian property boundaries.

He continued as Chief of the Chart Division until April 29, 1929, when he was commissioned by President Hoover as Director of the Coast and Geodetic Survey, succeeding E. Lester Jones, M. Am. Soc. C. E., who died on April 9, 1929. At that time, he held the relative rank of Captain in the Navy. In March, 1936, Captain Patton was promoted to the rank of Rear Admiral.

Under the leadership of Admiral Patton the modernization of survey methods and equipment, which had been initiated by Colonel Jones, was continued and accelerated. Through his own progressiveness and the encouragement he constantly gave to his colleagues, great advances were made in the efficiency of all branches of the Survey's work. Many of the improvements effected were at the start highly experimental and their final success was due in no small measure to his vision of their possibilities and his unfailing assistance in their development.

An outstanding achievement of this period was the expansion of the operations of the Coast and Geodetic Survey during the depression years from 1933 to 1935. From the beginning of the depression Admiral Patton realized that there was a vast amount of work of this character which could be carried on to advantage throughout the country and that it was well adapted to the relief of unemployment, especially in the case of men with engineering training. With the effective co-operation of the officers of the Society and others vitally interested in the conditions confronting the Engineering Profession at that time, he was able to secure substantial allotments of emergency funds which were utilized to provide employment for a large number of engineers and, at the same time, to accomplish an extensive amount of worth-while work much of which had been urgently needed for some time.

He derived great pleasure from the success of this undertaking and often expressed his gratification at the receipt of expressions of appreciation from engineers who were not only provided with employment during this trying period, but were enabled to utilize their ability in their chosen profession for the accomplishment of projects of lasting value to the nation.

Always modest and unassuming, Admiral Patton was a man of high ideals and integrity. In addition to being an able engineer and executive, he had to a marked degree the qualities which brought him the high esteem and affection of all who knew him. His career is admirably epitomized in the following tribute by Secretary of Commerce Daniel C. Roper:

"In the death of Admiral Patton the Government has lost one of its most capable officials and the engineering profession one of its outstanding leaders. He was held in the highest esteem by his associates and by Members of, and Committees in, the Congress with whom he came in contact, and was recognized as an authority in his work throughout the world. The Coast and Geodetic Survey over which he has been the head for 8½ years is one of the most efficient and progressive bureaus of our Government. Devotion to service by men of the character, integrity and standing of Admiral Patton gives a new assurance to American citizenship; it gives us greater confidence in the future of our country. We grieve over his passing, but we are thankful for his contribution to the service of the Department of Commerce and to the nation."

Admiral Patton was a Past-President of the Washington Society of Engineers; Life Trustee of the National Geographic Society; Trustee of the Woods Hole Oceanographic Institute; and a member of the National Research Council, Association of American Geographers, American Geophysical Union, and the American Astronomical Society. He was also a member of the Cosmos Club, of Washington. He was buried at Arlington National Cemetery with full military honors.

He was married on November 7, 1912, to Virginia Mitchell, of Seattle, Wash., who survives him. He also left a son, Raymond S. Patton; two daughters, Helen M. and Virginia M. Patton; his mother, Mrs. Oliver Patton; a sister, Mrs. J. C. Custenborder; and a brother, Paul C. Patton.

Admiral Patton was elected a Member of the American Society of Civil Engineers on July 11, 1921.

CARROLL PAUL, M. Am. Soc. C. E.[1]

DIED MAY 23, 1937

Carroll Paul was born on May 5, 1882, in Tokyo, Japan, the son of Henry Martyn Paul and Augusta Anna (Gray) Paul. Both parents came of old American stock. His father was descended from that Richard Paul who settled in Taunton, Mass., in 1636, and his mother traced her descent from Peregrine White of the *Mayflower*.

Henry Martyn Paul was graduated from Dartmouth College, at Hanover, N. H., in 1873, and did brilliant post-graduate work in astronomy under the late Professor Charles A. Young, who, later, recommended him for the position of Professor of Astronomy with the University of Tokyo, in Japan, when it was established in 1880. Henry Paul remained in Japan three years, and it was during this time that Carroll Paul, his only child, was born.

In 1884, upon returning to the United States, Professor Paul held a position in the Naval Observatory, at Washington, D. C. It was in the excellent public schools of this city that his son prepared for college. He entered Dartmouth in the Class of 1903, and, at the age of 21, was graduated with the degree of Bachelor of Science. The following year the Thayer School conferred on him the degree of Civil Engineer. While in college he became a member of Phi Delta Theta Fraternity.

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[1] Memoir prepared in the Longyear Office, Assisted by F. P. Burrall, President of the Longyear Corporations, Marquette, Mich.

In 1904, Mr. Paul entered the United States Reclamation Service, first as Engineering Aid, doing instrument work on the Roosevelt Dam, in Arizona; and then, in 1906, he was sent to the North Platte Project, in Nebraska, where many kinds of office work, such as monthly estimates, computation of costs, design of concrete drops in lateral canals, determination of irrigable lands, etc., were required of him in his capacity of Assistant to the Engineer in Charge of the Distribution System.

In August, 1907, as the result of a competitive examination in which he stood first, he was appointed Assistant Civil Engineer, U. S. Navy, with the rank of Ensign. He was then assigned to duty at the General Electric Company, in Schenectady, N. Y., for the purpose of making tests of electrical machinery for the Navy. In July, 1909, he was transferred to the Bureau of Yards and Docks, at Washington, where, under the Chief of Bureau, he had charge of the Power Plant Section, which handles all plans for operation, repair, extension, and new construction of central power plants in all U. S. Navy Yards and Naval Stations.

In September, 1910, Mr. Paul reported for duty at the Navy Yard, in Norfolk, Va., where, as Assistant to the Public Works Officer, he had charge of the general maintenance and repair of public works at the Yard.

In 1911, shortly after his marriage on March 25, to Helen Longyear, of Marquette, Mich., he was ordered to the Naval Station at Olongapo, in the Philippine Islands, where he acted as Public Works Officer in charge of all maintenance and repair, and, as such, served as a member of the Board for Special Investigation of the Condition of the Floating Dry Dock *Dewey*, which sank one night at its mooring in Subig Bay, apparently without cause. In August, 1911, he was raised to the rank of Lieutenant (Junior Grade).

In November, 1912, he was transferred from Olongapo to the Naval Station at Cavite, Philippine Islands, where he had charge of all public works; this included under-water repairs to a Marine Railway, where inspection had to be made in diving helmet and bathing suit.

At the expiration of his tour of duty in the tropics, Lieutenant and Mrs. Paul returned to the United States by way of India, Egypt, and Europe, using, in this way, leave accumulated during their long absence from home.

Lieutenant Paul, upon his return in September, 1913, was ordered to the Philadelphia (Pa.) Navy Yard as Senior Assistant to the Public Works Officer, who was often called away on other duty, leaving everything in charge of his Senior Assistant. Here, among other things, the work included the designing of ship-building ways, under the general supervision of the Public Works Officer.

In April, 1914, Mr. Paul was commissioned a Lieutenant. The following year he was sent to the Naval Station, at New Orleans, La., where he was in full charge of all public works. He also acted as Senior Assistant to the

Industrial Manager who had charge of all the other operations of the Station, except those of military administration.

During his tour of duty here, in February, 1916, a tropical hurricane came up from Cuba, destroying much shipping along the Mississippi River and jeopardizing a floating dry dock belonging to the Navy Yard. It was a desperate all-night battle to keep the unwieldy structure from breaking loose and smashing its way down river through the anchored shipping, but finally Lieutenant Paul and his men succeeded in getting it under control, saving not only the dry dock, but also the two ships laid up for repairs inside it.

In October, 1916, Lieutenant Paul reported for duty at the Navy Yard in Boston, Mass., but a month later he was sent to emergency duty at the Naval Training Sation, Great Lakes, Ill., to take over work on the building and testing of a mechanical filter plant for the Station.

This work completed, he returned to Boston just before the declaration of war with Germany in April, 1917, and was sent forthwith to Guam, in the center of the Pacific, to build a radio station 600 ft. high. Messages from Nauen, in Germany, were picked up by this sation—at that time considered a remarkable achievement.

In July, 1917, he was made Lieutenant-Commander, and upon his return to the United States in 1919, he went to Yorktown, Va., where he assumed charge of all work at the Navy Mine Depot, the Naval Fuel Oil Station, and the Naval Flying Field—a triple job, executed under trying post-war conditions.

From there, in January, 1922, he was transferred to Parris Island, S. C., as Public Works Officer for the Marine Barracks; in October of that year, following the death of his father-in-law, he resigned from the Navy to become Executor and Trustee of the Estate of John M. Longyear, of Marquette.

Three years later, on October 20, 1925, he joined the Naval Reserve with the rank of Lieutenant-Commander (CEC), which was his status at the time of his death.

With the change to civilian life Mr. Paul stepped into a different existence. His work, which up to that time had been of a more or less standard type, now became far more complex, a kind of "mental engineering," dealing with problems that had no ready solution, for the administration of a large property is no easy matter. Much of his work had to do with technical subjects in unfamiliar fields, such as lawsuits, investments, factory management, and mine leases, but for fifteen years he carried this burden without stumbling, and those who have taken it up now realize how much they owe to his keen analytical mind, his foresight, and his good judgment.

On Sunday, May 23, 1937, Mr. Paul rose early to go rifle shooting—a sport in which he took keen delight. He made an excellent score, returned home, and, while dressing for dinner, fell dead. It was with a profound sense of shock that his family and friends learned what had happened, for every one had thought him in the best of health.

In accordance with his express wish, his funeral in Marquette was followed on June 12, by interment of his ashes in the National Cemetery, at Arlington, Va.

Mr. Paul is survived by his widow, Helen Longyear Paul, and by four children, Beatrice, Howard, Philip Martyn, and Judith.

By virtue of his even disposition and kindly nature, Carroll Paul possessed a true flair for making friends, and his great natural gifts found expression in many ways outside his professional work. For years, he was President of the Local Choral Society, and also one of the pillars of the Marquette Music Club. He did much serious work in music, his fine baritone voice showing to special advantage in oratorios such as the "Messiah."

He found diversion in such sports as swimming and sailing and tennis, but the sport that held his deepest interest, and the one in which he excelled, was shooting. In 1924, he shot moose in Ontario, Canada, and, in 1927, he made a trip into the unmapped Peace River country, in Alberta, Canada, to get Rocky Mountain goats and big horn, bringing back by way of trophy, several beautiful heads. He was a member of a Shooting Club, in Louisiana and a Hunting and Fishing Club, in Michigan, and, as his interest in skeet shooting developed, he organized, in 1936, the Marquette Skeet Club of which he was President at the time of his death.

Other organizations to which he belonged were the Rotary Club, the University Club, of Chicago, Ill., the Army and Navy Club, of New York, N. Y., the Congressional Club, of Washington, D. C., and the Huron Mountain Club, of Michigan. He was also a member of the American Society of Naval Engineers, and of the Naval Institute.

Mr. Paul was elected a Member of the American Society of Civil Engineers on July 9, 1923.

FRANKLIN CALHOUN PILLSBURY, M. Am. Soc. C. E.[1]

Died May 15, 1937

Franklin Calhoun Pillsbury, whose career was so closely interwoven with public highway history in Massachusetts, was by his early connection with the State Highway Commission, a pioneer in the development of the modern highway.

Mr. Pillsbury was Project Engineer of the Massachusetts Department of Public Works. He had previously been Construction Engineer, and before that District Engineer of the former Massachusetts Highway Commission, which was the pioneer in American road-building from the time of the old bicycle days, through the automobile era. Previous ideals and methods had to be cast aside, and new plans devised to make a road surface that would withstand rubber-shod fast traffic.

Franklin Calhoun Pillsbury was born on August 13, 1869, at Chicago, Ill., the son of Samuel and Georgiana (Dix) Pillsbury. He came to Massachusetts as a young man. With a talent for art, he began courses of study and later, taught for a time, in the Eric Pape School of Art, in Boston, Mass. His artistic training proved of value to him when he began the study of engineering, and it was a factor in the selection and planning of highway locations. Not a few Massachusetts highways owe their attractiveness to his vision as an artist which instinctively took advantage of local features of the landscape.

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[1] Memoir prepared by G. H. Delano, Chf. Engr., State Dept. of Public Works, Boston, Mass.

His early technical experience was obtained in the employ of Mr. Ernest W. Bowditch, of Boston, particularly on surveys and various construction works. Subsequently, as Resident Engineer, he had the responsibility of large landscape, sewerage, and water-works projects, which involved numerous incidental structures of various kinds in many localities.

In 1895, he began his service with the Commonwealth of Massachusetts as Resident Engineer with the Massachusetts Highway Commission, shortly after its organization. In the early years of road-building his practical experience was of benefit in presenting to local communities reliable data and methods for building good roads.

Mr. Pillsbury became a Division Engineer in 1896, and from 1896 to 1920, he had charge of the work of the State Highway Commission in Essex, Middlesex, and Suffolk Counties, the eastern half of Worcester County, and the western half of Norfolk County.

In 1920, he was appointed Construction Engineer, and, in 1923, he was made Project Engineer, in charge of all surveys, designs, and estimates of highway costs (except bridges), the selection of locations and highway layouts, and the preparation of contracts and specifications for construction and reconstruction on all highways built by the Department. In this capacity he planned such great roads as the Worcester Turnpike, the Providence Turnpike, the Southwest Connection by-passing the City of Worcester, the highways connecting the East Boston Tunnel with the North Shore Road, Revere Beach Boulevard, and the Newburyport Turnpike, the Concord Road, Northern and Southern Arteries, and the circumferential highway—in fact, every important highway built by the State in recent years.

It was largely due to Mr. Pillsbury's skill and ingenuity that the old macadam road, the best in its day, was transformed into the bituminous macadam, which is still regarded by many as the best type of automobile highway. His invention of the vehicular pressure-spraying machine, and the method for the construction of bituminous macadam road surface, made a great advance in the art of road-building.

Commissioner of Public Works William F. Callahan has stated that:

> "The training which Mr. Pillsbury obtained as the foundation for his great professional work, was gained in practical experience. It is greatly to his credit, that without the advantages of attendance at college or technical school, he was able to develop correct principles of engineering, which, coupled with his unusual ability to visualize, not only the final completed highway but far beyond that and into the future in the selection of routes and trends of traffic, have resulted in a system of highways to meet the great demands of the rapidly growing volume of motor vehicle traffic. It may be said justly that the present great State Highway system is the result of his extraordinary ability and thorough mastery of highway design and location."

Mr. Pillsbury was a member of the Boston Art Club, Boston City Club, Boston Society of Civil Engineers, Massachusetts Highway Association, the American Road Builders Association, and the Massachusetts State Engineers Association.

He was married on January 14, 1897, to Bessie Wentworth, the daughter of the late Dr. Walter Wentworth, of Pittsfield, Mass. Mrs. Pillsbury died on August 20, 1932. He is survived by two sons, Samuel W. Pillsbury who lives at Saranac Lake, N. Y., and Franklin C. Pillsbury, Jr., of Pittsfield, Mass.

Mr. Pillsbury was elected a Member of the American Society of Civil Engineers on August 31, 1909.

CLARENCE DuBOIS POLLOCK, M. Am. Soc. C. E.[1]

DIED APRIL 30, 1937

Clarence DuBois Pollock was born in Washington, D. C., October 17, 1871, the son of Edie Stewart Pollock and Sallie (DuBois) Pollock. Following graduation from High School, in Washington, D. C., he entered the Massachusetts Institute of Technology, at Boston, Mass., from which he was graduated in 1894 with the degree of Bachelor of Science in Civil Engineering.

During his last two summer vacations Mr. Pollock was employed as Inspector of Sewers in Newton, Mass., and as Instrumentman on surveys along the coast of Maine for the United States Light House Establishment. Immediately following his graduation from college he was for six months Chief of Party on the construction of the Commonwealth Avenue extension through the Newtons (then called Newton Central Boulevard), under the late Henry D. Woods, M. Am. Soc. C. E., City Engineer.

In January, 1895, he came to New York City (where he spent most of the remainder of his life) as Assistant to the late Col. George E. Waring, Jr., at that time Commissioner of Street Cleaning of New York, testing garbage reduction, incineration, and snow-melting machinery. In July, 1895, Mr. Pollock became Assistant Engineer in the Bureau of Street Construction and Maintenance, Department of City Works, Brooklyn, N. Y. When that city became the Borough of Brooklyn of Greater New York he was made Assistant Engineer of the Department of Highways of that Borough, and was engaged in street paving, drainage, utility conduits, street railway reconstruction, etc.

From May, 1900, to March, 1902, with the title of Senior Assistant Engineer of Sewer Division, he was in charge of construction during the rebuilding of the sewers in connection with the building of the first subway (Lafayette Street, Fourth Avenue, 42d Street, and Broadway), with Calvin W. Hendrick, Engineer of Sewers of the New York Rapid Transit Railroad Commission. For the following ten years he served as Principal Assistant Engineer of

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[1] Memoir prepared by A. Prescott Folwell, M. Am. Soc. C. E.

Highways, in charge of paving construction, first in the Borough of Brooklyn, then in Manhattan Borough, and during the last two years, he was Acting Engineer in Charge of the Bureau.

In 1912, Mr. Pollock was employed by the Cuban Government as Chief Engineer of Pavement Construction in Havana. In the latter part of 1913, he returned to the United States as Consultant to the City of San Antonio, Tex., on street widening and paving under a $4 500 000 bond issue.

From July, 1914, until his death, Mr. Pollock conducted a private engineering practice at 15 Park Row, New York City, serving both municipalities and private corporations as Consulting Engineer on construction, litigation, appraisals, arbitrations, etc. During the World War he had charge, as Project Engineer, of three housing projects for the United States Housing Corporation, and one (at Groton, Conn.) for the United States Shipping Board, which included the construction of all the utilities—water supply, gas, electricity, sewerage, and paving. In 1929 and 1930, he made an engineering investigation of, and report on, the paving and highway work done in Cook County, Illinois, under a $24 500 000 bond issue; and was one of a board of three engineers appointed by Mayor Peters, of Boston, Mass., to conduct a paving investigation. For about fifteen years he was Consulting Engineer of the Granite Paving Block Association. From 1916 to 1921, inclusive, he was Senior Member of the firm of Pollock and Taber.

At the time of his death Mr. Pollock had supervision, as Chief Resident Engineer Inspector for the Public Works Administration, in New York City, of work valued at $18 000 000, including incinerators, sewers, grade crossing, bridges, court houses, jails, etc.

For many years he had been a regular or occasional Lecturer on Paving at several engineering schools, including the Massachusetts Institute of Technology, Johns Hopkins University, Dartmouth College, and the Universities of Wisconsin, Michigan, Ohio, and Chicago.

He was active in many technical societies, including the American Society for Testing Materials, Permanent International Association of Road Congresses, National Conference on City Planning, American Society of Municipal Engineers (of which he was a Director), Brooklyn Engineers Club (of which he was a Charter Member and Past-President), Technology Club of New York, Municipal Engineers Club (of which he was Secretary for several years), and one of the Founders and Past-President of the Technology Alumni Society. For several years he was a Trustee and Financial Secretary of the Bedford Presbyterian Church, in Brooklyn, N. Y.

It is for himself rather than for his professional accomplishments and services that those who knew him best will remember him most. He always saw the best in any one and made allowances for less admirable qualities which could not be ignored. A contractor once said: "I would not think of putting one over on Pollock—I would hate to hurt his feelings", which illustrates the effect of his lovable disposition on even the most "hard boiled." Those under him—assistants, foremen, and laborers—were more than friendly to him, for they knew that he was their friend. Nothing pleased him so much as giving pleasure to others; no trouble was too great if it would do this.

He was married in October, 1896, to Frances Brown, of Boston, Mass., who survives him, as does also a sister, Mrs. Gillett Wynkoop, of Princeton, N. J.

Mr. Pollock was elected an Associate Member of the American Society of Civil Engineers on January 8, 1902, and a Member on April 4, 1905.

HENRY IRWIN RANDALL, M. Am. Soc. C. E.[1]

DIED DECEMBER 29, 1937

Henry Irwin Randall was born at Pownal, Me., on August 20, 1863, the son of William and Helen (Sylvester) Randall. He was educated at the University of California, at Berkeley, Calif., and was graduated from its College of Civil Engineering in 1887.

Mr. Randall entered the employ of the Southern Pacific Company in July, 1887, and from that date until November, 1890, he was engaged on railroad location, construction, and maintenance of the lines of the Company.

From November, 1890, when he left the service of the Southern Pacific Company, until July, 1895, Mr. Randall was employed as Instructor in the College of Civil Engineering of the University of California, and from this latter date until July, 1903, he served as Assistant Professor. During many vacations and also during a short leave of absence from his college teaching, he was engaged by the Southern Pacific Company on railroad location, construction, and maintenance work.

In July, 1903, after resigning his Assistant Professorship at the University of California, Mr. Randall again entered the service of the Southern Pacific Company, in the Engineering Department, as Assistant Engineer, being employed principally on railroad location, construction, and on special work, under the direction of the Company's Chief Engineer, and remained there until December, 1914. During this period, Mr. Randall was engaged on difficult mountain railroad location on sections of the railroad between San Diego and the Imperial Valley, California.

From December, 1914, to January, 1916, Mr. Randall was engaged in private practice; and from January, 1916, until July, 1921, he again was engaged in the service of the Southern Pacific Company, this time as Assistant Engineer in connection with railroad construction and valuation work.

After he left the service of the Southern Pacific Company in July, 1921, and until the time of his death, he did not engage in any professional activities, except for the period from October 3, 1925, to January 15, 1926, when he was again called back by the Railroad Company for some special work on railroad valuation work.

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[1] Memoir prepared by W. H. Kirkbride, M. Am. Soc. C. E.

Mr. Randall had a high sense of responsibility and devotion to duty, and all work entrusted to him was completed with painstaking care. He was a skillful and accomplished engineer and a man of high principles and integrity.

He was a member of Durant Lodge, Free and Accepted Masons, Berkeley Commandery, Knights Templar, and the First Congregational Church, of Berkeley.

Mr. Randall was granted a life membership in the Society in 1934, having paid dues as a Corporate Member for thirty-five years.

He is survived by his widow, Mrs. Gertrude L. Randall; two sons, Richard Ray Randall, and Winslow H. Randall; and a daughter, Elizabeth Randall.

Mr. Randall was elected a Member of the American Society of Civil Engineers on February 7, 1906.

ARTHUR LOWRIE REEDER, M. Am. Soc. C. E.[1]

DIED MAY 24, 1937

Arthur Lowrie Reeder, the youngest of four sons, was born on June 8, 1879, at Camden, N. J. His ancestors were of English and Dutch stock—with a trace of Scotch. His father's great-great-great-grandfather, John Reeder, came from England to Salem, Mass., in 1652. His grandmother's great-great-grandfather came from England to Bristol, Pa., in 1705, and bought a farm of 475 acres in the vicinity. His mother was the great-granddaughter of Dr. Isaac Harris, of Southern New Jersey, who was a surgeon on General Washington's Staff during the Revolutionary War. She was also descended from Cornelius Van Nieukirk, of Holland, whose son came to this country in 1659.

One of Arthur Reeder's ancestors, when only two years old, was almost burned at the stake by Indians. Held in his mother's arms, he was saved by a rescue party before any serious damage had been done. Another ancestor ran an early ferry-boat between Brooklyn, N. Y., and New York, N. Y., near the site of the present Brooklyn Bridge.

Silas Andrews Reeder, his father, a graduate of the University of Pennsylvania Medical School, at Philadelphia, Pa., was a Surgeon in the United States Navy throughout the Civil War, being attached to various vessels of the Key West Squadron, engaged in the blockade of Mobile, Ala., and other Southern ports.

Arthur Lowrie Reeder obtained his education in the Central High School, of Philadelphia, and the University of Pennsylvania, where he obtained the degree of Bachelor of Science in Electrical Engineering, in June, 1900. He began work immediately with the New York Shipbuilding Company, of Camden, as a Special Apprentice. The following year he went to the William Cramp and Sons' Ship and Engine Building Company, of Philadelphia as a helper on installation of auxiliary machinery in ships.

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[1] Memoir prepared by William C. Reeder, M. Am. Soc. C. E.

In May, 1902, after a few months with the Baldwin Locomotive Works, he entered the service of the Filtration Bureau, of Philadelphia, on the design and construction of a $28 000 000 improvement to the Water System, including five filter plants, pumping stations, conduits, pipe lines, reservoirs, etc. He also had charge of the design, specifications, construction, and maintenance of all electrical work, and, later, represented the Water Bureau in an electrolysis survey of all water pipes in the City's system. This engagement lasted seven years.

After a year as an Instructor of Mathematics at Temple University, in Philadelphia, Mr. Reeder began, in 1910, a 5-yr engagement with the Water Department of the City of Baltimore, Md., as Designing Engineer, Chief Draftsman, and Principal Assistant Engineer on a $5 000 000 water supply improvement, including the design and construction of a dam, pumping station, and a 128 000 000-gal mechanical filter plant.

From March, 1916, to May, 1918, Mr. Reeder was, successively, Assistant Engineer with Hill and Ferguson, of New York, N. Y., on water supply, water purification, sewage treatment, etc.; with the late James H. Fuertes, M. Am. Soc. C. E., of New York City, on city planning, water supply, and design of sewage treatment plants; and with Day and Zimmerman, Incorporated, of New York City, on appraisal work.

For the next two years, he was with the Pennsylvania Department of Health, in the Design and Construction Section of the Engineering Division, as Assistant Engineer and Section Chief on investigations, plans, specifications, and supervision of construction, for water supply, sewage treatment plants, incinerators, and the preliminary design of two large hospitals for contagious diseases.

The following year Mr. Reeder was engaged with the Division of Water, of the City of Cleveland, Ohio, as Senior Assistant Engineer on underpinning, design of filter plants, and studies of chemical solutions.

From June, 1921, to March, 1924, Mr. Reeder began the first of a series of three engagements with the Bureau of Water, at Reading, Pa., at this time making a thorough study of the existing water facilities of the city and formulating a comprehensive plan for the extension and improvement of the water supply, at an estimated cost of $7 600 000.

During the remainder of 1924 and 1925, Mr. Reeder was with Gannett, Seelye and Fleming, Engineers, Incorporated, of Harrisburg, Pa., in charge of all designs and specifications for dams, filter plants, pumping stations, and sewerage. In 1926 and 1927, he was again with the Reading Water Bureau as Engineer in charge of the Improvement Division on the preparation of a reservoir site, the design and construction of a dam and reinforced concrete highway arch bridge, and the preliminary design of a 42 000 000-gal mechanical filter plant.

In 1928, Mr. Reeder was with the Mahoning Valley Sanitary District, of Youngstown, Ohio, in charge of all designs for a new water system for

Youngstown and Niles, Ohio, including force mains, clear-water reservoir, electric pumping station, and a 42 000 000-gal water purification and softening plant, the total cost of which was estimated at $9 150 000. From the fall of 1930 to March, 1932, he was again in Youngstown continuing this improvement. During 1929 and early in 1930, Mr. Reeder was with H. Burdett Cleveland, M. Am. Soc. C. E., of New York City, as Principal Assistant Engineer on the construction of a trunk sanitary sewer and treatment plant and the design of incinerator and water supply improvements.

From April, 1932, to December, 1935, he returned for his third engagement with the Reading Water Bureau, as Engineer in charge of the Design Division, on active resumption of the improvement work. This included the plans and specifications for twenty-three contracts for the construction of a pressure tunnel, two clear-water reservoirs, and the first unit consisting of one-half of the ultimate 42 000 000-gal mechanical filter plant.

The year 1936 was spent in extensive genealogical research. At the beginning of 1937, Mr. Reeder began an engagement with Morrell Vrooman, Incorporated, and was designing a water filtration system for Gloversville, N. Y., at the time of his sudden death.

His large-scale water supply improvements included the design, construction, and repair of eighteen water purification plants, having a combined daily capacity exceeding 970 000 000 gal.

Mr. Reeder was quiet and unassuming, but had a keen sense of humor. His reaction to casual events showed a mind acutely alive to the ridiculous. His wit was never unkind nor at the expense of others. He had many friends and no enemies. His engineering work was most painstaking and accurate, and he was punctilious in maintaining the highest ethical standards.

He was a member of the American Water Works Association, the Pennsylvania Water Works Operators' Association, and the National Geographic Society. He was also a member of the Presbyterian Church.

He was married on June 8, 1904, to Annie A. Helffrich, who died in Reading, Pa., in November, 1934. They had no children. He is survived by three older brothers, Franklin H. Reeder, William C. Reeder, M. Am. Soc. C. E., and Robert P. Reeder.

Mr. Reeder was elected a Member of the American Society of Civil Engineers on August 29, 1927.

THERON MONROE RIPLEY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 30, 1936

Theron Monroe Ripley was born at Macedon, Wayne County, N. Y., on September 28, 1868, the son of Theron Powell Ripley and Anne (Norton) Ripley. His career includes participation in two wars, construction of bridges and hydro-electric plants, rehabilitation of canals, and the building of main-line highways.

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[1] Memoir prepared by a Committee of the Buffalo, N. Y., Section, consisting of John H. Feigel, Assoc. M. Am. Soc. C. E., and George S. Minniss, and George F. Unger, Members, Am. Soc. C. E.

His college training was obtained at Marietta College, in Marietta, Ohio, from which he was graduated in the Arts Course and, having shown a special aptitude for mathematics, he obtained a position with the Montana Railroad Company at Helena, Mont., in September, 1893, and was connected with the running of sixty miles of preliminary surveys on railroad lines through unusually difficult topography.

Young Ripley then obtained a position with the Montana Power Company making surveys for a new hydro-electric plant on the Missouri River, on which he later served as Resident Engineer during construction.

In April, 1898, with the true patriotic fervor which he possessed, he left Montana to participate in the Spanish-American War, first as a Sergeant in the First Montana Volunteer Infantry, seeing service in the United States and, finally, in Manila, Philippine Islands. At the conclusion of the war with Spain, Mr. Ripley entered the employ of the Philippine Commission and served until 1901 as Superintendent of Parks for the City of Manila.

On his return to the United States in 1901, he became Supervising Engineer for the Ohio Bridge and Ferry Company on the construction of a new bridge across the Ohio River, at Marietta, Ohio. At that time he made a special study of the Ohio Canal and for a year was Chairman of the Legislative Committee of the Ohio Canal Association.

From 1903 to late in 1905, he carried on a private practice as a Consulting Engineer, at Marietta, during which time he made surveys for electric railroad lines and numerous other utilities. He also designed and built the sewage disposal plant for Washington County, Ohio.

Mr. Ripley returned to New York State in 1905, and served, successively, as Assistant, Senior Assistant, and Resident Engineer on the Barge Canal in New York State. During this time he designed locks, dams, prisms, retaining walls, etc., and collaborated in re-writing the Barge Canal Specifications; he made an inspection of the progress work on the Mohawk River, and, later, had charge of work on the Oswego River amounting to $5 000 000, including new and varied types of construction, and the installation of several Tainter gates.

In 1913, he became Division Engineer in the New York State Highway Department and had charge of the Watertown, N. Y., Office, with the supervision of surveys, design, and construction, as well as maintenance of that important construction division with its modern, improved roads through the Adirondack Mountains.

At the entrance of the United States into the World War, Mr. Ripley was commissioned a Major of the Corps of Engineers, U. S. Army. He reported for active duty on December 28, 1917, at Camp Lee, in Petersburg, Va., and on January 5, 1918, was made Chief Engineer at Camp Abraham Eustis, near Newport News, Va., and very shortly thereafter was made Construction Quartermaster at Chickamauga Park, Georgia, which included Camp Forest, Camp Greenleaf, Camp Oglethorpe, as well as Base Hospital No. 14 and Prison Barracks.

At the conclusion of the war the necessity for the settlement of claims, and other business connected with the Chickamauga Camp, held Major Ripley in service until June 24, 1919. At the termination of his military service he returned to the New York State Highway Department as Division Engineer at Watertown, and later, at Hornell, N. Y.

Early in 1921 he was made District Engineer of the New York Department of Public Works, in charge of surveys, design, construction, and maintenance of highways, maintenance of the Barge Canal, 3 000 miles of highways, and 20 miles of canal and terminals, with headquarters at Fulton, N. Y. In 1925, he was placed in charge of the Western Division, with headquarters in Buffalo, N. Y.

In May, 1926, Major Ripley resigned from his position with the State to accept an appointment by the Erie County Board of Supervisors as Chief Engineer of the Greater Motorways Project in the Erie County Highway Department. As Chief Engineer of this project he built a system of wide highways in the towns adjoining the City of Buffalo that involved more than $25 000 000 in construction cost. He initiated and had charge of 1 200 sq miles of aerial surveys and mapping of Erie County, New York.

The Greater Motorways System included the construction of Sheridan Drive in the Towns of Tonawanda and Amherst, and the Southwestern Boulevard through West Seneca, Hamburg, and several other Western New York towns.

In January, 1931, the task of completing the Greater Motorways of Erie County having been accomplished, Major Ripley retired from public service and established a private consulting engineering practice which he carried on until the time of his fatal illness. His practice consisted of appearances before the Public Service Commission of New York State as an expert on grade separation and grade-crossing elimination, as well as expert testimony before the New York State Court of Claims.

He also took a deep interest in the improvement of the New York State Barge Canal and, by his intense study of the St. Lawrence Waterway Project, he became one of the best informed engineers in the East of this project. During the three or four years preceding his death, he was Chairman of the Canals and Waterways Committee of the Buffalo Section of the Society and wrote exhaustively on some of the phases of the St. Lawrence Waterway.

On May 8, 1902, Major Ripley was married to Edith Mills, of Marietta, Ohio and the delightful companionship of these two former students at Marietta College continued during the Major's lifetime. His widow survives him, as does also a son, Mills N. Ripley, and a daughter, Mrs. Edwin F. Cellette.

Major Ripley was an active member of many professional and social organizations, among them being the Buffalo City Planning Association, Buffalo Chamber of Commerce, Liscom-Seyburn Post, Spanish-American War Veterans, American Legion, American Society of Military Engineers, Buffalo Athletic Club, Rotary Club, of Watertown, N. Y., Torch Club, of Buffalo, and the Delta Upsilon Fraternity.

As an indication of Major Ripley's patriotism he held, until the time of his death, a commission as Major in the Engineers' Reserve Corps in the United States Army.

He was a member of the First Presbyterian Church, of Buffalo. His funeral was held from Forest Lawn Chapel and conducted by his dear friend, Dr. Ralph B. Hindman.

Major Ripley was elected an Associate Member of the American Society of Civil Engineers on November 6, 1901, and a Member on February 28, 1911.

WILLIAM KERPER RUNYON, M. Am. Soc. C. E.[1]

Died February 28, 1938

Few men are privileged, during their lifetime, to travel as "Bill" Runyon did, to many of the remote and little known corners of the earth, there to find important work for their minds and hands to do, strange sights for their eyes to see, and interesting friends to share their joys and sorrows.

William Kerper Runyon was born in Newark, N. J., on January 11, 1877, the son of Enos and Rose (Kerper) Runyon. Both parents were descended from early American families, and were natives of Rahway, N. J., and Reading, Pa., respectively.

Mr. Runyon attended the Newark Academy and the Newark High School, from which he was graduated at the age of fifteen. He entered Lafayette College, at Easton, Pa., from which he was graduated in Civil Engineering in 1896, and received his Master's Degree in 1899.

Upon leaving college in 1896, he started his professional career as a Rodman for the Erie Railroad Company in New Jersey. He resigned from this position in 1898 to enlist for the Spanish-American War, during which he served as a private in the 2d New Jersey Volunteer Infantry.

Following the war, he obtained his Master's Degree and, from 1899 to 1901, held various positions as Rodman and Chief of Party on railroad valuation and construction work with the New Jersey State Board of Assessors, the North Jersey Title Guarantee Company, and the West Virginia Short Line Railroad Company.

November, 1901, found him in Cuba, where he was employed as Resident Engineer on railroad construction for the Cuba Company, at Camaguey, Cuba. Following this engagement he returned to the United States and the Erie Railroad Company for a few months, as Transitman.

The lure of foreign places was strong, however, and in October, 1902, he obtained an appointment as one of three American engineers employed by the America-China Development Company on the location and construction of the Canton-Hankow Railroad, with headquarters in Canton, China. His duties on this project carried him far into the interior of China, to provinces which had never before been visited by white men. It is interesting to read his accounts of experiences among the ignorant and poverty stricken people of this country, who regarded his every movement with curiosity, wonder, and trepidation. In recognition of his outstanding service on this project, he was made a " red button " Mandarin, and was supplied with interpreters, soldiers, and servants on all his trips.

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[1] Memoir prepared by W. F. Foster, M. Am. Soc. C. E.

While engaged on this work, he found himself at times regarded by people of the country as a benevolent foreign god, and, at other times, the object of rifle fire—a man with a price upon his head. Undaunted, he quietly and unswervingly carried to completion the location and construction of his division of the great railroad, accomplishing delicate diplomatic missions with high governmental officials and solving technical problems with equal facility. It was during this engagement that Mr. Runyon first became acquainted with Herbert Hoover, Hon. M. Am. Soc. C. E., for whom he always maintained the highest regard.

Leaving China in November, 1904, Mr. Runyon went to Tampico, Tamps., Mexico, where he made a survey of the asphalt deposits for the Pan-American Company. One year later, in November, 1905, he became Resident Engineer for the Cuba Eastern Railroad, at Guantanamo.

From July, 1906, to September, 1907, he was employed by the Bolivia Railway Company as Locating Engineer, with headquarters in La Paz, Bolivia, but was forced to leave when he contracted pneumonia while working in mountainous territory at an altitude of 13 000 ft above sea level.

In September, 1907, he was appointed a Division Engineer on the famous Madeira-Mamore Railway, at Porto Velho, Brazil. It is said of the construction of this railroad that "a man was buried with every tie." Repeated attacks of jungle fever finally forced Mr. Runyon to leave the project after two and one-half years of service, at which time he and the Surgeon General were the oldest men in point of service remaining on the project.

In June, 1910, he had recovered sufficiently from this experience to take a position as Assistant Engineer engaged in the physical valuation of the property of the New York Central Railroad Company, for the State of Massachusetts.

Returning to South America in March, 1911, Mr. Runyon was engaged by the Government of Peru and the Eddy Peruvian Company as Assistant Chief Engineer and, later, as Chief Engineer in the making of preliminary surveys and estimates for the construction of the Amazon Pacific Railroad, the Ucayali Railroad, and the Huancayo-Ayacucho Railroad, in that country.

Mr. Runyon next operated a copper mine at Moquequa, Peru, as General Manager and part owner, but disposed of his interests when the United States entered the World War.

Returning to this country, he served as Office Engineer in the Office of the Chief of Engineers, War Department, Washington, D. C., where he was principally engaged in the distribution of materials for military railways until the close of the war, and thereafter in the inventory and sale of this equipment.

Soon after coming to Washington, he was married to Minerva M. Byrne, in December, 1918, who passed away on December 27, 1931.

Upon the completion of his work with the War Department in August, 1920, Mr. Runyon returned to Peru as Chief of the Technical Division of Wessel, Duval and Company, importers and exporters of automobiles, tractors, trucks, and machinery, with headquarters at Lima.

In January, 1924, Mr. Runyon went to California and in August of that year was engaged as a Civil Engineer in the office of the Los Angeles County Surveyor. He became Deputy County Surveyor in charge of the Sanitary Sewer Permit Division, which was established under his direction. He made many friends among engineers, contractors, and property owners who appreciated the ability, tact, and courtesy with which this Division issued and inspected more than 30 000 permits for sewer connections during the fourteen years that preceded his death.

During this period he also supervised the preparation of sewer records and maps of more than 810 miles of sanitary sewers constructed under the supervision of the County Surveyor, and computed the quantities upon which payment to contractors was made for this work.

Mr. Runyon was a member of the Phi Kappa Psi Fraternity, and the Bone and Key, when in college, and remained an active member of his Alumni group. He was a member of South Pasadena Lodge of Masons, the Royal Arch, Knights Templar, a Thirty-second Degree Mason, and a member of the Al Malaikah Shrine, at Los Angeles, Calif.

He is survived by his widow, Mrs. Margaret Francis Runyon, to whom he was married in August, 1932.

Those of his friends who knew him best will always remember his quiet, kindly disposition, his courtesy and tact, his unswerving loyalty, his ready smile, and cheerful disposition.

At the insistence of friends, Mr. Runyon wrote a number of sparkling articles on his experiences in China, which were published and enjoyed by his many co-workers and others. Those stories revealed how greatly he enjoyed his life in all its fullness, and furnished some conception of the manner in which American engineers became known throughout the world as ambassadors of peace and good will.

Mr. Runyon was elected a Junior of the American Society of Civil Engineers on April 5, 1898; an Associate Member on October 5, 1904; and a Member on May 8, 1922.

HIRAM NEWTON SAVAGE, M. Am. Soc. C. E.[1]

DIED JUNE 24, 1934

Hiram Newton Savage was born in Lancaster, N. H., on October 6, 1861, the son of Hazen Nelson and Laura Ann (Newton) Savage. He was the son of a farmer. After his public school education, Mr. Savage worked his way through the New Hampshire College of Agriculture and the Mechanic Arts, at Durham, N. H., from which he was graduated in 1887, with the degree of Bachelor of Science. In 1891, he was graduated from the Thayer School of Civil Engineering, of Dartmouth College, with the degree of Civil Engineer. In 1913, the University of New Hampshire conferred upon him the honorary degree of Doctor of Science.

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[1] Memoir prepared by Charles P. Williams, M. Am. Soc. C. E.

In 1888, Mr. Savage was Assistant Engineer, and, later, Resident Engineer, of the East Tennessee and Georgia Railway, the Nashville and Tellico Railway, and the Athens (Tenn.) Improvement Company. From May to July, 1889, he served as Assistant Engineer of the Hydraulic Mining Company, in the San Pedro Mining District, New Mexico, and from July, 1889, to April, 1890, he was Chief Engineer of that Company and of the Rio Grande Water Company, in New Mexico, in charge of a survey of the Ortiz, San Pedro, and Tejon Grants, embracing 100 000 acres; placer-mine prospecting and locations; ditch location and construction; and preliminary location and estimate for a pipe line, 58 miles in length. From April to September, 1890, he was Engineer of "Billings Park", White River Junction, Vermont. In September of that year he located an extension of the sewerage system of West Randolph, Vt.

After his graduation from the Thayer School of Civil Engineering in 1891, Mr. Savage went to Southern California, and was employed by the San Diego Land and Town Company, of National City, Calif., as Chief Engineer, in charge of the construction of the Sweetwater Dam and Distribution System that was to be used to furnish irrigation and domestic supply. His work included the construction and reconstruction, operation, and maintenance of the System; the partition and subdivision of 40 000 acres of land; the location, construction, and maintenance of the National City and Otay Railway; and the construction of Sweetwater Park and Race Track, at National City.

In 1895, Mr. Savage became Consulting Engineer to the Southern California Mountain Water Company, in connection with the Morena, Upper Otay, and Lower Otay Dams, and the water-carrying system to San Diego, Calif.

From 1898 to 1903, he was Consulting Engineer for the San Diego and Cuyamaca Railway Company, the San Diego and La Jolla Railway Company, the Coronado Beach Railway Company, the Cuyamaca Water Company, and for the Contractor for the Zuninga Shoals Jetty, in San Diego Harbor, a Government project.

In 1903, Mr. Savage was appointed Consulting Engineer in the United States Reclamation Service, the organization of which had been begun the preceding year, and, from 1905 to 1915, he was Supervising Engineer of the Northern Division of that Service, including the States of Montana, North Dakota, and Wyoming. The District included eleven primary projects: The Huntley, Lower Yellowstone, Sun River, Milk River, St. Mary (storage), Flathead (Indian), Blackfeet (Indian), and Fort Peck (Indian) Projects, in Montana; the Williston and Buford-Trenton Projects, in North Dakota; and the Shoshone Project, in Wyoming; all of which were under construction and operation within the period of his service. He also investigated the irrigation possibilities for about ten secondary projects, with a view to their construction when and if funds were made available. Among the many important structures built on the primary projects were the Shoshone Dam, a concrete arch, 328 ft in height above the foundation, then the highest dam in the world, and the Corbett Tunnel, 11 ft in diameter and 3 miles long. In addition to his work in the Northern Division of the Reclamation Service, Mr. Savage at times served on Consulting Boards relative to the work in other Divisions.

He resigned from the U. S. Reclamation Service in 1915 and, in 1916, became Consulting and Supervising Engineer for the Sweetwater Water Company of California, engaged on the reconstruction and enlargement of the Sweetwater Dam, the spillway and abutments of which had been damaged by the record flood of 1916.

From 1917 to 1923, Mr. Savage was Hydraulic Engineer for the City of San Diego, Calif. He supervised the design and construction of the Barrett and the new Lower Otay Dams, the enlargement and reconstruction of Morena Dam and Spillway, and the extensions to the City's rapid sand filtration plants. During this period, he compiled all known hydrographic and climatological data for San Diego County, made estimates of the future water requirements for the City of San Diego and vicinity, investigated available water resources, and made a plan of future water development for the city and its environs, together with plans and estimates for the dams, reservoirs, carrying systems, and purification plants necessary for accomplishing this program. He made filings of water appropriations and intiated proceedings for the acquisition of the necessary rights of way. Notwithstanding the excellent and valuable work done by Mr. Savage during this engagement, his relations with the City Government of San Diego were not at all agreeable. The City Council was composed largely of politicians, who had little appreciation of his work, and whose actions were governed by political motives, rather than the good of the community. Finally, when attending a meeting of the hostile City Council, he was subjected to a tirade of criticism and abuse, and was informed that his services were terminated. Much to the chagrin of his opponents, who had expected a retort in kind, he replied calmly "very well", and with sedate dignity withdrew from the Council Chamber.

In 1923 and 1924, Mr. Savage made a trip around the world, visiting nineteen foreign countries, including Italy, Egypt, the Sudan, the Holy Land, Java, the Philippine Islands, China, and Japan. A second tour was made in 1924 and 1925. On this tour, he visited Hawaii and twenty foreign countries, including the South Sea Islands, New Zealand, Australia, the Philippine Islands, and India, where he explored the Ganges and Indus River Systems. He visited also Mesopotamia, where he reported to both King Feisal and to the British officials, on the irrigation and drainage requirements for the Government irrigation system at Irak. He traveled overland across the Holy Land, and reported on Zion and National Home movements; inspected the drainage basin in Egypt and the Sudan, traveling southward in Equatorial Africa more than 5 000 miles by railways, water routes, and overland on foot, conveyed by a force of negro porters. He went into Abyssinia, through the Sudan to the Belgian Congo, continuing through Uganda and across Lakes Albert Nyanza, Kioga, and Victoria Nyanza, to the head-waters of the Nile. Continuing southerly he went through Kenya, *via* Nairobi and Tanganyika, to Zanzibar and the Indian Ocean, thence returning by way of the Red Sea to Cairo, where he reported to the late King Fuad I, of Egypt, and to the Government officials of the Sudan, Egypt, and the British Empire, on the irrigation requirements of Egypt and the Sudan, on the water resources and on the control of the Nile, with recommendations to construct a third dam on the Nile at Nag Hamadi, and to increase the height of the Assuan Dam sufficiently to double the present storage capacity.

In 1925 and 1926, Mr. Savage inspected twenty-five National, State, and Corporate irrigation projects, in the western part of the United States and in Alberta, Canada, reporting to President Calvin Coolidge on their administrative, technical, social, and economic conditions and requirements.

In 1926 and 1927, he made a tour of inspection of the majority of the Latin-American Republics in the West Indies, Central America, and South America, going from New York, N. Y., to the West Indies, thence to Central America, through the Panama Canal, along the West Coast of South America, by the Transandine Railway, from Valparaiso, Chile, to Buenos Aires, Argentine Republic, and thence back to New York. Several trips were made into the Andes Mountains. From Buenos Aires he made a trip inland of more than 1 800 miles. From Rio de Janeiro, Brazil, a trip was made into the interior of Brazil, to and down the Alta Parana River, one of the principal tributaries of the Rio de la Plata. On this tour, studies were made of the natural resources of the several countries, with reference to the feasibility of their development and utilization.

In 1927, Mr. Savage again made inspections of the principal irrigation projects in Western United States, reporting thereon to President Coolidge. He also inspected irrigation and hydro-electric projects in Alberta and British Columbia, Canada.

In 1927 and 1928, he made a third trip around the world, visiting Latin-American Republics of South America, thence across South America by the Transandine Railway, across the Atlantic, traveling 4 000 miles in South Africa, from Cape Town to the Belgian Congo, continuing to Mozambique, Tanganyika, and Kenya, across the Indian Ocean, and to the interior of Northeast India, thence to Arabia, Egypt, and the controlling works of the Nile. From here, he went to the Holy Land, thence over Asia Minor to the Black Sea, continuing through Turkey, Greece, Italy, Switzerland, and France to England and Scotland. In these latter countries, he inspected the water supply and work at Manchester, the combined water supply project of Manchester and Liverpool, and the water supply of Glasgow, returning by way of Ireland, thence across the Atlantic to Canada and the United States.

For a time Mr. Savage was associated with Research Service, Inc., of Washington, D. C.

After five years of fruitless endeavor by the City of San Diego, to make some progress in the development of its water resources, within which time the Sutherland Dam was begun and later abandoned, the citizens had become so insistent in demanding the return of Mr. Savage that the City Council deemed it necessary to comply with their demands; accordingly, on July 2, 1928, he again took charge of the Municipal Bureau of Water Development, Operation, and Maintenance. From this time until his death he served the City with zeal and untiring energy. Even during his last sickness, he insisted on being informed daily regarding the progress of the work under his charge.

During this last engagement negotiations and acquisitions of rights of way and water rights progressed as rapidly as economically possible. Arrangement with the Federal Government was made relative to securing, when needed, an additional supply of water from the Colorado River, and plans were developed for works for the conveyance of this supply. The El Capitan Dam was constructed giving the city an additional supply of 10 000 000 gal per day.

Throughout his life Mr. Savage had an uncanny knowledge of human nature and exemplified great fortitude in overcoming obstacles and in pushing work. His ability to find the "ulterior motive", together with his unquestioned honesty of purpose and integrity, made him loved by his friends and hated by those who opposed him.

His motto was "the most good for the greatest number throughout the longest time". Stoical as an Indian, he was never swerved by praise or criticism.

In recognition of the valuable work he had done for the City of San Diego in the development of its water supply, the City Council on July 9, 1934, changed the name of Lower Otay Dam to Savage Dam.

His foresight and ability to anticipate future needs and plan accordingly were remarkable. Sweetwater, Barrett, Morena, El Capitan, and Savage Dams, which will serve the inhabitants of the San Diego area for ages to come, will stand as monuments to his ability as a man and as an engineer.

Mr. Savage was married in December, 1891, to Linna Bell Clough, of New Hampshire, who died in October, 1897. Two daughters were born to them, Lucy Eunice (Mrs. Robert L. Colthart) and Laura Ada (Mrs. Lawrence W. Hoppe). In 1927, Mr. Savage was married to Eugenia Hurlock, of Maryland, who, with his daughters, survives him.

He was a member of the University Club and the Rotary Club of San Diego. He was also a member of the Masonic Order.

Mr. Savage was elected an Associate Member of the American Society of Civil Engineers on March 7, 1894, and a Member on October 7, 1896.

WALTER JUSTIN SHERMAN, M. Am. Soc. C. E.[1]

Died April 15, 1937

Walter Justin Sherman was born at Florence, Ohio, on October 16, 1854, the son of Nathan Gould and Elizabeth (Otis) Sherman, and the grandson of Justin Sherman who settled at Wakeman, Huron County, Ohio, in 1822. He was a lineal descendant of Sarah Mitchell, the sister of an early President of Harvard University, and of Samuel Sherman who settled in Watertown, Mass., in 1634.

After his graduation from Norwalk (Ohio) High School in 1873, Mr. Sherman entered Cornell University, at Ithaca, N. Y., from which he was graduated in 1877, with the degree of Bachelor of Civil Engineering, which later was followed by his Civil Engineering degree. During Mr. Sherman's attendance at Cornell University he was a member of the Delta Kappa Epsilon Fraternity; he was also a member of a famous rowing crew.

Mr. Sherman's graduation was coincident with a great era of steam railroad construction, and, at once, he became Assistant Chief Engineer of the Lake Erie and Louisville Railroad. The following year he was appointed Chief Engineer of the Delphos and Kokomo Railroad and the Delphos, Bluffton and Frankfort Railroad. Then followed these milestones of his railroad and other engineering projects: 1879, Locating Engineer for the Chicago and Atlanta Railroad; 1879–1881, Chief Engineer for the Toledo, Delphos and Burlington Railroad; Chief Engineer for the Dayton and Southeastern Railroad; 1881–1884, Chief Engineer for the Louisville, Evansville and St. Louis Railroad; 1884–1888, Assistant Chief Engineer for the Gulf, Colorado and Santa Fé Railroad; 1889–1890, Chief Engineer for the Dallas, Pacific and Southeastern Railroad, for which Company he built 300 miles of railway in 300 working days; 1890, Chief Engineer for the Dallas (Tex.) Rapid Transit Railroad and the North Dallas Circuit Railroad; 1892–1893, Chief Engineer for the Galveston Construction Company, on the building of the Galveston, Tex., Harbor Works; 1894–1895, Contracting Engineer for the Toledo (Ohio) Bridge Company; and 1896–1897, Chief Engineer for the Wheeling and Lake Erie Railroad. The town of Justin, Tex., was named for Mr. Sherman.

In November, 1896, he formed a partnership with Henry E. Riggs, M. Am. Soc. C. E., under the firm name of The Riggs and Sherman Company, for the general practice of civil engineering. This partnership continued until April 30, 1912, when Mr. Riggs joined the Engineering Faculty of the University of Michigan, at Ann Arbor, Mich. Of this partnership, Professor Riggs states:

"The fact that two men formed an engineering partnership which continued for fifteen years and a half and during that time never had any argument or dispute over matters of policy, questions of design or finances, indicates that their friendship was very close. Sherman was one of the fairest, most honorable men I have ever known."

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[1] Memoir prepared by the late George Champe, M. Am. Soc. C. E.

Reorganizing under the name of The W. J. Sherman Company, practice was continued until 1927, when, celebrating fifty years of continuous and active practice of civil engineering, Mr. Sherman retired from the profession and devoted his time to personal affairs, studies and writings, and projects in behalf of the Ohio State Archæological and Historical Society and the Historical Society of Northwestern Ohio, of which he was President-Editor at the time of his death. The most beautiful monument erected in the State Park in the Maumee Valley commemorating the Battle of Fallen Timbers, is one of the direct results of his untiring efforts.

Mr. Sherman's important engineering assignments included municipal water-works systems and water purification works, sewerage systems and sewage treatment works, revetment and dock construction, railroad and highway bridges, a fire service pumping station for the City of Toledo, and many other projects.

He was the author of numerous technical papers, including: "The Ohio Centennial Exposition", read before the Michigan Society of Engineers, in 1900; "The Sewers of London", read before the Toledo Society of Engineers; "The Development of a Coal Mine", published in the *Cornell Civil Engineer*, 1907; "The Public Water Supply of the City of Philadelphia", read before the Toledo Society of Engineers; "Water Purification at Fostoria, Ohio", published in *The Southern Engineer*, Atlanta, Ga., 1907; reports on the water supply problems of Salem, Cambridge, Zanesville, Newark, Wellsville, Delaware, and Defiance, Ohio; Muskegon and Adrian, Mich., and many other places; "Valuation of Ann Arbor Water Works, with a View of Municipal Ownership"; "Some Observations on Electric Railways", read before the Michigan Society of Engineers; "The Galveston Harbor Works", read before the St. Louis Engineering Society; "Flood Prevention at Lima and Marietta", read before the Ohio Engineering Society; reports on "The Pollution of Swan Creek and Ottawa River", Toledo, Ohio, and "The Diversion of Swan Creek"; "Should Engineers in the Teaching Profession Engage in Private Practice"; "Old Fort Industry and Miami", published in Van Tassell's "History of the Maumee Valley"; "Fort Industry—an Historical Mystery", published in *Quarterly Bulletin* of the Historical Society of Northwestern Ohio; "Mad Anthony Wayne in the Maumee Valley"; "The Future of the Ohio Canal System"; "The Status of the Engineer under Ohio Laws", "Barge Canals and Canalized Rivers"; "Inland Waterways"; "Underground and Surface Water Supplies"; and "Flood Prevention at Mansfield, Ohio".

Mr. Sherman was a life member of the Ohio Archæological and Historical Society, The Firelands Historical Society, and the Ohio Society of Sons of the American Revolution.

On July 14, 1892, he was united in marriage with Catherine Bowmer, at Cloverport, Ky. Mrs. Sherman survives him with their two daughters, Mrs. Lynn (Margaret B.) Troxel, of Tiffin, Ohio, and Mrs. C. Barnes (Elizabeth M.) Walbridge, of Toledo, and seven grand-children.

It was the writer's privilege to know Mr. Sherman intimately for more than thirty years as professional associate and competitor, fellow club and church member, and neighbor, and to realize that the Engineering Profession has lost an outstanding member. Although he had not been in active practice for a number of years, his interest in engineering had continued unabated until the time of his death.

Mr. Sherman was elected a Member of the American Society of Civil Engineers on March 4, 1909.

JOHN KENNETH WEST SHIBLEY, M. Am. Soc. C. E.[1]

Died May 1, 1937

John Kenneth West Shibley was born on March 17, 1882, in Yorker, Ont., Canada. When he was three years old his parents moved to Colton, Calif., where he attended the public schools. He was graduated from the San Bernardino (Calif.), High School, and, in 1901, entered the University of Southern California, at Los Angeles. In 1903, he transferred to the University of California, at Berkeley, Calif., where he entered the Department of Civil Engineering.

In 1905, Mr. Shibley received the appointment of Assistant Engineer in the United States Reclamation Service, entering the Drafting Department, at Roosevelt, Ariz., during the construction of the Roosevelt Dam. He transferred to the Survey Division of the Reclamation Service, at Phoenix, Ariz., in 1907, in which position he had responsible charge of canal surveys for the Salt River Valley Project. In 1910, he was engaged in private practice of engineering, with an office in Mesa, Ariz. From 1911 to 1914, he served as Assistant Engineer in the U. S. Reclamation Service, with headquarters at Las Cruces, N. Mex., during the construction of the Elephant Butte Dam.

Mr. Shibley returned to Los Angeles, Calif., in 1914 to become affiliated with the Pacific Coast Office of the New York Continental Jewel Filtration Company, and had charge of the construction of the first large municipal filtration plant on the Pacific Coast, in San Diego, Calif. In 1915, with the organization of the California Filter Company, he moved to San Francisco, Calif., and became Manager of that Company. Mr. Shibley had the distinction of installing the first chlorinator for a domestic supply on the Pacific Coast.

In 1915 and 1916, he was the Agent of the Electro Bleaching Gas Company, of New York, N. Y., on the sale of chlorine and gas chlorinators for water supplies. In 1916 he became Agent of the Wallace and Tiernan Company, and continued in the field of water purification also as Manager and President of the California Filter Company. He continued in the latter position until 1933, when the Shibley Company, of Seattle, Wash., was organized.

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[1] Memoir prepared by Paul F. Bovard, Mgr., California Filter Co., Inc., San Francisco, Calif.

In the Northwest Territory, Mr. Shibley made detailed studies of water supplies with special reference to pulp and paper manufacture. His articles, which appeared in the *Journal* of the Technical Association of the Pulp and Paper Industry, testify to his knowledge of the water requirements for this industry. His engineering ability in designing plants which were automatic in operation as well as economical testified to his ability in studying these particular problems. Practically all the large pulp and paper mills in the Northwest are equipped with water purification plants designed by Mr. Shibley. He was not satisfied with orthodox methods used in automatic control and, in recent years, had devised a remote control push-bottom system which operated large valves and pumps, giving records of operation, the accuracy of which has not been duplicated in any other system.

Mr. Shibley had an outstanding reputation both as an engineer of design and for salesmanship. His manner was characterized by extraordinarily quick mental reactions and his social contacts made lifelong friends among his clients. His large circle of friends over a long period of years, particularly among his clients, testified to the high esteem in which he was held by them and by members of the Engineering Profession. He will be long remembered as a distinctive character who enjoyed the reputation of an able engineer.

Mr. Shibley was not the type that enjoyed athletic exercise to any great degree, but he more than made up for this deficiency by his interest in athletics, and by his congenial personality and his enjoyment of short periods of recreation.

In 1906, he was married to Marian Bovard, of Los Angeles, Calif., who with a son, Bovard, and two daughters, Jean and Lillian, survives him.

In 1917, Mr. Shibley received a commission as Captain of Engineers in the United States Army, and served for the duration of the World War.

He was a member of the American Water Works Association and had the distinction of being a Charter Member and assisting in the organization of both the California Section and the Northwest Section of the Association; he was also a member of the Technical Association of the Pulp and Paper Industry. He was a Scottish Rite Mason and a member of Islam Shrine, of San Francisco.

Mr. Shibley was elected an Associate Member of the American Society of Civil Engineers on October 9, 1917, and a member on November 26, 1923.

AUGUSTUS SMITH, M. Am. Soc. C.E.[1]

Died February 29, 1932

Augustus Smith was born in New York, N. Y., on July 30, 1868, the eldest son of Henry Atterbury Smith and Rosena Watson (Ryder) Smith. His father was born in Hickley, Leicestershire, England, emigrated to New York in 1838, and at the time of his death in 1891 was a prosperous manufacturer of wall-paper. The Smith family occupied a great brownstone house in the far West Forties of New York City. The four sons—Augustus, Henry Atterbury, Arthur, and Oliver—were "earmarked" early in life as engineer, architect, lawyer, and business man, and each lived to achieve success in his chosen calling. Their home training was free from supernatural sanctions, and in their leisure they played string quartets and chess, and painted in oils. The architect—Henry Atterbury Smith, M. Am. Soc. C. E.—is the sole survivor of this interesting quartet.

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[1] Memoir prepared by Alger C. Gildersleeve, M. Am. Soc. C. E.

Augustus Smith was educated in the Public Schools of New York City, received his Bachelor of Arts Degree from the College of the City of New York in 1887, entered the School of Mines, at Columbia University in New York City, in the Junior year, and was graduated in 1889 with the degree of Civil Engineer. He was a member of the Phi Beta Kappa Society.

Soon after his graduation from Columbia the Link Belt Company had an opening in its Philadelphia, Pa., Office for a Draftsman, and Mr. Smith applied for the job. As he entered the door, he encountered his classmate, Charles Piez, a rival applicant. The Link Belt Company showed its good judgment by hiring both boys. After not many years Mr. Piez was made President of the Company; and Mr. Smith became Head Draftsman in the New York Office, and then Engineering Salesman.

In 1893, he opened his own office in New York City as Engineer and Contractor, and, in 1906, incorporated the Bergen Point Iron Works of Bayonne, N. J. He was President of this Company until his death. As a Contracting Engineer his most important works were the two great coaling stations for the Panama Canal at Cristobal and Colon, the cost of which was about $2 000 000; and two others for the United States Navy—one at Narragansett Bay, R. I., and the other at the New York Navy Yard, in Brooklyn, N. Y.

Augustus Smith considered himself as a "plain man" and "of the people," his personal habits testifying to his natural asceticism; his beautiful residence, "Frogwood," in Roselle, N. J.; his unique summer home, "South Dunes," at Bay Head, N. J.; his motor cars and his yacht, were acquired against his natural inclination, and he always felt a bit uneasy over them.

In his youth, Augustus Smith was greatly impressed with the works of Jonathan Swift, and later in life he often quoted "Gulliver" in his talks and writings. Swift's bitter indictment of his fellow men roused an ambition in Mr. Smith to do what he could to remedy these evil manners of mankind, and to this end he applied himself diligently to the study of social concepts and problems. The origin and nature of the sense of duty, the validity of the concept of natural rights, and the meaning of justice, were themes that were always before him, and for which he was trying to develop workable formulas.

His philosophy, although he never claimed one, was essentially pragmatic. The mysteries of the stars and atoms, as elucidated by the modern physicists, meant as little to him as the promulgations of the mystics.

After profound study of the writings of Adam Smith, Karl Marx, François Fourier, and others, he made a laboratory of his Bergen Point Iron Works, testing a "Compensation Plan" by which the profits were equably divided between "Capital," "Talent," and "Labor." The plan operated successfully for many years, but failed to meet the problems of the "Great Depression."

In 1924, he was appointed to the New Jersey State Board of Commerce and Nagivation, and for many years inspected the waterways of his adopted State in his motor yacht, *The Gambolier*. This boat had been built from his own plans, and when he loaned it to the Government during the World War the engine was found to be of such unique construction that none of the personnel could fathom its intricacies, and it was returned to him with thanks.

He took an active interest in the civic affairs of his home town—Roselle—and was a member of the Borough Council there for three years, acting as Head of the Public Works Department.

Joining the American Economic Association, he was for a time an ardent disciple of Professor Irving Fisher, of Yale University, and supported the effort to make a science of economics with the "yardstick of value," the "money illusion," and other ideas. Although the two economists did not always agree, they had a great respect for each other's opinions—the Professor sometimes sending his manuscripts to the Engineer for comments, which were promptly returned on many closely typewritten pages. Professor Fisher's adherence to the "prohibition illusion" was a shock to Mr. Smith's conceptions of "natural rights."

Augustus Smith was a voluminous letter writer, and at times privately printed little pamphlets which he presented to his friends. In these essays he presented his views on various subjects of contemporaneous interest. His paper on the Narragansett Bay Coaling Station was published by the Society[2] in 1906. With the exception of discussions of various papers, he left no other published works. He sometimes tired of the pseudo-sciences, and his escapism—entertainment without any social purpose—was a correspondence on pure mathematics conducted with a few chosen friends. In one of his last letters he proposed a problem of which his death sealed his solution.

He was a Member of the American Society of Mechanical Engineers, the Franklin Institute, and the American Economic Association.

In 1903, he was married to Harriet Christine Davies, who died the following year; in 1907, he was married to Anna Barbara Such, who died in 1917. An only child—by his second wife—is Henry Such Smith.

Mr. Smith was elected a Member of the American Society of Civil Engineers on June 4, 1902.

BERNHARD ALEXANDER SMITH, M. Am. Soc. C. E.[1]

DIED MARCH 1, 1936

Bernhard Alexander Smith was born at Stawell, Victoria, Australia, on December 4, 1863, the son of Bernhard Smith, Police Magistrate, and Olivia

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[2] "The Narragansett Bay Coal Depot," by Augustus Smith, *Transactions*, Am. Soc. C. E., Vol. LVII (December, 1906), p. 204.

[1] Memoir prepared by George Higgins, M. Am. Soc. C. E.

Frances Josephine (Boyes) Smith. His father was a noted amateur artist of the pre-Raphaelite school.

Bernhard Alexander Smith was educated at the Stawell Grammar School, at Queen's College, St. Kilda, Melbourne, Victoria, Australia, and at the University of Melbourne, obtaining the Argus Scholarship in Engineering at the Final Honor Examination at the University.

His first professional engagement was that of Engineer to the contractors for the construction of a railway line between Melbourne and Kew. In the following year, 1886, he obtained a Surveyor's License for the Colony of Victoria. Between 1888 and 1894, he practised in partnership with Mr. T. C. Hodgson. In 1890, Mr. Smith obtained the Certificate of Qualification as an Engineer of Water Supply under the Victorian Water Act of 1890; and for part of that year he taught at the Scotch College, which is one of the six Associated Public Schools of Victoria. The University of Melbourne also appointed him Lecturer in Surveying, Leveling, and Practical Mensuration. In 1891, when the same University appointed him Lecturer on Hydraulic Engineering, he resigned his lectureship in Surveying.

During the years 1899 to 1901, Mr. Smith practised in partnership with Mr. A. G. M. Michell, who later acquired world-wide fame as the inventor of the Michell thrust bearing, the crankless engine, the turbo-pump, the tangent jet wheel, etc., and whose scientific writings are widely read.

In 1903, he obtained a Surveyor's License for the State of New South Wales, and, in 1904, he obtained a Special License under the Real Property Act of 1900, in that State.

Mr. Smith reported on, designed, and constructed numerous works, the variety of which will be evident from the following examples: The design and supervision of installations of sewerage works at Horsham and Swan Hill, and preliminary reports for Warragul, Bairnsdale, Seymour, and Wodonga. He was consulted in Court cases about flood damage (Vinnicombe *v.* Macgregor; Colonial Ammunition Company *v.* Commissioners of Railways; and Kanuluik *v.* Mayor of Hawthorn). He also made reports and, where necessary, designs for storm-water drains for the Cities of Caulfield and St. Kilda; designs for the repair of the damage done by floods at Kooyong Tennis Courts; and for a flood bank around Moyne Swamp. He also reported on the obstruction caused to the River Yarra by the railway bridge at South Yarra, and on the flooding due to overflow of Moonee Ponds Creek.

He constructed the arched dam, 77 ft high, at Mount Cole, Ararat; the pumping plant, water tower, and treatment works, at Bairnsdale; the arched dam, turbines, motors, etc., for a private hydro-electric project; and the new reticulation mains for Seymour. He made reports on the remodeling of the water supply, of Shepparton; the pumping plant for Swan Hill; the water supply for Tallarock; the Yuthong hydro-electric project; a water tower, pumping plant, weir, reticulation, etc., at Wangaratta; the pumps and 1 000 000-gal reinforced concrete tank, at Warragul; a complete pumping water-supply project for Wodonga; the hydro-electric project for Stawell; the water supply for Camperdown and Terang; the water-works for Lismore; the installation of a water supply at Lorne, Maffra, and Mansfield; and on the water supply for Murtoa, Sunbury, Trafalgar, and Glenrowan. (All these places are in the State of Victoria.)

Mr. Smith also installed a water supply system, at Devonport, Tasmania; and reported on drainage in the vicinity of certain creeks, in the Riverina District, New South Wales.

The surveys and plans for a steam tramway for a mining company, in Tasmania, were made by Mr. Smith, as well as the valuation of the Brisbane (Queensland) Tramways prior to their transfer from a private company to the Government; he also prepared the valuation of the Melbourne (Victoria) Tramways in a similar case.

Installations for heating and ventilation were made by Mr. Smith for a tannery, at Preston; a soap factory, at Ballarat; a biscuit factory, at South Melbourne; a brewery at Abbotsford; a coffee palace, in Melbourne; the Melbourne Hospital, and three other hospitals in Melbourne, namely; the Children's, the Women's, and the Queen's Memorial for Infectious Diseases. The installations for electricity supplies for the Towns of Maffra, Cowes (Philip Island), and Horsham, as well as for the Shire of Upper Murray, were also a part of Mr. Smith's work, which included the construction of a bridge over the Tambo River, at Swan Reach, Victoria and a concrete stand at the Melbourne Cricket Ground.

Mr. Smith was the author of the following technical papers, etc: "Dynamical Models"[2]; "Michell's Method of Testing Fans"[3]; "Transition Curves"[3]; "The Comparison of a Tape with the Melbourne Observatory Standard"[4]; "A Test for Heat Insulators"[5]; "Town Sewerage"[6]; "Ventilation of Buildings"[7]; "Arched Dams"[8] (for which he was awarded the J. James R. Croes Medal in 1920, by the American Society of Civil Engineers); and "Experimental Deformation of a Cylindrical Arched Dam"[9].

He became a member of the Victorian Institute of Surveyors in 1889. In 1914, Mr. Smith was elected a member of the Victorian Institute of Electrical Engineers, and of the Melbourne University Engineering Society. In 1918, on the inception of the Public Health Commission of Melbourne, he was elected a member of the Commission. The year, 1923, marked his election as Fellow of the Victorian Institute of Surveyors, a member of the Faculty of Engineering of the University of Melbourne, and a member of the Victorian Lifts Board. In 1925, Mr. Smith became Chairman of the Melbourne Division of the Institution of Engineers, Australia, and, in 1928, he served as President of that Institution. He was a member of both the Australasian and the British Associations for the Advancement of Science.

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[2] *Transactions*, Victorian Inst. of Surveyors, Vol. V.

[3] *Loc. cit.*, Vol. VI.

[4] *Transactions*, Victorian Inst. of Surveyors, Vol. IV.

[5] *Transactions*, Institution of Engineers of Australia, 1920.

[6] *Australian Municipal Journal*, November 15, 1926.

[7] *Health Bulletin, No. 12*, Comm. of Public Health, Victoria, October-December, 1927.

[8] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1920), p. 2027.

[9] *Loc. cit.*, Vol. 91 (1927), p. 705.

Mr. Smith's principal recreation was music, the piano being his favorite instrument. It was at his suggestion that arrangements were made for an excellent series of orchestral concerts in Melbourne, which were conducted by the late Professor Marshall Hall.

His friends will remember him for his cordiality and kindliness, his courtesy and consideration for the opinions of others. In the Engineering World he set an example by his patient conscientious search for accuracy.

His marriage to Agnes Janet Shields, the daughter of a prominent Melbourne physician, took place on December 13, 1899. He died at Armadale, Victoria, on March 1, 1936, retaining his intellectual keenness during a long period of bodily weakness, and is survived by his widow, one son, and two daughters. The son, Mr. D. Bernhard Smith, had been his partner for a number of years. A distinguished cousin of Mr. Smith who predeceased him by a few years, was Philip a' Morley Parker, M. Am. Soc. C. E., the author of the well-known treatise on "The Control of Water".

Mr. Smith was elected a Member of the American Society of Civil Engineers on June 18, 1918.

EUGENE RAYMOND SMITH, M. Am. Soc. C. E.[1]

Died January 14, 1938

Eugene Raymond Smith was born in Brooklyn, N. Y., on February 10, 1858, the son of Charles Carpenter and Adelia (Hawkins) Smith. His parents later removed to Islip, N. Y., where he received his early education in the public schools. Later, he entered Cornell University, at Ithaca, N. Y., from which he was graduated in 1877 with the degree of Bachelor in Civil Engineering.

From 1877 to 1880 Mr. Smith was employed as Assistant Engineer on the construction of the Burlington, Cedar Rapids and Northern Railroad; Assistant Inspector of dredging at Cherry Island Flats and Bulkhead Bar, on the Delaware River; and Assistant Engineer in charge of construction on the Denver and Rio Grande Railroad in Price River Canyon, Utah.

In 1880, he decided to go into private practice and opened an office in Islip as a Civil Engineer and Surveyor. His practice was of a highly varied character, calling for knowledge of engineering fundamentals and ingenuity. Among the prominent projects with which he was connected, were the following: The location and alignment of the Long Island Motor Parkway; Long Island State Park Commission in its early days; the formation of the Fire Island State Park as a park; the first grade-crossing elimination work in Western Suffolk County, New York, at the Cutting and Vanderbilt crossings of Merrick Road and the Long Island Railroad; and the development of the incorporated Village of Brightwaters and "Idlehour," formerly the estate of the late W. K. Vanderbilt, in Oakdale, N. Y. He was in charge of the construction of the trout preserve at Wantagh, N. Y., and also at Nissequogue, N. Y., for the Nissequogue Trout Club; of the field work at the New York City Insane Asylum, at Central Islip, on sewer work, etc.; location survey maps and profiles for Kings, Queens, and Suffolk Railroad Company from the Brooklyn City line to Far Rockaway, N. Y.; of the reclamation of 80 acres of salt marsh land at Islip, and of the reclamation of salt marsh land at Broad Channel, Jamaica Bay, New York; the extension of the distribution system of Northport Water Works Company, Northport, N. Y.; and several small dredging improvements on the south side of Long Island, New York. Mr. Smith's reputation as a Surveyor was such that in preparing the Hyde Atlas of the southern shore of Long Island, his collaboration was eagerly sought.

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[1] Memoir prepared by Robert J. Bartley, M. Am. Soc. C. E.

His varied experience with land reclamation on the south side of Long Island, led to his paper on the "The Compressibility of Salt Marsh Under the Weight of Earth Fill."[2]

Mr. Smith was always the student, well informed, independent, and liberal in his views. He was the kind of engineer who did not believe that his value to society ended with purely engineering work. Being of a kindly nature, his keen sense of humor was always a source of pleasure to his friends.

His community activities included: Treasurer of the Islip Methodist Episcopal Church and President of its Board of Trustees; Chief of the Islip Fire Department; President of the Library Board; President of the Islip Board of Education; President and Director of the First National Bank of Islip; and Director of the First National Bank of Bay Shore, N. Y. Mr. Smith was a Past-President of the Nassau-Suffolk Civil Engineers, Incorporated, and a member of the New York State Association for Professional Engineers and Land Surveyors.

He was married, in 1885, to Maywood Clock, of Islip, who, with a son, Rodney J. Smith; a daughter, Marian (Smith) Williams; a grand-daughter, one sister, and two brothers, survives him. He retired from active business in 1926, but retained his interest in banking until about 1933.

Mr. Smith was elected a Junior of the American Society of Civil Engineers on May 2, 1888, and a Member on May 1, 1901.

CHRISTOPHER HENRY SNYDER, M. Am. Soc. C. E.[1]

DIED MAY 27, 1937

Christopher Henry Snyder, the son of William Cowperthwait Snyder and Isyphene Caroline (Pearce) Snyder, was born in Fulton, Ill., on June 12, 1866. His father was engaged in shipping and warehousing in Fulton at a time when the Mississippi River was an important trade route and Fulton a distributing point for Northwestern Illinois.

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[2] *Transactions*, Am. Soc. C. E., Vol. XXXVII (June, 1897), p. 213.

[1] Memoir prepared by Frederick R. Muhs, M. Am. Soc. C. E.

As a boy, Mr. Snyder attended the public schools at Fulton, later entering the University of Illinois, at Urbana, Ill., from which he was graduated in 1890 with the degree of Bachelor of Science in Civil Engineering. After graduation Mr. Snyder spent three years as a Draftsman with the Keystone Bridge Company, two years of which were spent in the Chicago Office, and one year at the plant in Pittsburgh, Pa.

In 1893 he became associated with Milliken Brothers, Incorporated, of New York, N. Y., working for five years as a Draftsman, two years as Chief Draftsman, two years as Contracting Engineer at Honolulu, Hawaii, and ten years as Contracting Engineer, at San Francisco, Calif.

In 1912 he opened an office in San Francisco as a Consulting and Designing Engineer for structural work, and continued his practice until his death, on May 27, 1937. Some of the buildings for which he designed the structural frames are as follows: Department of Labor and Interstate Commerce Building, Washington, D. C.; City Hall, Federal Building, War Memorial Opera House, and Veterans Building, City and County Hospital, Relief Home, University of California Hospital, 450 Sutter Building, and the Pacific Gas and Electric Company Office Building, in San Francisco; Stanford University Library, Palo Alto, Calif.; Pasadena City Hall, Pasadena, Calif.; Makee Sugar Company Mill, and the American Sugar Company Mill, in Honolulu.

When concrete became an important material in structures, Mr. Snyder designed and patented a "flat slab-steel frame" construction for buildings, which was used in many of the modern buildings.

On October 14, 1899, Mr. Snyder was married to Harriet Runyon, to which union the following children were born: Henry R., William C., Helen J., and John R. Mrs. Snyder and the children survive him.

Mr. Snyder was interested in the raising of engineering standards and served on many committees having to do with the welfare of the Civil Engineering Profession. His professional affiliations included the San Francisco Section of the Society (President, 1914), the Western Society of Engineers, and Pacific Association of Consulting Engineers. He was a Charter Member and assisted in organizing the Structural Engineers Association of Northern California (President, 1934). He was also a member of the Engineers' Club of San Francisco (President, 1925–26); the Union League Club, Commercial Club, Commonwealth Club, Kiwanis Club, and the Olympic Club, all of San Francisco; the Claremont Country Club, of Oakland, Calif.; and the Mount Diablo Country Club, of Danville, Calif. His principal recreations were music and golf.

A fitting tribute[2] to Mr. Snyder was paid by Arthur Brown, Jr., a San Francisco architect, which is here quoted in full:

> "The architectural and engineering fraternity has lost one of its most distinguished and outstanding members by the death of Christopher H. Snyder. For the past thirty years he was intimately associated with the design and execution of much of the important engineering and architectural construction that has been done in the San Francisco Bay region, and the

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[2] *The Architect and Engineer,* June, 1937.

architects for whom he acted in the capacity of consulting and designing engineer will always be grateful to him for his splendid work in structural design, upon which they could confidently depend.

"C. H. Snyder was singularly fitted for his task as designing and consulting engineer for the architectural structures to which he gave so much of his professional attention. Thoroughly schooled in engineering theory at the University of Illinois, he passed a long apprenticeship in practical application in the steel manufacturing plant of the Keystone Bridge Company. Following his training in structural design from the producer's standpoint, he joined the organization of Milliken Brothers, contractors, where he had many years of experience and responsibility in the actual administration and execution of engineering enterprises of the first order.

"These three phases of his career rounded out and completed his highly competent professional equipment. His recommendations could be confidently depended upon for rational design and feasibility, and his estimates of cost were based on positive knowledge and experience. Thus he was an invaluable collaborator for those architects with whom he was associated. His designs were always simple and direct—as was his personal character. His brilliant engineering imagination was founded on precise knowledge and his ingenuity and resourcefulness followed naturally from a thorough familiarity with the details of his craft. He did not consciously concern himself with making engineering history, but concentrated his attention strictly to his prime objective of conceiving safe, practical and economical structures. His drawings were models of clearness and gave precise, systematic and complete information. Possessed of high ideals he never wavered in defending his convictions; nor was he known to compromise on safety and good practice.

"Mr. Snyder's professional qualities were a clear reflection of his temperament. He had little sympathy with the vague experiments of some of the innovators in either engineering, economics, or politics, but defended sturdily the fundamental convictions born of his long experience of life and professional endeavor. He believed in hard steel, hard dollars, and hard common sense—in spite of new trends. These high qualities were generally recognized and, consequently, he enjoyed the highest respect of every one in the community.

"His friends will long mourn his passing and will long cherish the memory of a stalwart and likable personality."

Mr. Snyder was elected a Member of the American Society of Civil Engineers on October 2, 1901.

EDWARD EVERETT STETSON, M. Am. Soc. C. E.[1]

DIED JULY 13, 1937

Edward Everett Stetson was born at Mattapoisett, Mass., on October 8, 1882. His parents were Everett Cortez and Annie (Hiller) Stetson. Both families were pioneers in New England, coming from England to Massachusetts in the early part of the Seventeenth Century (the Stetsons in 1634 and the Hillers in 1629). The stock was strong and hearty on both sides, and produced many people of note during the three centuries. Mr. Stetson's grandfather was the Captain of a whaler, and his father, still hale and hearty at the age of 84, is a land-owner and lumber-mill operator in Mattapoisett.

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[1] Memoir prepared by Joshua D'Esposito and Einar Weidemann, Members, Am. Soc. C. E.

Mr. Stetson had his early schooling at Mattapoisett and, later, went to High School, at Fair Haven, Mass. Between times he worked on his father's farm and in the mill. In High School he gave such a good account of himself as a student in mathematics that the Principal persuaded his father to permit him to study Engineering instead of returning to the farm and lumber mill as originally planned. He entered the Massachusetts Institute of Technology at Boston, Mass., and received his degree of Bachelor of Science in Civil Engineering in 1904. Immediately after his graduation, Mr. Stetson spent some time at survey work in Sheffield, Pa., but in September of the same year he was called to his Alma Mater as an Instructor of Railroad Engineering and remained in this position for a year.

His desire for practical railway work was realized in 1905, when he accepted a position as Levelman with the Pennsylvania Railroad Company, with headquarters at Pittsburgh, Pa. Mr. Stetson quickly impressed himself as a young man of outstanding character and ability, and as early as 1909 he had been advanced to the position of Engineer in Charge in the Office of the Chief Engineer, Maintenance of Way.

During these earlier years of his career, his clear analytical mind contributed some valuable research work, particularly on stresses in railroad tracks, at that time a subject about which information was greatly lacking.

During the period from 1909 to 1916, Mr. Stetson was actively engaged as Engineer in Charge of Construction of several track-elevation projects undertaken by the Pennsylvania Railroad Company, principally in Columbus and Cincinnati, Ohio. This difficult class of work which requires the maintenance of railroad operations simultaneously with the procedure of the construction work, and which is often made more complicated by the frequently diverging interests of the municipalities, contractors, and railroad company, gained for him valuable experience that stood him in good stead in his later career.

At the outbreak of the World War and particularly during the United States Railroad Administration, many new facilities were called for, and Mr. Stetson returned to Pittsburgh in 1916, to service as Assistant Engineer on the Staff of the Chief Engineer of Construction, directing various construction projects from Headquarters.

The Chicago Union Station Company was organized in 1915 and the construction of the new $100 000 000 passenger terminal was commenced in the fall of that year. This vast project, one of the greatest engineering undertakings of its time, called for the construction of new modern facilities on ground occupied by the old passenger and freight stations, without interruption to regular railroad traffic, a traffic which involved the handling of 250 to 300 passenger trains daily and a large amount of mail and express business. This work had scarcely been started when it had to be suspended temporarily on account of the war, and it was not before the early part of 1919 that construction work was resumed in a vigorous manner. That the choice then fell upon Mr. Stetson to fill the difficult position of Engineer of

Construction for this undertaking shows the high esteem in which he was held. Neither did he disappoint his superiors. His experience and ability as a construction engineer; his high degree of leadership; his fairness and sound judgment in all his dealings with the many contractors, railroad operating men, and the public in general, contributed in a large measure to the successful completion of the great project, the execution of which required a laboriously developed program for co-ordinated construction in piecemeal of miles of railroad tracks, viaducts, bridges, and buildings costing millions of dollars, temporary and permanent structures, all to be handled with the business of the railroad terminal going on as usual. It would be expected that such a great amount of work continuing for several years could not be completed without some serious accident. This was not the case, and it might be noted as a singular record that at the completion of the work there had been no single case of serious injury to passengers or public.

In 1923, Mr. Stetson was promoted to be Principal Assistant Engineer of the Station Project, but continued in direct charge of all construction work until its completion in 1926.

The late fall of 1926 found him returning to the Pennsylvania Railroad Company as Assistant to the Chief Engineer, with headquarters in Chicago, Ill. He remained in this position until his death and, during later years, was actively engaged in the direction and administration of various construction projects and grade separation work on the Western Region of the Railroad.

He was a member of the Western Society of Engineers and the American Railway Engineering Association.

His untimely and sudden death, after an operation, was a blow to his many friends who knew him as a strong and vigorous man who had never been on sick leave. Mr. Stetson was a man of quiet and unassuming manner, but one of energy and determination, and work entrusted to him would be sure of vigorous prosecution. He had little patience with delay and procrastination.

His many friends among the contractors will remember him as the practical and resourceful engineer, keenly interested in their work and always glad and ready to have them draw upon his engineering knowledge and experience. They will remember him as an eminently fair-minded arbiter, loyal to them, his company, and his conscience. The fact that his decisions were rendered with a characteristic smile only added to their effectiveness.

His more intimate friends will remember him as the strong and manly man he was—always poised, never excited. To the many younger engineers in charge of construction work under his direction, he was the safe haven they could rely upon when the sea became too turbulent. He was always their faithful friend and would always find a way.

He was married in 1908, to Hilda C. Amundsson, at Warren, Pa. His widow and two children survive him, Mrs. Helen (Stetson) Moore, of Dallas, Tex., and Everett Charles Stetson.

Mr. Stetson was elected a Member of the American Society of Civil Engineers on May 28, 1923.

HORACE STRINGFELLOW, M. Am. Soc. C. E.[1]

DIED JANUARY 1, 1937

Horace Stringfellow was born at Montgomery, Ala., on March 28, 1882. He was the son of the Rev. James Horace Stringfellow and Lula B. (Haskins) Stringfellow. For many years his father was Rector of Christ Church, at Tuscaloosa, Ala.

Mr. Stringfellow's boyhood was spent in Tuscaloosa, where he attended Verner's School. His family intended him for the ministry, and with that career in prospect he entered the University of Alabama, at University, Ala., in 1898. However, a previous six months as Rodman with the United States Corps of Engineers may have had a great deal to do with Mr. Stringfellow's change in profession to engineering. In any case he changed from the University of Alabama to the University of the South, at Sewanee, Tenn., from which he was graduated with the degree of Bachelor of Arts, in 1902.

During a summer vacation, and prior to his graduation, Mr. Stringfellow served as Topographer with the Mobile and Western Alabama Railroad Company. From January to May, 1903, he was employed as Transitman by the Mobile, Jackson and Kansas City Railroad Company; and from May, 1903, to April, 1905, he was engaged in the same capacity with the Southern Railway Company.

His service with the Southern Railway Company continued, with short interruptions in 1913 and 1918, for nearly twenty years. He was Assistant Engineer on construction from April, 1905, to September, 1913. From September, 1913, until January, 1914, he superintended the construction of concrete reservoirs at Lake Montebello for the City of Baltimore, Md.

From February to September, 1914, he was Field Engineer on valuation for the Atlanta, Birmingham and Atlantic Railroad Company, and from September, 1914, to December, 1917, he was Assistant Engineer on the Southern Railway engaged on surveys, reports, and construction, including the rebuilding of the Warrior River Bridge. From December, 1917, to August, 1918, he was District Engineer on the same railroad, in charge of double-track construction, which involved 42 miles of heavy work and the elimination of seven tunnels.

In August, 1918, Mr. Stringfellow was commissioned Captain of the 105th Engineers, serving at Camp Shelby, Mississippi. He remained there until his discharge from the Army on December 16, 1918.

He returned to the Southern Railway Company from January, 1919, to March, 1920, as District Engineer, and was in charge of double-track construction from Washington, D. C., to Central South Carolina. This work consisted of the construction of about 200 miles of additional main track, with important yard and terminal work.

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[1] Memoir prepared by a Committee of the St. Louis Section, consisting of W. J. Burton and E. M. Durham, Jr., Members, Am. Soc. C. E.

In March, 1920, Mr. Stringfellow became Regional Engineer of the United States Railroad Administration, engaged in the liquidation of claims growing out of Federal control of railroads. This work was completed in 1924.

From 1924, until his death on January 1, 1937, Mr. Stringfellow was employed by the Missouri Pacific Railroad Company—first, as Engineering Assistant in the Executive Department and, later, as Executive General Agent at Denver, Colo. His work included a wide variety of special investigations and reports, involving traffic and operating problems, as well as those of an engineering nature. As Executive Assistant, he represented the President of the Company in many important matters.

Mr. Stringfellow combined engineering ability with good common sense. His pleasing personality and his ability to make friends were great assets in his work.

He was a member of the American Railway Engineering Association, and of the Bellerive Country Club, in St. Louis, Mo. He was a member of the Protestant Episcopal Church, and was an independent in politics.

In 1913, he was married to Virginia Mauney, who died shortly thereafter. In 1928, he was married to Ethel Jones Edmonds, who survives him.

Mr. Stringfellow was elected an Associate Member of the American Society of Civil Engineers on September 12, 1916, and a Member on April 26, 1921.

HARRY RANDOLPH TALCOTT, M. Am. Soc. C. E.[1]

Died April 6, 1937

Harry Randolph Talcott was born in Richmond, Va., on November 9, 1861, the son of Charles Gratiot and Theodosia (Lawrence) Talcott. For the preceding two generations the family had been actively associated in engineering work in the United States. His grandfather, Colonel Andrew Talcott, who was graduated from the United States Military Academy, at West Point, N. Y., in July, 1818, was in charge of the construction of fortifications at Fortress Monroe, Virginia, during which time General Robert E. Lee, as a Junior Lieutenant, was a member of his staff. His father, Charles Gratiot Talcott, for whom General Lee developed a strong affection through his frequent visits to his home, during the War between the States, served as a Captain on the staff of General Lee, in charge of railroads throughout the Confederate States. Mr. Talcott, in childhood, had the privilege of intimate association with General Lee, after the conclusion of hostilities, and thus, with his passing, one of the few remaining links with that period is dissolved.

Mr. Talcott was educated at the Episcopal High School, in Alexandria, Va., and Rittenhouse Academy, in Washington, D. C., from which latter institution he was graduated in June, 1878. His active career began in 1879, when he entered the service of the Chesapeake and Ohio Railway Company in the capacity of Rodman on preliminary surveys. He remained a member of the Chesapeake and Ohio organization, rising to the position of Resident Engineer on Construction, until May, 1882, when he resigned to enter the service of the Western North Carolina Railroad Company (later a part of the Atlantic Coast Line Railroad Company) as Assistant Engineer on Location, serving in that capacity until November of that year, when he resigned to accept a position with the United States Fish Commission as Assistant Engineer on Topographic Surveys, and in February, 1883, was engaged with the United States Coast and Geodetic Survey on the St. Johns River reconnaissance. In May, 1883, he returned to the Western North Carolina Railroad Company as Assistant Engineer on Location, remaining in this position until December of that year. In January, 1884, he entered the organization of the Richmond, Fredericksburg and Potomac Railroad Company as Assistant Engineer, but remained with the Company for a very short period.

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[1] Memoir prepared by a Committee of the Maryland Section, consisting of Jenks B. Jenkins and H. A. Lane, Members, Am. Soc. C. E.

In February, 1884, Mr. Talcott entered the service of the Baltimore and Ohio Railroad Company as Assistant Engineer in charge of the Field Corps on the construction of the Susquehanna River Bridge, comprising a part of that railroad's extension to Philadelphia, Pa., which was then in process of construction. In January, 1885, he became Assistant Engineer in charge of the construction of the section of that line between Chester, Pa., and the Susquehanna River. In April, 1886, he resigned from the Baltimore and Ohio Railroad Company to enter the service of the Charleston, Cincinnati and Columbus Railroad Company as Transitman on Location. From July, 1886, to January, 1887, he was employed by Rice and Coleman, Contractors, as Superintendent on the construction of the Newbury and Augusta Railroad. Between February, 1887, and September of that year, Mr. Talcott was engaged by the East Lake Land Company as Assistant Engineer at Birmingham, Ala. From September, 1887, to May, 1888, he was engaged in a contracting business as a member of the firm of Haskell and Talcott, at Birmingham, Ala.; however, on the latter date, Mr. Talcott returned to railroad work, accepting a position as Transitman on Location with the Knoxville Southern Railroad Company, and in October, 1888, he became Resident Engineer on the construction of the Knoxville, Cumberland Gap and Louisville Railroad.

Between September, 1889, and July, 1892, he was employed by the Seattle, Lake Shore and Eastern Railroad Company (later a part of the Northern Pacific System), first as Assistant Engineer and then as Chief Engineer. From July, 1892, to March, 1894, he served as Second Vice-President of the Seattle Coal and Iron Company, in charge of that Company's mines and business in the State of Washington. Between the latter date and December, 1897, he acted as Agent for the Receiver of that Company and, at the same time, served as Chief Engineer of the Seattle, Lake Shore and Eastern Railroad Company. In June, 1898, he became Assistant to the Superintendent of Construction of the White Pass and Yukon Railroad, in Alaska; in December, 1899, he became Chief Engineer of the Seattle and International Railway Company and, in September, 1900, Resident Engineer in charge of the Seattle Division of the Northern Pacific Railway Company. Between December, 1901, and March, 1903, he was associated with his brother-in-law, the late Herman Hollerith. In this connection, he held the position of Treasurer of the old Taft and Pearce Machine Company, of Woonsocket, R. I.

In March, 1903, Mr. Talcott returned to the Baltimore and Ohio Railroad Company, with which Company he spent the remainder of his career. He was first employed as Assistant Engineer in charge of the office of the late Francis Lee Stuart, Past-President, Am. Soc. C. E., District Engineer, at Cumberland, Md., and, later, in charge of miscellaneous surveys in that office. In March, 1905, he was transferred to the main office at Baltimore, Md., as Engineer of Surveys.

In order to carry out the provisions of Section 19*a* of the Interstate Commerce Act, known as the "Valuation Act," approved March 1, 1913, a Valuation Committee was appointed for the Baltimore and Ohio System, with Mr. Talcott as its Chairman, about January 1, 1914, and in June, 1917, he was appointed Assistant Chief Engineer, which positions he occupied at the time of his death.

Although he was engaged at various times in railroad construction and operation, his principal activities since his return to the Baltimore and Ohio System in 1903 were in the fields of locations and surveys, and under his jurisdiction surveys for a number of important projects were made, embracing: Improvement of the grade and alignment over the Allegheny Mountains, which included the present double-track Sand Patch Tunnel, on the Chicago Line, having a length of 4 475 ft, at the summit of the Allegheny Mountains; improvement of the St. Louis Line on the west end of the Cumberland Division, which included the change of line and reduction of grade west of Cheat River, between Blaser and West End, W. Va., on which is located the present double-track Kingwood Tunnel having a length of 4 202 ft; and improvement of the east end of the Cumberland Division, which included the Magnolia Cut-Off east of Cumberland, Md., in the Potomac River Valley, where the river is deeply entrenched on a meandering course through the eastern front of the Allegheny Mountains; this cut-off is 13.2 miles long, has four tunnels aggregating 6 958 ft in length, two crossings of the Potomac River, and effects a saving of 5.7 miles in main-line distance.

Mr. Talcott was a member of the American Railway Engineering Association; a member of the Engineers Club of Baltimore; a life member of the National Geographic Society; and a member of the New York Historical Society. He was a member of the Protestant Episcopal Church.

He was never married. His entire life was dedicated to his work, which he performed with exceptional ability and energy. He was a great lover of the out-of-doors; philosophical by nature, and of the highest integrity and the finest character, a true and helpful friend who had the love, respect, and admiration of all his associates. He always showed great sympathy for those in trouble, and the help he gave to them was proverbial. Through the advice of Mr. Talcott, many a young man was aided and guided in his course in life. His passing is a great and distinct loss to his many friends and to the profession.

Mr. Talcott was elected a Member of the American Society of Civil Engineers on October 5, 1904.

ROBERT BRUCE TINSLEY, M. Am. Soc. C. E.[1]

Died February 24, 1937

Robert Bruce Tinsley was born in Danville, Va., on August 24, 1883. His parents were James Whitfield and Nannie (James) Tinsley. He was graduated from the Radford, Va., High School. From this institution, for outstanding work in his studies, he received a scholarship to St. Albans Preparatory School, at Radford, in 1900. On leaving St. Albans, he entered the Virginia Polytechnic Institute, at Blacksburg, Va., where he continued until 1905, pursuing a course in Civil and Mining Engineering. During his vacations, he gained practical experience in his chosen profession by working for a firm of consulting engineers engaged in railroad location, underground mine surveys, coke-oven construction, etc.

On the completion of his studies at the Virginia Polytechnic Institute, Mr. Tinsley accepted a position as Transitman and Assistant Resident Engineer with the Southern Railway Company, at Bristol, Va.-Tenn., where for a year, until January 1907, he was engaged on railroad location and construction in the adjacent mountain country. Next followed six months of work, for a firm of consulting engineers, devoted largely to field problems in connection with railroad location and mining developments. During the next year, he was engaged in underground mine surveys, and, as Resident Engineer for the Chicago, Indianapolis, and Evansville Railroad Company, he located a 65-mile line from Vincennes to Evansville, Ind. From July, 1908, to July, 1909, Mr. Tinsley was Chief Engineer of the New Mexico Midland Railroad Company at San Antonio, N. Mex., and its subsidiary, the Carthage Fuel Company, of Carthage, N. Mex. In this capacity he relocated and constructed sections of 16 miles of mountain railroad, and directed the underground surveys for three large coal mines, together with the essential diamond-core drill work.

From July, 1909, to May, 1914, Mr. Tinsley was engaged in several capacities in connection with the construction of the Panama Canal and was employed, successively, as Transitman, Junior Engineer, Assistant Engineer, and, finally, as Superintendent of Construction in the Pacific Division. During this time he was first in charge of the layout work for the Pedro Miguel Locks and Dams; then Superintendent of Construction on the Miraflores Lock, hydraulic core-dam, spillway dam, and electrical control houses for the Pedro Miguel and Miraflores Locks, the Cristobal Coaling Station, and later, directed the work of the hydraulic sluicing plant at Gold Hill, Culebra Cut. For his excellent work in connection with the performance of the aforementioned duties, he received letters of commendation from the late General George W. Goethals, U. S. Army, M. Am. Soc. C. E., and S. B. Williamson, M. Am. Soc. C. E., the Division Engineer.

In May, 1914, Mr. Tinsley accepted a position as Manager and Engineer for the General Asphalt Company, at Guanoco, Venezuela, and performed work involving railroad and channel maintenance, dock and road construction, camp sanitation, and water supply.

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[1] Memoir prepared by H. B. Smith, Engr., and I. H. Patty, Associate Engr., U. S. Engr. Office, Galveston, Tex.

He became Division Superintendent of Construction and Resident Engineer for the Chile Exploration Company, at Chiquicamata, Chile, in January 1916, and continued in this capacity for nearly two years. As such, he had direct charge of the construction of a $39 000 000 copper leaching plant, which involved the construction of railroads, highways, two 67-mile desert pipe lines, dams and storage reservoirs for water supply, and the installation of electric-generating machinery and equipment, and other construction activities. When he left the services of this Company, his associates and employees presented him with a fine gold watch.

With the outbreak of the World War, Mr. Tinsley volunteered his services in defense of his country. He entered the service in January, 1918, and served with distinction as Captain of Company C, 605th Engineers, with the American Expeditionary Force in France. At the completion of his military service in July, 1919, the men of his Company, as a token of their high esteem for him, presented him with a loving cup.

Soon after returning to private life, Mr. Tinsley became President and General Manager of the Crest Coal Company, of Big Stone Gap, Va., and continued in this capacity until April, 1924. During the summer of this year, he was again in Government service as Superintendent of Construction of Lock and Dam No. 50, Ohio River, reporting to the District Engineer, Louisville, Ky., District.

Retiring again from Government service he was for two years Vice-President and General Superintendent of the Smith Company, of Birmingham, Ala. With this firm he was in direct charge of the construction of steel and concrete bridges and important municipal utilities. In September, 1926, Mr. Tinsley became Resident Engineer for the City of Lexington, Va., and while holding this position he had charge of the engineering and construction of a large earth dam for the city water supply. In 1928, he became Consulting Engineer for the Town of Big Stone Gap, where he directed a water-shed survey and an investigation on water supply possibilities. At the conclusion of this work, in the same year, he constructed three highway bridges for the Jordan Construction Company, of Albany, N. Y.

In October, 1928, Mr. Tinsley accepted a position with the United States Engineer Department, New Orleans, La., where he was in charge of general channel rectification work and maintenance of navigation in South Pass and Southwest Pass at the mouth of the Mississippi River. This work included bank revetments, construction of jetties, bulkheads, spur-dikes, and hydrographic and hydrometric surveys. While engaged on this work he was also associated with the performance of three important hydraulic model studies for channel improvement at the Passes of the Mississippi River, which were made at the United States Waterways Experiment Station, at Vicksburg, Miss. As a result of these model studies, new works were designed and constructed which resulted in marked improvement of the channel.

Early in 1935, he accepted a transfer to the United States Engineer Office, at Galveston, Tex. In this office he held the position of Senior Engineer, in charge of the General Engineering Division. His work in this connection, until the time of his death, comprised among other things, the making of engineering studies and the preparation of reports in connection with river and harbor improvements for flood control and navigation.

Mr. Tinsley was married on October 23, 1907, to Sue Goodloe, daughter of John and Elizabeth (Ferguson) Goodloe. He is survived by his widow and an only son, John Goodloe Tinsley. He was a member of the American Institute of Mining and Metallurgical Engineers, and the Society of American Military Engineers.

He was a man of rare personal charm and magnetism, and always made for himself a large number of friends and admirers among those with whom he was associated. Apropos of this, it is fitting to quote the following from the pen of Lieutenant Colonel E. H. Marks, U. S. District Engineer, M. Am. Soc. C. E., at Galveston, under whom Mr. Tinsley last served:

> "Robert Bruce Tinsley had a great wealth of varied experience but perhaps of even greater value was his devotion to his duties and his kindly considerate treatment of others. To him no task was done unless it was well done. Always working regardless of hours he gave the most meticulous attention to details of estimates and designs and was unsparing of himself in carrying out his tasks. Those privileged to work with him were always mindful of his friendly advice and encouragement. He was a fine Virginia gentleman."

Mr. Tinsley was elected an Associate Member of the American Society of Civil Engineers on February 6, 1912, and a Member on October 9, 1917.

ARTHUR CARLING TONER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 4, 1937

Arthur Carling Toner was born in Baltimore, Md., on November 21, 1881, the son of John M. and Kate (Carling) Toner. His early education was obtained in the public schools of Baltimore. In 1898, he entered the University of Virginia, in Charlottesville, Va., where he studied for a little more than two years. Mr. Toner then studied for three years at the University of Pennsylvania, in Philadelphia, Pa., and was graduated in June, 1904, with the degree of Bachelor of Science in Civil Engineering.

Mr. Toner was closely connected with the construction industry throughout his life. His earliest employment after graduation was with the Latta and Terry Construction Company for which firm he had worked during college vacation periods as Instrumentman and Timekeeper. Between 1904 and 1906, he was Engineer and Assistant Superintendent on electric railway construction and improvements at the United States Naval Academy, in Annapolis, Md., for which the Latta and Terry Construction Company were the Contractors. Following the completion of the work at the Naval Academy, Mr. Toner entered the employ of the Baltimore Sewerage Commission, with which he remained until January, 1914. During this period, while Mr. Toner was Resident Engineer in charge of construction, extensive improvements were made in the system, including the construction of several large interceptors and the tunnel for Jones Falls Conduits.

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[1] Memoir prepared by Claude A. Webb and Leo H. Corning, Assoc. Members, Am. Soc. C. E.

The construction experience thus gained by Mr. Toner equipped him for his next position, taken in January, 1914, with the Association of American Portland Cement Manufacturers, later, the Portland Cement Association. With headquarters in Philadelphia, Mr. Toner was Field Representative in the southeastern part of the United States, doing educational and engineering work pertaining to various uses of cement and particularly concrete highway construction. Much of his work consisted of acting in a consulting capacity on the preparation of plans and specifications and on the construction of projects involving the use of concrete.

In August, 1916, Mr. Toner became District Engineer for the Portland Cement Association, with headquarters in Pittsburgh, Pa. In that position, he was in charge of the work of the Association in Ohio and Western Pennsylvania. With the outbreak of the World War in April, 1918, he was given a leave of absence by the Portland Cement Association and became Assistant General Manager of the Liberty Shipbuilding Company, in Boston, Mass., Contractors for the Emergency Fleet Corporation. He was at Wilmington, N. C., for about four months as General Assistant in the organization of a construction force and the building of a shipyard for the construction of concrete ships. Later, he was in charge of the Brunswick, Ga., Yard for about six months as Representative of the Contractors for the Emergency Fleet Corporation. While at the Brunswick Yard the concrete ship, *Atlantus,* was built under Mr. Toner's supervision.

In February, 1919, Mr. Toner again rejoined the staff of the Portland Cement Association as District Engineer in Pittsburgh. In 1925, he became Regional Manager of the Mid-Eastern Region of the Portland Cement Association, with headquarters in the same city. In June, 1929, he was transferred to the Washington, D. C., Office as Manager, where he remained until 1933. Ill-health forced his temporary retirement at that time, but, later, he was able to resume active work with the Association in its General Office, in Chicago, Ill., where he remained until his death. During President Hoover's administration, Mr. Toner served on the President's Committee for the Unemployed.

During the period of his connection with the cement industry from 1914 until 1937, Mr. Toner contributed much to its progress. He saw the general adoption of concrete highways, the extensive use of concrete in bridges and buildings, and an increase in the consumption of cement from 88 000 000 bbl per yr to a maximum of 176 000 000 bbl. At the time of his death, Mr. Toner was the oldest employee in the Portland Cement Association from the standpoint of years of service.

Two years after his graduation from college, on August 15, 1906, Mr. Toner was married to Margaret A. Murphy, who, with their son, Arthur C. Toner, Jr., survives him.

Earnestness and sincerity were Mr. Toner's outstanding characteristics. Never did he state a half-truth to accomplish his purpose, and he gave of himself without stint to every task undertaken. He had an analytical mind which caused him to study carefully all phases of a problem in an effort to be certain his conclusions were based upon facts. Mr. Toner's counsel and advice were valued by many who recognized him as a true friend. He was active in several societies and clubs, including the Engineers Society of Western Pennsylvania; the Engineers Club, of Philadelphia; the National Press Club; and the Cosmos Club, of Washington, D. C.

Mr. Toner was elected an Associate Member of the American Society of Civil Engineers on October 1, 1912, and a Member on October 3, 1922.

JOHN JERVIS VAIL, M. Am. Soc. C. E.[1]

Died December 2, 1936

John Jervis Vail, the son of Robert Bruce and Emily (Jervis) Vail, was born at Rahway, N. J., on September 14, 1877. His early education was secured in the public schools of Rahway and at the Rutgers Preparatory School, New Brunswick, N. J. He was graduated from Rutgers University with the degree of Bachelor of Science in 1898, and in 1902 was awarded the degree of Civil Engineer.

From December, 1898, until his death on December 2, 1936, Mr. Vail was connected with the Pennsylvania Railroad Company. Until April, 1900, he was a Rodman in the office of the Assistant Engineer at Jersey City, N. J. He was then transferred to the Chief Engineer's Department, Corps of W. C. Bowles, Engineer of Construction. He became second in rank to Mr. Bowles in 1901, and for the next seventeen years he was in charge in the field. In that time, he supervised the filling, dredging, and erection of bulkheads in New York Bay at the Greenville Terminal; was in charge of the extension of the Pennsylvania Railroad to New York, N. Y.; the elevation of the railroad through Rahway; and important rock excavation work in Jersey City. From 1918 to 1924, Mr. Vail was Assistant Engineer at Meadows Yard, in New Jersey; and was connected with the Valuation Department of the Pennsylvania Company, on appraisals of special structures and on estimates of cost of an automatic electric system, for the New York and New Jersey Power and Light Development.

In 1925, Mr. Vail became Engineer of Construction, Chief Engineer's Department, and supervised the construction of Pier K in Jersey City, the American Railway Express Building, in Long Island City, N. Y., the rebuilding of the Journal Square Station, Jersey City, for the Pennsylvania Railroad —Hudson and Manhattan Railroad Companies.

In 1928, he built the Hackensack River Freight Bridge, the foundations of the State Highway Bridge, and the Hackensack River Passenger Bridge. In 1930, he was engaged in water-front development in Jersey City.

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[1] Memoir prepared by Earl Reed Silvers, Esq., Dept. of Alumni and Public Relations, Rutgers Univ., New Brunswick, N. J.

In September, 1931, Mr. Vail undertook the preparation of claims for a large reduction of railroad property taxes in the State of New Jersey. Chiefly through his efforts, a reduction in tax assessment of $4 000 000 was obtained, and a reduction in annual tax payment of $100 000. For the next two years, he continued his tax surveys, his work resulting in a total reduction in assessments of $12 000 000.

He was a member of the American Society of Terminal Engineers; New Jersey Professional Engineers; and of Phi Beta Kappa, Zeta Psi, and Theta Nu Epsilon.

A lifelong resident of Rahway, Mr. Vail was continually active in city affairs. He served as a member of the Board of Education for twelve years, and for several years was Vice-President of the Board. As a member of the Common Council, he was the author of the first Building Code established by the City, and was the first to make a five-year plan for city streets and public improvements. In other municipal matters, his knowledge of engineering and experience in public affairs and large construction projects made his service to the City of inestimable value. Throughout his life he took a deep interest in its welfare and fostered many movements which tended to promote its growth along permanent and desirable lines.

Mr. Vail was a member of the Second Presbyterian Church, serving as Deacon for twenty-five years and also on its Board of Trustees. For a long period, he was a Director of the Rahway Young Men's Christian Association, being especially interested in athletic activities and character-building programs. He was President of the Rutgers Club, of Rahway, in its first year of existence, and a member of the Rutgers University Alumni Council.

All who knew him held him in the greatest respect. His rugged honesty, his modesty, his unselfish service to the community, his willingness to assume duties from which he might have excused himself, endeared him to a host of friends. Mr. Vail lived the life of a Christian gentleman; was always considerate of the feelings of others, and always tolerant of human failings and human weaknesses.

On October 19, 1905, he was married, in Rahway, to Anne Evelyn Baumann. He is survived by his widow, two daughters, Anna Marie Vail and Helen Camille (Vail) Hayes, and one son, John Jervis, Jr.

Mr. Vail was elected an Associate Member of the American Society of Civil Engineers on January 4, 1905, and a Member on October 26, 1931.

GEORGE REED WADSWORTH, M. Am. Soc. C. E.[1]

DIED NOVEMBER 1, 1937

George Reed Wadsworth was born in Keene, N. H., on August 22, 1875, the son of Samuel Wadsworth and Ella (Reed) Wadsworth. He was a descendant of Christopher Wadsworth who first came to this country in 1632 and settled at Duxbury, Mass. He attended the Massachusetts Institute of

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[1] Memoir prepared by Lester D. Gardner, Secretary, Inst. of the Aeronautical Sciences, Inc., New York, N. Y.

Technology, at Boston, Mass., and was graduated with the degree of Bachelor of Science in 1898. He was a leader in undergraduate activities, and at graduation he was First Marshal of his Class, the highest elective honor.

Immediately after his graduation Mr. Wadsworth joined the Engineering Department of the New York Central Railroad Company. He was promoted rapidly from Transitman to Acting Supervisor of Track. In 1901, he was given charge of yards and engine-house layouts, and bridge masonry and renewals, until he became Resident Engineer at Albany, N. Y., in responsible charge of contract work, substructure renewal of the freight bridge across the Hudson River, the passenger station at Troy, N. Y., and car shops at West Albany, N. Y. Coming to New York, N. Y., in 1902, Mr. Wadsworth became Assistant Engineer on terminal improvements in New York City, in charge of the Port Morris Depression, including a 500-ft double-track rock tunnel; 1 500 ft of double-track concrete arch-roof subway; and designs for four-tracking the Electrification Zone. In 1904, he was in charge of the contract work on terminal improvements from Mott Haven, New York City, to Croton-on-Hudson, N. Y.

In May, 1905, he joined the Construction Department of J. G. White and Company, Incorporated, and for two years was engaged in steam and electric railroad construction, irrigation projects, wharves, dams, etc.

In December, 1907, Mr. Wadsworth became Consulting Engineer to the Metropolitan Improvement Commission, Boston, Mass. He prepared a survey which was embodied in the Commission's report to the Commonwealth of Massachusetts, covering the proposed complete unification and operation under joint control of all passenger and freight steam railroad properties and facilities within the Boston Terminal District. He also made a report on the proposed development of Boston Harbor to provide additional water-front facilities.

In June, 1909, Mr. Wadsworth joined the East Boston Company, as Engineer in charge of the preparation of plans and estimates for proposed docks, wharves, railroads, etc., along the water-front of East Boston. From June, 1910, to July, 1913, he was in charge of the design of Peerless automobiles, as Engineering Manager and Chief Engineer of the Peerless Motor Car Company, in Cleveland, Ohio. In 1913, he joined the firm of Gray and Davis, Incorporated, of Boston, which manufactured automobile starting and lighting equipment.

Believing that the United States would soon join the Allies in their struggle with Germany, Mr. Wadsworth decided to acquire some preliminary military training. He attended the Plattsburgh Training Camp in the summer of 1916. This was the first of the Plattsburgh camps and was under the direct supervision of the late Major-General Leonard Wood, U. S. Army. Probably there has never been such a distinguished group of "rookies" as those who joined the First Provisional Training Regiment.

In April, 1917, Mr. Wadsworth became a member of the Staff of Major Henry Souther as a Civilian Engineer in the Aviation Section of the United States Signal Corps. This group was directly in charge of the huge engineering program required to establish hundreds of flying centers for training aviation cadets and construction companies for war service. On June 30, 1917, he was commissioned a Captain in the Aviation Section of the Signal Corps, and on October 3 of the same year he was promoted to the rank of Major.

In October, 1917, at the request of the Chief of the Bureau of Construction and Repair of the United States Navy Department, Major Wadsworth was detailed by the Chief Signal Officer of the Army to the Naval Aircraft Factory, at Philadelphia, Pa. He became Chief Engineer, and under his direction were manufactured many twin Liberty-engined, 7-ton flying boats for patrol and bombing by the Navy. He did an outstanding job of assembling an engineering organization for the factory and in carrying on, under the direction and in consultation with the Bureau of Construction and Repair of the U. S. Navy Department, the engineering, drafting, experimental, and inspection work at the factory.

After his discharge from the Service on December 30, 1918, Major Wadsworth, with a group of aviation engineers, formed The United Engineering Corporation, in New York. This firm was engaged in the development of air transport, airplane manufacturing, and the disposal of surplus war aircraft supplies.

In 1921, he returned to his engineering practice and made several investigations and reports on industrial and economic surveys principally within the Automotive Industry. In 1923, he became interested in the manufacture of electric washing machines and was made Vice-President and General Manager of the Conlon Corporation, in Chicago, Ill.

In 1924 he became an official of the State of New York. Starting with an appointment as Director of the Division of Operating and Planning Research of the Department of Architecture he conducted a study involving basic determination of requirements, diagrammatic layouts, and operating details, involving the future expenditure of $80 000 000 for new State Hospitals for the Insane. Transferring in 1928 to the Department of Correction as Assistant Commissioner he became Superintendent of Prison Industries, in charge of operations of industrial activities in all the correctional institutions of New York State. The production of these prison shops was limited to about twenty lines, but had an annual value of about $2 000 000. He retired from active service on June 30, 1937, due to ill health.

George Wadsworth was regarded by his associates, even from his undergraduate days, as a brilliant student of civic affairs as well as an experienced engineer. His studies in connection with public health and economic trends were an avocation which brought him in touch with many of the forward looking statisticians of the United States. His reports, which he usually preferred to draw and type himself, reflected the skill of presentation which he inherited from his father who during his later life became an amateur maker of many scientific instruments and whose 8-day astronomical clocks are worthy to be placed in scientific laboratories. In his personal relationships, Major Wadsworth attracted the friendship of a wide circle of talented men in many walks of life.

He was a member of the University Club, and the Engineers Club of New York, the Society of Automotive Engineers, and the American Society of Mechanical Engineers.

He was married at Albany, N. Y., to Alice Randie, on April 9, 1902, and is survived by his widow and a son, George Randie Wadsworth, of New York.

Mr. Wadsworth was elected an Associate Member of the American Society of Civil Engineers on November 7, 1906, and a Member on November 8, 1909.

ISAAC STANLEY WALKER, M. Am. Soc. C. E.[1]

Died January 23, 1938

Isaac Stanley Walker was born on July 10, 1883, at Philadelphia, Pa. He was the son of Isaac and Hannah (Singleton) Walker. His father was born in 1840 in Lancashire, England, and came to America in 1861, settling first in Canada, and, later, in Philadelphia.

Mr. Walker received his early education in the Public Schools of Philadelphia, and, later, attended Temple University and Drexel Institute of Technology in that city.

For a period of about five months in 1900, Mr. Walker served as an Apprentice in one of the Survey Districts of Philadelphia. For six months in 1901, he served as a Chainman and Rodman on the Survey Corps of the Cornwall and Lebanon Railroad Company and the Pennsylvania Railroad Company.

From August, 1901, to May, 1910, Mr. Walker was in the employ of the City of Philadelphia, engaged on the Water-Works System. During this time he filled successive positions as Chainman, Rodman, Transitman, and Assistant Engineer. Seven years of this time were spent on the construction and operation of the Torresdale, Belmont, and Queen Lane Filter Plants, and two years on the design and construction of pipe distribution systems.

From May, 1910, to October, 1916, Mr. Walker was engaged as a Staff Engineer with the firm of Hering and Fuller (the late Rudolph Hering, M. Am. Soc. C. E., and the late George Warren Fuller, M. Am. Soc. C. E.) and, later that of Hering and Gregory (the late John Herbert Gregory, M. Am. Soc. C. E.), Consulting Engineers of New York, N. Y. During this period, he served as Designer in preparation of reports, plans, and specifications for the sewerage systems of Vincennes, Ind., and Batavia, N. Y.; for the sewage treatment works of Atlanta, Ga., Batavia, Kings Park, Albany, Mamaroneck, and Larchmont, N. Y.; Chatham-Madison, Orange, Montclair, East Orange, and Trenton, N. J.; Bridgeport, Conn.; and Roland Park, Md.; for water-works and filter plants at Grand Rapids, Mich., Minneapolis, Minn., Montreal, Que., Canada, Zanesville, Ohio, Lynn, Mass., Morgan Estate, New

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[1] Memoir prepared by a Committee of the Philadelphia Section, Am. Soc. C. E., consisting of W. H. Chorlton, and H. M. Freeburn, Members, Am. Soc. C. E., and C. A. Howland, and L. L. Jenne, Assoc. Members, Am. Soc. C. E.

York, and Charleston, S. C. During 1915 and 1916 Mr. Walker was also an Instructor in Sanitary Engineering in the Extension School Course of Columbia University, in New York City.

From October, 1916, to September, 1918, he was Designing Engineer for sewage treatment works of the City of Philadelphia. During the last five months of this period, he was Division Engineer in charge of this work—a $35 000 000 project.

From September, 1918, to September, 1927, Mr. Walker was General Manager of four water properties—at Chester, Pa., Vincennes, and Greencastle, Ind., and the Delaware Water Supply Company. He was Managing Executive during this period, in charge of all engineering work, including a new filter plant with wharf, intakes, and pumping station at Chester. A valuation of the property of this Company made by him was the basis for a grant of increase in rates by the Public Service Commission of Pennsylvania. During the last two years of this period, Mr. Walker was President of the New Chester Water Company and Secretary of the Vincennes and Greencastle Companies.

From September, 1927, to January, 1930, he was Vice-President and General Manager of the Pennsylvania Water Service Company and the Scranton-Spring Brook Water Service Company, subsidiaries of the Federal Water Service Corporation, with headquarters at Wilkes-Barre, Pa.

From January, 1930, until his death, Mr. Walker had an office in Philadelphia as a Consulting Sanitary Engineer. During this time he visited Russia, as the Representative of a New York Contracting Firm and at the invitation of the Soviet Government, to study and report on a comprehensive plan of water supply, sewerage, and sewage treatment works for the City of Moscow. He also served as an Associate Water Consultant for the National Resources Committee of the United States Government and made a comprehensive study and report on the Delaware River Basin.

Mr. Walker gave freely of his time and effort for the betterment of his profession. By his high qualities of courage, uprightness, and helpfulness, he endeared himself to his associates.

On May 10, 1905, he was married to Lillie Margaret Winner, of Philadelphia. His widow and five children—Edward Stanley, Robert Winner, William Alan, Ruth Evelyn, and Helen Margaret—survive him.

Mr. Walker served as Director, Vice-President, and President of the Philadelphia Section of the Society. He was a member of the American Water Works Association, and served as Chairman of the Four State Section in 1935–1936, and Secretary of the Committee on Water Rates for three years. He was a member of the Pennsylvania Water Works Association, a member of the Executive Committee for four years, and Vice-President for two years. He was also a member of the American Public Works Association, the Pennsylvania Water Works Operators Association, the Pennsylvania Sewage Works Association, and the Engineers Club of Philadelphia.

Mr. Walker was elected an Associate Member of the American Society of Civil Engineers on September 3, 1913, and a Member on January 19, 1923.

JAMES GOOLD WARREN, M. Am. Soc. C. E.[1]

DIED NOVEMBER 2, 1937

James Goold Warren, the son of Joseph and Jane Vail (Goold) Warren, was born at Buffalo, N. Y., on September 12, 1858. He was descended from pioneer stock, the first of his family, John Warren, having arrived in this country from England in 1630. His father, leaving his native State of Vermont, as a young man, took up his residence in Buffalo where he was, for many years, one of its prominent citizens. At the time of his death, in 1876, he was Editor-in-Chief of the *Buffalo Daily Courier*.

James Warren received his early education in the public schools of Buffalo. Later attending Phillips-Exeter Academy, at Exeter, N. H., young Warren planned to continue an education in medicine. This plan was interrupted, however, by his father's death. On July 1, 1877, he entered the United States Military Academy, at West Point, N. Y., from which he was graduated on June 11, 1881, Number 5 in his Class. He was promoted to the rank of Additional Second Lieutenant, Corps of Engineers, U. S. Army.

Lieutenant Warren's first assignment was with the Engineer Battalion at Willets Point, N. Y., from which he was graduated in 1884. During this period he was promoted to the rank of First Lieutenant. From 1884 to 1885, he served as Assistant to the officer in charge of the Chattanooga, Tenn., District, Engineer Department-at-Large. From 1885 to 1887, he was detailed as Instructor in Civil and Military Engineering at the U. S. Military Academy. In 1887 he became Secretary and Executive Officer, Academic Staff of the Engineer School, and Adjutant of the Battalion of Engineers at Willets Point, where he continued on duty until 1891. From 1891 to 1893, he served as Assistant to the officer in charge of the construction of fortifications in New York Harbor.

After a short assignment as Secretary and Disbursing Officer of the Mississippi River Commission, at St. Louis, Mo., Lieutenant Warren became District Engineer at Louisville, Ky., and served as such until 1898. He was promoted to the rank of Captain in 1894.

In 1898, Captain Warren was assigned as District Engineer at Milwaukee, Wis., and, in that capacity as well as that of Engineer of the Ninth Lighthouse District, he served until 1905. He became a Major in the Corps of Engineers in 1901.

From 1905 to 1910, Major Warren was District Engineer at Cincinnati, Ohio, and was also Division Engineer, Central Division, from 1909 to 1910. He was promoted to the rank of Lieutenant Colonel in 1907.

In 1906, he was made a Member of the Mississippi River Commission and served continuously in that capacity until 1919.

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[1] Memoir prepared by R. B. Perry, Esq., under the direction of Col. E. H. Marks, Corps of Engrs., U. S. Army, M. Am. Soc. C. E.

In 1910, Lt.-Col. Warren became District Engineer at Buffalo, where he had charge of both the Buffalo and Oswego Engineer Districts. He was promoted to the rank of Colonel in 1912 and, in the same year, he was appointed Division Engineer of the Great Lakes Division, in which capacity he supervised all river and harbor activities of the Great Lakes and the St. Lawrence River. He continued to serve concurrently as District Engineer until 1915, and, again, assumed that responsibility from 1917 to 1919.

In 1917, Colonel Warren, by authority of a Joint Resolution of Congress, assumed charge of the "Investigation of Diversion of Water from Great Lakes and Niagara River." This investigation covered the various phases of water diversion, including its effect upon navigation, sanitary and power purposes, and the preservation of the scenic beauty of Niagara Falls and the rapids of Niagara River. The report of this investigation, published in 1920, is regarded as an authoritative text on this all-important subject. The then Senior Member of the Board of Engineers, the late Brigadier-General Harry Taylor, U. S. Army, M. Am. Soc. C. E., commenting for the Board upon Colonel Warren's report, stated:

> "The report is the only comprehensive and thorough investigation of all these subjects ever made and possesses great value, not only from the technical, but also from the very full historic, presentation of the matters with which it deals."

While still Division Engineer of the Great Lakes Division, Colonel Warren, having attained the age of 64 yr, was retired by operation of law on September 12, 1922. He continued to make his home in Buffalo until his death.

During his forty-five years of active service with the Corps of Engineers, Colonel Warren justly earned the reputation of being one of the outstanding members of that organization. As District Engineer at Louisville, Milwaukee, Cincinnati, and Buffalo; during his long term as Engineer of the Great Lakes Division; as a member, for so many years, of the Mississippi River Commission; and as a member of a large number of Special Boards of Engineers, charged with the investigation of a wide variety of problems, he gained invaluable experience and came to be regarded as an authority on all matters pertaining to navigation on the Great Lakes and the inland waterways of the United States.

In 1881, Colonel Warren was married to Sarah Clifton Wheeler, at West Point. Mrs. Warren died in Milwaukee in 1901. A son, Wheeler, and a daughter, Emily Warren, survive him.

Aside from his enviable record as an officer of the Corps of Engineers, Colonel Warren was well known for his unswerving devotion to duty and his unfailing loyalty to those who were associated with him. Commenting on his passing, Colonel John Millis, Corps of Engineers, U. S. Army (*Retired*), M. Am. Soc. C. E., a classmate of Colonel Warren at the Military Academy, stated:

> "I recall the affection and respect which the class of 1881 had for Jim Warren, our first Captain, and the strict fairness and devotion with which

he performed the duties of that youthful but responsible office. These characteristics he continued to display through all of his subsequent career."

Maj.-Gen. E. M. Markham, U. S. Army (*Retired*), M. Am. Soc. C. E., formerly Chief of Engineers, U. S. Army, added the following:

"I pride myself that I analyzed Colonel Warren in early days as a gentle, kindly, characterful man, inflexible of purpose, wishful that he might conceal his downright gentleness. His sturdiness of purpose and character as I observed them, gave me a lifelong inspiration."

In writing of Colonel Warren's career, Gen. Malin Craig, Chief of Staff, U. S. Army, concludes:

"Keenly interested in his profession, with special knowledge of civil engineering as applied to river and harbor improvements, Colonel Warren was on duty for many years in connection with engineering projects on the rivers and lakes of the middle west, and loyally and efficiently performed the duties of an Engineer of the Army throughout his active service covering a period of forty-five years. His death is deeply regretted."

Colonel Warren was elected a Member of the American Society of Civil Engineers on June 24, 1914.

LEONARD CHASE WASON, M. Am. Soc. C. E.[1]

Died April 30, 1937

Leonard Chase Wason was born on August 5, 1868, in Brookline, Mass., the son of Elbridge and Mary Isabella (Chase) Wason. On both the paternal and maternal sides his ancestors were among the earliest of the settlers in the United States.

After his preparatory education, Mr. Wason entered and was graduated from the Massachusetts Institute of Technology, at Boston, Mass., in 1891, with the degree of Bachelor of Science in Electrical Engineering. He began his professional career as a Draftsman in the office of the late Frederick S. Pearson, M. Am. Soc. C. E., of Boston, Consulting Engineer to the Brooklyn City Railway Company, of New York, N. Y., and the Dominion Coal Company, of Boston. For two years Mr. Wason was engaged in design and field supervision. He made all the plans for the Eastern Station, on Kent Avenue, of the Brooklyn Heights Street Railway. He designed a part of a power station at Ottawa, Ont., Canada, and nearly all the improvements made by the Dominion Coal Company for its mines at Cape Breton, Nova Scotia, and its terminals at Montreal, Que., Canada. In 1894, he was Superintendent of Construction of wooden coal-pockets and tower at Wellington (Ont.) Basin. He also designed and built the power plant for coal-handling towers at Hochelaga, Que., Canada.

About this time Mr. Wason became keenly interested in the latent possibilities of reinforced concrete which was then practically in its infancy. In

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[1] Memoir prepared by S. L. MacMillan, Vice-President, Aberthaw Co., Boston, Mass.

1884 the late Ernest L. Ransome, Affiliate, Am. Soc. C. E., had been granted a United States patent applying to the use of "iron bars, bands, or hoops, twisted, so as to form a continuous bond with the concrete, in which they are permanently imbedded", and during the following ten years a few structures of reinforced concrete had been built in California. In 1894, Mr. Wason, with Messrs. William Hartley Dennett, and Ross F. Tucker, the latter holding a license from the Ransome and Smith Company, of San Francisco, Calif., granting to him the use of various patents for concrete construction in the States of Massachusetts, Rhode Island, and part of Connecticut, associated themselves together for the purpose of forming a company to engage in construction work, and incorporated the Aberthaw Construction Company under the laws of the State of Maine. This name was later changed to Aberthaw Company, and a corporate charter was issued to the Company by the Commonwealth of Massachusetts.

During the first year, Mr. Wason served as Director and Engineer of the Company, but was elected President and Treasurer in 1895, which offices, together with that of Director, he held continuously until the time of his death.

As he devoted practically his entire business career to the development and management of this Company and lived to see it grow from a very small and obscure beginning into one of the leading construction companies in the United States, the following notes regarding the derivation of the name "Aberthaw" may be of interest.

As previously stated, Mr. Wason had become enthused over the possibilities of concrete. He and his associates were familiar with the experiments carried on by John Smeaton in 1756 when he was engaged in rebuilding the Eddystone Lighthouse off the English Coast. This lighthouse was built in an exposed and dangerous location where the difficulties of construction were enormous and where terrific storms not infrequently lashed the work with furious severity. Smeaton, therefore, deemed it necessary to use a cement that would be capable of withstanding the influence of sea water and possessing also the capacity of resisting the mechanical action of violent storms. With painstaking care he set out to discover, if possible, a cement that would fulfill these requirements. After numerous tests with the different materials available to him, and concerning which he has left detailed and complete accounts, he determined that all the limes which could set under water were obtained from the calcination of such limestone as contained a large proportion of clay in their composition, and it is universally acknowledged to-day that Smeaton originated the science of water cements. He demonstrated that the real cause of hydraulicity of cement was due to an argillaceous admixture and that the previously held theory that the purest and hardest limestone was the best material for such purposes was incorrect.

Of the different limes with which he experimented, a blue lias lime from Aberthaw, a little sea-coast town in Glamorganshire, Wales, when mixed with various proportions of tarras greatly excelled any of the other mixtures. Its behavior under water was satisfactory, and it continued to harden for months.

Using a cement compounded of this Aberthaw lime and of puzzolana brought from Civita Vecchia, near Rome, Italy, Smeaton constructed the Eddystone Lighthouse, which was to endure for many years as a beacon not only to storm-tossed sailors, but also to those who were endeavoring through research and experimentation to increase the technical knowledge pertaining to the Building and Engineering Professions. That the base of this lighthouse still stands is eloquent testimony to the ability of its constructor and the efficacy of the material with which he worked.

In his accounts, he states that by the use of Aberthaw lime and puzzolana he did not doubt "that they would make a cement that would equal the best merchantable Portland Stone in solidity and durability".

In 1824, Joseph Aspdin, a bricklayer of Leeds, England, was granted a patent for his invention of "an improvement in the Modes of Producing an Artificial Stone". He gave the name "Portland Cement" to the product and was the inventor of that application of the name. While he maintained a great deal of secrecy concerning the exact proportions of the various ingredients contained in the cement he manufactured, it is believed that he also made extensive use of "Aberthaw" lime in his product. It, therefore, was a happy inspiration and the result of studious research into the early history of the materials with which Mr. Wason and his associates planned to work and specialize, rather than mere chance that suggested the adoption by them of the name, "Aberthaw", as one peculiarly fitting to their enterprise. Under Mr. Wason's guidance and direction the Aberthaw Company was the pioneer company in New England to specialize in designing and building in reinforced concrete, later extending its activities to all types of construction work. He conducted extensive experiments and research in concrete materials, their uses, and application, and originated many devices useful in construction work.

Among the early noteworthy structures built by the Aberthaw Company under his direction were the Harvard Stadium; Fall River, Mass., Public Library; and the Attleboro, Mass., water stand-pipe, which at the time it was constructed, was the largest reinforced concrete stand-pipe in the world. During the World War, the Company built structures costing many millions of dollars, including buildings for the Winchester Repeating Arms Company, at New Haven, Conn., the Victory Plant of the United States Navy Department, at Squantum, Mass., and many other projects of similar nature. In more recent years the Company constructed the Christian Science Publishing Building, at Boston, at a cost of $4 000 000; re-built the Provident Institution for Savings Bank Building, at Boston; constructed the State Pier at Portland, Me.; the reinforced concrete grandstand, capable of seating 13 000, at Suffolk Downs, East Boston, Mass.; the Converse Memorial Building for the New England Baptist Hospital, at Boston; the Hanchett Memorial Building for the Lowell General Hospital, at Lowell, Mass., and numerous other projects.

During the period of Mr. Wason's Presidency, the Aberthaw Company built structures totaling more than $100 000 000 in value, and ranging geographically from the Gaspé Peninsula in Canada to the British West Indies, and from the Atlantic Coast to the Pacific Coast.

In 1905 Mr. Wason assisted in organizing the Federal Mutual Liability Insurance Company, and served as a Director in that Company, and, later, as a Director in the Lumberman's Mutual Casualty Company, of Chicago, Ill.

He was one of the very early members of the American Concrete Institute and served as President of that organization in 1915 and 1916. He was a donor of the American Concrete Institute Wason Award for research on concrete; also of the Wason Medal for the most meritorious paper on the subject presented annually.

He assisted in the founding of the Associated General Contractors of America, Inc., and served as Vice-President of the New England Branch of that organization. Upon learning of Mr. Wason's death, Mr. Edward J. Harding, Managing Director of the Associated General Contractors of America, Inc., wrote to the Aberthaw Company:

"Word of Mr. Wason's passing was received by the Association while its Executive Board was in session in Washington. All members present were visibly affected by this information and the Board, in behalf of the Association, endeavored to express its deep feeling by resolution. This resolution is here transmitted with the request that you convey these sentiments of the Association to the members of his family and to his business associates.

"RESOLUTION

Adopted by the Governing Board of The Associated General Contractors of America, Inc., May 1, 1937, Washington, D. C.

"With deep sorrow the Governing Board of the Associated General Contractors of America, in meeting assembled at Washington, D. C., May 1, 1937, learns of the passing of Leonard C. Wason on April 30, 1937. This Association, through its Governing Board, expresses its appreciation of his deep interest and activity in its founding and his continuous loyalty to its principles; now, therefore, be it

"*Resolved*, that the members of the Associated General Contractors of America record their positive knowledge, individually and collectively, that he has left to this Association and the Construction Industry a heritage in the record of his noble and unselfish life; his technical ability and counsel; his sterling character and high conception of duty, and his unfailing kindness have inspired the respect, admiration and affection of his associates and friends on whom his passing has produced profound grief and sorrow, with a realization that the memory of his character and accomplishments will serve as an incentive to those who will carry forward the constructive work for which he so tirelessly labored; and be it further

"*Resolved* that this expression be spread upon the permanent records of the Associated General Contractors of America and that copies be sent to his family and business associates."

In 1901, Mr. Wason prepared a Hand Book for the use of engineers, architects, and builders, giving data relative to reinforced concrete construction. He also wrote numerous articles for technical societies and the technical press concerning the design of reinforced concrete structures, the physical properties of buildings materials, methods and equipment for use in construction, cost accounting, estimating, etc.

He was awarded the Phebe Hobson Fowler Professional Award by the Society for meritorious achievement for first honor (1929), as a recognition of his work as Chairman of a Committee of the Northeastern Section which developed the Code of Practice adopted by the Society. For the Society, he served as a member of the Joint Committee on Specifications for Concrete and Reinforced Concrete. He also represented the Society as a member of the Sectional Committee working on the development of a Construction Safety Code sponsored by the American Standards Association. He was always keenly interested in the subject of accident prevention, and felt that every reasonable precaution should be taken that would ensure safe working conditions for employees. It was, therefore, a source of great pride to Mr. Wason when, in 1933, the Aberthaw Company won the Goble Contest for that year. This contest is held annually under the auspices of the Associated General Contractors. The award is made at the Annual Convention of the Associated General Contractors and consists of a sterling silver cup offered as a prize to the member (company) having the best accident prevention record during the contest period on man-hours lost against man-hours worked. Typical of his sincere desire to prevent accidents and the grief and loss caused to the family of the person injured is the notice Mr. Wason had printed on each pay envelope issued by Aberthaw Company which read:

> "Any man working on a runway or scaffold, or in, upon, or in connection with any ways, works or machinery which he considers unsafe or dangerous, must report the same to the foreman in charge. If the foreman fails to remedy the defects, you should refuse to work upon or near the place considered unsafe."

He was interested in the welfare of his workmen and always ready to lend his time to their problems and financial assistance in needy cases.

He was a member of the American Society for Testing Materials, the American Society of Mechanical Engineers, the Boston Society of Civil Engineers, in which body he held office as Director and Vice-President. He also belonged to Chi Phi Fraternity, the Algonquin, and Boston City Clubs, the Fraternity Club, of New York, and the Boston Chamber of Commerce.

Mr. Wason had always been active in civic affairs, having been Chairman of the Brookline Planning Board; a member of the Board of Appeal (Building Department); a member of the Brookline Town Meeting; and a member of the Boston Noise Abatement Committee. He was a life member of the Massachusetts Charitable Mechanics Association since 1928; a Director of the Master Builders Association of Boston during 1925, 1926, and 1927, and a member of the Business Men's Vocational Board of the College of Business Administration of Boston University. He served as a Director and as a member of the Executive Committee of the Massachusetts State Chamber of Commerce, and was also a member of the United States Chamber of Commerce. His favorite outdoor diversion was golf.

For many years prior to his death he had been a member of The First Church of Christ Scientist, in Boston.

Mr. Wason was a man who never sought public prominence, although he gave generously of his time to civic affairs and served on many committees.

He was always very much interested in pending legislation and firmly believed it was the duty of citizens to advise their legislative representatives concerning their views on current bills under consideration for enactment and to appear at public hearings for the purpose of either supporting or opposing measures of importance to the community. He was a liberal contributor to charitable enterprises.

He insisted on absolute honesty in all his transactions and maintained that a reputation for honesty and integrity was the most valuable asset that an individual or a company could possess. He possessed a very even disposition and no matter how trying or provocative a situation might be he never gave outward evidence that he was perturbed. He gave full consideration to all matters that came to his attention for decision and never rendered a hasty judgment. He never gave any indication of any knowledge of fear. He never spoke disparagingly of any one. He was abstemious in his habits and furnished an example of honest and upright living to his friends and associates.

Mr. Wason was imbued with a spirit of cheerfulness. This, together with a kindly nature, he manifested daily in his association with his employees. He always looked on the brighter side of things and maintained an optimistic attitude even during the trying period of the depression when the outlook for the Construction Industry was very uncertain.

He was twice married: First, on October 8, 1896, to Harriet C., daughter of James D. K. Willis, of Boston. Mrs. Wason died in 1915, leaving four children, Elbridge, Alfred Boyd, Raymond, and Lawrence Willis Wason. On June 24, 1916, Mr. Wason was married to Mrs. Annie B. (Atwood) Redlon, widow of Charles H. Redlon, of Portland, Me., who survives him.

Mr. Wason was elected an Associate Member of the American Society of Civil Engineers on April 3, 1901, and a Member on April 1, 1903.

SAMUEL C. WEISKOPF, M. Am. Soc. C. E.[1]

DIED DECEMBER 20, 1936

Samuel C. Weiskopf was born in New York, N. Y., on September 17, 1860. He was the fourth of six children, five boys and a girl. Most of his childhood was spent in Milwaukee, Wis., to which city the family moved when he was about twelve years old.

After the usual preparatory schooling he entered the University of Michigan, at Ann Arbor, Mich., and was graduated from the Engineering Department in June, 1882, with the degree of Bachelor of Science. Immediately thereafter he obtained employment as a Draftsman in the office of Rust and Coolidge, Bridge Builders, in Chicago, Ill., and fifteen months later received an appointment as Assistant Engineer in the office of the Chief Engineer of the Chicago, Milwaukee, and St. Paul Railroad Company. As such he assisted in the design of bridges and other structures until the fall of 1885, when he was put in charge of the shop work for all bridges of the road. By 1888, Mr. Weiskopf had become known as an authority, reporting on bridges for the Richmond and Allegheny Railroad, Western Railway of Alabama, and Kanawha and Ohio Railway Companies. The Cleveland and Mahoning Valley Railway Company had adopted his designs for roundhouses in competition with its own.

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[1] Memoir prepared by William H. Yates, and John W. Pickworth, Members, Am. Soc. C. E.

In 1898, Mr. Weiskopf changed his headquarters to New York City, and in private practice began to devote himself more to building than to bridge work. At that time cast iron was used generally for columns, and his details of bases and beam connections were adopted as standard and published, in 1901, by the American Bridge Company. He had a genius for simplification and standardization, and that helped him to attain a position of pioneering leadership in an ever-widening field of high building construction.

Quickly and thoroughly he adopted the philosophies of this new work. He realized that here he had to accommodate the engineering with safety to architectural requirements, and his successful accomplishment of this aim, together with a natural modesty, perhaps accounts for the fact that he did not receive even greater public recognition than he did. His devotion to the art of engineering made him take the greatest personal satisfaction in having the architecture of the finished building unspoiled by an obtrusive skeleton.

He retained the boldness of the bridge builder, and as he designed bigger and higher structures he did not hesitate to execute longer and heavier spans in order to create large rooms unobstructed by column supports. He was expert in all phases of building design, was one of the first to adopt the pneumatic caisson for foundations, and in intricate wind-bracing he contributed valuable details, making possible the elimination of unsightly brackets and braces without the sacrifice of rigidity. He had the vision to perceive that it was possible to erect a high frame without using diagonals but with the joints between columns and beams adequate in strength and stiffness, concealed within the depth of the floor and the width of the column. The Trinity Building, in New York City, was one of the early designs embodying this principle. Many other buildings in the Wall Street District gave him an opportunity to develop and apply this conception. Some of the most prominent buildings in that District are: The City Investing Building; Adams Express Building; American Telephone and Telegraph Building; The Bank of America; and on three corners of Wall and Broad Streets, the Bankers Trust Company, the New York Stock Exchange (where 24 stories are carried by trusses over the Board Room), and the Morgan Bank, part of which he later spanned to carry 33 stories of the Equitable Trust Company Building, the air above the bank having been leased for a long period of years.

Farther up-town in New York City, the American Radiator Building and the *Daily News* Building bear the stamp of his work. He designed many buildings in other parts of the United States, particularly in Pittsburgh, Pa., where the Mellon National Bank and the Gulf Oil Buildings stand. The

36-story narrow tower of the Travelers Insurance Company Building, in Hartford, Conn. (supported partly on cantilever trusses over an existing structure, and the highest building in New England), was perhaps one of the most difficult and involved problems in wind-bracing ever to be presented to an engineer. It is one of Mr. Weiskopf's most daring and successful accomplishments.

In 1920, he took his Principal Assistant into partnership to form the firm of Weiskopf and Pickworth and three years later, just after the earthquake disaster in Japan, he was called there to advise the Mitsui Bank on the problems of designing to withstand such shocks. The design of the main building in Tokyo and of four branch banks in other Japanese cities was the result. These structures attracted world-wide attention as the first thorough attempt at a solution to such a bracing problem, and were the crowning achievement to a career devoted to intricate structural design.

His kindly nature in all matters not only encouraged the young engineers who came in contact with him, but proved most helpful to the less experienced. His mind was always open to suggestions of his fellow engineers, but before any acceptance of such he would satisfy himself that the fundamental principles he so strongly advocated were not violated. He was so thorough in his analysis of any engineering problem that the practical execution of his design never failed to produce the desired results.

Mr. Weiskopf was for many years an active member of the Engineers Club in New York City, but he always found his greatest pleasure in his family life. In his later years he traveled extensively in Europe for recreation, and, in 1934, he retired from active practice to his home in Florida, where he died after a long illness.

His kind mildness of manner, his modesty, and the simple honesty of his thinking, endeared him to all who knew him, but denied him honors he deserved but never aggressively sought. He is survived by his widow, Stella C. Weiskopf, and his two sons, Walter H. Weiskopf, M. Am. Soc. C. E., who is active as a partner in carrying on the practice of his father's firm, and Edward F. Weiskopf, Assoc. M. Am. Soc. C. E.

The Structural Engineering Profession can rightfully be thankful that Samuel C. Weiskopf was a member of it.

Mr. Weiskopf was elected a Junior of the American Society of Civil Engineers on May 7, 1884, and a Member on December 5, 1888.

WILLARD OLNEY WHITE, M. Am. Soc. C. E.[1]

DIED MAY 19, 1937

Willard Olney White was born in Western, N. Y., on September 15, 1876. He was the son of the late Franklin and Harriet (Olney) White. His forebears were early settlers of Rhode Island who migrated to New York State and settled in the rich farming country there about 1800.

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[1] Memoir prepared by Chester M. Lingle, Esq., assisted by Albert J. Opperman, Esq., Uniontown, Pa.

As a boy Mr. White attended public school in Western and, subsequently, prepared for college at Cazenovia Seminary, Cazenovia, N. Y., from which he was graduated in 1896. He then entered the Engineering School of Cornell University, at Ithaca, N. Y., and was graduated in 1900 with the degree of Bachelor of Science.

From June to September of that year, he was engaged as a Chainman and Rodman for the Central Railroad Company of New Jersey. He then entered the Engineering Department of the American Coke Company, at Edenborn, Pa., as a Draftsman, later, becoming Chief Draftsman for the Company. This work consisted of the development of 9 500 acres of Pittsburgh coal, requiring six shafts, 800 coke ovens, houses, water system, streets, and all plant buildings.

In May, 1901, Mr. White joined the Engineering Department of the H. C. Frick Company, as Assistant Division Engineer, at Leckrone, Pa. Two years later, in May, 1903, he was transferred to the General Office of the same Company at Scottdale, Pa., in charge of design and construction of buildings, trestles, coke ovens, arches for permanent haulage ways in mines, mechanical haulage devices, etc. He also supervised a part of the construction of arch lining and permanent track arrangement for shaft bottoms at the Lambert and Ranes Mines.

From January to June, 1904, he served as a Draftsman in the Engineering Department of the Middle Division of New York State Canals, at Syracuse, N. Y. He was then appointed Assistant Engineer of the New York State Barge Canal, in charge of the Drafting Office, preparing plans and estimates for the canal construction. From February, 1905, to March, 1907, he was in charge of the Drafting Office on Barge Canal Residency No. 5, at Rome, N. Y., where he supervised the preparation of contract maps and estimates of quantities for 25 miles of canal.

During this time he qualified as Resident Engineer, but before receiving his appointment Mr. White accepted a position as Chief Engineer of the Tower-Hill Connellsville Coke Company, at Republic, Pa. As such he was in full charge of the construction of two coke plants, involving the sinking of four shafts, the installation of all power equipment, the building of tenement houses, the development of mines, and the construction of 680 coke ovens, together with railroad connections. At this time he also patented a door for coke ovens which the Tower-Hill Company used on many of its new ovens.

In October, 1910, Mr. White opened an office in Uniontown, Pa., as a Consulting Civil and Mining Engineer. Although his practice was successful, he preferred construction work, and in May, 1916, he entered the Engineering Department of the Youngstown Sheet and Tube Company, at Hubbard, Ohio, in charge of construction. In December of the same year he was made Chief Engineer of the Buckeye Coal Company, at Nemacolin, Pa., a subsidiary of the Youngstown Sheet and Tube Company. This work consisted of the complete development of 8 600 acres of Pittsburgh coal for a daily production of 6 000 tons. The plant at Nemacolin is one of the finest in the United States, with all modern improvements in the town plan. The mine with its concrete lined shafts, slope, and shaft bottom headings, required the most exacting detail and advance thinking and planning for such a large daily tonnage of coal.

In May, 1918, Mr. White was appointed Chief Engineer for W. J. Rainey, Incorporated, in Uniontown. This Company operated eight coal mines. The development of new mines and the modernization of old ones were under his supervision. He remained with this Company until his retirement in April, 1936.

Mr. White was a splendid citizen of unquestioned integrity. He won the respect of all who knew him, and made real friends among all the types of people with whom he came in contact. These friendships, once made, were lasting because his own high standards inspired confidence and loyalty in others.

Due to the exacting nature of his work, he had little time for recreation. Although fond of good music and books, his greatest joy was in fishing, and he never missed an opportunity to avail himself of this sport. He was a keen observer and student of Nature and spent many vacations in the Adirondacks where he could fully enjoy the woods and streams which he loved so much. In later years he turned to water-color painting as a hobby, and his years of detailed study of Nature showed in the accuracy and beauty of his painting.

Mr. White was a member of the Rod and Bob Club, of Cornell University; a member of the Presbyterian Church; and for many years an active Rotarian. He was also a member of the American Institute of Mining and Metallurgical Engineers.

In June, 1904, he was married to Mary Ann Neff, a daughter of Dr. and Mrs. George W. Neff, of Masontown, Pa. She and one daughter, Mary (White) Hubbard, survive him.

Mr. White was elected an Associate Member of the American Society of Civil Engineers on April 3, 1907, and a Member on March 3, 1911.

WINTER LINCOLN WILSON, M. Am. Soc. C. E.[1]

DIED JULY 15, 1937

Winter Lincoln Wilson was born in Cecil County, Maryland, on December 29, 1866, the son of John Emory and Hannah Dutton (Broomall) Wilson. His early education was obtained in the public schools of Elkton, and the Elkton Academy, prior to entering Lehigh University, at Bethlehem, Pa., from which he was graduated in 1888 with the degree of Civil Engineer.

Following his graduation Mr. Wilson entered the employ of the Pennsylvania Railroad Company in various capacities and was connected with numerous preliminary surveying and construction projects then being carried on by that Company. In 1891, he was engaged as a Topographer (Corps No. 3) by the Intercontinental Railway Company, making surveys through Ecuador and Peru, in South America, in entire charge of the topography. Returning in 1892 to Washington, D. C., he completed the mapping from Quito, Ecuador, to Lake Titicaca, Peru. Later, he took charge of completing maps and profiles of the Intercontinental Railway surveys from Quito, Ecuador, to Punta Arenas, Costa Rica.

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[1] Memoir prepared by a Committee of the Lehigh Valley Section, consisting of S. A. Becker and M. O. Fuller, Members, Am. Soc. C. E.

His experiences in surveying work impressed Mr. Wilson with the need of a knowledge of law and legal matters. Consequently, for several years he read and studied law, and, in 1894, was admitted to the bar in his home County.

Until 1896, his work as Assistant to Construction Engineers on various projects of the Pennsylvania Railroad Company gave him opportunities to enrich his experience in railroad work.

From 1896 to 1898 he had charge of mapping from the notes of thirteen years of field work by the Guatemala Commission for the Guatemala-Mexico boundary surveys.

Mr. Wilson was appointed Instructor in Civil Engineering at Lehigh University in 1898. During this appointment, he continued his theoretical studies and received the degree of Master of Science in 1901. His practical connections included a study of the virtual profile of the Lehigh Valley Railroad from Jersey City, N. J., to Mauch Chunk, Pa. In 1901, he accepted the headship of the Department of Civil Engineering at Tulane University, at New Orleans, La., which he held until 1904 when he returned to Lehigh University as Assistant Professor of Civil Engineering. Upon the retirement of the late Mansfield Merriman, M. Am. Soc. C. E., in 1906, Professor Wilson was appointed Acting Head of the Department, and, in 1907, was appointed to the Chair of the J. T. Stuart Professorship of Railroad Engineering, which position he held until he was obliged to retire on account of ill health in 1935. He was then made Professor Emeritus of Railroad Engineering. He was the author of the book entitled "Elements of Railroad Track and Construction," published in 1908.

Professor Wilson was active in practical engagements of civil engineering as well as civic government. He served as a member of the old Borough Council of Bethlehem for many years. As Chairman of the Street Committee, he was responsible for the permanency of street improvements made at the time. He served on a special committee appointed to study the water supply and was instrumental in increasing and improving the source of supply and the installation of an improved distribution works, the greater part of which is still a valuable and efficient part of the water supply system for the city. He took part in the campaign, and was connected with some of the preliminary work, which eventually resulted in the building of the city's "Hill-to-Hill" Bridge. He was active in the work of the reconstruction of the Broad Street Bridge and the widening of its approaches.

Professor Wilson served on various Faculty committees and was repeatedly formulating and supporting policies tending toward the improvement of existing conditions. As a member of the Lehigh University Athletic Association for many years, and also as its Secretary, he did much to improve and protect the physical and athletic training of the students.

He was Superintendent of Construction of the Alumni Memorial Building erected on the campus to serve the various Administration units of the University.

Professor Wilson was identified with numerous organizations and clubs, namely: Lehigh University Alumni Association; University Home Club; Bethlehem Club; Saucon Valley Country Club; Lehigh Country Club; and various others. He was a member of the American Society of Military Engineers; Society for the Promotion of Engineering Education; American Railway Engineering Association; and a Registered Civil Engineer and Surveyor.

He became a member of the Masonic Fraternity prior to coming to Bethlehem, and was affiliated with the Bethlehem Lodge No. 283, F. and A. M., being a Past Master; Zinzendorf Chapter No. 216, R. A. M., being a High Priest; Bethlehem Council No. 36, R. S. M.; Lehigh Consistory, A. S. S. R.; and a Charter Member of Bethlehem Forest No. 61, Tall Cedars of Lebanon.

He was a member of the staff engaged in the vocational training of enlisted men at Camp Coppee during the World War. He also volunteered on the Committee of Advisers to registrants for service overseas, and was a member of the Bethlehem Home Guards.

Professor Wilson was twice married. His first wife was Mary Mahala Turner, of Baltimore, Md., who died in 1928. They had two sons who survive him: Maris Wright Wilson, and Captain Alvin Turner Wilson, U. S. Army. His second wife, Mrs. Ann Fenner Anderson, preceded him to the grave in 1932.

Professor Wilson had a clear and vigorous mind which made him effective as a teacher and valuable as a counselor in many matters. He was generous, and had a warm friendship for students and colleagues. He was a patriot in his unselfish loyalty to his country, a doer of good deeds, and a timely help to the practical success of many a fellow alumnus.

Professor Wilson was elected a Member of the American Society of Civil Engineers on May 1, 1901.

ALLAN SHELDON WOODLE, JR., M. Am. Soc. C. E.[1]

Died March 20, 1937

Allan Sheldon Woodle, Jr., was born on March 17, 1878, in Altoona, Pa., the son of the Rev. Allan Sheldon Woodle, rector of St. Luke's Episcopal Church, at Altoona, and Abby Cary (Tisdale) Woodle. His early education was received in St. Paul's School, Concord, N. H., from which he was graduated in 1895. He attended Trinity College in Hartford, Conn., from which, in 1899, he was graduated with the degree of Bachelor of Arts. In the spring and summer of 1898, he worked as Chainman for the Pennsylvania Railroad Company.

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[1] Memoir prepared by C. B. Rose, Works Mgr., Baldwin Locomotive Works, Philadelphia, Pa.

Immediately upon his graduation from college, Mr. Woodle secured a position with the Pennsylvania Railroad Company, and was engaged until May, 1902, as a Rodman, Leveler, and Transitman in the Construction Department. He was then transferred to the Maintenance-of-Way Department and worked as Assistant Supervisor at Greensburg and Mifflin, Pa., for more than a year.

In May, 1903, he was employed by the Baltimore and Ohio Railroad Company as Assistant Engineer of Maintenance and Way. He was transferred after a year to Philadelphia, Pa., as Assistant Division Engineer. In August, 1905, he went to Wheeling, W. Va., where, until September, 1906, he served as Division Engineer.

During the next ten years Mr. Woodle was employed as Assistant Plant Engineer and Engineer in Charge of Construction for The Baldwin Locomotive Works, at Eddystone, Pa., where much large construction was under way to house the manufacture of locomotives.

For two years during the World War, Mr. Woodle served as Engineer of Plant for the Eddystone Ammunition Corporation. He then returned to The Baldwin Locomotive Works, with which Company he remained, as Plant Engineer, until his death.

Mr. Woodle was married to Alice Jenner, on August 30, 1911, and they made their home in Narberth and Ridley Park, Pa. He died on March 20, 1937, of pneumonia, after a short illness, and is survived by his widow and three children, Allan Sheldon, III, Abby Jane, and Mary Elizabeth.

He wrote a number of articles which were published in various magazines, on the subjects of reinforced concrete design and on steam-boiler efficiency.

Mr. Woodle will be remembered by all who knew him as a man of extraordinarily gracious disposition. His manner was quiet and dignified, and few people ever saw him upset or ruffled. He was a scholarly man and a good and efficient business executive.

He was a member of the Concrete Institute Club, and of the Psi Upsilon Fraternity.

Mr. Woodle was elected an Associate Member of the American Society of Civil Engineers on July 9, 1906, and a Member on March 12, 1918.

ROBERT JOSEPH HENDERSON WORCESTER, M. Am. Soc. C. E.[1]

Died September 27, 1937

Robert Joseph Henderson Worcester was born in Boston, Mass., on December 8, 1882, and died, after a protracted, distressing illness, at Elizabethton, Tenn.

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[1] Memoir prepared by J. R. Worcester, Hon. M. Am. Soc. C. E.

He became an engineer without the benefit of a technical education, through his ability to acquire and retain information and an ambition to improve every opportunity for learning. He was never satisfied to " rest on his oars," but was always wide awake and on the lookout for instruction. He had the benefit of a Correspondence School course in mapping and surveying, and was able to learn much by private reading. His only schooling was in primary schools near Newark, N. J., and in the United States School for Naval Apprentices, at Newport, R. I.

At the age of eighteen, Mr. Worcester enlisted as a Private in the United States Army, and was assigned to the 11th Cavalry, then being organized at Fort Myer, Virginia. In 1902, this unit was sent to the Philippine Islands, where he served for three years, as Private, Corporal, and Sergeant. Upon his discharge, he returned to Newark and found work as a Foreman for the Newark Paving Company.

In January, 1905, he entered the employ of Westinghouse, Church, Kerr, and Company, with which he was associated for about eight years, in various capacities, such as Rodman, Transitman, Foreman, Inspector, Party Chief, Engineer, Assistant Superintendent, Office Assistant, Computer, etc. The work included street railway construction and equipment, and an investigation of a water power site on the Kootenai River in Northwestern Montana. This investigation required making cross-sections of the river bottom and measurements of stream flow under great difficulties, owing to the swiftness of the current and the extremely cold weather. During this period, he also had some experience in concrete construction, as Foreman for the Eastern Concrete Construction Company, and the Aberthaw Company.

In May, 1913, Mr. Worcester was engaged by the Town of Concord, Mass., as Engineer and Superintendent of Roads and Bridges, which position he held, except during the World War, until May, 1920.

He was instrumental in organizing the " Concord Minute Men," and in August, 1917, he attended the Second Officers Training Camp, at Plattsburgh, N. Y. In December, 1917, he joined the 151st Depot Brigade, then stationed at Camp Devens, Massachusetts, as Brigade Engineer and Utilities Officer, and was in charge of drainage, road construction, and buildings, until June, 1918, when he was sent to France, having received a commission as First Lieutenant of Engineers. With the 317th (Rainbow) Division, he was in the St. Mihiel Salient, his work consisting largely in the construction of trenches, roads, and bridges. He was cited and received the Purple Cross for building a pontoon bridge, in the night and without tools, in record time. After the Armistice he was with the Army of Occupation on the Rhine until May, 1919. At his discharge he became a Captain in the Officers Reserve Corps, but was obliged to resign as he was unable to attend the summer encampments.

In May, 1920, Mr. Worcester joined the organization of Lockwood, Greene and Company, which he served as Resident Engineer on various projects throughout the country. Among these were: A cotton mill and steam power plant at Somersworth, N. H.; and a hydro-electric and cotton mill at New Braunfels, Tex. He was the Works Manager on the construction of the first worsted mill south of the Mason and Dixon Line, at Duncan, S. C., where a steam plant and a mill village were also built. In 1925, he had charge of the Charlotte, N. C., Office of the Company, from which he was transferred to Johnson City, Tenn., to supervise the construction of the rayon plant of the American Bemberg Corporation, and, later, at the same place, the plant of the American Glanstoff Corporation.

Upon the completion of these plants, Mr. Worcester entered the service of these Companies, as Plant Engineer, Consulting Engineer, and, finally, in 1930, as Personnel Director for the North American Rayon Corporation, which position he held until his death.

Mr. Worcester was beloved and respected by all who knew him. He was interested in all social and athletic activities; a leader of the Boy Scouts, from which organization he received the Silver Beaver decoration; a devoted member of his Church; a Past-President of the local Rotary Club; and a Director of the Chamber of Commerce of Elizabethton, where he had his residence. He was also a member of the Boston Society of Civil Engineers.

He was married, in 1909, to Sharlot Groshans, of East Liverpool, Ohio, who survives him. They had no children.

Mr. Worcester was elected a Member of the American Society of Civil Engineers on January 19, 1925.

DAVID LeROY YARNELL, M. Am. Soc. C. E.[1]

DIED MARCH 9, 1937

David LeRoy Yarnell, the son of Ervin and Catherine (Countryman) Yarnell, was born at Storm Lake, Iowa, on January 13, 1886, and died on March 9, 1937, of coronary occlusion, after spending the day at his office.

He was educated in the public schools in Iowa, was graduated from Iowa State College, at Ames, Iowa, in 1908 with the degree of Bachelor of Civil Engineering, and received the professional degree of Civil Engineer from that College in 1916. In 1926, he was awarded the degree of Master of Science in Hydraulic Engineering by the State University of Iowa, at Iowa City, Iowa.

The early years of Mr. Yarnell's professional life were spent in the field, chiefly in connection with drainage engineering. In April, 1909, he was appointed Assistant Drainage Engineer in the United States Department of Agriculture, and was engaged on preliminary investigations and designs of drainage works in Nebraska, Kansas, Virginia, North Carolina, Oklahoma, and Arkansas. Advancing to the grade of Drainage Engineer in February, 1911, he was placed in charge of surveys and designs of drainage systems, technical investigations on run-off from drained land, and investigations of ground-water in tiled land. He was a member of the Board of Engineers appointed to prepare a plan for the Cypress Creek Drainage District, in Arkansas, and during this period he also surveyed and prepared plans for reclamation work at Pond Creek, Ky.

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[1] Memoir prepared by S. M. Woodward, L. A. Jones, and F. T. Mavis, Members, Am. Soc. C. E.

In 1916, Mr. Yarnell became Senior Drainage Engineer of the U. S. Department of Agriculture, and it was then that he embarked upon his career in research which was to bring him world-wide recognition for careful, systematic, and authoritative contributions to experimental hydraulics.

During 1916 and 1917, at Arlington, Va., he conducted systematic tests on the flow of water in drain-tile and vitrified pipe in sizes from 4 in. to 12 in. in diameter. The report of these studies [2] foreshadowed the pattern of many of his later reports, and it is one of his most widely known publications. More than 800 distinct tests were conducted with an unrelaxing attention to detail which characterized all his work. No other report on this subject has commanded greater confidence from hydraulic engineers.

In 1922, hydraulic investigations were undertaken co-operatively by the U. S. Department of Agriculture and the State University of Iowa, and Mr. Yarnell was placed in charge of these investigations at Iowa City. His first major research at the Hydraulics Laboratory was on the flow of water through culverts,[3] and during a 2-yr period 3 300 tests of culverts ranging in size from 12 in. in diameter to 4 ft square were conducted at the Laboratory. He employed Senior and Graduate Engineering Students to assist him with the details of this investigation, and from that time until his death, his kindly influence and encouragement and his professional assistance and supervision were felt by virtually every student of hydraulic engineering at the University.

In 1929, he made tests by means of models to determine the benefits of proposed cut-offs of the Des Moines River, at Ottumwa, Iowa. This study was probably the first attempt in the United States to determine in advance of actual field construction the benefits to be derived from straightening rivers. Mr. Yarnell was among the first of American engineers to recognize the value of river-model studies which have since been accepted as a necessary part of virtually every major hydraulic engineering project in America.

The J. James R. Croes Medal of the Society was conferred jointly upon Mr. Yarnell and the late Floyd A. Nagler, M. Am. Soc. C. E., of the State University of Iowa, for a paper entitled " The Effect of Turbulence Upon the Registration of Current Meters." [4]

Other significant laboratory investigations conducted by Mr. Yarnell from 1928 to 1933 included the flow of water around bends in pipes of rectangular and circular cross-section, the back-water caused by pile trestles and bridge piers, and the hydraulics of flow over railway and highway embankments. The last of these studies was conducted on models and on full-scale highway and railway embankments tested in the river flume under conditions simulating those found in Nature.

Mr. Yarnell was the author of a *Bulletin* on rainfall frequency-intensity data, in which he made an analysis of 28 077 rain storms representing the combined records of every Weather Bureau Station equipped with automatic rain-gages in the continental United States from its earliest record to the end of 1933. He showed exceptional patience in his analysis of this staggering tangle of statistical data, and presented the material in such form that it is now widely used.[5]

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[2] " The Flow of Water in Drain Tile," by D. L. Yarnell and Sherman M. Woodward, M. Am. Soc. C. E., *Bulletin No. 854*, U. S. Dept. of Agriculture, Govt. Printing Office (1920).

[3] " Flow of Water Through Culverts," by David L. Yarnell, Floyd A. Nagler, and Sherman M. Woodward, Members, Am. Soc. C. E., *Bulletin No. 1*, Univ. of Iowa Studies in Eng. (June, 1926).

[4] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), pp. 766–795.

More than twenty-five papers and *Bulletins* are directly credited to Mr. Yarnell's authorship. Countless others bear acknowledgments to him for his kindly interest and assistance. To his co-workers and to young men entering upon the study of engineering he gave most generously of his wealth of knowledge and professional experience. His support of the Iowa Hydraulics Laboratory during its early years was a large factor in its rapid development, and his use of graduate students as assistants was extremely beneficial to scores of young men in enabling them to continue their studies while gaining practical experience in hydraulic engineering.

A sensitive man, courteous, mild-mannered, and retiring, "Dave" Yarnell was genuinely loved by those who came to know him well, and he was held in high esteem by all those who made his acquaintance. He exemplified the gentleman whom Julius Hare visioned nearly a century ago when he wrote:

> "The real gentleman should be gentle in everything, at least in everything that depends on himself—carriage, temper, constructions, aims, desires. He ought therefore to be mild, calm, quiet, even, temperate—not hasty in judgment, not exorbitant in ambition, not overbearing, not proud, not rapacious, nor oppressive; for these things are contrary to gentleness."

Mr. Yarnell was a member of the American Society of Agricultural Engineers; the American Geophysical Union; the Iowa Engineering Society; the Engineers Club of Iowa City; the Triangle Club of the University of Iowa; Tau Beta Pi (Honorary Engineering Fraternity); Sigma Xi (Honorary Scientific Society) and other professional and scientific organizations. He was a Mason and a member of the Congregational Church.

On August 24, 1914, Mr. Yarnell was married to Alice Lee Roche, of Washington, D. C. He is survived by his widow, two sisters, Mrs. W. P. Bair and Mrs. Henry Weber; and two brothers, Admiral Harry E. Yarnell, U. S. Navy, and Ross Yarnell Shearon.

Mr. Yarnell was elected a Junior of the American Society of Civil Engineers on April 5, 1910; an Associate Member on October 1, 1912; and a Member on April 21, 1920.

CHARLES ROLLIN ALLEN, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 29, 1937

Charles Rollin Allen was born on November 9, 1872, in Williamstown, Vt., the son of C. Rollin and Evelina (Bigelow) Allen. He attended the local schools and also Goddard Seminary, in Barre, Vt. His engineering training was acquired at Tufts College, in Medford, Mass., from which he was graduated in 1899, with the degree of Civil Engineer.

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[5] "Rainfall Intensity-Frequency Data," by David L. Yarnell, U. S. Dept. of Agriculture, *Miscellaneous Publication No. 204* (1935).

[1] Memoir prepared by C. Rollin Allen, Jr., Saratoga Springs, N. Y.

After his graduation Mr. Allen was first employed as Engineer for the Metropolitan Sewer District, of Boston, Mass., and then returned to Barre as City Engineer. In this position he supervised the installation of the granite block pavement on Main Street, which is still in use. In 1907 he accepted a position with the Highway Department of the State of New York as County Assistant and Location Engineer until 1914, when he set up his own office as Consulting Engineer on highway work.

As a Consulting Engineer for the contractors, he was connected with highway work from Vermont to Buffalo, N. Y., and from Canada to Pennsylvania. This work included many of the noted highways in New York State among which were Storm King, Whiteface, Crow's Nest, and, most recently, the Rensselaer Bridge over the New York Central and Boston and Albany Railroads. He did a great deal of work in the Court of Claims of the State as expert witness on highways, valuation of work done, and uncompleted contracts.

During the World War, Mr. Allen was employed by the Federal Government, first at Nitro, W. Va., and then in the shipyard at Hog Island, Philadelphia, Pa.

As a Consulting Engineer he had a good opportunity to show his infallible patience and good humor, dealing with contractors and representatives of the State, smoothing out their differences, and keeping his work running smoothly to a satisfactory conclusion.

He was married on October 29, 1904, to Mabel S. Calef, who survives him, together with three sons, Stanley C., C. Rollin, Jr., and Richard S., and one daughter, Lucy E. Allen.

Mr. Allen was elected an Associate Member of the American Society of Civil Engineers on June 6, 1911.

RUSSELL AVERILL ANDERSON, Assoc. M. Am. Soc. C. E.[1]

Died May 31, 1937

Russell Averill Anderson was born at Wrightsville Beach, N. C., on August 31, 1896, the son of Burdis Anderson and Ella (Butters) Anderson. He first attended school at Wilmington, N. C., but later his parents moved to Michigan where his mother's family was prominent in the lumbering industry of that State.

Mr. Anderson's parents settled in Sault Ste. Marie, Mich., and there he lived and received his public and High School training until 1914, when he entered Northwestern University, at Evanston, Ill. After three years of academic study, he left the University to join the United States Army Air Corps in which he attained the rank of Lieutenant.

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[1] Memoir prepared by James H. Cissel, and G. Donald Kennedy, Members, Am. Soc. C. E., and Frederick C. Taylor, M. Am. Soc. C. E.

When the close of the World War ended his military service, he began his technical education in the Engineering College of the University of Michigan, at Ann Arbor, Mich. During the entire period of his studies at Michigan, his class-room training was strengthened and given practical point by part-time employment in the offices of W. C. Hoad and Arthur J. Decker, Members, Am. Soc. C. E., Consulting Engineers, in Ann Arbor.

In 1920, an opportunity, both to gain engineering experience in a wider field and to earn money needed for the completion of his engineering course, resulted in an interruption of his college work. For nearly two years, Mr. Anderson was employed on appraisal work, first by Henry Earle Riggs, M. Am. Soc. C. E., in the United States and, later, by the J. G. White Company, in the Philippine Islands, in an appraisal of the properties of the Manila Light and Power Company.

Returning to the University of Michigan in the spring of 1922, Mr. Anderson was graduated with honors in Civil Engineering with the Class of that year, thus closing a scholastic career in which "he was rated very highly by his teachers, and showed a capacity for getting things done and for doing the job well."

During his college years, his work for Hoad and Decker had included a variety of problems both in the office and in the field. The firm's business was largely concerned with sewerage and water systems and other municipal work, and it was to this field that Mr. Anderson returned when he resumed his connection with the same office after his graduation.

From June, 1922, until October, 1923, he was engaged in preliminary studies, reports, and designs. Comprehensive plans for sewerage were developed by him for the cities of Muskegon, Royal Oak, Trenton, and Owosso, in Michigan. He planned extensive water-works improvements for several other cities in Michigan. From October, 1923, until February, 1924, he managed his firm's Branch Office at Jackson, Mich., where he had charge of the collection of field data pertinent to comprehensive sewerage and drainage plans for that city.

From February to September, 1924, Mr. Anderson made field surveys and office studies, and prepared a report on the disposal of industrial wastes from the automobile plants at Flint, Mich. His work for the next year was concerned principally with sewerage and sewage disposal problems for Oakland County, Michigan, and the City of Adrian, Mich.

Mr. Anderson left the consulting engineering field in October, 1925, and became associated with the R. D. Baker Construction Company, of Royal Oak, Mich. His first work with this Company was in connection with estimating, for bidding purposes, the cost of the Royal Oak Drain, an Oakland County project for the construction of a sewer approximately six miles long and ranging in diameter from 15 ft at the outlet to 6 ft at the upper end. The contract for this project, which was let at a later date, was awarded the R. D. Baker Construction Company for approximately $1 000 000. During

1926, Mr. Anderson was actively engaged in the supervision of the construction of this project.

From the time of his employment by the Baker Company to the middle of 1933, Mr. Anderson, as Vice-President and Secretary, was active in the management of the Company and in the prosecution of several other large projects in Michigan, namely, the construction of 1.5 miles of sea wall along the shores of Lake St. Clair in Grosse Pointe Shores; the construction of several miles of steel water main ranging in size from 72 in. to 30 in. in diameter in Royal Oak, Detroit, and Monroe; a complete water main system for the Village of Inkster, comprising 60 miles of mains; numerous projects and extensions on the Flint sewer system; and many other construction projects of similar nature, including a 16-ft storm sewer consisting of both tunnel and open-cut construction in Dallas, Tex.

In 1933, Mr. Anderson went with the Michigan State Highway Department as Resident Engineer on the project for widening Woodward Avenue, in Detroit. Soon after the completion of this important construction he joined the Administrative Staff of the Department as Director of its Finance Division. In that position Mr. Anderson assumed and admirably discharged the responsibilities connected with the financial, contractual, and procurement operations of the Department in a period not only of great construction activity, but of extreme fiscal uncertainty and confusion. During the four years that he filled this position, the practice of the Department, in relation to construction contracts, accounting, and purchasing was readjusted and practically re-organized under his direction.

So effectively did Mr. Anderson perform the duties of this position that he was counted among the most able administrators in the public service of the State, and, at the time of his death, was being considered for posts of still higher responsibility and trust. The following resolution adopted concurrently by both Chambers of the State Legislature of Michigan reflects the esteem in which he was held:

> "*Whereas,* Russell A. Anderson, finance director of the State Highway Department, died suddenly May 31 in the prime of his life; and
>
> "*Whereas,* He was an able leader in the engineering profession who had given distinguished service to the State Highway Department and to the state of Michigan; and
>
> "*Whereas,* Russell A. Anderson combined rare ability with a keen sense of humor, a pleasing personality and rugged integrity that always revealed him as a fine gentleman; and
>
> "*Whereas,* His untimely death has been a tremendous shock to his colleagues and his many friends who will never forget his beautiful character; now therefore be it
>
> "*Resolved by the House of Representatives* (The Senate concurring), That the members of the Michigan legislature express their profound sorrow on the death of Russell A. Anderson, Finance Director of the State Highway Department; and be it further
>
> "*Resolved,* That copies of this resolution be sent to the widow and to the State Highway Department.
>
> "Adopted June 4, 1937."

As an Engineer and as an Executive, Mr. Anderson exemplified the technical accuracy, the insistent energy, and the integrity of principle and practice which the profession demands. In his personal relations, the same keen intelligence and inherent loyalty which brought promise and fulfillment to his career as an Engineer, made him a friend whose genial companionship and genuine qualities were valued and are remembered by all who knew him.

Mr. Anderson was a Charter Member of the Engineering Society of Detroit, Treasurer of the Michigan Engineering Society, and President of the Michigan Good Roads Federation. In addition, he guided the activities of the Michigan Bureau for Highway Education.

He was married to Marian Van Valen, of Parma, Mich., in 1921, while he was still a student at the University of Michigan. Mrs. Anderson and two children, Suzanne and Russell, survive him.

Mr. Anderson was elected a Junior of the American Society of Civil Engineers on November 26, 1923, and an Associate Member on June 7, 1926.

ALBERT WORTHINGTON ATWATER, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 24, 1937

Albert Worthington Atwater, the oldest son of John A. and Clara (Worthington) Atwater, was born in Brooklyn, N. Y., on December 23, 1885. He was of early American stock.

He received his preparatory education in the Public Schools of Springfield, Mass. Entering the School of Mechanical Engineering of Purdue University, at Lafayette, Ind., in 1902, he earned the degree of Bachelor of Science in 1906. Subsequently (1909), he was awarded the professional degree of Mechanical Engineer on the presentation of a thesis on "The Design of the Hulett Patent Movable Car Dumper."

Early in his professional career, Mr. Atwater identified himself with the design of heavy mechanical equipment and, although he became active in other and varied fields of engineering, he never lost interest in this type of work. As a Designing Engineer and, later, as Sales Engineer, he kept in close touch with this important branch of engineering.

He began his professional work with the firm of Wellman-Seaver-Morgan Company, of Cleveland, Ohio, as an Apprentice Draftsman, shortly after graduation from High School. This firm was one of the leaders in the development of mechanical equipment for handling heavy freight, particularly in the rapid loading and unloading of Lake carriers. Mr. Atwater was connected with this Company for eight years,—during college vacations and for four years after his graduation. His work was principally in connection with the design of ore-unloading equipment and coal-loading equipment for boats engaged in Great Lakes transportation. He had much to do with the details of the development of the Hulett movable car dumper.

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[1] Memoir prepared by a Committee of the Tennessee Valley Section, consisting of Ned H. Sayford, M. Am. Soc. C. E., and J. G. Allen and F. C. Wardwell, Assoc. Members, Am. Soc. C. E.

In 1910, he resigned to accept an appointment as a teacher of Mechanical Drawing in East Technical High School, at Cleveland. He remained in this position for two years.

Mr. Atwater became Chief Draftsman for the Cleveland Illuminating Company in 1912, and, later, was promoted to the position of Construction Engineer in charge of all plant construction and maintenance. As such, he was in complete charge of the design, purchase of equipment, and construction of all steam projects of this Company, including complete plants of a generating capacity as high as 50 000 kw. At this time, he became interested in the Prime Movers Committee of the National Electric Light Association and for several years took an active part in its work. During the time that he was with the Illuminating Company, Mr. Atwater had much to do with the evaluation of physical property in connection with the Company's celebrated rate case with the State of Ohio.

For a short time between 1919 and 1921, Mr. Atwater was employed by Arthur G. McKee and Company on blast-furnace design, but in March, 1921, he again entered the service of the Cleveland Board of Education, this time as Electrical Engineer. The City of Cleveland was undergoing a very rapid expansion in its school-building program. During a period of two years, he supervised the design and construction of the electrical installation on a $15 000 000 school-building program.

In 1923, Mr. Atwater became connected with the E. W. Clark Engineering Corporation which was designing and constructing many large steam-power projects. As the Company's Mechanical Engineer, he was in charge of the first high-temperature, high-pressure electric steam plant. Later, he was in charge of the erection of the 40 000-kw steam plant of the Nashville Railway and Light Company (later, the Tennessee Electric Power Company), at Hales Bar, near Chattanooga, Tenn. In 1925, this plant was referred to as one of the three outstanding steam plants of the United States.

Upon the completion of the Hales Bar Steam Plant, Mr. Atwater became a partner in the contracting firm of Martin and Warlick Company. This firm contracted for and built many schools, factories, motion picture theaters, and garages, in Chattanooga and other cities of Tennessee. At this time he was a member of the Associated General Contractors of America and was President of the Chattanooga Chapter of the Association, and a Director of its State Branch.

In 1927, Mr. Atwater became Mechanical Engineer on the design of steam-power projects for Stone and Webster Engineering Corporation, of Boston, Mass.

Returning to Chattanooga in 1928, he re-entered contracting work and also engaged in sales engineering, acting as the East Tennessee Sales Representative for the Tennessee Products Corporation and several large equipment manufacturers. He also served in an advisory capacity to Street

Brothers Machine Company, in the design of cableways and other heavy construction equipment.

Upon the advent of the Tennesee Valley Authority (TVA), in Tennessee, Mr. Atwater entered the employ of the Authority as an Examining Engineer in the Personnel Division, where he interviewed and was instrumental in hiring a large percentage of the construction personnel for Norris Dam.

Later, he transferred first to the Design Division and then to the Construction Plant Division of the TVA, assisting in the design of construction plants for Chickamauga, Hiwassee, and Guntersville Dams.

With the start of construction of the Chickamauga Project, Mr. Atwater was transferred to that project as Assistant and afterward became Associate Mechanical Engineer. He was later promoted to the position of Progress Engineer, which he held at the time of his death.

In addition to a keen interest in his profession, Mr. Atwater found time to devote much study to history, particularly to the military history of his country. He was an authority on the subject of military operations in the Civil War and, frequently, was called upon to describe the battle formations in the district around Chattanooga, where several decisive battles were fought. His extensive collection of maps and books on the subject is of considerable interest to historians. At the time of his death, he was preparing a series of talks on Civil War movements to be given to the Army Officers studying the tactics and strategy of actions around Chattanooga.

Mr. Atwater was a man of quiet manner, extremely modest, conscientious, and of highest integrity. His dry wit made him an extremely interesting conversationalist.

He is survived by his widow, Ida (Grove) Atwater, to whom he was married at Cleveland, in 1906; also by one son, Grove Worthington Atwater.

He was a member of the Theta Xi Fraternity, a member of the Chattanooga Engineers Club, and was a licensed Professional Engineer in the State of Tennessee.

Mr. Atwater was elected an Associate Member of the American Society of Civil Engineers on September 9, 1919.

JOSEPH DUNTON BROWN, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 8, 1937

Joseph Dunton Brown, the oldest son of John Dunton and Josephine (Marshall) Brown, was born in Chelsea, Mass., on March 5, 1884. His ancestors were of the earlier settlers of New England. His father, born in Wiscasset, Me., was left an orphan at the age of ten years, his parents being lost at sea while making a voyage to Australia in the *Emily Dunton*. In Boston, Mass., John Dunton Brown met Josephine Marshall, and was married to her in January, 1881. He then moved to Chelsea where, as stated, Joseph Dunton Brown was born.

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[1] Memoir prepared by J. S. Lewis, Esq., Waterbury, Conn.

Mr. Brown's early life was spent in Chelsea and Everett, Mass., where he attended Grammar and High School. At the age of nineteen he secured a position with the John Hancock Insurance Company. While with this Company, he attended night classes at Lowell Institute, Lowell, Mass., where he studied Civil Engineering. Upon completing his course at the Lowell Institute he accepted a position as Engineer with The Connecticut Company, New Haven, Conn., under Charles Rufus Harte, M. Am. Soc. C. E.

From 1913 to 1918, Mr. Brown was an Engineer with the Connecticut State Highway Department. In 1918, he went to Stevenson, Conn., with the J. A. P. Crisfield Company, working directly under Mr. Crisfield on the construction of the Stevenson Dam for The Connecticut Light and Power Company. On the completion of this project, he went with the Lockwood Greene Company, of Boston, but was with this Company only for a short time, when he was recalled by The Connecticut Light and Power Company and made Real Estate Engineer. Under his superintendence most of the Company's transmission lines were laid out and rights of way surveyed, and purchased. He had a great deal to do with the successful completion of The Rocky River Development which includes the formation of Lake Candlewood, the largest lake in the State.

He was a great lover of his father's native State—Maine—his greatest pleasure being to fish her streams with congenial companions, a pleasure he was able to enjoy several times each year. It was always his wish that he could spend his old age in Maine and, at the time of his death, he was negotiating the purchase of an estate on the Maine Coast.

Mr. Brown was a member of the Board of Fire Commissioners and the Board of Water Commissioners of the Town of Southington, Conn., and a member of the Congregational Church, in New Milford, Conn. He was a member of the Connecticut Society of Civil Engineers. His Masonic affiliations included membership in the Blue Lodge, in Rockville, Conn., Knight Templars, in Danbury, Conn., and the Mystic Shrine, in Bridgeport, Conn.

In 1912, he was married to Gertrude E. Martin, of Rockville, Conn. From this union there are three children, Donald Marshall, Marjorie Janet, and Dorothy Gertrude, who, with Mrs. Brown, survive him. Donald is a graduate of the University of Maine; Marjorie, an undergraduate at the Connecticut College for Women, in New London; and Dorothy, a High School student, thus bearing out Mr. Brown's determination to give his children an easier start in life than he had.

He loved his home in Marion, Conn., and spent much of his spare time beautifying its surroundings, his hobby being forestry and wood-working. He was friendly and affable and took pleasure in doing favors for his friends. It is difficult to replace a man of Mr. Brown's character and attainments; his going is an irreparable loss to both his friends and The Connecticut Light and Power Company.

Mr. Brown was elected an Associate Member of the American Society of Civil Engineers on January 14, 1929.

VERNE LOUIS CONRAD, Assoc. M. Am. Soc. C. E.[1]

Died November 24, 1937

Verne Louis Conrad was born on January 6, 1879, in Bellaire, Ohio, the son of W. Harry Conrad and Elizabeth (Rinker) Conrad. His parents moved to Uhrichsville, Ohio, when he was four years of age, and here he grew up and attended school. It was in the local High School that he became acquainted with his future wife. He attended Ohio State University, at Columbus, Ohio, from which he was graduated in 1902 with the degree of Civil Engineer.

Immediately after leaving college, Mr. Conrad accepted a position with the Engineering Corps of the Pennsylvania Railroad Company, on the Logansport Division, where he was employed in the Maintenance-of-Way Office (June to October, 1902). He was then placed in charge of double-track construction from Logansport, Ind., to Chicago, Ill. (October, 1902, to November, 1903); after which he was transferred to the Cincinnati Division (November, 1903, to August, 1904). From August, 1904, to February, 1905, he was engaged on the design of bridges and grade separation, on the Logansport Division; and from February 1905, to October, 1907, he was in charge of construction for the elimination of Bulger Tunnel and 4-track change alignment, on the Pittsburgh (Pa.) Division.

Mr. Conrad was Superintendent for Butler Brothers Hoff Company, on the Detroit River Tunnel from October, 1907, to March, 1908, and Division Engineer for the Detroit River Tunnel Company, in charge of the Subaqueous Division, until January, 1910.

At that time, Mr. Conrad moved to Texas, locating near Donna, where he was engaged as Engineer on Surveys and Design for the Hidalgo County Drainage District for about one year. He then became Engineer for the Union Irrigation District, at Raymondville, Tex., and was engaged on irrigation work at San Saba, Tex., until November, 1913, when he moved to Cameron County, locating in Brownsville, Tex. His first major engagement in Brownsville was the resurvey of the County boundaries and topographical features, and the preparation of an Official Map of Cameron County, still (1938) in current use. This work required nearly three years.

From March, 1916, until his death, Mr. Conrad was engaged in extensive private engineering practice in Cameron and neighboring counties, with offices in Brownsville. He served as Special Highway Engineer for Cameron County from July, 1919, to January, 1920, and was Engineer for a large number of irrigation districts in Cameron County, among which were the La Feria Irrigation District and Cameron County Water Districts Nos. 8, 10, 11, 16, and 17. He made surveys for the relocation of the Point Isabel Railroad and right-of-way surveys for the Brownsville Navigation District. He also had charge of numerous other surveys, plants, and developments in his locality to the time of his death.

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[1] Memoir prepared by W. O. Washington, M. Am. Soc. C. E.

Mr. Conrad took an active interest in civic affairs. His keen sense of humor and genial personality made him a friend of all. He was a hard worker, but was always willing to help others.

At Uhrichsville, on October 6, 1904, he was married to Mary Ellen Parry, and theirs was a happy family throughout the years. Surviving him are his widow; his mother, Mrs. Elizabeth Conrad; a son, Verne Louis Conrad, Jr.; and a daughter, Sarita (Mrs. F. C. Whitmer).

Mr. Conrad was elected an Associate Member of the American Society of Civil Engineers on December 6, 1920.

ROSS ELROY HAMILTON, Assoc. M. Am. Soc. C. E.[1]

Died July 24, 1937

Ross Elroy Hamilton, the son of James and Jane (Leggett) Hamilton, was born in Coshocton County, Ohio, on September 1, 1879. He received his early education in the country schools of the County, taking his High School work at Coschocton High School from which he was graduated in June, 1899. In September of the same year he entered the Ohio State University, at Columbus, Ohio, from which he was graduated in 1903 with the degree of Civil Engineer.

Mr. Hamilton's engineering experience began in May, 1903, as Transitman with the Pennsylvania Railway Company. He continued in railroad work until 1908, being employed, successively, by the New York Central and Hudson River Railway Company on grade reduction work; as Assistant City Engineer with the City of Urbana, Ill.; as Instrumentman by the New York Central and Southwestern Railroad Companies; as Assistant Engineer, Department of Maintenance and Way, Baltimore and Ohio Railway Company; as Instrumentman on location and construction, Tidewater Railroad Company; and as Engineer in charge of masonry structures on line reconstruction work, by the Southern Railway Company, at Lynchburg, Va.

In February, 1908, Mr. Hamilton returned to Coshocton and entered the private practice of his profession. He supervised the field work on a 6-mile dam, acted as City Engineer, and designed and supervised the construction of the disposal plant, water-works, and storm-sewer system for West Lafayette, Ohio.

In September, 1909, he took office as County Engineer of Coshocton County, serving two terms of two years each. He then served two years as Division Engineer in the Department of Public Works, in charge of the Ohio and Erie Canal, and one year as Assistant Division Engineer in the Department of Highways, at New Philadelphia, Ohio. He was then re-elected as County Engineer of Coshocton County and served two terms. He resigned this office in 1920 to accept a position with the State Highway Department as Resident Engineer of Coshocton County. He continued as Resident Engineer until 1934 when he resigned to re-enter private practice, being employed by the City of Coshocton as Supervising Engineer in charge of the new water-works construction. He served a period with the Muskingum Conservancy District, at New Philadelphia, and returned home in 1936, resuming private work. He had several municipal projects under way when he became ill.

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[1] Memoir prepared by S. S. Senter, Esq., Canton, Ohio.

Mr. Hamilton was interested and active in civic life and its problems, wherever he was located. He was a life-long member of the Democratic party and took an active part in its councils. The confidence of the community in his ability and integrity was expressed in his election as County Engineer on four different occasions.

He was a member of the Masonic Fraternity, a Kiwanian, and an active member of the Presbyterian Church, serving frequently in various official capacities.

He stood high in his profession and had the respect of his professional associates and of all those who knew him. There was something about his appearance and conversation that gave the impression of honesty, which was his outstanding characteristic. He was fair and " square " in all his dealings and the profession has lost a man who followed in his work and every-day life, its highest ideals.

In his many years of service to the community of Coshocton County, Mr. Hamilton has left numerous monuments to his memory. He is sometimes spoken of as the " father " of the Coshocton County System of Highways because through his efforts there are many miles of highly improved roads which will be a reminder of his service for many years to come.

In his private life he was a splendid family man, a good companion, kindly, sincere, and honorable. He made and kept friends because of his genuine worth and stability, his lack of pretence, and his unconquerable spirit. His passing is a distinct loss to the profession and to the community, and his many friends and the surviving members of his family will miss him sorely.

He was married at Coshocton, on June 16, 1909, to Adeline Merrill and in his home life he was very fortunate and supremely happy. His widow and one son, Thomas, survive him. For them it may be said, " Love hath a tie which Time nor Death can sever."

Mr. Hamilton was elected an Associate Member of the American Society of Civil Engineers on April 1, 1914.

FOSTER WARREN HARVEY, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 11, 1937

Foster Warren Harvey was born in Nashua, Iowa, on September 5, 1899. He was educated in the public schools of Sioux Falls, S. Dak., and later attended the University of South Dakota, at Vermilion, from which he was graduated as an Honor Student, with the degree of Bachelor of Science in Civil Engineering, in the Class of 1923.

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[1] Memoir prepared by Ray Adams, Assoc. M. Am. Soc. C. E.

Immediately after his graduation, Mr. Harvey accepted employment with the Illinois State Division of Highways. After serving approximately two months as an Instrumentman and Inspector, he was appointed Resident Engineer on earth grading, bridges, and paving. In July 1925, he entered the service of Carr and McFadden, Consulting Engineers, of Palm Beach, Fla., as Chief of Party on development work for subdivisions and Designer of reinforced concrete and steel structures. The major part of his field work was in the swampy districts of Florida. He found the sub-tropical climate and conditions to be quite unsuited to him, and, therefore, he again sought employment in a more temperate locality. In October, 1925, he was engaged as a Designer by the firm of Harrington, Howard, and Ash, Consulting Engineers, of Kansas City, Mo. His duties with this firm consisted of design of details, and preparation of design drawings for fixed and movable bridges, the most notable of which was the railroad truss of the Mississippi River Bridge, at Vicksburg, Miss.

In March, 1928, he was employed by the Bureau of Bridges, Missouri State Highway Commission, as a Designer, and remained in this position until his death. Mr. Harvey's period of employment with this Bureau coincided with the largest program of highway bridge construction in the history of the State. He was engaged almost entirely in responsible charge of preliminary investigations, design, and the supervision of the preparation of design plans for bridges of major proportions, across navigable streams. The most notable of these is the highway bridge across the Missouri River on U. S. Route 40 TR, at Weldon Springs, Mo. This is a 32-ft roadway bridge of 2 614 ft over-all length. The river spans are two 2-span continuous trusses with a span of 513 ft, center to center of piers. This bridge was dedicated and formally opened on June 26, 1937.

Mr. Harvey was a member of the Protestant Episcopal Church, and a member of the Vestry of Grace Episcopal Church, in Jefferson City, Mo. He was active in lay affairs of his Church. In Masonic circles, he was a member of the Blue Lodge in Jefferson City. He was a member of the Independent Order of Odd Fellows.

Mr. Harvey served in the Army, in a training camp, during the latter part of 1918. Upon graduating from the University of South Dakota, he was appointed a Second Lieutenant in the Officers Reserve Corps, U. S. Army. He was later promoted to the grade of First Lieutenant, and, in August, 1933, to that of Captain, Infantry Reserve. He received a Certificate of Capacity for the grade of Major, Infantry Reserve, in May, 1937, and would have been promoted to that grade upon the occurrence of the first vacancy after August, 1938. He was a member of the Reserve Officers Association and participated in its activities.

He was stricken with meningitis on June 6, 1937, and succumbed to this illness on June 11, at his home in Jefferson City. Funeral services were held in Grace Episcopal Church in Jefferson City, and he was buried with military honors at Sioux Falls, S. Dak. He is survived by his mother, Mrs. Myrtle L. Harvey, with whom he had made his home for the past thirteen years.

In all his work, Mr. Harvey was outstanding for his thoroughness and ingenuity. He was singularly vigorous in his attack of difficult problems, and completed his solution with the same degree of energy that he displayed at the beginning. He often expressed his belief that the excellence of a bridge design depended largely upon the ingenuity and thoroughness exercised in the design of details. He was considered the leading authority in the Missouri Highway Department on the design of steel bridges. He was very patient with those of his professional associates whose experience did not equal his, and always proved himself willing to be of assistance to them. His associates feel that his death interrupted a career that was just beginning to ripen to its full potentiality.

Mr. Harvey was elected an Associate Member of the American Society of Civil Engineers on November 28, 1931.

CLARENCE SCOTT HOWELL, Assoc. M. Am. Soc. C. E.[1]

Died September 23, 1936

Clarence Scott Howell, the son of William Roe Howell and Fanny (Scott) Howell, was born in New York, N. Y., on August 11, 1875. He was in direct descent from Edward Howell, founder of Southampton, N. Y., and Captain Nathaniel Roe of the Continental Army. His father was a noted artist and photographer, receiving among other awards the American Institute Medal for the best full-length pastel in 1873, and the Great Medal of Merit from the Vienna Exposition in the same year.

As a boy, influenced by his father's vocation, Clarence Scott Howell showed proficiency in photography and studied art in New York City and Brooklyn schools. These studies were interrupted by the death of the father, and the boy had to seek remunerative employment. He was appointed a page in the United States House of Representatives during President Cleveland's Second Administration. In Washington, D. C., Mr. Howell continued his studies at Columbian College, the Corcoran Art School, and the Art Students' League.

In 1895, he received an appointment as Photographer in the Washington, D. C., Navy Yard, and, in 1896, he was transferred to the Engineering Department at the Norfolk (Va.) Navy Yard, where he remained until he resigned as Assistant Engineer, in 1906. It was at this time that he decided to make engineering and architecture his life work.

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[1] Memoir prepared by Herbert W. Ferris, Assoc. M. Am. Soc. C. E.

In 1907, Mr. Howell as Superintendent for the Raymond Concrete Pile Company, built the foundations for the Contagious Disease Hospital and the Baggage and Dormitory Buildings, at Ellis Island, New York Harbor.

From 1908 to 1916, in private consulting practice, with offices in Washington and New York City, as a member of the firm of Howell and Howell, Engineers and Architects, and as the Chief Engineer of the Giant Concrete Pile Company, Mr. Howell specialized in the design of heavy foundations, the more notable being: The substructure for the all-concrete ocean pier, at Long Beach, N. Y.; the foundation and superstructure for the reinforced concrete bridge and approaches across Reynolds Channel, also at Long Beach; and the foundation and superstructure for an elastic arch bridge spanning Popolon Creek and Valley, New York. At the request of Major J. M. Carson, U. S. Army, he was consulted on the design of the foundations for the West Academy Building of the United States Military Academy, at West Point, N. Y.

During this period Mr. Howell made a special study of concrete piles, and of pile-driving methods and apparatus. These studies resulted in his designing the "Giant Concrete Pile", a reinforced concrete pile driven from the point. This method of pile-driving was patented by Mr. Howell in the United States Patent Office (Patents No. 1,352,128, September 7, 1920, and No. 1,366,460, January 25, 1921). These piles were used in the foundations of the following buildings: Power House, Manhattan State Hospital, on Ward's Island, New York, 1916; United States Court House, Custom House and Appraisers Stores, at Wilmington, N. C., 1916; and Factory No. 4, Mutual Chemical Company of America, at Baltimore, Md., 1920.

In 1916 the Giant Concrete Pile Company became affiliated with the North-Eastern Construction Company of New York City. Mr. Howell became associated with the latter Company as Designing and Field Engineer until the re-organization of the Giant Concrete Pile Company in May, 1920. He became Vice-President and Chief Engineer in this new Company and also re-established himself in private practice in New York City, until 1930.

In addition to his achievements in heavy foundation work, Mr. Howell designed and supervised the construction of many types of structures, namely, reinforced concrete garages and industrial buildings, office buildings, piers, sea-walls, and private residences.

In 1930, he again became associated with the North-Eastern Construction Company, in New York City, as Designing and Field Engineer, which position he held at the time of his death.

On May 9, 1918, Mr. Howell was commissioned Captain, Engineers, United States Army. He commanded Company B, 551st Engineer Service Battalion, at Camp A. A. Humphreys, Virginia. His orders for duty overseas were cancelled at the time of the Armistice and he was honorably discharged from the Army, on January 10, 1919.

He was an indefatigable worker, yet he found time to contribute articles to engineering magazines, photography, painting, and collecting rare tropical plants. Mrs. Howell has presented his plant collection to the New York

Botanical Gardens, where it is exhibited as a memorial to him. Conspicuous among his contributions to engineering literature are the following: In *Engineering News:* "Straight or Tapered Piles", February 25, 1912; and in *Engineering News-Record:* "A New Concrete Pile", March 2, 1916; "Validity of Pile Formula", April 27, 1916; and "Bottom Driven Concrete Piles on Government Job", December 28, 1916. An article entitled "Concrete Pile Standards" was published in *Concrete,* in July, 1917.

At the time of his death Mr. Howell was engaged in writing the memoirs of his days as a page in the House of Representatives, and a novel, the locale of which was the Great Smokies of Tennessee. These works he left unfinished.

Taking an active part in civic and community affairs, he gave freely of his time and engineering knowledge toward the betterment of his community. He was held in high esteem by his fellow townsmen for he was a man of sterling character and quiet habits, and his judgment was solicited on all important movements.

On January 1, 1896, at Washington, D. C., he was married to Nannie G. Cook, daughter of General George W. Cook, veteran of the Civil and Indian Wars. He is survived by Mrs. Howell, two daughters, Mrs. Frederick R. Hawley and Mrs. Richard Carroll; three grandchildren, Nancy Carroll, Richard Carroll, Jr., and Virginia Hawley; and a brother, Frank Scott Howell.

He was licensed as a Professional Engineer of the State of New York on January 27, 1918.

Mr. Howell was elected an Associate Member of the American Society of Civil Engineers on November 27, 1917.

JACOB DUNCAN JAQUES, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 10, 1936

Jacob Duncan Jaques was born in Woodbury, N. J., on November 15, 1878, the son of Edwin Thomas and Charlotte Louise (Duncan) Jaques. From his parents he received an heritage of family tradition reaching back to early Colonial times. His first formal instruction was obtained in two schools rich in historic associations, the Friends Select School and the William Penn Charter School, in Philadelphia, Pa. To attend these schools he journeyed back and forth daily from Woodbury over the trains and ferries then in use, not foreseeing that he would have a part in providing the faster transportation of the present. At the age of 19 he entered the University of Pennsylvania to study Civil Engineering and was graduated in 1901 with the degree of Bachelor of Science in Civil Engineering.

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[1] Memoir prepared by a Committee of the Philadelphia Section, consisting of C. E. Myers and Charles H. Stevens, Members, Am. Soc. C. E., and Charles A. Howland, Assoc. M. Am. Soc. C. E.

Immediately after his graduation, Mr. Jaques secured a position as Transitman on the construction of a sewerage system for Woodbury, and, in September, 1901, was given charge of the work. He left Woodbury in January, 1902, to work as Transitman for Ford, Bacon, and Davis, on the construction of a street railway between Ardmore and Llanerch, Pa., but returned, in March, to Woodbury and resumed charge of the sewerage work as Resident Engineer. Leaving Woodbury in October, 1902, he was employed for six and one-half years by the Philadelphia Rapid Transit Company on the construction of the Market Street Subway and Elevated Railway. First, as Transitman in charge of a corps on general field work, he made the surveys, including through levels and base-line measurement for the West Market Street Elevated Line. He was then placed in charge of the location and construction of foundations in the more than three miles of this work and, in December, 1905, he was promoted to the position of Assistant Engineer and given charge of the construction of about a mile of the East Market Street Subway from Eighth Street to the Delaware River.

After the completion of the rapid transit work in March, 1909, Mr. Jaques was engaged by the United States Government as Inspector in charge of building the superstructure of the Cross Ledge Light Station, in Delaware Bay. In March, 1910, he assumed the duties of General Superintendent in charge of construction for Hungerford and Terry, a Philadelphia firm specializing in water-filtration construction. An opportunity came, however, to take charge of construction work in Western Pennsylvania for the Atlantic Refining Company and, in 1911 and 1912, he was engaged upon this work in Braddock and Ridgway, Pa. Returning in 1913 to Eastern Pennsylvania, he entered upon four years of important work for the New Jersey Zinc Company at the Palmerton, Pa., plant. As Assistant Chief of Construction, he was in charge of the Company's forces varying from 100 to 500 men, on plant improvements and additions costing $4 000 000. The principal structures erected included an oxide trestle, 1 200 ft long, of steel superstructure on concrete piers; a lithopone plant comprising frame and steel buildings on concrete foundations; a $1 000 000 addition, 900 ft long, to the existing oxide plant; and an acid plant also costing $1 000 000. This latter was a complete unit for the manufacture of fuming sulfuric acid and included buildings of frame, brick, and steel, overhead piping, machinery installation, sewers, water mains, and railroad trackage. One of the most difficult undertakings was the reconstruction of a blast furnace which necessitated foundation reconstruction of the furnace while in fire, rapid demolition of the old furnace, and erection of the new.

When the United States entered the World War, Mr. Jaques enlisted in the Civil Engineer Corps of the Naval Reserve Force, was commissioned Lieutenant (Senior Grade), and served at the Boston (Mass.) Navy Yard as Assistant to the District Public Works Officer. Placed in charge of construction contracts in the First Naval District, he saw erected under his supervision the Naval Radio School on Cambridge Common; the Naval Training School, on Bumkin Island, in Boston Harbor; additions to the Chelsea

Naval Hospital, at Chelsea, Mass.; additions to the Naval Air Station, at Chatham, Mass.; and a coal-handling plant in South Boston. When, in 1919, he received his discharge from the Federal Service, he returned to Philadelphia and entered the employ of the Department of City Transit where he was assigned to studies of the foundations of existing buildings to determine the underpinning which would be required for the proposed South Broad Street Subway. In 1920, however, he left the City service to take charge of the erection for the Empire Zinc Company of a zinc oxide plant, at Canon City, Colo. This work, costing $1 000 000, included, in addition to the oxide furnace building, a bag house, brick ore kilns, cooperage and fan buildings, and offices.

Returning in 1921 to Eastern Pennsylvania, Mr. Jaques was employed as Engineer by M. and T. E. Farrell, Contractors, on the construction of 5 000 ft of the reinforced concrete Lincoln Highway through Downingtown. Upon the completion of this work, he entered private practice in the contracting field and in the years 1922–1924, contracted for and built sewage disposal plants at West Collingswood, N. J., and Woodlynne, N. J.; a sea wall at National Park, N. J.; and a main pipe tunnel of concrete for the Camden County Tuberculosis Hospital, at Asyla, N. J. In 1925, he returned to the Department of City Transit, in Philadelphia, as Assistant Construction Engineer and was engaged for four years principally upon special investigations and reports for the Chief Engineer. These reports covered a variety of subjects, including paint best suited for concrete surfaces in subway stations, methods for stopping leaks in completed subway under back-fill, type of face brick most suitable for the terminal shop buildings, cost of ventilation wells, payment for installation of new gas mains, study to recommend a plan for handling traffic during subway construction, and an analysis of contractors' claims.

In 1930, he obtained a leave of absence from the Philadelphia Department of City Transit to enter the employ of the Shell Eastern Petroleum Products, Incorporated, upon the construction of bulk terminals. As District Engineer for the Company he had charge of building the terminal at 37th and Reed Streets, Philadelphia, and one in Trenton, N. J. The former construction involved hydrographic surveys followed by dredging in the Schuylkill River, erection of timber bulkhead, grading of plant area, and the installation of storage tanks with piping, pumps, and fire-walls. At Trenton, the work covered principally the installation of tanks with their auxiliary equipment.

In October, 1930, Mr. Jaques returned to the position of Assistant Construction Engineer in the Philadelphia Department of City Transit and was placed in immediate charge of subway and underground concourse construction beneath and east of the City Hall. Because of the tremendous loads to be supported under the continually occupied City Hall and the poor quality of the existing building foundations, the work presented great difficulties. It was necessary to proceed with the utmost care, and constant study and close supervision were required. From this work he went to take charge of constructing 7 200 ft of two-track subway in South Broad Street from Kater Street to Snyder Avenue. This construction included, in addition to the subway trench proper, three stations and their entrances, underpinning of buildings, relocation and restoration of underground structures, and paving of sidewalks and street surface. As the subway structure was situated in the east half of Broad Street and had a sub-grade depth of 30 to 35 ft below street level, it was necessary to underpin nearly all buildings along the eastern house line. Construction difficulties were increased by the large quantities of water encountered along the entire line and by stretches of bad ground which required excavation below plan sub-grade and the use of steel sheet-piling.

In 1933, the construction of a high-speed line was begun across the Delaware River between Philadelphia and Camden, N. J. Mr. Jaques was employed by the Delaware River Joint Commission first on the supervision of work under a $1 000 000 contract for a two-track subway to connect the existing subway in North Eighth Street, Philadelphia, with the west end of the bridge line. Later, he was transferred to the Design Division where he was engaged upon special problems in connection with the relocation of structures owned by public utilities and, in co-operation with the Commissioners of Fairmount Park, upon the restoration of historic Franklin Square.

To undertake the last work upon which he was to be engaged, he returned to the scene of an earlier activity. For the New Jersey Zinc Company, he took charge of engineering and working forces on the rapid construction of a large plant addition at Palmerton, Pa. The underpinning of foundations carrying heavy dead and live loads was an important part of an operation which included building foundations, erection of steel, and the installation of mills, conveyors, and other equipment. The work was carried on day and night in order to meet time requirements for completion. From Palmerton he returned to his home in Philadelphia and was with his family when stricken by the attack of acute appendicitis which, in a few days, caused his death.

Few engineers are privileged to achieve as largely as Mr. Jaques in their chosen field and, at the same time, to make and keep the friendship of so many with whom they come in contact through the years. He was highly regarded for his ability in the field of engineering, particularly in construction, and, to this, the works which his energy and resourcefulness brought to successful completion, testify. More than this, however, his genial personality, enduring friendliness, and keen interest endeared him to many. His circle of friends was broadened by a fondness which he had from early boyhood for trap-shooting. He grew to be an outstanding expert in the sport, and this avocation brought him into organizations of kindred spirits where he did much to develop the art.

He was interested in activities for the benefit of the profession and joined with other engineers in organizations for this purpose but, like others whose work takes them frequently to new locations, he could not always retain these memberships. He was a Registered Professional Engineer in Pennsylvania for many years, a member of the Sons of the American Revolution, and also a member of several gun clubs.

On February 8, 1921, he was married, in Oakmont, Pa., to Grace Vernon Hunter who, with his daughter, Louise, survives him.

Mr. Jaques was elected an Associate Member of the American Society of Civil Engineers on November 1, 1910.

WILLIAM EDWARD JOHNSON, Assoc. M. Am. Soc. C. E.[1]

Died June 30, 1937

William Edward Johnson was born at Glastonbury, Conn., on March 10, 1867, the son of Giles M. and Emily (Allen) Johnson. His education was received at Williston Academy, Easthampton, Mass., the Connecticut Literary Institute, Suffield, Conn., and Lehigh University, Bethlehem, Pa.

Shortly after leaving college (1890), Mr. Johnson entered the employ of the Board of Water Commissioners of the City of Hartford, Conn., as Assistant Engineer in charge of reservoirs. In 1895, he was appointed Engineer in charge of the Reservoir Division. This position he held until May 18, 1936, when due to illness, he was retired on pension. During his long period of service, he saw the growth of the Hartford Water Supply System (under the Board of Water Commissioners which, later, was succeeded by the Water Bureau of The Metropolitan District Commission of Hartford County), from several small reservoirs in the immediate vicinity of Hartford, to the present system embodying the Nepaug Reservoir, 15 miles from Hartford, completed in 1918, and the Barkhamsted Reservoir, 25 miles from Hartford, now (1938) nearing completion.

Mr. Johnson was in active charge of construction and surveys for additional water supply in the progressive improvement of the West Hartford Reservoir System. He gave to his work a sincere and absorbing interest which produced that quality of supervision and reliability essential to the proper administration of public service, and like many another, the value of his services was not fully realized until after his passing.

In 1892, Mr. Johnson was married to Florence May Alford. He is survived by three sons, Hubert Edward Johnson, Alfred William Johnson, and Giles Meigs Johnson, and one daughter, F. Ruth Johnson.

He was a member of Wyllys Lodge (Masons), Washington Commandery, Knights Templar, Jeremiah Wadsworth Chapter, Sons of the American Revolution, Connecticut Society of Civil Engineers, the Connecticut Historical Society, and during the World War he served in the Connecticut Home Guards.

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[1] Memoir prepared by Caleb Mills Saville, M. Am. Soc. C. E.

Mr. Johnson was elected an Associate Member of the American Society of Civil Engineers on April 3, 1901.

CHARLES WILLIAM HENRY NESSLER, Assoc. M. Am. Soc. C. E.[1]

DIED OCTOBER 7, 1937

Charles William Henry Nessler was born in Mountain View, Calif., on December 18, 1890, the son of Charles W. H. and Catherine (Hannon) Nessler.

His formal education ended with the completion of Grammar School in Mountain View and near-by San Francisco. Although Mr. Nessler was denied the opportunity to gain a college education, he acquired a thorough knowledge of topographic engineering by practical experience in the field and constant study at home.

He began his engineering experience as a Rodman and Recorder with a topographic field party of the United States Geological Survey during the summer of 1912 in the San Joaquin Valley of California. The outdoor life so appealed to him that he decided to make every effort to continue with the organization. However, he early recognized that without further education and study his progress would be limited. Lack of funds prevented formal attendance at college, and an intensive course of home study seemed the only solution. Soon, thereafter, he enrolled with the International Correspondence Schools, at Scranton, Pa., in a course of Surveying and Mapping.

During the next few years Mr. Nessler continued with the U. S. Geological Survey in the capacity of Rodman and Recorder, working with various field parties engaged in topographic mapping in the State of California. The winter of 1916–1917 was spent with the Interstate Commerce Commission, as Chainman and Rodman in connection with railroad valuation work.

With the completion of the course in Surveying and Mapping and the experience gained over a period of four or five years, he was prepared to enter the classified Civil Service. On May 14, 1917, Mr. Nessler was appointed a Junior Topographer in the U. S. Geological Survey, and with the exception of his World War service, he continued with this organization until his death.

His first assignment was to make a topographic survey of a part of the Santa Lucia Range on the coast of California. This was followed by geodetic and topographic surveys in various parts of the Pacific Coast States, including Arizona and Idaho. His enthusiasm and energy, combined with a natural aptitude for delineating topographic features, were quickly recognized and as a result he was given some of the more difficult and important assignments.

His work with the U. S. Geological Survey was temporarily interrupted by the World War. He enlisted on March 11, 1918, and served as Private

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[1] Memoir prepared by Conrad A. Ecklund, M. Am. Soc. C. E.

First Class with Company A, 29th Engineers. One year of his enlistment was spent overseas in France and while there he contracted rheumatic fever which left his heart in a weakened condition. After the Armistice he accompanied the Army of Occupation into Germany, returning to the United States in July, 1919.

On July 18, 1919, Mr. Nessler received his honorable discharge from the Army and two days later was reinstated in the U. S. Geological Survey as an Assistant Topographic Engineer. On July 11, 1924, he was promoted to the position of Associate Topographic Engineer, which title he held at the time of his death. Following his reinstatement with the Survey he again felt the need of further study and accordingly enrolled in special courses with the La Salle Extension University, of Chicago, Ill., and with the University of California, at Berkeley, Calif.

In 1922, when the U. S. Geological Survey entered into a co-operative agreement with the County of Los Angeles to make a topographic map of that County, Mr. Nessler was one of the first engineers to be selected for this very important work; he spent the next three years mapping a considerable part of the southern and more densely populated area. The original plan provided for mapping only the south one-third of the County. This was completed in the fall of 1925. Mr. Nessler was then assigned to river surveys in the State of Oregon, and for the next two years he was engaged in making river and dam-site surveys in the States of Oregon and Arizona, and in the office preparation of the data obtained in the field. In 1928, he had topographic mapping assignments on the Mt. Jefferson Quadrangle, in Oregon, and the Turret Peak Quadrangle, in Arizona.

When work was resumed in 1929 on the Los Angeles County Project, Mr. Nessler was again assigned to this work, remaining until the topographic map of Los Angeles County was completed in the spring of 1937. Then followed his last field assignment, the mapping of a mountainous area in Arizona. The recurrence of the ailment contracted during the World War brought an abrupt termination to his field career. Throughout his entire service he possessed not only marked topographic ability, but also those other qualities, both physical and personal, that are so necessary to the successful planning and execution of field assignments under unusual and often hazardous conditions.

Mr. Nessler was a member of American Society of Photogrammetry, and a member of the Sacramento Section of the Society. He also took an active interest as a member of the American Legion Post No. 127, of Glendale, Calif.

Untiring in work, strong in character, keen in intellect, and possessed of a high sense of personal responsibility for public service, Mr. Nessler leaves his organization and friends greatly indebted to his activities.

He was married on September 19, 1920, to Isabel Barcroft, of Madera, Calif., and is survived by his widow, a daughter, Evelyn M., a son, Harry J., and a brother, S. J. Nessler.

Mr. Nessler was elected an Associate Member of the American Society of Civil Engineers on June 15, 1936.

SVEN ALBERT NORLING, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 4, 1937

Sven Albert Norling was born in Söderhamm, on the Baltic Coast, in the northern part of Sweden, on March 21, 1894. A true Viking, he was descended on his father's side from a line of sea captains and on that of his mother from a family of artists, of whom his grandfather was a painter of note. He was one of a family of five children.

He attended the public schools and a private High School in Söderhamm. He obtained his first college degree at Ultuna Agricultural and Engineering College, at Upsala, Sweden, in 1917, after which he spent a year studying at Halle an der Saale, an old university town in Germany. Returning to Upsala on a scholarship in the fall of 1918, Mr. Norling received his Master's Degree the following year, majoring in land reclamation engineering. In 1919, he was a Teacher at Molkom High and Agricultural School, in the Province of Värmland. In May, 1920, he accepted a position with the Swedish Royal Water Commission as a Civil Engineer in Charge of Topographic Surveys in connection with regulation works on Lake Vänern and the Trollhättan Hydro-Electric developments.

Mr. Norling was one of ten successful applicants for as many Swedish-American scholarships out of a total of 200 and sailed for the United States in the fall of 1920 for a year's study at the University of Wisconsin, Madison, Wis. While there, he pursued special studies in hydraulic and drainage engineering and after the conclusion of his year's work received an appointment as Research Assistant at the University of Minnesota on investigations on the behavior of ground-water. In the fall of 1923, having finished the curriculum required by the University of Wisconsin, he received the degree of Doctor of Philosophy from that institution. His thesis dealt with the effects of the elevation of the ground-water table on yields of various kinds of crops, a field of research in which very little work had been done in the United States prior to that time.

In 1924, Mr. Norling established a consulting office in Minneapolis, Minn., and until the time of his death had acted in a consulting capacity in many parts of the United States. He was one of the Engineering Consultants on the Great Lakes Water Level Case, in which a number of the Lake States were involved. He was also a member of the Minneapolis Water Supply Commission appointed in 1932 to study ways and means of improving the water pressure in the southern part of the city and removing objectionable tastes and odors from the city supply.

From 1934 to the time of his death, Mr. Norling served, successively, as Consulting Hydraulic Engineer for the Civil Works Administration, State Emergency Relief Administration, and Works Progress Administration on a number of water conservation projects undertaken by the State of Minnesota in co-operation with these agencies.

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[1] Memoir prepared by a Committee of the Northwestern Section, consisting of E. V Willard and Adolph F. Meyer, Members, Am. Soc. C. E.

While engineering was his chosen field of work, he engaged extensively in farming. The possibilities of peat lands to produce intensively of vegetables and truck-farm products appealed to him, with the result that he always interested himself in peat-land farming. At the time of his death, he was operating a 500-acre truck farm near Forest Lake, Minn., where he employed from forty to fifty men during the summer seasons.

In 1921, Mr. Norling was married to Borgny Kirsten Myhr, herself a graduate of the University of Oslo, at Oslo, Norway, whom he had met before coming to the United States. Besides Mrs. Norling, he is survived in this country by two daughters, Karin and Celia, and one son, Gunnar.

He was a Registered Professional Engineer in Minnesota, a Scottish Rite Mason, and a member of "Svenska Sällskapet," a club made up mostly of Swedish professional men residing in Minneapolis and St. Paul.

Mr. Norling was elected an Associate Member of the American Society of Civil Engineers on June 6, 1927.

BEALE MELANCTHON SCHMUCKER, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 23, 1937

Beale Melancthon Schmucker, whose career closely paralleled the development of automotive highway transportation in the United States, contributed greatly to that development in Southern New Jersey.

Beale Melancthon Schmucker was born on September 22, 1888, the son of Dr. and Mrs. Samuel C. Schmucker. His father is a beloved Professor Emeritus at State Teachers' College, at West Chester, Pa., a well known lecturer on scientific subjects, and an author of note. His mother is also well known in her field, having illustrated a number of her husband's books. With such a background, it is not difficult to understand why Mr. Schmucker considered his position one of public trust, and discharged his duties in accordance with the best engineering principles. He was a graduate of the University of Pennsylvania, at Philadelphia, Pa., in the Class of 1911, with a degree of Bachelor of Science in Civil Engineering.

For several years Mr. Schmucker was County Engineer for the County of Camden, in New Jersey. He came to Camden County, in 1912, after one year in the employ of the City Engineer of Ocean City, N. J. In 1915, he became the Assistant County Engineer and in 1929, the County Engineer.

During the twenty-six years of his service to Camden County, automobile registration in the United States increased from 900 000 to 24 000 000, or more than twenty-six times. New Jersey had more than its proportionate share of this growth. Situated between Philadelphia and Atlantic City, Camden County early received its quota of very heavy traffic. To solve this problem the then County Engineer, the late John J. Albertson, M. Am. Soc. C. E., and his Assistant, Mr. Schmucker, early adopted a durable and safe hard surface concrete, for their new and re-constructed highways. Of this material they constructed the Camden County Section of the famed White Horse Pike, and, later, Mr. Schmucker, as County Engineer, built the Camden County Section of the Black Horse Pike. These highways are known throughout the East and at the time of their construction embodied the most advanced thought in highway design.

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[1] Memoir prepared by S. Herbert Taylor, Esq., Camden, N. J.

Mr. Schmucker was one of the pioneers in the adoption of the lane idea in highway design. On the Camden County Section of the Black Horse Pike, constructed in 1929, the lanes, in addition to being built into the highway proper, are built into the intersections. This was the first such use of this method in New Jersey.

During the period of the World War, Mr. Schmucker relinquished his County duties to serve with the 104th Engineers, 29th Division. He was swiftly promoted from Private to Captain, being commissioned First Lieutenant in command of his Company in August, 1917, and Captain in March, 1918. As Captain, he took part in both the Argonne and Meuse Offenses.

Besides serving Camden County and his country, Mr. Schmucker served well his fellow engineers. Long an active member of the Society, he also took an interest in the affairs of the Licensed Engineers' Association. He served as President of the South Jersey Chapter and was serving as President of the parent body, the New Jersey Association of Professional Engineers and Land Surveyors, at the time of his death. Many of his engineering associates will long remember his courage, sacrifice, and unfailing devotion to this trust during his long illness. Many a time his wife, Jean Roser Schmucker, drove him 80 or 100 miles to a meeting, and then put him to bed in a hotel room for a few hours, in order that he might gain strength enough to conduct the meeting.

He served as President of the New Jersey Association of County Engineers for one term. He was a member of the Camden Club, the Camden Lodge of the Benevolent and Protective Order of Elks, and the Tavistock Country Club.

Like his distinguished father, he was a lover of the out-of-doors, and all his hobbies—hunting, fishing, and golf—were connected with it. His personality was forcible and direct, happily relieved by a jovial sense of humor.

Mr. Schmucker was elected an Associate Member of the American Society of Civil Engineers on October 9, 1917. He was an active member of the Philadelphia Section, serving on various Committees, and as a Director from 1929 to 1931.

WILLIAM LEE SELMER, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 11, 1937

William Lee Selmer was born in Boston, Mass., on August 29, 1885, the son of William and Hulda (Lee) Selmer. His parents moved to Brooklyn,

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[1] Memoir prepared by Charles J. Slipper, Asst. Engr., Transit Comm., New York, N. Y.

N. Y., during his early years and he lived in New York, N. Y., and its environs, for the remainder of his life. He received his education in its public schools and at Cooper Union, from which he obtained the degree of Bachelor of Science in 1907, and that of Civil Engineer in 1911.

Probably due to his father's influence, early in life he became interested in topographical surveying and other engineering work and began his career with a city surveyor on title insurance surveys before he was seventeen.

While completing his education, Mr. Selmer earned his living and obtained experience in various engineering positions. After a competitive examination, he entered the service of New York City in 1903 as a Rodman with the Board of Rapid Transit Railroad Commissioners, in which position he was engaged in the construction of the city's first subway.

After a year and a half of this subway work, he spent two and one-half years in the Topographical Bureau of the Borough of Richmond. He was engaged on the mapping of the city plan for that Borough, which work was then at its height. He returned to the Rapid Transit Board just before it was replaced by the Public Service Commission for the First District. This Commission was created by New York State's first Public Utility Regulation Act, and its members assumed office on July 1, 1907. Mr. Selmer spent the remainder of his life in the employ of the State of New York with this Commission and its successors.

Soon after taking office, the members of the Commission directed their attention, under their regulatory powers over steam railroads, to conditions at highway-railroad grade crossings on the trunk railroads in New York City. With a view to separating their grades or improving protection at the crossings, a staff was organized to make engineering analyses of the various situations and problems at the 400 or more such crossings in the city and to report to the Commission, at first under the direction of Daniel L. Turner, M. Am. Soc. C. E. From 1911 to the time of his death, on July 11, 1937, such matters were largely under Mr. Selmer's immediate charge. Although his engineering activities were concerned principally with grade-crossing elimination in New York City, they also covered regulatory work for the Commission in rapid transit operation, maintenance, and inspection of track and structures to insure safety in operation and of passengers. During his time, grade crossings in New York City were reduced in number by more than one-half. More than $35 000 000 was expended, and an elimination program adopted covering the remaining crossings over which high-speed railroad trains operate, which will bring the total cost to more than $50 000 000.

Mr. Selmer's chief interests centered in his family and in his engineering work—to them he was ever faithful, above all else. His predominating characteristics—modesty and sincerity—were immediately recognized by those who had contact with him.

Mr. Turner, in a letter of condolence to the family, said of him:

> "Always he could be depended upon. He had my confidence from the beginning. It was a great pleasure to have him as an associate. His record with the Commission has been outstanding, and a fine example of a most praiseworthy and unselfish service. It will not be forgotten soon."

Robert Ridgway, Past-President, and Hon. M. Am. Soc. C. E., in a similar communication, said:

"Having been associated so long, directly and indirectly, with Mr. Selmer, I feel keenly his loss. A faithful public servant, I wish his work was better known to the public at large. We who knew how ably and conscientiously he worked and how he gave his best to his duties, will miss him."

On his death the Transit Commission caused to be spread upon its Minutes the following resolution which admirably epitomizes the man, his career, and the feelings of those who knew him:

"It is with painful feelings that the Commission notes on its records the death of William L. Selmer, which, on July 11, 1937, brought to an untimely close his faithful and fruitful public service of thirty years, during which he fulfilled to the utmost every duty laid upon him, and they were many. As engineer in charge of grade crossing eliminations he brought to the varied and difficult problems involved in securing public safety an ability, professional skill, resourcefulness, tact, and unlimited devotion to duty which left nothing to be desired. All over the great City of New York various structures of earth, steel and concrete stand today as enduring monuments to his useful and efficient life.

"To his host of friends the poignancy of grief is sharpened by remembrance of those lovable human qualities which he possessed to an unusual degree and which made him in every sense a complete man liked and respected of men.

"To his surviving wife and family, the Commission extends its deepest sympathy in their great sorrow for the loss of a faithful and loving husband and father who leaves after him as a source of solemn pride the fruits of work well done, an unblemished record and a spotless character."

On April 28, 1914, he was married to Emily Josephine Whaley, who, with two daughters, Olive and Joan, are left to mourn his passing.

Mr. Selmer was elected a Junior of the American Society of Civil Engineers on March 31, 1908, and an Associate Member on March 2, 1915. He was also a member of the Municipal Engineers of the City of New York.

PHILIP SCOTT TYRE, Assoc. M. Am. Soc. C. E.[1]

DIED AUGUST 25, 1937

Philip Scott Tyre was born in Wilmington, Del., on July 14, 1881. He was the oldest of three children of James and Eliza (Bow) Tyre, whose families were early settlers in the State of Delaware.

At the age of six, young Philip entered the Wilmington Public Schools, and, in 1899, he was graduated with honors from the Wilmington High School. During this time he helped meet the family budget by working Saturdays and selling newspapers on the streets of Wilmington. During his High School days he took a special course in draftsmanship, and upon graduation he entered the employ of the Edge Moor Bridge Works, as a Draftsman.

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[1] Memoir prepared by Reynold H. Greenberg, Esq., Philadelphia, Pa.

In 1900, Mr. Tyre associated himself with the New York Shipbuilding Company as an Engineer. In 1903, in order that he might study art, he accepted a position with the Patterson Contracting Company, of Denver, Colo., and attended night classes in the Denver School of Art. Mr. Tyre returned to the New York Shipbuilding Company in 1904. In 1907 he was made Chief Engineer of the Belmont Iron Works, of Philadelphia, Pa. Once again, Mr. Tyre furthered his art studies by attending the Pennsylvania Academy of The Fine Arts in Philadelphia, and after his graduation he was awarded a Fellowship in the Academy.

In 1908, he became associated with George F. Pawling and Company, acting, successively, as Structural Draftsman, Chief Draftsman, and, later, as Chief Engineer of the Company. In this position he had responsible charge of designing office, mill, mercantile, and factory buildings, as well as tank towers, bridges, etc.

In 1914, he left the Pawling Company to enter private practice as an Architect and Engineer, but when the United States entered the World War, because of his experience, particularly in marine engineering, Mr. Tyre was called to the Emergency Fleet Corporation. In the spring of 1918, he was appointed Progress Engineer in charge of all Districts of the Corporation; and, later, he became Assistant Head of the Inspection and Production Department, Division of Ship Construction. In 1919, he was made Assistant to the Vice-President, Mr. J. L. Ackerson, in complete charge of ship construction. During his engagement with the Emergency Fleet Corporation he organized, systematized, and invented master records of ship construction for the entire Emergency Fleet Corporation. He inaugurated and planned the Fourth of July Launching Program of the Emergency Fleet Corporation in 1918, under which one hundred ships were launched in one day, a feat never before attempted. In 1920, Mr. Tyre was honorably discharged from the Emergency Fleet Corporation and resumed his private practice as an Architect and Engineer. He maintained his offices in the Architects Building, in Philadelphia, until his death.

The story of Philip Scott Tyre's life should really be written in two chapters, because when, in February, 1929, he lost the use of his right arm through paralysis, with an indomitable will seldom seen in any man, he refused to give up his activities in engineering and architecture; and, further, he refused to give up the hobby that he loved so dearly—painting. He learned to use his left hand, and one of the greatest tributes ever paid to any man was when the Art Club of Philadelphia announced a One-Man Exhibit in 1935 of the works of Philip Scott Tyre. All the paintings on exhibition had been completed after his affliction.

Mr. Tyre was particularly active in the design and construction of automobile buildings, and to-day, in Philadelphia, there are scores of such buildings that remain as monuments to his originality and architectural genius. Among the many structures he designed in Philadelphia were buildings for the Packard Motor Company, Cadillac Motor Company, Nash Motor Company, Hupmobile Company, Graham-Paige, and many others. Upon the completion of the Nash Building, in Philadelphia, Mr. C. W. Nash, President of the Nash Motor Company, publicly stated that in all his automobile experience, he had never visited any automobile sales and service building that was quite as complete in detail as the Nash Philadelphia Building. Mr. Tyre was an Honorary Member of the Philadelphia Automobile Trade Association, and all the National Automobile Shows held in Philadelphia, were designed and operated under his direction.

As an Architect and Engineer, Philip Scott Tyre never followed, but always led. He was the first man in Philadelphia to construct a large building utilizing concrete for all exterior facades, and he received nation-wide commendation for his handling of the Lasher Printing Company Building, in which the front facade was treated architecturally in poured concrete.

In 1907, he was married to Mabel C. Marcy, of Philadelphia, who survives him, as do his three daughters—Jean Campbell (Mrs. Gerry Mason), who was the Assistant Dean of Skidmore College; Marjorie Cole (Mrs. Robert McGinnis), harpist with the Philadelphia Symphony Orchestra; and Sarah Marcy, a teacher at the Episcopal Academy, in Philadelphia.

At his death Mr. Tyre was a member of the Philadelphia Section of the Society; a member of the American Institute of Architects, Philadelphia Chapter; Associate Member of the American Society of Naval Architects and Marine Engineers; Art Club of Philadelphia; a Fellow of the Pennsylvania Academy of The Fine Arts; member of the Engineers Club of Philadelphia; American Federation of Art; Pennsylvania Museum; and School of Industrial Art. He was also a member of the Union League and North Hills Country Club, in which he was a Director and Chairman of the Building Committee.

In recent years, Mr. Tyre enjoyed nothing more than to go to his country home in Erwinna, Bucks County, Pa., where he did most of his painting and reading. He was particularly proud of his fine library of engineering and architectural editions. He was also a prolific worker in oil; and it was while sketching at Erwinna that he became ill. On August 25, 1937, the Philadelphia daily papers, as well as the Associated Press, carried the news of his death to his friends and associates all over the United States. So ended a beautiful life that, while not full when measured by years, was truly full when measured by achievement.

Mr. Tyre was elected an Associate Member of the American Society of Civil Engineers on July 9, 1912.

HENRY RADCLYFFE ST. ARVANS WALTERS, Assoc. M. Am. Soc. C. E.[1]

Died February 23, 1937

Henry Radclyffe St. Arvans Walters was born at Ashby, Suffolk County, England, on October 24, 1875, the son of the Rev. Edmond Walters and Anne

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[1] Memoir prepared by W. A. Hazard, Assoc. M. Am. Soc. C. E.

Elizabeth (Browne) Walters. He received his early education at King's School, Warwick, England.

In 1892, when Mr. Walters was seventeen years of age, his father brought the family to the United States and located in California. In March, 1896, Mr. Walters entered the employ of the Llewellyn Iron Works at Los Angeles, Calif., as Special Apprentice, where he remained for two years, or until March, 1898, when he entered the shops of the Santa Fé, Prescott and Phoenix Railway Company, at Prescott, Ariz., remaining there until August, 1899.

In the fall of 1899, Mr. Walters came East and entered Lehigh University, at Bethlehem, Pa., from which he was graduated in 1903 with the degree of Civil Engineer. During his college years, he was employed in the summer vacation periods, as follows: In 1900 in the Ordnance Shops of the Bethlehem Steel Company, at Bethlehem; in 1901 he was employed in the Easton, Pa., Shops of the Lehigh Valley Railroad Company; and, in 1902, he was in charge of the assembly and erection of structural steelwork for the Guerber Engineering Company, of Bethlehem, which work included the erection of a 360-ft cantilever single-track railroad span at Beavers Run, Pa.

After his graduation from college, Mr. Walters again became associated with the Guerber Engineering Company in successive capacities of Chief Draftsman, Chief Engineer, and Superintendent of Manufacture. He remained with this firm until 1908, when he entered the employ of the Bethlehem Steel Company, as Superintendent of the Fabricating Shops, Saucon Division, of the Bethlehem Plant. In this capacity, he served for about two years, and, in 1910, was placed in executive charge of the Fabrication Department as Engineer of Fabrication, which position he occupied until 1916.

For the succeeding years, covering the period of the World War, Mr. Walters' duties were of a very special and often of a very confidential nature. He was given charge of construction, except plant construction, which included principally the development of housing projects, and was thus engaged until 1922. At the time of the consolidation of the Cambria Steel Company and the Lackawanna Steel Company with the Bethlehem Steel Company, Mr. Walters was assigned the duty of co-ordinating the Operating Departments of the three Companies, and, finally, re-organizing the Fabricating and Structural Sales Departments into the Fabrication and Erection Department, of which he became Manager, and in which capacity he continued until 1931.

On January 1, 1931, the McClintic-Marshall Company, of Pittsburgh, Pa., was consolidated with the Bethlehem Steel Company, and Mr. Walters became Assistant to the President of the McClintic-Marshall Corporation, which position he occupied until 1936, when this Corporation was dissolved. From that time until his death, he was a member of the General Sales Department of the Bethlehem Steel Company in a consulting capacity, and with special assignments in connection with the development of sales activities.

Through his long association with the Bethlehem Steel Company and his intimate connection with its development, Mr. Walters' services were of inestimable value, and his advice and opinions were constantly being sought in the development and enlargement of the Company's facilities. He enjoyed a wide circle of friends and acquaintances, and was recognized as a man of sterling qualities, fair in all his dealings with his fellow men, an able executive, and a true friend to all his associates.

Mr. Walters is survived by his widow, Dorothy Griffin Walters, four sisters and four brothers.

He was a member of Minsi Lodge No. 733 F. and A. M., and Ezra Chapter No. 201, Royal Arch Masons, Red Cross of Constantine, Mary Conclave No. 5. He was also a member of Trinity Episcopal Church, of Bethlehem, Pa.

He was a past member of the Board of Governors of the Bethlehem Chamber of Commerce, a member of the Bethlehem Club, Northampton Country Club, and Lehigh Valley Engineers' Club.

Mr. Walters was elected a Junior of the American Society of Civil Engineers on October 6, 1903, and an Associate Member on April 5, 1905.

WARREN WITHEE, Assoc. M. Am. Soc. C. E.[1]

Died February 3, 1938

Warren Withee was born at Minneapolis, Minn., on April 4, 1893, the son of H. W. Withee and Harriett (Brewer) Withee. After finishing his studies in the local grade schools, he attended the University of Minnesota from which he received the degree of Bachelor of Science in Engineering in 1915.

After graduation from the University, Mr. Withee took the position of Assistant Engineer with George A. Ralph, Consulting Engineer, of St. Paul, Minn., acting as Chief of Party and Office Engineer on surveying and office computations for drainage ditches, etc. He held this position from June, 1915, until October, 1917, when he entered the United States Army and served, successively, as Private, Corporal, and Sergeant in the 29th and 74th Engineers, in France. His work consisted of locating enemy artillery by sound ranging, surveying base lines, and locating positions.

Upon his return to the United States in March, 1919, Mr. Withee became Chief of Party for the City of St. Paul on measuring and cross-sectioning streets for paving assessments. From October, 1919, to April, 1920, he acted as Chief of Party in preliminary surveys for drainage ditches for S. B. Gardner, Consulting Engineer, of Benson, Minn. During 1920, he served for a time as Valuation Engineer for the Chicago, Burlington, and Quincy Railroad Company.

From November 18, 1920, until his death, Mr. Withee was Principal Assistant to the District Engineer in the Water Resources Branch of the United States Geological Survey, at Chattanooga, Tenn., entering this service as Assistant Engineer and progressing to the grade of Hydraulic Engineer.

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[1] Memoir prepared by C. E. McCashin, M. Am. Soc. C. E.

Mr. Withee assisted in all the water resources investigations in co-operation with the U. S. Army Engineers, during the period when the Tennessee River survey was made and continued in this work with the advent of the Tennessee Valley Authority in the construction of projects for the development of the Tennessee Valley. He assisted also in compiling water resources data in the Cumberland Valley and in the Mississippi Valley, in Western Tennessee.

For several years Mr. Withee was a member of the Chattanooga Engineers' Club, the Tennessee Valley Section of the Society, and the American Geophysical Union.

On August 9, 1922, he was married to Nelle Dorsey, of Chattanooga, who survives him.

He had a charming personality which made his friends legion. He was an able hydraulic engineer and an excellent executive. He was a gentleman, a friend, a soldier, and an untiring worker whose place both personally and officially will be difficult to fill. His genial disposition, his kindly assistance and loyalty to all his friends and associates will always be a cherished memory to those who knew him.

Mr. Withee was elected an Associate Member of the American Society of Civil Engineers on September 9, 1935.

WALTER McILHANEY WOLFE, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 11, 1937

Walter McIlhaney Wolfe was born on December 4, 1887, at Chatham, N. J., the son of William James Wolfe, M.D., and Clara (McIlhaney) Wolfe. Dr. Wolfe was a distinguished man who, by choice, through the years had maintained an unique position as minister to the bodily ailments, as well as the spiritual and economic problems, of his neighbors and friends in his section of New Jersey, without thought for his own gain.

Walter McIlhaney Wolfe's ancestors were equally distinguished, having come to Pennsylvania from Alsace in 1751, taking a prominent place in the government and the upbuilding of that and of the neighboring sections—one, George Wolfe, having served as Governor of Pennsylvania from 1829 to 1835.

After preparation in local schools, Walter Wolfe was matriculated on September 5, 1904, at the Virginia Military Institute, at Lexington, Va. During each year of his 4-yr course at the Institute, he stood among the first ten in his class in scholarship. He was graduated on June 24, 1908, with the degree of Bachelor of Science in Civil Engineering ranking third among the candidates for this degree.

After graduation, he entered the employ of the North-Eastern Construction Company, of New York, N. Y., serving as Inspector of concrete pile foundations for the seventeen Isolation Wards of the United States Immigrant Station, at Ellis Island, New York Harbor, and as Office Engineer in connection with the construction of the buildings proper.

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[1] Memoir prepared by Mason Garber, M. Am. Soc. C. E.

In August, 1909, Mr. Wolfe was appointed Assistant Engineer for the Gore-Meenan Company, in connection with the construction of the Garrison Tunnel, of the Catskill Aqueduct, later becoming Engineer for C. M. Bolton and Company on the Millwood Tunnel and Open Cut, and the Putman Siphon of the Catskill Aqueduct, of the water supply for New York City.

After a term as Assistant Engineer and Superintendent, with the Litchfield Construction Company on heavy construction work, in November, 1912, he became associated with Frederick L. Cranford, Incorporated, on the construction of part of the Dual Subway System for the City of New York, including Church Street Subway, Vesey Street Tunnel, Beekman Street Tunnel, Park Place Station, and crossings under existing Broadway and Park Row subways. His work in collaboration with the late James C. Meem, M. Am. Soc. C. E., of the Cranford Company, served as a model for later subway construction methods.

In 1914, Mr. Wolfe was granted the degree of Civil Engineer by the Virginia Military Institute.

On May 8, 1917, he resigned from the Cranford Company and entered service with the 15th Company (Engineers) at Plattsburgh Barracks, N. Y., and on May 16 was commissioned Captain of Engineers, United States Reserve. Assigned to 6th U. S. Engineers, Captain Wolfe was appointed Battalion Adjutant at Camp Belvoir, Virginia, in July, 1917, and in September, he was made Regimental Adjutant, later serving as Post Adjutant, at Washington Barracks, Washington, D. C. On December 4, 1917, he sailed with his regiment on board the U. S. S. *George Washington*. With the 6th Engineers he participated in the Somme Defensive (5th Army, British Expeditionary Force); in the defense of Amiens, with the Australian Corps (4th Army, B. E. F.); and with the Third Division of the American Expeditionary Force, in the actions of the Champagne-Marne and the Aisne-Marne during July, 1918.

He was returned to the United States in September, 1918, with a cadre of the Third Division, A. E. F., to train a new Sapper Regiment. This same month he was promoted to be Major of Engineers, U. S. Army, with rank from July 30, 1918.

After assignments with the Training Section, Office of Chief of Engineers, in Washington, and as Assistant Professor of Military Science and Tactics (Engineers) at the Massachusetts Institute of Technology, in Cambridge, Mass., he was discharged, at his own request, at the Office of the Chief of Engineers, Washington, D. C., on August 4, 1919.

In 1923, Major Wolfe was commissioned Lieutenant Colonel, Engineers Reserve, and from 1931 to 1935, served as Colonel of the 376th Reserve Regiment of Engineers. From 1935, until his retirement in December, 1936, he served as Commanding Officer of the 375th Reserve Regiment of Engineers.

On his release from active service with the Army, he again became associated with the North-Eastern Construction Company, directing its construction operations at the Aberdeen Proving Grounds and the Edgewood Arsenal, in Maryland. He was later put in complete charge of that Company's operations in Delaware, Maryland, and the District of Columbia, and maintained his District Office, in Baltimore. In 1924, he was elected a Vice-President of the Company and remained active in that capacity until his death, which occurred suddenly from a heart attack on May 11, 1937, at the University Club, in Baltimore. He was buried in Arlington National Cemetery.

During his eighteen years of service as District Manager and Resident Vice-President with the North-Eastern Construction Company, a great number of large and creditably executed construction projects were completed under his immediate direction and supervision. Among the more important were the McComas Street Terminal for the Western Maryland Railroad; the Nurses Home for the Baltimore City Hospital; the Guilford Pumping Station for the Bureau of Water Supply, City of Baltimore; a building group for the Franciscan Novitiate, at Folly Quarter, Md.; the Federal Land Bank, at Baltimore; the Reptile House for the National Zoological Park in Washington, D. C.; a filtration plant for Hagerstown, Md.; and a great many other building and engineering projects in that vicinity.

Colonel Wolfe was at all times active in the affairs of his profession, serving as a Director of the Baltimore Building Congress and Exchange, as well as Secretary of the Baltimore Builders' Chapter of the Associated General Contractors of America.

He maintained his active interest in his military connections, serving as Vice-President of the Third Corps Area of the Society of the Third Division, was a Past Commander of the Military Order of Foreign Wars, Maryland Commanders, a member of the Military Order of the World War, the German H. H. Emory Post of the American Legion, the Society of American Military Engineers, and the Sojourners.

He also held membership in the University Club, of Baltimore; the Torch Club; Scottish Rite, A. F. and A. M., and the Mystic Shrine; the Junior Order of United American Mechanics, of Madison, N. J.; the Rotary Club, of Baltimore; a member of the Executive Committee, Virginia Military Institute Alumni Association, and a Past-President of the Baltimore Chapter.

The recognition accorded Colonel Wolfe's efforts in his chosen vocation, his military service, and his civic activities, best indicate the measure of the man.

In 1914, he was married to Mary Douglas MacNaughtan, daughter of the Rev. Dr. John MacNaughtan, of Chatham, N. J., and is survived by his widow, a daughter, Margaret Douglas Wolfe, and a son, Walter McIlhaney Wolfe, Jr., a sister, Katharine M. Wolfe, and one brother, William C. Wolfe.

Colonel Wolfe was elected an Associate Member of the American Society of Civil Engineers on November 25, 1919.

EDWIN BRIGHTMAN, Jun. Am. Soc. C. E.[1]

Died October 6, 1937

Edwin Brightman, the son of Charles L. Brightman and Charlotte (Clarkson) Brightman, was born at Holyoke, Mass., on October 23, 1911. His father was elected a member of the Faculty of Syracuse University, at Syracuse, N. Y., in 1916, and the children received their early education in the Syracuse Public Schools.

Mr. Brightman was graduated from Nottingham High School in 1929, and entered the College of Applied Science, at Syracuse University, in September of that year. He was enrolled in the course in Civil Engineering and was graduated from that Division with the degree of Bachelor of Science in Civil Engineering in June, 1933. He made an excellent scholastic record in his college course. He was a member of Tau Beta Pi, Theta Tau, and Sigma Phi Epsilon Fraternities, and was active in other matters pertaining to student affairs. His friendship was enjoyed by his classmates and Faculty associates.

Although he completed his college course during the industrial depression, Mr. Brightman immediately found employment, and was engaged for about two years on part-time engineering projects, including geodetic surveying for the Civil Works Administration in up-State New York, a drainage survey for the Smith Canastota Company, at Canastota, N. Y., and design work for the Paper and Pulp Department of the New York State College of Forestry at Syracuse. On March 1, 1935, he secured a position with the Guardian Life Insurance Company of New York, N. Y., as Inspector and Appraiser, which position he was holding at the time of his death.

Mr. Brightman was married to Carol Marion, of Stamford, Conn., on April 7, 1937. He died on October 7, 1937, at his home in Brooklyn, N. Y., after a short illness due to typhoid fever. He is survived by his widow, his parents, and two sisters.

His untimely death cut short a career of unusual promise, and he will be missed by a host of friends who loved and admired him.

Mr. Brightman was elected a Junior of the American Society of Civil Engineers on October 30, 1933.

JAMES DANIEL MICKEY, Jun. Am. Soc. C. E.[2]

Died July 6, 1937

James Daniel Mickey was born in Lincoln, Nebr., on May 15, 1913, the son of Clark Edwin and Irene (Eddy) Mickey. He received his education in the public schools of his native city from which he was graduated in 1931. He then entered the College of Engineering of the University of Nebraska, at Lincoln, from which he received the degree of Bachelor of Science in Civil Engineering, in June, 1935.

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[1] Memoir prepared by a Committee of the Syracuse Section, consisting of Louis Mitchell and Edward F. Berry, Members, Am. Soc. C. E.

[2] Memoir prepared by Clark E. Mickey, M. Am. Soc. C. E.

While attending High School, he became interested in preparing for an engineering career. During the vacation periods, he was employed by the Abel Construction Company on highway construction and by the Lancaster County Engineer's Office on survey parties.

During the period of his college education Mr. Mickey was employed part time by his father, Clark E. Mickey, M. Am. Soc. C. E., as Rodman, Draftsman, and Computer. During the summer of 1934 he was engaged in the Maintenance-of-Way Department of the Chicago, Burlington and Quincy Railroad Company, in Lincoln, as Rodman, Draftsman, and Computer on construction and maintenance surveys.

After his graduation from the University of Nebraska, Mr. Mickey was employed from June to November, 1935, in the Engineering Division of the Chicago, Burlington and Quincy Railroad Company as Rodman, Levelman, and Computer, during the reconstruction of railway lines which had been destroyed by a major flood of the Republican River in Colorado and Nebraska. When this work was completed, he was engaged from November, 1935, to January, 1937, by the Central Nebraska Public Power and Irrigation District, of Hastings, Nebr., as Levelman, Instrumentman, Detailer, Designer, and Assistant Office Engineer during the surveys, design, and preliminary construction of irrigation canals, structures, and a large earth dam. During the following month, he served as Instrumentman and Party Chief with the Loup River Public Power District, of Columbus, Nebr., on surveys for the location of high-tension electric power transmission lines. He severed his connection with the District in order to accept a more permanent position in the Engineering Department of the Phillips Petroleum Company, at Bartlesville, Okla. While engaged in this work, he was stricken with a virulent attack of poliomyelitis and passed away in the Menorah Hospital, in Kansas City, Mo. He was buried beside his twin sister, Justine R. Mickey, in the family plot in Wyuka Cemetery, in Lincoln.

Mr. Mickey's sudden and untimely death was a severe shock to his family, friends, and associates. He is survived by his father and mother and two sisters, Janice E. Mickey and Jeannette C. Mickey.

While attending the University of Nebraska, Mr. Mickey was a member of the Freshman football squad, Reserve Officers Training Corps, Pershing Rifle Company (Military), Nebraska State National Guard, Scabbard and Blade, and the Student Chapter of the American Society of Civil Engineers. He maintained a high scholarship and was an earnest and excellent student. Much of his time and ability was devoted to his many friends, associates, and college activities. His manner was straightforward, but he was kindly, courteous, and thoughtful toward others.

Throughout his brief career as an Engineer, he was appreciated and respected by his many friends and engineering associates and received many commendations from his employers and superiors. His character, personality, demeanor, and actions won him the confidence and trust of those with whom he was associated. He was a member of the Westminster Presbyterian Church of Lincoln.

Mr. Mickey was elected a Junior of the American Society of Civil Engineers on August 17, 1936.

LAWRENCE KING SNYDER, Jun. Am. Soc. C. E.[1]

DIED NOVEMBER 14, 1937

Lawrence King Snyder was born in Linn Creek, Mo., on October 5, 1906. He received his early education in the local schools and at Fayette High School, at Fayette, Mo. Later he attended Central College, at Fayette, and the Missouri School of Mines and Metallurgy, at Rolla, Mo., from which he was graduated in 1929 with the degree of Bachelor of Science in Civil Engineering.

Mr. Snyder had worked as Rodman and as Concrete Paving Plant Inspector, at Sikeston, Mo., for the Missouri State Highway Department during summer months previous to his graduation. Following graduation, he began work in his chosen profession as Bridge Designer, having served in that capacity, successively, for the Cleveland, Cincinnati, Chicago, and St. Louis Railway Company, at Cincinnati, Ohio, the Missouri State Highway Department, at Sikeston, and the St. Louis-San Francisco Railway Company, at St. Louis, Mo., continuing in the latter position until 1931, when he engaged in private engineering practice, making surveys and preparing plans and estimates for county highway bridges in Missouri.

Mr. Snyder returned to the Missouri School of Mines and Metallurgy in September, 1931, for graduate study in Structural Engineering. He prepared his thesis on the use of "Influence Lines in Calculation of Bridge Stresses," and received the degree of Master of Science in Civil Engineering in 1932. He was elected to membership in the Honor Society of Phi Kappa Phi, and the Engineering Society of Theta Tau.

He then was employed by the Missouri State Highway Department in the Bridge Department, at Jefferson City, Mo., during the years 1933 and 1934, and up to April, 1935, when he was re-employed by the St. Louis-San Francisco Railway Company as Bridge Designer, with headquarters at Springfield, Mo. Mr. Snyder continued in this position, working principally on the design of grade separation structures, until his health failed in June, 1937.

He was a very popular young man, of pleasing personality, and his many friends will regret his untimely passing. He was a member of the Sigma Nu Fraternity. He was not married. His nearest surviving relative is his grandmother, Mrs. L. S. Snyder, of Eldom, Mo.

Mr. Snyder was elected a Junior of the American Society of Civil Engineers on February 23, 1932.

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[1] Memoir prepared by R. B. Hennessy, Bridge Engr., St. Louis-San Francisco Ry., Springfield, Mo.

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